

计算方法第一次作业

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2023218155-计算机 23-3 班-汪博

1. 第六题

6. 解: (1) $\because f(x) = x^3$, 当 $x_0 = -1$ 时 $f(-1) = -1$, 当 $x_0 = 1$ 时 $f(1) = 1$

$$\therefore p_1(x) = f(-1) \cdot \frac{x-1}{-1-1} + f(1) \cdot \frac{x-(-1)}{1-(-1)}$$

$$= -1 \cdot \frac{x-1}{-2} + 1 \cdot \frac{x+1}{2}$$

$$= \frac{1}{2}x + \frac{1}{2} + \frac{1}{2}x - \frac{1}{2} = x$$

\therefore 线性插值 $p_1(x) = x$

(2) $f(-1) = -1, f(0) = 0, f(1) = 1$

$$p_2(x) = f(-1) \cdot \frac{(x-0)(x-1)}{(-1-0)(-1-1)} + f(0) \cdot \frac{(x-(-1))(x-1)}{(0-(-1))(0-1)} + f(1) \cdot \frac{(x-(-1))(x-0)}{(1-(-1))(1-0)}$$

$$= -\frac{x(x-1)}{2} + \frac{x(x+1)}{2}$$

$$= -\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}x^2 + \frac{1}{2}x = x$$

\therefore 二次插值 $p_2(x) = x$

(3) $f(-1) = -1, f(0) = 0, f(1) = 1, f(2) = 8$

$$p_3(x) = f(-1) \cdot \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} + f(0) \cdot \frac{(x-(-1))(x-1)(x-2)}{(0-(-1))(0-1)(0-2)} +$$

$$f(1) \cdot \frac{(x-(-1))(x-0)(x-2)}{(1-(-1))(1-0)(1-2)} + f(2) \cdot \frac{(x-(-1))(x-0)(x-1)}{(2-(-1))(2-0)(2-1)}$$

$$= -\frac{x(x-1)(x-2)}{-6} + \frac{x(x+1)(x-2)}{-2} + 8 \cdot \frac{x(x-1)(x-1)}{6} = \frac{x^3 - 3x^2 + 2x}{-6} - \frac{x^3 + x^2 - 2x}{2} + \frac{8x^3 - 8x}{6}$$

$$= \frac{6x^3}{6} = x^3$$

\therefore 三次插值 $p_3(x) = x^3$

2. 第十一题

11. 解: (1)

$$p_2(x) = f(x_0) \cdot \frac{(x-0.47)(x-0.48)}{(0.46-0.47)(0.46-0.48)} + f(x_1) \cdot \frac{(x-0.46)(x-0.48)}{(0.47-0.46)(0.47-0.48)} + f(x_2) \cdot \frac{(x-0.46)(x-0.47)}{(0.48-0.46)(0.48-0.47)}$$

$$= -0.4785x^2 - 41.30467x - 0.0284638$$

$$p_2(0.472) = 1.49355588$$

(2) $p_2(x) = 0.5$

$$x = 0.476934$$

3. 第十二题

12. 解: (1) $P_3(x) = f(1) \cdot \frac{(x-1)(x-3)(x-4)}{(1-1)(1-3)(1-4)} + f(1) \cdot \frac{(x-1)(x-3)(x-4)}{(1-1)(1-3)(1-4)} + \dots$

$f(1)=3, f(4)=1$
 $f(3)=10, f(4)=44$

$= 1 \cdot \frac{(x-1)(x-3)(x-4)}{-40} + 3 \cdot \frac{(x-1)(x-3)(x-4)}{12} + 10 \cdot \frac{(x-1)(x-3)(x-4)}{-8} + 44 \cdot \frac{(x-1)(x-3)(x-4)}{15}$

$= 4x^3 - 3x + 2$

$R_3(x) = f(x) - P_3(x) = 0$

(2) $P_3(x) = f(1) \cdot \frac{(x-1)(x-3)(x-4)}{(1-1)(1-3)(1-4)} + f(1) \cdot \frac{(x-1)(x-3)(x-4)}{(1-1)(1-3)(1-4)} + \dots$

$f(1)=3, f(1)=-1$
 $f(3)=27, f(4)=128$

$= 5x^3 - 11x^2 - 7x + 12$

$R_3(x) = f(x) - P_3(x) = x^4 - 3x^3 + 11x^2 + 7x - 12 = (x+1)(x-3)(x-4)$

4. 第十三题

13. 解: 线性插值: $P_1(x) = f(0.32) \cdot \frac{(x-0.34)}{0.32-0.34} + f(0.34) \cdot \frac{(x-0.32)}{0.34-0.32}$

