Case Study 3: Election Prediction

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1. Introduction

The outcome of the 2016 election not only stunned the nation, but also sent shockwaves through the statistical and polling communities. Over the course of the election year in 2016 up until the week of the election, poll predictions of Hillary Clinton's likelihood of beating Donald Trump ranged from 71%- 99% probability [1]. So when Trump beat these odds, the polling industry lost a lot of trust from the general public [2].

This small, specialized industry that is the political polling business, has a great deal of influence on how the election is portrayed in the news media, voter decisions, and candidate partisan policy initiatives. [1]. Million dollar decisions on advertising and campaign strategy are dictated by polls.

Polls conducted early in the election year are a weak predictor of election outcomes because the general public has paid less attention to the race and is less knowledgeable of the candidates' platforms. Voters that tend to sway between parties often report that they are undecided, however these are arguably the most important opinions in predicting the election outcomes. [3] As the election nears, polls become more accurate, but that is where historical prediction models have been lacking – they fail to take into account the differential accuracy in poll results. Historical models, regression based models that rely on outcomes from past elections and structural factors, predict election outcomes at a single point in time which renders high levels of uncertainty [4].

Drew A. Linzer developed a dynamic bayesian forecasting model to predict the U.S. presidential election at the national and state level that combines these historical models with everchanging poll updates. Linzer's model uses hierarchical specification to handle states being polled on different days and takes into account sampling errors of the polls and national campaign effects [4].

In this report we aim to:

- Predict the outcome of the presidential election and the electoral college vote using the Linzer model to predict swing state outcomes in combination
- Predict whether the US Senate remains in Republican control by using an adaptation of the Linzer model and FiveThirtyEight Senate poll data for each state
- Predict the outcomes of all 13 NC Congressional elections using Linzer model with input from FiveThirtyEight Senate poll data for North Carolina along with our model that predicts who will vote in North Carolina
- Predict the outcome of the NC Senate election and the associated uncertainty using the Linzer model.

Taking into account the anomaly that was the 2016 election, we chose to use the Linzer model to best account for slight and unexpected changes leading up to election day. In Section 2 of this report we will discuss datasets used for all tasks, including a brief exploratory data analysis. In Section 3, we formulate models and methodologies that answer all the research questions. Section 4 will present model diagnostic and sanity check validation of prediction. Then, in section 5 we present the major results of our analysis. Section 6 will be focused on conducting sensitivity analysis to test data imputation hypothesis and prior choices.

2. Data Source and EDA

2.1 Description of Data

In order to answer these questions, we used a total of four datasets: senate polls, house polls, 2020 US presidential election polls, and North Carolina voter registration history snapshot dataset. Both the senate and house polls dataset comes from the fivethirtyeight website [5], while the presidential election polls dataset comes from the Economist website [6].Senate and house polls have 38 variables and 4061 and 2655 observations respectively. The presidential election polls have 1447 observations and 17 variables which are: poll state, pollster, sponsor, start and end date, entry time, number of observations, population, method, Biden, Trump, Biden margin, other, undecided, URL, include, and notes.

The North Carolina voter history dataset contains information on the 2016 elections and about how voters voted (method, election, county, party affiliation, precinct). This will help us model and identify potential voters in 2020 (Interim report). On top of the created model, we used the 2020 voter registration snapshot dataset to identify potential voters, and use this information as additional input to model house polls results.

2.2 Exploratory Data Analysis

To get a basic understanding of the four datasets we were working with, we went through the data to understand what we were working with. In the senate and the house polls, we explored the variables of: methodology of the poll, the state in which the poll was conducted, which party the candidate was, and the percentage of the poll. In the presidential election polls, we explored similar variables of: method of poll and state, in addition to the margin between Biden and Trump. Because all three of these datasets had similar variables, we are able to compare them to each other. In Figure 1, we can see how the methods of polling were distributed between the house, senate, and presidential polls.

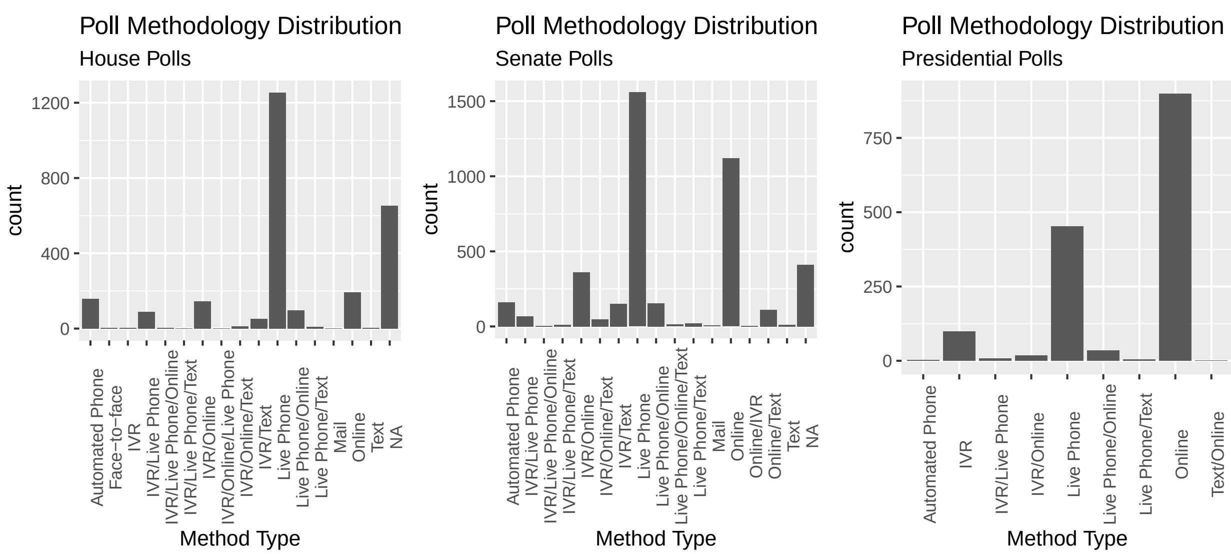

Figure 1: Poll Methodology

In Figure 2, we can see how each poll's distribution of which state was polled, and in Figure 3, we can see how the house and senate polls' candidate party are different from each other. Finally, in Figure 4, we can see the distribution of the margin between Biden and Trump.

