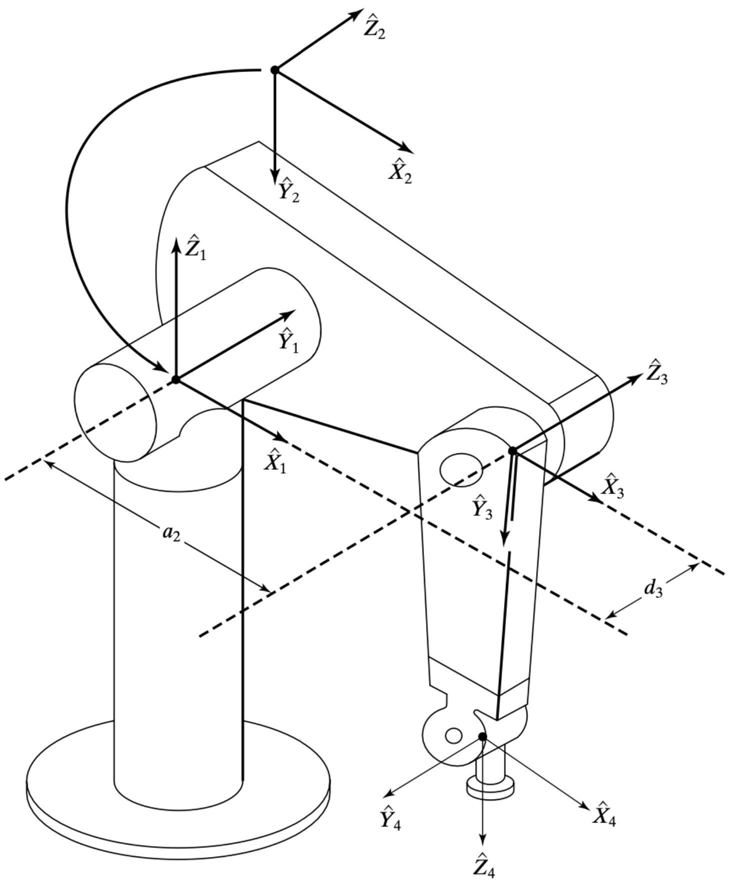
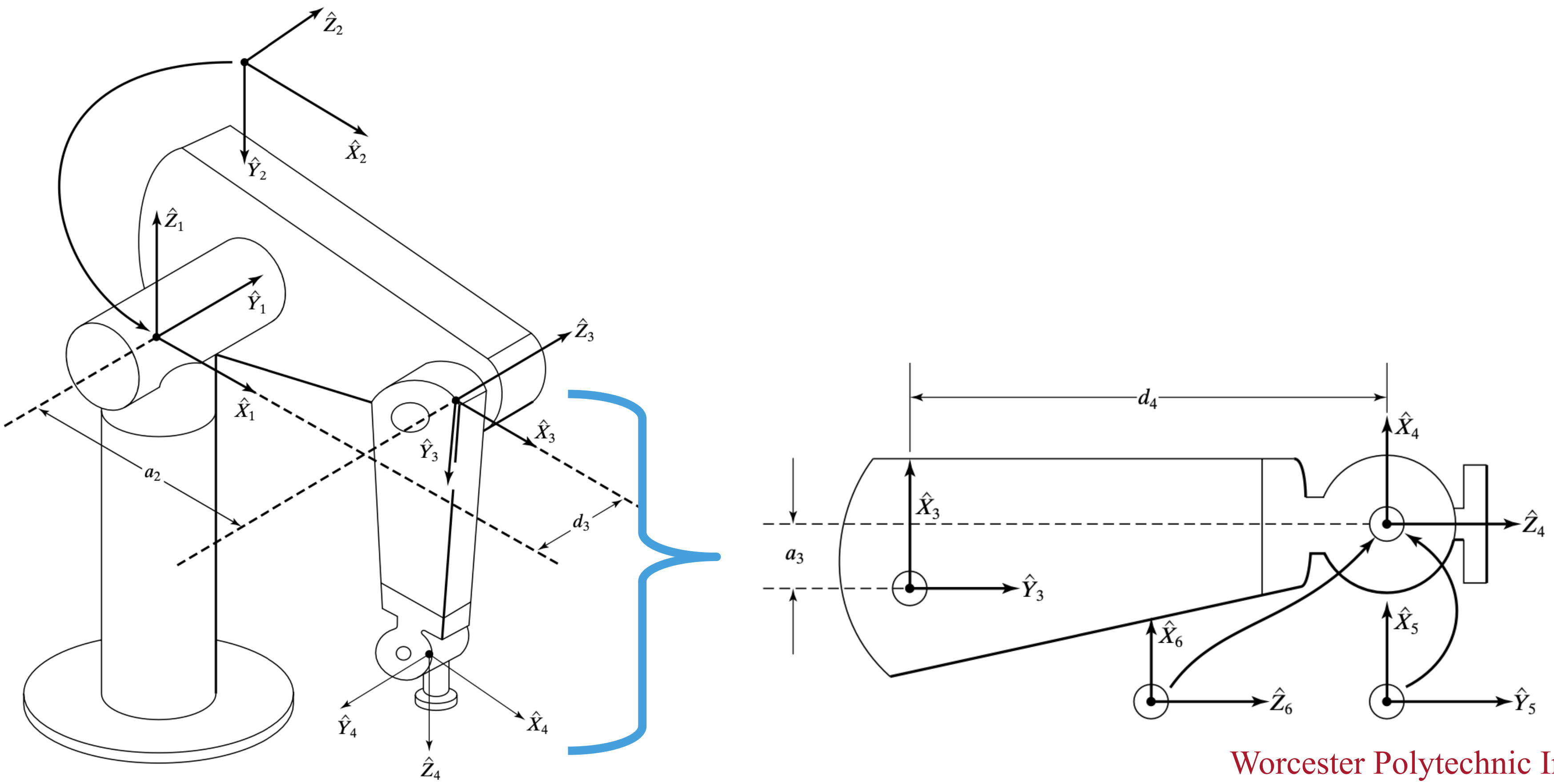
PUMA 560 - Unimate robot

