Why be so Critical?
Nineteenth Century Mathematics and the Origins of Analysis

Janet Heine Barnett*

August 18, 2017

One striking feature of nineteenth century mathematics, as contrasted with that of previous eras, is the higher degree of rigor and precision demanded by its practitioners. This tendency was especially noticeable in *analysis*, a field of mathematics that essentially began with the "invention" of calculus by Leibniz and Newton in the mid-17th century. Unlike the calculus studied in an undergraduate course today, however, the calculus of Newton, Leibniz and their immediate followers focused entirely on the study of geometric *curves*, using algebra (or 'analysis') as an aid in their work. This situation changed dramatically in the 18th century when the focus of calculus shifted instead to the study of *functions*, a change due largely to the influence of the Swiss mathematician and physicist Leonhard Euler (1707–1783). In the hands of Euler and his contemporaries, functions became a powerful problem solving and modelling tool in physics, astronomy, and related mathematical fields such as differential equations and the calculus of variations. Why then, after nearly 200 years of success in the development and application of calculus techniques, did 19th-century mathematicians feel the need to bring a more critical perspective to the study of calculus? This project explores this question through selected excerpts from the writings of the 19th century mathematicians who led the initiative to raise the level of rigor in the field of analysis.

1 The Problem with Analysis: Bolzano, Cauchy and Dedekind

To begin to get a feel for what mathematicians felt was wrong with the state of analysis at the start of the 19th century, we will read excerpts from three well-known analysts of the time: Bernard Bolzano (1781–1848), Augustin-Louis Cauchy (1789–1857) and Richard Dedekind (1831–1916). In these excerpts, these mathematicians expressed their concerns about the relation of calculus (analysis) to geometry, and also about the state of calculus (analysis) in general. As you read what they each had to say, consider how their concerns seem to be the same or different. The project questions that follow these excerpts will then ask you about these comparisons, and also direct your attention towards certain specific aspects of the excerpts.[1]

*Department of Mathematics and Physics, Colorado State University-Pueblo, Pueblo, CO 81001-4901; janet.barnett@csupueblo.edu.

[1]To set them apart from the project narrative, all original source excerpts are set in sans serif font and bracketed by the following symbol at their beginning and end: ∞∞

Bernard Bolzano, 1817, Rein analytischer Beweis des Lehrsatzes, dass zwischen je zwey Werthen, die ein entgegengesetzes Resultat gewähren, wenigstens eine reele Wurzel der Gleichung liege[2]

There are two propositions in the theory of equations of which it could still be said, until recently, that a completely correct proof was unknown. One is the proposition: *that between any two values of the unknown quantity which give results of opposite signs there must always lie at least one real root of the equation*. The other is: *that every algebraic rational integral function of one variable quantity can be divided into real factors of first or second degree*. After several unsuccessful attempts by d'Alembert, Euler, de Foncenex, Lagrange, Laplace, Klügel, and others at proving the latter proposition Gauss finally supplied, last year, two proofs which leave very little to be desired. Indeed, this outstanding scholar had already presented us with a proof of this proposition in 1799, but it had, as he admitted, the defect that it proved a purely analytic truth on the basis of a geometrical consideration. But his two most recent proofs are quite free of this defect; the trigonometric functions which occur in them can, and must, be understood in a purely analytic sense.

The other proposition mentioned above is not one which so far has concerned scholars to any great extent. Nevertheless, we do find mathematicians of great repute concerned with the proposition, and already different kinds of proof have been attempted. To be convinced of this one need only compare the various treatments of the proposition which have been given by, for example, Kästner, Clairaut, Lacroix, Metternich, Klügel, Lagrange, Rösling, and several others.

However, a more careful examination very soon shows that none of these proofs can be viewed as adequate. The most common kind of proof depends on a truth borrowed from *geometry*, namely, *that every continuous line of simple curvature of which the ordinates are first positive and then negative (or conversely) must necessarily intersect the x-axis somewhere at a point that lies in between those ordinates*. There is certainly no questions concerning the correctness, nor the indeed the obviousness, of this geometrical proposition. But it is clear that it is an intolerable offense against correct method to derive truths of pure (or general) mathematics (i.e., arithmetic[3], algebra, analysis) from considerations which belong to a merely applied (or special) part, namely, *geometry*. [...]

