POLITECNICO
MILANO 1863

Data analysis for mechanical system identification

FRF – random

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Human-structure interaction

Problem: **people** can be considered and modelled as mechanical systems therefore it is reasonable to think that they **interact with the mechanical/civil structures** on which they stand

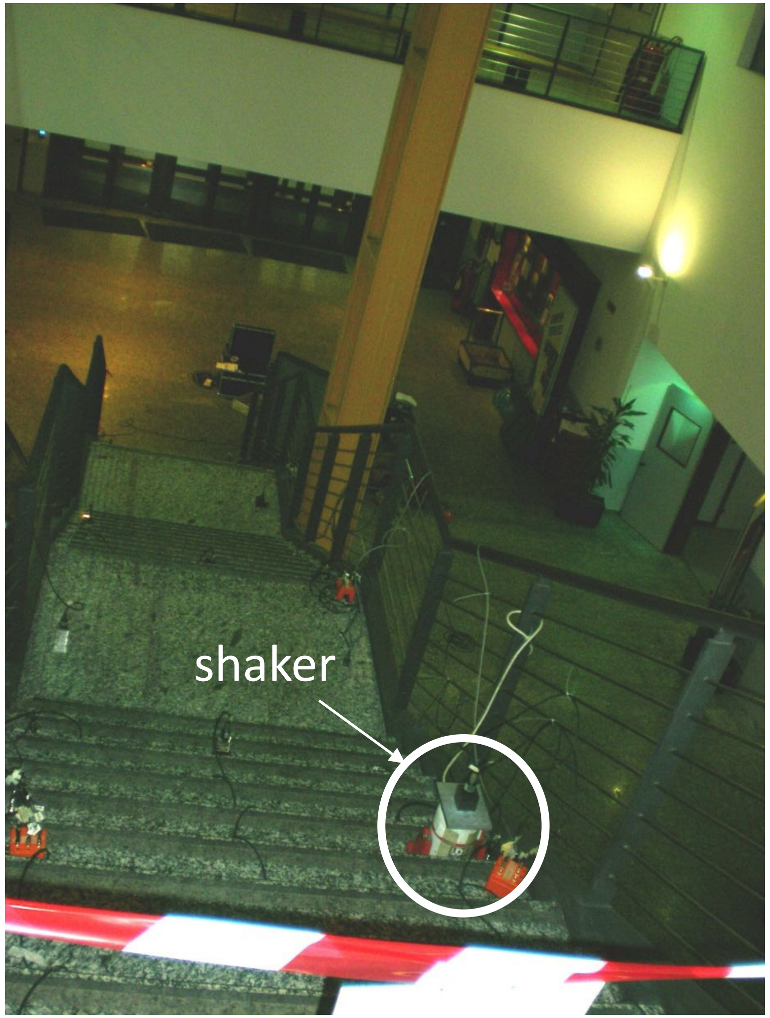
Goal: to **investigate** the effect of the human-structure interaction by looking at **how the dynamical characteristics** of the structure **change** when humans are on it.

⇩

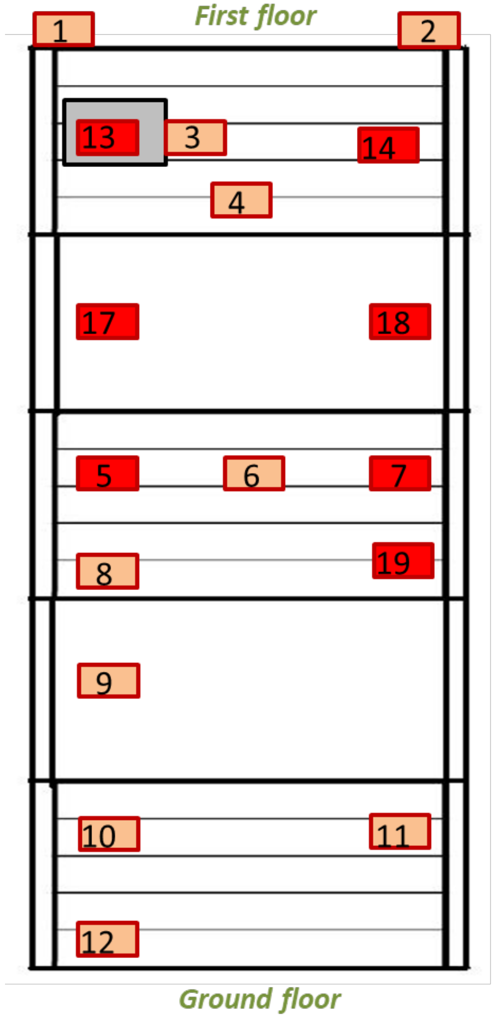
The response of the structure to a known input should be studied when people are or not on the structure itself.

