Forma Newton a polinomului de interpolare Lagrange; schema Aitken de calcul al diferențelor divizate

x	-1	1	2	3
f	0	2	6	20

$\bar{x} = 0, \ f(\bar{x}) = ?$

$n = 3 \quad x_0 = -1 \quad x_1 = 1 \quad x_2 = 2 \quad x_3 = 3$

$y_0 = 0 \quad y_1 = 2 \quad y_2 = 6 \quad y_3 = 20$

$f(x_i) = y_i \quad i = \overline{0,3}$

Forma Newton a polinomului de interpolare Lagrange:

$$l_3(x) = y_0 + [x_0, x_1]_f (x - x_0) + [x_0, x_1, x_2](x - x_0)(x - x_1)$$
$$+ [x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$
$$= 0 + [-1, 1](x+1) + [-1, 1, 2](x+1)(x-1) +$$
$$+ [-1, 1, 2, 3](x+1)(x-1)(x-2)$$

$$f(0) \simeq l_3(0)$$

$$[x_i, x_j] = \frac{y_j - y_i}{x_j - x_i}$$

$$[x_{i_0}, x_{i_1}, \dots, x_{i_k}] = \frac{[x_{i_1}, \dots, x_{i_k}] - [x_{i_0}, \dots x_{i_{k-1}}]}{x_{i_k} - x_{i_0}}$$

Schema Aitken de calcul al diferențelor divizate

Pas 1 Pas 2

$-1 \quad 0$

$1 \quad y_1 \; 2 \leftarrow [-1,1] = \dfrac{y_1-y_0}{x_1-x_0} = \dfrac{2-0}{1-(-1)} = 1$

$2 \quad y_2 \; 6 \leftarrow [1,2] = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{6-2}{2-1} = 4 \leftarrow [-1,1,2]$

$3 \quad y_3 \; 20 \leftarrow [2,3] = \dfrac{y_3-y_2}{x_3-x_2} = \dfrac{20-6}{3-2} = 14 \leftarrow [1,2,3]$

Pas 2: $[-1,1,2] = \dfrac{[1,2]-[-1,1]}{2-(-1)} = \dfrac{4-1}{3} = 1$

$$[1,2,3] = \dfrac{[2,3]-[1,2]}{3-1} = \dfrac{14-4}{2} = 5$$

Pas 3 $[-1,1,2,3] = \dfrac{[1,2,3]-[-1,1,2]}{3-(-1)} = \dfrac{5-1}{4} = 1$

$$l_3(x) = (x+1) + (x+1)(x-1) + (x+1)(x-1)(x-2)$$

$$f_3(0) \simeq l_3(0) = 2$$

Interpolare în sensul celor mai mici pătrate
(Least Squares Interpolation)

$m = 1 \qquad P_1(x) = a_1 x + a_0$

$\{a_0, a_1\}$ soluția problemei de optimizare

$$\min \{ g(a_0, a_1) ;\ a_0, a_1 \in \mathbb{R} \} \qquad (LSP)$$

$$g(a_0, a_1) = \sum_{r=0}^{n} (a_1 x_r + a_0 - y_r)^2$$

x	x_0	x_1	$\dots$	x_n
f	y_0	y_1	$\dots$	y_n

$y_i = f(x_i)$

$$f(x) \simeq P_1(x)$$

Soluția problemei (LSP) se găsește printre soluțiile sistemului:

$$\begin{cases} \dfrac{\partial g}{\partial a_0} = 0 \\[2ex] \dfrac{\partial g}{\partial a_1} = 0 \end{cases}$$

$$\frac{\partial g}{\partial a_0}(a_0, a_1) = \sum_{r=0}^{n} 2\,(a_1 x_r + a_0 - y_r)$$

$$\frac{\partial g}{\partial a_1}(a_0, a_1) = \sum_{r=0}^{n} 2\,(a_1 x_r + a_0 - y_r)\cdot x_r$$

