NAME:_____

MATH 322 EXAM 1

- Print your name clearly in the space provided.
- You may use your textbook and class notes only.
- You may not consult with anyone other than me.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	20	
2	18	
3	18	
4	20	
5	22	
6	2	
Total:	100	

1. [20 points] Let line, point, and lie on be undefined terms and consider the following set of axioms.

   a) Every line lies on at least two points.
   b) If x, y are two points, then x, y lie on precisely one line.
   c) Any two lines share a point in common.
   d) There exists a set of four points, no three of which are collinear.

   Find a model for this geometry using exactly 7 points. Be sure to say what you mean by line, point, and lies on.

2. [18 points] Consider the following axiom: "Given a line and a point not on the line, there exists exactly one line through the given point that does not intersect the given line." Use this axiom to prove that if a line intersects (but does not equal) one of two parallel lines, then it must intersect the other one.

3. [18 points] Let ΔABC be a right Euclidean triangle with right angle at A. Let D be a line through A perpendicular to the hypotenuse BC. Prove that each of the two smaller triangles thus formed are similar to each other and to the original triangle.

4. In our definition of semi-regular tessellation, we said that any transformation should be **vertex transitive** i.e. if T is an is a transformation and V is the vertex set of a semi-regular tessellation, then $T(V) = V$ so that T sends vertices to vertices (but not necessarily edges to edges). Consider the six vertices of a hexagon (without edges). Give an example of a geometric object on those six vertices by constructing edges between them which has

   (a) [10 points] rotational and reflective vertex transitive symmetry but only reflective edge symmetry.

   (b) [10 points] rotational and reflective vertex transitive symmetry with neither rotational nor reflective edge symmetry.

5. Construct a fractal as follows. Begin with an equilateral triangle with perimeter length 1. Divide each line segment into three equal pieces, and construct the base of an equilateral triangle on the middle segment of each line segment pointing inside the triangle. Remove the base of each triangle. This completes one iteration. Repeat. We construct a fractal by taking the number of iterations to infinity (See pictures for further clarification).

   (a) [12 points] What is the perimeter after n iterations? What is the perimeter of the fractal?

   (b) [10 points] What is the Hausdorff dimension of the fractal?

6. [2 points] Draw a point.