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Implementation of Particle Swarm Optimization in Construction of Optimal Risky Portfolios

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Abstract - **Since Markowitz's substantial work, the mean-variance model has revolutionized the way people think about portfolio of assets. According to the modern portfolio theory, the fundamental principle of financial investments is a diversification where investors diversify their investments into different types of assets. Constructing an optimal risky portfolio is a high-dimensional constrained optimization problem where financial investors look for an optimal combination of their investments among different financial assets with the aim of achieving a maximum reward-to-variability ratio. Among the various methodologies suggested, the most popular one is based on maximizing the well-known Sharpe ratio.**

In this study, we apply particle swarm optimization (PSO) for constructing optimal risky portfolios based on Sharpe ratio for financial investments. A particle swarm solver is developed and tested on a risky investment portfolio. The method is applied to a sample of stocks in Tehran Stock Exchange. Experimental results reveal that the proposed PSO algorithm provides a very feasible and useful tool to assist the investors in planning their investment strategy and constructing their portfolio.

Keyword - **modern portfolio theory, optimal risky portfolio, PSO, Sharpe ratio**

I. INTRODUCTION

Since Markowitz's pioneering work [1] was published, the mean-variance model has revolutionized the way people think about portfolio of assets, and has gained widespread acceptance as a practical tool for portfolio optimization. But Markowitz's modern portfolio theory only provides a solution to asset allocation among the pre-determined assets. Investors have difficulties to find out those good quality assets because of information asymmetry and asset price fluctuations. The suitable way of constructing a portfolio is to select some good quality assets first and then to optimize asset allocation using modern portfolio theory.

With focus on the business computing, applying artificial intelligence methods to portfolio selection and optimization is a good way to meet the challenge. Some studies have been presented to solve asset selection problem. Reference [2] applied artificial neural network (ANN) to select valuable stocks. For instance, reference [3] proposed a fuzzy rule-base stock selection model with rate of return, current ratio, and yield rate as input factors. This model uses genetic algorithm to find each company's appraisal grade and employs a multi-period random capital allocation model, empirical results indicate that investment portfolios constructed using this method perform well in terms of predicted rate of return, variance, and utility value. Reference [4] proposed a Genetic Algorithm Model for portfolio selection. This model considers both equity and debt securities and vice versa. The computerized dynamic portfolio has outperformed the senses throughout the testing period.

In addition, reference [5] utilized support vector machine (SVM) to train universal feedforward neural networks (FNN) to perform stock selection. Some researchers, such as [6-7], trained neural networks to predict asset behavior and used the neural network to make the asset allocation decisions. Reference [8] applied dynamic programming to construct a multi-stage stochastic model for solving asset allocation problem. Recently, reference [9] designed a double-stage genetic optimization algorithm to identify good quality assets.

However, these approaches have some drawbacks in solving the portfolio selection problem. For example, fuzzy approach [3] usually lacks learning ability, while neural network approach [2], [5], [6], [7] has over-fitting problem and is often easy to trap into local minima.

In order to overcome these shortcomings, we use PSO to solve the portfolio selection and optimization problem. PSO is a population based stochastic optimization technique developed in 1995. The underlying motivation for development of PSO algorithm was social behavior of bird flocking or fish schooling [10]. PSO has become a popular optimization method as they often succeed in finding the best optimum by global search in contrast with most common optimization algorithms. In comparison with the dynamic programming [8], PSO allows the users to get the sub-optimal solution while dynamic programming cannot, which is very important for some financial problems. Since the time is a limit in financial world, the investors often use a sub-optimal but acceptable solution to allocate assets. Due to these advantages, we use PSO to perform portfolio selection.

The main motivation of this study is to employ a PSO algorithm for portfolio selection. First, we identify good quality assets in terms of asset ranking. Then asset allocation in the selected good quality assets is optimized using a PSO algorithm based on Markowitz's theory. Through the PSO process, an optimal portfolio can be determined. The rest of the paper is organized as follows.

Section 2 describes optimization process based on the PSO in detail. In section 3, PSO for optimal portfolio is discussed. In order to test the efficiency of the proposed algorithm, a simulation study is performed in Section 4. And Section 5 is devoted to conclusions of the paper.

II. PARTICLE SWARM OPTIMIZATION

As mentioned earlier, PSO is one of the population based optimization technique inspired by social behavior of bird flocking and fish schooling. It is a simulation of the behaviors of swarm such as bird flocking or fish schooling. Suppose the following scenario: a group of birds is randomly searching for food in an area, where there is only one piece of food available and none of them knows where it is, but they can estimate how far it would be at each iteration. The problem here is "what is the best strategy to find and get that food".

