Interpolare în sensul celor mai mici pătrate (Least Squares Interpolation)

$m = 1 \qquad P_1(x) = a_1 x + a_0$

$\{a_0, a_1\}$ soluția problemei de optimizare

$$\min \{ g(a_0, a_1) ; \ a_0, a_1 \in \mathbb{R} \} \qquad (LSP)$$

$$g(a_0, a_1) = \sum_{r=0}^{n} (a_1 x_r + a_0 - y_r)^2$$

x	x_0	x_1	$\dots$	x_n
f	y_0	y_1	$\dots$	y_n

$y_i = f(x_i)$

$$f(x) \simeq P_1(x)$$

Soluția problemei (LSP) se găsește printre soluțiile sistemului:

$$\begin{cases} \dfrac{\partial g}{\partial a_0} = 0 \\[2ex] \dfrac{\partial g}{\partial a_1} = 0 \end{cases}$$

$$\frac{\partial g}{\partial a_0}(a_0, a_1) = \sum_{r=0}^{n} 2\,(a_1 x_r + a_0 - y_r)$$

$$\frac{\partial g}{\partial a_1}(a_0, a_1) = \sum_{r=0}^{n} 2\,(a_1 x_r + a_0 - y_r)\cdot x_r$$

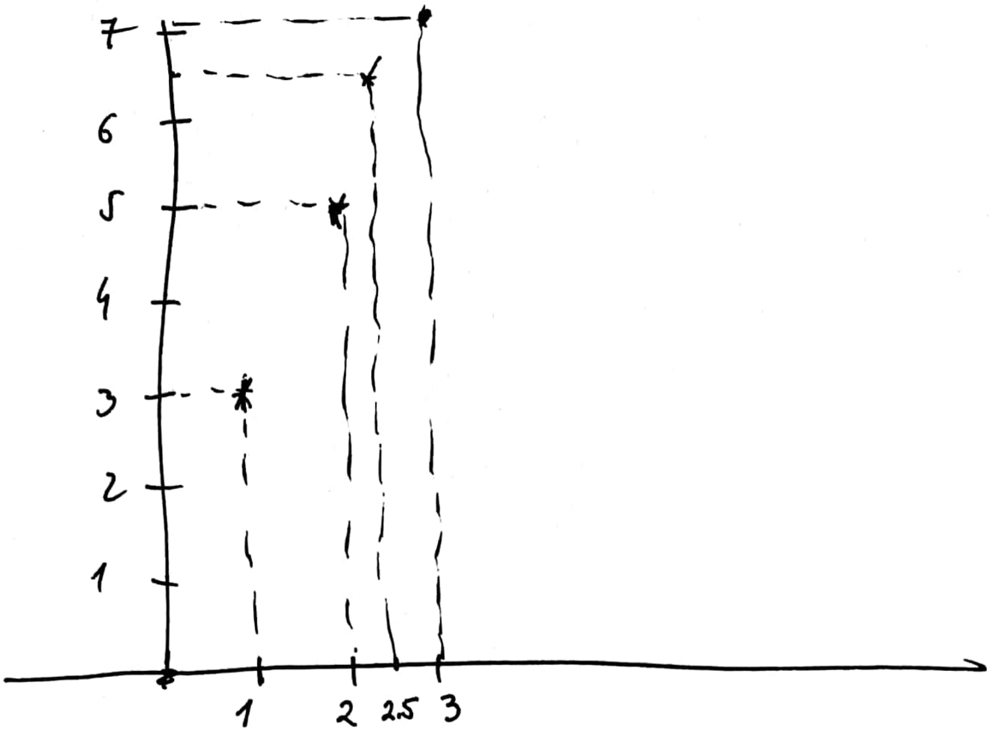
$$\Rightarrow \begin{cases} a_0 \cdot \sum_{r=0}^{n} 1 + a_1 \sum_{r=0}^{n} x_r = \sum_{r=0}^{n} y_r \\ a_0 \sum_{r=0}^{n} x_r + a_1 \sum_{r=0}^{n} x_r^2 = \sum_{r=0}^{n} x_r y_r \end{cases}$$

Soluția sistemului de mai sus este soluția problemei (LSP) dacă matricea

$$\begin{bmatrix} \frac{\partial^2 g}{\partial a_0^2} & \frac{\partial^2 g}{\partial a_0 \partial a_1} \\ \frac{\partial^2 g}{\partial a_1 \partial a_0} & \frac{\partial^2 g}{\partial a_1^2} \end{bmatrix} = 2 \begin{bmatrix} \sum_{r=0}^{n} 1 & \sum_{r=0}^{n} x_r \\ \sum_{r=0}^{n} x_r & \sum_{r=0}^{n} x_r^2 \end{bmatrix}$$

este pozitiv definită.

