Descompunerea Cholesky

Să se calculeze descompunerea Cholesky cu $l_{ii} > 0 \; \forall i$ pentru matricea

$$A = \begin{pmatrix} 4 & 2 & 8 \\ 2 & 10 & 10 \\ 8 & 10 & 21 \end{pmatrix}$$

- $A = A^T$
- $A > 0 \iff \det A_p > 0 \; \forall p = \overline{1,n}$

$A > 0 \iff \forall \lambda$ valoare proprie pentru A, $\underline{\lambda > 0}$

$\underline{A > 0}$ $\quad A_1 = [a_{11}] = 4 \quad \det A_1 = 4 > 0$

$A_2 = \begin{bmatrix} 4 & 2 \\ 2 & 10 \end{bmatrix} \quad \det A_2 = 36 > 0$

$A_3 = A \quad \det A_3 = 36 > 0$

$$\begin{pmatrix} 4 & 2 & 8 \\ 2 & 10 & 10 \\ 8 & 10 & 21 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} * \begin{pmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{pmatrix}$$

$$= \begin{pmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} l_{31} \\ l_{21} l_{11} & l_{21}^2 + l_{22}^2 & l_{21} l_{31} + l_{22} l_{32} \\ l_{31} l_{11} & l_{31} l_{21} + l_{32} l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{pmatrix}$$

Pas 1 : l_{11}, l_{21}, l_{31}

l_{11} : $a_{11} = 4 = (LL^T)_{11} = l_{11}^2 \Rightarrow l_{11} = 2$

l_{21} : $a_{21} = 2 = l_{21} l_{11} = 2 \cdot l_{21} \Rightarrow l_{21} = 1$

l_{31} : $a_{31} = 8 = (LL^T)_{31} = l_{31} l_{11} \Rightarrow l_{31} = 4$

Pas 2 : l_{22}, l_{32}

l_{22} : $a_{22} = 10 = (LL^T)_{22} = l_{21}^2 + l_{22}^2 = 1 + l_{22}^2 \Rightarrow l_{22} = 3$

l_{32} : $a_{32} = 10 = l_{31} l_{21} + l_{32} l_{22} = 4 \cdot 1 + 3 l_{32} \Rightarrow$

$l_{32} = 2$

Pas 3 : l_{33}

l_{33} : $a_{33} = 21 = l_{31}^2 + l_{32}^2 + l_{33}^2 = 4^2 + 2^2 + l_{33}^2 \Rightarrow$

$l_{33} = 1$

$$\begin{pmatrix} 4 & 2 & 8 \\ 2 & 10 & 10 \\ 8 & 10 & 21 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 2 & 1 \end{pmatrix}}_{L} * \underbrace{\begin{pmatrix} 2 & 1 & 4 \\ 0 & 3 & 2 \\ 0 & 0 & 1 \end{pmatrix}}_{L^T}$$