

# Nonlinear control problem (direct approach)

Optimization of the shuttle's thrust, based on direct transcription method with free-end state, fixed final time.

$$\begin{aligned}
 m\dot{v} &= T - C_D v^2 - mg \sin \gamma \\
 m\dot{\gamma} &= C_L v^2 - mg \cos \gamma \\
 \dot{h} &= v \sin \gamma \\
 \dot{m} &= -\alpha T
 \end{aligned}
 \quad
 \mathbf{x} = \begin{bmatrix} h \\ v \\ m \\ \alpha \end{bmatrix}
 \quad
 u = T
 \quad
 A(\mathbf{x}, u) = \begin{bmatrix} 0 & \sin x_4 & 0 & x_2 \cos x_4 \\ 0 & \frac{-2C_D x_2}{x_3} & -\frac{u - C_D x_2^2}{x_3^2} & -g \cos x_4 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{C_L}{x_3} + \frac{g \cos x_4}{x_2^2} & -\frac{C_L x_2}{x_3^2} & \frac{g \sin x_4}{x_2} \end{bmatrix}
 \quad
 B(\mathbf{x}, u) = \begin{bmatrix} 0 \\ \frac{1}{x_3} \\ -\alpha \\ 0 \end{bmatrix}$$

NASA space shuttle has been taken as reference for parameters and trajectory data [1].

	Initial( $\mathbf{x}_0$ )	Final( $\mathbf{x}_f$ )	Unit
$x_1$	0	$50 \cdot 10^3$	[m]
$x_2$	50	1200	[m/s]
$x_3$	$2050 \cdot 10^3$	$880 \cdot 10^3$	[kg]
$x_4$	89	30	[deg]

Cost function matrices chosen to prioritize final state accuracy while also limiting control effort. Upper bound on control given by maximum available thrust force ( $u \leq 31,25 \cdot 10^6$  [N]).

$$J(\mathbf{x}, \mathbf{u}) = \frac{1}{2}(\mathbf{x}(t_f) - \mathbf{x}_f)^T \mathbf{P}(\mathbf{x}(t_f) - \mathbf{x}_f) + \frac{1}{2} \int_{t_0}^{t_f} \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} dt$$

$$\begin{aligned}
 \mathbf{Q} &= \text{diag}([\text{zeros}(4, 1)]); \\
 \mathbf{R} &= \text{diag}([1e-6]); \\
 \mathbf{P} &= \text{diag}([300 \ 1000 \ 0.01 \ 1]);
 \end{aligned}$$

# Space Shuttle Simulation

Direct transcription (direct)

