

C-3.40

Show that $\log_b f(n)$ is $\Theta(\log_2 f(n))$ if $b > 1$ is a constant.

Proof. This is trivial

$$\log_2 f(n) = \frac{\log_b f(n)}{\log_b 2}$$

where b is const

so

$\log_b 2$ is also const

and we can write whole thing $\log_2 f(n)$ times a const

$$c = \frac{1}{\log_b 2}$$
$$\log_2 f(n) = c \cdot \log_b f(n)$$

and it is $\Theta(\log_2 f(n))$

so

$$\log_b f(n) \text{ is } \Theta(\log_2 f(n))$$