

## Part 1:

1.)

$$\begin{array}{cc}
 X = \begin{pmatrix} -2 \\ -5 \\ -3 \\ 0 \\ -8 \\ -2 \\ 1 \\ 5 \\ -1 \\ 6 \end{pmatrix} & Y = \begin{pmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{pmatrix} & \text{add bias to } X & X = \begin{pmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{pmatrix} & Y = \begin{pmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{pmatrix}
 \end{array}$$

The formula to compute thetas is,  $\theta = (X^T X)^{-1} (X^T Y)$

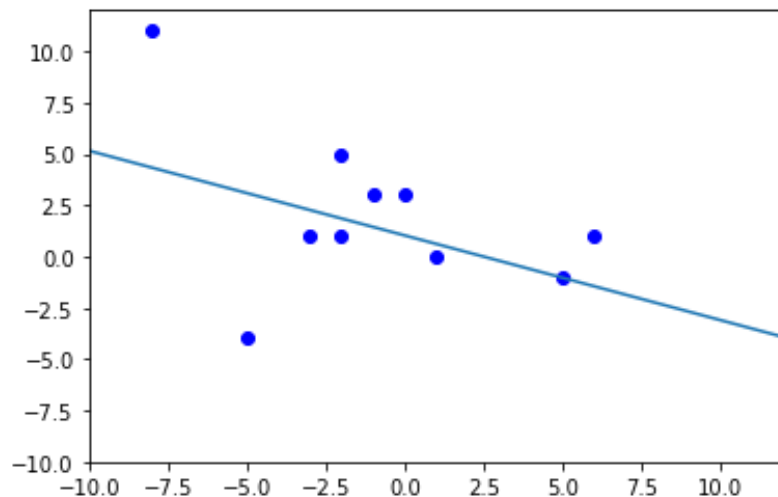
Using this with our X and Y matrices gives us the following  $\theta$  matrix

$$\begin{pmatrix} 1.028589 \\ -0.4126787 \end{pmatrix}$$

$$\theta_0 = 1.028589, \theta_1 = -0.4126787$$

The equation for our line of best fit is:  $y = (-0.4126787)x + 1.028589$

The line with the dataset points are plotted below



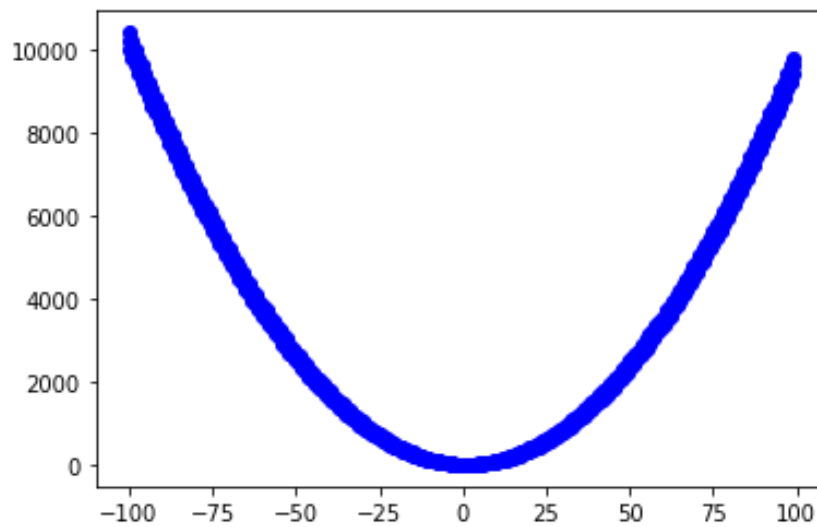
2.)

a. Given  $J = (x_1 + x_2 - 2)^2$

$$\frac{\partial J}{\partial x_1} = 2x_1 + 2x_2 - 4 \quad \frac{\partial J}{\partial x_2} = 2x_1 + 2x_2 - 4$$

For both  $x_1$  and  $x_2$ :  $\frac{\partial J}{\partial x_{1,2}} (x_1 + x_2 - 2)^2 = 2(x_1 + x_2 - 2)(1) = 2x_1 + 2x_2 - 4$

b.



c.  $x_1, x_2 = 1$  gives us the minimum of  $J$  which is 0

Part 2:

Final Model:

$$\theta_0 = 2986.896551724138$$

$$\theta_1 = 1088.4862496885312$$

$$\theta_2 = -341.67869902828295$$

$$y = 2986.89655 + (1088.4862497)x_1 + (-341.6786990)x_2$$

RMSE=709.897098

Part 3:

RMSE=351.6779056674261

Part 4:

Final Model:

$$\theta_0 = 2964.90782605$$

$$\theta_1 = 1278.04815434$$

$$\theta_2 = -200.89824176$$

$$y = 2964.90782605 + (1278.04815434)x_1 + (-200.89824176)x_2$$

RMSE=570.170565711668

From the plot we can see the training RMSE is reaching the convergent point faster than the testing RMSE.