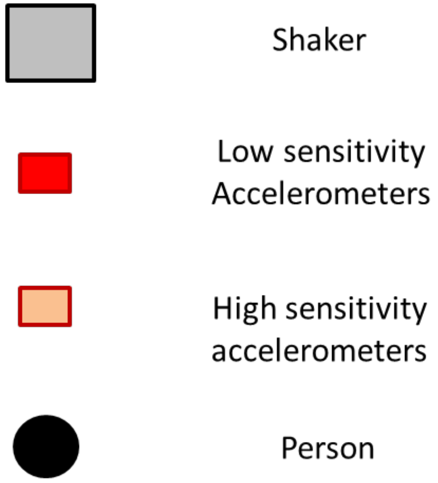
<u>As a first step the **cross-spectrum** between the input and the output of the system has to be analysed.</u>

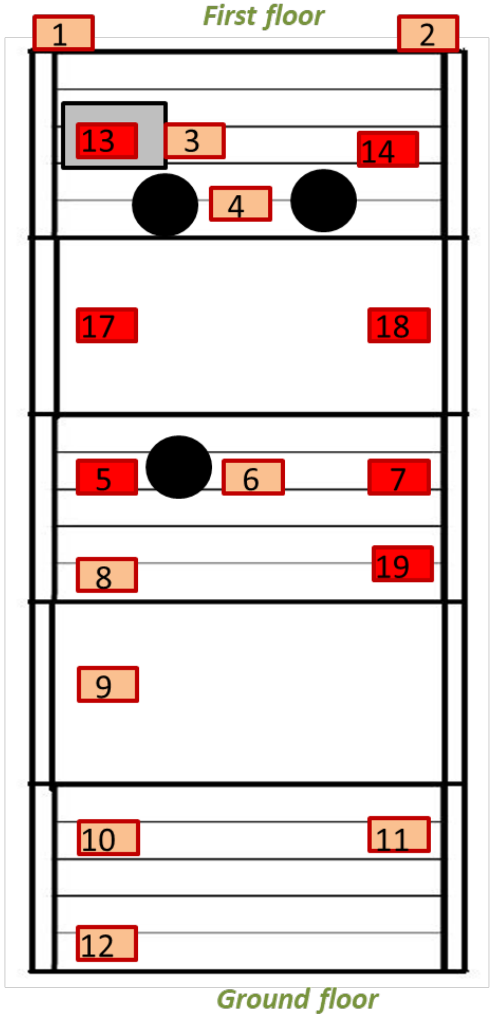
Test structure:
Staircase of B12 Building in Bovisa Campus (Milano)



Measurement set-up







Power spectrum

Problem: the spectrum of random signals cannot be easily interpreted as it is

⇩

It is possible to perform an **averaging** process on several sub-records of the original signal.

Power spectrum:

$$S_{AA}(f_k) = E\left[\hat{A}_j^*(f_k)\hat{A}_j(f_k)\right] =$$

$$= \lim_{n_d \to \infty} \frac{1}{n_d} \sum_{i=1}^{n_d} \hat{A}_i^*(f_k)\hat{A}_i(f_k)$$

Cross-spectrum

Which are the **main information** related to the cross-spectrum?

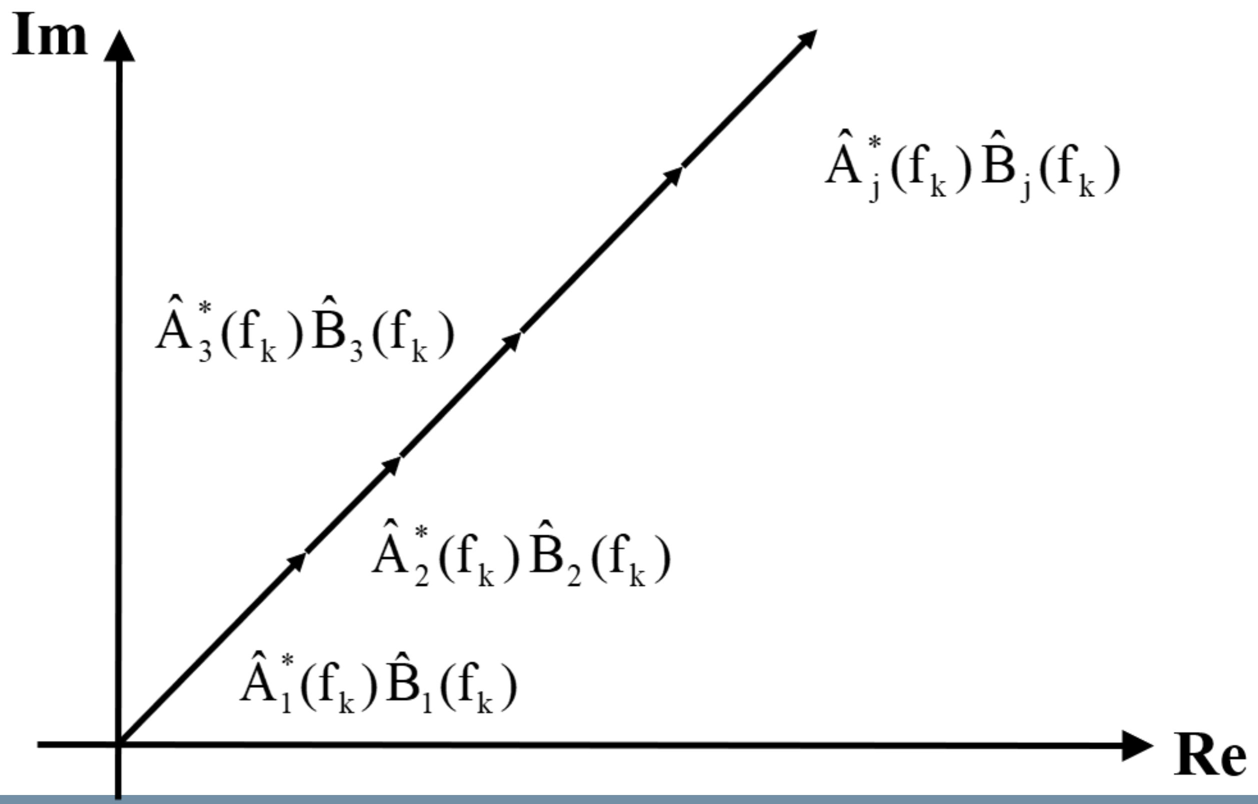
The cross-spectrum indicates the **correlation level** of two signals in a given frequency range

