Mitchell's Book: Ex. 3.3 DTs

Machine Learning 2024-25 Course Activity

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True or false: If decision tree $D2$ is an elaboration of tree $D1$, then $D1$ is *more-general-than* $D2$. Assume $D1$ and $D2$ are decision trees representing arbitrary boolean functions, and that $D2$ is an elaboration of $D1$ if ID3 could extend $D1$ into $D2$. If true, give a proof; if false, a counterexample.

More-general-than definition

The definition of *more-general-than* from the book is the following:

Given two hypotheses h_j and h_k, h_j is *more-general-than-or-equal-to* h_k if and only if any instance that satisfies h_k also satisfies h_i.
We can write it as: $h_j \geq_g h_k$ if and only if

$$\big((h_k(x) = 1) \Rightarrow \big(h_j(x) = 1\big), \forall x \in X\big)$$

Examples of $D1$ & $D2$

Let $D1$ be a simple boolean decision tree, based on a single feature x. The behavior of the tree is represented by the following function:

$$f_{D1}(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Now, consider $D2$, an elaboration of $D1$: $D2$ is a decision tree based on two features, x and y. The behavior of the tree is defined by the following function:

$$f_{D2}(x, y) = \begin{cases} 0 & \text{if } x = 0 \text{ and } y = 0 \\ 1 & \text{otherwise} \end{cases}$$

Truth table

x	y	D1	D2
1	1	**1**	**1**
1	0	**1**	**1**
0	1	**0**	**1**
0	0	**0**	**0**

We can observe that:

- $D1$ makes decisions based on the value of x, ignoring y.
- $D2$ is more specific because it considers both x and y; for example it splits when $x = 1$ and $y = 0$.

We can say that $D2$ is an *elaboration* of $D1$ because it extends it: in fact $D2$ has the same feature x that is part of $D1$, plus an additional feature y. Therefore, $D2$ is a more detailed version of $D1$ and $\forall x \in X$ the property $D1$ *more-general-than* $D2$ holds.

Example of $D2'$

Now, let's consider $D2'$, an elaboration of $D1$: $D2'$ is a decision tree based again on two features x and y. The behavior of the tree is represented by the following function:

$$f_{D2'}(x, y) = \begin{cases} 1 & \text{if } x = 1 \text{ and } y = 1 \\ 0 & \text{otherwise} \end{cases}$$

In this way, $D2'$ makes a split when the input $x = 1$ and $y = 1$.

Truth table

x	y	D1	D2'
1	1	**1**	**1**
1	0	**1**	**0**
0	1	**0**	**0**
0	0	**0**	**0**

True or False?

Now, can we say that $D1$ is *more-general-than* $D2'$? **No**. In fact, we can see that the property of *more-general-than-or-equal-to* does not hold $\forall x \in X$. For example, with features $x = 1$ and $y = 0$, $D1$ returns True, while $D2'$ returns False; hence, they label the features differently.

This shows that the property *more-general-than* does not always hold when one decision tree is an elaboration of another.