# ME621 - Advanced Finite Element Methods
# Assignment 4

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# 1 Requests

The goal of this assignment is to write an elastic-plastic stress update algorithm for simple shear deformation using MATLAB. It is important to note that this algorithm is independent of the finite element analysis and is strictly on the implementation of computational plasticity. This implementation is similar to writing a custom user-defined material subroutine (UMAT) for commercial FE software, with the exception that you must obtain the kinematic variables across time.

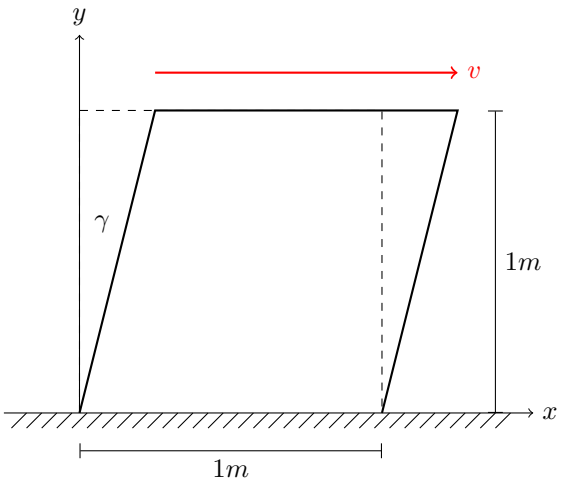


Figure 1: Problem representation in its initial (dashed lines) and final (solid lines) configuration.

Consider a square-shape aluminum plate under simple shear deformation. The deformation gradient for simple shear is:

$$F = \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} \tag{1}$$

- Obtain infinitesimal strain in the simple shear problem when the shear deformation is linearly increased from zero to $\gamma = 1$. Use constant time increments.
- Implement von-Mises plasticity according to the radial return algorithm described in the lecture notes. Use the following elastic constitutive equation in modeling of the elastic-plastic deformation. Do not consider kinematic hardening.

$$\dot{\sigma} = C : \dot{\varepsilon} \tag{2}$$

Plot the shear stress as a function of shear strain, $\gamma$.
Assume the material has a Young's modulus $E = 70\text{GPa}$ and a Poisson's ratio of $\nu = 0.3$. During plastic deformation, the material hardens according to the following isotropic hardening equation

$$\bar{\sigma} = \sigma_{Y0} + K(\bar{\varepsilon}^p)^n \tag{3}$$

where $\sigma_{Y0} = 200\text{MPa}$, $K = 325\text{MPa}$, and $n = 0.125$.

- Use the following elastic constitutive equation instead of the one at the second request and compare the results

$$\sigma^{oT} = C : D \tag{4}$$

Is there any significant difference between the results? Why?
$\sigma^{oT}$ is the Truesdell rate of Cauchy stress and $D$ is the rate-of-deformation tensor.

# 2 Methodology

To solve the given problem, we will build our code algorithm on top of the one developed for the previous assignment.
In particular, inside our Updated Lagrangian Formulation, we will check if the current configuration is in the elastic or plastic regime, and we will update the stress and strain tensors accordingly:

- If the current strain is below the yield strain, we will update the stress tensor using the constitutive model for the elastic regime.
- If the current strain is above the yield strain, we will update the stress tensor using the constitutive model for the plastic regime.

In the plastic region, we will neglect any hardening due to kinematic and will assume the material to follow the isotropic hardening model.

# 3 Solution

The majority of the code for this assignment is based on the one developed for the previous assignment.
In the following, we limit our self to describe how the radial return algorithm works and how we implemented it in our code.

