Three Parameter Weibull Distribution Estimation Based on Particle Swarm Optimization

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***Abstract*—Through the simulation data, maximum likelihood estimation (M-L), ant colony algorithm (ACO) and particle swarm optimization (PSO) are used to compare the parameters estimation of three parameter Weibull distribution, the shortage of the maximum likelihood estimation in the three parameter solution process is analyzed, For the defects of ACO algorithm optimization result that it is easy to cause the local optimum (producing "premature" phenomenon), a solution based on the whole area iteration method proposed. By comparing the performance of the three algorithms under the index of fitness, efficiency, correlation, AD test and so on, the conclusion that ant colony algorithm and particle swarm optimization algorithm have better applicability in parameter estimation of three parameter Weibull distribution and particle swarm optimization algorithm is superior to ant colony algorithm and maximum likelihood estimation is drawn.**

Keywords—Weibull distribution; Parameters estimation; Ant colony algorithm; Fault bearing

I. INTRODUCTION

This Weibull distribution is one of the most common and important life distribution functions in the theory of reliability [1]. With its strong adaptability and robustness, the Weibull distribution is widely used as a probabilistic statistical model describing the distribution of mechanical life. The three parameter Weibull distribution increases the minimum failure time compared with the two parameter Weibull distribution, which can describe the reliability distribution rule of the product life cycle more accurately. But the corresponding complexity is also higher. At present, the method of parameter estimation for the three parameter Weibull distribution mainly includes moment estimation, least square method, linear regression estimation and maximum likelihood estimation. Yan Xiaodong et al. [2] compare the various estimation methods of Weibull distribution, and point out the scope of application of various algorithms. Yang Zhizhong et al. [3] use the least square method to obtain the shape parameters by linear regression and get the scale parameter and the position parameter with the interpolation method, however the solving process is relatively complex. Yang Moucun et al. [4] solve the three nonlinear transcendental equations of maximum likelihood method by the combination of the reduced order method and the dichotomous method, But there is a complicated calculation process and a higher requirement for the initial value.

In recent years, with the development of artificial intelligence algorithms, many scholars have used some intelligent algorithms for this problem, such as ant colony algorithm, genetic algorithm, simulated annealing method and so on. Although the precision of the ant colony algorithm is improved, the stability of the ant colony algorithm is poor, and the efficiency of the genetic algorithm is low, and the phenomenon of premature convergence is easy to appear. For example, Wang Hui et al. [5] use ant colony algorithm to estimate the parameters of the two parameter Weibull distribution model. Compared with particle swarm optimization and maximum likelihood estimation, he finds that the ant colony algorithm has good adaptability to the Weibull distribution model. It solved the difficulty of convergence when applying the traditional numerical method. At the same time, it is found that the convergence rate of ant colony algorithm is more than twice as fast as the particle swarm optimization algorithm. Dong Sheng et al. [6] use particle swarm optimization (PSO) to estimate the parameters of Weibull. And compared with other algorithms, it is shown that the particle swarm optimization is an effective method to estimate Weibull parameters in marine engineering environmental factors statistics.

Weibull distribution has always been a hot topic in the research of reliability theory, and in this theory, the parameter estimation and reliability evaluation index of the Weibull distribution are studied more. The main content of this paper is the application of particle swarm optimization and ant colony algorithm to the estimation of the three parameters in the Weibull distribution model and compared with the conventional maximum likelihood estimation (M-L). Through the simulation data, the comparison and analysis of the fitting accuracy and operation efficiency under the evaluation indexes such as fitness, running time and AD test are carried out. The results show that ant colony algorithm and particle algorithm are effective algorithms for estimating the three parameters of Weibull distribution, and particle swarm optimization algorithm is superior to ant colony algorithm and traditional maximum likelihood estimation.

II. ESTIMATION OF WEIBULL DISTRIBUTION PARAMETERS

Weibull distribution is one of the most commonly used distribution patterns describing the distribution law of life data of mechanical systems and their parts [5,6]. Because the number of parameters is different, it can be divided into single parameter, two parameter and three parameter Weibull distribution. The three parameters of the Weibull distribution and the probability density can be modeled as [7,8].

$$F(x) = 1 - \exp\left\{-(\frac{x-\gamma}{\beta})^{\alpha}\right\} \tag{1}$$

$$f(x)=(\frac{\alpha}{\beta})(\frac{x-\gamma}{\beta})^{\alpha-1}\exp\left\{-(\frac{x-\gamma}{\beta})^{\alpha}\right\} \tag{2}$$

where γ (>0) is the shape parameter, β (≥ 0) is the scale parameter, α (>0) is the shape parameter, and x the sample data.

