Lab - Mechatronics
Modelling and control of a Magnetic Levitation System

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Listings

Acronyms

MISO Multiple Input Single Output. 4

MLS Magnetic Levitation System. 2, 3, 4, 5, 6

SISO Single Input Single Output. 4

1 Where we are now

For now, we are ignoring the second coil and have a (possibly wrong) modelling of both the electromagnetic and mechanical subsystems of the MLS.
Keep always in mind that:

- What we want to control is the position of the ball, z.
- The input of the overall system, will always be the target postion z_{ref}.
- The output of the overall system, will always be the actual position z.
- Our control logic, should then received the some kind of position error, $e = z_{\text{ref}} - z$ and output the voltage to apply to the coil.

$$\begin{cases} \dot{i} = -\frac{R}{L} \cdot i + \frac{\Delta V}{L} \\ \dot{z} = v \\ \dot{v} = g - \frac{F_{em}}{m} \end{cases} \tag{1}$$

From other online resources and thesis from others, we have seen that the magnetic force have been modelled as:

$$F_{em} = k_v \frac{i^2}{z^2} \tag{2}$$

However, this simple model doesn't take into account any inductive perturbation that an object immersed in a magnetic field produces on the field itself. For example, the manual on the MLS2EM, has some formulas (never seen before) that somehow take into account this effect.

State space representation

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{3}$$

$$x = \begin{bmatrix} i \\ z \\ v \end{bmatrix} \quad u = \Delta V \quad y = z \tag{4}$$

$$A = \begin{bmatrix} -\frac{R}{L} & 0 & 0 \\ 0 & 0 & 1 \\ \text{Taylor} & \text{expansion} & \text{here} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tag{5}$$

2 Introduction

The aim of this laboratory experience is to precisely control the levitation of a ferromagnetic object immerse in a magnetic field. This kind of system is commonly referred to as a Magnetic Levitation System (MLS).
The work has been splitted into two main phases:

- **Modelling and parameters identification**: in this phase, the system has been modelled by means of both differential equations and state space representation, and the parameters of the model have been identified through experimental data performed directly on the real system. Some preliminary consideration about stability and controllability have also been made.
- **Control design**: in this phase, many different control techniques have been implemented and tested. The main goal was to compare the performances of different controllers in terms of stability, robustness and tracking capabilities.

Report structure This report covers all the aspects of the laboratory experience, from the theoretical background to the practical implementation of the control algorithms. In particular, in Section 3 the system is described in detail, a model is derived and the parameters are identified. In Section 4 a set of SISO controllers are designed to work on a reduced model of the system, where only one coil is active. On the other hand, in Section 5 a set of MISO controllers are designed to work on the full model of the system, where both coils are actively used to control the position of the magnet. In Section 6 the performances of the different controllers are compared, and finally in Section 7 some conclusions are drawn.

Tools An extensive use of `MATLAB` and `Simulink` has been made to implement the controllers and to simulate the system. All the source code and simulations used for this report can be found on the GitHub repository at the following link: `https://github.com/Bocchio01/062020-Lab-Mechatronics`.

3 Magnetic Levitation System

As stated in the introduction, the system under study is the Magnetic Levitation System (MLS) provided by `Inteco` (product vebsite: `https://www.inteco.com.pl/products/magnetic-levitation-systems/`). In Figure 1 a picture of the system present in the laboratory is shown.

Figure 1: Magnetic Levitation System

As it can be seen quite clearly, the system is composed of a simple mechanical structure that is used to support two electromagnets and an optical infrared sensor. Along with the mechanical structure, a ferromagnetic ball and a control unit are present.
At its core principle, the system uses the interaction between the magnetic field generated by the electromagnets and the ferromagnetic ball to keep the ball in a desired position. The optical sensor is used to measure the position of the ball and provide feedback to the control unit that, in turn, adjusts the current flowing through the electromagnets to keep the ball in a desired position. In Figure 2 a schematic representation of the upper half of the system is shown.

