The Study of Model for Portfolio Investment Based on Ant Colony Algorithm

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***Abstract*—The risk and benefits are consider synthesizely in portforio investment based on the Markowitz portfolio theory. A multi-objective programming model of portforio investment is established and studied the model solution with the ant group algorithm, then obtained a better result compared to using the Lingo model. Unified the ant group algorithm and the modern computer's formidable operational capability, making the investor to be more convenient in the actual operation**

***Keywords-* multi-objective programming, portfolio investment, ant colony algorithm, Lingo1**

I. INTRODUCTION

China's stock market is full of vigor and vitality, and also full of confusion and risk. Speeding up the research step in the field of financial and investment will provide certain theoretical support for the development of capital market and investors to improve the investment activities,so it is imperative to explore operational mechanism which adapts to Chinese socialist market economy.for a long time, the investor's decision-making depends on the experiences in the practice and has not risen to the theory altitude. Since the 1950s Markowitz founded the modern portfolio theory, many scholars underwent the difficult research work to carry on and have enriched and developed this theory,so they obtained many effective methods in solving many problems of the portfolio, but how to make the portfolio model become optimization decision-making model is an urgent problem that needs to be solved.

In recent years, with the development of artificial intelligence, applied the intelligent optimization method in the investment profolio question has become the broader research area. This paper used the continuous optimization ant algorithm to solve the optimal solution problem of the portfolio model, the experimental result has indicated that this model is accuracy and validity.

II. MULTI OBJECTIVE DECISION-MAKING MODEL OF PORTFOLIO INVESTMENT

Supposed invest n kinds securities, the kind of i securities's returns rate is $r_i\,(i=1,2,\cdots,n)$. Due to r_i receives the influence of stock market's various factors, thus r_i could be regarded as a random variability. $R_i = E(r_i)$ is the mean of r_i , $\sigma_i^2 = E(r_i - R_i)^2$ is the variance of r_i , x_i indicates t he proportion of portfolio investment which i security invests in($\sum_{i=1}^{n} x_i = 1$).Then the portfolio investment's expectation returns ratio and thevariance respe-

ctively are: $$R = \sum_{i=1}^{n} x_i R_i, \sigma^2 = \sum_{i=1}^{n}\sum_{i=1}^{n} x_i x_j \sigma_{ij} \qquad (1)$$

Here $\sigma_{ij} = E(r_i - R_j)(r_j - R_j)$ is the covariance of i-securities and j-securities then multi-objective program-ming model of portfolio investment can be reveived:

$$\max R = \sum_{i=1}^{n} x_i R_i$$
$$\min \sigma^2 = \sum_{i=1}^{n}\sum_{i=1}^{n} x_i x_j \sigma_{ij} \qquad (2)$$
$$s.t \begin{cases} \sum_{i=1}^{n} x_i = 1 \\ x_i \ge 0 \quad i = 1,2,\ldots,n \end{cases}$$

$x_i \ge 0$ expressed that our country does not permit short selling.

$X = (x_1, x_2, \ldots, x_n)^T, R = (R_1, R_2, \ldots, R_n)^T, C = (\sigma_{ij})_{n\times n}$

C is symmetric positive definite matrices,R is covariance matrix , $E = (1,1,\ldots,1)^T$

and (2) expresses the matrix: max $R=R^TX$

$$\max R = R^T X$$
$$\min \sigma^2 = X^T C X \qquad (3)$$
$$s.t \begin{cases} E^T X = 1 \\ X \ge 0 \end{cases}$$

$$\text{Namely: } \min F(x) = \left\{ \left[f_1(x), f_2(x) \right]^T \Big| \sum_{i=1}^{n} x_i = 1 \right\} \qquad (4)$$
$$f_1(x) = -R^T X, f_2(x) = X^T C X$$

Hence constructed a multi-objective decision making model to be able to optimize the income and the risk simultaneously. The solution of multi-objectives optimize emerges one after another incessantly, but transforming the multi-objective questions as the simple target question is the basic philosophy, with the aim that using a maturer simple-target optimize method to obtain optimal solution. Each kind of solution may divide into two kinds approximately: One kind aims at optimizing one component of the multi-objective functions, but takes other components become the constraints, or constructs a sequence simple target to optimize; Another kind make the multi-objective function

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$$\min F(\sigma^2,-R)=\mu\sigma^2-(1-\mu)R$$
$$s.t\begin{cases}E^T X=1\\X\geq 0\end{cases} \quad (5)$$

Aversion coefficient μ 's scope is $0 \leq \mu \leq 1$ .The great number showed that the investor cannot accept the risk, when $\mu=1$ the investors can completely avoid risks.

