Ex12 Sem3

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$$T(n) = \begin{cases} \Theta(1), & n=1 \\ 2T(n/2)+\Theta(n), & n>1 \end{cases}$$

Arătați că $T(n) = \Theta(n \log(n))$

$\Theta(f(n)) = \{ g(n) \mid \exists c_1, c_2 > 0, n_0 \geq 0$ a.î. $c_1 \cdot f(n) \leq g(n) \leq c_2 \cdot f(n), \forall n \geq n_0 \}$

$n = 1 \Rightarrow T(n) = \Theta(1) = \Theta(\underbrace{n \log(n)}_{1 \log 1})$

$n > 1 \rightarrow$ pp. că $T(n/2) = \Theta(n/2 \cdot \log n/2)$ și

dem $T(n) = \Theta(n \log n)$

$T(n) = 2T(n/2) + \Theta(n) = 2 \cdot \Theta(n/2 \cdot \log n/2) + \Theta(n)$

$\Rightarrow T(n) = \{ 2 \cdot g(n) + h(n) \mid g(n) \in \Theta(n/2 \log n/2)$, și $h(n) \in \Theta(n) \}$

$g(n) \in \Theta\left(\frac{n}{2} \log\left(\frac{n}{2}\right)\right) \Leftrightarrow \exists c_1, c_2 > 0, n_0 \geq 0$ a.î.

$c_1 \cdot \frac{n}{2} \log\left(\frac{n}{2}\right) \leq g(n) \leq c_2 \cdot \frac{n}{2} \log\left(\frac{n}{2}\right) \quad \forall n \geq n_0 \Leftrightarrow$

$\Leftrightarrow \cancel{2} \cdot c_1 \cdot \frac{n}{\cancel{2}} \log\left(\frac{n}{2}\right) \leq 2g(n) \leq \cancel{2} \cdot c_2 \cdot \frac{n}{\cancel{2}} \log\left(\frac{n}{2}\right) \quad \forall n \geq n_0 \Leftrightarrow$

$\Leftrightarrow c_1 \cdot n \log\left(\frac{n}{2}\right) \leq 2g(n) \leq c_2 \cdot n \cdot \log\left(\frac{n}{2}\right) \quad \forall n \geq n_0$ (1)

$h(n) \in \Theta(n) \Leftrightarrow \exists c_3, c_4 > 0, n_1 \geq 0$ a.î.

$c_3 \cdot n \leq h(n) \leq c_4 \cdot n \quad \forall n \geq n_1$ (2)

(1), (2) $\Rightarrow$

$\Rightarrow c_1 \cdot n \cdot \log\left(\frac{n}{2}\right) + c_3 \cdot n \leq 2g(n) + h(n) \leq c_2 \cdot n \cdot \log\left(\frac{n}{2}\right) + c_4 \cdot n \quad \forall n \geq \max(n_0, n_1)$

$\Rightarrow c_1 \cdot n (\log n - \log 2) + c_3 \cdot n \leq 2g(n) + h(n) \leq c_2 \cdot n \cdot (\log n - \log 2) + c_4 \cdot n$

~~dar~~ Fie $c_1' = \min(c_1, c_3) \Rightarrow c_1' \cdot n (\log n - \log 2) + c_1' \cdot n \leq c_1 \cdot n \cdot (\log n - \log 2) + c_3 n$

$c_2' = \max(c_2, c_4) \Rightarrow c_2 \cdot n \cdot (\log n - \log 2) + c_4 \cdot n \leq c_2' \cdot n (\log n - \log 2) + c_2' \cdot n$

$\Rightarrow$

$\Rightarrow c_1' n (\log n - \log 2 + 1) \leq 2g(n) + h(n) \leq c_2' \cdot n (\log n - \log 2 + 1) \Rightarrow$

$\forall n \geq \max(n_0, n_1)$

$\Rightarrow c_1' \cdot n \cdot \log n \leq 2g(n) + h(n) \leq c_2' \cdot n \cdot \log n$

$\Rightarrow \exists c_1', c_2' > 0, n_0' > 0$ a.î.

$c_1' = \min(c_1, c_3)$, $c_2' = \max(c_2, c_4)$, $n_0' = \max(n_0, n_1)$

$\forall g(n) \in \Theta\left(\frac{n}{2} \log \frac{n}{2}\right)$ și $h(n) \in \Theta(n)$

$\Rightarrow 2g(n) + h(n) \in \Theta(n \log n)$

$\Rightarrow T(n) = \Theta(n \log n)$