Augustin Cauchy, 1821, Cours d'Analyse[4]

I have sought to give to the methods [in this text] all the rigour which is demanded in geometry, in such a way as never to refer to reasons drawn from the generality of algebra. ...... [Reasons drawn from the generality of algebra] tend to cause an indefinite validity to be attributed to the algebraic formulae, while in reality the majority of these formulae hold only under certain conditions, and for certain values of the variables which they contain. By determining these conditions and values, and by fixing precisely the meaning of the notations I shall make use of, I shall dispel all uncertainty.

[2] English translation of title: *Purely analytic proof of the theorem that between any two values which give results of opposite sign there lies at least one real root of the equation*

[3] As was not uncommon in the nineteenth century, Bolzano's use of the word 'arithmetic' here referred to the mathematical discipline that is today called 'number theory.'

[4] English translation of title: *Course on Analysis*

Augustin Cauchy, 1823, Résumé des leçons sur le calcul infinitésimal[5]

> My principal aim has been to reconcile rigor ... with the simplicity which results from the direct consideration of infinitesimals. I therefore rejected divergent series expansions, and deferred Taylor's formula to the integral calculus [because its remainder is given by an integral formula]. I am aware that [Lagrange] used Taylor's formula as the basis of this theory of the derivative. But ... most geometers[6] are now dubious about the use of divergent series; moreover in some cases when the Taylor series converges, its sum differs from the given function.

Richard Dedekind, 1872, Stetigkeit und irrationale Zahlen[7]

> My attention was first directed toward the considerations which form the subject of this pamphlet in the autumn of 1858. As professor in the Polytechnic School in Zürich I found myself for the first time obliged to lecture upon the elements of the differential calculus and felt more keenly than ever before the lack of a really scientific foundation for arithmetic[8]. In discussing the notion of the approach of a variable magnitude to a fixed limiting value, and especially in proving the theorem that every magnitude which grows continually but not beyond all limits, must certainly approach a limiting value, I had recourse to geometric evidences. Even now such resort to geometric intuition in a first presentation of the differential calculus, I regard as exceedingly useful, from the didactic standpoint, and indeed indispensable, if one does not wish to lose too much time. But that this form of introduction into the differential calculus can make no claim to being scientific, no one will deny. For myself this feeling of dissatisfaction was so overpowering that I made the fixed resolve to keep meditating on the question until I should find a purely arithmetic and perfectly rigorous foundation for the principles of infinitesimal analysis.

Question 1

In what way do the concerns of these three mathematicians about the relation of calculus (analysis) to geometry, and about the state of calculus (analysis) in general, seem to be the same/different?

[5] English translation of title: *Summary of lessons on the infinitesimal calculus*

[6] The meaning of the word 'geometer' also changed over time; in Cauchy's time, this word referred to any mathematician (and not just someone who worked in geometry.)

[7] English translation of title: *Continuity of irrational numbers*

[8] Unlike Bolzano's use of the word 'arithmetic' to mean 'number theory', Dedekind's use of the expression 'scientific foundation for arithmetic' was related to the set of real numbers and its underlying structure.

Question 2

This question looks at some of the mathematical results mentioned by Bolzano, Cauchy and Dedekind.

(a) Note that:

- Bolzano discussed two specific theorems — identify or write these theorems here:

- Dedekind discussed one specific theorem — identify or write that theorem here:

- Cauchy made reference to the Taylor formula and related results — look back to see what he has to say, and briefly describe his concerns.

(b) Which of the results in part (a) are familiar to you?
For each that is, try to state it in "modern" terms, or give its "modern name".

(c) Which of the results in part (a), if any, do you believe to be true (and why)?

2 Niels Abel: *Can you keep from laughing, friends?*

In this section, we will examine an excerpt from a letter written by young Norwegian mathematician Niels Abel (1802–1829) to his high school teacher, Bernt Michael Holmboe, on January 26, 1826. Abel is often remembered for his celebrated impossibility proof in the theory of equations in which he proved that a 'quintic formula' for the general fifth degree polynomial equation does not exist — a proof that marked an important step in the mathematical quest for algebraic solutions to polynomial equations which began with the development of Babylonian procedures for solving quadratic equations in 1700 BCE. Abel is equally well known for his work in analysis, and especially the theory of elliptic functions. In his letter to Holmboe, written during a study-abroad trip to Paris, Abel described some of his concerns about the state of analysis in general, and particularly about the use of infinite series. The **letter itself (in English translation) appears on pages 8 – 9** of this project; after reading it, complete your responses to questions 3 – 6 below.

