Modelling and control of a Magnetic Levitation System

Mid-project presentation

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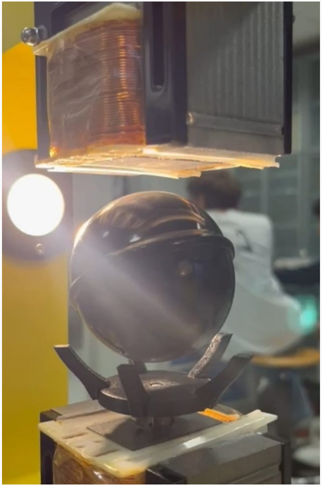
Agenda

1. What we have done
2. What we are working on
3. What we would like to do
4. Open questions

Figure 1: Magnetic Levitation System and its components

Project objectives

Magnetic Levitation System (MLS) it's an electromechanical system that enhances magnetic fields to levitate a ferromagnetic object. It's known for its non-linear behavior and its instability.



Project objectives:
Make the ball levitate.

What we have done

Modelling

We derived the **equations of motion** of the Magnetic Levitation System (MLS) based on a Lagrangian approach.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot{\mathbf{u}}}\right) - \frac{\partial \mathcal{T}}{\partial \mathbf{u}} + \frac{\partial \mathcal{D}}{\partial \dot{\mathbf{u}}} + \frac{\partial \mathcal{U}}{\partial \mathbf{u}} = \mathcal{Q} \tag{1}$$

$$\begin{aligned}
\mathcal{T} &= \frac{1}{2}m\dot{z}^2 + \frac{1}{2}L_1(z, \dot{q}_1)\dot{q}_1{}^2 + \frac{1}{2}L_2(z, \dot{q}_2)\dot{q}_2{}^2 \\
\mathcal{D} &= \int_{\dot{z}(\cdot)} \frac{1}{2}C_d A\rho\dot{z}^2 d\dot{z} + \int_{\dot{q}_1(\cdot)} R_1(\dot{q}_1)\dot{q}_1 d\dot{q}_1 + \int_{\dot{q}_2(\cdot)} R_2(\dot{q}_2)\dot{q}_2 d\dot{q}_2 \\
\mathcal{U} &= -mgz - q_1 V_1 - q_2 V_2 \\
\mathcal{Q} &= 0
\end{aligned} \tag{2}$$

We derived the **equations of motion** of the Magnetic Levitation System (MLS) based on a Lagrangian approach.

$$
\begin{cases}
m\ddot{z} - \frac{1}{2}\frac{\partial L_1}{\partial z}\dot{q_1}^2 - \frac{1}{2}\frac{\partial L_2}{\partial z}\dot{q_2}^2 + \frac{1}{2}C_d A\rho\dot{z}|\dot{z}| - mg = 0 \\
\frac{1}{2}\left(\frac{\partial^2 L_1}{\partial\dot{q_1}\partial z}\dot{z} + \frac{\partial^2 L_1}{\partial\dot{q_1}^2}\ddot{q_1}\right)\dot{q_1}^2 + \frac{\partial L_1}{\partial\dot{q_1}}\dot{q_1}\ddot{q_1} + \left(\frac{\partial L_1}{\partial z}\dot{z} + \frac{\partial L_1}{\partial\dot{q_1}}\ddot{q_1}\right)\dot{q_1} + L_1\ddot{q_1} + R_1\dot{q_1} - V_1 = 0 \\
\frac{1}{2}\left(\frac{\partial^2 L_2}{\partial\dot{q_2}\partial z}\dot{z} + \frac{\partial^2 L_2}{\partial\dot{q_2}^2}\ddot{q_2}\right)\dot{q_2}^2 + \frac{\partial L_2}{\partial\dot{q_2}}\dot{q_2}\ddot{q_2} + \left(\frac{\partial L_2}{\partial z}\dot{z} + \frac{\partial L_2}{\partial\dot{q_2}}\ddot{q_2}\right)\dot{q_2} + L_2\ddot{q_2} + R_2\dot{q_2} - V_2 = 0
\end{cases}
\tag{1}
$$

Modelling

In order to simplify the model, we have **neglected the effect of the current on the value of the inductances (strong assumption)**. We also have neglected any velocity linearly dependent terms in the equations of motion.

$$\begin{cases} \frac{\partial L}{\partial I} & \approx 0 \\ \frac{\partial^2 L}{\partial I^2} & \approx 0 \\ \dot{z} & \approx 0 \end{cases} \tag{1}$$

From literature, we also have found an experimental based model for the inductances.

$$L = L(z) = L_0 + L_z e^{-a_z z} \tag{2}$$

Modelling

The current model is a simplified version of the original one, but from experimental data we can see that it's still **able to capture the main dynamics of the system**.

$$
\begin{cases}
\dot{z} = v \\
\dot{v} = m^{-1} \left(\frac{1}{2} \frac{\partial L_1}{\partial z} I_1^2 + \frac{1}{2} \frac{\partial L_2}{\partial z} I_2^2 + mg \right) \\
\dot{I}_1 = L_1^{-1} \left(-R_1 I_1 + (k_1 U_1 + c_1) \right) \\
\dot{I}_2 = L_2^{-1} \left(-R_2 I_2 + (k_2 U_2 + c_2) \right)
\end{cases}
\tag{1}
$$

Notice that z is the position of the ball (what we want to control), while U_1 and U_2 are the inputs of the system (what we can control).