A. Maximum likelihood estimation (M-L) of three parameter Weibull model

First, Suppose that the whole is $X \sim Weibull(\lambda,\beta,\alpha)$, $x_1, x_2, \cdots x_n$ is the n independent sample from the whole. Test data x_i are arranged in a small to large order. Likelihood function of Weibull distribution can be modeled as

$$\begin{aligned} L(\gamma,\beta,\alpha) &= \prod_{i=1}^{n} f(x_i;\gamma,\beta,\alpha) \\ &= \prod_{i=1}^{n}\frac{\alpha}{\beta}(\frac{x_i-\gamma}{\beta})^{\alpha-1}\exp(-(\frac{x_i-\gamma}{\beta})^{\alpha}) \\ &= \frac{\alpha^n}{\beta^{n\alpha}}\prod_{i=1}^{n}(x_i-\gamma)^{\alpha-1}\exp(-\sum_{i=1}^{n}(\frac{x_i-\gamma}{\beta})^{\alpha}) \end{aligned} \tag{3}$$

where $\gamma < \min(x_i)$, Taking logarithm in both sides of Eq. (3), yields

$$\begin{aligned} \ln L = n\ln\alpha - n\alpha\ln\beta + (\alpha-1)\sum_{i=1}^{n}\ln(x_i-\gamma) \\ -\sum_{i=1}^{n}(\frac{x_i-\gamma}{\beta})^{\alpha} \end{aligned} \tag{4}$$

Calculating the partial derivative of α、β、γ in Eq. (4), and making it equal to 0, yields

$$\begin{cases} \frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - n\ln\beta + \sum_{i=1}^{n}\ln(x_i-\gamma) \\ \quad -\sum_{i=1}^{n}(\frac{x_i-\gamma}{\beta})^{\alpha}\ln\frac{x_i-\gamma}{\beta} = 0 \\ \frac{\partial \ln L}{\partial \beta} = -\frac{n\alpha}{\beta} + \sum_{i=1}^{n}(\frac{x_i-\gamma}{\beta})^{\alpha}\frac{\alpha}{\beta} = 0 \\ \frac{\partial \ln L}{\partial \gamma} = -\sum_{i=1}^{n}\frac{\alpha-1}{x_i-\gamma} + \alpha\sum_{i=1}^{n}(\frac{x_i-\gamma}{\beta})^{\alpha-1} = 0 \end{cases} \tag{5}$$

The Eq. (5) is a nonlinear transcendental equation set. In solving the maximum likelihood estimation equations of Weibull distribution, three parameters are generally converted to two parameters. The following steps are as follows:

First, the location parameter γ is taken to a specified value, as shown below

$$\gamma_j = \frac{\min(x_i)}{m}\cdot j \tag{6}$$

where m is the decomposed number of the minimum value of the sample. Due to $\min(x_i)-\gamma_j > 0$, $j = 1,2,\cdots,m-1$. Then the three parameter equation of Eq. (5) can be transformed into a two parameter equation, as shown below

$$\begin{cases} \frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - n\ln\beta + \sum_{i=1}^{n}\ln(x_i-\gamma_i) \\ \quad -\sum_{i=1}^{n}(\frac{x_i-\gamma_i}{\beta})^{\alpha}\ln\frac{x_i-\gamma_i}{\beta} = 0 \\ \frac{\partial \ln L}{\partial \beta} = -\frac{n\alpha}{\beta} + \sum_{i=1}^{n}(\frac{x_i-\gamma_i}{\beta})^{\alpha}\frac{\alpha}{\beta} = 0 \end{cases} \tag{7}$$

The parameter β of the second function of the Eq. (7) is expunged and brought into the first function. the first function of the Eq. (7) can be expressed as

$$\frac{n}{\alpha} + \sum_{i=1}^{n}\ln(x_i-\gamma_i) - \frac{n}{\sum_{i=1}^{n}(x_i-\gamma_i)^{\alpha}}\sum_{i=1}^{n}(x_i-\gamma_i)^{\alpha}\ln(x_i-\gamma_i) = 0 \tag{8}$$

Eq. (8) is a function on the monotonous decreasing of α, When the parameter n of the function is expunged, $f(\alpha)$ can be obtained. A solution of α is calculated by the following dichotomy:

Step 1: Let σ_y represent the standard deviation of $y_1, y_2, \cdots y_n$, and $y_i = \ln(x_i - \gamma_i)$, n is the sample size. Let $I = 1.28/\sigma_y$ and calculate $f(I)$.

Step 2: If $f(I)>0$, find the smallest integer k that makes $f(I/2^k)>0$, and let $L=I/2^k$ $H=I/2^{k+1}$; If $f(I)<0$, find the smallest integer k that makes $f(2^k I)>0$, and let $L=2^{k-1}I$, $H=2^k I$.

