Logic(s) for computer science - Week 13
Normal forms for First Order Logic - Part II
Tutorialr

1. Show that the following formulae in CSNF are unsatisfiable using binary resolution:
   (a) $\forall x.\forall y.\forall z.\Big(\big(\neg P(x,z) \vee R(x,x,z)\big) \wedge \big(\neg R(e,x,e)\big) \wedge \big(P(e,y)\big)\Big)$;
   (b) $\forall x.\forall y.\Big(\big(\neg P(x,y) \vee Q(x) \vee Q(y)\big) \wedge \big(\neg Q(i(i(e)))\big) \wedge \big(P(i(x),i(x))\big)\Big)$.
2. Establish using the binary resolution that the following formulae are valid:
   (a) $\Big(\big(\forall x.\forall y.\forall z(P(x,y) \wedge P(y,z) \rightarrow P(x,z))\big) \wedge P(x,y) \wedge P(y,x)\Big) \rightarrow P(x,x)$;
   (b) $\big(\forall x.Q(x)\big) \rightarrow \big(\exists x.Q(x)\big)$;
   (c) $\big(\neg\forall x.Q(x)\big) \leftrightarrow \big(\exists x.\neg Q(x)\big)$;
   (d) $\big(\neg\exists x.Q(x)\big) \leftrightarrow \big(\forall x.\neg Q(x)\big)$;
   (e) $\big(\exists y.\forall x.P(x,y)\big) \rightarrow \big(\forall x.\exists y.P(x,y)\big)$;
   (f) $\big(\forall x.(P(x,x) \leftrightarrow Q(x))\big) \rightarrow \big(P(e,e) \rightarrow Q(e)\big)$.