CHAPTER 1

Introduction to magnetic levitation

1.1 Basics concepts on electromagnetism

Magnetism is a well-known expression of one of the four fundamental interactions (the electromagnetic force). It was discovered in ancient world and studied through all ages to this day. It is now considered as a quantum mechanical interaction, but classical magnetism is able to excellently describe it, when relevant length scale is large enough to neglect quantum mechanics. Basics of this description are Maxwell's equations [1]:

1. $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$
2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$
3. $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{dt}$
4. $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} + \mu_0 i_{enc}$

They summarize all the work made by 19th century scientists like Oersted, Biot, Savart, Ampere and Faraday in describing electrical and magnetic fields.

Magnetic field is generated by moving electric charges or by intrinsic properties of some specific materials. As far as mean of propagation influences its properties, two fields are considered: magnetic field intensity (accounted as H) and its density field in any kind of mean (called B). In vacuum, they are proportional by the so called *magnetic permeability of the vacuum* (equal to 4pie-7):

$$B = \mu_0 H$$

In most of practical applications, field source is a current flowing into a conductor, generally a wire, as shown in fig 1a. According to Ampere's law for H (4^{th} Maxwell equation):

$$\oint_L \vec{H}\, d\vec{r} = \sum_k I$$

With L as a generic closed path (of versor r) and I the currents encircled into this path.

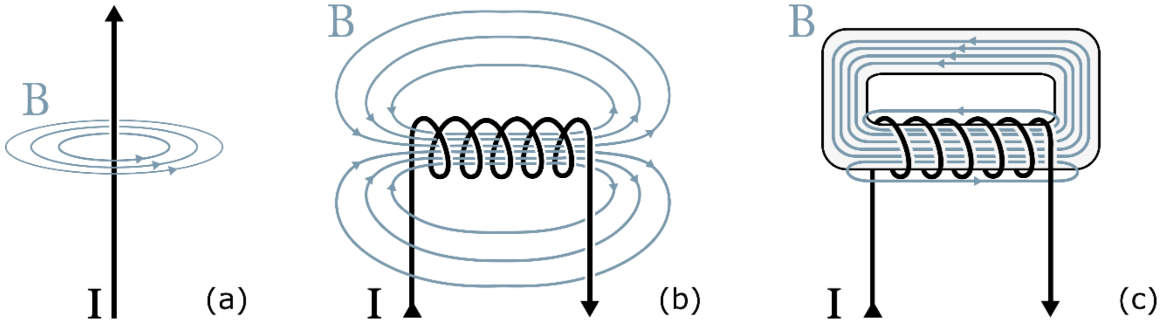

Figure 1: basics of magnetism application. Current inducing a magnetic field (a), solenoid(b), magnetism in material (c)

One of the most common arrangement for magnetic field generation is the solenoid, a coil which returns a quasi-homogeneous field in its core (fig 1b) and acts as a magnetic dipole outside. If an iron-like material is placed into solenoid core, it modifies field line, as shown in figure 1c, deviating it along its own geometry. This effect is strictly related to other magnetic properties of materials. In general, material influence on field is expressed by its permeability μ. It allows to write

$$B = \mu H$$

This parameter regards both intensity and direction of the field. A Snell-like law holds regarding a flux travelling among two different media. It is not so trivial to apply, as far as lines are not straight as in optics.

$$\frac{tg\alpha_1}{tg\alpha_2} = \frac{\mu_1}{\mu_2}$$

With respect to the way they produce fields and interact with external ones, materials are generally accounted as diamagnetic, paramagnetic or ferromagnetic materials. Diamagnetism regards materials which create a weak induced field in a direction opposite to an external one, while paramagnetism accounts materials that form an internal field in the same direction of the external one. Therefore, diamagnetic are repelled and paramagnetic attracted by external source. Ferromagnetism regards material which do not only generate an internal field which is consistently stronger than the one by diamagnetic or paramagnetic materials, but they also remain magnetized even if external field is removed Their permeability is thousand time the one of vacuum. It is due to the orientation of atomic magnetic dipoles (by the fact that electrons are elementary charges in motion) which return the magnetization density field M. This one combines with the external field into overall magnetic flux which is order of magnitude higher than the one reachable with other material classes. It returns an hysteretic behaviour, which accounts wide studies in licterature [17].