Modelling

The model has also been linearized and transformed in state-space form.

$$\begin{aligned} \dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= g(\mathbf{x}, \mathbf{u}) \end{aligned} \quad \rightarrow \quad \begin{aligned} \dot{\delta\mathbf{x}} &\approx A\delta\mathbf{x} + B\delta\mathbf{u} \\ \delta\mathbf{y} &\approx C\delta\mathbf{x} + D\delta\mathbf{u} \end{aligned} \tag{1}$$

Parameters identification

In order to control the system, we had to **identify the parameters of the system**. To do so, many experiments have been conducted.

Control to Voltage mapping.

As predictable, the control to voltage mapping is a linear function outside the 'no control zone'.

$$V = \begin{cases} V_{min} & \text{if } U < U_{min} \\ kU + c & \text{if } U \geq U_{min} \end{cases} \quad (2)$$

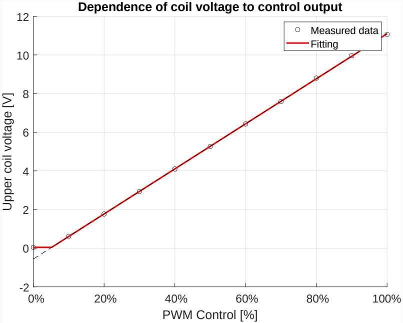


Figure 2: Voltage as a function of U

Electromagnetic force characterizations.

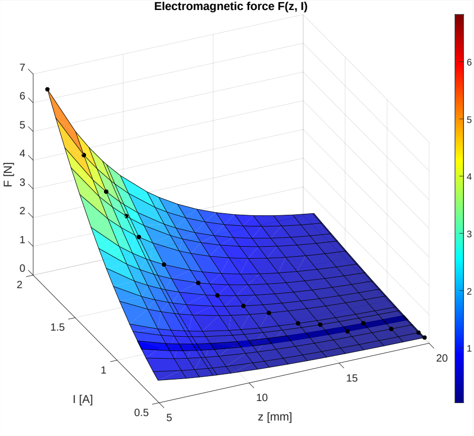


Figure 2: Electromagnetic force as a function of z and I

Notice that from the theoretical model, we have found the electromagnetic force acting on the ball to be:

$$F_{em} = \frac{1}{2}\frac{\partial L}{\partial z} I^2 \qquad (2)$$

We have implemented a Simulink model of the system.

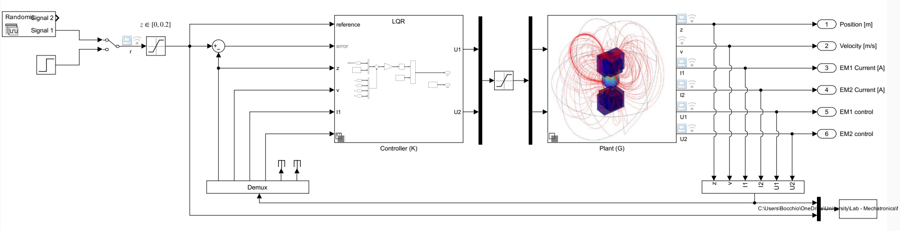


Figure 3: Simulink root model of the Magnetic Levitation System

Some controllers have also been implemented and tested.

Controlling

PID (both with and without anti-windup) controllers have been tested.

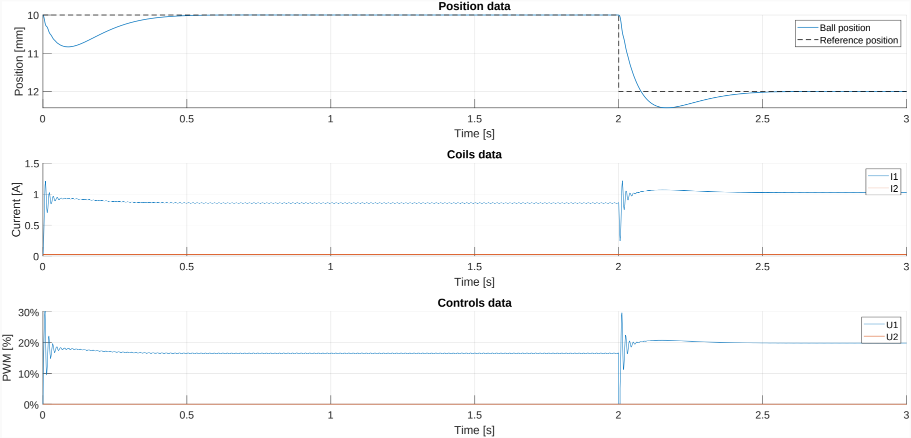


Figure 3: PID with anti-windup controller

A PID controller without anti-windup has also been tested but with a clearly worse performance (strong oscillations around the reference).

