Nucleul ?

572h) $h_8(\mathbf{x}) = x_1x_2 - x_1x_3 + x_2x_3 - 1,\ \mathbf{x} = (x_1, x_2, x_3) \in \mathbb{R}^3.$

dar apar termeni de ordin 2

Când nu apar pătrate trebuie să facem apriori o transformare

i.e. apar produse $x_i x_j$

$$\begin{cases} x_1 = y_1 + y_2 \\ x_2 = y_1 - y_2 \\ x_3 = y_3 \end{cases}$$

$(a+b)(a-b) = a^2 - b^2$

$h_8(y) = (y_1 + y_2)(y_1 - y_2) -$

$(y_1 + y_2)y_3 + (y_1 - y_2)y_3 - 1$

$= y_1^2 - y_2^2 - \cancel{y_1y_3} - y_2y_3 +$

$\cancel{y_1y_3} - y_2y_3 - 1 =$

$y_1^2 - y_2^2 - 2y_2y_3 - 1 =$

$$y_1^2 - (y_2^2 + 2y_2y_3 + y_3^2) + y_3^2$$

$$-1 =$$

$$y_1^2 - (y_2 + y_3)^2 + y_3^2 - 1$$

$$\begin{cases} z_1 = y_1 \\ z_2 = y_2 + y_3 \\ z_3 = y_3 \end{cases}$$

$$h_g(z) = z_1^2 - z_2^2 + z_3^2 - 1$$

Ker $h_g(z)$

$$\Rightarrow z_1^2 - z_2^2 + z_3^2 - 1 = 0$$

hiperboloid cu o pânză.

Nucleul

S7.5 a) $h(\mathbf{x}) = 3x_1 - 4x_2 + x_3 - x_4 + 2$, $\mathbf{x} = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4$.

$\sum_{i=1}^{n} a_i x_i + a_0 = 0$ hiperplan = spațiu liniar cu dim = dim sp - 1

$(a_1, \dots, a_n) \neq (0, 0, \dots 0)$

Nu toți coef sunt 0

ex: $\mathbb{R}^3$ $3-1=2$

$\Rightarrow$ hiperplan = plan

$\mathbb{R}^2$ $2-1=1$

$\Rightarrow$ hiperplan = dreaptă

$3x_1 - 4x_2 + x_3 - x_4 + 2 = 0$

1 sistem de 1 ec cu 4 nec

Mat sist: $\begin{pmatrix} 3 & -4 & 1 & -1 \end{pmatrix}$

rang mat = 1

$\Rightarrow$ rang op = 1

$\Rightarrow$ dim Ker = dim sp − rang =

$4 - 1 = 3$

$$\text{a)}\ f(x,y) := \begin{cases} \dfrac{xy^2}{x^2+y^4}, & (x,y) \neq (0,0); \\ 0, & (x,y) = (0,0); \end{cases}$$

Cont în fiec var, dar nu sunt cont global.

În afara lui $(0,0)$ $x^2+y^4 \neq 0 \Rightarrow$

$\dfrac{xy^2}{x^2+y^4}$ fct cont (raport de fct cont care nu se anulează)

Cont în x în $(0,0)$

$\Updownarrow$

$\lim\limits_{x \to 0} f(x,0) = f(0,0)$?

$\quad$ //

$$\lim_{x \to 0} \frac{x \cdot 0^2}{x^2 + 0^4} = \lim_{x \to 0} \frac{0}{x^2} = \lim_{x \to 0} 0$$

$= 0 = f(0,0) \Rightarrow f$ cont în x

Cont în y în $(0,0)$

$$\lim_{y \to 0} f(0,y) = f(0,0)?$$

$$\lim_{y \to 0} \frac{0 \cdot y^2}{0 + y^4} = \lim_{y \to 0} \frac{0}{y^4} = \lim_{y \to 0} 0 = 0$$

$= f(0,0) \Rightarrow f$ cont în y.

$$\lim_{(x,y) \to (0,0)} f(x,y)$$

Var I Studiem limitele direcționale (poate fi de ajutor)

$$\overbrace{l = \lim_{t \to 0} f((0,0) + t(u,v))}^{(u,v) \neq (0,0)} =$$

$$\lim_{t \to 0} f(tu, tv) = \lim_{t \to 0} \frac{tu\, t^2 v^2}{t^2 u^2 + t^4 v^4} =$$

$$\lim_{t \to 0} \frac{t^3 u v^2}{t^2(u^2 + t^2 v^4)} \ (t^2 =$$

$$\lim_{t \to 0} \frac{t u v^2}{u^2 + t^2 v^4}$$

Dacă $u \neq 0$ $l = \lim_{t \to 0} \frac{t u v^2}{u^2 + t^2 v^4} = 0$

$u = 0 \Rightarrow \underline{v \neq 0}$

$$\lim_{t \to 0} \frac{t^3 u v^2}{t^2(u^2 + t^2 v^4)} \overset{u=0}{=} \lim_{t \to 0} \frac{0}{t^4 v^4} =$$

$$\lim_{t \to 0} 0 = 0$$

$$\Rightarrow \lim_{t \to 0} f(tu, tv) = f(0,0)$$

$\to$ În acest caz studiind limitelor direcționale pt a deduce informații despre cont globală în $(0,0)$ nu ne-a fost de ajutor.

