

Exercice 4:  $100^{1000} \equiv ? \pmod{13}$

$$\begin{array}{r} 100 \overline{) 13} \\ - 91 \phantom{00} \\ \hline 9 \phantom{00} \end{array}$$

$$100 \equiv 9 \pmod{13}$$

$$1000 \equiv 0 \pmod{100}$$

Donc :  $100^{1000} \equiv 9 \pmod{13}$

$$\Rightarrow 100^{1000} \bmod 13 = 9$$

Exercice 5:

$$\frac{5+2i}{1-2i} = \frac{(5+2i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{5 + 12i - 4}{5} = \frac{1}{5} + \frac{12i}{5}$$

$$\begin{aligned} \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^3 &= \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= \left(\frac{1}{4} - i\frac{\sqrt{3}}{4} - i\frac{\sqrt{3}}{4} - \frac{3}{4}\right)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= \left(-\frac{2}{4} - 2i\frac{\sqrt{3}}{4}\right)\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \\ &= \left(\frac{2}{8} - 2i\frac{\sqrt{3}}{8} + 2i\frac{\sqrt{3}}{8} + 2\frac{\sqrt{3}}{8}\right) \\ &= \left(\frac{2}{8} + \frac{2\sqrt{3}}{8}\right) = \frac{\sqrt{3}}{4} + \frac{1}{4} \end{aligned}$$

$$\frac{(1+i)^9}{(1-i)^7} = \frac{16+16i}{8+8i}$$

$$= \frac{(16+16i)(8-8i)}{(8+8i)(8-8i)} = \frac{128+128i}{64} = 2$$