CHAPTER 3

Simplified approach to modeling

3.1 Variable inductance circuit model

A complete electromechanical model is provided in chapter 2. It is suitable for modelling a low-frequency magnetic machines, specifically an AMB device. As already stated, it is made for control logics design. First step in this part will be cascade control (it is fully discussed in chapter 5), so that an only-electrical model is required for inner loop. It can be directly extracted from full model, but it can be also rewritten in a simpler formulation. The one here developed is specifically made for soft hysteresis, because it introduces the following hypothesis: B-H hysteretic curve can be approximated as a mean non-linear curve. If no hysteretic behaviour is introduced, M is expressed by an algebraic equation as $M = M(x, I)$. As far as it holds, the following simplified electrical model provides a useful tool to reach these goals:

- Fast control implementation. Design one logic to implement in more magnets, collecting a library of ready-to-use controls
- Fast model implementation. Approach exposed does not require experimental field validation, so that it is simpler also to validate
- Alternative sensing techniques. Implement specimen position and velocity as modelled disturbs. If accurate enough, it may be useful in sensor-less estimation of gap position and velocity

The aim is, so, to develop a model of kind:

$$\dot{I} = \dot{I}(x, \dot{x}, I, V) \tag{3.1}$$

With I as state variable, V as input and x, $\dot{x}$ as disturbs. Consider usual electric circuit equation as 1.11 on page 6. It can be rearranged to isolate differential inductance (introduced in section 1.2 on page 5), assuming it varies only by position and current, as far as M is removed from state:

$$
\begin{aligned}
V &= IR + \frac{\mathrm{d}(LI)}{\mathrm{d}t} \\
V &= IR + \frac{\mathrm{d}L}{\mathrm{d}t}I + L\dot{I} \\
V &= IR + \left(\frac{\mathrm{d}L}{\mathrm{d}I}\dot{I} + \frac{\mathrm{d}L}{\mathrm{d}x}\dot{x}\right)I + L\dot{I} \\
V &= IR + \left(\frac{\mathrm{d}L}{\mathrm{d}I}I + L\right)\dot{I} + \frac{\mathrm{d}L}{\mathrm{d}x}I\dot{x} \\
V &= IR + L_d\dot{I} + \frac{\mathrm{d}L}{\mathrm{d}x}I\dot{x} \qquad (3.2)
\end{aligned}
$$

This formulation requires both L_d and L expressions as function of current and position. They can be obtained, indeed, from the main model (the one in Chapter 2), but they can also be estimated directly. Consider the system in generic steady-state (I_0, x_0). As far as electrical dynamics is stable and position is assumed as a disturb, it is an equilibrium point.

$$
V = IR + \left(\left.\frac{\mathrm{d}L}{\mathrm{d}I}\right|_{I_0} I_0 + L\right)\dot{I} + \cancel{\left.\frac{\mathrm{d}L}{\mathrm{d}x}\right|_{x_0} I_0\dot{x}} \quad \rightarrow \quad V = IR + L_d(I_0, x_0)\dot{I} \qquad (3.3)
$$

By this last formulation, it is locally a common RL circuit. The idea is so to evaluate local τ at each equilibrium point by a sequence of small steps. A staircase input returns a series of transients from which evaluate the constant. It is possible to build a $\tau - I$ function per each position. To generalize, regardless the gap, it si possible to asses that current behaviour is always of the same type: slow system for low currents, faster system above a certain current threshold, with a narrow transition between these two sections. The step-series estimation approach is in line with the goals of this simplified model: provide a fast and reliable approach to deal with variable inductance without modelling and experimental validating dynamics of magnetization and field. It is advisable to plan an huge number of trials, with both ascending and descending stairs, in order to average hysteretic effects. The result is a Linear Parameter Varying system (LPV)[26], whose varying parameter (τ) can be directly expressed by a grid or with a fitting function [21].
In order to properly build a reliable model, also the "absolute" inductance L has to

be evaluated. One quick way to do it is to consider τ definition, applied in the case of constant position x_0. For the sake of clearness, consider current derivatives as $f'(x)$:

