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Multi-Objective Optimization and Multi-Criteria Decision Aid
Applied to the Design of 3D-Stacked Integrated Circuits

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I thank you all! I love you all!

Abstract

In the past decades, the microelectronic industry has been following the Moore's law to improve the performance of integrated circuits (IC). However, it will probably be impossible to follow this law in the future due to physical limitations appearing with the miniaturization of the transistors below a certain threshold. In order to overcome this problem, new technologies have emerged, and among them the 3D-Stacked Integrated Circuits (3D-SIC) have been proposed to keep the Moore's momentum alive. 3D-SICs can bring numerous advantages in the design of future ICs but at the cost of additional design complexity due to their highly combinatorial nature, and the optimization of several conflicting criteria. In this paper, we present a first study of tools that can help the design of 3D-SICs, using multiobjective optimization (MOO). Our study has targeted one of the main issues in the design of 3D-SICs: the floorplanning. This thesis shows that the use of a multi-criteria paradigm can provide relevant and objective analysis of the problem that may not be feasible with the current design methods. This can allow a quick design space exploration and an improvement of the current design flows. Also, with its flexibility, MOO can cope with the multiple degrees of freedom of 3D-SICs, which enables more design possibilities.

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Introduction

2D architecture, current design flows and their limitations

In order to continuously improve the performance of integrated circuits (IC), technologists deploy enormous efforts to produce IC manufacturing process that is compelling to follow the well-known Moore's Law (see Figure 1). This empirical law predicts a doubling of the transistors' integration each 18 months and therefore increasing logic capacity of the circuit per unit area.

The improvements of 2D architectures are primarily driven by the reduction of the transistor size. By reducing transistor dimensions, the switching speed is increased thanks to the shorter distance between the source and the drain, implying an improvement of the overall speed of the designs.

However, as the transistor size is decreasing, the observed improvement is also getting smaller. Indeed, a smaller transistor allows higher device density but will slightly decrease the dynamic and increase the total delay (sum of gate and interconnection delays) at the level of the complete circuit. Also, power consumption is increased due to higher leakage and increasing interconnection wire length [8]. In Figure 2 is shown the trends in transistor gate delay and interconnect delay with IC fabrication technology where the crossover point represents the interconnect bottleneck [2].

With the miniaturization, quantum effects such as quantum tunnelling will significantly affect how a transistor behave [9].

In addition to these physical aspects, economical considerations that will hinder the IC evolution beyond 20nm have to be taken into account [8, 10].

In order to overcome these limitations, new technologies have been proposed such as the carbon nanotubes [11], the nanowire transistors [12], the single-electron transistors [13], but also the 3D-Stacked Integrated Circuits (3D-SIC) proposed by the academic and industrial communities. The latter has been often cited as the most prominent one [3].

Fast evolution of IC manufacturing technologies makes even the design of 2D-ICs a complex and tedious task with the growing number of design choices at the system level (e.g. number and type of functional units and memories, type and topology of

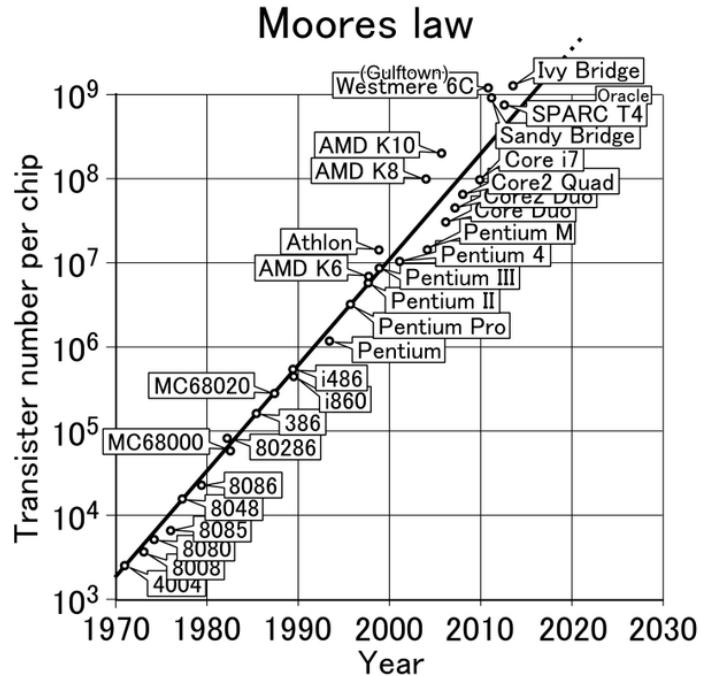


Figure 1: Moore's law [1]

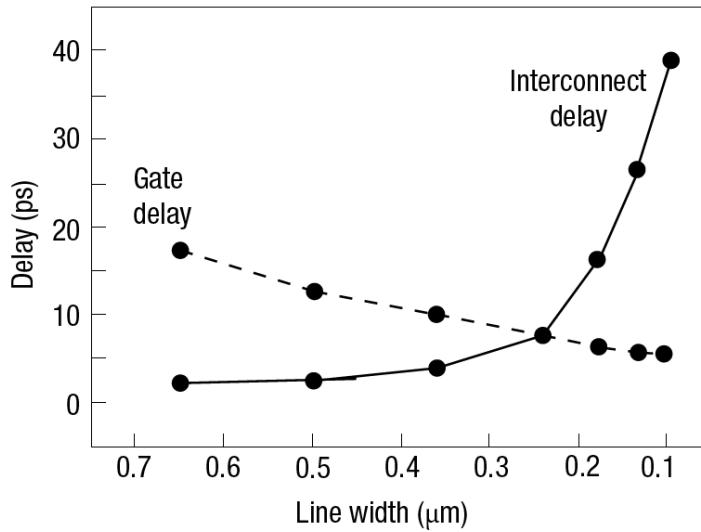


Figure 2: Trends in transistor gate delay and interconnect delay with IC fabrication technology [2]

the interconnection system, etc.) and physical level (respecting area/timing/power constraints). Using 3D-SICs introduces even more degrees of freedom: number of tiers, choices for manufacturing technology (e.g. full 3D integration, silicon interposer, face-to-face, back-to-face, etc.), 3D partitioning and placement strategies etc. These new degrees of freedom will contribute to the combinatorial explosion of already huge design spaces. Moreover, practice and 2D design experience cannot be fully exploited with 3D technology, since 3D-SICs change considerably the way ICs are implemented. The current design flows, which already showed their limits with conventional 2D-ICs, may thus need improvements to be able to deal with the increased complexity of emerging 3D-SICs [10, 21].

One of the solutions to face this problem is to develop high-level tools which can quickly explore design spaces and give early and reasonably accurate performance estimations based on physical prototyping of the 3D circuits [10].

In addition, performance estimation/optimization and the selection of the most-suitable solutions usually implies to take several objectives in account (e.g. maximization of the performance, minimization of the cost, minimization of the package size, etc.).

Currently, the design tools can be considered to follow a uni-criterion paradigm. Indeed, they have sequential development steps and each criterion is optimized without considering the impact on other criteria. This can lead to several rollbacks in the design flow since the achievement of the requirements can be time consuming (typical design iterations are measured in weeks).

On the other hand, multi-criteria approaches have been developed to optimize all the criteria simultaneously. Designing 3D-SICs inherently implies a huge design space and numerous degrees of freedom and criteria. This is the reason why we propose to apply this paradigm for the design of 3D-SICs.

Research questions

Multi-objective optimization and multi-criteria decision aid were developed from the need of taking into account several criteria simultaneously. These tools from the operations research field have shown their abilities in solving similar problems in other fields, which also have a large solution space and applying metaheuristics have shown interesting results [37].

In this thesis, we will show the applicability of a multi-criteria paradigm for the design of 3D-SICs:

- how a 3D circuit can be modelled to apply multi-objective optimization
- what kind of information can be provided to a designer
- how multi-criteria decision aid can exploit these results to assist a designer

Outline

In the first chapter, we will take an overview of the design and manufacturing of 3D-SICs. We will explain the limitations of current design flows and present the developments that have been carried out to overcome these problems. We will discuss why they should be improved and introduce how a multi-criteria paradigm can be useful.

In the chapter two, we will present an overview of the main tools in the MCDA fields where some of the most-used methods will be presented.

In the third chapter, we will define the problem we tackle (the 3D floorplanning) with the considered criteria. We will then show how a 3D-SIC can be modelled in order to apply multi-objective optimization. Simulations will be ran on a case study and show what kind of information can be provided to a designer. The methodology will then be validated with a realistic case study to show the added value of a multi-criteria paradigm compared to a uni-criterion approach.

In the chapter four, we will show the robustness of the methodology and the associated algorithms. We will use classical indicators of the fields to analyse the convergence and diversity properties.

In the fifth chapter, we will explain how the obtained results can be exploited using multi-criteria decision aid. We will discuss on how such a paradigm can be used for designing circuits and what needs to be done in order to integrate it to actual design flows.

Finally, we will conclude on the results of the thesis and express some possible perspectives.

Publications

The works described in this thesis have led to several publications in refereed journals and conference proceedings:

TO ADD

1

Review of the literature

Part I: Microelectronics design

Chapter abstract

In this chapter, we present an overview of the 3D-stacked integrated circuits (3D-SIC) that have been proposed by the industrial and academic communities to overcome 2D-IC's limitations. We show how 3D-SICs can be designed and manufactured and explain why current design flows should be improved to deal these new challenges.

1.1 Introduction

In this chapter we will

1.2 3D integration

Most of the current ICs are designed with electronic components (i.e. transistors) that are planar (although multi-gate transistors, such as finFETs tends to extend in the 3rd dimension) interconnected using up to a maximum of 12 (also planar) wiring (metal) layers per circuit. Those conventional ICs can thus be considered to be two-dimensional (2D)-ICs since the interconnections are predominantly made in a planar

fashion [14, 15]. As a major evolution of 2D-ICs, 3D-SICs are designed with multiple traditional 2D-ICs (that are manufactured independently, using standard CMOS technology) that are assembled (stacked) vertically in 3D-tiers. Different 2D circuits communicate between tiers using vertical interconnections that need to connect front side of the chip and the backside, i.e. they need to traverse bulk silicon. These connections can be Through Silicon Vias (TSV), micro bumps (μ Bump) or copper pads (CuPad) and they can be today manufactured with satisfactory geometrical properties, namely their diameter, pitch and height, allowing efficient integration of real-world systems [16, 17]. This is shown in Figure 1.1, where 2 dies, oriented face down are connected. An active component (i.e. logic gate) of the T1 is connected to the T2 using a TSV, back side metallization layer (to enable TSV placement anywhere in the T1 die), and μ bump on the top layer of the T2, that is then connected, through a series of metal layers of the T2, to the active component of the top tier (T2).

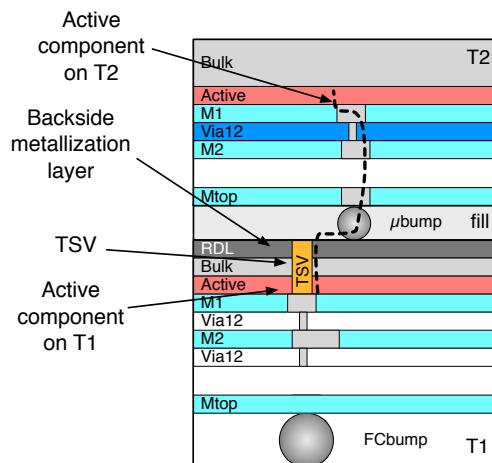


Figure 1.1: Illustration of the wiring properties of a 3D-SIC

1.2.1 Manufacturing technologies

Several 3D manufacturing technologies have been proposed and have been used to implement complete systems. Among the existing possibilities, four major categories of methods that illustrate 3D integration can be cited [3, 18].

Transistor stacking The transistor stacking consists in creating several transistors level on one substrate. This should be the better way to manufacture 3D circuits although the success rate are currently limited due to thermal issues among the different limitations. The required temperatures to create a layer of high-performance

transistors would provoke the destruction of the copper and aluminium already laid down on the previous layer [3].

Chip stacking This methods consists in stacking components that have been designed and tested separately to produce a system-in-package (SiP). The vertically-stacked chips are interconnected with traditional wirings (lateral wire bondings). The principal advantage of this method is an improvement in terms of size. The wirings are shorter however the components integration density is not increased compared to a 2D system.

Die-on-wafer stacking In this method, known good dies (KGD), which are functional tested chips, are connected to a host wafer containing other KGDs. These KGDs can be interconnected with organic glues, oxide or metal bonding. The wafer and the bonded KGDs are then shaped to create the interconnections. Different substrates can be combined if the required temperature is low enough to minimize non-homogeneous expansion effects.

The die-on-wafer stacking can use interconnections on the edges of the chips or through-die. Depending on the interconnection type, this method can produce a better integration level than the chip stacking, with a better cost per connection ratio and a higher interconnection density, while holding the advantages of the KGDs.

The quality of the stacking depends on the pick-and-place equipment which is used to position the dies on the wafer. The placement accuracy will determine the possible interconnection density. Also, current equipments are supposed to handle fully buffered chips, not naked circuits so it does not provide protection to static discharge.

Wafer-level stacking This methods consists in bonding entire wafers into a stack. The vertical through-wafer connections are made directly through each substrate to the next wafer and its transistors layer. Similarly to the previous method, the interconnection density rely on the precision of the alignment, which is however currently better than the die-on-wafer stacking. This greater accuracy implies a better cost per connection ratio and a higher interconnection density compared to the die-on-wafer stacking.

The use of mixed substrates is also possible, only limited by the process temperatures. All the processing is done at the wafer level so wafer handling equipments are used. Since these provide protection to static discharge so there is no need to include buffering between the layers. The methods to bind two wafers are the same that are available for the die-on-wafer method.

One drawback to wafer-level stacking is its efficiency, since the chips on a wafer are not all KGDs.

1.2.2 3D-SIC advantages

Interconnection length The 3D integration allows to design circuits with components closer to each other. Wire of a few millimetres long can be replaced by TSV of a few tens of microns, as shown in Figure 1.2. These shorter interconnections will introduce shorter delays, hence allowing higher working frequencies [3, 19].

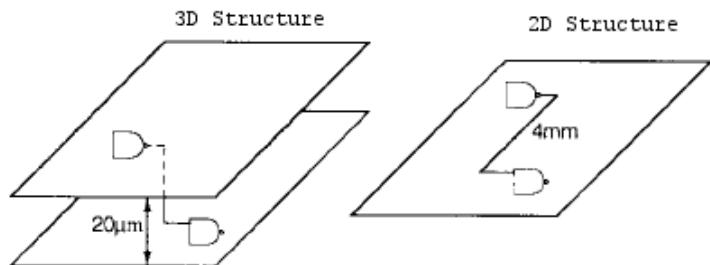


Figure 1.2: Shorter interconnections [3]

Silicon efficiency and accessibility Adding a vertical dimension allows to increase the integration density. It is therefore possible to have more logic gates than a 2D-IC for the same footprint, hence a more efficient use of the silicon as shown in Figure 1.3. For instance, compared to the footprint of a 2D-IC, the 3D-SICs can double the integration for a 50% use of a 2D footprint [3].

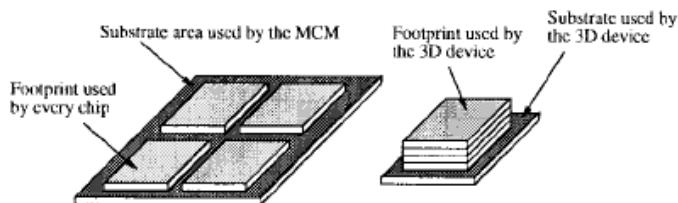


Figure 1.3: Silicon efficiency [3]

In addition, the 3D integration allows a better accessibility for the components, as shown in Figure 1.4. Indeed, for a 2D structure, 8 accessible neighbours can be considered for a central element (Figure 1.4 (a)), whereas for a 3D structure, the number of accessible neighbours can reach 116 with through-tiers interconnections (Figure 1.4 (b)) [3].

Bandwidth The use of TSVs on 3D-SIC can significantly increase the bandwidth of a circuit. Indeed, as shown in Figure 1.5, the interconnections are not only limited

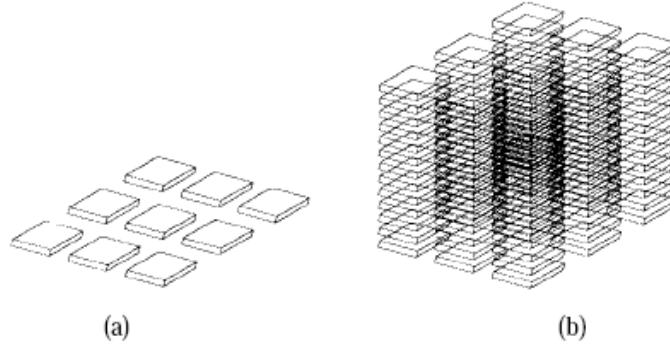


Figure 1.4: Components accessibility [3]

to peripheral connections but can also make use of the circuit's surface. At a same working frequencies, this allows more bandwidth while at lower frequencies, the same bandwidth usage will require less power.

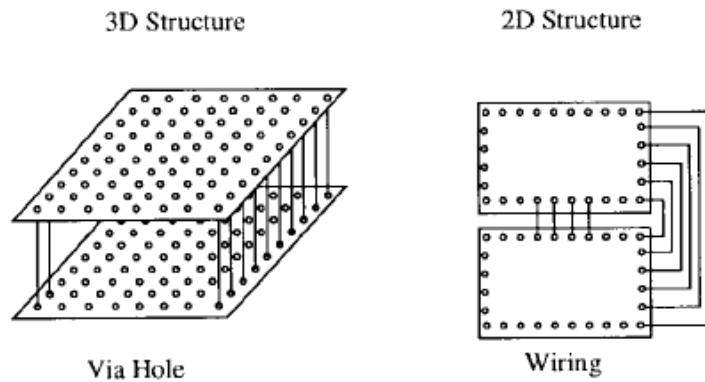


Figure 1.5: Bandwidth improvement [3]

Consumption and noise Shorter interconnections generally translates into lower capacitance and inductance parasitics. This means a decrease of the numbers of repeaters, hence a better consumption, less noise and less jitter hence lower delays and power consumption.

Heterogeneous circuits The 3D technologies allow truly heterogeneous designs. For instance, it is possible to integrate, in addition to traditional digital circuits of different technologies, analogical circuits such as sensors or antennas, as well as power supply, which give 3D-SIC a high degree diversity [20]. The Fig. 1.6 shows

a schematic view of a 3D-SIC developed by IMEC for biomedical purposes that contains antennas, DSPs, EEG/ECG sensors, a power supply and solar cells [4].

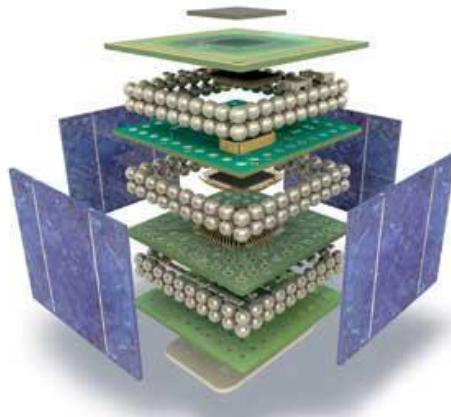


Figure 1.6: Schematic illustration of an heterogeneous 3D-SIC (developed by IMEC) [4]

1.2.3 3D-SIC design challenges

As explained, 3D-SICs offer numerous design perspectives thanks to their advantages. However there are drawbacks that need to be taken into account and that will be discussed in the following paragraphs.

Thermal dissipation The power density has increased exponentially over the past decades for the 2D-ICs and it appears that this trend will continue in the near future. As for 3D-SICs, due to their higher component density, they will also be subject to higher power density so thermal management should be considered carefully [3]. A simplified model of thermal dissipation has been developed in this thesis and will be presented in Chapter 3.

Cost With the appearance of a new technology, the involvement of a high cost should often be expected. In the case of 3D technology, the cost is currently high due to the lack of infrastructure and the reluctance of manufacturers who do not want to risk to change to new technologies [3].

Design complexity and design software A large number of systems have been implemented using the 2D technologies which means that current tools can cope with 2D design complexity even if they show more and more their limits [10, 21]. As for

3D-SICs, the increased complexity can be tackled by developing adapted software [3]. However, to the best of our knowledge, few 3D dedicated software currently exist and they are mainly developed for and owned by particular manufacturers and are based on 2D design tools which does not allow to tackle the complexity of 3D designs integrally.

1.3 2.5D-ICs by Xilinx

Now that the 3D integration has been introduced, let us give some notes that are worth mentioning about the 2.5D-ICs introduced by Xilinx [5]. 2.5D integration can be considered as a stepping stone to 3D design, as illustrated in Figure 1.7. Dies are placed on a silicon interposer where are located the interconnections required to bind the dies, as shown in Figure 1.8. Compared to classical 2D-ICs, this allow higher interconnect density while being less challenging than 3D-SICs in terms of design flow, thermal issues, reliability, testing and cost as the technology is already existing.

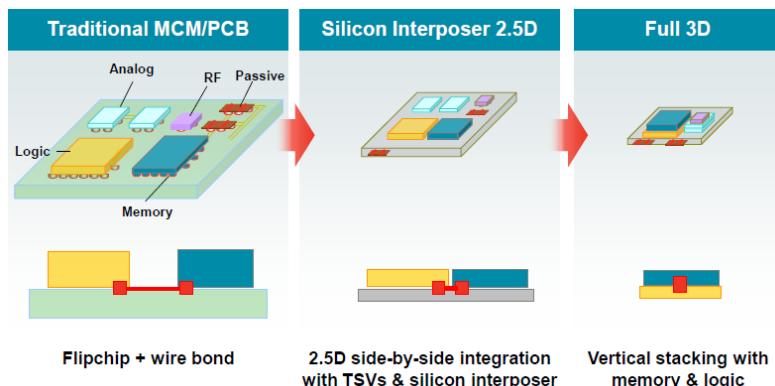


Figure 1.7: 2.5D as a stepping stone to 3D integration [5]

In the following section, we will have an overview about these software tools and generally about the design flows used to design integrated circuits.

1.4 Current design flows and their limitations

Design flows are the combination of electronic design automation (EDA) tools used to produce an integrated circuit. These flows can generally be summarized in 4 main steps [22], as shown in Figure 1.9.

As one can observe, the design flows are sequential. The process goes from one step to the other with local optimization loops. In practice, it is not unusual to

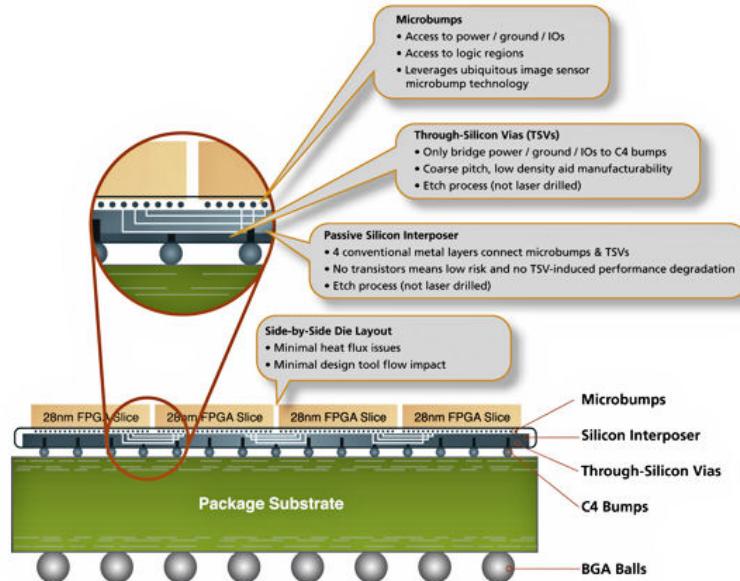


Figure 1.8: Illustration of the 2.5D integration with a silicon interposer [5]

Global view

Detailed view

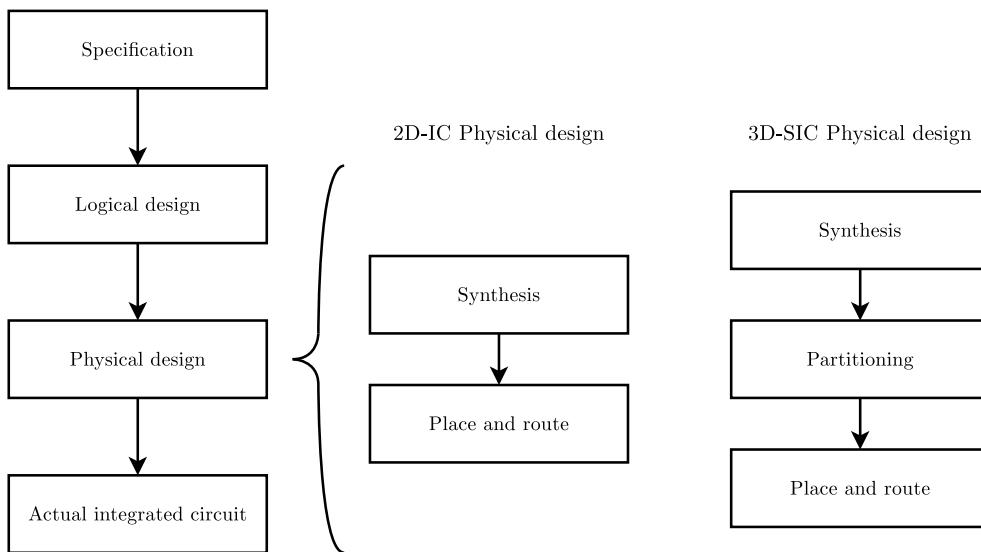


Figure 1.9: General classical design flow

have several rollbacks to the previous steps due to inconsistency in the optimization process. As explained previously, designing ICs implies numerous choices. At the moment, with this growing complexity, the current design flows can already show their limits. For instance, most of the time, the designers will be likely to freeze a certain amount of choices on basis of their experience, and then begin the optimization process with the remaining parameters. This will therefore limit the exploration of the design space and other good solutions may be ignored. In addition, the fixed choices can be questionable since they are based on the designer's experience though they could also be based on more objective facts.

The current design flows, which already showed their limits with conventional 2D-ICs, may thus need improvements to be able to deal with the increased complexity of emerging 3D-SICs [10, 21].

