L1

$\varphi \in LP$

LP = cea mai mică mulțime a.î.

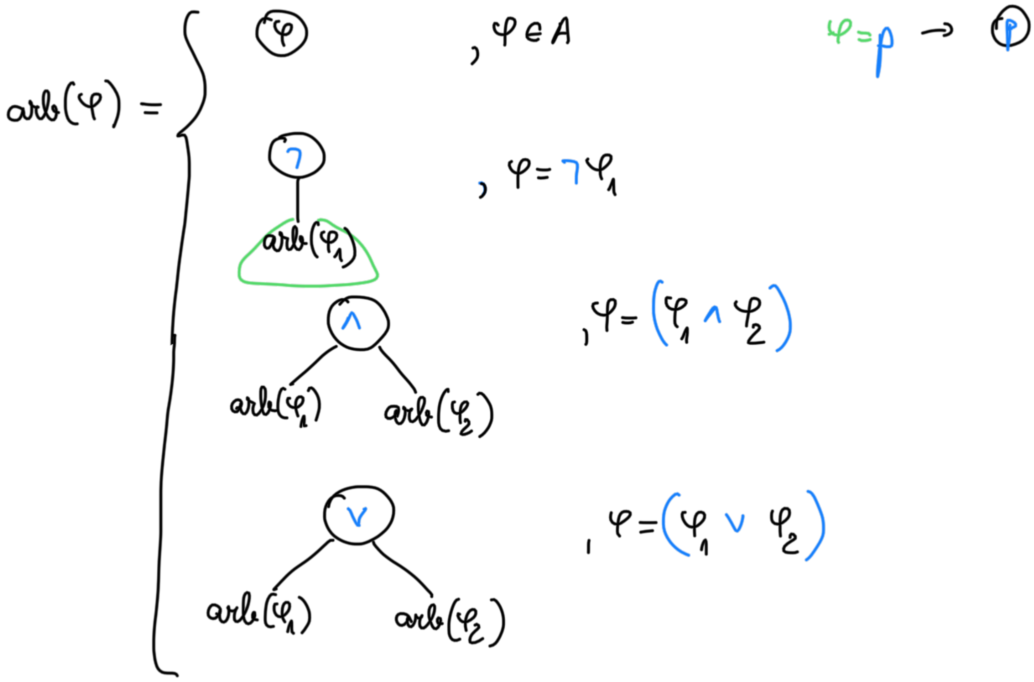
CB : $A \subseteq LP$ $\quad A = \{p, q, r, p_1 \ldots\}$ ← var prop

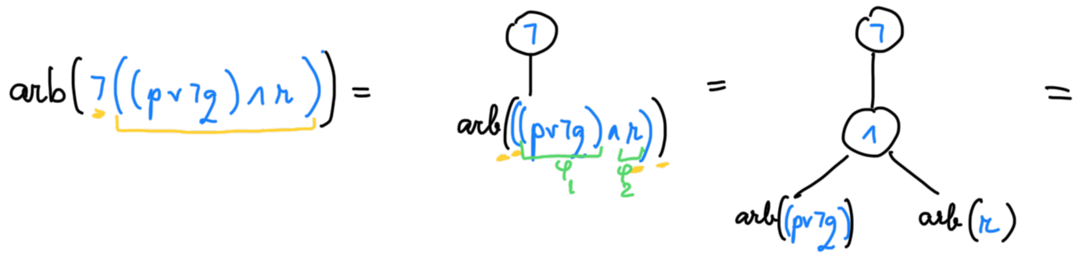
CI 1 : Dacă $\varphi_1 \in LP$, atunci $\neg\varphi_1 \in LP$