$$G_{xy}(f_k) = \frac{1}{N} * \sum_{i=1}^{N} (conj(X(f_k)) * Y(f_k))$$

The **average process** allows to increase statistical reliability of the result

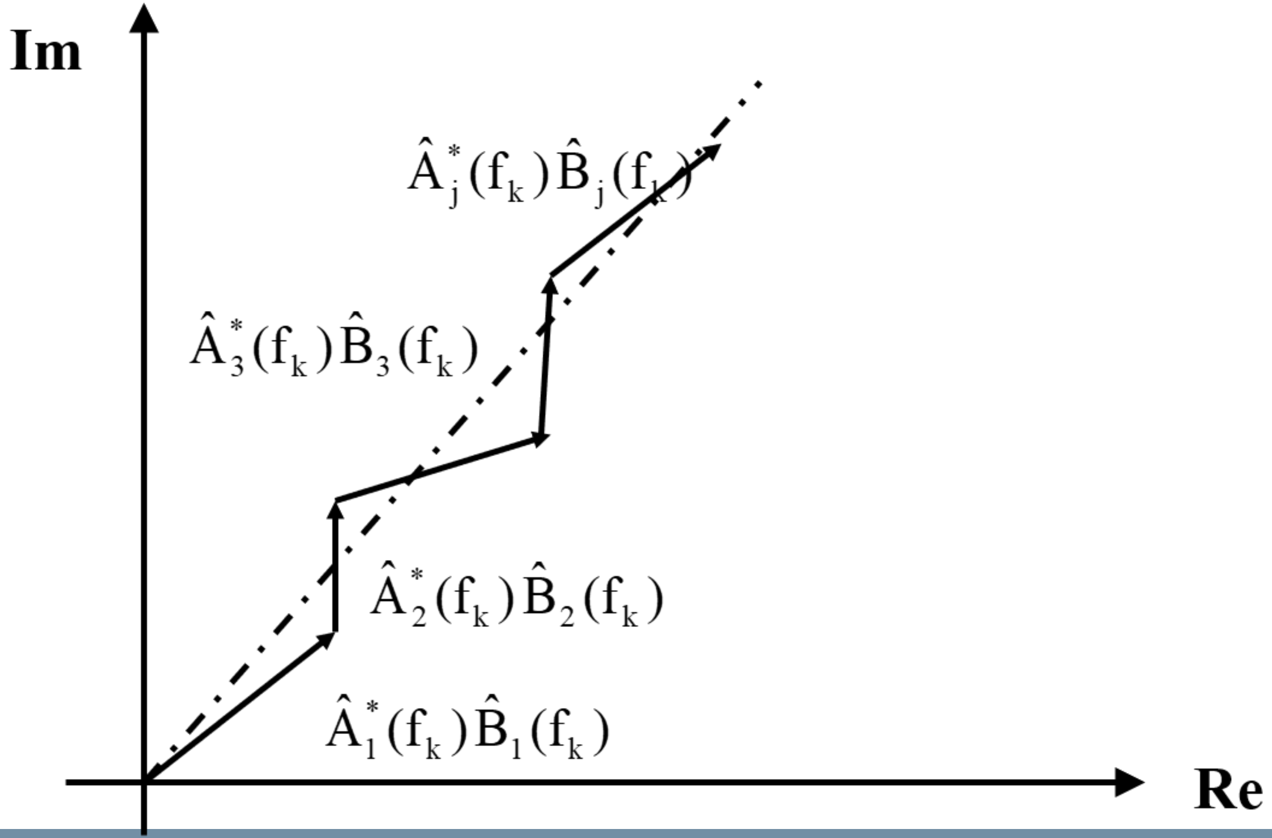
Complete correlation between the two signals:

The **phase shift** between the signals **doesn't change** with varying the considered time history

