S6. 6

S6.6 Fie $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ endomorfismul care, în raport cu baza alcătuită din $\overbrace{b_1 = (1, -1) \text{ şi } b_2 = (0, 1)}^{B}$, are matricea

$$A^{T_1} = \begin{bmatrix} 3 & 1 \\ -1 & 3 \end{bmatrix}.$$

De asemenea, fie $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ un endomorfism care, faţă de vectorii $\overbrace{v_1 = (2, 1) \text{ şi } v_2 = (-1, 1)}^{B'}$, are matricea

$$A^{T_2} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

Să se determine matricea endomorfismului $T_2 - T_1$ în raport cu sistemul de vectori $\{v_1, v_2\}$, precum şi matricea lui $T_2 \circ T_1$ faţă de baza canonică a lui $\mathbb{R}^2$.

Verificăm că v_1, v_2 formează bază

$\operatorname{card} B' = \dim \mathbb{R}^2$

$\Rightarrow$ Pt. a verif că B' bază e suficient să verif $\det[v_1, v_2] \neq 0$

$$\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 3 \neq 0$$

$$\begin{array}{ccc} \mathbb{R}^2 & & \mathbb{R}^2 \\ C & \longrightarrow & C \\ B & \xrightarrow{T_1} & B \\ B' & \xrightarrow{T_2} & B' \end{array}$$

$T_2 - T_1$ în raport cu B'

$\Rightarrow$ Vrem să exprimăm T_1 în raport cu B'.

$$S_{B|B'} = \left(S_{C|B}\right)^{-1} \cdot S_{C|B'}$$

$$(S_{C|B'})^{-1} = ?$$

$$\begin{tikzcd} C \arrow[r, "S_{CB}"] \arrow[dr, "S_{CB'}"'] & B \arrow[d, "S_{BB'}"] \\ & B' \end{tikzcd}$$

$$S_{C|B'} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$S_{CB} \cdot S_{BB'} = S_{CB'}$$

$$S_{BB'} = (S_{CB})^{-1} S_{CB'}$$

$$(S_{C|B'})^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$S_{B'|B} = \frac{1}{3}\begin{pmatrix} 0 & 1 \\ -3 & 2 \end{pmatrix}$$

$$S_{B|B'} = (S_{B'B})^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$$

$$A^{T'}_{B'} = (S_{B|B'})^{-1} \cdot A^{T'} \cdot S_{B|B'}$$

$$A^{T'}_{B'} = \frac{1}{3}\begin{pmatrix} 0 & 1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} -1 & 3 \\ -11 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 7 & 1 \\ -13 & 11 \end{pmatrix}$$

$$A_{B'}^{T_2-T_1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 7 & 1 \\ -13 & 11 \end{pmatrix} = \boxed{\frac{1}{3}\begin{pmatrix} -1 & 2 \\ 16 & -8 \end{pmatrix}}$$

$$A_B^{T_1} = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix} = S_{CB}^{-1} \cdot A_C^{T_1} \cdot S_{CB}$$

$$A_C^{T_1} = S_{CB} \cdot A_B^{T_1} \cdot (S_{CB})^{-1}$$

$$S_{CB} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$(S_{CB})^{-1} = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right) \underline{\underline{L_2 = L_2 + L_1}}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right)$$

$$(S_{CB})^{-1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$A^{T_1}_{C} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -2 & 2 \end{pmatrix}$$

$$A^{T_2}_{C} = S_{CB'} \cdot A^{T_2}_{B'} \cdot (S_{CB'})^{-1}$$

$$S_{C|B'} = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$(S_{C|B'})^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$$

$$A^{T_2}_{B'} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$A^{T_2}_{C} = \frac{1}{3} \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} =$$

$$\frac{1}{3}\begin{pmatrix} 3 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} =$$

$$\frac{1}{3}\begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix}$$

$$A^{T_2-T_1}_{C} = \frac{1}{3}\begin{pmatrix} 2 & 5 \\ 1 & 7 \end{pmatrix} - \begin{pmatrix} 4 & 1 \\ -2 & 2 \end{pmatrix} =$$