- Courtesy of Unimation Incorporated
- 6 DoFs
- All rotational joints
- RRRRRR mechanism

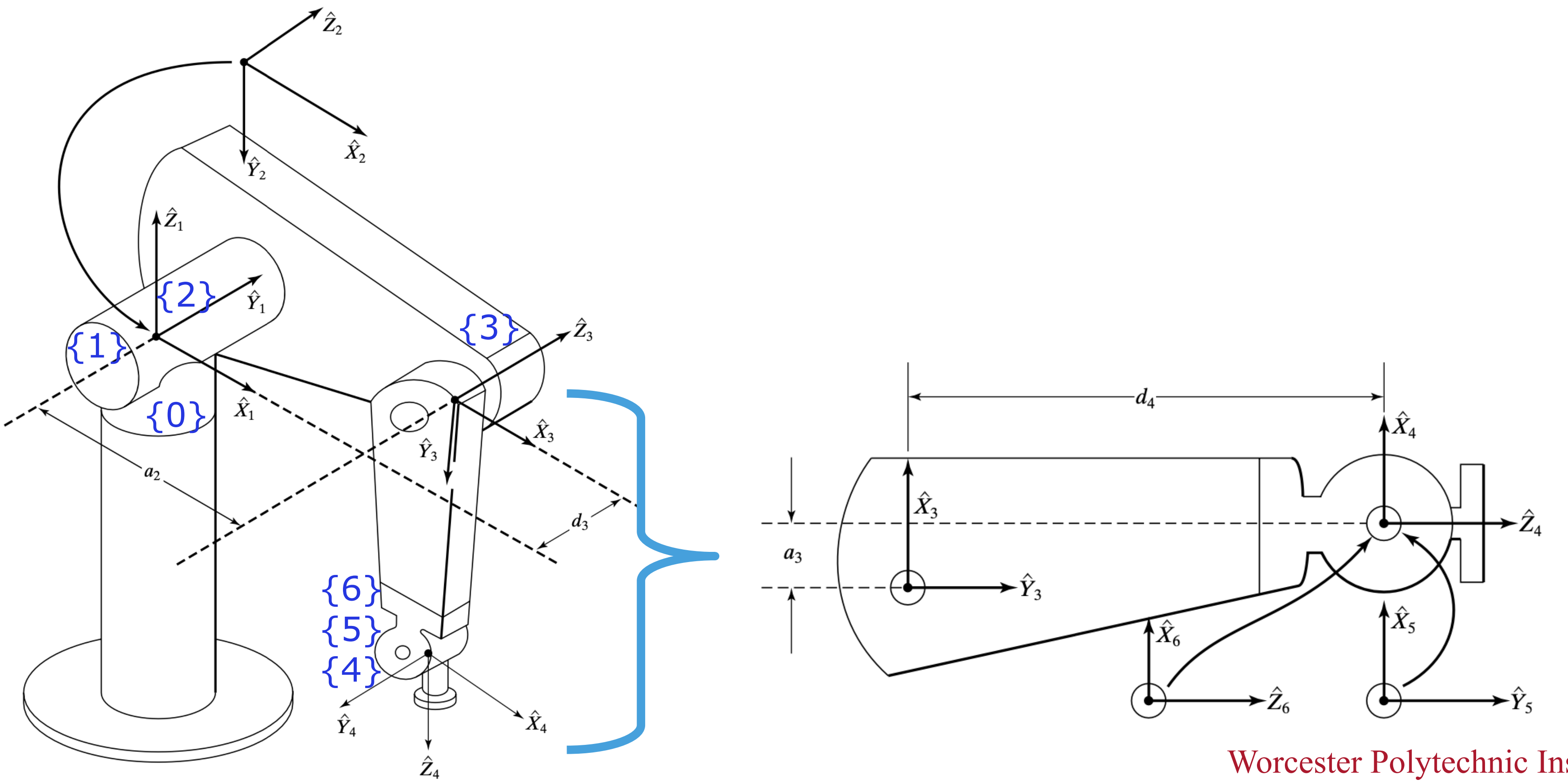
PUMA 560 - Unimate robot - Mechanical chain



