1. Express each of the following statements as a conditional statement in "if-then" form or as a universally quantified statement.

   a) Every odd number is prime.

   b) The sum of the angles of a triangle is 180 degrees.

   c) Passing the test requires solving all of the problems.

   d) Lockers must be turned in by the last day of class.

   e) Haste makes waste.

2. Prove that if $0 < a < b$, then $a^2 < ab < b^2$ and $0 < \sqrt{a} < \sqrt{b}$.

3. Using a truth table, show that an implication is logically equivalent to the negation of the converse.

4. Let $P(x)$ be the assertion "x is odd", and let $Q(x)$ be the assertion "x is twice an integer." Determine whether the following statements are true:

   a) $(\forall x \in \mathbb{Z})[P(x) \implies Q(x)]$.

   b) $(\forall x \in \mathbb{Z})[Q(x) \implies P(x)]$.

5. Show that the following statement is false: "If a and b are integers, then there are integers m, n such that $a = m + n$ and $b = m - n$." What can be added to the hypothesis of the statement to make it true?

6. The statement below is not always true for $x, y \in \mathbb{R}$. Give an example where it is false, and add a hypothesis on y that makes it a true statement.
   "If x and y are nonzero real numbers and $x > y$, then $(-1/x) > (-1/y)$."

7. Prove that if x and y are distinct real numbers, then $(x+1)^2 = (y+1)^2$ if and only if $x + y = -2$. How does the conclusion change if we allow $x = y$?

8. Two opposite squares corner squares are deleted from an eight by eight checkerboard. Prove that the remaining squares cannot be covered exactly by dominoes (rectangles consisting of two adjacent squares). [Hint: Use proof by contradiction.]

9. Prove that the statements $P \Longrightarrow Q$ and $Q \Longrightarrow R$ imply $P \Longrightarrow R$. This property of implication is called *transitivity*.