NAME: ____________________

MATH 112 EXAM 2

October 15, 2010

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- Print your name clearly in the space provided.
- You may use a calculator.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	72	
2	10	
3	10	
4	8	
Total:	100	

You may use the following formulas (if applicable):

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\frac{d}{dx}(secx) = \sec x tanx$$

$$\int \sin^n x dx = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$\int \sin^2 x dx = \frac{x}{2} - \frac{\sin 2x}{4} + C = \frac{x}{2} - \frac{1}{2} \sin x \cos x + C$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2} \sin x \cos x + C$$

$$\int \frac{dx}{x^2+1} = \tan^{-1} x + C$$

1. [72 points] Choose any 4 of the following integrals to solve. 18 points each. You may choose up to 2 of any of the remaining 5 to solve for extra credit for 4 points each. (no partial credit on the extra credit problems-all or nothing) **Please clearly mark which problems you wish to turn in for the 4 test problems and which you would like to turn in for extra credit.**

   (a) $\int \frac{3dx}{(x+1)(x^2+x)} = \int \frac{3\,dx}{(x+1)(x)(x+1)}$

$$\frac{3}{(x+1)^2 x} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x}$$

$$3 = Ax(x+1) + Bx + C(x+1)^2$$

Let $x = 0$

$\cancel{3}$ $3 = C$

Let $x = -1$

$-3 = B$

$3 = A(2) - 3 + 3 \cdot 4$

$A = -3$

$$\int \frac{3\,dx}{(x+1)(x^2+x)} = 3\int \frac{-1}{x+1} - \frac{1}{(x+1)^2} + \frac{1}{x}\,dx$$

$$= 3\left[-\ln|x+1| + \frac{1}{x+1} + \ln|x|\right] + C$$

(b) $\int_0^4 x\sqrt{4-x}dx$

$$u = 4-x$$
$$x = 4-u$$
$$du = -dx$$

$$\int_0^4 x\sqrt{4-x}\,dx = \int_4^0 (4-u)u^{1/2}(-du) = -\int_4^0 (4u^{1/2} - u^{3/2})\,du$$

$$= 4\left(\frac{2u^{3/2}}{3} - \frac{2u^{5/2}}{5}\right)\Bigg|_0^4 = \frac{8}{3}(4^{3/2}) - \frac{2}{5}(4^{5/2})$$

$$= \frac{64}{3} - \frac{64}{5} = \frac{128}{15}$$

(c) $\int \frac{dx}{(x^2+1)^3}$

$$x = \tan\theta$$
$$dx = \sec^2\theta\,d\theta$$

$$= \int \frac{\sec^2\theta\,d\theta}{\sec^6\theta} = \int \cos^4\theta\,d\theta$$

$$= \frac{\cos^3\theta\sin\theta}{4} + \frac{3}{4}\int \cos^2\theta\,d\theta = \frac{1}{4}\cos^3\theta\sin\theta + \frac{3}{4}\left(\frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta\right)$$

$$\sin\theta = \frac{x}{\sqrt{x^2+1}} \qquad \cos\theta = \frac{1}{\sqrt{x^2+1}}$$

$$\therefore \frac{1}{4}\left(\frac{1}{\sqrt{x^2+1}}\right)^3\left(\frac{x}{\sqrt{x^2+1}}\right) + \frac{3}{8}\tan^{-1}x + \frac{3}{8}\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) + C$$

(d) $\int \frac{3x^3 \tan x + 2x^2}{x^2 \sec x} dx$

$$= \int \frac{3x^3 \tan x}{x^2 \sec x} dx + \int \frac{2x^2}{x^2 \sec x} dx$$

$$= 3\int (\sin x) x \, dx + 2\int \cos x \, dx$$

$u = x \quad dv = \sin x$
$du = dx \quad v = -\cos x$

$$= 3\left[-x\cos x + \int \cos x \, dx\right] + 2\sin x + C$$

$$= -3x\cos x + 5\sin x + C$$

(e) $\int \cos x \sin^5 x dx$

$$= \int \cos x (1-\cos^2 x)^2 \sin x \, dx$$

$u = \cos x$
$du = -\sin x \, dx$

$$= -\int u(1-u^2)^2 du = \int (-u + 2u^3 + u^5) du$$

$$= -\frac{1}{2}u^2 + \frac{1}{2}u^4 - \frac{1}{6}u^6 + C$$

$$= -\frac{1}{2}\cos^2 x + \frac{1}{2}\cos^4 x + \frac{1}{6}\cos^6 x + C$$

(f) $\int \frac{dx}{\sqrt{x^2+x}}$

$$x^2+x = x^2+x+\frac{1}{4}-\frac{1}{4} = \left(x+\frac{1}{2}\right)^2-\frac{1}{4}$$