$= 0.314567 \cdot \frac{(x-0.34)}{-0.02} + 0.333487 \cdot \frac{(x-0.32)}{0.02}$

$= 0.946x + 0.01847$

$P_1(0.3367) = 0.333652$

$R_1(x) = 0.92 \times 10^5$

抛物插值: $P_2(x) = f(0.32) \cdot \frac{(x-0.34)(x-0.36)}{(0.32-0.34)(0.32-0.36)} + f(0.34) \cdot \frac{(x-0.32)(x-0.36)}{(0.34-0.32)(0.34-0.36)} + f(0.36) \cdot \frac{(x-0.32)(x-0.34)}{(0.36-0.32)(0.36-0.34)}$

$= -0.16625x^2 + 1.05572x - 0.006241$

$\therefore P_2(0.3367) = 0.333726785375$

$R_2(x) = 0.24 \times 10^6$

5. 第十六题

16. 解: 构造差商表:

	0阶差商	1阶差商	2阶差商	3阶差商	4阶差商
1	7				
2	21	14			
3	59	38	12		
4	133	74	18	(2)	
5	255	122	24	2	0

\therefore 可求 $f[1,2,3,4] = 2$
 $f[1,2,3,4,5] = 0$

6. 第十七题

17. 解: 构造差商表:

x_i	0阶	1阶	2阶	3阶	4阶	5阶
$x_0=1$	-3					
$x_1=2$	0	3				
$x_2=3$	15	15	6			
$x_3=4$	48	33	9	1		
$x_4=5$	105	57	12	1	0	
$x_5=6$	192	87	15	1	0	0

$\therefore p_5(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2) + f[x_0, x_1, x_2, x_3, x_4](x-x_0)(x-x_1)(x-x_2)(x-x_3) + f[x_0, x_1, x_2, x_3, x_4, x_5](x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

$= -3 + 3(x-1) + 6(x-1)(x-2) + 1(x-1)(x-2)(x-3) + 0(x-1)(x-2)(x-3)(x-4) + 0(x-1)(x-2)(x-3)(x-4)(x-5)$

$= x^3 - 4x$

7. 第三十一题

31. 解: 考虑每一段上, 有:

$$M = f''(\xi)_{\max}$$

$$\therefore \frac{M}{8} h^2 \leq 0.5 \times 10^{-5}$$

$$M = 1$$

$$\therefore h \leq 2\sqrt{0.5 \times 10^{-5}}$$

\therefore 最大取 $2\sqrt{0.5 \times 10^{-5}}$

8. 第三十六题

36. 解: $N=9$ 时:

$$\begin{cases} aN + b \sum_{i=1}^N x_i = \sum_{i=1}^N y_i \\ a \sum_{i=1}^N x_i + b \sum_{i=1}^N x_i^2 = \sum_{i=1}^N x_i y_i \end{cases} \Rightarrow \begin{cases} a = 2.0314 \\ b = 2.25165 \end{cases}$$

$y = bx + a = 2.25165x + 2.0314$

37. 解: $N=5$ 时:

$$\begin{cases} a_0 N + a_1 \sum_{i=1}^N x_i + a_2 \sum_{i=1}^N x_i^2 = \sum_{i=1}^N y_i \\ a_0 \sum_{i=1}^N x_i + a_1 \sum_{i=1}^N x_i^2 + a_2 \sum_{i=1}^N x_i^3 = \sum_{i=1}^N x_i y_i \\ a_0 \sum_{i=1}^N x_i^2 + a_1 \sum_{i=1}^N x_i^3 + a_2 \sum_{i=1}^N x_i^4 = \sum_{i=1}^N x_i^2 y_i \end{cases} \Rightarrow \begin{cases} a = 2.0001 \\ b = 2.25165 \\ c = 0.0313 \end{cases}$$

$y = 0.0313x^2 + 2.25165x + 2.0001$

9. 第三十七题

37. 解: $N=5$ 时:

$$\begin{cases} aN + b \sum_{i=1}^N x_i^2 = \sum_{i=1}^N y_i \\ a \sum_{i=1}^N x_i^2 + b \sum_{i=1}^N x_i^4 = \sum_{i=1}^N x_i^2 y_i \end{cases} \Rightarrow \begin{cases} a = 0.9732361 \\ b = 0.0503451 \end{cases}$$

$y = 0.0503451x^2 + 0.9732361$