TRIUMPHS PSPs Available for Classroom Use (Spring 2022)

Descriptions of all PSPs available at: http://webpages.ursinus.edu/nscoville/TRIUMPHS project descriptions.pdf

Complete versions of most of the PSPs listed below are available at: https://digitalcommons.ursinus.edu/triumphs/ The *Notes to Instructors* section at the end of each PSP includes further information about its goals and design.

To obtain a preliminary copy of any PSP not yet posted on the TRIUMPHS website, contact: <u>janet.barnett@csupueblo.edu</u>

* indicates a PSP that is suitable for use in History of Mathematics Courses and/or Capstone Courses for Pre-service Secondary Teachers.

| Full-PSPs (numbers correlate with posted PSP Descriptions) | Intended Course(s) | Author |
|---|---|--------------------------------|
| F 01. A Genetic Context for Understanding the Trigonometric Functions (Also available as six independent mini-versions; see M 41–M 46.) | Pre-calculus /Trigonometry* | Danny Otero |
| F 02. Determining the Determinant | Linear Algebra | Danny Otero |
| F 03. Solving a System of Linear Equations Using the Ancient Chinese Methods | Linear Algebra * | Mary Flagg |
| F 04. Investigating Difference Equations | Discrete Mathematics | Dave Ruch |
| F 05. Quantifying Certainty: the p-value | Statistics | Dominic Klyve |
| F 06. Pythagorean Theorem and Exigency of Parallel Postulate | Geometry * | Jerry Lodder |
| F 07. Failure of the Parallel Postulate | Geometry | Jerry Lodder |
| F 08. Dedekind and the Creation of Ideals | Abstract Algebra | Janet Barnett |
| F 09. Primes, Divisibility & Factoring | Number Theory * | Dominic Klyve |
| F 10. The Pell Equation in Indian Mathematics | Number Theory * | Toke Knudsen & Keith Jones |
| F 11. Greatest Common Divisor: Algorithm and Proof | Introduction to Proof / Number Theory/ Discrete Mathematics /Abstract Algebra | Mary Flagg |
| F 12. The Möbius Function and Möbius Inversion | Number Theory | Carl Lienert |
| F 13. Bolzano on Continuity and the Intermediate Value Theorem | Introductory Analysis | Dave Ruch |
| F 14. Rigorous Debates over Debatable Rigor in Analysis: Monster Functions in Introductory Analysis | Introductory Analysis | Janet Barnett |
| F 15. An Introduction to Algebra and Geometry in the Complex Plane | Complex Variables | Diana White & Nick Scoville |
| F 16. Nearness without Distance | Topology | Nick Scoville |
| F 17. Connectedness - Its Evolution and Applications | Topology | Nick Scoville |
| F 18. Construction of Figurate Numbers | General Education * | Jerry Lodder |
| F 19. Pascal's Triangle and Mathematical Induction | General Education * | Jerry Lodder |

