In Search of Emergent Phenomena in Empirically Constructed Model of Shoaling Fish

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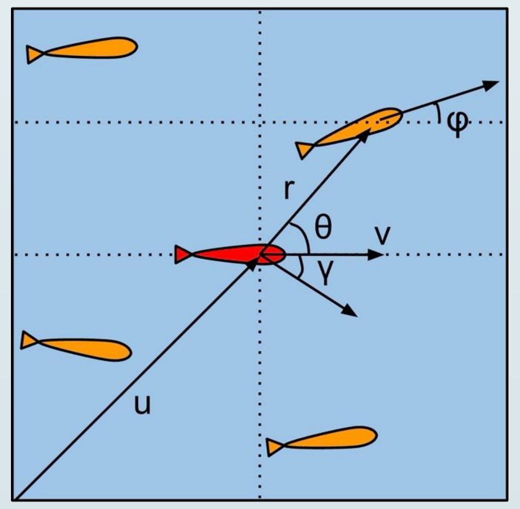
BACKGROUND

- Complex collective behavior emerge in the movement of groups of animals governed by local principles: flocks of birds, schools of fish, etc. Computational methods allow us to simulate and study these emergent behavior to verify and elucidate empirical observations
- Classical models such as the Vicsek model of self-propelled particles were simple but used aprioristic assumptions that were not grounded in empirical evidence [1]
- Recent studies and methods provide new clues into the rules that govern local interactions between animals [2-4] and the resulting emergent phenomena [1,7,8]
- **In this project, I attempt to simulate the shoaling of fish in 2D using empirically consistent local interaction rules and explore emergent phenomena across variation in simulation parameters**

INTERACTION RULES AND METHODS

- The local interaction rules were determined according to two papers that describe the dynamics of shoaling/schooling fish [2,3]:
- Fish interact by modulating speed and and aligning with their nearest neighbor [2,3]. Adjustments are primarily governed by distance |r| and relative orientation to the neighbor θ



1. Speed: Long range attraction, short-range repulsion

$$\frac{d\mathrm{v}}{d\mathrm{t}} = \alpha_{\max}\cos(\theta)\tanh(|\vec{\mathrm{r}}|-2)+\delta_\alpha$$

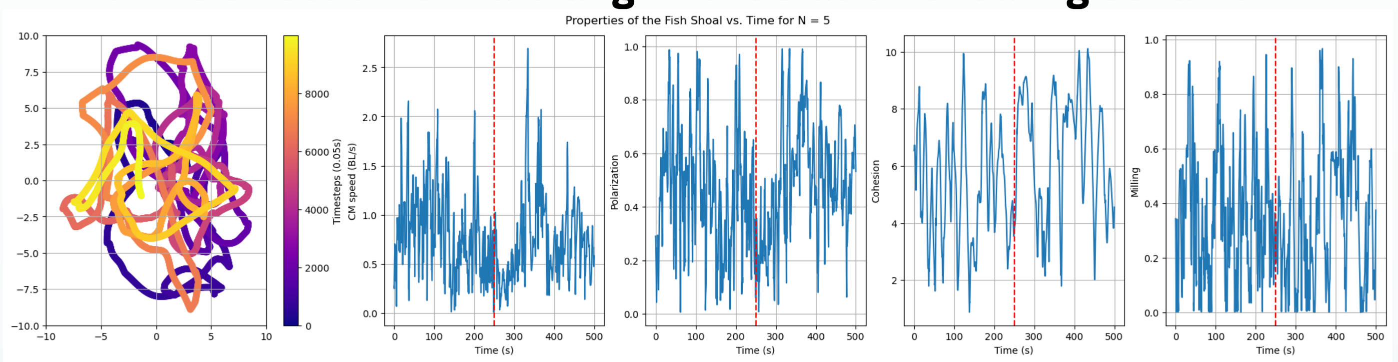
2. Rotation: Turn to the nearest neighbor

$$\frac{d\gamma}{d\mathrm{t}} = \begin{cases} \omega_{\max}|\vec{\mathrm{r}}|\sin(\theta)e^{-|\vec{\mathrm{r}}|/\ell_\omega}+\delta_\omega & \text{if } |\vec{\mathrm{r}}| \geq 1\text{ BL} \\ -\omega_{\max}\sin(\theta)+\delta_\omega & \text{if } |\vec{\mathrm{r}}| < 1\text{ BL} \end{cases}$$

