STAMATE VALENTIN

$$A = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 2 & 0 & 1.5 \end{pmatrix} \end{array} \qquad b = \begin{pmatrix} 1 \\ 15 \\ 6.5 \end{pmatrix}$$

1. $M = B^{-1}C$

$A = B - C \Rightarrow C = B - A$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 2 & 0 & 1.5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -5 \\ 0 & 0 & -0.5 \end{pmatrix}$$

$$\cancel{B^{-1}B =} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2+2 & 0 & 1 \end{pmatrix} = I_3$$

$$B^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -5 \\ 0 & 0 & -0.5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -5 \\ 0 & 0 & -0.5 \end{pmatrix}$$

$$\alpha = B^{-1}b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 15 \\ 6.5 \end{pmatrix} = \begin{pmatrix} 1 \\ 15 \\ \cancel{6.5}\ 4.5 \end{pmatrix}$$

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• Studierea convergentei

$x^{(k)} \to x^*$, $k \to \infty$ $\forall x^{(0)} \Leftrightarrow \rho(M) < 1$

$\rho(M) = \max\{|\lambda|; \lambda$ - valoare proprie a matricii $M\}$.

Valorile proprii se pot găsi rezolvând ecuația: ~~det~~

$\det(\lambda I_3 - M) = 0$

$$\det(\lambda I_3 - M) = \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 5 \\ 0 & 0 & \lambda+0.5 \end{vmatrix} = \lambda^2(\lambda+0.5) = 0$$

$\lambda_{1,2} = 0$, $\lambda_2 = -0.5$ $\Rightarrow \rho(M) = \cancel{0.5 \Rightarrow con}$

$0.5 < 1 \Rightarrow x^{(k)} \to x^*$, $k \to \infty$

$$x^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$x^{(1)} = M x^0 + d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -5 \\ 0 & 0 & -0.5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 15 \\ 5.5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -5 \\ -0.5 \end{pmatrix} + \begin{pmatrix} 1 \\ 15 \\ 5.5 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ 5 \end{pmatrix}$$

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2. $x_0 = -2$ $\quad y_0 = -2$

$x_1 = 0$ $\quad y_1 = -4$

$x_2 = 2$ $\quad y_2 = 2$

$x_3 = 4$ $\quad y_3 = 16$

Polinomul de interpolare Lagrange $L_3(x)$, polinom de grad 3.

$L_3(x_i) = y_i \quad \forall i = \overline{0,3}$

Forma newton a polinomului de interpolare Lagrange: schema lui Aitken

$$L_3(x) = y_0 + [x_0, x_1](x - x_0) + [x_0, x_1, x_2](x - x_0)(x - x_1) +$$
$$+ [x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$
$$= -2 + [-2, 0](x+2) + [-2, 0, 2](x+2)x +$$
$$+ [-2, 0, 2, 4](x+2)x(x-2)\cancel{(x-4)})$$

$$[x_0, x_1, \dots x_k] \overset{def}{=} \sum_{i=0}^{k} \frac{y_i}{\prod\limits_{\substack{j=0 \\ j \neq i}} (x_i - x_j)}$$

$$\cancel{[-2,0] = \frac{-2}{-2} + 0 = 1}$$

$$\cancel{[-2,0,2] = \frac{-2}{(-2)\cdot(-4)}} + $$

$$[-2,0] = \frac{-2}{-2} + \frac{-4}{2} = 1 + (-2) = -1$$

$$[-2,0,2] = \frac{-2}{(-2)(-4)} + \frac{-4}{2\cdot(-2)} + \frac{2}{4\cdot 2} = -\frac{1}{4} + 1 + \frac{1}{4} = 1$$

$$[-2,0,2,4] = \frac{-2}{(-2)(-4)\cdot(-6)} + \frac{-4}{2\cdot(-2)(-4)} + \frac{2}{4\cdot 2\cdot(-2)} + \frac{16}{6\cdot 4\cdot 2} =$$

STAMATE VALENTIN

$f[-2,0,2,4] = \frac{1}{24} - \frac{1}{4} - \frac{1}{8} + \frac{1}{3} = \frac{1-6-3+8}{24} = 0$

$L_3(x) = \cancel{-2+} -2 -(x+2) + (x+2)x + 0$

$L_3(x) = -2 -(x+2) + (x+2)x$

$L_3(1) = -2 - 3 + 3 = -2$