Obviously, the simplest strategy is to follow the bird known as the nearest one to the food. PSO inventers were inspired of such natural process based scenarios to solve the optimization problems. In PSO, each single solution, called a particle, is considered as a bird, the group becomes a swarm (population) and the search space is the area to explore. Each particle has a fitness value calculated by a fitness function, and a velocity of flying towards the optimum, food.

All particles fly across the problem space following the particle nearest to the optimum. PSO starts with initial population of solutions, which is updated iteration-by-iteration. Therefore, PSO can be counted as an evolutionary algorithm besides being a meta-heuristics method, which allows exploiting the searching experience of a single particle as well as the best of the whole swarm.

The original PSO algorithm is initialized with a population of K particles to fly in the D-dimensional problem space. Each particle is consisted of a solution, called "position" of the particle, and a randomized set of D dimensions that is used for evolving a new position, called "velocity". The velocity of each particle is dynamically adjusted by the flying experiences of its neighbors and its own. At every iteration, each particle updates to a new velocity by using old velocity and its own "experiences" and that of its neighboring particles. The new velocity is then used to move the particle to a new position.

In other words, a brief description of how the algorithm works is as follow: Initially, some particle is identified as the best particle in a neighborhood of particles, based on its fitness. All the particles are then accelerated in the direction of this particle, but also in direction of their own best solutions that they have discovered previously.

Occasionally the particles will overshoot their target, exploring the search space beyond the current best particles. All particles have the opportunity to discover better particles en route, in which case the other particles will change direction and head towards the new best particle. Since most functions have some continuity, chances are that a good solution will be surrounded by equally good, or better, solutions. By approaching the current best solution from different direction in search space, the chances are good that these neighboring solutions will be discovered by some of the particles [11].

A. Variations and Practicalities

The PSO seems well-suited for some of the most pressing computational problems in many fields. Many computational problems involve search through a huge number of possibilities for solutions. One example is searching for a set of rules that will predict the ups and downs of a financial market such as stock prices. Such search problems can often benefit from an effective use of parallelism, in which many deferent possibilities are explored simultaneously in an efficient way.

There are a number of considerations in using PSO in practice**:**

- PSO is a robust stochastic optimization technique based on the movement and intelligence of swarms.
- PSO applies the concept of social interaction to problem solving.
- It uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution.
- Each particle is treated as a point in a D-dimensional space which adjusts its "flying" according to its own flying experience as well as the flying experience of other particles
- Each particle keeps track of its coordinates in the solution space which are associated with the best solution (fitness) that has achieved so far by that particle. This value is called personal best, ***pbest***.
- Another best value that is tracked by the PSO is the best value obtained so far by any particle in the neighborhood of that particle. This value is called ***gbest***.
- Unlike in genetic algorithms, in PSO, there is no selection operation.
- All particles in PSO are kept as members of the population through the course of the run
- PSO is the only algorithm that does not implement the survival of the fittest.
- No crossover operation in PSO.
- The basic concept of PSO lies in accelerating each particle toward its *pbest* and the *gbest* locations, with a random weighted acceleration at each time step as shown in Fig.1

Each particle tries to modify its position using the following information:

- the current positions,
- the current velocities,
- the distance between the current position and pbest,
- the distance between the current position and the gbest.

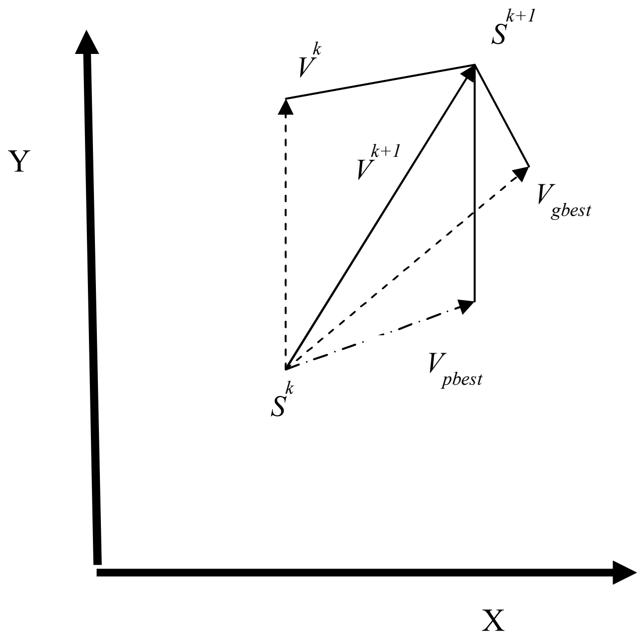

Fig. 1. Concept of modification of a searching point by PSO

where,

S^k	: current searching point.
S^{k+1}	: modified searching point.
V^k	: current velocity.
V^{k+1}	: modified velocity.
V_{pbest}	: velocity based on pbest.
V_{gbest}	: velocity based on gbest

