Barem

Examen - Restanta / – Matematică – Lucrarea 2 - nr. 1

(10.02.2020)

Subiectul 1 .... **45 puncte**

a) $\frac{\partial f}{\partial x} = -4x + 2y + 6$ ....

$\frac{\partial f}{\partial y} = 2x - 10y + 6$ ....

$\frac{\partial f}{\partial z} = -2z + 2$ ....

$\nabla f(1,-1,1) = \left(\frac{\partial f}{\partial x}(1,-1,1), \frac{\partial f}{\partial y}(1,-1,1), \frac{\partial f}{\partial z}(1,-1,1)\right) = (0, 18, 0)$ **15**

b) Abordarea subiectului ....

Rezolvarea sistemului $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0$: $(x, y, z) = (2, 1, 1)$ **10**

c) $\frac{\partial^2 f}{\partial x^2} = -4$, $\frac{\partial^2 f}{\partial y^2} = -10$, $\frac{\partial^2 f}{\partial z^2} = -2$, ...

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = 2$, $\frac{\partial^2 f}{\partial x \partial z} = \frac{\partial^2 f}{\partial z \partial x} = 0$, $\frac{\partial^2 f}{\partial y \partial z} = \frac{\partial^2 f}{\partial z \partial y} = 0$. ....

Determinarea Hessianului în punctul critic: $H_f(2,1,1) = \begin{bmatrix} -4 & 2 & 0 \\ 2 & -10 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ ....

$(-1)^{k+1}\Delta_k < 0, \forall k = \overline{1,n}$, unde Δ_k sunt minorii principali ai hessianului $H_f(2,1,1)$. ....

Concluzie: $(2,1,1)$ punct de maxim local **20**

Subiectul 2 .... **45 puncte**

a) Calculul limitelor iterate

$\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} f(x,y)\right) = \lim\limits_{x\to 0}\left(\lim\limits_{y\to 0}\frac{|x|^\alpha \cdot |y|^\beta}{\sqrt{x^2+y^2}}\right) = \lim\limits_{x\to 0}\left(\frac{0}{\sqrt{x^2}}\right) = 0$ ....

$\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} f(x,y)\right) = \lim\limits_{y\to 0}\left(\lim\limits_{x\to 0}\frac{|x|^\alpha \cdot |y|^\beta}{\sqrt{x^2+y^2}}\right) = \lim\limits_{y\to 0}\left(\frac{0}{\sqrt{y^2}}\right) = 0$ **10**

b) Abordarea subiectului ....

Funcţia f este continuă pe $\mathbb{R}^2 \setminus \{(0,0)\}$ ....

Studiul continuităţii în $(0,0)$:

Avem $|f(x,y)| = \left|\frac{|x|^\alpha \cdot |y|^\beta}{\sqrt{x^2+y^2}}\right| \le \left|\frac{|x|^\alpha \cdot |y|^\beta}{\sqrt{2|x|\cdot|y|}}\right| = \frac{1}{\sqrt{2}}|x|^{\alpha-\frac{1}{2}} \cdot |y|^{\beta-\frac{1}{2}} \overset{not}{=} g(x,y)$ (1).

$\lim\limits_{(x,y)\to(0,0)} g(x,y) = \lim\limits_{(x,y)\to(0,0)} \frac{1}{\sqrt{2}}|x|^{\alpha-\frac{1}{2}} \cdot |y|^{\beta-\frac{1}{2}} = 0,\ \alpha, \beta > \frac{1}{2}$ (2)

Din (1) şi (2) rezultă $\lim\limits_{(x,y)\to(0,0)} \frac{|x|^\alpha \cdot |y|^\beta}{\sqrt{x^2+y^2}} = 0 \Rightarrow f$ este continuă pe $\mathbb{R}^2$. **15**

c) $\iint_D \left(x(1-y) + \frac{1}{x^2+1}\right) dxdy = \int_0^1 \left(\int_{-x^2}^{x^2+1} \left(x(1-y) + \frac{1}{x^2+1}\right) dy\right) dx = \int_0^1 \left(\left(-\frac{x(1-y)^2}{2} + \frac{1}{x^2+1}\cdot y\right)\Bigg|_{y=-x^2}^{y=x^2+1}\right) dx$

$= \int_0^1 \left(-\frac{x^5}{2} + \frac{x}{2}\cdot(1+x^2)^2 + \frac{2x^2+1}{x^2+1}\right) dx = \left(-\frac{x^6}{12} + \frac{x^2}{4} + \frac{x^4}{4} + \frac{x^6}{12}\right)\Bigg|_{x=0}^{x=1} + \int_0^1 \frac{2x^2+1}{x^2+1} dx = \frac{1}{2} + 2 - \int_0^1 \frac{1}{x^2+1} dx$

$= \frac{5}{2} - \operatorname{arctg} x\Big|_0^1 = \frac{5}{2} - \frac{\pi}{4}$ **20**

Puncte din oficiu: .... **10 puncte**