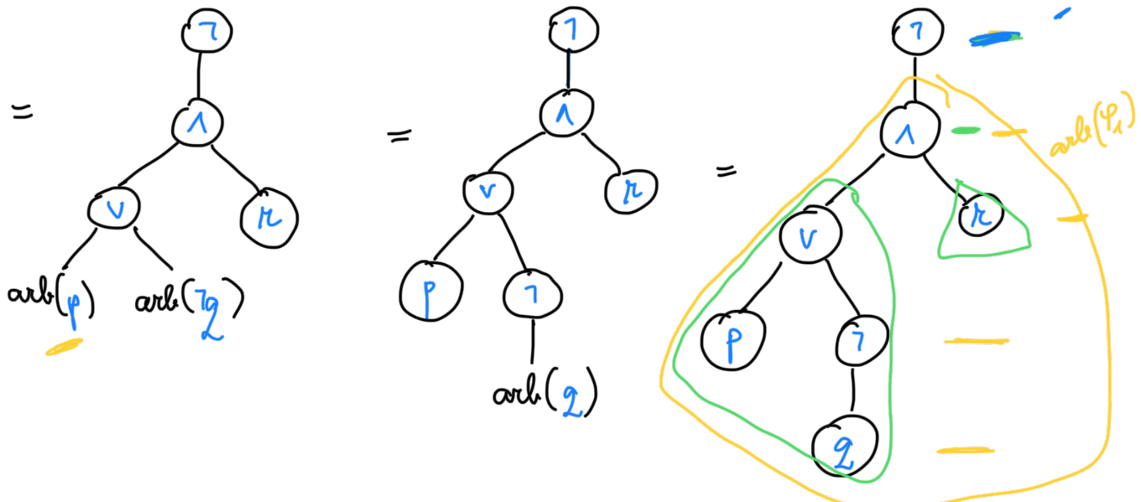
CI 2 : Dacă $\varphi_1, \varphi_2 \in LP$, atunci $(\varphi_1 \wedge \varphi_2) \in LP$

CI 3 : Dacă $\varphi_1, \varphi_2 \in LP$, atunci $(\varphi_1 \vee \varphi_2) \in LP$

arb : $LP \to$ Trees







$\text{height}(\varphi) = 5$

$\text{size}(\varphi) = 7$

$\text{height} : LP \to \mathbb{N}$

$$\text{height}(\varphi) = \begin{cases} 1 & , \varphi \in A \\ 1 + \text{height}(\varphi_1) & , \varphi = \neg \varphi_1 \\ 1 + \max(\text{height}(\varphi_1), \text{height}(\varphi_2)) & , \varphi = (\varphi_1 \wedge \varphi_2) \text{ sau } \varphi = (\varphi_1 \vee \varphi_2) \end{cases}$$

$\text{size} : LP \to \mathbb{N}$

$$\text{size}(\varphi) = \begin{cases} 1 & , \varphi \in A \\ 1 + \text{size}(\varphi_1) & , \varphi = \neg \varphi_1 \\ 1 + \text{size}(\varphi_1) + \text{size}(\varphi_2) & , \varphi = (\varphi_1 \wedge \varphi_2) \text{ sau } \varphi = (\varphi_1 \vee \varphi_2) \end{cases}$$

Ex2h: pt orice $\varphi \in LP$, avem că

$P(\varphi)$: $\text{height}(\varphi) < \text{size}(\varphi) + 1$.

pt orice $n \in \mathbb{N}$ avem

$P(n): n+1>0$

$\mathbb{N}$ = cea mai mică mulțime a.î.

CB : $0 \in \mathbb{N}$

CB : $n=0$

$P(0): 0+1>0$ "A"

CI : dacă $k \in \mathbb{N}$, atunci $\overbrace{succ(k)}^{k+1} \in \mathbb{N}$

CI : $n>0$, pp. $P(k)$ "A" și dem $P(k+1)$

ip. ind : $P(k): k+1>0$ "A"

$P(k+1): k+1+1>0$

$\mathbb{N} = \{ \overset{k=0}{0}, 1, \ldots \}$

$P(0)$ ✓ $P(1)$ ✓

$\mathbb{LP}$ - cea mai mică mulțime a.î.

CB : $A \subseteq \mathbb{LP}$ (pt orice $\varphi \in A$, $\varphi \in \mathbb{LP}$)

CB : $P(\varphi)$ "A".

CI1 : dacă $\varphi \in \mathbb{LP}$, atunci $\neg\varphi \in \mathbb{LP}$

CI1 : pp. $P(\varphi)$ "A" și dem $P(\neg\varphi)$

CI2 : dacă $\varphi_1, \varphi_2 \in \mathbb{LP}$, atunci $(\varphi_1 \wedge \varphi_2) \in \mathbb{LP}$

CI2 : pp. $P(\varphi_1), P(\varphi_2)$ "A" și dem $P((\varphi_1 \wedge \varphi_2))$

CI3 : similar.