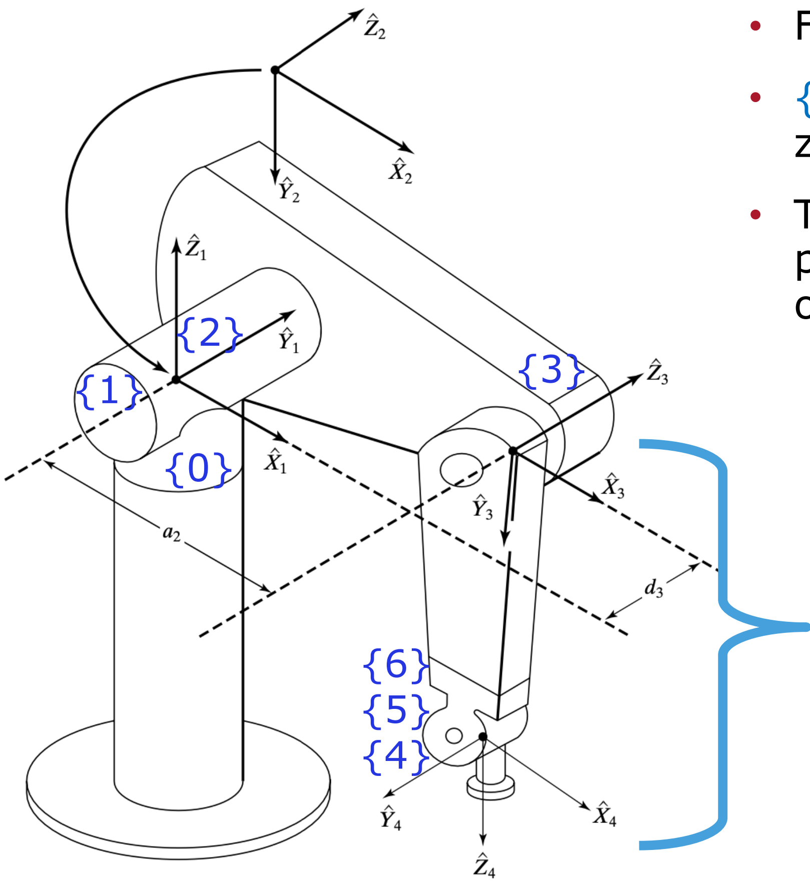
PUMA 560 - Unimate robot - Mechanical chain



