Modeling Hemodynamics During Venous Occlusion

Achala Punase
Angélica Chanvoedou
Afonso Ferreira
Benjamin Ogwang
Sofia Pagoaga

Supervisor: David Romero i Sànchez

July 11, 2025

[math-in].net Red Española Matemática-Industria
ECMI EUROPEAN CONSORTIUM FOR MATHEMATICS IN INDUSTRY
PT MATHS IN rede portuguesa de matemática para a indústria e inovação
CRM CENTRE DE RECERCA MATEMÀTICA
EXCELENCIA MARÍA DE MAEZTU 2022 - 2025

Motivation

Biomedical Relevance:

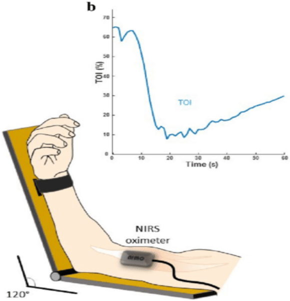
Understanding hemodynamic responses (such as blood pooling and oxygenation changes) is critical for developing non-invasive diagnostics in vascular health.

Clinical Applications:

- Early detection of vascular insufficiency
- Monitoring microcirculation
- Improving interpretation of near-infrared spectroscopy (NIRS) data

Scientific Challenge:

Translating experimental measurements into interpretable physiological parameters requires robust mathematical modeling.



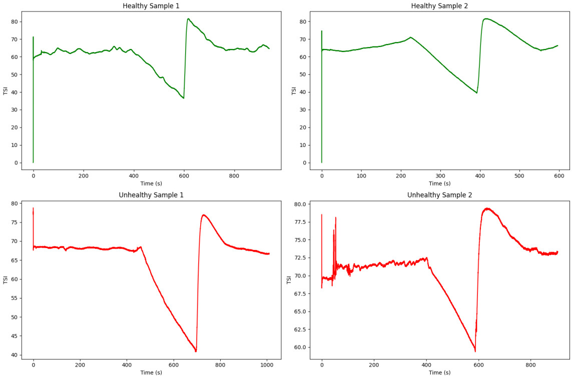
Objectives

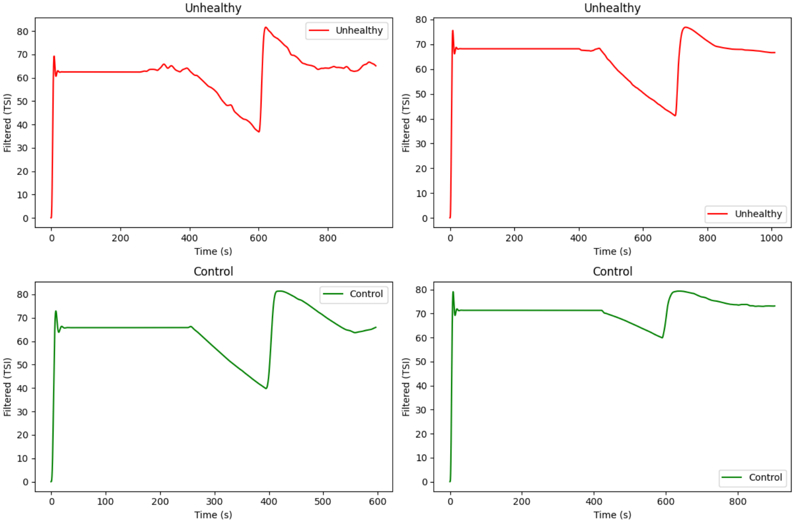
- Model Hemodynamic Responses: Capture blood volume increase, flow reduction, and oxygen saturation changes during venous occlusion.
- Identify Physiological Parameters: Extract time constants, slopes, and model parameters that characterize the response.
- Evaluate Diagnostic Potential: Investigate how modeling outputs and parameter estimates can inform or enhance non-invasive diagnostic techniques for assessing vascular health or microcirculatory function.

Data Visualization & Preprocessing

TSI (Tissue Saturation Index): A measure of local oxygen saturation in tissue derived from near-infrared spectroscopy (NIRS).

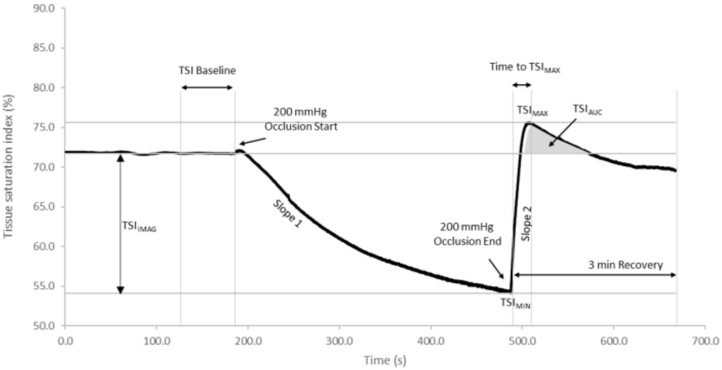
1. Plot raw NIRS time series (e.g., TSI—Tissue Saturation Index).
2. Identify baseline, occlusion, and recovery phases.

