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To cite this article: Deepak Sharma *et al* 2018 *IOP Conf. Ser.: Mater. Sci. Eng.* **377** 012156

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# Modelling and state estimation for control of magnetic levitation system via a state feedback based full order observer approach

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**Abstract.** This work proposes to develop mathematical model and event trigger control using a state observer, which precisely tracks position of the solid steel ball of magnetically actuated levitation system. The lumped uncertainties i.e. non-linearities exist in the magnetic levitation system, overwhelmed by the state observer. State space model was derived by the system equation. Based on desired specification required poles are placed using pole placement method to stabilize the closed loop system in response to a small size step input then full order observer estimate the state variables. Here ball position and coil current is directly measurable, but velocity is not directly measurable, which has been investigated in this work.

## 1. Introduction

Magnetic levitation is an electromechanical coupling concept which enables the contactless motion and levitation of an object with the help of magnetic field. The magnetic force overwhelms the gravitational force. The magnetic field is created by the electromagnetic coil and current in the coil controls the magnitude of the magnetic force. This force can be attractive or repulsive type. Magnetic levitation (Maglev) is a non-contact technology; it has a wide area of application such as high speed maglev train [1], magnetic bearing, launching rocket, electromagnetic aircraft system, maglev wind turbine and maglev micro robot etc. It offers many advantages such as friction less movement, high speed, isolation of environment and low maintenance. The modelling and control of this system is difficult due to the system behaviour, which is highly non-linear in nature. Consequently, the system becomes unstable with fast dynamics. A lot of control techniques [2] and methods discussed to overcome nonlinearity such as feedback linearization [3], Gain Scheduling linear controller design, feed forward control, linear and nonlinear state space control [4-5].

In this paper, a single axis (1DOF) magnetic levitation system is considered as shown in Figure 1. The system consists of an electromagnetic coil which is an upper part of apparatus, analogue and digital interface, power amplifier, one inch diameter solid steel ball and position sensor. This study develops non-linear dynamic model of the system which is linearized about the operating point by Taylor's series approximation method [6]. The state variables considered for this system are ball position, velocity of the ball and current through the electromagnet. Mathematical modelling of the system is developed by the study of electrical and mechanical subsystem. According to design specification, a state observer [7] is proposed, which estimate the state variables of the system. Position sensor of the apparatus tracks the ball position directly but there is no device to measure the



velocity of the ball so that the full order observer [8] is considered to estimate velocity of the ball and the other state variables.

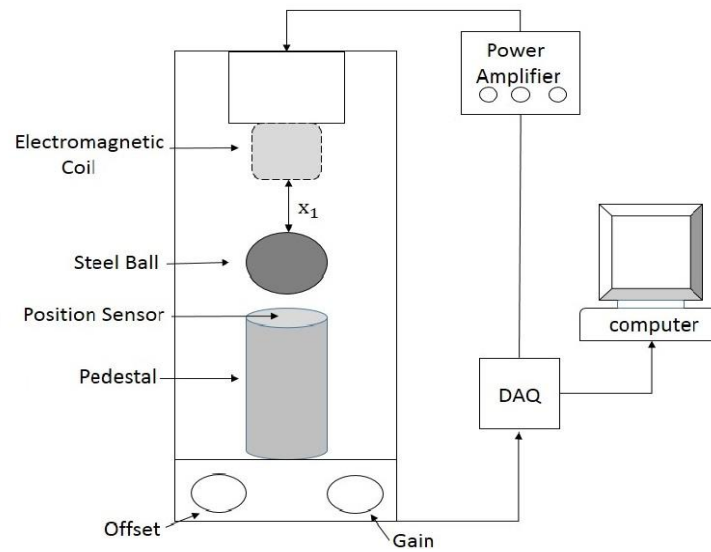


Figure 1. Magnetic levitation system set up

## 2. Dynamic modelling of maglev system

The schematic diagram of the maglev system is shown in Figure 2. The ball is levitating in free space has a six degree of freedom but only vertical axis of the system is control. The system used L-R circuit for control action which is divided into two following systems i.e. an electrical and an electromechanical [9].