Step 3: No matter what happens in step 2, there is always a solution of $f(\alpha)=0$ in the interval [L, H]. Let $M=(L+H)/2$ and calculate $f(M)$, if $f(M)>0$, $H=M$, otherwise $L=M$.

Step 4: Repeat calculation of step 3 by dichotomy, until $H-M<2I/10^{6}$.

The estimation of the shape parameters can be obtained by the above iterative process. The estimation value of the scale parameters can be obtained by substituting the shape parameters into Eq. (7). And then bringing $\hat{\alpha}$、$\hat{\beta}$ and $\hat{\gamma}_j$ into the equation (4), The function as shown below can be obtained:

$$\ln L_j = n\ln\hat{\alpha} - n\hat{\alpha}\ln\hat{\beta} + (\hat{\alpha}-1)\sum_{i=1}^{n}\ln(x_i-\hat{\gamma}_j) - \sum_{i=1}^{n}\left(\frac{x_i-\hat{\gamma}_j}{\hat{\beta}}\right)^{\hat{\alpha}} \tag{9}$$

Due to $j=1,2,\cdots,m-1$, We know, the number of $\ln L_j$ is m-1. Compare the value of $\ln L_j$, the position parameters $\hat{\gamma}_j$ corresponding to the maximum function value can be obtained. That is the estimated value of the position parameter $\hat{\gamma}$. It can be seen that the method of maximum likelihood estimation is to give the value of position parameters γ beforehand, and then the method of elimination and order reduction is used to solve the problem. This algorithm is simple and fast. But the computational accuracy and time consuming are related to the number of decomposed portions of the position parameter γ. It is known from the Eq. (6) that the greater the value of m is, the higher the precision and the longer the time it takes.

B. Ant colony algorithm (ACO) for three parameter Weibull model

Ant colony algorithm is a kind of "natural" algorithm inspired by ants' foraging behavior. It has good characteristics in solving nonlinear optimization problems. It combines the distributed computing and the positive feedback mechanism, which can quickly find out the better solution and avoid the premature convergence [6,7]. At the same time, the ant colony algorithm also has strong robustness, and is easy to combine with other algorithms. Therefore, ant colony algorithm is introduced into the Weibull distribution parameter optimization problem.

First, the objective function of the ant colony algorithm is determined, in order to keep the consistency of the algorithm, the objective function is treated with maximum likelihood estimation, as shown in Eq. (4). M ants are randomly placed on the solution space [-U1, U2]. Because the objective function contains three variables and the range of values is quite different (especially the bearing life samples), the solution space should be set up separately. In the solution space, the current position of the i ants $\boldsymbol{x}_i^1$ and the next position $\boldsymbol{x}_i^2$ are initialized, and the target values $f(\boldsymbol{x}_i^1)$ and $f(\boldsymbol{x}_i^2)$ are calculated. The minimum position $\boldsymbol{x}_{min}$ and the minimum function value $f_{\min}$ in this cycle are recorded. According to the following function, $\Delta\tau_i$、τ_i^{k+1}、ξ_{ij} and p_{ij} is successively calculated.

The increment of the individual pheromone of the i ants $\Delta\tau_i$ can be obtained by the following equation

$$\Delta\tau_i = \begin{cases} f(\boldsymbol{x}^1)-f(\boldsymbol{x}^2), & f(\boldsymbol{x}^1)>f(\boldsymbol{x}^2) \\ 0, & \text{el se} \end{cases} \tag{10}$$

The update of the i ants' individual pheromone can be expressed as

$$\tau_i^{k+1} = \rho\tau_i^k + \Delta\tau_i \tag{11}$$

The expectation between i and j ants in this cycle can be obtained by the following equation

$$\xi_{ij} = \begin{cases} f(\boldsymbol{x}_i)-f(\boldsymbol{x}_j), & f(\boldsymbol{x}_i)>f(\boldsymbol{x}_j) \\ 0, & f(\boldsymbol{x}_i)\le f(\boldsymbol{x}_j) \end{cases} \tag{12}$$

The expectation of the transfer of i ants to j ants can be obtained by the following equation

$$p_{ij} = \begin{cases} \dfrac{\tau_j^u\xi_{ij}^v}{\sum_{j=1}^{m}\tau_j^u\xi_{ij}^v}, & \xi_{ij}>0 \\ 0, & \xi_{ij}\le 0 \end{cases} \tag{13}$$

