NAME: ______________________________

MATH 236 EXAM 1

- Print your name clearly in the space provided.
- You may use your textbook and class notes only.
- You may not consult with anyone other than me.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

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Signature

Question	Points	Score
1	14	
2	20	
3	16	
4	8	
5	16	
6	12	
7	14	
Total:	100	

1. [14 points] Show that $P \Rightarrow [Q \wedge R]$ if and only if $(P \Rightarrow Q) \wedge (P \Rightarrow R)$.

   Solution 1 *The following truth table proves the assertion:*

P	Q	R	$Q \wedge R$	$P \Rightarrow Q$	$P \Rightarrow R$	$P \Rightarrow Q \wedge R$	$[P \Rightarrow Q] \wedge [P \Rightarrow R]$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	T	T	T	T
F	T	T	F	T	F	F	T
T	F	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	F	T	F	T	F	T

2. [20 points] Prove that an integer n is odd if and only if n^3 is odd.

   Solution 2 ($\Rightarrow$) *Let n be odd. Then $n = 2k + 1$ for some $k \in \mathbb{Z}$. We wish to show that n^3 is odd. We have*

$$n^3 = (2k + 1)^3 = 8k^3 + 12k^2 + 6k + 1 = 2(4k^3 + 6k^2 + 3k) + 1.$$

   Since $k \in \mathbb{Z}$, $4k^3 + 6k^2 + 3k \in \mathbb{Z}$ so that n^3 is odd.

   ($\Leftarrow$) *We prove the contrapositive; that is, we show that if n is even, then n^3 is even. Let $n = 2k$ for some $k \in \mathbb{Z}$. We show that n^3 is also odd. We have*

$$n^3 = (2k)^3 = 8k^3 = 2(4k^3).$$

   Since $k \in \mathbb{Z}$, $4k^3 \in \mathbb{Z}$ so that n^3 is even. Thus if n^3 is odd, then n is odd.

3. [16 points] Prove that if $x + y$ is irrational, then either x is irrational or y is irrational.

   Solution 3 *We proceed by proving the contrapositive; that is, if x and y are both rational, then $x + y$ is rational. Write $x = \frac{a}{b}, y = \frac{c}{d}$ in lowest terms with $a, b, c, d \in \mathbb{Z}$, $b, d \neq 0$. Then $x + y = \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ which is a rational number since $ad + bc, bd \in \mathbb{Z}$ and $bd \neq 0$ since $b, d \neq 0$. Hence if $x + y$ is irrational, then either x is irrational or y is irrational.*

4. [8 points] Write out "if-then" statement(s) that need to be proved in order to prove the following:

   A map of CW complexes $f\colon X \to Y$ is a homotopy equivalence if and only if $f_*\colon \pi_i(X) \to \pi_i(Y)$ is a group isomorphism for all i.

   Solution 4 *If a map of CW complexes $f\colon X \to Y$ is a homotopy equivalence, then $f_*\colon \pi_i(X) \to \pi_i(Y)$ is a group isomorphism for all i.*

   If $f_*\colon \pi_i(X) \to \pi_i(Y)$ is a group isomorphism for all i, then a map of CW complexes $f\colon X \to Y$ is a homotopy equivalence.

5. [16 points] Which of the following is equal to the set $\{x \in \mathbb{R} : x(x+2)^2(x-\frac{3}{2}) = 0\}$?

   (a) $\{-2, 0, 3\}$

   (b) $\{\frac{3}{2}, -2, 0\}$

   (c) $\{-2, -2, 0, \frac{3}{2}\}$

   (d) $\{-2, 3\}$

   Justify your answers.

   Solution 5 *Let $a \in A = \{x \in \mathbb{R} : x(x+2)^2(x-\frac{3}{2}) = 0\}$. Then $a(a+2)^2(a-\frac{3}{2}) = 0$ and hence either $a = 0, a+2 = 0$ or $(a-\frac{3}{2}) = 0$. This implies that either $a = 0, a = -2$, or $a = \frac{3}{2}$. Thus both the sets $\{\frac{3}{2}, -2, 0\}$ and $\{-2, -2, 0, \frac{3}{2}\}$ are contained in A. The other direction is similar.*

6. [12 points] Prove or give a counterexample:

   If $A \cup C \subseteq B \cup C$, then $A \subseteq B$.

   Solution 6 *See Homework number 2 solution.*

7. [14 points] Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

   Solution 7 *Let $x \in A \cap (B \cup C)$. We have*
   $\Rightarrow x \in A$ and [$x \in B$ or $x \in C$]
   $\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \in B$
   $\Rightarrow x \in (A \cap B) \cup (A \cap C)$.

   Thus $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

   To show the other direction, let $x \in (A \cap B) \cup (A \cap C)$. Then $\Rightarrow x \in (A \cap B) \cup (A \cap C)$
   $\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \in B$
   $\Rightarrow x \in A$ and [$x \in B$ or $x \in C$]
   $\Rightarrow x \in A \cap (B \cup C)$.

   We conclude that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.