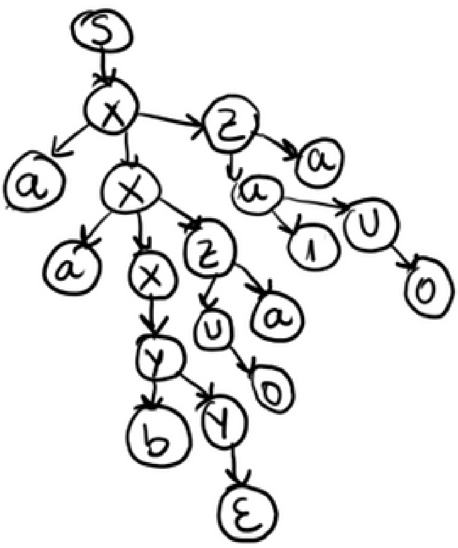
Rezolvare model de exercitii TS1

1. Fie L = {$a^n b^m u_1 a u_2 a....u_n a$, $u_i \in \{0,1\}+$, u_i contine cel putin un 0, $m \geq 0$, $n \geq 0$}.
   a) Construiti o gramatica de un tip cat mai mare pentru L (precizati tipul)
   b) Construiti o derivare extrem stanga pentru a^2b^m0a10a
   c) Construiti un arbore de derivare pentru $a^2b0a10a$

a)
$S \to X$
$X \to aXZ \mid Y$
$Y \to bY \mid \varepsilon$
$Z \to Ua$
$U \to 0U \mid 1U \mid 0$

b) $a^2b0a10a$
$S \to X \to aXZ \to aaXZZ \to aaYZZ \to aabYZZ \to aabZZ \to aabUaZ \to aab0aZ \to aab0aUa \to aab0a1Ua \to aab0a10a$

c)


2. Fie L = {$va^{n1}u_1a^{n2}u_2..............a^{nk}u_k$, $u_i \in \{0,1\}^*$, u_i se termina cu 11 sau 00, $n_i \geq 0$, pentru orice i, $k \geq 0$, $v \in \{b,c\}+$ }
   a) L se poate scrie ca: $L = L_1(L_2)^*$. Precizati cine sunt L1 si L2 si construiti gramaticile G1 si G2 care sa genereze cele doua limbaje
   b) Construiti o gramatica G care sa genereze L
   c) Construiti un arbore de derivare in gramatica G pentru $ba^200a011$
   d) Construiti o expresie regulata care sa descrie L

a) $L = L_1 (L_2)^*$

$L_1 = \{ v, \ v = \{b, c\}^+ \}$

$L_2 = \{ a^{n_1} u_1 a^{n_2} u_2 \dots a^{n_k} u_k, \ u_i \in \{0,1\}^*$, u_i se termină cu 00 sau 11,

$n_i \geq 0, \ \forall i, \ k \geq 0 \}$

$L(G_1)$: $S \to bX \mid cX$

$X \to bX \mid cX \mid \varepsilon$

$L(G_2)$: $S \to XS \mid X$

$X \to aX \mid Y$

$Y \to Z00 \mid Z11$

$Z \to 0Z \mid 1Z \mid \varepsilon$

b) $L(G)$: $S \to bXY \mid cXY$

$X \to bX \mid cX \mid \varepsilon$

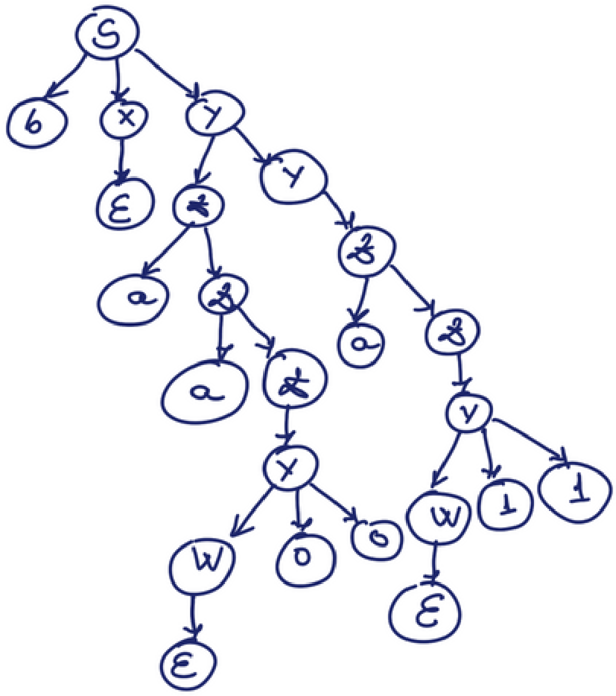
$Y \to ZY \mid Z \mid \varepsilon$

$Z \to aZ \mid V$

$V \to W00 \mid W11$

$W \to 0W \mid 1W \mid \varepsilon$

c) $ba^2 00a11$.



