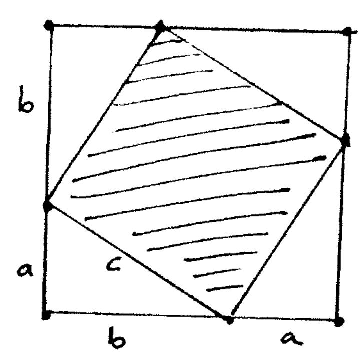
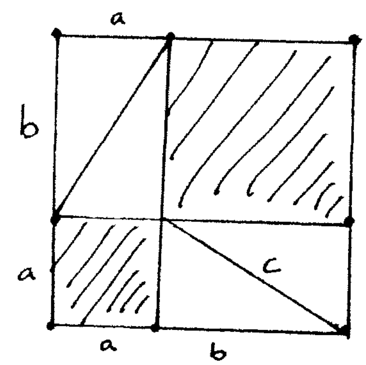
1. The Pythagorean Theorem states that if ΔABC is a right triangle with right angle at vertex C and a, b, c are the lengths of the sides opposite vertices A, B, C respectively, $a^2 + b^2 = c^2$. Show how the following diagrams can be used to prove the Pythagorean Theorem.





2. Explain how to construct the following using only compass and straightedge.

   a) Given a line ℓ and a point p not on ℓ, construct a line through P that is perpendicular to ℓ.

   b) Given an angle $\angle BAC$, construct the angle bisector.

3. Using only Euclid's postulates (page 1) and common notions (page 2), is it possible to show that

   a) Every line has at least two points lying on it?

   b) For every line there is at least one point that does not lie on the line?

4. Use the parallel postulate to show that the sum of the interior angles of any triangle is always 180°. This, along with Theorem 1.2.1 in the book shows that the parallel postulate is equivalent to saying that the sum of the interior angles of any triangle is always 180°. [Hint: You will need to use the famous "alternate interior angles are equal" fact from high school geometry.]

5. According to ancient Egyptian sources, the volume of a truncated pyramid with square base is given by $V = \frac{h}{3}(a^2 + ab + b^2)$ where the base of the pyramid is an $a \times a$ square, the top is a $b \times b$ square, and the height measured perpendicular to the base is h. Use the fact that the volume of a (non-truncated) pyramid is $\frac{1}{3}b \cdot h$ to verify the Egyptian formula. [Hint: Use similar triangle.]

6. Define *one-point geometry* to be the geometry consisting of just one point and no line. Which incidence axioms and parallel postulates does one-point geometry satisfy? Explain.

7. Find a finite model for incidence geometry in which there is one line that has exactly three points lying on it and there are other lines that have exactly two points lying on them.

8. a) Construct a five-point geometry in an analogous manner to our construction of a three and four-point geometry in class.

   b) Show that this is an incidence geometry.

   c) A geometry is said to satisfy the *hyperbolic parallel postulate* if for every line ℓ and every point P that does not lie on ℓ, there are at least two lines m and n such that P lies on both m and n and both m and n are parallel to ℓ. Show that the five-point geometry satisfies the hyperbolic parallel postulate.