UNIVERSITATEA TEHNICĂ
DIN CLUJ-NAPOCA

# Proiect

# Teoria Sistemelor


Student

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Anul 2

Grupa 30121

Profesori îndrumători

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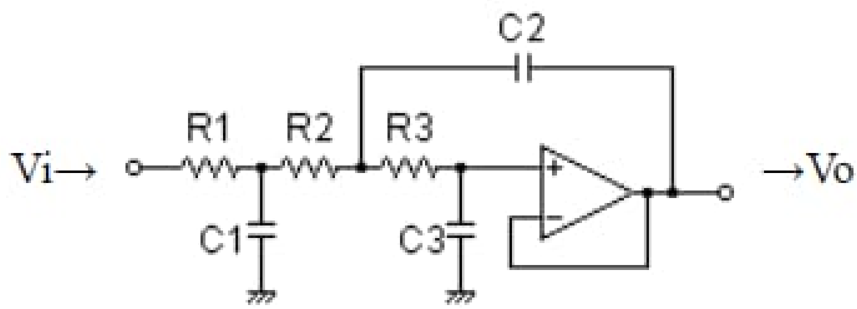
Data  Mai 2021

# Cuprins:

## 1. Circuitul filtru trece-jos de tip Sallen-Key




Valori initiale:

$R_1 = 100\Omega$

$R_2 = 100\Omega$

$R_3 = 5 * 10^6\Omega$

$C_1 = 10^{-6}F$

$C_2 = 10^{-6}F$

$C_3 = 5 * 10^{-6}F$

Componentele active sunt: $C_1$ , $C_2$ , $C_3$

Componentele pasive sunt: $R_1$, $R_2$ , $R_3$

## 2. Modelul matematic u/x/y al sistemului

(L1/2) Sa se obtina modelul matematic u/x/y al sistemului ales

$\mathrm{x}1 = u_{\mathrm{c}1}$

$\mathrm{x}2 = u_{\mathrm{c}2}$

$\mathrm{x}3 = u_{\mathrm{c}3}$

$i_{\mathrm{R}1} = i_{\mathrm{C}1} + i_{\mathrm{R}2}$

$i_{\mathrm{R}2} = i_{\mathrm{R}3} + i_{\mathrm{C}2}$

$i_{\mathrm{R}3} = i_{\mathrm{C}3}$

$u_{\mathrm{C}1} = u_{\mathrm{R}2} + u_{\mathrm{R}3} + u_{\mathrm{C}3}$

$u_{\mathrm{C}2} = u_{\mathrm{R}3}$

$y = u_{\mathrm{c}3} = \mathrm{x}3$

$$\left.\begin{aligned}&u = u_{\mathrm{R1}} + u_{\mathrm{C1}}\\&u = \mathrm{R1} * i_{\mathrm{R1}} + \mathrm{x1}\\&u_{\mathrm{C2}} = u_{\mathrm{R3}}\end{aligned}\right\} \Rightarrow \mathrm{x2} = i_{\mathrm{C3}} * \mathrm{R3} = \mathrm{C3} * \mathrm{x3}\,' * \mathrm{R3} \Rightarrow x_3\,' = \frac{\mathrm{x2}}{\mathrm{R3} * \mathrm{C3}}$$

$$\mathrm{x2} = i_{\mathrm{R3}} * \mathrm{R3}$$

$$\left.\begin{aligned}&u_{\mathrm{R1}} = \mathrm{x1} - \mathrm{x2} - \mathrm{x3}\\&\mathrm{x1} = u_{\mathrm{R2}} + \mathrm{x2} + \mathrm{x3}\\&i_{\mathrm{R2}} = i_{\mathrm{R3}} + i_{\mathrm{C2}}\end{aligned}\right\} \Rightarrow \frac{u_{\mathrm{R2}}}{\mathrm{R2}} = \frac{u_{\mathrm{R3}}}{\mathrm{R3}} + \mathrm{C2} * \mathrm{X2}' \Rightarrow \frac{(\mathrm{X1} - \mathrm{X2} - \mathrm{X3})}{\mathrm{R2}} = \mathrm{C2} * \mathrm{X2}' + \frac{\mathrm{X2}}{\mathrm{R3}} \Rightarrow$$

$$\Rightarrow \mathrm{x2}\,' = \frac{\mathrm{x1}}{\mathrm{C2} * \mathrm{R2}} - \frac{\mathrm{x2}}{\mathrm{C2}}\left(\frac{1}{\mathrm{R3}} + \frac{1}{\mathrm{R2}}\right) - \frac{\mathrm{x3}}{\mathrm{C2} * \mathrm{R2}}$$

$$u = u_{R1} + u_{\mathrm{C1}}$$

$$u = \mathrm{R1} * i_{\mathrm{R1}} + \mathrm{x1}$$

$$u = \mathrm{R1} * \left(C_1 * \dot{x}_1 + \frac{U_{\mathrm{R2}}}{R_2}\right) + \mathrm{x1}$$

$$u = R_1 * C_1 * \dot{x}_1 + \frac{R_1}{R_2}(x_1 - x_2 - x_3) + x_1$$

$$\dot{x}_1 = \frac{1}{\mathrm{R1} * \mathrm{C1}} - \mathrm{x1}\left(\frac{1}{\mathrm{R1} * \mathrm{C1}} + \frac{1}{\mathrm{R1} * \mathrm{C1}}\right) + \frac{\mathrm{x2}}{\mathrm{R2} * \mathrm{C1}} + \frac{\mathrm{x3}}{\mathrm{R2} * \mathrm{C1}}$$

$$\dot{x} = \begin{bmatrix} -\left(\frac{1}{\mathrm{R1} * \mathrm{C1}} + \frac{1}{\mathrm{R1} * \mathrm{C1}}\right) & \frac{1}{\mathrm{R2} * \mathrm{C1}} & \frac{1}{\mathrm{R2} * \mathrm{C1}} \\ \frac{1}{\mathrm{C2} * \mathrm{R2}} & -\frac{1}{\mathrm{C2}}\left(\frac{1}{\mathrm{R2}} + \frac{1}{\mathrm{R3}}\right) & -\frac{1}{\mathrm{C2} * \mathrm{R2}} \\ 0 & \frac{1}{\mathrm{R3} * \mathrm{C3}} & 0 \end{bmatrix} x\{= A\} + \begin{bmatrix} \frac{1}{\mathrm{R1} * \mathrm{C1}} \\ 0 \\ 0 \end{bmatrix}\{= B\} * u$$

$$y = [0 \quad 0 \quad 1]x\{= C\} + [0]\{= D\} * u$$

$$\Rightarrow A = 10^4 \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 4 * 10^{-6} & 0 \end{bmatrix} \quad B = 10^4 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = [0 \quad 0 \quad 1] \quad D = [0]$$

## 3. Modelul intrare-iesire si functia de transfer

(L3/4) Sa se determine modelul intrare-iesire ¸si sa se deduca functia de transfer. Sa se verifice rezultatul obtinut prin intermediul relatiei dintre spatiul starilor si functia de transfer;

$$H(s) = \frac{y_s}{u_s}$$

syms r1 r2 r3 c1 c2 c3 $s$

$\mathrm{A1} = [-(1/(\mathrm{r2} * \mathrm{c1}) + 1/(\mathrm{r1} * \mathrm{c1})), 1/(\mathrm{r2} * \mathrm{c1}), 1/(\mathrm{c1} * \mathrm{r2}); 1/(\mathrm{r2} * \mathrm{c2}), -1/\mathrm{c2} * (1/\mathrm{r2} + 1/\mathrm{r3}), -1/(\mathrm{r2} * \mathrm{c2}); 0, 1/(\mathrm{r3} * \mathrm{c3}), 0]$