The modification of the particle's position can be mathematically modeled according the following equation:

$$V_i^{k+1} = wV_i^k + c_1 rand_1(....)\times(pbest_i - S_i^k)$$
$$+ c_2 rand_2(....)\times(gbest_i - S_i^k) + ...(1)$$

where,

V_i^k velocity of agent i at iteration k,

w weighting function which is updated by the recursive formula (2), below

c_j weighting factors usually constant during the execution of the algorithm,

rand() uniformly distributed random number between 0 and 1,

S_i^k current position of agent i at iteration k,

$pbest_i$ pbest of agent i,

gbest gbest of the group.

The following weighting function is usually utilized in Eq. (1)

$$w = w_{Max} - \left(\frac{w_{Max} - w_{Min}}{Iter_{Max}} \times Iter\right) \qquad (2)$$

where w_{Max} denotes initial weights, w_{Min} indicates final weights, $Iter_{Max}$ means maximum iteration number and *Iter* is a sign of current iteration number. Finally, the new position of particle *i*, S_i^{k+1} , is calculated as shown in (3)

$$S_i^{k+1} = S_i^k + V_i^{k+1} \qquad (3)$$

III. PSO FOR OPTIMIZING PORTFOLIO

The modern portfolio theory (MPT) portfolio optimization problem is an ex-ante model of portfolio analysis, so to obtain the best performance in an ex-ante context the mean-variance model requires accurate forecasts of the future return and risk structure. The simplest and most widely used method to generate such future inputs is to rely on their historic ex-post values. However, the use of a portfolio selection procedure based on historical parameters that ignores this fact is likely to produce sub-optimal results in subsequent periods [12].
However, portfolio management does not only focus on the maximizing return but also on risk minimization. Therefore, good stock ranking is not enough for portfolio management; risk and return factors must be taken into account in terms of modern portfolio theory.

Modern portfolio theory is based on a reasonable trade-off between expected return and risk. Portfolio optimization model can be solved by quadratic programming (QP). But it can also be solved by PSO. Since it is a typical optimization problem, PSO is suitable for this task.

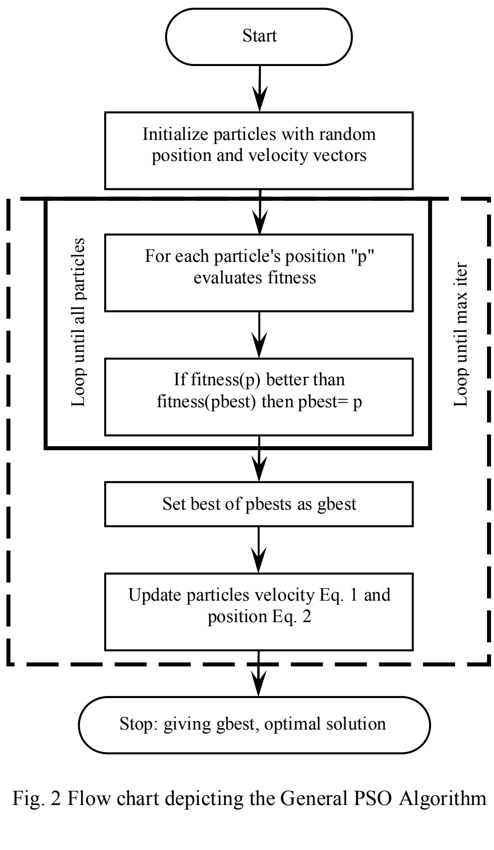

Fig. 2 Flow chart depicting the General PSO Algorithm

The major feature of PSO algorithms is their simplicity in implementation and high computational efficiency in solving optimization problems. We implement a PSO solver for constructing optimal risky portfolios using the basic form of the PSO algorithm as described in [10]. For the basic procedure of PSO for this problem a suitable particle representation is presented to encode its solution space and an appropriate objective function is designed.

The objective function is an important issue in PSO for solving the problem. In the portfolio optimization, the objective function must make a rational tradeoff between minimizing risk and maximizing return.

Among the various methodologies suggested, the most popular one is based on maximizing the well-known Sharpe ratio, a quantity which represents a measure of the amount of additional return (above the risk-free rate) a portfolio provides compared to the risk it carries [13]. This is done because most investors have a fundamental interest in achieving the best portfolio risk/return trade-off; the Sharpe tangency portfolio is particularly attractive as this portfolio is the one offering the highest ex-post mean return per unit risk. Therefore, the PSO can be performed by maximizing this objective function. The objective function for each particle is the indicator for PSO to perform the selection.

