Consider the following justification that the Fibonacci function, $F(n)$ (see Proposition 3.20) is $O(n)$:
Base case ($n \geq 2$):
$F(1) = 1$ and $F(2) = 2$.
Induction step ($n > 2$):
Assume claim true for $n' < n$. Consider n.
$F(n) = F(n-2) + F(n-1)$.
By induction, $F(n-2)$ is $O(n-2)$ and $F(n-1)$ is $O(n-1)$.
Then, $F(n)$ is $O((n-2)+(n-1))$, by the identity presented in Exercise R-3.11.
Therefore, $F(n)$ is $O(n)$.
What is wrong with this justification?

I will show by example that Fibonacci function, $F(n)$ is not $O(n)$.
For now lets try to proof that $F(n)$ is $O(n)$ and that means:

$$F(n) \geq c \cdot n, \text{ for } c = \{c \in \mathbb{R} \mid c > 0\},\ n \geq 1.$$

Firstly we need to find a constant that matches to our function.
$F(1) = 1$
$F(2) = 1$
$F(3) = 2$
$F(4) = 3$
Let c be 120.
Base cases:

n	$F(n)$		$120n$
1	1	$<$	120
2	1	$<$	230
3	2	$<$	360

Inductive hipotessis:
$F(k) \geq ck$ for some $k \geq n_0$,
and we want to show $F(k) \geq ck$ for every k

$$\begin{aligned}
F(k) &= F(k-1) + F(k-2) \\
&\leq c(k-1) + c(k-2), \text{ by inductive hipotessis} \\
&= c(k+k-1-2) \\
&= c(2k-3) \\
&= 2ck - 3c \\
&< 2ck, \text{ since } 0 > -3c \\
&\nleq ck
\end{aligned}$$

This shows that $F(n)$ can not be $O(n)$ because i have proved that by example? ■