$\mathrm{B1} = [1/(\mathrm{r1} * \mathrm{c1}); 0; 0]$

$\mathrm{C1} = [001]$

$\mathrm{D1} = 0;$

$I = \mathrm{eye}(3)$

$\mathrm{H1} = \mathrm{C1} * (s * I - \mathrm{A1})\hat{}(-1) * \mathrm{B1} + \mathrm{D1}$

$$H(s) = \frac{1}{(\mathrm{c1} * \mathrm{r1} * s + \mathrm{c3} * \mathrm{r1} * s + \mathrm{c3} * \mathrm{r2} * s + \mathrm{c3} * \mathrm{r3} * s + \mathrm{c1} * \mathrm{c3} * \mathrm{r1} * \mathrm{r2} * s\hat{}2 + \mathrm{c1} * \mathrm{c3} * \mathrm{r1} * \mathrm{r3} * s\hat{}2}$$

$$\frac{}{+\mathrm{c2} * \mathrm{c3} * \mathrm{r1} * \mathrm{r3} * s\hat{}2 + \mathrm{c2} * \mathrm{c3} * \mathrm{r2} * \mathrm{r3} * s\hat{}2 + \mathrm{c1} * \mathrm{c2} * \mathrm{c3} * \mathrm{r1} * \mathrm{r2} * \mathrm{r3} * s\hat{}3 + 1)}$$

$$\Rightarrow u_s = y_s\left(1 + (C_{12} * R_{12} + 2C_3R_{12} + C_3R_3)s + \left(C_{12}C_3R_{12}^2 + 3C_{12}C_3R_{12}R_3\right)s^2 + \left(C_{12}^2C_3R_{12}^2R_3\right)s^3\right)$$

$$u_s = y_s + y_s^{*}(C_{12} * R_{12} + 2C_3R_{12} + C_3R_3) + y_s^{**}\left(C_{12}C_3R_{12}^2 + 3C_{12}C_3R_{12}R_3\right) + y_s^{***}\left(C_{12}^2C_3R_{12}^2R_3\right)$$

$$H(s) = \frac{1}{c_{12}\hat{}2 * c_3 * r_{12}\hat{}2 * r_3\left(\frac{c_{12}r_{12} + 2c_3r_{12} + c_3r_3}{c_{12}^2c_3r_{12}^2r_3}s + \frac{c_{12}c_3r_{12}^2 + 3c_{12}c_3r_{12}r_3}{c_{12}^2c_3r_{12}^2r_3}s^2 + \frac{1}{c_{12}^2c_3r_{12}^2r_3} + s^3\right)} \Rightarrow$$

$$\Rightarrow H(s) = \frac{\frac{1}{c_{12}\hat{}2 * c_3 * r_{12}\hat{}2 * r_3}}{\frac{s}{c_{12}c_3r_{12}r_3} + \frac{2s}{c_{12}^2r_{12}r_3} + \frac{s}{c_{12}^2r_{12}^2} + \frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}} + s^3 + \frac{1}{c_{12}^2c_3r_{12}^2r_3}}$$

$$H(s) = \frac{4 * 10^6}{s^3 + 3 * 10^4 * s^2 + 10^8\, s + 4 * 10^6}$$

## 4. Singularitatile sistemului

(L3) Sa se evidentieze simbolic singularitatile sistemului, apoi sa se particularizeze pentru valorile fiecarui student¸si sa se figureze singularitatile in planul complex;

$$H(s) = \frac{\frac{1}{c_{12}\hat{}2 * c_3 * r_{12}\hat{}2 * r_3}}{\frac{s}{c_{12}c_3r_{12}r_3} + \frac{2s}{c_{12}^2r_{12}r_3} + \frac{s}{c_{12}^2r_{12}^2} + \frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}} + s^3 + \frac{1}{c_{12}^2c_3r_{12}^2r_3}}$$

In cazul nostru zerourile sistemului nu se gasesc deoarece in ecuatia numaratorului nu exista "s", iar polii sistemului se gasesc prin rezolvarea ecuatiei numitorului.

Polii sistemului:

$\widehat{s_1}$ =-2.6180*$10^4$

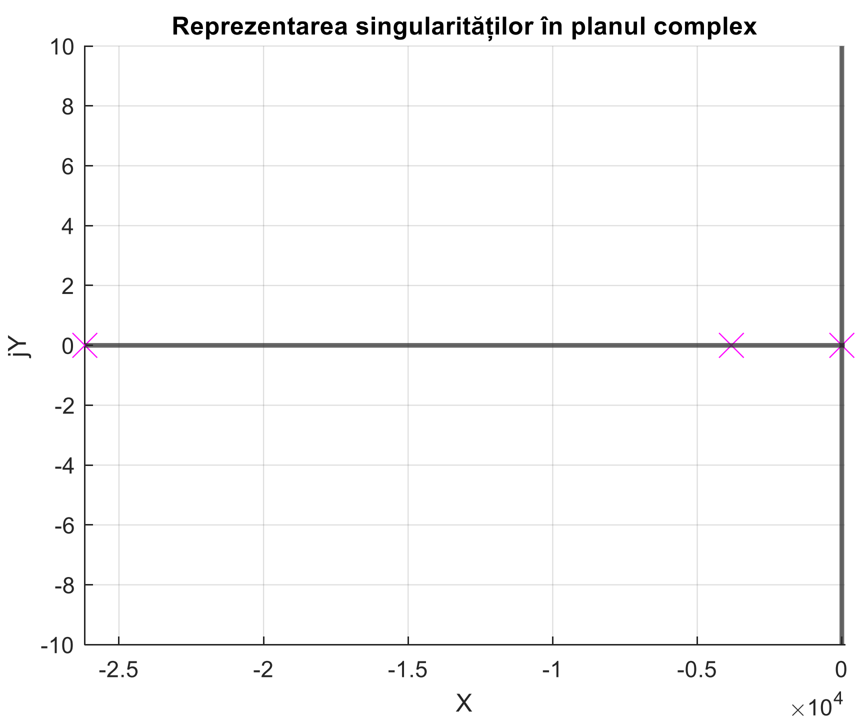
$\widehat{s_2}$ =-0.3820*$10^4$

$\widehat{s_3}$ =0.04

Zerourile sistemului:

```
y =

  0×1 empty double column vector
```



# 5. FCC si FCO

(L5) Sa se determine realizarile de stare corespunzatoare formelor canonice de control (FCC) ¸si de observare (FCO). Sa se realizeze o schema Simulink in care sa se implementeze aceste realizari de stare;