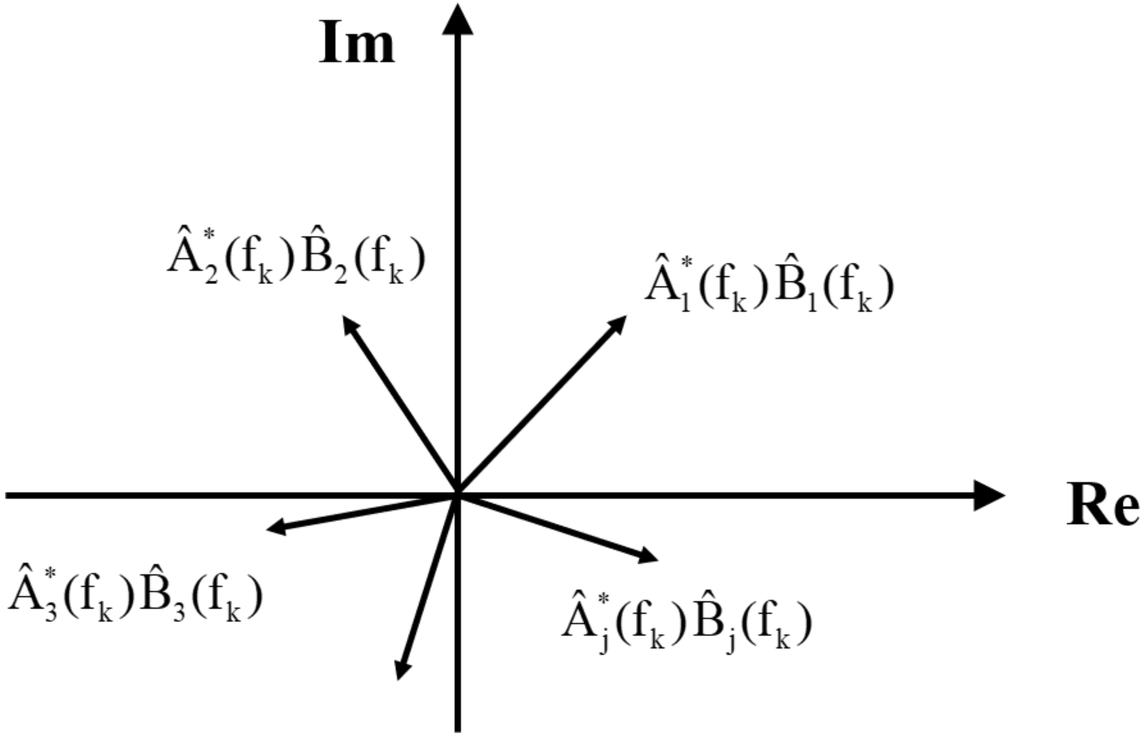


Effect of the averaging process on the cross-spectrum evaluation

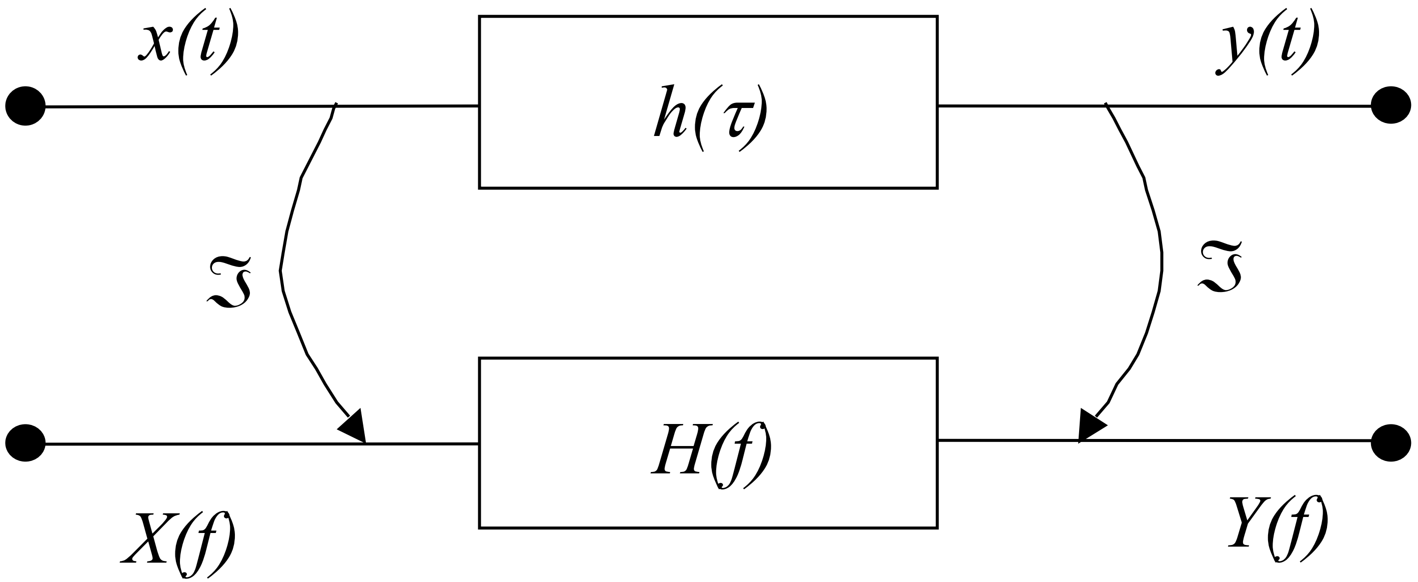
Partial correlation between the two signals:
The **phase shift** between the signals **change** with varying the considered time history