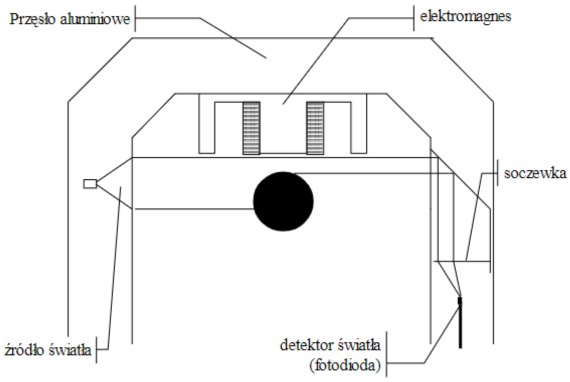

Figure 2: Schematic representation of the MLS system.

Real world application Despiste the fact that our system is a simplified version of a real-world application, the magnetic levitation principle is used in many real-world applications.
One of the most common applications is the magnetic levitation trains, also known as 'MagLev' trains. These trains use the magnetic levitation principle to lift the train off the tracks and propel it forward using the magnetic field generated by the tracks. The main advantage of this technology is the absence of friction between the

train and the tracks, which allows the train to reach higher speeds and reduce the noise characteristic of the traditional trains. Some of the fastest (operating) trains in the world are MagLev trains, with the Shanghai MagLev train being the fastest, reaching a top speed of $623km/h$ [1].
Another application of the magnetic levitation principle is the magnetic bearings. These bearings use the magnetic field generated by electromagnets to levitate a rotor and keep it in a desired position. The main advantage of this technology is the absence of mechanical contact between the rotor and the stator, which allows the rotor to reach higher speeds and reduce the wear of the components.

3.1 Model derivation

The MLS is a complex system that can be divided into two main subsystems:

- **Electromagnetic subsystem**: it takes into account the electrical components of the system from the power supply to the generation of the magnetic field by the coils;
- **Mechanical subsystem**: it takes into account the dynamics of the ball and all the forces acting on it, including the electromagnetic forces generated by the electromagnetic subsystem.

Due to the presence of the ball that moves inside a magnetic field, a complex connection between the two subsystems that goes beyond the simple force balance exists. For this reason, it's almost impossible to derive a complete model of the system without considering both subsystems at the same time.
In the following sections, we will derive the equations that governs the MLS system, adopting an energetic approach that takes into account the energy conservation principle.

3.1.1 Mathematical model

In Figure 3, a schematic representation of all the components of the MLS system is shown.

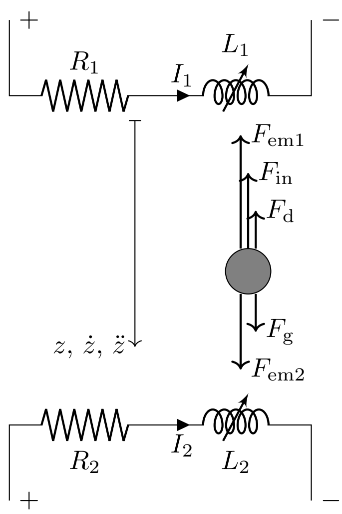

Figure 3: Schematic representation of the MLS system.

Before proceeding with the derivation of the equations, we can give a brief description of the components of the system:

Component	Description	Units
R_1	Resistance of the coil 1	Ω
L_1	Inductance of the coil 1	H
I_1	Current flowing through the coil 1	A
F_{em1}	Electromagnetic force generated by the coil 1	N
F_{in}	Inertial force due to the acceleration of the ball	N
F_{d}	Drag force due to the air resistance	N
F_{g}	Gravitational force acting on the ball	N
F_{em2}	Electromagnetic force generated by the coil 2	N
R_2	Resistance of the coil 2	Ω
L_2	Inductance of the coil 2	H
I_2	Current flowing through the coil 2	A

We can now proceed with the derivation of the equations that govern the system. At first, we can recall the energy conservation principle that states that the sum of the kinetic and potential energy of the system is equivalent to the dissipated energy. Thanks to the Lagrange's equation, we write the following equation encapsulating the energy conservation principle:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{\mathbf{u}}}\right) - \frac{\partial \mathcal{T}}{\partial \mathbf{u}} + \frac{\partial \mathcal{D}}{\partial \dot{\mathbf{u}}} + \frac{\partial \mathcal{U}}{\partial \mathbf{u}} = \mathcal{Q} \tag{6}$$