<u>Forma canonică de control</u>

$$\left(\begin{array}{c|c} A_{FCC} & B_{FCC} \\ \hline C_{FCC} & D \end{array}\right) = \left(\begin{array}{ccccc|c} -a_{n-1} & -a_{n-2} & \dots & -a_1 & -a_0 & 1 \\ 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 \\ \hline b_{n-1} & b_{n-2} & \dots & b_1 & b_0 & d \end{array}\right)$$

$$H(s) = 1 + \frac{-\frac{s}{c_{12}c_3r_{12}r_3} - \frac{2s}{c_{12}^2r_{12}r_3} - \frac{s}{c_{12}^2r_{12}^2} - \frac{s^2}{c_{12}r_3} - \frac{3s^2}{c_{12}r_{12}} - s^3}{\frac{s}{c_{12}c_3r_{12}r_3} + \frac{2s}{c_{12}^2r_{12}r_3} + \frac{s}{c_{12}^2r_{12}^2} + \frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}} + s^3 + \frac{1}{c_{12}^2c_3r_{12}^2r_3}}$$

$$H(s) = 1 + \frac{-s^3 - 3*10^4*s^2 - 10^8s}{s^3 + 3*10^4*s^2 + 10^8*s + 4*10^6}$$

$$A_{\text{FCC}} = \begin{bmatrix} -3*10^4 & -10^8 & -4*10^6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B_{\text{FCC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C_{\text{FCC}} = \begin{bmatrix} -3*10^4 & -10^8 & 0 \end{bmatrix}$$

$$D_{\text{FCC}} = [1]$$

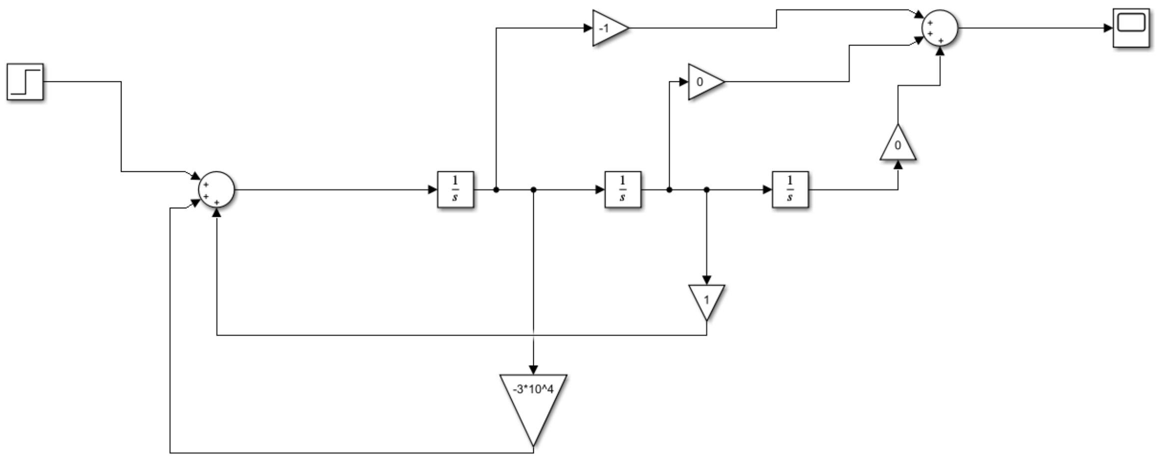
$$\dot{x}_1 = -\left(\frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}}\right)*x_1 - \left(\frac{s}{c_{12}c_3r_{12}r_3} + \frac{2s}{c_{12}^2r_{12}r_3} + \frac{s}{c_{12}^2r_{12}^2}\right)*x_2 - \frac{1}{c_{12}^2c_3r_{12}^2r_3}*x_3 - u$$

$$\dot{x}_1 = -3*10^4*x_1 - 10^8*x_2 - 4*10^6*x_3 - u$$

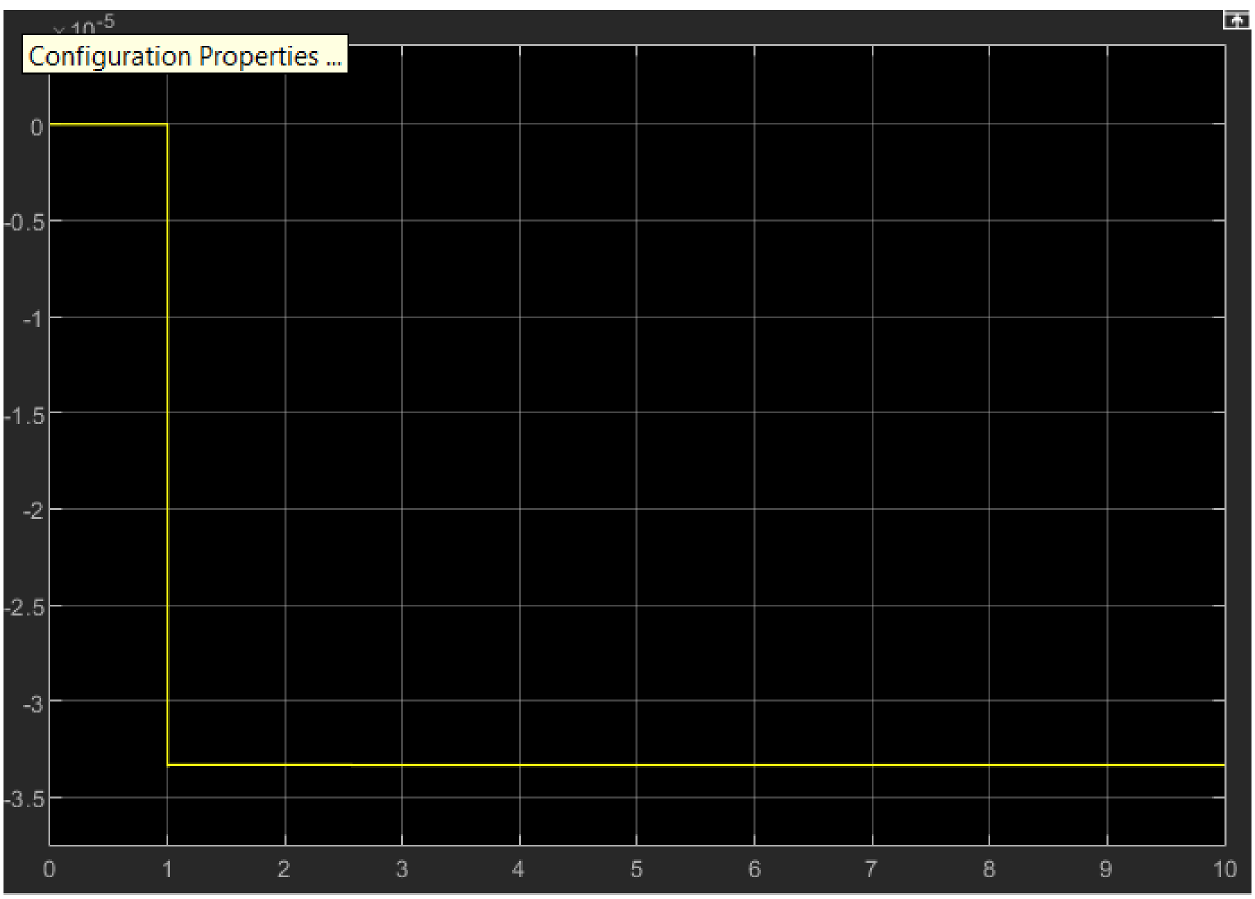
$$\dot{x}_2 = \text{x1}$$

$$\dot{x}_3 = \text{x2}$$

$$y = -3*10^4*x_1 + -10^8*x_2 + u$$



-1
0
0
1
-3*10^4

× 10^-5
Configuration Properties ...
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
1
2
3
4
5
6
7
8
9
10

$$A_{\mathrm{FCO}} = \begin{bmatrix} -3*10^4 & 1 & 0 \\ -10^8 & 0 & 1 \\ -4*10^6 & 0 & 0 \end{bmatrix}$$

$$B_{\mathrm{FCO}} = \begin{bmatrix} -3*10^4 \\ -10^8 \\ 0 \end{bmatrix}$$

$$C_{\mathrm{FCO}} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$D_{\mathrm{FCO}} = \begin{bmatrix} 1 \end{bmatrix}$$