$$\begin{pmatrix} -\frac{10}{3} & \frac{2}{3} \\ \frac{7}{3} & \frac{1}{3} \end{pmatrix}$$

$$A^{T_2-T_1}_{B'} = \left(S_{C|B'}\right)^{-1} \cdot A^{T_2-T_1}_{C} \cdot S_{C|B'}$$

$$= \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} -\frac{10}{3} & \frac{2}{3} \\ \frac{7}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{9}\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -10 & 2 \\ 7 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} =$$

$$\frac{1}{9}\begin{pmatrix} -3 & 3 \\ 24 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} =$$

$$\frac{1}{3}\begin{pmatrix} -1 & 1 \\ 8 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} =$$

$$\boxed{\frac{1}{3}\begin{pmatrix} -1 & 2 \\ 16 & -8 \end{pmatrix}}$$

$$A^{T_2 \circ T_1}_{c} = A^{T_2}_{c} \cdot A^{T_1}_{c} =$$

$$\frac{1}{3}\begin{pmatrix} 2 & 1 \\ 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} 4 & 1 \\ -2 & 2 \end{pmatrix}$$

$A_C \qquad A_B$

$$A_B = (S_{C|B})^{-1} \cdot A_C \cdot S_{C|B} \quad | (S_{C|B})^{-1}$$

Met I

$$S_{C|B} | \quad A_B \cdot (S_{C|B})^{-1} = (S_{C|B})^{-1} \cdot A_C$$

$$S_{C|B} \cdot A_B \cdot (S_{C|B})^{-1} = A_C$$

Met II

$$A_C = (S_{B|C})^{-1} \cdot A_B \cdot S_{B|C}$$

$$(S_{B|C})^{-1} = S_{C|B}$$

$$A_C = S_{C|B} \cdot A_B \cdot (S_{C|B})^{-1}$$

$$T_1(x_1,x_2,x_3,x_4) = (x_1+x_2+x_3+x_4, x_1+x_2-x_3-x_4, x_1-x_2+x_3-x_4, x_1-x_2-x_3+x_4),$$
$$\forall\ (x_1,x_2,x_3,x_4) \in \mathbb{R}^4$$

şi

$$T_2(x_1,x_2,x_3) = (2x_1+x_2+x_3, 2x_1+3x_2+x_3, 3x_1+3x_2+4x_3)\,,\ \forall\ (x_1,x_2,x_3) \in \mathbb{R}^3$$

a) Să se afle valorile proprii şi vectorii proprii corespunzători;

b) Să se afle subspaţiile proprii şi dimensiunile lor;

c) Să se analizeze posibilitatea diagonalizării lui T_1 şi T_2. În caz afirmativ, să se afle baza în care se manifestă forma diagonală, matricea schimbării de bază în cauză, precum şi forma diagonală ca atare.

T_1

1. Matricea operatorului

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

2. Polinomul caracteristic

$$\det(A - \lambda I_4) = \begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & -1 & -1 \\ 1 & -1 & 1-\lambda & -1 \\ 1 & -1 & -1 & 1-\lambda \end{vmatrix}$$

$$(1-\lambda)\cdot(-1)^{1+1}\begin{vmatrix} 1-\lambda & -1 & -1 \\ -1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} +$$

$$1\cdot(-1)^{1+2}\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} +$$

$$1\cdot(-1)^{1+3}\begin{vmatrix} 1 & 1 & 1 \\ 1-\lambda & -1 & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} +$$

$$1\,(-1)^{1+4}\begin{vmatrix} 1 & 1 & 1 \\ 1-\lambda & -1 & -1 \\ -1 & 1-\lambda & -1 \end{vmatrix}$$

$$\det(A - \lambda I_4) = \begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1-\lambda & -1 & -1 \\ 1 & -1 & 1-\lambda & -1 \\ 1 & -1 & -1 & 1-\lambda \end{vmatrix}$$