$\mathbb{LP} = \{ \overset{\varphi}{p}, q, \overset{\neg\varphi}{\neg p}, \overset{\neg\varphi}{\neg\neg p}, (p \wedge q) \}$

$P(p)$ ✓ $P(q)$ ✓ $P(\neg p)$ ✓ $P(\neg\neg p)$

Dem prin ind structurală :

CB : $\varphi \in A$

$P(\varphi): 1 < 1+1 \iff 1<2$ "A".

CI1 : $\varphi = \neg\varphi_1$

(ip. inductiva) pp. $P(\varphi_1)$ "A" $\iff height(\varphi_1) < size(\varphi_1)+1$ (1) "A" ip.

și dem $P(\neg\varphi_1)$

Tb. dem $P(\neg\varphi_1)$: $height(\neg\varphi_1) < size(\neg\varphi_1)+1$

$\iff 1+height(\varphi_1) < 1+size(\varphi_1)+1 \quad / -1$

$\iff height(\varphi_1) < size(\varphi_1)+1$ "A" din ip. ind.

CI2 : $\varphi = (\varphi_1 \wedge \varphi_2)$

ip. ind. pp. $P(\varphi_1)$ "A" $\iff height(\varphi_1) < size(\varphi_1)+1$ (2) "A" din

$P(\varphi_2)$ "A" $\iff height(\varphi_2) < size(\varphi_2)+1$ (3) "A" ip. ind.

Dem. $P((\varphi_1 \wedge \varphi_2))$

Tb dem că $P((\varphi_1 \wedge \varphi_2))$: $height((\varphi_1 \wedge \varphi_2)) < size((\varphi_1 \wedge \varphi_2)) + 1$

$\Leftrightarrow 1 + \max(height(\varphi_1), height(\varphi_2)) < 1 + size(\varphi_1) + size(\varphi_2) + 1 \ /_{-1}$

$\Leftrightarrow \max(height(\varphi_1), height(\varphi_2)) < size(\varphi_1) + size(\varphi_2) + 1$

tb dem.

pt orice $x, y \in \mathbb{N}^*$

$\max(x, y) < x + y$

$height(\varphi_1), height(\varphi_2) \in \mathbb{N}^*$ (>0 din def.)

$\underbrace{\max(height(\varphi_1), height(\varphi_2))}_{a} < height(\varphi_1) + height(\varphi_2)$

$\Rightarrow \underbrace{\max(height(\varphi_1), height(\varphi_2))}_{a} \leq \underbrace{height(\varphi_1) + height(\varphi_2) - 1}_{b}$ (4) "A"

Din (2)+(3) $\Rightarrow height(\varphi_1) + height(\varphi_2) < size(\varphi_1) + size(\varphi_2) + 1 + 1 \ /_{-1}$

$\Rightarrow \underbrace{height(\varphi_1) + height(\varphi_2) - 1}_{b} < \underbrace{size(\varphi_1) + size(\varphi_2) + 1}_{c}$ (5) "A"

Din (4),(5) $\Rightarrow \max(height(\varphi_1), height(\varphi_2)) < size(\varphi_1) + size(\varphi_2) + 1$

$a \leq b < c \Rightarrow a < c$

$\Rightarrow P((\varphi_1 \wedge \varphi_2))$

CI 3 : $\varphi = (\varphi_1 \vee \varphi_2)$

Dem aproape identică cu CI2

Din CB, CI1, CI2, CI3 $\Rightarrow P(\varphi)$ "A" pt orice $\varphi \in LP$

Ex 58   5) $\varphi = (\underbrace{(p \vee \neg q)}_{\varphi_1} \wedge \underbrace{(\neg p \vee r)}_{\varphi_2})$ satisfiabilă ?

φ satisfiabilă ddacă există o atribuire $\tau : A \to B$ a.î. $\hat{\tau}(\varphi) = 1$

$\hat{\tau}(\varphi) = \hat{\tau}(\underbrace{(p \vee \neg q)}_{\varphi_1 \in \{0,1\}}) * \hat{\tau}(\underbrace{(\neg p \vee r)}_{\varphi_2 \in \{0,1\}})$

$* : B \times B \to B$

$1 * 1 = 1$

$1 * 0 = 0$

$0 * 1 = 0$

$0 * 0 = 0$

$$= \left(\hat{\tau}(p) + \hat{\tau}(\neg q)\right) * \left(\hat{\tau}(\neg p) + \hat{\tau}(r)\right)$$
$$= \left(\tau(p) + \overline{\tau(q)}\right) * \left(\overline{\tau(p)} + \tau(r)\right) \overset{?}{=} 1$$