For the moment, most 3D design flows adapt classical flows to include 3D specificities, in particular 3D partitioning and 3D place & route (see Figure 1.9). We can observe that these two steps are separated: the circuits are first (manually) partitioned, then the place & route occurs for each layer. However, one can guess that the performances of a 3D-SIC will depend on the position of a component, considering simultaneously its position on a layer ((X,Y)-coordinates) and the layer where it lies (Z-coordinate). 3D design flows therefore need improvements to take into account these three coordinates at the same time.

1.5 Design space exploration tools

In order to cope with the increasing complexity of integrated circuits and the limitations of the current design flows, numerous tools have been proposed, in particular works about design space exploration (DSE) that have been developed to quickly suggest possible interesting solutions to a designer and speed up the design processes. In this section, we will describe different DSE tools that have been proposed in the literature.

1.5.1 2D-IC tools

MILAN The MILAN (Model based Integrated simuLAtioN) framework [23] aims to simplify the optimization and the exploration of design spaces for SoC platforms. This tool works on the component level and allows the users to choose a compromise between the simulation speed and the results accuracy. The exploration and optimization process is done in two phases: first it searches for possible combinations between the architecture, the application and the mapping and second it estimates the performances (power, latency) depending the precision asked by the users.

SoC Architecture Explorer SoC Architecture Explorer [24] is a multi-objective optimization and exploration tool that aims the design of SoC architectures by evaluating the compromises between the footprint and the execution time. The exploration process focuses on the application and the system architecture where the tool analyses the data flow and estimates the data transfers to determine a number of possible architectures.

modeFRONTIER (ESTECO) modeFRONTIER [25] is a proprietary development environment developed by ESTECO. It is a multi-objective optimization tool that aims parallel SoC architectures. modeFRONTIER allows to deal with up to one million different design configurations thanks to statistical analysis tools and data mining techniques.

MULTICUBE The MULTICUBE project (MULTI³) [26,27] is a European project started in 2008 and dedicated to the multi-objective exploration of MPSoC architectures for multimedia embedded systems. The aims is to developed a framework that allows a quick and automated exploration of the design space to improve the performances of a MPSoC with metrics such as power, latency, computing performance, bandwidth, QoS, etc. This project is based on several heuristics and optimization algorithms that reduce the exploration time and allow a quick selection of the best solutions of a Pareto-optimal frontier. In addition, MULTICUBE also aims to define an application-oriented framework based on the results of the multi-objective exploration to optimize the resources allocation and the tasks scheduling of the applications. The exploration is done at the system level, using the SystemC language. The project includes proprietary and open-source tools whose development targets the industry. Among the developed prototyping tools, Multicube explorer and Multicube-SCoPE can be cited.

Multicube Explorer Multicube Explorer [28] is a design space exploration framework for supporting platform-based design. This tools allows a fast optimization of a system with objective functions such as power, delays, surface, etc. by means of a system simulator. Multicube explorer proposes several multi-objective optimization methods that aim to propose the best compromises.

Multicube-SCoPE Multicube-SCoPE [29] is an evolution of the SCoPE tool [30] oriented to design space exploration. It is a fast system performance and power simulator providing metrics associated with a system in order to drive the DSE process.

1.5.2 3D-SIC tools

DSE for 3D-stacked DRAMs by Weis *et al.* Design space exploration for 3D-stacked DRAMs has been developed by Weis *et al.* [31]. They defined a 3D-DRAM based on a SystemC model with a 3D channel controller and also considered a wiring model for the TSVs. The used metrics are area, performance and energy efficiency evaluated for different DRAM architectures and technologies. 3D thermal issues have been kept out of the scope of the study. The simulation results allowed them to have a trade-off analysis of horizontal wirings against vertical wirings in terms of energy and cell efficiency. They could show quantitatively how a 3D-DRAM can perform better than a classical DRAM.

Observation This work is really interesting as it shows the stakes of using the 3D technology for DRAM. However, since it is based on DRAMs, the tools work with a memory structure that is repeated in the 3D-DRAM, which does not take into consideration more heterogeneous architectures. Also, only trade-off analyses are performed, which does not give a more global multi-criteria insight of the results as it will be illustrated in Chapter 2.

DSE for 3D architecture and DSE for 3D integrated circuits by Xie *et al.* Design space exploration for 3D architecture and design space exploration for 3D integrated circuits are two works proposed by Xie *et al.* [32,33]. In the first study, they combine several tools to perform a DSE:

- for the 3D cache partitioning, two strategies have been proposed at the subarrays granularity level
- the area, the delay and the energy of a 3D cache are assessed following a cost function
- 3DCacti, a tool developed to explore various 3D partitioning options of caches
- thermal-aware 3D floorplanning based on simulated annealing

With the DSE, they are able to propose different possible architectures for 3D microprocessor design by performing trade-off analyses of the criteria. The second study is an extension where a cost analysis is added.

Observation These works seem to be among the most integrated study in the literature with cache partitioning and microprocessor floorplanning, and considering several criteria including thermal issues. However, the partitioning and the floorplanning are separated while a more 3D approach should consider both dimensions simultaneously. Also, the criteria are aggregated with a cost function which can lead to inconsistency as it will be explained in Chapter 2 and only trade-off analyses are performed.

Automated design flow for 3D microarchitecture evaluation by Cong *et al.* An automated design flow for 3D microarchitecture evaluation has been proposed by Cong *et al.* [34]. They propose an evaluation flow for performance assessment and thermal management. This allows them to perform thermal-aware 3D floorplanning.

Observation This work is worth mentioning as it proposes a quick way to evaluate temperature issues. However, it only deals with the thermal criterion.

PathFinding flow The PathFinding flow is a project led by IMEC and Milojevic *et al.* in collaboration with Atrenta [35, 36]. The aim of this work is to be able to produce a specification for the architecture and for the technology with assessment of performance, power and cost. The methodology is divided in 3 steps:

1. 3D system level design exploration with a rough estimation of the performance, power and cost parameters. The designer will be able to focus on the 2D design issues while manually considering the 3D specificities.
2. RTL (Register Transfer Level) elaboration, which links the system level to the physical design by producing RTL models.
3. 3D physical design prototyping, which allows fast exploration of the physical design impact of alternative design/technology options on the performance, power and cost parameters.

Observation This work is also among the most integrated study in the literature. However, the criteria optimization is done following a uni-criterion approach which does not allow to explore quickly several possibilities.

1.6 Conclusion

In this chapter, we have presented an overview of the evolution of IC design. Manufacturers have pushed back the limitations of the silicon for the past decades and are now facing new challenges due mainly to quantum effects. 3D-SICs have been proposed to face these problems and we have shown a quick review of this promising technology.

With the 3D integration, design flows have evolved and integrate 3D partitioning and 3D place and route. However, these two steps are performed separately while they should be considered simultaneously as the circuits' performances will depend on the position of a component on a layer and the layer where it lies.

We have then presented researches that aim to deal with these challenges by making use of multi-objective optimization. To the best of our knowledge, all these tools use a uni-criterion approach or deal with a limited set of criteria while performing

only trade-off analyses from a Pareto front. The goal of this research is to show that a more multi-criteria-oriented optimization could be more suitable to take into account the many aspects of a design and that a more global multi-criteria analysis can provide more information.

In the next chapter, we will describe a short overview of the tools coming from the operations research that will allow to take into account multiple criteria simultaneously.

2

Review of the literature

Part II: Operations research

Chapter abstract

In this chapter, we briefly present the basics of multi-objective optimization and multi-criteria decision aid, in order to justify our choice to use such a paradigm. We will explain when a multi-criteria approach should be used and present some of the most-used methods of the field.

2.1 Introduction

In this chapter, we will briefly present the basics of multi-objective optimization and multi-criteria decision aid, in order to justify our choice to use such a paradigm. As stated in Chapter 1, the 3D integration can offer new perspectives but designing 3D-SICs includes two major distinctive features: multiple criteria and a huge number of possible solutions. When facing such problems, two main approaches exist: the uni-criterion paradigm and the multi-criteria paradigm. For optimization problems, these paradigm will refer to the terminology mono-objective/multi-objective optimization while for decision aid, the terminology uni-criterion/multi-criteria will be used.

In the following, we will briefly describe each paradigm, showing some of the main approaches alongside illustrative examples. We will first present the uni-criterion methodology, show why it can be limited in our context and explain why a multi-criteria paradigm can be more suitable.

2.2 The uni-criterion paradigm

2.2.1 Problem formulation

An optimization problem can be formulated, without loss of generality, as [7]

$$\begin{aligned} \min f(x) \\ x \in A \end{aligned} \tag{2.1}$$

where f is a real-valued function evaluating the solutions denoted x , and A is the set of solutions, f is also called the *criterion* on which x is evaluated. Let us note that the equation 2.1 expresses a *minimization* problem. A *maximization* problem can be seen as a minimization problem with the identity

$$\max_{x \in A} f(x) = -\min_{x \in A} (-f(x))$$

so that there is no loss of generality by using only *minimization* formulation.

In order to give a more precise idea of what an optimization problem is, we will describe in the next section some typical examples taken from the reference book [7, 37].

2.2.2 Examples of typical optimization problems

Linear programming

Linear programming (LP) is a problem formulation where the aim is to optimize a linear function, subject to linear inequality constraints. This can be formulated as follows:

$$\min \mathbf{c}^\top \mathbf{x} \tag{2.2}$$

subject to

$$A\mathbf{x} \leq \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

where \mathbf{x} is a vector of continuous, integer or boolean variables to be determined, \mathbf{c} and \mathbf{b} are vectors of coefficients, A is a matrix of coefficients.

Efficient exact methods for solving LP problem exist such as, among the most knowns, the simplex algorithm [38] or the interior point method [39].

Example 2.1 (Linear programming). A given company produces two electronic boards $Board_1$ and $Board_2$ based on two kinds of memories M_1 and M_2 . The objective consists in finding the most profitable product mix, given the availability of each memory M_1 and M_2 , and the amount of memory used as well as the profit per board, as shown in Table 2.1. The decision variables are x_1 and x_2 that represent respectively

the amount of $Board_1$ and $Board_2$ produced. The objective is to maximize the profit. The problem can be formulated as an LP:

$$\max \text{ profit} = 5x_1 + 4x_2$$

subject to the constraints

$$\begin{aligned} 192x_1 + 128x_2 &\leq 1024 \\ 32x_1 + 64x_2 &\leq 192 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Table 2.1: Data associated with the LP problem

	Usage for $Board_1$	Usage for $Board_2$	Availability
M_1	192	128	1024
M_2	32	64	192
Profit per unit	€5	€4	

Integer linear programming

Integer linear programming deals with linear problems where the variables are restricted to be integers:

$$\min \mathbf{c}^\top \mathbf{x} \tag{2.3}$$

subject to

$$\begin{aligned} A\mathbf{x} &\leq \mathbf{b} \\ \mathbf{x} &\in \mathbb{N} \end{aligned}$$

where \mathbf{c} and \mathbf{b} are vectors and A is a matrix of coefficients.

When the set of decision variables contains both discrete and continuous variables, the problem refers to **mixed integer programs** (MILP).

Other particular ILP problems which deals with variables that are restricted to be either 0 or 1 are called **0-1 linear programming**.

Example 2.2 (Travelling salesman problem (TSP) [37]). This is one of the most known optimization problem. It can be formulated as follows: given n cities and the distance between each pair of cities, we have to find the shortest tour that visits each city once and returns to the origin city. This problem can be formulated as an ILP problem.

Let d_{ij} be the distance between the city i and the city j , S be the set of solutions (tours) and define:

$$x_{ij} = \begin{cases} 1 & \text{if the path goes from city } i \text{ to city } j \\ 0 & \text{otherwise} \end{cases}$$

The ILP formulation is then:

$$\min \sum_{i=0}^n \sum_{j=0, j \neq i}^n d_{ij} x_{ij}$$

s.t.

$$\begin{aligned} \sum_{i=0, i \neq j}^n x_{ij} &= 1 & j = 0, \dots, n \\ \sum_{j=0, j \neq i}^n x_{ij} &= 1 & i = 0, \dots, n \\ \sum_{i \in S, j \notin S} x_{ij} &\geq 1 & \forall S \subset \{1, \dots, n\} \\ 0 \leq x_{ij} \leq 1 & & \forall i, j \\ x_{ij} &\in \mathbb{N} & \forall i, j \end{aligned}$$

■

Non-linear programming

Non-linear programming (NLP) models deal with mathematical problems where some of the constraints and/or the objective function are non linear:

$$\min f(x) \quad (2.4)$$

where

$$\begin{aligned} f : \mathbb{R}^n &\rightarrow \mathbb{R} \\ x &\in \mathbb{R}^n \end{aligned}$$

subject to

$$g_i(x) \leq 0, i \in J = 1, \dots, m$$

where $g_i : X \rightarrow \mathbb{R}^n$ are the inequality constraints.

NLP are generally more difficult to solve than LP [37] and metaheuristics (see Section 2.4.2) are commonly used to solve this class of problems.

2.3 From the uni-criterion paradigm to the multi-criteria paradigm

With a uni-criterion paradigm, the optimization of one criterion is generally performed while considering that this single criterion synthesizes all the characteristics

of the problems or that the other criteria already satisfy an acceptable level. This methodology will search for a solution which is supposed to be optimal according to this criterion. However, most problems encountered in the field of IC design, and more generally in other industrial fields, contains several conflicting criteria as it will be illustrated in Chapter 3. Finding a solution that simultaneously optimizes all the criteria is only possible in rare cases and if optimality can be reached.

For instance, when designing ICs, a manufacturer will try to simultaneously maximize the performance while minimize the cost of the circuit. However, we can already guess that those two objectives are conflicting. Also, producing high-end ICs can be subject to more difficulties in terms of thermal dissipation. In addition, a criterion based on ecological standards may have impacts on the cost and the performance of an IC.

This example shows that a uni-criterion approach cannot always be applied since there is no achievable optimum, as several criteria have to be simultaneously taken into account. A solution that optimizes one criterion will likely affect another.

In order to deal with the multiple criteria of a problem, another paradigm consists in taking into account all the criteria simultaneously. This is the goal of the multi-criteria paradigm which aims to:

1. find the solutions that are efficient on all the criteria simultaneously with the multi-objective optimization;
2. provide support to a decision maker facing several conflicting solutions with multi-criteria decision aid (MCDA) that allows to highlight such conflicts and therefore obtain a compromise with a transparent process.

2.4 The multi-criteria paradigm

2.4.1 Problem formulation

A multi-criteria problem can be formulated without loss of generality as follows [7]:

$$\begin{aligned} & \min\{f_1(x), f_2(x), \dots, f_m(x)\} \\ & x \in \mathcal{A} \end{aligned} \tag{2.5}$$

where $\{f_1(x), f_2(x), \dots, f_m(x)\}$ is a set denoted \mathcal{F} of m evaluation criteria that needs to be minimized and x is a solution of the set $\mathcal{A} = \{a_1, a_2, \dots, a_n\}$.

As explained in Section 2.3, an optimal solution can be impossible to find for a multi-criteria problem. However, compromise solutions can exist and in order to identify them, a dominance relation has been defined [7]:

Definition 2.3 (Dominance). A solution a_1 dominates a solution a_2 if:

- a_1 is at least as good as a_2 on all criteria;

- a_1 is strictly better than a_2 on at least one criterion.

From this dominance relation, it is then possible to filter the solutions in order to keep only the non-dominated ones. This set of *efficient* solutions is called the Pareto frontier. Let us note that the *efficient* solutions refer to the decision space while the Pareto frontier refers to the evaluation space.

Two approaches can be used to establish this set [40]:

- *Exact methods* which aims to compute the Pareto frontier directly [41, 42].
- *Approximate methods* which are based on metaheuristics to quickly explore the solution space and approach as best as possible the Pareto optimal frontier [37].

As explained in Chapter 1, designing 3D-SICs involves a huge solution space to deal with in the optimization process. The solution (that is to say the most-suitable 3D-SIC architecture) is unknown and an exhaustive search would take a prohibitive time. Also, due to the nature of the criteria (discrete and continuous variables, linear and non-linear criteria) that will be defined in Chapter 3, we have few hopes to be able to develop an exact method. For those reasons, approximate methods with metaheuristics for multi-objective optimization will be used. Let us also remind that the aim of this thesis is to evaluate the applicability of a multi-criteria paradigm to the design of 3D circuits. Therefore developing exact methods has been kept out of the scope of this work.

2.4.2 Metaheuristics for multi-objective optimization

Metaheuristics are a family of approximate optimization methods. They aim to provide "acceptable" solutions in reasonable time for solving complex problems [37]. As stated previously, the optimal solution of a multi-objective optimization problem (MOP) is not a single solution but a set of solutions defined as Pareto optimal solutions. The main goal is therefore to obtain this set.

In our study, due to the heterogeneous nature of the criteria, there are few hopes to find the exact Pareto optimal solutions. In such cases, metaheuristics are commonly used and the goal is then to find an approximation of this set. Two properties have to be respected in order to ensure good approximations: convergence to the Pareto optimal front and uniform diversity. The first property allows to have solutions that are closed to the Pareto set whereas the second property shows a good distribution around the Pareto front.

Numerous metaheuristics have been developed since the 50s. Among the most known, let us cite genetic algorithm [43], scatter search [44], simulated annealing [45], tabu search [46], memetic algorithms [47] and ant colony optimization [48].

In this work, we will focus on genetic algorithms (GA) as they are quick to implement for a first approach and are suitable to heterogeneous variables problems. More details about other metaheuristics can be found in reference books such as [37, 49, 50].

General description of genetic algorithms

Genetic algorithms have been developed by Holland in the 1970s [43]. They are metaheuristics that reproduce the properties of a natural selection process as described by Charles Darwin. GAs are based on the principle of improvement of gene pool of a population over generations. GAs will mimic the natural evolution with techniques such as selection, crossover and mutation. In the following, we will briefly describe the general methodology of a GA without considering a multi-objective case since the key steps are similar. Afterwards, we will describe one of the most popular multi-objective genetic algorithms: NSGA-II (Non-dominated Sorting Genetic Algorithm).

Genetic algorithms rely on a population that is evolved toward better solutions or individuals. The evolution is an iterative process and starts usually with randomly-generated solutions. At each iteration, every individual is evaluated to define its fitness. The fitter ones are more likely to be selected for genetic modifications (crossover and possibly mutation). The produced solutions constitute the new generation that will be used for the next iteration. The algorithm is commonly terminated when a maximum number of generations has been produced or when a certain fitness level has been satisfied. The general pseudo-code for genetic algorithms is shown in Algorithm 1.

Algorithm 1: General pseudo-code for genetic algorithms

```

1 CHOOSE initial population;
2 EVALUATE each individual's fitness;
3 repeat
4   | SELECT parents;
5   | CROSSOVER pairs of parents;
6   | MUTATE the resulting offspring;
7   | EVALUATE the new candidates;
8   | SELECT individuals for the next generation;
9 until TERMINATION CONDITION satisfied;
```

Representation of a solution The representation or encoding of a solution is called a chromosome and depends on the problem. Several examples of problems show binary encodings however, in our study we will use a real-valued matrix that will be detailed in Chapter 3. Nevertheless, without loss of generality, we will illustrate the principles of a genetic algorithm by using binary-coded solutions.

Initialization Initially many solutions are generated, usually randomly to form the initial population. Depending on the problem, the generation of the initial population can be guided (seeded) to areas where optimal solutions are likely to be found.

Selection The selection is a stochastic process usually planned so that the fitter solutions have a higher probability of being selected. This aims to ensure the convergence of the algorithm.

In particular, one can mention the roulette wheel selection method where the fitness level is used to associate a probability of selection to each candidate. If f_i is the fitness of the individual i , its probability to be selected is $p_i = \frac{f_i}{\sum_{j=0}^n f_j}$ where n is the number of individuals in the population.

Crossover Once a pair of individuals has been selected, they will be crossed-over. Typically, two children are created from each set of parents. One method of crossover (one-point crossover) will be explained here but other approaches exists []. A random crossover point will be selected on both parents. Beyond that point, the data will be swapped with the information of the other parent as show in Example 2.4.

Example 2.4 (Crossover example). Let us consider two individuals x and y of the population:

$$\begin{array}{rcccccccccc} x & = & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ y & = & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{array}$$

If the randomly-chosen crossover point is 2 then the obtained offspring is:

$$\begin{array}{rcccc|cccccc} x' & = & 0 & 1 & | & 0 & 0 & 1 & 0 & 1 & 0 \\ y' & = & 1 & 1 & | & 1 & 0 & 1 & 1 & 0 & 0 \end{array} \blacksquare$$

Mutation Mutation is a genetic operation used to ensure diversity in the generated populations. It changes one piece or more information in the chromosome of an individual. This alteration depends on how the solution is encoded. If it is a bit string, the most common operation is to apply a bit flip (see Example 2.5) while for float chromosomes, new values can be generated following user-defined rules (see detailed illustration in Chapter 3).

Example 2.5 (Mutation example). Let us consider one individual x' of the population:

If the randomly-chosen mutation point is 3 then x' becomes:

$$x' = 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0$$

$$x' = 0 \ 1 \boxed{1} \ 0 \ 1 \ 0 \ 1 \ 0$$

Termination The generational process is repeated until a termination condition has been encountered. Common conditions are:

- a certain level of fitness reached;
- fixed number of generations reached;
- simulation elapsed time reached;
- no better results produced after several generations.

Multi-objective genetic algorithm: NSGA-II

While the original genetic algorithms have been developed for mono-objective purposes, they have also been extended to multi-objective optimization and among the most known, one can cite NSGA-II.

NSGA-II stands for Non-dominated Sorting Genetic Algorithm and has been developed by Deb [51] to provide a multi-objective version for genetic algorithms. It is an evolution of the original NSGA proposed in [52]. NSGA-II follows the same steps as a classical GA and additionally implements techniques, particularly in the selection step, to take into account several objectives simultaneously.

NSGA-II selection The selection is based on the Pareto dominance principle, particularly the Pareto rank which allows to sort all the solutions of a set following an extended Pareto principle and the crowding distance which estimates how dense the surrounding of a solution is.

Definition 2.6 (Pareto rank [51]). *From a given pool of solutions, the Pareto optimal ones are of rank 1. For the higher ranks the following process is repeated iteratively: to find the solutions of rank $i \geq 2$, the solutions of rank $i - 1$ are removed and the Pareto solutions from this subset are of rank i .*

Definition 2.7 (Crowding distance [51]). *The crowding distance is a measure of the density of solutions surrounding a particular point in the population. It is computed by taking the average distance of the two points on either side of this point along each of the objectives (see. Algorithm 2).*

The number of solutions per generation is fixed as constant. Between two solutions with different Pareto ranks, the lower rank will be preferred. Otherwise, if both solutions have the same Pareto rank then the one located in a lesser crowded region will be preferred.

Algorithm 2: Crowding distance for the set of solutions A

```

1  $l = |A|;$ 
2 foreach  $i$  do
3   | set  $A[i]_{distance} = 0$ ;
4 end
5 foreach objective  $m$  do
6   |  $A = \text{sort}(A, m)$ ;
7   |  $A[1]_{distance} = A[l]_{distance} = \infty$ ;
8   | for  $i = 2$  to  $(l - 1)$  do
9     |   |  $A[i]_{distance} = A[i]_{distance} + (A[i + 1] \cdot m - A[i - 1] \cdot m)$ 
10    | end
11 end

```

2.4.3 Multi-criteria decision aid

Once the Pareto frontier is obtained or approximated, the compromise solutions can be found by establishing a preference model of the decision maker facing several conflicting solutions. Those models can be classified into three broad categories [40, 53] whose methods will be detailed in Section 2.4.5:

1. *Aggregation methods*: numerical scores are calculated by aggregating the criteria to determine the level of preference for a solution. The most known aggregation methods are the Multi-Attribute Utility Theory (MAUT) [54] and the Analytic Hierarchy Process [55].
2. *Interactive methods*: it is a sequential process composed by alternating computation steps and dialogue with the decision maker. A first compromise is submitted to the decision maker who can accept or deny it. If the solution is denied, the DM can give extra information (e.g. releasing a constraint) about his preferences (dialogue) and a new solution can be calculated, so a new decision process begins. Otherwise, no better solution can be found and the process stops. Among the most known interactive methods, the STEP Method (STEM) [56] or the Satisficing Trade-Off Method (STOM) [57] can be cited.
3. *Outranking methods*: the solutions are compared pairwise which enables the possibility to identify the relationship between the solutions. This shows the preference for a solution in comparison to another one. PROMETHEE [58] and ELECTRE [59] are among the most known outranking methods.

Generally, the purpose of MCDA is to provide answers for three main problematic [60]:

1. *The choice problematic ($P.\alpha$):* the aid aims the selection of a small number of good solutions in such way that one or several compromise solutions can be chosen.

Example 2.8. *In circuit design, the objective would be to choose the best compromise CPU in terms of performance and price.* ■

2. *The sorting problematic ($P.\beta$):* the aid aims the assignment of each solution to a predefined (ordered) category.

Example 2.9. *Depending on performance, price, radiation resistance, thermal operational range, electronic components can be sorted for commercial, industrial or military and spatial purposes.* ■

3. *The ranking problematic ($P.\gamma$):* the aid aims the complete or partial preorder of all the solutions.

Example 2.10. *With a preorder for CPUs based on an assessment of their performance, it is possible to associate a price to each processor depending on their ranking.* ■

2.4.4 Preference modelling definitions

Before introducing some important MCDA methods, let us first define some definitions about preference modelling in order to ease the understanding of the following sections.

When modelling the decision maker's preferences, three binary relations which result from the comparison of two alternatives a_i and $a_j \in \mathcal{A}$ are defined [40]:

$$\begin{cases} a_i P a_j & \text{if } a_i \text{ is preferred to } a_j \\ a_i I a_j & \text{if } a_i \text{ is indifferent to } a_j \\ a_i R a_j & \text{if } a_i \text{ is incomparable to } a_j \end{cases} \quad (2.6)$$

These relations translate situations of preference, indifference and incomparability and it can be assumed that they satisfy the following properties:

$$\forall a_i, a_j \in \mathcal{A} \quad \begin{cases} a_i P a_j \Rightarrow a_i \neg P a_j : & P \text{ is asymmetric} \\ a_i I a_i : & I \text{ is reflexive} \\ a_i I a_j \Rightarrow a_j I a_i : & I \text{ is symmetric} \\ a_i \neg R a_i : & R \text{ is irreflexive} \\ a_i R a_j \Rightarrow a_j R a_i : & R \text{ is symmetric} \end{cases} \quad (2.7)$$

Intuitively:

- $a_i P a_j$ corresponds to the existence of clear and positive reasons that justify significant preference in favour of a_j

- aIb corresponds to the existence of clear and positive reasons that justify equivalence between the two alternatives
- aRb corresponds to an absence of clear and positive reasons that justify any of the two preceding relations

2.4.5 Some important multi-criteria methods

Multi-Attribute Utility Theory

Multi-Attribute Utility Theory (MAUT) has been introduced by Fishburn [61] and Keeney and Raiffa [62]. This method belongs to the family of aggregation methods that consist in substituting the initial multi-criteria problem

$$\min\{f_1(x), f_2(x), \dots, f_m(x) | x \in \mathcal{A}\} \quad (2.8)$$

the following uni-criterion problem:

$$\min\{U(x) | x \in \mathcal{A}\} \quad (2.9)$$

where $U(x)$ is called the utility function that aggregates all the criteria to a single criterion:

$$U(x) = U[f_1(x), f_2(x), \dots, f_m(x)] \quad (2.10)$$

One of the most used utility function is the weighted sum:

$$U(x) = \sum_{j=1}^m w_j f_j(x) \quad (2.11)$$

where w_j is the weight associated to the criterion j .