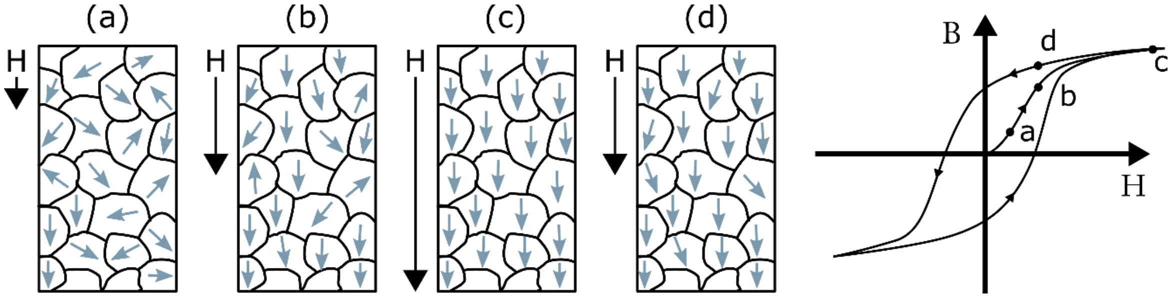

Figure 2 hysteresis in ferromagnetic materials. Microscopic behaviour of magnetic dipoles

A brief qualitative description of this behaviour is illustrated in fig 2. It is stated that the more H field is strong, the more internal dipoles orientate in its direction (A). It goes on until all dipoles are oriented. At this point (B), the material "saturates", so that magnetic density rise only by H effect (as it happens in vacuum). If H is lowered (C), part of internal dipoles does not come back to their original orientation, so that B doesn't go to zero even after H is removed. It is called saturation remanence (Br). To bring it back to zero, a certain H in opposite direction must be applied. It is called coercive field (Hc). The general formulation for field in ferromagnetic materials is:

$$B = \mu_0(H + M)$$

It is generally easier to calculate or estimate H by external field. For this reason, also M can be expressed as function of H by a non-linear term called susceptibility:

$$M = \chi H$$

It is a strongh non-linear relation. Permeability of ferromagnetic material is so not constant at all. Each material has its own curve, but they can be roughtly classified as "soft" or "hard" hysteretic curves. Soft exibit narrow loop and S-shape, hard have large squared loops.

As far as magnetization regards moving dipoles, temperature infuences their dynamics. Rising it, magnetization field is weaker and there's also a limit value, the Curie temperature, above which material loses its ferromagnetic property.

Ampere's law principle, which states how current generate a magnetic field, has its own dual in Faraday's law, which describes how a magnetic field induces an electromotive force into a conductor:

$$e.m.f. = -\frac{d\phi}{dt}$$

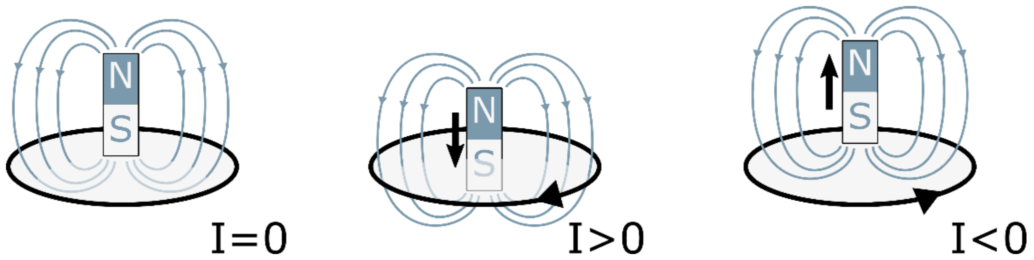

Figure 3: Faraday's law acting by moving magnet

With ϕ as flux linkage to the conductor. Figure 3 illustrates the basics case of one single spire and a permanent magnet in motion.

