

*A Concise Linear Encoding of
Crystallographic Space Group
Settings and Group-Subgroup
Relationships (CLEG)*

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Introduction: Space group settings

The International Tables identifies the 230 space groups that are possible for crystal structures.

1	<i>P</i> 1	2	<i>P</i> -1	3	<i>P</i> 2	4	<i>P</i> 2 ₁	5	<i>C</i> 2
6	<i>P</i> m	7	<i>P</i> c	8	<i>C</i> m	9	<i>C</i> c	10	<i>P</i> 2/ <i>m</i>
11	<i>P</i> 2 ₁ / <i>m</i>	12	<i>C</i> 2/ <i>m</i>	13	<i>P</i> 2/ <i>c</i>	14	<i>P</i> 2 ₁ / <i>c</i>	15	<i>C</i> 2/ <i>c</i>
16	<i>P</i> 222	17	<i>P</i> 222 ₁	18	<i>P</i> 2 ₁ 2 ₁ 2	19	<i>P</i> 2 ₁ 2 ₁ 2 ₁	20	<i>C</i> 222 ₁
21	<i>C</i> 222	22	<i>F</i> 222	23	<i>I</i> 222	24	<i>I</i> 2 ₁ 2 ₁ 2 ₁	25	<i>P</i> mm2
26	<i>P</i> mc2 ₁	27	<i>P</i> cc2	28	<i>P</i> ma2	29	<i>P</i> ca2 ₁	30	<i>P</i> nc2
31	<i>P</i> mn2 ₁	32	<i>P</i> ba2	33	<i>P</i> na2 ₁	34	<i>P</i> nn2	35	<i>C</i> mm2
36	<i>C</i> mc2 ₁	37	<i>C</i> cc2	38	<i>A</i> mm2	39	<i>A</i> em2	40	<i>A</i> ma2
41	<i>A</i> ea2	42	<i>F</i> mm2	43	<i>F</i> dd2	44	<i>I</i> mm2	45	<i>I</i> ba2
46	<i>I</i> ma2	47	<i>P</i> mmm	48	<i>P</i> nnn	49	<i>P</i> ccm	50	<i>P</i> ban
51	<i>P</i> mma	52	<i>P</i> nna	53	<i>P</i> mna	54	<i>P</i> cca	55	<i>P</i> bam
56	<i>P</i> ccn	57	<i>P</i> bcm	58	<i>P</i> nnm	59	<i>P</i> mmn	60	<i>P</i> bcn
61	<i>P</i> bca	62	<i>P</i> nma	63	<i>C</i> mcm	64	<i>C</i> mce	65	<i>C</i> mmm
66	<i>C</i> ccm	67	<i>C</i> mme	68	<i>C</i> cce	69	<i>F</i> mmm	70	<i>F</i> ddd
71	<i>I</i> mmm	72	<i>I</i> bam	73	<i>I</i> bca	74	<i>I</i> mma	75	<i>P</i> 4
76	<i>P</i> 4 ₁	77	<i>P</i> 4 ₂	78	<i>P</i> 4 ₃	79	<i>I</i> 4	80	<i>I</i> 4 ₁
81	<i>P</i> -4	82	<i>I</i> -4	83	<i>P</i> 4/ <i>m</i>	84	<i>P</i> 4 ₂ / <i>m</i>	85	<i>P</i> 4/ <i>n</i>
86	<i>P</i> 4 ₂ / <i>n</i>	87	<i>I</i> 4/ <i>m</i>	88	<i>I</i> 4 ₁ / <i>a</i>	89	<i>P</i> 422	90	<i>P</i> 42 ₁ 2
91	<i>P</i> 4 ₁ 22	92	<i>P</i> 4 ₁ 2 ₁ 2	93	<i>P</i> 4 ₂ 22	94	<i>P</i> 4 ₂ 2 ₁ 2	95	<i>P</i> 4 ₃ 22
96	<i>P</i> 4 ₃ 2 ₁ 2	97	<i>I</i> 422	98	<i>I</i> 4 ₁ 22	99	<i>P</i> 4mm	100	<i>P</i> 4bm
101	<i>P</i> 4 ₂ cm	102	<i>P</i> 4 ₂ nm	103	<i>P</i> 4cc	104	<i>P</i> 4nc	105	<i>P</i> 4 ₂ mc
106	<i>P</i> 4 ₂ bc	107	<i>I</i> 4mm	108	<i>I</i> 4cm	109	<i>I</i> 4 ₁ md	110	<i>I</i> 4 ₁ cd
111	<i>P</i> -42m	112	<i>P</i> -42c	113	<i>P</i> -42 ₁ m	114	<i>P</i> -42 ₁ c	115	<i>P</i> -4m2

116	<i>P</i> -4c2	117	<i>P</i> -4b2	118	<i>P</i> -4n2	119	<i>I</i> -4m2	120	<i>I</i> -4c2
121	<i>I</i> -42m	122	<i>I</i> -42d	123	<i>P</i> 4/ <i>mmm</i>	124	<i>P</i> 4/ <i>mcc</i>	125	<i>P</i> 4/ <i>nbm</i>
126	<i>P</i> 4/ <i>nnc</i>	127	<i>P</i> 4/ <i>mbm</i>	128	<i>P</i> 4/ <i>mnc</i>	129	<i>P</i> 4/ <i>nmm</i>	130	<i>P</i> 4/ <i>ncc</i>
131	<i>P</i> 4 ₂ / <i>mmc</i>	132	<i>P</i> 4 ₂ / <i>mcm</i>	133	<i>P</i> 4 ₂ / <i>nbc</i>	134	<i>P</i> 4 ₂ / <i>nnm</i>	135	<i>P</i> 4 ₂ / <i>mbc</i>
136	<i>P</i> 4 ₂ / <i>mnm</i>	137	<i>P</i> 4 ₂ / <i>nmc</i>	138	<i>P</i> 4 ₂ / <i>ncm</i>	139	<i>I</i> 4/ <i>mmm</i>	140	<i>I</i> 4/ <i>mcm</i>
141	<i>I</i> 4 ₁ / <i>amd</i>	142	<i>I</i> 4 ₁ / <i>acd</i>	143	<i>P</i> 3	144	<i>P</i> 3 ₁	145	<i>P</i> 3 ₂
146	<i>R</i> 3	147	<i>P</i> -3	148	<i>R</i> -3	149	<i>P</i> 312	150	<i>P</i> 321
151	<i>P</i> 3 ₁ 12	152	<i>P</i> 3 ₁ 21	153	<i>P</i> 3 ₂ 12	154	<i>P</i> 3 ₂ 21	155	<i>R</i> 32
156	<i>P</i> 3m1	157	<i>P</i> 31m	158	<i>P</i> 3c1	159	<i>P</i> 31c	160	<i>R</i> 3m
161	<i>R</i> 3c	162	<i>P</i> -31m	163	<i>P</i> -31c	164	<i>P</i> -3m1	165	<i>P</i> -3c1
166	<i>R</i> -3m	167	<i>R</i> -3c	168	<i>P</i> 6	169	<i>P</i> 6 ₁	170	<i>P</i> 6 ₅
171	<i>P</i> 6 ₂	172	<i>P</i> 6 ₄	173	<i>P</i> 6 ₃	174	<i>P</i> -6	175	<i>P</i> 6/ <i>m</i>
176	<i>P</i> 6 ₃ / <i>m</i>	177	<i>P</i> 622	178	<i>P</i> 6 ₁ 22	179	<i>P</i> 6 ₅ 22	180	<i>P</i> 6 ₂ 22
181	<i>P</i> 6 ₄ 22	182	<i>P</i> 6 ₃ 22	183	<i>P</i> 6mm	184	<i>P</i> 6cc	185	<i>P</i> 6 ₃ cm
186	<i>P</i> 6 ₃ mc	187	<i>P</i> -6m2	188	<i>P</i> -6c2	189	<i>P</i> -62m	190	<i>P</i> -62c
191	<i>P</i> 6/ <i>mmm</i>	192	<i>P</i> 6/ <i>mcc</i>	193	<i>P</i> 6 ₃ / <i>mcm</i>	194	<i>P</i> 6 ₃ / <i>mmc</i>	195	<i>P</i> 23
196	<i>F</i> 23	197	<i>I</i> 23	198	<i>P</i> 2 ₁ 3	199	<i>I</i> 2 ₁ 3	200	<i>P</i> m-3
201	<i>P</i> n-3	202	<i>F</i> m-3	203	<i>F</i> d-3	204	<i>I</i> m-3	205	<i>P</i> a-3
206	<i>I</i> a-3	207	<i>P</i> 432	208	<i>P</i> 4 ₂ 32	209	<i>F</i> 432	210	<i>F</i> 4 ₁ 32
211	<i>I</i> 432	212	<i>P</i> 4 ₃ 32	213	<i>P</i> 4 ₁ 32	214	<i>I</i> 4 ₁ 32	215	<i>P</i> -43m
216	<i>F</i> -43m	217	<i>I</i> -43m	218	<i>P</i> -43n	219	<i>F</i> -43c	220	<i>I</i> -43d
221	<i>P</i> m-3m	222	<i>P</i> n-3n	223	<i>P</i> m-3n	224	<i>P</i> n-3m	225	<i>F</i> m-3m
226	<i>F</i> m-3c	227	<i>F</i> d-3m	228	<i>F</i> d-3c	229	<i>I</i> m-3m	230	<i>I</i> a-3d

Introduction: Space group settings

Each space group can be described using multiple settings.

The BCS identifies 611 “conventional” space group settings.

ITA-settings for the space group *P2/c* (No. 13) [unique axis *b*]

ITA number	Setting
13	<i>P 1 2/c 1</i>
13	<i>P 1 2/n 1</i>
13	<i>P 1 2/a 1</i>
13	<i>P 1 1 2/a</i>
13	<i>P 1 1 2/n</i>
13	<i>P 1 1 2/b</i>
13	<i>P 2/b 1 1</i>
13	<i>P 2/n 1 1</i>
13	<i>P 2/c 1 1</i>

Introduction: Space group settings

One setting is the “standard” setting.