x	1	2	2.5	3
f	3	5	6.5	7



a_0, a_1 soluția sistemului

$$\begin{cases} 4a_0 + 8.5a_1 = 21.5 \\ 8.5a_0 + 20.25a_1 = 50.25 \end{cases}$$

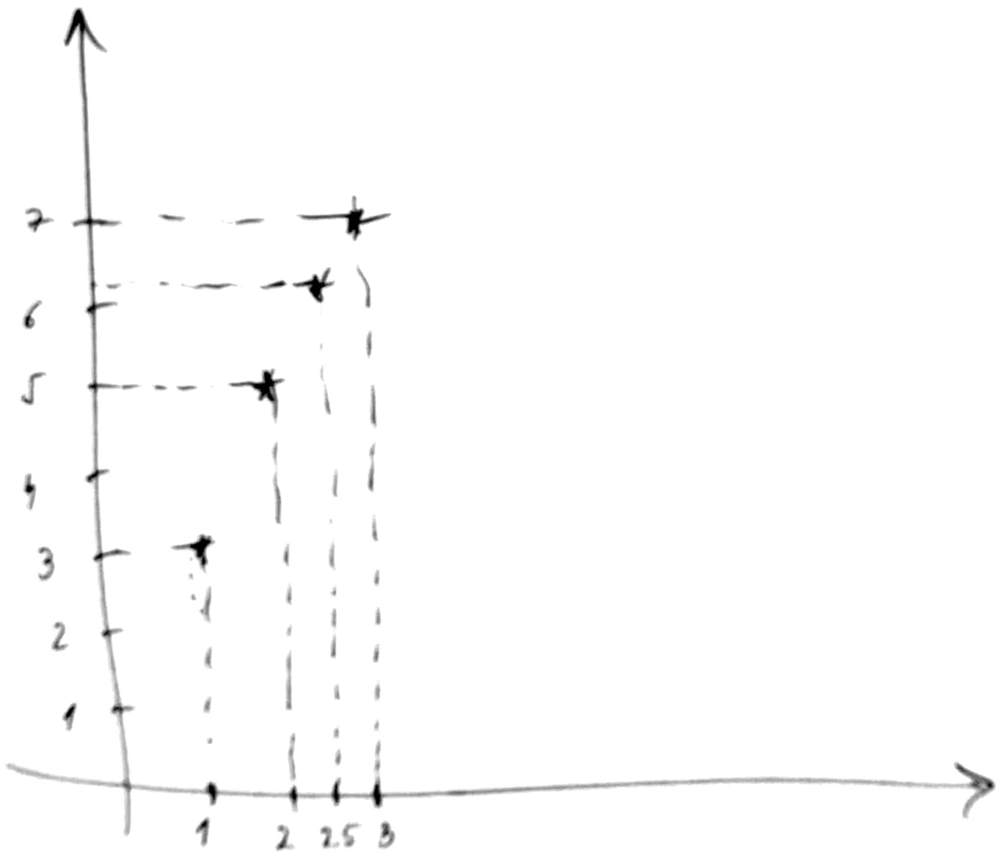
$a_0 = 0{,}9429 \qquad a_1 = 2.0857$

x	1	2	2.5	3
f	3	5	6.1	7

a_0, a_1 soluția sistemului:

$$4a_0 + 8.5a_1 = 21.1$$
$$8.5a_0 + 20.25a_1 = 49.25$$

$$a_0 = 0.9886 \qquad a_1 = 2.0171$$



(a_0, a_1) soluția sistemului

$$Ba = f$$

$$\sum_{j=0}^{m} \left(\sum_{k=0}^{n} x_k^{i+j} \right) a_j = \sum_{k=0}^{n} y_k x_k^i$$

$m = 1$

$i = 0$
$$\sum_{j=0}^{1} \left(\sum_{k=0}^{n} x_k^{0+j} \right) a_j = \sum_{k=0}^{n} y_k x_k^0 \Rightarrow$$

$$\left(\sum_{k=0}^{n} x_k^0 \right) a_0 + \left(\sum_{k=0}^{n} x_k \right) a_1 = \sum_{k=0}^{n} y_k$$

$i = 1$
$$\sum_{j=0}^{1} \left(\sum_{k=0}^{n} x_k^{1+j} \right) a_j = \sum_{k=0}^{n} y_k x_k^1 \Rightarrow$$

$$\left(\sum_{k=0}^{n} x_k^{1+0} \right) \cdot a_0 + \left(\sum_{k=0}^{n} x_k^{1+1} \right) a_1 = \sum_{k=0}^{n} y_k x_k^1$$

$$\begin{cases} \left(\sum_{k=0}^{n} 1 \right) a_0 + \left(\sum_{k=0}^{n} x_k \right) a_1 = \sum_{y=0}^{n} y_k \\ \left(\sum_{k=0}^{n} x_k \right) a_0 + \left(\sum_{k=0}^{n} x_k^2 \right) a_1 = \sum_{y=0}^{n} y_k x_k \end{cases}$$