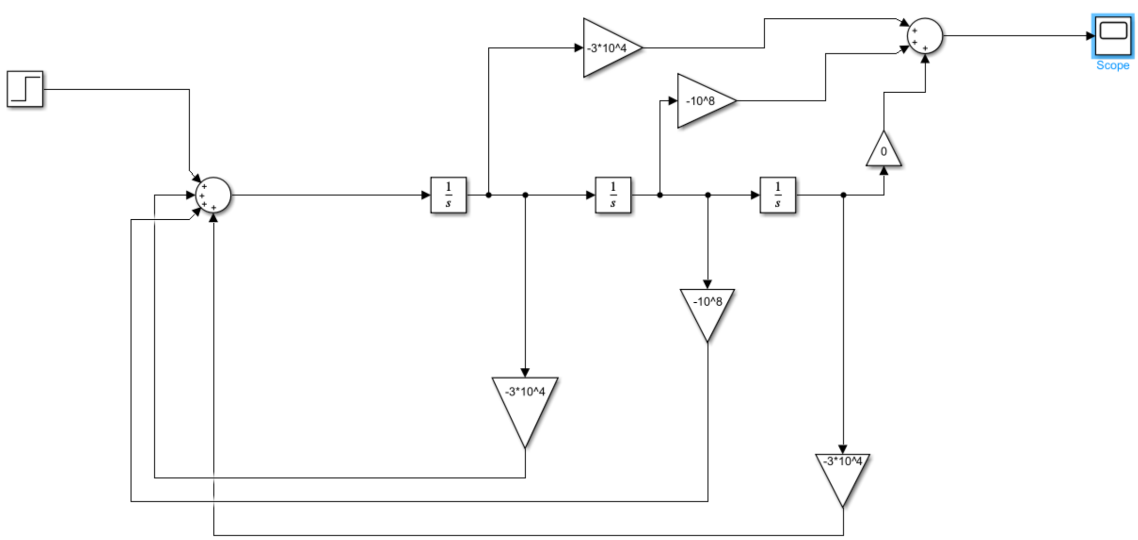
$$\dot{x}_1 = -\left(\frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}}\right) * x_1 + x_2 - \left(\frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}}\right)u$$

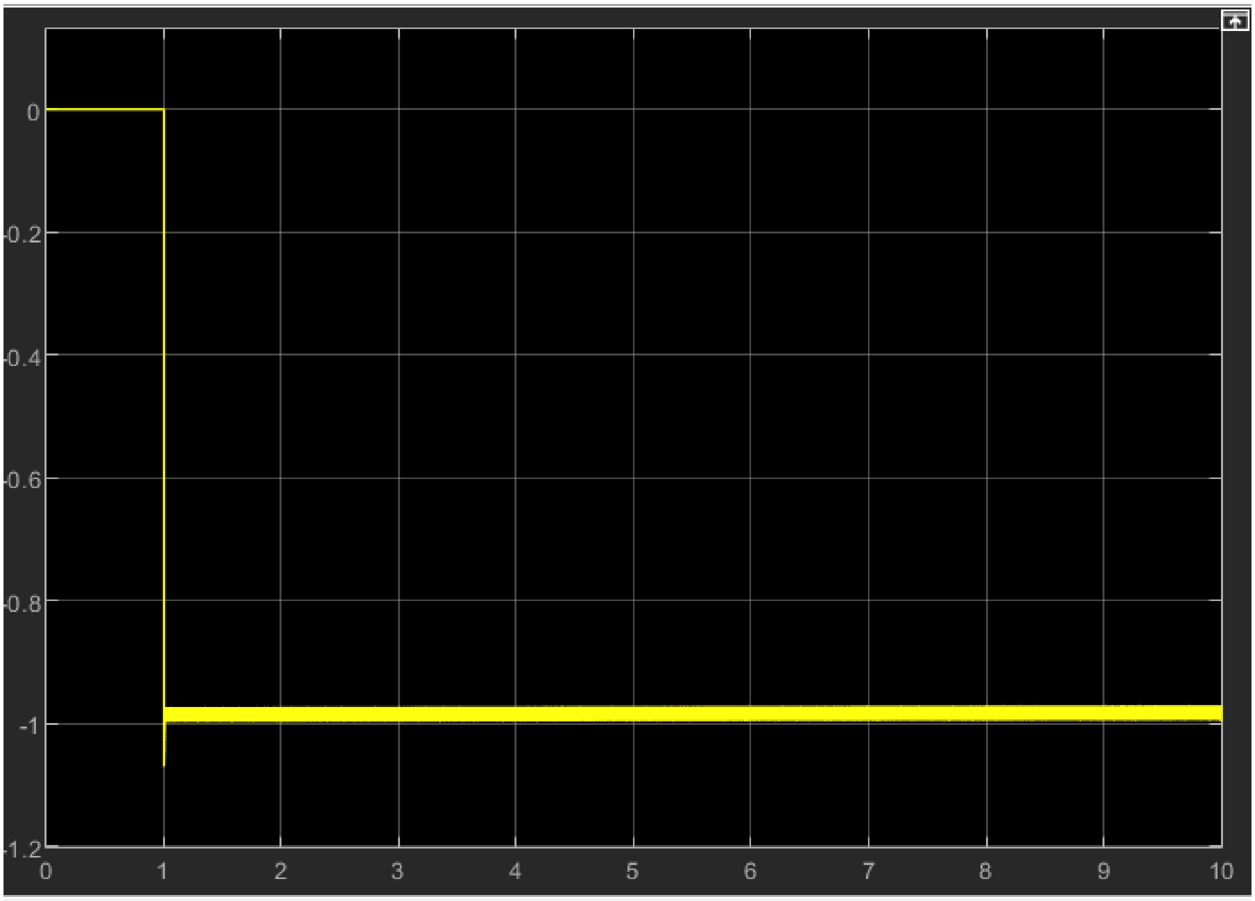
$$\dot{x}_1 = -3*10^4 * x_1 + x_2 - 3*10^4 * u$$

$$\dot{x}_2 = -10^8 * x_1 + x_3 - 10^8 * u$$

$$\dot{x}_3 = -4*10^6 * x_1$$

$$y = \mathrm{x1} + u$$






# 6. Functia de transfer in forma minimala

(L5) Sa se determine functia de transfer in forma minimala;

$$H(s) = \frac{4 * 10^6}{s^3 + 3 * 10^4 * s^2 + 10^8 s + 4 * 10^6}$$

markov=deconv([numzeros(1,5)],den)

