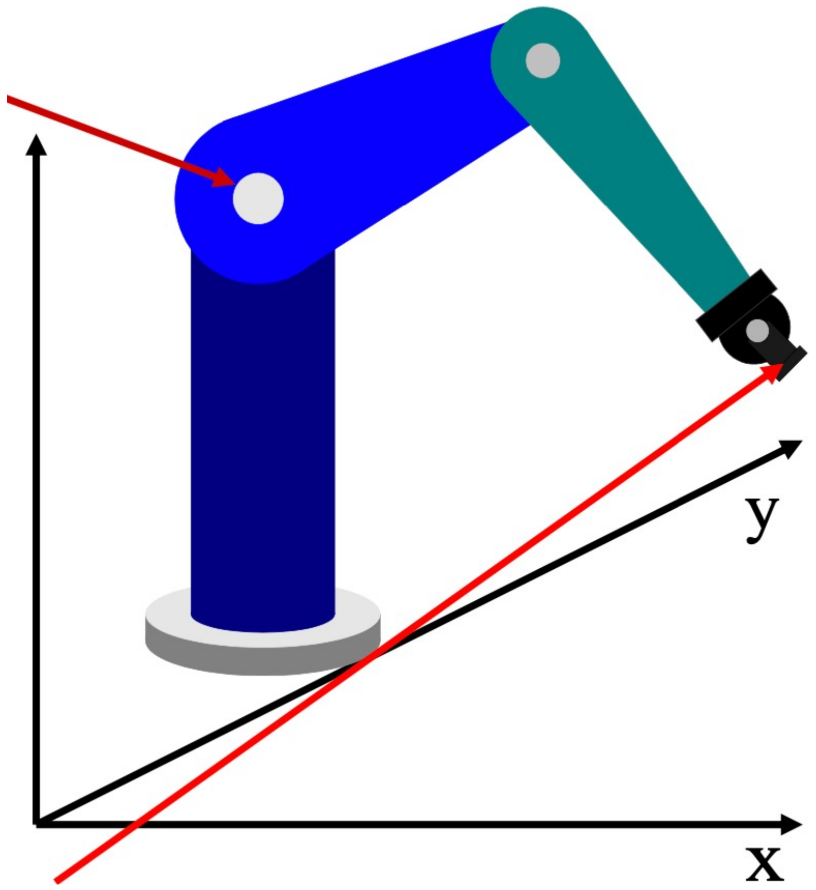
Lecture 6

Cont. Forward Kinematics (FK) for Industrial Manipulators

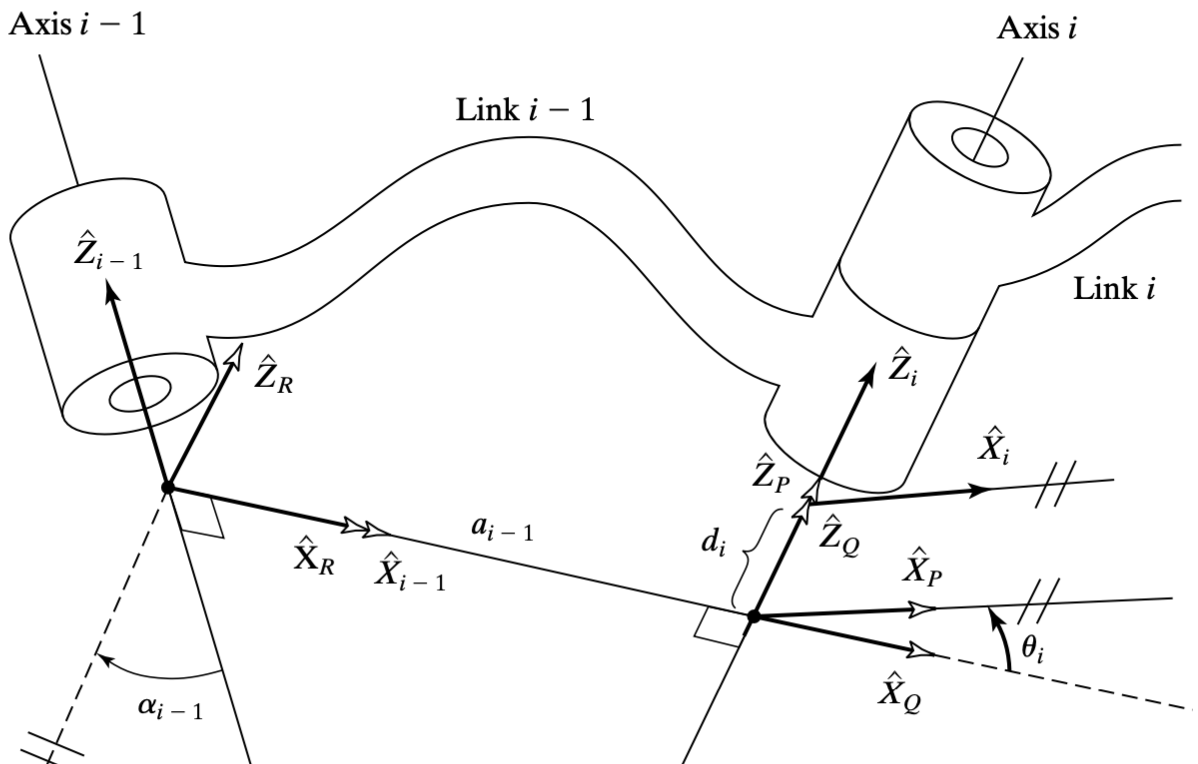
Task #2 - Derivation of link transformations

- **Task #1**: Attach frames to the end of each link where the joints are located. We also attach a frame to the end of the end effector link right at the tip of the robot and we attach a frame to the base of the robot.
- **Task #2**: Find transformation matrix between each two consecutive frames starting from the base going to the end effector frame successively.
- **Task #3**: post multiply transformation matrices successively to derive the intended 4x4 homogeneous transformation matrix from the base to the end effector frame.



Task #2 - Derivation of link transformations

- Construct the transform that defines frame {i} relative to the frame {i−1}
- This transformation will be a function of the four link parameters



Task #2 - Derivation of link transformations

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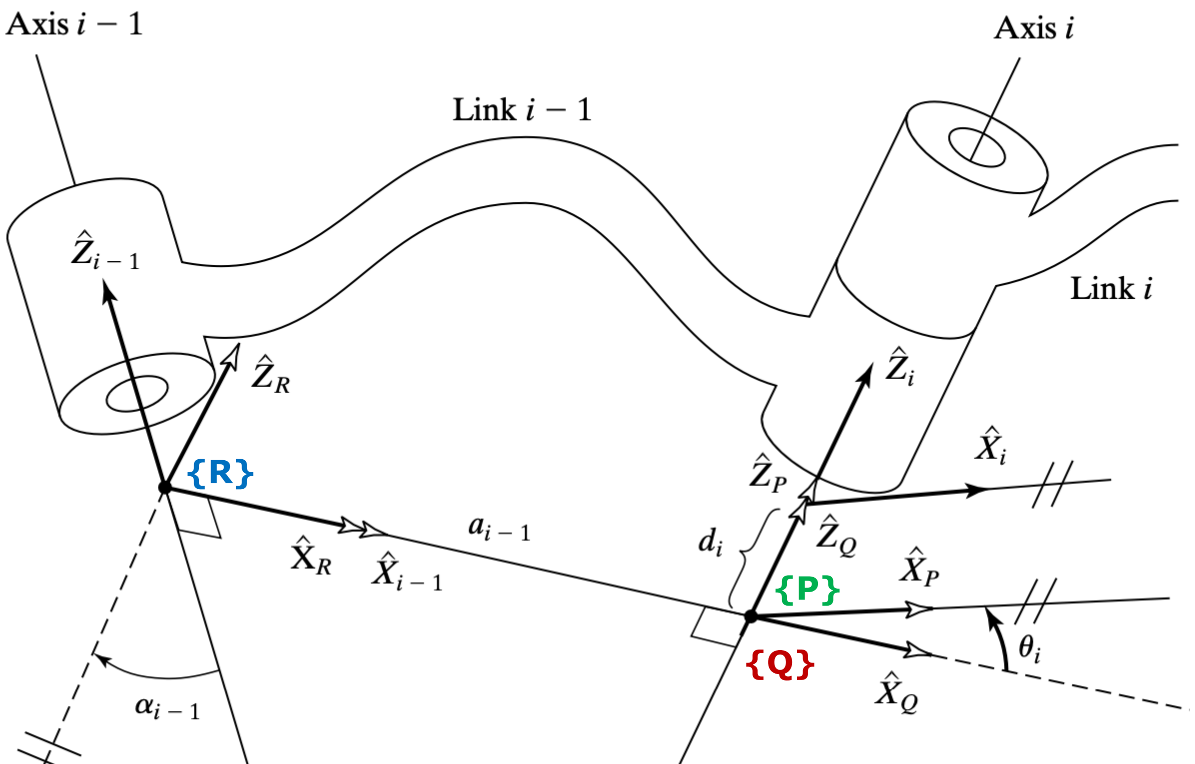

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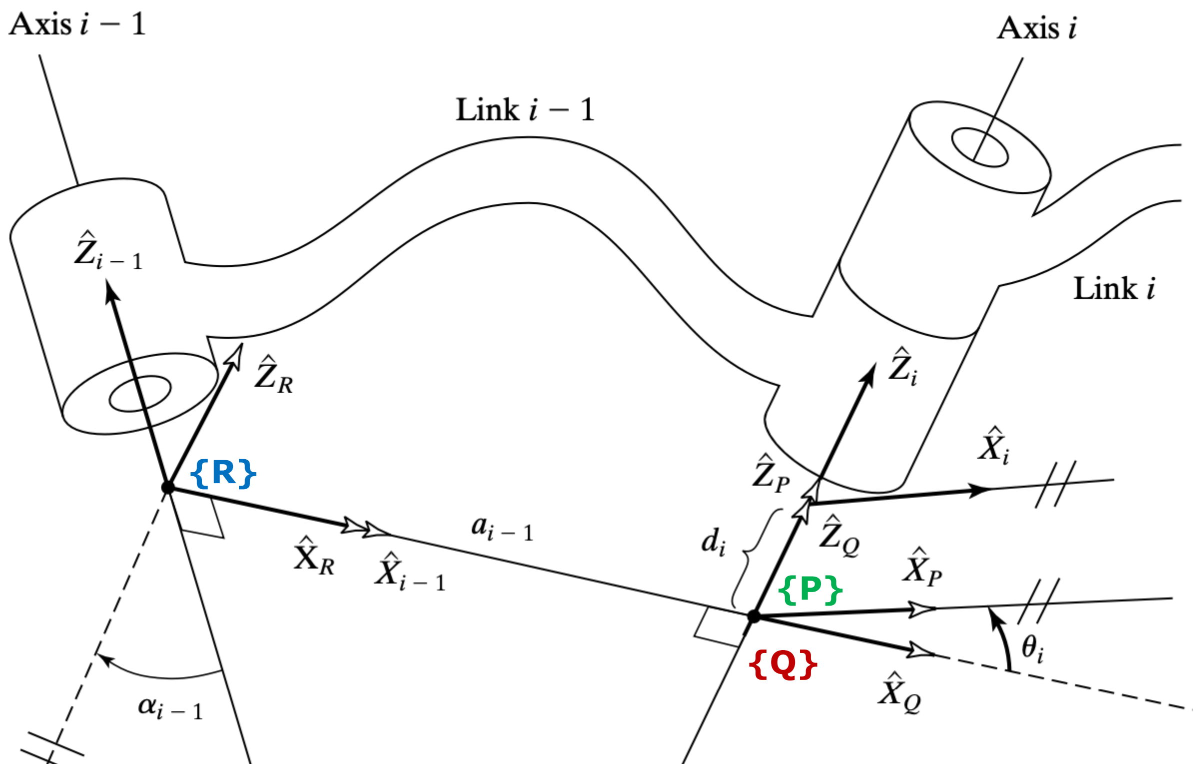
- Write the transformation that transforms vectors defined in {i} to their description in {i−1}



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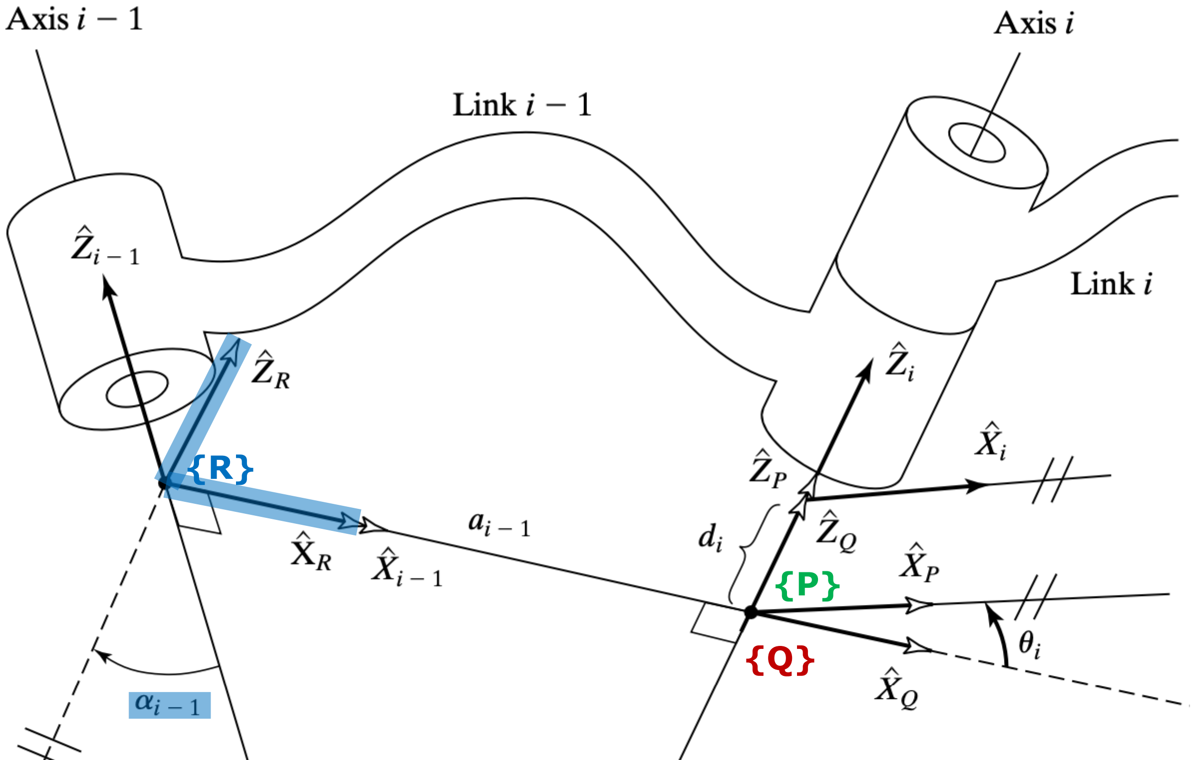

