EXERCISES (INSTRUMENTAL VARIABLES)

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Provide succinct arguments to address the following questions:

(1) Evaluate the truth of following statement: "In the linear regression $y = X\beta + u$ the usual identifying assumption $\mathbb{E}(u|X) = 0$ implies $\mathbb{E}(h(X) \cdot u) = 0$ for any function h satisfying some regularity conditions related to measurability."
(2) Consider the same linear regression $y = X\beta + u$, but now suppose an alternative identifying assumption $\mathbb{E}(X|u) = 0$. Construct a simple estimator based on this alternative. Compare the usual and alternative identifying assumptions; are they equivalent? Is one stronger than the other?
(3) Suppose that $y = f(X) + u$ for some unknown but continuous function f. Suppose we want to use observed data on X to predict outcomes y, and seek a predictor $\hat{y}(X)$ which is "best" in the sense that the mean squared prediction error $\mathbb{E}(y - \hat{y}(X)|X)^2$ is minimized. What can we say about $\hat{y}$ and its relation to the conditional expectation $\mathbb{E}(y|X)$? Its relation to u?
(4) Let $y = X\beta + u$, and let D be a binary random variable. with $\mathbb{E}(u|D) = 0$ and $\mathbb{E}(X|D) \neq \mathbb{E}(X)$. Establish that D is a valid instrument, and work out a particularly simple expression for the IV estimator in this case. Discuss.
(5) Write out the two causal diagrams which justify, respectively, the least squares estimator and the IV estimator. What would it mean for one model to be correct, but not the other? How could you test this?

. *Date*: Due April 20, 2022.