MAT 141 Homework #3

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6.1 Question 26 — Grade:

Given the set $S_i = \{x \in \mathbb{R} | 1 < x < 1 + \frac{1}{i}\}$, then:

(a) $\bigcup_{i=1}^{4} S_i = \{x \in \mathbb{R} | 1 < x < 2\}$

(b) $\bigcap_{i=1}^{4} S_i = \{x \in \mathbb{R} | 1 < x < \frac{5}{4}\}$

(c) This set is not mutually disjoint, because it does not yield $\emptyset$ when intersected.

(d) $\bigcup_{i=1}^{n} S_i = \{x \in \mathbb{R} | 1 < x < 2\}$

(e) $\bigcap_{i=1}^{n} S_i = \{x \in \mathbb{R} | 1 < x < \frac{1+n}{n}\}$

(f) $\bigcup_{i=1}^{\infty} S_i = \{x \in \mathbb{R} | 1 < x < 2\}$

(g) $\bigcap_{i=1}^{\infty} S_i = \{x \in \mathbb{R} | 1 < x < 1 + \text{some incredibly tiny number}\}$

Faculty Comments

7.2 Question 56 — Grade:

Faculty Comments

Given our set X, we can create a one-to-one correspondance between the powerset of X and the set of all binary strings. To start, we would have the subsets of X, a few examples being:

$$\emptyset$$

$$\{x_1, x_3, x_4\}$$

$$\{x_2, x_4\}$$

$$\{x_1, x_2, x_3, x_4\}$$

Using the subscripts assigned to each element of X, we can create a binary string that represents each subset. from our first example, we would have a binary string of 0000, since there are no elements. The second set would be 1011, since we have x_1, x_3 and x_4, but not x_2. The third set would be 0110, by the same logic, and the final example would be 1111, since x_1, x_2, x_3 and x_4 are all present. This could be expanded and repeated for any number of elements, and thus we have a one-to-one correspondance between the powerset of X and the set of all binary strings.

7.3 Question 20 — Grade:

Faculty Comments

Given that $f : X \to Y$ and $g : Y \to Z$, and $g \circ f$ is onto, then f does not have to be onto. For example, if we have 3 elements in X, 5 elements in Y, and 2 elements in Z, then f can map each element in X to at least one element in Y, but not all. This would mean that f was not onto. However, g can map all elements from Y to both elements from Z, which would allow $g \circ f$ to be onto. Therefore, f does not have to be onto in order for $g \circ f$ to be onto as well.

8.2 Question 26 — Grade:

Faculty Comments

To define a relation R where sRt and the sum of all elements of s is equal to the sum of all elements of t, then we can define some classes as follows:

$$\begin{aligned}
[0] = &\{[0,0,0,0]\} \\
[1] = &\{[0,0,0,1],[0,0,1,0],[0,1,0,0],[1,0,0,0]\} \\
[2] = &\{[0,0,0,2],[0,0,1,1],[0,0,2,0],[0,1,0,1],[0,1,1,0], \\
&[0,2,0,0],[1,0,0,1],[1,0,1,0],[1,1,0,0],[2,0,0,0]\}
\end{aligned}$$