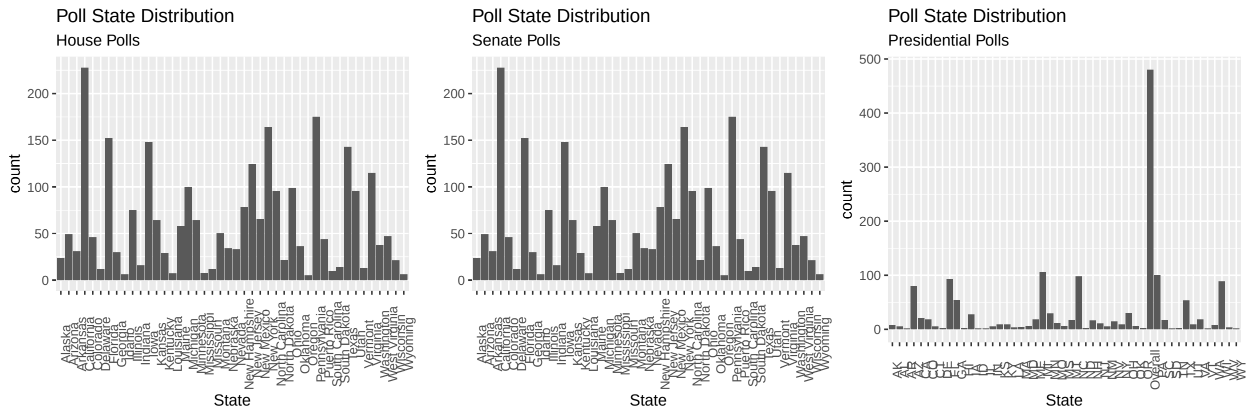

Figure 2: Polls State Distribution

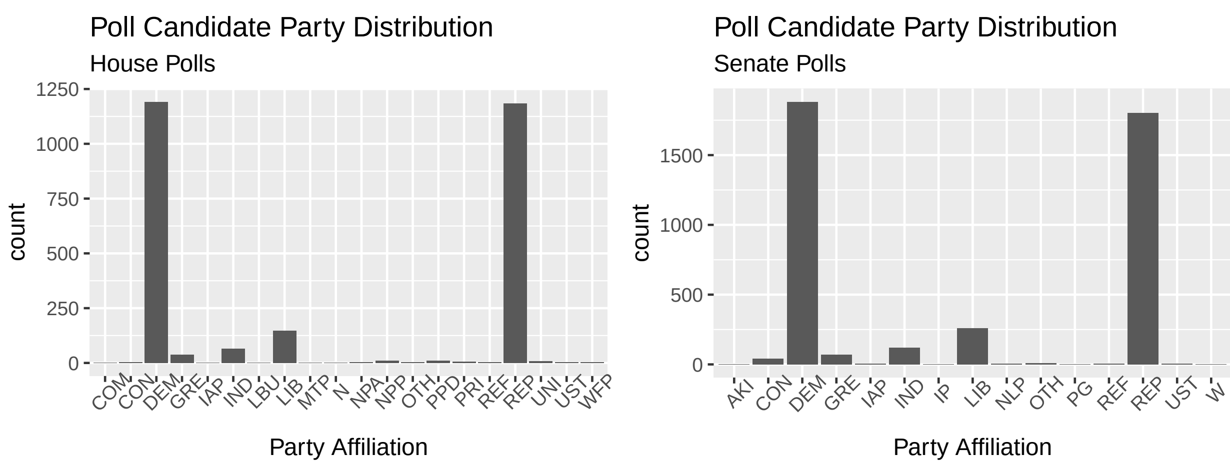

Figure 3: Candidate Party Polls

To explore the North Carolina voter history dataset, we looked at variables such as method, county, and party affiliation of the voter.

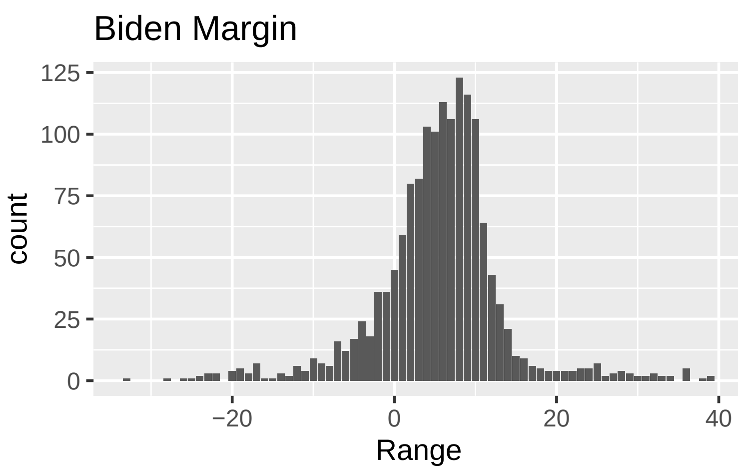

Figure 4: Biden Margin

We can see in Figure 5 that the majority of North Carolina voters are from two counties, Wake and Mecklenberg. Party affiliation and the method that the voters used are included in our appendix.

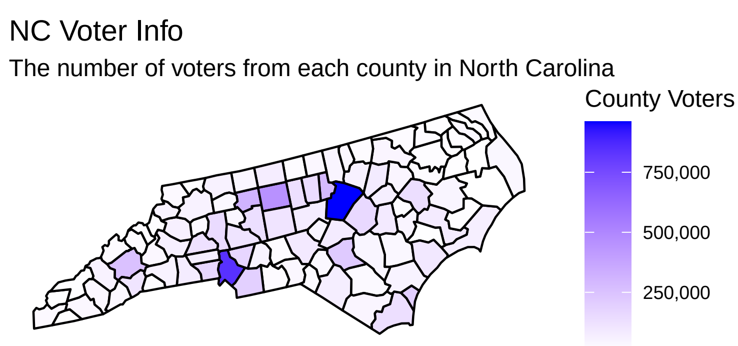

Figure 5: North Carolina Map of Voter Distribution by County

3. Model Formulation

3.1 Model for Question 1

In this section, we build a model to answer our first research question: Predict the outcome of the presidential election and the electoral college vote using the Linzer model to predict swing state outcomes in combination. To determine swing states, we've relied on worldpopulationreview's partisan index [7]. These states are: FL, GA, IA ,NC ,OH ,TX ,AZ ,MI ,MN ,NV ,NH ,PA ,and WI. We tend to incorporate more states as swing states to avoid strong assumptions that any state is going to win. We follow the route of Linzer and build our model. Specifically, the motivation is that we want to model Joe Biden's percentage of EC vote in the state specific level of detail. We believe that throughout time, each state's preference for Joe Biden is randomly varying and follows a random walk process. Besides, each state's uncertainty of their attitude towards Joe Biden is also state-widely different. We make the assumption that such uncertainty is constant throughout time. Therefore, we create the following extension of Linzer Model.

$$
\begin{aligned}
y_k &\sim N\left(\beta_{i[k]t[k]}, (\sigma_y^2)_{i[k]}\right) \\
\text{for } t > 1 : \beta_{it} &\sim N\left(\beta_{i,t-1}, (\sigma_\beta^2)\right) \\
\text{for } t = 1 : \beta_{i1} &\sim N\left(\mu_0, \sigma_0^2\right) \\
(\sigma_y^2)_{i[k]} &\sim \text{InvGamma}(\nu_y, \tau_y) \\
(\sigma_\beta^2) &\sim \text{InvGamma}(\nu_\beta, \tau_\beta)
\end{aligned}
$$

$$
\begin{aligned}
\mu_0 &\sim N(50, 17) \\
\sigma_0^2 &\sim \text{InvGamma}(\frac{1}{2}, \frac{1}{2}) \\
\nu_y &\sim \text{Uni}(0, 100) \\
\tau_y &\sim \text{Uni}(0, 100) \\
\nu_\beta &\sim \text{Uni}(0, 100) \\
\tau_\beta &\sim \text{Uni}(0, 100)
\end{aligned}
$$

Where k index the polls, i index the states, and t index the date. $i[k]$ represents the k^{th} poll's corresponding state index, and $t[k]$ represents the k^{th} poll's corresponding date index. We model samples from the economist poll data of 2020 and therefore are able to sample the posterior distribution of 1) Joe Biden's chance of winning, and 2) total electoral vote. The sampled posterior distribution y (percentage of supportance within each state) will be calculated by comparing to rounding by 50% to simulate the "winner takes all" procedure. After such adjustment, we can simply multiply the number of electoral college votes of each state to obtain a posterior distribution of electoral college votes of the entire nation. Thus, 2) can be obtained. By comparing to 270 again provides the probability of Joe Biden being elected.

```
## [1] 303.4
```

3.2 Model for Question 2, 4

In this section, we're answering question 2 and 4 jointly: Predict whether the US Senate remains in republican control and predict the outcome of the NC Senate election. Similarly, by using the above model with exactly the same notation, and by switching Economist dataset to FiveThirtyEight senate polls data, the same inference procedure can be produced. This time, considering there are some states not running senator elections (Wisconsin, Ohio", Washington, Maryland, Pennsylvania, California, New York, Hawaii, Connecticut, Nevada, Indiana, North Dakota, Missouri, Utah, Vermont, Florida), we only model the states that are running senator elections. After modeling and predicting the Republican's support percentage in each of these"electing states", we can predict whether the Republican or Democrat party wins the state majority in that state. This procedure also includes North Carolina. Furthermore, by summing the posterior samples of Republican senators in each state on election day, we can obtain the posterior distribution of US Senate remains in Republican Party's control.

3.3 Model for Question 3

In this section, we answer research question 3: Predict the outcomes of all 13 NC Congressional elections. The dataset we've used is the FiveThirtyEight house poll data. However, the dataset's sample sizes on each NC congressional district are small. This is due to the fact house polls happen less frequently compared to either senate polls or presidential polls. Besides, not all congressional districts are competitive because there is no term limit for representatives. FiveThirtyEight has only recorded poll history data for competitive districts including districts 2 (already conclusive), 3, 7, 8, 9, 11, 13. Thus, correlation between non-competitive districts cannot be captured without inputting extra datasets. Thus, this section's modeling procedure will rely on the output of the interim reports' "who vote" results.

