

Exercice 1

$$\varphi = \{ \{P, Q, R\}, \{P, \neg Q, \neg R\}, \{P, \neg S\}, \{\neg Q, \neg R, \neg S\}, \{\neg P, \neg Q, R\}, \{T, U\}, \{T, \neg U\}, \{Q, \neg T\}, \{\neg R, \neg T\} \}$$

↓  $[0/S]$   $\neg S \in \text{Pure}(F)$

$$\{ \{P, Q, R\}, \{P, \neg Q, \neg R\}, \{\neg P, \neg Q, R\}, \{T, U\}, \{T, \neg U\}, \{Q, \neg T\}, \{\neg R, \neg T\} \}$$

split(T)  $[1/T]$

split( $\neg T$ )  $[0/T]$

$$\{ \{P, Q, R\}, \{P, \neg Q, \neg R\}, \{\neg P, \neg Q, R\}, \{Q\}, \{\neg R\} \}$$

$$\{ \{P, Q, R\}, \{P, \neg Q, \neg R\}, \{\neg P, \neg Q, R\}, \{U\}, \{\neg U\}, \{\neg R, \neg T\} \}$$

↓  $[1/Q]$   $Q \in \text{unit}(F)$

↓  $[1/U]$   $U \in \text{unit}(F)$

$$\{ \{P, \neg R\}, \{\neg P, R\}, \{\neg R\} \}$$

$$\{ \{P, Q, R\}, \{P, \neg Q, \neg R\}, \{\neg P, \neg Q, R\}, \{\}, \{\neg R, \neg T\} \}$$

↓  $[0/R]$   $\neg R \in \text{unit}(F)$

On a  $\{\}$  donc cette branche n'est pas SAT.

$$\{ \{\neg P\} \}$$

↓  $[0/P]$   $\neg P \in \text{unit}(F)$

$$\emptyset$$

$$\text{SAT} \rightarrow \text{modèle } [0/P][1/Q][0/R][0/S][1/T][1/U]$$

↳  $U = 0$  ou  $1$   
sans importance.

Exercice 2

a)  $[[1; -2; -3]; [-1; 2; -3]; [-1; -2; 3]]$  ✓

b) let rec is\_clause f:  
math f with:  
| Prop P | Neg(Prop P) → true  
| Or (F1, F2) → is\_clause (F1) && is\_clause (F2)  
| \_ → false ✓

e) let clauses\_of\_cnf f =  
math f with  
| And (F1, F2) → List.rev\_append clauses\_of\_cnf F1 clauses\_of\_cnf F2  
| F1 → if not is\_clause F1  
then failwith "not CNF"  
else [F1] ✓

c) let rec is\_cnf f:  
math f with:  
| Prop P | Neg(Prop P) → true  
| And (F1, F2) → is\_cnf F1 && is\_cnf F2  
| Or (F1, F2) → is\_clause (F1) && is\_clause F2  
| \_ → false ✓

f) let clauses\_of\_cnf f:  
if not is\_cnf f then None  
else Some List.map List\_of\_clause (clauses\_of\_cnf f) ✓

d) let List\_of\_clause f =  
math f with:  
| Prop P → [P]  
| Neg(P) → [-P]  
| Or (F1, F2) → List.rev\_append (List\_of\_clause F1) (List\_of\_clause F2)  
| \_ → failwith "not a clause" ✓

