

Matematică - Calcul diferențial și integral

Seminar - Săptămâna 4

***Exerciții recomandate:** 4.1(a-f), 4.2(a-f)

***Rezerve:** 4.1(g,j,k,l), 4.2(g,i,k)

S4.1 Folosind diverse criterii de convergență, să se stabilească natura seriilor:

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|--|---|
| a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cdot n}{2^{n-1}};$ | h) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+1)};$ |
| b) $\sum_{n=0}^{\infty} (-1)^n \frac{\ln 2 + 3^n}{\ln 3 + 2^n};$ | i) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \ln(n)};$ |
| c) $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1};$ | j) $\sum_{n=1}^{\infty} \frac{1}{n^{+1} \sqrt{\ln(n+1)}};$ |
| d) $\sum_{n=1}^{\infty} \frac{\sin n \cdot \cos n^2}{\sqrt{n}}, n \in \mathbb{N}^*;$ | k) $\sum_{n=1}^{\infty} \frac{n+1}{n} \cdot \frac{\sin \frac{n\pi}{6}}{\sqrt{n^3+1}};$ |
| e) $\sum_{n=1}^{\infty} (-1)^n \frac{(2n+1)!!}{2^n \cdot n!};$ | l) $\sum_{n=0}^{\infty} \frac{a^n + \operatorname{sh} n}{3^n} \cdot b^n, a, b \in \mathbb{R};$ |
| f) $\sum_{n=1}^{\infty} (-1)^{n-1} \ln \left(\frac{n^2+2}{n^2+1} \right);$ | m) $\sum_{n=1}^{\infty} \operatorname{tg}^n \left(a + \frac{b}{n} \right), a, b \in \left(0, \frac{\pi}{2} \right);$ |
| g) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^{n+1}}{n^{n+2}};$ | n) $\sum_{n=1}^{\infty} (-1)^{n-1} n^{\alpha} \left(\ln \left(\frac{n+2}{n} \right) \right)^{\beta}, \alpha, \beta \in \mathbb{R}.$ |

S4.2 Să se studieze natura următoarelor serii de puteri:

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|---|---|
| a) $\sum_{n=0}^{\infty} [2 + (-1)^n] x^n, x \in \mathbb{R};$ | g) $\sum_{n=1}^{\infty} \frac{2^n (x+1)^{2n}}{(4n+1)^2}, x \in \mathbb{R};$ |
| b) $\sum_{n=0}^{\infty} \frac{n+1}{\sqrt{n^4+n^3+1}} \left(\frac{x+1}{2x+3} \right)^n, x \in \mathbb{R} \setminus \left\{ \frac{3}{2} \right\};$ | h) $\sum_{n=1}^{\infty} (\sqrt{n}-1)^n \cdot x^n, x \in \mathbb{R};$ |
| c) $\sum_{n=1}^{\infty} \left(\cos \frac{1}{n} \right)^{\frac{n^2+2}{n+2}} \cdot x^n, x \in \mathbb{R};$ | i) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^n \left(\frac{1-x}{1-2x} \right)^n, x \in \mathbb{R} \setminus \left\{ \frac{1}{2} \right\};$ |
| d) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-4)^n}{n \cdot 3^n}, x \in \mathbb{R};$ | j) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n} \left(\frac{1-x^2}{1+x^2} \right)^n, x \in \mathbb{R};$ |
| e) $\sum_{n=1}^{\infty} \frac{x^n}{n^p}, p \in \mathbb{R};$ | k) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{\frac{n}{2}} \sqrt{1+n^2}} \operatorname{tg}^n x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right);$ |
| f) $\sum_{n=2}^{\infty} \frac{x^n}{3^n \cdot n \cdot \ln n} x \in \mathbb{R};$ | l) $\sum_{n=1}^{\infty} \frac{n!}{(a+1)(a+2) \cdot \dots \cdot (a+n)} x^n, a > 0, x \in \mathbb{R}.$ |

Bibliografie recomandată

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