Interpolare pe noduri echidistante
formulele lui Newton

x	-1	1	3	5
f	6	2	6	18

$x_0 = -1 \quad y_0 = f(x_0) = 6$
$x_1 = 1 \quad y_1 = f(x_1) = 2$
$x_2 = 3 \quad y_2 = f(x_2) = 6$
$x_3 = 5 \quad y_3 = f(x_3) = 18$

$\bar{x} = 0 \qquad f(\bar{x}) = ?$ $\qquad \bar{x} = x_0 + \bar{t}h = -1 + \bar{t}\cdot 2$

$h = x_{i+1} - x_i = 2 \quad \forall i = \overline{0,2}$

$f(\bar{x}) \simeq L_3(\bar{x}) = l_3(\bar{t}) = L_3(x_0 + \bar{t}h)$

$\bar{x} = 0 = -1 + \bar{t}\cdot 2 \Rightarrow \bar{t} = \frac{1}{2}$

$$l_3(\bar{t}) = y_0 + \Delta f(x_0)\,\bar{t} + \Delta^2 f(x_0)\frac{\bar{t}(\bar{t}-1)}{2!} + \Delta^3 f(x_0)\frac{\bar{t}(\bar{t}-1)(\bar{t}-2)}{3!}$$

$$= 6 + \Delta f(-1)\,\bar{t} + \Delta^2 f(-1)\frac{\bar{t}(\bar{t}-1)}{2} + \Delta^3 f(-1)\frac{\bar{t}(\bar{t}-1)(\bar{t}-2)}{6}$$ – formula Newton progresivă

$\Delta f(x) = f(x+h) - f(x)$

$\Delta^k f(x) = \Delta(\Delta^{k-1} f(x)) = \Delta^{k-1} f(x+h) - \Delta^{k-1} f(x)$

$$\Delta^k f(x) = \sum_{i=0}^{k} C_k^i (-1)^{k-i} f(x+ih)$$

Calcul $\Delta^k f(-1)$: direct și recursiv

$$\Delta f(-1) = y_1 - y_0 = 2 - 6 = -4$$

$$\Delta^2 f(-1) = C_2^0 (-1)^{2-0} f(x_0) + C_2^1 \cdot (-1)^{2-1} f(x_1) + C_2^2 (-1)^{2-2} f(x_2) = 6 - 2\cdot 2 + 6 = 8$$

$$\Delta^3 f(-1) = C_3^0 (-1)^{3-0} \cdot y_0 + C_3^1 (-1)^{3-1} \cdot y_1 + C_3^2 (-1)^{3-2} y_2 + C_3^3 (-1)^{3-3} y_3 = -6 + 3\cdot 2 - 3\cdot 6 + 18 = 0$$

6

2 $\quad \Delta f(-1) = y_1 - y_0 = 2 - 6 = -4$

6 $\quad \Delta f(1) = y_2 - y_1 = 6 - 2 = 4 \qquad \Delta^2 f(-1) = \Delta f(1) - \Delta f(-1) = 4 - (-4) = 8$

18 $\quad \Delta f(3) = y_3 - y_2 = 18 - 6 = 12 \qquad \Delta^2 f(1) = \Delta f(3) - \Delta f(1) = 12 - 4 = 8$

$$\Delta^3 f(-1) = \Delta^2 f(1) - \Delta^2 f(-1) = 8 - 8 = 0$$

$$f(0) \simeq l_3\left(\tfrac{1}{2}\right) = 6 - 4 \cdot \tfrac{1}{2} + 8 \cdot \tfrac{1}{2}\left(\tfrac{1}{2} - 1\right) \cdot \tfrac{1}{2} + 0 \cdot \tfrac{1}{2}\left(\tfrac{1}{2} - 1\right)\left(\tfrac{1}{2} - 2\right) \cdot \tfrac{1}{6} = 3$$

