Probability and Measure – Tutorial 4

1. Let $\Omega = \{a, b, c\}$ be a set consisting of three distinct elements and let $\mu^* : P(\Omega) \to [0, \infty]$ be defined by
$$\mu^*(\emptyset) = 0, \quad \mu^*(\Omega) = 2, \quad \mu^*(A) = 1, \quad \text{otherwise.}$$
Show that μ^* is an outer measure on Ω and determine the μ^*-measurable subsets of Ω.

2. Let μ^* be an outer measure on Ω and assume that $\mu^*(A) = 0$ for some $A \subset \Omega$. Show that $\mu^*(A \cup B) = \mu^*(B)$ for any $B \subset \Omega$.

3. Let Ω be a set with an outer measure μ^*. Let $A \subset \Omega$ be measurable. Let $B \subset \Omega$ be such that $A \subset B$, $\mu^*(A) = \mu^*(B)$, and $\mu^*(B) < \infty$. Show that B is measurable.

4. Let Ω be a set with an outer measure μ^*. Let $A \subset \Omega$ be measurable and let $B \subset \Omega$ with $\mu^*(B) < \infty$. Show that
$$\mu^*(A \cup B) = \mu^*(A) + \mu^*(B) - \mu^*(A \cap B).$$