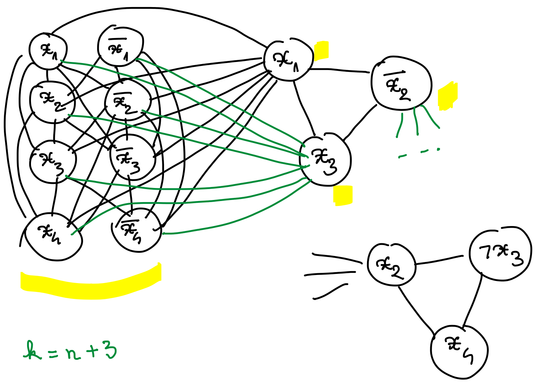
Reduceri

Thursday, 28 May 2020 14:50

(1) 3DNF $\propto$ CLIQUE

$\varphi = (x_1 \wedge \neg x_2 \wedge x_3) \vee (x_2 \wedge \neg x_3 \wedge x_4)$



$k = n + 3$

(2) Independent set $\propto$ Vertex cover

$(G, k) \longrightarrow (G', k')$

Vertex cover

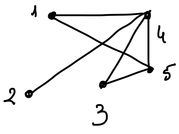
INPUT: $G = (V, E)$, $k \in \mathbb{N}$

OUTPUT: Da ddacă $\exists S \subseteq V$ a.î.

$|S| \leq k$ și

$\forall (i,j) \in E$, $i \in S$ sau $j \in S$

G, k



$\longrightarrow$

$G' = G$

$k' = n - k$

$k = 3$

$A = \{1,2,3\} \longrightarrow S = \{4,5\}$

$S = V - A$ – vertex cover

$\exists A \subseteq V$ ind. set cu $|A| \geq k \iff \exists S = V - A$ vertex cover în G

$|S| \leq n - k$

"$\Rightarrow$" Fie $A \subseteq V$ ind set $\Rightarrow \forall (i,j) \in E \Rightarrow$
$|A| \geq k$

$\Rightarrow i$ sau $j \notin A$ (altfel A nu este ind. set

$\Rightarrow i \in V \setminus A$ sau $j \in V - A \Rightarrow$

$\Rightarrow S = V \setminus A$ vertex cover.

$|A| \geq k \Rightarrow -|A| \leq -k \Rightarrow |S| \leq n - k$

$|S| = |V| - |A|$

"$\Leftarrow$" Fie S vertex cover $\Rightarrow \forall (i,j) \in E$ avem
$|S| \leq k'$ $i \in S$ sau $j \in S \Rightarrow$

$\Rightarrow \forall v, w \in V \setminus S \Rightarrow$

$\Rightarrow (v,w) \notin E \Rightarrow$

$\Rightarrow A = V \setminus S$ ind set.

$|A| = |V| - |S| \geq n - k'$

$|S| \leq k' \Rightarrow -|S| \geq -k'$