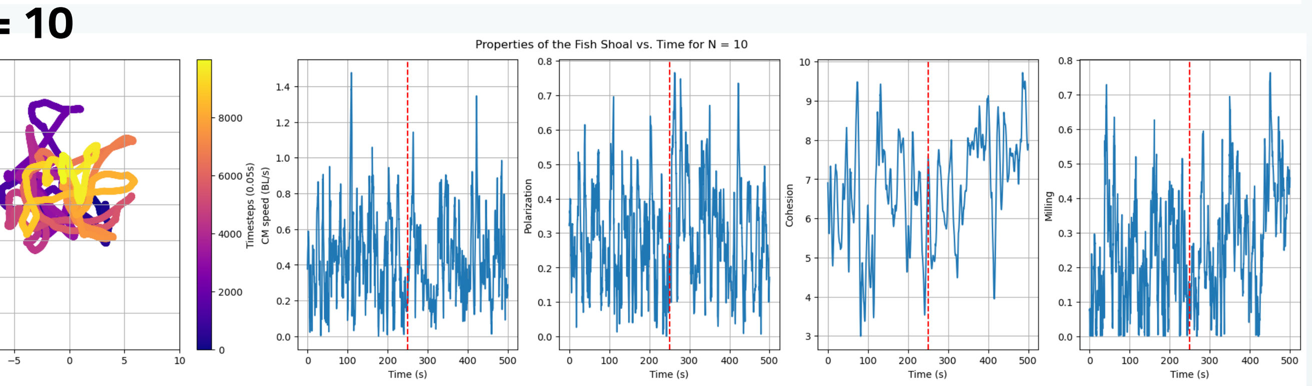
For each timestep, $|\vec{\mathrm{v}}_{\mathrm{t+1}}| = |\vec{\mathrm{v}}_{\mathrm{t}}| + (\mathrm{dv}/\mathrm{dt}_{\mathrm{t+1}})\tau$
$\gamma_{\mathrm{t+1}} = \gamma_{\mathrm{t}} + (\mathrm{d}\gamma/\mathrm{dt}_{\mathrm{t+1}})\tau$ and $|\mathrm{u}_{\mathrm{t+1}}| = |\mathrm{u}_{\mathrm{t}}| + \vec{\mathrm{v}}_{\mathrm{t}}\tau$

SIMULATIONS

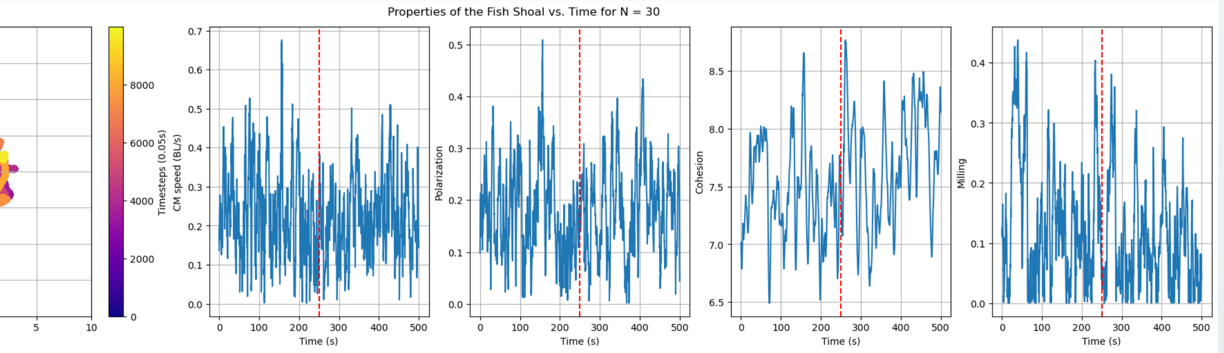
N = 5 The effect of forward alignment bias on shoaling behavior