ITA-settings for the space group *P2/c* (No. 13) [unique axis *b*]

ITA number	Setting
13	<i>P 1 2/c 1</i>
13	<i>P 1 2/n 1</i>
13	<i>P 1 2/a 1</i>
13	<i>P 1 1 2/a</i>
13	<i>P 1 1 2/n</i>
13	<i>P 1 1 2/b</i>
13	<i>P 2/b 1 1</i>
13	<i>P 2/n 1 1</i>
13	<i>P 2/c 1 1</i>

Introduction: Space group settings

The BSC describes settings using matrix transformations in relation to the standard setting.


The inline “(P,p)” notation describes the matrix in a convenient linear column-based format.

The general position of the group $P 1 1 2/n$ (No. 13)

The calculations are performed applying the following transformation matrix

Transformation matrix (\mathbf{P}, \mathbf{p}): $\mathbf{a}, -\mathbf{a}-\mathbf{c}, \mathbf{b}$; 0,0,0

$$\text{Matrix form: } (\mathbf{P}, \mathbf{p}) = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$



ITA number	Setting
13	$P 1 2/c 1$
13	$P 1 2/n 1$
13	$P 1 2/a 1$
13	$P 1 1 2/a$
13	$P 1 1 2/n$
13	$P 1 1 2/b$
13	$P 2/b 1 1$
13	$P 2/n 1 1$
13	$P 2/c 1 1$

Introduction: Group-Subgroup relationships

When comparing crystal structures, we are often interested in how their symmetries differ.

How do their lattices compare?

What symmetry elements have been lost or gained?

What sort of distortions relate one structure to another?

Introduction: Group-Subgroup relationships

The general position of the group $P2/c$ (No. 13) [unique axis b]
in the basis of its supergroup $P2/m$ (No. 10) [unique axis b]

The BSC also describes
subgroup relationships using
(P,p) matrices.

Transformation matrix (P, p): $a-c, b, 2c$; 0,0,1/2

$$\text{Matrix form: } (P, p) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 1/2 \end{pmatrix}$$

[Click here to get the general position of the standard/default setting in text format](#)
[Click here to get the general position of the transformed setting in text format](#)

N	Standard/Default Setting		Transformed	
	(x,y,z) form	matrix form	(x,y,z) form	matrix form
(0,0,0) + set				
1	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
2	-x,y,-z+1/2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	-x,y,-z+2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \end{pmatrix}$

Introduction: Group-Subgroup relationships

Bilbao Crystallographic Server → Transformations → Special Setting → Transformation matrix

Help

Transformation matrix

To change the basis of the group general positions is used the transformation matrix P :

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1/4 \\ 0 \end{bmatrix}$$

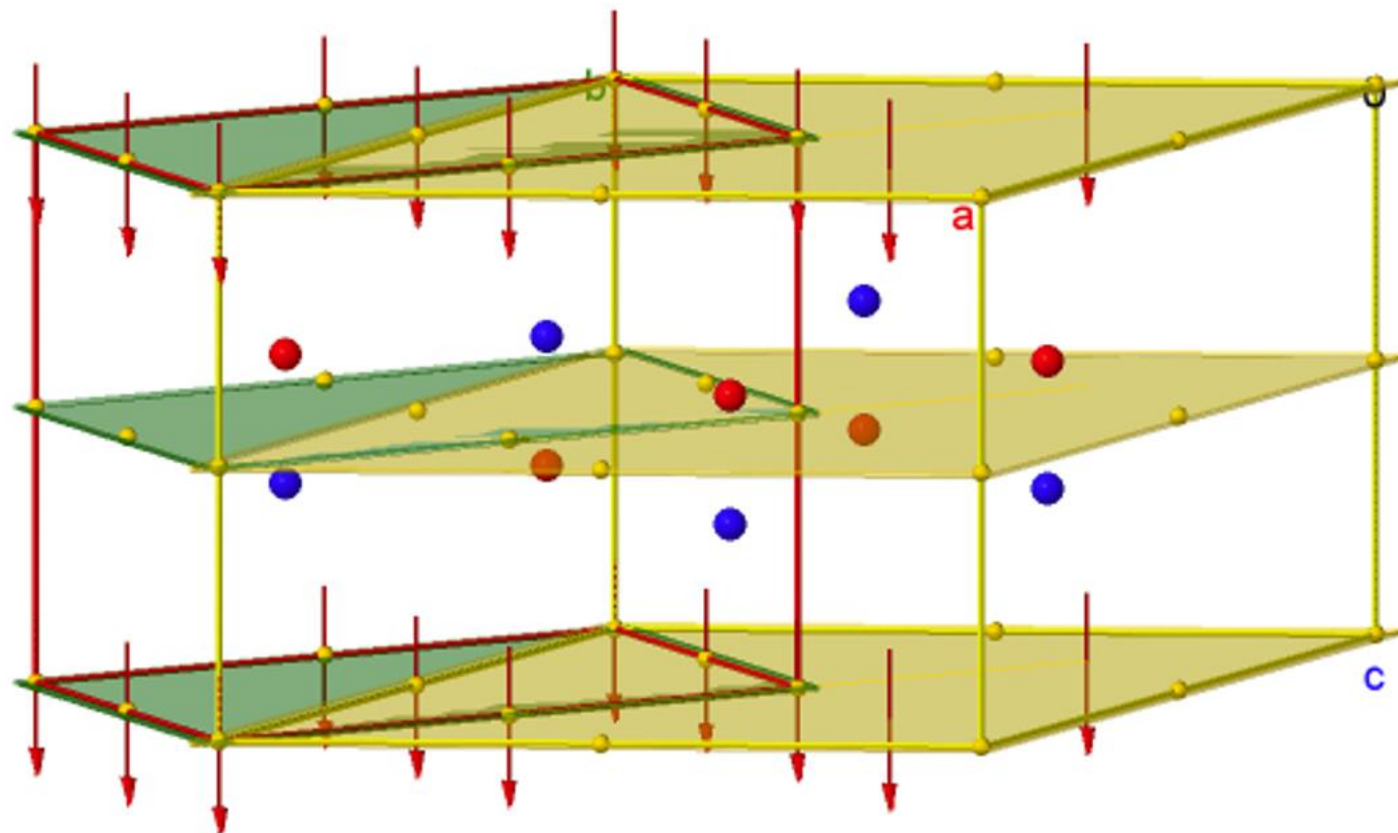
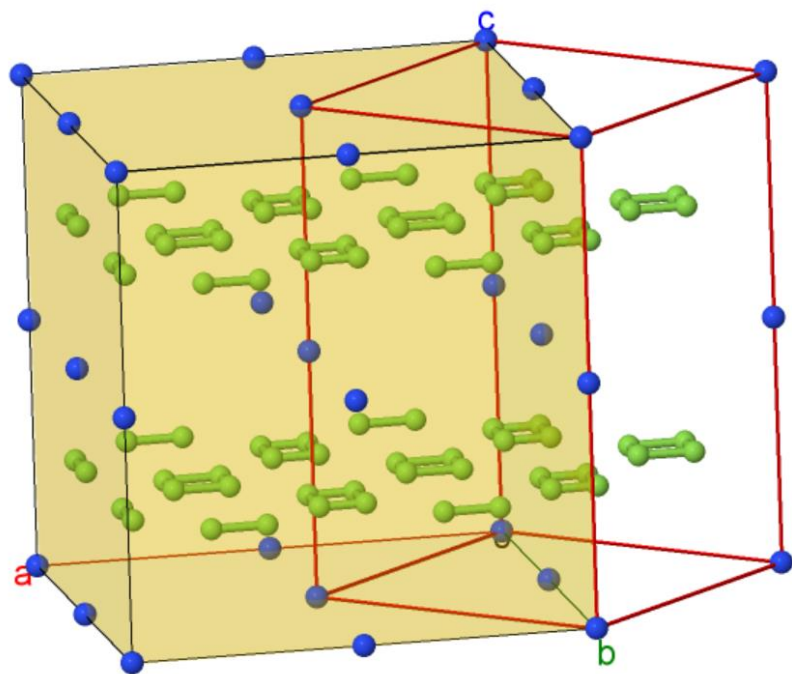
Inverse transformation $Q=P^{-1}$:

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ -1/4 \\ 0 \end{bmatrix}$$

The linear part of the transformation $P=(P,p)$ implies the change of basis vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$: $(\mathbf{a}', \mathbf{b}', \mathbf{c}') = (\mathbf{a}, \mathbf{b}, \mathbf{c}) P$

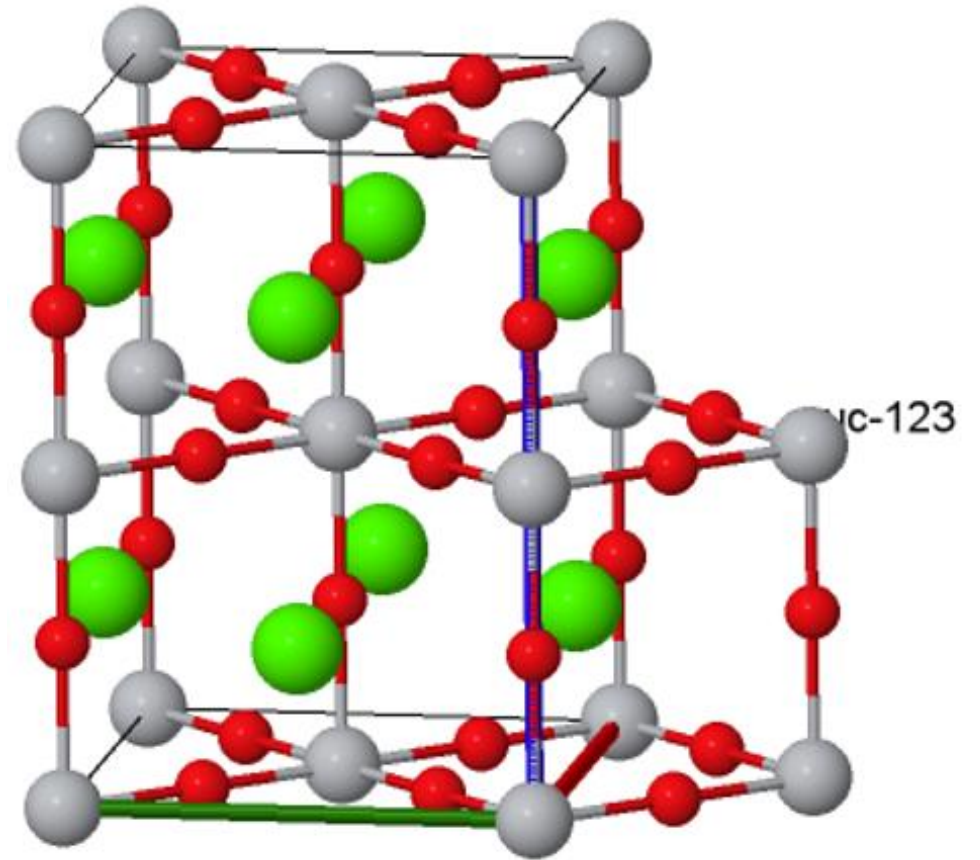
P is the transformation matrix from the basis of the supergroup to the basis of the subgroup. Because we are going to the basis of supergroup, each representative (R) is transformed with the transformation matrix P using: R (in new basis) = $P * R * Q$

