Семинар 14

Зад. 1) Да се док., че $\varphi_1 \& \varphi_2 \& \varphi_3 \Rightarrow \varphi_4$ чрез Язна Ф, където:

$\varphi_1 \leftrightharpoons \exists z \forall x [\forall y\, p(f(x), y) \vee q(z, f(x))]$

$\varphi_2 \leftrightharpoons \exists z \forall y [q(y, f(f(y))) \Rightarrow \forall x\, p(x, f(z))]$

$\varphi_3 \leftrightharpoons \forall x \forall y [p(x, f(y)) \Rightarrow (p(f(y), x) \vee \neg \exists y \exists x\, p(x, y))]$

$\varphi_4 \leftrightharpoons \exists y (\exists x\, p(f(x), y) \& \exists x\, p(y, x))$

Д-во:/

"тавтология" $\Leftrightarrow$ ¬ е "неизпълнимо"

$\neg(\varphi_1 \& \varphi_2 \& \varphi_3 \Rightarrow \varphi_4) \vDash \varphi_1 \& \varphi_2 \& \varphi_3 \& \neg\varphi_4$

step 1: Отрицания само пред атомарни ф-ли (възможно най-навътре ги избутваме и се оправяме с "$\Rightarrow$" и "$\Leftrightarrow$") /* 2 стъпки от КНФ */

step 2: ПНФ

step 3: СНФ

step 4: КНФ /* Последната стъпка - прилагане дистрибутивния закон $(\varphi \& \psi) \vee \chi \vDash (\varphi \vee \chi) \& (\psi \vee \chi)$

$\varphi_1' \leftrightharpoons \overset{a}{\exists z} \forall x \forall y [p(f(x), y) \vee q(z, f(x))]$

$\varphi_1^S \leftrightharpoons \forall x \forall y [\underbrace{p(f(x), y) \vee q(a, f(x))}_{D_1}]$

$\varphi_1^{final} \leftrightharpoons \varphi_1^S$

b

$\varphi_2' \leftrightharpoons \exists z \forall y \forall x [\neg q(y, f(f(y))) \vee p(x, f(z))]$

$\varphi_2^S \leftrightharpoons \forall y \forall x [\underbrace{\neg q(y, f(f(y))) \vee p(x, f(b))}_{D_2}]$

$\varphi_2^{final} \leftrightharpoons \varphi_2^S$

генаме t v

$$\varphi_3' \leftrightharpoons \forall x \forall y [\neg p(x, f(y)) \vee p(f(y), x) \vee \forall y \forall x \neg p(x, y)] \models$$

нямат никир общо

$$\models \forall x \forall y \forall t \forall v [\underbrace{\neg p(x, f(y)) \vee p(f(y), x) \vee \neg p(v, t)}_{D_3}]$$

$$\varphi_3^{final} \leftrightharpoons \varphi_3'$$

$$\varphi_4' \leftrightharpoons \neg \varphi_4 \models \forall y (\forall x \neg p(f(x), y) \vee \forall x^{z} \neg p(y, x)) \models$$

$$\forall y \forall x \forall z (\underbrace{\neg p(f(x), y) \vee \neg p(y, z)}_{D_4})$$

$D_1 = \{ p(f(x), y), q(a, f(x)) \}$
$D_2 = \{ \neg q(y, f(f(y))), p(x, f(b)) \}$
$D_3 = \{ \neg p(x, f(y)), p(f(y), x), \neg p(v, t) \}$
$D_4 = \{ \neg p(f(x), y), \neg p(y, z) \}$

Let the game begin:

Rule 1: Най-обща резолвента

$D_1 = \{L\} \cup D_1'$

нямат общи променливи!

$D_2 = \{\neg \tilde{L}\} \cup D_2'$

Тогава намираме субституция σ, така че
$$L\sigma = \tilde{L}\sigma$$
$$Res_L(D_1, D_2) = D_1'\sigma \cup D_2'\sigma$$

Rule 2: Колапс

$D_1 = \{L, \tilde{L}\} \cup D_1'$ /* Разгл. случая когато са 2 дизюнкта */

Намираме субституция σ, така че $L\sigma = \tilde{L}\sigma$; $Collapse(D_1) = D_1'\sigma \cup \{L\sigma\}$

① Разгл. D_3

$\sigma = \{ t/x,\ t/f(y) \}$

$Collapse(D_3) = \{ \neg p(x, f(y)),\ p(f(y), x) \} = D_5$

② Разгл. D_4

$\sigma = \{ y/f(x),\ z/f(x) \}$

$Collapse(D_4) = \{ \neg p(f(x), f(x)) \} = D_6$

③ Разгл. D_1 и D_2

$$\left| \begin{array}{l} a = y_2 \\ f(x_1) = f(f(y_2)) \end{array} \right. \Leftrightarrow \left| \begin{array}{l} y_2 = a \\ x_1 = f(a) \end{array} \right.$$