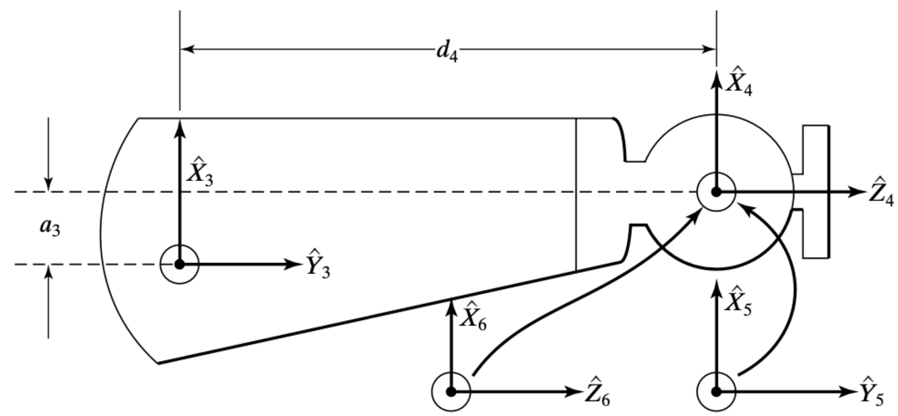
PUMA 560 - Unimate robot - Attach frames



PUMA 560 - Unimate robot - Attach frames

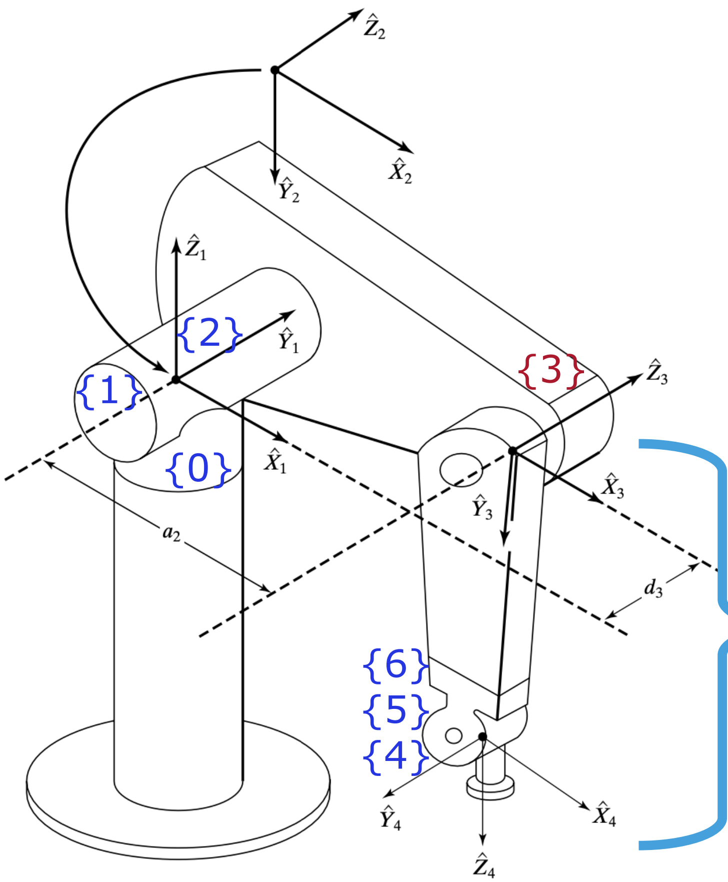


- Figure → All joints angles equal to zero
- {0} not shown - coincident with frame {1} when θ_1 is zero
- The joint axes of joints 4,5,6 all intersect at a common point and this point of intersection coincides with the origin of frames {4},{5},{6}. Also, they are orthogonal

