

# Multi-criteria ordered clustering

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## 1 Definitions

- $\mathcal{A}$ : the set of  $n$  alternatives  $\mathcal{A} = \{a_1, a_2, \dots, a_n\}$  (notation:  $a_i$  or  $a_j$ ,  $i, j = 1, 2, \dots, n$ )
- $\mathcal{F}$ : the set of  $m$  criteria  $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$  (notation:  $f_k$ ,  $k = 1, 2, \dots, m$ )
- $\mathcal{R}$ : the set of  $l$  clusters  $\mathcal{R} = \{r_1, r_2, \dots, r_l\}$  (notation:  $r_h$ ,  $h = 1, 2, \dots, l$ )

## 2 Input data

- Evaluation table:

$a$	$f_1(\cdot)$	$f_2(\cdot)$	$\dots$	$f_k(\cdot)$	$\dots$	$f_m(\cdot)$
$a_1$	$f_1(a_1)$	$f_2(a_1)$	$\dots$	$f_k(a_1)$	$\dots$	$f_m(a_1)$
$a_2$	$f_1(a_2)$	$f_2(a_2)$	$\dots$	$f_k(a_2)$	$\dots$	$f_m(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$a_i$	$f_1(a_i)$	$f_2(a_i)$	$\dots$	$f_k(a_i)$	$\dots$	$f_m(a_i)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
$a_n$	$f_1(a_n)$	$f_2(a_n)$	$\dots$	$f_k(a_n)$	$\dots$	$f_m(a_n)$
- $\mathcal{W}$ : the set of  $m$  weights for the criteria:  $\mathcal{W} = \{w_1, w_2, \dots, w_m\}$  (notation:  $w_k$ ,  $k = 1, 2, \dots, m$ )
- $N$ : the number of clusters
- Big  $M$  (large value, see Note 1)

## 3 Decision variables

- $c_{ih} = \begin{cases} 1 & \text{if } a_i \in r_h \\ 0 & \text{otherwise} \end{cases}$ ,  $c_{ih} \in \{0, 1\}$
- $r_{kh}$ : position of the cluster  $h$  on the criterion  $k$ ,  $f_k(r_h)$

## 4 Equations

$$\max z = \text{s.t.} \tag{1}$$

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i) \tag{netflow} \tag{2}$$

$$\phi^+(a_i) = \frac{1}{n-1} \sum_{j=1, j \neq i}^n \sum_{k=1}^m w_k \beta_{ijk} \tag{positive flow} \tag{3}$$

$$\phi^-(a_i) = \frac{1}{n-1} \sum_{j=1, j \neq i}^n \sum_{k=1}^m w_k \beta_{jik} \tag{negative flow} \tag{4}$$

$$\beta_{ijk} \geq \frac{f_k(a_i) - f_k(a_j)}{M} \quad \forall i, j = 1, 2, \dots, n; \quad k = 1, 2, \dots, m \tag{linearization, see Note 2} \tag{5}$$

$$\beta_{ijk} < \frac{f_k(a_i) - f_k(a_j)}{M} + 1 \quad \forall i, j = 1, 2, \dots, n; \quad k = 1, 2, \dots, m \tag{6}$$

$$\beta_{ijk} \in \{0, 1\} \quad \forall i, j = 1, 2, \dots, n; \quad k = 1, 2, \dots, m \tag{7}$$

$$c_{ih} \in \{0, 1\} \quad \forall i = 1, 2, \dots, n; \quad h = 1, 2, \dots, l \tag{decision variables} \tag{8}$$

## 5 Notes

1. Big  $M$  chosen so that  $\frac{f_k(a_i) - f_k(a_j)}{M} \in ]-1; 1[$

2.  $\beta_{ijk} = \begin{cases} 1 & \text{if } f_k(a_i) > f_k(a_j) \\ 0 & \text{otherwise} \end{cases}, \quad \beta_{ijk} \in \{0, 1\}$