

On refactoriser par $(x+1)$:

$$\begin{array}{r|l} X^4 + X^3 - X - 1 & (X+1) \\ - (X^4 + X^3) & X^3 - 1 \\ \hline & -X - 1 \\ & - (-X - 1) \\ \hline & 0 \end{array}$$

Alors: $P(X) = (X^3 - 1)(X+1)^2$

On refactorise cette fois par $(X-1)$

$$\begin{array}{r|l} X^3 - 1 & (X-1) \\ - (X^3 - X^2) & X^2 + X + 1 \\ \hline & X^2 - 1 \\ & - (X^2 - X) \\ \hline & X - 1 \\ & - (X - 1) \\ \hline & 0 \end{array}$$

Donc: $P(X) = (X-1)(X+1)^2(X^2 + X + 1)$

Dans $[X]$: $X^2 + X + 1$

$\Delta = b^2 - 4ac = 1 - 4 \times 1 \times 1 = -3$

$\Delta < 0$, alors: $x_1 = \frac{-b - i\sqrt{\Delta}}{2a}$

$x_2 = \frac{-b + i\sqrt{\Delta}}{2a}$

Donc:
$$\begin{cases} x_1 = \frac{-1 - \sqrt{3}i}{2} \\ x_2 = \frac{-1 + \sqrt{3}i}{2} \end{cases}$$