N = 10



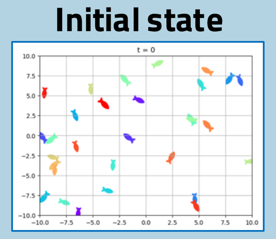
N = 30



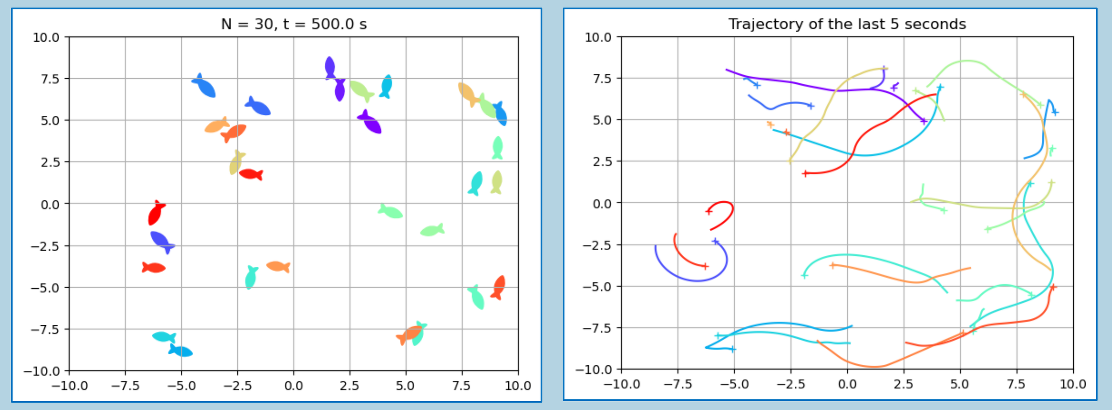
Example of fish shoaling simulation:

Simulations were conducted with timestep = 0.05s for 10,000 iterations. The mean velocity was around 1 BL/s. The average distance to neighbor per simulation was between 1.5 and 3 BL.