$$R\tau(I) = L'(I)I + L(I) \tag{3.4}$$

Which is an ODE of the kind:

$$y'(x) = \frac{f(x) - y(x)}{x} \tag{3.5}$$

As far as $f(x)$ is provided numerically, also the equation has to be solved with a proper method. A typical variable step Runge-Kutta Method solver, as ODE45 is able to provide the solution. Each gap position results in a different curve, indeed. They are so merged in a surface as function of current and position. Absolute inductance allows to evaluate two fundamental quantities: velocity disturbs and core magnetic field. The first one is made by evaluating the remaining term of eq. 3.2 on the facing page. The second by inductance definition (equation 1.11 on page 6). Validity and limitations of this approach are assessed in the following unit, together with experimental validation of main model. Application in control logics design is a topic of Chapter 5. In order to provide a few references, starting from what is

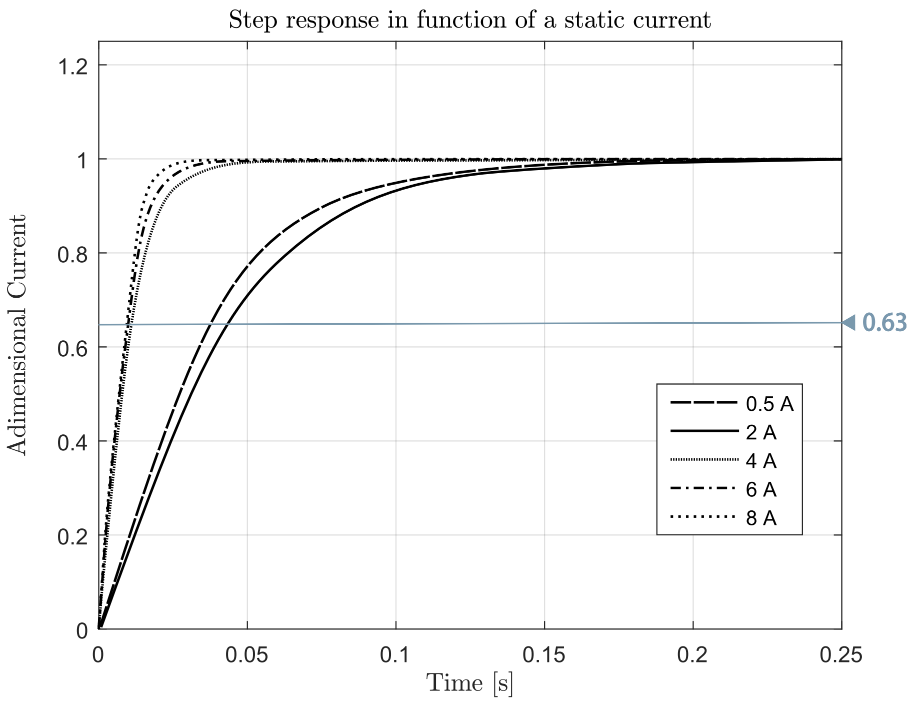

Figure 3.1: Current steps normalized for a generic fixed gap. Below saturation, fast field growth results in a higher flux variation (and so inductance). Over saturation, field is slow and so it is its flux derivative, returning faster current dynamics

asserted in figure 1.4 on page 6, final inductance returns a shape like the one in figure 3.2. This map is not so uncommon in transformers theory, related to DC bias effect[5][27]. Nevertheless, saturation is generally not considered, so that the final plateau of this map is cut off in Inductance vs DC-bias curves. For informational purpose, a good function to fit this surface is:

$$L(x, I) = p_{x,1} \operatorname{atan}(p_{x,2} I + p_{x,3}) + p_{x,4} \tag{3.6}$$

Where $p_{x,i}$ are polynomial functions in x. Generally, in fact, trigonometrical functions and affine are used to express relations about saturations and hysteresis. Jiles-Atherton anhysteretic curve (compare 2.30 on page 27) is based on $\coth(x)$. As far as it holds that:

$$B(x, I) = \frac{L(x, I)I}{A} \tag{3.7}$$

Also total magnetic field density in core can be evaluated with the same function.