d) $(b|c)^+ \cdot ((a^*) \cdot (0|1)^* 00|11)^*$

3. Fie L = $\{a^n b^m u, u \in \{0,1\}^*, m \geq 0, n \geq 0\}$.

a) Construiti o gramatica G de tip 3 cu maxim 7 reguli pentru limbajul L
b) Construiti o gramatica G' in forma normala echivalenta cu G.
c) Construiti un automat nedeterminist echivalent cu gramatica G'.
d) Construiti o expresie regulata care sa descrie L

a) $S \to aS \mid X$

$X \to bX \mid Y$

$Y \to 0Y \mid 1Y \mid \varepsilon$

b) Mai întâi, vom elimina regulile de ștergere.

$N_0 = \{A \mid A \in N,\ A \to \epsilon \in P\};\ i = 0;$
do {
  $i = i + 1;$
  $N_i = N_{i-1} \cup \{X \mid X \in N,\ \exists X \to \alpha \in P, \alpha \in N_{i-1}^*\};$
} while $N_i \neq N_{i-1};$
$N_\epsilon = N_i;$

Plecăm de la N0. În N0 punem neterminalul care trece în E, adică Y. $N_0 = \{Y\}$; $i=0$;

do { $i=1$;
$N_1 = N_0 \cup \{X\} = \{X, Y\}$
} while $N_0 \neq N_1$ (A)

do { $i=2$;
$N_2 = N_1 \cup \{S\} = \{S, X, Y\}$
} while $N_1 \neq N_2$ (A)

do { $i=3$;
$N_3 = N_2 \cup \{S\} = \{S, X, Y\}$
} while $N_2 \neq N_3$ (F)

Rescriem regulile după eliminarea ștergerilor.

$S \to aS \mid X \mid a \mid \varepsilon$

$X \to bX \mid Y \mid b$

$Y \to 0Y \mid 1Y \mid 0 \mid 1$

Vom elimina redenumirile

```
for (A ∈ N) {
    N_0 = {A}; i = 0;
    do{
        i = i + 1;
        N_i = N_{i-1} ∪ {C | C ∈ N, ∃B → C ∈ P, B ∈ N_{i-1}};
    } while N_i ≠ N_{i-1};
    N_A = N_i; //N_A = {X ∈ N | A ⇒* X}
}
```

$N_0 = \{S\};\ i=0;$

do $\{\ i=1;$

$N_1 = N_0 \cup \{X\} = \{S, X\}\ \}$ while $N_1 \neq N_0$ (A)

do $\{\ i=2;$

$N_2 = N_1 \cup \{Y\} = \{S, X, Y\}\ \}$ while $N_2 \neq N_1$ (A)

do $\{\ i=3;$

$N_3 = N_2 \cup \emptyset = N_2\ \}$ while $N_3 \neq N_2$ (F)

$N_S = \{S, X, Y\}$

$N_0 = \{X\};\ i=0;$

do $\{\ i=1;$

$N_1 = N_0 \cup \{Y\} = \{X, Y\}\ \}$ while $N_0 \neq N_1$ (A)

do $\{\ i=2;$

$N_2 = N_1 \cup \emptyset = N_1\ \}$ while $N_1 \neq N_2$ (F)

$N_X = \{X, Y\}$

$N_0 = \{Y\}$; $i=0$;

do { $i = 1$;

$N_1 = N_0 \cup \emptyset = N_0$ } while $N_0 \neq N_1$ (F)

