Nonlinear control problem (indirect approach)

Optimization of the shuttle's thrust, based on gradient descent algorithm with free-end state, fixed final time.

$$
\begin{aligned}
m\dot{v} &= T - C_D v^2 - mg\sin\gamma \\
m\dot{\gamma}v &= C_L v^2 - mg\cos\gamma \\
\dot{h} &= v\sin\gamma \\
\dot{m} &= -\alpha T
\end{aligned}
$$

$$
\mathbf{x} = \begin{bmatrix} h \\ v \\ m \\ \alpha \end{bmatrix} \qquad u = T
$$

$$
A(\mathbf{x}, u) = \begin{bmatrix} 0 & \sin x_4 & 0 & x_2\cos x_4 \\ 0 & \frac{-2C_D x_2}{x_3} & -\frac{u - C_D x_2^2}{x_3^2} & -g\cos x_4 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{C_L}{x_3} + \frac{g\cos x_4}{x_2^2} & -\frac{C_L x_2}{x_3^2} & \frac{g\sin x_4}{x_2} \end{bmatrix} \qquad B(\mathbf{x}, u) = \begin{bmatrix} 0 \\ \frac{1}{x_3} \\ -\alpha \\ 0 \end{bmatrix}
$$

NASA space shuttle has been taken as reference for parameters and trajectory data [1].

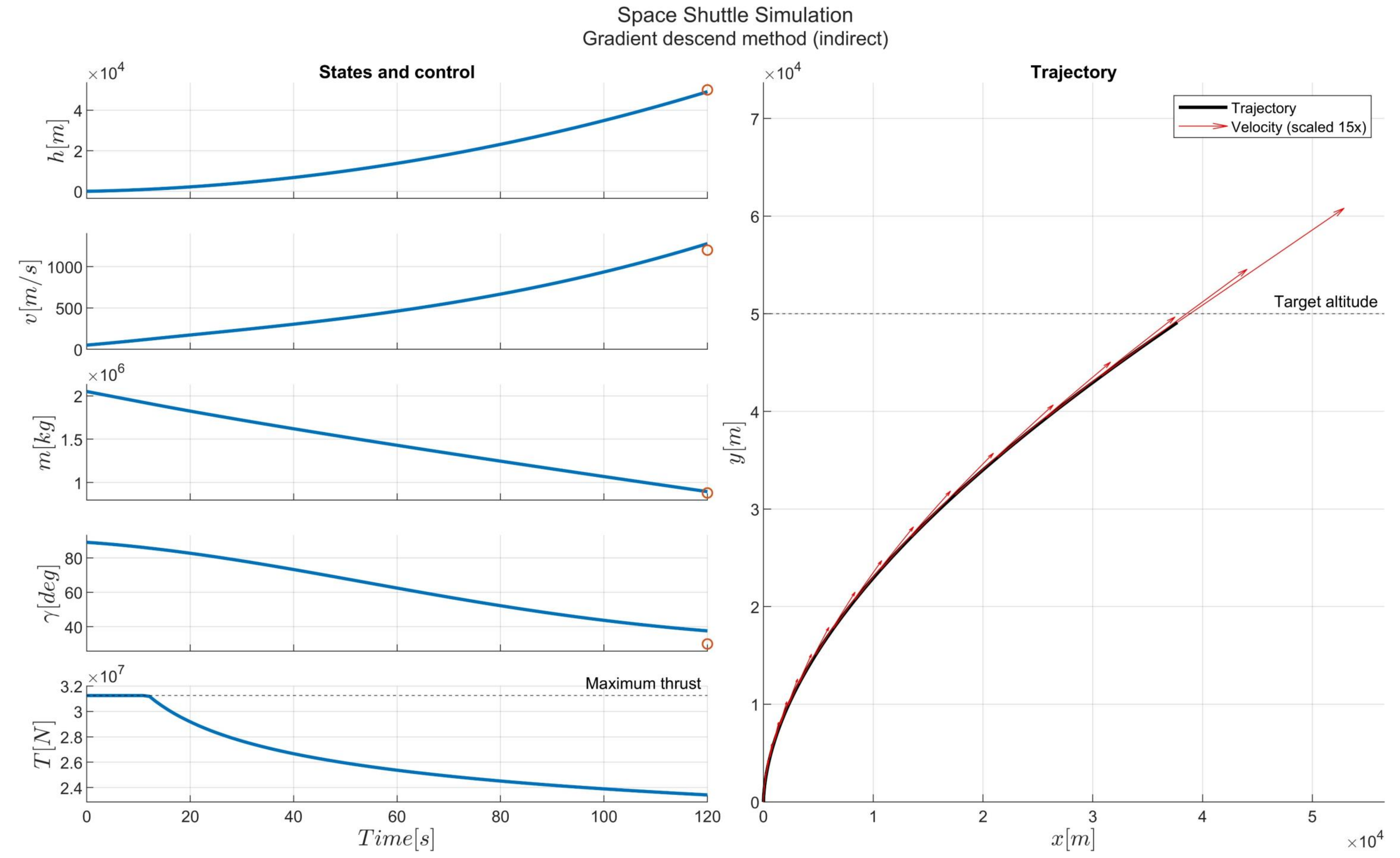
	Initial$(\mathbf{x}_0)$	**Final**$(\mathbf{x}_f)$	**Unit**
x_1	0	$50 \cdot 10^3$	[m]
x_2	50	1200	[m/s]
x_3	$2050 \cdot 10^3$	$880 \cdot 10^3$	[kg]
x_4	89	30	[deg]

Cost function matrices chosen to prioritize final state accuracy while also limiting control effort. Upper bound on control given by maximum available thrust force ($u \leq 31{,}25 \cdot 10^6\ [N]$).

$$
J(\mathbf{x}, \mathbf{u}) = \frac{1}{2}(\mathbf{x}(t_f) - \mathbf{x}_f)^T \mathbf{P}(\mathbf{x}(t_f) - \mathbf{x}_f) + \frac{1}{2}\int_{t_0}^{t_f} \mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{u}^T\mathbf{R}\mathbf{u}dt
$$

```
Q = diag([zeros(4, 1)]);
R = diag([1e-6]);
P = diag([300 1000 0.01 1]);
```

[1] Velocity-and-Acceleration-Profiles-of-Space-Shuttles_Kwan-Jie-Lee.pdf



Space Shuttle Simulation
Gradient descend method (indirect)
States and control
$\times 10^4$
$h[m]$
4
2
0
$v[m/s]$
1000
500
0
$\times 10^6$
$m[kg]$
2
1.5
1
$\gamma[deg]$
80
60
40
$\times 10^7$
3.2
3
2.8
2.6
2.4
$T[N]$
Maximum thrust
0
20
40
60
80
100
120
$Time[s]$
Trajectory
$\times 10^4$
7
6
5
4
3
2
1
0
$y[m]$
Trajectory
Velocity (scaled 15x)
Target altitude
0
1
2
3
4
5
$x[m]$
$\times 10^4$