Where $\mathbf{u}$ is the generalized coordinates of the system, $\mathbf{Q}$ is the generalized input to the system, T is the kinetic energy of the system, U is the potential energy of the system, and D is the dissipated energy of the system.
In the following paragraphs, we will report only the main steps taken to derive the equations of motion of the system. Many passages have been omitted for brevity, but the complete derivation can be found in the Appendix A.
At first, we can define all the energetic terms of the system. Notice that with respect to traditional purely mechanical systems, we also have to consider the stored energy in the coils (inductance), the dissipation due to the resistance of the coils, and the potential energy given by the external power supply.
By doing so, we can write the kinetic energy of the system as:

$$\mathcal{T} = \frac{1}{2}m\dot{z}^2 + \frac{1}{2}L_1(z, T_1)\dot{q_1}^2 + \frac{1}{2}L_2(z, T_2)\dot{q_2}^2 \tag{7}$$

The potential energy of the system as:

$$\mathcal{U} = -mgz - q_1 V_1 - q_2 V_2 \tag{8}$$

The dissipated energy of the system as:

$$\mathcal{D} = \int_0^t \frac{1}{2} C_d A \rho d\dot{z} + \int_0^t R_1 d\dot{q_1} + \int_0^t R_2 d\dot{q_2} \tag{9}$$

And the generalized input to the system as:

$$\mathcal{Q} = 0 \tag{10}$$

As can be clearly seen, we have chosen to consider the external power supplied to the coils as a potential energy term and not as a generalized input. Notice also the minus sign in the potential energy term, which is due to the fact that the gravitational force increases the potential energy with respect to the chosen reference frame (positive downwards).
Before proceeding, it's necessary to explicitly the dependence of the inductance terms on the generalized coordinates. In particular, we can write the inductance terms as:

$$\begin{aligned} L_1 &= L_1(z, T_1) = L_0 + L_d e^{-az} \\ L_2 &= L_2(z, T_2) = L_0 + L_d e^{-a(d-z)} \end{aligned} \tag{11}$$

How do we model the inductance-temperature dependency? And what about the resistance-temperature dependency?
Where L_0 is the inductance when no objects are present in the magnetic field volume, while the exponential terms account for the presence of the ball and its distance from the coils.

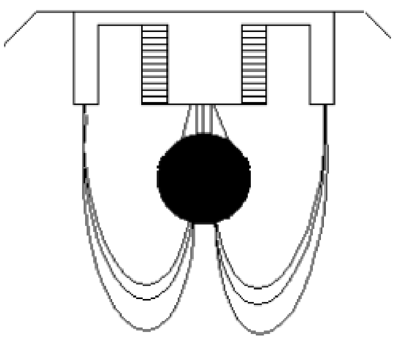

Figure 4: Representation of the object in the magnetic field.

By substituting the kinetic, potential, and dissipated energy terms into the Lagrange's equation, we can derive the equations of motion of the system.

$$\begin{cases} \dot{z} = v_z \\ \dot{v_z} = \frac{1}{m}\left(\frac{1}{2}aL_d e^{-az} i_1^2 - \frac{1}{2}aL_d e^{-a(d-z)} i_2^2 - \frac{1}{2}C_d A\rho\dot{z} + mg\right) \\ \dot{i_1} = \frac{1}{L_0 + L_d e^{-az}}\left(aL_d e^{-az}\dot{z}i_1 - R_1 i_1 + V_1\right) \\ \dot{i_2} = \frac{1}{L_0 + L_d e^{-a(d-z)}}\left(-aL_d e^{-a(d-z)}\dot{z}i_2 - R_2 i_2 + V_2\right) \\ \dot{T_1} = \text{To be worked out} \\ \dot{T_2} = \text{To be worked out} \end{cases} \tag{12}$$

Where v_z is the velocity of the ball, i_1 and i_2 are the currents flowing through the coils, and T_1 and T_2 are the temperatures of the coils.
The equations of motion of the system are highly non-linear and coupled, making the system hard to control.

3.2 Parameters identification

Based on the MLS2EM datasheet/manual (pages 6-18), the procedure to identify the parameters of the system should be as follows.
Launch `mls2em-usb2-main` script and open tools/identification window and then:

1. Optical sensor calibration: it's the curve the mesured voltage from the optical sensor to the effective height of the ball. It's used internally by the `MLS-SIM-BLOCK`. Output: mls2em-usb2-Sensor.mat-¿SensorData.[Distance-mm, Sensor-V], $z = f(\Delta V)$.