$N_Y = \{Y\}$

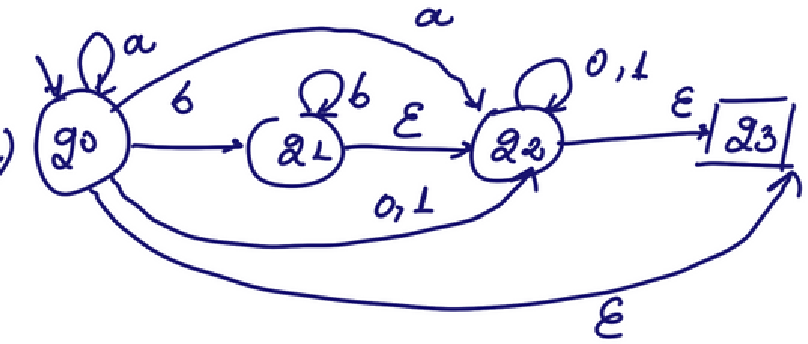
Vom rescrie regulile eliminând redenumirile.

G': $S \to aS \mid bX \mid b \mid 0Y \mid 1Y \mid 0 \mid 1 \mid \varepsilon$

$X \to bX \mid b \mid 0Y \mid 1Y \mid 0 \mid 1$

$Y \to 0Y \mid 1Y \mid 0 \mid 1$

c)



d) $(a^*) \cdot (b^*) \cdot (0|1)^*$

4. Fie urmatoarea gramatica:

S → 0S | 1A

A → 1A |0A | 11B

B → 1B | 2B | ε

a) Precizati limbajul generat de gramatica
b) Construiti o expresie regulata echivalenta
c) Aduceti gramatica la forma normala

b) $(0^*) \cdot 1 \cdot (0|1)^* \cdot 11 \cdot (1|2)^*$

a) $S \to 1\underbrace{A}_{11B} \to 111\underbrace{B}_{\varepsilon} \to 111$

$S \to 0\underbrace{S}_{0S} \to 00\underbrace{S}_{1A} \to 001\underbrace{A}_{0A} \to 0010\underbrace{A}_{1A} \to 00101\underbrace{A}_{11B} \to 0010111\underbrace{B}_{1B}$

$\to 00101111\underbrace{B}_{2B} \to 001011112\underbrace{B}_{\varepsilon} \to 001011112.$

$\mathcal{L} = \{ w\,11\,u \mid w = \{0,1\}^*,\ w$ conține cel puțin un 1, $|w| \geq 1$
$u = \{1,2\}^*,\ |u| \geq 0\}.$

c) Vom elimina regulile de ștergere.

$N_0 = \{A | A \in N,\ A \to \epsilon \in P\};\ i = 0;$
do {
    $i = i + 1;$
    $N_i = N_{i-1} \cup \{X | X \in N,\ \exists X \to \alpha \in P, \alpha \in N_{i-1}^*\};$
} while $N_i \neq N_{i-1};$
$N_\epsilon = N_i;$

$N_0 = \{B\};\ i = 0;$
do { $i = 1$
$N_1 = N_0 \cup \emptyset = \{B\}$
} while $N_1 \neq N_0$ (F)

$S \to 0S \mid 1A$
$A \to 1A \mid 11B \mid 11$
$B \to 1B \mid 2B \mid 1 \mid 2$

Apoi, vom elimina redenumirile:

for ($A \in N$) {
    $N_0 = \{A\};\ i = 0;$
    do{
        $i = i + 1;$
        $N_i = N_{i-1} \cup \{C | C \in N,\ \exists B \to C \in P, B \in N_{i-1}\};$
    } while $N_i \neq N_{i-1};$
    $N_A = N_i; // N_A = \{X \in N | A \Rightarrow^* X\}$
}

Nu se pot face redenumiri pt că nu avem reguli de forma $N \to N$,
N= neterminal.

5. Fie expresia regulata: (a*|b*)*(aa)*ba*.
   a) Simplificati expresia (construiti o expresie echivalenta cu numar minim de operatori, precizand cum ati simplificat fiecare subexpresie)
   b) Precizati limbajul descris de expresie
   c) Construiti o gramatica echivalenta cu expresia

a) $(a^*/b^*)^*(aa)^*ba^* = (a^*/b^*)(aa)^*ba^* =$

$(a/b)^*(aa)^*ba^*$

b) $L = \{ wa^n b a^m \mid w = \{a,b\}^*, |w| \geq 0;\ n \geq 0\ ;n = 2k;$
$m = \{0,1\};\ k \in N$

c) G: $S \to aS/bS/X$

$X \to aaX/bY$

$Y \to aY/\varepsilon$

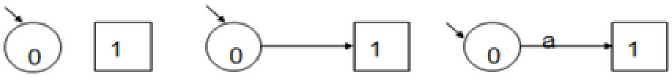
6. Fie expresia regulata: $00^*(0^*1|1)^*1^*$
   a) Simplificati expresia
   b) Precizati limbajul descris de expresie
   c) Construiti automatul cu epsilon-tranzitii echivalent cu expresia