$$\frac{xy^2}{x^2 + y^4},$$
$$0,$$

$$x_n = \frac{1}{n^2} \to 0$$
$$y_n = \frac{1}{n} \to 0$$

$$\lim_{n \to \infty} f(x_n, y_n) = \lim_{n \to \infty} \frac{\frac{1}{n^2} \cdot \left(\frac{1}{n}\right)^2}{\left(\frac{1}{n^2}\right)^2 + \left(\frac{1}{n}\right)^4}$$

$$= \lim_{n \to \infty} \frac{\frac{1}{n^2} \cdot \frac{1}{n^2}}{\frac{1}{n^4} + \frac{1}{n^4}} = \lim_{n \to \infty} \frac{\frac{1}{\cancel{n^4}}}{\frac{2}{\cancel{n^4}}} = \frac{1}{2} \quad (1)$$

$$x_n' = \frac{2}{n^2} \to 0$$

$$y_n' = \frac{1}{n} \to 0$$

$$\lim_{n\to\infty} f(x_n', y_n') = \lim_{n\to\infty} \frac{\frac{2}{n^2}\cdot\left(\frac{1}{n}\right)^2}{\left(\frac{2}{n^2}\right)^2 + \left(\frac{1}{n}\right)^4}$$

$$= \lim_{n\to\infty} \frac{\frac{2}{n^2}\cdot\frac{1}{n^2}}{\frac{4}{n^4}+\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{2}{\cancel{n^4}}}{\frac{5}{\cancel{n^4}}} = \frac{2}{5} \quad (2)$$

Din (1), (2) $\Rightarrow$

$$\left.\begin{array}{l} \lim\limits_{n\to\infty} f(x_n, y_n) \neq \lim\limits_{n\to\infty} f(x_n', y_n') \\ (x_n, y_n) \to (0,0) \\ (x_n', y_n') \to (0,0) \end{array}\right\} \Rightarrow$$

$$\nexists \lim_{(x,y)\to(0,0)} f(x,y) \to$$

f nu este cont

S8 2 i)

$$f(x,y) := \frac{x^2-y^2}{|x|+|y|};$$

$x^2 = |x|^2$

$$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{x^2-0}{|x|+0} =$$

$$\lim_{x\to 0} \frac{x^2}{|x|} = \lim_{x\to 0} \frac{|x|^{\cancel{2}}}{\cancel{|x|}} = \lim_{x\to 0} |x| = 0$$

$$\lim_{y\to 0} f(0,y) = \lim_{y\to 0} \frac{0^2-y^2}{|0|+|-y|} =$$

$$\lim_{y\to 0} \frac{-y^2}{|y|} = \lim_{y\to 0} -\frac{|y|^{\cancel{2}}}{\cancel{|y|}} = \lim_{y\to 0} -|y| = 0$$

$$\lim_{x\to 0} \lim_{y\to 0} f(x,y) = \lim_{x\to 0} \underbrace{\lim_{y\to 0} \frac{x^2-y^2}{|x|+|y|}}$$

$$\lim_{x\to 0} \frac{x^2}{|x|} = \lim_{x\to 0} \frac{|x|^{\cancel{2}}}{\cancel{|x|}} = \lim_{x\to 0} |x| = 0$$

$$\lim_{y\to 0}\lim_{x\to 0} f(x,y) = \lim_{y\to 0}\lim_{x\to 0}\frac{x^2-y^2}{|x|+|y|} =$$

$$\lim_{y\to 0} \frac{-y^2}{|y|} = \lim_{y\to 0}\frac{-|y|^{\cancel{2}}}{\cancel{|y|}} = \lim_{y\to 0} -|y| = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{|x|+|y|} =$$