2. Static characterization (coil): it's the curve of the current in the coil due to a given applied voltage. It's assumed to be linear $i = f(\Delta V) = k_i\Delta V + c_i$, but with a deadzone at the beginning $i_{MIN} = f(\Delta V_{MIN})$. Output: $k_i[\frac{A}{V}]$, $c_i[A]$, $i_{MIN}[A]$, $V_{MIN}[V]$.

3. Minimal effort to move the ball: from this we can have a relation between the current in the coil and the force applied to the ball.

4. Dynamic characterization (coil): as we know, obkect moving in a magnetic field generate distorsion in the field itself/changes in the coil's current. All these effects are taken into account by K_i and f_i. Basically, the plot on page 18, shows the solution for a classical RL circuit $i(t) = i_0 + i_\infty(1 - e^{-\frac{t}{\tau}})$, derived from the differential equation $\dot{i} = \frac{V-Ri}{L}$. It's unclear what K_i and f_i really are.

So, brief recap of the parameters that the manual uses in its mathematical model:

Parameter	Description	Unit	Notes
m	Ball mass	kg	
g	Gravity acceleration	$\frac{m}{s^2}$	
F_{emP1}	Magnetic modelling related	H	To be clarified
F_{emP2}	Magnetic modelling related	m	To be clarified
f_{iP1}	Inductance modelling related	$m \cdot s$	To be clarified
f_{iP2}	Inductance modelling related	m	To be clarified
k_i	Basically it's the conductance $1/R$	$\frac{A}{V}$	
c_i	Coil offset	A	
i_{MIN}	Minimum current (deadzone of control)	A	
V_{MIN}	Minimum voltage (deadzone of control)	V	
K_i	Current rise modelling in coil		
f_i	Current rise modelling in coil		

Table 1: Parameters of the MLS2EM system

Function	Description	Unit
F_g	Gravity force acting on the ball	N
F_{em1}	Electromagnetic force from upper coil	N
F_{em2}	Electromagnetic force from lower coil	N
$f_i(x_1)$	Function to model the variation of the inductance in the coils due to the ball position	s

Table 2: Functions of the MLS2EM system

4 Single Coil Control

5 Double Coil Control

6 Control Techniques Comparison

Based on the MLS2EM datasheet/manual, some suggestions are (page 5):

- SISO, MISO, BIBO controllers design
- Intelligent/Adaptive Control
- Frequency analysis
- Nonlinear control
- Hardware-in-the-Loop
- Real-Time control
- Closed Loop PID Control

7 Conclusions

References

[1] Wikipedia contributors. Scmaglev — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=SCMaglev&oldid=1243224393, 2024. [Online; accessed 28-September-2024].

A Complete mathematical model derivation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{u}}}\right) - \frac{\partial T}{\partial \mathbf{u}} + \frac{\partial D}{\partial \dot{\mathbf{u}}} + \frac{\partial U}{\partial \mathbf{u}} = \mathbf{Q} \tag{13}$$

$$\mathcal{T} = \frac{1}{2}m\dot{z}^2 + \frac{1}{2}L_1(z, T_1)\dot{q_1}^2 + \frac{1}{2}L_2(z, T_2)\dot{q_2}^2 \tag{14}$$

$$\mathcal{U} = -mgz - q_1V_1 - q_2V_2 \tag{15}$$

$$\mathcal{D} = \int_0^t \frac{1}{2}C_dA\rho d\dot{z} + \int_0^t R_1 d\dot{q_1} + \int_0^t R_2 d\dot{q_2} \tag{16}$$

