Mitchell's Book: Ex. 4.3 NN

Machine Learning 2024-25 Course Activity

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Consider two perceptrons defined by the threshold expression $w_0 + w_1 x_1 + w_2 x_2 > 0$. Perceptron A has weight values

$$w_0 = 1,\ w_1 = 2,\ w_2 = 1$$

and perceptron B has the weight values

$$w_0 = 0,\ w_1 = 2,\ w_2 = 1$$

True or false? Perceptron A is *more-general-than* perceptron B.

More-general-than definition

The definition of *more-general-than* from the book is the following:

Given two hypotheses h_j and h_k, h_j is *more-general-than-or-equal-to* h_k if and only if any instance that satisfies h_k also satisfies h_j.

We can write it as: $h_j \geq_g h_k$ if and only if

$$\big((h_k(x) = 1) \Rightarrow \big(h_j(x) = 1\big), \forall x \in X\big)$$

Input examples

Let's consider all the possible inputs for perceptrons A and B. If we find an input where B outputs 1 and A outputs 0 we can say that A is not *more-general-than* B.

1. $x_1 = 1, x_2 = 1$

Perceptron A:

The treeshold expression is the following:

$$w_0 + w_1 \cdot 1 + w_2 \cdot 1 = 1 + 2 \cdot 1 + 1 \cdot 1 = 4$$

hence the step function is activated.

Perceptron B:

The treeshold expression is the following:

$$w_0 + w_1 \cdot 1 + w_2 \cdot 1 = 0 + 2 \cdot 1 + 1 \cdot 1 = 3$$

hence the step function is activated.

Until now, we can say that Perceptron A is *more-general-than* Perceptron B.

2. $x_1 = 1, x_2 = 0$

Perceptron A:

The treeshold expression is the following:

$$w_0 + w_1 \cdot 1 + w_2 \cdot 0 = 1 + 2 \cdot 1 + 1 \cdot 0 = 3$$

hence the step function is activated.

Perceptron B:
The treeshold expression is the following:

$$w_0 + w_1 \cdot 1 + w_2 \cdot 0 = 0 + 2 \cdot 1 + 1 \cdot 0 = 2$$

hence the step function is activated.

Until now, we can say that Perceptron A is *more-general-than* Perceptron B.

3. $x_1 = 0, x_2 = 1$

Perceptron A:
The treeshold expression is the following:

$$w_0 + w_1 \cdot 0 + w_2 \cdot 1 = 1 + 2 \cdot 0 + 1 \cdot 1 = 2$$

hence the step function is activated.

Perceptron B:
The treeshold expression is the following:

$$w_0 + w_1 \cdot 0 + w_2 \cdot 1 = 0 + 2 \cdot 0 + 1 \cdot 1 = 1$$

hence the step function is activated.

Until now, we can say that Perceptron A is *more-general-than* Perceptron B.

4. $x_1 = 0, x_2 = 0$

Perceptron A:
The treeshold expression is the following:

$$w_0 + w_1 \cdot 0 + w_2 \cdot 0 = 1 + 2 \cdot 0 + 1 \cdot 0 = 1$$

hence the step function is activated.

Perceptron B:
The treeshold expression is the following:

$$w_0 + w_1 \cdot 0 + w_2 \cdot 0 = 0 + 2 \cdot 0 + 1 \cdot 0 = 0$$

hence the step function is **not activated**.

Conclusion: True

We can say that Perceptron A is *more-general-than* Perceptron B, because for all the inputs where B outputs 1 $\Rightarrow$ A outputs 1.

We cannot say the opposite, in fact Perceptron B is *more-general-than* A is **False** because there is one case where A outputs 1 $\not\Rightarrow$ B outputs 1.