Exercice 11 : Montrer que pour tout $n \in \mathbb{N}^*$ et tout $z \in \mathbb{C}$, on a :

$$(z-1)(1+z+z^2+\dots+z^{n-1}) = z^n - 1$$

$$\Leftrightarrow \cancel{z}+\cancel{z^2}+\cancel{z^3}+\dots+\cancel{z^{n-1}}+z^n-1-\cancel{z}-\cancel{z^2}-\cancel{z^3}-\dots-\cancel{z^{n-1}} = z^n-1$$

$$\Leftrightarrow z^n - 1 = z^n - 1$$

Exercice 12 :

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & -1 \\ -2 & -2 & -1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 2 & -1 \\ -2 & -2 & -1 \end{vmatrix} = 1 \times \begin{vmatrix} 2 & -1 \\ -2 & -1 \end{vmatrix} - 2 \times \begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 2 \\ -2 & -2 \end{vmatrix}$$

$$= 1 \times (2 \times (-1) - (-2) \times (-1)) - 2 \times (1 \times (-1) - (-2) \times (-1)) + 1 \times (1 \times (-2) - (-2) \times 2)$$

$$= -2 - 2 + 2 + 4 - 2 + 4 = 4$$

Donc, $\det(A) = 4$

$$(A \mid I_3) = \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ -2 & -2 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 & 1 & 0 \\ 0 & 2 & 1 & 2 & 0 & 1 \end{array}\right) \Leftrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -2 & -1 & 1 & 0 \end{array}\right)$$