## 3.1 Radial Return Algorithm

The radial return algorithm is a method used to update the stress tensor in the plastic regime.
The algorithm is based on the assumption that the stress tensor is always on the yield surface, and the plastic strain is always normal to the yield surface.
The algorithm can be summarized in the following steps:

1. Compute the trial stress tensor $\boldsymbol{\sigma}^{\text{trial}}$ using the constitutive model for the elastic regime.
2. Compute the trial deviatoric stress tensor $\boldsymbol{s}^{\text{trial}}$.
3. Compute the trial yield function $f^{\text{trial}}$.
4. If $f^{\text{trial}} \leq 0$, the stress tensor is already on the yield surface, and the algorithm can proceed to the next step.
5. If $f^{\text{trial}} > 0$, the stress tensor is outside the yield surface, and the algorithm needs to be corrected.
6. Compute the plastic multiplier $\lambda$.
7. Update the stress tensor using the plastic multiplier.
8. Update the plastic strain tensor.
9. Update the back stress tensor.
10. Update the hardening variable.
11. Update the stress tensor using the updated hardening variable.

## 3.2 Flow Charts

To better understand the flow used in the solution of the problem, we leave in the Appendix section (Section A) a couple of flow charts that represent the main steps involved in the solution of the problem.

# 4 Results

The numerical results for the problem described in Section 1 are reported in the following.

# A Flow Charts

The following flow chart mimics the structure of the MATLAB code used to solve the problem.
The code is structured in two nested loops: the main loop (**Explicit Time Integration Algorithm**) and the inner loop (**Stress Update Algorithm**).
The Explicit Time Integration Algorithm is the main loop that iterates until the convergence criterion is met. Inside the main loop, the Stress Update Algorithm is called to update the stress state of each element (at each integration point).

