

# 计算方法第一次作业

2024年12月15日

2023218155-计算机 23-3 班-汪博

## 1. 第六题

$$6. \text{Q: (1) } \because f(x_0)=x^3, \quad \frac{1}{2}x_0=-1 \Rightarrow f(-1)=-1, \quad \frac{1}{3}x_0=1 \Rightarrow f(1)=1$$

$$\therefore P_1(x_0) = f(-1) \cdot \frac{x-1}{-1-1} + f(1) \cdot \frac{x+1}{1-1}$$

$$= -1 \cdot \frac{x-1}{2} + 1 \cdot \frac{x+1}{2}$$

$$= \frac{1}{2}x + \frac{1}{2} + \frac{1}{2}x - \frac{1}{2} = x$$

∴ 1次插值  $P_1(x_0)=x$

$$(2) \quad f(-1)=-1, \quad f(0)=0, \quad f(1)=1$$

$$P_2(x_0) = f(-1) \cdot \frac{(x-0)(x-1)}{(-1-0)(-1-1)} + f(0) \cdot \frac{(x-1)(x+1)}{(0-1)(0+1)} + f(1) \cdot \frac{(x-0)(x+1)}{(1-0)(1+1)}$$

$$= -\frac{x(x+1)}{2} + \frac{x(x+1)}{2}$$

$$= -\frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}x^2 + \frac{1}{2}x = x$$

∴ 2次插值  $P_2(x_0)=x$

$$(3) \quad f(-1)=-1, \quad f(0)=0, \quad f(1)=1, \quad f(2)=8$$

$$P_3(x_0) = f(-1) \cdot \frac{(x-0)(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} + f(0) \cdot \frac{(x-1)(x-0)(x-2)}{(0-1)(0-1)(0-2)} +$$

$$f(1) \cdot \frac{(x-1)(x-0)(x-1)}{(1-0)(1-1)(1-2)} + f(2) \cdot \frac{(x-1)(x-0)(x-1)}{(2-1)(2-1)(2-2)}$$

$$= -\frac{b(b-1)(b-2)}{-6} + \frac{b(b+1)(b-2)}{-2} + 8 \frac{b(b-1)(b+1)}{6} = \frac{x^3 - 3x^2 + 2x - 3x^2 + 3x + 6x + 8x^3 - 8x}{6}$$

$$= \frac{6x^3}{6} = x^3$$

∴ 3次插值  $P_3(x_0)=x^3$

## 2. 第十一题

$$11. \text{Q: (1)}$$

$$P_2(x_0) = f(x_0) - \frac{(x-0.4)(x-0.8)}{(0.4-0.4)(0.4-0.8)} + f(0.4) \cdot \frac{(x-0.4)(x-0.8)}{(0.4-0.4)(0.4-0.8)} - f(x_0) \cdot \frac{(x-0.4)(x-0.8)}{(0.8-0.4)(0.8-0.4)}$$

$$= -0.476934x^2 + 1.30469x - 0.476934$$

$$P_2(0.472) = 1.49355588$$

$$(2) \quad P_2(x_0) = 0.5$$

$$x = 0.476934$$

### 3. 第十二題

$$\begin{aligned}
 & 12. \text{ 計算} \quad P_3(x) = f(-1) \cdot \frac{(x-1)(x-3)(x-4)}{(-1-1)(-1-3)(-1-4)} + f(1) \cdot \frac{(x+1)(x-3)(x-4)}{(1+1)(1-3)(1-4)} + f(3) \cdot \frac{(x+1)(x-1)(x-4)}{(3+1)(3-1)(3-4)} + f(4) \cdot \frac{(x+1)(x-1)(x-3)}{(4+1)(4-1)(4-3)} \\
 & = 1 \cdot \frac{(x-1)(x-3)(x-4)}{-40} + 3 \cdot \frac{(x+1)(x-3)(x-4)}{12} + 10 \cdot \frac{(x+1)(x-1)(x-4)}{-8} + 24 \cdot \frac{(x+1)(x-1)(x-3)}{15} \\
 & = 4x^3 - 3x^2 + 2 \\
 R_3(x) &= f(x) - P_3(x) = 0
 \end{aligned}$$
  

$$\begin{aligned}
 & 13. \quad P_3(x) = f(-1) \cdot \frac{(x+1)(x-3)(x-4)}{(-2+1)(-2-3)(-2-4)} + f(1) \cdot \frac{(x+4)(x-3)(x-4)}{2+1)(2-3)(2-4)} + f(3) \cdot \frac{(x+1)(x-1)(x-4)}{4 \cdot 2 \cdot 1+1)5 \cdot 3 \cdot 1} \\
 & = 5x^3 - 11x^2 - 7x + 12 \\
 R_3(x) &= f(x) - P_3(x) = x^4 - 3x^3 + 11x^2 - 7x + 12 = (x+1) \times (x-3) \times (x-4)
 \end{aligned}$$