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| F 20. The French Connection: Borda, Condorcet and the Mathematics of Voting Theory | Math for Liberal Arts / General Education * | Janet Barnett |
| F 21. An Introduction to a Rigorous Definition of Derivative | Introductory Analysis | Dave Ruch |
| F 22. Investigations into Bolzano's Bounded Set Theorem | Introductory Analysis | Dave Ruch |
| F 23. The Mean Value Theorem | Introductory Analysis | Dave Ruch |
| F 24. Abel and Cauchy on a Rigorous Approach to Infinite Series | Introductory Analysis | Dave Ruch |
| F 25. The Definite Integrals of Cauchy and Riemann | Introductory Analysis | Dave Ruch |
| F 26. Gaussian Integers and Dedekind Ideals: A Number Theory Project | Number Theory * | Janet Barnett |
| F 27. Otto Hölder's Formal Christening of the Quotient Group Concept | Abstract Algebra | Janet Barnett |
| F 28. Roots of Early Group Theory in the Works of Lagrange | Abstract Algebra | Janet Barnett |
| F 29. Radius of Curvature According to Christiaan Huygens | Vector Calculus | Jerry Lodder |
| F 31. Cross Cultural Comparisons: The Art of Computing the Greatest Common Divisor | Elementary Education Courses | Mary Flagg |
| F 32. A Look at Desargues' Theorem from Dual Perspectives | Geometry / Introduction to Proof | Carl Lienert |
| F 33. Solving Equations and Completing the Square: From the Roots of Algebra (Also available in a mini-version; see M 28.) | Pre-calculus * | Danny Otero |
| F 34. Argand's Development of the Complex Plane | Complex Variables * | Diana White & Nick Scoville |
| F 35. Riemann's Development of the Cauchy-Riemann Equations | Complex Variables | Dave Ruch |
| F 36. Gauss and Cauchy on Complex Integration | Complex Variables | Dave Ruch |
| F 37. Representing and Interpreting Data with Playfair (Also available as three independent mini-versions; see M 31–M 33.) | Statistics * | Diana White, River Bond, Joshua Eastes & Negar Janani |
| F 38. Runge-Kutta 4 (and Other Numerical Methods for ODE's) | Differential Equations | Adam Parker |
| F 39. Stitching Dedekind Cuts to Construct the Real Numbers | Introductory Analysis | Michael Saclolo |
| F 40. The Fermat-Torricelli Point and Cauchy's Method of Gradient Descent | Multivariable Calculus | Kenneth Monks |
| F 41. Stained Glass, Windmills and the Edge of the Universe: An Exploration of Green's Theorem | Multivariable Calculus | Abe Edwards |
| F 42. Finding Exact Sums of Infinite Series (Available in two versions, a shorter version more suitable for Calculus 2 and a two-week version suitable for a capstone course.) | Calculus 2; Capstone/enrichment courses * | Danny Otero & James Sellers |
| F 43. Deciphering the Calculations on Some Old Babylonian Tablets | Elementary Education * | Zoë Misiewicz |
| F 44: Fourier's Heat Equations and the Birth of Fourier Series (Also available in a mini-version; see M 40.) | Differential Equations / Multivariable Calculus | Kenneth Monks |

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| Mini-PSPs (numbers correlate with posted PSP Descriptions) | Intended Course(s) | Author |
|---|--|---------------|
| M 01.Babylonian Numeration | General Education / Elementary Education Courses * | Dominic Klyve |
| M 02. L'Hôpital's Rule | Calculus * | Danny Otero |
| M 03. Derivatives of The Sine and Cosine Function | Calculus I * | Dominic Klyve |
| M 04. Beyond Riemann Sums | Calculus I * | Dominic Klyve |
| M 05. Fermat's Method for Finding Maxima and Minima | Calculus 1 * | Kenneth Monks |
| M 06. Euler's Calculation of the Sum of the Reciprocals of Squares | Calculus 2 * | Kenneth Monks |
| M 07. Braess' Paradox in City Planning: An Application of Multivariable Optimization | Multivariable Calculus | Kenneth Monks |
| M 08. The Origin of the Prime Number Theorem | Number Theory | Dominic Klyve |
| M 09. How to Calculate π: Machin's Inverse Tangents | Calculus 2 * | Dominic Klyve |
| M 10. How to calculate π: Buffon's Needle (Two versions available, one with no calculus pre-requisite.) | Calculus 2 ; Precalculus / Courses for Middle School Teachers* | Dominic Klyve |
| M 11. Bhāskara's Approximation and Mādhava's Infinite Series for Sine | Calculus 2 * | Kenneth Monks |
| M 12. Fourier's Proof of the Irrationality of <i>e</i> | Calculus 2 * | Kenneth Monks |
| M 13. Gaussian Guesswork: Elliptic Integrals and Integration by Substitution | Calculus 2 | Janet Barnett |
| M 14. Gaussian Guesswork: Polar Coordinates, Arc Length and the Lemniscate Curve | Calculus 2 | Janet Barnett |
| M 15. Gaussian Guesswork: Sequences & the Arithmetic-Geometric Mean | Calculus 2 | Janet Barnett |
| M 16. The logarithm of –1 | Complex Variables | Dominic Klyve |
| M 17.Why be so critical? Origins of Analysis in 19th Century Mathematics | Introductory Analysis * | Janet Barnett |
| M 18. Topology from Analysis: Making the Connection | Topology / Introductory Analysis | Nick Scoville |
| M 19. Connecting Connectedness | Topology | Nick Scoville |
| M 20. The Cantor Set before Cantor | Topology | Nick Scoville |
| M 21. A Compact Introduction to a Generalized Extreme Value Theorem | Topology | Nick Scoville |
| M 22. From Sets to Metric Spaces to Topological Spaces | Topology | Nick Scoville |
| M 23. The Closure Operation as the Foundation of Topology | Topology | Nick Scoville |
| M 24. Euler's Rediscovery of <i>e</i> | Introductory Analysis / Calculus 2 | Dave Ruch |
| M 25. Henri Lebesgue and the Integral Concept | Introductory Analysis | Janet Barnett |