This phenomenon influences circuit dynamics, as far as minus sign means that the induced e.m.f. opposes to the current flowing in the circuit, introducing the problem of having transitions in current behaviour. It happens even with no external field: variable current flows in a circuit (as a solenoid), so that a variable field is created. This field is linked to the conductor itself, so that its variations induce an e.m.f. on the source circuit. This specific case is called self-induction and it is evaluated through a parameter called induction:

$$L = \frac{\phi}{I} \quad \rightarrow \quad e.m.f. = -\frac{d(LI)}{dt}$$

In most common applications, it turns out to be a constant, related only to conductor geometry. But in applications which rely on ferromagnetic materials, even a simple arrangement as Fig.1c, it is not constant at all. As far as current generates H, which enters in B-H field and returns B (so its flux ϕ), inductance in circuit depends on its own current. For voltage calculation, differential inductance is generally referred, as

$$L_d = \frac{\partial \phi}{\partial I}$$

It is all summarized in figure 3:

[TO BE DONE]

Another phenomenon which is commonly experienced in magnetism is the generation of forces. It may be commonly referred to three cases. The first one relies on Lorentz force on a moving charge (fig 4a). Considering only the magnetic component, it is a force which acts perpendicular to charges motion.

$$\vec{F} = \int I\vec{dl} \times \vec{B}$$

The second one is the reluctance force. It is based on the same principle which drives flux into iron-like core. As far as field concentrates in regions with higher permeability, if its path is interrupted by gaps in the material (as in figure 4b), it exerts a force to minimize the gap and so the stored energy. It is so an only attractive force, in field's direction:

$$\vec{F} = \frac{dU_m}{\overrightarrow{dx}}$$

The third one is the interaction among two magnets, permanents or electromagnets (as in figure 4c). It has more different formulations, depending on some hypothesis that may be needed or neglectable case by case. One of the most general case regards two dipoles which are small (or far) enough to neglect their geometry. Force is evaluated by first magnet's field effect on the second, this one regarded as a magnetic dipole:

$$\mathbf{F} = \nabla\left(\mathbf{m}_2 \cdot \mathbf{B}_1\right)$$

This force may be attractive or repulsive. It is an advantage in terms of magnetic levitation, because if both the actuator and the object have their own field, force can act in both directions (in place of being only attractive, as pure reluctance one).

Most applications, anyway, deals with systems in which levitating objects have not their own field. For this reason, the main comparison regards Lorentz and reluctance forces. By the way, the second one is able to produce stronger forces.

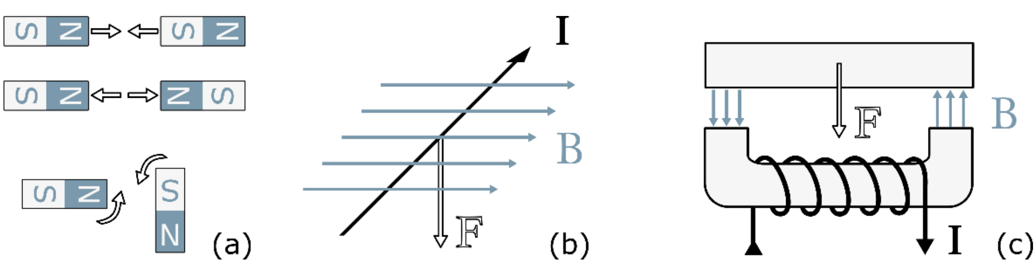

Figure 4: magnetic forces. Between magnets (a), on moving charges (b) or among magnetic circuit (c)

1.2 General approach to electromagnetic devices

Magnetism is exploited in a wide range of devices. Motors, transformers and bearings are just the most common applications. An electric circuit generates a magnetic field which moves a rotor, induces another current or keeps a body in its position.

As far as Maxwell equations are quite complex, a simpler analytical approach is generally adopted, the so called magnetic circuit. It is based on Ampere's law (eq.3) and on analogy with electrical circuit. It works only by similarities in formulations, as far as from a physical point of view, the two phenomena are consistently different. The basics of this approach are summarized in figure 5 and table 1

As far as equivalent Ohm's law and Kirchhoff's laws hold, it is possible to write:

$$NI = \sum \phi_i \Re_i$$

Rewriting it in H form, it returns Ampere's law (eq.3).

$$NI = \sum \phi_i \Re_i = \sum A_i B_i \frac{l_i}{\mu_i A_i} = \sum \frac{B_i}{\mu_i} l_i = \sum H_i l_i$$

Therefore, the main aspect which regards magnetic circuit building is to correctly identify all the paths followed by flux lines and evaluate their length, cross section, permeability and field. It is deeply discussed in the following chapter. Another aspect to be discussed is the implementation of non-linearities and hysteresis as reluctance-like elements in circuit.