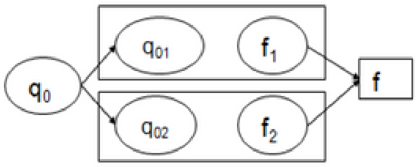
a) $00^*(0^*1/1)^*1 = 00^*(0^*1)^*1$

$r/r = r$

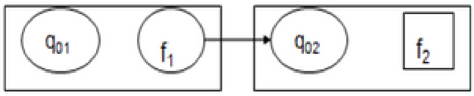
$r \cdot r^* = r^+$

b) $L = \{ 0^n w 1 \mid w = \{01\};\ |w| \geq 0 \}$.

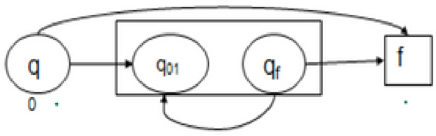




- $E = E_1 | E_2$



- $E = E_1 E_2$



- $E = E_1^*$

7. Fie gramatica:
   G = ({S,A,B,C},S,{a,b,c},S,P) cu P:
   S → AaA
   A → B
   B → bBc|C
   C → dC|ε
   A → aA|bA| a | b

   a) Sa se descrie limbajul generat de gramatica
   b) Construiti o derivare extrem dreapta si un arbore de derivare pentru un cuvant de lungime cel putin 6
   c) Sa se aduca gramatica la forma normala Chomsky

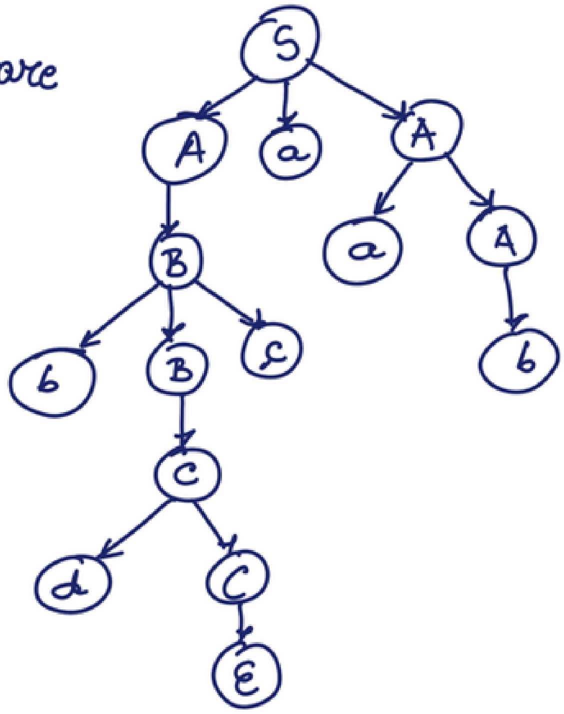
a) $\mathcal{L} = \{ w\ a\ v \mid w, v = \{a,b,c,d\}^*;$ w, v sunt de forma $b^n d^m c^n,\ n, m \ge 0$ SAU $a^{n_1} b^{m_1} a^{n_2} b^{m_2} \dots a^{n_k} b^{m_j},$ $n_i, m_i \ge 1;\ k, j \ge 1 \}$.