Controlling

LQR controllers with (limited) tracking capabilities have been tested.

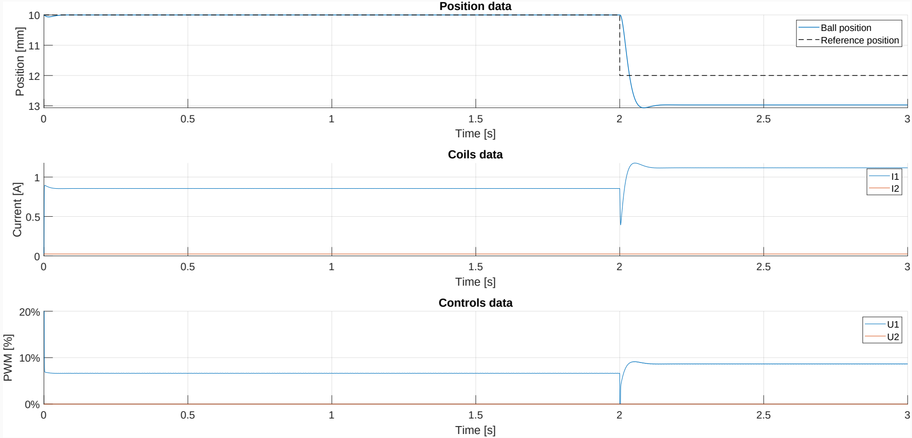


Figure 3: LQR controller

What we are working on

Further inductance characterization

Even if maybe not relevant for control purposes, we are trying to **study the dependence of the inductance on both the ball position and the current**.

$$L = L(z, I) = L_0 + L_z e^{-a_z z} + L_I \tanh(-a_I I) \tag{3}$$

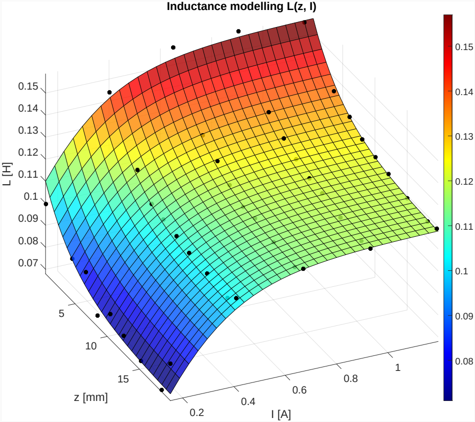


Figure 4: Fitted model for $L(z, I)$

Higher current values are needed to obtain experimental data over all the possible operating regions.

MCP controller

We are currently trying to implement a Model Predictive Controller (MPC).

We want to switch from classical linearized restricted state-space controllers to a more advanced and robust controller.

However, looks like Simulink doesn't really like the way we are doing it... many implementation issues.

What we would like to do

Implement more advanced controllers

Nonlinear controllers are particularly interesting because of the highly nonlinear nature of the system. However, they look like they are going to be a challenge to implement.

Linear Controllers:

- Model Predictive Control
- Cascaded Control
- Pole Placement
- Linear Quadratic Integrator

Nonlinear Controllers:

- Backstepping
- Feedback Linearization

When possible, we would also like to implement logics such as Kalman Filters or gain scheduling.

Study the effects of approximations

Once the characterization of the inductances is complete, we would like to study the effects of the approximations we have made in the model.

For example by comparing:

- Linearized vs nonlinear model with respect to the real world system;
- Effects of simplified ($L(z)$) vs complete ($L(z, I)$) inductance modelling.

Controllers comparison

We would like to compare the performance of the controllers we have implemented.

To do so, we will probably create a 'Race of Controllers[1]' where we will compare various performance indices.

[1] More on this in the final presentation.

Open questions

Questions

1. **Influence of current on inductance**: is our assumption of neglecting the influence of current on inductance valid?
2. **Use of a single coil**: implications and limitations of using a single vs double coil to control the system.
3. **Discretization**: impact of discretization over controller's performance.
4. **MPC linearization**: problems of linearization within the Model Predictive Control.

References i

P. Balko and D. Rosinova.
Modeling of magnetic levitation system.
pages 252–257, 06 2017.

INTECO.
Magnetic Levitation System 2EM, User manual.

MATLAB.
MATLAB Documentation.

Unkown.
Simplified approach to modeling.

Questions?

Thank you!