$\sigma = \{ y_2/a,\ x_1/f(a) \}$

$Res(D_1, D_2) = \{ p(f(f(a)), y_1),\ p(x_2, f(b)) \} = D_7$

④ Разгл. D_6 и D_7

$\sigma = \{ x/b,\ x_2/f(b) \}$

$Res(D_6, D_7) = \{ p(f(f(a)), y_1) \} = D_8$

⑤ Разгл. D_8 и D_6

$\sigma = \{ x/f(a),\ y_1/f(f(a)) \}$

$Res(D_6, D_8) = \blacksquare$

Зад ② Да се док. чрез МнФ, че $\varphi_1 \Rightarrow \varphi_2$

$\varphi_1 \leftrightharpoons$ "който пие получава махмурлук"

$\varphi_2 \leftrightharpoons$ "хората пият заради махмурлука"

Д-во/

$\varphi_1 \leftrightharpoons \forall x (drink(x) \Rightarrow hangover(x))$

$\varphi_2 \leftrightharpoons \forall x (\neg hangover(x) \Rightarrow \neg drink(x))$

- /* ако нямат махмурлук, то няма да пият */

$\varphi_1 \Rightarrow \varphi_2 \xrightarrow{\neg} \varphi_1 \& \neg\varphi_2$

$\varphi_1^f \leftrightharpoons \forall x (\neg drink(x) \vee hangover(x))$

$\varphi_2' \leftrightharpoons \exists x (\neg hangover(x) \& drink(x))$

$\varphi_2^f \leftrightharpoons (\neg hangover(a) \& drink(a))$

$D_1 = \{\neg drink(x), hangover(x)\}$
$D_2 = \{\neg hangover(a)\}$
$D_3 = \{drink(a)\}$

<u>Разгл. D_1 и D_3</u>

$\sigma = \{x/a\}$

$Res(D_1; D_3) = \{hangover(a)\} = D_4$

<u>Разгл. D_2 и D_4</u>

$Res(D_2, D_4) = \square$.

Зад 3) Да се док. по ФМР, че
$\varphi_1 \,\&\, \varphi_2 \,\&\, \varphi_3 \Rightarrow \varphi_4$, където:

$\varphi_1 \rightleftharpoons \forall x \exists y [q(y,x) \,\&\, \forall z (q(y,z) \Rightarrow r(z,x))]$

$\varphi_2 \rightleftharpoons \forall x [\exists y\, q(x,y) \Rightarrow \exists y (q(x,y) \,\&\, \neg \exists z (q(y,z) \,\&\, q(x,z)))]$

$\varphi_3 \rightleftharpoons \forall x \forall y \forall z [q(y,x) \,\&\, r(z,y) \Rightarrow q(z,x)]$

$\varphi_4 \rightleftharpoons \forall x \neg q(x,x)$

П-во/

$\varphi_1' \rightleftharpoons \forall x \exists y [q(y,x) \,\&\, \forall z (\neg q(y,z) \vee r(z,x))] \dashv\vdash$

$\dashv\vdash \forall x \underset{f(x)}{\exists y} \forall z [q(y,x) \,\&\, (\neg q(y,z) \vee r(z,x))]$

$\varphi_1^f \rightleftharpoons \varphi_1^S \rightleftharpoons \forall x \forall z [\underbrace{q(f(x),x)}_{D_1} \,\&\, \underbrace{(\neg q(f(x),z) \vee r(z,x))}_{D_2}]$

$\varphi_2' \rightleftharpoons \forall x [\forall y \neg q(x,y) \vee \overset{t}{\exists y} (q(x,y) \,\&\, \forall z (\neg q(y,z) \vee \neg q(x,z)))] \dashv\vdash$

$\dashv\vdash \forall x \underset{g(x)}{\exists t} \forall y \forall z [\neg q(x,y) \vee (q(x,t) \,\&\, (\neg q(t,z) \vee \neg q(x,z)))]$

$\varphi_2^S \dashv\vdash \forall x \forall y \forall z [\neg q(x,y) \vee (q(x,g(x)) \,\&\, (\neg q(g(x),z) \vee \neg q(x,z)))]$

$\varphi_2^f \dashv\vdash \forall x \forall y \forall z [\overbrace{(\neg q(x,y) \vee q(x,g(x)))}^{D_3} \,\&\, \underbrace{(\neg q(x,y) \vee \neg q(g(x),z) \vee \neg q(x,z))}_{D_4}]$

$$\varphi_3^f \leftrightharpoons \forall x \forall y \forall z (\underbrace{\neg q(y,x) \vee \neg r(z,y) \vee q(z,x)}_{D_5})$$

$$\varphi_4' \leftrightharpoons \neg \varphi_4 \dashv\vdash \exists x_a\, q(x,x)$$

$$\varphi_4^f \leftrightharpoons \varphi_4^s \leftrightharpoons q(a,a) \quad D_6$$