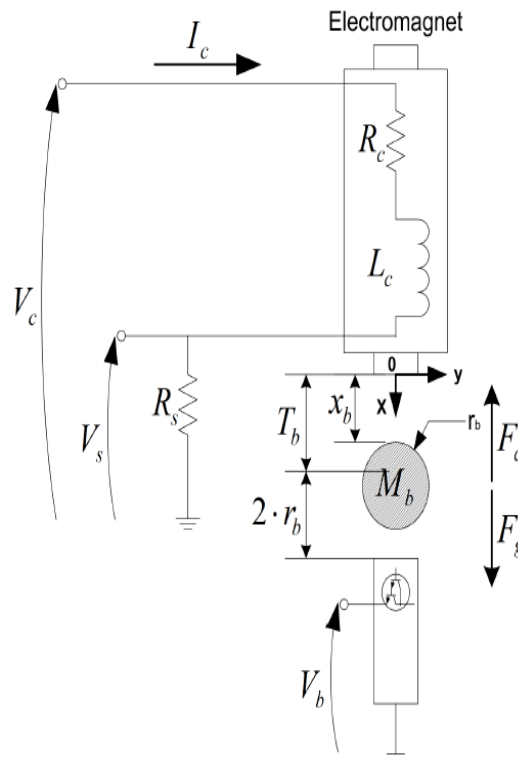


Figure 2. Schematic view of maglev plant

Table 1. System parameters

Notations	Description	Numerical value	Notations	Description	Numerical value
$L_c$	Inductance of the coil	412.5mH	$r_b$	Radius of the ball	0.0127m
$R_c$	Coil resistance	10 $\Omega$	$l_c$	Length of the coil	0.0825
$g$	Acceleration due to gravity	9.81 m/s <sup>2</sup>	$M_b$	Mass of the ball	0.068kg
$K_m$	Force constant of electromagnet	6.5308 $\times 10^{-5}$ Nm <sup>2</sup> /A <sup>2</sup>	$K_b$	Ball position sensor sensitivity	0.0028
$R_s$	Current sense resistance	1 $\Omega$	$N_c$	Coil wire turns	2450
$r_c$	Coil core radius	0.008m			

The voltage sense ( $V_s$ ) is used to measure the current in the coil. The coil current can be computed using the following relationship

$$V_s = i(t)_s R_s \quad (1)$$

The first-order differential equation is obtained by Kirchhoff's voltage law,

$$V_c(t) = (R_s + R_c)i_c(t) + \frac{d(L(x)i_c(t))}{dt} \quad (2)$$

Where,  $V_c$  is applied coiled voltage and  $i_c$  is the coil current. The force induced by the electromagnet acting on the ball,

$$F_c = \frac{K_m i_c(t)^2}{2x_b^2} \quad (3)$$

Where,  $x_b > 0$  is the air gap between the ball and the face of the electromagnet and  $K_m$  is the electromagnetic force constant. The force due to gravity on the ball is given by,

$$F_b = M_b g \quad (4)$$

Applying the Newton's second law of motion on the ball,

$$M_b \ddot{x}_b = -\frac{K_m i_c(t)^2}{2x_b(t)^2} + M_b g \quad (5)$$

Assuming  $x_b = x_1$ ;  $v = x_2$ ;  $i_c = x_3$ ;  $u = V_c$ .

Non-linear state-space model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ g - \frac{K_m}{2M_b} \left(\frac{x_3}{x_1}\right)^2 \\ -\frac{R_c + R_s}{L_c} x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_c} \end{bmatrix} u \quad (6)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (7)$$

Where,  $v$  is the ball velocity and position of ball ( $x_b$ ) is taken as output.

This nonlinear model is linearized about an operating point (equilibrium position). By Taylor's approximation about  $x_b = x_{10} = x_{b0}$ ,  $i_c = i_{c0} = x_{30}$ .

