Seminar 4

***Exerciţii recomandate:** 4.1(a-f), 4.2(a), 4.3(a-f)

***Rezerve:** 4.1(g,j,k,l), 4.3(g,i,k)

S4.1 Folosind diverse criterii de convergenţă, să se stabilească natura seriilor:

$$\text{a) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot n}{2^{n-1}}; \quad \text{b) } \sum_{n=0}^{\infty} (-1)^n \frac{\ln 2 + 3^n}{\ln 3 + 2^n}; \quad \text{c) } \sum_{n=0}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1}; \quad \text{d) } \sum_{n=1}^{\infty} \frac{\sin n \cdot \cos n^2}{\sqrt{n}}, n \in \mathbb{N}^*;$$

$$\text{e) } \sum_{n=1}^{\infty} (-1)^n \frac{(2n+1)!!}{2^n \cdot n!}; \quad \text{f) } \sum_{n=1}^{\infty} (-1)^{n-1} \ln\left(\frac{n^2+2}{n^2+1}\right); \text{g) } \sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^{n+1}}{n^{n+2}}; \quad \text{h) } \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)};$$

$$\text{i) } \sum_{n=1}^{\infty} \frac{(-1)^n}{n - \ln(n)}; \quad \text{j) } \sum_{n=1}^{\infty} \frac{1}{\sqrt[n+1]{\ln(n+1)}}; \quad \text{k) } \sum_{n=1}^{\infty} \frac{n+1}{n} \cdot \frac{\sin \frac{n\pi}{6}}{\sqrt{n^3+1}}; \quad \text{l) } \sum_{n=0}^{\infty} \frac{a^n + \operatorname{sh} n}{3^n} \cdot b^n, a, b \in \mathbb{R};$$

$$\text{m) } \sum_{n=1}^{\infty} \operatorname{tg}^n\left(a + \frac{b}{n}\right), a, b \in \left(0, \frac{\pi}{2}\right); \quad \text{n) } \sum_{n=1}^{\infty} (-1)^{n-1} n^{\alpha} \left(\ln\left(\frac{n+2}{n}\right)\right)^{\beta}, \alpha, \beta \in \mathbb{R}.$$

S4.2 Să studieze convergenţa produsului Cauchy al următoarelor serii:

$$\text{a) } \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \text{ şi } \sum_{n=1}^{\infty} \frac{n}{2^n}; \quad \text{b) } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n+1}} \text{ şi } \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n+1}}.$$

S4.3 Să se studieze natura următoarelor serii de puteri:

$$\text{a) } \sum_{n=0}^{\infty} [2 + (-1)^n] x^n, \ x \in \mathbb{R}; \quad \text{b) } \sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^4+n^3+1}} \left(\frac{x+1}{2x+3}\right)^n, \ x \in \mathbb{R} \setminus \left\{\frac{3}{2}\right\};$$

$$\text{c) } \sum_{n=1}^{\infty} \left(\cos\frac{1}{n}\right)^{\frac{n^2+2}{n+2}} \cdot x^n, x \in \mathbb{R}; \quad \text{d) } \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-4)^n}{n \cdot 3^n}, x \in \mathbb{R}; \quad \text{e) } \sum_{n=1}^{\infty} \frac{x^n}{n^p}, \ p \in \mathbb{R};$$

$$\text{f) } \sum_{n=2}^{\infty} \frac{x^n}{3^n \cdot n \cdot \ln n} \ x \in \mathbb{R}; \quad \text{g) } \sum_{n=1}^{\infty} \frac{2^n (x+1)^{2n}}{(4n+1)^2}, \ x \in \mathbb{R}; \quad \text{h) } \sum_{n=1}^{\infty} \left(\sqrt{n} - 1\right)^n \cdot x^n, x \in \mathbb{R};$$

$$\text{i) } \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n \left(\frac{1-x}{1-2x}\right)^n, x \in \mathbb{R} \setminus \left\{\frac{1}{2}\right\}; \quad \text{j) } \sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n} \left(\frac{1-x^2}{1+x^2}\right)^n, x \in \mathbb{R};$$

$$\text{k) } \sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{\frac{n}{2}} \sqrt{1+n^2}} \operatorname{tg}^n x, \ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right); \quad \text{l) } \sum_{n=1}^{\infty} \frac{n!}{(a+1)(a+2) \cdot \ldots \cdot (a+n)} x^n, a > 0, x \in \mathbb{R}.$$

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