Question 3
Find at least two references in Abel's letter to infinite series as an important concept or issue in mathematics.

To what degree do the concerns that Cauchy expressed about series agree with Abel's view of series?

Question 4

What was it that Abel thought was "exceedingly surprising" about the "current" state of mathematics? Be specific here!

Do you agree with his reaction to this state of affairs? Explain.

Question 5

Towards the end of this excerpt, Abel remarked that a series of the following form can be convergent for 'x less than 1', but divergent for $x = 1$:

$$\phi(x) = a_0 + a_1 x + a_2 x^2 + \dots$$

(a) Provide an example in which this occurs, specifying both the series (by giving values for the coefficients $a_0, a_1, \dots$) and the function $\phi(x)$ to which that series converges for 'x less than 1'. (Note: You don't really need to work too hard to do this.)

(b) Notice that Abel went on to speculate that an even worse situation might occur. Namely, he proposed the possibility that a series $\phi(x) = a_0 + a_1 x + a_2 x^2 + \dots$ might be convergent for 'x less than 1' *and* convergent for $x = 1$, but in such a way that $\lim\limits_{x \to 1} \phi(x)$ is not equal to $\phi(1)$.

What mathematical concept is involved here? That is, if such a function ϕ does in fact exist, what function property is ϕ lacking?

Question 6

Consider the following series discussed by Abel at the end of this extract:

$$\frac{x}{2} = \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \text{etc.}$$

(a) Describe how this series is different from a power series.

(b) Now complete Abel's arguments concerning the numerical aspects of this series by determining what is absurd about this formula for $x = \pi$.

(c) Next complete Abel's comments about the differential aspects of this series by differentiating the formula term-by-term in order to show what can go wrong when one "applies operations to infinite series as if they were finite". [Be sure to say what is wrong with the differentiation results!]

Heinrik Abel, 1826, Letter to Holmboe

Since my arrival in Berlin I have also occupied myself with the solution of the following general problem: *To find all the equations which can be solved algebraically.* I have not yet solved it, but as nearly as I can judge, I am close. When the degree of the equation is prime, it is not too difficult, but when this number is composite, the devil mixes in. I have found a large number of solvable equations, in addition to those already known. When I have finished the paper which I hope to write, I flatter myself that it will be good. It will be general, and one will find there a method, which seems to be the most essential point.

Another problem with which I have occupied myself a lot is the summation of the series

$$\cos mx + m\cos(m-2)x + \frac{m(m-1)}{2}\cos(m-4)x + \ldots$$

When m is a positive integer, the sum of this series as you know, is $(2\cos x)^m$, but when m is not an integer, this is no longer the case, except when x is less than $\pi/2$.

There is no other problem which has occupied mathematicians in recent times as much as this one. Poisson, Poinsot, Plana, Crelle and an enormous number of others have tried to solve it, and Poinsot is the first to have found the correct sum, but his reasoning is totally false. Until this time no one has been able to get to the end with this [problem]. I am happy that I quite rigorously have arrived to this [end]. A memoir about this will appear in the Journal, and another I will soon send to France to appear in Gergonne's *Annales de Mathematiques*.

[There follows a discussion, omitted here, of some results concerning the above series which Abel has found.]

Divergent series are on the whole devilish, and it is a shame that one dares to base any demonstration on them. One can obtain whatever one wants, when one uses them. It is they which have created so much disaster and so many paradoxes. Can one imagine anything more appalling than to say

$$0 = 1 - 2^n + 3^n - 4^n + etc.$$

where n is a positive integer? *Risum teneatis amici!*[9]

I have in general had my eyes opened in a most astonishing manner: Because when one excludes the most simple cases, for ex. the geometric series, then in the whole of mathematics there is almost no infinite series whose sum is determined in a strict way. In other words, the most important part of mathematics stands there without foundation. Most of it is correct, that is true, which is exceedingly surprising. I am working hard to search for the reason behind this. A very interesting task. I do not think you will be able to propose to me many theorems in which there are infinite series, against whose proof I shall not provide reasoned objections. Do it, and I will answer you.

[9] Latin for "Can you keep from laughing, friends!"

[There follows a discussion, omitted here, about the Binomial Series, about which Abel has derived certain results.]

