# Statistics 831: Probability and Statistical Models

Homework #1
Due 1/22/2020

## 1 Problems:

1. (RC 1$^{\text{st}}$ ed: 2.5) The RANDU generator, once popular on IBM machines, is based on the recursion:

$$X_{n+1} = 65539X_n \mod 2^{31}.$$

Illustrate the undesirable behavior of this generator with a computer experiment, and produce a 3D plot which demonstrates the problem clearly. (*Hint:* show $X_{t+1} = (6X_t - 9X_{t-1}) \mod 2^{31}$

2. In class we discussed the Box-Muller algorithm for generating pairs of iid $N(0,1)$ random variates.

   (a) Show that if $X_1$ and $X_2$ are independent standard normal random variables then their polar coordinates $R = \sqrt{X_1^2 + X_2^2}$ and $\theta = \tan^{-1} X_1/X_2$ are also independent and derive their distributions.

   (b) Show how to generate $R$ and $\theta$ by the inverse cdf method, and a pair of standard normals via the substitution $X_1 = R\cos\theta$ and $X_2 = R\sin\theta$.

   (c) Implement the Box-Muller algorithm, and compare with the method described in class based on $k = 12$ independent uniform $[0,1]$ r.v.'s and the CLT. Are the two "statistically indistinguishable"? How do the respective computational costs compare?

3. RC 2.23 Skip (c) and part ii of (d). You may use available routines to generate the Gamma random variables.

4. RC 2.30 Skip (f).

5. Given a normal distribution $N(0,1)$ restricted to $\mathbb{R}^+$, construct an Accept-Reject algorithm based on $Exp(\lambda)$ and optimize in $\lambda$. Repeat for intervals $[1,\infty)$, $[2,\infty)$, $[4,\infty)$. For each interval, compare the efficiency to direct rejection sampling from the normal.