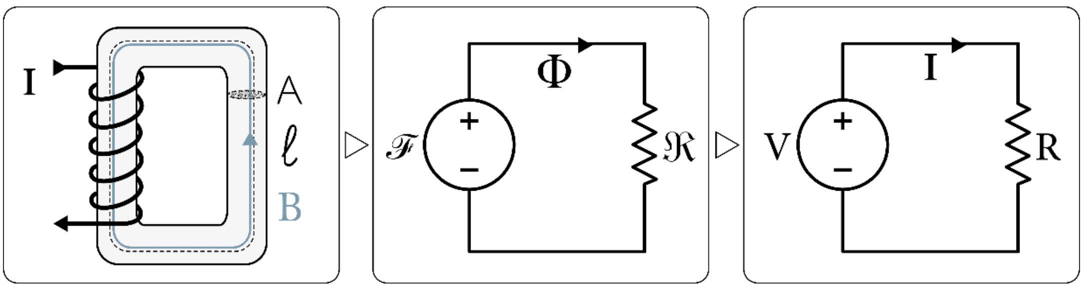

Figure 5: from magnetic system to equivalent circuit

Equivalent parameters of magnetic and electric circuits

Magnetic circuit		Electric circuit	
Definition	Symbol and relation	Symbol and relation	Definition
Magnetic flux density or induction	$\mathbf{B} = \mu\mathbf{H}$	$\mathbf{j} = \sigma\mathbf{E}$	Current density (Ohm's law)
Magnetic permeability	$\mu = \mu_0\mu_R$	σ	Electric conductivity
Magnetic field	$\mathbf{H}$	$\mathbf{E}$	Electric field
Magnetomotive force (mmf)	$V_H = \oint H \cdot \mathrm{d}\mathbf{l} = N_c I_c$	$V = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{l}$	Voltage
Total magnetic flux	$\psi = BS = \frac{V_H}{R_H}$	$I = jS = \frac{V}{R}$	Total current
	$(= \int \mathbf{B} \cdot ds)$	$(= \int \mathbf{j} \cdot ds)$	
Magnetic reluctance	$R_H = \frac{l}{\mu S}$	$R = \frac{l}{\sigma S}$	Resistance

Magnetic circuit takes current as an input, but the overall system includes also electrical dynamics. This is described by usual electric circuit theory, with a peculiarity: inductance is not constant at all. Referring to circuit in figure 6, it is possible to write:

$$V = IR + \frac{d\phi}{dt}$$

Last series of phenomena to consider, dealing with an electromagnetic device, are the losses. They are based on already mentioned physical principles, but they are not so trivial to understand and evaluate. As far as they are part of the dynamics to model, it is provided here just a brief qualitative introduction, to deal with them slighter in the following chapter:

a. Eddy current losses: Faraday's law (eq.?) holds also for the iron core. It means that magnetic field induces an e.m.f. also on it, resulting in energy loss. They are quite