III. ANT COLONY ALGORITHM

To determine the fitness function: In this paper, the constraints will join the objective function as the the penalty factorization, the fitness function will be:

$$\min F(\sigma^2,-R)=\mu\sigma^2-(1-\mu)R+M\left|E^T X-1\right| \quad (6)$$

In order to achieve the ant colony algorithm's search process, the structure of the transition probability criteria are as follows: Supposed there are m groups artificial ant, each group has n artificial ants, starts to put the stochastically division region of the solution space $[0,1]$'s $l\times n$ in certain positions randomly, each regional ant's condition transition probability is:

$$p_{ij}=\begin{cases}(\tau_j)^{\alpha}(\eta_{ij})^{\beta},\eta_{ij}<0 \quad \cdot \cdot \\0, \quad 其 \quad \text{else}\end{cases} \quad =1,2,\ldots,l \quad (7)$$

Therefore used the group ant's unceasingly moves in $l\times n$ division regional in $[0,1]$, as well as in some region's part random searching carries on, ant k's shift and tsearch's rule in the region i is:

$$\begin{cases}\arg\max\{\ p\ ij\ \},\ \text{Shift region j to conduct random search } j = i, j=1,2,\ldots,l\\ \text{else}, \quad \text{Conduct random search}\end{cases} \quad (8)$$

The renewal equation of the egion j 's information element is:

$$\tau_j(t+1)=\rho\tau_j(t)+\sum_{k=1}^{m}\Delta\tau_j^k\ ,\quad j=1,2,\ldots,l \quad (9)$$

$$\Delta\tau_j^k=\begin{cases}QL_j^k, L_j^k>0\\0, \quad L_j^k\leq 0\end{cases} \quad j=1,2,\ldots,l$$

In the formula, $\Delta\tau_j^k$ reflected the ant k's increasing attract intensity in this circulation in the region j 's partial search; L_j^k expressed the objective function's change quantity of this circulation ant k's partial search in the region j, the definition is:

$$L_j^k=f(x_{j0}^k)-f(x_j^k) \quad (10)$$

$$x_{ij}=x_{i0}+\frac{1}{l}(j-1+\gamma_j^k) \quad (11)$$

$$x_{ij0}=x_{i0}+\frac{1}{l}(j-1+\gamma_{j0}^k) \quad (12)$$

$$i=1,2,\ldots,n \quad j=1,2\ldots,l$$

Among them γ_j^k, γ_{j0}^k are the random number which obey the uniform distribution in $[0,1/l]$ and the random number which express the ant k 's partial search in this circulation in the region j,

$X_j^k=[x_{1j}^k,x_{2j}^k,\ldots,x_{nj}^k]$ and $X_{j0}^k=[x_{1j0}^k,x_{2j0}^k,\ldots,x_{nj0}^k]$ are the search point 's position vector of the n Uygur space.

IV. COMPUTATION EXAMPLE

This article gave a combination about two kinds of securities to confirm this model, each securities' returns ratio vector was: $R=(0.151,0.137)$. The two securities' returns ratio shown in Table 1.

Table 1 securities returns

Economic conditions	Possible returns ratio		Probability
	Securities 1	Securities 2	
1	-0.188	0.188	0.2
2	0.26	-0.248	0.1
3	0.221	0.238	0.4
4	0.246	0.27	0.2
5	0.253	-0.246	0.1
Mean	0.151	0.137	-

In the experiment various algorithms parameter's value respectively is: Ant's Information density $Q=1$ which releases in various region's search, the various region's attraction intensity durable coeffi cient $\rho=0.7$ the attraction intensity heuristic factor $\alpha=1$ the expectation heuristic factor $\beta=1.5$, the operation stop condition is the difference between optium value which the ant algorithm achieved and the corresponding theory optimum value of optimized question namely: $\delta=0.001$; The district number of function solution space $l=10$, and the number of ants group is $m=9$. In this paper, based on different risk preferences, separately obtained different combinations when $\mu=0.3$, $\mu=0.4$, $\mu=0.5$, $\mu=0.6$, $\mu=0.7$, $\mu=0.8$.

Table 2 two model's income portfolio in different risk preference

risk preferences	Algorithm				
	Securities 1	Securities 2	σ^2	R	Objective Function Value
0.3	0.834	0.1659	0.023	0.149	-0.1009
0.4	0.754	0.2458	0.0187	0.1479	-0.085
0.5	0.706	0.2938	0.017	0.1472	-0.0693
0.6	0.674	0.3257	0.0163	0.1468	-0.0538
0.7	0.651	0.3486	0.016	0.1465	-0.0348
0.8	0.634	0.3657	0.0158	0.1462	-0.0229
risk preferences	Lingo				
	Securities 1	Securities 2	σ^2	R	Objective Function Value
0.3	0.723	0.277	0.0175	0.1474	-0.0977
0.4	0.683	0.3172	0.0165	0.1469	-0.0814
0.5	0.659	0.3414	0.016	0.1464	-0.0651
0.6	0.643	0.3575	0.0159	0.1463	-0.0489
0.7	0.631	0.369	0.0158	0.1462	-0.0327
0.8	0.622	0.3776	0.0157	0.1461	-0.0166

The above result showed that under each kind of risk condition, the objective function value obtained by this article's algorithm is smaller than the objective function value obtained by lingo, which indicated that the algorithm in solving the multi-objective programming question is better than lingo.

V. CONCLUSION

According to the present securities-investment market's situation of China(do not allow short-selling secure-Ties), proposed a objective model of the securities investment combination optimization under conditions of the nonnegative investment ratio, and designed ant group algorithm to solve this model's continual optimazation. Through the example of computer simulation, we can see that this algorithm is effective in solving the multi-objective programmings and in optimizing portfolio investment's application. Unified the ant group algorithm and the modern computer's formidable operational capability, making the investor to be more convenient in the actual operation.

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