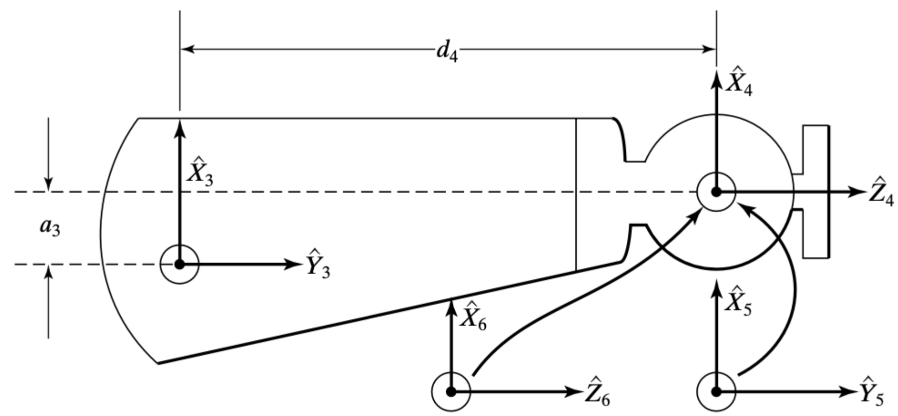


*** Unimation has used a slightly different assignment of zero location of the joints, such that $\theta_3^* = \theta_3 - 180°$, where θ_3^* is the position of joint 3 in Unimation's convention.

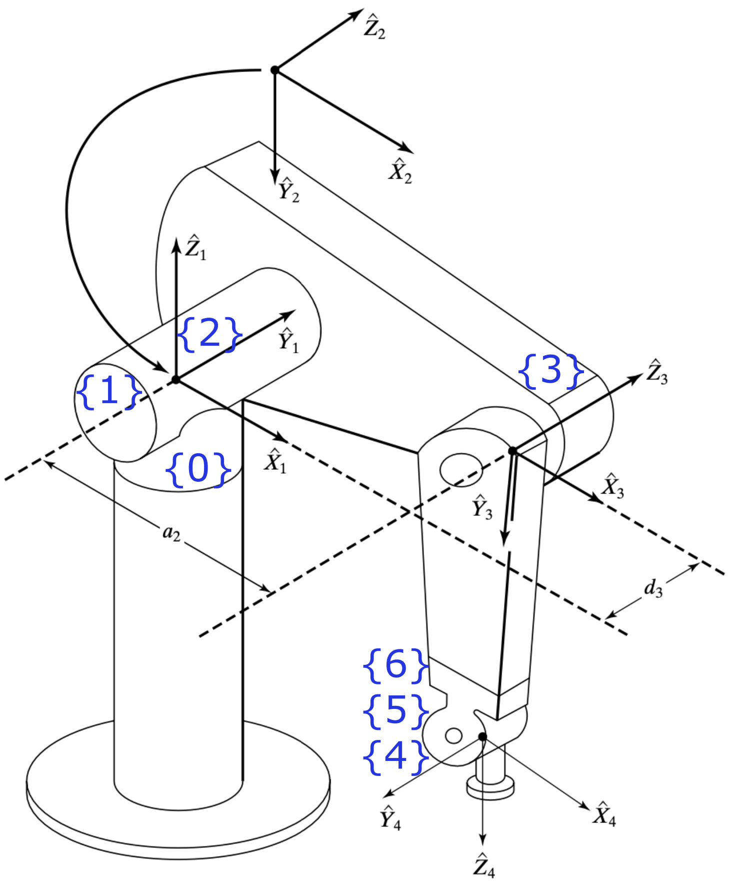
PUMA 560 - Unimate robot - Attach frames



- Figure → All joints angles equal to zero***
- {0} not shown - coincident with frame {1} when θ_1 is zero
- The joint axes of joints 4,5,6 all intersect at a common point and this point of intersection coincides with the origin of frames {4},{5},{6}. Also, they are orthogonal

