Alpaydin's Book: Ex. 11.14.3 NN

Machine Learning 2024-25 Course Activity

Furno Francesco - francesco.furno@studenti.unipd.it - 2139507

December 18, 2024

Exercise 11.14.3

Show the perceptron that calculates the parity of its three inputs.

Parity function

Let's see what is a parity function. There exist two types of it:

- **even parity**: outputs 1 when the number of values set to 1 is even. For example:
  - $1, 0, 1 \Rightarrow 1$
  - $1, 0, 0 \Rightarrow 0$
- **odd parity**: outputs 1 when the number of values set to 1 is odd. For example:
  - $1, 0, 1 \Rightarrow 0$
  - $1, 1, 1 \Rightarrow 1$

Now, let's consider three inputs, A, B, C and the *odd parity* function.

$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$	$\textbf{Parity}(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Table 1: $\text{Parity}(A, B, C)$ truth table

Note that the *even parity* function can be obtained as $\neg\ \text{Parity}(A, B, C)$.

Relation between Parity and XOR

The XOR function is strictly related to the parity function. The XOR function directly computes the *odd parity* of its input.

A	B	C	$A \oplus B$	$A \oplus B \oplus C$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

Table 2: $A \oplus B \oplus C$ truth table

Hence,

- the *odd parity* of (A, B, C) can be seen as $(A \oplus B \oplus C)$.
- the *even parity* of (A, B, C) can be seen as $\neg(A \oplus B \oplus C)$

Perceptron implementation? Not with a single layer

We already saw that the *odd parity* function can be seen as a combination of two XOR functions. Is it possible to implement a perceptron for the *odd parity* function? Not with a single layer, in fact, there is no hyperplane that linearly separates the positive and negative values in the XOR function.

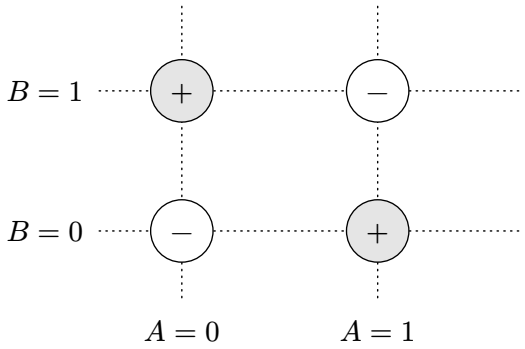

Figure 1: Visual representation of $A \oplus B$

Since the XOR function, which forms the basis of the parity function, is not linearly separable, it is impossible to construct a hyperplane that separates the positive and negative values of the parity function in a three-dimensional space. Consequently, a single-layer perceptron cannot implement the parity function for three inputs.