$a|b \Rightarrow \exists c$ a.î. $b = a c$

1) 0 divide doar 0

2) $a|0$

$a|a$

3) $1|a$

4) $a|b \Leftrightarrow a|-b$

5) $a|b$ și $b|c \Rightarrow a|c$

6) $a|b+c$ și $a|b \Rightarrow a|c$

7) $a|b \Rightarrow ac|bc$

$c \neq 0 \wedge ac|bc \Rightarrow a|b$

8) $a|b$, $a|c \Rightarrow a|\beta b + \gamma c$

Comb. lin. a,b $\forall_d = (\alpha, \beta)$ $\alpha \cdot a + \beta \cdot b = d$ $d = (a,b)$

p

11) $(a,b)=1$, $a|c \wedge b|c \Rightarrow ab|c$

$(a,b)=1 \Rightarrow \exists \alpha_1, \beta_1$, $\alpha_1 a + \beta_1 b = 1 \,/\cdot c$ $ac\alpha_1 + bc\beta_1 = c$

$a|c \Rightarrow \exists \alpha_2 \in \mathbb{Z}$, $c = a \cdot \alpha_2$

$b|c \Rightarrow \exists \beta_2 \in \mathbb{Z}$, $c = b \cdot \beta_2$ $c = a\, b\beta_2 \alpha_1 + b\, a\alpha_2 \beta_1 = ab\underbrace{(\alpha_1\beta_2 + \alpha_2\beta_1)}_{\omega}$

(?) $c = ab \cdot \boxed{\omega}$

12) $(b, a_1) = 1$, $b \mid a_1 \cdot a_2 \Rightarrow b \mid a_2$

13) p prim, $p \mid ab \Rightarrow$ fie $p \mid a$ fie $p \mid b$

Algoritmul lui Euclid

$\boxed{(a,b) = r_m}$ $a = r_{-1}$, $b = r_0$

$r_{-1} = r_0 \cdot q_1 + r_1$

$r_0 = r_1 \cdot q_2 + r_2$

$\vdots$

$r_{m-2} = r_{m-1} q_m + \boxed{r_m}$

$r_{m-1} = r_m \cdot q_{m+1} + 0$

ex: $a = 24$, $b = 7$

$\underset{a}{24} = \underset{b}{7} \cdot \underset{q}{3} + \underset{r}{3}$

$7 = 3 \cdot 2 + 1$

$3 = 1 \cdot 3 + 0$

$V_3 = V_{24} - 3V_7$

Algoritmul Extins al lui Euclid

$(a,b) = \alpha \cdot a + \beta \cdot b$ $\quad V_a = (1, 0)$ $\quad V_b = (0, 1)$

$V_3 = V_{24} - \overset{q}{3} V_7 = (1,0) - 3(0,1) = (1,0) - (3 \cdot 0, 3 \cdot 1) = (1, -3)$

$V_1 = V_7 - \underset{q}{2} V_3 = (0,1) - 2 \cdot (1,-3) = (0 - 2 \cdot 1, 1 - 2 \cdot (-3)) = (\underset{\alpha}{-2}, \underset{\beta}{7})$

$V_{(a,b)} = (-2, 7)$ $\quad \alpha = -2;\ \beta = 7.$

Ecuații liniare diofantice

$$ax + by = c$$

$\boxed{!}$ $\exists$ soluție în $\mathbb{Z} \Leftrightarrow (a,b) \mid c$ → Algoritmul de calcul pt x și y:

① Alg. Ext. Euclid: $(a,b) = \alpha \cdot a + \beta \cdot b$

② $x = \alpha \cdot \frac{c}{(a,b)}$; $y = \beta \cdot \frac{c}{(a,b)}$

ex: $24x + 7y = 8$ $\exists$ sol în $\mathbb{Z}$? $(24,7) = 1 \mid 8$ ✓

$x = (-2) \cdot \frac{8}{1} = -16$ $y = 7 \cdot \frac{8}{1} = 56$

Vf. $\underbrace{24 \cdot (-16)}_{-384} + \underbrace{56 \cdot 7}_{392} = 8$ ✓

$a = 7$ $b = 24$ $V_a = V_7 = (1,0)$ $V_b = V_{24} = (0,1)$

$7 = 24 \cdot 0 + 7$ $V_7 = (1,0)$

$24 = 7 \cdot 3 + 3$ $V_3 = V_{24} - 3 \cdot V_7 = (0,1) - 3 \cdot (1,0) = (-3,1)$

$7 = 3 \cdot 2 + \boxed{1}$ $V_1 = V_7 - 2 \cdot V_3 = (1,0) - 2 \cdot (-3,1) = (\underset{\alpha}{7}, \underset{\beta}{-2})$

$3 = 1 \cdot 3 + 0$

$a = 7$ $b = 24$ ⟶ $\alpha = 7$; $\beta = -2$

$\boxed{?} \cdot a + \boxed{?} \cdot b = (a,b)$ $7 \cdot \underset{a}{7} + (-2) \cdot \underset{b}{24} = 1$