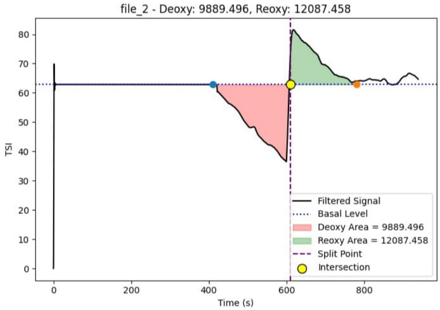


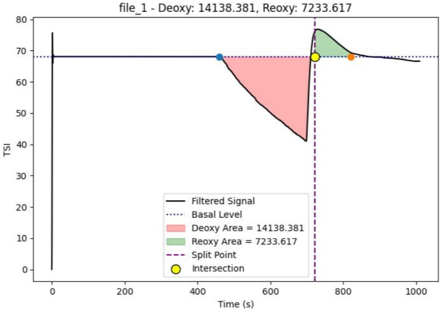

Unhealthy
Unhealthy
Filtered (TSI)
Time (s)
Unhealthy
Unhealthy
Filtered (TSI)
Time (s)
Control
Control
Filtered (TSI)
Time (s)
Control
Control
Filtered (TSI)
Time (s)

Data Visualization & Preprocessing

1. Smoothing and detrending.
2. Quantify increasing slopes (e.g., blood pooling).
3. Quantify decreasing slopes (e.g., recovery).




file_2 - Deoxy: 9889.496, Reoxy: 12087.458
TSI
Time (s)
Filtered Signal
Basal Level
Deoxy Area = 9889.496
Reoxy Area = 12087.458
Split Point
Intersection


file_1 - Deoxy: 14138.381, Reoxy: 7233.617
TSI
Time (s)
Filtered Signal
Basal Level
Deoxy Area = 14138.381
Reoxy Area = 7233.617
Split Point
Intersection

Physical Modeling of TSI Dynamics

Initial Model: Windkessel Model

- Simple representation of compliance and resistance
- Captures volume accumulation and outflow resistance

$$dP(t)/dt = \frac{1}{C}\left(Q(t) - \frac{P}{R}\right)$$

rearranging terms for Q(t)

$$Q(t) = CdP(t)/dt + \frac{P}{R}$$

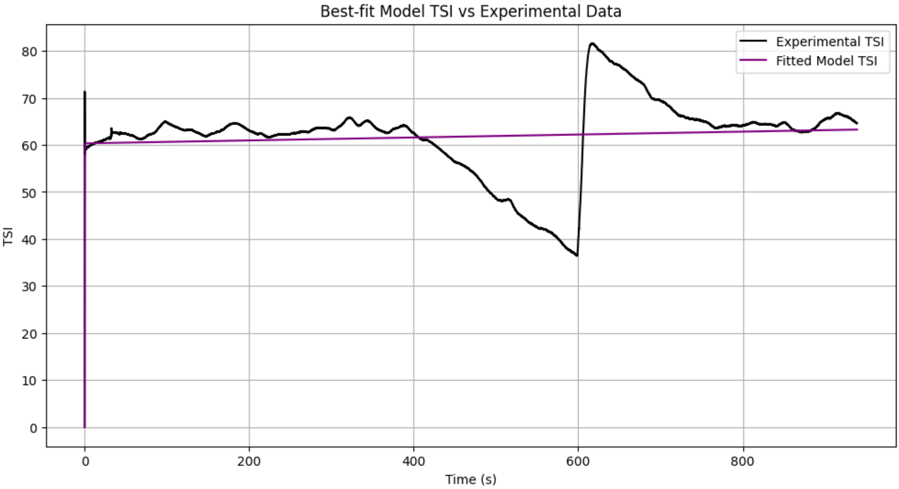
$$dTSI/dt = aQ(t) - b$$

Plugging in Q(t) and solving the ODE,

$$TSI(t) = TSI(0) + a\int_0^t Q(s)\,ds \ - \ bt$$

Choice for P(t): $A(1 - e^{-kt})$

WindKessel Model



Physical Modeling of TSI Dynamics

Observation

- While intuitive, the Windkessel model may not fully capture oscillatory features seen in the data.