$L_2 = L_2 + L_1$

$L_4 = L_4 - L_3$

$-1-1+\lambda$

$$\begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 2-\lambda & 2-\lambda & 0 & 0 \\ 1 & -1 & 1-\lambda & -1 \\ 0 & 0 & -2+\lambda & 2-\lambda \end{vmatrix} =$$

$$(2-\lambda)\begin{vmatrix} 1-\lambda & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 1-\lambda & -1 \\ 0 & 0 & -2+\lambda & 2-\lambda \end{vmatrix}$$

$C_1 = C_1 - C_2$

$$(2-\lambda)\begin{vmatrix} -\lambda & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & -1 & 1-\lambda & -1 \\ 0 & 0 & -2+\lambda & 2-\lambda \end{vmatrix} =$$

$$(2-\lambda)\cdot(-1)^{2+2}\begin{vmatrix} -\lambda & 1 & 1 \\ 2 & 1-\lambda & -1 \\ 0 & -2+\lambda & 2-\lambda \end{vmatrix} =$$

$$(2-\lambda)^2\begin{vmatrix} -\lambda & 1 & 1 \\ 2 & 1-\lambda & -1 \\ 0 & -1 & 1 \end{vmatrix} \underset{C_2 = C_2 + C_3}{=}$$

$$(2-\lambda)^2\begin{vmatrix} -\lambda & 2 & 1 \\ 2 & -\lambda & -1 \\ 0 & 0 & 1 \end{vmatrix} =$$

$$(2-\lambda)^2\begin{vmatrix} -\lambda & 2 \\ 2 & -\lambda \end{vmatrix} =$$

$$(2-\lambda)^2(\lambda^2-4) =$$

$$(\lambda-2)^3(\lambda+2)$$

3. Valorile proprii

$\lambda_1 = 2 \qquad m_{\lambda_1} = 3$

$\lambda_2 = -2 \qquad m_{\lambda_2} = 1$

4. Subspații proprii

$\lambda_1 = 2$

$$(A - 2I_4)\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2 & 1 & 1 & 1 \\ 1 & 1-2 & -1 & -1 \\ 1 & -1 & 1-2 & -1 \\ 1 & -1 & -1 & 1-2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

S $L_1 = L_1$

$$\begin{pmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 \end{pmatrix}$$

$\Rightarrow$ rang $(A - 2 I_4) = 1$

$\Delta = |1| \neq 0$

Nec pp x_1

Nec sec $x_2 = \alpha$ $x_3 = \beta$ $x_4 = \mu$

$$-x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 = \alpha + \beta + \mu$$

$$V_{\lambda_1} = \{ (\alpha + \beta + \mu, \alpha, \beta, \mu) \mid \alpha, \beta, \mu \in \mathbb{R} \}$$

$\dim V_{\lambda_1} = 3$

$$v_{\lambda_1}^1 \xrightarrow[\beta=\gamma=0]{\alpha=1} (1, 1, 0, 0)$$

$$v_{\lambda_1}^2 \xrightarrow[\alpha=\gamma=0]{\beta=1} (1, 0, 1, 0)$$

$$v_{\lambda_1}^3 \xrightarrow[\alpha=\beta=0]{\gamma=1} (1, 0, 0, 1)$$

$$B_{\lambda_1} = \{ v_{\lambda_1}^1, v_{\lambda_1}^2, v_{\lambda_1}^3 \}$$

$\lambda_2 = -2$

$$(A + 2I_2) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1+2 & 1 & 1 & 1 \\ 1 & 1+2 & -1 & -1 \\ 1 & -1 & 1+2 & -1 \\ 1 & -1 & -1 & 1+2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & -1 & -1 \\ 1 & -1 & 3 & -1 \\ 1 & -1 & -1 & 3 \end{pmatrix}}_{\det = 0} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} \quad L_1 = L_1 + L_2 + L_3$$