The associated state vector  $X_0 = [x_{10} \ 0 \ x_{30}]$ . Equilibrium position of ball  $x_{b0} = 6$  mm and equilibrium current  $i_{c0} = 0.86$  A.

Linearized model in state space form as under,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2g}{x_{10}} & 0 & -\frac{2g}{x_{30}} \\ 0 & 0 & -\frac{R_c + R_s}{L_c} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_c} \end{bmatrix} u \quad (8)$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2g}{x_{10}} & 0 & -\frac{2g}{x_{30}} \\ 0 & 0 & -\frac{R_c + R_s}{L_c} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_c} \end{bmatrix} \quad C = [1 \ 0 \ 0] \quad (9)$$

### 3. Stability

Maglev open loop system is unstable because one of the Eigen values of  $\det(sI - A) = 0$ ,  $[p_1 = 57.1839, p_2 = -57.1839, p_3 = -26.6700]$  lies in positive half of the s-plane. The response of open loop for small amount of step is shown below,

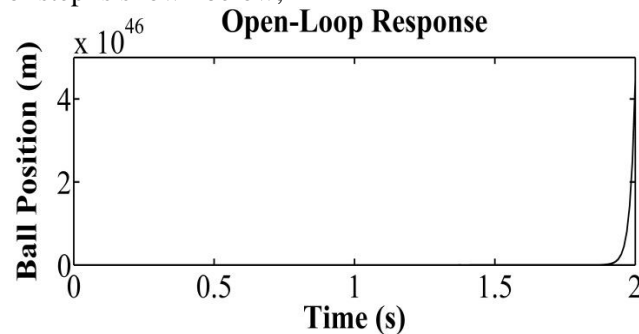


Figure 3. Open loop response ball position

The response shows that the ball is going outside the range of linearization validity that means the ball hits the electromagnet or the pedestal.

According to design specification, the ball position behaviour should satisfy the following criteria: The settling time and the overshoot should be less than 0.5 sec and 5% respectively, based on that following 2nd order parameter  $\zeta=0.7$ ,  $\omega_n=20$  rad/s are needed, the two dominant poles at  $-14 \pm 14i$  and the third pole is placed at  $-80$  and feedback gain matrix  $K$  is calculated in matlab.

$$K = [6569 \ 106.9 \ -33.6]$$

For a small amount of step input, the value of  $\bar{N} = 568.1139$  is calculated using matlab commands, run the simulation to check whether it is able to track the trajectory or not.

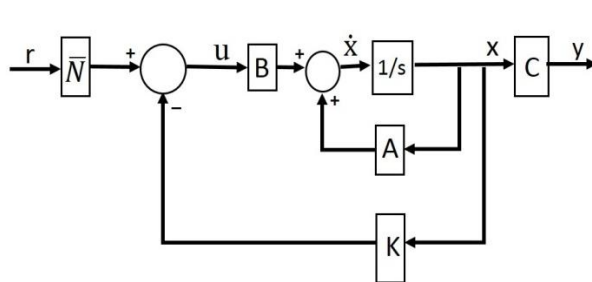


Figure 4. Block diagram of feedback control system

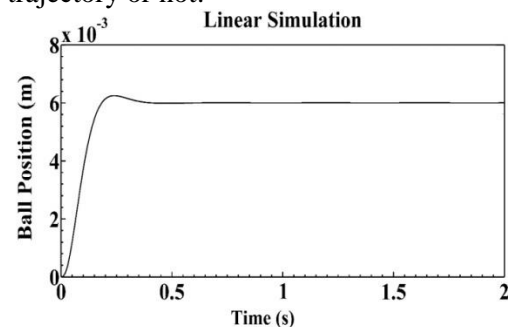


Figure 5. Closed loop response ball position

The closed loop response of ball position is shown in Figure 5. If all the states variables are not measurable, design an observer to estimate them. In maglev system only ball position and coil current can be measurable directly but velocity cannot be measurable directly, so full order observer is used and three new estimated states added.