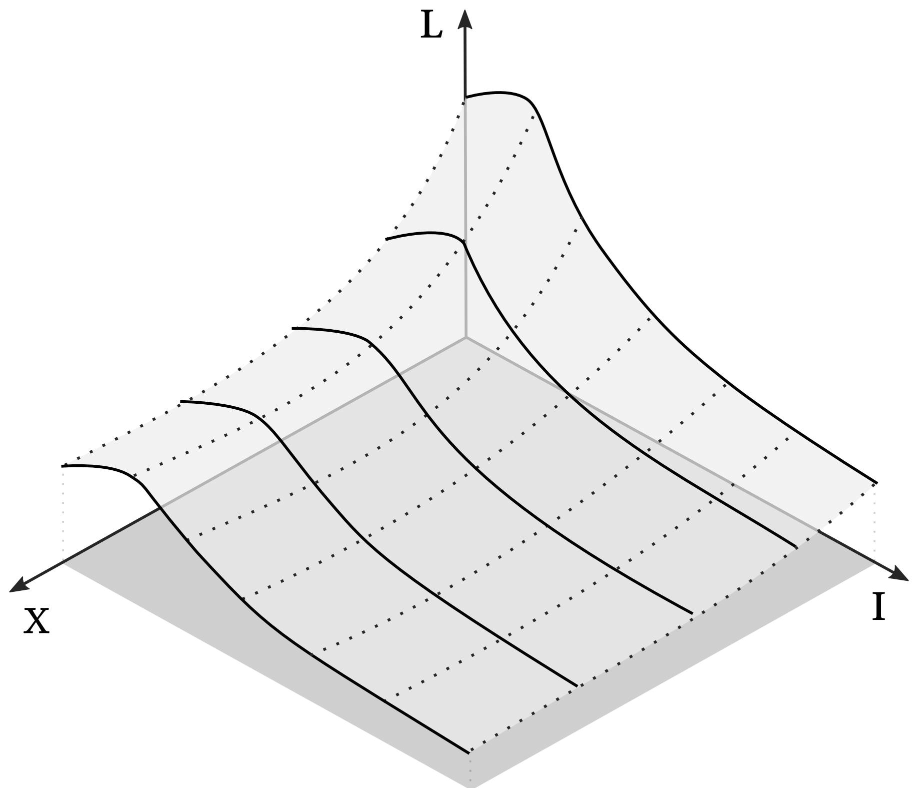

Figure 3.2: Inductance as function of current and position, obtained by evaluation of curves for different gap (continuous lines), to be merged (dotted lines) in a 3D surface. They are generally made by a first hight plateau (depending on initial permeability), a decreasing part and a final low plateau (saturation reached)

3.2 Force-current mechanical model

Consider the final equation for force (eq. 2.27 on page 25). It can be rewritten in an even more compact form, as:

$$F = K(x)\frac{B(x,I)^2}{\mu_0} \tag{3.8}$$

As made for current dynamics, the first step is to get free from magnetization. It means to get the so-called "anhysteretic curve". There's no need of B-H curve, B-I curve in function of gap is enough. It can be done by experimentally measure it[28], if its possible to record also magnetic field in gap. A simpler way, but with higher errors, is to estimate B as exposed in previous section. As far as a function for B is obtained, it is possible to estimate also $K(x)$ function. The idea is that classical maglev theory is not completely useless in this application. It holds, indeed, in a very small interval around an equilibrium position. If current dynamics is already assessed (as suggested in previous chapter), a cascade control with constant gains should be enough to keep the specimen in a specific position. PI control for current can be directly designed (refer to Chapter 5 if necessary), while a trial and error approach for a simple P control in position takes not too much time to work. The goal is to return a $x_0 - I_0$ equilibrium relation. To be more specific, the result is a function as $I_0 = I_0(x_0)$, because at each position corresponds its current. The next step is to evaluate $K(x)$ for each equilibrium point. It is takes for guaranteed that specimen weight is known. So that, equation 3.8 can be rewritten as:

$$mg = K(x_0)\frac{B(x_0, I_0(x_0))^2}{\mu_0} \quad \rightarrow \quad K(x_0) = \frac{\mu_0 mg}{B(x_0)^2} \tag{3.9}$$

