

Consider the Fibonacci function, $F(n)$ (see Proposition 3.20). Show by induction that $F(n)$ is $\Omega((3/2)^n)$.

Lets find c and n_0 such that $F(n) \geq c(\frac{3}{2})^n$
for $c = \frac{4}{9}$ and $n_0 = 1$ everything works great

Inductive Hypothesis:

$$\begin{aligned} F(k) &\geq c\left(\frac{3}{2}\right)^k \\ &= F(k+1) + F(k-1) \\ &\geq c\left(\frac{3}{2}\right)^k + c\left(\frac{3}{2}\right)^{k-1} \\ &= c\left(\frac{3}{2}\right)^{k-1} \cdot \frac{3}{2} + c\left(\frac{3}{2}\right)^{k-1} \\ &= c\left(\frac{3}{2}\right)^{k-1} \left(\frac{3}{2} + 1\right) \\ &> c\left(\frac{3}{2}\right)^{k-1} \left(\frac{3}{2}\right)^2, \text{ since } \frac{5}{2} > \frac{9}{4} \\ &= c\left(\frac{3}{2}\right)^{k+1} \end{aligned}$$



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