ME663 Assignment #1: Navier-Stokes Solver for an Incompressible Fluid Flow

Handout #2
Due Date: March 1, 2024 (Friday)

Write a computer program using Fortran 77, Fortran 90, C, C++, Python, Matlab, or JAX to solve the incompressible Navier-Stokes equation based on Vanka's [1] Symmetric Coupled Gauss-Seidel (SCGS) method. The objective here is to find out the numerical solutions for a flow in a square cavity with the top wall moving at a constant velocity U_{lid}.

1. Use UDS, Hybrid, and/or QUICK convection schemes on a mesh of 40x40 and 80x80 nodes at Re=400 and 1000, where $Re= U_{lid} L/\nu$ and L is the length of the cavity.
2. Set the convergence criterion to 10^{-4}.
3. Report the wall-clock time and total number of iterations for each case.
4. Compare your results in terms of u(y) at x=0.5L and v(x) at y=0.5L with results in Tables I and II from Ghia et al.'s [2] paper.
5. What is the optimal under-relaxation factor for each case?
6. If you try more than one convection scheme**,** which scheme is more accurate compared to Ghia et al.'s 'exact solutions'?
7. Use 2nd-order and 4th-order schemes to approximate the diffusion term $\nu[\partial^2( )/\partial x^2 + \partial^2( )/\partial y^2]$ in the Navier-Stokes equation.
8. The report should contain at least the following 3 parts:
    a. Derivation of your discretized equation including boundary condition;
    b. Results and discussion;
    c. Printout of your computer program with brief explanation as Appendix.
9. Provide me your source code(s) via Email with a simple README file about how to run your code and read your output files.

Note: Submitting your report and source code(s) to Dropbox on Learn.

References:

[1] S.P. Vanka (1986), Block-Implicit Multigrid Solution of Navier-Stokes Equations in Primitive Variables, *Journal of Computational Physics*, Vol. 65, 138-158.

[2] U. Ghia, K.N. Ghia, C.T. Shin (1982), High-Re Solutions for Incompressible Flow Using the Navier-Stokes Equations and a Multigrid Method, *Journal of Computational Physics*, Vol. 48, 387-411.