NAME:_____

MATH 112 EXAM 1

September 22, 2010

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- Print your name clearly in the space provided.
- You may use a calculator.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	48	
2	12	
3	42	
Total:	102	

You may use the following formulas (if applicable):

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{x^2+1} = \tan^{-1} x + C$$

$$\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1} x + C$$

1. Compute the following integrals.

(a) [12 points] $\int \frac{\cos(\ln x)}{x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int \cos u \, du = \sin(u) + C = \sin(\ln x) + C$

(b) [12 points] $\int \frac{dx}{x\sqrt{4x^2-1}}$

$u = 2x$

$du = 2dx$

$\frac{1}{2} \int \frac{du}{\frac{u}{2}\sqrt{u^2-1}} = \sec^{-1} u + C$

$= \sec^{-1}(2x) + C$

(c) [12 points] $\int_0^2 \frac{dt}{4t+12}$ $u = 4t+12$

$du = 4\,dt$

$$\frac{1}{4}\int_{12}^{20} \frac{du}{u} = \frac{1}{4} \ln u \Big|_{12}^{20} = \frac{1}{4}\ln(20) - \frac{1}{4}\ln(12)$$

$$= \frac{\ln\left(\frac{20}{12}\right)}{4} = \frac{\ln(5/3)}{4}$$

(d) [12 points] $\int \frac{(x^2-1)e^{x^2-2x}}{x+1} dx$

$u = x^2-2x$

$du = 2x-2\,dx$

$$\int \frac{(x+1)(x-1)\,e^{x^2-2x}}{(x+1)}\,dx$$

$$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2-2x} + C$$

2. The isotope Thorium-234 has a half-life of 24.5 days.

   (a) [6 points] Find the equation which models the amount $y(t)$ of Thorium-234 at time t.

$$P(t) = P_0 e^{kt}$$

$$\frac{1}{2} = e^{k24.5}$$

$$\ln\left(\frac{1}{2}\right) = 24.5k$$

$$-.028 = \frac{\ln(1/2)}{24.5} = k$$

   (b) [6 points] At $t = 0$, a sample contains 2 kg of Thorium-234. How much remains after 365 days?

$$2e^{(\ln(1/2)/24.5)\,365} \approx 7.29 \times 10^{-5}$$

3. Find the volume of the solid obtained by rotating the region enclosed by the curves about the given axis.

   (a) [14 points] $y = 2x, y = 0, x = 8$ about $x - axis$

$$\pi \int_0^8 (2x)^2 \, dx = \pi \int_0^8 4x^2 \, dx = \pi \left[\frac{4x^3}{3} \right]_0^8$$

$$= \frac{\pi \, 4 \cdot 512}{3}$$

$$= \frac{2048}{3} \pi$$

(b) [14 points] $y = -x^2 + 4x - 3, y = 0$ about $y = -1$

$$\pi \int_1^3 (-x^2+4x-3)^2 - 1 \; dx = \frac{56\pi}{15}$$

(c) [14 points] $y^2 = 4x, y = x$ about $y = 8$

$$\pi \int_0^4 \left((8-x)^2 - (8-\sqrt{4x})^2 \right) dx = 32\pi$$