Using the model to predict who will vote in 2020, we ran this binary output model on all the registered voters in NC to predict whether they'll vote or not. In the NC voter registration profile 2020 snapshot dataset, there is a column indicating party affiliation of each potential voter. Thus, we can predict the percentage of Republican voters for each congressional district and use this percentage as an imputed value for polls percentage. In this way, we're essentially imputing polling results y for non-competitive districts, which can be also fed into the Linzer Model defined below. A detailed description of how the imputation process work will be discussed after we introduced the Linzer model below:

$$
\begin{aligned}
y_k &\sim N\left(\beta_{i[k]t[k]}, (\sigma_y^2)_{i[k]}\right) \\
\text{for } t > 1: \beta_{it} &\sim N\left(\beta_{i,t-1}, (\sigma_\beta^2)\right) \\
\text{for } t = 1: \beta_{i1} &\sim N\left(\mu_0, \sigma_0^2\right) \\
(\sigma_y^2)_{i[k]} &\sim \text{InvGamma}(\nu_y, \tau_y) \\
(\sigma_\beta^2) &\sim \text{InvGamma}(\nu_\beta, \tau_\beta)
\end{aligned}
$$

$$
\begin{aligned}
\mu_0 &\sim N(50, 17) \\
\sigma_0^2 &\sim \text{InvGamma}(\frac{1}{2}, \frac{1}{2}) \\
\nu_y &\sim \text{Uni}(0, 100) \\
\tau_y &\sim \text{Uni}(0, 100) \\
\nu_\beta &\sim \text{Uni}(0, 100) \\
\tau_\beta &\sim \text{Uni}(0, 100)
\end{aligned}
$$

Where k index the polls or the imputed polls, i index the congressional district, and t index the polls date. $i[k]$ represents the k^{th} poll's corresponding congressional district index, and $t[k]$ represents the k^{th} poll's corresponding date index.

Now we can start discussion of the imputation. To impute y, we are essentially "creating" more polls for those unobserved congressional districts. Suppose we want to create an y_{k+1} that happened on October 10 for district 1, a non-competitive district, we look into all the voters who've registered to vote in congressional district 1 prior to October based on the 2020 voter registration file snapshot dataset. Suppose n_{k+1} voters have registered, we use the model in our interim report to predict who among these n_{k+1} voters are likely to vote. Suppose there are n^*_{k+1} likely voters. Then we calculate the percentage of voters who are republicans. Such percentage is the value we've imputed for y_{k+1}, which will be served as an additional "poll" to be fed into the Linzer model. By imputing $y_{k+1}, y_{k+2}\cdots$ for all the non-competitive districts on everyday following the above procedure, we've obtained a complete "polls record" for non-competitive districts. This can efficiently increase effective sample size for MCMC and also share mutual information on unobserved competitive districts via estimated correlation matrix.

Though efficient and reasonable, the imputation process above still makes strong assumptions. Thus, sensitivity analysis of imputed percentage will be further explored later in section 6.2.

4 Diagnostic and Validation

4.1 Model Diagnostic

Due to the fact that the parameter set is sparse – that is, β matrix has very few observations due to the lack of poll data, it is necessary to test the convergence of MCMC. We've run 6 MCMC chains on the input data for all the 3 models. Below in appendix (******) is some randomly selected model parameters' traceplot. The reason for only illustrating some of them is that the models have too many parameters to keep track of, thus it is not feasible to illustrate them all. Trace plot shows no significant convergent issues. Besides, the Gelman Rubin test has returned estimated Rhat to be 1.042, 1051, and 1.003, showing sufficient mixing. This holds for all the 3 MCMC models. Thus, we can deduce that MCMC has sufficiently converged.

4.2 Results Validation and Sanity Check

5 Modeling Result

5.1 Presidential Election and Electoral Vote

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## [1] "biden wins 0.674"
```

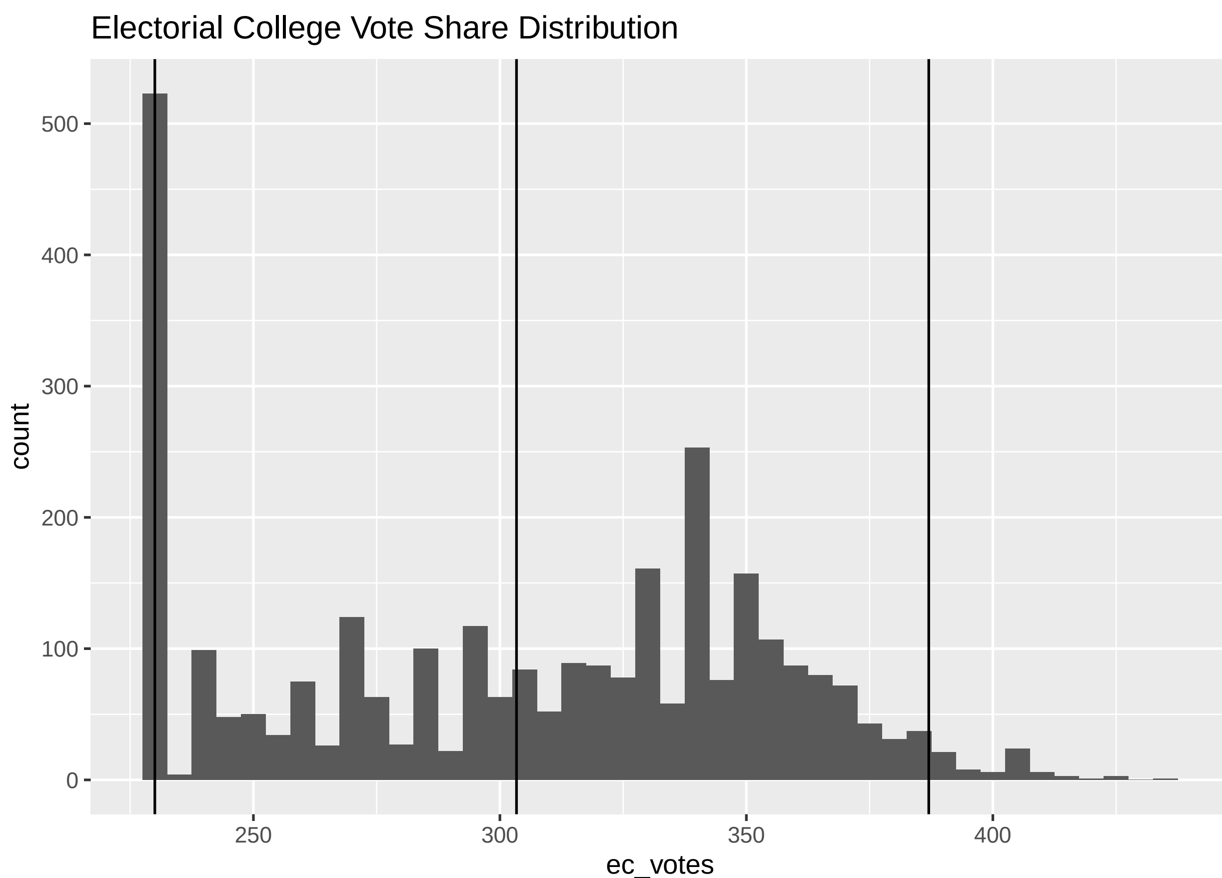

Table 1: EC Vote Total (Biden)

	x
2.5%	230
97.5%	387

5.2 Whether Senate Remains in Republican Control