With this utility function, it is then possible to compute an aggregated score for each solutions and rank them in order to choose among the best ones.

MAUT has been applied in numerous cases and developments have been provided to axiomatize this method and justify its use [54].

Analytical Hierarchy Process (AHP)

Analytical Hierarchy Process (AHP) has been developed by Saaty [55]. This multi-criteria method is based on mathematics and psychology and allows to face structurally complex choices by decomposing the problem in several sub-problems that can be analysed independently and are easier to understand. Similarly to PROMETHEE and ELECTRE, AHP proceeds by making pairwise comparisons of the alternatives, but on basis of a ordinal scale from 1 to 9. Indeed, one of the distinctive features of this methods is to build a matrix by asking the decision maker to compare all pairs of alternatives and criteria. Therefore, the input for AHP is not an evaluation table

but the DM's preference matrix. The normalized right-hand eigenvector of this matrix is then used to compute the score associated to each alternative and the weight associated to each criterion.

In order to illustrate AHP, we will give more details on a particular case where only the criteria are compared. The decision maker will make pairwise comparisons and give an ordinal scale of preference for the criteria. The following matrix can be obtained:

$$A = \begin{pmatrix} 1 & a_{12} & \dots & a_{1j} & \dots & a_{1m} \\ \frac{1}{a_{12}} & 1 & \dots & a_{2j} & \dots & a_{2m} \\ \vdots & & & & & \\ \frac{1}{a_{1j}} & \frac{1}{a_{2j}} & \dots & a_{ij} & \dots & a_{im} \\ \vdots & & & & & \\ \frac{1}{a_{1m}} & \frac{1}{a_{2m}} & \dots & a_{im} & \dots & 1 \end{pmatrix} \quad (2.12)$$

where a_{ij} is expresses the relative importance of the criterion i over the criterion j .

From this matrix, AHP uses a method based on eigenvector to extract the related weights of each criterion that can be used, for instance, as input data for MAUT in a weighted sum.

A comparison matrix is said to be consistent if $a_{ij}a_{jk} = a_{ik} \forall i, j, k$. However, consistency cannot always be reached and AHP's developers have defined a Consistency Index (CI):

$$CI = \frac{\lambda_{max} - m}{m - 1} \quad (2.13)$$

where λ_{max} is the largest eigenvalue of the matrix and m is the matrix size.

This Consistency Index is then compared to Random (consistency) Index (RI) which are considered to be appropriate CIs. These RIs are obtained by randomly generating matrices and taking the average CI values.

A Consistency Ratio (CR) then is defined:

$$CR = \frac{CI}{RI} \quad (2.14)$$

If the value of the Consistency Ratio is lower or equal to 10%, the inconsistency is considered to be acceptable. Otherwise, the decision maker has to revise judgements.

STEP Method (STEM)

The STEP Method has been proposed by Benayoun [56]. STEM is an interactive and iterative exploration procedure that aims to reach the best compromise according the decision maker after a certain number of cycles. Each cycle is composed of a calculation phase and a decision-making phase (discussion with the decision maker):

1. An efficient compromise solution is determined.

2. This solution is submitted to the decision maker. Three cases can then happen:
 - (a) The decision maker is satisfied and the procedure ends;
 - (b) The decision maker wants to simultaneously improve all the evaluations. This is impossible since the proposed solution is efficient. The procedure ends and cannot help the decision maker.
 - (c) The decision maker identifies a particular criterion on which a concession can be made in order to improve other criteria. A new efficient solution can then be determined.
 - (d) This new solution is submitted. Go to step 2.

Satisficing Trade-Off Method (STOM)

The STOM method has been proposed by Nakayama [57]. Similarly to STEM, it relies on a discussion with the decision maker but is based on the setting of an ideal point defined as follows:

Definition 2.11 (Ideal point). *The ideal point $f^* = (f_1^*, f_2^*, \dots, f_m^*)$ is defined such that $f_i^* = \min\{f_i(x), \forall i = 1, 2, \dots, m, \forall x \in \mathcal{A}\}$.*

The ideal point possesses as coordinates the best values that can be achieved for each criterion separately.

STOM can be summarized in four steps:

1. The first step is to set the ideal point.
2. Then the aspiration level for each criterion is asked to the decision maker; this is the reference point for each criterion of the decision maker.
3. A Pareto solution nearest to the aspiration level is determined.
4. This solution is submitted to the decision maker. If it is satisfactory, the procedure ends. Otherwise, the decision maker is asked to trade off to define another aspiration level. Go to step 3.

The PROMETHEE methods

PROMETHEE (Preference Ranking Organisation METHod for Enrichment Evaluations) has been initiated by Brans [58] and developed with Mareschal [63] and Vincke [64]. In this section, we will only describe the basics of PROMETHEE. More details can be found in [65].

The PROMETHEE methods are based on the three following steps:

- Enriching the preference structure: a preference function is introduced.
- Enriching the dominance relation: a valued outranking relation is determined.

- Decision aid: the valued outranking relations are exploited.

1. Preference function

Since the dominance relation is really poor (binary relation), a preference function $P_k(a_i, a_j)$ will be introduced to enrich it. This function gives the preference degree of an alternative a_i over an alternative a_j with respect to the function $d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$ which is the difference between the evaluation of a_i and a_j for the criterion k , assuming a non decreasing function.

Consequently, it is therefore possible to define several types of preference functions based on preference (P) or indifference (Q) thresholds, as shown in Table 2.2. Below the indifference threshold, the decision maker will consider having no preference while above the preference threshold, the decision maker will have no more difference in its preference.

2. Valued outranking relation

Multi-criteria preference index

The multi-criteria preference index is defined as follows:

$$\pi(a_i, a_j) = \sum_{k=1}^m P_k(a_i, a_j) \cdot w_k, \forall i \neq j \text{ with } \sum_{k=1}^m w_k = 1 \quad (2.15)$$

where $w_k > 0, k = 1, 2, \dots, m$ are the weights on each criterion. $\pi(a_i, a_j)$ represents a measure of the preference of a_i over a_j on all the criteria.

Let us note the following properties of the preference index:

$$\pi(a_i, a_i) = 0 \quad (2.16)$$

$$0 \leq \pi(a_i, a_j) \quad (2.17)$$

$$\pi(a_i, a_j) + \pi(a_j, a_i) \leq 1 \quad (2.18)$$

Outranking flow

An “outranking flow” is then defined on the basis of the preference index. That allows to compare alternatives with each others. Three types of flow are formulated:

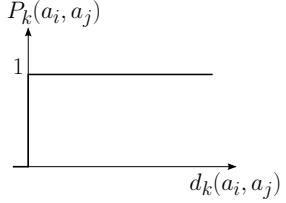
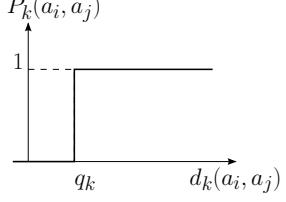
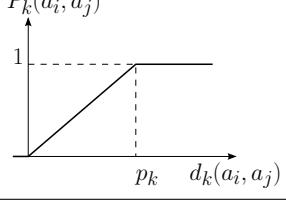
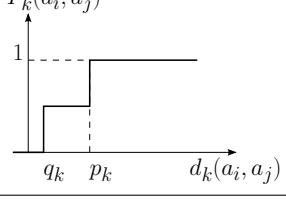
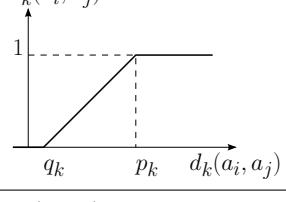
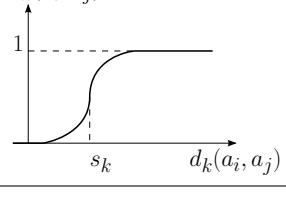
- The positive outranking flow: $\phi^+ = \frac{1}{n-1} \sum_{j \neq i} \pi(a_i, a_j)$. This flow expresses how a_i outranks all the other alternatives.
- The negative outranking flow: $\phi^- = \frac{1}{n-1} \sum_{j \neq i} \pi(a_j, a_i)$. This flow expresses how a_i is outranked by all the other alternatives.
- The net flow: $\phi(a) = \phi^+(a_i) - \phi^-(a_i)$. This flow expresses the balance between the positive and negative flows of a_i .

Let us note the following properties for these flows:

$$\phi^+, \phi^- \in [0; 1] \quad (2.19)$$

$$\phi \in [-1; 1] \quad (2.20)$$

Table 2.2: Preference functions (reproduced from [7])

Usual		Strict preference
U-shape		Q: indifference threshold
V-shape		P: preference threshold
Level		Q: indifference threshold P: preference threshold
Linear		Q: indifference threshold P: preference threshold
Gaussian		S: preference threshold

Based on these flows, the PROMETHEE methods will establish an outranking.

3. *PROMETHEE I*

The positive and negative flows allow to sort the alternatives of A . Let (S^+, I^+) and (S^-, I^-) be the two complete pre-orders obtained from these flows:

$$\begin{cases} a_i S^+ a_j \Leftrightarrow \phi^+(a_i) > \phi^+(a_j) \\ a_i I^+ a_j \Leftrightarrow \phi^+(a_i) = \phi^+(a_j) \end{cases} \quad (2.21)$$

This means that the higher the positive flow is, the better the alternative.

$$\begin{cases} a_i S^- a_j \Leftrightarrow \phi^-(a_i) < \phi^-(a_j) \\ a_i I^- a_j \Leftrightarrow \phi^-(a_i) = \phi^-(a_j) \end{cases} \quad (2.22)$$

This means that the lower the negative flow is, the better the alternative.

PROMETHEE I establishes a partial ranking by taking the intersection of these two pre-orders:

$$\begin{cases} a_i P^{(1)} a_j \Leftrightarrow \begin{cases} a_i S^+ a_j \text{ and } a_i S^- a_j \\ a_i S^+ a_j \text{ and } a_i I^- a_j \\ a_i I^+ a_j \text{ and } a_i S^- a_j \end{cases} \\ a_i I^{(1)} a_j \Leftrightarrow a_i I^+ a_j \text{ and } a_i I^- a_j \\ a_i R^{(1)} a_j \text{ otherwise} \end{cases} \quad (2.23)$$

where $(P^{(1)}, I^{(1)}, R^{(1)})$ represent respectively the preference, the indifference and the incomparability in PROMETHEE I.

- $a_i P^{(1)} a_j$ (“ a_i is preferred to a_j ”): a_i is simultaneously better and less worse than a_j .
- $a_i I^{(1)} a_j$ (“ a_i and a_j are indifferent”): a_i is neither better nor worse than a_j .
- $a_i R^{(1)} a_j$ (“ a_i and a_j are incomparable”): a_i is better than a_j on some criteria while a_j is better than a_i on other criteria.

4. *PROMETHEE II*

In order to obtain a complete ranking, the net flow will be considered:

$$\begin{cases} a_i P^{(2)} a_j \Leftrightarrow \phi(a_i) > \phi(a_j) \\ a_i I^{(2)} a_j \Leftrightarrow \phi(a_i) = \phi(a_j) \end{cases} \quad (2.24)$$

where $P^{(2)}$ et $I^{(2)}$ represent respectively the preference and the indifference in PROMETHEE II. This means that the higher the net flow is, the better the alternative.

Let us note that, unlike PROMETHEE I, PROMETHEE II does not give place to incomparability and a complete ranking can directly be obtained.

5. *The GAIA plane*

While it is impossible to have a visual representation of the solution space when there are more than three criteria, the GAIA (Geometrical Analysis for Interactive Assistance) plane can give a visualization even if there are more than three criteria, by means of the principal component analysis (PCA) of the net flows on the decision maker's preferences for each criterion.

The PCA allows a projection of the alternatives on a plane that minimizes the loss of information induced by this projection.

This plan allows to have a visual descriptive analysis with several criteria. It can highlight the conflict between criteria and show the profiles of the alternatives. This will help to identify the potential compromise solutions.

In order to illustrate the use of the GAIA plane, we will take a simple example taken from the PROMETHEE-based software D-Sight developed by Q. Hayez: the purchase of a car. Six vehicles are evaluated on 5 criteria: price, horse power, 0-100 km/h acceleration time, consumption and CO₂ emission. The associated GAIA plane is given in Figure 2.1.

Four distinctive visual information are shown:

- (a) The green axes that represent the projections of each criterion's axis.
- (b) The blue dots that represent the projection of each solution's uni-criterion net flow. The value of the uni-criterion net flow is read by projecting the point on the related criterion axis.
- (c) The red axis that represents the *decision stick* which is the projection of the set of weights and gives the decision direction.
- (d) The *delta* value that represents the percentage of kept information since there are projection errors.

From the GAIA plane, we can observe how the criteria are related between each other. Indeed, criteria axes that have opposite directions are conflicting, whereas criteria with the same direction share the same optimization trend. In this case, we can see that the price is conflicting with the power and the 0-100 km/h acceleration time, which is to be expected. Also, the CO₂ and the consumption criteria have the same direction which also reflects the reality.

As for the decision stick, it allows a decision maker to know in which direction is located his preferences. Indeed, the alternatives with the highest net flow score will have their furthest projection on that axis, in the direction of that axis. This visually represents the PROMETHEE II ranking, provided that the *delta* value shows that enough information has been kept with the projection.

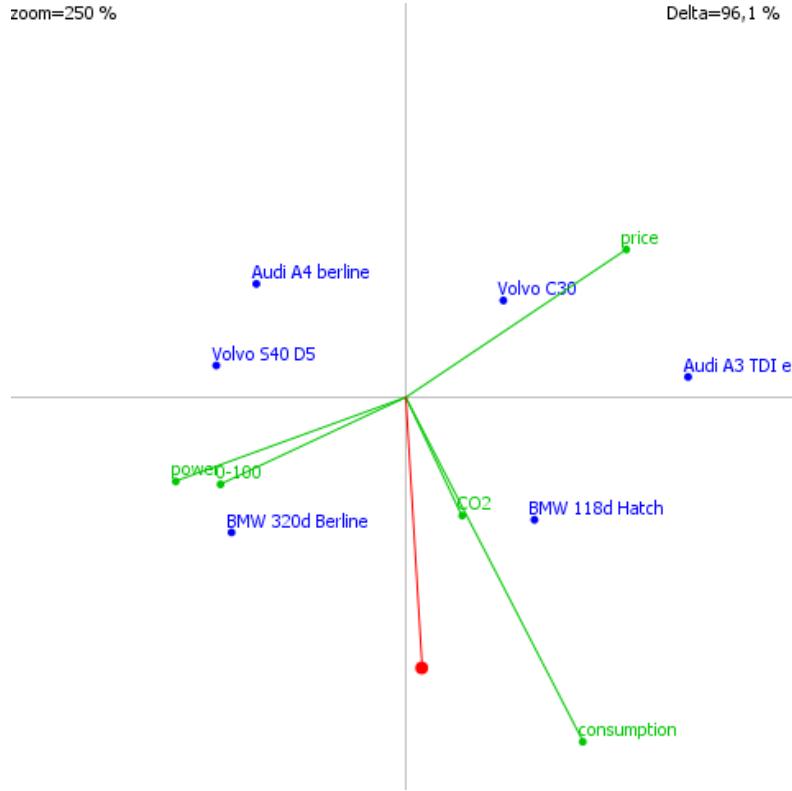


Figure 2.1: Purchase of a car - GAIA plane

The ELECTRE methods

ELECTRE (*ELimination Et Choix Traduisant la REalité*, or ELimination and Choice Translating REality) has been developed by Roy [59]. In this section, we will only describe the basics of ELECTRE. More details can be found in [66].

1. ELECTRE I

ELECTRE I is a method linked to the $P.\alpha$ problematic that aims to obtain a subset N of alternatives such that all the solutions that do not belong to this set is outranked by at least one alternative of N and the solutions of N do not outrank each other. N is therefore not the set of good alternatives but rather the set where the best compromise can certainly be found.

The outranking relation is obtained by establishing a weight w_k for each criterion. A concordance index is the associated to each pair (a_i, a_j) of alterna-

tives:

$$c(a_i, a_j) = \frac{1}{W} \sum_{j: f_k(a_i) \leq f_k(a_j)} w_k, \text{ where } W = \sum_{k=1}^m w_k, w_k > 0 \quad (2.25)$$

The concordance index represents a measure of the arguments favourable to the statement “ a_i outranks a_j ”.

A discordance index can also be defined:

$$d(a_i, a_j) = \begin{cases} 0 & \text{if } f_k(a_i) \geq f_k(a_j), \forall k \\ \frac{1}{\delta} \max_k [f_k(a_j) - f_k(a_i)] & \text{otherwise} \end{cases} \quad (2.26)$$

The discordance index is therefore higher if the preference of a_j over a_i is strong on at least one criterion.

Then concordance \hat{c} and discordance \hat{d} thresholds are defined alongside the outranking relation S :

$$\forall i \neq j, a_i S a_j \text{ iff } \begin{cases} c(a_i, a_j) \geq \hat{c} \\ d(a_i, a_j) \leq \hat{d} \end{cases} \quad (2.27)$$

From this definition, a subset N of alternatives is established such that:

$$\begin{cases} \forall a_j \in A \setminus N, \exists a_i \in N : a_i S a_j \\ \forall a_i, a_j \in N, a_i \overline{S} a_j \end{cases} \quad (2.28)$$

A subset N of alternatives is established such that all the alternatives that do not belong to this set is outranked by at least one alternative of N and the alternatives of N are incomparable. The decision process will therefore take place within the set N .

2. ELECTRE II

This method aims to rank the alternatives. The outranking relation is defined by fixing two concordance thresholds \hat{c}_1 and \hat{c}_2 such that $\hat{c}_1 > \hat{c}_2$ and by building a strong outranking relation S^F and a weak outranking relation S^f based on these two thresholds:

$$a_i S^F a_j \text{ iff } \begin{cases} c(a_i, a_j) \geq \hat{c}_1 \\ \sum_{j: f_k(a_i) > f_k(a_j)} w_k > \sum_{j: g_k(a_i) < g_k(a_j)} w_k \\ (f_k(a_i), f_k(a_j)) \notin D_k, \forall k \end{cases} \quad (2.29)$$

$$a_i S^f a_j \text{ iff } \begin{cases} c(a_i, a_j) \geq \hat{c}_2 \\ \sum_{j: f_k(a_i) > f_k(a_j)} w_k > \sum_{j: f_k(a_i) < f_k(a_j)} w_k \\ (f_k(a_i), f_k(a_j)) \notin D_k, \forall k \end{cases} \quad (2.30)$$

The discordance can also induce two levels of relations by building two sets of discordance for each criterion.

In order to obtain the ranking, a set is determined from S^F . This set B contains the alternatives that are not strongly outranked by any others. From B and S^f , the set A^1 of alternatives that are not weakly outranked by any alternatives of B is determined. The set A^1 constitutes the best alternatives class. A^1 is then removed and the process is repeated to find A^2 and so on until a complete pre-order is obtained.

Let us note that a second complete pre-order can be obtained by applying the process first with the less good alternatives class and then the best ones.

3. ELECTRE III

This method takes into account the indifference and preference thresholds. It is based on a valued outranking relation that is less sensible to data and parameters variabilities.

In ELECTRE III, an outranking degree $S(a_i, a_j)$ associated to each pair (a_i, a_j) of alternatives is defined. It can be understood as an “degree of credibility of outranking” of a_i over a_j .

A weight w_k is associated to each criterion and for each pair (a_i, a_j) of alternatives the concordance index is computed as follows:

$$c(a_i, a_j) = \frac{1}{W} \sum_{k=1}^m w_k c_k(a_i, a_j), \text{ where } W = \sum_{k=1}^m w_k \quad (2.31)$$

with

$$c_k(a_i, a_j) = \begin{cases} 1 & \text{if } f_k(a_i) + q_k(f_k(a_i)) \geq f_k(a_j) \\ 0 & \text{if } f_k(a_i) + p_k(f_k(a_i)) \leq f_k(a_j) \\ \text{linear} & \text{if } f_k(a_i) + q_k(f_k(a_i)) \leq f_k(a_j) \\ & \leq f_k(a_i) + p_k(f_k(a_i)) \end{cases} \quad (2.32)$$

where q_k et p_k represent respectively the indifference and preference thresholds.

The definition of discordance is then enriched by the introduction of a veto threshold $v_k(f_k(a_i))$ for each criterion k such that any credibility for the outranking of a_j by a_i is refused if $f_k(a_j) \geq f_k(a_i) + v_k(f_k(a_i))$.

A discordance index is then defined:

$$D_k(a_i, a_j) = \begin{cases} 0 & \text{if } f_k(a_j) \leq f_k(a_i) + p_k(f_k(a_i)) \\ 1 & \text{if } f_k(a_j) \geq f_k(a_i) + v_k(f_k(a_i)) \\ \text{linear} & \text{if } f_k(a_i) + p_k(f_k(a_i)) \leq f_k(a_j) \\ & \leq f_k(a_i) + v_k(f_k(a_i)) \end{cases} \quad (2.33)$$

The degree of outranking is finally defined:

$$S(a_i, a_j) = \begin{cases} c(a_i, a_j) & \text{if } D_k(a_i, a_j) \leq c(a_i, a_j) \\ c(a_i, a_j) \prod_{k \in \mathcal{F}(a_i, a_j)} \frac{1 - D_k(a_i, a_j)}{1 - c(a_i, a_j)} & \text{otherwise} \end{cases} \quad \forall k \quad (2.34)$$

where $\mathcal{F}(a_i, a_j)$ is the set of criteria for which $D_k(a_i, a_j) > c(a_i, a_j)$. The degree of outranking is thus equal to the concordance index when no criterion is discordant, otherwise the concordance index is decreased proportionally depending on the importance of the discordances.

A value $\lambda = \max_{a_i, a_j \in \mathcal{A}, i \neq j} S(a_i, a_j)$ is determined and only the outranking degree that have a value greater or equal to $\lambda - s(\lambda)$, where $s(\lambda)$ is a threshold to be determined, are considered. A ranking can then be determined from a qualification index $Q(a)$ for each alternative a that represents the difference between the number of outranked alternatives by a and the number of alternatives that outrank a . The set of actions having the largest qualification will be called the first distillate D_1 .

If D_1 contains only one alternative, the previous procedure is repeated with $\mathcal{A} \setminus D_1$. Otherwise the same procedure is applied for D_1 and if the obtained distillate D_2 contains only one alternative, the procedure is repeated with $D_1 \setminus D_2$. Otherwise, it is applied for D_2 , and so on until D_1 is completely used, before starting with $\mathcal{A} \setminus D_1$. This procedure produces a first complete preorder.

A second complete preorder can be obtained by applying the opposite procedure where the alternatives with the smallest qualification are first used.

2.5 Conclusion

In this chapter, we have given an overview of the basics of multi-objective optimization and multi-criteria decision aid. We have explained when a multi-criteria paradigm should be applied and have presented some of the commonly-used method in the field.

In the next chapter, we will define the problem we tackle and show how a 3D-SIC can be modelled in order to apply multi-objective optimization.

3

Problem definition and 3D-stacked integrated circuit model

Chapter abstract

In this chapter, we define the design problem we tackle: the 3D floorplanning. We then define the criteria we consider and show how a 3D-SIC can be modelled in order to apply multi-objective optimization. Simulations are ran based on a 3MF MPSoC platform and the obtained results can provide qualitative and quantitative information to a designer that would not be available with current tools. Finally, the methodology is validated with a more realistic case study to show that a multi-criteria paradigm does give added value compared to a uni-criterion approach.

Associated publications:

3.1 Introduction

In Chapter 1, we have presented a review of the literature about the field of microelectronics design. We have highlighted some limitations of the current tools that already occur for 2D-ICs. In this chapter, we will define the 3D floorplanning problem, which is the issue we tackle and show how we model it in order to propose improvements to design flows.

3.2 Problem definition

As stated in Chapter 1, the limitations of the current design flows can be summarized in three points:

- Limitation of the design space exploration
- Unicriterion optimization or trade-off analysis on a limited number of criteria
- Few 3D-SIC dedicated tools

In order to address these limitations, we propose in this thesis a methodology based on multi-objective/criteria tools and taking into account 3D-SIC specificities to explore the design space.

While this methodology could be applied at different levels in a design flow, we have focused our development in the logical design step and the virtual prototyping flow, more specifically the floorplanning with performance assessments.

3.2.1 Designing an IC

In order to meet the specifications, a design has first to make a choice at a physical level:

- Targeted architecture, e.g. ASIC, FPGA
- Number of functional units
- Number of memories and their size
- The general layout
- ...

Since the 3D-SICs are based on conventional circuits, the options and degrees of freedom coming from 2D-ICs are still present:

- Process technology, e.g. 180 nm to 22 nm CMOS
- Memories technology, e.g. SRAM, DRAM, FLASH
- Communication infrastructure, e.g. bus, Network-on-Chip
- ...

In addition to those options and degrees of freedom coming from 2D-ICs, numerous 3D-SIC's parameters appear [3]:

- Number of tiers to use
- Place and route of the functional units between the tiers
- Technology to use per tiers (heterogeneity)
- Interconnection and geometry between tiers
- 3D-SIC integration technology
- 3D-SIC assembly technology
- ...

The above mentioned parameters illustrate the numerous possibilities for designing a circuit and how the design space for 2D-ICs becomes much larger when considering 3D-SICs. The main issue is therefore to choose the most efficient combination

among all those options. This can thus be compared to a combinatorial optimization problem. Also, given the multi-criteria nature of designing 3D-SIC, we choose to take into account all the criteria simultaneously for the optimization. In our case, due to the heterogeneous nature of the criteria (see Section 3.3), we have few hopes to successfully adopt an exact method and we will therefore use metaheuristics which are commonly-used tools for such kinds of problems. In the next section will define the criteria that a designer can consider.

