

Consider the Fibonacci function, $F(n)$ (see Proposition 3.20). Show by induction that $F(n)$ is $\Omega((3/2)^n)$.

Lets find c and n_0 such that $F(n) \geq c(\frac{3}{2})^n$ for constant c and $n \geq n_0$
for $c = \frac{4}{9}$ and $n_0 = 1$ everything works great

n	$F(n)$	$\frac{4}{9}(\frac{3}{2})^n$
1	1	$\frac{2}{3}$
2	1	1
3	2	$\frac{3}{2}$
4	3	$2\frac{1}{4}$

Inductive Hypothesis:

$F(k) \geq c(\frac{3}{2})^k$, for some $k \geq n_0$

Lets show that $F(k+1) \geq c(\frac{3}{2})^{k+1}$

$$F(k+1) = F(k) + F(k-1)$$

Lets apply out Hypothesis

$$\begin{aligned}
 F(k+1) &\geq c(\frac{3}{2})^k + c(\frac{3}{2})^{k-1} \\
 &= c(\frac{3}{2})^{k-1} \cdot \frac{3}{2} + c(\frac{3}{2})^{k-1} \\
 &= c(\frac{3}{2})^{k-1}(\frac{3}{2} + 1) \\
 &> c(\frac{3}{2})^{k-1}(\frac{3}{2})^2, \text{ since } \frac{5}{2} > \frac{9}{4} \\
 &= c(\frac{3}{2})^{k+1}
 \end{aligned}$$



<http://www.simonfoucher.com/McGill/COMP250/Lectures/Math/lecture13.pdf>