```
## [1] "republican control senate 0.200333333333333"
```

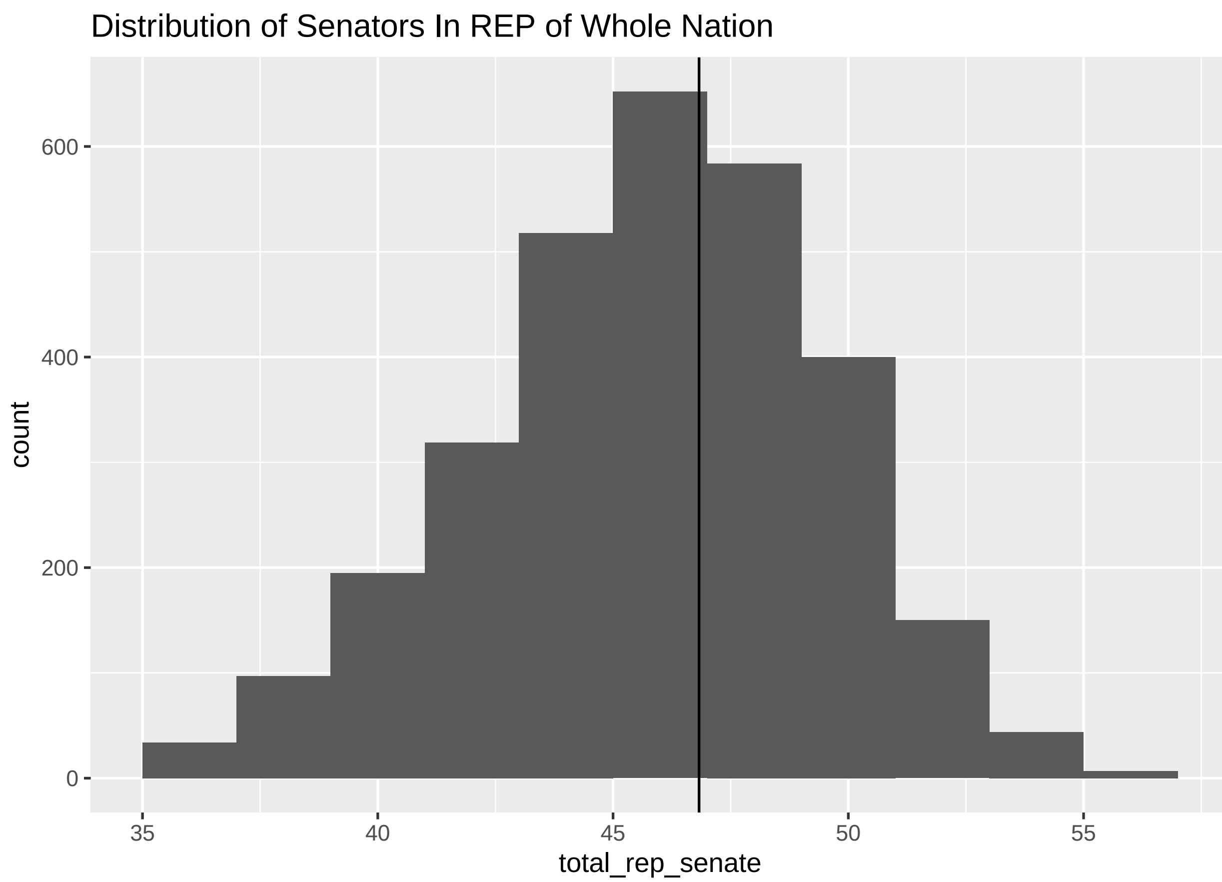

Table 2: House All Congressional District Winning Probability (REP)

	Official Model	Informative Prior	Imputation Perturbation
District 9	0.4017	0.1960	0.2727
District 11	0.3657	0.3963	0.4260
District 8	0.6433	0.2310	0.3357
District 3	0.6677	0.4683	0.6873
District 13	0.6897	0.4643	0.6010
District 2	0.7193	0.7687	0.4877
District 7	1.0000	0.6803	0.6437
district 1	0.0000	0.0000	0.0000
district 4	0.0000	0.0000	0.0000
district 5	1.0000	1.0000	1.0000
district 6	0.0000	0.0000	0.0000
district 10	1.0000	1.0000	1.0000
district 12	0.0000	0.0000	0.0000

Table 4: North Carolina Vote Share Interval Estimate

	x
2.5%	45.65
50%	48.27
97.5%	50.71

5.3 13 NC Congressional District Result

Table 3: House All Congressional District Winning Probability (REP)

	Official Model			Informative Prior			Imputation Perturbation		
	2.5%	50%	97.5%	2.5%	50%	97.5%	2.5%	50%	97.5%
District 9	46.10	49.55	53.44	45.58	48.66	51.73	45.73	48.97	51.97
District 11	41.47	48.41	55.71	44.63	48.80	61.46	44.57	49.45	53.98
District 8	40.74	54.34	71.66	38.52	47.43	53.10	39.25	45.08	57.74
District 3	39.70	54.16	61.31	40.64	49.68	56.30	45.03	52.17	64.02
District 13	46.52	52.91	60.37	41.38	49.44	58.86	45.72	50.91	58.35
District 2	41.23	52.33	61.45	44.85	52.88	58.20	43.50	49.88	57.10
District 7	53.73	57.52	67.91	42.90	57.15	69.12	32.32	51.88	56.09
district 1	26.12	26.83	27.59	26.56	26.84	27.10	26.66	26.84	27.01
district 4	22.53	22.78	23.03	22.45	22.79	23.13	22.57	22.78	23.00
district 5	68.37	68.55	68.74	68.38	68.56	68.74	68.38	68.56	68.73
district 6	45.59	45.72	45.85	45.59	45.72	45.85	45.60	45.72	45.85
district 10	69.72	70.02	70.30	69.53	70.02	70.47	69.85	70.02	70.18
district 12	36.11	36.27	36.43	36.12	36.27	36.41	36.12	36.27	36.41

5.4 NC Senator Election

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## ‘stat_bin()‘ using ‘bins = 30‘. Pick better value with ‘binwidth‘.
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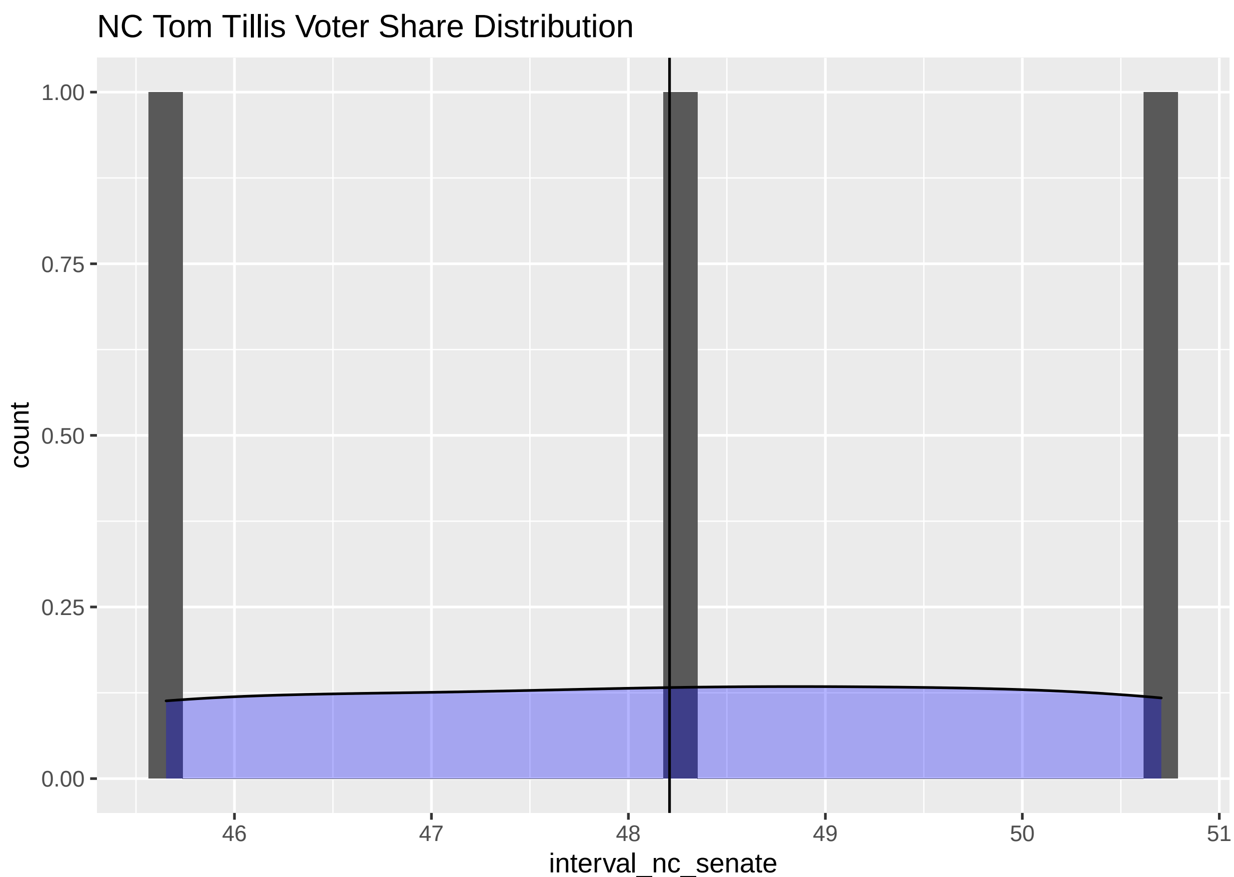