Lack of correlation between the two signals:
The **phase shift changes randomly** with varying the considered time history ($n_d \rightarrow \infty$ $S_{AB} \rightarrow 0$)



For any system, **the transfer function** is defined as



$$y(t) = \int_0^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Transfer Functions

Convolution in the time domain

$\Downarrow$

Product in the frequency domain

$$\boxed{Y(f) = X(f) \cdot H(f)}$$

Frequency Responce Function of the system

$$\boxed{Y(f) = X(f) \cdot H(f)}$$

$$X^*(f)\,Y(f) = H(f)\;\;X^*(f)\;\;X(f)$$

$$G_{xy}(f) = H(f)\;\;G_{xx}(f)$$

$$H(f) = \frac{Y(f)}{X(f)} \equiv \frac{G_{xy}(f)}{G_{xx}(f)} = H_1(f)$$

H1 estimator

- is not affected by the uncorrelated noise on the output measurement;
- underestimate the frequency responce function in the case of noise on the input measurement

FRF - H_2

$$\boxed{Y(f) = X(f) \cdot H(f)}$$

$$Y^*(f)\,Y(f) = H(f)\;\;Y^*(f)\;\;X(f)$$

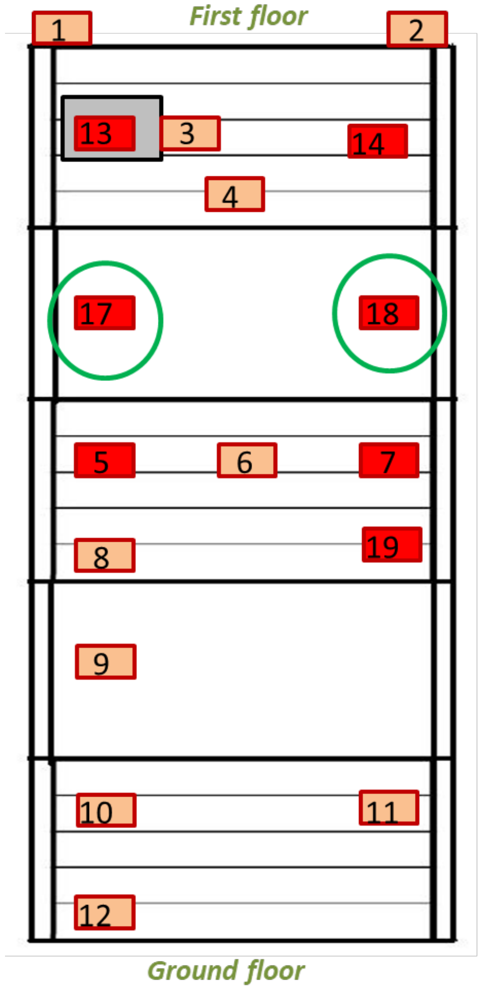
$$G_{yy}(f) = H\;(f)\,G_{yx}(f)$$

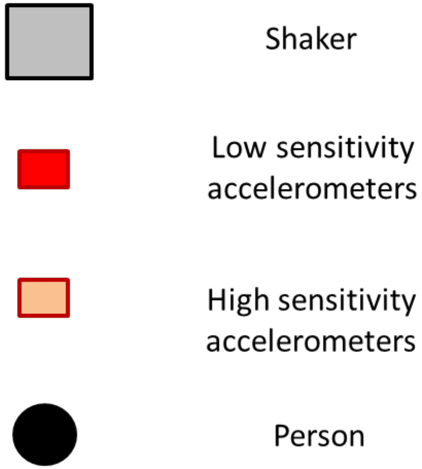
$$H(f)=\frac{Y(f)}{X(f)}\equiv\frac{G_{yy}(f)}{G_{yx}(f)}=H_2(f)$$