At the end, mechanical dynamics is expressed by an equation like:

$$\ddot{x} = \frac{f(x,I)}{m} - g \tag{3.10}$$

Which can be linearised around each equilibrium position as

$$\ddot{x} = \frac{1}{m}\frac{\partial f(x,I)}{\partial x}\bigg|_{x_0}(x - x_0) + \frac{1}{m}\frac{\partial f(x,I)}{\partial I}\bigg|_{x_0,}(I - I_0(x_0)) \tag{3.11}$$

As fas ar this linearisation holds only at equilibrium position, this time the resulting LPV system is scheduled only in one dimension.
A rough method to expand the LPV system also in that regions in which specimen is not at equilibrium is to approximate derivatives with different quotients of currents

with respect to equilibrium current:

$$\ddot{x} = \frac{1}{m}\frac{\partial f(x,I)}{\partial x}\bigg|_{x_0}(x-x_0) + \frac{1}{m}\frac{\Delta f(x,I)}{\Delta I}\bigg|_{x_0}(I - I_0(x_0)) \tag{3.12}$$

This formulation returns a system with two state variables $(x, \dot{x})$, one input I and typically position as output. Despite all, it is not so useful in control design for this specific reason: current dynamics (compare figure 3.1 on page 45) is not so fast with respect to specimen one. It affects classical cascade hypothesis, which says that current dynamics (inner loop) is seen as an algebraic expression from position (outer loop). This hypothesis, indeed, holds in actual case only with a really fast inner loop. In turn, it requires an high-speed control, with proper saturation limits. If performances are not good enough, controlled dynamics must be implemented even in a cascade design. This is better explained in Chapter 5. Up to this point, it may be enough to consider the last step of this simplified approach, which is a a full LPV system, embedding both dynamics.