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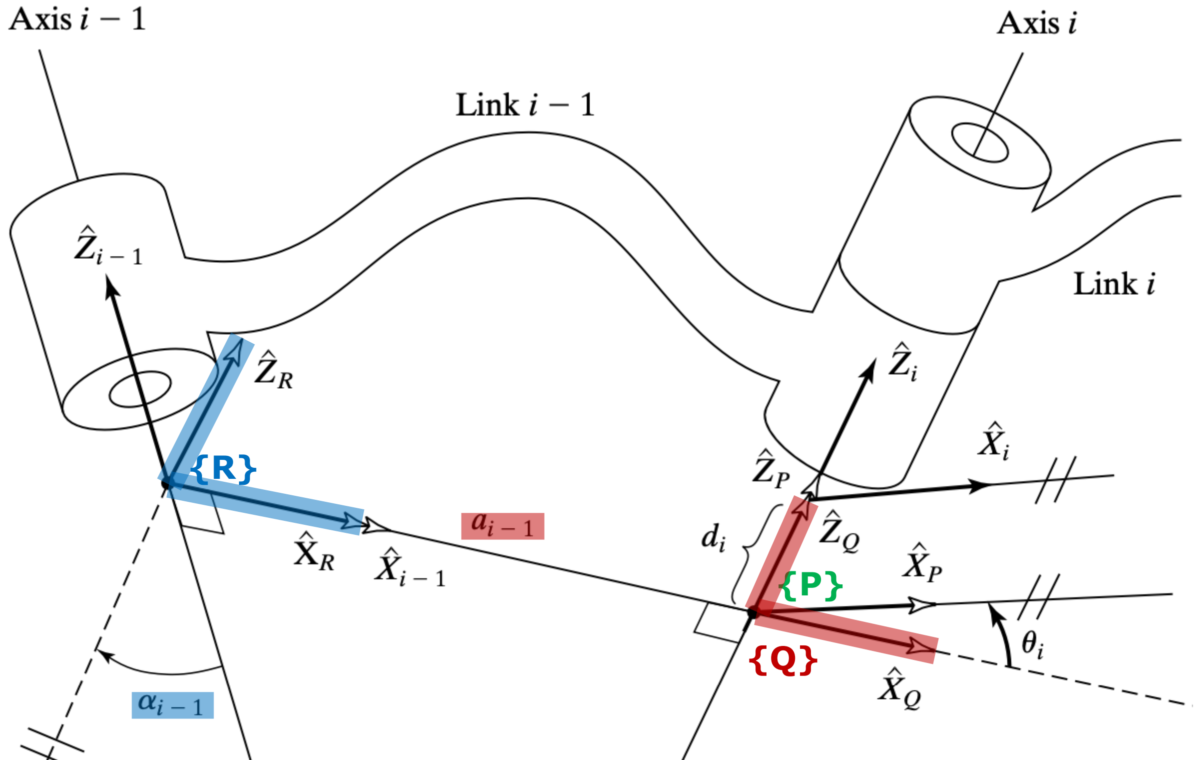

- Frame {R} differs from frame {i-1} only by a rotation of α_{i-1}
- Frame {Q} differs from {R} by a translation a_{i-1}

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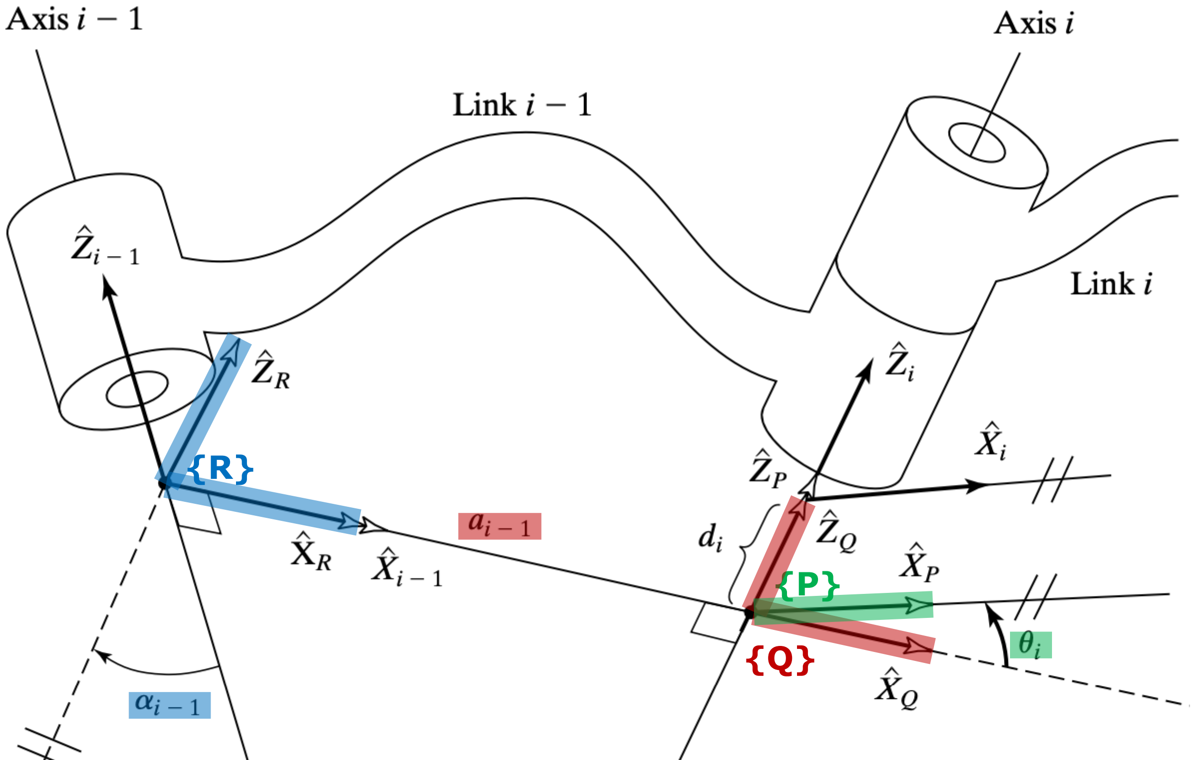

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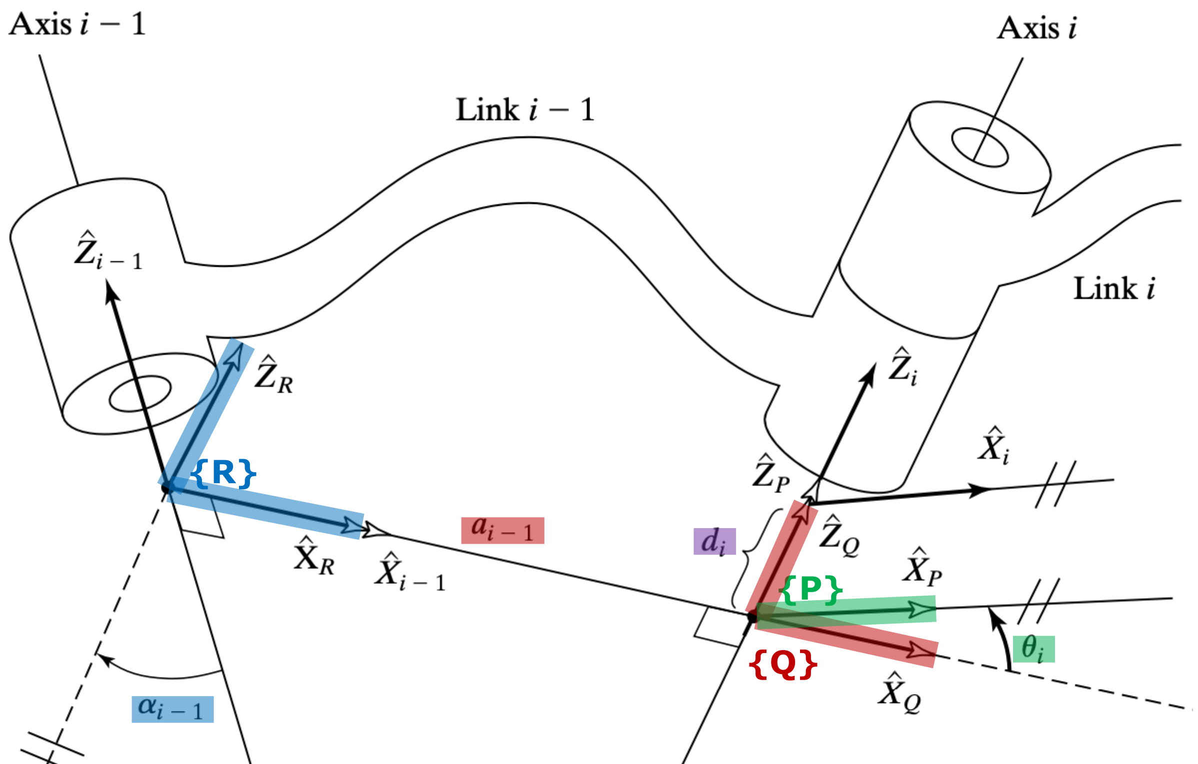

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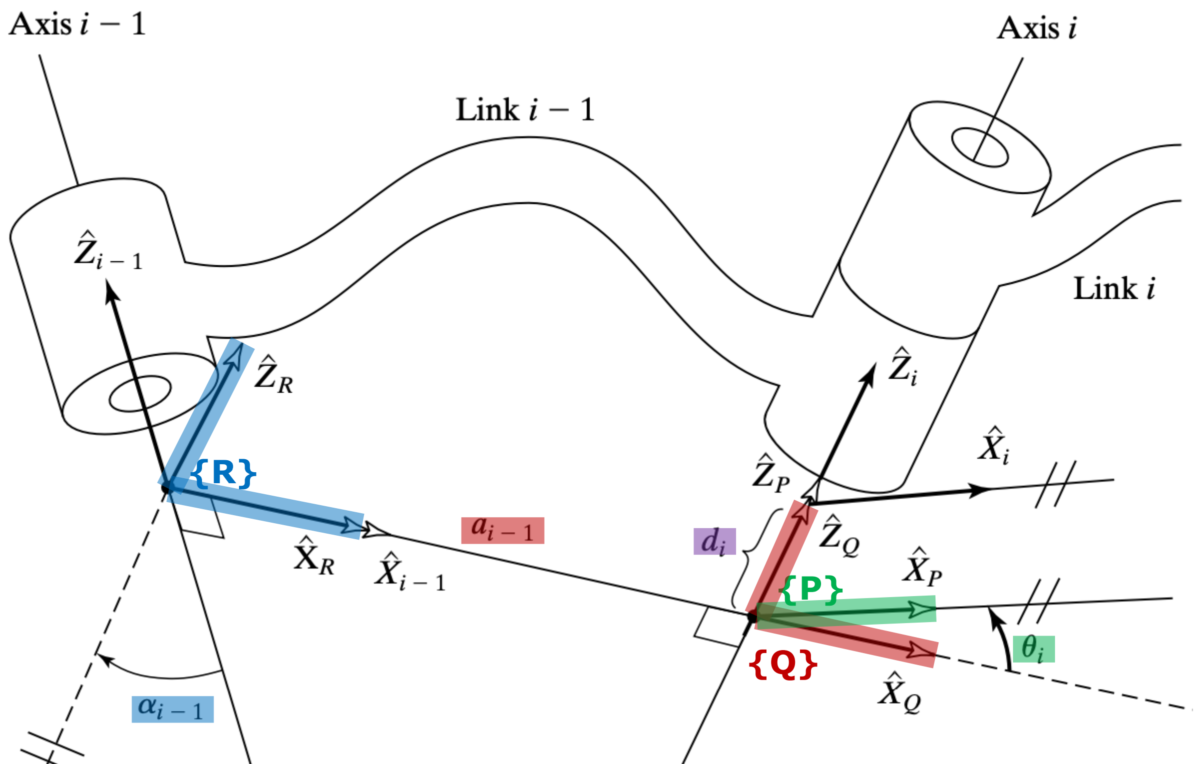
Task #2 - Derivation of link transformations

- $^{i-1}P = {}^{i-1}_{R}T \, {}^{R}_{Q}T \, {}^{Q}_{P}T \, {}^{P}_{i}T \, {}^{i}P = {}^{i-1}_{i}T \, {}^{i}P$



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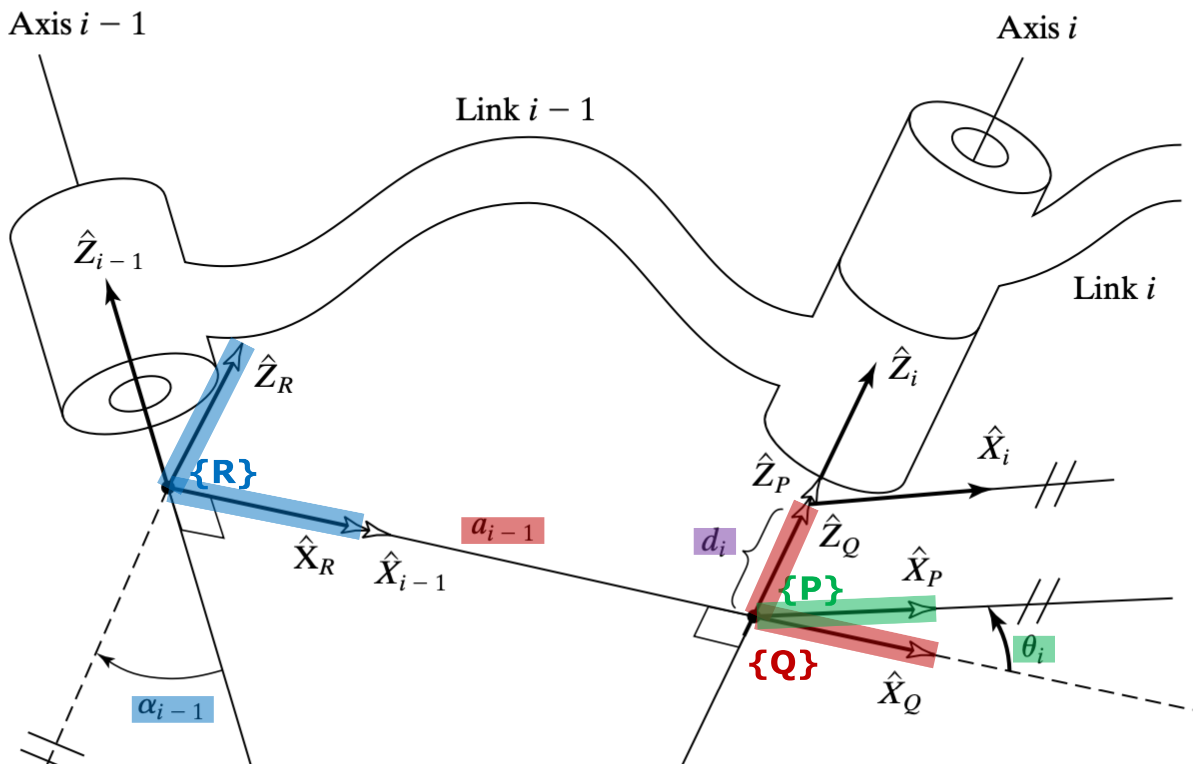
- $^{i-1}_{i}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i)$