My challenge: modeling settings and subgroup relationships in Jmol



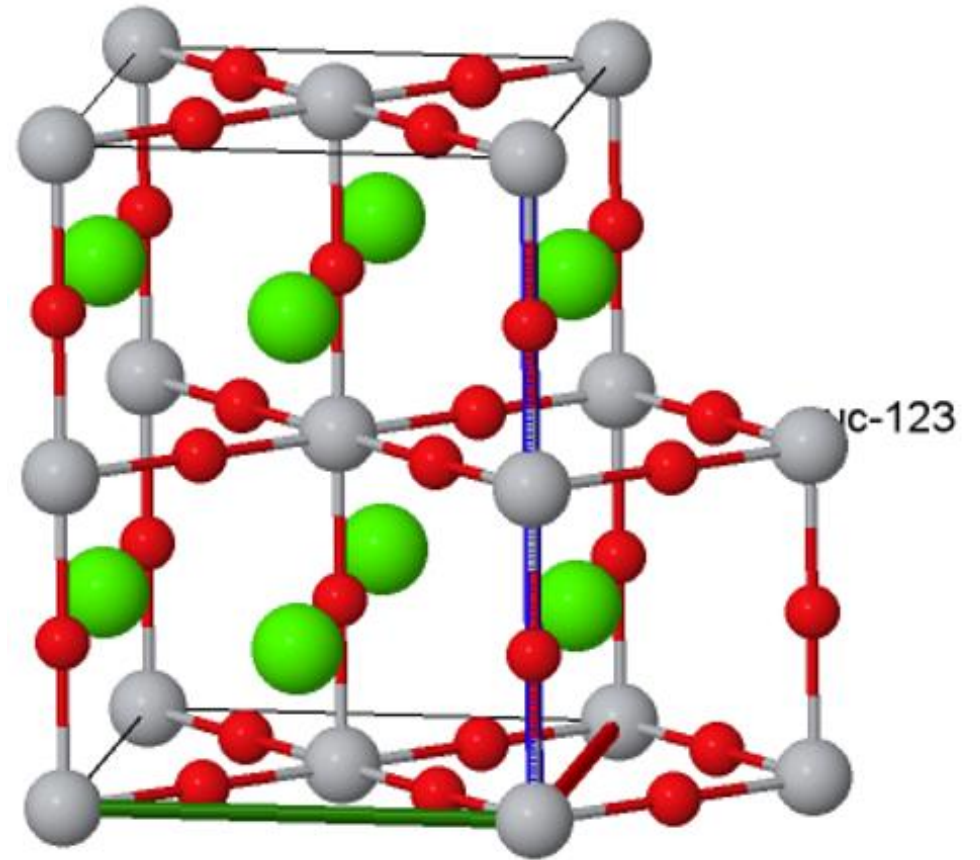
The Problem: How to express relationships concisely

- Simple, concise
- Comprehensive
- Flexible



The Problem: How to express relationships concisely

- Simple, concise
- Comprehensive
- Flexible
- Linear string of characters
- Scriptable



The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Settings: Combine ITA space group number with BCS (P,p) notation:

P 1 2/m 1

10:a,b,c;0,0,0

P 1 2/c 1

13:a,b,c;0,0,0

P 1 1 2/m

10:c,a,b;0,0,0

P 1 1 2/n

13:a,-a-c,b;0,0,0

P 2/m 1 1

10:b,c,a;0,0,0

P 2/b 1 1

13:b,c,a;0,0,0

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Settings: Combine ITA space group number with BCS (P,p) notation:

P 1 2/m 1

10:a,b,c;0,0,0

P 1 2/c 1

13:a,b,c;0,0,0

P 1 1 2/m

10:c,a,b;0,0,0

P 1 1 2/n

13:a,-a-c,b;0,0,0

P 2/m 1 1

10:b,c,a;0,0,0

P 2/b 1 1

13:b,c,a;0,0,0

Simple!

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Settings: Allow for defaults “a,b,c” and “;0,0,0” :

P 1 2/m 1	10
P 1 1 2/m	10:c,a,b
P 2/m 1 1	10:b,c,a

P 1 2/c 1	13
P 1 1 2/n	13:a,-a-c,b
P 2/b 1 1	13:b,c,a

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Settings: Allow for defaults “a,b,c” and “;0,0,0” :

P 1 2/m 1	10
P 1 1 2/m	10:c,a,b
P 2/m 1 1	10:b,c,a


P 1 2/c 1	13
P 1 1 2/n	13:a,-a-c,b
P 2/b 1 1	13:b,c,a

Concise!

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Subgroups: Expand ITA “P2/m > P2/c” notation:


10 > a-c,b,2c;0,0,1/2 > 13

Transformation matrix (P, p): a-c,b,2c ; 0,0,1/2

Matrix form: $(\mathbf{P}, \mathbf{p}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 1/2 \end{pmatrix}$

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Subgroups: Add “!” meaning “not” or “inverse of”:

10:b,c,a > **!b,c,a** > 10

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Subgroups: Allow chaining:

10:b,c,a > !b,c,a > 10

10 > a-c,b,2c;0,0,1/2 > 13

13 > a,-a-c,b > 13:a,-a-c,b

?

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Subgroups: Allow chaining by :

10:b,c,a > !b,c,a > 10

10 > a-c,b,2c;0,0,1/2 > 13

13 > a,-a-c,b > 13:a,-a-c,b

\$ P1 = matrix("!b,c,a")

\$ P2 = matrix("a-c,b,2c;0,0,1/2")

\$ P3 = matrix("a,-a-c,b")

\$ P = \$P1 * \$P2 * \$P3

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Subgroups: Allow chaining:

10:b,c,a > **!b,c,a** > 10

10 > **a-c,b,2c;0,0,1/2** > 13

13 > **a,-a-c,b** > 13:a,-a-c,b

10:b,c,a > **-b+c,-b-c,a;0,1/2,0** > 13:a,-a-c,b

\$ print matrix("!b,c,a > a-c,b,2c;0,0,1/2 > a,-a-c,b", "abc")

-b+c,-b-c,a;0,1/2,0

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

10:b,c,a > !b,c,a > 10

nonstandard to standard setting

10 > a-c,b,2c;0,0,1/2 > 13

group to subgroup (standard settings)

13 > a,-a-c,b > 13:a,-a-c,b

standard to nonstandard

10:b,c,a > -b+c,-b-c,a:0,1/2,0 > 13:a,-a-c,b

nonstandard group to nonstandard subgroup

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Examples

10:b,c,a > !b,c,a > 10

nonstandard to standard setting

10 > a-c,b,2c;0,0,1/2 > 13

group to subgroup (standard settings)

13 > a,-a-c,b > 13:a,-a-c,b

standard to nonstandard

10:b,c,a > -b+c,-b-c,a:0,1/2,0 > 13:a,-a-c,b

nonstandard group to nonstandard subgroup

Comprehensive!

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Allow Hermann-Mauguin notation?

$P\ 2/m\ 1\ 1 > !b,c,a > P\ 2/m$

nonstandard to standard setting

$P\ 2/m > a-c,b,2c;0,0,1/2 > P\ 2/c$

group to subgroup (standard settings)

$P\ 2/c > a,-a-c,b > P\ 1\ 1\ 2/n$

standard to nonstandard

$P\ 2/m\ 1\ 1 > -b+c,-b-c,a;0,1/2,0 > P\ 1\ 1\ 2/n$

nonstandard group to nonstandard subgroup

(for human consumption only!)

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

*Q: Allow Hall notation (**for standard setting only**)*

$(-P\ 2y):b,c,a > !b,c,a > (-P\ 2y)$

nonstandard to standard setting

$(-P\ 2y) > a-c,b,2c;0,0,1/2 > (-P\ 2cy)$

group to subgroup (standard settings)

$(-P\ 2cy) > a,-a-c,b > (-P\ 2cy):a,-a-c,b$

standard to nonstandard

$(-P\ 2y):b,c,a > -b+c,-b-c,a;0,1/2,0 > (-P\ 2cy):a,-a-c,b$

nonstandard group to nonstandard subgroup

(completely machine readable; no ITA referencing)

The Solution: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

*Q: Allow Hall notation (**for standard setting only**)*

$(-P\ 2y):b,c,a > !b,c,a > (-P\ 2y)$

nonstandard to standard setting

$(-P\ 2y) > a-c,b,2c;0,0,1/2 > (-P\ 2cy)$

group to subgroup (standard settings)

$(-P\ 2cy) > a,-a-c,b > (-P\ 2cy):a,-a-c,b$

standard to nonstandard

$(-P\ 2y):b,c,a > -b+c,-b-c,a;0,1/2,0 > (-P\ 2cy):a,-a-c,b$

nonstandard group to nonstandard subgroup

(completely machine readable; no ITA referencing)

Flexible!