If $p_{ij}>0$, The position of the i ants is updated to the position of the j ants. The subscript i, j is the ant label, $1\le i, j\le m$; m represents total number of ants. Label 1, 2 indicate the before and after of the ant moves. ρ is the volatilization factor of pheromone ,the value is 0.1~0.9. u is the weight of the size of the pheromone's effect on the selected path, the value is 0~5. v is the weight of self – heuristics, the value is 0~5. $\boldsymbol{U}$ is interval length; rand is a random number between 0~1; k is the number of iterations. The next position of the m ants is updated by Eq. (14) or (15)

$$\boldsymbol{x}_i^2 = \boldsymbol{U}_1 + (\boldsymbol{U}_1 - \boldsymbol{U}_2)\times rand \tag{14}$$

where $\boldsymbol{U}_1$ is the lower limit of the interval, and the $\boldsymbol{U}_2$ is the upper limit of the interval.

$$x_i^2 = x_i^1 + (U_1/2 + (U_1 - U_2)/2) \times rand \times r^k \tag{15}$$

where x_i^1 is the best position of the two positions in the last iteration or the position after the transfer, r is the interval reduction factor, and k is the iterative step.

There are two ways to update the next location of the ant colony algorithm. Eq.(14) is the update method called full area search iteration. Eq. (15) is a traditional way of moving in the x_i^1 neighborhood.

If the following terminating conditions are satisfied:(1) The maximum number of iterations is satisfied;(2) The function value of the two time is less than the given value $\varepsilon : |f_{k+1} - f_k| \le \varepsilon$, then stop iterating. Continue to iterate if the terminating condition is not satisfied. Finally get the x_{k+1}, where $x_{k+1}(1)$ is the estimated value $\hat{\alpha}$, $x_{k+1}(2)$ is the estimated value $\hat{\beta}$, $x_{k+1}(3)$ is the estimated value $\hat{\gamma}$.

When the next position is updated in the ant colony algorithm, Compared with the traditional way of moving in the x_i^1 neighborhood, the full area search iteration has the advantages of less risk of falling into the local optimal value and high computing efficiency.

C. Particle swarm optimization (PSO) for three parameter Weibull model

Particle swarm optimization algorithm is an intelligent optimization algorithm based on Group, it has excellent characteristics in solving nonlinear problems. In the iterative search process, the location and speed of the iterative process are corrected by the local optimal solution *pbest* and the global optimal solution *gbest*, thus completing the final optimization. Particle swarm optimization is not a high requirement for the initial value and the following iterative equation is used to iterate the velocity and position of the particle:

$$\begin{cases} v^{t+1}(i,k) = wv^t(i,k) + c_1 rand\left[pbest^t(i,k) - x^t(i,k)\right] \\ \qquad + c_2 rand\left[gbest^t(i,k) - x^t(i,k)\right] \\ x^{t+1}(i,k) = x^t(i,k) + v^{t+1}(i,k) \end{cases} \tag{16}$$

where i is the sequence number of a particle; k is the number of unknowns and the k coordinate components of the space in which the particle is located. t is the T iteration; w is the inertia weight, the value is 0.1~0.9; c1, c2 is the acceleration factor, the value is 1~2, the rand is the random number, and the value is 0~1.

The estimation of Weibull distribution parameters by particle swarm optimization can be converted to the optimization problem of the following functions:

$$\begin{cases} \min f(\boldsymbol{x}) = f(x_i \mid \alpha, \beta, \gamma) = -\ln L \\ \quad = -\{n\ln\alpha - n\alpha\ln\beta + (\alpha-1)\sum_{i=1}^{n}\ln(x_i-\gamma) \\ \quad -\sum_{i=1}^{n}(\frac{x_i-\gamma}{\beta})^{\alpha}\} \\ s.t.\ \ \alpha,\beta,\gamma > 0,\ \gamma < \min(x_i) \end{cases} \tag{17}$$

The steps of the particle swarm optimization algorithm for the estimation of the Weibull distribution parameters are as follows:

Step 1: Set the number of iterations, the number of initial populations, the values of c1 and c2. Because the range of three variables in the objective function varies greatly, the solution space and initial velocity are set up respectively. Then the initial position is taken as the initial best position of each particle, pbest0. The fitness value of each particle is calculated, and the corresponding position of the minimum target function is found as the initial global optimal position $gbest^0$.

Step 2: The iteration loop is entered, and the speed and location of each particle is updated by the Eq. (16). The fitness value after iteration is compared with its best value. If the particle's current fitness value is smaller than its optimal value, then the new fitness value is used to replace the best value of the previous round, that is, $pbest^i=x_i$.