```
## [1] "Tome Tillis Win Prob 0.578"
```

6 Sensitivity Analysis

6.1 Informative Prior Check

As indicated in the model above in section 3, compared to the size of the β matrix, the total number of samples to y_k is comparably small. Thus, we're concerned that variation of prior values of parameters can lead to drastically different results. As we've already defined in section 3, we've set the prior mean to be 50%. This prior mean value is arguably the most reasonable value. Thus, this sensitivity check section will not test sensitivity of μ_0. However, all the other parameters, including $\nu_y, \nu_\beta, \tau_y, \tau_\beta, \sigma^2$ controls the prior population (inverse of uncertainty) of belief that the vote share percentage is 50%. Augmenting their values will make the prior less informative, whereas decreasing their values will make the Linzer model's posterior harder to deviate from generating 50% vote share percentage. Thus, we're testing the sensitivity to such uncertainty preset in our model by comparing the set of prior defined in section 3 to the following set of priors:

$$
\begin{aligned}
\mu_0 &\sim N(50, 1) \\
\sigma_0^2 &\sim \text{InvGamma}(\frac{1}{2}, \frac{1}{2}) \\
\nu_y &\sim \text{Uni}(0, 1) \\
\tau_y &\sim \text{Uni}(0, 1) \\
\nu_\beta &\sim \text{Uni}(0, 1) \\
\tau_\beta &\sim \text{Uni}(0, 1)
\end{aligned}
$$

The result is also attached in Table (——) (——) (——) (——) (——) (——) in the appendix. In all these tables, the "Official Model" column gives point or interval estimates using the prior in section 3, the model section, whereas the "Informative Prior" column gives all the same results using the above set of prior. Based on the observation, though the range of uncertainty parameters have been adjusted to 2 digits of magnitude, both estimates are still very similar. Thus, this justifies 2 facts:

** The sample size in the Linzer Model is sufficient in providing enough information (rather than only drawing information from the priors) to inform posterior distributions

** Our model generates convergent results that are insusceptible from prior choice.

Thus, our model outputs are valid and trustworthy

6.2 Imputation Perturbation Check

As mentioned before in section 3.3, the congressional district prediction utilizes manual imputation to generate forged poll data y for noncompetitive districts. We're highly concerned about whether such imputation is valid and whether any trivial deviation from truth will introduce overwhelming errors. Thus, we've also conducted the perturbation check on imputed values.