CLEG applications

tabular description of
BCS group/maximal
subgroup relationships
through index 4:

i	cleg	tr_type	tr_subtype	det	index	conj_class
1	$1 > 2a, a+b, a+c > 1$	k	eu	2	2	a
2	$1 > 2a, a+b, c > 1$	k	eu	2	2	b
3	$1 > 2a, b, a+c > 1$	k	eu	2	2	c
191	$8 > a-2c, b, 2c > 9$	k	eu	2	2	a
192	$8 > a, b, 2c > 9$	k	eu	2	2	b
193	$9 > 1/2a - 1/2b, 1/2a + 1/2b, c > 1$	t		0.5	2	a
194	$9 > a, b, -a+c; 0, 1/4, 0 > 7$	k	ct	1	2	a
202	$9 > a, b, 3c > 9$	k	eu	3	3	e
203	$10 > a, b, c > 2$	t		1	2	a
204	$10 > a, b, c > 3$	t		1	2	a
205	$10 > a, b, c > 6$	t		1	2	a
206	$10 > a-c, b, 2c; 0, 0, 1/2 > 10$	k	eu	2	2	a
1558	$69 > -b, a, c > 64$	k	ct	1	2	c
1559	$69 > b, c, a > 64$	k	ct	1	2	d
1560	$69 > c, a, b > 64$	k	ct	1	2	e
1561	$69 > c, b, -a > 64$	k	ct	1	2	f
1562	$69 > a, b, c > 65$	k	ct	1	2	a
1563	$69 > c, a, b > 65$	k	ct	1	2	b
3764	$230 > a-b, b+c, -1/2a - 1/2b + 1/2c; 1/2, 0, 1/2 > 167$	t		1.5	4	a
3765	$230 > a, b, c > 206$	t		1	2	a
3766	$230 > a, b, c > 214$	t		1	2	a
3767	$230 > a, b, c > 220$	t		1	2	a

CLEG applications

JSON description of ITA space group settings for Jmol:

```
▼ 226:
  clegId:  "47:a,b,c"
▼ 227:
  clegId:  "48:a,b,c"
▼ 228:
  clegId:  "48:a,b,c;1/4,1/4,1/4"
▼ 229:
  clegId:  "49:a,b,c"
▼ 230:
  clegId:  "49:c,a,b"
▼ 231:
  clegId:  "49:b,c,a"
```

CLEG applications

Jmol scripting for calculation:

Transformation matrix (**P**, **p**): **a-c,b,2c** ; 0,0,1/2

$$\text{Matrix form: } (\mathbf{P}, \mathbf{p}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 1/2 \end{pmatrix}$$

[Click here to get the general position of the standard/default setting in text format](#)

[Click here to get the general position of the transformed setting in text format](#)

N	Standard/Default Setting		Transformed	
	(x,y,z) form	matrix form	(x,y,z) form	matrix form
(0,0,0) + set				
1	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
2	-x,y,-z+1/2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	-x,y,-z+2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \end{pmatrix}$

CLEG applications

Jmol scripting for calculation:

```
$ P = matrix("a-c,b,2c;0,0,1/2")  
$ R = symop("-x,y,-z+1/2")  
$ Q = !P  
$ print matrix(P * R * Q, "xyz")
```

-x,y,-z+2

Transformation matrix (P, p): a-c,b,2c ; 0,0,1/2

$$\text{Matrix form: } (P, p) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 2 & 1/2 \end{pmatrix}$$

[Click here to get the general position of the standard/default setting in text format](#)
[Click here to get the general position of the transformed setting in text format](#)

N	Standard/Default Setting		Transformed	
	(x,y,z) form	matrix form	(x,y,z) form	matrix form
(0,0,0) + set				
1	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	x,y,z	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
2	-x,y,-z+1/2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1/2 \end{pmatrix}$	-x,y,-z+2	$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 \end{pmatrix}$

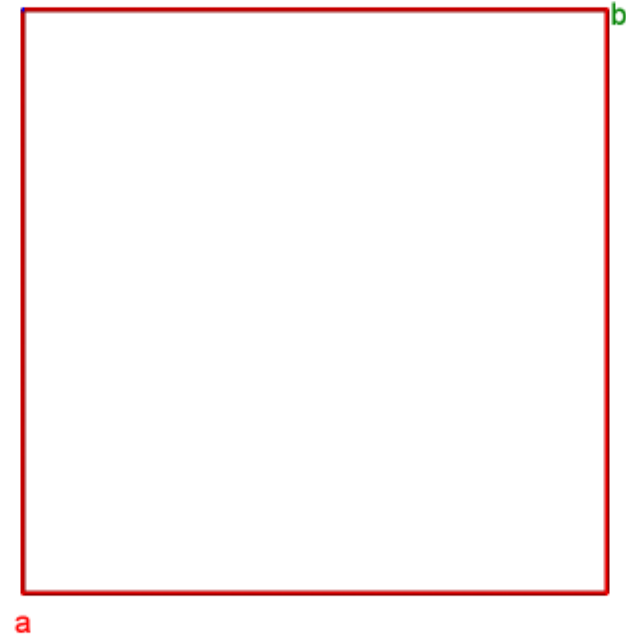
CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup 140
```

```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup 140
```

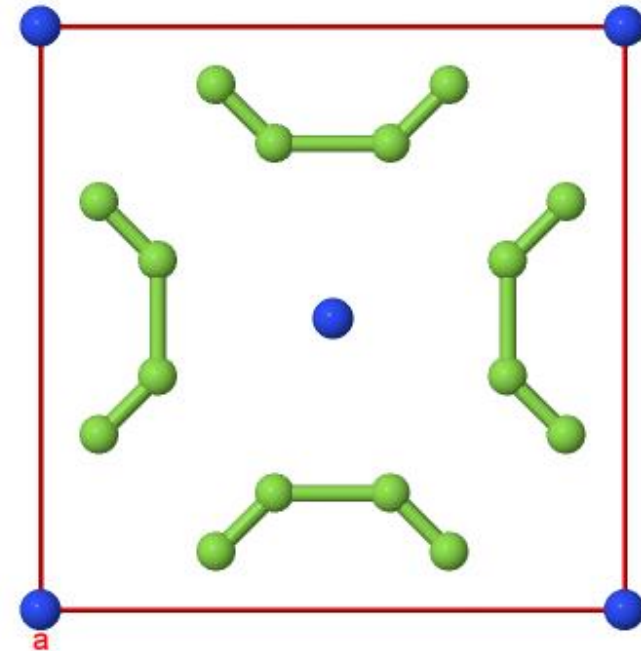
```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```

```
modelkit add F {0.60 0.20 0.25/1}
```

```
modelkit add N wyckoff c packed
```

```
connect 1.0 2.1
```



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup 140
```

```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```

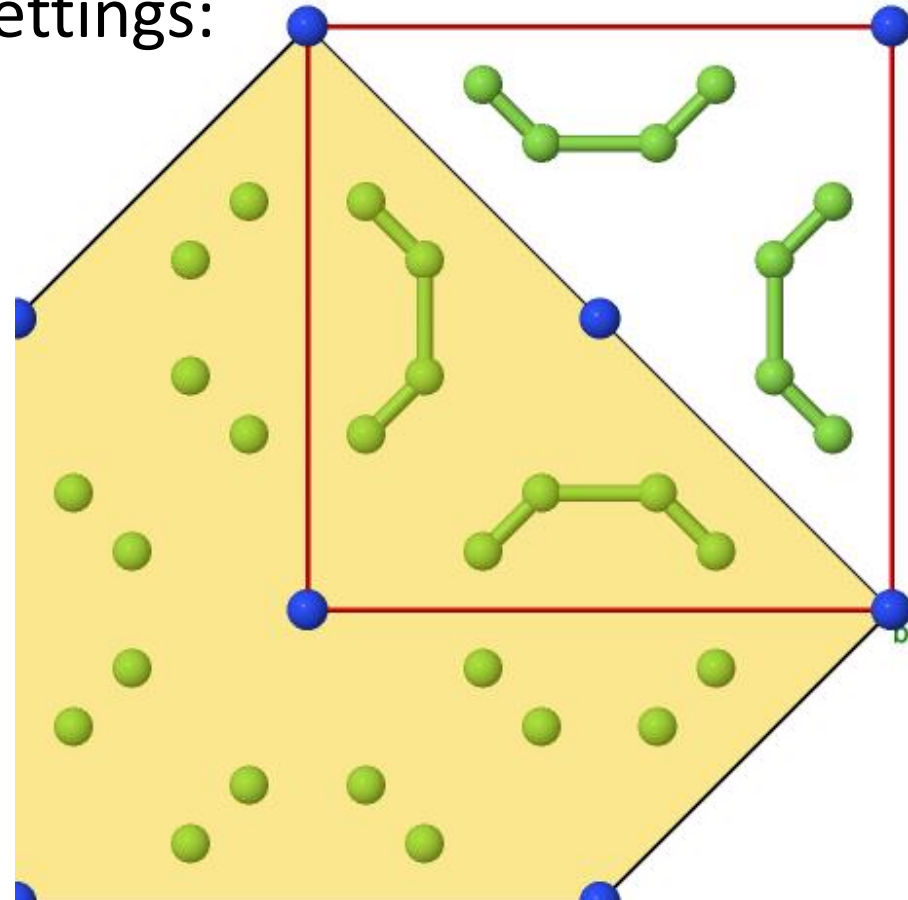
```
modelkit add F {0.60 0.20 0.25/1}
```

```
modelkit add N wyckoff c packed
```

```
connect 1.0 2.1
```

```
modelkit spacegroup "140:a-b,a+b,c" packed
```

```
draw ID uc2 unitcell fill nomesh color translucent 0.8
```



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup 140
```

```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```

```
modelkit add F {0.60 0.20 0.25/1}
```

