RUNSER Laure — Mathématiques : Algèbre TD 1 — 02.11.2020

Rappels sur les systèmes linéaires

correction en vert

Exercice 1

a) $\begin{cases} 8x_1 + 3x_2 = 0 \\ 2x_1 + 6x_2 = 0 \end{cases} \Rightarrow \begin{cases} 8x_1 + 3x_2 = 0 \\ -14x_1 = 0 \end{cases}$ $L_2 - 2L_1$ $\Rightarrow \begin{cases} x_2 = 0 \\ x_1 = 0 \end{cases}$ Donc $\boxed{\mathcal{S} = \{(0,0)\}}$

b) $\begin{cases} 8x_1 + 3x_2 = 3 \\ 2x_1 + 6x_2 = -1 \end{cases} \Rightarrow \begin{cases} 8x_1 + 3x_2 = 3 \\ -14x_1 = -7 \end{cases}$ $L_2 - 2L_1$ $\Rightarrow \begin{cases} 8x_1 + 3x_2 = 3 \\ x_1 = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} 3x_2 = 3 - 8\times\frac{1}{2} \\ x_1 = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} x_2 = -1/3 \\ x_1 = 1/2 \end{cases}$

Donc $\boxed{\mathcal{S} = \{(1/2, -1/3)\}}$

Exercice 2

a) $\left(\begin{array}{cccc|c} 2 & 1 & 2 & 3 & 0 \\ 1 & 1 & 3 & 0 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 3 & 0 & 0 \\ 0 & -1 & -4 & 3 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 0 \\ 0 & 1 & 4 & -3 & 0 \end{array}\right)$

$\boxed{\mathcal{S} = \{(x_3 - 3x_4, -4x_3 + 3x_4, x_3, x_4), x_3, x_4 \in \mathbb{R}\}}$

b) $\begin{cases} 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ x_1 + x_2 + 3x_3 = 1 \end{cases} \Rightarrow \begin{cases} 0 - x_2 - 4x_3 + 3x_4 = -1 \\ x_1 + x_2 + 3x_3 = 1 \end{cases}$ $L_1 - 2L_2$ $\Rightarrow \begin{cases} x_2 = -4x_3 + 3x_4 - 1 \\ x_1 = x_3 - 3x_4 + 1 \end{cases}$

Donc $\boxed{\mathcal{S} = \{(x_3 - 3x_4 + 1, -4x_3 + 3x_4 - 1, x_3, x_4), x_3, x_4 \in \mathbb{R}\}}$

c) $\begin{cases} 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ x_1 + x_2 + 3x_3 = -1 \end{cases} \Rightarrow \begin{cases} 0 - x_2 - 4x_3 + 3x_4 = 3 \\ x_1 + x_2 + 3x_3 = -1 \end{cases}$ $L_1 - 2L_2$ $\Rightarrow \begin{cases} x_2 = -4x_3 + 3x_4 + 3 \\ x_1 = x_3 - 3x_4 - 1 \end{cases}$

Donc $\boxed{\mathcal{S} = \{(x_3 - 3x_4 - 1, -4x_3 + 3x_4 + 3, x_3, x_4), x_3, x_4 \in \mathbb{R}\}}$

Exercice 3

a) $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 3 & -1 & 0 \\ -2 & & 2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 4 & 2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 2/3 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$

Donc $\boxed{\mathcal{S} = \{(0,0,0)\}}$

b) $\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 3 & -1 & -1 \\ -2 & 0 & 2 & 1 \end{array}\right) \to \begin{array}{c} L_1 \\ 1/3\, L_2 \\ L_3 + 2L_1 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & -1/3 & -1/3 \\ 0 & 4 & 2 & 3 \end{array}\right) \to \begin{array}{c} L_1 \\ L_2 \\ L_3 - 4L_2 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & -1/3 & -1/3 \\ 0 & 0 & 10/3 & 13/3 \end{array}\right)$

$\to \begin{array}{c} L_1 - 2L_2 \\ L_2 + 1/3\, L_3 \\ 3/10\, L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4/5 \\ 0 & 1 & 0 & 1/10 \\ 0 & 0 & 1 & 13/10 \end{array}\right)$ Donc $\boxed{\mathcal{S} = \left\{\left(\frac{4}{5}, \frac{1}{10}, \frac{13}{10}\right)\right\}}$

Exercice 4

a) $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & -1 & 0 \end{array}\right) \to \begin{array}{c} L_1 \\ -(L_2 - L_1) \\ -(L_3 - L_1) \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right) \to \begin{array}{c} L_1 \\ L_2 \\ -(L_3 - 2L_2) \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$

$\to \begin{array}{c} L_1 - L_2 \\ L_2 - L_3 \\ L_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$ Donc $\boxed{\mathcal{S} = \{(0,0,0)\}}$

Exercice 14

on ne met pas les nombres à droite parce que c'est que des 0

$$\begin{pmatrix} u & 1 & 1 & 1 \\ 1 & 1+u & 1 & 1 \\ 1 & 1 & 2+u & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} u-1 & 0 & 0 & 0 \\ 0 & u & 0 & 0 \\ 0 & 0 & 1+u & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Si $u = 1$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$x = -t$ $\quad$ $y = z = 0$ $\quad$ t libre

$$\boxed{\mathcal{S} = \{(t, 0, 0, t),\ t \in \mathbb{R}\}}$$

Si $u = 0$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\mathcal{S} = \{(0, t, 0, t),\ t \in \mathbb{R}\}}$$

Si $u = -1$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{\mathcal{S} = \{(0, 0, t, t),\ t \in \mathbb{R}\}}$$

Sinon on échelonne et on trouve une matrice diagonale échelonnée réduite

$$\boxed{\mathcal{S} = \{(0,0,0,0)\}}$$