EXERCISES (GMM)

ETHAN LIGON

When we approach a new estimation problem from a GMM perspective there's a simple set of steps we can follow.

(1) Describe the parameter space B;
(2) Describe a function $g_j(b)$ such that $\mathbb{E}g_j(\beta) = 0$;
(3) Describe an estimator for the covariance matrix $\mathbb{E}g_j(\beta)g_j(\beta)^\top$.

1. Questions

(1) Explain how the the set of steps outlined above can be used to construct an optimally weighted GMM estimator.
(2) Consider the following models. For each, provide a causal diagram; construct the optimally weighted GMM estimator of the unknown parameters (various Greek letters); and give an estimator for the covariance matrix of your estimates. If any additional assumptions are required for your estimator to be identified please provide these.
   (a) $\mathbb{E}y = \mu$; $\mathbb{E}(y-\mu)^2 = \sigma^2$; $\mathbb{E}(y-\mu)^3 = 0$.
   (b) $y = \alpha + X\beta + u$; with $\mathbb{E}(X^\top u) = \mathbb{E}u = 0$.
   (c) $y = \alpha + X\beta + u$; with $\mathbb{E}(X^\top u) = \mathbb{E}u = 0$, and $\mathbb{E}(u^2) = \sigma^2$.
   (d) $y = \alpha + X\beta + u$; with $\mathbb{E}(X^\top u) = \mathbb{E}u = 0$, and $\mathbb{E}(u^2) = e^{X\sigma}$.
   (e) $y = \alpha + X\beta + u$; with $\mathbb{E}(Z^\top u) = \mathbb{E}u = 0$ and $\mathbb{E}Z^\top X = \boldsymbol{Q}$.
   (f) $y = f(X\beta) + u$; with f a known scalar function and with $\mathbb{E}(Z^\top u) = \mathbb{E}u = 0$ and $\mathbb{E}Z^\top X f'(X\beta) = \boldsymbol{Q}(\beta)$. (Bonus question: where does this last restriction come from, and what role does it play?)
   (g) $y = f(X, \beta) + u$; with f a known function and with $\mathbb{E}(Z^\top u) = \mathbb{E}u = 0$ and $\mathbb{E}Z^\top \frac{\partial f}{\partial \beta^\top}(X, \beta) = \boldsymbol{Q}(\beta)$.
   (h) $y^\gamma = \alpha + u$, with $y > 0$ and γ a scalar, and $\mathbb{E}(Z^\top u) = \mathbb{E}u = 0$ and $\mathbb{E}Z^\top \begin{bmatrix} \gamma y^{\gamma-1} \\ -1 \end{bmatrix} = \boldsymbol{Q}(\gamma)$.

. *Date*: April 25, 2022.

2. Computing

(1) Select three of the models above, and for each of these write a data-generating process in `python`. Your function `dgp` should take as arguments a sample size `N` and a vector of "true" parameters `b0`, and return a dataset (y, X).

(2) Select the most interesting of the data generating processes you developed, and using the code in `gmm.py` or `GMM_class.py` (see `https://github.com/ligonteaching/ARE212_Materials/`) use data from your `dgp` to analyze the finite sample performance of the corresponding GMM estimator you've constructed. Of particular interest is the distribution of your estimator using a sample size N and how this distribution compares with the limiting distribution as $N \to \infty$.