$$\mathcal{Q} = 0 \tag{17}$$

$$\begin{cases} \frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{z}}\right) - \frac{\partial \mathcal{T}}{\partial z} + \frac{\partial \mathcal{D}}{\partial \dot{z}} + \frac{\partial \mathcal{U}}{\partial z} = \mathcal{Q} \\ \frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{q_1}}\right) - \frac{\partial \mathcal{T}}{\partial q_1} + \frac{\partial \mathcal{D}}{\partial \dot{q_1}} + \frac{\partial \mathcal{U}}{\partial q_1} = \mathcal{Q} \\ \frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{q_2}}\right) - \frac{\partial \mathcal{T}}{\partial q_2} + \frac{\partial \mathcal{D}}{\partial \dot{q_2}} + \frac{\partial \mathcal{U}}{\partial q_2} = \mathcal{Q} \\ \frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{T_1}}\right) - \frac{\partial \mathcal{T}}{\partial T_1} + \frac{\partial \mathcal{D}}{\partial \dot{T_1}} + \frac{\partial \mathcal{U}}{\partial T_1} = \mathcal{Q} \\ \frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{T_2}}\right) - \frac{\partial \mathcal{T}}{\partial T_2} + \frac{\partial \mathcal{D}}{\partial \dot{T_2}} + \frac{\partial \mathcal{U}}{\partial T_2} = \mathcal{Q} \end{cases} \tag{18}$$

$$\begin{cases} m\ddot{z} - \frac{1}{2}\frac{\partial L_1}{\partial z}\dot{q_1}^2 - \frac{1}{2}\frac{\partial L_2}{\partial z}\dot{q_2}^2 + \frac{1}{2}C_dA\rho\dot{z} - mg = 0 \\ L_1\ddot{q_1} + \frac{\partial L_1}{\partial z}\dot{z}\dot{q_1} + \frac{\partial L_1}{\partial T_1}\dot{T_1}\dot{q_1} + R_1\dot{q_1} - V_1 = 0 \\ L_2\ddot{q_2} + \frac{\partial L_2}{\partial z}\dot{z}\dot{q_2} + \frac{\partial L_2}{\partial T_2}\dot{T_2}\dot{q_2} + R_2\dot{q_2} - V_2 = 0 \\ \text{To be worked out} \\ \text{To be worked out} \end{cases} \tag{19}$$

$$\begin{aligned} L_1 &= L_1(z, T_1) = L_0 + L_d e^{-az} \\ L_2 &= L_2(z, T_2) = L_0 + L_d e^{-a(d-z)} \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{\partial L_1}{\partial z} &= -aL_d e^{-az} \\ \frac{\partial L_2}{\partial z} &= aL_d e^{-a(d-z)} \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{\partial L_1}{\partial T_1} &= 0 \\ \frac{\partial L_2}{\partial T_2} &= 0 \end{aligned} \tag{22}$$

$$\begin{cases} \dot{z} = v_z \\ \dot{v_z} = \frac{1}{m}\left(\frac{1}{2}aL_d e^{-az}i_1^2 - \frac{1}{2}aL_d e^{-a(d-z)}i_2^2 - \frac{1}{2}C_dA\rho\dot{z} + mg\right) \\ \dot{i_1} = \frac{1}{L_0 + L_d e^{-az}}\left(aL_d e^{-az}\dot{z}i_1 - R_1 i_1 + V_1\right) \\ \dot{i_2} = \frac{1}{L_0 + L_d e^{-a(d-z)}}\left(-aL_d e^{-a(d-z)}\dot{z}i_2 - R_2 i_2 + V_2\right) \\ \dot{T_1} = \text{To be worked out} \\ \dot{T_2} = \text{To be worked out} \end{cases} \tag{23}$$

Suggested by Copilot:

$$\begin{cases} \dot{z} = v_z \\ \dot{v_z} = \frac{1}{m}\left(\frac{1}{2}aL_d e^{-az}i_1^2 - \frac{1}{2}aL_d e^{-a(d-z)}i_2^2 - \frac{1}{2}C_dA\rho\dot{z} + mg\right) \\ \dot{i_1} = \frac{1}{L_0 + L_d e^{-az}}\left(aL_d e^{-az}\dot{z}i_1 - R_1 i_1 + V_1\right) \\ \dot{i_2} = \frac{1}{L_0 + L_d e^{-a(d-z)}}\left(-aL_d e^{-a(d-z)}\dot{z}i_2 - R_2 i_2 + V_2\right) \\ \dot{T_1} = \frac{1}{mc_1}\left(\frac{1}{2}aL_d e^{-az}i_1^2 - R_1 i_1 + V_1\right) \\ \dot{T_2} = \frac{1}{mc_2}\left(\frac{1}{2}aL_d e^{-a(d-z)}i_2^2 - R_2 i_2 + V_2\right) \end{cases} \tag{24}$$