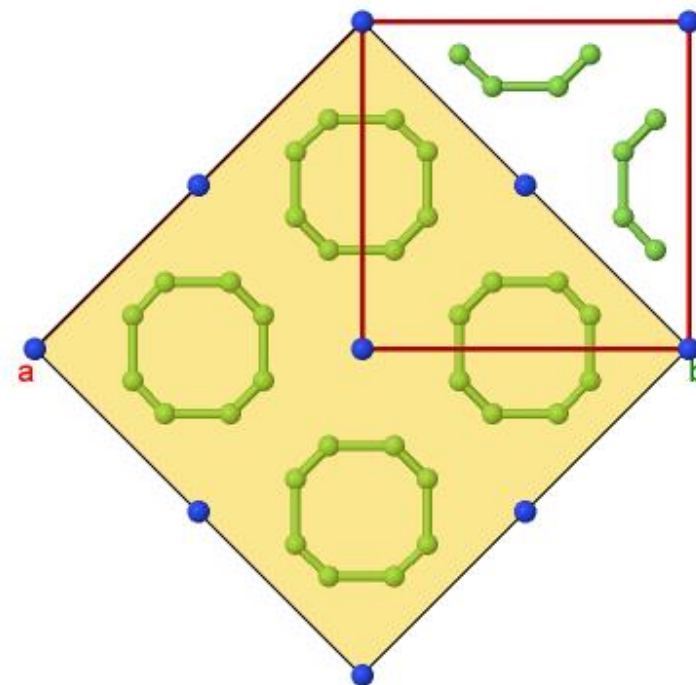
```
modelkit add N wyckoff c packed
```

```
connect 1.0 2.1
```

```
modelkit spacegroup "140:a-b,a+b,c" packed
```

```
draw ID uc2 unitcell fill nomesh color translucent 0.8
```

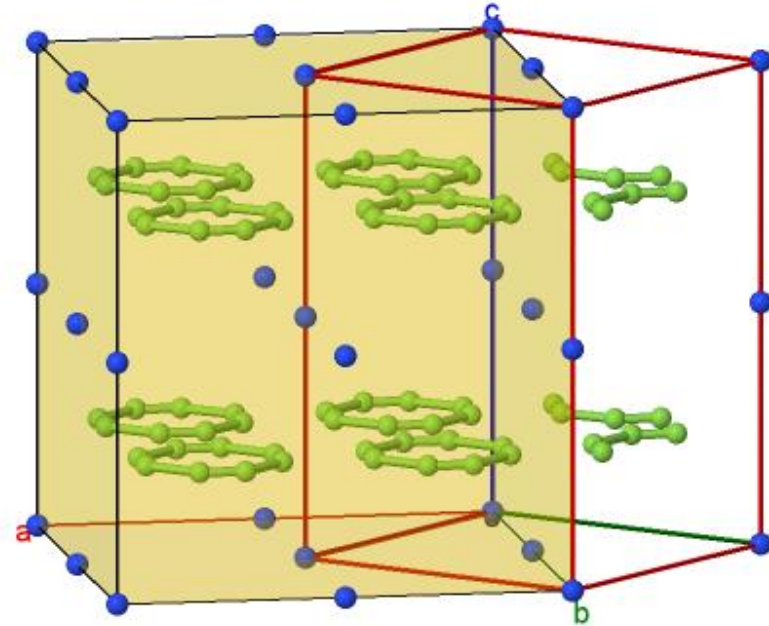
```
connect 1.0 2.1; center unitcell; zoom 100
```



CLEG applications

Jmol scripting for visualization of settings:

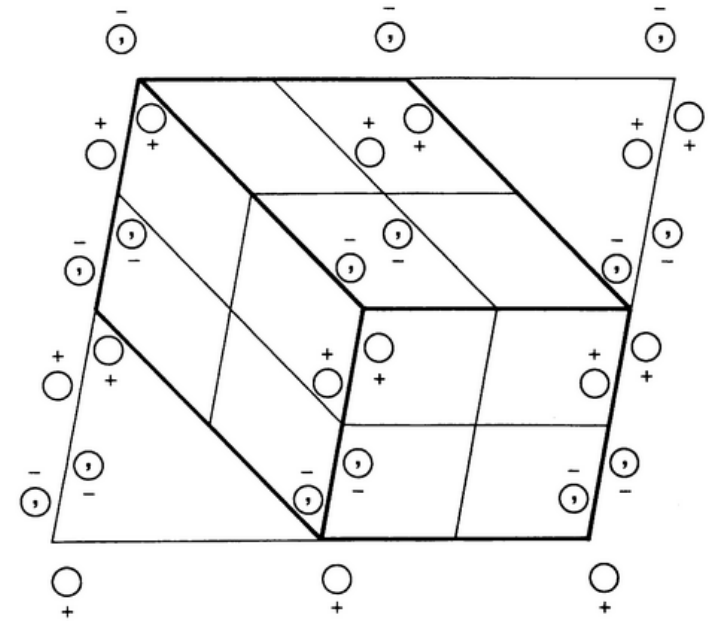
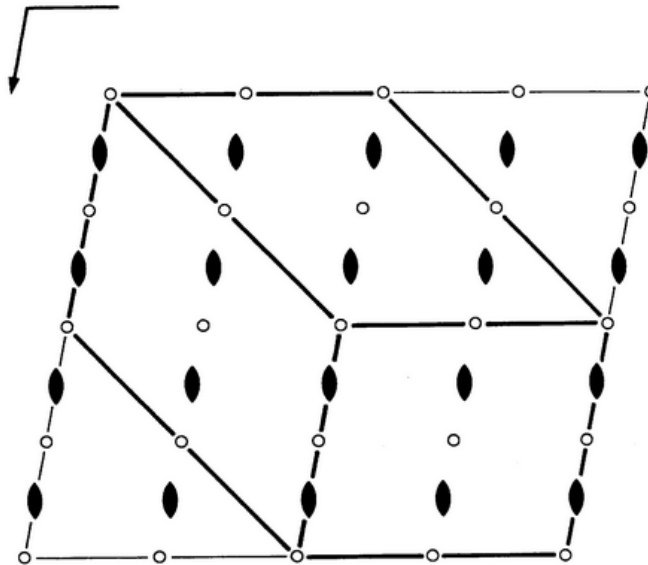
```
modelkit zap spacegroup 140  
draw ID uc1 unitcell color red  
draw ID ax1 axes  
modelkit add F {0.60 0.20 0.25/1}  
modelkit add N wyckoff c packed  
connect 1.0 2.1  
modelkit spacegroup "140:a-b,a+b,c" packed  
draw ID uc2 unitcell fill nomesh color translucent 0.8  
connect 1.0 2.1; center unitcell; zoom 100  
rotate z -35; rotate x -80
```



CLEG applications

Jmol scripting for visualization of settings:

Can someone
explain this to me?



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup "13:a,b,c"
```

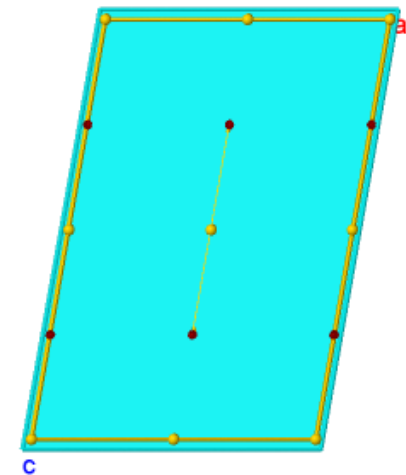
```
moveto axis b1
```

```
draw uc1 unitcell
```

```
draw sg1 spacegroup all
```

```
show symops
```

1	x,y,z	identity
2	$-x,y,-z+1/2$	2 axis
3	$-x,-y,-z$	Ci: 0 0 0
4	$x,-y,z+1/2$	c-glide plane translation: 0 0 1/2



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup "13:a,b,c"
```

```
moveto axis b1
```

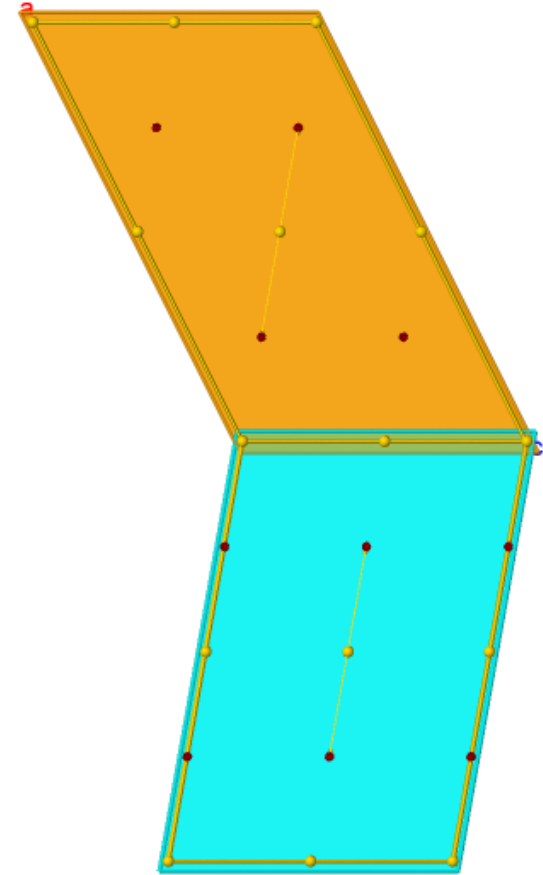
```
draw uc1 unitcell
```

```
draw sg1 spacegroup all
```

```
modelkit spacegroup "13:-a-c,b,a"
```

```
draw uc2 unitcell
```

```
draw sg2 spacegroup all
```



