[redacted] - 13

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8.5

S I. $h: \mathbb{R}^2 \to \mathbb{R}$ $h(x) = x_1^2 + 2x_1x_2 - 5x_2^2$

a) h. la f. m.

$h(x) = x_1^2 + 2x_1x_2 + x_2^2 - 6x_2^2 = (x_1 + x_2)^2 - 6x_2^2$

$\begin{cases} y_1 = x_1 + x_2 \\ y_2 = \sqrt{6}\, x_2 \end{cases} \Rightarrow h(y) = y_1^2 - y_2^2$

$y_1^2 - y_2^2 = 0 \Rightarrow y_1^2 = y_2^2 \Rightarrow y_1 = y_2$

$y_1 = \pm y_2$

c) Natura geometrică: parabolă ✗ 2 drepte

b) signatura: $(1, 1, 0)$ ✓

$\Rightarrow \begin{cases} z_1 = x_1 + x_2 \\ z_2 = \sqrt{6}\, x_2 \end{cases}$ $X_{B_c} = S^{-1} \cdot X_B$ $S^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & \sqrt{2} \end{pmatrix}$

$\det(S^{-1}) = \sqrt{2}$, $S^{-t} = \begin{pmatrix} 1 & 0 \\ 1 & \sqrt{2} \end{pmatrix}$ $S^{*} = \begin{pmatrix} (-1)^{1+1} \cdot \sqrt{2} & (-1)^{1+2} \cdot 1 \\ 0 & (-1)^{2+2} \cdot 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & -1 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow (S^{-1})^{-1} = S = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix} =$

$\Rightarrow S = \begin{pmatrix} 1 & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$

$\Rightarrow$ B_c în care are ~~baza~~ forma canonică:

$B_c = \{ \bar{v}_1 = 1\bar{e}_1, \bar{v}_2 = -\frac{\sqrt{2}}{2} e_1 + \frac{\sqrt{2}}{2} \bar{e}_2 \}$

S II $f:\mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = \begin{cases} \dfrac{(xy)^2}{x^4+y^4}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

a) $\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} \dfrac{(xy)^2}{x^4+y^4}\right) = \cancel{\lim\limits_{x\to 0}\lim\limits_{y\to 0}} = \lim\limits_{x\to 0} \dfrac{(0)^2}{x^4+0^4} =$

$= \lim\limits_{x\to 0} 0 = 0$

$\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} \dfrac{(xy)^2}{x^4+y^4}\right) = \lim\limits_{y\to 0} \dfrac{(0)^2}{0^4+y^4} = \lim\limits_{y\to 0} 0 = 0$

b) existența lim. globale $\lim\limits_{(x,y)\to(0,0)} \dfrac{(xy)^2}{x^4+y^4}$

Fie $(x_n, y_n) \to (0,0)$ $\quad x_n = \frac{1}{n}$, $y_n = \frac{1}{n^2}$

$\Rightarrow \cancel{\lim\limits_{(x,y)\to(0,0)}} \lim\limits_{n\to\infty} \dfrac{\left(\frac{1}{n}\cdot\frac{1}{n^2}\right)^2}{\frac{1}{n^4}+\frac{1}{n^8}} = \lim\limits_{n\to\infty} \dfrac{\frac{1}{n^6}}{\frac{n^4+1}{n^8}} = \lim\limits_{n\to\infty} \dfrac{n^2}{n^4+1}$

$= \lim\limits_{n\to\infty} \dfrac{1}{n^2+\frac{1}{n^2}} = 0$

$\cancel{\lim\limits_{n\to\infty}}$ $\quad x_n = \frac{1}{n}$, $y_n = \frac{2}{n}$

$\lim\limits_{n\to\infty} \dfrac{\left(\frac{1}{n}\cdot\frac{2}{n}\right)^2}{\frac{1}{n^4}+\frac{1}{n^4}} = \lim\limits_{n\to\infty} \dfrac{\frac{4}{n^4}}{\frac{2}{n^4}} = \frac{4}{2} = 2$

$x_n = \frac{2}{n^2}$, $y_n = \frac{1}{n^2}$

$\lim\limits_{n\to\infty} \dfrac{\left(\frac{2}{n^2}\cdot\frac{1}{n^2}\right)^2}{\frac{16}{n^4}+\frac{1}{n^4}} = \lim\limits_{n\to\infty} \dfrac{\frac{4}{n^8}}{\frac{17}{n^4}} = \lim\limits_{n\to\infty} \dfrac{\frac{4}{17}}{n^4} = 0$

$\Rightarrow \nexists$ limitei globale în $(0,0)$

c) $\dfrac{\partial f}{\partial x}(x,y) = \dfrac{2y^2x(x^4+y^4) - x^2y^2\cdot 4x^3}{(x^4+y^4)^2}$

$= \dfrac{2y^2x^5 + 2xy^6 - 4x^5y^2}{(x^4+y^4)^2} = \dfrac{2xy^6 - 2x^5y^2}{(x^4+y^4)^2}$

$\dfrac{\partial f}{\partial y}(x,y) = \dfrac{2x^2y(x^4+y^4) - x^2y^2\cdot 4y^3}{(x^4+y^4)^2} = \dfrac{2x^6y - 2x^2y^5}{(x^4+y^4)^2}$

EXISTĂ DERIVATE PARȚIALE într-un pct oarecare $(x,y) \neq (0,0)$

$$\frac{\partial f}{\partial x}(0,0) = \lim_{x \to 0} \frac{f(x,0) - f(0,0)}{x - 0} = \lim_{x \to 0} \frac{\frac{(x \cdot 0)^2}{x^4 + 0^4} - 0}{x} = \lim_{x \to 0} \frac{0}{x} = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{y \to 0} \frac{f(0,y) - f(0,0)}{y - 0} = \lim_{y \to 0} \frac{\frac{(0 \cdot y)^2}{0^4 + y^4} - 0}{y} = \lim_{y \to 0} \frac{0}{y} = 0$$

$\Rightarrow$ EXISTĂ DERIVATE PARȚIALE DE ORD 1 ÎN $(0,0)$