$$\lim_{(x,y)\to(0,0)} \frac{|x|^2-|y|^2}{|x|+|y|} =$$

$$\lim_{(x,y)\to(0,0)} \frac{(|x|-|y|)\cancel{(|x|+|y|)}}{\cancel{|x|+|y|}} =$$

$$\lim_{(x,y)\to(0,0)} |x|-|y| =$$

$$\lim_{(x,y)\to(0,0)} |x| - \lim_{(x,y)\to(0,0)} |y| =$$

$$\lim_{x\to 0} |x| - \lim_{y\to 0} |y| = 0-0=0$$

$$f(x,y) := (x^2+y^2)^{x^2y^2};$$

$$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} (x^2+0^2)^{x^2\cdot 0} =$$

$$\lim_{x\to 0} x^{2^0} = \lim_{x\to 0} 1 = 1$$

$$0^0$$

$$f(x,y) = e^{\ln (x^2+y^2)^{x^2y^2}} =$$

$$e^{\underbrace{x^2y^2 \ln (x^2+y^2)}_{x,y\to 0}}$$

$$\sqrt{\frac{x^2+y^2}{2}} \geq \sqrt{xy} \quad \Big|^2$$

$$x^2+y^2 \geq 2xy$$

$$\underbrace{x^2y^2}\ln \underbrace{(x^2+y^2)}_{\downarrow\, 0} \leq 0$$

ln monoton

$$0 \geq x^2 y^2 \ln(x^2+y^2) \geq x^2 y^2 \ln(xy)$$

$$\geq (xy)^2 \ln(xy)$$

$$0$$

$$\lim_{(x,y)\to(0,0)} (xy)^2 \ln(xy) \overset{z=xy}{=}$$

$$\lim_{z\to 0} z^2 \ln z = \lim_{z\to 0} \frac{\ln z}{\frac{1}{z^2}} \overset{l'H}{\underset{\text{Dacă } \exists}{=}}$$

$$\lim_{z\to 0} \frac{\frac{1}{\cancel{z}}}{-\frac{2}{z^{\cancel{3}2}}} = \lim_{z\to 0} -\frac{z^2}{2} = 0$$

$$\lim_{(x,y)\to(0,0)} x^2 y^2 \ln(x^2+y^2) = 0$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = 1$$

$$f(x,y) := \frac{\sin(xy)}{\sqrt{x^2+y^2}};$$

$$\frac{\sin z}{z} \xrightarrow{z \to 0} 1$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{\sqrt{x^2+y^2}} =$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} \cdot \frac{xy}{\sqrt{x^2+y^2}}$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} \cdot \lim_{(x,y)\to(0,0)} \underline{\frac{xy}{\sqrt{x^2+y^2}}}$$

$$= 0$$

$$0 \le \left|\frac{xy}{\sqrt{x^2+y^2}}\right| \le \frac{x^2+y^2}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2}$$

$$0$$

$$\lim_{(x,y)\to(0,0)} \left|\frac{xy}{\sqrt{x^2+y^2}}\right| = 0$$

a) $$\lim_{(x,y)\to(0,0)} \left(\frac{xy}{\sqrt{1+xy}-1}, \frac{\sin(x^3+y^3)}{\sqrt{x^2+y^2+1}-1} \right);$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{1+xy}-1} \quad \underline{z = xy}$$

$$\lim_{z\to 0} \frac{z \quad (\sqrt{1+z}+1)}{\sqrt{1+z}-1} = \lim_{z\to 0} \frac{\cancel{z}(\sqrt{1+z}+1)}{\cancel{1+z-1}} =$$

$$\lim_{z\to 0} \left(\sqrt{1+z}+1\right) = 2$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^3+y^3)}{\sqrt{x^2+y^2+1}-1} =$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(x^3+y^3)}{x^3+y^3} \cdot \frac{x^3+y^3}{\sqrt{x^2+y^2+1}-1}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{\sqrt{x^2+y^2+1}-1} \cdot \frac{\sqrt{x^2+y^2+1}+1)}{} =$$

$$\lim_{(x,y)\to(0,0)} \frac{(x^3+y^3)(\sqrt{x^2+y^2+1}+1)}{x^2+y^2+\cancel{1}-\cancel{1}}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$$

$$\frac{|x^3+y^3|}{x^2+y^2} \le \frac{2|x^3+y^3|}{(|x|+|y|)^2} \le$$

$\to 0$

$$x^2+y^2 \ge 2|xy|$$

$+$

$$x^2+y^2 = x^2+y^2$$

$$\overline{2(x^2+y^2) \ge (|x|+|y|)^2}$$

$$\frac{1}{(x^2+y^2)} \le \frac{2}{(|x|+|y|)^2}$$

$$\frac{2\left(|x|^3+|y|^3\right)}{\left(|x|+|y|\right)^2} =$$

$$\frac{2\cancel{\left(|x|+|y|\right)}\left(\underline{|x|^2+|y|^2}-|xy|\right)}{\left(|x|+|y|\right)^{\cancel{2}}}$$

$$\leq \frac{2\left(|x|^2+|y|^2\right)}{|x|+|y|} \leq$$

$$\frac{2\left(|x|+|y|\right)^{\cancel{2}}}{\cancel{|x|+|y|}} \longrightarrow 0$$

$$\frac{x^2y^2z^2}{(x-y)^2+(y-z)^2+(x-z)^2}$$

$$f(x,y,z) := \begin{cases} (x^2+y^2+z^2)^{1/3}\ln(x^2+y^2+z^2), & (x,y,z) \neq (0,0,0); \\ 1/3, & (x,y,z) = (0,0,0). \end{cases}$$