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup "13:a,b,c"
```

```
moveto axis b1
```

```
draw uc1 unitcell
```

```
draw sg1 spacegroup all
```

```
modelkit spacegroup "13:-a-c,b,a"
```

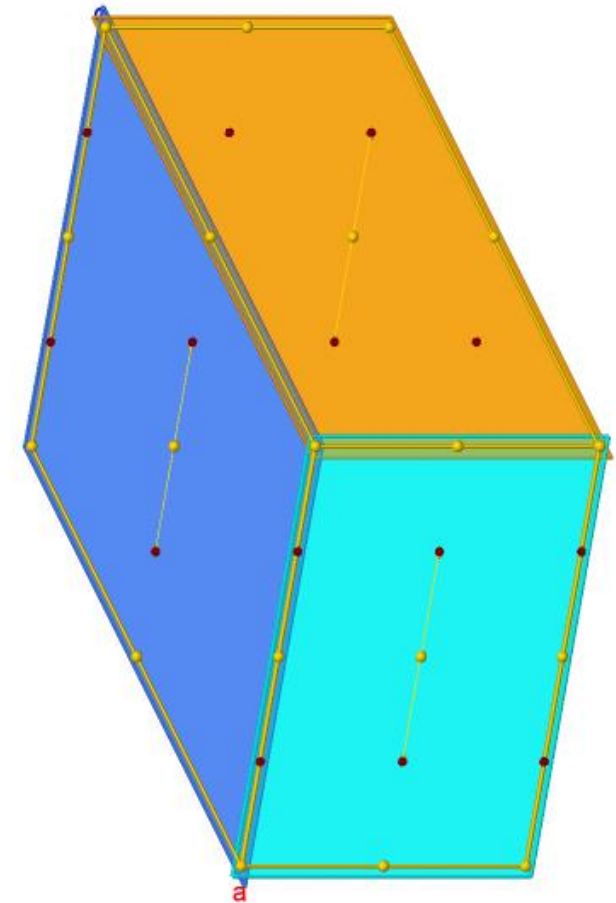
```
draw uc2 unitcell
```

```
draw sg2 spacegroup all
```

```
modelkit spacegroup "13:c,b,-a-c"
```

```
draw uc3 unitcell
```

```
draw sg3 spacegroup all
```



CLEG applications

Jmol scripting for visualization of settings:

modelkit zap spacegroup 13.1

moveto axis b1

draw uc1 unitcell

draw sg1 spacegroup all

modelkit spacegroup 13.2

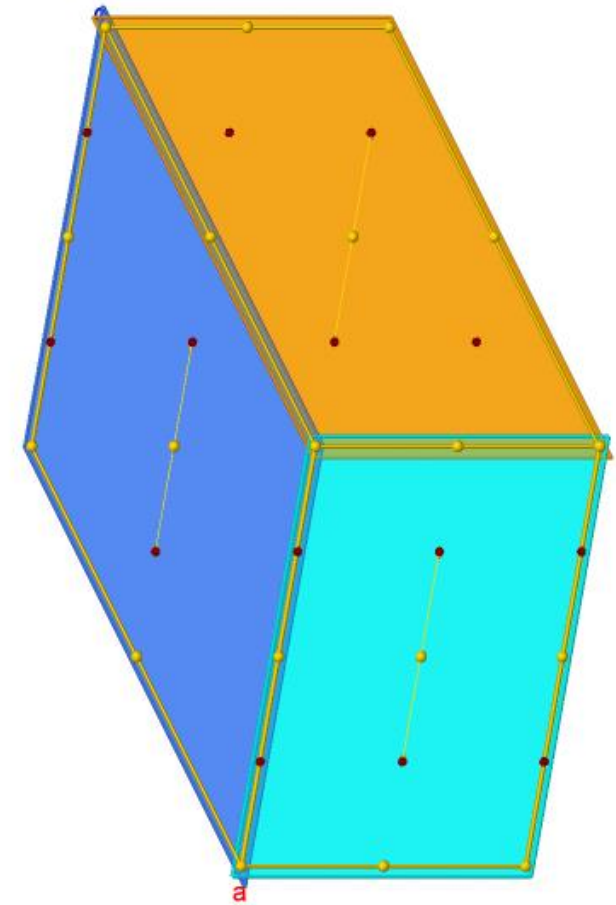
draw uc2 unitcell

draw sg2 spacegroup all

modelkit spacegroup 13.3

draw uc3 unitcell

draw sg3 spacegroup all



CLEG applications

Jmol scripting for visualization of settings:

```
modelkit zap spacegroup "13:a,b,c"
```

```
moveto axis b1
```

```
draw uc1 unitcell
```

```
draw sg1 spacegroup all
```

```
modelkit spacegroup "13:-a-c,b,a"
```

```
draw uc2 unitcell
```

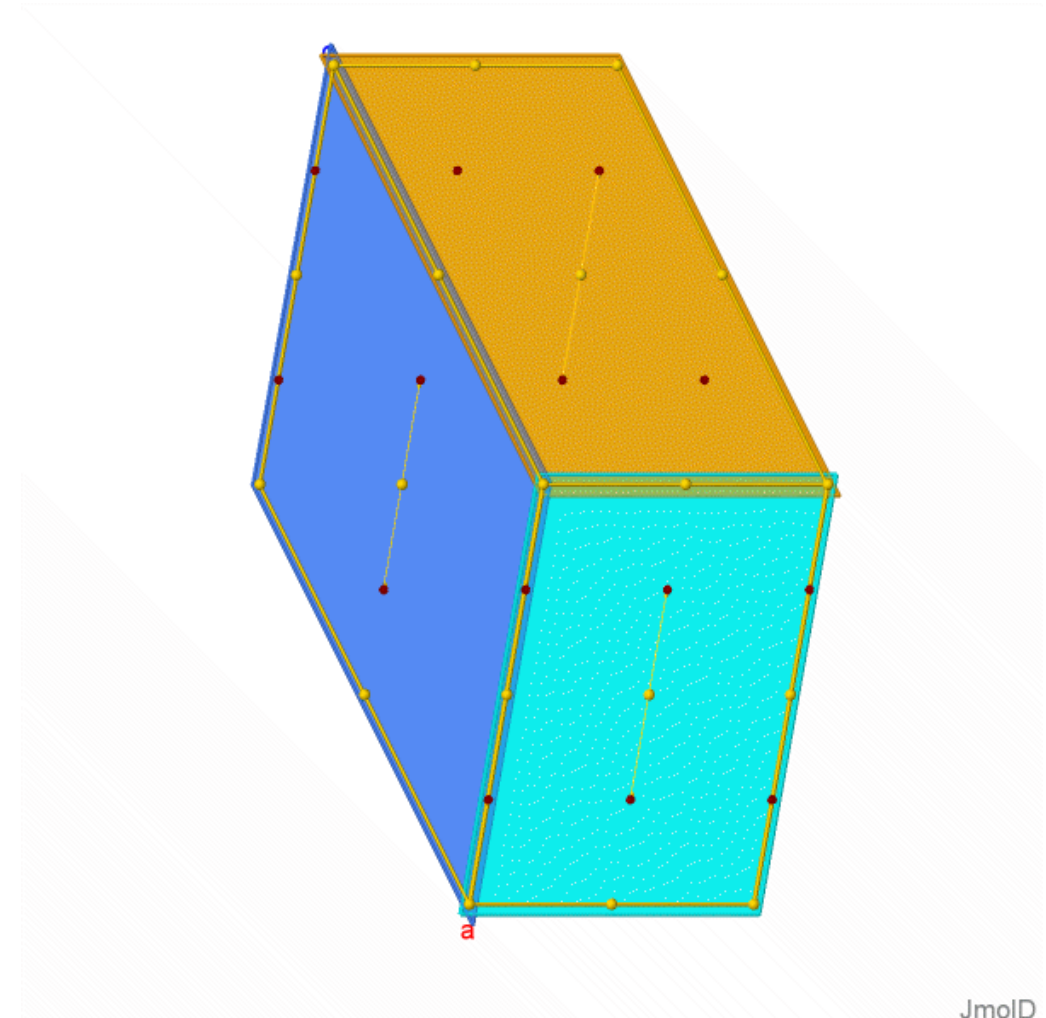
```
draw sg2 spacegroup all
```

```
modelkit spacegroup "13:c,b,-a-c"
```

```
draw uc3 unitcell
```

```
draw sg3 spacegroup all
```

<https://github.com/BobHanson/CLEG>



CLEG applications

Jmol scripting for visualization of subgroups:

```
modelkit zap spacegroup 230
```

```
zoom 60
```

```
draw ID uc1 unitcell color red
```

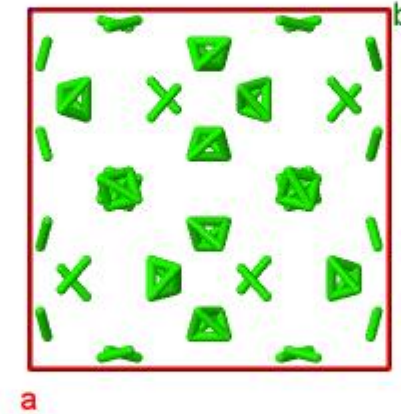
```
draw ID ax1 axes
```