### 4. 第十三題

$$\begin{aligned}
 & 13. \text{ 計算} \quad P_1(0.0) = f(0.32) \cdot \frac{(x-0.34)}{0.32-0.34} + f(0.34) \cdot \frac{(x-0.32)}{0.34-0.32} \\
 & = 0.314567 \cdot \frac{(x-0.34)}{-0.02} + 0.333487 \cdot \frac{(x-0.32)}{0.02} \\
 & = 0.9466x + 0.011847 \\
 P_1(0.367) &= 0.3303652 \\
 R_1(x) &= 0.92 \times 10^{-5}
 \end{aligned}$$

$$\begin{aligned}
 & \text{地圖插值: } P_2(x) = f(0.32) \cdot \frac{(x-0.34)(x-0.36)}{(0.32-0.34)(0.32-0.36)} + f(0.34) \cdot \frac{(x-0.32)(x-0.36)}{(0.34-0.32)(0.34-0.36)} \\
 & + f(0.36) \cdot \frac{(x-0.32)(x-0.34)}{(0.36-0.32)(0.36-0.34)}
 \end{aligned}$$

$$= -0.1662x^2 + 1.05572x - 0.006241$$

$$\therefore P_2(0.367) = 0.3303726785375$$

$$R_2(x) = 0.241 \times 10^{-6}$$

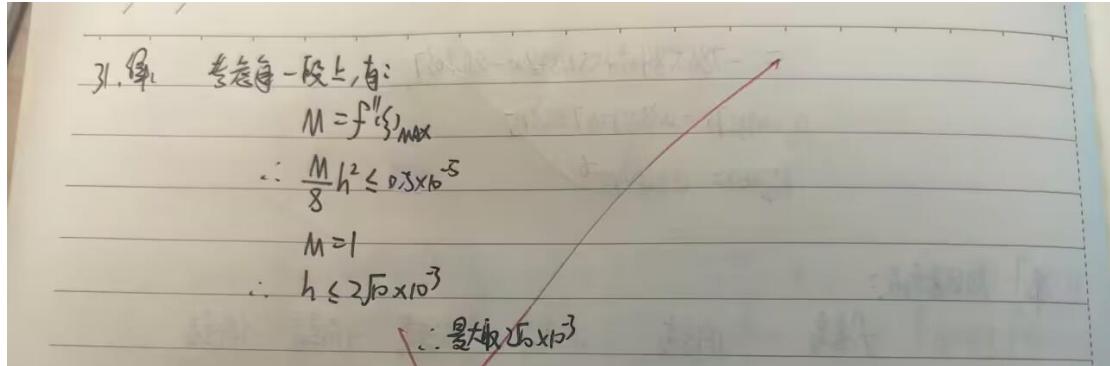
### 5. 第十六題

16. 順序表:		0阶高	1阶高	2阶高	3阶高	4阶高	
1	7						
2	21	14					
3	59	38	12				
4	133	74	18	②			
5	253	122	24	2	0		
可解		$f[1, 2, 3, 4] = 2$					
		$f[1, 2, 3, 4, 5] = 0$					

### 6. 第十七題

17. 亂序表:		0阶	1阶	2阶	3阶	4阶	5阶	
$b_0=1$	3							
$b_1=2$	0	3						
$b_2=3$	15	15	6					
$b_3=4$	48	33	9	1				
$b_4=5$	105	57	12	1	0			
$b_5=6$	192	87	15	1	0	0		
		$\therefore P_3(x) = f[b_0] + f[b_0, b_1](x - b_0) + f[b_0, b_1, b_2](x - b_0)(x - b_1) +$ $f[b_0, b_1, b_2, b_3](x - b_0)(x - b_1)(x - b_2)$						
		$= 3 + 3(x - 1) + 6(x - 1)(x - 2) + 1(x - 1)(x - 2)(x - 3)$						
		$= x^3 - 4x$						

### 7. 第三十一題



8. 第三十六题

$$\begin{aligned}
 & 36. \text{ 例: } \begin{cases} aN + b \sum_{i=1}^N x_i = \sum_{i=1}^N y_i \\ a \sum_{i=1}^N x_i + b \sum_{i=1}^N x_i^2 = \sum_{i=1}^N x_i y_i \end{cases} \Rightarrow \begin{cases} a = 2.01314 \\ b = 2.25165 \end{cases} \\
 & N=9 \text{ 时: } \begin{cases} a_0 N + a_1 \sum_{i=1}^N x_i + a_2 \sum_{i=1}^N x_i^2 = \sum_{i=1}^N y_i \\ a_0 \sum_{i=1}^N x_i + a_1 \sum_{i=1}^N x_i^2 + a_2 \sum_{i=1}^N x_i^3 = \sum_{i=1}^N x_i y_i \\ a_0 \sum_{i=1}^N x_i^3 + a_1 \sum_{i=1}^N x_i^4 + a_2 \sum_{i=1}^N x_i^5 = \sum_{i=1}^N x_i^2 y_i \end{cases} \Rightarrow \begin{cases} a = 2.0001 \\ b = 2.25165 \\ c = 0.0313 \end{cases} \\
 & y = 0.0313x^2 + 2.25165x + 2.0001
 \end{aligned}$$

9. 第三十七题

$$\begin{aligned}
 & 37. \text{ 例: } \begin{cases} N=5 \text{ 时: } \begin{array}{l} aN + b \sum_{i=1}^N x_i^2 = \sum_{i=1}^N y_i \\ a \sum_{i=1}^N x_i^2 + b \sum_{i=1}^N x_i^4 = \frac{N}{2} x_i^2 y_i \end{array} \Rightarrow \begin{cases} a = 0.973236 \\ b = 0.000345 \end{cases} \\ y = 0.000345x^2 + 0.973236 \end{cases}
 \end{aligned}$$