$\gamma_0 = 0$

$\gamma_1 = 0$

$\gamma_2 = 0$

$\gamma_3 = 0$

$\gamma_4 = -10^{-11}$

$\gamma_5 = 3.2 * 10^{15}$

hankel=[markov(2)markov(3)markov(4);markov(3)markov(4)markov(5);markov(4)markov(5)markov(6)];

$$H=\begin{bmatrix}\gamma_1 & \gamma_2 & \gamma_3\\ \gamma_2 & \gamma_3 & \gamma_4\\ \gamma_3 & \gamma_4 & \gamma_5\end{bmatrix}=10^{15}*\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -0.0001\\ 0 & -0.0001 & 3.2\end{bmatrix}$$

detH= $-6.4*10^{19}$ => rank=2

# 7. Stabilitatea interna si externa

$$H(s)=\frac{4*10^6}{s^3+3*10^4*s^2+10^8 s+4*10^6}$$

$$s^3+3*10^4*s^2+10^8 s+4*10^6=0$$

$$\hat{s}_1=-2.6180*10^4$$

$$\hat{s}_2=-0.382*10^4$$

$$\hat{s}_3=0.04$$

$$\dot{x}=\begin{bmatrix}-\left(\frac{1}{\mathrm{R1}*\mathrm{C1}}+\frac{1}{\mathrm{R1}*\mathrm{C1}}\right) & \frac{1}{\mathrm{R2}*\mathrm{C1}} & \frac{1}{\mathrm{R2}*\mathrm{C1}}\\ \frac{1}{\mathrm{C2}*\mathrm{R2}} & -\frac{1}{\mathrm{C2}}\left(\frac{1}{\mathrm{R2}}+\frac{1}{\mathrm{R3}}\right) & -\frac{1}{\mathrm{C2}*\mathrm{R2}}\\ 0 & \frac{1}{\mathrm{R3}*\mathrm{C3}} & 0\end{bmatrix}*x\{=A\}+\begin{bmatrix}\frac{1}{\mathrm{R1}*\mathrm{C1}}\\ 0\\ 0\end{bmatrix}\{=B\}*u$$

$$y=[0\quad 0\quad 1]x\{=C\}+[0]\{=D\}*u$$

$$=10^4\begin{bmatrix}-2 & 1 & 1\\ 1 & -1 & -1\\ 0 & 4*10^{-6} & 0\end{bmatrix}*x+10^4\begin{bmatrix}1\\0\\0\end{bmatrix}*u$$

$$\det(\mathrm{sI}-A)=0\Rightarrow\det(\lambda I_3-A)=0$$

$$10^4\begin{bmatrix}2+\lambda & -1 & -1\\ -1 & 1+\lambda & +1\\ 0 & -4*10^{-6} & \lambda\end{bmatrix}=0$$

Tabelul Routh − Hurwitz

$\lambda^3+3*\lambda^2+(4*10^{-6}+2)*\lambda+4*10^{-6}+1=0$ polinom caracteristic

$\lambda^3\ \ 1\quad (4*10^{-6}+2)$

$\lambda^2\ \ 3\quad 4*10^{-6}+1$

− − − − − − − − − − −

$\lambda^1\ \ (4*10^{-6}+2)\ \ 0$

$\lambda^0\ \ (4*10^{-6}+1)\ \ 0$

$$\frac{-\begin{bmatrix}1 & (4*10^{-6}+2)\\ 3 & 0\end{bmatrix}}{3}=(4*10^{-6}+2)$$

$$\frac{\begin{bmatrix}3 & (4*10^{-6}+1)\\ (4*10^{-6}+2) & 0\end{bmatrix}}{(4*10^{-6}+2)}=(4*10^{-6}+1)$$

$\Rightarrow$ Sistemul este stabil intern $\Rightarrow$ sistemul este stabil extern

## 8. Stabilitatea cu ajutorul functiei Lyapunov

$$A = 10^4 \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 4*10^{-6} & 0 \end{bmatrix}$$

$$A^t \cdot P + P \cdot A = -Q \text{ (ecuatia algebrica Lyapunov)}$$

$$Q = \text{eye}(length(A))$$

$$Q \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = lyap(A', Q)$$

$$lyap(A', Q) = \begin{bmatrix} 0 & 0 & 0.0001 \\ 0 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & 25.0001 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0.0001 \\ 0 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & 25.0001 \end{bmatrix}$$

$$V_{(x)} = x^t \cdot P \cdot x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \cdot P \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leftarrow \text{functia de energie}$$

$$V_{(x)} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0.0001 \\ 0 & 0.0001 & 0.0001 \\ 0.0001 & 0.0001 & 25.0001 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$V_{(x)} = \begin{bmatrix} 0.0001 * x_3 & 0.0001 * x_2 + 0.0001 * x_3 & 0.0001 * x_1 + 0.0001 * x_2 + 25.0001 * x_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$V_{(x)} = 0.0001 * x_1 * x_3 + 0.0001 * x_2^2 + 0.0001 * x_2 * x_3 + 0.0001 * x_1 * x_3 + 0.0001 * x_2 * x_3 + 25.0001 * x_3^2$$

$$V_{(x)} = 0.0002 * x_1 * x_3 + 0.0001 * x_2^2 + 0.0002 * x_2 * x_3 + 25.0001 * x_3^2 \leftarrow \text{functia de energie}$$

$$\text{eig}(P) = \{0; 0.0001; 25.0001\} \rightarrow \text{valorile proprii ale matricei } P \text{ sunt pozitive}$$

$$\Rightarrow \text{sistemul este intern asimptotic stabil}$$

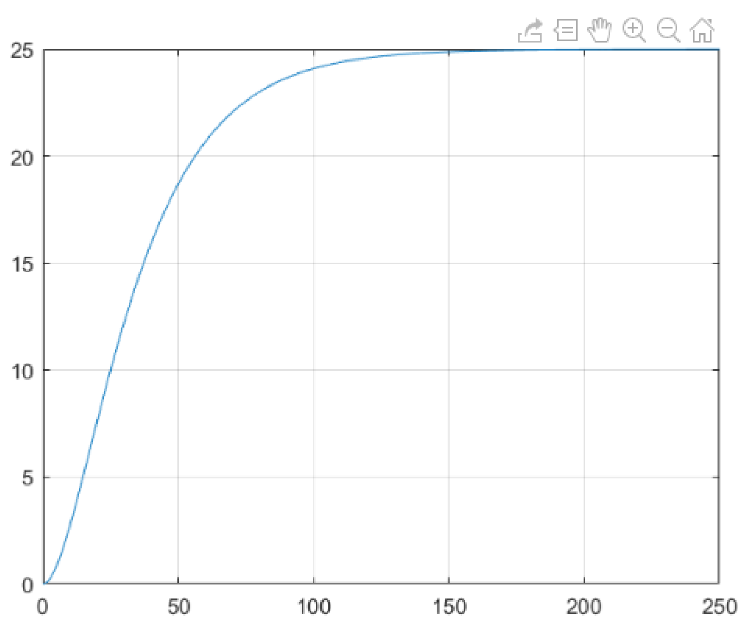
```
r1=100 ;
r2=100;
r3=5*10^6;
c1=10^(-6);
```

```
c2=10^(-6) ;
c3=5*10^(-6);


A=[-(1/(r2*c1) + 1/(r1*c1)) 1/(r2*c1) 1/(c1*r2);
   1/(r2*c2) -1/c2*(1/r2 + 1/r3) -1/(r2*c2) ;
   0 1/(r3*c3) 0];
B=[1/(r1*c1);0;0];
C=[0 0 1];
D=0;
sys = ss(A, B, C, D);
t = 0:0.1:250;
step_f = @(t)(t>=0);
st = step_f(t);
[~, time, x] = lsim(sys, st, t);


Vx = zeros(length(t), 1);
for i = 1:length(t)
    Vx(i) = x(i,:) * P * x(i, :)';
end


figure;
plot(t, Vx);
grid;
```