în afara lui (0,0,0) f cont

$$\lim_{(x,y,z)\to(0,0,0)} (x^2+y^2+z^2)^{1/3} \ln(x^2+y^2+z^2)$$

$$\underset{t>0}{\underline{x^2+y^2+z^2=t}} \quad \lim_{t\to 0} t^{1/3} \ln t =$$

$$\lim_{t\to 0} \frac{\ln t}{t^{-1/3}} \underset{\text{Dacă } \exists}{\overset{L'H}{=}} \lim_{t\to 0} \frac{\frac{1}{t}}{-\frac{1}{3} t^{-\frac{4}{3}}} =$$

$$-3 \lim_{t\to 0} \frac{t^{-1}}{t^{-\frac{4}{3}}} = -3 \lim_{t\to 0} t^{-1+\frac{4}{3}} =$$

$$-3 \lim_{t\to 0} t^{\frac{1}{3}} =$$

$$-3\lim_{t\to 0} \sqrt[3]{t} = 0 \nexists$$

$$f(0,0,0)$$

) $f(x,y) := \begin{cases} \frac{|x|}{y} e^{-|x|y^{-2}}, & y \neq 0; \\ 1, & y = 0; \end{cases}$

$$\lim_{y\to 0} \frac{|x|}{y} e^{\underline{-|x|y^{-2}}}$$

$$y \to 0 \Rightarrow y^{-2} = \frac{1}{y^2} \to \infty$$

$$e^{\underbrace{-|x|\cdot\infty}} = e^{-\infty} \to 0$$

$$t = y^{-2} \qquad y = \pm\sqrt{\frac{1}{t}} = \pm\frac{1}{\sqrt{t}}$$

$$\lim_{t\to\infty} \pm|x|\sqrt{t}\, e^{-|x|t} =$$

$$\lim_{t\to\infty} \pm \frac{|x|\sqrt{t}}{e^{|x|t}} =$$

$$|x| \lim_{t\to\infty} \pm \frac{\sqrt{t}}{e^{|x|t}}$$

$$\pm |x| \lim_{t\to\infty} \frac{\sqrt{t}}{e^{|x|t}} \quad \frac{\overline{l'H}}{\text{Dacă}}\}$$

$$\pm |x| \lim_{t\to\infty} \frac{\frac{1}{2\sqrt{t}}}{|x| e^{|x|t}} = 0 \neq$$

$f(x, 0)$

$\Rightarrow f$ disc în $(x, 0)$

$x \neq 0$

pt $x = 0$ $f(x,y) = \begin{cases} 0, & y \neq 0 \\ 1, & y = 0 \end{cases}$

f dis în $(0,0)$

$$\lim_{n\to\infty} \frac{4^n \cdot n! \cdot n!}{(2n)!} =$$

$$4^n \cdot n! \cdot n! =$$

$$2^n \cdot 2^n \cdot n! \cdot n! =$$

$$\underbrace{(2^n \cdot n!)}_{(2n)!!} \cdot (2^n \cdot n!)$$

$$2 \cdot 1 \cdot 2 \cdot 2 \cdot \ldots \cdot 2 \cdot n$$

$$\lim_{n\to\infty} \frac{2^n \cdot n!}{1 \cdot 3 \cdot \ldots \cdot (2n-1)} =$$

$$\lim_{n\to\infty} \frac{2}{1} \cdot \frac{4}{3} \cdot \ldots \cdot \frac{2n}{2n-1} =$$

$$\lim_{n\to\infty} \prod_{k=1}^{n} \frac{2k}{2k-1} =$$

$$\lim_{n\to\infty} \prod_{k=1}^{n} \left(\frac{2k-1}{2k-1} + \frac{1}{2k-1} \right)$$

$$\lim_{n\to\infty} \prod_{k=1}^{n} \left(1 + \frac{1}{2k-1} \right) =$$

$$\left(1+\frac{1}{1}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{5}\right)\left(1+\frac{1}{7}\right)$$

$$= 1 + \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} +$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2k-1} \qquad \infty$$