$$\bar{x} = 4 \qquad f(\bar{x}) \simeq L_3(\bar{x}) = \tilde{l}_3(\bar{t}) = L_3(x_n + \bar{t}h)$$

$$\tilde{l}_3(\bar{t}) = y_3 + \Delta f(x_2)\,\bar{t} + \Delta^2 f(x_1)\,\frac{\bar{t}(\bar{t}+1)}{2!} +$$

$$+ \Delta^3 f(x_0)\,\frac{\bar{t}(\bar{t}+1)(\bar{t}+2)}{3!} =$$

$$= 18 + \Delta f(3)\,\bar{t} + \Delta^2 f(1)\,\frac{\bar{t}(\bar{t}+1)}{2!} +$$

$$+ \Delta^3 f(-1)\,\frac{\bar{t}(\bar{t}+1)(\bar{t}+2)}{6}$$

formula Newton regresivă

$$\Delta f(3) = y_3 - y_2 = 18 - 6 = 12$$

$$\Delta^2 f(1) = C_2^0 (-1)^{2-0} f(1) + C_2^1 (-1)^{2-1} f(3) + C_2^2 (-1)^{2-2} f(5)$$

$$= 2 - 2\cdot 6 + 18 = 12$$

$$\Delta^3 f(-1) = 0$$

6	$\Delta f(-1) = -4$	$\Delta^2 f(-1) = 8$	$\Delta^3 f(-1) = 0$
2	$\Delta f(1) = 4$	$\Delta^2 f(1) = 8$	
6	$\Delta f(3) = 12$		
18			

$$\bar{x} = 4 = 5 + 2\cdot\bar{t} \Rightarrow \bar{t} = -\frac{1}{2}$$

$$f(4) \simeq \tilde{l}_3\left(-\frac{1}{2}\right) = 18 + 12\cdot\left(-\frac{1}{2}\right) + 8\cdot\left(-\frac{1}{2}\right)\left(-\frac{1}{2}+1\right)\cdot\frac{1}{2} +$$

$$+ 0\cdot\left(-\frac{1}{2}\right)\left(-\frac{1}{2}+1\right)\cdot\left(-\frac{1}{2}+2\right)\cdot\frac{1}{6} = 11$$

Funcții spline liniare continue

x	-1	0	1	3
f	6	3	2	6

$\bar{x} = 2$, $f(\bar{x}) = ?$

$$f(\bar{x}) \simeq S(\bar{x}) = \begin{cases} P_0(\bar{x}) & \text{dacă } \bar{x} \in [x_0, x_1) \\ P_1(\bar{x}) & \text{dacă } \bar{x} \in [x_1, x_2) \\ P_2(\bar{x}) & \text{dacă } \bar{x} \in [x_2, x_3] \end{cases}$$

P_0, P_1, P_2 polinoame de gradul 1, S funcție continuă $\left(P_0(x_1) = P_1(x_1), P_1(x_2) = P_2(x_2) \right)$ a.î.

$$S(x_i) = y_i \quad i = 0, 1, 2, 3$$

$$P_i(x) = \frac{x - x_i}{x_{i+1} - x_i} y_{i+1} + \frac{x_{i+1} - x}{x_{i+1} - x} y_i \quad i = \overline{0, 2}$$

$$P_0(x) = \frac{x - (-1)}{0 - (-1)} \cdot 3 + \frac{0 - x}{0 - (-1)} \cdot 6 = 3(1 - x) \ , x \in [-1, 0)$$

$$P_1(x) = \frac{x - 0}{1 - 0} \cdot 2 + \frac{1 - x}{1 - 0} \cdot 3 = 3 - x \ , \ x \in [0, 1)$$

$$P_2(x) = \frac{x - 1}{3 - 1} \cdot 6 + \frac{3 - x}{3 - 1} \cdot 2 = 2x \ , \ x \in [1, 3]$$

$$f(2) \simeq S(2) = P_2(2) = 4$$