Step 3: Global optimal value is updated. The optimal fitness value of the whole particle (the global optimal value of the present wheel) is compared with the best fitness of each particle (local optimal value). If the best fitness value of each particle is less than the global fitness value, the global optimal value is updated, that is, $gbest^i=pbest^i$.

Step 4: After completing the 2 and 3 steps, then the speed and position of each particle is updated according to Eq. (16) until the number of iterations that are set is completed or meet the predetermined accuracy requirements.

Step 5: Finally, x_{k+1} is obtained, where $x_{k+1}(1)$ is the estimated value $\hat{\alpha}$, $x_{k+1}(2)$ is the estimated value $\hat{\beta}$, and $x_{k+1}(3)$)is the estimated value $\hat{\gamma}$.

III. CASE AND RESULT ANALYSIS

A. Evaluation index of Weibull distribution

Define (1)The correlation coefficient ρ is an important index for evaluating the correlation degree of the sample and fitting equation, can be described as

$$\rho = \frac{\sum_{i=1}^{n} Y_i v_i - n\overline{Y}\cdot\overline{v}}{\sqrt{\sum_{i=1}^{n}(Y_i-\overline{Y})^2}\sqrt{\sum_{i=1}^{n}(v_i-\overline{v})^2}} \tag{18}$$

where

$$\begin{cases} Y_i = \hat{\alpha}\ln(x_i - \hat{\gamma}) - \hat{\alpha}\ln\hat{\beta} \\ v_i = \ln\{-\ln(1-F(x_i))\} \end{cases} \quad (19)$$

where x_i is sample data, n is data length, $\hat{\gamma}$ is the estimated value of shape parameter γ , $\hat{\beta}$ is the estimated value of scale parameter β, $\hat{\alpha}$ is the estimated value of shape parameter α, $\overline{Y}$ is the mean value of Y_i , and $\overline{v}$ is the mean value of v_i . $F(x_i)$ can be calculated by the medium rank calculation Eq. (12)

$$F(x_i) = (i-0.3)/(n+0.4) \quad (20)$$

where i is the serial number of the sample data, and n is the number of the sample data.

(2) AD test is a common test method in reliability analysis. It is more accurate than W^2 test method and is suitable for small sample. Where the better the fitting effect of the distribution function and the sample data, the smaller the value of AD is.

$$AD = \sum_{i=1}^{n}\frac{1-2i}{n}\{\ln[F(x_i)+\ln[1-F(x_{n+1-i})]]\}-n \quad (21)$$

where

$$F(x_i) = 1 - e^{-(\frac{x_i-\hat{\gamma}}{\hat{\beta}})^{\hat{\alpha}}} \quad (22)$$

The observational significant level is:

$$OSL = 1/(1+e^{[-0.10+1.24\ln(AD^*)+4.48AD^*]}) \quad (23)$$

where

$$AD^* = (1+\frac{0.2}{\sqrt{n}})AD \quad (24)$$

If $OSL \le 0.05$, the overall sample can be determined (95% confidence) to be inconsistent with the three parameter Weibull distribution, otherwise, the overall sample conform to the three parameter Weibull distribution.

(3)In order to display the fitting degree of the sample data set intuitively, the P-P graph can be used to test whether the data conforms to the Weibull distribution. Generally, when the data conforms to the Weibull distribution, each point in the P-P graph is approximately a straight line. If the sample point is located near the straight line, the fitting effect is good. The methods are as follows:

First, the sample x_i is order from small to large, the Weibull distribution is linearized, Eq. (3) is rewritten as:

$$1-F(x_i) = \exp\left\{-(\frac{x_i-\hat{\gamma}}{\hat{\beta}})^{\hat{\alpha}}\right\} \quad (25)$$

Then, the two sides of the Eq. (25) are taken logarithms twice:

$$\ln\{-\ln(1-F(x_i))\} = \hat{\alpha}\ln(x_i-\hat{\gamma}) - \hat{\alpha}\ln\hat{\beta} \quad (26)$$

Finally, the estimated values $\hat{\gamma}$, $\hat{\beta}$ and $\hat{\alpha}$ are taken into the Eq. (26), $\ln(x_i-\hat{\gamma})$ is taken as a horizontal coordinate, denoted by X. $\hat{\alpha}\ln(x_i-\hat{\gamma})-\hat{\alpha}\ln\hat{\beta}$ is taken as a ordinate, denoted by Y. And the linear equation $Y=aX+b$ is drawn, where $a = \hat{\alpha}$, $b = -\hat{\alpha}\ln\hat{\beta}$.

At the same time, the corresponding sample point (u_i, v_i) is also drawn on the probability distribution map. The value of (u_i , v_i) can be modeled by the following function

$$\begin{cases} u_i = \ln(x_i-\hat{\gamma}) \\ v_i = \ln\{-\ln(1-F(x_i))\} \end{cases} \quad (27)$$

where, the cumulative frequency $F(x_i)$ is calculated by the median rank.

