1.a $A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 2 & 5 \\ 1 & 1 & 7 \end{pmatrix}$ $\quad b = \begin{pmatrix} 2 \\ 3 \\ 10 \end{pmatrix}$

$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}$ $\quad U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$

$$LU = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{pmatrix}$$

Pasul 1. Linia 1 din L, Coloana 1 din U

u_{11}

$(LU)_{11} = A_{11} \Rightarrow u_{11} = 2$

Pasul 2. Linia 2 din L, Coloana 2 din U

l_{21} u_{12} u_{22}

$(LU)_{21} = A_{21} \Rightarrow \underline{l_{21}} \cdot u_{11} = 1 \Rightarrow l_{21} = 0.5$

$(LU)_{12} = A_{12} \Rightarrow u_{12} = 0$

$(LU)_{22} = A_{22} \Rightarrow l_{21} \cdot u_{12} + \underline{u_{22}} = 2 \Rightarrow u_{22} = 2$

Pasul 3. Linia 3 din L, Coloana 3 din U

l_{31} l_{32} u_{13} u_{23} u_{33}

$(LU)_{31} = A_{31} \Rightarrow l_{31} u_{11} = 1 \Rightarrow l_{31} = 0.5$

$(LU)_{32} = A_{32} \Rightarrow l_{31} u_{12} + \underline{l_{32}} u_{22} = 1 \Rightarrow l_{32} = 0.5$

$(LU)_{13} = A_{13} \Rightarrow u_{13} = 3$;

$(LU)_{23} = A_{23} \Rightarrow l_{21}u_{13} + u_{23} = 5 \Rightarrow u_{23} = 3.5$

$(LU)_{33} = A_{33} \Rightarrow l_{31}u_{13} + l_{32}u_{23} + u_{33} = 7 \Rightarrow u_{33} = 3.75$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.5 & 0.5 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & 3.5 \\ 0 & 0 & 3.75 \end{pmatrix}$$

$Ax = b \Leftrightarrow LUx = b \Leftrightarrow \begin{cases} Ly = b \Rightarrow \text{sol } y^* \\ Ux = y^* \Rightarrow \text{sol } x^* \end{cases}$; $x^* = A^{-1}b$

$$y = \begin{pmatrix} y_{11} \\ y_{21} \\ y_{31} \end{pmatrix} \qquad x = \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix} \qquad b = \begin{pmatrix} 2 \\ 3 \\ 10 \end{pmatrix}$$

- $Ly = b$

$y_{11} = 2$

$y_{21} = 2$

$y_{31} = 8$

$$Ly = \begin{pmatrix} y_{11} \\ 0.5y_{11} + y_{21} \\ 0.5y_{11} + 0.5y_{21} + y_{31} \end{pmatrix}$$

- $Ux = y^*$

$x_{31} = 2.13$

$x_{21} = -2.72$

$x_{11} = -2.2$

$$Ux = \begin{pmatrix} 2x_{11} + 3x_{31} \\ 2x_{21} + 3.5x_{31} \\ 3.75 \cdot x_{31} \end{pmatrix}$$

$$x = \begin{pmatrix} -2.2 \\ -2.72 \\ 2.13 \end{pmatrix}$$

(Verificat online)

2.

$$A = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \qquad L = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$$

a. $M = (m_{ij}) \in \mathbb{R}^{n \times n}$ $\quad m_{ij} = \begin{cases} -\frac{a_{ij}}{a_{ii}} & i \neq j \\ 0 & i = j \end{cases}$

$$M = \begin{pmatrix} 0 & 0 & -1.5 \\ -1 & 0 & 0 \\ 0 & -0.5 & 0 \end{pmatrix}$$

b. Sirul converge cand $k \to \infty$ ~~atunci cand~~ ~~daca~~ daca $\|M\| < 1$

$\|M\| = -0.75$ $\not<$ 1 $\Rightarrow$ sirul converge

c. $x^0 = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ $\quad B^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

$B = \text{diag}[a_{11}, a_{22} \dots a_{nn}]$ $\quad B^{-1} = \text{diag}\left[\frac{1}{a_{11}}, \frac{1}{a_{22}} \dots \frac{1}{a_{nn}}\right]$

$\alpha = B^{-1} \cdot L$ $\quad \alpha = \begin{pmatrix} 1.5 \\ 2 \\ 1.5 \end{pmatrix}$

$x^{(k+1)} = M x^{(k)} + \alpha = g_{k+1} + \alpha$

$x^1 = M x^0 + \alpha = g_1 + \alpha \qquad g_1 = \begin{pmatrix} -3 \\ -2 \\ -2 \end{pmatrix}$

$x^1 = \begin{pmatrix} -1.5 \\ 0 \\ -0.5 \end{pmatrix}$

3. $\neg(\exists)\Gamma$.