

Dedekinds Ideals: Prime Mathematics for Todays Students

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As with other structures in modern abstract algebra, the ring concept has deep historical roots in several nineteenth century mathematical developments. These include especially the work on algebraic number theory by Richard Dedekind (1831-1916), in which the concept of an ideal appeared for the first time. Between 1871 and 1894, Dedekind published four different versions of his theory of ideals, none of which was simply a revision of an earlier paper. Instead, each publication described a new version of the theory in which Dedekind reformulated the underlying concepts in clearer and more abstract terms. Both the brilliant mathematical insights resulting from these patient years of re-working his ideas, and the precision and clarity with which expressed those ideas, have justifiably earned Dedekind renown as one of the most influential mathematicians of the nineteenth century.

In this talk, we describe a “Primary Source Project” (PSP) based on Dedekind’s original writings that can be used to teach elementary ring and ideal theory in today’s abstract algebra classroom. A series of tasks interspersed between original source excerpts provides students with the means to consolidate their understanding of these important algebraic structures, while extending their ability to write proofs based on today’s mathematical standards, a standard that Dedekind helped to bring into being. Through guided reading of excerpts from Dedekind, students are also exposed to the original motivation behind his definition of an ideal: to restore the essential properties of integer divisibility, especially unique prime factorization, back to algebraic number fields.