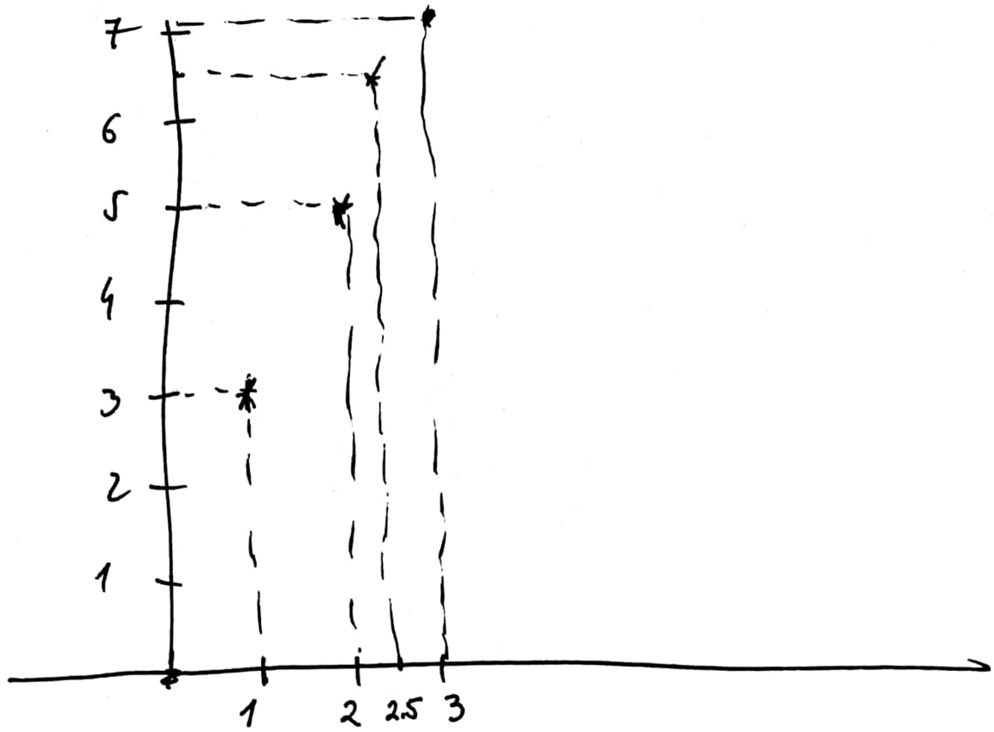
$$\Rightarrow \begin{cases} a_0 \cdot \sum_{r=0}^{n} 1 + a_1 \sum_{r=0}^{n} x_r = \sum_{r=0}^{n} y_r \\ a_0 \sum_{r=0}^{n} x_r + a_1 \sum_{r=0}^{n} x_r^2 = \sum_{r=0}^{n} x_r y_r \end{cases}$$

Soluția sistemului de mai sus este soluția problemei (LSP) dacă matricea

$$\begin{bmatrix} \dfrac{\partial^2 g}{\partial a_0^2} & \dfrac{\partial^2 g}{\partial a_0 \partial a_1} \\ \dfrac{\partial^2 g}{\partial a_1 \partial a_0} & \dfrac{\partial^2 g}{\partial a_1^2} \end{bmatrix} = 2 \begin{bmatrix} \sum_{r=0}^{n} 1 & \sum_{r=0}^{n} x_r \\ \sum_{r=0}^{n} x_r & \sum_{r=0}^{n} x_r^2 \end{bmatrix}$$

este pozitiv definită.