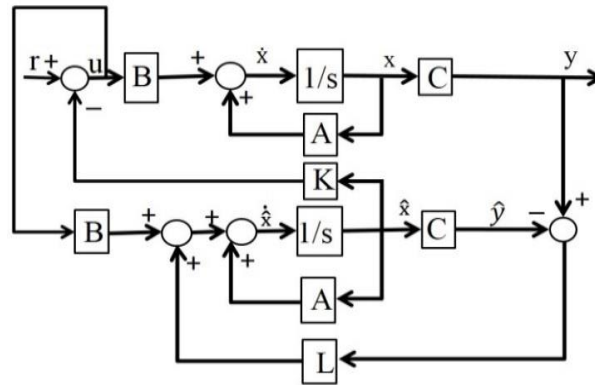


Figure 6. Block diagram of full order state observer

The observed matrices for the dynamics of the system is given below,

$$A(t) = \begin{bmatrix} A - B * K & B * K \\ \text{zeros}(\text{size}(A)) & A - L * C \end{bmatrix}, B(t) = \begin{bmatrix} B * \bar{N} \\ \text{zeros}(\text{size}(B)) \end{bmatrix}, C(t) = [C \quad \text{zeros}(\text{size}(C))] ]$$

The poles (Eigen value of matrices) of  $(A-LC)$  give the error dynamics of the observer [8]. The behaviour of the observer should be faster than that of the system. In order to get this, the poles are placed much far away from the dominant poles in the left half of s-plane [7].

Here observer poles are placed at  $op_1 = -100$ ,  $op_2 = -105$ ,  $op_3 = -110$ . By using full state feedback observer, the error dynamics of the state is illustrated as:

$$e = x - x_{\text{est}}.$$

(10)

$$\dot{e} = (A-LC) e$$

Assuming the initial condition of the observer to be zero as a result of which the state's initial condition and the error's initial condition will same. If matrix is an asymptotically stable matrix, the error vector will converge to zero for any initial error vector  $e(0)$ . The error dynamics tends to zero as  $t \rightarrow \infty$  [5] [8]. Plot the responses for all the states  $x$  and respective estimated states  $x_{\text{est}}$ . Using matlab, simulation responses are plotted. All the state variables ( $x$ ) are represented by solid line and estimated states ( $x_{\text{est}}$ ) are represented by dotted line. The behaviour of the state can be estimated by the observer quite perfectly and faster.

