The nonlinear dynamics of an electromechanical actuator with nonlinear spring and inductance is:

$$m\ddot{x} + c\dot{x} + k_1x + k_3x^3 = \alpha i$$

$$(L_0 + \beta_1 T + \beta_2 T^2) \dot{i} + Ri = u$$

$$\dot{T} = \frac{1}{C_T} \left[Ri^2 - h \left(T - T_{env} \right) \right]$$

$$T_{env}$$

$$k(x) \qquad i$$

$$L(T)$$

$$x_1 = x$$
 Piston displacement

$$x_2 = \dot{x}$$
 Piston velocity

$$x_3 = i$$
 Current

$$x_4 = T$$
 Temperature

$$u = V$$
 Voltage (control input)

 C_T Thermal capacity h Thermal dissipation coefficient

 T_{env} Ambient temperature m Mass of the piston

 L_0 Inductance at a reference temperature T=0

 $eta_{1,2}$ Coefficients for the T-dependent inductance

 $k_{1,3}$ Coefficients for the x-dependent stiffness

lpha Force constant R Resistance

The nonlinear dynamics of an electromechanical actuator with nonlinear spring and inductance is:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1}{m}x_1 - \frac{k_3}{m}x_1^3 - \frac{c}{m}x_2 + \frac{\alpha}{m}x_3$$

$$\dot{x}_3 = -\frac{R}{L(T)}x_3 + \frac{1}{L(T)}u$$

$$x_4 = \frac{1}{C_T}\left[Rx_3^2 - h\left(x_4 - T_{env}\right)\right]$$

$$x_1 = x$$
 Piston displacement
$$x_2 = \dot{x}$$
 Piston velocity
$$x_3 = i$$
 Current
$$x_4 = T$$
 Temperature
$$u = V$$
 Voltage (control input)
$$T_{env}$$

Design a LQR that operates around a certain reference voltage and employs a KF for estimate the full state. In addition to the full state feedback, provide an estimate of one of the characteristics constants starting from uncertain initial conditions.