3.3 Model and criteria definition

Typically, the criteria that have to be optimized simultaneously can be the performance, the power consumption, the cost, the package size, the heat dissipation, etc. In this model, we will define five criteria which are among the most important parameters while designing a circuit [36]:

1. *The interconnection global length:* this parameter can reflect the global performance of a system. The objective is to minimize it in order to have, for instance, a short delay and low power consumption. It will be calculated using the Manhattan distance [67]:

$$d_{i,j} = |x_i - x_j| + |y_i - y_j| \quad (3.1)$$

where (x_k, y_k) is the geometrical coordinates of the k^{th} block. As a first approximation, the center point of each block will be selected as reference coordinate. Also, since it is more interesting to place close to each other two blocks that require a large bandwidth (BW) to communicate, we will balance the values as follows:

$$d'_{i,j} = \frac{d_{i,j}}{BW_{ij}} \quad (3.2)$$

where BW_{ij} is the bandwidth required between the block i and the block j . The global interconnection length D will be the sum of $d'_{i,j}$ for all communicating blocks:

$$D = \sum d'_{i,j} \quad (3.3)$$

2. *The cost:* an economical factor is obviously an important criteria for a design. This criteria has been estimated with the aid of an expert in 3D-SIC manufacturing. While a circuit can be more efficient with many layers, it will also be more expensive. This criteria has to be minimized. Due to the confidential nature of the cost of a 3D-SIC, we will consider a simplified model where the cost is proportional to the area and increasing exponentially with the number of tiers:

$$\text{cost} = a(\text{tech}).S + b(\text{tech})^{\text{layer number}} \quad (3.4)$$

where $a(tech)$ and $b(tech)$ are coefficient depending on the technology assigned. Let us note that this criterion includes both discrete and continuous variables.

3. *The package volume:* this can be an important criteria when designing embedded circuits. The package volume is calculated as follows:

$$\text{volume} = \text{largest layer size} * \text{stack thickness} * \text{number of tiers} \quad (3.5)$$

A large approximation of $200 \mu\text{m}$ will be made for the thickness of one tiers. Let us note that this criterion includes both discrete (number of tiers) and continuous variables (layer size).

4. *The clock tree position:* in this model, we consider a synchronous system so the objective is to minimize the distance between each block and the clock tree in order to have a high frequency. We choose arbitrarily to approximate the reference point as a fixed point located at the upper left corner of the middle tier of the 3D-SIC.
5. *The thermal dissipation:* thermal dissipation is one of the major issues when designing 3D-SICs. It can be more appropriate to place two blocks underneath each other in successive tiers but a high heat dissipation may happen in intensive computational process. This criterion is a research topic on its own [68, 69]. Here we will use a simplified evaluation model with finite elements. This model will consider that the dissipated power, intra- or inter-tiers, is inversely proportional to the distance to the heat source:

$$P_{diss} = P_{comp} \sum_{i=1}^n \frac{1}{R_{th,i} \cdot r} \quad (3.6)$$

where r the distance to the heat source and $R_{th,i}$ the thermal resistance depending on whether the dissipation is intra- or inter- tiers. This criterion is still on early development stage in our research and we can generate thermal maps of a floorplan as shown in Figure 3.1 but this is currently based on finite elements [70] which require quite a long computational time even for a simplified thermal model. This criterion in its current development stage is difficult to integrate to the exploration process, due to the computation time of finite elements methods. In the current work, we will simply compute the peak power of a circuit which can be done more quickly.

At first, we will focus on the three first criteria in order to be able to have a visualization of the design space. We will also arbitrarily introduce some limitations in term of degrees of freedom to analyse what happens if we release a constraint. This will be done while considering the three same criteria, in order to keep a visualization and show how the flexibility of MOO will improve the information and the results. Then we will analyse our methodology with the five criteria that have been presented.

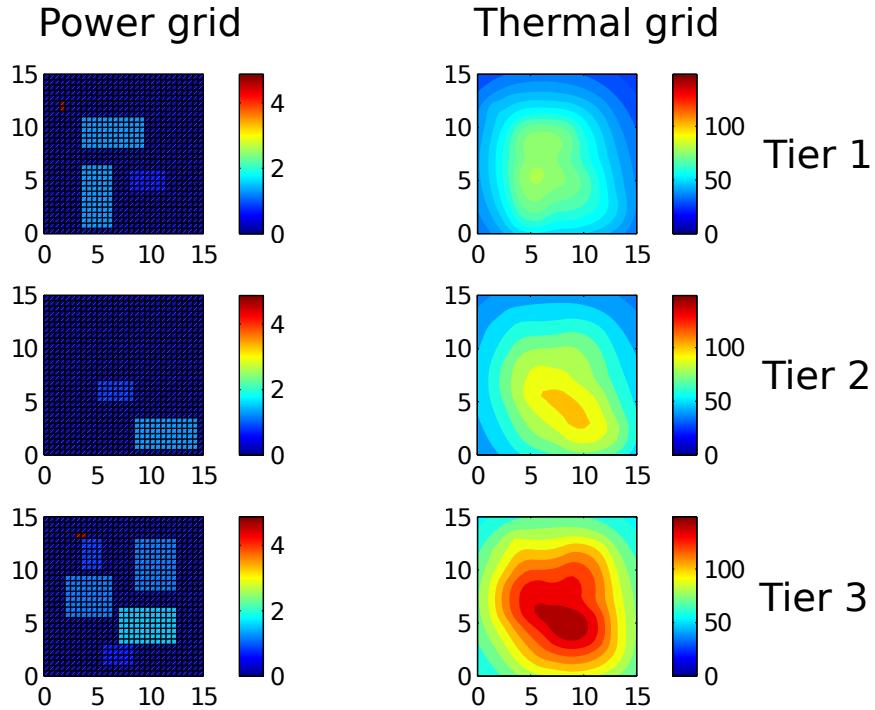


Figure 3.1: Power grid and thermal map of a floorplan (3 tiers)

3.4 Design methodology

In summary, the problem we are facing is to place several blocks that have to be assigned in many tiers while considering multiple conflicting criteria. Now that the criteria have been defined, we will present a proposition of a new design methodology based on multi-objective optimization, with the related model.

As explained in Chapter 1, designing ICs implies numerous choices and designers are likely to freeze a certain amount of choices on basis of their experience. This will therefore limit the exploration of the design space and good solutions may be ignored.

In order to enable an efficient design space exploration, we propose a method in four steps based on MCDA which is illustrated in Figure 3.2. The implementation will be briefly presented in the next section.

For the problem we consider, the input data will contain the information about the scenario:

- Type and number of blocks: computational units, memories, etc.
- Size of the blocks: inherent to the block.
- Minimum aspect ratio: we consider a degree of freedom where a block can

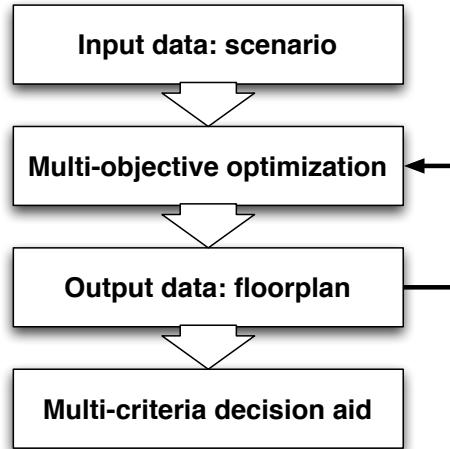


Figure 3.2: MCDA-based design methodology for 3D DSE floorplanning

have its dimensions varying within a given aspect ratio range. This means that a block does not have to be square, as shown in Figure 3.3. This parameter can influence the delay in a block.

- Size variability of a block: we add this degree of freedom considering that the specified size of a block can be fixed by the designer but this fixed size can restrict the design space exploration. The variability of a block's size can impact the performances and the global footprint.

In addition, the bandwidth requirements are needed as they will indicate which are the important interconnections and prevent two blocks that require a large bandwidth from being too far from each other. The available manufacturing technologies are also useful to enable the design of heterogeneous systems.

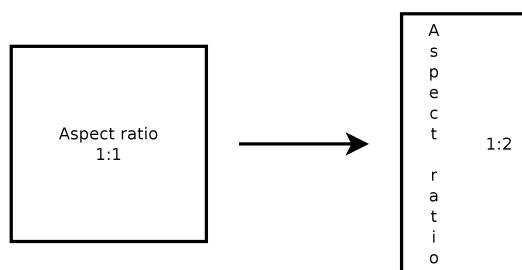


Figure 3.3: Example of aspect ratio degree of freedom

The combination of all the parameters described in the model are the possible alternatives for a 3D-SIC design and will provide output data after design space exploration. For a floorplanning problem the required output data are generally the geometrical layout of the circuit [36]:

- The geometrical coordinates for each block and the assigned layer.
- The size of each block (if it can vary from the specified size).
- The aspect ratio for each block.
- The technology assigned to each tier: a thinner technology will reduce the size of each block. The size of a block will define the number of transistors inside using a given technology, for example 180 nm. For a constant number of transistors, if the block is manufactured with a smaller technology, let us say 45 nm, then its size will be divided by a $(180/45)^2$ factor, as shown in Figure 3.4. Please note, that this factor is a rough approximation which is not always met with real physical design and of course this accuracy can be improved.

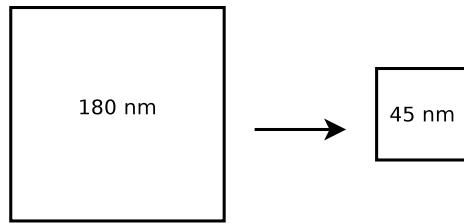


Figure 3.4: The use of different manufacturing technologies results in a size variation

3.5 Case study and implementation

In this section, we will briefly explain the implementation of our design method and show some experimental results based on this first approach. Due to the huge size of the design space, an explicit enumeration of the possible solutions will take considerable time. For instance, for a simplified problem of 3 tiers of 10x10 mm and 5 blocks of 2x2 mm to place, there are 75 possible positions for each block. The number of permutations is given by $\frac{n!}{(n-k)!}$, where n is the number of possible positions and k the number of blocks. For this simplified problem, there are therefore 2 071 126 800 combinations. For 10 blocks to place, the number of possibilities increases to more than $3 \cdot 10^{18}$. Besides, this small calculation does not take into account the numerous other possible choices.

We will therefore apply a metaheuristic and more specifically a NSGA-II algo-

Table 3.1: Scenario input matrix example

Component	ID	Size (90 nm)	Min aspect ratio	Size variability
ADRES 1~6	1~6	18.6 mm ²	0.5	±20%
FIFO	7	0.54 mm ²	0.5	0
L2D1-2	8-9	6.74 mm ²	0.5	+30%
L2Is1-2	10-11	6.62 mm ²	0.5	+30%
EMIF	12	0.66 mm ²	0.5	0.1
ARM	13	0.89 mm ²	0.5	0

ID: Component identification number

rithm. We consider a case study based on the 3MF MPSoC platform developed at IMEC [6].

3.5.1 Description of the case study and modeling

The 3MF MPSoC is made of 13 blocks as shown on Figure 3.5:

- 6 ADRES processors ([71])
- 2 data memories (L2D#)
- 2 instruction memories (L2Is#)
- 1 external memory interface (EMIF)
- 1 input/output processor (FIFO)
- 1 ARM processor (ARM)

Details about the area required for each component is given in Table 3.1 for a 90 nm technology. This table is also the input matrix required to specify the scenario.

The 3MF MPSoC can be configured for three use cases which have specific bandwidth requirements. For the following results, we will base our simulation on the "data split scenario" configuration which possesses the communication specifications shown in Table 3.2. This information is implemented, as shown in Table 3.3, in an input matrix which is built by specifying the communication structure: the first column will contain the ID of the source block and each next pairs of columns will contain the ID of the target blocks and the bandwidth required.

The input data are thus shown in Tables 3.1 and 3.3. The available technologies are also needed to take advantage of the heterogeneity. An example matrix for this input data is given in Table 3.4. Since no information is given about the ARM unit bandwidth usage, we will simplify our problem and not include it in the implementation. We consider therefore 12 blocks to assign.

In summary, the problem we consider is to place 12 blocks while taking into account several (5) criteria. We will also consider a scenario where the blocks can be placed on 1 up to 5 tiers. The input data will be processed to generate floorplans.

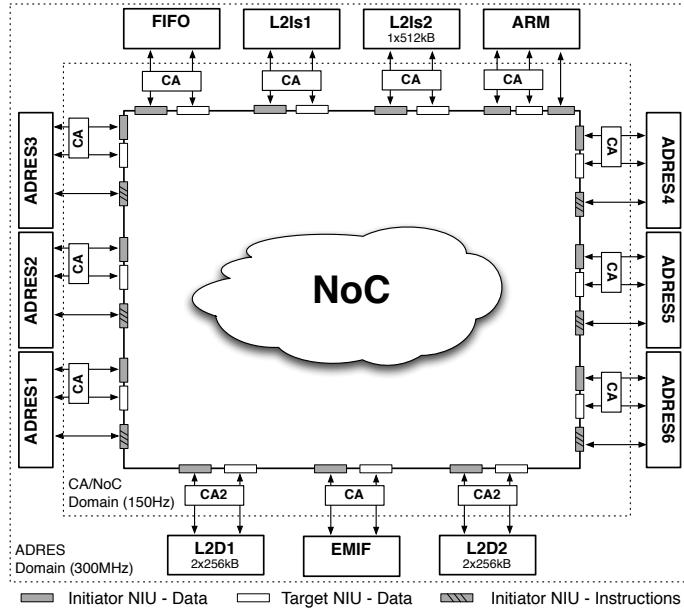


Figure 3.5: Architecture of the 3MF MPSoC platform [6]

Table 3.2: "Data split scenario" bandwidth requirements

Source	Target	Bandwidth (MB/s)
FIFO	EMIF	39.6
EMIF	ADRES <i>i</i>	6.6
L2D1	ADRES <i>i</i>	26.4
L2D2	L2D1	52.7
ADRES <i>i</i>	FIFO	1.2
ADRES <i>i</i>	L2D2	6.6
ADRES <i>j</i>	L2Is1	300
ADRES <i>k</i>	L2Is2	300

Index: $i, j, k \in \mathbb{N}^+$;
 $1 \leq i \leq 6; 1 \leq j \leq 3; 4 \leq k \leq 6$

Those output data will be encoded using the matrix model following the example shown in Table 3.5. They will be generated through a multi-objective optimization.

As explained earlier, we will use a metaheuristic to approximate the Pareto optimal frontier. For that purpose, we choose to use NSGA-II [51] as a proof of concept. The algorithm was run from a sample of 10 000 generated solutions from 1 up to 5 tiers. This size of random solutions is chosen arbitrarily since it is actually quite difficult to estimate the size of solution space, due to the heterogeneous nature of the

Table 3.3: Bandwidth input matrix

S	T	B	T	B	T	B	T	B	T	B	T	B	T	B
1	7	1.2	9	6.6	10	300	0	0	0	0	0	0	0	0
2	7	1.2	9	6.6	10	300	0	0	0	0	0	0	0	0
3	7	1.2	9	6.6	10	300	0	0	0	0	0	0	0	0
4	7	1.2	9	6.6	11	300	0	0	0	0	0	0	0	0
5	7	1.2	9	6.6	11	300	0	0	0	0	0	0	0	0
6	7	1.2	9	6.6	11	300	0	0	0	0	0	0	0	0
7	12	39.6	0	0	0	0	0	0	0	0	0	0	0	0
8	1	26.4	2	26.4	3	26.4	4	26.4	5	26.4	6	26.4		
9	8	52.7	0	0	0	0	0	0	0	0	0	0	0	
12	1	6.6	2	6.6	3	6.6	4	6.6	5	6.6	6	6.6		

S: source block ID

(T, B): target block ID and required bandwidth

Table 3.4: Available technologies input matrix example

Technology (nm)				
90	60	45	32	22

criteria. Also, taking too few solutions (e.g. 100) is not interesting since we have empirically observed that our algorithm will take a longer time to begin to converge. 10 000 randomly-generated solutions seems to us a good compromise of time and workable solutions.

In the following section, we will present some details about the implementation of the NSGA-II algorithm.

3.5.2 Implementation of the exploration algorithm: NSGA-II

As shown in Table 3.5, we choose to encode our data in real or integer values, so that they can be used directly by design tools:

- The component identification number (ID) is a fixed integer value linked to the component.
- The assigned layer (L) is a discrete value ranging from 1 to 5 in the case study.
- The geometrical coordinates (X,Y) are real values that depends on the dimension of the circuits and the aspect ratio of a block, so that the component cannot be placed outside the chip.
- The size (S) is a fixed real value linked to the component.
- The aspect ratio (AR) is a real value ranging from AR_{min} to $1/AR_{min}$ where AR_{min} is given as a specification as explained in Section 4.

- The length in X and Y axis (LX, LY) are real values computed from the size and the aspect ratio.
- The assigned technology per layer is a discrete value taking one of the specified technology (see Table 3.4).

This matrix will be our full chromosome for the NSGA-II algorithm (see example in Table 3.5).

Table 3.5: Output matrix template

ID	L	X	Y	S	AR	LX	LY	T
1	2	4.5	6	18.6	1	4.3128	4.3128	90
2	2	4	0.4	18.6	1	4.3128	4.3128	90
3	3	3.1	6.9	18.6	1	4.3128	4.3128	90
4	3	8.4	10.1	18.6	1	4.3128	4.3128	90
5	3	6.6	2.2	18.6	1	4.3128	4.3128	90
6	1	9	5.7	18.6	1	4.3128	4.3128	90
7	1	10	3.5	0.54	1	0.7348	0.7348	90
8	1	7.5	11	6.74	1	2.5962	2.5962	90
9	2	9	5	6.74	1	2.5962	2.5962	90
10	1	4.5	8	6.62	1	2.5729	2.5729	90
11	2	8.6	0.4	6.62	1	2.5729	2.5729	90
12	3	8.3	7.4	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

We implemented our design space exploration following the steps of the NSGA-II which can be summarized by the diagram shown in Figure 3.6.

Initialization (the initial population)

We will work with a minimum size of population, namely 50, which is a common value in GAs [72]. The initial population will be a set of at least 50 solutions with the best Pareto ranks from a randomly-generated set of 10 000 solutions. The produced set places the blocks randomly (using a uniform distribution) and does not allow overlapping between the blocks (these incorrect solutions are simply removed). We could of course use a greedy algorithm as well as a more advanced method such as GRASP (Greedy Randomized Adaptive Search Procedure) [73]. This can be done as future work for comparison purposes.

Of course, having at least 50 Pareto solutions does not always happen. Actually, the selection is based on the Pareto rank so it does not include only the Pareto solutions (rank 1), but also the solutions with lower ranks until there are enough solutions.

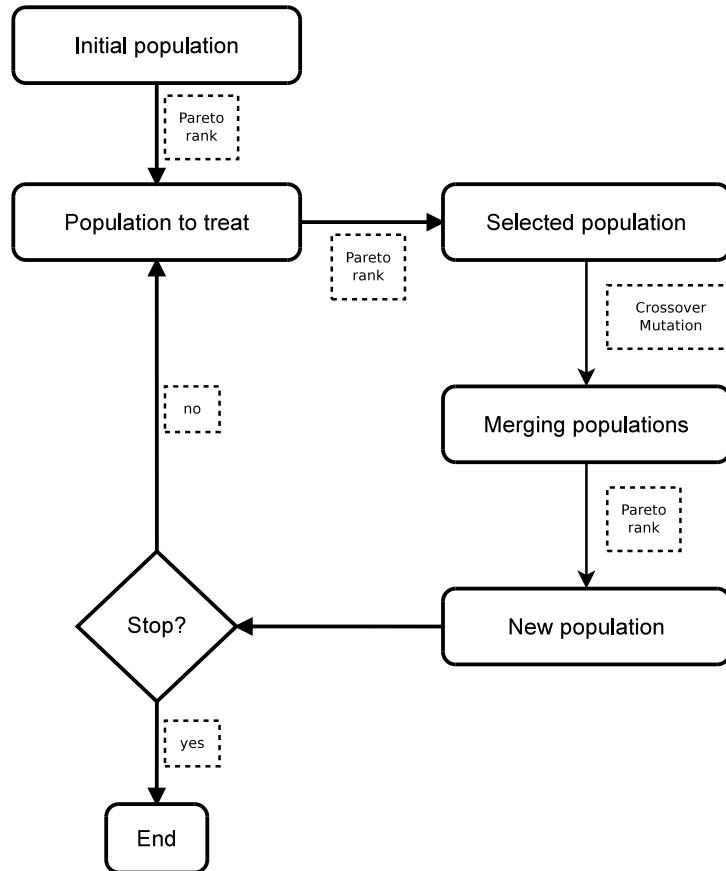


Figure 3.6: General NSGA-II steps

Selection for crossover

For the selection step, two solutions will be allowed to make a crossover depending on a roulette wheel where the probability is proportional to the normalized Euclidean distance between the solutions ordered by their Pareto rank in the objective space. The normalization is done as follows :

$$\frac{g_i(a_j)}{\max_{a_j \in A} g_i(a_j)} \quad (3.7)$$

where A is the set of alternatives in the Pareto front with $a_j \in A$ and $g_i(a_j)$ is the evaluation of the alternative a_j on the criterion i .

The probability for two solutions to do a crossover will vary linearly with the Euclidean distance between them, as shown in Figure 3.7. The distance between the two furthest alternatives ($d_{furthest}$) will be associated with a probability $P_{c,min}$ while the distance between the two closest alternatives ($d_{closest}$) will be associated with a probability $P_{c,max}$. If two solutions are close to each other, they will have more chance to reproduce than if they are distant. This is to ensure the intensification properties of our algorithm. Therefore, we will have to specify a lower bound ($P_{c,min}$) and an upper bound ($P_{c,max}$) for the crossover probability. $P_{c,min}$ is set for the solutions which are the furthest to each other while $P_{c,max}$ is set for those which are the closest. In between, the probability will vary linearly inside these bounds.

These values will be fixed as $[P_{c,min} = 0.6; P_{c,max} = 1.0]$ since these seem to be common values [72].

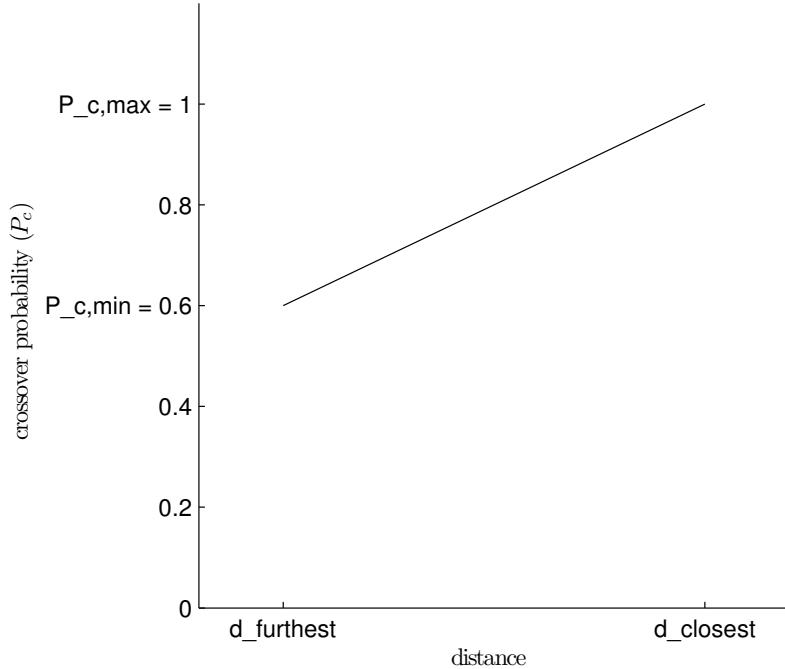


Figure 3.7: Evolution of the crossover probability as a function of the distance between two solutions

Crossover

Let us now see how does the crossover occur. First, let us remark that it does not have limitations for the exploration process since the information contained in the matrix spans the whole circuit.

Second, we have to analyze how the chromosome is coded in order to see how we will apply the crossover step. For instance, let us choose the Layer (L) column as indicator for the crossover. If we order the matrix in Table 3.5 following this column, we will have the Table 3.6 and the Table 3.7 for another solution that we will use for the crossover.

Now, without loss of generality, let us suppose that the crossover happens (randomly) on line 7. One of the child will be the Table 3.8 and we see that the original scenario is not preserved since the first column (in bold) contains the same ID several times.

Table 3.6: First parent, ordered by L column; the line specifies the crossover cut

ID	L	X	Y	S	AR	LX	LY	T
6	1	9	5.7	18.6	1	4.3128	4.3128	90
7	1	10	3.5	0.54	1	0.7348	0.7348	90
8	1	7.5	11	6.74	1	2.5962	2.5962	90
10	1	4.5	8	6.62	1	2.5729	2.5729	90
1	2	4.5	6	18.6	1	4.3128	4.3128	90
2	2	4	0.4	18.6	1	4.3128	4.3128	90
9	2	9	5	6.74	1	2.5962	2.5962	90
11	2	8.6	0.4	6.62	1	2.5729	2.5729	90
3	3	3.1	6.9	18.6	1	4.3128	4.3128	90
4	3	8.4	10.1	18.6	1	4.3128	4.3128	90
5	3	6.6	2.2	18.6	1	4.3128	4.3128	90
12	3	8.3	7.4	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

We observe that the only possible indicator for the crossover step is the ID column. Indeed if we order the two parents following the ID column, we have the Tables 3.9 and 3.10. If we still consider that the crossover occurs on line 7, we can have the child shown in Table 3.11. We see that there is no inconsistency since the scenario is still respected.