Extended Approach:Harmonic Oscillator Model

- Incorporates inertia, compliance, and damping.
- Allows modeling of underdamped or overdamped behavior in blood flow and oxygenation recovery.

$$\ddot{x} + 2\beta \cdot \dot{x} + \omega_0^2 \cdot x - a \cdot F = 0$$

where

$$\omega_0 \in [0.005, 0.5]$$

and

$$\beta \in [0.0016, 0.16]$$

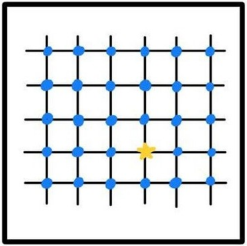
Parameter Estimation

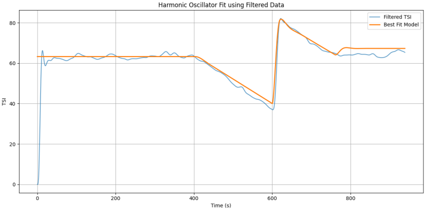
Optimization Strategy:

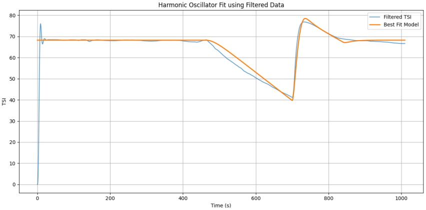
We aim to minimize the difference between the experimental TSI data and the solution of the harmonic oscillator model. To identify the optimal parameters, we implement a grid search.

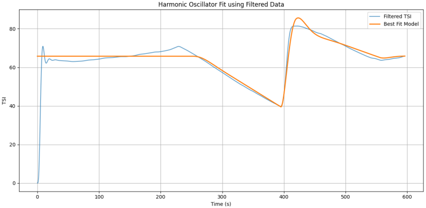
Specifically:

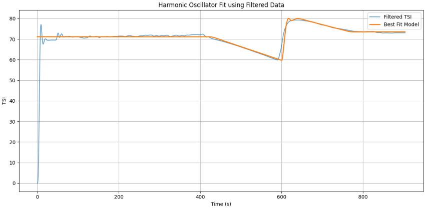
- Cost Function: Mean Squared Error between the observed TSI and the modeled TSI.
- Parameter Estimation: A grid search is conducted to find parameter combinations that yield the smallest discrepancy




Harmonic Oscillator Fit using Filtered Data
Filtered TSI
Best Fit Model
TSI
Time (s)


Harmonic Oscillator Fit using Filtered Data
Filtered TSI
Best Fit Model
TSI
Time (s)


Harmonic Oscillator Fit using Filtered Data
Filtered TSI
Best Fit Model
TSI
Time (s)


Harmonic Oscillator Fit using Filtered Data
Filtered TSI
Best Fit Model
TSI
Time (s)

Results

Feature Comparison

Feature	Healthy	Unhealthy
TSI AUC	Larger	Smaller
Variability	Low	Higher
Post-release recovery	Rapid	Delayed
Recovery Slope	Steep positive slope	Shallow

- AUC: quantifies the excess oxygen saturation delivered to the tissue after the period of restricted blood flow.
- β – Damping parameter: Resistance or friction opposing re-equilibration of flow.
- ω_0 – natural frequency: reflects intrinsic microvascular responsiveness (how quickly the system returns to equilibrium)

Conclusions

- The combination of (NIRS)-derived TSI time series with mathematical modeling frameworks, including both Windkessel and damped harmonic oscillator models, enables quantitative characterization of microcirculatory dynamics during and after venous occlusion.
- Specific biomarkers derived from TSI curves—namely baseline oxygen saturation, deoxygenation slope, reoxygenation slope, TSI area under the curve, damping coefficient (β),etc.—provide complementary information reflecting tissue oxygen consumption, vascular compliance, and the efficiency of reactive hyperemia.
- The comparative analysis of these biomarkers demonstrates clear differentiation between healthy and unhealthy vascular profiles. Specifically, lower baseline TSI, reduced post-occlusion AUC, higher variability, delayed recovery slopes, increased damping, and diminished natural frequency are consistently associated with impaired microcirculatory function.

Future Work & Improvements

- Develop more sophisticated models with multiple compartments representing different tissue layers or vascular beds.
- Apply the models across a larger cohort with diverse clinical conditions to validate the parameters as robust biomarkers of vascular health.
- Explore machine learning methods to predict vascular dysfunction

Bibliography

Hou-Jen Chen and Graham A. Wright.
A physiological model for interpretation of arterial spin labeling reactive hyperemia of calf muscles.
PLoS ONE, 12(8):e0183259, 2017.

Toi Van Vo, Peter E. Hammer, Matthew L. Hoimes, Shalini Nadgir, Sergio Fantini.
Mathematical model for the hemodynamic response to venous occlusion measured with near-infrared spectroscopy in the human forearm.
IEEE Transactions on Biomedical Engineering, 54(4):573–584, 2007.

Questions & Discussion

Thank You!