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} \quad C_1 = C_1 - C_2$$

$$\begin{vmatrix} 0 & 1 & 1 \\ -4 & 3 & -1 \\ 0 & -1 & 3 \end{vmatrix} = 4 \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} =$$

$$16 \neq 0$$

x_1 var sec

$x_1 = \alpha$ $\quad$ x_2, x_3, x_4 var pp

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -\alpha \\ -\alpha \\ -\alpha \end{pmatrix} \quad L_1 = L_1 + L_2 + L_3$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -3\alpha \\ -\alpha \\ -\alpha \end{pmatrix}$$

$L_2 = L_2 + L_1$

$L_3 = L_3 + L_1$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -3\alpha \\ -4\alpha \\ -4\alpha \end{pmatrix}$$

$$4x_3 = -4\alpha \rightarrow x_3 = -\alpha$$

$$4x_4 = -4\alpha \rightarrow x_4 = -\alpha$$

$$x_2 + x_3 + x_4 = -3\alpha \rightarrow x_2 = -\alpha$$

$$V_{\lambda_2} = \{ (\alpha, -\alpha, -\alpha, -\alpha) \mid \alpha \in \mathbb{R} \}$$

$$v_{\lambda_2} = (1, -1, -1, -1)$$

5. Este diag?

$$m_{\lambda_1} + m_{\lambda_2} = \dim \mathbb{R}_4 ?$$

$$3 + 1 = 4 \quad \checkmark$$

$\underbrace{m_{\lambda_1}} = \dim V_{\lambda_1}$?

$3 = 3$ ✓

$m_{\lambda_2} = \dim V_{\lambda_2}$?

$1 = 1$ ✓

$\Rightarrow$ Este diag?

$$A_D = \begin{pmatrix} \lambda_1 & & & \\ & \ddots & & \\ & & \lambda_1 & \\ & & & \lambda_2 \end{pmatrix}$$

m_{λ_1}

$$A_D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$D = \{ (1,1,0,0), (1,0,1,0), (1,0,0,1), (1,-1,-1,-1) \}$$

$$\boxed{A_D = (S_{CD})^{-1} \cdot A \cdot S_{CD}}$$

$$S_{CD} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$T_2(x_1, x_2, x_3) = (2x_1 + x_2 + x_3, 2x_1 + 3x_2 + x_3, 3x_1 + 3x_2 + 4x_3)\,,\ \forall\ (x_1, x_2, x_3) \in \mathbb{R}^3$

1. $A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 3 & 4 \end{pmatrix}$

2. $\begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 1 \\ 3 & 3 & 4-\lambda \end{vmatrix}$ $\underline{\underline{C_1 = C_1 - C_2}}$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ -1+\lambda & 3-\lambda & 1 \\ 0 & 3 & 4-\lambda \end{vmatrix} =$$

$$(1-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ -1 & 3-\lambda & 1 \\ 0 & 3 & 4-\lambda \end{vmatrix} \quad \underline{\underline{C_2 = C_2 + C_1}}$$

$$(1-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 4-\lambda & 2 \\ 0 & 3 & 4-\lambda \end{vmatrix} =$$

$$(1-\lambda)\begin{vmatrix} 4-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} =$$

$$(1-\lambda)\left((4-\lambda)^2 - 6\right) =$$

$$(1-\lambda)(\lambda^2 - 8\lambda + 16 - 6)$$

$$(1-\lambda)(\lambda^2 - 8\lambda + 10)$$

$$\lambda^2 - 8\lambda + 10 = 0$$

3. $\lambda_{1,2} = \dfrac{8 \pm \sqrt{64-40}}{2}$

$\lambda_{1,2} = \dfrac{8 \pm \sqrt{24}}{2} =$

$\dfrac{8 \pm 2\sqrt{6}}{2} = 4 \pm \sqrt{6}$

$\lambda_1 = 4 - \sqrt{6}$ $\qquad m_{\lambda_1} = 1$

$\lambda_2 = 4 + \sqrt{6}$ $\qquad m_{\lambda_2} = 1$

$\lambda_3 = 1$ $\qquad m_{\lambda_3} = 1$

4.