$D_1 = \{q(f(x_1), x_1)\}$
$D_2 = \{\neg q(f(x_2), z_2), r(z_2, x_2)\}$
$D_3 = \{\neg q(x_3, y_3), q(x_3, g(x_3))\}$
$D_4 = \{\neg q(x_4, y_4), \neg q(g(y_4), z_4), \neg q(x_4, z_4)\}$
$D_5 = \{\neg q(y_5, x_5), \neg r(z_5, y_5), q(z_5, x_5)\}$
$D_6 = \{q(a,a)\}$

① Разбл. D_2 и D_5
$\sigma = \{y_2/y_5, z_2/z_5\}$
$Res(D_2, D_5) = \underbrace{\{\neg q(f(x_2), z_5), \neg q(y_5, x_5), q(z_5, x_5)\}}_{D_7}$

② Разбл. D_3 и D_7
$\sigma = \{x_3/f(x_2), z_5/g(x_3)\}$
$Res(D_3, D_7) = \{\neg q(f(x_2), y_3), \neg q(y_5, x_5),$
$q(g(f(x_3)), x_5)\} = D_8$

③ Разбл. D_4 и D_8
$\sigma = \{x_4/f(x_3), z_4/x_5\}$
$Res(D_4, D_8) = \{\neg q(f(x_3), y_4), \neg q(f(x_3), z_5),$
$\neg q(f(x_2), y_3), \neg q(y_5, x_5)\} = D_9$

④ Разбл. D_9 ; $\sigma = \{x_1/x_3, y_4/x_3\}$

$Collapse(D_9) = \{\neg q(f(x_3), y_4), \neg q(y_5, x_5)\} = D_{10}$

⑤ Разгл. D_1 и D_{10}

$\sigma = \{x_3/x_1, y_4/x_1\}$

$Res(D_1, D_{10}) = \{\neg q(y_5, x_5)\} = D_{11}$

⑥ Разгл. D_6 и D_{11}

$\sigma = \{y_5/a, x_5/a\}$

$Res(D_6, D_{11}) = \blacksquare$.

Допълнителни задачки

Зад. 1 -||- $\varphi_1 \& \varphi_2 \Rightarrow \varphi_3$, където:

$\varphi_1 \leftrightharpoons \forall x [\forall y (s(y) \Rightarrow p(y,x)) \Rightarrow \neg r(x)]$

$\varphi_2 \leftrightharpoons \exists y \forall x [\neg s(x) \vee \neg q(x,y)]$

$\varphi_3 \leftrightharpoons \exists x \forall y [r(y) \Rightarrow \exists z (\neg p(z,y) \& \neg q(z,x))]$

Частично решение:

Трябва да стигнете до:

$D_1 = \{s(f(x)), \neg r(x)\}$

$D_2 = \{\neg p(f(x), x), \neg r(x)\}$

$D_3 = \{\neg s(x), \neg q(x, a)\}$

$D_4 = \{r(g(x))\}$

$D_5 = \{p(z, g(x)), q(z, x)\}$

Примерен извод на $\blacksquare$

① Разгл. D_1 и $D_4 \Rightarrow D_6$

② Разгл. D_2 и $D_4 \Rightarrow D_7$

③ Разгл. D_3 и $D_5 \Rightarrow D_8$

④ Разгл. D_8 и $D_7 \Rightarrow D_9$

⑤ Разгл. D_6 и $D_9 \Rightarrow \blacksquare$

зад ② -||- $\varphi_1 \& \varphi_2 \Rightarrow \varphi_3$ където

$\varphi_1 \leftrightharpoons \exists z \forall y [s(y) \vee q(z,y)]$

$\varphi_2 \leftrightharpoons \forall y \exists z [\neg r(z) \& \forall t (p(z,t) \Rightarrow \neg q(y,t))]$

$\varphi_3 \leftrightharpoons \exists y [\forall z (\neg s(z) \Rightarrow \neg p(y,z)) \& \neg r(y)]$

Частично решение:

Получаваме:

$D_1 = \{s(y), q(a,y)\}$

$D_2 = \{\neg r(f(y))\}$

$D_3 = \{\neg p(f(y),t), \neg q(y,t)\}$

$D_4 = \{\neg s(g(y)), r(y)\}$

$D_5 = \{ , p(y,g(y)), r(y)\}$

Примерен извод на $\blacksquare$:

① Разгл. D_2 и $D_4 \Rightarrow D_6$

② Разгл. D_2 и $D_5 \Rightarrow D_7$

③ Разгл. D_1 и $D_3 \Rightarrow D_8$

④ Разгл. D_8 и $D_7 \Rightarrow D_9$

⑤ Разгл. D_6 и $D_9 \Rightarrow \blacksquare$

Успех! :)