When the PSO model is adopted for portfolio selection, portfolio weights adjustment can be made under the control of optimizing Sharpe ratio. In the sequel, we illustrate how this can be achieved under the condition that both short sale and borrowing money are not permitted. Consequently, we consider the return of a typical portfolio which is given by [14]:

$$R_t = (1-\alpha_t)r^f + \alpha_t \sum_{j=1}^{m} \beta_t^{(j)} x_t^{(j)}, \quad (4)$$

Subject to: $$\begin{cases} 0 \le \alpha_t \le 1, \\ \sum_{j=1}^{m} \beta_t^{(j)} = 1, \\ 0 \le \beta_t \le 1, \end{cases}$$

where r^f, denotes the risk-free rate of return, x_t denotes returns of risky securities, α_t the proportion of total capital to be invested in the *j*-th risky securities and $\beta_t^{(j)}$ the proportion of α_t to be invested in the *jth* risky asset. Instead of focusing on the mean variance efficient frontier, we seek to optimize the portfolio Sharpe ratio (S_p) [15], with $S_p = M(R_T)/\sqrt{V(R_T)}$ given by [14]. In other words, the objective function to maximize is:

$$\max S_p = \frac{M(R_T)}{\sqrt{V(R_T)}} \quad (5)$$

where $M(R_T) = \frac{1}{T}\sum_{t=1}^{T} R_t$, is the conditional expected return and $V(R_T) = \frac{1}{T}\sum_{t=1}^{T}[R_t - M(R_T)]^2$, is a measure of risk or volatility, $\{x_t\}_{t=1}^{T}$ is the time series of the observed return series, in turn, adjusts the portfolio weights α_t and $\beta_t^{(j)}$, respectively. Maximizing the portfolio Sharpe ratio in effect balances the trade-off between maximizing the expected return and at the same time minimizing the risk. In implementation, we can simply use the PSO. Through the optimization process of PSO, the most valuable portfolio, i.e., good stock combination with optimal asset allocation can be mined and discovered to support investors' decision-making.

IV. DATA DESCRIPTION AND SIMULATION

For simulation, some good quality stocks with the highest rank are selected. These stocks can be revealed in terms of stock ranking. Every three months the top 50 stocks are determined by Tehran Stock Exchange Services Company (TSESC). Their ranking is based on harmonic mean of several variables such as liquidity, market share, EPS, P/E, and profitability ratio. It is reasonable to consider the common part of these lists as selected stocks. The closing prices of risky securities are needed to calculate for each stock by accumulating the return of each day. The daily data used in this study is a sample of stock the closing price obtained from the top 50 companies of Tehran Stock Exchange (**http://www.tsesc.com**). This sample consists of 12 securities. Risk-free rate of return comes from report of Central Bank of Iran (**http://www.cbi.ir**). The sample data span the period from January 1, 2003 to December, 31 2005 and consist of 484 observations (*T=484*).

The portfolio optimization is then performed for asset allocation. Consequently, the portfolio allocation, weight of each stock in the portfolio, will be obtained from PSO process by maximizing the objective function (Sharpe ratio). Therefore, the most valuable portfolio can be mined and discovered by PSO algorithm. For the sake of simplicity in demonstrations we calculate the Sharpe ratio of an equal weight portfolio for comparing the outcome of these two methods.

V. EXPERIMENTAL RESULTS

In this paper, a PSO algorithm applied to optimizing a risky portfolio. The algorithm proposed is a recent meta-heuristic. The algorithm has been tested on a typical instance and optimal result is obtained in a reasonable computing time. Simulation result demonstrates that the proposed PSO optimization algorithm is an effective (giving a better Sharpe Ratio) portfolio optimization approach. Thus this approach can mine the most valuable portfolio for investors. Table 1 shows the performance of optimal and equal weighted portfolios [5].

TABLE I
THE RETURN COMPARISON BETWEEN OPTIMAL PORTFOLIO AND EQUALLY WEIGHTED PORTFOLIO

	Risk-free rate	Excess Return	Risk	Sharpe ratio
Optimal portfolio	15%	165.3%	0.27	6.12
Equal weighted portfolio	15%	53.4%	0.23	2.325

The comparison between the optimal portfolio and equal weighted portfolio shows that the PSO approach to portfolio selection problem with a view towards the maximization of Sharp Ratio gives a more dependable and better result in a reasonable method and in a feasible implementation and practically applicable time for execution of the algorithm.

Further extensions or variations of PSO, such as lbest or Von Neumann PSO algorithms, can be considered as next proposals of this study.

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