The Taylor theorem, the basis of all of Higher Mathematics, is equally poorly demonstrated. I have found only one strict proof, and that is by Cauchy in his *Leçons sur le calcul infinitésimal* (of 1823). He there proves that one has:

$$\phi(x+\alpha) = \phi x + \alpha\phi' x + \frac{1}{2}\alpha^2\phi'' x + \ldots$$

as often as the series is convergent (but one uses it easily in all cases). To show by a general example how poorly one is reasoning and how careful one ought to be, I will choose the following example: Let

$$a_0 + a_1 + a_2 + a_3 + a_4 + \text{etc.}$$

be any infinite series. Then you know that a very useful way to sum this series is to search for the sum of the following:

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + etc.$$

and after that to put $x = 1$ in the result. This may be correct, but to me it seems one cannot assume it without proof, because even if one proves that

$$\phi(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$

for all values of x less than 1, it cannot be said because of this that the same thing happens for $x = 1$. It could very well be possible that the series $a_0 + a_1 x + a_2 x^2 + \ldots$ approaches a quite different quantity than $a_0 + a_1 + a_2 + \ldots$ when x approaches more and more to 1. This is clear in the general case when the series $a_0 + a_1 + a_2 + \ldots$ is divergent, because then it has no sum. I have proved that it is correct when the series is convergent. The following example shows how one can cheat oneself. It can be strictly proved for all values of x less than π that

$$\frac{x}{2} = \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - etc.$$

From this it seems to follow that the same formula should hold for $x = \pi$, but then we would obtain (an absurdity).

. . . .

One applies all operations to infinite series as if they were finite, but is this permissible? I think not, — Where is it proved that one gets the differential of an infinite series by differentiating each term? It is easy to give an example for which this is not true; e.g,,

$$\frac{x}{2} = \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - etc.$$

. . . .

3 Concluding Questions and Comments

The concerns expressed by Abel, Bolzano, Cauchy and Dedekind in the excerpts we have read in this project were emblematic of the state of analysis at the turn of the nineteenth century. Ultimately, mathematicians of the nineteenth century responded to this set of concerns by moving to the requirement of *formal proof* as a way to certify knowledge via the *rigorous use of inequalities* intended to capture the notion of two real numbers 'being close' that underlies the limit concept. Other factors that influenced this direction included new teaching and research situations, such as the École Polytechnique in Paris, that required mathematicians to think carefully about their ideas in order to explain them to others. Today, this nineteenth century response remains at the core of the study and practice of real analysis. The final question in this project takes another look back at the motivations of those who led the way in formulating this response, as they expressed it in their own words.

Question 7
Look back at the excerpts from the works of Abel, Bolzano, Cauchy and Dedekind that we have read in this project. What questions or comments would you address to these mathematicians about aspects of their concerns that are not addressed in the earlier questions? (Write at least one question and at least one comment, please!)

References

[1] Abel, N., "Breve fra og til Abel," in *Festkrift ved Hundredeaarsjubilæet for Niels Henrik Abels Fødsel* (editors E. Holst, C. Stømer and L. Sylow), Kristiana: Jacob Dybwad, 1902.

[2] Bolzano, B., *Rein analytischer Beweis des Lehrsatzes, dass zwischen je zwey Werthen, die ein entgegengesetzes Resultat gewähren, wenigstens eine reele Wurzel der Gleichung liege*, Leipzig: W. Engelmann, 1817.

[3] Cauchy, A.,*Cours d'Analyse de L'Ecole Royale Polytechnique*, Paris: Debure, 1821.

[4] Cauchy, A., *Résumé des Leçons sur le calcul infinitésimal*, Paris: Debure, 1823.

[5] Dedekind, R., *Stetigkeit und irrationale Zahlen*, Braunschweig: F. Vieweg und sohn, 1872.

[6] Sørensen, H.K. Exceptions and counterexamples: Understanding Abel's comment on *Cauchy's Theorem*, *Historia Mathematica* 32 (2005), 453 – 480.

Notes to Instructors

The goal of this Primary Source Project (PSP) is to provide context for the use of rigorous proofs and precise ϵ-inequalities that developed out of concerns about the state of analysis that first arose in the nineteenth century, but which remain defining characteristics of today's analysis. Both these tools of the current trade (i.e., rigorous proof, precise inequalities) offer challenges to students of introductory analysis, who have typically encountered calculus only as a procedural and applied discipline up to this point in their mathematical studies. By offering a glimpse into the problems that motivated nineteenth century mathematicians to shift towards a more formal and abstract study of the concepts underlying these procedures and applications, the readings in this PSP provide students with a context for making a similar shift in their own understanding of these concepts. Completing this PSP early in the course can also provide students and instructors with a basis for reflection on and discussion of current standards of proof and rigor throughout the course.