The perturbation is conducted as follows. For all the imputed y, we add independent yet identical Gaussian noise to them.

$$y := y + \epsilon \quad \epsilon \sim N(0, \kappa^2)$$

We're concerned whether 10% deviation of imputed vote share percentage from the truth vote share percentage will generate completely different results. Therefore, we chose $\kappa = 5\%$, that 10% range is plus or minus 2 standard deviations from the mean. This is the perturbation process we've conducted.

Similarly, before and after perturbation, the results are all attached in Table (——) (——) (——) (——) (——) (——). This time, the "Imputation Perturbation" column contains all the estimated results. We still observe no major difference between the estimated probability or vote share interval estimates. Thus, small errors introduced in or imputation won't affect the final prediction too much. Thus, the results generated upon imputed values are credible.

Bibliography

[1] https://blueprint.ucla.edu/feature/are-polls-reliable/

[2] https://newrepublic.com/article/154124/polling-industry-crisis

[3] https://projects.economist.com/us-2020-forecast/president/how-this-works?fbclid=IwAR2DIgVtL4uVjyKiVrpj9f3KMiUy

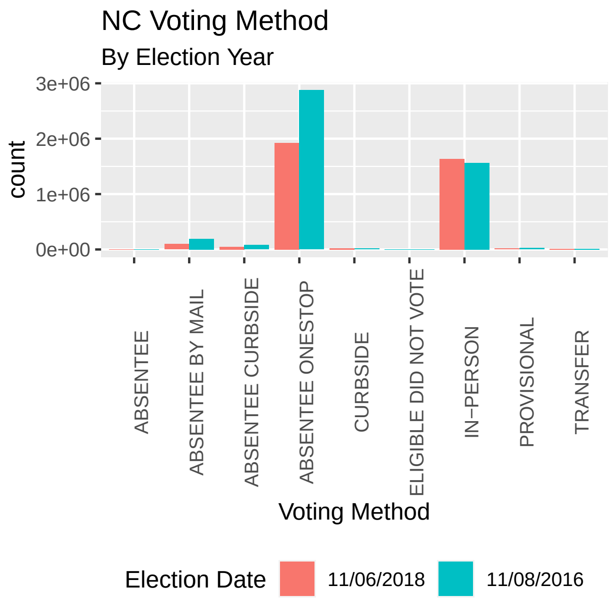
[4] Linzer Paper (https://doi.org/10.1080/01621459.2012.737735)

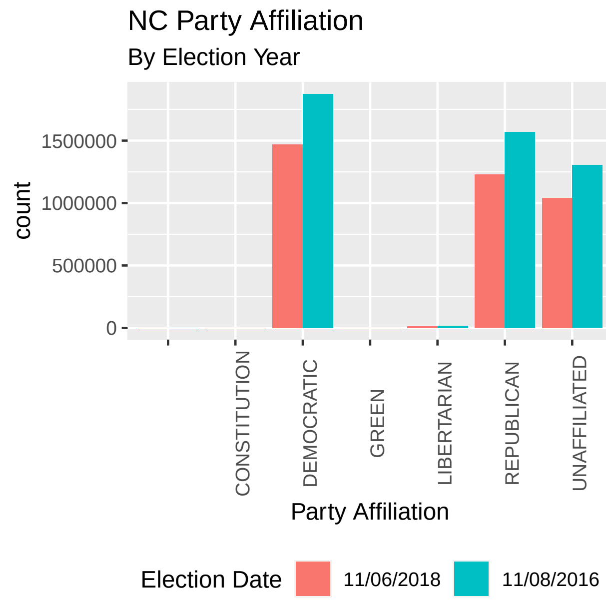
[5] https://projects.fivethirtyeight.com/polls/senate/

[6] https://projects.economist.com/us-2020-forecast/president/how-this-works.

[7] https://worldpopulationreview.com/state-rankings/blue-states

Appendix



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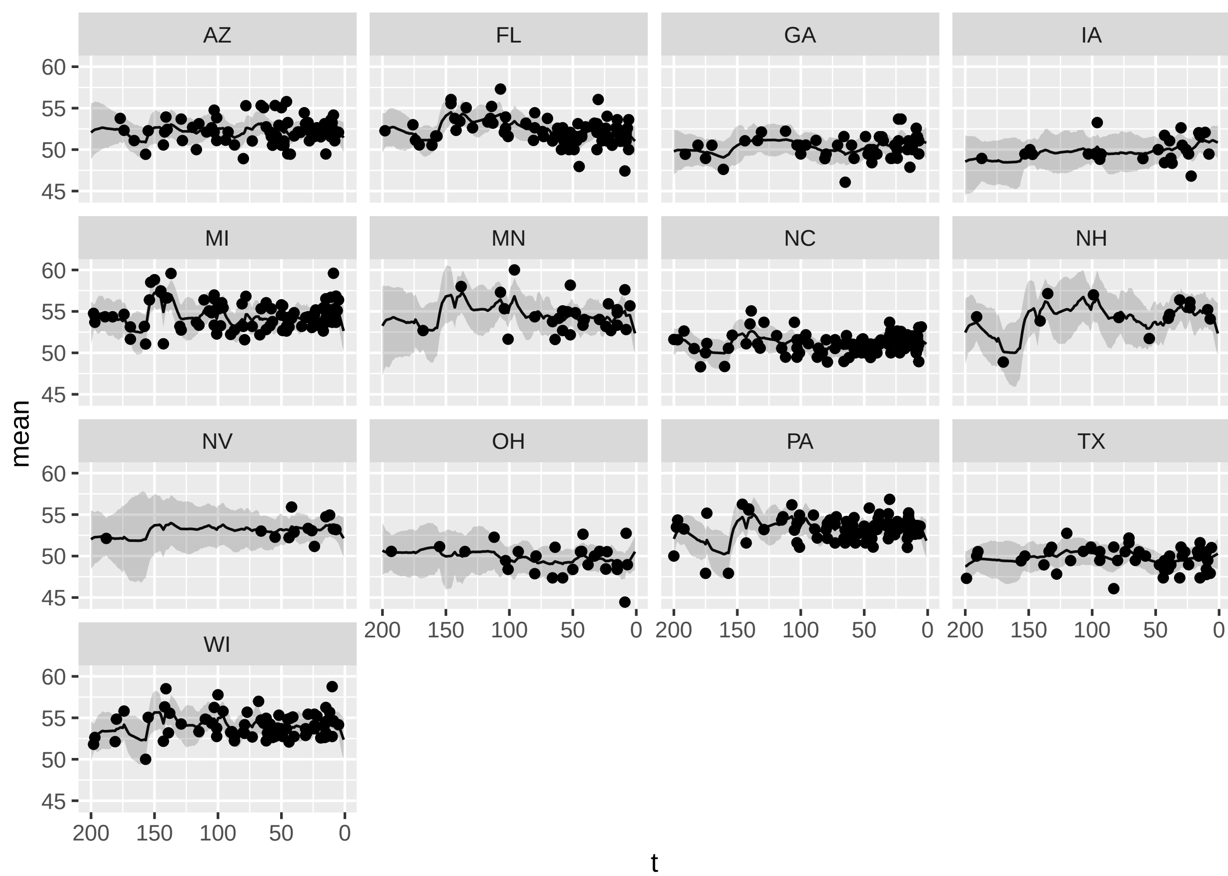

Table 5: Swing State Winning Probability (Biden)

	Official Model	Informative Prior	Imputation Perturbation
AZ	0.6860	0.6677	0.7753
NC	0.5443	0.5367	0.5643
MI	0.8897	0.9047	0.9467
WI	0.8517	0.8730	0.9173
MN	0.8313	0.8457	0.8867
TX	0.2127	0.2497	0.1847
FL	0.5093	0.5007	0.5937
PA	0.7557	0.7370	0.8123
GA	0.4390	0.4093	0.3990
OH	0.3317	0.3143	0.2817
NV	0.8210	0.8550	0.8807
NH	0.8067	0.8210	0.8843
IA	0.4363	0.4813	0.4867

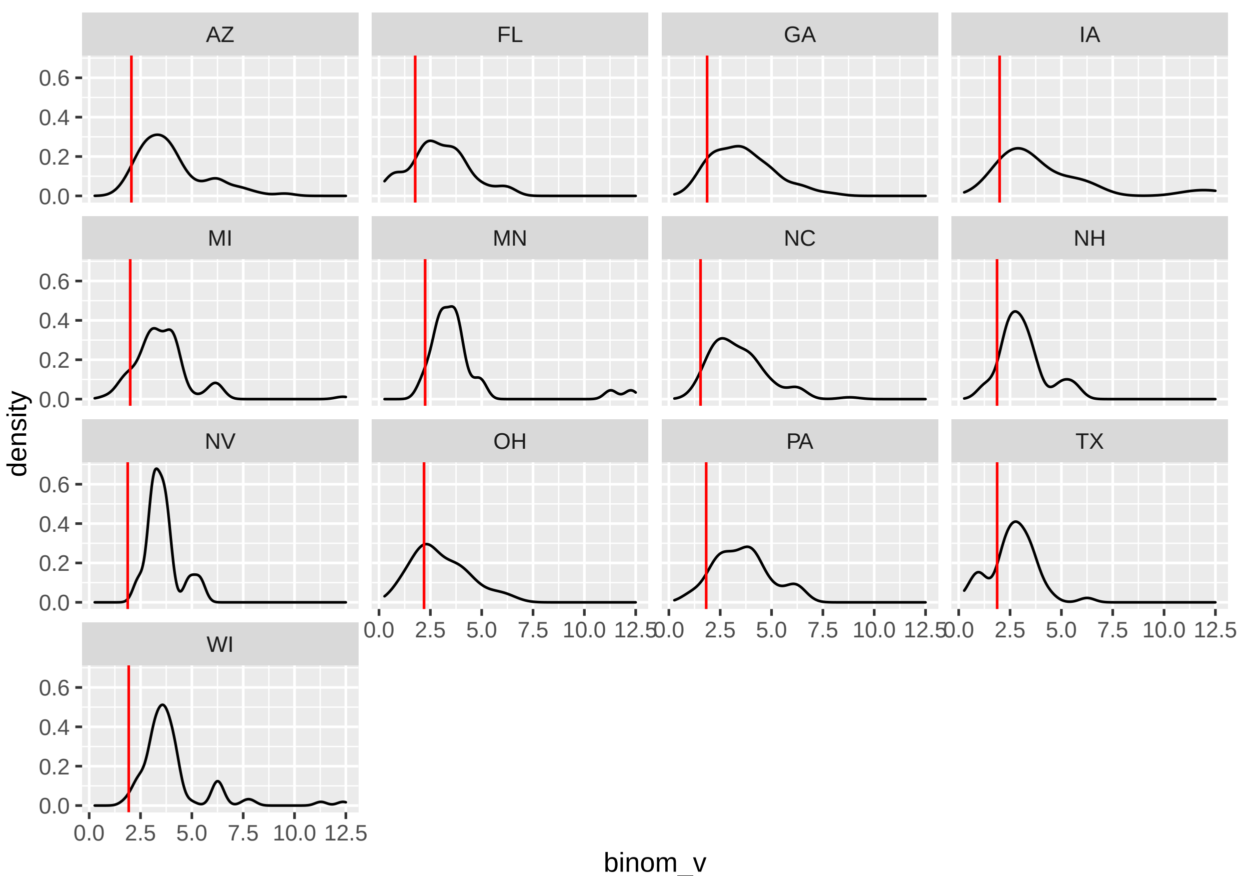

Table 6: Swing State Share Percentage Interval Estimate (Biden)

	Official Model			Informative Prior			Imputation Perturbation		
	2.5%	50%	97.5%	2.5%	50%	97.5%	2.5%	50%	97.5%
AZ	48.73	50.44	52.32	48.73	50.44	52.32	48.73	50.44	52.32
