Logic for Computer Science. Week 3 - Exercise Sheet

1. Let $A = \{\mathtt{p}, \mathtt{q}, \mathtt{r}, \ldots\}$ be the set of propositional variables. Let $\tau : A \to B$ be the truth assignment defined as follows: $\tau(\mathtt{p}) = 1$, $\tau(\mathtt{q}) = 0$, $\tau(\mathtt{r}) = 0$, $\tau(a) = 0$ for any other propositional variable $a \in A \setminus \{\mathtt{p}, \mathtt{q}, \mathtt{r}\}$.

   Find the truth value of the following formulae in the assignment τ:

   (a) $(\mathtt{p} \wedge \mathtt{q})$; (b) $(\mathtt{q} \wedge \mathtt{p})$; (c) $\neg\mathtt{q}$; (d) $(\neg\mathtt{q} \wedge \mathtt{r})$; (e) $((\neg\mathtt{q} \wedge \mathtt{r}) \vee \neg p)$.

2. Find an assignment τ that is a model for the following formulae (one assignment for each formula):

   (a) $(\mathtt{p} \wedge \mathtt{q})$; (b) $(\mathtt{p} \wedge \neg\mathtt{q})$; (c) $((\mathtt{p} \wedge \neg\mathtt{q}) \vee \mathtt{q})$.

3. Find an assignment τ in which the following formulae are false (one assignment per formula):

   (a) $(\mathtt{p} \vee \mathtt{q})$; (b) $(\mathtt{q} \wedge (\mathtt{p} \vee \neg\mathtt{q}))$; (c) $((\mathtt{p} \wedge \neg\mathtt{q}) \vee \mathtt{q})$.

4. Which of the following formulae are satisfiable?

   (a) $(\mathtt{p}\wedge\neg\mathtt{p})$; (b) $(\mathtt{p}\vee\neg\mathtt{p})$; (c) $((\mathtt{p}\vee\neg\mathtt{p})\wedge\neg\mathtt{q})$; (d) $((\mathtt{p}\vee\neg\mathtt{p})\wedge(\neg\mathtt{p}\wedge\mathtt{q}))$; (e) $((\mathtt{p}\vee\neg\mathtt{q})\wedge(\neg\mathtt{p}\vee\mathtt{r}))$.

5. Which of the following formulae are valid?

   (a) $(\mathtt{p}\wedge\neg\mathtt{p})$; (b) $(\mathtt{p}\vee\neg\mathtt{p})$; (c) $\mathtt{p}$; (d) $((\mathtt{p}\vee\neg\mathtt{p})\wedge\neg\mathtt{q})$; (e) $(\mathtt{p} \to \neg\mathtt{p})$; (f) $((\mathtt{p} \wedge \mathtt{q}) \vee (\neg\mathtt{p} \wedge \mathtt{r}))$.

6. Associate to each of the following statements a formula in *PL* that models its meaning in English.

   (a) If it rains outside, I stay inside or go to the mall. I don't stay inside unless I'm bored. It rains and I'm not bored.

   (b) I study logic only if it is not possible to go outside. It is possible to go outside if it is not raining and if it is hot. As I am not studying logic and outside is hot, it means it is raining.

   (c) Thing go well in the country if the leaders of the country are not thiefs and if the economy is healthy. People go abroad if and only if things do not go well in the country. The economy is healthy, but people leave abroad.

7. Give 5 examples of contradictions.

8. Give 5 examples of tautologies.

9. Prove that, for any formulae $\varphi_1, \varphi_2, \varphi_3 \in PL$, the following equivalences hold:

   (a) $(\varphi_1 \wedge (\varphi_2 \wedge \varphi_3)) \equiv ((\varphi_1 \wedge \varphi_2) \wedge \varphi_3)$; (b) $(\varphi_1 \wedge \varphi_2) \equiv (\varphi_2 \wedge \varphi_1)$; (c) $(\varphi_1 \vee (\varphi_2 \vee \varphi_3)) \equiv ((\varphi_1 \vee \varphi_2) \vee \varphi_3)$; (d) $(\varphi_1 \vee \varphi_2) \equiv (\varphi_2 \vee \varphi_1)$; (e) $(\neg(\neg\varphi_1)) \equiv \varphi_1$; (f) $(\neg(\varphi_1 \wedge \varphi_2)) \equiv ((\neg\varphi_1) \vee (\neg\varphi_2))$; (g) $(\neg(\varphi_1 \vee \varphi_2)) \equiv ((\neg\varphi_1) \wedge (\neg\varphi$

10. Prove that, for any formulae $\varphi_1, \varphi_2 \in PL$, the following equivalence hold if and only if $\varphi_1 \in PL$ is a tautology:

    (a) $\varphi_1 \vee \varphi_2 \equiv \varphi_1$; (b) $\varphi_1 \wedge \varphi_2 \equiv \varphi_2$.

11. Prove that, for any formulae $\varphi_1, \varphi_2 \in PL$, the following equivalences hold if and only if $\varphi_1 \in PL$ is a contradiction:

    (a) $\varphi_1 \wedge \varphi_2 \equiv \varphi_1$; (b) $\varphi_1 \vee \varphi_2 \equiv \varphi_2$.

12. Show that $\neg\mathtt{p}$ is a tautological consequence of $(\mathtt{p} \to \neg\mathtt{p})$.

13. Show that $\mathtt{p}$ is not a semantical consequence of $(\mathtt{q} \to \mathtt{p})$.

14. Show that $\mathtt{p}$ is a semantical consequence of $(\mathtt{q} \to \mathtt{p})$ and $\mathtt{q}$.

15. Show that $\mathtt{p}_3$ is a logical consequence of $(\mathtt{p}_1 \to (\mathtt{p}_2 \vee \mathtt{p}_3))$, $(\neg\mathtt{p}_2 \leftrightarrow \neg\mathtt{p}_4)$ and $(\mathtt{p}_1 \wedge \neg\mathtt{p}_4)$.