

3. Assume by way of contradiction that $K_{3,3}$ is planar. Then the graph satisfies Euler's thm

$$v - e + f = 2. \quad \text{Since } |V(K_{3,3})| = 6 \quad \text{and}$$

$$|E(K_{3,3})| = 9$$

$$\begin{aligned} 6 - 9 + f &= 2 \\ f &= 5 \end{aligned}$$

and $K_{3,3}$ has 5 faces.

Observe that a face of $K_{3,3}$ corresponds to a cycle and since the minimum cycle length in $K_{3,3}$ is 4, and ~~each~~ the number of edges per face is counted (possibly) twice, we have that

$$4f \leq 2e.$$

$$\text{But then} \quad 4(5) \leq 2(9)$$

$$20 \leq 18 \quad \text{which is a}$$

contradiction. □