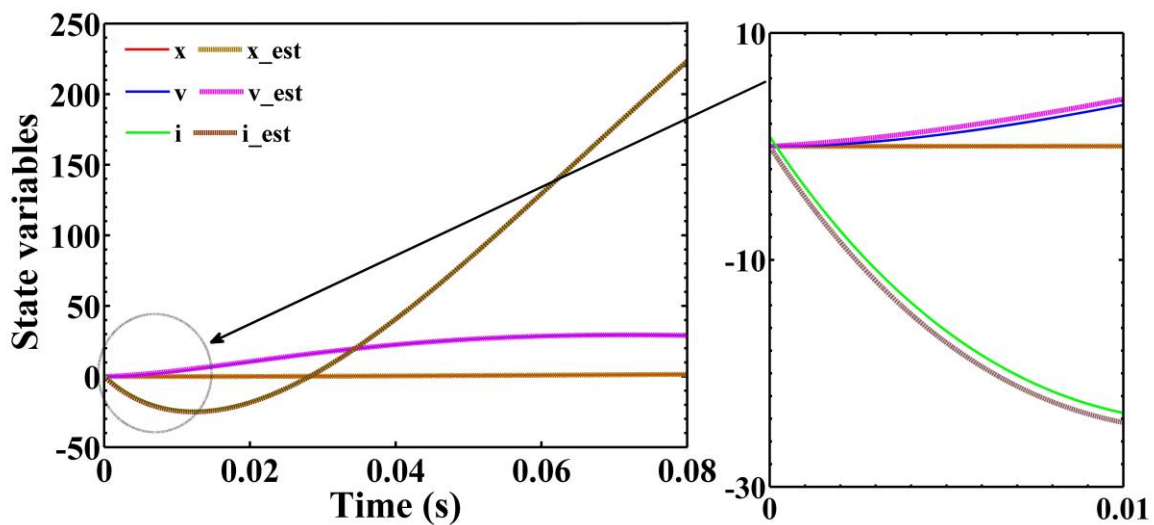


Figure 7. Response of state observed variable

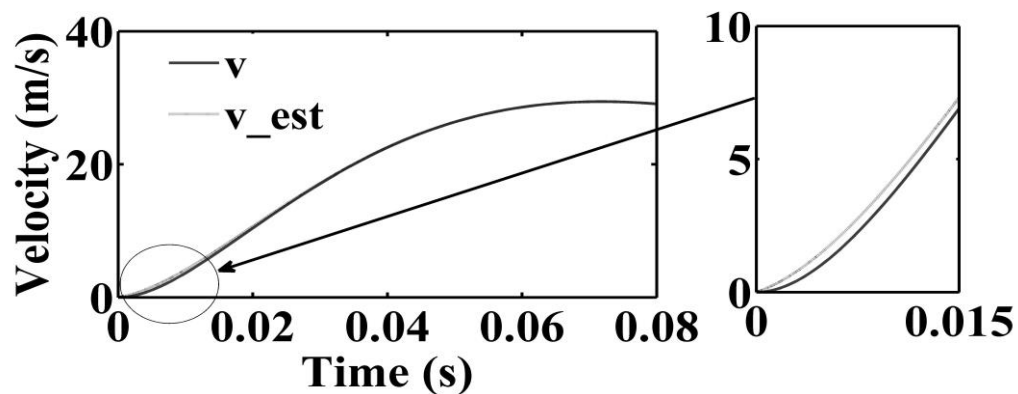


Figure 8. Actual velocity and estimated velocity of ball

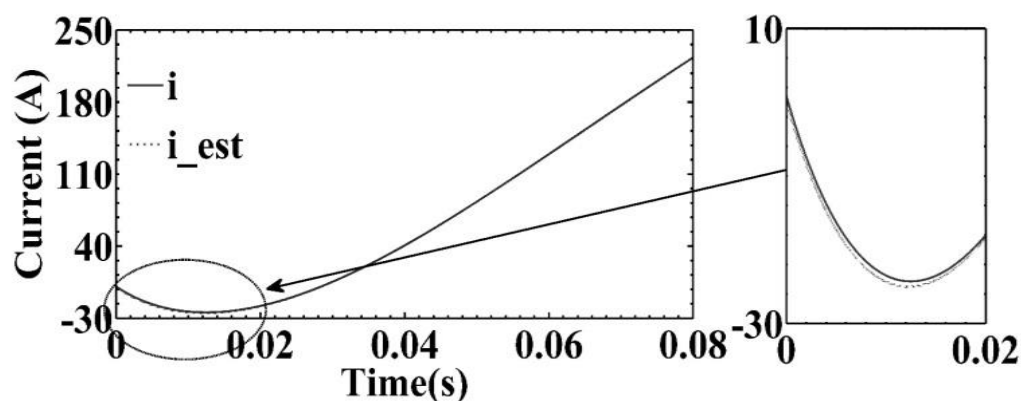


Figure 9. Actual current and estimated current

#### 4. Conclusions

The modelling and control of 1DOF magnetic levitation system has been investigated in this paper. The mathematical models of the plant are derived by the general physical laws of the system and the non-linearity is linearized by Taylor's approach about an operating range. Based on design criteria desired poles are placed to obtain a stable response of the system. A full order state observer is designed for the systems, the result is able to track the reference step input quite accurately. Simulation results illustrates the stabilisation of the ball and estimate the unknown states effectively.

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