9. Inversa lui Laplace pentru functia pondere,indicial si rampa

$$H(s) = \frac{4*10^6}{s^3 + 3*10^4*s^2 + 10^8 s + 4*10^6}$$

$$h(t) = L^{-1}\{H(s)\} - \text{functia pondere}$$

$$y(t) = L^{-1}\left\{H(s)\cdot\frac{1}{s}\right\} - \text{raspuns indicial}$$

$$\text{yr}(t) = L^{-1}\left\{H(s)\cdot\frac{1}{s^2}\right\} - \text{raspuns la rampa}$$

$$L^{-1}\{H(s)\} = L^{-1}$$

$$\left\{\frac{4*10^6}{(s+2.618*10^4)(s+3820)(s+0.04)}\right\} = 4*10^6\left(L^{-1}\left\{\frac{1}{(s+2.618*10^4)}\right\} + L^{-1}\left\{\frac{1}{(s+3820)}\right\} + L^{-1}\left\{\frac{1}{(s+0.04)}\right\}\right) = e^{-2.618*10^4 t} + e^{-3820t} + e^{-0.04t}$$

$$L^{-1}\left\{H(s)\cdot\frac{1}{s}\right\} = L^{-1}$$

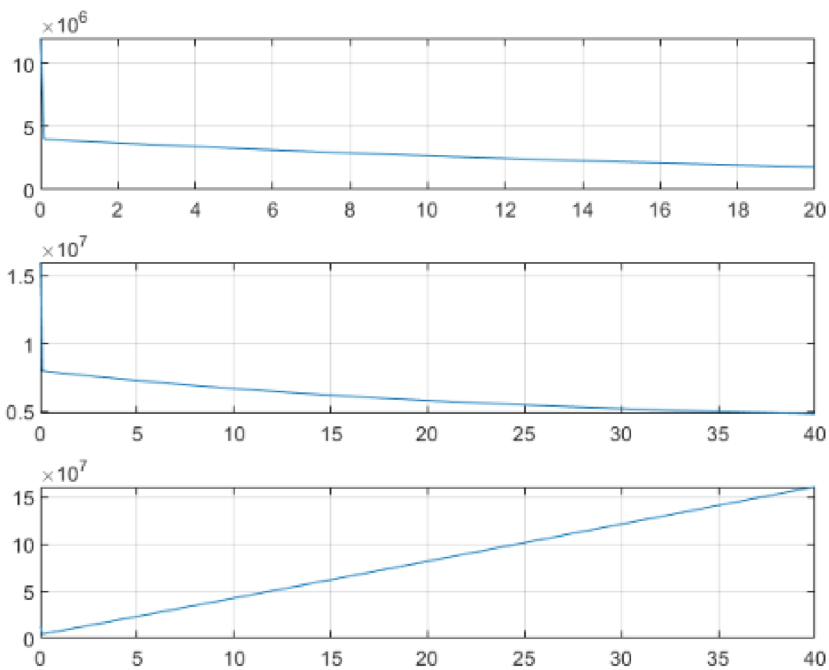
$$\left\{\frac{4*10^6}{(s+2.618*10^4)(s+3820)(s+0.04)s}\right\} = 4*10^6\left(L^{-1}\left\{\frac{1}{(s+2.618*10^4)}\right\} + L^{-1}\left\{\frac{1}{(s+3820)}\right\} + L^{-1}\left\{\frac{1}{(s+0.04)}\right\} + L^{-1}\left\{\frac{1}{s}\right\}\right) =$$

$$e^{-2.618*10^4 t} + e^{-3820t} + e^{-0.04t} + 1$$

$$L^{-1}\left\{H(s)\cdot\frac{1}{s^2}\right\} = L^{-1}$$

$$\left\{\frac{4*10^6}{(s+2.618*10^4)(s+3820)(s+0.04)s}\right\} = 4*10^6\left(L^{-1}\left\{\frac{1}{(s+2.618*10^4)}\right\} + L^{-1}\left\{\frac{1}{(s+3820)}\right\} + L^{-1}\left\{\frac{1}{(s+0.04)}\right\} + L^{-1}\left\{\frac{1}{s^2}\right\}\right) =$$

$$e^{-2.618*10^4 t} + e^{-3820t} + e^{-0.04t} + t$$



# 10.Performantele sistemului

$$H(s) = \frac{4 * 10^6}{s^3 \ + 3 * 10^4 * s^2 + 10^8 \, s + 4 * 10^6}$$

$$H(s) = \frac{4 * 10^6}{(s + 2.618 * 10^4)(s + 3820)(s + 0.04)}$$

$$H(s) = \frac{4 * 10^6}{(s + 2.618 * 10^4)(s^2 + 3820.04s + 152.8)}$$

$$\frac{\left(\overset{0}{T}_S + 1\right)\left(s^2 + 2w_n\zeta s + w_n^2\right)}{\left(\overset{\wedge}{T}_S + 1\right)\left(s^2 + 2w_n\zeta s + w_n^2\right)}$$

$$\overset{\wedge}{T}_S = \frac{1}{2.618 * 10^4}$$

$$\overset{0}{w}_n = 2 * 10^3$$

$$\overset{\wedge}{w}_n = \sqrt{152.8}$$

$$2\overset{0}{\zeta}\overset{0}{w}_n = 0 \Rightarrow \overset{0}{\zeta} = 0$$

$$2\overset{\wedge}{\zeta}\overset{\wedge}{w}_n = 3820.04 \Rightarrow \overset{\wedge}{\zeta} = 154.51$$

$$k = H(0) = 1$$

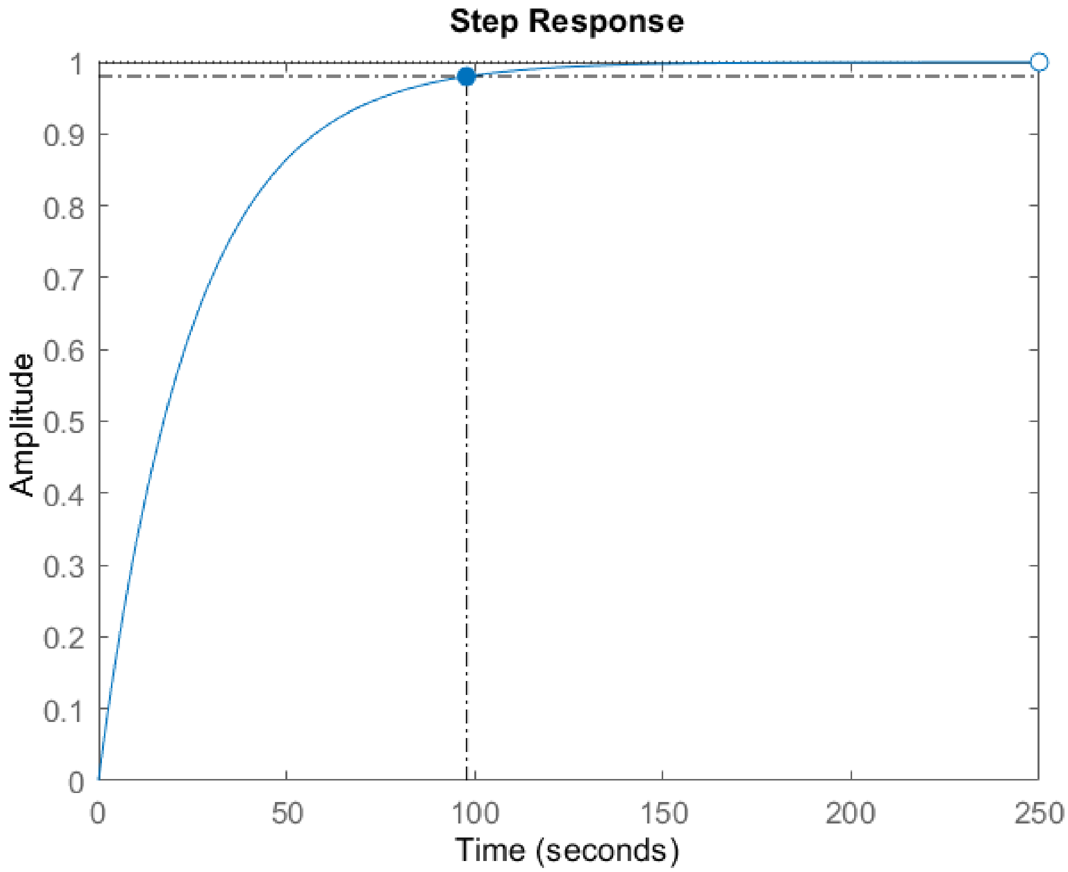
$$\text{Essp} = \lim_{s \to 0} 1 - H(s) = 1 - H(0) = 1 - 1 = 0$$