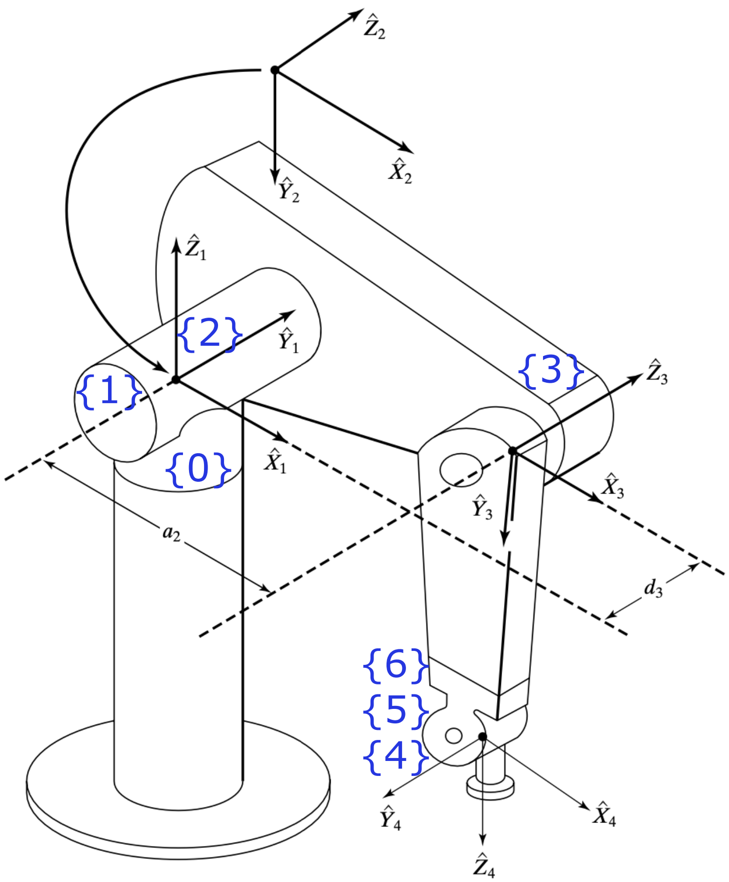


PUMA 560 - Unimate robot - DH parameters

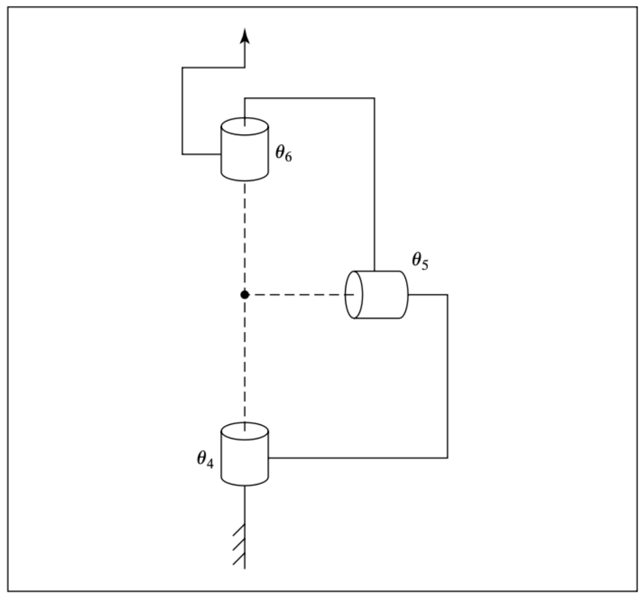


i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

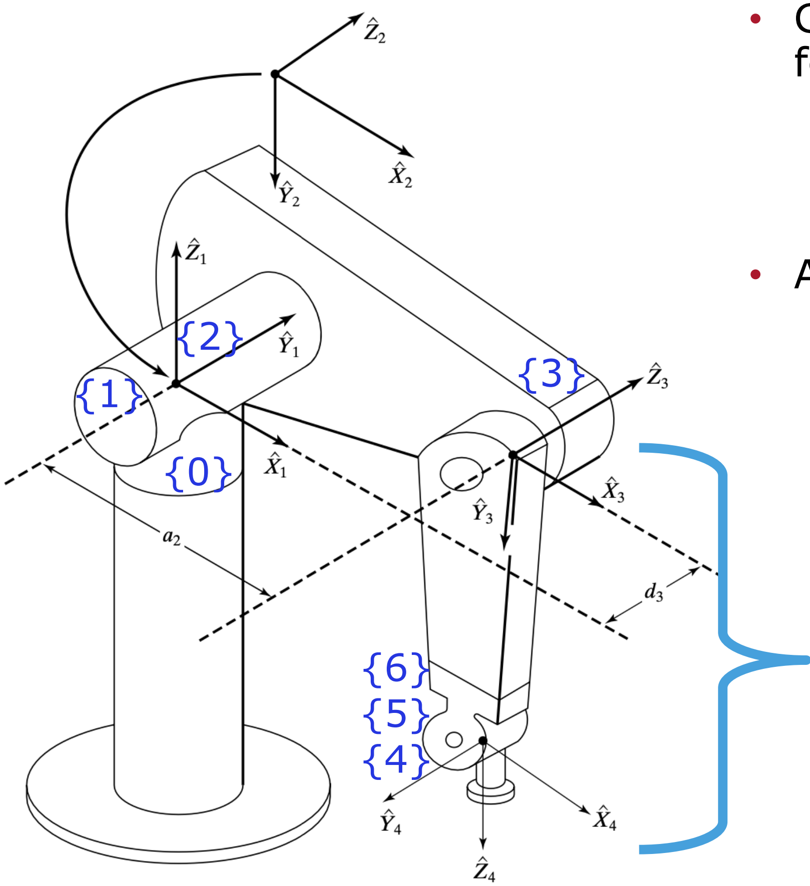
PUMA 560 - Unimate robot - DH parameters



- Gearing arrangement in the wrist couples together the motions of joints 4, 5, and 6.
- We must make a distinction between joint space and actuator space and solve the complete kinematics in two steps



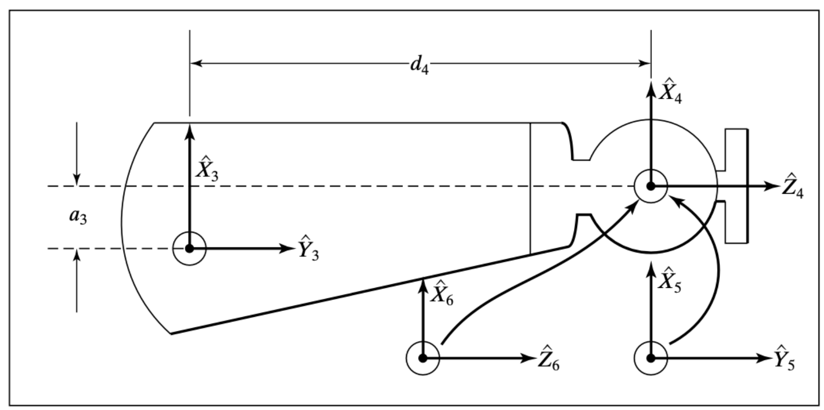
PUMA 560 - Example 2



- Given the DH parameters and the link transformations formular above, **find the transformations** below:

$$ {}^0_1T,\ {}^1_2T,\ {}^2_3T,\ {}^3_4T,\ {}^4_5T,\ {}^5_6T $$

- Also, find the: ${}^0_6T$



PUMA 560 - Example 2 - Solution

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

PUMA 560 - Example 2 - Solution - T_0_1

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1

$$
{}^{i-1}_{\ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

PUMA 560 - Example 2 - Solution - T_1_2

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2

$$
{}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

PUMA 560 - Example 2 - Solution - T_2_3

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

PUMA 560 - Example 2 - Solution - T_3_4

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^{3}_{4}T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

PUMA 560 - Example 2 - Solution - T_4_5

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5

$${}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^{3}_{4}T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad {}^{4}_{5}T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$${}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

PUMA 560 - Example 2 - Solution - T_5_6

i	$\alpha_i - 1$	$a_i - 1$	d_i	θi
1	0	0	0	θ_1
2	$-90°$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-90°$	a_3	d_4	θ_4
5	$90°$	0	0	θ_5
6	$-90°$	0	0	θ_6

$$^{i-1}_{\ \ i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0_1T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^3_4T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^1_2T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^4_5T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$^2_3T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad ^5_6T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

PUMA 560 - Example 2 - Solution - T_1_6

- Form the ${}^0_6T$ by multiplication of the individual link matrices

$$
{}^4_6T = {}^4_5T\,{}^5_6T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^3_6T = {}^3_4T\,{}^4_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1_3T = {}^1_2T\,{}^2_3T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

