Sem3-A5

Monday, 19 October 2020 10:04

Ex 66

2.

Var. Prop	Prop atomică
p	Lucrez la logică
q	Se poate ieși afară
r	Plouă
s	Este cald

$\neg q$ (nu se poate ieși afară)

$p \to \neg q$

$(\neg r \wedge s) \to q$

$(\neg p \wedge s) \to r$

Ex 99

$\underbrace{\{((q \wedge r) \wedge q), (p \wedge p)\}}_{\Gamma} \vdash (p \wedge r)$ validă

$\wedge_{e_1} \dfrac{\Gamma \vdash (\varphi_1 \wedge \varphi_2)}{\Gamma \vdash \varphi_1}$

1. $\Gamma \vdash (p \wedge p)$ (ipoteză)
2. $\Gamma \vdash p$ $(\wedge_{e_1}, 1)$
3. $\Gamma \vdash ((q \wedge r) \wedge q)$ (ip)
4. $\Gamma \vdash (q \wedge r)$ $(\wedge_{e_1}, 3)$

$\wedge_i \dfrac{\Gamma \vdash \varphi_1 \quad \Gamma \vdash \varphi_2}{\Gamma \vdash (\varphi_1 \wedge \varphi_2)}$

5. $\Gamma \vdash r$ $(\wedge_{e_2}, 4)$
6. $\Gamma \vdash (p \wedge r)$ $(\wedge_i, 2, 5)$

Ex 100

(1)

1. $\Gamma \vdash (p \wedge q)$ (ip)
2. $\Gamma \vdash p$ $(\wedge_{e_1}, 1)$
3. $\Gamma \vdash r$ (ip)

$\vee_{i_1} \dfrac{\Gamma \vdash \varphi_1}{\Gamma \vdash (\varphi_1 \vee \varphi_2)}$

4. $\Gamma \vdash (r \vee r')$ $(\vee_{i_1}, 3)$
5. $\underbrace{(p \wedge q), r}_{\Gamma} \vdash (p \wedge (r \vee r'))$ $(\wedge_i, 2, 4)$

(2)

$\to_e \dfrac{\overset{S_1}{\Gamma' \vdash \varphi_1 \to \varphi_2} \quad \overset{S_2}{\Gamma' \vdash \varphi_1}}{\Gamma' \vdash \varphi_2}$

S_1 1. $\Gamma, (p \wedge q) \vdash (\overset{\varphi_1}{p} \to \overset{\varphi_2}{(q \to r)})$ (ip)

2. $\Gamma, (p \wedge q) \vdash (p \wedge q)$ (ip)

3. $\Gamma, (p \wedge q) \vdash p$ $(\wedge_{e_1}, 2)$

S_1 4. $\Gamma, (p \wedge q) \vdash (\overset{\varphi_1}{q} \to \overset{\varphi_2}{r})$ $(\to_e, 1, 3)$

S_2 5. $\Gamma, (p \wedge q) \vdash q$ $(\wedge_{e_2}, 2)$

$\to_i \dfrac{\Gamma, \varphi_1 \vdash \varphi_2}{\Gamma \vdash (\varphi_1 \to \varphi_2)}$

6. $\overbrace{\Gamma, (p \wedge q)}^{\Gamma'} \vdash r$ $(\to_e, 4, 5)$

7. $\underbrace{(p \to (q \to r))}_{\Gamma} \vdash ((p \wedge q) \to r)$ $(\to_i, 6)$

(3)

1. $\Gamma, \neg r \vdash \neg q$ (ip)
2. $\Gamma, \neg r \vdash ((p \wedge \neg r) \to q)$ (ip)
3. $\Gamma, \neg r \vdash p$ (ip)
4. $\Gamma, \neg r \vdash \neg r$ (ip)
5. $\Gamma, \neg r \vdash (p \wedge \neg r)$ $(\wedge_i, 3, 4)$

$\dfrac{\overset{S_1}{\Gamma' \vdash \varphi} \quad \overset{S_2}{\Gamma' \vdash \neg\varphi}}{\Gamma' \vdash \bot}$

S_1 6. $\Gamma, \neg r \vdash q$ $(\to_e, 2, 5)$

7. $\Gamma, \neg r \vdash \bot$ $(\neg_e, 6, 1)$

8. $\Gamma \vdash \neg\overbrace{\neg r}^{\varphi}$ $(\neg_i, 7)$

9. $\underbrace{((p \wedge \neg r) \to q), \neg q, p}_{\Gamma} \vdash r$ $(\neg\neg_e, 8)$

Ex 102

(1) $\neg\neg_i \dfrac{\Gamma \vdash \varphi}{\Gamma \vdash \neg\neg\varphi}$

1. $\Gamma \vdash \varphi$ (ip regulii)

⋮ ?

j. $\Gamma \vdash \neg\neg\varphi$

(4) 1. $\Gamma \vdash (\varphi \to \varphi')$ (ip regulii)

2. $\Gamma \vdash \neg\varphi'$ (ip ...)

⋮ ?

j. $\Gamma \vdash \neg\varphi$

Ex 103

$\Gamma \vdash \varphi$ atunci $\Gamma \models \varphi$

pp. $\underbrace{\Gamma \vdash \varphi \text{ validă}}$ și dem că $\Gamma \models \varphi$

↳ există o dem formală pt ea.

1.
2.
3.
⋮
n. $\Gamma \vdash \varphi$

CB: $n = 1$: Dem este.

1. $\Gamma \vdash \varphi$ (ip)

↳ condiția este $\varphi \in \Gamma$

$\Gamma \models \varphi$ ddacă pt. orice $\tau: A \to B$ a.î. $\hat{\tau}(\varphi') = 1$ pt. orice $\varphi' \in \Gamma$ at. avem și $\hat{\tau}(\varphi) = 1$

↳ adevărat deoarece $\varphi \in \Gamma$ ⇒

⇒ $\Gamma \models \varphi$

CI.

1. S_1
2. S_2
3. S_3
4. S_4

⋮

n. $\Gamma \vdash \varphi$ (regula, i, j)

pp. că teorema are loc pt secvențele $S_1 \dots S_{n-1}$

dem că are loc pt. $S_n = \Gamma \vdash \varphi$

Caz. 1 regula este $\wedge_i$ $(\varphi = (\varphi_1 \wedge \varphi_2))$

i: $\Gamma \vdash \varphi_1 \overset{\text{ip. ind}}{\Longrightarrow} \Gamma \models \varphi_1$ (1)

j: $\Gamma \vdash \varphi_2 \overset{\text{ip. ind}}{\Longrightarrow} \Gamma \models \varphi_2$ (2)

n: $\Gamma \vdash (\varphi_1 \wedge \varphi_2)$ $(\wedge_i, i, j)$

Dem că $\Gamma \models (\varphi_1 \wedge \varphi_2)$ folosind (1), (2)