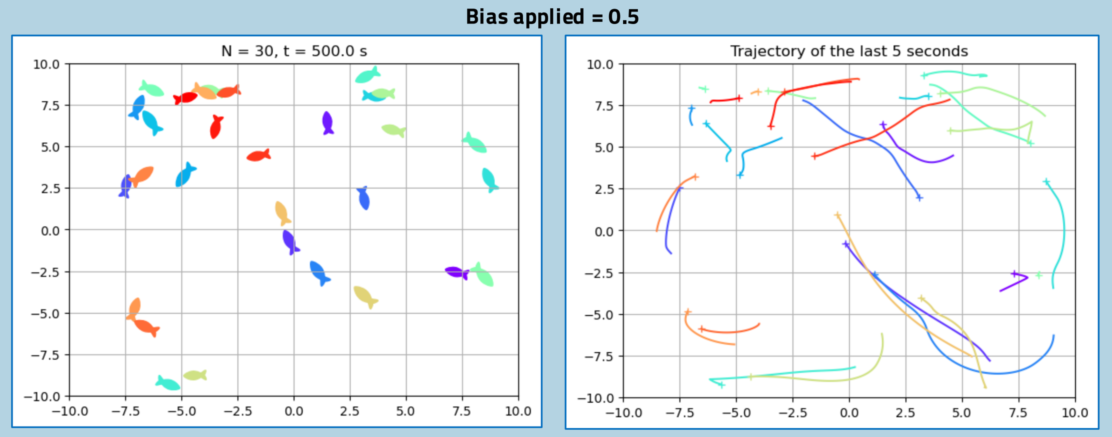
Initial state



No bias applied



Bias applied = 0.5

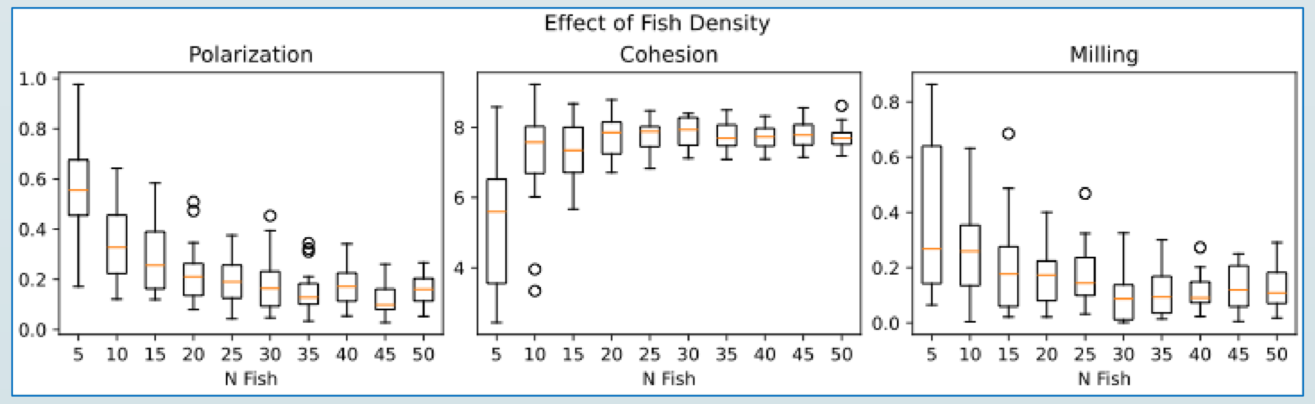


ANALYSIS AND RESULTS

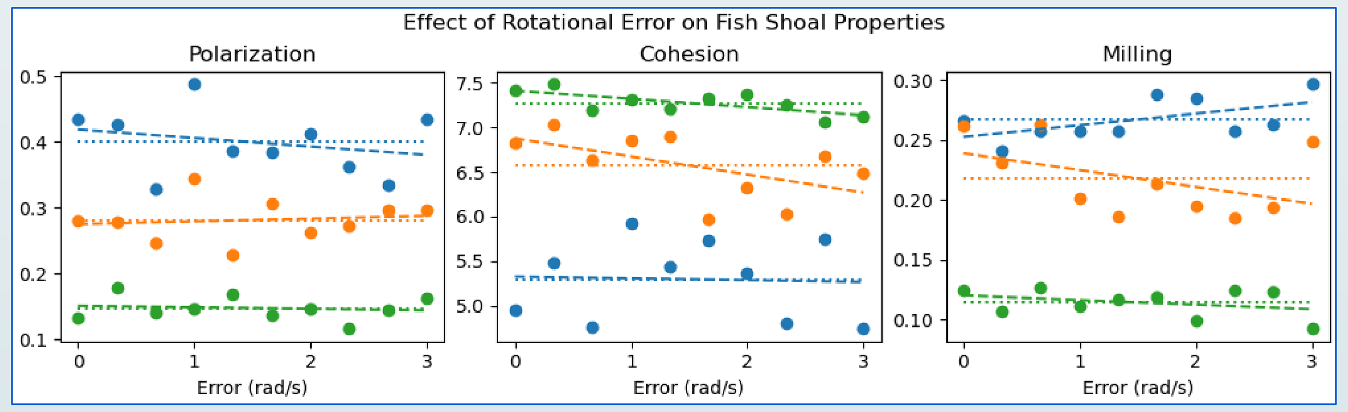
- Simulations were analyzed for their cohesion (C), polarization (P), and milling (M) across changes in parameters

$$\mathrm{C}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\|\vec{u_i}-\vec{u}_{cm}\|^2} \qquad \mathrm{P}=\frac{1}{N}\left\|\sum_{i=1}^{N}\frac{\vec{v_i}}{|\vec{v_i}|}\right\| \qquad \mathrm{M}=\frac{1}{N}\left\|\sum_{i=1}^{N}\frac{\vec{r_i}\times\vec{v_i}}{|\vec{r_i}||\vec{v_i}|}\right\|$$

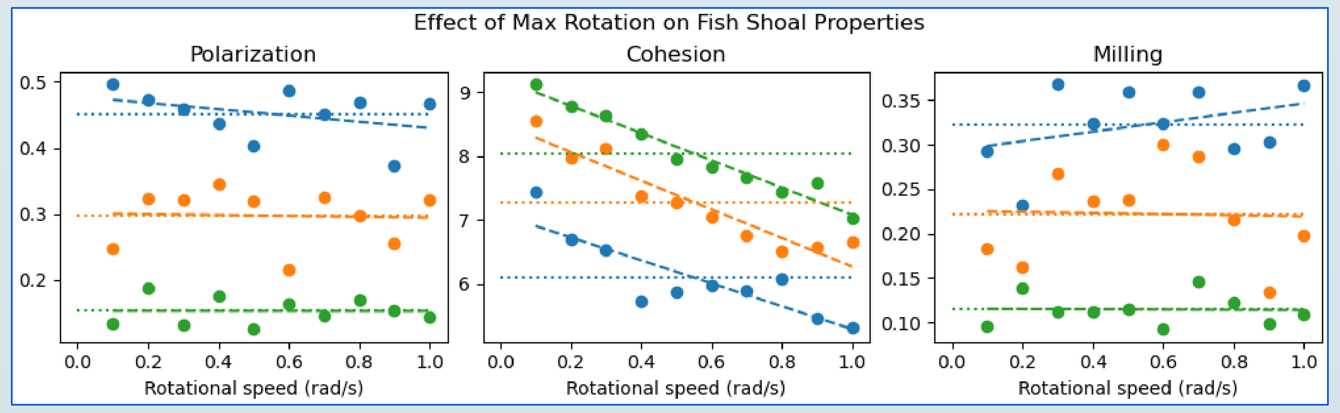
- There was a consistent increase in cohesion, milling, and polarization with respect to the number of fish



