# EXERCISES (GMM)

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When we approach a new estimation problem from a GMM perspective there's a simple set of steps we can follow.

(1) Describe the parameter space $B$;
(2) Describe a function $g_j(b)$ such that $\mathbb{E}g_j(\beta) = 0$;
(3) Describe an estimator for the covariance matrix $\mathbb{E}g_j(\beta)g_j(\beta)^\top$.

## 1. Questions

(1) Explain how the the set of steps outlined above can be used to construct an optimally weighted GMM estimator.
(2) Consider the following models. For each, provide a causal diagram; construct the optimally weighted GMM estimator of the unknown parameters (various Greek letters); and give an estimator for the covariance matrix of your estimates. If any additional assumptions are required for your estimator to be identified please provide these.
   (a) $\mathbb{E}y = \mu$; $\mathbb{E}(y - \mu)^2 = \sigma^2$; $\mathbb{E}(y - \mu)^3 = 0$.
   (b) $y = \alpha + X\beta + u$; with $\mathbb{E}(X^\top u) = 0$.
   (c) $y = \alpha + X\beta + u$; with $\mathbb{E}(X^\top u) = 0$, and $\mathbb{E}(u^2) = \sigma^2$.
   (d) $y = \alpha + X\beta + u$; with $\mathbb{E}(X^\top u) = 0$, and $\mathbb{E}(u^2) = e^{X\sigma}$.
   (e) $y = \alpha + X\beta + u$; with $\mathbb{E}(Z^\top u) = 0$ and $\mathbb{E}Z^\top X = \boldsymbol{Q}$.
   (f) $y = f(X\beta) + u$; with $f$ a known scalar function and with $\mathbb{E}(Z^\top u) = 0$ and $\mathbb{E}Z^\top X = \boldsymbol{Q}$.
   (g) $y = f(X, \beta) + u$; with $f$ a known function and with $\mathbb{E}(Z^\top u) = 0$ and $\mathbb{E}Z^\top X = \boldsymbol{Q}$.
   (h) $y^\gamma = \alpha + u$, with $y > 0$ and $\gamma$ a scalar.

## 2. Computing

(1) Select three of the models above, and for each of these write a data-generating process in `python`. Your function `dgp` should take as arguments a sample size $N$ and a vector of "true" parameters $b0$, and return a dataset $(y, X)$.

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(2) Select the most interesting of the data generating processes you developed, and using the code in `gmm.py` or `GMM_class.py` (see `https://github.com/ligonteaching/ARE212_Materials/`) use data from your `dgp` to analyze the finite sample performance of the corresponding GMM estimator you've constructed. Of particular interest is the distribution of your estimator using a sample size $N$ and how this distribution compares with the limiting distribution as $N \to \infty$.