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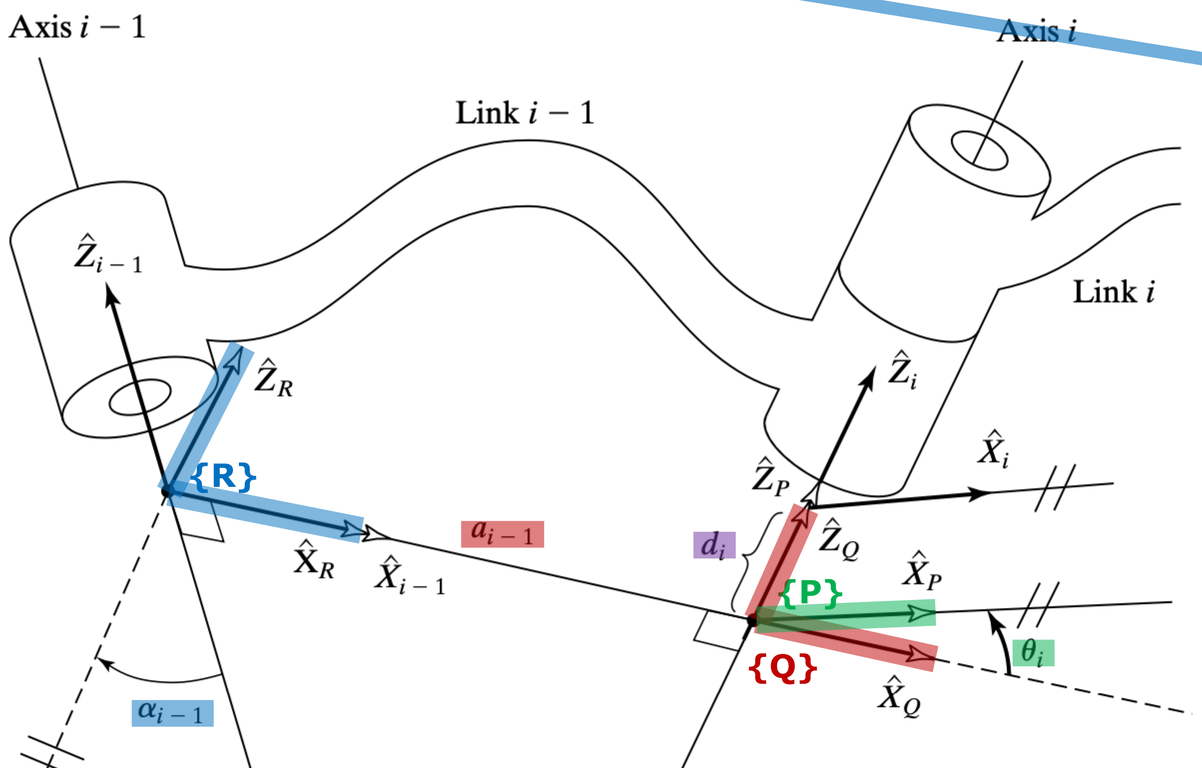

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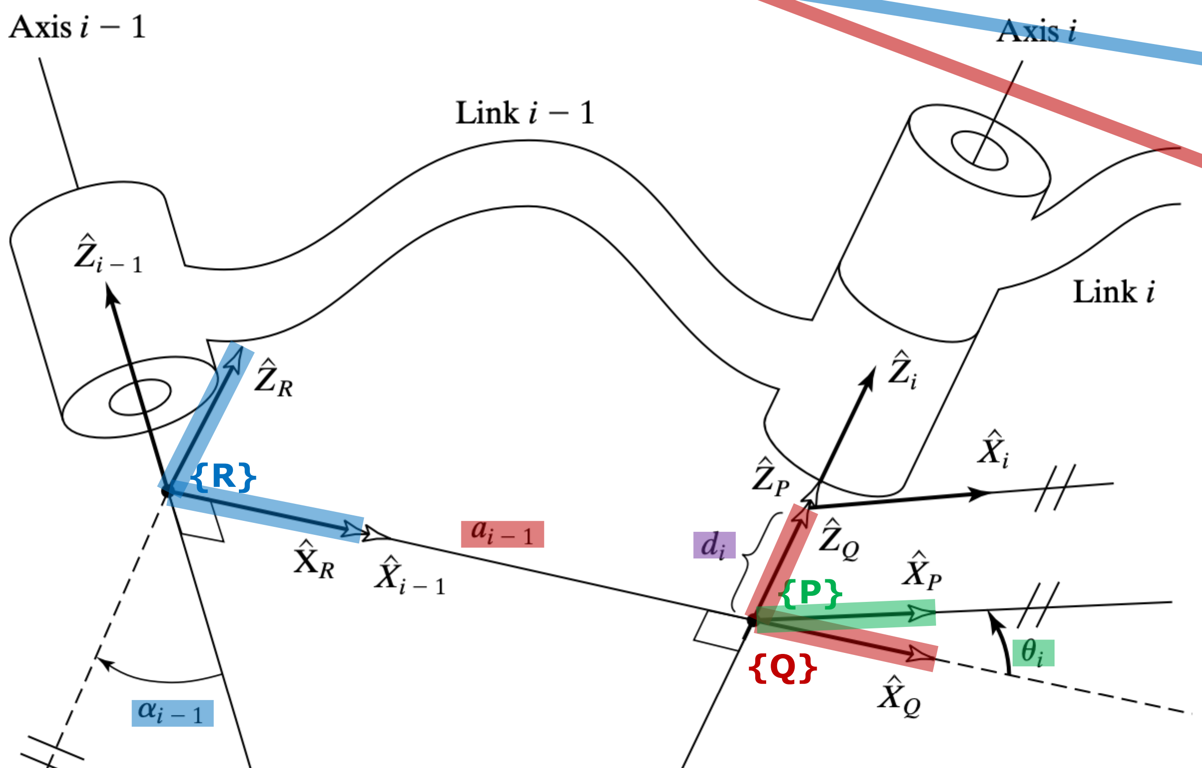

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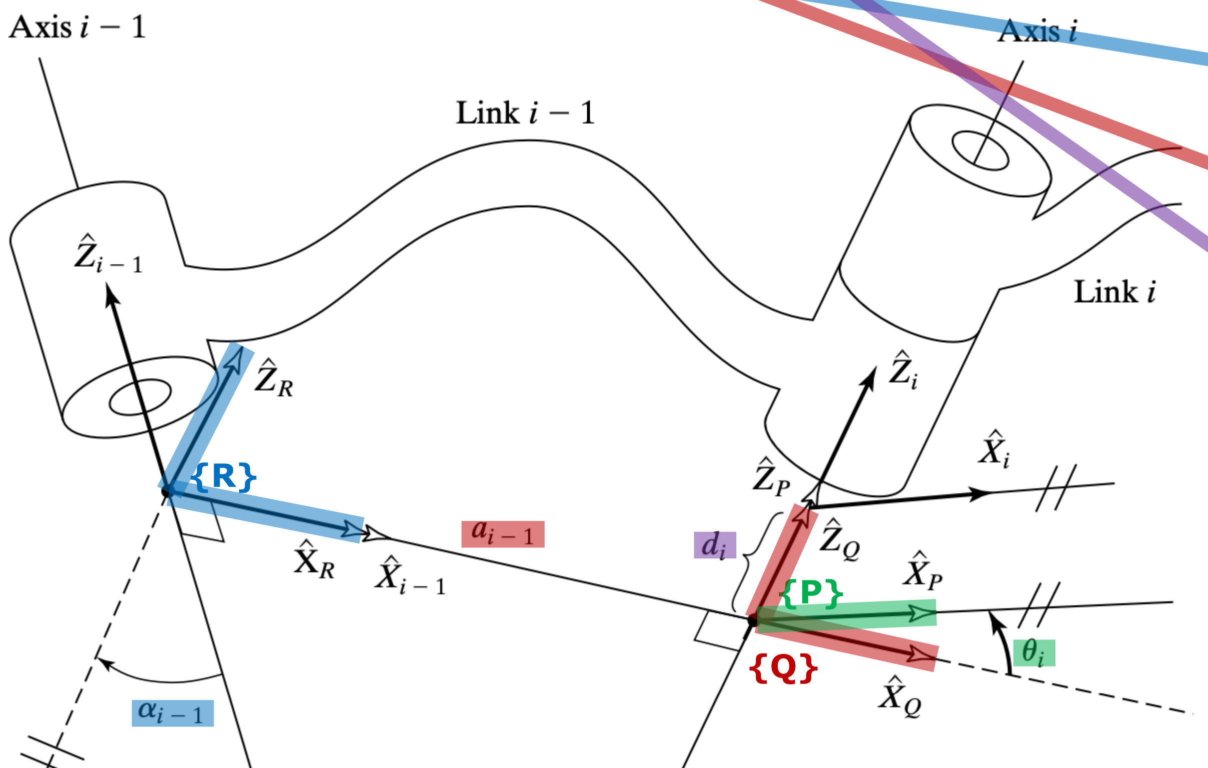

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- $${}^{i-1}P = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T \; {}^{i}P = {}^{i-1}_{i}T \; {}^{i}P$$
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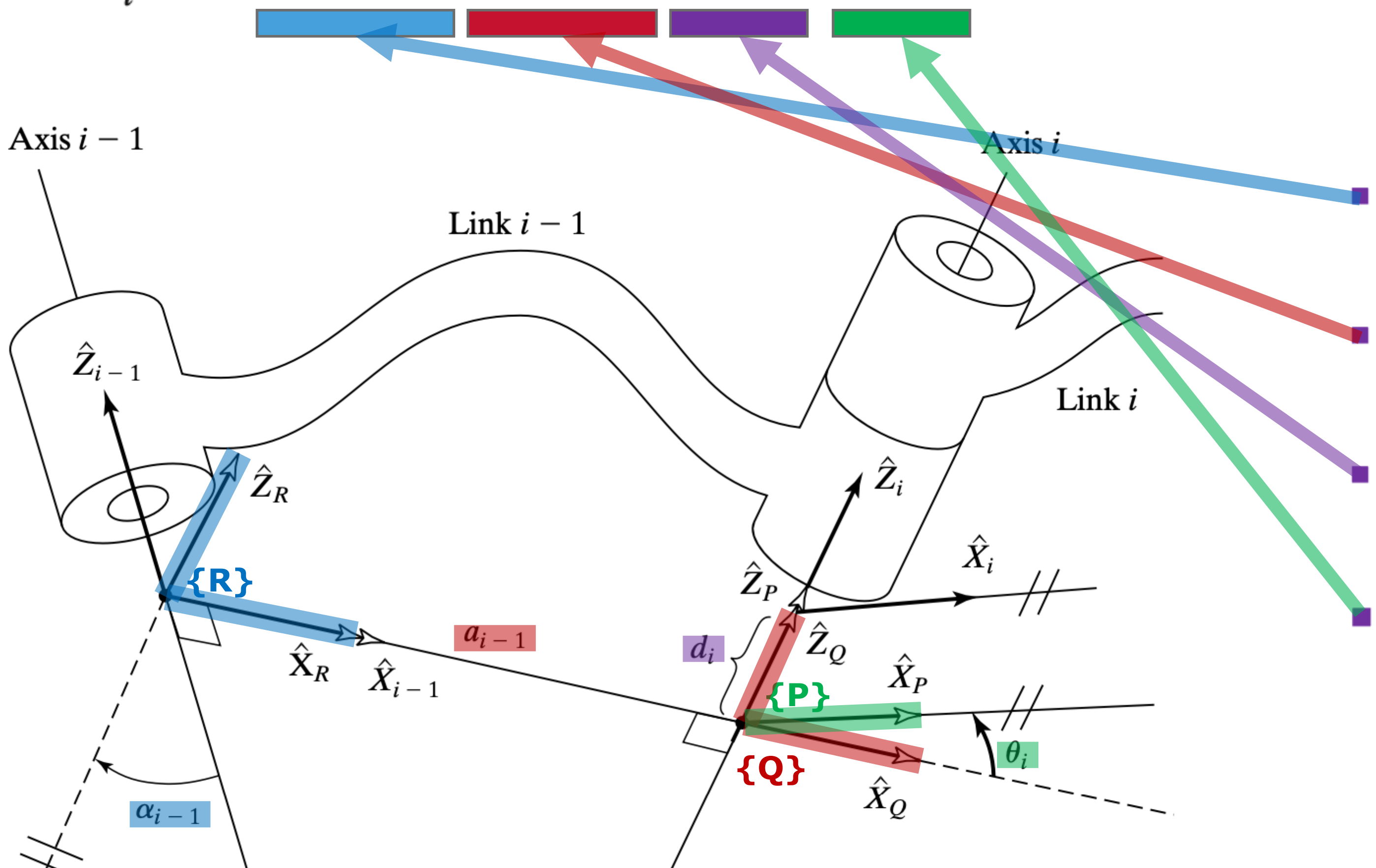

