NAME: ______________________

MATH 322 EXAM 2

- Print your name clearly in the space provided.
- You may use your textbook, class notes, and one linear algebra book of your choosing.
- You may not consult with anyone other than me.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	15	
2	15	
3	15	
4	20	
5	10	
6	25	
Total:	100	

1. [15 points] Let $S = \{z \in \mathbb{C} : \mathrm{im}(z) = 17\}$. This is a line in $\mathbb{C} = \mathbb{R}^2$. If $f(z) = \frac{-1}{z}$, show that $f(S)$ is a circle tangent to $\mathbb{R}$ at the origin.

2. Let $f(z) = \frac{3z+1}{2z+4}$.
   (a) [5 points] Compute $f(1+2i)$. Write your answer in the form $a+bi$.

   (b) [10 points] Find all fixed points of the isometry f.

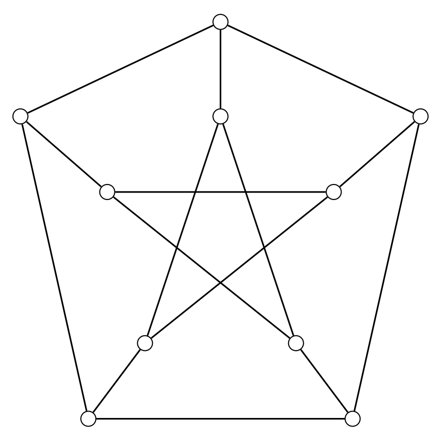
3. [15 points] Let O be the center of the Euclidean circle γ that is used to define the Poincaré disk. If P is a point in the Poincaré disk such that the Euclidean distance from O to P is r, find a formula for the Poincaré distance $d(O, P)$.

4. Let S^2 be the unit sphere. Under stereographic projection, find the subset of $\mathbb{C}$ that H is sent to for:
   (a) [10 points] $H = \{(X, Y, Z) \in S^2 : X \geq 0\}$

   (b) [10 points] $H = \{(X, Y, Z) \in S^2 : Z = \frac{1}{2}\}$

   Justify your answers.

5. [10 points] Show that the following graph is not planar.



6. We may view Möbius transformations as matrices; that is, if $H(z) = \frac{az+b}{cz+d}$, write $H(z) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Then if H and G are two Möbius transformations, their product may be computed by taking the regula matrix multiplication of H and G. In light of this, we say that two Möbius transformations G_1 and G_2 are **conjugate** if there exists a Möbius transformation T such that $G_1 = TG_2T^{-1}$.
   (a) [10 points] Show that conjugation is an equivalence relation.
   (b) [15 points] Define the **trace** of a Möbius transformation H by $\tau(H) = (a+d)^2$. Show that if G_1 and G_2 are conjugate, then $\tau(G_1) = \tau(G_2)$ [Hint: First prove that $\tau(AB) = \tau(BA)$ for any 2×2 matrices.]