If the sample is obtained by simulation, Then the u_i calculation in the sample point (u_i, v_i) can take the $\ln(x_i)$ directly. Because x_i is the sample data before the translation, the v_i remains the same.

By drawing the P-P diagram, the degree of deviation between the sample point and the estimate can be observed directly.

B. Case and result analysis

In order to further illustrate the superiority of particle swarm optimization and ant colony algorithm to the traditional algorithm, the numerical simulation method is used to verify it, The specific practice is as follows:

In Matlab, the analog data of the three parameter Weibull distribution with a certain number and subject to the set parameters is generated by the method of random simulation. And maximum likelihood estimation, ant colony algorithm and particle swarm optimization algorithm are used for parameter estimation. Then correlation degree, AD goodness of fit and operation efficiency are compared, and P-P diagram is drawn.

Example 1, with parameters of $\alpha = 2$,β=1000,γ=100, 100 samples that conform to the Weibull distribution are produced. In this simulated data, the minimum data is x_1=151 , and $-\ln L$= -746.6422 . When maximum likelihood estimation is used, γ takes a step length of $151/2000$, from 0 to 151. The calculation is carried out in the Eq. (6) method, and the corresponding shape parameter α_i , the scale parameter β_i and the displacement parameter γ_i , and the value of the $-\ln L(x_i \mid \alpha,\beta,\gamma)$ are recorded.

When the ant colony algorithm is used to calculate, the initial values of the shape parameter α, the scale parameter β and the position parameter γ are the random numbers of [0,4], [500,1500], [50,150] respectively. When the position of ant colony algorithm is updated, the update method of Eq. (14) is recorded as ACO1, the number of iterations is set to 60 times, and the number of ants is 300.The update method of formula (15) is recorded as ACO2, the number of iterations is set to 600 times, and the number of ants is 200.

When the particle swarm optimization is used, the number of iterations is set to 60 times, and the number of particles is set to 50. The initial values of the shape parameter α, the scale parameter β and the position parameter γ are the random numbers of [0,4], [500,1500], [50150] respectively. The inertia weight w is set to 0.8, the acceleration factor c1, and the c2 is set to 1, 12 respectively.

The results are as follows:

TABLE 1. FITTING RESULTS OF THE FIRST GROUP OF DATA

EM	M-L	ACO1	ACO2	PSO
IV	-	Rand	-	Rand
$\hat{\alpha}$	1.9213	1.9453	1.9413	1.9207
$\hat{\beta}$	930.9240	953.5423	966.0059	930.8861
$\hat{\gamma}$	111.3717	109.3451	107.8958	110.4021
$\ln L$	-746.3066	-746.3192	-746.5024	-746.3066
ρ	0.9420	0.9437	0.9329	0.9421
AD	0.2458	0.2699	0.4204	0.2458
OSL	0.6665	0.6902	0.3162	0.6666
T/s	2.0655	1.7056	9.7259	0.3122

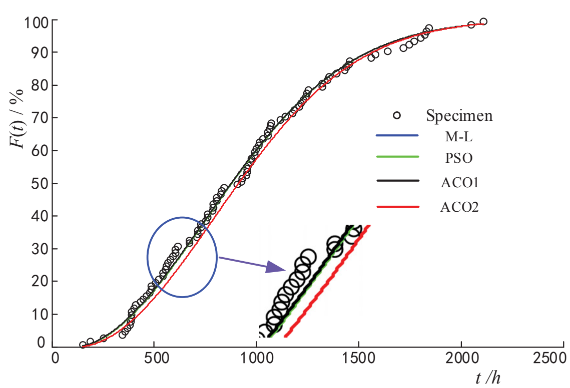

Fig. 1 cumulative distribution curve of Weibull distribution

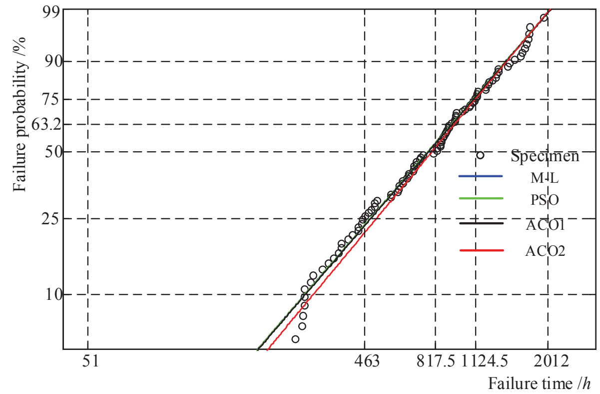

Fig.2. P-P diagram of Weibull distribution

Through the cumulative distribution curves and P-P diagrams of four algorithms shown in Figures 1 and 2, we can see that the 4 algorithms are more effective and the sample points are evenly distributed on both sides of the curve. But the fitting effect of ACO2 is relatively poor.