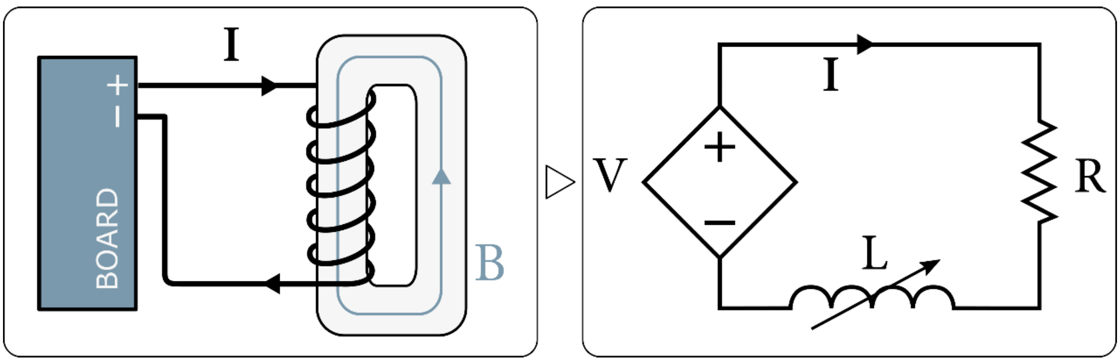

Figure 6: electrical circuit

relevant in high frequency application and they can be reduced by core lamination or by higher resistivity materials for core. Ferrite ($10\,\Omega \cdot m$) in place of iron ($10^{-8}\,\Omega \cdot m$) is the most common way for second solution.

b. Hysteresis losses: remaining of spontaneous magnetism requires extra m.m.f. in opposite direction to restore initial condition. For this reason, a complete work cycle of the device results in a loop into B-H curve whose area is the loss per unit volume.
c. Copper losses: caused by current flowing in the coil, in bearings they are dominant. This kind of losses produces a large amount of heat, which is able to rise system temperature enough to modify its dynamics. It is generally reduced by cooling down coils.
d. Windage losses: friction between rotor and air, relevant at very high speed application or in devices which deal with high pressure gasses.
e. Flux losses: iron cores confine magnetic flux within them. It works for most of flux lines, but a relatively small part of flux flows outside. For this reason, only a percentage of the generated flux is useful for the device. This behaviour is called leakage. Similar irregularities appear also in the presence of a small gap, as far as flux is not homogeneous and straight as supposed. Flux spreads out into the surrounding medium to reduce overall reluctance. It produces higher inductance and extra eddy currents in the near parts of core and coil. Figure 7 reports an example of both.

Most common expressions for power losses are [18]:

$$P_{\mathrm{ir,\,sin}} = afB_p^x + bf^2B_p^2 + ef^{1.5}B_p^{1.5}.$$

A few worlds may be spent about working with commercial products. Manufacturer generally provide a datasheet with all the main magnetic properties (B-H curve, saturation, remanence and coercive fields) and a few empirical expressions to easily evaluate power losses. Regarding the first kind of data, they are fully reliable only for the specific (or very similar) working conditions, as geometry, frequency and temperature. For this reason, they can be assumed as a reference, but it is better to not trust blindly them.

Empirical losses evaluation generally regards energy conversion for transformers. It is often possible to model these phenomena in a deeper way, regarding other quantities than power.

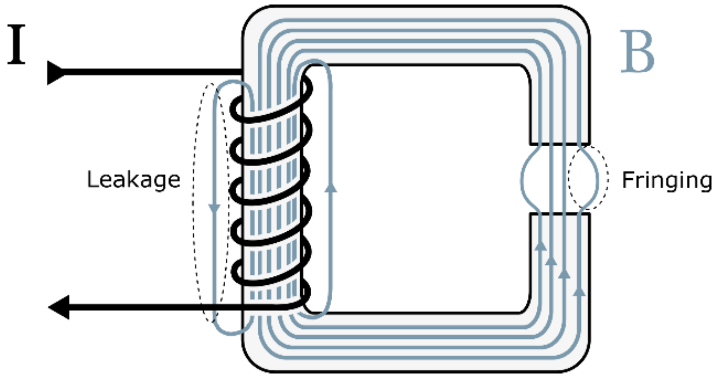

Figure 7: flux losses in a generic gapped device

1.3 Notes on hysteresis loop alterations

As far as most devices, like transformers, work at a certain frequency, an assessment of frequency effect on magnetic properties is necessary, and generally even provided [6], [14], The most common one is the modification of B-H curve as in Figure 8. It is due to eddy currents [18], which produce an extra H field opposite to external one. Its expression is:

$$\nabla^2 \mathbf{H}_{\mathrm{eddy}} = \sigma \frac{\partial \mathbf{B}}{\partial t}$$