$\mathcal{L} = \{ wu\ a\ wu \mid u = b^n d^m c^n,\ n, m \ge 0,\ |u| \ge 0;$ $w \in \{a,b\}^+;\ |w| \ge 1 \}$.

b) $S \to Aa\underbrace{A}_{aA} \to Aaa\underbrace{A}_{b} \to \underbrace{A}_{B}aab \to \underbrace{B}_{bBc}aab \to b\underbrace{B}_{C}caab \to b\underbrace{C}_{dC}caab$

$\to bd\underbrace{C}_{\varepsilon}caab \to bdcaab$

Arborele de derivare



c) S → AaA
A → B
B → bBc|C
C → dC|ε
A → aA|bA| a | b

Vom elimina regulile de ștergere (cele cu ε)

$N_0 = \{C\}; i=0;$

do{ $i=1;$
$N_1 = \{C\} \cup \{B\} = \{B, C\}$ } while $N_1 \neq N_0$ (A)

do { $i=2;$
$N_2 = \{B, C\} \cup \{A\} = \{A, B, C\}$ } while $N_2 \neq N_1$ (A)

Rescriem regulile: S → AaA / a / Aa / aA
A → B
B → bBc / bc / C
C → d / dC
A → aA / bA / a/b

Vom elimina redenumirile

$N = \{S, A, B, C\}$

$N_0 = \{S\}$; $i=0$

do { $i=1$; $N_1 = \{S\} \cup \emptyset = \{S\}$ } while $N_0 \neq N_1$ (F)

$N_S = \{S\}$

$N_0 = \{A\}$; $i=0$;

do { $i=1$; $N_1 = \{A\} \cup \{B\} = \{A,B\}$ } while $N_0 \neq N_1$ (A)

do { $i=2$; $N_2 = \{A,B\} \cup \{C\} = \{A,B,C\}$ } while $N_1 \neq N_2$ (A)

do { $i=3$; $N_3 = \{A,B,C\} \cup \emptyset = \{A,B,C\}$ } while $N_2 \neq N_3$ (F)

$N_A = \{A,B,C\}$.

$N_0 = \{B\}$; $i=0$;

do { $i=1$; $N_1 = \{B\} \cup \{C\} = \{B,C\}$ } while $N_0 \neq N_1$ (A)

do { $i=2$; $N_2 = \{B,C\} \cup \emptyset = \{B,C\}$ } while $N_2 \neq N_1$ (F)

$N_B = \{B,C\}$.

$N_0 = \{C\}$.

do { $i=1$; $N_1 = \{C\} \cup \emptyset = \{C\}$ } while $N_0 \neq N_1$ (F)

$N_C = \{C\}$

Rescriem regulile:

$S \to AaA \,/\, a \,/\, Aa \,/\, aA$

$A \to bBc \,/\, bc \,/\, d \,/\, dC$

$B \to bBc \,/\, bc \,/\, d \,/\, dC$

$C \to d \,/\, dC$

$A \to aA \,/\, bA \,/\, a \,/\, b$

Aducem în FNC (Forma Normală Chomsky, având toate regulile de genul $\begin{cases} A \to BC \\ B \to d \end{cases}$

$S \to A_1 A \,/\, a \,/\, AA_0 \,/\, A_0 A$

$A \to B_1 C_0 \,/\, B_0 C_0 \,/\, d \,/\, D_0 C_0$

$B \to B_1 C_0 \,/\, B_0 C_0 \,/\, d \,/\, D_0 C_0$

$C \to d \,/\, D_0 C$

$A \to A_0 A \,/\, B_0 A \,/\, a \,/\, b$

$A_1 \to AA_0$

$A_0 \to a$

$B_0 \to b$

$C_0 \to c$

$D_0 \to d$

$B_1 \to B_0 B$

8. Fie urmatoarea gramatica:
   G = ({S,A,B,C},S,{a,b,c},S,P) cu P:
   S → AB
   A → AB | a
   B → BC | b
   C → CC | c
   a) Precizati limbajul generat de gramatica.
   b) Folosind algoritmul CYK, verificati daca abbc ∈ L(G).
   c) Construiti un arbore de derivare pentru un cuvant la alegere de lungime 4.

a) $S \to \underset{AB}{\breve{A}B} \to \breve{A}BB \to a\breve{B}\breve{B} \longrightarrow a\breve{B}\breve{C}b \to ab\breve{C}\breve{C}b \to abccb.$

$\quad\; AB \qquad a \qquad BC\; b \qquad b\; CC \qquad c\; c$

Vom avea un singur $\underline{\underline{a}}$ și măcar un $\underline{\underline{b}}$

$L = \{ab^{n_1}c^{m_1}b^{n_2}c^{m_2}\ldots b^{n_k}c^{m_l} \mid k \geq 1; l \geq 0;\ n_i, m_j \geq 1;\ 1 \leq i \leq k;\ 0 \leq j \leq l\}$

b) Alg lui CYK (Cocke Younger Kasami) → se ocupă cu recunoașterea unui cuvânt în gramatici în FNC (Forma Normală Chomsky)

Avem cuvântul abbc, vrem să verificăm dacă abbc ∈ L(G)

Facem tabelul

$abbc \in L(G)$?