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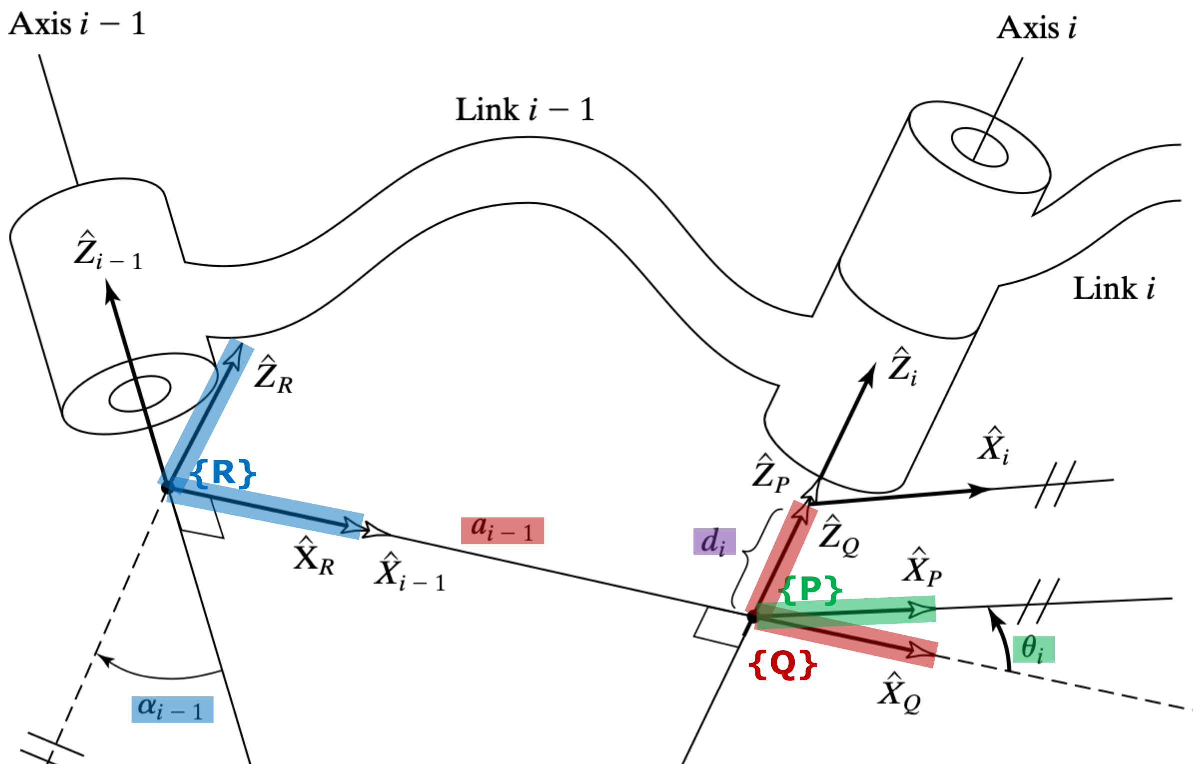
- $^{i-1}_{i}T = R_X(\alpha_{i-1}) D_X(a_{i-1}) R_Z(\theta_i) D_Z(d_i)$

$$^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link transformations - MATLAB

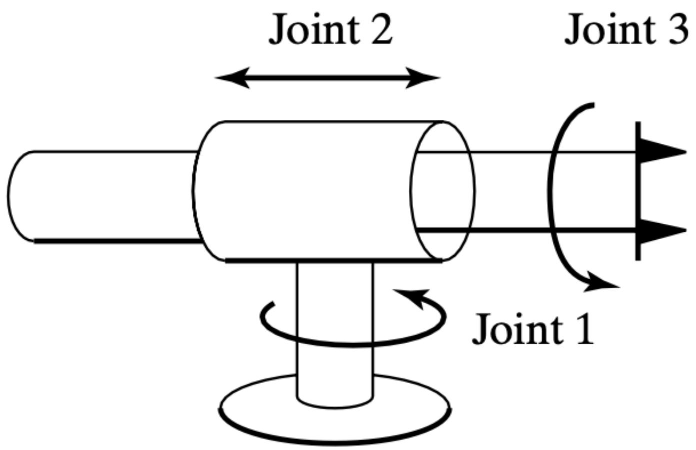
$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

```
Transformation(alphai_1, ai_1, di, thi) =

[               cos(thi),               -sin(thi),               0,               ai_1]
[cos(alphai_1)*sin(thi), cos(alphai_1)*cos(thi), -sin(alphai_1), -di*sin(alphai_1)]
[sin(alphai_1)*sin(thi), sin(alphai_1)*cos(thi),  cos(alphai_1),  di*cos(alphai_1)]
[                      0,                      0,              0,                 1]
```

RPR manipulator - Example 1

- Consider the RPR cylindrical manipulator
- Known DH parameters
- Calculate the ${}^{0}_{1}T$, ${}^{1}_{2}T$, ${}^{2}_{3}T$



RPR manipulator - Example 1 - Solution

- Substituting the DH parameters into link transformation equations → we get:

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

RPR manipulator - Example 1 - Solution

- Substituting the DH parameters into link transformation equations → we get:

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1

Substitute parameters

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

RPR manipulator - Example 1 - Solution

- Substituting the DH parameters into link transformation equations → we get:

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0

Substitute parameters

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

RPR manipulator - Example 1 - Solution

- Substituting the DH parameters into link transformation equations → we get:

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}. $$

Substitute parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

RPR manipulator - Example 1 - Solution - MATLAB

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1

- $${}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

- `Transformation(alphai_1, ai_1, di, thi)`
- `T_0_1 = Transformation(0, 0, 0, th1);`
- `T_0_1 =`

```
[cos(th1), -sin(th1), 0, 0]
[sin(th1),  cos(th1), 0, 0]
[       0,         0, 1, 0]
[       0,         0, 0, 1]
```

RPR manipulator - Example 1 - Solution - MATLAB

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0

- `Transformation(alphai_1, ai_1, di, thi)`
- `T_1_2 = Transformation(pi/2, 0, d2, 0);`
- `T_1_2 =`

```
[1, 0,  0,   0]
[0, 0, -1, -d2]
[0, 1,  0,   0]
[0, 0,  0,   1]
```

- $$ {}^{1}_{2}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

RPR manipulator - Example 1 - Solution - MATLAB

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

- `Transformation(alphai_1, ai_1, di, thi)`
- `T_2_3 = Transformation(0, 0, L2, th3);`
- T_2_3 =

```
[cos(th3), -sin(th3), 0,  0]
[sin(th3),  cos(th3), 0,  0]
[       0,         0, 1, L2]
[       0,         0, 0,  1]
```

- $${}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Example 1 - Final transformation - T_0_3

- Transformation matrix between the base frame {0} and the end-effector frame {n}:

```
T_0_1 =

[cos(th1), -sin(th1), 0, 0]
[sin(th1),  cos(th1), 0, 0]
[       0,         0, 1, 0]
[       0,         0, 0, 1]
```

```
T_1_2 =

[1, 0,  0,   0]
[0, 0, -1, -d2]
[0, 1,  0,   0]
[0, 0,  0,   1]
```

```
T_2_3 =

[cos(th3), -sin(th3), 0,  0]
[sin(th3),  cos(th3), 0,  0]
[       0,         0, 1, L2]
[       0,         0, 0,  1]
```

$$ {}^{0}_{n}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T \cdot \ldots \cdot {}^{n-1}_{n}T $$

Example 1 - Final transformation - T_0_3

$$ {}^{0}_{n}T = {}^{0}_{1}T \cdot {}^{1}_{2}T \cdot {}^{2}_{3}T \cdot \ldots \cdot {}^{n-1}_{n}T $$

- `T_0_3 = T_0_1 * T_1_2 * T_2_3;`

```
T_0_3 =

[cos(th1)*cos(th3), -cos(th1)*sin(th3),  sin(th1),   L2*sin(th1) + d2*sin(th1)]
[cos(th3)*sin(th1), -sin(th1)*sin(th3), -cos(th1), - L2*cos(th1) - d2*cos(th1)]
[         sin(th3),           cos(th3),         0,                           0]
[                0,                  0,         0,                           1]
```

- $${}^{0}_{n}T = \begin{bmatrix} & {}^{0}_{n}\mathbf{R} & & \begin{matrix} x \\ y \\ z \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}^{0}_{n}\mathbf{R}$ - rotation of {n} with respect to {0}

$[x \quad y \quad z]^T$ - coordinates of the end-effector in {0}

Example 1 - Final transformation - T_0_3 - RTB

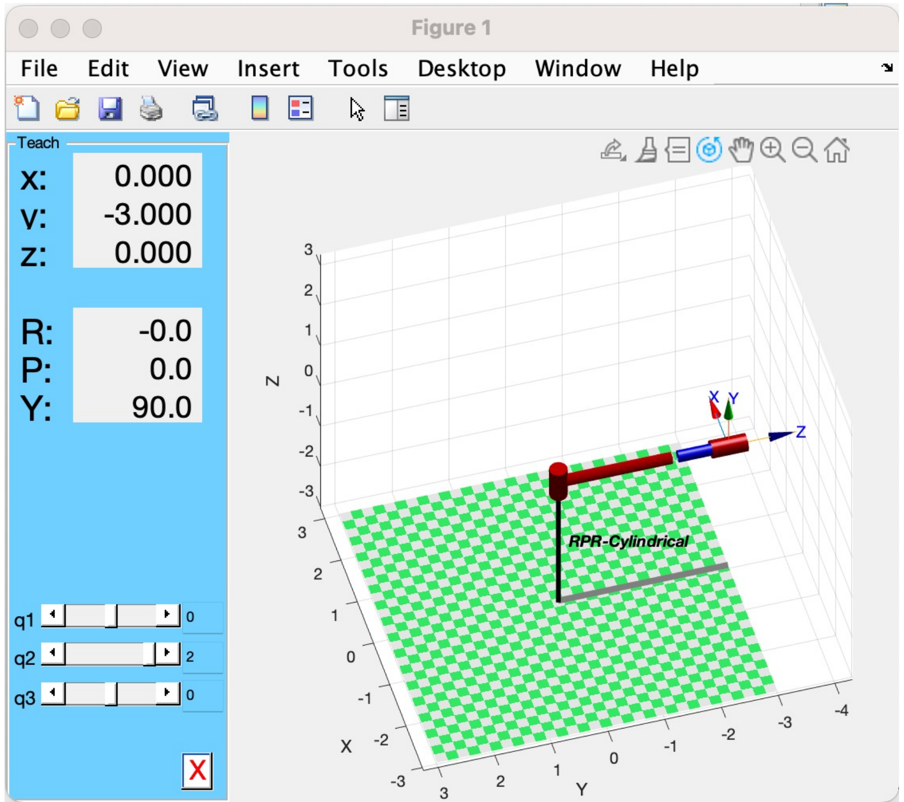
- T_0_3 =

```
[cos(th1)*cos(th3), -cos(th1)*sin(th3),  sin(th1),   L2*sin(th1) + d2*sin(th1)]
[cos(th3)*sin(th1), -sin(th1)*sin(th3), -cos(th1), - L2*cos(th1) - d2*cos(th1)]
[         sin(th3),           cos(th3),         0,                           0]
[                0,                  0,         0,                           1]
```

```
th1 = 0;
th3 = 0;
L2 = 1;
d2 = 2;
```

```
T_0_3 =

     1     0     0     0
     0     0    -1    -3
     0     1     0     0
     0     0     0     1
```



Example 1 - Final transformation - T_0_3 - RTB

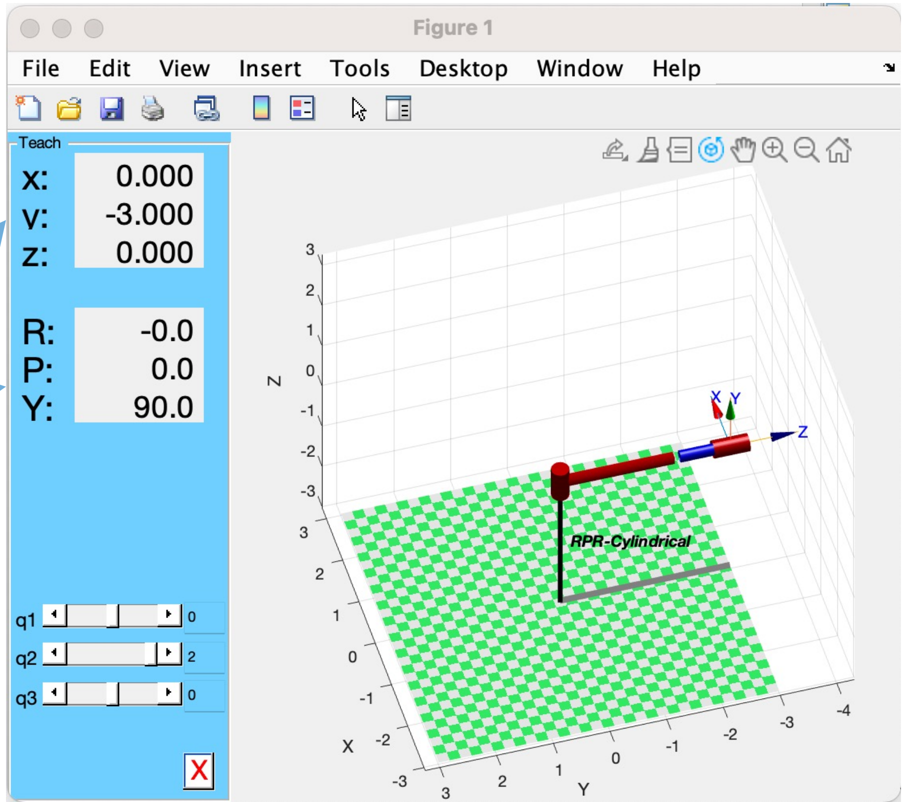
- T_0_3 =

```
[cos(th1)*cos(th3), -cos(th1)*sin(th3),  sin(th1),   L2*sin(th1) + d2*sin(th1)]
[cos(th3)*sin(th1), -sin(th1)*sin(th3), -cos(th1), - L2*cos(th1) - d2*cos(th1)]
[         sin(th3),           cos(th3),         0,                           0]
[                0,                  0,         0,                           1]
```

- th1 = 0;
- th3 = 0;
- L2 = 1;
- d2 = 2;

- T_0_3 =

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$



Example 1 - Final transformation - T_0_3 - RTB

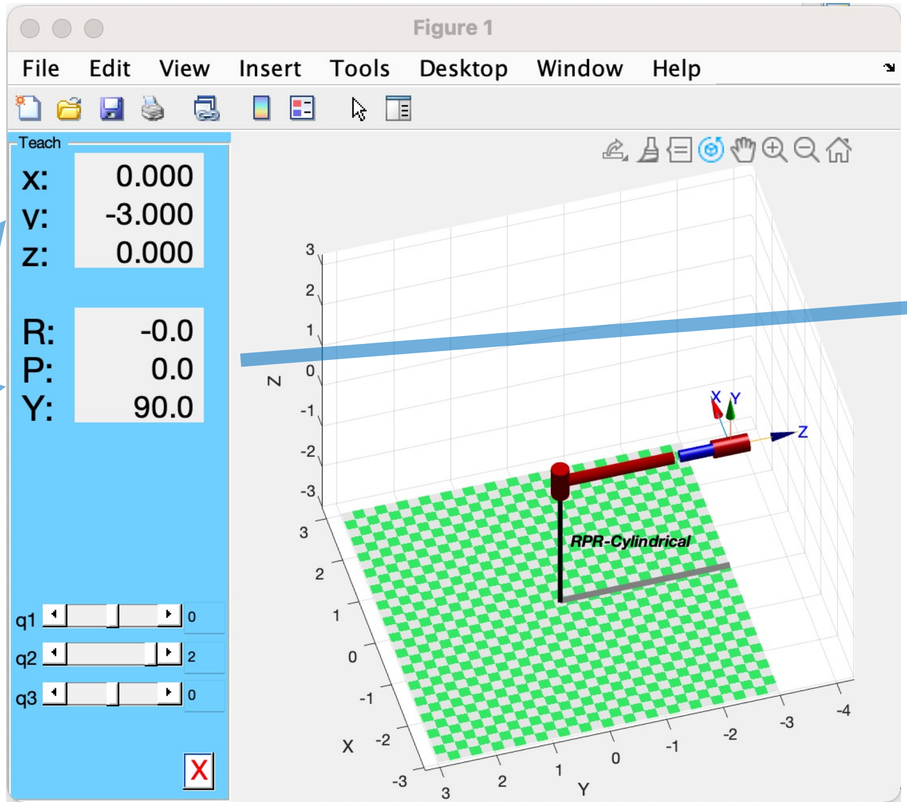
- T_0_3 =

```
[cos(th1)*cos(th3), -cos(th1)*sin(th3),  sin(th1),   L2*sin(th1) + d2*sin(th1)]
[cos(th3)*sin(th1), -sin(th1)*sin(th3), -cos(th1), - L2*cos(th1) - d2*cos(th1)]
[         sin(th3),           cos(th3),         0,                           0]
[                0,                  0,         0,                           1]
```