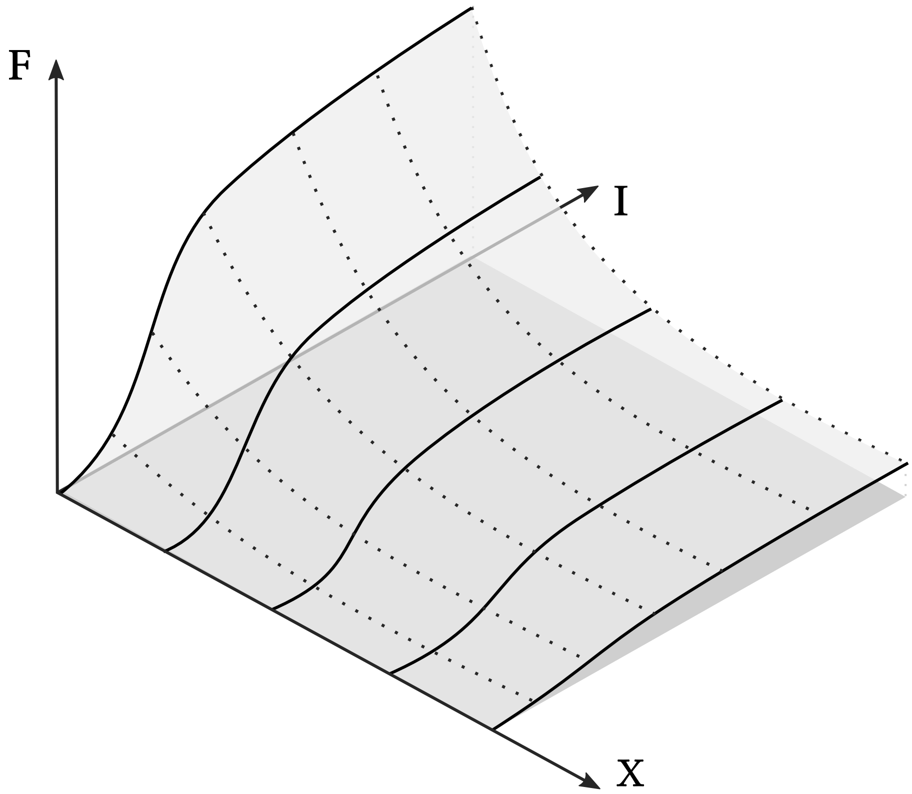

Figure 3.3: Force as function of current and position, obtained by evaluation of curves for different gap (continuous lines), to be merged (dotted lines) in a 3D surface. They are generally made by a first hight-slope segment (depending on initial permeability), a decreasing part and a final small-slope region (saturation reached)

3.3 Complete anhysteretic model

Equations developed in previous sections can be merged in a full LPV non-hysteretic model, defined by the following state space form:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{m}\frac{\partial f(x,I)}{\partial x}\Big|_{x_0} & \frac{1}{m}\frac{\Delta f(x,I)}{\Delta I}\Big|_{x_0} \\ 1 & 0 & 0 \\ -\frac{1}{R\tau}\frac{\mathrm{d}L}{\mathrm{d}x}\Big|_{x_0} I_0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{R\tau} \end{bmatrix} V \quad (3.13)$$

This formulation can be a reliable tool for control design of a maglev system. It takes into account both coupling of electrical and mechanical worlds, in therms of force and inductance. Moreover, it can be build not only as a simplification of the main hysteretic model, but also with a limited set of data, even with no gauss-meter, no load cell or neither. Main parameters to be scheduled are τ and f derivatives, which can be expressed by a proper function or collected in a table-like structure. In first case, it allows to work with also non-linear control logics, while the second one is suitable for scheduled linear controls. Regarding simulation phase, it is always better to rely on the full model, by its more completeness. At the end, a visual framework, as figure 3.4, is provided in order to summarize what can be done as a simplified approach.

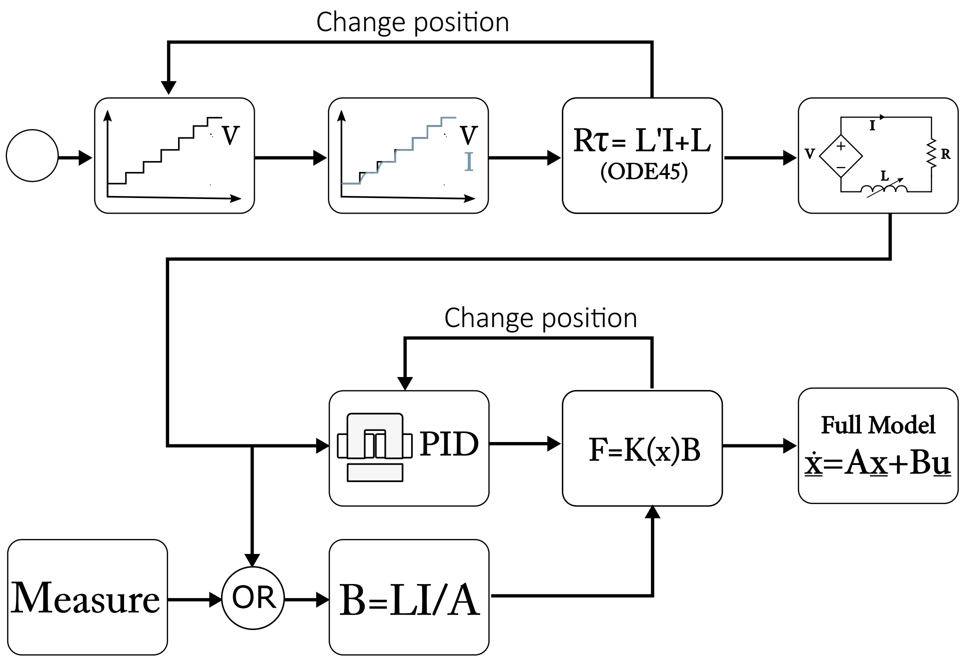

Figure 3.4: Simplified approach in modellization flowchart