Mutation

A mutation cannot happen anywhere in the matrix. Indeed, if we take the conclusion about the choice of the crossover row indicator, all the elements except the ID

Table 3.7: Second parent, ordered by L column; the line specifies the crossover cut

ID	L	X	Y	S	AR	LX	LY	T
4	1	1.6	8.5	18.6	1	4.3128	4.3128	90
5	1	1.2	1.3	18.6	1	4.3128	4.3128	90
6	1	0.6	4.7	18.6	1	4.3128	4.3128	90
3	2	5.9	4	18.6	1	4.3128	4.3128	90
9	2	5.4	8	6.74	1	2.5962	2.5962	90
10	2	8.5	8.1	6.62	1	2.5729	2.5729	90
11	2	2.8	4.6	6.62	1	2.5729	2.5729	90
1	3	7	6.3	18.6	1	4.3128	4.3128	90
2	3	7.4	9.8	18.6	1	4.3128	4.3128	90
7	3	5.6	5.5	0.54	1	0.7348	0.7348	90
8	3	2.8	5.5	6.74	1	2.5962	2.5962	90
12	3	5.7	7.5	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

Table 3.8: Possible child, ordered by L column

ID	L	X	Y	S	AR	LX	LY	T
6	1	9	5.7	18.6	1	4.3128	4.3128	90
7	1	10	3.5	0.54	1	0.7348	0.7348	90
8	1	7.5	11	6.74	1	2.5962	2.5962	90
10	1	4.5	8	6.62	1	2.5729	2.5729	90
1	2	4.5	6	18.6	1	4.3128	4.3128	90
2	2	4	0.4	18.6	1	4.3128	4.3128	90
9	2	9	5	6.74	1	2.5962	2.5962	90
1	3	7	6.3	18.6	1	4.3128	4.3128	90
2	3	7.4	9.8	18.6	1	4.3128	4.3128	90
7	3	5.6	5.5	0.54	1	0.7348	0.7348	90
8	3	2.8	5.5	6.74	1	2.5962	2.5962	90
12	3	5.7	7.5	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

column can mutate.

The mutation used is a random uniform distribution $U([a, b])$, where $[a, b]$ is the interval of values allowed for the mutation. For the discrete values, we use equidistributed probabilities. The mutation probability of a child will be set as $P_m = 0.3$. Empirical observations have shown that smaller mutation probability can easily lead to a local optimum. This can be explained by the fact that we choose that only one single element of a line can mutate instead of the whole line. If a child is forced to

Table 3.9: First parent, ordered by ID column; the line specifies the crossover cut

ID	L	X	Y	S	AR	LX	LY	T
1	2	4.5	6	18.6	1	4.3128	4.3128	90
2	2	4	0.4	18.6	1	4.3128	4.3128	90
3	3	3.1	6.9	18.6	1	4.3128	4.3128	90
4	3	8.4	10.1	18.6	1	4.3128	4.3128	90
5	3	6.6	2.2	18.6	1	4.3128	4.3128	90
6	1	9	5.7	18.6	1	4.3128	4.3128	90
7	1	10	3.5	0.54	1	0.7348	0.7348	90
8	1	7.5	11	6.74	1	2.5962	2.5962	90
9	2	9	5	6.74	1	2.5962	2.5962	90
10	1	4.5	8	6.62	1	2.5729	2.5729	90
11	2	8.6	0.4	6.62	1	2.5729	2.5729	90
12	3	8.3	7.4	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

Table 3.10: Second parent, ordered by ID column; the line specifies the crossover cut

ID	L	X	Y	S	AR	LX	LY	T
1	3	7	6.3	18.6	1	4.3128	4.3128	90
2	3	7.4	9.8	18.6	1	4.3128	4.3128	90
3	2	5.9	4	18.6	1	4.3128	4.3128	90
4	1	1.6	8.5	18.6	1	4.3128	4.3128	90
5	1	1.2	1.3	18.6	1	4.3128	4.3128	90
6	1	0.6	4.7	18.6	1	4.3128	4.3128	90
7	3	5.6	5.5	0.54	1	0.7348	0.73485	90
8	3	2.8	5.5	6.74	1	2.5962	2.5962	90
9	2	5.4	8	6.74	1	2.5962	2.5962	90
10	2	8.5	8.1	6.62	1	2.5729	2.5729	90
11	2	2.8	4.6	6.62	1	2.5729	2.5729	90
12	3	5.7	7.5	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

mutate, then one randomly-chosen value of the whole matrix will mutate within the range of values it is allowed to take.

A Gaussian mutation is also a common operator but it has not been chosen since it will produce a solution which is not far from the original one. This is not really interesting to have similar solutions when exploring the design space for integrated circuits. Of course, a large standard deviation value can be chosen but this will be likely to produce solution which are out of the feasible bounds.

Table 3.11: Possible child, ordered by ID column

ID	L	X	Y	S	AR	LX	LY	T
1	2	4.5	6	18.6	1	4.3128	4.3128	90
2	2	4	0.4	18.6	1	4.3128	4.3128	90
3	3	3.1	6.9	18.6	1	4.3128	4.3128	90
4	3	8.4	10.1	18.6	1	4.3128	4.3128	90
5	3	6.6	2.2	18.6	1	4.3128	4.3128	90
6	1	9	5.7	18.6	1	4.3128	4.3128	90
7	1	10	3.5	0.54	1	0.7348	0.7348	90
8	3	2.8	5.5	6.74	1	2.5962	2.5962	90
9	2	5.4	8	6.74	1	2.5962	2.5962	90
10	2	8.5	8.1	6.62	1	2.5729	2.5729	90
11	2	2.8	4.6	6.62	1	2.5729	2.5729	90
12	3	5.7	7.5	0.66	1	0.8124	0.8124	90

ID: component identification number; L: assigned layer;
(X, Y): geometrical coordinate; S: size (mm^2); AR: aspect ratio;
(LX, LY): length in X and Y axis; T: assigned technology for the layer

Consistency test

Of course, infeasible solutions may appear after the crossover/mutation step, since these operations are made with randomness. In order to verify that, we perform a test on each new solution to check if there is overlapping between the blocks. Currently, the solutions which are infeasible will be discarded. Of course, it is possible to apply some repair mechanism but it is to be investigated as future work even if we already produce feasible solutions.

Termination

Three stop conditions have been implemented and are based on what is commonly used:

- Maximum number of iterations, set to 100.
- Maximum elapsed time, set to 60 minutes.
- Maximum number of iterations with an unchanged population, set to 10.

The maximum elapsed time has been chosen arbitrarily for quick testing purposes. As illustrated in Section 3.5.2, the design space is huge and finding an accurate Pareto frontier can be time consuming. On other hand, NSGA-II has shown that it can quickly produce good approximations [51]. Having a simulation time of a few hours is therefore enough, considering that, in practice, the optimization of one single architecture can take from several days to several weeks with the current design tools. Also, due to the approximation in the model, trying to find a really accurate Pareto front would not have real added value either.

3.6 Results and their use for a designer

The optimization was done for three criteria (so that we can visualize the design space) and the main results are given in Figure 3.6 (interconnection length-cost projection) and Figure 3.6 (3D plot). Two conclusions can be drawn from that figure:

- The [10; 20] range values for the IL criteria: a small enhancement of the IL value leads to a large increase of the cost so the interest for a design with more than 4 tiers seems low.
- The [260; 280] range values for the cost criteria: a small increase of the price can give a large enhancement of the performance. A designer might consider accepting a slightly higher price for a sensitively better performance, knowing that this information can be quantified with an accurate model. Indeed, with the estimate model that we propose, a small 10% increase of the cost can decrease the IL by 60%.

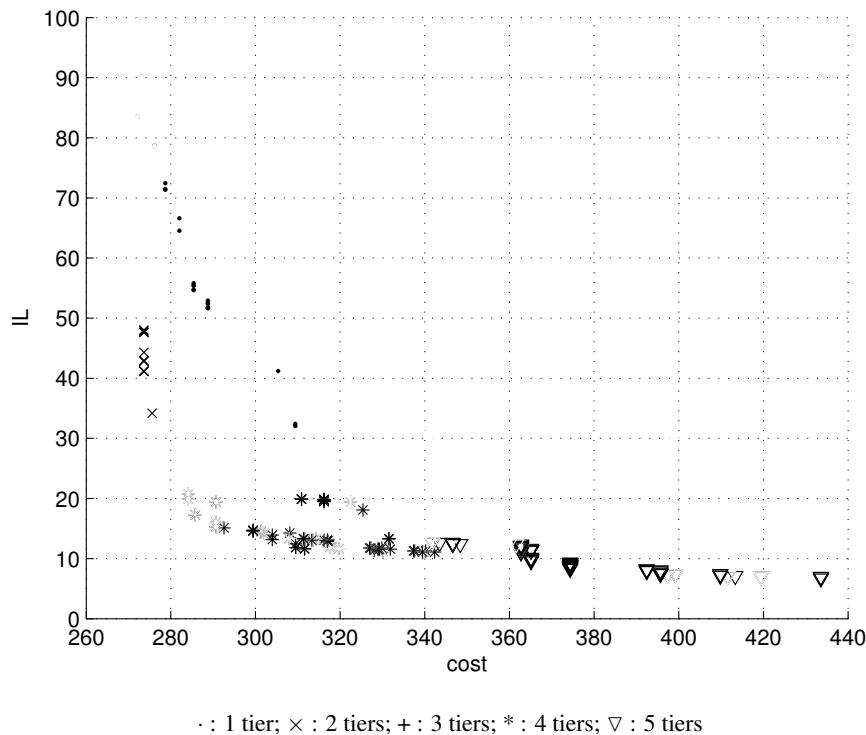


Figure 3.8: IL-cost projection view of the Pareto frontier

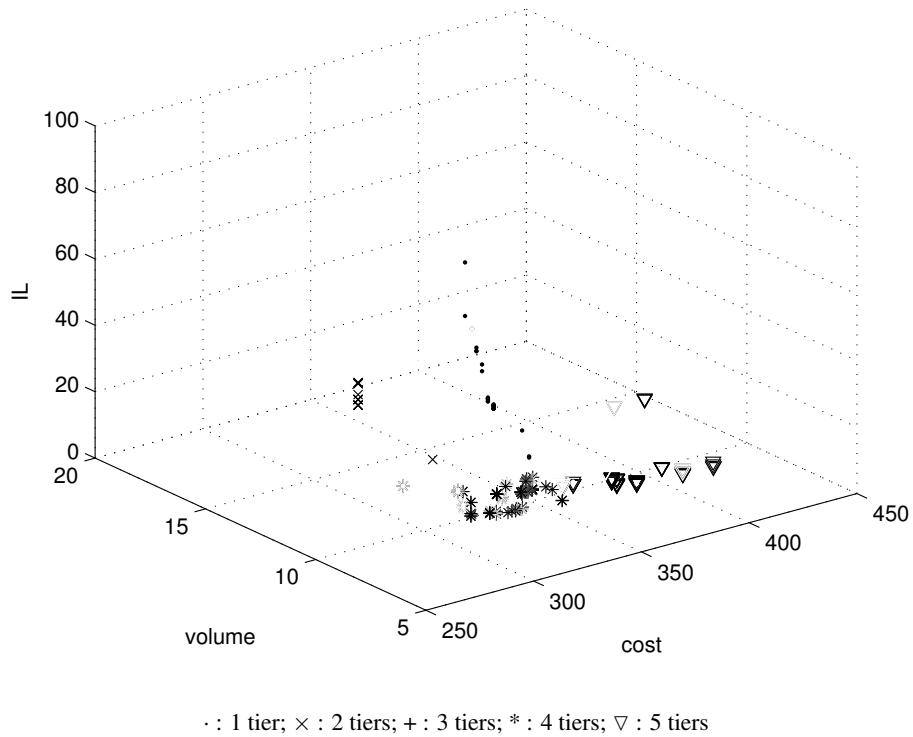


Figure 3.9: 3D view (interconnection length-volume-cost) of the Pareto frontier

These results did not take into account the degree of freedom of aspect ratio. If we go further by releasing a degree of freedom and allowing varying aspect ratios, we can have the Pareto front shown in Figure 3.10. This figure shows the Pareto front from Figure 3.6 (without aspect ratio, symbol: \cdot) alongside with a new Pareto front (with aspect ratio, symbol: $+$).

As expected, the Pareto front given when considering varying aspect ratios is globally better. Furthermore, by comparing the two graphs, we can see an interesting area where the two frontiers begin to merge at the cost value 350. This means that, in that area, it is not interesting to take the aspect ratio into account as the solutions will not necessarily be better. Once again, these kind of information can be important in the design of an IC and yet they would not be available with the current design flows since only a small number of possibilities are explored. Indeed, due to the sequential nature of the current design flows, such degrees of freedom are not even tried since they dramatically increase the duration of each optimization loop.

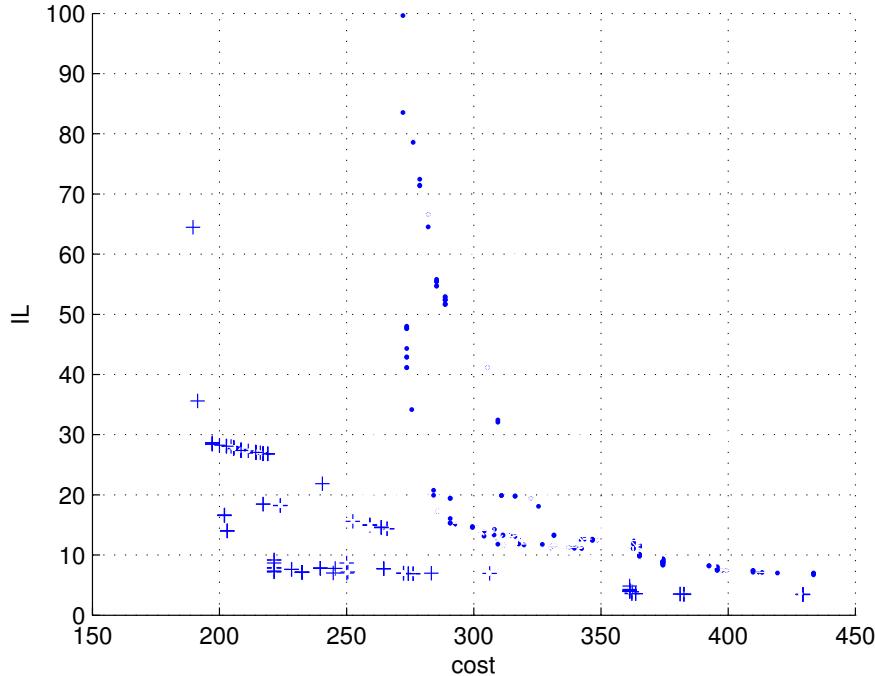


Figure 3.10: IL-cost projection view of the Pareto frontier (with and without aspect ratio)

· : Pareto front without aspect ratio; + : Pareto front with aspect ratio

3.7 Validation with a more realistic case study

In the previous section, we have shown how a circuit can be modelled in 3D-SIC to apply a multi-objective optimization and ran simulations based on the 3MF platform. This case study remains however limited as it contains only 12 blocks. In order to validate the results obtained previously, we will use a more realistic case study. It will be based on the 3MF platform where we will apply a scalability effect.

We will consider 24 ADRES processors instead of 6 and also separate the L1 cache memory from each ADRES core into L1 data cache memories and L1 instruction memories (total of 48 L1 memories blocks), assuming their size will be 10% of the related L2 memories. Since there is one L2D1 and one L2D2 data cache memory for 6 cores and one L2IIs instruction memory for 3 cores in the original design, we will have 16 blocks of L2 memories. We will keep the FIFO and EMIF blocks from the original model. This will thus increase the total number of functional blocks to

90, which is a realistic number to work with. In addition, we will also consider size variability that will bring a form of functional heterogeneity to the circuit: $\pm 30\%$ for the ADRES cores and $+100\%$ for the L1 and L2 memories. This will give the input matrix given in Table 3.12 (the full matrix model can be found in Appendix II).

Table 3.12: Validation case input matrix

Component	ID	Size (90 nm)	Min aspect ratio	Size variability
ADRES core 1~24	1~24	17.27 mm ²	0.5	$\pm 30\%$
L1D	25~48	0.67 mm ²	0.5	$+100\%$
L1Is	49~72	0.66 mm ²	0.5	$+100\%$
L2D1#1~4	73~76	6.74 mm ²	0.5	$+100\%$
L2D2#1~4	77~80	6.74 mm ²	0.5	$+100\%$
L2Is1~8	81~88	6.62 mm ²	0.5	$+100\%$
FIFO	89	0.54 mm ²	0.5	0
EMIF	90	0.66 mm ²	0.5	0

ID: Component identification number

For the communication requirements, the only change compared to the original case is that the exchanges between the ADRES processors and the L2 cache memories will now transit through the L1 cache memories. In order to simplify the necessary bandwidth between the ADRES cores and the L1 memories, we will assume a bandwidth 64 times bigger than for the L2. These data are summarized in Table 3.13 (the full matrix model can be found in Appendix III).

3.7.1 Simulation and validation procedure

In order to validate the methodology, we will run the simulations of this case study to produce floorplans with 1 to 3 layers. First, we will mimic the current design flow by performing a mono-objective optimization (minimizing the interconnection length). Then we will compare the obtained solution to other 2- and 3-tiers 3D-SIC that have been produced with a multi-objective approach.

After simulations, we obtain the following results for the mono-objective optimization of a 2D-IC and a 2-tiers circuit (we will denote them by A , B and C):

Alternative	Number of layers	Interconnection length score
A	1	6.2737
B	2	5.2357
C	3	4.0438

Without surprise, we observe that the 3D-SICs perform better on that criterion

Table 3.13: Validation case bandwidth requirements

Source	Target	Bandwidth (MB/s)
FIFO	EMIF	39.6
EMIF	ADRES core 1~24	6.6
ADRES core 1~24	L1D1~24	1690
ADRES core 1~24	L1Is1~24	19200
L2D1#1	L1D1~6	26.4
L2D1#2	L1D7~12	26.4
L2D1#3	L1D13~18	26.4
L2D1#4	L1D19~24	26.4
L2D2#1~4	L2D1#1~4	52.7
ADRES core 1~24	FIFO	1.2
L1D1~6	L2D2#1	6.6
L1D7~12	L2D2#2	6.6
L1D13~18	L2D2#3	6.6
L1D19~24	L2D2#4	6.6
L1Is1~3	L2Is1	300
L1Is4~6	L2Is2	300
L1Is7~9	L2Is3	300
L1Is10~12	L2Is4	300
L1Is13~15	L2Is5	300
L1Is16~18	L2Is6	300
L1Is19~21	L2Is7	300
L1Is22~24	L2Is8	300

than a 2D-IC, which validates the consistency of the model compare to what is expected in reality.

Now let us compare *A*, *B* and *C* to the 2- and 3-tiers circuits (respectively denoted *D* and *E*) obtained with a multi-objective optimization and that are the closest to *A* with respect to the interconnection length criterion:

Alternative	Number of layers	Interconnection length	Cost	Volume	Clock position	Power
<i>A</i>	1	6.2737	$1.2178 \cdot 10^9$	$2.4356 \cdot 10^7$	$3.3584 \cdot 10^4$	81
<i>B</i>	2	5.2357	$1.2267 \cdot 10^9$	$2.4914 \cdot 10^7$	$4.5687 \cdot 10^4$	351
<i>C</i>	3	4.0438	$1.1495 \cdot 10^9$	$2.3681 \cdot 10^7$	$6.7234 \cdot 10^4$	593
<i>D</i>	2	6.2099	$1.2304 \cdot 10^9$	$2.4877 \cdot 10^7$	$3.8250 \cdot 10^4$	280
<i>E</i>	3	6.2205	$1.1495 \cdot 10^9$	$2.3589 \cdot 10^7$	$3.1095 \cdot 10^4$	459

As expected, except on the interconnection length criterion, the alternative *D* is dominated by *A* on all the criteria. Analysing the circuit *E* is more interesting though as we can observe that *E* dominates *A* on all criteria except for the thermal dissipation. This may seem surprising, especially for the cost and the volume criteria,

however it can be explained by looking at the circuits themselves. In Figure 3.11, we can see the alternative *A* with a surface of $\sim 35 \times 35 \text{ mm}^2$ whereas for *E*, in Figure 3.12, the surface is $\sim 20 \times 20 \text{ mm}^2$ on 3 layers which is in total less than $35 \times 35 \text{ mm}^2$. The clock tree criterion is also better thanks to the smaller surface while the thermal dissipation is unsurprisingly worse due to the inherent greater power density of 3D-SICs.

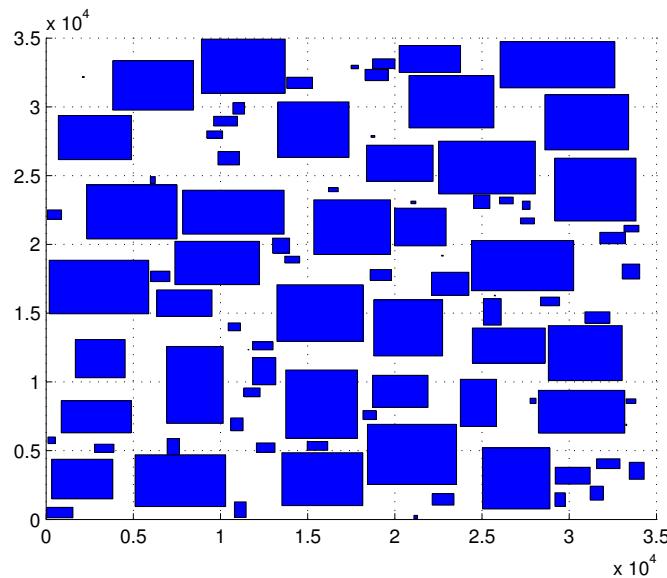


Figure 3.11: Extended 3MF platform (1 layer)

On the other hand, by following a mono-objective optimization, only the interconnection length would be considered and one would choose the circuit *C* as it holds the best score for this criterion while it performs rather badly on the clock position and thermal dissipation. Of course, *E* has better score in terms of volume, clock position and thermal dissipation but is not as good on the interconnection length criterion. Of course, these analyses have to be moderated within the limits of our model. Nevertheless, the choice is not trivial and we will show in Chapter 5 how these results can be exploited in order to help a designer choose in a transparent process.

3.8 Conclusion

The results have thus shown interesting analyses that can be relevant for a designer. First, using a multi-objective optimization methodology does not only consider all the criteria at the same time but also proceed to an extensive design space

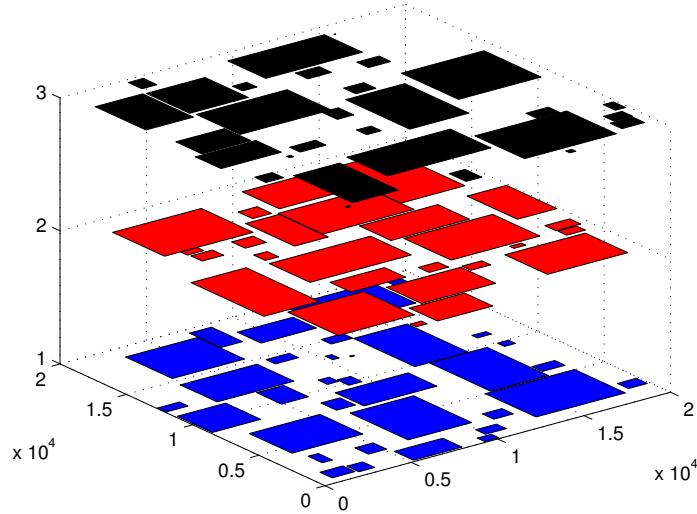


Figure 3.12: Extended 3MF platform (3 layers)

exploration which is rarely done with current tools. Second, the qualitative results shown here can give relevant information to the designer and they can be quantified with a more accurate model. Third, the flexibility of MOO allows to easily consider new degrees of freedom without having to change the paradigm. Finally, we have validated the methodology with a more realistic case study and shown that a multi-objective optimization gives added values compared to a mono-objective paradigm.

In the next chapter, we will show that this methodology and the associated algorithms are robust as they show good indicators of convergence and diversity.

4

Robustness of the methodology

Chapter abstract

In this chapter, we will show that the proposed methodology with the associated algorithms are robust. We will use classical indicator of the field to show that convergence and diversity properties are respected despite the complexity of designing 3D-SIC and the heterogeneous nature of the criteria.

Associated publication:

4.1 Introduction

In Chapter 3, we have shown how a 3D-SIC can be modelled as an optimization problem and we have applied a NSGA-II metaheuristic. The obtained results indicate that multi-objective optimization can give qualitative and quantitative information to a designer that would not be available with current design tools.

In this chapter we will take a deeper look at the results of the multi-objective optimization steps. We will analyse the properties of the design space in order to have the view over the convergence and the robustness of our methodology. We will use classical performance indicators such as the contribution indicator, the spread indicator, the binary ϵ -indicator, the unary hypervolume indicator and the density of the Pareto-front which are presented in [37, 74]. These indicators can be grouped in 3 categories defined in [37]:

1. The convergence-based indicators:

"The convergence metrics evaluate the effectiveness of the solutions in terms of the closeness to the optimal Pareto front."

2. The diversity-based indicators:

"Diversity indicators measure the uniformity of distribution of the obtained solutions in terms of dispersion and extension. In general, the diversity is researched in the objective space."

3. The hybrid indicators: that combine both convergence and diversity measures.

The following results have been obtained with an Intel Core i5 2.30 GHz, 4 GB DDR3 SDRAM for 5 independent experiments. The set of non-dominated solutions over all the runs will constitute the reference set R for the *epsilon* and the hypervolume indicators. Also, these results have been simulated with all the five criteria presented in Section 3.3 instead of only the three first in the case study of Chapter 3.

4.2 Contribution indicator

The contribution is a convergence-based binary indicator. The contribution of an approximation PO_1 relatively to another approximation PO_2 is the ratio of non-dominated solutions produced by PO_1 in PO^* , which is the set of Pareto solutions of $PO_1 \cup PO_2$:

$$Cont(PO_1/PO_2) = \frac{\frac{\|PO\|}{2} + \|W_1\| + \|N_1\|}{\|PO^*\|} \quad (4.1)$$

where PO is the set of solutions in $PO_1 \cap PO_2$, W_1 the set of solutions in PO_1 that dominate some solutions of PO_2 and N_1 the set of non-comparable solutions of PO_1 . This value has to be greater than 0.5 to indicate that PO_1 is better than PO_2 in terms of convergence to the Pareto front.

The Table 4.1 and the Figure 4.1 show the evolution of the averaged contribution indicator over the iterations for the 5 experiments. We see that for the first iterations, $Cont(PO_i/PO_{i-1})$ is greater than 0.5, which means that the algorithm does indeed improve the solutions, then for the last iterations, the indicators are lower than 0.5 which means that there is a convergence.

