Logic(s) for Computer Science - Week 9
The Syntax of First-Order Logic
Tutorial Exercises

November 27, 2018

1. Define the following notions: set, relation, function.
2. Give 3 examples of binary relations.
3. Give 3 examples of functions.
4. Give 3 examples of relations that are not functions.
5. What is a structure? What about a signature?
6. Identify a signature for the following statements and model the statements as formulae in first-order logic over that signature.

   John is a student. Any student learns Logic. Anyone learning Logic passes the exam. Any student is a person. There is a person who did not pass the exam. Therefore: not all persons are students.

7. Consider the structure $S = (\mathbb{R}, \{Nat, Int, Prime, Even, >\}, \{+, 0, 1, 2\})$, where *Nat*, *Int*, *Prime*, *Even* are unary predicates with the following meaning: $Nat(u)$ = "u is a natural number, $Int(u)$ = "u is an integer number", $Prime(u)$ = "u is a prime" and $Even(u)$ = "u is an even number". The binary predicate $>$ is the "greater than" relation over real numbers. The function $+$ is the addition function for real numbers. The constants $0, 1, 2$ are what you would expect.

   Model the following statements as first-order formulae in the signature associated to the structure S above:

   (a) Any natural number is also an integer.

   (b) The sum of any two natural numbers is a natural number.

   (c) No matter how we would choose a natural number, there is prime number that is greater than the number we chose.

   (d) If any natural number is a prime number, then zero is a prime number.

(e) No matter how we choose a prime number, there is a prime number greater than it.

(f) The sum of two even numbers is an even number.

(g) Any prime number greater than 2 is odd.

(h) Any prime number can be written as the sum of four prime numbers.

(i) The sum of two even numbers is an odd number.

(j) Any even number is the sum of two primes.

8. Give examples of 5 terms over the signature in Exercise 7 and compute their abstract syntax tree.

9. Give examples of 5 formulae over the signature in Exercise 7 and compute their abstract syntax tree.

10. Compute the abstract syntax tree of the following formulae (hint: place brackets around subformulae, in the priority order of the logical connectives):

    (a) $P(x) \lor P(y) \land \neg P(z)$;

    (b) $\neg\neg P(x) \lor P(y) \rightarrow P(x) \land \neg P(z)$;

    (c) $\forall x.\forall y.\neg\neg P(x) \lor P(y) \rightarrow P(x) \land \neg P(z)$;

    (d) $\forall x.\forall y.\neg\neg P(x) \lor P(y) \rightarrow \exists z.P(x) \land \neg P(z)$;

    (e) $\forall x'.\neg\forall x.P(x) \land \exists y.Q(x,y) \lor \neg Q(z,z) \rightarrow \exists z'.P(z')$.