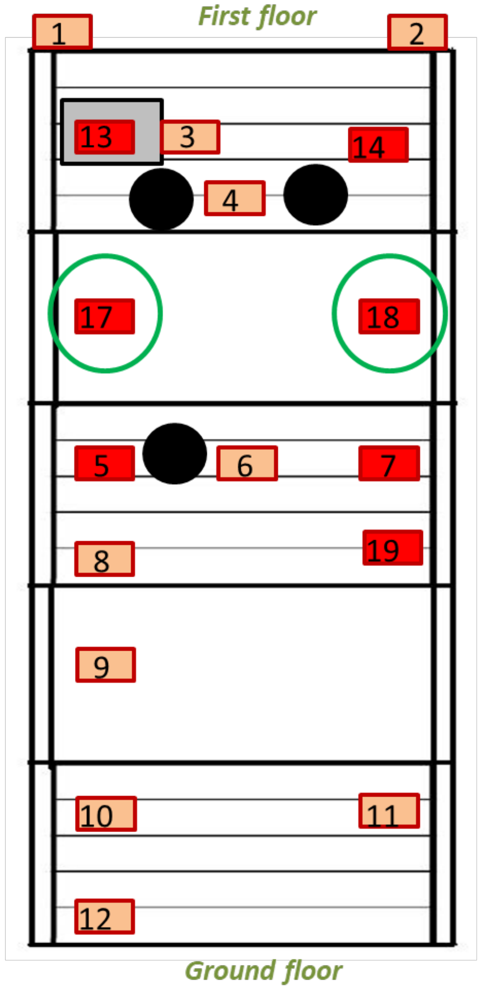
H2 estimator

- Is not affected by the uncorrelated noise on the input measurement;
- Overestimates the transfer function when ther is noise on the output measurement

Test set-up


First floor
1
2
13
3
14
4
17
18
5
6
7
8
19
9
10
11
12
Ground floor


Shaker
Low sensitivity accelerometers
High sensitivity accelerometers
Person


First floor
1
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13
3
14
4
17
18
5
6
7
8
19
9
10
11
12
Ground floor

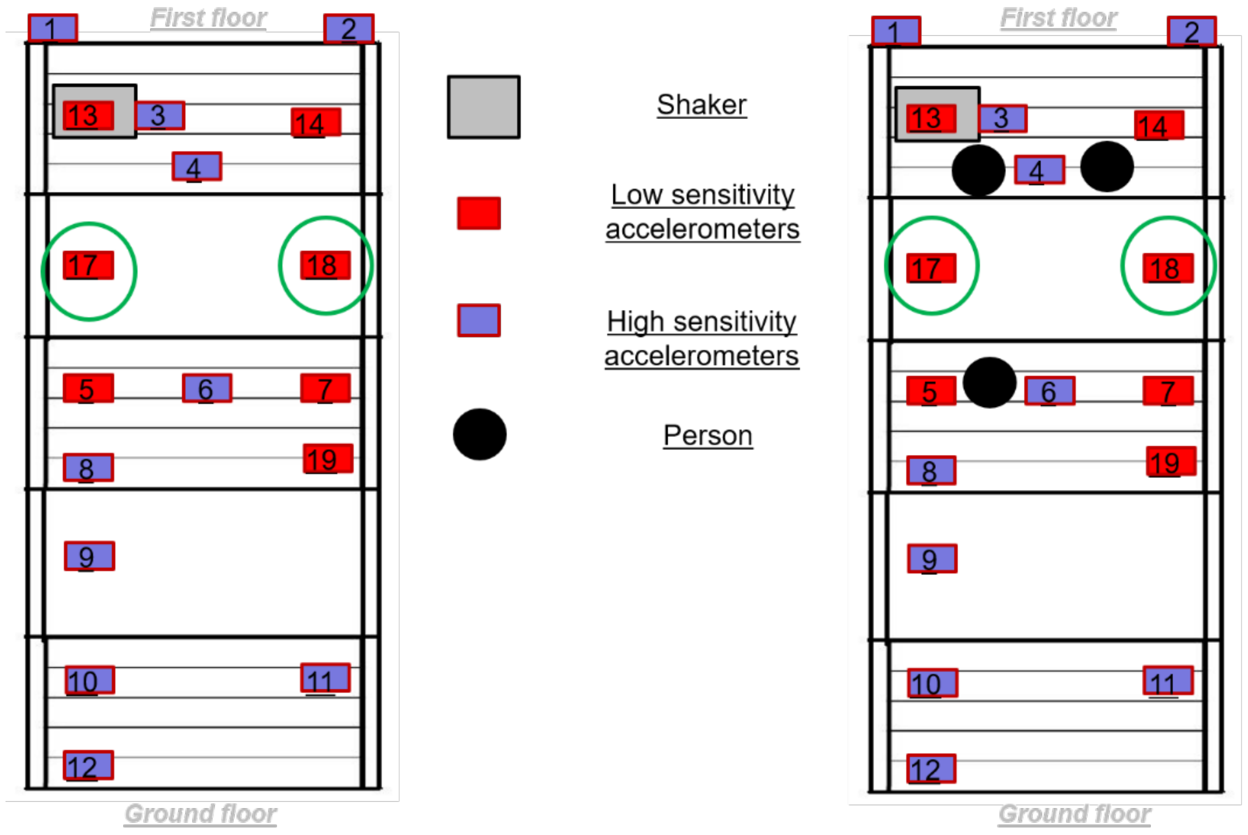
To do list

The data sets are related to 2 different tests on the staircase:

- «Lab7_staircase_empty»: no people on the staircase
- «Lab7_staircase_people»: 3 people on the staircase

Each file contains the sampling frequency (fsamp) and the time histories of:

- The forcing term (random input) in Data(:,1)
- Acceleration measured at point 17 in Data(:,2)
- Acceleration measured at point 18 in Data(:,3)



To do list

1. Evaluate, for both the cases (with and without people):
   - ✓ Power-spectra of the input and the output (both accelerometer 17 and 18), averaged using time histories of 30 s and 60 s
   - ✓ Cross-spectrum averaged using time histories of 30 s and 60 s.