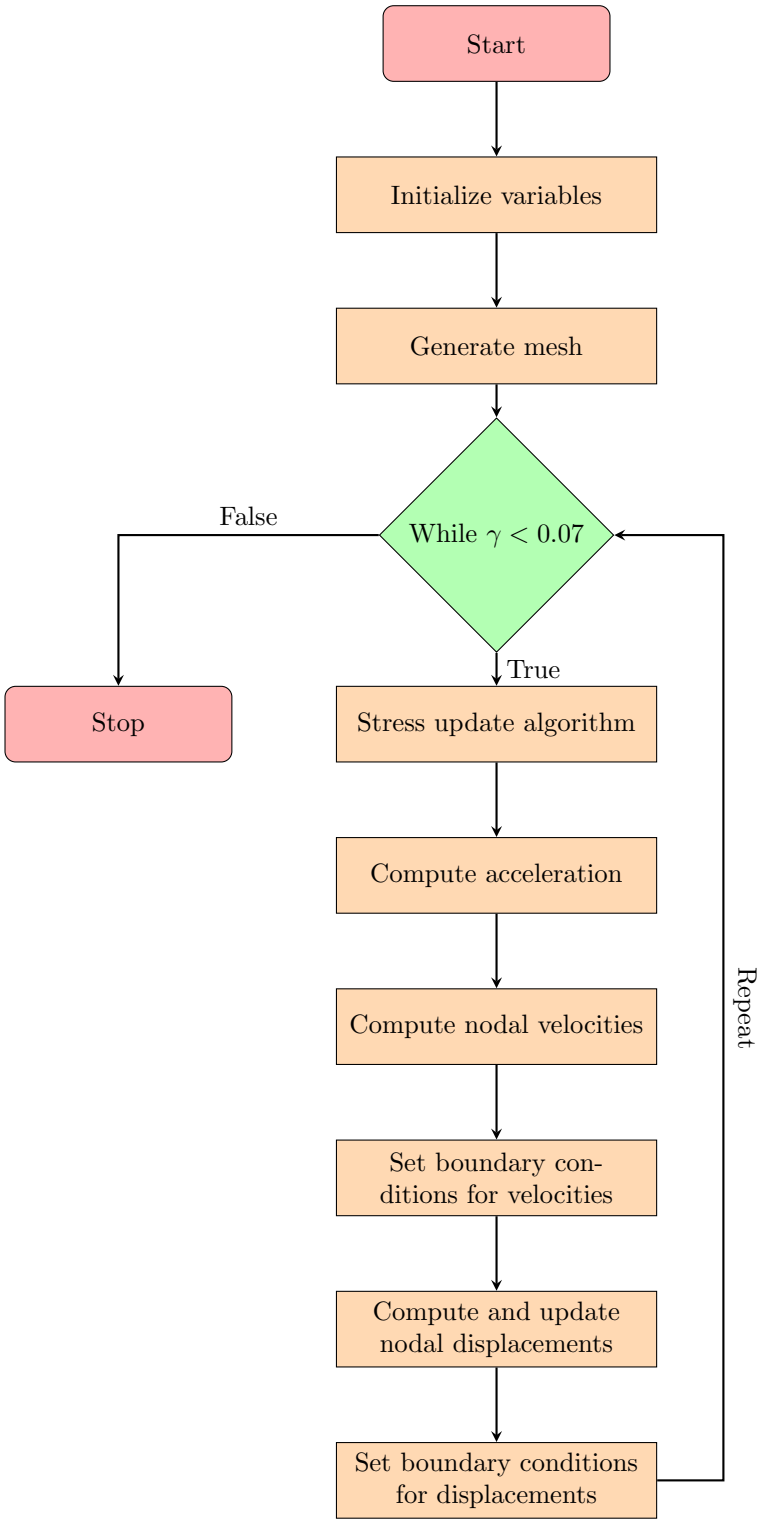


Figure 2: Flowchart for the **Explicit Time Integration Algorithm**. The convergence criterion $\gamma < 0.07$ is relative to the given problem, while the rest of the algorithm is general.