4.3 Spread indicator

The spread indicator I_s combines the distribution and cardinality to measure the dispersion of the approximated Pareto set A :

$$I_s = \frac{\sum_{u \in A} |\{u' \in A : \|F(u) - F(u')\| > \sigma\}|}{|A| - 1} \quad (4.2)$$

Iteration	$Cont(PO_i/PO_{i-1})$
1	0.7626
2	0.8510
3	0.8917
4	0.8788
5	0.8295
...	...
38	0.4522
39	0.3870
40	0.2369

Table 4.1: Evolution of the contribution indicator

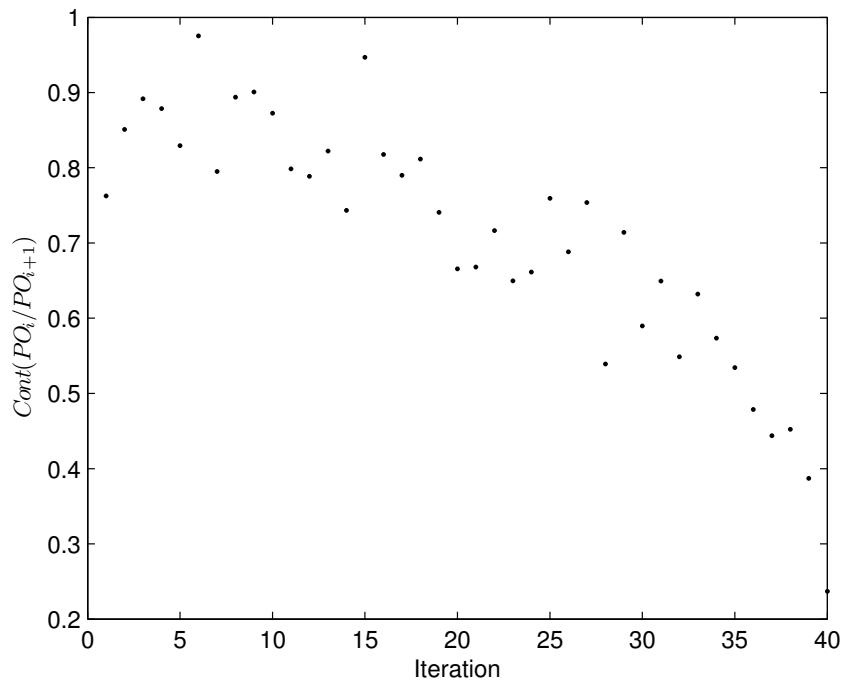


Figure 4.1: Evolution of the contribution indicator

where $F(u)$ is a fitness function and $\sigma > 0$ a neighborhood parameter. The closer is the measure to 1, the better is the spread of the approximated set A .

The Figure 4.2 shows the results of the spread indicator I_s function of the neighborhood indicator σ (all the 5 experiments share the same graph shape). We see that

the Pareto front is well spread: if we consider $I_s \geq 0.9$ we have $\sigma < 0.35$ in average for the 5 runs (normalized values), so we can consider that the algorithm produces a well-spread approximation of the Pareto front.

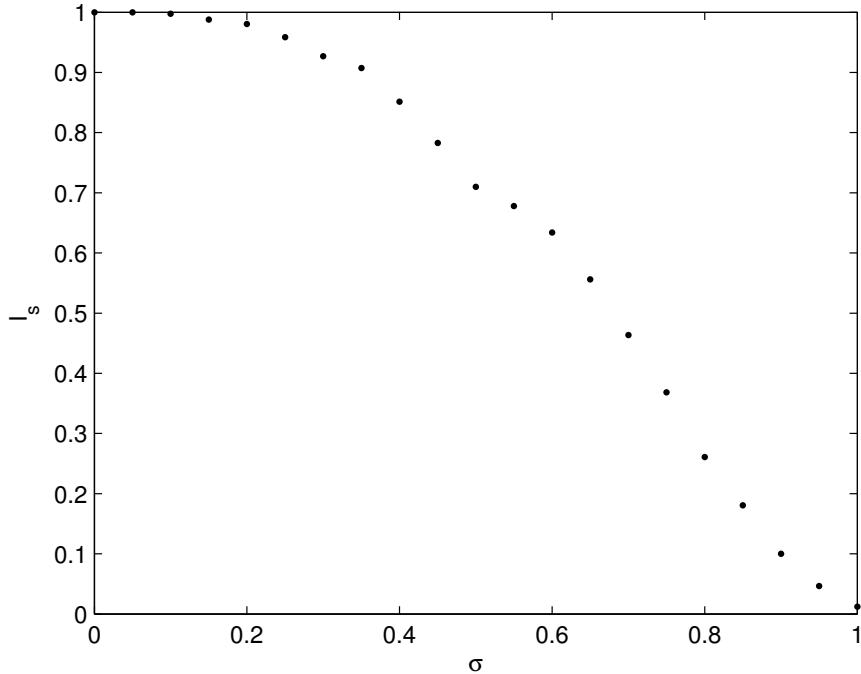


Figure 4.2: Spread indicator I_s function of the neighborhood parameter σ

4.4 Binary ϵ -indicator

The binary ϵ -indicator is a convergence-based indicator. It will give the quality of a solution front in comparison with another set, with regards to all objectives. Let us consider a minimization problem with n positive objectives. An objective vector $f^1 = (z_1^1, z_2^1, \dots, z_n^1)$ is said to ϵ -dominate another objective vector $f^2 = (z_1^2, z_2^2, \dots, z_n^2)$ if $\forall 1 \leq i \leq n : z_i^1 \leq \epsilon \cdot z_i^2$, for a given $\epsilon > 0$. A binary ϵ -indicator $I_\epsilon(A, B)$ gives the factor ϵ such that for any solution in B there is at least one solution in A that is not worse by a factor of ϵ in all objectives. $I_\epsilon(A, B)$ can be calculated as follows [74]:

$$I_\epsilon(A, B) = \max_{z^2 \in B} \min_{z^1 \in A} \max_{1 \leq i \leq n} \frac{z_i^1}{z_i^2} \quad (4.3)$$

The reference set R computed from all the experiments will serve to show the evolution of the ϵ -indicator over time. This evolution is shown in Table 4.2 (for averaged values after each 10 iterations). We can see that in the first iterations, $I_\epsilon(A, R) > 1$ and $I_\epsilon(R, A) \approx 1$ which means that the front is improved while in the last iterations, $I_\epsilon(A, R) > 1$ and $I_\epsilon(R, A) > 1$ which shows convergence.

A comparison of the binary ϵ -indicators between each experiment is also given, in Table 4.3. We can see that $I_\epsilon(A, B) > 1$ and $I_\epsilon(B, A) > 1$ which indicates that neither A weakly dominates B nor B weakly dominates S . This means that the generated front is consistent from one experiment to another.

Also, in Table 4.4 are given the ϵ -indicator between iterations of an experiment (the same observations apply for the other runs). We can see that in the first iterations, the front is always improved ($I_\epsilon(A_i, A_{i-1}) > 1$ and $I_\epsilon(A_{i-1}, A_i) \leq 1$) while in the last iterations, it begins to converge ($I_\epsilon(A_i, A_{i-1}) > 1$ and $I_\epsilon(A_{i-1}, A_i) > 1$).

Iteration	Averaged $I_\epsilon(A, R)$	Averaged $I_\epsilon(R, A)$
1	5.5255	1.0178
10	4.4307	1.0235
20	3.8102	1.1023
30	2.3614	1.1234
40	1.6569	1.2381

Table 4.2: Evolution of the binary ϵ -indicator (averaged values compared to the reference set R) over time

$I_\epsilon(A, B)$	Run 1	Run 2	Run 3	Run 4	Run 5
Run 1	1	1.7270	1.3594	1.8664	1.2542
Run 2	1.5713	1	1.4122	1.7791	1.3420
Run 3	1.4737	1.8638	1	1.9268	1.3069
Run 4	1.3436	1.4564	1.2843	1	1.2365
Run 5	1.4214	1.7650	1.3918	1.7545	1

Table 4.3: Comparison of the binary ϵ -indicators for each experiment

4.5 Unary hypervolume indicator

The hypervolume is an hybrid indicator. Since we already used a binary indicator (*epsilon*), we will use the hypervolume indicator I_H in its unary form. I_H , associated with an approximation set A is given by the volume of the space portion that is weakly dominated by the set A [37].

Iteration	$I_\epsilon(A_i, A_{i-1})$	$I_\epsilon(A_{i-1}, A_i)$
1	1.6674	1.1053
2	1.7223	1
3	2.4439	1
4	1.7477	1
5	2.0577	1
...
38	1.8788	1.4916
39	1.5344	1.8065
40	1.9862	1.6609

Table 4.4: Comparison of the binary ϵ -indicators between iterations of the same experiment

The evolution of the hypervolume (averaged values) is given in Table 4.5 and in Figure 4.3. We can see that the value is (linearly) increasing over time.

The result for each experiment is also given, in Table 4.6 and the used reference point is the worst point computed from all the sets for normalized data. As we can see, the values are rather consistent from one run to another.

Iteration	Averaged hypervolume
1	0.0574
10	0.0701
20	0.0876
30	0.0931
40	0.1036

Table 4.5: Evolution of the unary hypervolume indicator (averaged values compared to the reference set R) over time

Run	I_H
1	0.1171
2	0.1105
3	0.1015
4	0.1185
5	0.0939

Table 4.6: Hypervolume for each experiment

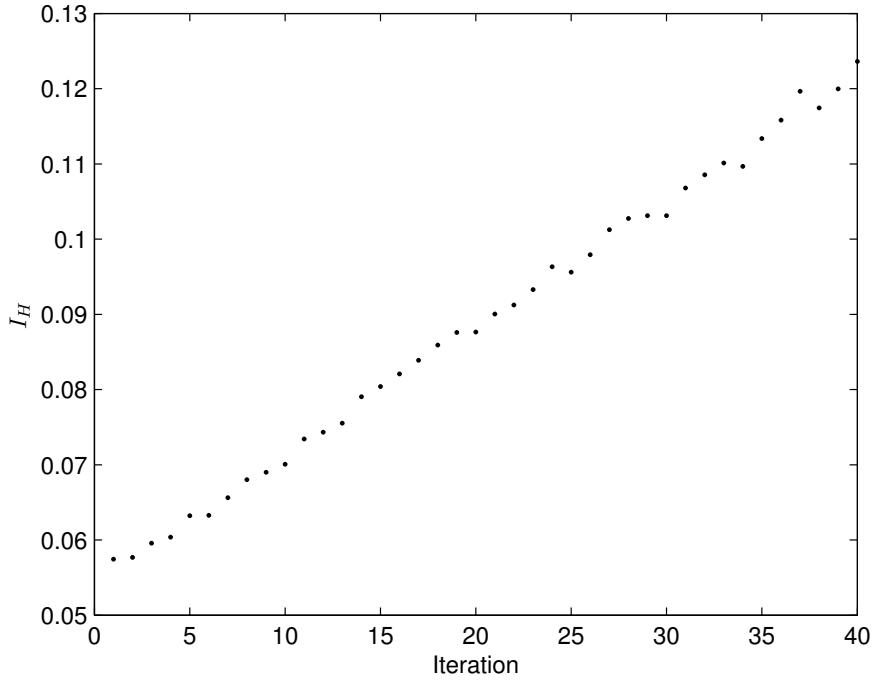


Figure 4.3: Evolution of the unary hypervolume indicator (averaged values compared to the reference set R over time)

4.6 Density of the Pareto front - gaps in the frontier

Another indicator of the Pareto front structure is its density. Here we will measure the density by finding gaps in the frontier. This will be done by counting the number of solutions in the neighborhood of another solution. Since the extreme distance between two solutions is 450.364 in average for the 5 runs (non-normalized), we consider that an acceptable neighborhood is twice the distance between two solutions if all the solutions were equidistant. We have thus a neighborhood of about 2. This test has shown that there was always at least one solution near another one, even for a neighborhood of 1, meaning that the algorithm can produce a sufficiently dense frontier.

4.7 Conclusion

In this chapter, we have shown that the proposed methodology has proved to be robust even if the problem contains criteria of heterogeneous nature. With the several

indicators that we have analysed, we can conclude that the algorithm we used can show good properties of convergence, spread and density.

Also, analyses have been performed to determine the shape of the Pareto front. Globally, the Pareto front is not convex, as one may expect since the heterogeneous nature of the criteria. This is probably due to some correlation between the criteria but this is still to be investigated.

In the next chapter, we will show how these results can be exploited, with a multi-criteria point of view, to help a designer.

5

Results exploitation

Chapter abstract

In this chapter, we show how the obtained results with multi-objective optimization can be used to help a designer, using multi-criteria decision aid. We present how the solutions can be analysed, with a multi-criteria point of view. Then we discuss on their possible actual use and how difficult it is to make the industry adopt it. We then propose some hints to make it integrated into design flows as we do believe that a multi-criteria paradigm can help in design integrated circuits and that it can progressively be of use since designers will have to face greater and greater challenges.

5.1 Introduction

In Chapter 3 we have shown how a 3D-SIC can be modelled and the problem we are dealing with in this work, as well as simulation results based on multi-objective optimization. In Chapter 4, we have shown that the methodology can show good convergence and diversity properties even if the problem contains criteria of heterogeneous nature. In this Chapter, we will discuss about how a designer could use these results and take advantage of a multi-criteria oriented methodology in the process of producing a 3D-SIC to, for instance, make a choice among the solutions of the Pareto frontier.

5.2 Preference modelling

As explained in Chapter 2, once a Pareto front has been determined or approximated, the next step is to choose among this set of solutions. One way to help decision makers to make their choice is to model their preferences, for instance with an outranking method. In the scope of this work, we will present the use of the PROMETHEE methodology as it has been developed in our department and has also shown good results in different fields [65].

5.2.1 Using the PROMETHEE methods

Building a PROMETHEE model

In order to use the PROMETHEE method, the decision maker has to inform about his preferences on the criteria, these being preference functions, indifference and preference thresholds and weights on the criteria (see Chapter 2). To illustrate this, we will use a PROMETHEE software called D-Sight that has been developed by Quantin Hayez and use the results of the simulations presented in Chapter 4. In those results, there are 804 alternatives in the Pareto front, evaluated on 5 criteria. The evaluation table can be found in Appendix I. Let us take a simple example of preference modelling in order to illustrate what kind of aid a multi-criteria analysis can provide.

First, we have to define preference functions for each criterion. For the interconnection distance, cost, volume and clock position, we will choose a V-shape function. With this function, the preference index will increase linearly until a preference threshold is reached which can be the case for these criteria. For the power dissipation, we will choose a U-shape function where there is no preference until an indifference threshold is reached. Indeed there can be no real problem if the difference between two circuits in terms of heat dissipation is low, and it can be directly problematic when this difference is high.

The thresholds will be set at a difference of 10% in the evaluations. For the weights, we will consider that the interconnection distance, the cost and the power dissipation are more important than the volume and the clock position, with the volume less important than the clock position. We will choose arbitrarily the weights and also suppose that the three first cited criteria share the same importance (25% each) and two remaining ones taking the last 25% (14% and 11% respectively). Let us note that it is possible to elicit the weights by answering simple questions, for instance with AHP.

A summary of all these data is given in Table 5.1. Of course, a more accurate model could be defined, however let us remind that the purpose here is to show how a multi-criteria analysis can give added information to designers.

Table 5.1: PROMETHEE model

Criterion	Preference function	Indifference threshold	Preference threshold	Weight
Interconnection distance	V-shape	x	10%	25%
Cost	V-shape	x	10%	25%
Volume	V-shape	x	10%	11%
Clock position	V-shape	x	10%	14%
Power dissipation	U-shape	10%	x	25%

Now that a model has been proposed, let us analyse the results produced by D-Sight.

Multi-criteria analysis

PROMETHEE rankings D-Sight will do all the computations of the flows and PROMETHEE (I and II) rankings can be obtained, based on the preferences. A decision maker can make a choice based on these rankings, for example by choosing the solution ranked first. In addition, other tools are available, that allow to have a transparent decision process and analyse the set of solutions to know why a given ranking is obtained. One of the most useful one is the GAIA plane which is illustrated in Figure 5.1 (for the sake of readability, the alternatives' name has been removed).

As a reminder from Chapter 2, the GAIA plane is based on the principal component analysis of the unicriterion net flows of the solutions and minimises the projection error of each alternative on it. Four distinctive visual information are shown:

1. The green axes that represent the projections of each criterion's axis.
2. The blue dots that represent the projection of each solution's uni-criterion net flow. The value of the uni-criterion net flow is read by projecting the point on the related criterion axis.
3. The red axis that represents the *decision stick* which is the projection of the set of weights and gives the decision direction.
4. The *delta* value that represents the percentage of kept information since there are projection errors.

The first observation that can be drawn is that the blue dots are at the same time well-spread and dense, which illustrate the conclusions of Chapter 4. This also means that each criterion is well-represented in terms of solutions.

Second, let us take a look at the information that is provided by the criteria axes (green). This is shown in Figure 5.2.

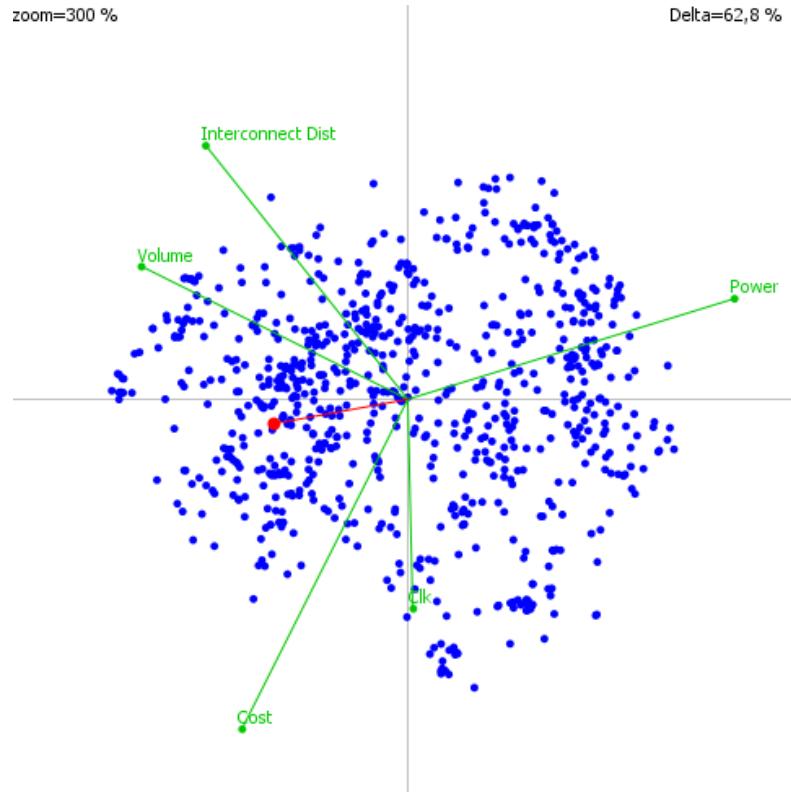


Figure 5.1: GAIA plane of the case study

From the GAIA plane, we can observe how the criteria are related between each other. Indeed, criteria axes that have opposite directions are conflicting, whereas criteria with the same direction share the same optimization trend. In the present case, we can see that the criteria of interconnection distance, power dissipation and cost are conflicting, which reflects the design reality. Also, the volume criterion shares the same direction as the interconnection distance criterion which is normal as reducing the interconnection distance will also tend to reduce the circuit volume. These observations also confirm that the defined model is indeed consistent with the reality.

Finally, let us have a view at the information provided by the decision stick. As explained previously, the decision stick represent the criteria weights and therefore gives the decision direction. Indeed, the alternatives with the highest net flow score will have their furthest projection on that axis, in the direction of that axis (see Figure 5.3). This visually represents the PROMETHEE II ranking, provided that the *delta* value shows that enough information has been kept with the projection. In this case,

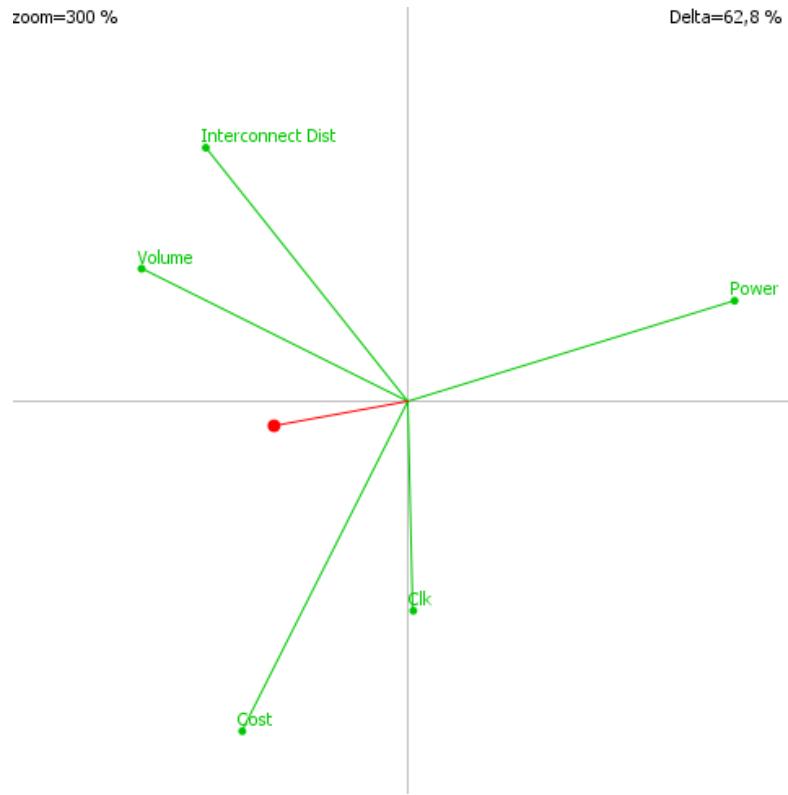


Figure 5.2: GAIA plane of the case study (criteria axis only)

this value is relatively low (62.8%) which means that a lot of information has been lost with the projection, that can lead, for instance, to PROMETHEE II ranking errors with the visual projections compared to the ranking obtained with the computed net flows.

Robustness analysis with stability intervals Another tool that can help decision makers is the robustness analysis that will allow them to know how stable a solution is, given the provided preference model. It is based on stability intervals on the weights where the first-ranked alternative will not change. This tool can be useful as there can be uncertainties on the values given for the weights. For the considered model, the stability intervals are shown in Table 5.2. We can observe that the first-ranked solution is relatively robust with all the criteria weights spanning on rather large intervals. This means that small uncertainties will not affect the ranking of the first alternative.

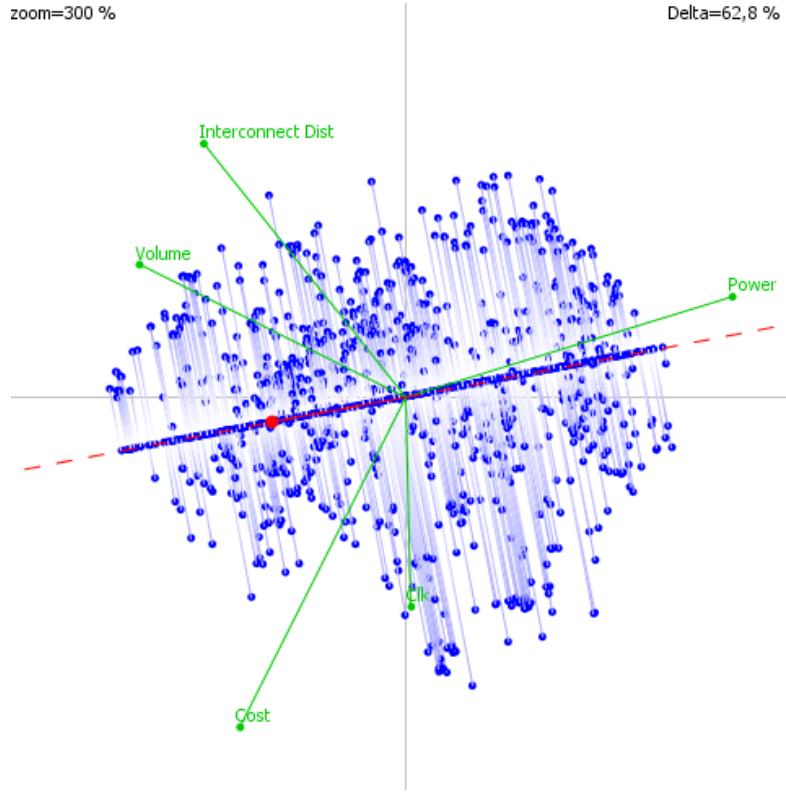


Figure 5.3: GAIA plane of the case study (with decision stick projection)

Table 5.2: Stability intervals (level 1)

Criterion	Min weight	Value	Max weight
Interconnection distance	5.73%	25%	50.00%
Cost	3.37%	25%	36.38%
Volume	0.00%	11%	23.85%
Clock position	2.06%	14%	43.06%
Power dissipation	17.85%	25%	68.21%

5.3 Constraint modelling

To ease decision making, it is also possible to model constraints in order to eliminate unrequired alternatives and reduce the number of solutions, which will ease the choice process. For that purpose, we have developed a visual interface where a de-

5.4. Pertinently representing multi-criteria information in evaluation tables 81

cision maker can introduce constraints to be fulfilled and see directly the remaining solutions that fit these requirements. The general interface is shown in Figure 5.4.

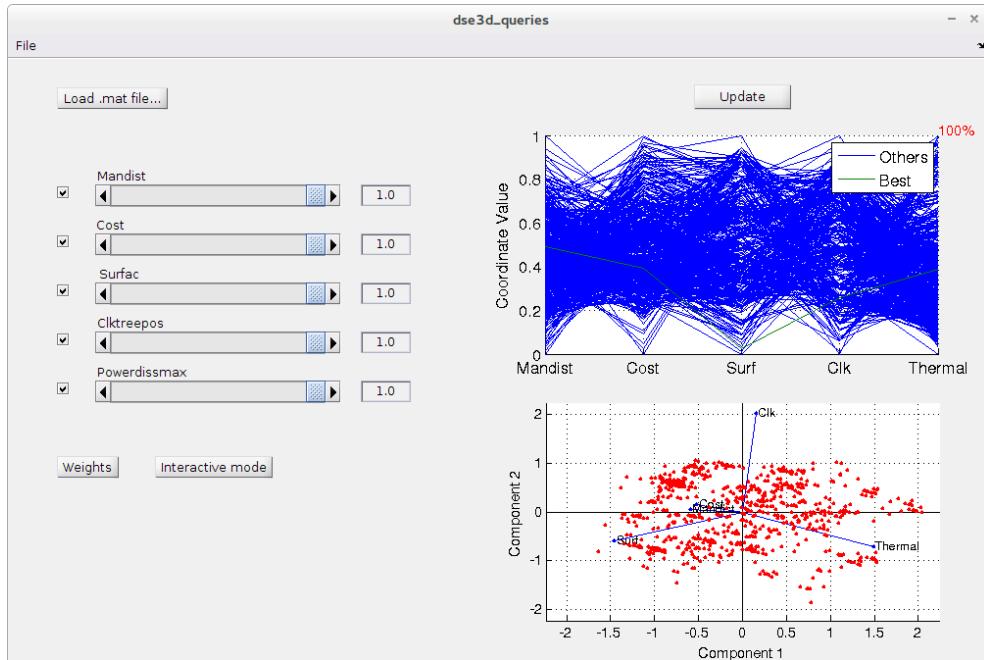


Figure 5.4: Constraints modelling (without filtered alternatives)

The sliders are used to define the constraints. Two graphs are represented to visualise the solutions. The first one uses parallel axes. Let us note that, in order to use this representation effectively, we have normalised the evaluations between 0 and 1. The second graph is a modified GAIA plane where the evaluations are projected (not the uni-criterion net flows). It can help a decision maker to easier see in which direction the solutions are.

