

Consider the Fibonacci function,  $F(n)$  (see Proposition 3.20). Show by induction that  $F(n)$  is  $\Omega((3/2)^n)$ .

Lets find  $c$  and  $n_0$  such that  $F(n) \geq c(\frac{3}{2})^n$  for constant  $c$  and  $n \geq n_0$ .  
For  $c = \frac{4}{9}$  and  $n_0 = 1$  everything works great

$n$	$F(n)$	$\frac{4}{9}(\frac{3}{2})^n$
1	1	$\frac{2}{3}$
2	1	1
3	2	$\frac{3}{2}$
4	3	$2\frac{1}{4}$

Inductive Hypothesis:

$$F(k) \geq c(\frac{3}{2})^k, \text{ for some } k \geq n_0$$

Lets show that  $F(k+1) \geq c(\frac{3}{2})^{k+1}$

$$F(k+1) = F(k) + F(k-1)$$

Lets apply out Hypothesis

$$\begin{aligned}
 F(k+1) &\geq c(\frac{3}{2})^k + c(\frac{3}{2})^{k-1} \\
 &= c(\frac{3}{2})^{k-1} \cdot \frac{3}{2} + c(\frac{3}{2})^{k-1} \\
 &= c(\frac{3}{2})^{k-1}(\frac{3}{2} + 1) \\
 &> c(\frac{3}{2})^{k-1}(\frac{3}{2})^2, \text{ since } \frac{5}{2} > \frac{9}{4} \\
 &= c(\frac{3}{2})^{k+1}
 \end{aligned}$$