- th1 = 0;
- th3 = 0;
- L2 = 1;
- d2 = 2;

- T_0_3 =

```
     1     0     0     0
     0     0    -1    -3
     0     1     0     0
     0     0     0     1
```




```
>> rotx(90)

ans =

     1     0     0
     0     0    -1
     0     1     0
```

Example 1 - Final transformation - T_0_3 - RTB

- T_0_3 =

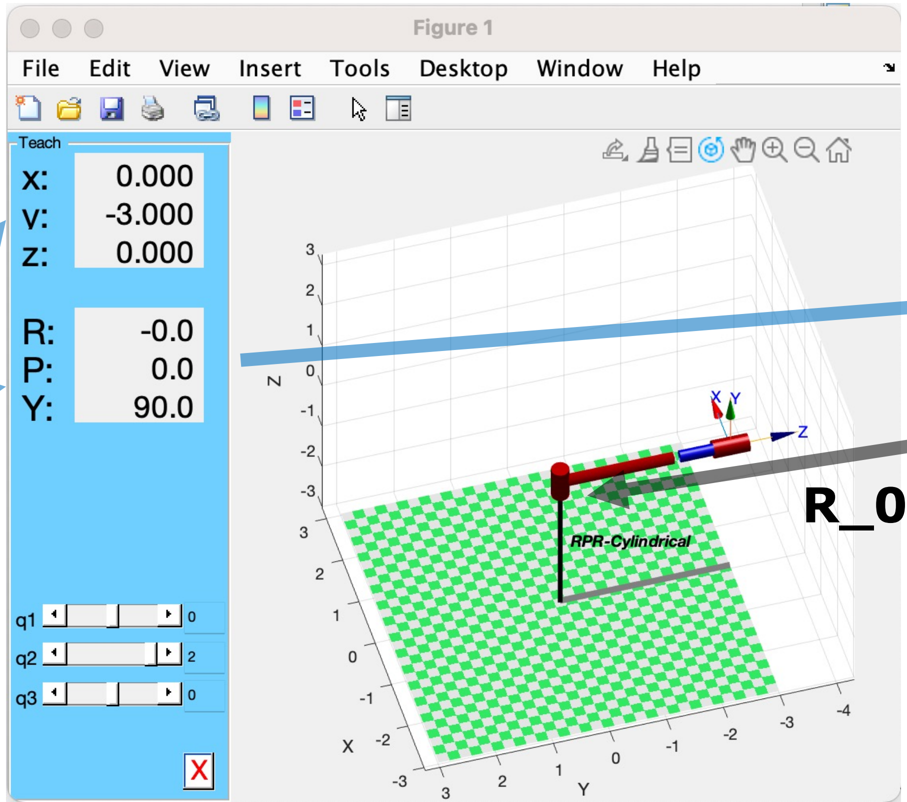
```
[cos(th1)*cos(th3), -cos(th1)*sin(th3),  sin(th1),   L2*sin(th1) + d2*sin(th1)]
[cos(th3)*sin(th1), -sin(th1)*sin(th3), -cos(th1), - L2*cos(th1) - d2*cos(th1)]
[         sin(th3),           cos(th3),         0,                           0]
[                0,                  0,         0,                           1]
```

```
th1 = 0;
th3 = 0;
L2 = 1;
d2 = 2;
```

```
>> eval(T_0_3)
```

```
T_0_3 =

     1     0     0     0
     0     0    -1    -3
     0     1     0     0
     0     0     0     1
```



R_0_3

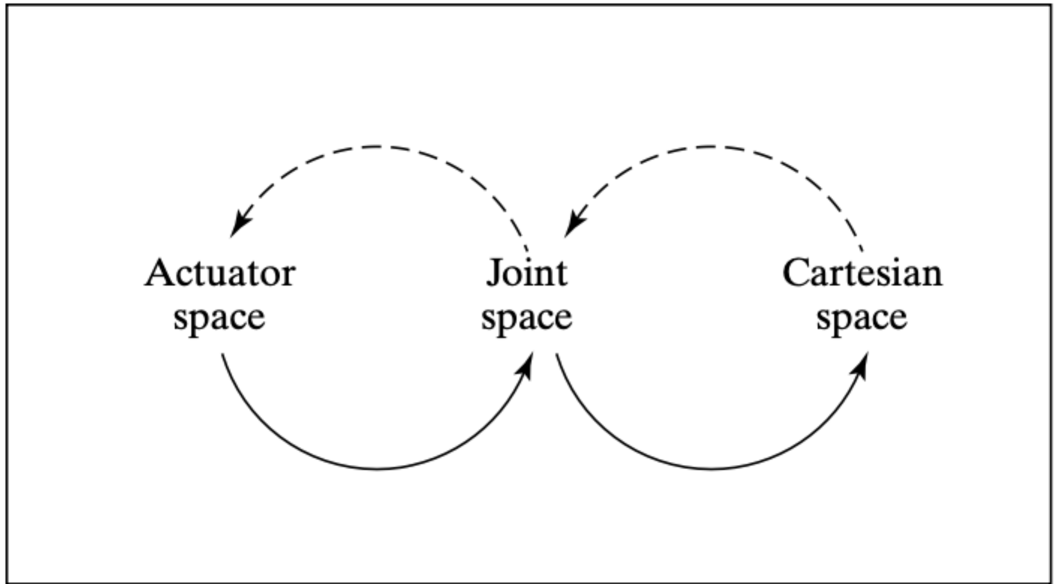
```
>> rotx(90)

ans =

     1     0     0
     0     0    -1
     0     1     0
```

Actuator - Joint - Cartesian spaces

- **Joint space**: the space of all joint vectors indicating the manipulator's position of the links - joint variables
- **Cartesian space**: the positions are measured along orthogonal axes and the orientation is measured according to any rotational conventions (i.e., ZYX Euler angles)
- **Actuator space**: the boundaries of a function that maps the joint vectors with the actuator values received from sensors measurements.



Actuator - Joint - Cartesian spaces - WAM robot

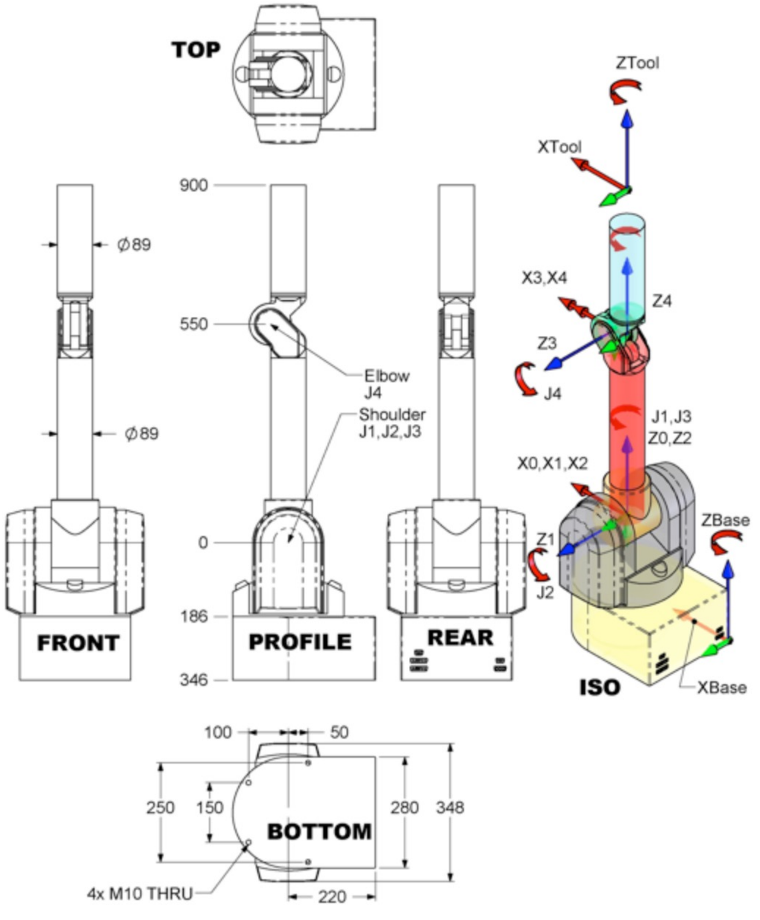


Figure 35 – WAM 4-DOF dimensions and D-H frames

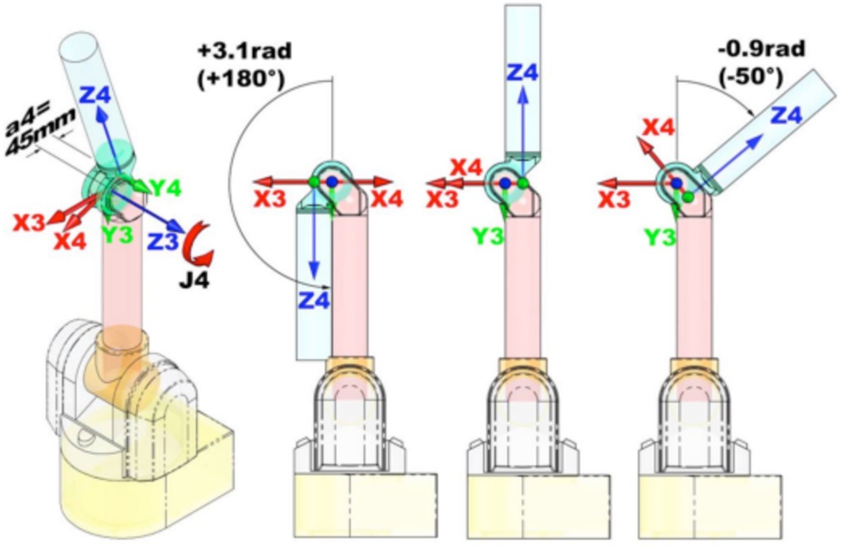


Figure 40: WAM Arm Joint 4 Frames and Limits

i	a_i	α_i	d_i	θ_i
1	0	$-\pi/2$	0	θ_1
2	0	$\pi/2$	0	θ_2
3	0.045	$-\pi/2$	0.55	θ_3
4	−0.045	$\pi/2$	0	θ_4
T	0	0	0.35	5

Table 8: Arm Transmission Ratios

Parameter	Value
N_1	42.0
N_2	28.25
N_3	28.25
n_3	1.68
N_4	18.0
N_5	9.48
N_6	9.48
N_7	14.93
n_6	1

$$\begin{bmatrix} J\theta_1 \\ J\theta_2 \\ J\theta_3 \\ J\theta_4 \end{bmatrix} = \begin{bmatrix} \frac{-1}{N_1} & 0 & 0 & 0 \\ 0 & \frac{1}{2N_2} & \frac{-1}{2N_2} & 0 \\ 0 & \frac{-n_3}{2N_2} & \frac{-n_3}{2N_2} & 0 \\ 0 & 0 & 0 & \frac{-1}{N_4} \end{bmatrix} \begin{bmatrix} M\theta_1 \\ M\theta_2 \\ M\theta_3 \\ M\theta_4 \end{bmatrix}$$

Equation 6: WAM Motor-to-Joint position transformations

$$\begin{bmatrix} J\theta_5 \\ J\theta_6 \\ J\theta_7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2N_5} & \frac{1}{2N_5} & 0 \\ \frac{-n_6}{2N_5} & \frac{n_6}{2N_5} & 0 \\ 0 & 0 & \frac{1}{N_7} \end{bmatrix} \begin{bmatrix} M\theta_5 \\ M\theta_6 \\ M\theta_7 \end{bmatrix}$$

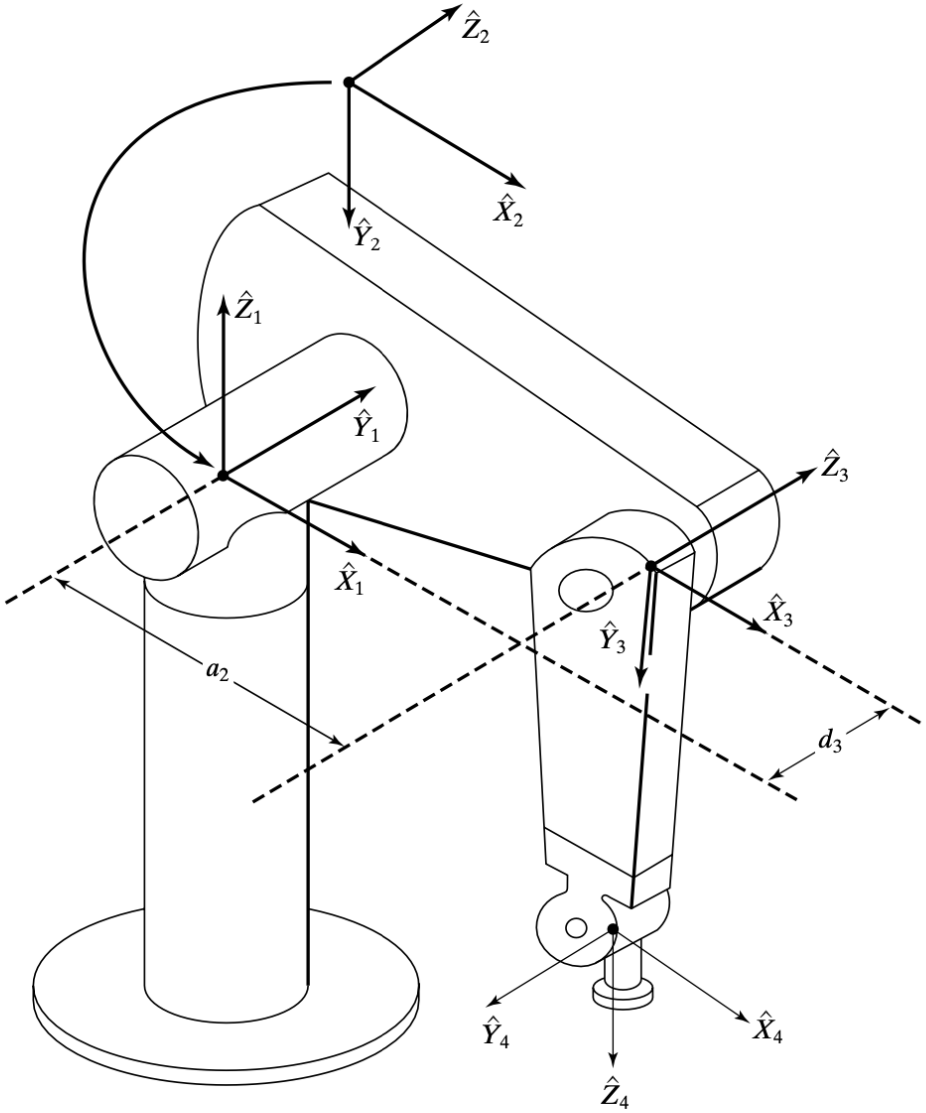
Equation 7: Wrist Motor-to-Joint position transformations

https://support.barrett.com/wiki/WAM/KinematicsJointRangesConversionFactors #Motor-to-JointTransformations