In Figure 5.5, an example is shown where alternatives have been filtered. We can see that from 804 solutions, there are 5 remaining possibilities which can ease the choice process.

5.4 Pertinently representing multi-criteria information in evaluation tables

Another way to help decision making could be to enrich evaluation tables with multi-criteria information. We have proposed a contribution with that purpose in

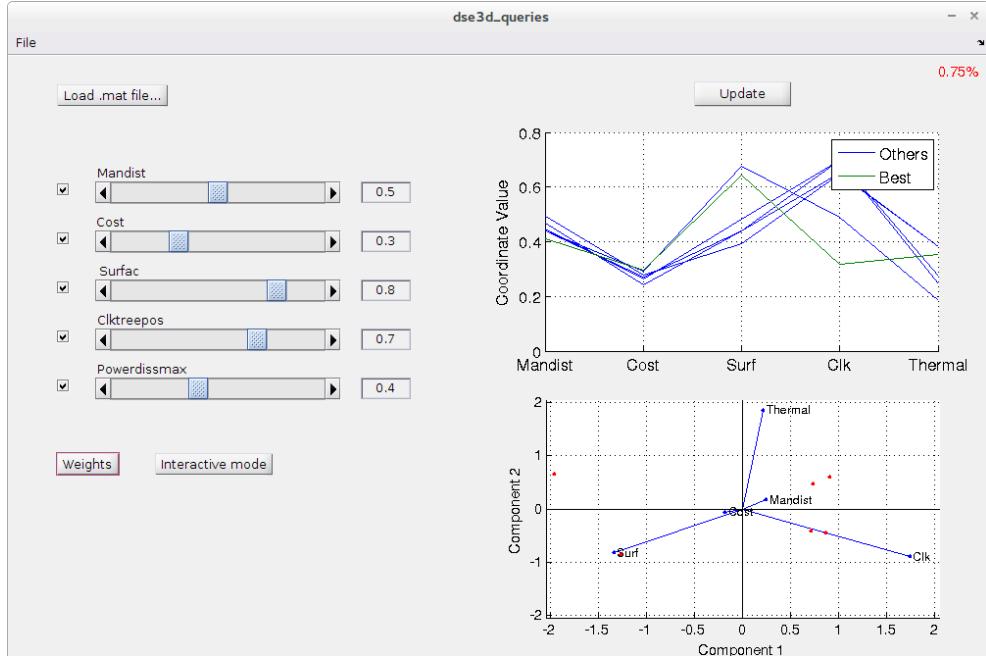


Figure 5.5: Constraints modelling (with filtered alternatives)

[75]. However, as it is difficult to find a direct application of this methodology in microelectronic design, this work will only be cited and reproduced in Appendix IV.

5.5 On the use of a multi-criteria paradigm in microelectronic design

As described in Chapter 1, the multi-criteria paradigm is rarely used in the field of IC design; at best, trade-off analyses are performed. To our knowledge, more global multi-criteria analyses have not been carried out yet.

When discussing with design experts, it is quite interesting to see how they easily understand the stake of the MCDA paradigm and how it would be able to help designers facing IC development challenges. However, it is more difficult to make them adopt this approach for three main reasons that have appeared throughout several discussions:

1. "It is not how we optimize circuits" seems to be one of the most frequent statements. Indeed, the industry follows a uni-criterion paradigm and is not used to first explore several possible solutions and then determine good com-

promise solutions. The designers will generally decide about an architecture and try to optimize it (following a uni-criterion paradigm) to fulfil the specifications.

2. Designers can understand how preference modelling work, however they are not used to answer questions about indifference/preference thresholds or criteria weights as they receive specifications to achieve. This would need a change in how the design of a circuit is approached and how specifications are formulated.
3. As a consequence of the fact that design space exploration is based on performance assessment, preference modelling would be based on estimated metrics. While it will provide relevant ordered information, this will not necessarily match the values of real specifications. Therefore, this adds a level of difficulty for the modelling.

As we can observe, the main reason of difficulties to adopt a multi-criteria paradigm lies in the lack of knowledge about this approach. This will need a deep work in all the steps in a design flow, from how the specifications are defined to the optimization processes. Specifications are currently more and more difficult to fulfil as the industry is nearing the limits of the present technologies. Changing how they are formulated, with therefore adequate methodologies, might help overcome this problem.

Also, (uni-criterion) optimization processes are nowadays more and more time-consuming (weeks to months). This can be seriously problematic in economical terms as a circuit will require more man-years. With this work, we have shown that applying a multi-objective optimization for design space exploration can shorten the design time. These simulations only last hours to days and can already give to designer assessments about the optimization of a circuit, which might lead to shorter optimization processes.

Finally, with the results we have obtained, we do believe that the multi-criteria analysis can aid designers when facing design challenges and allow them to make more transparent choices.

5.6 Conclusion

In this chapter, we have presented how the results of a multi-criteria approach can be exploited for designers. We have shown two ways to help in a decision process and cited a third work. Then we have discussed about the adoption of this paradigm in the field of microelectronic design where we have proposed some possible hints. Although the results we have obtained can provide relevant information to a designer that would not be available with current tools, their exploitation seems to be more complicated as it would require a change in how the industry works. Nevertheless, we do believe that a multi-criteria paradigm can help in design integrated circuits and

that it can progressively be integrated into design flows since designers will have to face greater and greater challenges.

6

Conclusions

In this thesis, we have studied the applicability of multi-objective optimization and multi-criteria decision aid in the context of 3D-stacked integrated circuits design.

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Appendix

I Case study - evaluation table

TO ADD

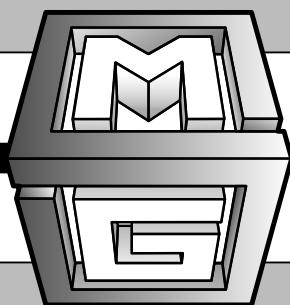
II Extended 3MF case study - input matrix

TO ADD

III Extended 3MF case study - bandwidth input matrix

TO ADD

IV PROMETHEE-compatible presentations of multicriteria evaluation tables



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**PROMETHEE-compatible presentations of
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PROMETHEE-compatible presentations of multicriteria evaluation tables

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Abstract: Most decision problems involve the simultaneous optimisation of several conflicting criteria. Generally, the first step to solve such problems is to identify the set of alternatives and the criteria they will be evaluated on, leading to the construction of an evaluation table. Of course, there are numerous ways to build such a table. For a problem of n alternatives and m criteria, there are $n! \cdot m!$ possibilities of representation. However, from a multicriteria point of view some of them can be more interesting than the others. In this article, we will focus on the PROMETHEE and GAIA methods from which the extracted information will serve to build tables. In order to evaluate the properties of these PROMETHEE-based representations, an indicator will be defined that uses only ordinal information of the values contained in a given table. This measure will also serve as a fitness function for a genetic algorithm that will find good – if not the best – tables. These will allow to draw comparisons with PROMETHEE-based representations.

Keywords: multicriteria decision aid, PROMETHEE, GAIA, evaluation table, visualisation, genetic algorithm

Reference to this paper should be made as follows: Lidouh, K., Doan, N.A.V and De Smet, Y. (xxxx) 'PROMETHEE-compatible presentations of multicriteria evaluation tables', *Int. J. Multicriteria Decision Making*, Vol. x, No. x, pp.XXX–XXX.

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1 Introduction

Most strategic decision problems involve the simultaneous optimisation of several conflicting criteria. For instance, in a procurement conducted by a transport company, the buyer (looking for new trucks) wants to simultaneously optimize: the investment and operational costs, both the quality of the vehicle and the supplier, the time of delivery, the mean time before failure, etc.

In a multicriteria analysis, the first step is to identify the set of alternatives, denoted $\mathcal{A} = \{a_1, a_2, \dots, a_n\}$ and evaluation criteria, denoted $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$. This leads to the construction of an evaluation table (see Table 1).

For the last 50 years several works have been proposed for the visual exploration of data tables or matrices. The first works dealt with reorderable matrices as a tool to represent structures and relationships [1, 2]. Later, other approaches such as block clustering were considered [3, 4] before the use of colour matrix visualisation [5, 6]. Throughout the years these techniques have been used to highlight trends and interesting displays in several cases such as the famous traveling salesman and shortest path problems [7, 8]. They have also been used conjointly with other visualisation tools such as scatterplot matrices and parallel coordinates [9]. In all of these contributions, the authors all agree on the fact that reordering rows and columns in data tables is an essential part in the graphical exploration of quantitative or qualitative data [10–12].

Table 1 Evaluation table

a	$f_1(\cdot)$	$f_2(\cdot)$	\dots	$f_j(\cdot)$	\dots	$f_m(\cdot)$
a_1	$f_1(a_1)$	$f_2(a_1)$	\dots	$f_j(a_1)$	\dots	$f_m(a_1)$
a_2	$f_1(a_2)$	$f_2(a_2)$	\dots	$f_j(a_2)$	\dots	$f_m(a_2)$
\vdots	\vdots	\vdots	\ddots	\vdots	\ddots	\vdots
a_i	$f_1(a_i)$	$f_2(a_i)$	\dots	$f_j(a_i)$	\dots	$f_m(a_i)$
\vdots	\vdots	\vdots	\ddots	\vdots	\ddots	\vdots
a_n	$f_1(a_n)$	$f_2(a_n)$	\dots	$f_j(a_n)$	\dots	$f_m(a_n)$

From a multicriteria decision aid viewpoint, there are plenty of ways to represent evaluation tables. For instance, one may list the set of alternatives and criteria in an alphabetic order. Potentially, given a set of n alternatives and m criteria, $n! \cdot m!$ different evaluation tables can be built. For instance, for a limited problem with only 10 alternatives and 6 criteria, already $2.6127 \cdot 10^9$ different evaluation tables can be displayed. From a multicriteria point of view, some of them are more interesting than others. The aim of this paper is to investigate how to represent the evaluation tables in order to display as much multicriteria information as possible.

Since the late sixties, researchers working in Multicriteria Decision Aid (MCDA) have developed original methods to address these situations. For instance, we can mention Multi-Attribute Utility Theory (MAUT) [13], Analytical Hierarchy Process (AHP) [14], ELECTRE [15], PROMETHEE [16], MACBETH [17], etc.

In this contribution, we will focus ourselves on PROMETHEE methods. These have been applied in hundreds of applications in finance, health care, environmental management, transport, sports, hydrology and water management, production, etc. [18]. This success is due on their simplicity and the existence of user-friendly software.

By making use of information extracted using the PROMETHEE methodology, we will be able to build evaluation tables to convey additional characteristics of the problem. In most cases, these representations will focus on gathering similar alternatives and rearranging the criteria such that their strong and weak characteristics appear more clearly. Furthermore, out of the many possibilities that stem from using the PROMETHEE methodology, we will identify those that yield the most relevant results. We will do so using a subset of the ranking of best cities [19]. This subset, composed of 14 cities is given in table 2. Each of these cities has been evaluated using six criteria, the details of which are described in a later section where we make use of the full ranking. Even with a small set like this one, we have $14! \cdot 6!$ (i.e. $6.2768 \cdot 10^{13}$) possible representations of this evaluation table.

Table 2 Best cities ranking subset - Evaluation table

Perm		1	2	3	4	5	6
	City	Stability	Healthcare	Culture and Environment	Education	Infrastructure	Spatial Characteristics
1	Hong Kong	95	87.5	85.9	100	96.4	75
2	Stockholm	95	95.8	91.2	100	96.4	58.9
3	Rome	80	87.5	91.7	100	92.9	67.3
4	New York	70	91.7	91.7	100	89.3	65.2
5	Atlanta	85	91.7	91.7	100	92.9	42.9
6	Buenos Aires	70	87.5	85.9	100	85.7	42.3
7	Santiago	75	70.8	89.1	83.3	85.7	35.1
8	Sao Paulo	60	70.8	80.3	66.7	66.1	52.4
9	Mexico City	45	66.7	82.4	75	46.4	65.8
10	New Delhi	55	58.3	55.6	75	58.9	58.6
11	Istanbul	55	50	68.8	58.3	67.9	47.5
12	Jakarta	50	45.8	59.3	66.7	57.1	42.3
13	Tehran	50	62.5	35.9	50	33.9	53.6
14	Dakar	50	41.7	59.7	50	37.5	22.6

Source: [19].

Of course, one can imagine that these representations are not all interesting. Therefore, we need to evaluate the tables in order to choose the best representation(s). For that

purpose, we have defined an indicator that only uses the ordinal properties of the values contained in a table: the ∇ -indicator. This will be described in Section 3.1.

With this measure, it will be possible to find the best permutations on the alternatives and the criteria. However, since the number of possibilities can be huge even with a small dataset, it may be impossible to find the best representation in reasonable time. Therefore we have decided to use a genetic algorithm (GA) for the optimisation of the ∇ -indicator, this family of algorithms having shown good properties for similar situations [20]. GAs belong to the class of evolutionary algorithms which generate solutions to optimisation problems using techniques inspired by natural evolution. Details about the implementation will be given in Section 3.2.

We will apply these two approaches on two case studies: the best cities ranking by the Economist Intelligence Unit and the Environmental Performance Index by two research centres of the Columbia University.

This paper is organized as follows: in Section 2 we will give a brief description of the PROMETHEE and GAIA methodologies and identify the possible evaluation tables that can be derived from them. Next, in Section 3, we will define the ∇ -indicator that will allow to evaluate the different representations. This measure will also be used as a fitness function for the genetic algorithm that will be applied. Finally, in Section 4 we will illustrate the two approaches using the previously-described case studies.

2 Constructivist approach

2.1 PROMETHEE and GAIA

In this subsection we recall the basics of the PROMETHEE and GAIA methods. Of course, a detailed description of these approaches goes beyond the scope of this contribution. Therefore we refer the interested reader to [21] for a detailed analysis.

Let $\mathcal{A} = \{a_1, a_2, \dots, a_n\}$ be a set of n alternatives and $\mathcal{F} = \{f_1, f_2, \dots, f_m\}$ be a set of m criteria. Without loss of generality, we assume that all criteria have to be maximized. The PROMETHEE methods are based on pairwize comparisons. At first, each pair of alternatives $a_i, a_j \in \mathcal{A}$ is compared on every criterion f_k :

$$d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$$

The quantity $d_k(a_i, a_j)$ represents the *advantage* of a_i over a_j for criterion f_k . On the one hand, when $d_k(a_i, a_j)$ is small enough, there is no good reason to say that a_i is better than a_j regarding criterion f_k . On the other hand, when $d_k(a_i, a_j)$ exceeds a certain limit, the decision maker may express that a_i is strictly preferred to a_j for f_k . In order to model these statements, the difference $d_k(a_i, a_j)$ is transformed into a unicriterion preference degree, denoted $P_k(a_i, a_j)$, by using a non-decreasing function H_k :

$$P_k(a_i, a_j) = H_k(d_k(a_i, a_j)), \quad \forall a_i, a_j \in \mathcal{A}$$

The quantity $P_k(a_i, a_j) \in [0, 1]$ and $P_k(a_i, a_j) = 0$ when $d_k(a_i, a_j) < 0$. There are plenty of functions that can be considered to compute the unicriterion preference degrees. In most software implementing the PROMETHEE method, 6 main functions are considered [22]. Figure 1 represents the so-called linear preference function. Two thresholds characterize it:

- q_k plays the role of an *indifference* threshold. When the difference $d_k(a_i, a_j) \leq q_k$, it is considered to be so small that the unicriterion preference is equal to zero;
- p_k plays the role of a *preference* threshold. When the difference $d_k(a_i, a_j) \geq p_k$, it is considered to be important enough to state that a_i is strongly preferred to a_j for this criterion.

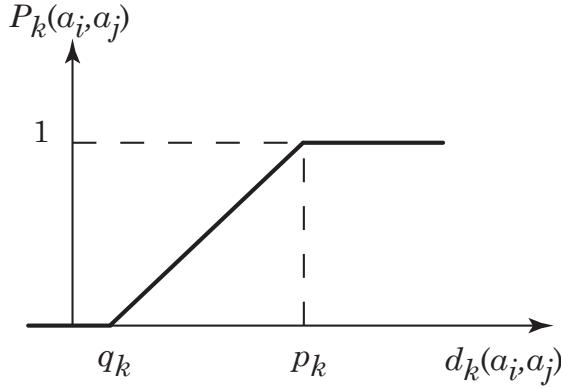


Figure 1: Generalized criterion of type 5

Once the unicriterion preference degrees between two actions a_i and a_j have been computed for every criterion, one has to aggregate these marginal contributions to obtain $P(a_i, a_j)$ i.e. a global measure of the preference of a_i over a_j :

$$P(a_i, a_j) = \sum_{k=1}^m \omega_k \cdot P_k(a_i, a_j)$$

where ω_k represents the relative importance of criterion f_k . These weights are assumed to be positive and normalized. Obviously, we have $P(a_i, a_j) \geq 0$ and $P(a_i, a_j) + P(a_j, a_i) \leq 1$.

The PROMETHEE I and II rankings are based on the exploitation of the matrix P . Therefore, three flows are built.; the positive flow ϕ^+ , the negative flow ϕ^- and the net flow ϕ :

$$\phi^+(a_i) = \frac{1}{n-1} \sum_{a_j \in \mathcal{A}, i \neq j} P(a_i, a_j)$$

$$\phi^-(a_i) = \frac{1}{n-1} \sum_{a_j \in \mathcal{A}, i \neq j} P(a_j, a_i)$$

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i)$$

The PROMETHEE I ranking is obtained as the intersection of the rankings induced by ϕ^+ and ϕ^- . The PROMETHEE II ranking is given by the ranking given by ϕ .

Finally, it is worth noting that:

$$\phi(a_i) = \frac{1}{n-1} \sum_{k=1}^m \sum_{a_j \in A} [P_k(a_i, a_j) - P_k(a_j, a_i)] \cdot \omega_k = \sum_{k=1}^m \phi_k(a_i) \cdot \omega_k$$

where $\phi_k(a_i)$ is called the k^{th} unicriterion net flow assigned to action a_i .

The PROMETHEE I and II ranking provide prescriptive tools for decision making. The GAIA [23] tool complements them with a descriptive approach. The idea is to represent each alternative by its evaluations in the unicriterion net flow space:

$$\Phi(a_i) = [\phi_1(a_i), \phi_2(a_i), \dots, \phi_m(a_i)]$$

GAIA is the result of a principal component analysis applied on this dataset. Therefore, the decision maker is able to visualize the decision problem on a plane and compare:

- the relative positions of alternatives (in order to identify groups of similar or distinct alternatives profiles);
- the relative positions of criteria (in order to identify conflicts or redundancies);
- the relative positions of alternatives with respect to a given criterion (in order to identify the best and worst alternatives for the different points of views);
- the relative positions of alternatives with respect to the so-called *decision stick* (in order to identify the best compromise solutions).

2.2 Visualisation possibilities

To illustrate the different combinations of orders we could use to rearrange an evaluation table, let us consider the subset of cities we introduced in Section 1. Using this table of 14 alternatives and 6 criteria, we will apply the PROMETHEE methodology by setting some arbitrary values for the parameters that the method requires. To keep a ranking close to the one obtained by the Economist Intelligence Unit, we will use the same weight values as they did for their model (see Section 4.2). Then, for the sake of simplicity, we will make use of usual preference functions. These are generalized preference functions for which both thresholds are equal to 0. By doing so, we will only make use of the ordinal data extracted from our table. Of course, a detailed discussion on the parameters goes beyond the scope of this contribution. The following example is just used for illustration purposes. Figure 2 shows the GAIA plane we obtain for this case.

Ordering the alternatives

One obvious choice to order the alternatives would be to use the PROMETHEE II net flows we obtain. This would allow us to order the alternatives from best to worst thereby ensuring that the best profiles are at the top of the table while the ones with the less desirable ones are at the end. Grouping the alternatives with similar global scores would inevitably serve to gather alternatives with similar profiles at the top and bottom of the table. In problems where the few best and few worst alternatives are of interest to the user, this representation could give interesting insights.

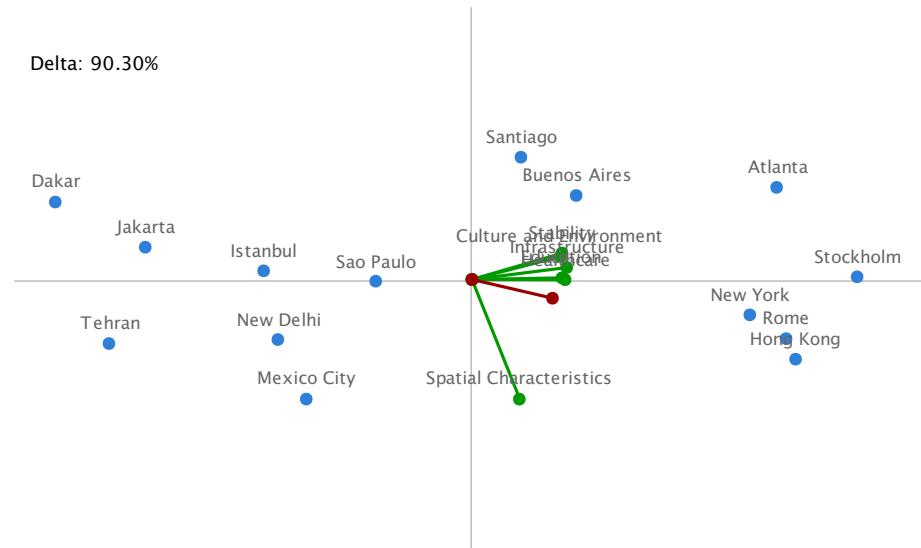


Figure 2: GAIA plane for the best cities subset

Two other options to order the alternatives can be found by using the GAIA plane. Indeed, in this projection, the alternatives have been positioned such that similar alternatives are closer to each other. By using the angle on which each alternative is positioned and scanning the entire plane we would order the alternatives by selecting them based on the types of strong points (or weak points) they have. The information that will be highlighted in this table are the profiles of the alternatives.

One other use of the GAIA plane would be to select alternatives based on their accordance with the decision axis. To do so easily we can compute the angles between the decision axis and the alternatives and select the alternatives from the smallest to the greatest angle. This would generate an order that is similar to the PROMETHEE II ranking obtained using the net flows. When considering the GAIA plane we obtained in Figure 2, we can see that the first alternatives would be Hong Kong, Rome, and New York. The last ones encountered would be Istanbul, Jakarta, and Dakar.

Ordering the criteria

To order the criteria, we also have different options. One would be to order them based on their weights. This would make sure that the first criteria are the ones that hold the greatest importance. This however will not necessarily order the criteria in a way that gathers similar characteristics of the profiles. Therefore, unless the profiles are already similar due to the nature of the alternatives and their order, the obtained table would not be so easy to read.

Other orders involve the use of the GAIA plane once more. The first one consists in choosing the criteria in the order they appear when scanning the projections. Since the criteria are positioned based on their correlation, choosing this order would allow us

to group strong and weak points that usually appear simultaneously. This type of order would work best if coupled with the similar way of ordering the alternatives (i.e. based on the angle of their position). Let us note that there are several options for the use of this technique. Indeed the starting angle has to be chosen but also the direction by which we will scan the plane.

Another possibility would be to select criteria based on their proximity with the decision axis. Once again, an easy way to do so, would be to compute the angle between the criteria axis and the decision axis and then to order them from smallest to greatest angle. The results using this technique however could be rather unpredictable.

Illustration

Among all the combinations of orders we generated only four drew our attention:

- Netflow - Angle: The first one consists in ordering the alternatives based on their net flow and the alternatives based on the angle of their axis on the GAIA plane. This representation has proven after several simulations to give us the most expected results by grouping all the good and bad alternatives and displaying their profiles such that the variations in their evaluations are smoother and easier to compare.
- Netflow - Weight: The second combination orders the alternatives based on their net flow and the criteria based on their weight. Even if this representation's aim is not to display smoother profiles like the previous one, it can be useful to attract the reader's attention on the characteristics that will have a greater impact on the final decision. Therefore it was only natural that the alternatives be ordered according to the final ranking we obtain.
- Angle - Angle: Among the combinations that use the angle at which the alternatives are located, the only one that gave us meaningful results were the ones where the criteria as well were ordered according to the angle of their axes. By choosing to start at the position of the decision axis and scanning the plane in an anti-clockwise motion, the alternatives appear from best to worst to best based on the characteristics of their profiles.
- Proximity - Proximity: Finally ordering the criteria and alternatives based on their proximity to the decision axis can sometimes give us interesting representations. However in most cases the results do not reflect any particular relationship between the alternatives or the criteria aside from the approximate ranking from best to worst.

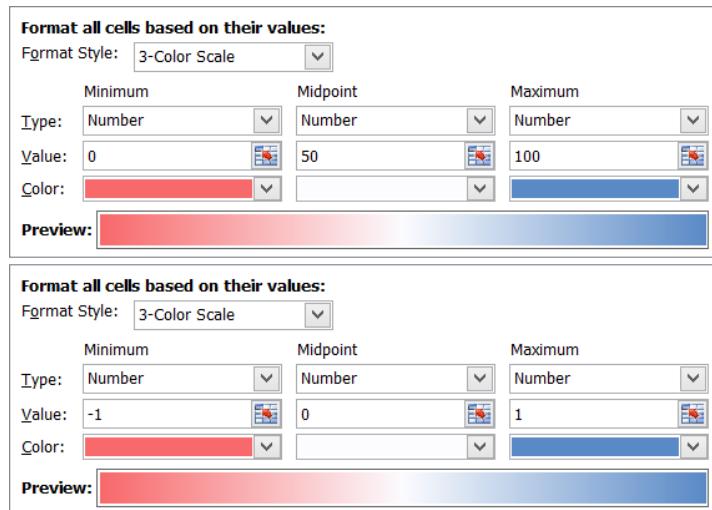
Table 3 shows the possible combinations of orders that can be applied to our evaluation tables where \bigcirc indicates the four chosen combinations and \times indicates the other possibilities that have not been kept for the study.