Another point which requires a few words is H definition. As far it is accounted as external field, it is associated with inducing current. It is quite simple to evaluate in most applications. In magnetic circuits like one in Figure 5, It is trivial to apply Ampere's law with a good approximation to obtain:

$$H = \frac{NI}{L}$$

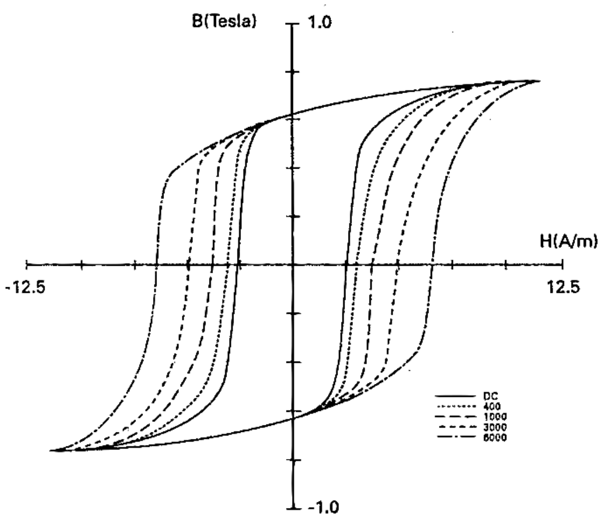

Figure 8 Measured hysteresis curves of Pennalloy 80 at dc, 400, 1000, 3000, and 6000 Hz (see Ref. 15). Reprinted from Design Manual for Tape Wound Cores (Magnet& Inc., 1987), p. 37.

In transformers theory, the result of this formula is accounted as H even in presence of discontinuities, such air gaps. The presence of such features, in reality, influences it, as H is not homogeneous nor continuous (continuity is a property of B field) and it would require a further analysis by flux real behaviour. Instead, this phenomenon is generally accounted as B-H curve shearing [6], [15]. It works considering “applied H” H_a in place of actual in-core H. For a generic gapped circuit, it is:

$$NI = H_c L_c + H_g L_g$$

$$H_a\left(L_c + L_g\right) = H_c L_c + H_g L_g$$

$$\frac{H_c}{H_a} = 1 + \left(1 - \frac{H_g}{H_a}\right)\frac{L_g}{L_c}$$

As far as B is continuous, $\mu_0 H_g = \mu_{nom} H_a$ where nominal permeability is the equivalent one for core and gap, so that $\mu_{nom} \gg \mu_0$ returns $H_g \gg H_a$. By these consideration, it is proved that right-side term in eq.?? is lower than 1. Its effect is shown in figure 9.

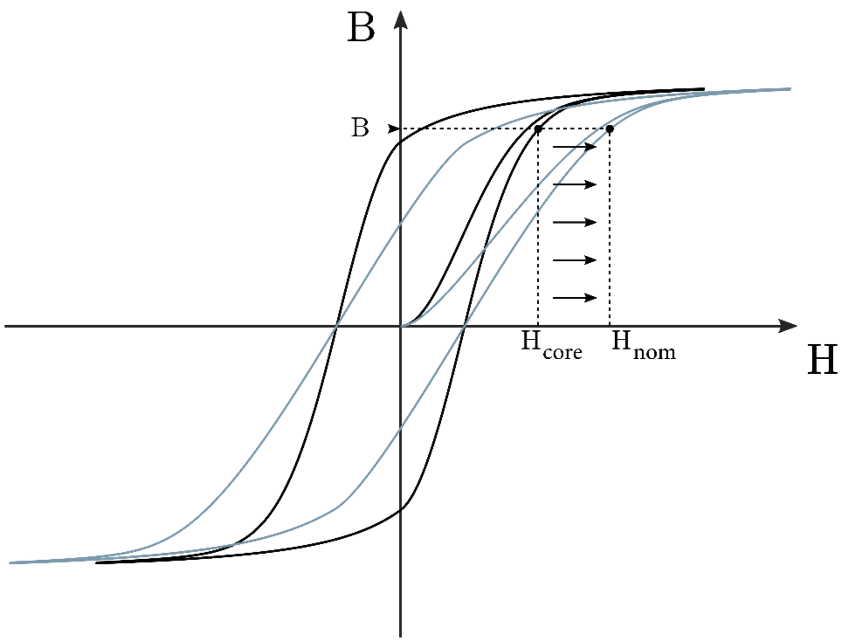

Figure 9 Shearing of B-H curve by gap introducing