Propositional Logic. Week 4 - Exercise Sheet

1. Give a formal proof of $\mathtt{p} \land \mathtt{r}$ from $(\mathtt{q} \land \mathtt{r}) \land \mathtt{q}$ and $\mathtt{p} \land \mathtt{p}$.

2. Show the validity of the following sequents:

   (a) $\mathtt{p} \land \mathtt{q}, \mathtt{r} \vdash \mathtt{p} \land (\mathtt{r} \lor \mathtt{r}')$;

   (b) $\mathtt{p} \to (\mathtt{q} \to \mathtt{r}) \vdash \mathtt{p} \land \mathtt{q} \to \mathtt{r}$;

   (c) $\mathtt{p} \land \neg\mathtt{r} \to \mathtt{q}, \neg\mathtt{q}, \mathtt{p} \vdash \mathtt{r}$;;

3. Finish the game at `https://profs.info.uaic.ro/~stefan.ciobaca/lnd.html`. Do not cheat. It is considered cheating if you change the JavaScript source code, if someone else solves a level for you or if you prove the derived rules using the derived rules themselves.

4. Prove that the following inference rules are derivable:

   (a) $\neg\neg i$;

   (b) LEM (law of excluded middle): $\textsc{Lem}\,\dfrac{}{\Gamma \vdash \varphi \lor \neg\varphi;}$

   (c) PBC (proof by contradiction): $\textsc{Pbc}\,\dfrac{\Gamma, \neg\varphi \vdash \bot}{\Gamma \vdash \varphi;}$

   (d) MT (modus tollens): $\mathrm{MT}\,\dfrac{\Gamma \vdash \varphi \to \varphi' \qquad \Gamma \vdash \neg\varphi'}{\Gamma \vdash \neg\varphi.}$

5. Prove the soundness theorem for natural deduction (by induction on the number of sequences in the formal proof).

6. Show that the rule $\neg\neg e$ is derivable using the LEM (i.e. you may use LEM in the derivation, but not $\neg\neg e$).

7. Prove, using the soundness and completeness theorems, that $\varphi_1 \dashv\vdash \varphi_2$ if and only if $\varphi_1 \equiv \varphi_2$.