$$V_{\lambda_1} \begin{pmatrix} 2-4+\sqrt{6} & 1 & 1 \\ 2 & 3-4+\sqrt{6} & 1 \\ 3 & 3 & 4-4+\sqrt{6} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2+\sqrt{6} & 1 & 1 \\ 2 & -1+\sqrt{6} & 1 \\ 3 & 3 & \sqrt{6} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 = \alpha$ nec sec

$$\begin{pmatrix} 1 & 1 \\ -1+\sqrt{6} & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} 2-\sqrt{6} \\ -2 \end{pmatrix}$$

$$x_2 + x_3 = (2-\sqrt{6})\alpha$$

$$(-1+\sqrt{6})x_2 + x_3 = -2\alpha \quad -$$

$$(2-\sqrt{6}) \cdot x_2 = (4-\sqrt{6})\alpha$$

$2+\sqrt{6}$,

$$x_2 = \frac{4-\sqrt{6}}{2-\sqrt{6}} \alpha$$

$$x_2 = \frac{8+4\sqrt{6}-2\sqrt{6}-6}{-2} \alpha$$

$$x_2 = \frac{2 + 2\sqrt{6}}{-2}\alpha$$

$$x_2 = (-1 - \sqrt{6})\alpha$$

$$x_3 = (2 - \cancel{\sqrt{6}})\alpha + (1 + \cancel{\sqrt{6}})\alpha$$

$$x_3 = 3\alpha$$

$$V_{\lambda_1} = \{(\alpha, (-1-\sqrt{6})\alpha, 3\alpha) \mid \alpha \in \mathbb{R}\}$$

$$v_{\lambda_1} = (1, -1-\sqrt{6}, 3)$$

V_{λ_2}

$$\begin{pmatrix} -2-\sqrt{6} & 1 & 1 \\ 2 & -1-\sqrt{6} & 1 \\ 3 & 3 & -\sqrt{6} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 = \alpha$

$$\begin{pmatrix} 1 & 1 \\ -1-\sqrt{6} & 1 \end{pmatrix}\begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \alpha\begin{pmatrix} 2+\sqrt{6} \\ -2 \end{pmatrix}$$

$$x_2 + x_3 = (2+\sqrt{6})\alpha$$

$$x_2(-1-\sqrt{6}) + x_3 = -2\alpha$$ $-$

$$x_2(2+\sqrt{6}) = (4+\sqrt{6})\alpha$$

$(\sqrt{6}-2)$

$$x_2 = \frac{4+\sqrt{6}}{2+\sqrt{6}}\alpha$$

$$x_2 = \frac{4\sqrt{6}-8+6-2\sqrt{6}}{6-4}\alpha$$

$$x_2 = \frac{2\sqrt{6}-2}{2}\alpha$$

$$x_2 = (\sqrt{6}-1)\alpha$$

$$x_3 = (2+\sqrt{6})\alpha - (\sqrt{6}-1)\alpha$$

$$x_3 = 3\alpha$$

$$V_{\lambda_2} = \{ (\alpha, (\sqrt{6}-1)\alpha, 3\alpha) \mid \alpha \in \mathbb{R} \}$$

$$v_{\lambda_2} = (1, \sqrt{6}-1, 3)$$

V_{λ_3} $\quad$ $\lambda_3 = 1$

$$\begin{pmatrix} 2-1 & 1 & 1 \\ 2 & 3-1 & 1 \\ 3 & 3 & 4-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 = \alpha$ $\quad \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ -2 \end{pmatrix}$ $\quad L_2 = L_2 - L_1$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad L_1 = L_1 - L_2$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$x_3 = 0$

$x_2 = -\alpha$

$V_{\lambda_3} = \{(\alpha, -\alpha, 0) \mid \alpha \in \mathbb{R}\}$

$v_{\lambda_3} = (1, -1, 0)$

Diagonalizabil

$$D = \begin{pmatrix} 4-\sqrt{6} & 0 & 0 \\ 0 & 4+\sqrt{6} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \left\{ (1, -1-\sqrt{6}, 3),\ (1, -1+\sqrt{6}, 3),\ (-1, -1, 0) \right\}$$