V_{ij}	$j=1$	$j=2$	$j=3$	$j=4$
$i=1$	$V_a=\{A\}$	$V_{ab}=\{S,A\}$	$V_{abb}=\{S,A\}$	$V_{abbc}=\{S,A\}$
$i=2$	$V_b=\{B\}$	$V_{bb}=\emptyset$	$V_{bbc}=\emptyset$	—
$i=3$	$V_b=\{B\}$	$V_{bc}=\{B\}$	—	—
$i=4$	$V_c=\{C\}$	—	—	—

V_{ij} = subcuvântul de la poziția $\underline{i}$ de lungime j

$V_{11}=V_a$; $V_{21}=V_b$; $V_{31}=V_b$; $V_{41}=V_c$

$V_a \Rightarrow$ Ce variabilă generează $\underline{\underline{a}}$? Răspuns: $\underline{\underline{A}}$

$V_b \Rightarrow$ Ce variabilă generează $\underline{\underline{b}}$? Răspuns: $\underline{\underline{B}}$

$V_c \Rightarrow$ Ce variabilă îl generează pe $\underline{\underline{c}}$?. Răspuns: $\underline{\underline{C}}$

$V_{12}=V_{ab}$; $V_{22}=V_{bb}$; $V_{32}=V_{bc}$

$$V_{ab}=V_a \circ V_b = \underbrace{\{M \mid M \to AB, A\in V_a ; B \in V_b\}}_{\text{formal}}$$

Ne vom uita care sunt variabilele care îl generează pe A, variabilele care îl generează pe C, iar apoi la regulile care trec prin B·C.

În cazul nostru, $V_a=\{A\}$; $V_b=\{B\}$, deci vom vedea cine trece în AB (vom avea pe $\{S,A\}$)

$V_{bb}=V_b \circ V_b=\{M/M \to BB, B\in V_b\}$ $(\emptyset)$

$V_{bc}=V_b \circ V_c=\{M/M \to BC, B\in V_b, C\in V_c\}=\{B\}$

$V_{13}=V_{abb}$; $V_{23}=V_{bbc}$

Avem următoarea formulă: $V_{ij}=\bigcup_{k=1}^{j-1} V_{ik} \circ V_{i+k\, j-k}$

- $\bigcup_{k=1}^{3-1} V_{ik} \circ V_{i+k\; j-k} = (\underbrace{V_{11}}_{V_a} \circ \underbrace{V_{22}}_{V_{bb}}) \cup (\underbrace{V_{12}}_{V_{ab}} \circ \underbrace{V_{31}}_{V_b}) =$

$=(V_a \circ V_{bc}) \cup (V_{ab} \cup V_c) = (\{A\} \circ \emptyset) \cup (\{S,A\} \circ \{B\}) =$

$= \{A\} \cup \{SB, AB\} = \{A, SB, AB\} \Rightarrow V_{13} = \{S,A\}$

- $\bigcup_{k=1}^{3-1} V_{ik} \circ V_{i+k\; j-k} = (V_b \circ V_{bc}) \cup (V_{bb} \circ V_c) =$

$= (\{B\} \circ \{B\}) \cup (\emptyset \circ \{C\}) = \{BB\} \cup \{C\} = \{BB, C\}$ $V_{23} = \emptyset$

- $\bigcup_{k=1}^{4-1} V_{ik} \circ V_{i+k\; j-k} = (V_{11} \circ V_{23}) \cup (V_{12} \circ V_{32}) \cup$
$\cup (V_{13} \circ V_{41}) =$

$= (\{A\} \circ \emptyset) \cup (\{S,A\} \circ \{B\}) \cup (\{S,A\} \circ \{C\}) = \{A, SB, AB, SC, AC\}$

$V_{14} = V_{abbc} = \{S, A\}$

if $(S \in V_{14})(A) \Rightarrow abbc \in L(G)$

c) Alegem cuvântul abbc (de la subpunctul b)