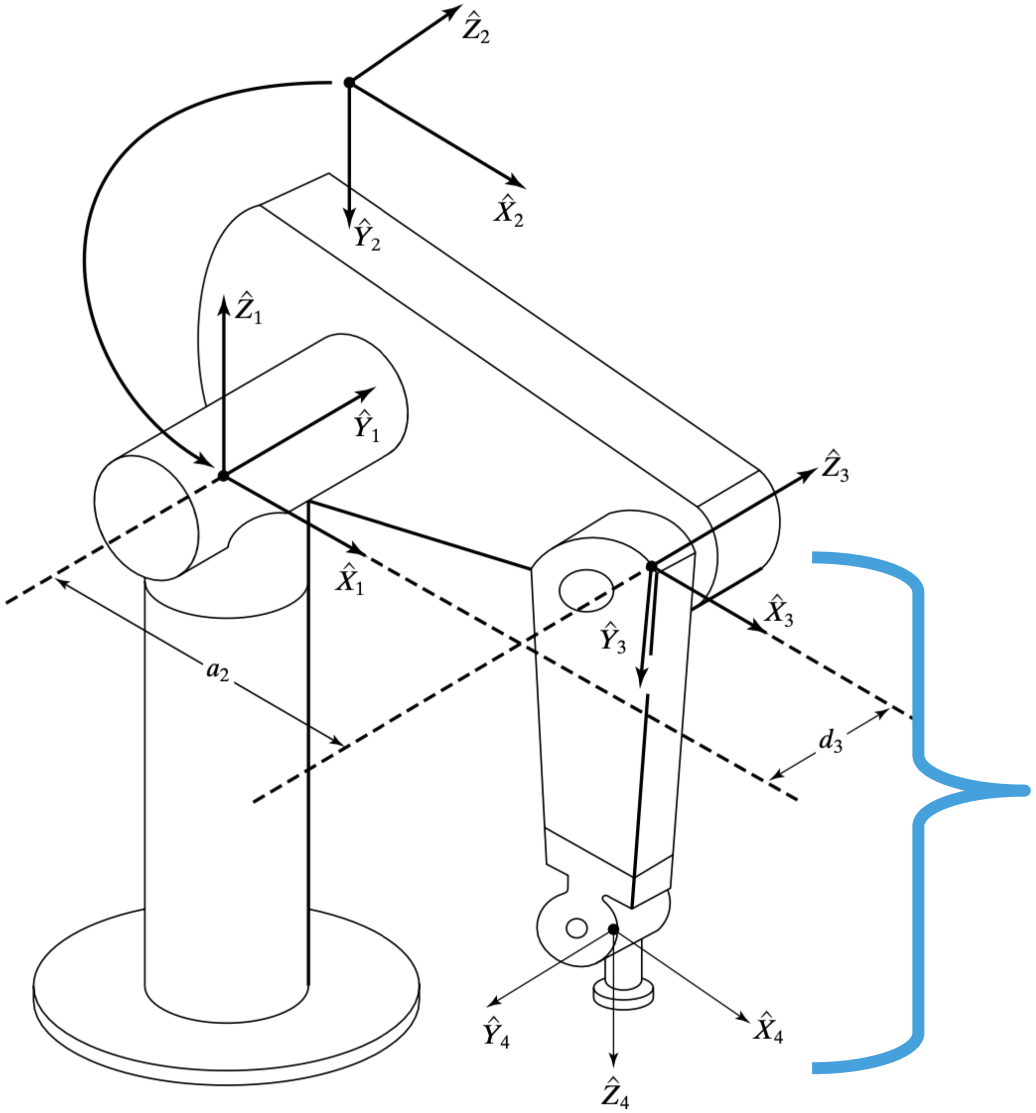
PUMA 560 - Unimate robot

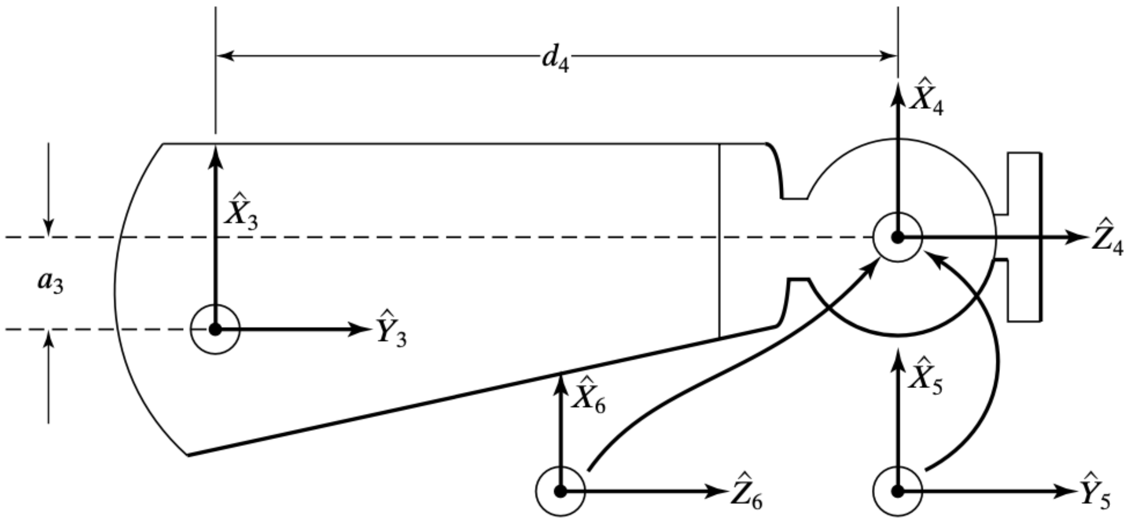
- Courtesy of Unimation Incorporated
- 6 DoFs
- All rotational joints
- RRRRRR mechanism

PUMA 560 - Unimate robot - Mechanical chain

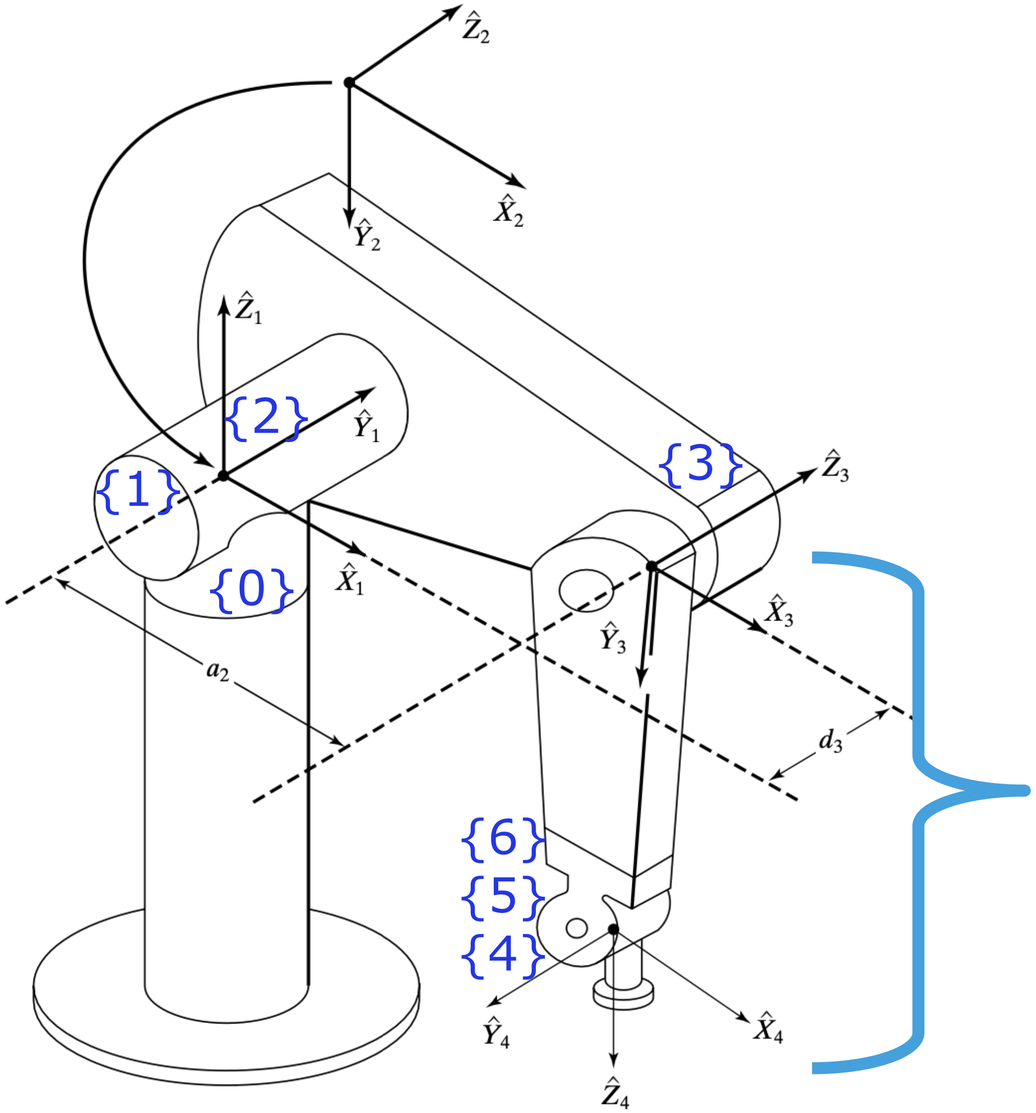


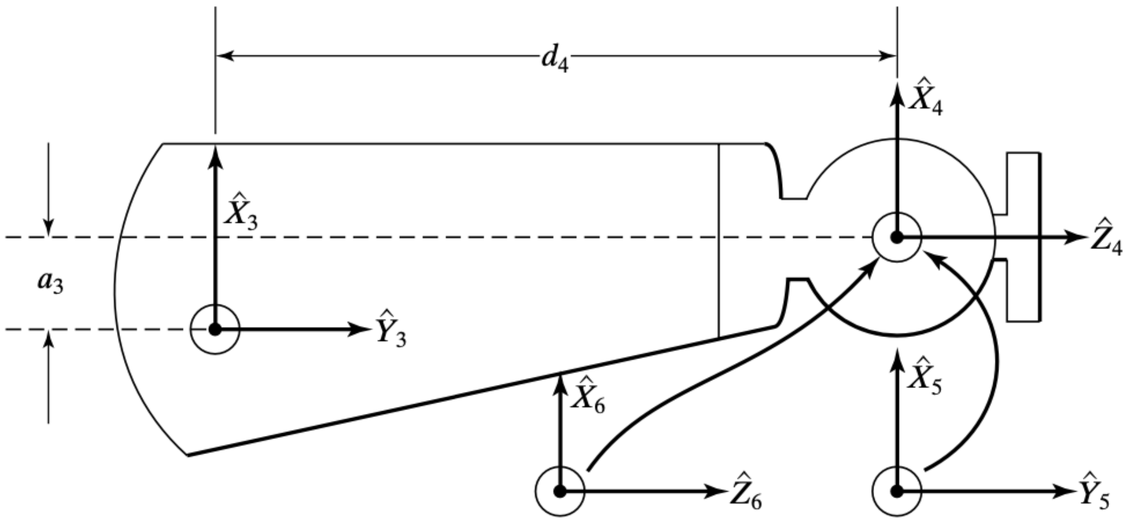
PUMA 560 - Unimate robot - Mechanical chain