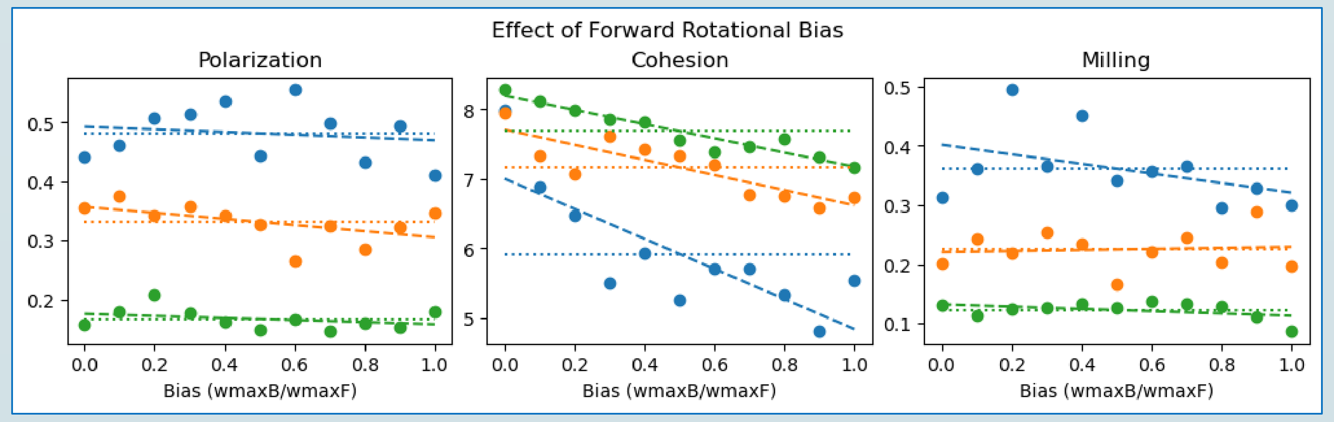
- The acceleration constant $\alpha_{\max}$ and error in acceleration δ_α did not produce significant or consistent influence on the shoaling properties across the fish group sizes
- Front rotational alignment bias had a slight effect on polarization and milling, while cohesion greatly increased with more bias with a strong linear relationship. Cohesion had a strong decreasing effect with increasing max rotation $\omega_{\max}$



$r_5=-0.26$ $r_{10}=0.14$ $r_{30}=-0.13$ $r_5=-0.05$ $r_{10}=-0.56$ $r_{30}=-0.69$ $r_5=0.57$ $r_{10}=-0.46$ $r_{30}=-0.33$



$r_5=-0.38$ $r_{10}=-0.05$ $r_{30}=-0.01$ $r_5=-0.85$ $r_{10}=-0.95$ $r_{30}=-0.98$ $r_5=0.37$ $r_{10}=-0.03$ $r_{30}=-0.03$



$r_5=-0.16$ $r_{10}=-0.53$ $r_{30}=-0.33$ $r_5=-0.80$ $r_{10}=-0.85$ $r_{30}=-0.96$ $r_5=-0.44$ $r_{10}=0.09$ $r_{30}=-0.44$

CONCLUSION and DISCUSSION

- The simulations often varied from empirical data, mainly in terms of lack of strong group cohesion and absence of equilibrium states, showing that the model was likely insufficient at capturing the full interactions between fish [2,3]
- Other studies have shown that phase transitions can be produced by adjusting the noise in the local interactions or strength in alignments [1,7,8]. These phase transitions are characterized by an inverse relation between polarization vs. milling and cohesion [8]. However, the empirically constructed model did not show large phase transitions across changes in single parameters. Thus, this phenomenon could be a result of multiple parameters acting together or suggest an inadequacy in the model
- Since the empirical evidence showed little to no alignment effect towards its neighbor's relative orientation, polarization was speculated to be mediated by asymmetry between front and back rotational effects [2,3]. This is partially supported by the slight increase of polarization in the forward rotational bias
- This analysis was challenging because the system seemed to never approach equilibrium. This could be a result of the wall boundary condition or other factors like an insufficient or convoluted empirical model. Many sets of local interaction rules can produce similar macroscopic behavior – it is difficult to determine the most general model that accounts for all observable interactions
- Possible next steps include adding stochasticity and intrinsic individual behavior [4], simultaneous updates, continuous rotational repulsion, improving boundary conditions, fit to empirical data for additional feedback

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This project was conducted as part of the course PHYS250 at the University of Chicago. For more information, visit https://github.com/Bobomoto/PHYS250-Final-Project