# Multi-criteria hierarchical clustering

N.A.V. Doan, J. Rosenfeld, Y. De Smet

## 1 Definitions

- $\mathcal{A}$: the set of $n$ alternatives $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$ (notation: $a_i$ or $a_j$, $i, j = 1, 2, \ldots, n$)

- $\mathcal{F}$: the set of $m$ criteria $\mathcal{F} = \{f_1, f_2, \ldots, f_m\}$ (notation: $f_k$, $k = 1, 2, \ldots, m$)

- $\mathcal{R}$: the set of $l$ clusters $\mathcal{R} = \{r_1, r_2, \ldots, r_l\}$ (notation: $r_h$, $h = 1, 2, \ldots, l$)

## 2 Input data

- Evaluation table:

| $a$ | $f_1(\cdot)$ | $f_2(\cdot)$ | $\ldots$ | $f_j(\cdot)$ | $\ldots$ | $f_m(\cdot)$ |
|---|---|---|---|---|---|---|
| $a_1$ | $f_1(a_1)$ | $f_2(a_1)$ | $\ldots$ | $f_j(a_1)$ | $\ldots$ | $f_m(a_1)$ |
| $a_2$ | $f_1(a_2)$ | $f_2(a_2)$ | $\ldots$ | $f_j(a_2)$ | $\ldots$ | $f_m(a_2)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $a_i$ | $f_(a_i)$ | $f_2(a_i)$ | $\ldots$ | $f_j(a_i)$ | $\ldots$ | $f_m(a_i)$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\ddots$ | $\vdots$ |
| $a_n$ | $f_1(a_n)$ | $f_2(a_n)$ | $\ldots$ | $f_j(a_n)$ | $\ldots$ | $f_m(a_n)$ |

- $\mathcal{W}$: the set of $m$ weights for the criteria: $\mathcal{W} = \{w_1, w_2, \ldots, w_m\}$ (notation: $w_k$, $k = 1, 2, \ldots, m$)

## 3 Decision variables

- $c_{ih} = \begin{cases} 1 & \text{if } a_i \in r_h \\ 0 & \text{otherwise} \end{cases}$, $\quad c_{ih} \in \{0, 1\}$

- Big $M$

## 4 Equations

$$\max z = \pi - \pi \tag{1}$$

$$\text{s.t.} \tag{2}$$

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i) \qquad \text{(netflow)} \tag{3}$$

$$\phi^+(a_i) = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} \sum_{k=1}^{m} w_k \beta_{ijk} \qquad \text{(positive flow)} \tag{4}$$

$$\phi^-(a_i) = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} \sum_{k=1}^{m} w_k \beta_{jik} \qquad \text{(negative flow)} \tag{5}$$

$$\beta_{ijk} \geq \frac{f_k(a_i) - f_k(a_j)}{M} \qquad \text{(linearization Note 1)} \tag{6}$$

$$\beta_{ijk} < \frac{f_k(a_i) - f_k(a_j)}{M} + 1 \tag{7}$$

$$\beta_{ijk} \in \{0, 1\} \tag{8}$$

$$c_{ik} \in \{0, 1\} \qquad \text{(decision variables)} \tag{9}$$

# 5 Notes

1. $\beta_{ijk} = \begin{cases} 1 & \text{if } f_k(a_i) > f_k(a_j) \\ 0 & otherwise \end{cases}$ , $\quad \beta_{ijk} \in \{0, 1\}$

2. Big $M$ chosen so that $\dfrac{f_k(a_i) - f_k(a_j)}{M} \in \, ]-1; 1[$