

Multi-criteria hierarchical clustering

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Definitions:

- \mathcal{A} : the set of n alternatives $\mathcal{A} = \{a_1, a_2, \dots, a_n\}$ ($a_i, i = 1, 2, \dots, n$)
- \mathcal{F} : the set of m criteria $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$ ($f_j, j = 1, 2, \dots, m$)
- \mathcal{R} : the set of l clusters $\mathcal{R} = \{r_1, r_2, \dots, r_l\}$ ($r_k, k = 1, 2, \dots, l$)

Decision variables:

- $c_{ik} = \begin{cases} 1 & \text{if } a_i \in r_k \\ 0 & \text{otherwise} \end{cases}, \quad c_{ik} \in \{0, 1\}$
- Big M

Equations:

$$\begin{aligned}
 \max z &= \pi - \pi \\
 \text{s.t.} \\
 \phi(a_i) &= \phi^+(a_i) - \phi^-(a_i) \text{ (netflow)} \\
 \phi^+(a_i) &= \frac{1}{n-1} \sum_{i=1, i \neq j}^n \sum_{k=1}^m w_k \beta_{ijk} \\
 \beta_{ijk} &< \frac{f_k(a_i) - f_k(a_j)}{M} + 1 \\
 \beta_{ijk} &\geq \frac{f_k(a_i) - f_k(a_j)}{M} \\
 \beta_{ijk} &\in \{0, 1\} \\
 c_{ik} &\in \{0, 1\} \text{ (decision variables)}
 \end{aligned}$$

Notes:

- $\beta_{ijk} = \begin{cases} 1 & \text{if } f_k(a_i) > f_k(a_j) \\ 0 & \text{otherwise} \end{cases}, \quad \beta_{ijk} \in \{0, 1\}$
- Big M chosen so that $\frac{f_k(a_i) - f_k(a_j)}{M} \in]-1; 1[$