Fie $\tau : A \to B$, $\tau(a) = 0$ pt orice $a \in A$

Verificare : $\left(0 + \overline{0}\right) * \left(\overline{0} + 0\right) = (0+1) * (1+0) =$
$= 1 * 1 = 1$

$\Rightarrow$ ~~am gasit~~ există o atribuire $\tau : A \to B$ a.î. $\hat{\tau}(\varphi) = 1$

$\Rightarrow$ φ este satisfiabilă

Ex 59 5) $\varphi = \underbrace{((p \wedge q)}_{\varphi_1} \vee \underbrace{(\neg p \wedge r))}_{\varphi_2}$ validă ?

φ validă ddacă pt orice $\tau : A \to B$, avem $\hat{\tau}(\varphi) = 1$.

$$\hat{\tau}(\varphi) = \hat{\tau}((p \wedge q)) + \hat{\tau}((\neg p \wedge r)) =$$
$$= \left(\hat{\tau}(p) * \hat{\tau}(q)\right) + \left(\hat{\tau}(\neg p) * \hat{\tau}(r)\right)$$
$$= \left(\tau(p) * \tau(q)\right) + \left(\overline{\tau(p)} * \tau(r)\right) = 1 \text{ pt orice } \tau \text{ ?}$$

Fie $\tau : A \to B$, $\tau(q) = 0$
$\tau(r) = 0$
$\tau(a) = 0$, pt orice $a \in A \setminus \{q, r\}$

Verificare : $\hat{\tau}(\varphi) = (0 * 0) + (\overline{0} * 0)$
$= 0 + (1 * 0) = 0 + 0 = 0$

$\Rightarrow$ am găsit o atribuire $\tau : A \to B$ a.î. $\hat{\tau}(\varphi) = 0$

$\Rightarrow$ nu pt orice $\tau : A \to B$ avem $\hat{\tau}(\varphi) = 1$

$\Rightarrow$ φ nu este validă

b) $\varphi = ((p \vee q) \vee \neg p)$ validă ?

φ validă ddacă pt orice $\tau : A \to B$, $\hat{\tau}(\varphi) = 1$

$\hat{\tau}(\varphi) = \hat{\tau}((p \vee q)) + \hat{\tau}(\neg p) = (\tau(p) + \tau(q)) + \overline{\tau(p)}$

Caz 1 : Fie $\tau : A \to B$ arbitrar a.î. $\tau(p) = 0$ (toate atribuirile care au $\tau(p)=0$)

$\hat{\tau}(\varphi) = (0 + \tau(q)) + \overline{0} =$

$= \tau(q) + 1 = 1$

Caz 2 : Fie $\tau : A \to B$ abitrar a.î. $\tau(p) = 1$

$\hat{\tau}(\varphi) = (1 + \tau(q)) + \overline{1} =$

$= 1 + 0 = 1$

Din Caz 1, Caz 2 $\Rightarrow$ pt orice $\tau : A \to B$ avem $\hat{\tau}(\varphi) = 1 \Rightarrow$

$\Rightarrow \varphi$ este validă.

Ex 58 : $\varphi = (p \wedge \neg p)$ satisfiabilă ?

φ satisfiabilă ddacă există $\tau : A \to B$ a.î. $\hat{\tau}(\varphi) = 1$

$\hat{\tau}(\varphi) = \tau(p) * \overline{\tau(p)}$

φ nu este satisfiabilă ddacă pt orice $\tau : A \to B$, $\hat{\tau}(\varphi) = 0$

nu există $\tau : A \to B$ a.î. $\hat{\tau}(\varphi) = 1$.

Caz 1 : Fie $\tau : A \to B$ arbitrar a.î. $\tau(p) = 0$

$\hat{\tau}(\varphi) = 0 * \overline{0} = 0 * 1 = 0$

Caz 2 : Fie $\tau : A \to B$ arbitrar a.î. $\tau(p) = 1$

$\hat{\tau}(\varphi) = 1 * \overline{1} = 1 * 0 = 0$

Din Caz 1, Caz 2 $\Rightarrow$ pt orice $\tau : A \to B$ avem $\hat{\tau}(\varphi) = 0 \Rightarrow$

$\Rightarrow$ nu există $\tau : A \to B$ a.î. $\hat{\tau}(\varphi) = 1 \Rightarrow$

$\Rightarrow \varphi$ nu este satisfiabilă.