Schema Aitken Neville

x	0	1	3	4
f	1	-2	10	49

$f(2) = ?$

Formula Neville:

$$l_k(x; x_0, x_1, \dots, x_k) = \frac{(x - x_0)\, l_{k-1}(x; x_1, \dots, x_k) - (x - x_k)\, l_{k-1}(x; x_0, \dots, x_{k-1})}{x_k - x}$$

Schema Aitken Neville

$$\begin{array}{llllll}
0 & 1 & & & \\
1 & -2 & l_1(2;0,1) = -5 & & \\
3 & 10 & l_1(2;1,3) = 4 & l_2(2;0,1,3) = 1 & \\
4 & 49 & l_1(2;3,4) = -29 & l_2(2;1,3,4) = -7 & l_3(2;0,1,3,4) = -3
\end{array}$$

$$l_1(2;0,1) = \frac{(2-0)\cdot(-2) - (2-1)\cdot 1}{1-0} = -5$$

$$l_1(2;1,3) = \frac{(2-1)\cdot 10 - (2-3)\cdot(-2)}{3-1} = 4$$

$$l_1(2;3,4) = \frac{(2-3)\cdot 49 - (2-4)\cdot 10}{4-3} = -29$$

$$l_2(2;0,1,3) = \frac{(2-0)\cdot l_1(2;1,3) - (2-3)\, l_1(2;0,1)}{3-0} =$$

$$= \frac{2\cdot 4 - (-1)(-5)}{3} = 1$$

$$l_2(2;1,3,4) = \frac{(2-1)\, l_1(2;3,4) - (2-4)\, l_1(2;1,3)}{4-1}$$

$$= \frac{1\cdot(-29) - (-2)\cdot 4}{3} = -7$$

$$l_3(2;0,1,3,4) = \frac{(2-0)\, l_2(2;1,3,4) - (2-4)\, l_2(2;0,1,3)}{4-0}$$

$$= \frac{2\cdot(-7) - (-2)\cdot 1}{4} = -3$$

Forma Newton a polinomului Lagrange

0 1

1 −2 $[0,1] = \frac{-2-1}{1-0} = -3$

3 10 $[1,3] = \frac{10-(-2)}{3-1} = 6$ $[0,1,3] = \frac{[1,3]-[0,1]}{3-0} = 3$

4 49 $[3,4] = \frac{49-10}{4-3} = 39$ $[1,3,4] = \frac{[3,4]-[1,3]}{4-1} = 11$ $[0,1,3,4]$

$$[0,1,3,4] = \frac{[1,3,4]-[0,1,3]}{4-0} = 2$$

$$l_3(x) = 1 - 3x + 3x(x-1) + 2x(x-1)(x-3)$$

$$l_3(2) = -3$$

Formule Newton pe noduri echidistante

x	0	2	4	6
f	1	-3	49	253

$n = 3 \quad x_0 = 0,\ x_1 = 2,\ x_2 = 4,\ x_3 = 6$

$h = 2 \quad y_0 = 1,\ y_1 = -3,\ y_2 = 49,\ y_3 = 253$

Calculul diferențelor finite cu o schemă de tip Aitken.

1			
-3	$\Delta f(0) = -4$		
49	$\Delta f(2) = 52$	$\Delta^2 f(0) = 56$	
253	$\Delta f(4) = 204$	$\Delta^2 f(2) = 152$	$\Delta^3 f(0) = 96$

$$\Delta f(0) = y_1 - y_0 = -3 - 1 = -4$$

$$\Delta f(2) = y_2 - y_1 = 49 - (-3) = 52$$

$$\Delta f(4) = y_3 - y_2 = 253 - 49 = 204$$

$$\Delta^2 f(0) = \Delta f(2) - \Delta f(0) = 52 - (-4) = 56$$

$$\Delta^2 f(2) = \Delta f(4) - \Delta f(2) = 204 - 52 = 152$$

$$\Delta^3 f(0) = \Delta^2 f(2) - \Delta^2 f(0) = 152 - 56 = 96$$

Formula Newton progresivă: $f(1) = ?$

$$l_n(x_0 + th) = y_0 + t\Delta f(x_0) + \frac{t(t-1)}{2!}\Delta^2 f(x_0)$$
$$+ \dots + \frac{t(t-1)\cdots(t-n+1)}{n!}\Delta^n f(x_0)$$

