Handout #5

ME663 Assignment #2

Compressible Flow

Due Date: April 12, 2024

Q1

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho E \end{bmatrix}, \ F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ (\rho E + p)u \end{bmatrix}$$

Derive the Jacobian matrix $A = \frac{\partial F}{\partial U}$ (see Handout #4, p. 14, Eq. (2.31.a)). **Note the page number in this assignment is referring to the page number in Laney's book: Computational Gasdynamics.**

Q2

Derive the right eigenvectors r_1, r_2, r_3 for matrix A in Q1. Note that

$$Q_A = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}$$

where Q_A is defined in Handout #4, p. 371, Eq. (3.23).

Q3

Derive the following flux-vector splitting

$$F^{\pm} = \left(\frac{\rho}{2\gamma}\right) \begin{bmatrix} 2(\gamma-1)\lambda_1^{\pm} + \lambda_2^{\pm} + \lambda_3^{\pm} \\ 2(\gamma-1)\lambda_1^{\pm} u + \lambda_2^{\pm}(u+a) + \lambda_3^{\pm}(u-a) \\ (\gamma-1)\lambda_1^{\pm} u^2 + \frac{\lambda_2^{\pm}}{2}(u+a)^2 + \frac{\lambda_3^{\pm}}{2}(u-a)^2 + \frac{3-\gamma}{2(\gamma-1)}(\lambda_2^{\pm} + \lambda_3^{\pm})a^2 \end{bmatrix}$$

or

$$F^{\pm} = \frac{1}{\gamma} Q_A \begin{bmatrix} (\gamma-1)\rho\lambda_1^{\pm} \\ a\lambda_2^{\pm} \\ -a\lambda_3^{\pm} \end{bmatrix}$$

Q4

Following Q3, show that $\lambda^{\pm}$ in $F^{\pm}$ corresponding to van Leer's flux-vector splitting method are (see Handout #4, p. 377, Eq. (18.31)):

$$
\begin{aligned}
\lambda_1^+ = & \quad \frac{a}{4}(M+1)^2\left[1-\frac{(M-1)^2}{\gamma+1}\right] \\
\lambda_2^+ = & \quad \frac{a}{4}(M+1)^2\left[3-M+\frac{\gamma-1}{\gamma+1}(M-1)^2\right] \\
\lambda_3^+ = & \quad \frac{a}{2}(M+1)^2\left(\frac{M-1}{\gamma+1}\right)\left[1+\frac{\gamma-1}{2}M\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\lambda_1^-(M) = & \quad -\lambda_1^+(-M) \\
\lambda_2^-(M) = & \quad -\lambda_3^+(-M) \\
\lambda_3^-(M) = & \quad -\lambda_2^+(-M)
\end{aligned}
$$

Q5

Write a computer code for Test Case 1 (Handout #4, p. 352) using both Steger-Warming and van Leer flux-vector splitting methods, and compare your numerical results with the exact solutions at $t = 0.01$ for $-10 \leq x \leq 10$ in terms of density, velocity, pressure, Mach number. Provide me your source code(s) via Email or Dropbox in Learn with a simple README file about how to run your code and read your output files.

References:

1) J.D. Anderson (2003), Modern Compressible Flow with Historical Perspective, McGraw-Hill.

2) C. Hirsch (1990), Numerical Computation of Internal and External Flows, John Wiley & Sons.

3) C.B. Laney (1998), Computational Gasdynamics, Cambridge University Press.

4) J.L. Steger and R.F. Warming (1981), "Flux Vector Splitting of the Inviscid Gasdynamic Equations with Applications to Finite-Difference Methods", Journal of Computational Physics, 40, pp. 263-293.

5) B. van Leer (1982), "Flux-Vector Splitting for the Euler Equations", Lecture Notes in Physics, 170, pp. 507-512.