Primary Source Projects in an Undergraduate Mathematics Classroom: A Pilot Case in a Topology Course

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The TRIUMPHS Project

***Tr**ansforming **I**nstruction in **U**ndergraduate **M**athematics via **P**rimary **H**istorical **S**ources*

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The TRIUMPHS Project

- Funded by the National Science Foundation
- Five-year, $1.5 million, seven-institution effort in the United States
- Funding Period: August 2015 – July 2020

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Colorado State University Pueblo

Denver

TRIUMPHS Project Website

[Project Descriptions](#)

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Who we are

Janet Barnett
Kathleen Clark

[Photos from September workshop in Denver now online!](#)

Collaborative Research: Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS)

Mathematics faculty members and educational researchers are increasingly recognizing the value of the history of mathematics as a support to student learning. This collaborative project, involving seven diverse institutions of higher education, will help students learn and develop a deeper interest in, and appreciation and understanding of, fundamental mathematical concepts and ideas by utilizing primary sources - original historical writings by mathematicians on topics in mathematics. Educational materials for students will be developed at all levels of undergraduate mathematics courses, and will be designed to capture the spark of discovery and to motivate subsequent lines of inquiry. In particular, the student projects to be developed will be built around primary source material to guide students, including pre-service teachers, mathematics majors, and other STEM discipline majors, to explore the mathematics of the original discovery in order to develop their own understanding of that discovery. Mathematics faculty and graduate students from over forty (40) institutions will participate in the development and testing process, thereby ensuring a large national network of faculty with expertise on the use of these educational materials. The impacts of the materials and approaches to implementing them will be investigated in terms of teaching, student learning, and departmental and institutional change.

Sponsor

National Science Foundation
WHERE DISCOVERIES BEGIN

This material is based upon work supported in part by the National Science Foundation under

Aims and Scope of TRIUMPHS

- Aim: to **design, develop**, **write, test/implement**, **evaluate,** and **disseminate** classroom materials based on primary sources ("Primary Source Projects" = PSPs) for teaching undergraduate mathematics courses
  - Plan to create 25 full-length PSPs and 30 "mini" PSPs over the course of grant project

- Scope of Topics: PSPs address topics ranging from **pre-calculus / trigonometry** and **elementary statistics** to **abstract algebra, analysis,** and **topology**

Aims and Scope, *continued*

- The materials allow instructors to replace traditional classroom instruction with PSPs that **directly engage students with the mathematics** they are studying

- Students **read source texts**, and through a **series of exercises** that are woven throughout the project, **develop a fuller understanding** of the mathematics they are studying

Full-Length PSPs (14)
A Genetic Context for Understanding the Trigonometric Functions
The Exigency of the Euclidean Parallel Postulate and the Pythagorean Theorem
Failure of the Euclidean Parallel Postulate and Distance in Hyperbolic Geometry
Richard Dedekind and the Creation of an Ideal: Early Developments in Ring Theory
Rigorous Debates over Debatable Rigor in Analysis (Darboux's Monster Function)
Primes, Divisibility & Fermat Numbers
Bolzano's Definition of Continuity, his Bounded Set Theorem, and an Application to Continuous Functions
Investigating Difference Equations
Investigations Into d'Alembert's Definition of Limit
An Introduction to a Rigorous Definition of Derivative
Investigations Into Bolzano's Formulation of the Least Upper Bound Property
The Mean Value Theorem
Abel and Cauchy on a Rigorous Approach to Infinite Series
The Definite Integrals of Cauchy and Riemann

Mini-PSPs (9)
Babylonian Numeration
Why Be So Critical? Nineteenth Century Mathematics and the Origins of Analysis
Topology from Analysis: Making the Connection
Connecting Connectedness
The Cantor Set Before Cantor
Henri Lebesgue and the Integral Concept
Euler's Rediscovery of *e*
Euler's Derivatives of the Sine and Cosine Functions

Topology Course Pilot Case Study

- Setting: Ursinus College
  - A small liberal arts, bachelor's degree granting institution near Philadelphia, Pennsylvania
- Topology Course: 8 students
  - full data profile for 5
- Three mini-PSPs implemented
  - PSPs help to provide students with motivation for subtle and nuanced mathematical definitions (in contrast to what typically occurs in traditional classrooms) – and the texts used exhibit the rationale behind these definitions

Research Questions

1. As a result of engaging with PSPs, what changes do students report in their attitudes and beliefs about learning mathematics?
2. As a result of engaging with PSPs, do students report any change in their mathematical worldview, and if so, what is the predominant view change?
3. In what ways does the use of PSPs influence mathematics (or mathematics-related major) students' beliefs and perceptions about mathematics?
4. As a result of engaging with PSPs, what do students report as challenges and benefits of learning from primary sources?

