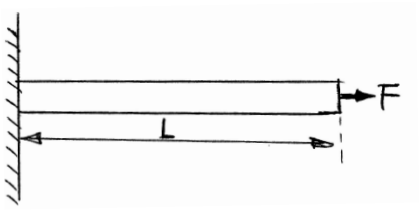
ME-621 Advanced Finite Element Method

Assignment 2

Due Date: Wed Feb 28, 2024 10:00am

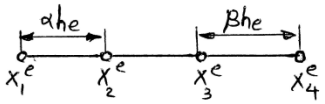
Problem: A tensile force *F* is applied to the end of the rod as shown in the following figure.



Discretize the rod into five 4-node elements to compute the displacement of the rod throughout time. Use an explicit integration scheme with Total Lagrangian finite element formulation. Use the parent (element) system of coordinates.

The rod is made of aluminum with density $\rho_0 = 2700\ kg/m^3$ and Young's modulus $E = 70\ GPa$. The initial cross-sectional area and length of the rod are $A_0 = 300\ mm^2$ and $L_0 = 200\ mm$, respectively. Use a linear constitutive equation of the form $P = E\frac{du}{dX}$ where *P* is the First Piola Kirchhoff stress.

I. Write out the shape functions for the following 4-node element with total length h_e and $\alpha = \beta = \frac{1}{3}$ in terms of the parent coordinate system, $-1 \leq \zeta \leq 1$.



II. Obtain $\boldsymbol{B}_0^e$ matrix for a general 4-node element and write the elemental force vectors $\boldsymbol{f}_{int}^e$, $\boldsymbol{f}_{ext}^e$, $\boldsymbol{f}_{kin}^e$, and elemental mass matrix $\boldsymbol{M}^e$ in discretized integral form. No need to integrate.

III. Calculate the displacement of the node on the very right end after 0.01s assuming the force is linearly increased at 100 000 kN/s from 0 kN to 1000 kN. Plot the deformation of the right end of the bar vs time. The force is intentionally high and rapid in order to allow for a visual representation of the deformation behavior (ignore yielding and assume elasticity).

IV. Is there a difference in the deformation behavior if the 1000 kN force is applied instantaneously at the beginning of the simulation (i.e. at t=0)? Plot the deformation of the right end vs time.

Note: Use Gaussian Quadrature method for integration and do not integrate by hand!

Hint: Make reasonable assumptions, if needed, and state your assumptions in your solution.