Logic for Computer Science - Tutorial 1

Week 1

Some of the following exercises are from the book *Peter Smith. An Introduction to Formal Logic. Cambridge University Press, 2003.*

Propositions. A *proposition* is a statement that can be true or false. Propositions are sometimes called *sentences*. Here are examples of propositions:

1. "You own a laptop computer."
2. "Snow is white."
3. "Snow is not white."
4. "My father goes to work and I go to school."
5. "It is raining outside, but I have an umbrella."
6. "Either it will rain tomorrow, or it won't rain."
7. "If I get a passing grade in Logic, I will celebrate."
8. "$2 + 2 = 4$." ("Two plus two is four.")

```
int sum(int n)
{
   int i   = 0;
   int sum = 0;
   while (i < n)
     {
        i   = i   + 1;
        sum = sum + i;
     }
   return sum;
}
```

Figure 1: The `sum` program.

Exercise 0.1. *Consider the program* `sum` *shown in Figure 1. Are the following propositions true or false?*

1. *"The program* `sum` *computes the sum of the first* `n` *natural numbers."*
2. *"If we change the condition* `i < n` *to* `i <= n` *then the program* `sum` *computes the sum of the first* `n` *natural numbers."*
3. *"If we swap the two instructions in the body of the* `while` *loop, then* `sum` *does not compute the sum of the first* `n` *natural numbers."*
4. *"If the input* `n` *is smaller than 0 then* `sum` *returns 0."*

The following are not examples of propositions since they are not either true or false.

- "Red and Black." (not a statement)
- "π." (not a statement)
- "Is it raining?" (question)
- "Let's go fishing!" (imperative)
- "x is greater than 7." (cannot tell unless I know who x is)
- "This sentence is false."

Exercise 0.2. *Can you formulate a statement about the* `sum` *program that is not a proposition?*

Arguments. An *argument* is a sequence of propositions. The last proposition is called the *conclusion* of the argument, while the other propositions (all except the last) are its *premisses*. Typically the conclusion of an argument is preceded by a word such as "therefore" or "so". Examples:
A1:

1. "John runs faster than Mary."
2. "Mary runs faster then Michael."
3. Therefore, "John runs faster then Michael."

A2:

1. "All men are mortal."
2. "Socrates is a man."
3. So, "Socrates is mortal."

Exercise 0.3. *Answer the following questions:*

- *What are the* premisses *and the* conclusion *of A1? What about A2?*
- *Are these arguments convincing to you?*

Of course, we can build arguments which are not convincing at all:

1. "All students are smart."
2. "John is a man."
3. So, "The Earth is round."

What makes an argument convincing? There are arguably two things that makes an argument compelling:

1. The conclusion follows without failure from the premisses.
2. The premisses are true.

Any argument satisfying the first item is called *deductively valid*. An argument satisfying both items is call *sound*.

Exercise 0.4. *Are the arguments below deductively valid? Are they sound?*

- *John is taller than Mary. Jane is shorter than Mary. So, John is taller than Jane.*
- *Some nerds are trainspotters. Some nerds wear parkas. So, some trainspotters wear parkas.*
- *Many politicians accept bribes. Most politicians have illegal affairs. So many people who accept bribes have illegal affairs.*
- *John plays either football or basketball. So, if John does not play football then he plays basketball.*
- *All students are smart. Some students are lazy. So, some lazy students are smart.*
- *None of the students is lazy. No lazy student wakes up early. So, some students don't wake up early.*
- *The input n for the* `sum` *program is less than 0. If the input* `n` *the* `sum` *program is less than 0 then the returned value of* `sum` *is 0. So, the returned value is of* `sum` *is 0.*
- *The input n for the* `sum` *program is 1. If the input* `n` *the* `sum` *program is 1 then the body of the loop is executed once. If the body of the loop is executed once then the value of* `i` *is 1 and the value of* `sum` *is 1. So, the returned value is of* `sum` *is 1.*