$$x_0 + th = 0 + t \cdot 2 = 1 \Rightarrow t = \frac{1}{2}$$

$$l_3(0 + t \cdot 2) = 1 + t\Delta f(0) + \frac{t(t-1)}{2}\Delta^2 f(0) +$$
$$+ \frac{t(t-1)(t-2)}{6}\Delta^3 f(0)$$

$$f(1) \simeq l_3\left(0 + \frac{1}{2} \cdot 2\right)$$

$$l_3\left(0 + \frac{1}{2} \cdot 2\right) = 1 + \frac{1}{2}(-4) + \frac{1}{2} \cdot \frac{1}{2}\left(\frac{1}{2} - 1\right) \cdot 56$$
$$+ \frac{1}{6} \cdot \frac{1}{2}\left(\frac{1}{2} - 1\right)\left(\frac{1}{2} - 2\right) \cdot 96 = -2$$

Formula Newton regresivă: $f(5) = ?$

$$l_n(x_n + t\cdot h) = y_n + t\,\Delta f(x_{n-1}) + \frac{1}{2}t(t+1)\Delta^2 f(x_{n-2})$$
$$+ \cdots + \frac{1}{n!}t(t+1)\cdots(t+n-1)\Delta^n f(x_0)$$

$$l_3(6+2t) = 253 + t\,\Delta f(4) + \frac{1}{2}t(t+1)\Delta^2 f(2)$$
$$+ \frac{1}{6}t(t+1)(t+2)\Delta^3 f(0)$$

$$= 253 + 204t + 152\cdot\frac{1}{2}t(t+1) +$$
$$+ 96\cdot\frac{1}{6}t(t+1)(t+2)$$

$$6+2t = 5 \Rightarrow t = -\frac{1}{2}$$

$$l_3\left(6+2\cdot\left(-\frac{1}{2}\right)\right) = 253 + 204\left(-\frac{1}{2}\right) +$$
$$+ 152\cdot\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}+1\right) + 96\cdot\frac{1}{6}\left(-\frac{1}{2}\right)\left(-\frac{1}{2}+1\right)\left(-\frac{1}{2}+2\right)$$
$$= 126$$

$$f(5) \simeq l_3\left(6+2\cdot\left(-\frac{1}{2}\right)\right) = 126$$

5

Funcții spline liniare continue

x	0	1	3	4
f	1	-2	10	49

$f(2) = ?$

$$S(x) = \begin{cases} P_0(x) & x \in [x_0, x_1] \\ P_1(x) & x \in [x_1, x_2] \\ \dots & \\ P_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

P_i - polinoame de gradul 1, $S \in C[x_0, x_n]$

$n = 3 \qquad x_0 = 0, x_1 = 1, x_2 = 3, x_3 = 4$

$y_0 = 1, y_1 = -2, y_2 = 10, y_3 = 49$

$$P_i(x) = \frac{x - x_i}{x_{i+1} - x_i} \cdot y_{i+1} + \frac{x_{i+1} - x}{x_{i+1} - x_i} y_i \qquad i = \overline{0, n-1}$$

(formula din curs)

$$= \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i) + y_i$$

(formula alternativă)

$$S(x)=\begin{cases} P_0(x) & x\in[0,1] \\ P_1(x) & x\in[1,3] \\ P_2(x) & x\in[3,4] \end{cases}$$

$$P_0(x)=\frac{x-0}{1-0}\cdot(-2)+\frac{1-x}{1-0}\cdot 1=-3x+1$$

(cu formula alternativă)

$$=\frac{-2-1}{1-0}(x-0)+1=-3x+1$$

$$P_1(x)=\frac{x-1}{3-1}\cdot 10+\frac{3-x}{3-1}\cdot(-2)=6x-8$$

(cu formula alternativă)

$$=\frac{10-(-2)}{3-1}\cdot(x-1)-2=6x-8$$

$$P_2(x)=\frac{x-3}{4-3}\cdot 49+\frac{4-x}{4-3}\cdot 10=39x-107$$

(cu formula alternativă)

$$=\frac{49-10}{4-3}(x-3)+10=39x-107$$

$$f(2)\simeq S(2)=P_1(2)=4$$