MAT 141 Homework 4

Lucas Vas

December 3, 2023

Section 9.2 Question 28 — Grade:

These are nested for loops in a language that I don't personally know (looks like Lua?) but I assume the logic follows the same. The total number of times that this loop will iterate is found by doing $(a - b + 1) * (c - d + 1)$. The reason we add the "+1" is because the bounds are inclusive of the values of b and d.

Faculty Comments

Section 9.4 Question 8 — Grade:

To solve this problem, we can create a set of sets that sum to 10 using the original numbers that we're given. That would look like this:

$$S = \{\{1, 9\}, \{2, 8\}, \{3, 7\}, \{4, 6\}, \{5\}\}$$

If we were to pick 5 values from this set, where those values are the first value in every pair, then we can see that there isn't necessarily a sum of any 2 values that would result in 10. If we chose 6, then we would be guaranteed a sum of 10.

Faculty Comments

Section 9.5 Question 17 | Grade:

For some reason, I just cannot visualize this problem and I don't know why that is.

(a) The straight lines can be found by doing $\binom{10}{2}$ This is equivalent to 45. This is because every line is a combination of 2 points, and there are 10 points in total.

(b) The number of straight lines that do not pass through A is found similarly - you're simply reducing the number of points that can be chosen from. This would be $\binom{9}{2}$, which is equivalent to 36.

(c) The number of triangles that are present in this figure is found by doing $\binom{10}{3}$, which is equivalent to 120. Triangles, by definition, have 3 points that are connected by straight lines, so we choose 3 points from the 10 that are possible.

(d) The number of triangles that do not pass through A is found similarly to the previous problem - you're simply reducing the number of points that can be chosen from. This would be $\binom{9}{3}$, which is equivalent to 84.

Faculty Comments

Section 9.7 Question 16 | Grade:

The problem that we're supposed to prove looks very similar to the binomial theorem. I noticed that the first section looks like the total number of combinations that are possible from the addition of sets m and n, where you choose r elements, where $r \leq m \vee n$. This looks like the same issue that's solved with the binomial theorem, as your coefficient from $(m+n)^x$ where $m^r n^{s-r}$ will yield the same answer. The equation that's presented in the problem deals with iterating through the value r so that you can choose r values from $m+n$. The first iteration will choose r values from m, which leaves no choices for n. The second iteration will choose $r-1$ values from m, which leaves a single choice for set n, and so on. This will eventually result in r values being chosen from n, which would sum to all combinations of $\binom{m+n}{r}$.

Faculty Comments

(BONUS) Section 9.3 Question 26 — Grade:

Faculty Comments

Using the set of all strings of a's, b's and c's:

(a) The list of all strings of lengths 0 through 3 that don't contain aa is:

s_0: $\emptyset$ - the empty string.

s_1: a, b, c

s_2: $ab, ac, ba, bb, bc, ca, cb, cc$

s_3: $aba, abb, abc, aca, acb, acc, bab, bac, bba, bbb, bbc, bca, bcb,$ $bcc, cab, cac, cba, cbb, cbc, cca, ccb, ccc$

(b) For all $n \geq 0$, the number of strings of length n that don't contain aa is:

s_0: 1 - just the empty string.

s_1: $3 = 3 - 0$

s_2: $8 = 9 - (3 * 0 + 1)$

s_3: $22 = 27 - (3 * 1 + 2)$

s_4: $61 = 81 - (3 * 5 + 6)$

(c) The recurrence relation for the number of strings of length n that don't contain aa is:

$$s_n = 3^n - 3^{n-2}$$

(d) Using this relation, the number of strings that exist (without aa) of length 4 is:

$$s_4 = 3^4 - 3^3 = 54$$