Multi-criteria hierarchical clustering

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Definitions:

- $\mathcal{A}$: the set of n alternatives $\mathcal{A} = \{a_1, a_2, \ldots, a_n\} \quad (a_i,\ i = 1, 2, \ldots, n)$
- $\mathcal{F}$: the set of m criteria $\mathcal{F} = \{f_1, f_2, \ldots, f_m\} \quad (f_k,\ k = 1, 2, \ldots, m)$
- $\mathcal{R}$: the set of l clusters $\mathcal{R} = \{r_1, r_2, \ldots, r_l\} \quad (r_h,\ h = 1, 2, \ldots, l)$

Input data:

- Evaluation table:

a	$f_1(\cdot)$	$f_2(\cdot)$	$\ldots$	$f_j(\cdot)$	$\ldots$	$f_m(\cdot)$
a_1	$f_1(a_1)$	$f_2(a_1)$	$\ldots$	$f_j(a_1)$	$\ldots$	$f_m(a_1)$
a_2	$f_1(a_2)$	$f_2(a_2)$	$\ldots$	$f_j(a_2)$	$\ldots$	$f_m(a_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_i	$f_(a_i)$	$f_2(a_i)$	$\ldots$	$f_j(a_i)$	$\ldots$	$f_m(a_i)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$
a_n	$f_1(a_n)$	$f_2(a_n)$	$\ldots$	$f_j(a_n)$	$\ldots$	$f_m(a_n)$

- $\mathcal{W}$: the set of m weights for the criteria: $\mathcal{W} = \{w_1, w_2, \ldots, w_m\} \quad (w_k,\ k = 1, 2, \ldots, m)$

Decision variables:

- $c_{ih} = \begin{cases} 1 & \text{if } a_i \in r_h \\ 0 & \text{otherwise} \end{cases}, \quad c_{ih} \in \{0, 1\}$
- Big M

Equations:

$$\begin{aligned}
\max z &= \pi - \pi \\
\text{s.t.} & \\
\phi(a_i) &= \phi^+(a_i) - \phi^-(a_i) \text{ (netflow)} \\
\phi^+(a_i) &= \frac{1}{n-1} \sum_{i=1, i\neq j}^{n} \sum_{k=1}^{m} w_k \beta_{ijk} \\
\beta_{ijk} &< \frac{f_k(a_i) - f_k(a_j)}{M} + 1 \\
\beta_{ijk} &\geq \frac{f_k(a_i) - f_k(a_j)}{M} \\
\beta_{ijk} &\in \{0, 1\} \\
c_{ik} &\in \{0, 1\} \text{ (decision variables)}
\end{aligned}$$

Notes:

- $\beta_{ijk} = \begin{cases} 1 & \text{if } f_k(a_i) > f_k(a_j) \\ 0 & \textit{otherwise} \end{cases}, \quad \beta_{ijk} \in \{0, 1\}$
- Big M chosen so that $\dfrac{f_k(a_i) - f_k(a_j)}{M} \in \,]{-1}; 1[$