Is the structure response correlated to the input?
Is the information given by the **cross-spectrum enough** to answer the previous question?
Otherwise, **what should we look at**?

2. Evaluate the Frequency response function between the input and the output as the Ratio of the Fourier transforms (**H**=Y/X) of the output and input

To do list

3. Evaluate the Frequency response function between the input and the output as **H1**, using time windows of 30 s and 60 s
4. Evaluate the Frequency response function between the input and the output as **H2**, using time windows of 30 s and 60 s

Which is the correct way to represent the actual FRF of the system?

Is it possible to identify which one of the proposed methods to calculate the FRF is the best one to represent the actual FRF of the system under analysis?

Plot of the spectrum

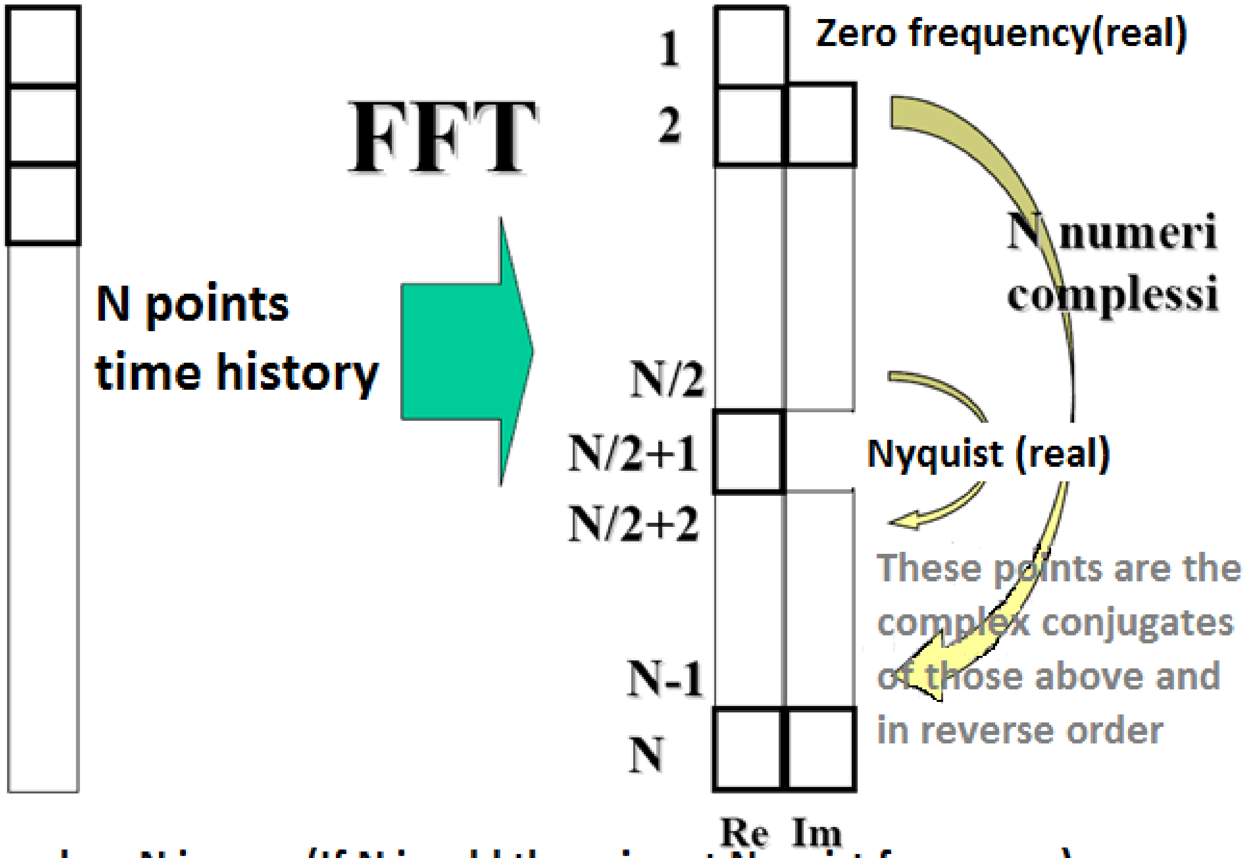
The spectrum is a **complex quantity**

It can be represented in different ways:

- Modulus and phase
- Real and imaginary part
- Complex plane Re-Im

As function of frequency

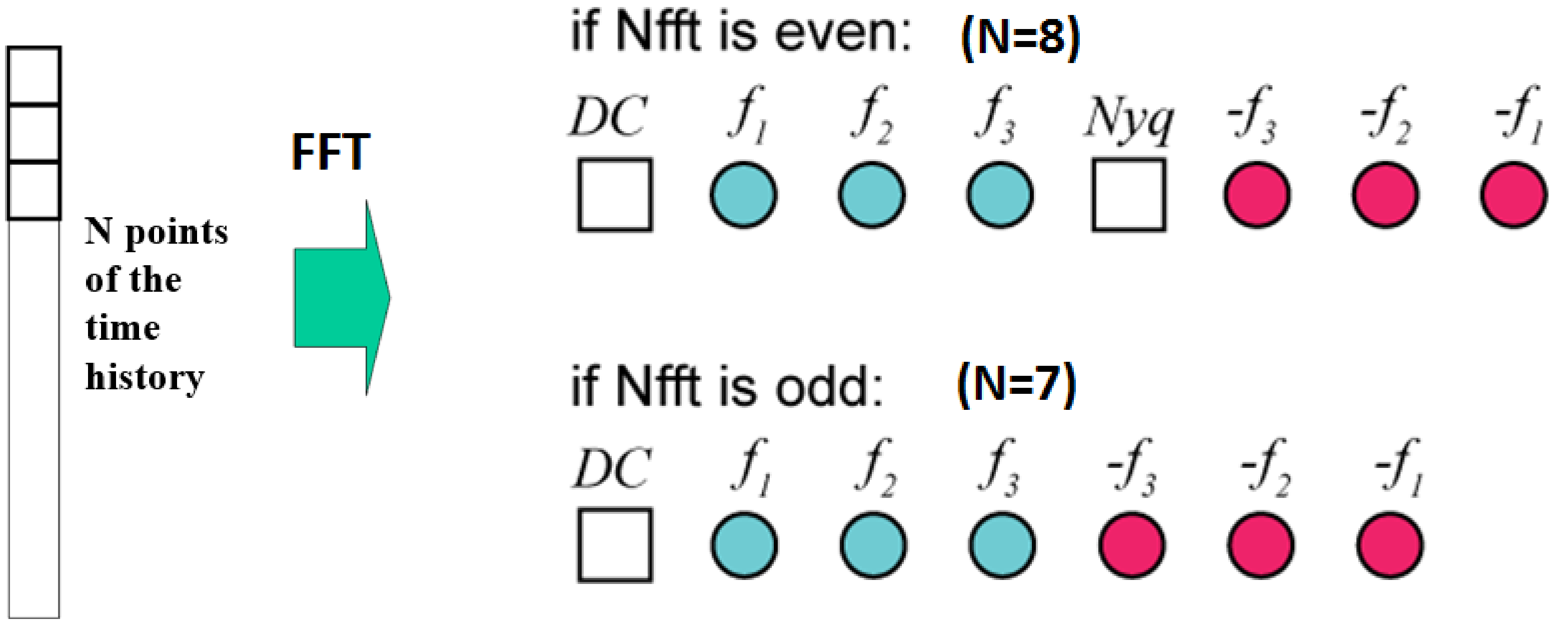
Pay attention:



when N is even(If N is odd,there is not Nyquist frequency)

Discrete fourier transform:
FFT Matlab

Pay attention:



Results from a MATLAB fft. Each symbol represents an element in the vector, squares are unique (DC and Nyquist if present), circles are positive (blue) and negative (red) frequencies

Discrete fourier transform: FFT Matlab

Pay attention:

It is possible to consider just the positive frequencies but…

N even →
consider (N/2+1) points
f_{max} = Nyquist

N odd →
consider ((N+1)/2) points
f_{max} = Nyquist-df/2

We need to normalize in the right way

N even:

```
y(1)=y(1)/N
y(2:N/2)= y(2:N/2)*2/N
y(N/2+1)=y(N/2+1)/N
```

N odd:

```
y(1)=y(1)/N
y(2:(N+1)/2)= y(2:(N+1)/2)*2/N
```

Fourier Transform: the Leakage error

Problem

For a given signal it is not always possible to identify a period and therefore sample it for an integer number of periods

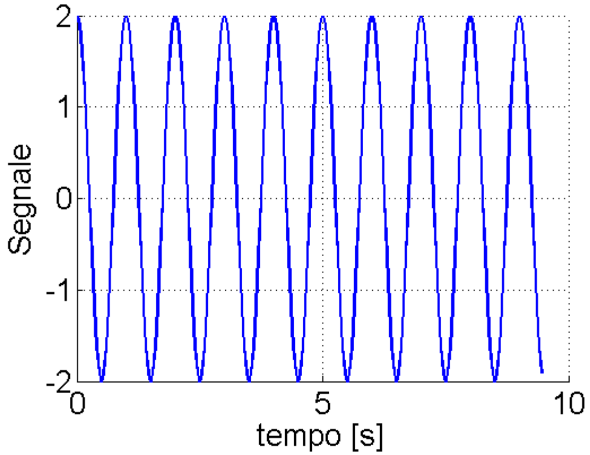
Leakage

Problem mitigation

It is possible to use windows different from the rectangular one. Each window modifies the signal and its spectrum in a different way.

The choice of the best window depends on the signal under analysis and on the goals of the measurement

Windows



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