NC	48.37	50.09	51.70	48.37	50.09	51.70	48.37	50.09	51.70
MI	49.29	51.50	54.63	49.29	51.50	54.63	49.29	51.50	54.63
WI	49.16	51.29	53.97	49.16	51.29	53.97	49.16	51.29	53.97
MN	48.97	51.18	54.47	48.97	51.18	54.47	48.97	51.18	54.47
TX	47.44	49.31	51.12	47.44	49.31	51.12	47.44	49.31	51.12
FL	48.29	50.02	51.80	48.29	50.02	51.80	48.29	50.02	51.80
PA	48.80	50.74	53.37	48.80	50.74	53.37	48.80	50.74	53.37
GA	47.91	49.88	51.64	47.91	49.88	51.64	47.91	49.88	51.64
OH	47.25	49.59	51.39	47.25	49.59	51.39	47.25	49.59	51.39
NV	48.89	51.07	53.66	48.89	51.07	53.66	48.89	51.07	53.66
NH	48.77	51.16	54.96	48.77	51.16	54.96	48.77	51.16	54.96
IA	47.96	49.85	51.92	47.96	49.85	51.92	47.96	49.85	51.92

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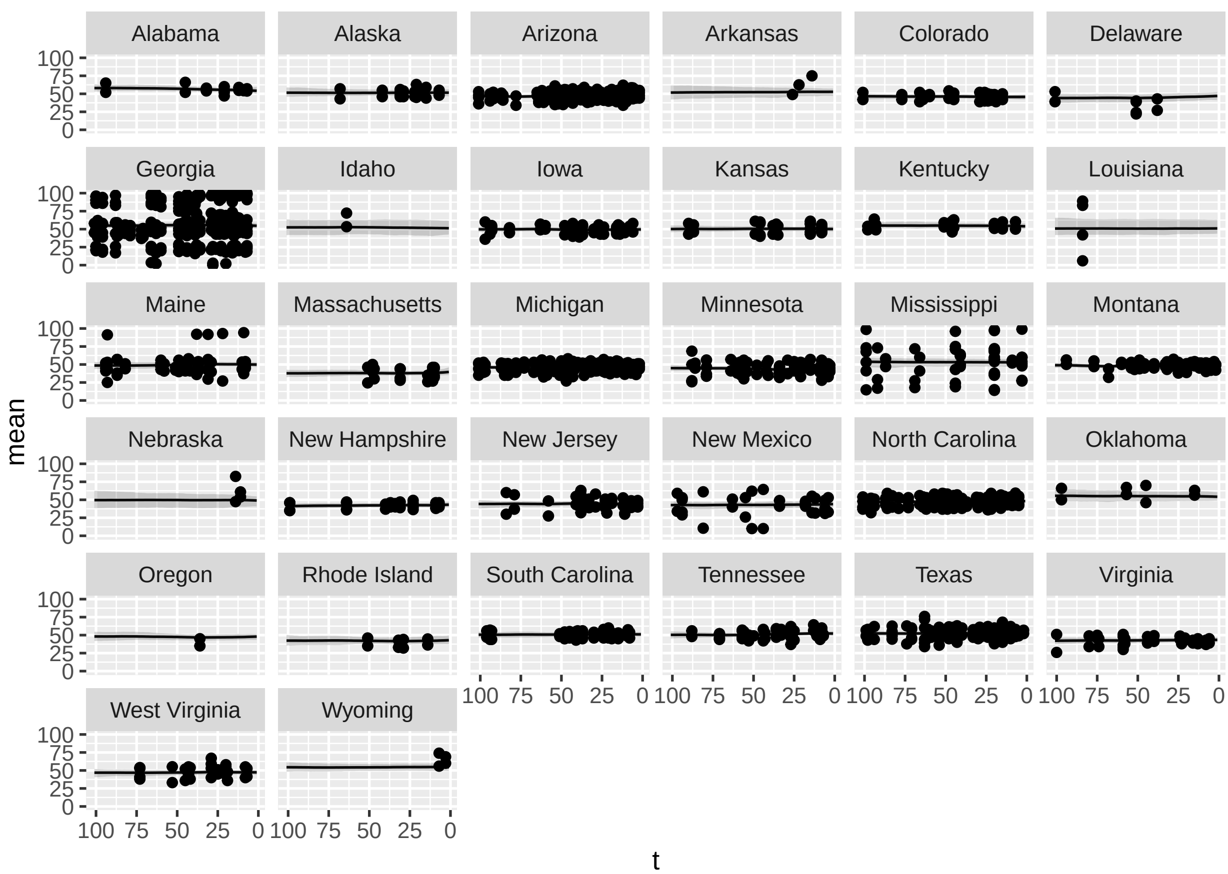


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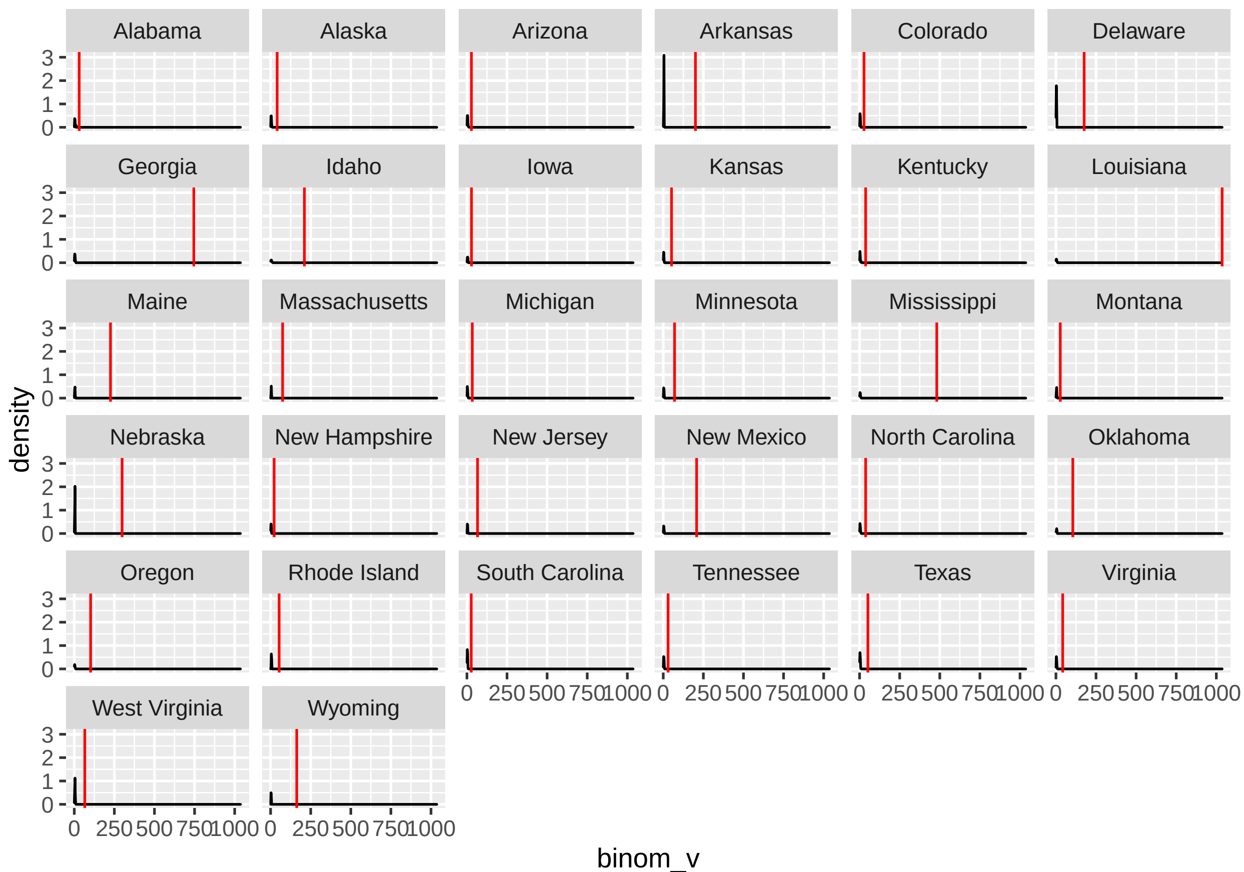
Alabama
Alaska
Arizona
Arkansas
Colorado
Delaware
Georgia
Idaho
Iowa
Kansas
Kentucky
Louisiana
Maine
Massachusetts
Michigan
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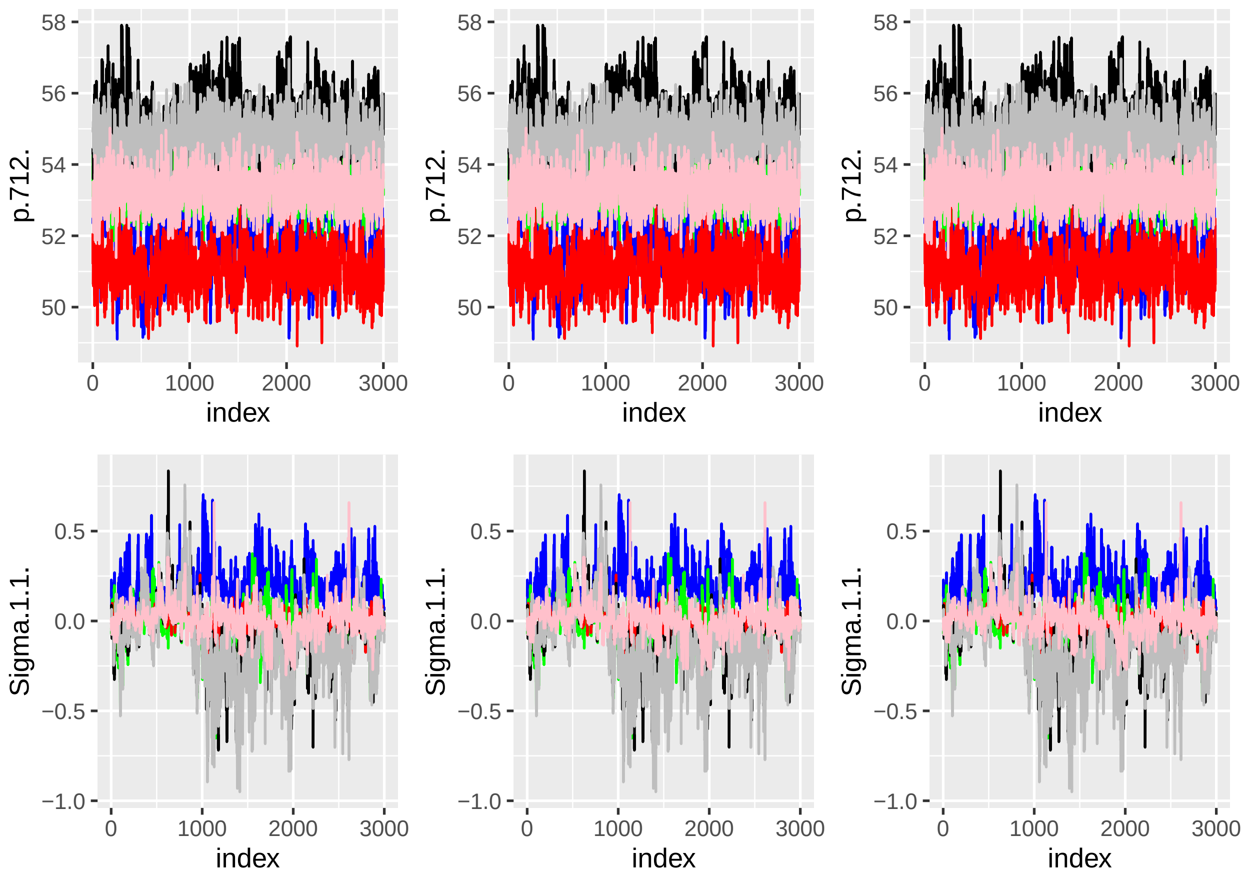
Mississippi
Montana
Nebraska
New Hampshire
New Jersey
New Mexico
North Carolina
Oklahoma
Oregon
Rhode Island
South Carolina
Tennessee
Texas
Virginia
West Virginia
Wyoming
density
binom_v
0
1
2
3
0
250
500
750
1000

Table 7: Senator All State Winning Probability (REP)

	Official Model	Informative Prior	Imputation Perturbation
Michigan	0.0577	0.0397	0.0690
Minnesota	0.0467	0.0073	0.0513
Arizona	0.8143	0.8043	0.8683
North Carolina	0.5780	0.5623	0.6040
Iowa	0.7930	0.9060	0.8613
Virginia	0.0043	0.0040	0.0200
Georgia	1.0000	0.9993	0.9980
Texas	0.9970	0.9940	0.9960
Alaska	0.9150	0.9263	0.9623
New Hampshire	0.0033	0.0000	0.0037
Alabama	0.9953	1.0000	0.9883
Kentucky	0.9993	1.0000	0.9997
Wyoming	0.9887	0.7537	0.9420
Montana	0.5300	0.4943	0.6280
Kansas	0.8897	0.8627	0.9120
Maine	0.8650	0.8497	0.7253
South Carolina	0.9550	0.9750	0.9657
Mississippi	0.9150	0.9947	0.9507
New Jersey	0.0387	0.0297	0.0320
Tennessee	0.9940	0.9870	0.9720
Nebraska	0.6857	0.5463	0.4890
Massachusetts	0.0000	0.0000	0.0063
Arkansas	0.9353	0.8677	0.8507
Oklahoma	0.9983	0.9057	0.9610
Colorado	0.0910	0.1747	0.2160
New Mexico	0.0327	0.3550	0.1250
West Virginia	0.3583	0.5640	0.5983
Oregon	0.5097	0.3760	0.6117
Delaware	0.4080	0.0903	0.3253
Idaho	0.7337	0.5697	0.4727
Louisiana	0.4960	0.5467	0.5997
Rhode Island	0.0707	0.0460	0.1300

Table 8: Senator All State Vote Share Percentage Interval Estimate (REP

	Official Model			Informative Prior			Imputation Perturbation		
