MAT 141 Homework 5

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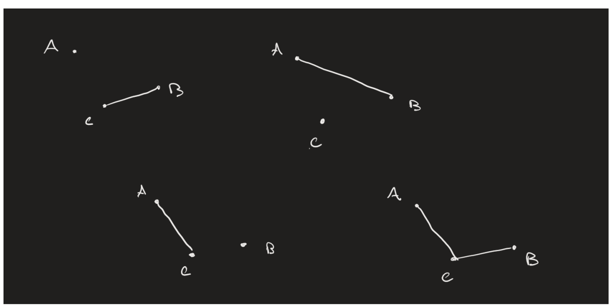
Chapter 10.1 Question 28 | Grade:

Faculty Comments

When there are at least 2 people in a party, that party will have at least 2 mutual friends or strangers. We can draw a graph showing this, where each person is a vertex and each ”friendship” is symbolized as an edge:



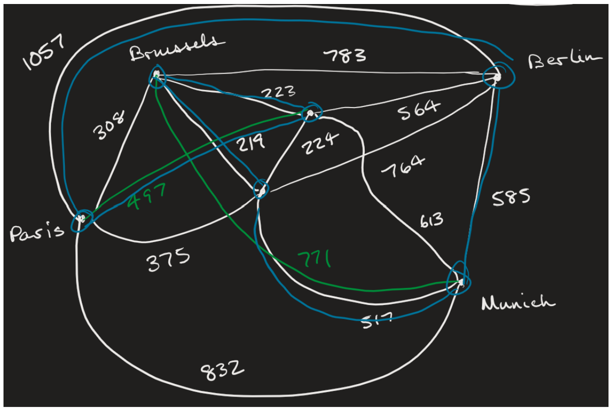
In these graphs, we can see that there are some isolated vertices, indicating that there are some people who have no friends (at this party, of course). We can also see that there are some vertices with degree 1 or more, indicating that there are some people with at least 1 friend. There are always at least 2 people that are friends in all of these graphs, so the statement is true.
I could have also drawn a grpah in which none of the vertices are connected, which would indicate that there are no mutual friends within said party. However, the statement still holds true because at this point there are at least 2 mutual strangers.

Chapter 10.1 Question 42

Grade:

Faculty Comments

In this problem, we are asked to find a Hamiltonian circuit in a weighted graph which minimizes the total distance travelled. My implementation of this question is shown in this graph:

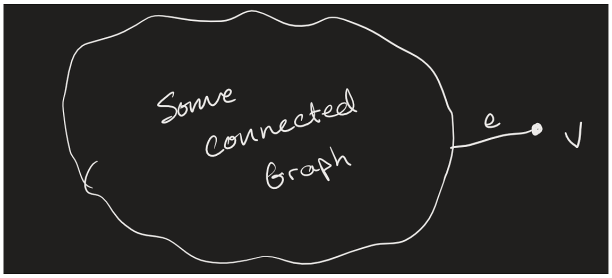


The total distance travelled is 3098. I found the circuit by hand, by choosing the shortest edge at each vertex towards another that has not been visited yet. I started at Berlin. The path is:

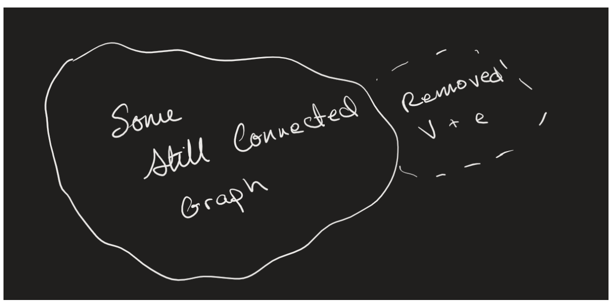
Berlin → Munich → Luxembourg → Brussels → Dusseldorf → Paris → Berlin

Chapter 10.4 Question 24 Grade:

Given a connected graph with a leaf vertex, we can remove that leaf and its edge and still have a connected graph. The first graph would look roughly like this:



I have abbreviated the graph due to the fact that most of it is irrelevant. The leaf vertex is labelled as V and the edge is labelled as e. The second graph would look roughly like this:



As we can see, the graph would still be connected. This is because the leaf vertex, by definition of being a leaf, only has one edge. Removing that edge would remove the leaf vertex, but the graph would still be connected because the leaf vertex was only connected to one other vertex.

Faculty Comments