Data Sources

- Student Change
  - Pre- / Post-Course Survey
  - Post-PSP Survey(s) [began Fall 2016]
  - Student Work [from each PSP]
  - Student Interviews [planned to begin Spring 2017]
  - Daily video capture, small-group audio, additional student work, and pre- / post-interviews:
    - Evidence of students' progress in "figuring out" (Sfard, 2014, p. 201) the meta-level rules that govern a new mathematical discourse [began Spring 2017]
    - The extent to which students' (verbal / written / other) actions both during and after engagement with the PSPs provide evidence of their acceptance of a new discourse [began Spring 2017]

Of Interest for this Pilot

- What student populations are being served in courses where PSPs are implemented / tested?
- **What do students discuss regarding learning of mathematics?**
- What do students observe about the use of PSPs in their undergraduate mathematics courses (in general; for their own learning)?
- **What are students views on mathematics (pre-/post)?**

UC: Topology Spring 2016 (PRE)

Comments on Pre to Post:	*Interesting case: Shifted from Process to Formalism*	***stable***	***stable***	*Shifted from Schema to Process*	***stable***
	AA12MI	AE15SM	JC09MC	JS08FE	MC18BA
Schema	**3.4**	3.2	3.4	**4.2**	1.6
Formal	**3.8**	3.4	3.8	4	4.4
Process	**4**	4.6	5	3.6	4.8
Applic.	**3.6**	3.4	3.6	3.4	3.4

Dominant view: Process aspect

UC: Topology Spring 2016 (POST)

	AA12MI	AE15SM	JC09MC	JS08FE	MC18BA
Schema	**3.4**	2.6	4.2	**3.6**	1.4
Formal	**4.8**	3.4	4	3.8	4
Process	**4.2**	4.4	5	**4**	4.4
Applic.	**3.8**	3.4	4.4	3.8	3.4

Dominant view: Process aspect

Definitions of Törner's Aspects

- ***Formalism-aspect*** of mathematics: "mathematics is characterised by **strictness, exactness, and precision** on the terminological and language levels concerning thinking ("logical," "objective," and "flawless thinking"), **argumentation, giving reasons and proof** of statements as well as theoretical systematology (**axiomatics** and the **strict deductive method**)" (Törner, p. 126)

Aspects, *continued*

- ***Application-aspect***: "…of the immediate relevance to **application or the practical use of mathematics**. The pupils' knowledge of mathematics is **important to** their **future life**: mathematics either helps to solve everyday tasks and problems or it is useful to one's occupation. Apart from that, mathematics is **of a general, fundamental use to society**" (Törner, p. 127)

Aspects, *continued*

- ***Process-aspect***: "Mathematics is…a process and…an activity in thinking about problems and gaining knowledge. On the one hand, this cognitive process is about **creating, inventing or re-inventing** (re-discovering) mathematics. On the other hand, it also includes the **comprehension of facts** and **understanding connections**. This problem, the oriented cognitive process of understanding, decisively requires thinking and arguing in regard to content as well as **sudden and new ideas**, **intuition** and **experimenting**. This [aspect] expresses the **dynamic view of mathematics**" (Törner, pp. 127-128)

Aspects, *continued*

- ***Schema-aspect***: "…operationally define a view of mathematics which is seen as a "**tool-box and bundle of formulas**" and an idea oriented with **algorithm and schemes**. Mathematics is characterised by a **collection of methods and rules** which precisely determine how to solve a task. The consequence with dealing with mathematics is: doing mathematics consists of **remembering** and **applying definitions, rules, formulas, facts** and **methods**. Mathematics consists of learning (and teaching!), practising and the remembering and **applying of routines**, **schemata,** and **applications**" (Törner, p. 128)

Next Steps

- Currently concentrating efforts to expand testing of projects in classrooms. We are:
  - Acquiring appropriate data to better inform the first four research questions (survey refinement; interviews)
  - Developing an intensive qualitative study on the role of PSPs in learning of meta-discursive rules
- Spring 2017:
  - 15 instructors
  - 17 courses
  - 10 states
  - ~334 students involved in courses using projects; 259 consenting students

Thank you for your attention!

We look forward to your questions and subsequent discussion.

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  http://webpages.ursinus.edu/nscoville/TRIUMPHS.html

The following slides are not…

…part of the presentation.

Three Key Project Activities

- **Develop and rigorously test classroom materials** to teach standard topics in the university undergraduate mathematics curriculum through primary historical sources,
- **Train faculty to implement PSPs** and **promote their use as widely as possible** through widespread dissemination via conference talks and training workshops, and
- **Study diverse aspects of their implementation and efficacy**.

Evaluation-with-Research Study

- Designed to provide both formative and summative evaluation of key project activities:
  - **Student Change**
  - Faculty Expertise
  - Development Cycle (development of PSPs)

Sites & Data Collection Thus Far

Fall 2015	Spring 2016	Fall 2016	Spring 2017
Colorado 1	**Colorado 1**	**Colorado 1**	**Colorado 1**
New Mexico 1	Pennsylvania 1	**New Mexico 1**	Colorado 2 (2 courses)
		Ohio 1 (2 instructors)	Colorado 3 (2 courses)
		Tennessee 1 (2 sections)	Colorado 4
			Alabama
			Ohio 1
			California 1
			Wisconsin 1
			Missouri
			Kansas
			Pennsylvania 2
			Pennsylvania 3
			New York 1
			New York 2
			Georgia (2 sections)

Meta-Discursive Rules
(intensive pilot study, Fall 2016 – Spring 2017)

Research Questions

1. When studying *number theoretic ideas focused on prime numbers* using primary source projects, what is the evidence of students' progress in "figuring out" (Sfard, 2014, p. 201) the meta-level rules that govern a new mathematical discourse?

2. To what extent do students' (verbal / written / other) actions both during and after engagement with the primary source projects provide evidence of their acceptance of a new discourse?