POLITECNICO
MILANO 1863

Structural Health Monitoring (SHM) as a multivariate outlier detection problem

Tie-rods case study

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Date to be defined

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Agenda

1. Motivations
2. Methods
   - Mahalanobis Square Distance (MSD)
   - Principal Component Analysis (PCA)

Figure 1: Steel tie-rods connect the buttresses of the Cathedral of Saint Peter of Beauvais in France.

Motivations

Disturbances effect of environmental condition on SHM

In case of axial-load beams (tie-rods), studies has highlighted that temperature variations can cause greater changes to structural vibration than the presence of damage itself.

The transverse vibration in a tensioned beam is described by the following equation[1]:

$$w(\xi, t) = [A\sin(\gamma_1\xi) + B\cos(\gamma_1\xi) + C\sin(\gamma_2\xi) + D\cos(\gamma_2\xi)]\, E\cos(\omega t + \phi) \qquad (1)$$

Where:

$$\gamma_1 = \sqrt{\frac{N - \sqrt{N^2 + 4EJ\rho A\omega^2}}{2EJ}} \qquad \gamma_2 = \sqrt{\frac{N + \sqrt{N^2 + 4EJ\rho A\omega^2}}{2EJ}} \qquad (2)$$

Notice that $N = N(Temperature) = N_0 + k(T - T_0)$, with $k \approx -60\frac{N}{^\circ C}$.

[1] A full derivation of the equation can be found in the appendix.

Methods

Multivariate outlier detection

For the reasons explained in the previous slides, it's important to isolate the effect of environmental conditions on the eigenfrequencies of the structure, in order to detect the presence of damage.

In the following slides, two methods are presented, both based on the concept of multivariate outlier detection in the frequency domain. In particular, the following approaches are considered:

- Mahalanobis Square Distance (MSD)
- Principal Component Analysis (PCA)

Mahalanobis Square Distance (MSD) approach

The Mahalanobis Square Distance (MSD) is a measure of the distance between a point and a distribution. It is defined as:

$$D_M^2 = (\mathbf{x} - \mu)^T \mathbf{\Sigma}^{-1} (\mathbf{x} - \mu) \tag{3}$$

Where:

- $\mathbf{x}$ is the vector of the observations
- μ is the mean of the observations
- $\mathbf{\Sigma}$ is the covariance matrix of the observations

The MSD is used to detect outliers in the data, by comparing the distance of each observation from the mean of the distribution.


M. Berardengo, F. Lucà, M. Vanali, and G. Annesi.
Short-training damage detection method for axially loaded beams subject to seasonal thermal variations.
Sensors, 23(3), 2023.


F. Lucà, S. Manzoni, A. Cigada, and L. Frate.
A vibration-based approach for health monitoring of tie-rods under uncertain environmental conditions.
Mechanical Systems and Signal Processing, 167:108547, 2022.

Questions?

Thank you!