Interpolare numerică

Fie tabelul

x	-1	0	1	3
f	6	3	2	6

$x_0 = -1 \quad y_0 = f(x_0) = 6$

$x_1 = 0 \quad y_1 = f(x_1) = 3$

$x_2 = 1 \quad y_2 = f(x_2) = 2$

$x_3 = 3 \quad y_3 = f(x_3) = 6$

Să se aproximeze $f(\bar{x})$, $\bar{x} = 2$

Polinomul de Interpolare Lagrange

$L_3(x)$ polinom de grad 3 cu proprietatea

$$L_3(x_i) = y_i \quad i = \overline{0,3}$$

$$f(\bar{x}) \simeq L_3(\bar{x})$$

$$L_3(x) = \sum_{i=0}^{3} y_i \prod_{\substack{j=0 \\ j\neq i}}^{3} \left(\frac{x - x_j}{x_i - x_j}\right)$$

$$L_3(x) = 6 \cdot \frac{(x-0)(x-1)(x-3)}{(-1-0)(-1-1)(-1-3)} + 3 \cdot \frac{(x+1)(x-1)(x-3)}{(0+1)(0-1)(0-3)} +$$

$$+ 2 \cdot \frac{(x+1) \cdot x\,(x-3)}{(1+1) \cdot 1 \cdot (1-3)} + 6\, \frac{(x+1) \cdot x\,(x-1)}{(3+1) \cdot 3\,(3-1)}$$

$$= -\frac{3}{4} x (x-1)(x-3) + (x+1)(x-1)(x-3)$$

$$- \frac{1}{2} (x+1) \cdot x (x-3) + \frac{1}{4} (x+1) x (x-1)$$

$$L_3(2) = -\frac{3}{4}\cdot 2(2-1)(2-3) + (2+1)(2-1)(2-3)$$
$$- \frac{1}{2}(2+1)\cdot 2(2-3) + \frac{1}{4}(2+1)\cdot 2(2-1) = 3$$

$$f(\bar{x}) \simeq 3$$

Forma Newton a polinomului de interpolare Lagrange; scheme de tip Aitken

$$L_3(x) = y_0 + [x_0, x_1](x-x_0) + [x_0, x_1, x_2](x-x_0)(x-x_1)$$
$$+ [x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2)$$
$$= 6 + [-1, 0](x+1) + [-1, 0, 1](x+1)\cdot x$$
$$+ [-1, 0, 1, 3](x+1)\cdot x(x-1)$$

$$[x_0, x_1, \ldots, x_k] \underset{\text{directă}}{\overset{\text{def}}{=}} \sum_{i=0}^{k} \frac{y_i}{\prod\limits_{\substack{j=0 \\ j\neq i}}^{k} (x_i - x_j)}$$

$$[x_0, x_1, \ldots, x_k] \underset{\text{recursivă}}{\overset{\text{def.}}{=}} \frac{[x_1, \ldots, x_k] - [x_0, \ldots, x_{k-1}]}{x_k - x_0}$$

$$[x_0] = y_0$$

$$[-1, 0, 1, 3] \underset{\text{directă}}{\overset{\text{def}}{=}} \frac{y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + \frac{y_1}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$
$$+ \frac{y_2}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + \frac{y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} =$$

$$[-1,0,1,3]=\frac{6}{(-1-0)(-1-1)(-1-3)}+\frac{3}{(0-(-1))(0-1)(0-3)}+$$

$$+\frac{2}{(1-(-1))(1-0)(1-3)}+\frac{6}{(3-(-1))(3-0)(3-1)}=0$$

Schema Aitken de calcul a diferențelor divizate

x	y	$[x_i,x_{i+1}]$	$[x_i,x_{i+1},x_{i+2}]$	
-1	6			
0	3	$[-1,0]=\frac{3-6}{0-(-1)}=-3$		
1	2	$[0,1]=\frac{2-3}{1-0}=-1$	$[-1,0,1]=\frac{[0,1]-[-1,0]}{1-(-1)}=\frac{-1-(-3)}{2}=1$	...
3	6	$[1,3]=\frac{6-2}{3-1}=2$	$[0,1,3]=\frac{[1,3]-[0,1]}{3-0}=\frac{2-(-1)}{3}=1$	

$[x_i,x_{i+1},x_{i+2},x_{i+3}]$:

$$[-1,0,1,3]=\frac{[0,1,3]-[-1,0,1]}{3-(-1)}=0$$

$$L_3(x)=6-3(x+1)+(x+1)\cdot x$$

$$L_3(2)=6-3(2+1)+(2+1)\cdot 2=3$$

Schema Aitken - Neville

L_{k-1}

$$L_k(x;x_0,\dots,x_k) = \frac{(x-x_0)\,L_{k-1}(x;x_1,\dots,x_k) - (x-x_k)\,l_{k-1}(x;x_0,\dots,x_{k-1})}{x_k-x_0}$$

$-1 \quad 6$

$0 \quad 3 \qquad L_{01}(2) = \dfrac{(2-(-1))\cdot 3 - (2-0)\cdot 6}{0-(-1)} = -3$

$1 \quad 2 \qquad L_{12}(2) = \dfrac{(2-0)\cdot 2 - (2-1)\cdot 3}{1-0} = 1 \qquad L_{012} = \dfrac{(2-(-1))\cdot L_{12} - (2-1)\cdot L_{01}}{1-(-1)} = 3$

$3 \quad 6 \qquad L_{23}(2) = \dfrac{(2-1)\cdot 6 - (2-3)\cdot 2}{3-1} = 4 \qquad L_{123} = \dfrac{(2-0)\cdot L_{23} - (2-3)L_{12}}{3-0} = 3$

$$L_{0123}(2) = L_3(2) = \frac{(2-(-1))\cdot L_{123} - (2-3)L_{012}}{3-(-1)} = \frac{3\cdot 3 - (-1)\cdot 3}{4} = 3$$