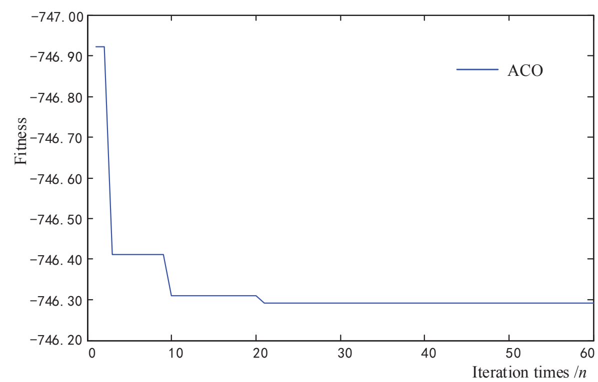

Fig.3. the iterative process of the ant colony algorithm using Eq. (13)

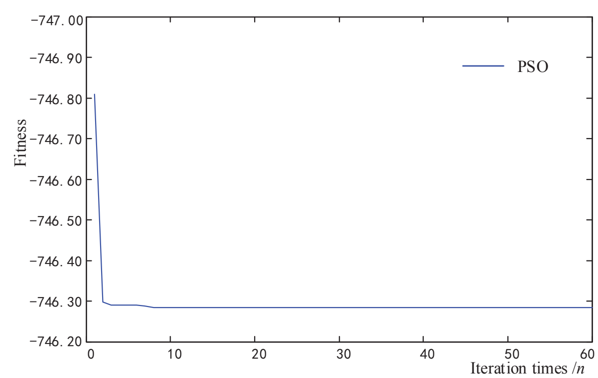

Fig.4. the iterative process of particle swarm optimization

From the comparison between figure 3 and Figure 4, we can see that the iterative convergence speed of particle swarm optimization algorithm is obviously better than that of ant colony algorithm. From the contrast of Table 1, we can see that the computation efficiency of particle swarm optimization algorithm is higher than that of ant colony algorithm.

Example 2, a group of bearing life test simulation data, parameters is set as follows: $\alpha = 1.3$, $\beta = 2200$, $\gamma = 50$, analog sample capacity is 20. In this simulated data, the minimum data is x_1=56, $-\ln L = $-176.6984.

When the maximum likelihood estimation is adopted, the step length γ is $106/2000$. Because of the number of samples is small, so in order to obtain higher accuracy, the number of iterations of ant colony algorithm (ACO1) is 100 times and the number of ants is 500. The number of iterations of ACO2 is set to 700 times, and the number of ants is 200. The number of iterations of particle swarm optimization algorithm is set to 100 times, the number of particles is 100, the initial values of the shape parameter α, the scale parameter β and the position parameter γ are the random numbers of [0,3], [1500,2500], [30,100] respectively. The results are as follows:

TABLE 2. FITTING RESULTS OF SECOND SETS OF DATA

EM	M-L	ACO1	ACO2	PSO
IV	-	Rand	Rand	Rand
$\hat{\alpha}$	1.1.3843	1.3688	1.4342	1.3843
$\hat{\beta}$	1895.0	1877.7	1903.5	1895.0
$\hat{\gamma}$	48.3675	50.8031	48.0524	48.3692
$\ln L$	-176.1752	-176.1969	-176.1760	-176.1752
ρ	0.9633	0.9645	0.9606	0.9633
AD	0.2458	0.2699	0.4204	0.2458
OSL	0.8151	0.8099	0.7702	0.8151
T/s	1.9825	5.4965	9.4285	0.4589

Compared with table 1, table 2, figure 1, and figure 2, we can draw the following conclusions:

When the 4 fitting algorithm are used to estimate the parameters, the shape parameter $\hat{\alpha}$, scale parameter $\hat{\beta}$, location parameter $\hat{\gamma}$, fitness value and correlation value estimated by M-L, ACO1, ACO2 and PSO algorithm are basically close to each other. It shows that particle swarm optimization and ant colony algorithm are suitable for the three parameter estimation of Weibull distribution.

In terms of operational efficiency, particle algorithm is superior to M-L algorithm, and M-L algorithm is better than ant colony algorithm. From the point of view of accuracy, ant colony algorithm is slightly better than particle algorithm and M-L algorithm. At the same time, we can also find that particle algorithm can get better accuracy through fewer particles and times of iteration, while the ant colony needs to set larger population number and iteration number.

