Alpaydin's Book: Ex. 11.14.2 NN

Machine Learning 2024-25 Course Activity

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Exercise 11.14.2

Show the perceptron that calculates NAND of its two inputs.

A NAND B

A NAND B is a boolean function that can be rewritten as $\neg(A \wedge B)$.

The following is the truth table for this boolean function:

A	B	$A \wedge B$	$\neg(A \wedge B)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

Table 1: $\neg(A \wedge B)$ truth table

Perceptron implementation

We consider two inputs A and B. We know that the perceptron should return 0 only if both A and B are set to 1, 1 otherwise.

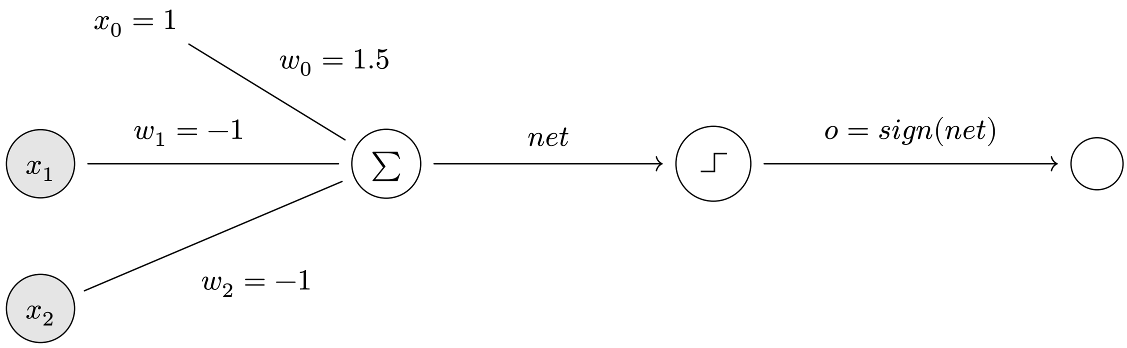
Note that it is possible to implement a perceptron for the NAND operator because it is a linearly separable function, making it perfectly suited for a perceptron

We set the input layer as follows:

- $x_0 = 1$
- $x_1 = A$
- $x_2 = B$

Now, let's assign the weights. We know that the function should output 0 when $x_1 = 1 \wedge x_2 = 1$, so when their sum is higher than 1.5. Then, the function should output 1 in all the other cases,so when $x_1 + x_2$ is lower than 1.5. We can set the weights as follows:

- $w_0 = 1.5$
- $w_1 = -1$
- $w_2 = -1$



where $net = \sum_{i=0}^{n} w_i x_i$ and $o = \sigma(net) = sign(net)$.

Now, let's consider all the possible values for inputs x_1 and x_2.

$\boldsymbol{x_1 = 0, x_2 = 0}$

The threshold expression is the following:

$$\begin{aligned} sign(net) &= sign(w_0 \cdot 1 + w_1 \cdot 0 + w_2 \cdot 0) \\ &= sign(1.5 \cdot 1 + (-1) \cdot 0 + (-1) \cdot 0) \\ &= sign(1.5) = 1 \end{aligned}$$

$\boldsymbol{x_1 = 0, x_2 = 1}$

The threshold expression is the following:

$$\begin{aligned} sign(net) &= sign(w_0 \cdot 1 + w_1 \cdot 0 + w_2 \cdot 1) \\ &= sign(1.5 \cdot 1 + (-1) \cdot 0 + (-1) \cdot 1) \\ &= sign(0.5) = 1 \end{aligned}$$

$\boldsymbol{x_1 = 1, x_2 = 0}$

The threshold expression is the following:

$$\begin{aligned} sign(net) &= sign(w_0 \cdot 1 + w_1 \cdot 1 + w_2 \cdot 0) \\ &= sign(1.5 \cdot 1 + (-1) \cdot 1 + (-1) \cdot 0) \\ &= sign(0.5) = 1 \end{aligned}$$

$\boldsymbol{x_1 = 1, x_2 = 1}$

The threshold expression is the following:

$$\begin{aligned} sign(net) &= sign(w_0 \cdot 1 + w_1 \cdot 1 + w_2 \cdot 1) \\ &= sign(1.5 \cdot 1 + (-1) \cdot 1 + (-1) \cdot 1) \\ &= sign(-0.5) = -1 \Rightarrow 0 \end{aligned}$$

Hence, the perceptron correctly implements the NAND operator. This demonstrates that the NAND operator is linearly separable and can be represented by a single-layer perceptron.