$$\text{Essv} = \lim_{s \to 0} \ \frac{1}{s}\left(1 - H(s)\right) = \lim_{s \to 0} \ \frac{1}{s}\left(1 - \frac{4 * 10^6}{s^3 \ + 3 * 10^4 * s^2 + 10^8 \, s + 4 * 10^6}\right)$$

$$= \lim_{s \to 0} \ \frac{1}{s}\left(\frac{s^3 \ + 3 * 10^4 * s^2 + 10^8 \, s}{s^3 \ + 3 * 10^4 * s^2 + 10^8 \, s + 4 * 10^6}\right) =$$

$$= \lim_{s \to 0} \ \frac{s^3 \ + 3 * 10^4 * s^2 + 10^8 \, s}{s^4 \ + 3 * 10^4 * s^3 + 10^8 \, s^2 + 4 * 10^6 s} \overset{l'H}{=}$$

$$= \lim_{s \to 0} \frac{3s^2 + 6 * 10^4 s + 10^8}{4s^3 + 9 * 10^4 s^2 + 2 * 10^8 s + 4 * 10^6} = \frac{10^8}{4 * 10^6} = 25$$




Timpul de raspuns tr = 97.8

Suprareglajul $\sigma = e^{\frac{-n\zeta}{\sqrt{1-\zeta^2}}} = 0\%$

Pulsatia de oscilatie $w_{\text{osc}} = w_n\sqrt{1-\zeta^2} = \text{Im}\left\{\hat{s}_{1,2}\right\} = 0$

11.Problema 10

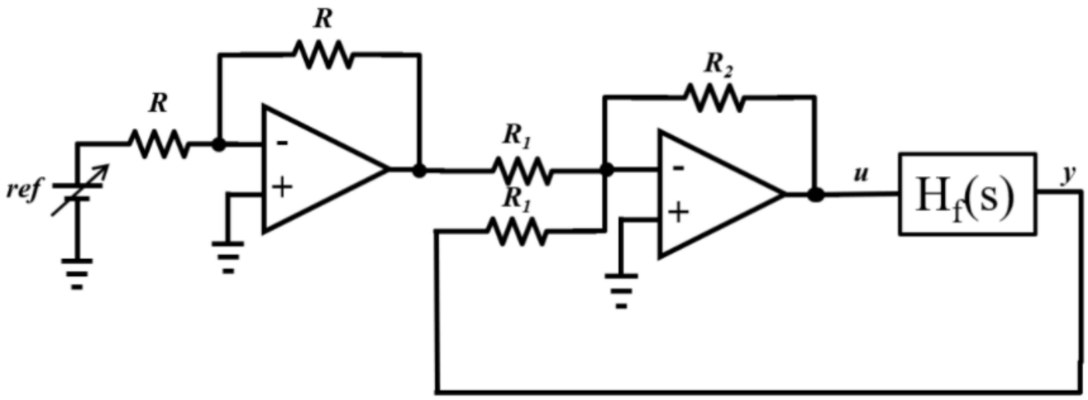
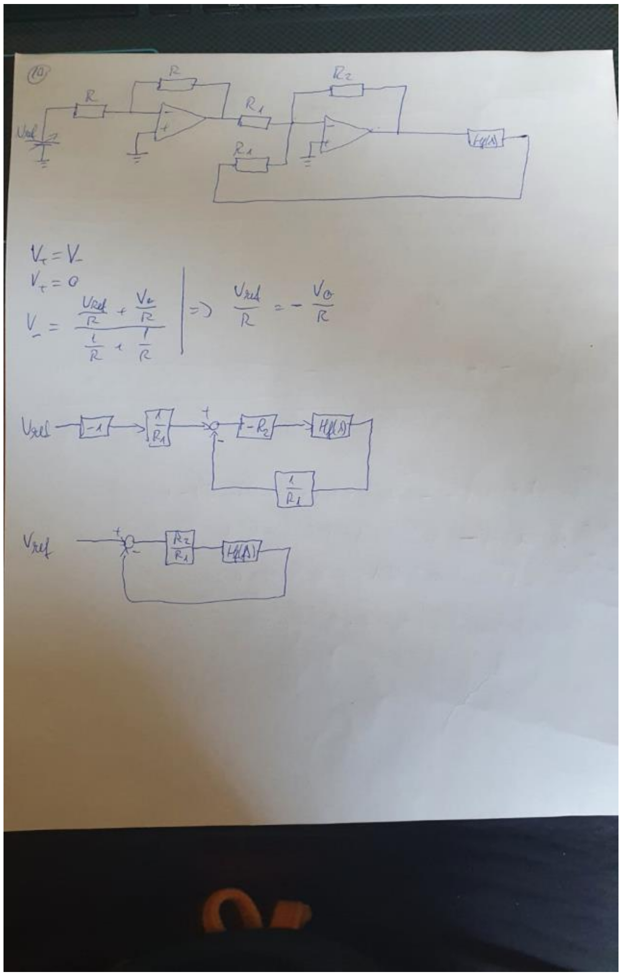


Figure 1: Structura unui sistem de reglare cu regulator proporţional

$$H(s) = \frac{\frac{1}{c_{12}\hat{}2 * c_3 * r_{12}\hat{}2 * r_3}}{\frac{s}{c_{12}c_3r_{12}r_3} + \frac{2s}{c_{12}^2r_{12}r_3} + \frac{s}{c_{12}^2r_{12}^2} + \frac{s^2}{c_{12}r_3} + \frac{3s^2}{c_{12}r_{12}} + s^3 + \frac{1}{c_{12}^2c_3r_{12}^2r_3}}$$

$$H(s) = \frac{4 * 10^6}{s^3 + 3 * 10^4 * s^2 + 10^8 s + 4 * 10^6}$$



Functia de transfer pe calea de reactie este: $H_r(s) = 1$

Functia de transfer pe calea directa este: $H_d(s) = \frac{R_2}{R_1} * H_f(s)$

Functia de transfer in bucla inchisa este: $H_0(s) = \frac{\frac{R_2}{R_1} * H_f(s)}{1+\frac{R_2}{R_1} * H_f(s)}$

b) Sa se determine functia de transfer a sistemului in bucla inchisa, unde Hf(s) reprezinta modelul matematic al procesului cu o intrare si o iesire ales la cerintele anterioare.

