

# MAT 141 HW 1

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## 1 Section 2.3, Question 38

### 1.1 Part B

Native C is a knave, and native D is a knight.

1. Suppose C is a knight.
2. What C says is true.
3. C and D must be knaves.
4. *Contradiction:* If C is a knight, then C has said that it is a knave. This doesn't work.
5. C is a knave.
6. D is not a knave.
7. D is a knight.

As we can see, C saying that both C and D are knaves would be a contradiction. C cannot make this assertion due to the fact that it, itself, is a knave. We then negate C's original statement, and get that D is not a knave but instead a knight.

### 1.2 Part C

There should be a single knave, and a single knight. In this case, we show what happens when you take either E or F's statement that the other is a knave as being true:

Suppose that what E says is true:

1. F is a knave. (That's what E says.)
2. E is not a knave. (Negation of F's statement.)
3. E is a knight. (By definition of being a knight)

The same is true if we take F's statement as being true:

1. E is a knave. (That's what F says.)
2. F is not a knave. (Negation of E's statement.)
3. F is a knight. (By definition of being a knight.)

No matter which native we take as being the true statement, we end up with one knave and one knight.

## 2 Section 3.3, Question 57

The first statement is  $(\forall x \in D) : (P(x) \vee Q(x))$ , and the second is  $((\forall x \in D) : P(x)) \vee ((\forall x \in D) : Q(x))$ . Should the first statement be true, then every  $x \in D$  *must* satisfy either  $P(x)$  or  $Q(x)$ . This means that the second statement will also always be true, since no matter which  $x$  you may choose, the chosen  $x$  *will* satisfy either  $P(x)$  or  $Q(x)$ , therefore making the overall statement true. Should the first statement be false, then  $x$  is neither  $P(x)$  or  $Q(x)$ , making the second statement false as well.

## 3 Section 3.3, Question 58

The first statement is  $(\exists x \in D) : (P(x) \vee Q(x))$ . The second statement is  $((\exists x \in D) : P(x)) \vee ((\exists x \in D) : Q(x))$ . I believe that the same logic applies here as for the previous question. No matter which  $x$  you choose, it will be guaranteed to satisfy either  $P(x)$  or  $Q(x)$ , or neither of them. This will give you the same truth value in both the first and second statements.

## 4 Section 3.4, Question 34

We can reorder our premises like so, rewriting them all in "if-then" format:

1. If you wrote *Hamlet*, then you are a true poet.
2. Shakespeare wrote *Hamlet*.
3. If a writer is a true poet, then they can stir the human heart.
4. If a writer can stir the human heart, then they understand human nature.
5. If a writer understands human nature, then they are clever.

We can also draw a conclusion from this - "therefore Shakespeare is clever."