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| M 26. Generating Pythagorean Triples via Gnomons (Two versions available, one more open-ended.) | Number Theory / Elementary Education Courses | Janet Barnett |
| M 27. Seeing and Understanding Data | Statistics /General Education / Elementary Education Courses * | Beverly Wood & Charlotte Bolch |
| M 28. Completing the Square: From the Roots of Algebra | Pre-calculus * | Danny Otero |
| M 29. Euler's Square Root Laws for Negative Numbers | Complex Variables * | Dave Ruch |
| M 30. Investigations Into d'Alembert's Definition of Limit (Two versions available, one lower-division, one upper-division.) | Calculus 2 / Introductory Analysis | Dave Ruch |
| M 31. Playfair's Introduction of Bar and Pie Charts to Represent Data | Statistics * | Diana White, River Bond, Joshua Eastes & Negar Janani |
| M 32. Playfair's Introduction of Time Series to Represent Data | Statistics * | Diana White, River Bond, Joshua Eastes & Negar Janani |
| M 33. Playfair's Novel Visual Displays of Data | Statistics * | Diana White, River Bond, Joshua Eastes & Negar Janani |
| M 34. Regression to the Mean | Statistics * | Dominic Klyve |
| M 35. Solving Linear First Order Differential Equations: Gottfried Leibniz' Change of Variables | Differential Equations | Adam Parker |
| M 36. Solving Linear First Order Differential Equations: Johann Bernoulli's Variation of Parameters | Differential Equations | Adam Parker |
| M 37. Solving Linear First Order Differential Equations: Leonard Euler's Integrating Factor | Differential Equations | Adam Parker |
| M 38. Wronskians and Linear Independence: A Theorem Misunderstood by Many | Differential Equations | Adam Parker |
| M 39. Leonhard Euler and Johann Bernoulli on Solving Higher Order Differential Equations with Constant Coefficients | Differential Equations | Adam Parker |
| M 40. Fourier's Heat Equation and the Birth of Modern Climate Science (Also available in a full-length version; see F 44.) | Differential Equations / Multivariable Calculus | Kenneth Monks |
| M 41. The Trigonometric Functions Through Their Origins: Babylonian Astronomy and Sexagesimal Numeration (See also F 01.) | Pre-calculus /Trigonometry * | Danny Otero |
| M 42. The Trigonometric Functions Through Their Origins: Hipparchus' Table of Chords (See also F 01.) | Pre-calculus /Trigonometry * | Danny Otero |
| M 43. The Trigonometric Functions Through Their Origins: Ptolemy Finds High Noon in Chords of Circles (See also F 01.) | Pre-calculus /Trigonometry * | Danny Otero |
| M 44. The Trigonometric Functions Through Their Origins: Varāhamihira and the Poetry of Sines (See also F 01.) | Pre-calculus /Trigonometry * | Danny Otero |
| M 45. The Trigonometric Functions Through Their Origins: al-Bīrūnī Does Trigonometry in the Shadows (See also F 01.) | Pre-calculus /Trigonometry * | Danny Otero |
| M 46. The Trigonometric Functions Through Their Origins: Regiomontanus and the Beginnings of Modern (See also F 01.) | Pre-calculus /Trigonometry * | Danny Otero |

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