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Exercice 9 :

1). $\begin{cases} U_0 = 6 \\ U_1 = 4 \\ \forall n \in \mathbb{N},\ U_{n+2} = \frac{5}{4} U_{n+1} - \frac{3}{8} U_n \end{cases}$

$U_{n+2} - \frac{5}{4} U_{n+1} + \frac{3}{8} U_n = 0 \quad \forall n \in \mathbb{N}$

$x^2 - \frac{5}{4} x + \frac{3}{8} = 0$

$\Delta = b^2 - 4ac = \left(-\frac{5}{4}\right)^2 - 4 \times 1 \times \frac{3}{8}$

$= \frac{25}{16} - \frac{12}{8} = \frac{1}{16}$

$x_1 = \frac{\frac{5}{4} - \sqrt{\frac{1}{16}}}{2} = \frac{\frac{5}{4} - \frac{1}{4}}{2} = \frac{1}{2}$

$x_2 = \frac{\frac{5}{4} + \sqrt{\frac{1}{16}}}{2} = \frac{3}{4}$

$U_n = \alpha \left(\frac{3}{4}\right)^n + n\beta \left(\frac{1}{2}\right)^n \quad (\alpha, \beta) \in \mathbb{R}^2$

$\begin{cases} U_0 = 6 = \alpha^0 + \beta^0 = 6 \\ U_1 = 4 = \alpha \left(\frac{3}{4}\right)^1 + \beta \left(\frac{1}{2}\right)^1 = 4 \end{cases}$

Donc : $\forall n \in \mathbb{N},\ U_n = 4 \times \left(\frac{3}{4}\right)^n + 2 \times \left(\frac{1}{2}\right)^n$