```
modelkit add N Wyckoff G
```

```
connect 0.5 1.5
```

```
spacefill off
```

```
color property site
```



JmolD

CLEG applications

Jmol scripting for visualization of subgroups:

modelkit zap spacegroup 230

draw ID uc1 unitcell color red

draw ID ax1 axes

modelkit add N Wyckoff G

modelkit spacegroup "230>a-b,b+c,-1/2a-1/2b+1/2c;1/2,1,-1/2>167" packed

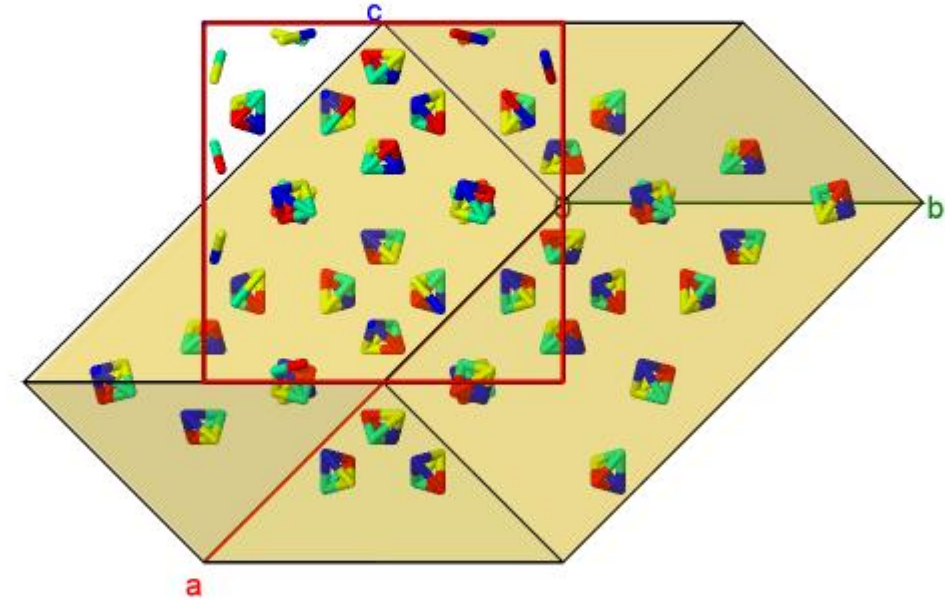
draw ID uc2 unitcell fill nomesh color translucent 0.8

connect 0.5 1.5

spacefill off

color property site

spin on



JmolD

CLEG applications

Jmol scripting for visualization of subgroups:

modelkit zap spacegroup 230

draw ID uc1 unitcell color red

draw ID ax1 axes

modelkit add N Wyckoff G

modelkit spacegroup "230>a-b,b+c,-1/2a-1/2b+1/2c;1/2,1,-1/2>167" packed

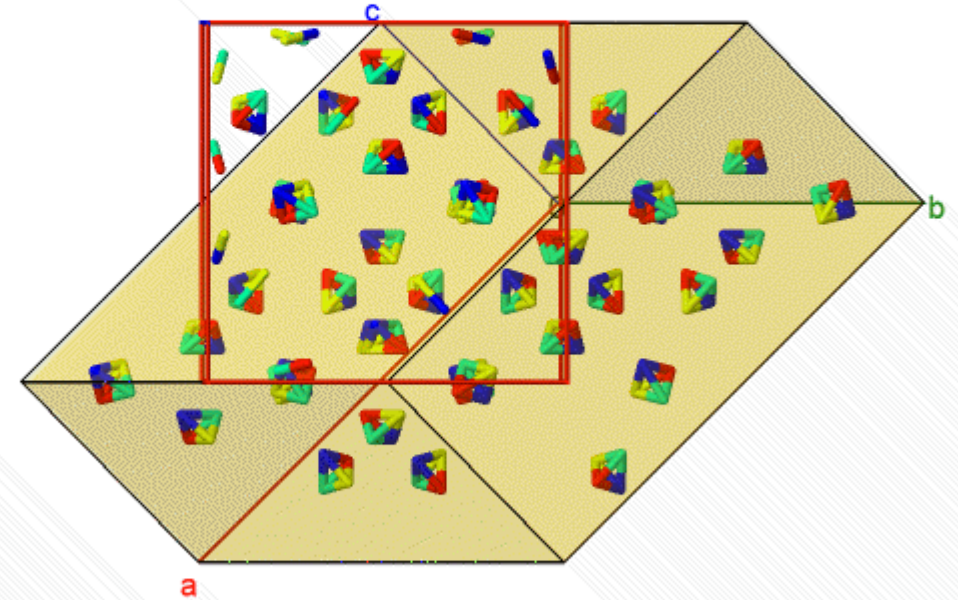
draw ID uc2 unitcell fill nomesh color translucent 0.8

connect 0.5 1.5

spacefill off

color property site

spin on



JmolD

CLEG applications

ISODISTORT/Jmol scripting for visualization of subgroups:

ISODISTORT: subgroup tree

Subgroup 1

221 Pm-3m, basis={{(0,-1,0),(-1,0,0),(0,0,-1)}, origin=(0,0,0), s=1, i=1

Maximal subgroups: 2

a=4.20000,b=4.20000,c=4.20000,alpha=90.00000,beta=90.00000,gamma=90.00000

Order parameters: GM1+ (a)

Active k vectors: (0,0,0)

This subgroup does not produce any selected distortions

Subgroup 2

123 P4/mmm, basis={{(1,0,0),(0,1,0),(0,0,1)}, origin=(0,0,0), s=1, i=3

Maximal subgroups: 3

a=4.20000,b=4.20000,c=4.20000,alpha=90.00000,beta=90.00000,gamma=90.00000

Order parameters: GM1+ (a) GM3+ (a,0)

Active k vectors: (0,0,0)

Subgroup 3

140 I4/mcm, basis={{(-1,1,0),(-1,-1,0),(0,0,2)}, origin=(0,0,0), s=2, i=6

Maximal subgroups:

a=5.93970,b=5.93970,c=8.40000,alpha=90.00000,beta=90.00000,gamma=90.00000

Order parameters: R4+ (a,0,0) GM1+ (a) GM3+ (a,0)

Active k vectors: (1/2,1/2,1/2), (0,0,0)

CLEG applications

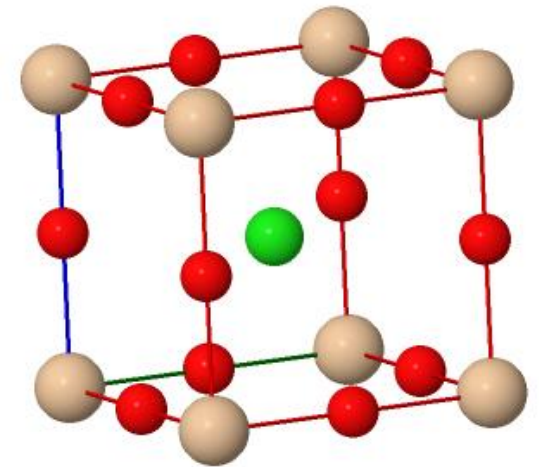
ISODISTORT/Jmol scripting for visualization of subgroups:

```
modelkit zap spacegroup "221:-b,-a,-c" unitcell [4.2 4.2 4.2 90 90 90]
```

```
modelkit add Si Wyckoff a packed
```

```
modelkit add Cl Wyckoff b packed
```

```
modelkit add O Wyckoff d packed
```



CLEG applications

ISODISTORT/Jmol scripting for visualization

```
modelkit zap spacegroup "221:-b,-a,-c" unitcell [4.2 4.2 4.2 90 90 90]
```

```
modelkit add Si Wyckoff a packed
```

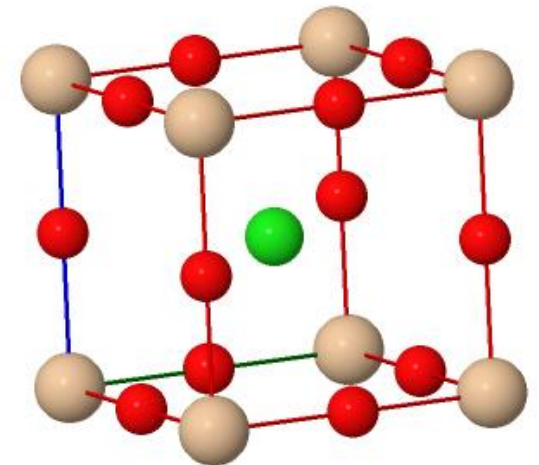
```
modelkit add Cl Wyckoff b packed
```

```
modelkit add O Wyckoff d packed
```

```
modelkit spacegroup "221:-b,-a,-c > a,b,c > 123" packed
```

```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```



CLEG applications

ISODISTORT/Jmol scripting for visualization of subgroups:

```
modelkit zap spacegroup "221:-b,-a,-c" unitcell [4.2 4.2 4.2 90 90 90]
```

```
modelkit add Si Wyckoff a packed
```

```
modelkit add Cl Wyckoff b packed
```

```
modelkit add O Wyckoff d packed
```

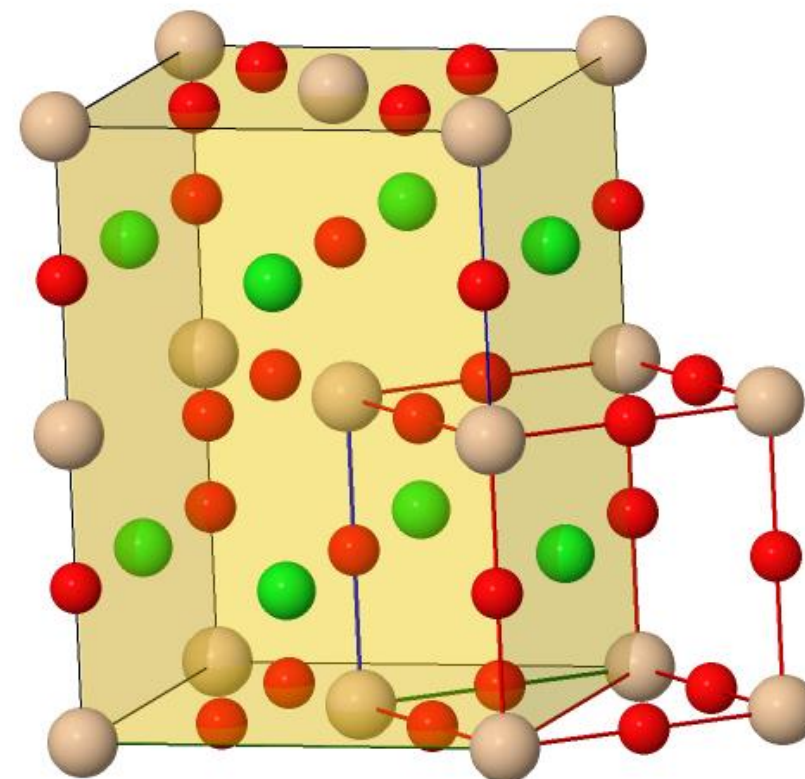
```
modelkit spacegroup "221:-b,-a,-c > a,b,c > 123" packed
```

```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```

```
modelkit spacegroup "123 > -a+b,-a-b,2c;1,0,0 > 140" packed
```

```
draw ID uc2 unitcell fill nomesh translucent 0.8
```



CLEG applications

ISODISTORT/Jmol scripting for visualization of subgroups:

