Estimating Labour Supply Elasticities for Married Women

Bob Wen

Jan, 2019

Cobb-Douglas Model

$$
\begin{aligned}
&Max\;\; U = \alpha_c lnc + \alpha_l lnl + \alpha_f lnf \\
&s.t.\; p_c c + p_f f = (1-\tau)w(1-l-f) + v
\end{aligned}
\tag{1}
$$

where

- c: composite consumption;
- l: leisure time;
- f: time spent on housework, care of family, community improvement;
- p_c: price of consumption;
- p_f: price of housework
- τ: tax rate
- $(1-\tau)w$: effective wage rate, the opportunity cost of both leisure and housework
- v: non-labour income and wealth

Cobb-Douglas Model

$$
\begin{aligned}
\mathcal{L} =& \alpha_c lnc + \alpha_l lnl + \alpha_f lnf \\
&+ \lambda[(1-\tau)w + v - p_c c - p_f f - (1-\tau)wl - (1-\tau)wf]
\end{aligned}
\tag{2}
$$

FOCs:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c} &= 0 \Rightarrow c = \frac{\alpha_c}{\lambda p_c} \\
\frac{\partial \mathcal{L}}{\partial l} &= 0 \Rightarrow l = \frac{\alpha_l}{\lambda(1-\tau)w} \\
\frac{\partial \mathcal{L}}{\partial f} &= 0 \Rightarrow f = \frac{\alpha_f}{\lambda(p_f + (1-\tau)w)} \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \Rightarrow budget\ constraint
\end{aligned}
\tag{3}
$$

The Interior Solution

$$
\begin{aligned}
c^* &= \theta_c \frac{(1-\tau)w + v}{p_c} \\
l^* &= \theta_l \frac{(1-\tau)w + v}{(1-\tau)w} \\
f^* &= \theta_f \frac{(1-\tau)w + v}{p_f + (1-\tau)w}
\end{aligned}
\tag{4}
$$

where $\theta_c, \theta_l, \theta_f$ are the budget shares of consumption, leisure time, and housework.

Thus, the faction of hours of work

$$
h^* = 1 - l^* - f^* = \theta_c - \theta_l \frac{v}{(1-\tau)w} - \theta_f \frac{v - p_f}{p_f + (1-\tau)w}
$$

(5)

Comparative Static Analysis

- $\frac{\partial h^*}{\partial \theta_c} > 0$
- $\frac{\partial h^*}{\partial \theta_l} < 0$
- $\frac{\partial h^*}{\partial v} < 0$
- $\frac{\partial h^*}{\partial \theta_f}$ uncertain, if $v > p_f$ then negative
- $\frac{\partial h^*}{\partial w}$ uncertain, if $v > p_f$ then positive
- $\frac{\partial h^*}{\partial \tau}$ uncertain, if $v > p_f$ then negative

Corner Solution

The fraction of hours of work h^* could be zero, i.e. not willing to work in labour market, if

- wage rate is low,
- budget share of consumption is low
- cost of housework and charitable work is low, or the benefit of them is high
- budget share of leisure is high
- budget share of housework or charitable work is high
- non-labour income, transfer, or wealth are high
- tax rate is high

It is about the comparison of the costs and benefits of working.

The corresponding empirical model

The willingness to work is affected by factors from individual level:

- Individual/family level: non-labour income v (including spouse income, transfers from government, and help from others), housework or family responsibility (number of children, age of the children), wage rate, preference, education, health, age, race, region

or two types of incentives:

- monetary incentives: own labour income, spouse labour income, transfers,
- non-monetary incentives: family responsibility and community charitable work; education; health; age; race, region

Estimate Model for Interior Solution

$$\begin{aligned} \mathit{ln\_hours}^{s}_{it} = \beta_0 + \beta_1 \mathit{ln\_labour\_income}^{s}_{it} \\ + \mathbf{Z_1}\alpha + \mathit{year\_dummies}\gamma \\ + u^{s}_{it} \end{aligned} \tag{6}$$

The exogenous covariates $\mathbf{Z_1}$=($\mathit{labour\_income\_type}_{it}$, $\mathit{ln\_spouse\_labour\_income}_{it}$, $\mathit{ln\_non\_labour\_income}_{it}$, edu_i, health_{it}, age_{it}, race_i, region_{it}).

Endogeneity Challenge I: OVB

Omitted Variable Bias (OVB): $Cov(ln\_labour\_income^{s}_{it}, u^{s}_{it}) \neq 0$
The omitted variables in the error component include unobserved individual ability and family traditions that affect both labour income required and hours of work.