In the ant colony algorithm, the updating method of Eq. (14) is better than that of Eq. (15) in terms of computation efficiency and computation accuracy. This shows that the whole area search and iteration update method is better than the update on the basis of the last iteration.

The M-L algorithm of maximum likelihood estimation is to transform three parameters into two parameters. When the location parameter $\Delta\gamma$ is large, the computation time is small and the computation accuracy is low. When the value of $\Delta\gamma$ is small, the computation time will increase exponentially, and the computation accuracy will increase. The ACO and PSO algorithms can avoid this contradiction. The ant colony algorithm and particle swarm optimization algorithm are different from the priority location parameter estimation, and then estimate the shape parameters and scale parameters. These two methods provide a new way for the three parameter Weibull distribution.

In large sample cases, such as example 1, in small sample cases, such as instance 2. Comparing two examples, we can find that if we increase the number of population and number of iterations properly, we can get higher computation accuracy. Meanwhile, we can also find that the time consumption of ant colony algorithm increases significantly. The time-consuming increase of the particle swarm is smaller.

In the method of parameter estimation of Weibull distribution, Compared with the maximum likelihood estimation, the advantage of ant colony algorithm and particle swarm optimization is that when the parameter estimation of four parameter and other complex Weibull distribution forms, such as competitive Weibull distribution, we can simply add variables and it is robust, which is difficult to achieve by maximum likelihood estimation. At the same time, the advantage of ant colony algorithm and particle swarm optimization is that it is convenient to change the objective function. If we use the least squares method as the objective function, we only need to simply replace the objective function.

IV. CONCLUSIONS

In this paper, the ant colony algorithm and particle swarm optimization algorithm are applied to the three parameter estimation of Weibull distribution. Through simulation data, we compare the fitting effect of maximum likelihood estimation, two kind of ant colony algorithm and particle swarm optimization algorithm in Weibull distribution parameter estimation. The results show that the ant colony algorithm and particle swarm optimization algorithm are effective algorithms for the three parameter Weibull distribution parameter estimation, and have the advantages of fast computation speed, high accuracy and robustness. The results also show that the particle swarm optimization is better than the ant colony algorithm and the maximum likelihood estimation. The full area search iteration updating method of ant colony algorithm is better than the traditional way of moving in the x_i^1 neighborhood.

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References

[1] Q.G. LI. Data Processing Method of Rolling Bearing Life and Fatigue Test [J]. BEARING, 1978, (03):33-52+79-13.

[2] X.D. Yan, X. Ma, R.T. Deng, et al. Comparison of the Parameters Estimation Methods for 3-Parameter Weibull Distribution [J]. JOURNAL OF NING BO UNIVERSITY (NSEE), 2005, (03):301-305.

[3] Z. Z. Yang, R. Y. Liu. Improvement of Parameter Estimation of Three Parameter Weibull Distribution [J]. Chinese Journal of Engineering Mathematics, 2004, (02):281-284+267.

[4] M.C. Yang, H. Nie, Advanced Algorithm for Maximum Likelihood Estimation of Three Parameter Weibull Distribution [J]. Journal of Nanjing University of Aeronautics & Astronautics, 2007, 39(1):31-34.

[5] B. Jie, X.H. Chen. The Equipment Life Prediction Based on Weibull Distribution and Genetic Algorithm [J]. J. of Math, 2016, 36(02):385-392.

[6] H. Wang, X. P. Meng, X. Ji, et al. The application of ant colony algorithm in parameter estimation of Weibull distribution model [J].Changchun Inst. Tech. (Nat.Sci.Edi.), 2014, 15(03):116-119.

[7] S. Dong, Y. Han, S. S. Tao, et al. Parameters Estimation for Weibull Distribution with Particle Swarm Optimization [J]. PERIODICAL OF OCEAN UNIVERSITY OF CHINA. 2012, 42(06):120-125.

[8] C.Y. Zheng, X. M. Liu, S. Huang. Estimating parameters of software reliability models by ant colony algorithm [J].Journal of Computer Applications, 2012, 32(04):1147-1151.

[9] Y. Deng. Parameter Estimation of Three-Parameter Weibull Distribution and Its Application in Reliability Analysis [J].JOURNAL OF VIBRATION AND SHOCK, 2015, 34(05):78-81.

[10] W. X. LIU The mechanical reliability design [M]. Beijing: Tsinghua University Press, 1996.

[11] D. Ling. Research on Weibull Distribution and Its Applicat-ion in Mechanical Reliability Engineering. [D]. University of Electronic Science and Technology of China, 2011.