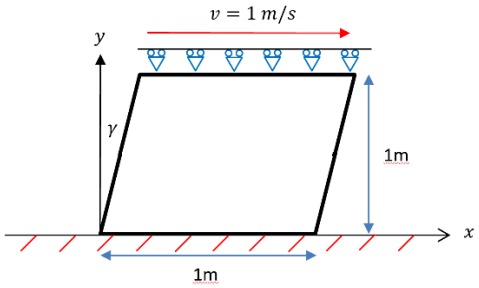
ME-621 Advanced Finite Element Method

Assignment 4

Due Date: **Mon Apr 15, 2024 11:59 pm**

Problem

The goal of this assignment is to write an elastic-plastic stress update algorithm for simple shear deformation using MATLAB. It is important to note that this algorithm is independent from the finite element analysis and is strictly on the implementation of computational plasticity. This implementation is similar to writing a custom user-defined material subroutine (UMAT) for commercial FE software, with the exception that you must obtain the kinematic variables across time.



Consider a square-shape aluminum plate under simple shear deformation. The deformation gradient for simple shear is:

$$F = \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix}$$

I. Obtain infinitesimal strain in the simple shear problem when the shear deformation is linearly increased from zero to $\gamma = 1$. Use constant time increments.

II. Implement von-Mises plasticity according to the radial return algorithm described in the lecture notes. Use the following elastic constitutive equation in modeling of the elastic-plastic deformation. Do not consider kinematic hardening.

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{C} : \dot{\boldsymbol{\varepsilon}}$$

Plot the shear stress as a function of shear strain, γ.

Assume the material has a Young's modulus of $E = 70$ GPa and a Poisson's ratio of $v = 0.3$. During plastic deformation, the material hardens according to the following isotropic hardening equation.

$$\bar{\sigma} = \sigma_{Y0} + K(\bar{\epsilon}^p)^n$$

where $\sigma_{Y0} = 200\ MPa$, $K = 325\ MPa$, and $n = 0.125$.

III. Use the following elastic constitutive equation instead of the one in part II and compare the results

$$\boldsymbol{\sigma}^{oT} = \boldsymbol{C} : \boldsymbol{D}$$

Is there any significant difference between the results? Why?

$\boldsymbol{\sigma}^{oT}$ is the Truesdell rate of Cauchy stress and **D** is the rate-of-deformation tensor.

Hint: The trial stress should be modified based on the following stress rate:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{C} : \boldsymbol{D} + \boldsymbol{L} \cdot \boldsymbol{\sigma} + \boldsymbol{\sigma} \cdot \boldsymbol{L}^T - \boldsymbol{tr}(\boldsymbol{L})\boldsymbol{\sigma}$$

Project Evaluation

This is a graduate level project/assignment and you need to submit a reasonable report explaining your solution process and code must be properly commented.

- **This is an individual assignment.**
- You must write proper and enough comment lines in your code to be understandable for the grader. DO NOT use MATLAB to take derivatives, this is a numerical course. And avoid using symbolic MATLAB as well. FE codes run super slow using sym.
- In your report, explain the process briefly and efficiently to be understandable for the reader.
- The report can be either typed or legibly handwritten (preferably using apps on tablets and laptops) and submitted electronically.
- The report file must be either in PDF or MS Word format.
- Upload your files to the dropbox folder in LEARN.

References for code commenting

- https://www.elegantthemes.com/blog/wordpress/how-to-comment-your-code-like-a-pro-best-practices-and-good-habits
- http://www.devtopics.com/13-tips-to-comment-your-code/
- https://www.hongkiat.com/blog/source-code-comment-styling-tips/