Runser Laure

Maths 3 TD 04

12.10.2020

Partie 1

Exercice 3

1) $f(x) = \sin(x)\cos(x)$

or $\sin(x) = x - \frac{1}{6}x^3 + o_0(x^4)$

$\cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o_0(x^4)$

donc $f(x) = x - \frac{1}{6}x^3 + o(x^4)$

$+ \left(-\frac{1}{2}\right)x^3 + \cancel{\frac{1}{12}x^5} + o_0(x^4)$

$+ \cancel{\frac{1}{24}x^5} - \cancel{\frac{1}{6\cdot 24}x^7} + o_0(x^4)$

[pas la peine de mettre les termes de degré > 4]

$$\boxed{f(x) = x - \frac{2}{3}x^3 + o_0(x^4)}$$

2) $f(x) = \sin(x + x^3)$ en 0 à l'ordre 5

chgt de variable $t = x + x^3$

comme $t = x + x^3 \Rightarrow t \underset{0}{\sim} x \Rightarrow o_0(t^5) = o_0(x^5)$

$\sin(t) = t - \frac{1}{6}t^3 + \frac{1}{120}t^5 + o_0(t^5)$

donc $f(x) = (x + x^3) - \frac{1}{6}(x + x^3)^3 + \frac{1}{120}(x + x^3)^5 + o_0(x^5)$

on ne met pas les termes de degré > 5

$f(x) = (x + x^3) - \frac{1}{6}\left(x^3 + 3x^5 + o(x^5)\right) + \frac{1}{120}\left(x^5 + o(x^5)\right) + o_0(x^5)$

$$\boxed{f(x) = x + \frac{5}{6}x^3 - \frac{59}{120}x^5 + o_0(x^5)}$$

3) $f(x) = \dfrac{(1 + e^x)^n}{2}$ en 0 à l'ordre 2

$f(x) = \frac{1}{2}\Big(1 + \underbrace{\left(1 + x + \frac{1}{2}x^2 + o_0(x^2)\right)}_{\text{DL de } e^x}\Big)^n$

$f(x) = \frac{1}{2}\left(2 + x + \frac{1}{2}x^2 + o_0(x^2)\right)^n$

$f(x) = \frac{2^n}{2}\Big(1 + \underbrace{\frac{1}{2}x + \frac{1}{4}x^2 + o_0(x^2)}_{\to 0 \text{ qd } x \to 0}\Big)^n$

changement de variable $t = \frac{1}{2}x + \frac{1}{4}x^2 + o_0(x^2)$ appliquer le DL de $(1+t)^m$ en 0 à l'ordre 2

donc $t \underset{0}{\sim} \frac{1}{2}x$ donc $o_0(t^2) = o_0(x^2)$

$$f(t) = 2^{m-1}\left(1 + m.t - \frac{m(m-1)}{2}t^2 + o_0(t^2)\right)$$

on va jusqu'à l'ordre 2 donc on ne garde que $\left(\frac{1}{2}x\right)^2$ → le reste a un ordre trop gd

$$f(x) = 2^{m-1}\left(1 + m\left(\frac{1}{2}x + \frac{1}{4}x^2 + o_0(x^2)\right) + \frac{m(m-1)}{2}\left(\frac{1}{2}x + \frac{1}{4}x^2 + o_0(x^2)\right)^2 + o_0(x^2)\right)$$

$$\boxed{f(x) = 2^{m-1}\left(1 + \frac{m}{2}x + \frac{m(m+1)}{8}x^2 + o_0(x^2)\right)}$$

5) $f(x) = \frac{\tan(x)}{\cos(x)}$ en 0 à l'ordre 4.

a) Trouvons le DL de $\tan(x)$ en 0 à l'ordre 4

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o_0(x^5)\right) \times \frac{1}{\underbrace{1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + o_0(x^5)}_{\to 0 \text{ qd } x \to 0}}$$

changement de variable :

$$t = \frac{x^2}{2} - \frac{1}{24}x^4 + o_0(x^5) \Rightarrow t \underset{0}{\sim} \frac{x^2}{2} \Rightarrow t^3 = \frac{1}{8}x^6 = o_0(x^5)$$ [il suffit d'aller jusqu'à l'ordre 2]

$$\tan(x) = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o_0(x^5)\right)\left(1 + t + t^2 + o_0(x^5)\right)$$

on ne garde que le premier terme

$$\tan(x) = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o_0(x^5)\right)\left(1 + \frac{x^2}{2} - \frac{x^4}{24} + o_0(x^5) + \left(\frac{x^2}{2} - \frac{x^4}{24} + o_0(x^5)\right)^2 + o_0(x^5)\right)$$

$$\tan(x) = \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o_0(x_5)\right)\left(1 + \frac{x^2}{2} - \frac{x^4}{24} + \frac{x^4}{4} + o_0(x^5)\right)$$

$$\boxed{\tan(x) = x + \frac{x^3}{3} + \frac{2}{15}x^5 + o_0(x^5)}$$

b) idée : $f(x) = \frac{\tan(x)}{\cos(x)} = \frac{\sin(x)}{\cos^2(x)} = \sin(x) \times \frac{1}{\left(1 - \frac{x^2}{2} + \cdots\right)^2} = \sin(x)\left(1 + (x^2 + \cdots) + (x^2 + \cdots)^2 + \cdots\right)$