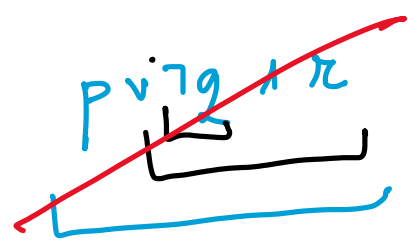


$\perp, \neg, \wedge, \vee, \rightarrow, \leftrightarrow$ Ex 126

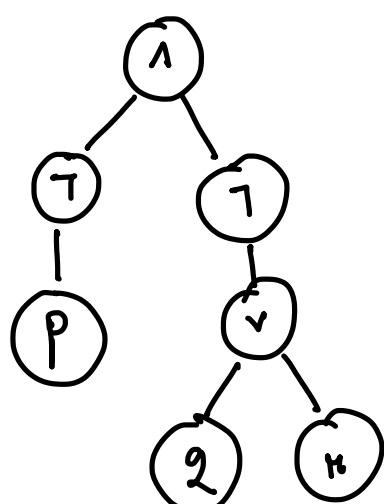
$$\textcircled{2} \quad (p \vee \neg q) \wedge r \qquad (2 + -3) \cdot 5$$



$$\textcircled{3} \quad \neg((p \vee \neg q) \wedge r)$$

Ex 127

$$\textcircled{4} \quad (\neg p \wedge \neg(q \vee r))$$



$$\textcircled{5} \quad (p \vee (\neg p \wedge \neg(q \vee r))) \qquad 2 + -2 \cdot -(\underbrace{5+6}_{11})$$

Ex 128

$$\textcircled{3} \quad \varphi = \neg((\overbrace{p \vee q}^{\varphi_1}) \wedge \overbrace{r}^{\varphi_2}) \equiv \overline{\neg(p \vee q)} \vee \neg r$$

$$\neg(\varphi_1 \wedge \varphi_2) \equiv \neg \varphi_1 \vee \neg \varphi_2$$

$$\equiv (\neg p \wedge \neg q) \vee \neg r$$

$$(\varphi_1 \wedge \varphi_2) \vee \varphi_3 \equiv (\varphi_1 \vee \varphi_3) \wedge (\varphi_2 \vee \varphi_3)$$

$$\equiv (\neg p \vee \neg r) \wedge (\neg q \vee \neg r) \quad \text{FNC.}$$

$$\begin{aligned} \varphi_1^c &= ((\neg p \vee \neg r) \wedge (\neg q \vee \neg r))^c = (\neg p \vee \neg r)^c \vee (\neg q \vee \neg r)^c = \\ &= (\neg p)^c \wedge (\neg r)^c \vee (\neg q)^c \wedge (\neg r)^c = \\ &= (p \wedge r) \vee (q \wedge r) \quad \text{FND} \end{aligned}$$

Ex 129

$$\varphi = (p \vee q) \rightarrow (p \wedge \neg r) \equiv \neg(p \vee q) \vee (p \wedge \neg r)$$

$$\varphi_1 \rightarrow \varphi_2 \equiv \neg \varphi_1 \vee \varphi_2$$

$$\equiv (\neg p \wedge \neg q) \vee (p \wedge \neg r)$$

$$\varphi_1 \vee (\varphi_2 \wedge \varphi_3) \equiv (\varphi_1 \vee \varphi_2) \wedge (\varphi_1 \vee \varphi_3)$$

$$\equiv ((\neg p \wedge \neg q) \vee p) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$(\varphi_1 \wedge \varphi_2) \vee \varphi_3 \equiv (\varphi_1 \vee \varphi_3) \wedge (\varphi_2 \vee \varphi_3)$$

$$\equiv ((\neg p \vee p) \wedge (\neg q \vee p)) \wedge ((\neg p \wedge \neg q) \vee \neg r)$$

$$\equiv (\neg p \vee p) \wedge (\neg q \vee p) \wedge (\neg p \vee \neg r) \wedge (\neg q \vee \neg r)$$

Ex 128

$$\begin{aligned} \text{II. } (p_1 \wedge q_1) \vee (p_2 \wedge q_2) &\equiv ((p_1 \wedge q_1) \vee p_2) \wedge ((p_1 \wedge q_1) \vee q_2) \\ &\equiv (\underline{p_1 \vee p_2}) \wedge (\underline{q_1 \vee p_2}) \wedge (\underline{p_1 \vee q_2}) \wedge (\underline{q_1 \vee q_2}) \\ &\equiv (p_1 \wedge q_1) \vee (p_2 \wedge q_2) \vee (p_3 \wedge q_3) \end{aligned}$$

Ex 131

$$\varphi \xrightarrow{\text{FNC}} \varphi^c \equiv \neg \varphi \quad \text{X}$$

$$\boxed{\begin{aligned} \neg \varphi \xrightarrow{\text{FNC}} (\neg \varphi)^c &\xrightarrow{\text{FND}} \\ (\neg \varphi)^c &\equiv \varphi \end{aligned}}$$

$$(p \vee \neg p) \wedge \varphi \equiv \varphi$$

$\perp = \tau(p \vee \neg p)$ for some $\tau: A \rightarrow B$.

$$9) \quad \varphi = \underbrace{p}_{\varphi_1} \leftrightarrow \underbrace{(q \rightarrow (\neg p \wedge \neg q))}_{\varphi_2}$$

$$\equiv (\underbrace{p}_{\varphi_1} \rightarrow \underbrace{(q \rightarrow (\neg p \wedge \neg q))}_{\varphi_2}) \wedge (\underbrace{(q \rightarrow (\neg p \wedge \neg q))}_{\varphi_2} \rightarrow \underbrace{p}_{\varphi_1})$$

$$\equiv (\neg p \vee (\neg q \vee (\neg p \wedge \neg q))) \wedge (\neg(q \rightarrow (\neg p \wedge \neg q)) \vee p)$$

$$\equiv (\underbrace{\neg p \vee \neg q}_{\varphi_1} \vee \underbrace{(\neg p \wedge \neg q)}_{\varphi_2 \wedge \varphi_3}) \wedge (\underbrace{(q \wedge \neg(\neg p \wedge \neg q))}_{\varphi_2} \vee p)$$

$$\equiv (\neg p \vee \neg q \vee \neg p) \wedge (\neg p \vee \neg q \vee \neg q) \wedge (\underbrace{(q \wedge \underbrace{(p \vee q)}_{\varphi_2})}_{\varphi_1} \vee \underbrace{p}_{\varphi_3})$$

$$\equiv (\neg p \vee \neg q \vee \neg p) \wedge (\neg p \vee \neg q \vee \neg q) \wedge (q \vee p) \wedge (p \vee q \vee p) \quad \text{FNC}$$