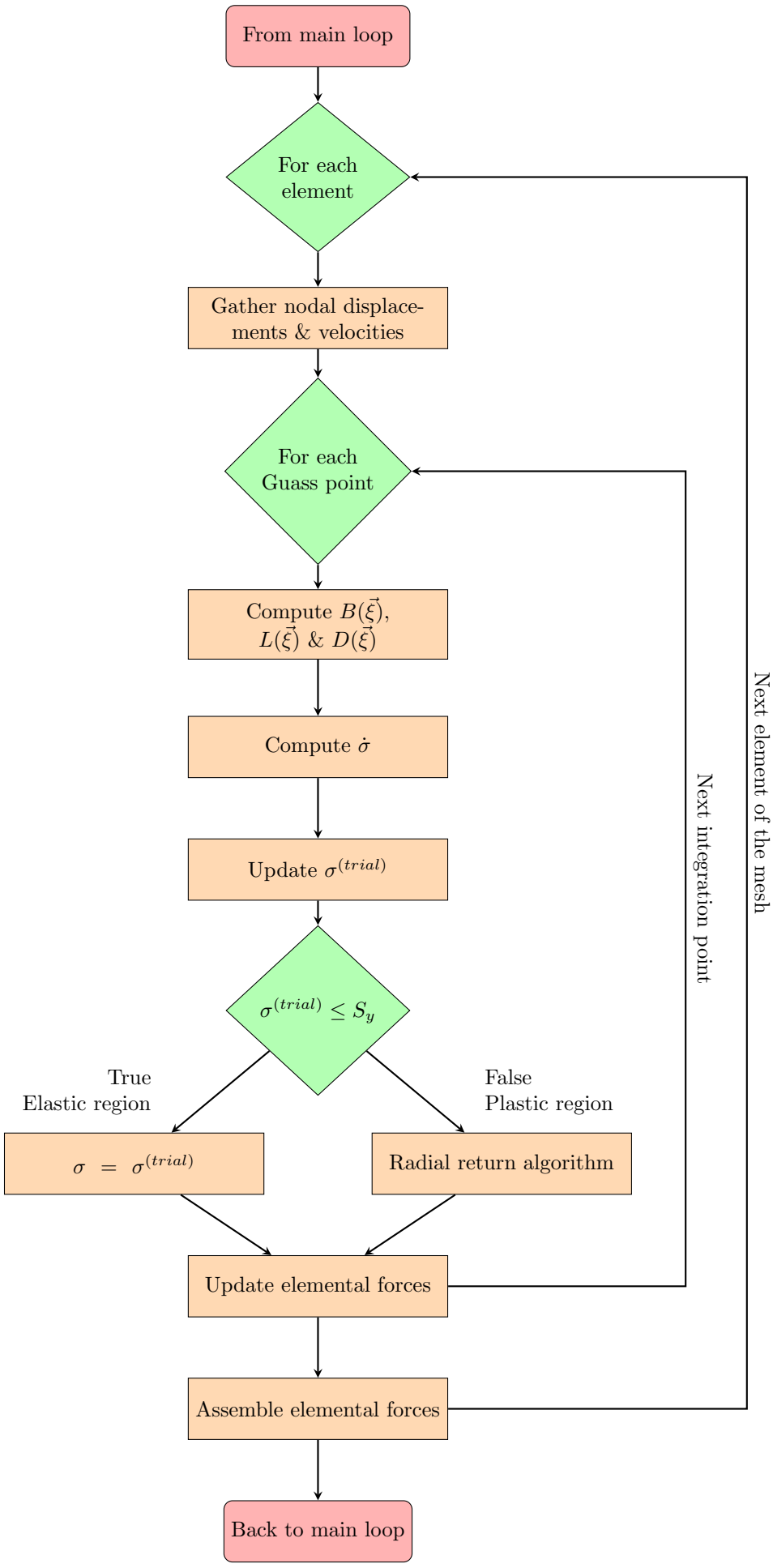


Figure 3: Flowchart for the **Stress Update Algorithm**. The decision blocks in this case represent for loops over the elements and the integration points.

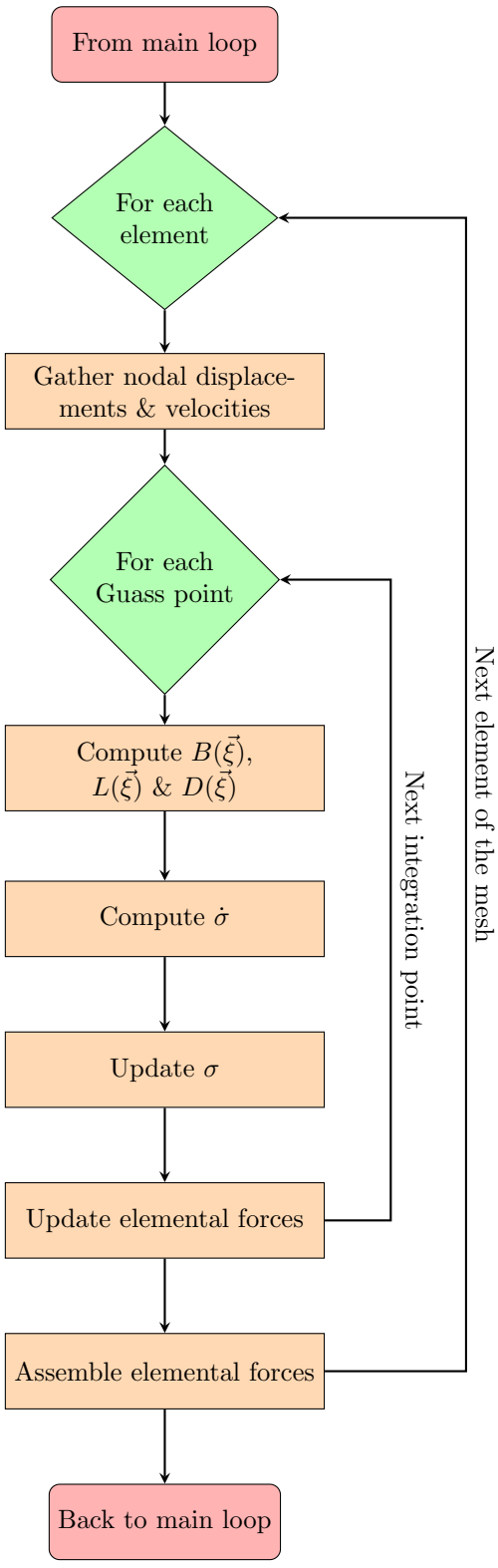


Figure 4: Flowchart for the **Stress Update Algorithm**. The decision blocks in this case represent for loops over the elements and the integration points.