The project assumes that students are familiar with fundamental concepts from a first year calculus course, including basic results about limits and power series. However, no prior study of analysis or experience with formal proof writing is needed.

Classroom implementation of this project can be accomplished by way of one of the two following basic approaches; hybrids of these two methods are, of course, also possible.

> *IMPLEMENTATION METHOD I*
>
> Students are assigned to read the entire PSP and respond (in writing) to the questions therein prior to class discussion. Typically, the author assigns this reading one week prior to a class discussion of it; other instructors have confirmed that sufficient time for careful advance reading is important for high quality in-class discussions. Students are encouraged to discuss the readings and PSP questions with each other or with the instructor (outside of class time) before the assigned due date (provided their written responses are their own). While there is no prohibition against using additional resources to complete the PSP (e.g., a calculus text), it is important to assure students that there is no need to do any historical research in order to complete it.
>
> On the assignment due date, a whole class discussion (45 - 50 minutes) of the reading is conducted by the instructor, with student responses to various PSP questions elicited during that discussion. An instructor-prepared handout containing solutions to select questions (especially Question #2) can be helpful during this discussion. The completed written work is typically collected at the close of that class period; however, the discussion could also be conducted after the instructor has collected and read students' written PSP work. The author does evaluate students' individual written work for a grade. That evaluation and grade is based primarily on completeness, but also takes into account both presentation (e.g., use of complete sentences) and accuracy (particularly with regard to the mathematical details in Questions # 2, 5, 6).
>
> A brief set of summary notes that could be used by an instructor during a whole class discussion of the PSP is offered below (pages 13–14). Although some type of summarizing discussion is highly recommended, that discussion need not adhere to the notes provided here.

IMPLEMENTATION METHOD II

Students are assigned to read only the primary source excerpts in the project as preparation for small group work on project during class time. Depending on the course and the class period length, this implementation plan may take up to 2 days to complete, with the excerpts from Sections 1 and 2 assigned for advance reading on two separate days.

During class time, students then work together in small groups to write their answers to the PSP questions, with the instructor circulating between groups to facilitate that work. The completed written work is then either collected from each group at the close of that class period (and possibly evaluated for a grade), or students can be asked to write formal responses to some or all of the questions on an individual basis (again, possibly evaluated for a grade).

Instructors opting for implementation in small groups may also wish to conduct a whole-group discussion, based on select portions of the attached Summary Discussion Notes, at one or more junctures during implementation.

LaTeX code for the attached Summary Discussion Notes, and for the entire PSP, are available from the author upon request; both can be modified by instructors as desired to better suit their course goals.

Acknowledgments

The development of this project has been partially supported by the National Science Foundation's Improving Undergraduate STEM Education Program under Grants No. 1523494, 1523561, 1523747, 1523753, 1523898, 1524065, and 1524098. Any opinions, findings, and conclusions or recommendations expressed in this project are those of the author and do not necessarily reflect the views of the National Science Foundation.

For more information about TRIUMPHS, visit `http://webpages.ursinus.edu/nscoville/TRIUMPHS.html`.

Summary Discussion Notes: ***Why be so Critical?***

- Caution that one of the difficulties with historical readings is that the meanings of words change over time; for example, 'geometer' referred to any mathematician (not just someone who worked with geometry)
- Overview of pre-nineteenth century calculus themes

Time Period	Focus *What objects should we study?*	Primary justification of "correctness" *How do we know our mathematics is "true"?*
17th century	Calculus of CURVES (using algebra as a tool)	New methods produce results that matched "old" (known) results (obtained from geometry)
18th century	Calculus of FUNCTIONS (with physics as primary motivation) NEW QUESTION: What is a function really? Related historical controversies: Fourier Series Convergence Vibrating String Problem	Methods produce correct predictions (in physics) NEW CONCERN: Is it valid to borrow "truths" from one domain (e.g., geometry, physics) to justify truths in another (e.g., mathematics)?

- Overview of the situation at the end of 18th/start of 19th century *(Four main points, I – IV)*

  I. Increasing mistrust of "geometric" intuition as valid proof method for "analytic" truths
  (and more general frustration that analytic "truths" are being verified by non-analytic 'proofs")
  Ask for evidence of this in the assigned reading.