```
modelkit zap spacegroup "221:-b,-a,-c" unitcell [4.2 4.2 4.2 90 90 90]
```

```
modelkit add Si Wyckoff a packed
```

```
modelkit add Cl Wyckoff b packed
```

```
modelkit add O Wyckoff d packed
```

```
modelkit spacegroup "221:-b,-a,-c > a,b,c > 123" packed
```

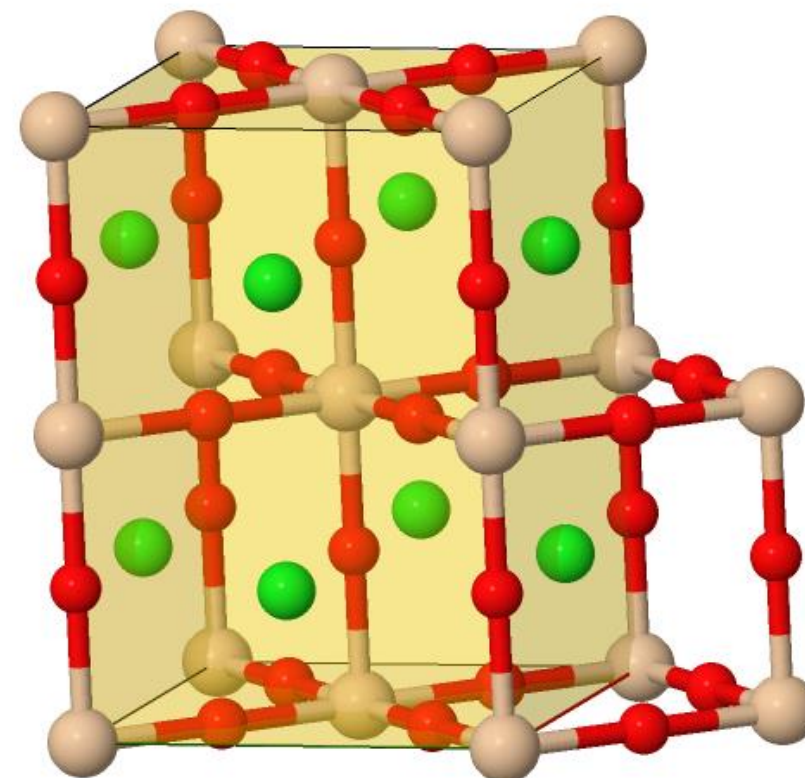
```
draw ID uc1 unitcell color red
```

```
draw ID ax1 axes
```

```
modelkit spacegroup "123 > -a+b,-a-b,2c;1,0,0 > 140" packed
```

```
draw ID uc2 unitcell fill nomesh translucent 0.8
```

```
connect 2.0 2.2
```



Summary: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

- **Simple**

P 1 2/m 1

10:a,b,c;0,0,0

P 1 2/c 1

13:a,b,c;0,0,0

P 1 1 2/m

10:c,a,b;0,0,0

P 1 1 2/n

13:a,-a-c,b;0,0,0

P 2/m 1 1

10:b,c,a;0,0,0

P 2/b 1 1

13:b,c,a;0,0,0

Summary: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

- Simple

P 1 2/m 1

10

P 1 2/c 1

13:a,b,c

- **Concise**

P 1 1 2/m

10:c,a,b

P 1 1 2/n

13:a,-a-c,b

P 2/m 1 1

10:b,c,a

P 2/b 1 1

13:b,c,a

Summary: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

- Simple
- Concise
- **Comprehensive**

$10:b,c,a > !b,c,a > 10$

$10 > a-c,b,2c;0,0,1/2 > 13$

$13 > a,-a-c,b > 13:a,-a-c,b$

$10:b,c,a > -b+c,-b-c,a;0,1/2,0 > 13:a,-a-c,b$

nonstandard to standard setting

group to subgroup (standard settings)

standard to nonstandard

nonstandard group to nonstandard subgroup

Summary: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

- Simple
- Concise
- Comprehensive
- **Flexible**

$10 > -b+c, -b-c, a:0, 1/2, 0 > 13$ (for human or machine consumption)

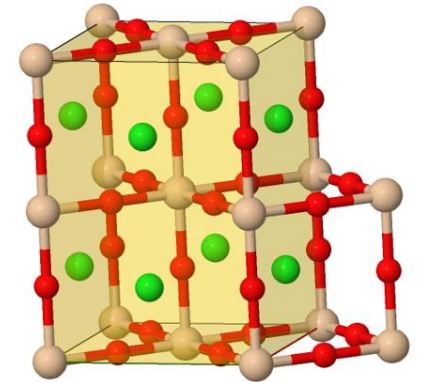
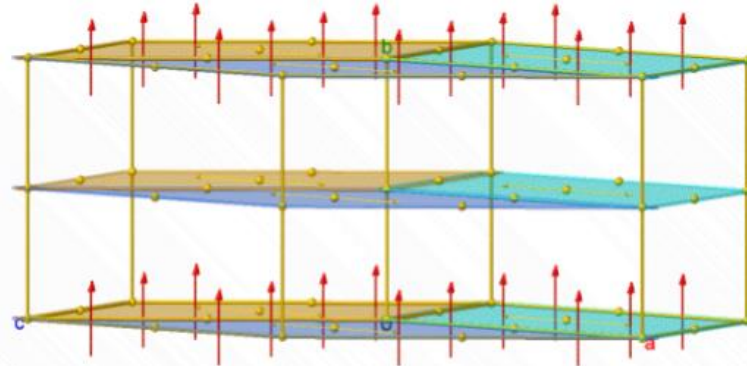
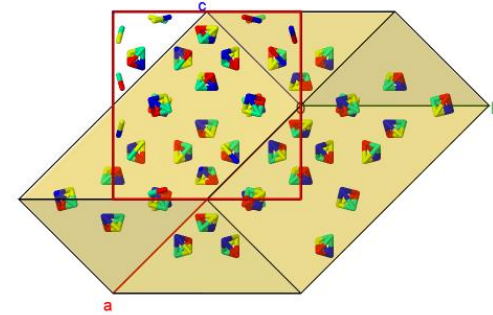
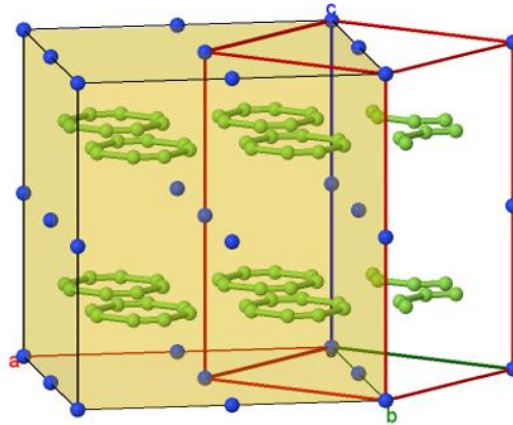
$P 2/m 1 1 > -b+c, -b-c, a:0, 1/2, 0 > P 1 1 2/n$ (for human consumption)

$(-P 2y):b, c, a > -b+c, -b-c, a:0, 1/2, 0 > (-P 2cy):a, -a-c, b$ (for machine consumption)

Summary: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

- Simple
- Concise
- Comprehensive
- Flexible
- **And it works!**

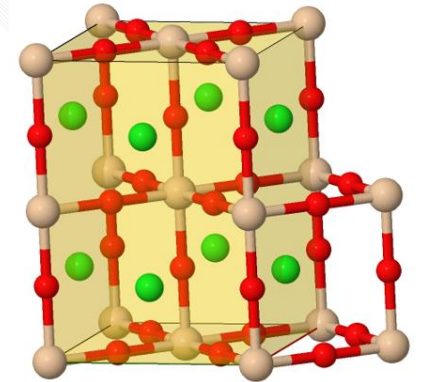
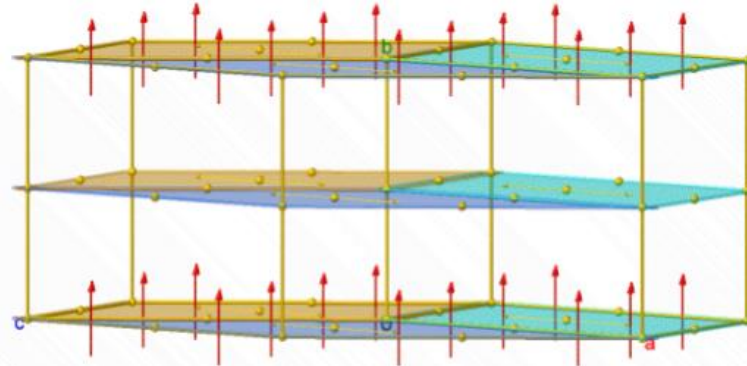
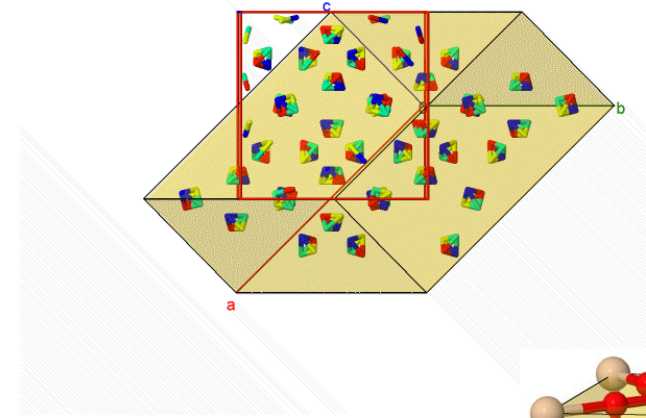
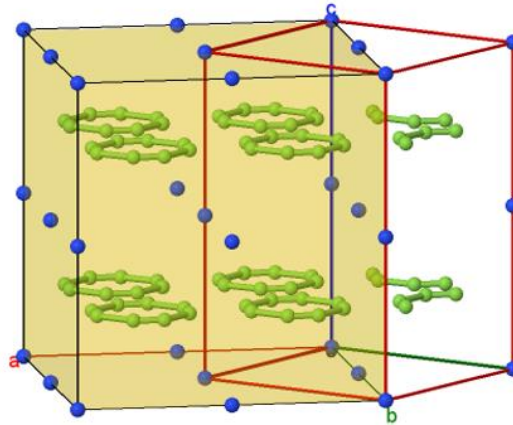


Summary: CLEG

A Concise Linear Encoding of Crystallographic Space Group Settings and Group-Subgroup Relationships (CLEG)

Many thanks to Branton Campbell, Harold Stokes, Mois Aroyo, and Gotzon Madariaga for their patience, support, and many great conversations.

Thank you!



More Jmol implementations this afternoon!

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Using Jmol to engage students and teach crystallography and crystallographic symmetry

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