$u = x+\frac{1}{2}$
$du = dx$

$$= \int \frac{dx}{\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}} = \int \frac{du}{\sqrt{u^2-\frac{1}{4}}}$$

$u = \frac{1}{2}\sec\theta$
$du = \frac{1}{2}\sec\theta\tan\theta\, d\theta$

$$= \int \frac{\frac{1}{2}\sec\theta\tan\theta}{1/2\tan\theta}\, d\theta = \int \sec\theta\, d\theta = \ln|\sec\theta+\tan\theta|+C$$

$$= \ln\left|2x+1+\sqrt{4(x^2+x+\tfrac{1}{4})-1}\right|+C$$

(g) $\int \frac{x^2}{(x+1)(x^2+1)}dx$

$$\frac{x^2}{(x+1)(x^2+1)} = \frac{A}{x+1}+\frac{Bx+C}{x^2+1}$$

$x^2 = A(x^2+1)+(Bx+C)(x+1)$

$A = 1/2$

$x^2 = (B+1/2)x^2+(B+C)x+(C+1/2)$

$B = 1/2$

$C = -1/2$

$$1/2\left[\int \frac{1}{x+1}+\frac{x}{x^2+1}-\frac{1}{x^2+1}\, dx\right] = \frac{1}{2}\ln|x+1|+\frac{1}{4}\ln|x^2+1|-\frac{1}{2}\tan^{-1}x+C$$

(h) $\int \frac{x^2+x+3}{(x-1)^3}dx$

$$\frac{x^2+x+3}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$x^2+x+3 = A(x-1)^2 + B(x-1) + C \qquad C=5$$

$$x^2+x+3 = Ax^2 + (B-2A)x + (A-B+5) \qquad A=1$$

$$B=3$$

$$\int \frac{1}{x-1} + \frac{3}{(x-1)^2} + \frac{5}{(x-1)^3}\,dx = \ln|x-1| - \frac{3}{x-1} - \frac{5}{2(x-1)^2} + C$$

(i) $\int x^2\sin(3x+1)dx$

$$u = x^2 \qquad dv = \sin(3x+1)$$
$$du = 2x\,dx \qquad v = -\tfrac{1}{3}\cos(3x+1)$$

$$x^2\left(-\tfrac{1}{3}\cos(3x+1)\right) - \int 2x\left(-\tfrac{1}{3}\cos(3x+1)\right)dx$$

$$u = x \qquad dv = \cos(3x+1$$
$$du = dx \qquad v = \frac{\sin(3x+1)}{3}$$

$$\tfrac{1}{3}x\sin(3x+1) - \int \sin(3x-1) \Rightarrow \tfrac{1}{3}x\sin(3x+1) + \tfrac{1}{9}\cos(3x+1) + C$$

$$\therefore\ -\tfrac{1}{3}x^2\cos(3x+1) + \tfrac{2}{9}x\sin(3x+1) + \tfrac{2}{27}\cos(3x+1) + C$$

2. [10 points] Prove or provide a counter-example. If f and g are two integrable functions, then

$$\int f(x) \cdot g(x)dx = \int f(x)dx \cdot \int g(x)dx.$$

False

$$\int x^2 dx \neq \left[\int x\, dx\right]\left[\int x\, dx\right]$$

$$\frac{x^3}{3} \neq \left[\frac{x^2}{2}\right]\left[\frac{x^2}{2}\right] = \frac{x^4}{4}$$

3. [10 points] Suppose that $\int f(x)dx = \ln x + \sqrt{x+1} + C$. Can $f(x)$ be a rational function? Explain.

No since the antiderivative of a rational can never be irrational i.e. have $\sqrt{x+1}$ term

4. [8 points] Which integral requires more work to evaluate? (You do not need to evaluate either one.)

$$\int \sin^{798}(x)\cos(x)dx \text{ or } \int \sin^{6}(x)\cos^{6}(x)dx.$$

Explain.

u-sub.