  II. Concern that existing 'algebraic' proof methods lack adequate rigor
  Ask for evidence of this in the assigned reading; two subthemes to elicit here:
  - Euclid had long been a model of rigor; nineteenth century mathematicians express desire to bring back something like an axiomatic approach as a foundation for certain knowledge
  - algebra allows too much generality (e.g., unrestricted)
    Makes it too easy to assume that properties (e.g., continuity, rationality) that hold at all "lower" values will also hold in the limit (*elicit or mention Abel power series example here*)

  III. Use of power series (in particular) lacks firm foundation
  Ask for evidence of this in the assigned reading; two mathematical points to elicit in particular:
  - Discuss current views about $\sum_{n=1}^{\infty} x^n$ (converges for $-1 < x < 1$ but diverges for $x = \pm 1$)
    Discuss Abel's use of the phrase 'x less than 1*' here (where today we would write '$|x| < 1$').*
  - Abel mentions we could also have convergence for $|x| \leq 1$ with $\lim\limits_{x \to 1} \phi(x) \neq \phi(1)$.
    Ask students for their answers to Question 4 and 5 here.

  IV. General concerns about foundations: *If we don't base calculus on power series, what do we use instead?*
  - Some possibilities (and early proponents of each):
    Fluxions (Newton) ; Infinitesimals (Leibniz) ; **Limits** (d'Alembert) $\leftarrow$ **The "winner"!**
  - Chosen option of 'limit' raises yet another new question: What is a limit really??

- Ultimate nineteenth century response to this set of concerns:

  Require $\underbrace{\text{FORMAL PROOFS}}_{\text{as way to certify knowledge}}$ via $\underbrace{\text{RIGOROUS use of INEQUALITIES}}_{\text{as way to talk about 'being close'}}$.

  – *Historical Aside:* Another factor that influenced this direction were new teaching & research situations (École Polytechnique) that required thinking carefully about ideas in order to explain them to others.

  – This nineteenth century response, which forms the basis of the work we will do together throughout this course, is often described as 'the arithmetization of analysis'.

An optional historical aside related to item III

The use of series and power series itself was NOT new in the nineteenth century!

- Power series had been around well before the invention of calculus; they were also part of 'pre-calculus' in the sense that, at least through the eighteenth century, understanding power series was considered a *pre-requisite* to the study of calculus.

- Newton (and others) used power series extensively as infinite polynomials that are easy to integrate and differentiate.

- An infinite series example from the 18th century: $1 - 1 + 1 - 1 \ldots = \frac{1}{2}$

  - A first "proof":
    $(1-1) + (1-1) + \ldots = 0 \quad ; \quad 1 - (1-1) + (1-1) + \ldots = 1$
    Series value is the average: $\frac{0+1}{2} = \frac{1}{2}$.

  - A second "proof" (endorsed by Euler, among others):

$$\sum_{n=1}^{\infty} (-1)^n = \frac{1}{1-(-1)} = \frac{1}{2}$$

For more about this and other divergent series in the 17th century, see the June 2006 MAA On-line column *How Euler Did It* by Ed Sandifer (available at *http://eulerarchive.maa.org/hedi/HEDI-2006-06.pdf*).

An optional historical aside related to nineteenth century mathematicians

Commenting on his experience following a visit to Paris in 1826, Abel wrote the following to Holmboe:

> Legendre is an exceedingly courteous man, but unfortunately as old as the stones.
>
> Cauchy is mad, and you cannot get anywhere with him, although he is the mathematician who knows at the moment how to treat mathematics. Cauchy is extremely Catholic and bigoted. A very strange thing in a mathematician ...
>
> Poisson is a short man with a nice little belly. He carries himself with dignity. Likewise Fourier. Lacroix is terribly bald and extremely old. On Monday I am going to be introduced to several of these gentlemen by Hachette.
>
> Otherwise I do not like the Frenchman as much as the German, the Frenchman is uncommonly reserved towards foreigners. It is difficult to make his close acquaintance. And I dare not count on such a thing. Everyone wants to teach and nobody to learn. The most absolute egotism prevails everywhere. The only things that the Frenchman seeks from foreigners are the practical. He is the only one who can create something theoretical. You can imagine that it is difficult to become noticed, especially for a beginner.