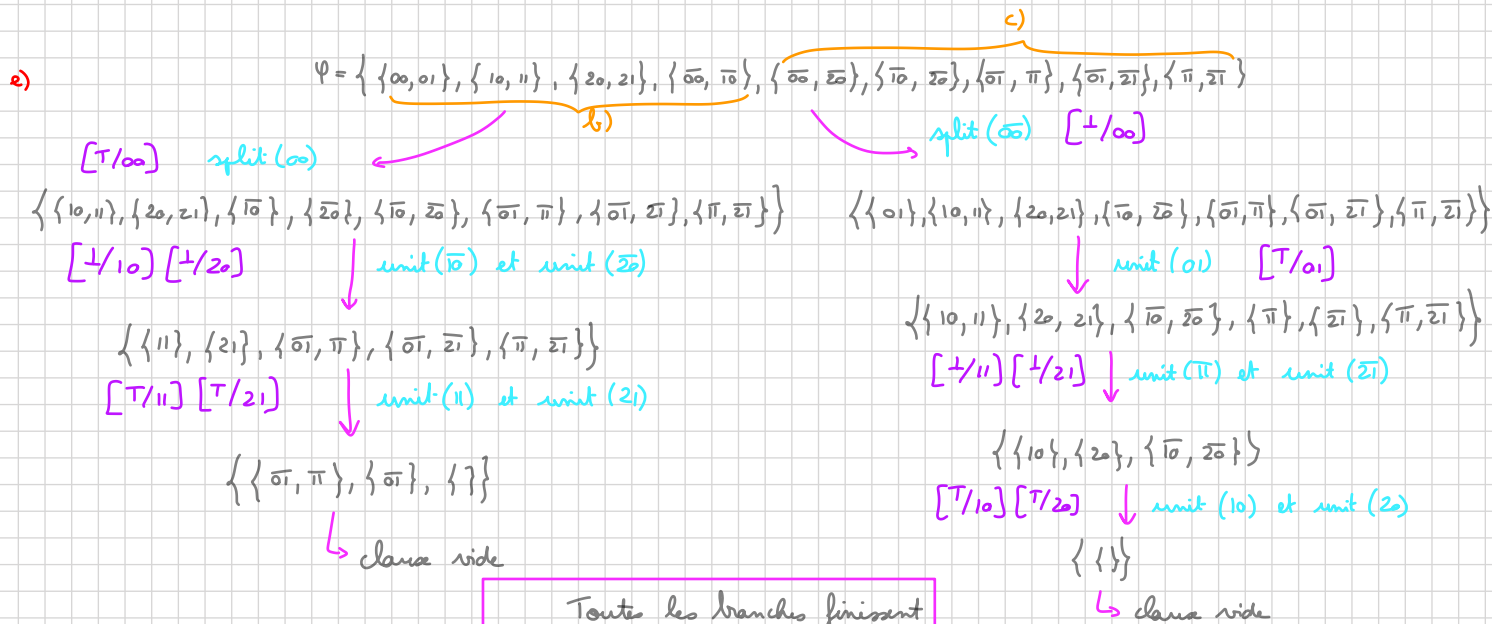
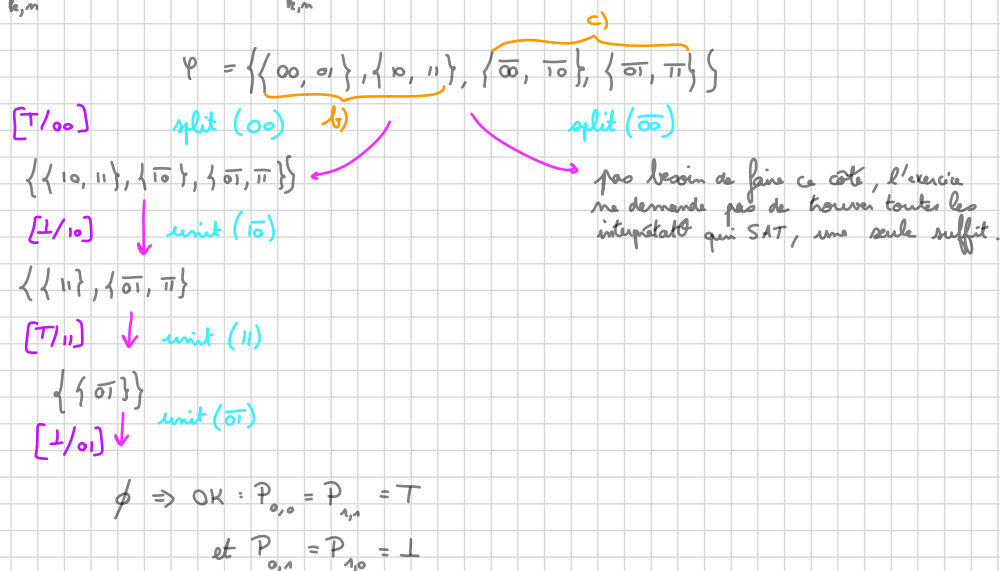
### Exercice 3

a)  $P_{k,m}$  est vraie si le  $k$ -ième pigeon est dans le  $m$ -ième trou.

b)  $\bigwedge_{i=1}^k \bigvee_{j=1}^m P_{ij}$  ✓

c)  $\bigwedge_{j=1}^m \bigvee_{1 \leq i_1 < i_2 \leq k} \neg P_{i_1 j} \vee \neg P_{i_2 j}$  ✓

d) On écrit  $P_{k,m}$  comme  $k m$  et  $\neg P_{k,m}$  comme  $\bar{k} \bar{m}$



Toutes les branches finissent sur une classe vide  $\Rightarrow \Psi \notin \text{SAT}$ .

#### Exercice 4

a) Non

b)