PUMA 560 - Example 2 - Solution - T_1_6

- Form the ${}^{0}_{6}T$ by multiplication of the individual link matrices

$$ {}^{4}_{6}T = {}^{4}_{5}T\,{}^{5}_{6}T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{6}T = {}^{3}_{4}T\,{}^{4}_{6}T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & -c_4s_5 & a_3 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{3}T = {}^{1}_{2}T\,{}^{2}_{3}T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{6}T = {}^{1}_{3}T\,{}^{3}_{6}T = \begin{bmatrix} {}^{1}r_{11} & {}^{1}r_{12} & {}^{1}r_{13} & {}^{1}p_x \\ {}^{1}r_{21} & {}^{1}r_{22} & {}^{1}r_{23} & {}^{1}p_y \\ {}^{1}r_{31} & {}^{1}r_{32} & {}^{1}r_{33} & {}^{1}p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

where

$$ \begin{aligned} {}^{1}r_{11} &= c_{23}[c_4c_5c_6 - s_4s_6] - s_{23}s_5s_6, \\ {}^{1}r_{21} &= -s_4c_5c_6 - c_4s_6, \\ {}^{1}r_{31} &= -s_{23}[c_4c_5c_6 - s_4s_6] - c_{23}s_5c_6, \\ {}^{1}r_{12} &= -c_{23}[c_4c_5s_6 + s_4c_6] + s_{23}s_5s_6, \\ {}^{1}r_{22} &= s_4c_5s_6 - c_4c_6, \\ {}^{1}r_{32} &= s_{23}[c_4c_5s_6 + s_4c_6] + c_{23}s_5s_6, \\ {}^{1}r_{13} &= -c_{23}c_4s_5 - s_{23}c_5, \\ {}^{1}r_{23} &= s_4s_5, \\ {}^{1}r_{33} &= s_{23}c_4s_5 - c_{23}c_5, \\ {}^{1}p_x &= a_2c_2 + a_3c_{23} - d_4s_{23}, \\ {}^{1}p_y &= d_3, \\ {}^{1}p_x &= -a_3s_{23} - a_2s_2 - d_4c_{23}. \end{aligned} $$

PUMA 560 - Example 2 - Solution - T_0_6

$$ {}^{0}_{6}T = {}^{0}_{1}T \; {}^{1}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Here,

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), $$

$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6), $$

$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, $$

$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), $$

$$ r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), $$

$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, $$

$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, $$

$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, $$

$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5, $$

$$ p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1, $$

$$ p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1, $$

$$ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}. $$

PUMA 560 - Example 2 - Solution - T_0_6 - RTB

$$ {}^0_6T = {}^0_1T \, {}^1_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Here,

$$ r_{11} = c_1[c_{23}(c_4c_5c_6 - s_4s_5) - s_{23}s_5c_5] + s_1(s_4c_5c_6 + c_4s_6), $$
$$ r_{21} = s_1[c_{23}(c_4c_5c_6 - s_4s_6) - s_{23}s_5c_6 - c_1(s_4c_5c_6 + c_4s_6), $$
$$ r_{31} = -s_{23}(c_4c_5c_6 - s_4s_6) - c_{23}s_5c_6, $$

$$ r_{12} = c_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] + s_1(c_4c_6 - s_4c_5s_6), $$
$$ r_{22} = s_1[c_{23}(-c_4c_5s_6 - s_4c_6) + s_{23}s_5s_6] - c_1(c_4c_6 - s_4c_5s_6), $$
$$ r_{32} = -s_{23}(-c_4c_5s_6 - s_4c_6) + c_{23}s_5s_6, $$

$$ r_{13} = -c_1(c_{23}c_4s_5 + s_{23}c_5) - s_1s_4s_5, $$
$$ r_{23} = -s_1(c_{23}c_4s_5 + s_{23}c_5) + c_1s_4s_5, $$
$$ r_{33} = s_{23}c_4s_5 - c_{23}c_5, $$

$$ p_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_3s_1, $$
$$ p_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_3c_1, $$
$$ p_z = -a_3s_{23} - a_2s_2 - d_4c_{23}. $$