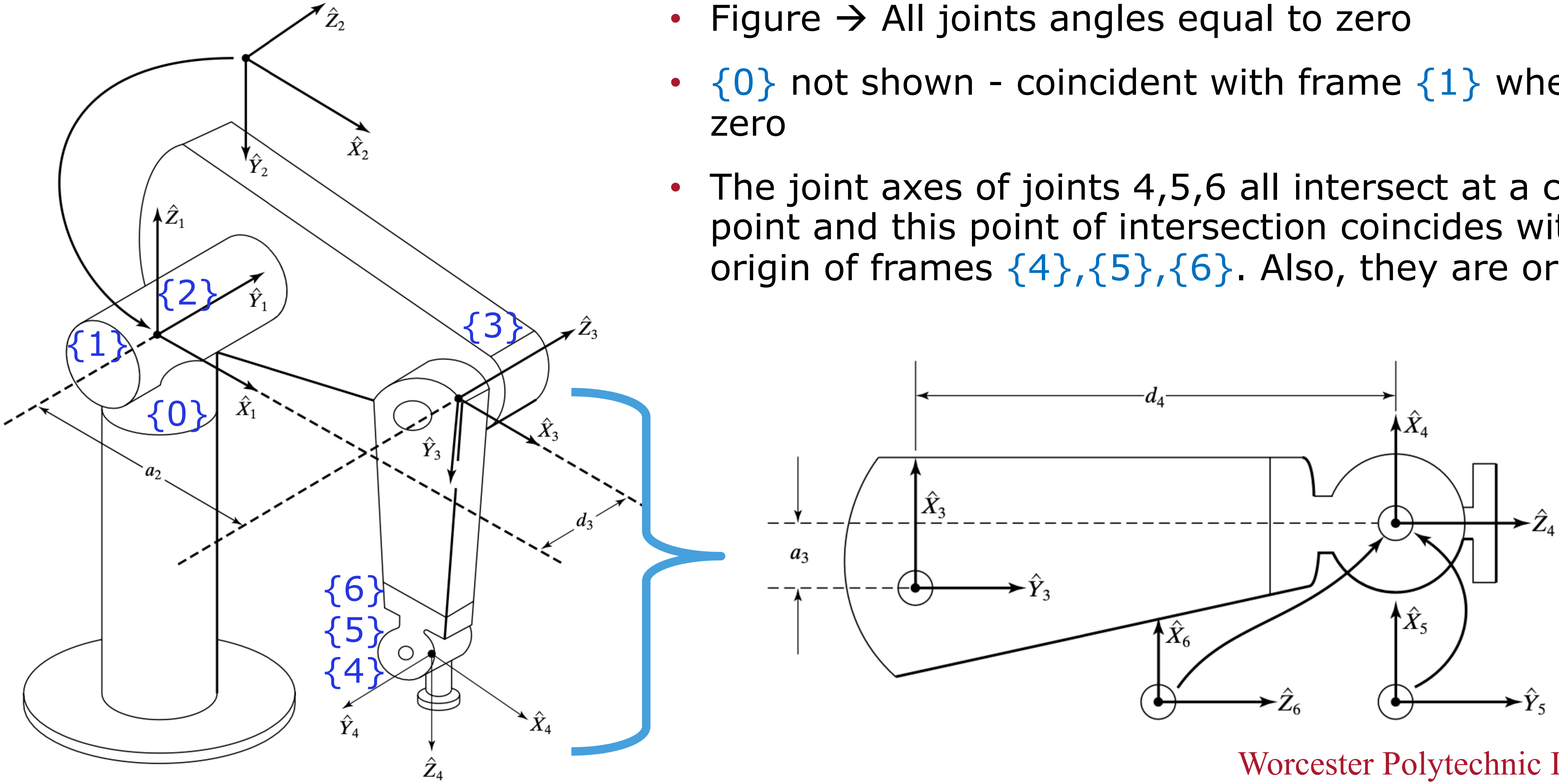


PUMA 560 - Unimate robot - Attach frames





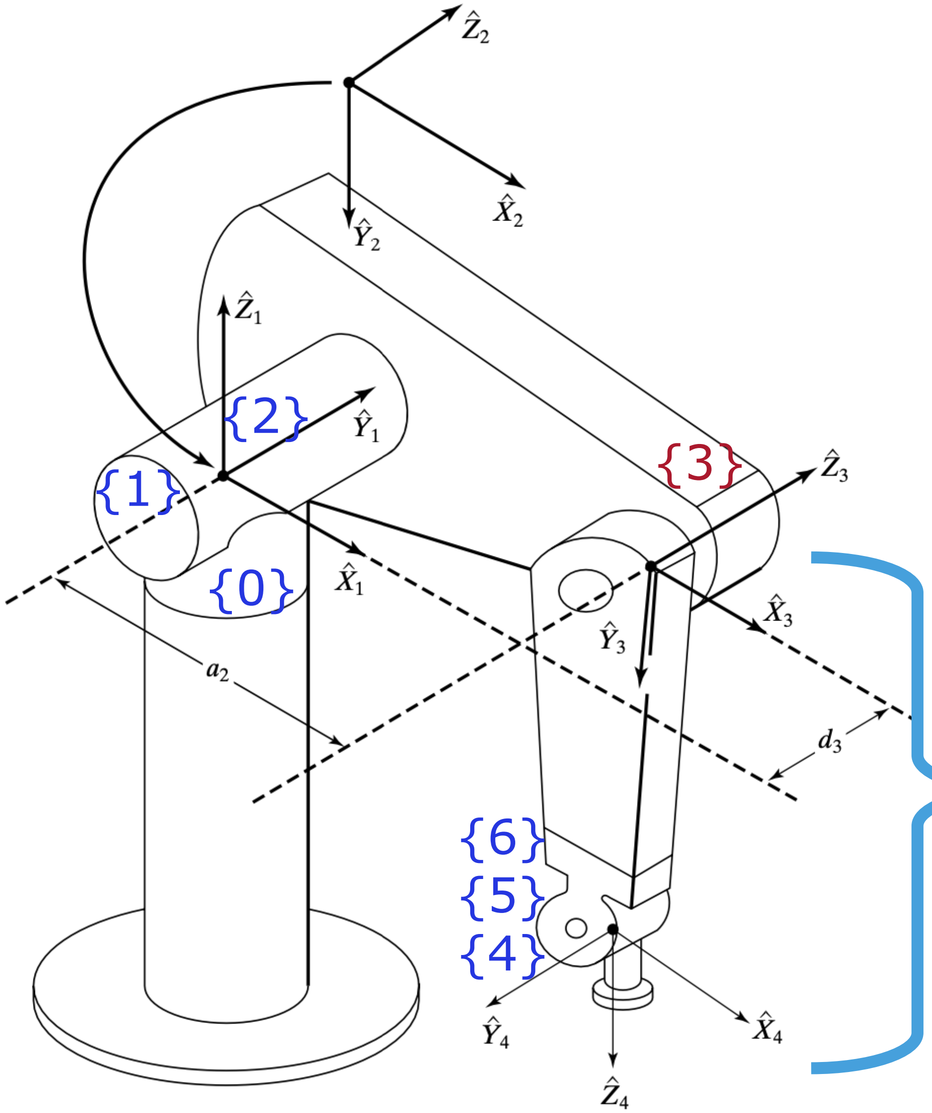
PUMA 560 - Unimate robot - Attach frames



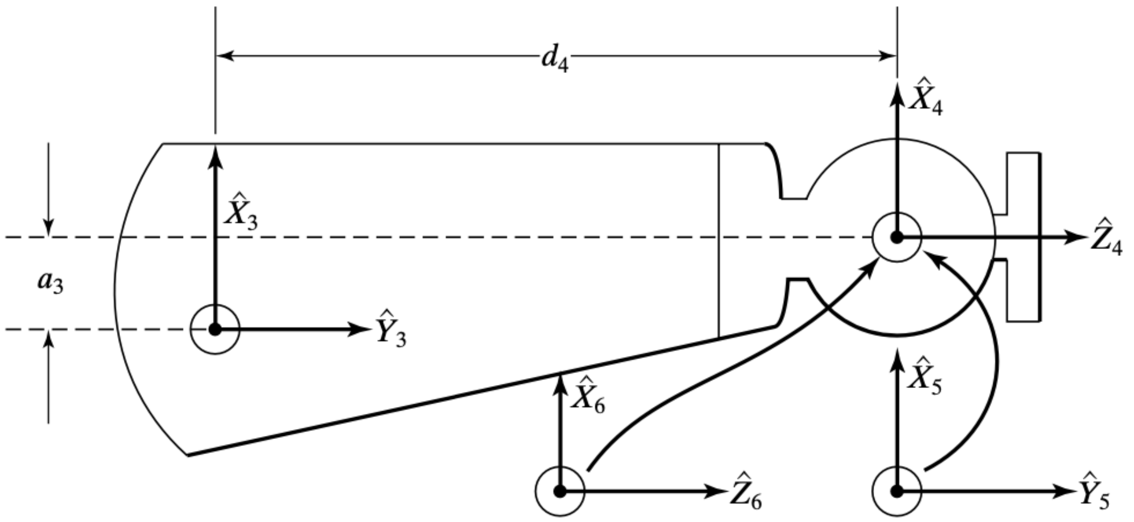
- Figure → All joints angles equal to zero
- {0} not shown - coincident with frame {1} when θ_1 is zero
- The joint axes of joints 4,5,6 all intersect at a common point and this point of intersection coincides with the origin of frames {4},{5},{6}. Also, they are orthogonal

*** Unimation has used a slightly different assignment of zero location of the joints, such that $\theta_3^* = \theta_3 - 180°$, where θ_3^* is the position of joint 3 in Unimation's convention.

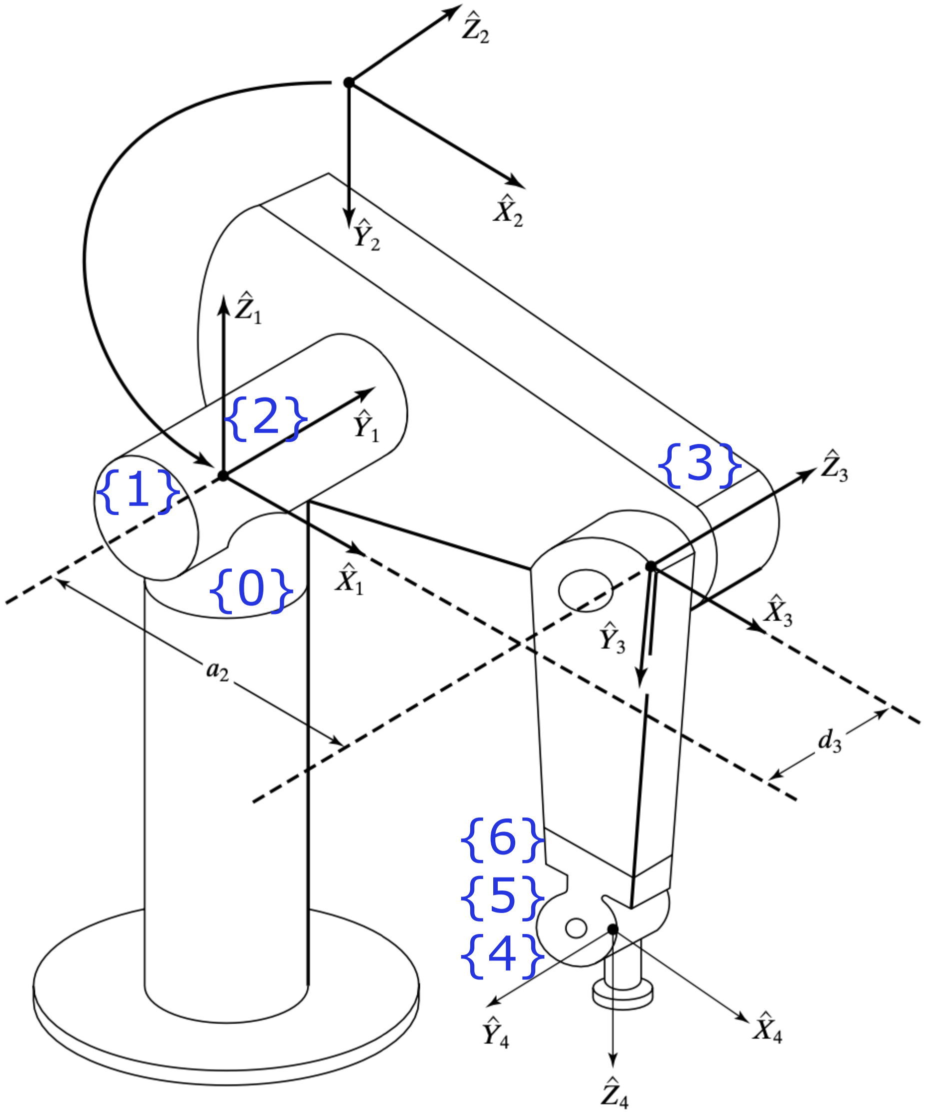
PUMA 560 - Unimate robot - Attach frames



- Figure → All joints angles equal to zero***
- {0} not shown - coincident with frame {1} when θ_1 is zero
- The joint axes of joints 4,5,6 all intersect at a common point and this point of intersection coincides with the origin of frames {4},{5},{6}. Also, they are orthogonal