x	1	2	2.5	3
f	3	5	6.5	7



a_0, a_1 soluția sistemului

$$\begin{cases} 4a_0 + 8.5a_1 = 21.5 \\ 8.5a_0 + 20.25a_1 = 50.25 \end{cases}$$

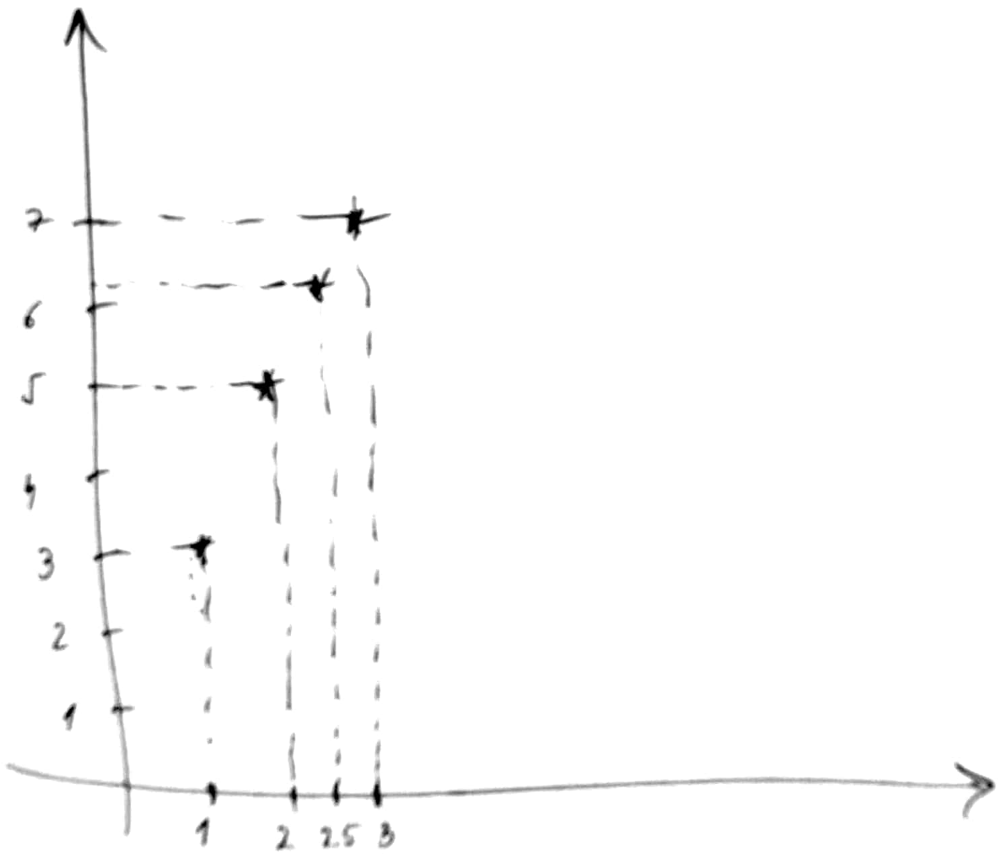
$a_0 = 0,9429 \qquad a_1 = 2.0857$

x	1	2	2.5	3
f	3	5	6.1	7

a_0, a_1 soluția sistemului:

$$4a_0 + 8.5a_1 = 21.1$$
$$8.5a_0 + 20.25a_1 = 49.25$$

$$a_0 = 0.9886 \qquad a_1 = 2.0171$$



(a_0, a_1) soluția sistemului

$$Ba = f$$

$$\sum_{j=0}^{m}\left(\sum_{k=0}^{n} x_k^{i+j}\right) a_j = \sum_{k=0}^{n} y_k x_k^{i}$$

$m = 1$

$i = 0$ $$\sum_{j=0}^{1}\left(\sum_{k=0}^{n} x_k^{0+j}\right) a_j = \sum_{k=0}^{n} y_k x_k^{0} \Rightarrow$$

$$\left(\sum_{k=0}^{n} x_k^{0}\right) a_0 + \left(\sum_{k=0}^{n} x_k\right) a_1 = \sum_{k=0}^{n} y_k$$

$i = 1$ $$\sum_{j=0}^{1}\left(\sum_{k=0}^{n} x_k^{1+j}\right) a_j = \sum_{k=0}^{n} y_k x_k^{1} \Rightarrow$$

$$\left(\sum_{k=0}^{n} x_k^{1+0}\right) \cdot a_0 + \left(\sum_{k=0}^{n} x_k^{1+1}\right) a_1 = \sum_{k=0}^{n} y_k x_k^{1}$$

$$\begin{cases} \left(\sum_{k=0}^{n} 1\right) a_0 + \left(\sum_{k=0}^{n} x_k\right) a_1 = \sum_{y=0}^{n} y_k \\ \left(\sum_{k=0}^{n} x_k\right) a_0 + \left(\sum_{k=0}^{n} x_k^{2}\right) a_1 = \sum_{y=0}^{n} y_k x_k \end{cases}$$