	2.5%	50%	97.5%	2.5%	50%	97.5%	2.5%	50%	97.5%
Michigan	43.75	46.11	48.54	43.72	45.98	48.23	44.06	46.32	48.56
Minnesota	41.23	44.35	48.71	41.83	44.24	47.15	41.52	44.55	48.60
Arizona	46.83	48.93	51.01	47.04	48.83	51.00	47.15	49.08	51.18
North Carolina	45.65	48.27	50.71	45.88	48.19	50.74	45.93	48.32	51.01
Iowa	46.35	49.66	53.17	47.02	49.71	52.33	46.76	49.76	52.46
Virginia	40.22	43.48	47.01	39.51	42.67	46.37	40.48	44.38	47.82
Georgia	50.60	54.55	58.68	50.80	55.18	59.48	49.72	53.59	56.93
Texas	49.51	52.33	54.81	48.74	51.99	55.10	49.13	52.11	54.62
Alaska	46.40	51.68	56.20	47.30	50.63	55.10	47.62	51.29	54.42
New Hampshire	40.28	43.03	47.03	39.93	43.02	46.26	40.31	43.27	46.78
Alabama	49.58	54.03	59.48	50.68	54.95	58.34	48.75	53.58	56.73
Kentucky	49.69	54.14	57.92	49.98	53.89	57.04	49.56	52.93	57.23
Wyoming	48.88	54.38	59.94	45.33	50.57	62.02	47.28	51.40	57.44
Montana	45.74	48.09	50.56	45.68	47.99	50.23	46.12	48.38	50.76
Kansas	46.65	50.13	53.86	46.40	50.36	54.54	46.76	50.32	53.70
Maine	46.30	49.84	53.28	45.14	50.91	58.11	44.77	49.76	54.70
South Carolina	47.67	51.17	54.32	48.00	50.79	54.36	47.85	50.78	53.63
Mississippi	46.58	52.72	57.82	49.07	53.69	59.03	47.21	52.05	57.59
New Jersey	41.43	44.51	48.44	40.51	44.49	48.15	42.12	44.64	48.27
Tennessee	48.90	52.18	55.46	48.44	52.28	54.89	47.87	51.68	54.67
Nebraska	40.74	49.62	54.79	43.87	48.31	55.77	42.43	47.88	57.26
Massachusetts	35.33	39.09	46.01	34.64	39.56	44.54	35.24	41.11	46.87
Arkansas	47.00	53.08	57.24	46.28	50.80	56.40	46.03	50.43	57.14
Oklahoma	48.73	54.69	60.64	46.11	54.57	60.37	47.64	52.54	56.24
Colorado	42.11	45.86	49.04	42.85	46.52	49.51	42.68	46.36	49.71
New Mexico	40.01	44.16	48.19	41.88	46.86	51.59	40.62	45.20	49.61
West Virginia	43.89	47.42	50.83	43.69	48.41	52.37	43.89	48.53	52.61
Oregon	43.24	48.04	51.72	40.83	46.74	55.17	42.42	49.17	54.09
Delaware	41.19	47.35	52.80	38.46	44.38	49.21	42.31	46.94	50.82
Idaho	42.58	50.87	61.65	42.04	48.58	55.44	41.83	47.73	58.65
Louisiana	43.76	47.97	63.16	42.80	48.43	60.13	44.08	48.72	52.81
Rhode Island	37.33	42.96	49.73	36.69	44.26	48.34	40.12	44.88	50.19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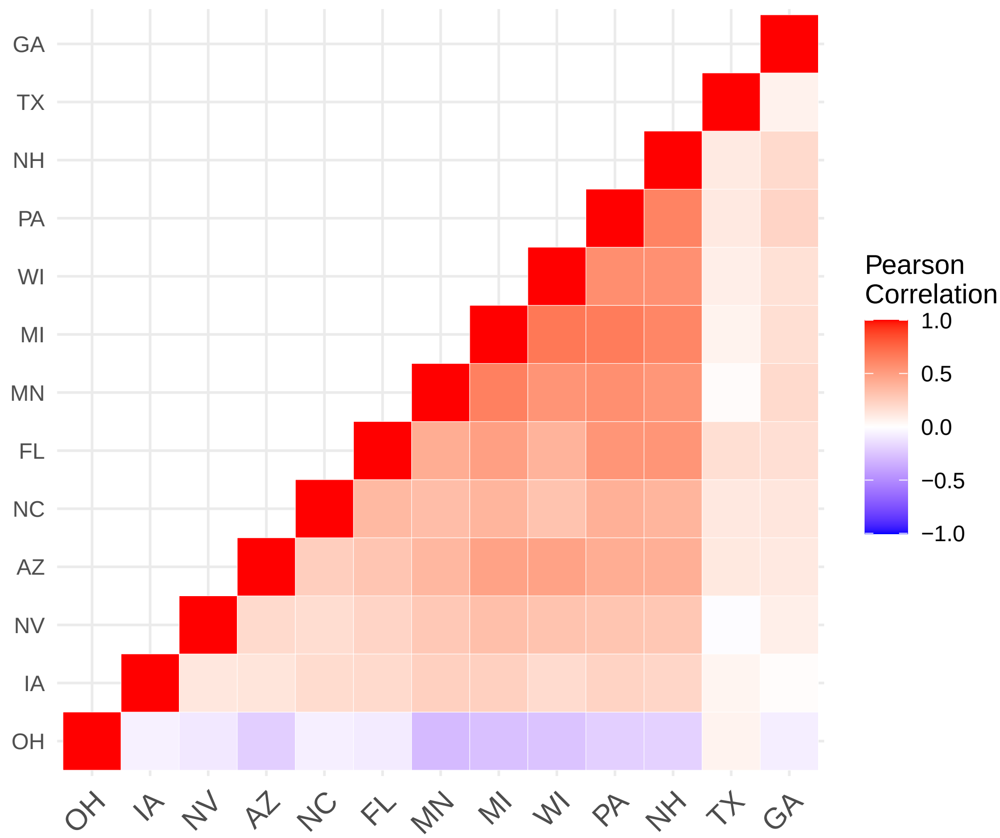


```r
mean(as.data.frame(jags_sims_mv$BUGSoutput$summary)$Rhat)
```

```
## [1] 1.035
```

```r
mean(as.data.frame(jags_sims_senator$BUGSoutput$summary)$Rhat)
```

```
## [1] 1.338
```

```r
mean(as.data.frame(jags_sims_house$BUGSoutput$summary)$Rhat)
```

```
## [1] 2.457
```