MAT 141 Practice Exam 2

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Problem # 1	**Grade:**
The first four terms of the sequence $b_j = \frac{5-j}{5+j}$ are: $b_0 = \frac{5}{5} = 1$ $b_1 = \frac{5-1}{5+1} = \frac{2}{3}$ $b_2 = \frac{5-2}{5+2} = \frac{3}{7}$ $b_3 = \frac{5-3}{5+3} = \frac{1}{4}$	***Faculty Comments***

Problem # 2	**Grade:**
The explicit formula for the given sequence is $a_n = \frac{1}{n} - \frac{1}{n+1}$, for any n such that $n > 0$.	***Faculty Comments***

Problem # 3 Grade:

Given the sum $\sum_{k=0}^{5} 2k+1$, the terms are:

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$k_0 = 2(0) + 1 = 1$

$k_1 = 2(1) + 1 = 3$

$k_2 = 2(2) + 1 = 5$

$k_3 = 2(3) + 1 = 7$

$k_4 = 2(4) + 1 = 9$

$k_5 = 2(5) + 1 = 11$

We would then sum these together, giving us a value of 36.

Problem # 4 Grade:

Given the product $\prod_{i=1}^{4} (\frac{1}{2})^i$, the terms are:

Faculty Comments

$i_1 = (\frac{1}{2})^1 = \frac{1}{2}$

$i_2 = (\frac{1}{2})^2 = \frac{1}{2^2}$

$i_3 = (\frac{1}{2})^3 = \frac{1}{2^3}$

$i_4 = (\frac{1}{2})^4 = \frac{1}{2^4}$

We then multiply these terms together, giving us a product of $\frac{1}{2^{10}}$.

Problem # 5 Grade:

Given the sequence $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2$, we can rewrite this as:

$$\sum_{i=1}^{7}(-1)^{i+1}(i^2)$$

Faculty Comments

Problem # 6 Grade:

Since our first term is a 3, and not a 1, we need to reindex our summation to fit it. That will look like this, for values $j = i + 3$ and $i = j - 3$:

$$\sum_{j=3}^{n+3} i + 3 = \frac{(n+3)(n+4)}{2} - (n+3)(3)$$

After plugging in our values, we get that:

$$S = \frac{(1000+3)(1000+4)}{2} - (1000+3)(3) = 500497$$

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Problem # 7 Grade:

The final term of the summation can be found by doing:

$$\frac{1}{(n+1)^2+1} = \frac{1}{n^2+2n+2}$$

Then we rewrite the summation to separate off the final term:

$$(\sum_{i=1}^{n} \frac{1}{n^2+1}) + \frac{1}{n^2+2n+2}$$

Faculty Comments

Problem # 8 Grade:

Given our original formula, we can reindex it like so, using $i = j - 1$:

$$\sum_{j=2}^{n+2} \frac{j^2}{(j-1) * n}$$

Faculty Comments

Problem # 9 Grade:

Given our recursive sequence with first terms $s_0 = 1, s_1 = 1$, we find the first 5 terms of the sequence as:

$$s_0 = 1$$

$$s_1 = 1$$

$$s_2 = \frac{1}{s_1 + s_0} = \frac{1}{2}$$

$$s_3 = \frac{1}{s_2 + s_1} = \frac{2}{3}$$

$$s_4 = \frac{1}{s_3 + s_2} = \frac{3}{4}$$

Faculty Comments

Problem # 10 **Grade:**

Given the relation $h_k = 2^k - h_{k-1}$ and first term $h_0 = 1$, we get h_3 like so:

$$\begin{aligned} h_3 &= 2^3 - h_2 \\ &= 2^3 - 2^2 + h_1 \\ &= 2^3 - 2^2 + 2^1 - h_0 \\ &= 2^3 - 2^2 + 2^1 - 2^0 \end{aligned}$$

We can see that our relation looks like this:

$$h_n = 2^n - 2^{n-1} + 2^{n-2} - 2^{n-3} + \dots$$

Which can be rewritten, using summation notation:

$$(\sum_{i=0}^{n-1} -2^i) + 2^n$$

We then use the general formula for geometric summations, and get our explicit equation:

$$(\frac{1 - (-2)^n}{1 + 2}) + 2^n$$

Faculty Comments

Problem # 11 Grade:

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Our base case is that $n = 4$. We use the assertion about the Fibonacci Sequence to show this:

$$\begin{aligned} F_4 &= 3F_1 + 2F_0 \\ &= F_1 + (2F_1 + 2F_0) \\ &= F_1 + 2F_2 \\ &= F_1 + F_2 + F_2 \\ &= F_3 + F_2 \\ &= F_4 \end{aligned}$$

We then assume that $n = k$, and that $F_n = 3F_{k-3} + 2F_{k-4}$. It then follows that:

$$\begin{aligned} F_{k+1} &= 3F_{k-2} + 2F_{k-3} \\ &= 3(F_{k-3} + F_{k-4}) + 2F_{k-3} \\ &= (3F_{k-3} + 2F_{k-4}) + F_{k-4} + 2F_{k-3} \\ &= F_k + (F_{k-4} + F_{k-3}) + F_{k-3} \\ &= F_k + F_{k-2} + F_{k-3} \\ &= F_k + F_{k-1} \\ &= F_{k+1} \checkmark \end{aligned}$$

The definition of the Fibonacci Sequence holds true, and therefore
$F_k = 3F_{k-3} + 2F_{k-4}$ for all $n \geq 4$.