Multi-criteria hierarchical clustering

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Definitions:

- $\mathcal{A}$: the set of n alternatives $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$ $(a_i,\ i = 1, 2, \ldots, n)$
- $\mathcal{F}$: the set of m criteria $\mathcal{F} = \{f_1, f_2, \ldots, f_m\}$ $(f_j,\ j = 1, 2, \ldots, m)$
- $\mathcal{R}$: the set of l clusters $\mathcal{R} = \{r_1, r_2, \ldots, r_l\}$ $(r_k,\ k = 1, 2, \ldots, l)$

Decision variables:

- $c_{ik} = \begin{cases} 1 & \text{if } a_i \in r_k \\ 0 & \text{otherwise} \end{cases}, \quad c_{ik} \in \{0, 1\}$
- Big M

Equations:

$$
\begin{aligned}
\max z &= objective\_function \\
\text{s.t.} & \\
\phi(a_i) &= \phi^+(a_i) - \phi^-(a_i) \text{ (netflow)} \\
\phi^+(a_i) &= \frac{1}{n-1} \sum_{i=1, i\neq j}^{n} \sum_{k=1}^{m} w_k \beta_{ijk} \\
\beta_{ijk} &< \frac{f_k(a_i) - f_k(a_j)}{M} + 1 \\
\beta_{ijk} &\geq \frac{f_k(a_i) - f_k(a_j)}{M} \\
\beta_{ijk} &\in \{0, 1\} \\
c_{ik} &\in \{0, 1\} \text{ (decision variables)}
\end{aligned}
$$

Notes:

- $\beta_{ijk} = \begin{cases} 1 & \text{if } f_k(a_i) > f_k(a_j) \\ 0 & otherwise \end{cases}, \quad \beta_{ijk} \in \{0, 1\}$