$H_d(s) = \frac{R_2}{R_1} * H_f(s)$

$k = \frac{R2}{R1} H_f(s)$ $\qquad k_{cr} = 7.47 * 10^5$

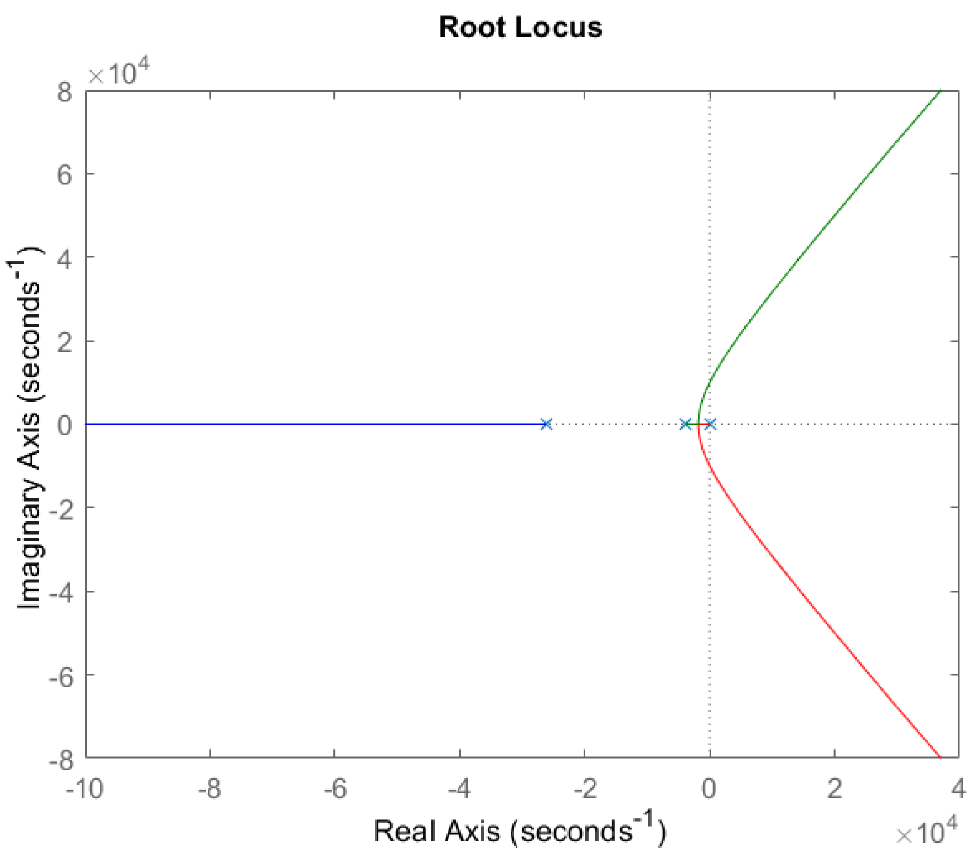
Avem n=3 -> Polii sistemului sunt:

$\overset{\wedge}{s}_1 = -2.6180 * 10^4$

$\overset{\wedge}{s}_2 = -0.382 * 10^4$

$\overset{\wedge}{s}_3 = 0.04$

m=0 -> nu avem zerouri (asemeni subpunctului 3)




$$k\epsilon(0{,}2.22 * 10^4) \rightarrow \ s_{\widehat{01,02,03}}\varepsilon R_- \rightarrow regim\ aperiodic\ amortizat \rightarrow moduriile\ e^{-2.618*10^\wedge 4t}\ , e^{\ -0.382*10^\wedge 4t}, e^{0.04*t}$$

$$k = 2.22 * 10^4 \rightarrow \ \widehat{s_{01}} = \ \widehat{s_{02}} = \ -36.1\ , R_- \rightarrow regim\ aperiodic\ critic\ amortizat \rightarrow \ te^{2.22*10^\wedge 4t}, te^{2.22*10^\wedge 4t}, e^{0.04*t}$$

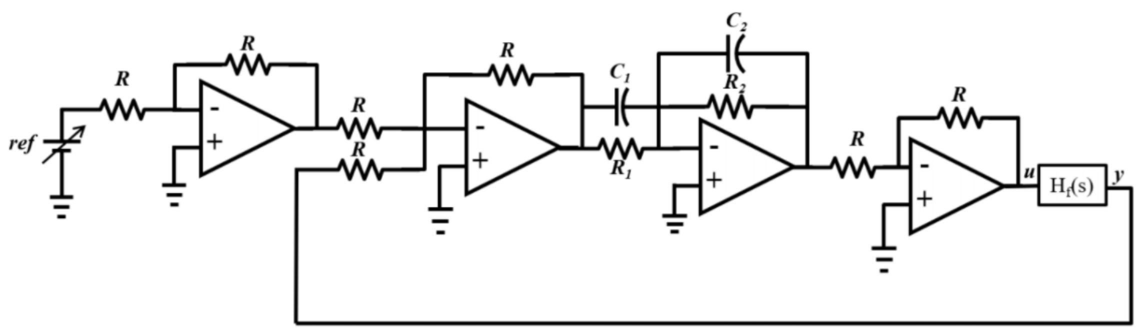
$$k \in (2.22 * 10^4, 7.47 * 10^5) \rightarrow \ \widehat{s_{01,02}} \in C_- \rightarrow regim\ oscilant\ neamortizant \rightarrow modurile\colon e^{71*t}\sin(1.02i * t)\ , e^{0.04*t}$$

$k = 7.47 * 10^5 \rightarrow \ \widehat{s_{01,02}} = 71\ , s_{03}\varepsilon R_- \rightarrow regim\ aperiodic\ critic\ neamortizat \rightarrow modurile\ te^{71t}\ , te^{\ 71t}, e^{0.04*t}$

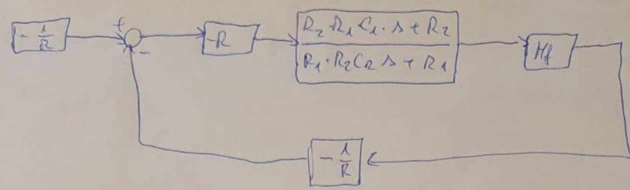
$k \in (7.47 * 10^5, inf) \rightarrow \ s_{01,02}\varepsilon R_+, s_{03}\varepsilon R_- \rightarrow regim\ aperiodic\ neamortizat \rightarrow e^{-2.618*10^\wedge 4t}\ , e^{\ -0.382*10^\wedge 4t}, e^{0.04*t}$

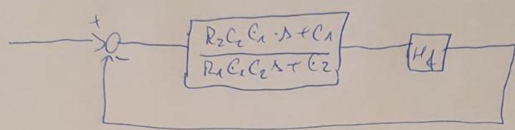
## 12. Problema 11.A

Pentru structura de reglare din figura 2, avand un regulator de tip Lead/Lag, iar Hf(s) reprezinta modelul matematic al procesului cu o intrare si o iesire ales la cerintele anterioare



11




a)

$$H_R(s) = \frac{R_2 R_1 C_1 s + R_2}{R_1 C_1 C_2 s + R_1} = \frac{R_2}{R_1} \cdot \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$$

b)

$$H(s) = \frac{\dfrac{R_2 C_2 C_1 \cdot s + C_1}{R_1 C_1 C_2 s + C_2} H_f}{1 + \dfrac{R_2 C_2 C_1 s + C_1}{R_1 C_1 C_2 s + C_2} H_f}$$