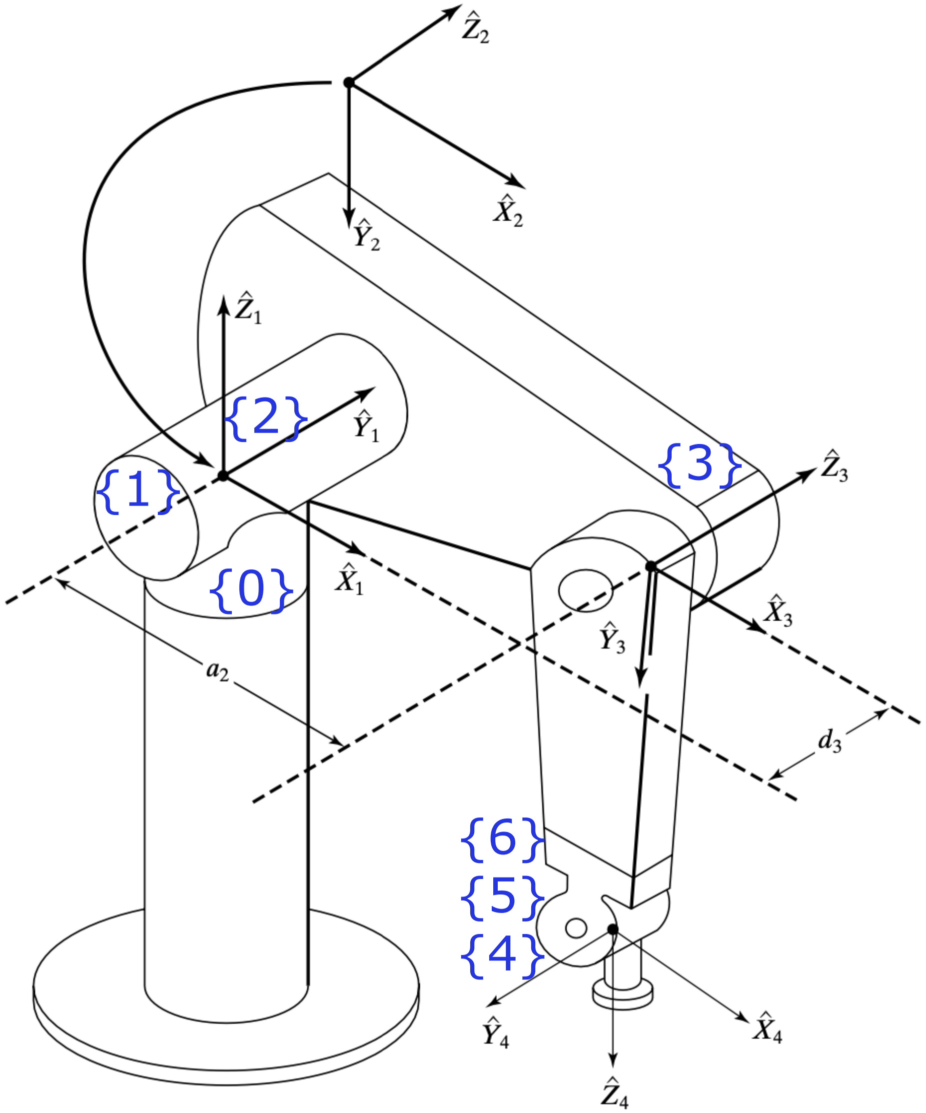


PUMA 560 - Unimate robot - DH parameters

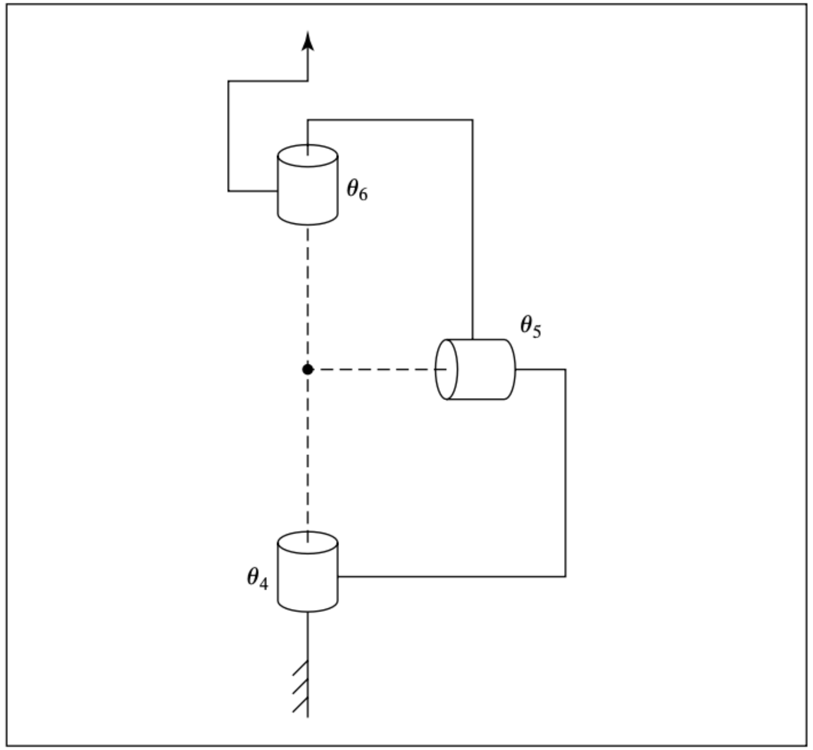


i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

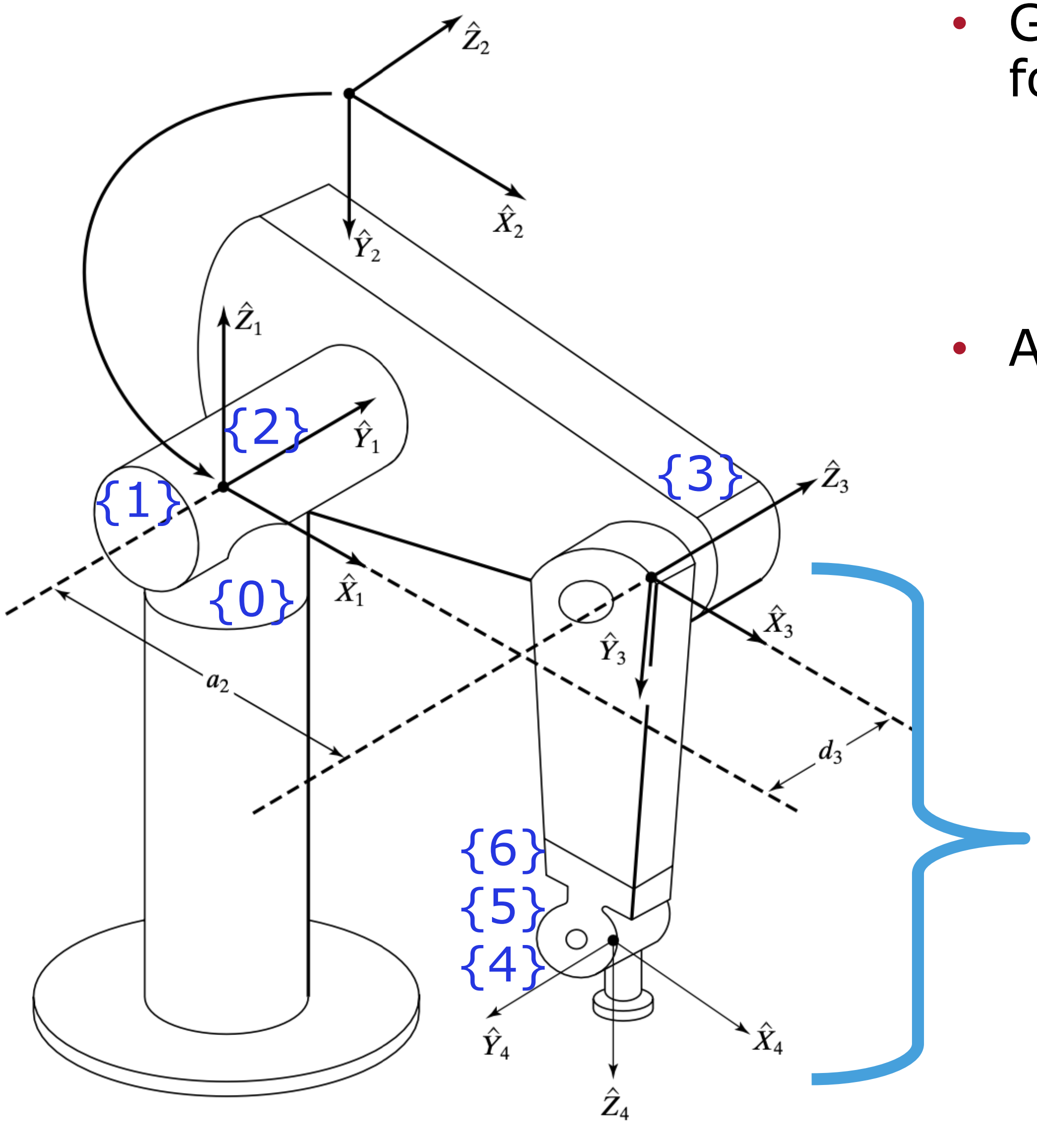
PUMA 560 - Unimate robot - DH parameters



- Gearing arrangement in the wrist couples together the motions of joints 4, 5, and 6.
- We must make a distinction between joint space and actuator space and solve the complete kinematics in two steps



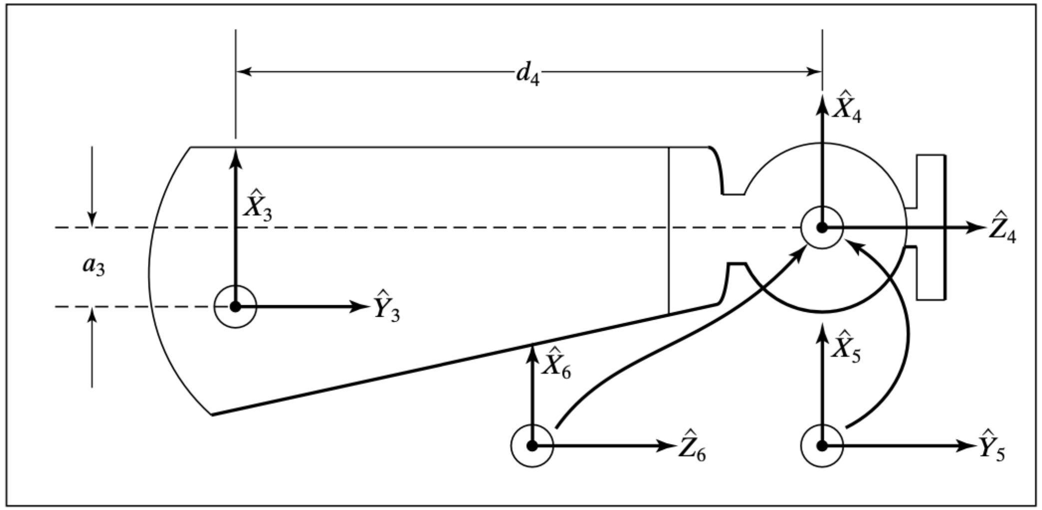
PUMA 560 - Example 2



- Given the DH parameters and the link transformations formular above, **find the transformations** below:

$$ {}^{0}_{1}T,\ {}^{1}_{2}T,\ {}^{2}_{3}T,\ {}^{3}_{4}T,\ {}^{4}_{5}T,\ {}^{5}_{6}T $$

- Also, find the: ${}^{0}_{6}T$



PUMA 560 - Example 2 - Solution

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

$$ {}^{i-1}_{\;\;i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

PUMA 560 - Example 2 - Solution - T_0_1

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1

$$ {}^{i-1}_{\ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

PUMA 560 - Example 2 - Solution - T_1_2

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

PUMA 560 - Example 2 - Solution - T_2_3

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	-90°	0	0	θ_2
3	0	a_2	d_3	θ_3

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

PUMA 560 - Example 2 - Solution - T_3_4

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4

$$
{}^{i-1}_{\;\;i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

PUMA 560 - Example 2 - Solution - T_4_5

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5

$$ {}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

PUMA 560 - Example 2 - Solution - T_5_6

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	-90°	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

$$
{}^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad
{}^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
$$

PUMA 560 - Example 2 - Solution - T_1_6

- Form the ${}^{0}_{6}T$ by multiplication of the individual link matrices

$$
{}^{4}_{6}T = {}^{4}_{5}T\,{}^{5}_{6}T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{6}T = {}^{3}_{4}T\,{}^{4}_{6}T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{3}T = {}^{1}_{2}T\,{}^{2}_{3}T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

PUMA 560 - Example 2 - Solution - T_1_6

- Form the ${}^0_6T$ by multiplication of the individual link matrices

$$ {}^4_6T = {}^4_5T\,{}^5_6T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_6T = {}^3_4T\,{}^4_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_3T = {}^1_2T\,{}^2_3T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_6T = {}^1_3T\,{}^3_6T = \begin{bmatrix} {}^1r_{11} & {}^1r_{12} & {}^1r_{13} & {}^1p_x \\ {}^1r_{21} & {}^1r_{22} & {}^1r_{23} & {}^1p_y \\ {}^1r_{31} & {}^1r_{32} & {}^1r_{33} & {}^1p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

where

$$ \begin{aligned} {}^1r_{11} &= c_{23}[c_4c_5c_6 - s_4s_6] - s_{23}s_5s_6, \\ {}^1r_{21} &= -s_4c_5c_6 - c_4s_6, \\ {}^1r_{31} &= -s_{23}[c_4c_5c_6 - s_4s_6] - c_{23}s_5c_6, \\ {}^1r_{12} &= -c_{23}[c_4c_5s_6 + s_4c_6] + s_{23}s_5s_6, \\ {}^1r_{22} &= s_4c_5s_6 - c_4c_6, \\ {}^1r_{32} &= s_{23}[c_4c_5s_6 + s_4c_6] + c_{23}s_5s_6, \\ {}^1r_{13} &= -c_{23}c_4s_5 - s_{23}c_5, \\ {}^1r_{23} &= s_4s_5, \\ {}^1r_{33} &= s_{23}c_4s_5 - c_{23}c_5, \\ {}^1p_x &= a_2c_2 + a_3c_{23} - d_4s_{23}, \\ {}^1p_y &= d_3, \\ {}^1p_x &= -a_3s_{23} - a_2s_2 - d_4c_{23}. \end{aligned} $$

PUMA 560 - Example 2 - Solution - T_0_6

$$ {}^0_6T = {}^0_1T\, {}^1_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Here,

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), $$

$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6), $$

$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, $$

$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), $$

$$ r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), $$

$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, $$

$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, $$

$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, $$

$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5, $$

$$ p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1, $$

$$ p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1, $$

$$ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}. $$

PUMA 560 - Example 2 - Solution - T_0_6 - RTB

$$ {}^0_6T = {}^0_1T\, {}^1_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

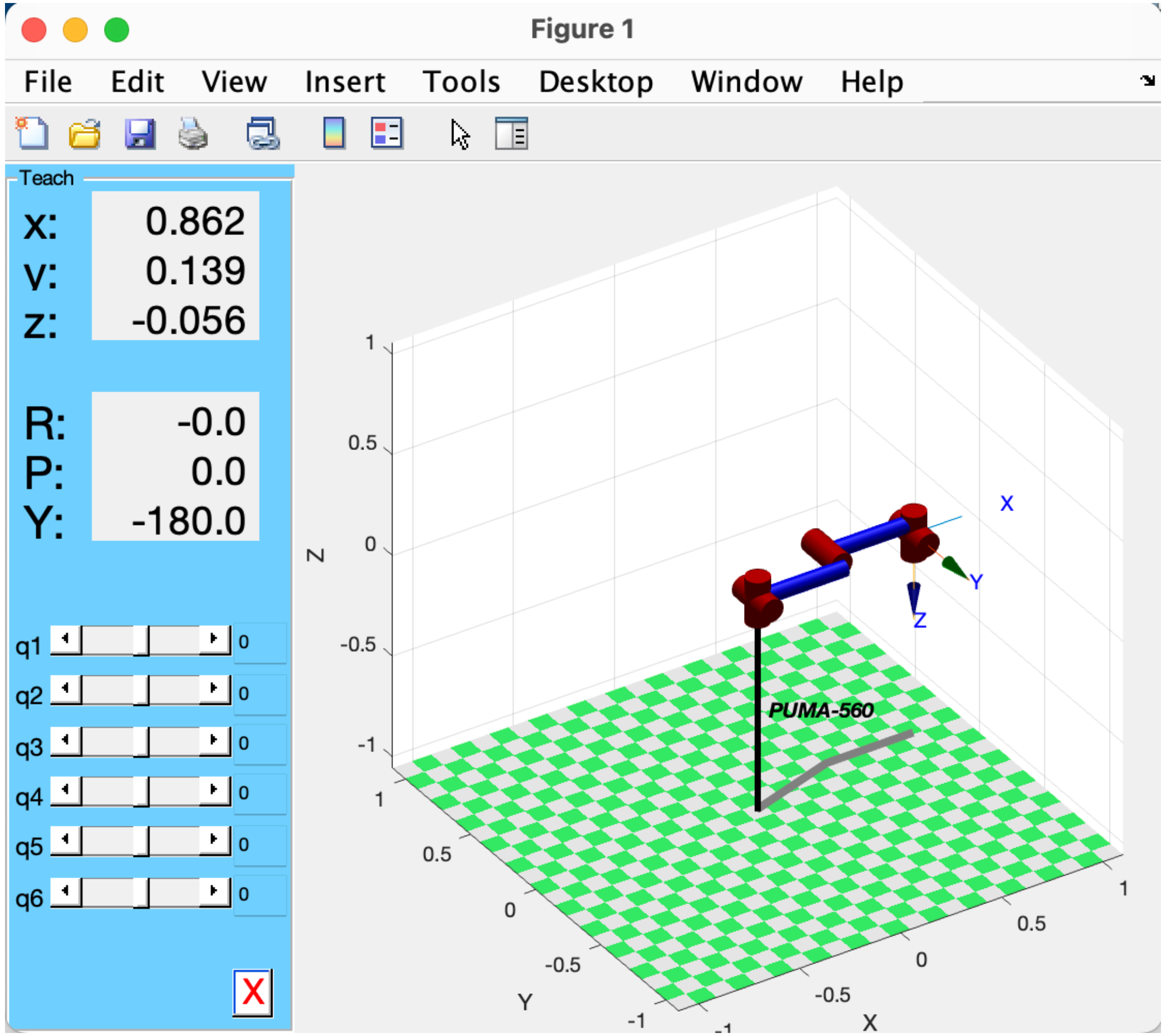
Here,

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), $$
$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6), $$
$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, $$

$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), $$
$$ r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), $$
$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, $$

$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, $$
$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, $$
$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5, $$

$$ p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1, $$
$$ p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1, $$
$$ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}. $$



… end of Lecture 6