Solutions

Fixed effects a_i^s:

$$\begin{aligned} ln\_hours_{it}^s = \beta_0 + \beta_1 ln\_labour\_income_{it}^s \\ + \mathbf{Z_1}\alpha + year\_dummies\gamma \\ + a_i^s + \varepsilon_{it}^s \end{aligned} \tag{7}$$

- a_i^s: Unobserved individual heterogeneity include family traditions and individual characteristics and preferences that are different among individuals but time-invariant.
- It can be cancelled out using within estimator or first differencing estimator.

Solution II

Instrumental Variables (IV):

- Relevance condition: $Cov(\mathbf{z}_2, \mathit{ln\_labour\_income}) \neq 0$;
- Exclusion restriction: $Cov(\mathbf{z}_2, \varepsilon) = 0$
- IVs ($\mathbf{z}_2$) are the demand shifters that come from wage offering equation determined by labour demand: experience, occupations, union status, economic environment measured by average state unemployment rate. These factors influence the equilibrium wage rates, and they affect individual's decision to work (the labour supply) only through this channel. They are not correlated with the unobserved individual ability or family traditions that affect individual's labour supply choice. They do not directly affect individual labour supply either.

2SLS

- First stage:
  Regress log wage rate on the demand shifters z_2 and exogenous covariates z_1, and then obtain the fitted value of log wage rate.
- Second stage:

$$\begin{aligned} ln\_hours^{s}_{it} = \beta_0 + \beta_1 ln\_lab\hat{our}\_income_{it} \\ + \mathbf{Z_1}\alpha + year\_dummies\gamma \\ + a^{s}_{i} + \varepsilon^{s}_{it} \end{aligned} \tag{8}$$

Endogeneity Challenge II: Sample Selection

Even though we have controlled for the OVB, there is still another endogenous problem from sample selection. If we intend to investigate the causal relationship between labour supply of the population instead of only those already employed, we should consider the factors that lead to self-selection into employment.

$$ln\_hours^s_{it} = \begin{cases} \beta_0 + \beta_1 ln\_labour\_income^s_{it} \\ +\mathbf{Z_1}\alpha + year\_dummies\gamma \\ +a^s_i + \varepsilon^s_{it} & if \;\; lfp = 1 \\ \\ missing & if \;\; lfp = 0 \end{cases}$$

Solution

Include the probability of willingness to work of the employed in the hours of work labour supply model.

$$\begin{aligned} ln\_hours^s_{it} = \beta_0 + \beta_1 ln\_lab\hat{our}\_income_{it} \\ + \mathbf{Z_1}\alpha + year\_dummies\gamma \\ + \beta_2 willingness_{it} + a^s_i + \varepsilon^s_{it} \end{aligned} \quad (9)$$

where *willingness* is the probability of willingness to work for each individual in the sample. It comes from the willingness to work (labour force participation) model for all population.

Estimate Model for Corner Solution: The willingness to work regression

Willingness to Work -Labour Force Participation of Women -Logit Panel Data Model

$$
\begin{aligned}
Pr(wife\_lfp_{it} = 1|\mathbf{x}) = \Lambda(\gamma_0 &+ \gamma_1 lnhusband\_labourincome_{it} \\
&+ \gamma_2 non\_labour\_income_{it} \\
&+ \gamma_3 num\_children_{it} + \gamma_4 age\_youngestchil_{it} \\
&+ \gamma_5 edu_i + \gamma_6 health_{it} + \gamma_7 age_{it})
\end{aligned} \tag{10}
$$

Then, compute the probability of willingness to work for each individual.

OVB in the Willingness to Work Logit Model and Solutions

Number of children could be endogenous, because same unobserved factors may affect fertility and willingness to work at the same time.

- Panel data method (within estimator or first differencing) to control for time-invariant fixed effects
- Use exogenous variables that are correlated with fertility but have no direct effects on willingness to work from outside the logit equation. These individual level variables include
  - Recreation and living condition variables, such as TV ownership, expenditure on recreation, and air conditioning.

2SLS for Willingness to Work Model

- The first stage:

$$num\_children_{it} = J_{1it}\theta_1 + J_{2it}\theta_2 + \omega_{it} \tag{11}$$

  where J_{1it} are the exogenous explanatory variables from the logit willingness to work equation; J_{2it} are the IVs from outside the model (the recreation variables and living conditions variables).

- The second stage:
  Replace $num\_children_{it}$ with its fitted value in (10).