NAME:________________________

MATH 322 FINAL EXAM

- Print your name clearly in the space provided.
- You may use your textbook, one linear algebra book, and class notes only.
- You may not consult with anyone other than me.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	15	
2	30	
3	10	
4	20	
5	15	
6	15	
7	10	
8	35	
Total:	150	

1. [15 points] Consider a right triangle ABC with M the midpoint of line segment $\overline{AB}$. If $AM = MC$ (that is, the length of the line segment from A to M) show that angle ACB is a right angle. Conclude that if the vertices of triangle ABC lie on a circle and $\overline{AB}$ is a diameter, then angle ACB is a right angle.

2. Consider the following set of axioms:

   1. There exists at least one line.
   2. There are exactly three points on every line.
   3. Not all points are on the same line.
   4. There is exactly one line on any two distinct points.
   5. For each line ℓ and each point P not on ℓ, there exists exactly one line on P which is not on any point of ℓ.

   Show that

   (a) [15 points] For every point, there is a line not on that point.

   (b) [15 points] For every point, there are at least four lines on that point.

3. [10 points] Compute the rotation index of the following figure:

4. [20 points] A (filled in) triangle whose vertices are elements of $\mathbb{Z} \times \mathbb{Z}$ (called integral vertices) and which does not contain any other points of $\mathbb{Z} \times \mathbb{Z}$ is called **elementary**. Using linear algebra, it can be shown that the area of such a triangle is always $\frac{1}{2}$. Use this fact and Euler's theorem to deduce the following Theorem:

   The area of any polygon P in $\mathbb{R}^2$ with integral vertices is given by $A = v_i + \frac{1}{2}v_b - 1$ where v_i is the number of integral points on the interior of P and v_b is the number of integral points on the boundary of P.

   Hints: Write $v = v_i + v_b$ and $e = e_i + e_b$ for the number of vertices on the boundary and interior, respectively, and edges of the boundary and interior, respectively. Triangulate the polygon using the lattice points. Find a formula for the area in terms of the number of faces, and then find a formula for the number of faces in terms of e_i and e_b. Use algebra and Euler's theorem to prove the result.

5. [15 points] What is the largest possible angle sum for a triangle in spherical geometry? Justify and illustrate your answer.

6. [15 points] Explain why the concept of similar triangles (with the same angles but different side lengths) does not make sense in spherical geometry.

7. [10 points] Let $\vec{u}$ be a unit vector in $\mathbb{R}^3$ and p any point in $\mathbb{R}^3$. Using the matrix formula for a rotation of p about $\vec{u}$, show that if $\theta = 2n\pi$ where $n \in \mathbb{Z}$, then the image of p under the rotation is p. For extra credit, either prove or give a counterexample to the converse.

8. Consider a rectangular box with unitless dimensions 2×2 on each end and 2×4 on the top, bottom, and sides.
   (a) [10 points] Find the shortest path on this surface from the center of one end to the center of the other end.
   (b) [10 points] Find the shortest path on this surface from one corner to the opposite corner.
   (c) [15 points] Find the shortest path on this surface from the point A halfway up one edge to the point B halfway up the opposite edge.