NAME: ______________________________

MATH 112 EXAM 3

December 8, 2010

INSTRUCTIONS: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- Print your name clearly in the space provided.
- You may use a calculator.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	10	
2	15	
3	10	
4	12	
5	12	
6	21	
7	20	
8	0	
Total:	100	

You may use

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \dots \text{ for} |x| < 1.$$

1. [10 points] Choose either (a) or (b).

   (a) Determine whether or not $\sum_{n=2}^{\infty} \frac{1}{n^2-\sqrt{n}}$ converges.

   (b) Determine whether or not $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges.

2. [15 points] Use the fact that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ to find the exact value of $\sum_{n=2}^{\infty} \left(2^{n+1} + \frac{3^n}{n^2}\right) \frac{1}{3^n}$.

3. [10 points] Use the ratio test to determine whether or not $\sum_{n=1}^{\infty} n! e^{-n^2}$ converges.

4. [12 points] Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(n!)^3}{(3n)!} x^n$.

5. [12 points] Find the first three terms of the Maclaurin series for $f(x) = e^x \cos(x)$.

6. (a) [5 points] Find the Maclaurin series for $\frac{1}{1+x}$.

(b) [10 points] Find the Maclaurin series for $\ln(1+x)$.

(c) [6 points] Use your answer in (b) to show that

$$\ln\frac{3}{2} = \frac{1}{2} - \frac{1}{2 \cdot 2^2} + \frac{1}{3 \cdot 2^3} - \frac{1}{4 \cdot 2^4} + \dots.$$

7. [20 points] Label each statement as true or false (no ambiguous letters that look like both a T and an F please). Below, a_n and b_n are sequences.

(a) ____ If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ diverges.

(b) ____ If $a_n \le b_n$ for all n, then $\sum_{n=0}^{\infty} a_n \le \sum_{n=0}^{\infty} b_n$

(c) ____ A convergent sequence is monotonic.

(d) ____ The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for $p > 1$ and diverges otherwise.

(e) ____ If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

(f) ____ The series $\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln n}$ converges absolutely.

(g) ____ $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$

(h) ____ The ratio test can be used to determine whether $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges.

(i) ____ If a_n and b_n are positive monotone-decreasing to 0, then $\sum_{n=1}^{\infty} a_n + b_n = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$.

(j) ____ If $\sum_{n=1}^{\infty} a_n 6^n$ is convergent, then $\sum_{n=1}^{\infty} a_n (-6)^n$ is convergent.

8. (Extra Credit) Show that $e^{\pi i} + 1 = 0$ where $i = \sqrt{-1}$. [Hint: Use Maclaurin series]