To better compare these tables and understand the impact of changing the orders we applied a colouring process to the value cells based on two colourmaps. A spreadsheet software such as Microsoft Excel allows us to use conditional formatting rules to achieve this result. The colourmaps we used are displayed in Figure 3. The lowest values will be coloured in red, the middle values in white and the highest values in blue. The evaluations in our examples range from 0 to 100. For the net flows, these values range from -1 to +1. Furthermore, we did not use pure red and blue colours for the extreme values as these would have rendered the data unreadable. Instead we chose the threshold colours

Table 3 Combinations of orders and chosen representations

		Order of criteria		
		Weights	Angle	Proximity
Order of alternatives	Netflows	○	○	×
	Angle	×	○	×
	Proximity	×	×	○

commonly proposed by Excel for readability purposes: the red we use is [248, 105, 107] and the blue is [90, 138, 198] in the RGB colour system.

**Figure 3:** Colourmaps for the conditional formatting rules

Thus, for the best cities ranking subset, we have generated the four chosen combinations. These are given in Figures 4, 5, and 6.

The first table (see Figure 4) shows the table for alternatives ordered by the net flow and criteria ordered by their angle relative to the decision stick. As can be seen due to the coloring, the choice of starting at the position of the decision stick seems wrong in this case. Indeed criterion 2 would have been better off as the first in this table.

The second table (see Figure 5) which uses the weights to order the criteria gives surprisingly good results. Indeed, as the criteria with the greatest weight are also the ones with the smallest evaluation values, it seems as though the best values are located in the top right corner and the lowest in the bottom left.

Unsurprisingly, the table in Figure 6a shows the best values in the four corners and the worst in the middle of the table. This is the natural result when ordering the alternatives and the criteria based on the angle of their position on the GAIA plane.

	Crit4	Crit5	Crit3	Crit1	Crit6	Crit2	NetFlows
A2	100	96,4	91,2	95	58,9	95,8	0,704808
A1	100	96,4	85,9	95	75	87,5	0,694231
A3	100	92,9	91,7	80	67,3	87,5	0,667308
A4	100	89,3	91,7	70	65,2	91,7	0,541346
A5	100	92,9	91,7	85	42,9	91,7	0,446154
A6	100	85,7	85,9	70	42,3	87,5	0,030769
A7	83,3	85,7	89,1	75	35,1	70,8	-0,03846
A8	66,7	66,1	80,3	60	52,4	70,8	-0,14615
A9	75	46,4	82,4	45	65,8	66,7	-0,17885
A10	75	58,9	55,6	55	58,6	58,3	-0,30865
A11	58,3	67,9	68,8	55	47,5	50	-0,35481
A13	50	33,9	35,9	50	53,6	62,5	-0,575
A12	66,7	57,1	59,3	50	42,3	45,8	-0,65577
A14	50	37,5	59,7	50	22,6	41,7	-0,82692

Figure 4: Visualisation - Best cities ranking subset - Netflow-angle

	Crit6	Crit1	Crit3	Crit2	Crit5	Crit4	NetFlows
A2	58,9	95	91,2	95,8	96,4	100	0,704808
A1	75	95	85,9	87,5	96,4	100	0,694231
A3	67,3	80	91,7	87,5	92,9	100	0,667308
A4	65,2	70	91,7	91,7	89,3	100	0,541346
A5	42,9	85	91,7	91,7	92,9	100	0,446154
A6	42,3	70	85,9	87,5	85,7	100	0,030769
A7	35,1	75	89,1	70,8	85,7	83,3	-0,03846
A8	52,4	60	80,3	70,8	66,1	66,7	-0,14615
A9	65,8	45	82,4	66,7	46,4	75	-0,17885
A10	58,6	55	55,6	58,3	58,9	75	-0,30865
A11	47,5	55	68,8	50	67,9	58,3	-0,35481
A13	53,6	50	35,9	62,5	33,9	50	-0,575
A12	42,3	50	59,3	45,8	57,1	66,7	-0,65577
A14	22,6	50	59,7	41,7	37,5	50	-0,82692

Figure 5: Visualisation - Best cities ranking subset - Netflow-Weight

	Crit4	Crit5	Crit3	Crit1	Crit6	Crit2	NetFlows
A2	100	96,4	91,2	95	58,9	95,8	0,704808
A5	100	92,9	91,7	85	42,9	91,7	0,446154
A6	100	85,7	85,9	70	42,3	87,5	0,030769
A7	83,3	85,7	89,1	75	35,1	70,8	-0,03846
A14	50	37,5	59,7	50	22,6	41,7	-0,82692
A12	66,7	57,1	59,3	50	42,3	45,8	-0,65577
A11	58,3	67,9	68,8	55	47,5	50	-0,35481
A8	66,7	66,1	80,3	60	52,4	70,8	-0,14615
A13	50	33,9	35,9	50	53,6	62,5	-0,575
A10	75	58,9	55,6	55	58,6	66,7	-0,30865
A9	75	46,4	82,4	45	65,8	66,7	-0,17885
A1	100	96,4	85,9	95	75	87,5	0,694231
A3	100	92,9	91,7	80	67,3	87,5	0,667308
A4	100	89,3	91,7	70	65,2	91,7	0,541346

(a) Angle-Angle

(b) Proximity-Proximity

Figure 6: Visualisation - Best cities ranking subset

As for Figure 6b, it shows the table that is obtained when both the alternatives and the criteria are ordered based on their proximity to the decision axis. The result shows us that the highest values are gathered in the top left corner. Let us note that a very similar result would have been obtained had we used the net flows to order the alternatives.

3 Optimisation approach

3.1 Development of an optimisation indicator: the ∇ -indicator

In order to compare the different possibilities of visualisation, we need an indicator that will evaluate the presentation quality of a table. We arbitrarily chose to verify that the best values are located at the top left of the table while the worst values are located at the bottom right. We have therefore developed such an indicator that will only use the ordinal information of the values contained in the table. We will denote such indicator ∇ that is the total number of ordered pairs of values where the first value is greater than or equal to the second, for each row and column (in other words, the number of pairs that are compatible with our convention).

A computation example of the ∇ -indicator is shown in Table 4 where:

- $\nabla_{i \cdot}$ is the ∇ value of the i -th row:

$$\nabla_{i \cdot} = \sum_{k=1}^m \sum_{l=k+1}^m \mathbb{1}_{k < l} \mathbb{1}_{f_k(a_i) \geq f_l(a_i)}$$

- $\nabla_{\cdot j}$ is the ∇ value of the j -th column:

$$\nabla_{\cdot j} = \sum_{k=1}^n \sum_{l=k+1}^n \mathbb{1}_{k < l} \mathbb{1}_{f_j(a_k) \geq f_j(a_l)}$$

- The total $\nabla = \sum_{i=1}^n \nabla_{i \cdot} + \sum_{j=1}^m \nabla_{\cdot j}$

We can use this indicator as a value to compare the different representations of an evaluation table.

3.2 Optimizing the ∇ -indicator with a genetic algorithm

Now that we have defined an indicator that can evaluate the presentation quality of an evaluation table, we can use it in order to find the best permutations of alternatives and criteria that will maximize it. As stated in the introduction, finding the optimal solution might be tedious as the number of solutions can be huge: $n! \cdot m!$ possibilities.

Finding the best solution table can thus be seen as a combinatorial optimisation problem. Up to now, we have not found an exact method to solve it, therefore we will use a genetic algorithm to find a good solution in reasonable time. In what follows we describe its main steps.

Table 4 Best cities ranking subset - Evaluation table - ∇ -indicator computation

City	Stability	Healthcare	Culture and Environment	Education	Infrastructure	Spatial Characteristics	∇_i
Hong Kong	95	87.5	85.9	100	96.4	75	9
Stockholm	95	95.8	91.2	100	96.4	58.9	8
Rome	80	87.5	91.7	100	92.9	67.3	6
New York	70	91.7	91.7	100	89.3	65.2	9
Atlanta	85	91.7	91.7	100	92.9	42.9	7
Buenos Aires	70	87.5	85.9	100	85.7	42.3	9
Santiago	75	70.8	89.1	83.3	85.7	35.1	8
Sao Paulo	60	70.8	80.3	66.7	66.1	52.4	10
Mexico City	45	66.7	82.4	75	46.4	65.8	7
New Delhi	55	58.3	55.6	75	58.9	58.6	4
Istanbul	55	50	68.8	58.3	67.9	47.5	8
Jakarta	50	45.8	59.3	66.7	57.1	42.3	8
Tehran	50	62.5	35.9	50	33.9	53.6	9
Dakar	50	41.7	59.7	50	37.5	22.6	12
$\nabla \cdot j$	82	83	76	88	84	66	$\nabla = 593$

Source: [19].

Selection

A solution table is composed of two informations: the permutation on the alternatives and the permutation on the criteria. These will thus constitute a gene. For example, the gene $\{[7, 1, 13, 2, 14, 6, 10, 12, 11, 4, 8, 3, 9, 5], [3, 5, 6, 1, 4, 2]\}$ applied to the example of Table 2 will produce the Table 5.

Table 5 Best cities ranking subset - Evaluation table - Permutation example

Perm	City	3	5	6	1	4	2
		Culture and Environment	Infrastructure	Spatial Characteristics	Stability	Education	Healthcare
7	Santiago	75	70.8	89.1	83.3	85.7	35.1
1	Hong Kong	95	87.5	85.9	100	96.4	75
13	Tehran	50	62.5	35.9	50	33.9	53.6
2	Stockholm	95	95.8	91.2	100	96.4	58.9
14	Dakar	50	41.7	59.7	50	37.5	22.6
6	Buenos Aires	70	87.5	85.9	100	85.7	42.3
10	New Delhi	55	58.3	55.6	75	58.9	58.6
12	Jakarta	50	45.8	59.3	66.7	57.1	42.3
11	Istanbul	55	50	68.8	58.3	67.9	47.5
4	New York	70	91.7	91.7	100	89.3	65.2
8	Sao Paulo	60	70.8	80.3	66.7	66.1	52.4
3	Rome	80	87.5	91.7	100	92.9	67.3
9	Mexico City	45	66.7	82.4	75	46.4	65.8
5	Atlanta	85	91.7	91.7	100	92.9	42.9

From this point, a pool of random solutions can be generated to initialize the algorithm (by using a uniform distribution). From a set of 10000 randomly-generated solutions, the 100 best will be selected and will constitute the initial population. This selection rule will be used for each generation.

Crossover

For the crossover, a classical method has been used: the one-point crossover. A random crossover point is selected on both parents. Beyond that point, the data will be completed following the order of appearance in the other parent, as shown in Figure 7. This method will be used separately both for the alternatives and criteria permutations. The crossover probability has been set as $p_c = 1$.

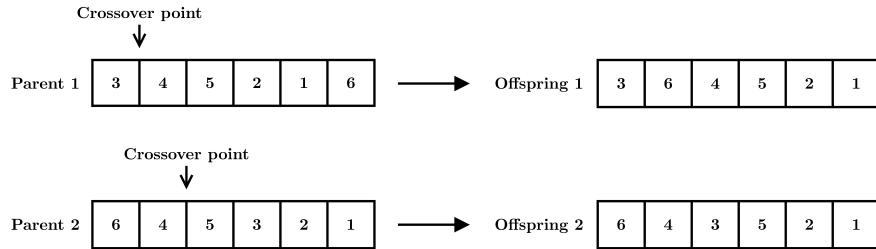


Figure 7: Crossover example

Mutation

For the mutation, two random data of an offspring will be swapped, as shown in Figure 8. This method will be used separately both for the alternatives and criteria permutations. The mutation probability has been set as $p_m = 0.1$.

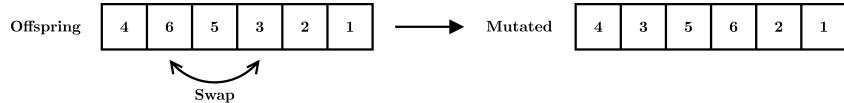


Figure 8: Mutation example

Termination conditions

Common termination conditions have been used: fixed number of generations (50) and maximum computation time (30 min).

Simulation environment and performance

The genetic algorithm have been implemented on MATLAB and the simulations have been carried out on an Intel® Core™ i5-2410M Processor (dual core, 2.3 GHz).

The average running time for 50 generations, as a function of the problem size (using our case studies as examples) is given in Table 6. Figure 9 shows the evolution of the ∇ -indicator over 50 iterations for the best cities ranking subset. The other case studies have similar convergence shape.

Table 6 Genetic algorithm - Average running time

	Table size (n alt. \times m crit.)		
	14×6	70×6	19×9
Time (min.)	5.75	25.01	10.56

Figure 9: Evolution of the ∇ -indicator over the iterations for the best cities ranking subset

4 Case studies

4.1 Comparison of the approaches

4.1.1 Comparison of the tables: defining a ratio

When comparing different representations, we can compare their ∇ -indicators. However, depending on the size of a table, this comparison is not that trivial. Indeed, a difference of, for instance, 20 between two ∇ -indicators can be important for a small table but insignificant for a big table.

Therefore, in order to keep the comparison as objective as possible, we introduce a ratio denoted R :

$$R = \frac{\nabla - \nabla_{\text{worst}}}{\nabla_{\text{best}} - \nabla_{\text{worst}}}$$

where ∇_{best} is the best ∇ found with our genetic algorithm and ∇_{worst} is the value associated to the worst table found by taking the opposite permutations of the alternatives and criteria of the best table.

Let us note that it is possible to find a theoretical maximum value for the ∇ -indicator of a given table for n alternatives and m criteria:

$$\nabla_{\text{max}} = \frac{n \cdot m \cdot (m - 1)}{2} + \frac{m \cdot n \cdot (n - 1)}{2}$$

However it would not be realistic to use it as a reference. Indeed this ∇_{max} can only be reached with well-chosen values which would not be the case for real multicriteria decision problems.

4.2 Best cities ranking

Our first case study is based on the best cities ranking by the Economist Intelligence Unit [19]. This dataset is composed of 70 cities evaluated on 6 criteria: stability, healthcare, culture and environment, education, infrastructure, and spatial characteristics (see Table 7).

This study consists in an update of the existing EIU Liveability index to which a sixth criterion has been added to take into account spatial characteristics. This added factor carries a weight of 25% and seeks to account for spatial aspects such as urban form, the geographical situation of the city, cultural assets and pollution.

Just like we illustrated our approach in Section 2.2, we make use of usual preference functions. The weights for the criteria are taken from the analysis by the EIU (see Table 7).

Table 7 Best cities ranking - Evaluation table

Perm	City	1	2	3	4	5	6	Aggregate
		Stability (18.75%)	Healthcare (15%)	Culture and Environment (18.75%)	Education (7.5%)	Infrastructure (15%)	Spatial Characteristics (25%)	
1	Hong Kong	95	87.5	85.9	100	96.4	75	87.8
2	Amsterdam	80	100	97.2	91.7	96.4	71.3	87.4
3	Osaka	90	100	93.5	100	96.4	64	87.4
4	Paris	85	100	97.2	100	96.4	63.7	87.1
5	Sydney	90	100	94.4	100	100	55.7	86
6	Stockholm	95	95.8	91.2	100	96.4	58.9	86
7	Berlin	85	100	97.2	91.7	96.4	61.7	85.9
8	Toronto	100	100	97.2	100	89.3	50	85.4
9	Munich	85	100	97.2	91.7	89.3	62.5	85.1
10	Tokyo	90	100	94.4	100	92.9	53.3	84.3
11	Rome	80	87.5	91.7	100	92.9	67.3	83.6
12	London	70	87.5	97.2	100	89.3	72.6	83.5
13	Madrid	85	87.5	94.4	100	92.9	61.3	83.5
14	Washington DC	80	91.7	94.4	100	96.4	55.1	82.2
15	Chicago	85	91.7	91.7	100	92.9	52.7	81.5
16	New York	70	91.7	91.7	100	89.3	65.2	81.3
17	Los Angeles	80	91.7	94.4	100	89.3	50.3	79.9
18	San Francisco	85	91.7	94.4	83.3	85.7	53	79.7
19	Boston	80	91.7	91.7	100	96.4	46.7	79.6
20	Seoul	80	83.3	85.6	100	89.3	58.8	79.1
21	Atlanta	85	91.7	91.7	100	92.9	42.9	79
22	Singapore	95	87.5	76.6	83.3	100	46.7	78.2
23	Miami	85	91.7	91.7	100	92.9	39.3	78.1
24	Budapest	85	91.7	90	100	83.9	43	77.4
25	Lisbon	80	87.5	95.1	91.7	80.4	41.7	75.3
26	Buenos Aires	70	87.5	85.9	100	85.7	42.3	73.3
27	Moscow	65	79.2	81.5	91.7	83.9	54.2	72.3
28	St Petersburg	65	87.5	81.5	83.3	80.4	48.2	70.9
29	Athens	75	83.3	83.1	75	75	47.3	70.8
30	Beijing	80	66.7	72.2	83.3	85.7	51.5	70.5
31	Santiago	75	70.8	89.1	83.3	85.7	35.1	69.3
32	Warsaw	80	70.8	80.3	75	82.1	39	68.4
33	Shanghai	80	62.5	75	75	75	46.1	66.8
34	Shenzhen	85	62.5	63.7	66.7	82.1	48.5	66.7
35	Lima	60	66.7	81.7	91.7	75	47.3	66.5
36	Sao Paulo	60	70.8	80.3	66.7	66.1	52.4	64.9
37	Kuala Lumpur	80	62.5	67.8	91.7	76.8	36.6	64.6
38	Tianjin	90	66.7	65.3	66.7	82.1	27.7	63.4
39	Guangzhou	80	62.5	61.1	66.7	76.8	42.9	63.1
40	Johannesburg	50	58.3	90.5	83.3	69.6	44.9	63
41	Mexico City	45	66.7	82.4	75	46.4	65.8	62.9
42	Rio de Janeiro	55	66.7	77.5	83.3	71.4	44	62.8
43	Bucharest	80	66.7	74.3	66.7	66.1	34.7	62.5
44	Kiev	70	75	73.4	83.3	50	33.3	60.2
45	Belgrade	60	75	73.1	75	57.1	36	59.4
46	New Delhi	55	58.3	55.6	75	58.9	58.6	58.6
47	Dalian	85	62.5	62	66.7	75	21	58.4
48	Manila	60	58.3	63.2	66.7	64.3	46.1	58
49	Bangkok	50	62.5	64.4	100	69.6	36.3	57.8
50	Bogota	35	62.5	75.2	66.7	64.3	50.6	57.3
51	Istanbul	55	50	68.8	58.3	67.9	47.5	57.1
52	Mumbai	60	54.2	56.3	66.7	51.8	52.1	55.7
53	Casablanca	65	45.8	60.9	58.3	60.7	43.8	54.9
54	Caracas	30	41.7	76.6	75	60.7	52.1	54
55	Cairo	55	45.8	54.9	58.3	53.6	48.2	51.9
56	Jakarta	50	45.8	59.3	66.7	57.1	42.3	51.5
57	Hanoi	55	54.2	53.7	58.3	51.8	38.4	50.2
58	Tashkent	50	58.3	55.3	75	51.8	26.8	48.6
59	Damascus	55	50	54.2	41.7	55.4	36.5	48.5
60	Ho Chi Minh City	55	50	49.5	66.7	48.2	35.1	48.1
61	Tehran	50	62.5	35.9	50	33.9	53.6	47.7
62	Nairobi	40	45.8	69.9	66.7	42.9	33.9	47.4
63	Lusaka	60	33.3	59.7	41.7	55.4	23.2	44.7
64	Phnom Penh	60	37.5	49.3	58.3	53.6	24.1	44.6
65	Karachi	20	45.8	38.7	66.7	51.8	48.5	42.8
66	Dakar	50	41.7	59.7	50	37.5	22.6	41.9
67	Abidjan	25	45.8	54.2	50	53.6	30.1	41
68	Dhaka	50	29.2	43.3	41.7	26.8	35.7	37.9
69	Lagos	25	33.3	52.3	33.3	48.2	22.3	34.8
70	Harare	30	20.8	53	66.7	35.7	17.3	33.4

Source: [19].

The GAIA plane obtained using this parametrisation is shown in Figure 10. We can observe that all the five initial criteria of this ranking seem to be correlated whereas the newly-added category discriminates the cities in a different way.

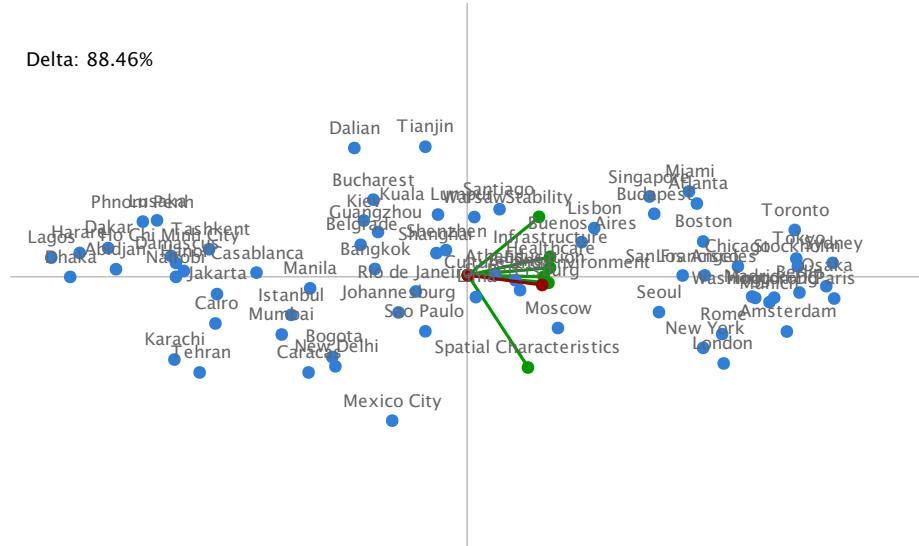


Figure 10: GAIA plane for the best cities ranking

Two PROMETHEE-based tables are given in Figures 11 and 12, respectively for the evaluations and the unicriterion netflows, alongside the two best tables found with our genetic algorithm.

We can see in Figure 11a a representation of the evaluation table using the Netflow-Angle combination as described in Section 2.2. This illustration is similar to the one obtained for the best cities subset (see Figure 4). Once again, since the starting point for the scanning process has been arbitrarily set as the decision axis, criterion 6 (spatial characteristics) ends up in the middle of the table at the fourth position. The computed ∇ -indicator for this representation is 13149 which is unsurprisingly lower than the one for the best table (13535, see Table 11b) due to the incurred penalty of the sixth criterion's position. However, this representation still holds good ordinal properties: if we evaluate the ratio R of this table, as defined in Section 4.1.1, the value found is 96.35%. That means that ∇ -indicator is at 96.35% of the possible range for this evaluation table.

The Figure 12a illustrates the Angle-Angle combination for the unicriterion netflow table of this case. The same observations as for Figure 6a applies: by ordering both alternatives and criteria based on their angle relatively to the decision axis, the lowest unicriterion netflow values will lie in the center of the table while the highest will be located at the top and bottom. That is why the ∇ -indicator (7722) is low compared to the best found (13289). As expected, the ratio R is close to 50% (44.83%).

	Crit2	Crit5	Crit1	Crit6	Crit3	Crit4	NetFlows		Crit4	Crit3	Crit5	Crit2	Crit1	Crit6	Netflows
A4	100	96,4	85	63,7	97,2	100	0,804167	A1	100	85,9	96,4	87,5	95	75	0,716848
A3	100	96,4	90	64	93,5	100	0,795109	A2	91,7	97,2	96,4	100	80	71,3	0,744746
A5	100	100	90	55,7	94,4	100	0,777899	A3	100	93,5	96,4	100	90	64	0,795109
A7	100	96,4	85	61,7	97,2	91,7	0,759239	A4	100	97,2	96,4	100	85	63,7	0,804167
A2	100	96,4	80	71,3	97,2	91,7	0,744746	A5	100	94,4	100	100	90	55,7	0,777899
A6	95,8	96,4	95	58,9	91,2	100	0,72029	A6	100	91,2	96,4	95,8	95	58,9	0,72029
A1	87,5	96,4	95	75	85,9	100	0,716848	A7	91,7	97,2	96,4	100	85	61,7	0,759239
A9	100	89,3	85	62,5	97,2	91,7	0,709964	A8	100	97,2	89,3	100	100	50	0,67337
A10	100	92,9	90	53,3	94,4	100	0,696739	A9	91,7	97,2	89,3	100	85	62,5	0,709964
A8	100	89,3	100	50	97,2	100	0,67337	A10	100	94,4	92,9	100	90	53,3	0,696739
A13	87,5	92,9	85	61,3	94,4	100	0,635688	A11	100	91,7	92,9	87,5	80	67,3	0,583152
A14	91,7	96,4	80	55,1	94,4	100	0,60163	A12	100	97,2	89,3	87,5	70	72,6	0,59058
A12	87,5	89,3	70	72,6	97,2	100	0,59058	A13	100	94,4	92,9	87,5	85	61,3	0,635688
A11	87,5	92,9	80	67,3	91,7	100	0,583152	A14	100	94,4	96,4	91,7	80	55,1	0,60163
A15	91,7	92,9	85	52,7	91,7	100	0,562138	A15	100	91,7	92,9	91,7	85	52,7	0,562138
A16	91,7	89,3	70	65,2	91,7	100	0,522464	A16	100	91,7	89,3	91,7	70	65,2	0,522464
A18	91,7	85,7	85	53	94,4	83,3	0,512862	A17	100	94,4	89,3	91,7	80	50,3	0,465399
A17	91,7	89,3	80	50,3	94,4	100	0,465399	A18	83,3	94,4	85,7	91,7	85	53	0,512862
A20	83,3	89,3	80	58,8	85,6	100	0,418478	A19	100	91,7	96,4	91,7	80	46,7	0,415036
A19	91,7	96,4	80	46,7	91,7	100	0,415036	A20	100	85,6	89,3	83,3	80	58,8	0,418478
A21	91,7	92,9	85	42,9	91,7	100	0,391848	A21	100	91,7	92,9	91,7	85	42,9	0,391848
A22	87,5	100	95	46,7	76,6	83,3	0,374819	A22	83,3	76,6	100	87,5	95	46,7	0,374819
A23	91,7	92,9	85	39,3	91,7	100	0,359239	A23	100	91,7	92,9	91,7	85	39,3	0,359239
A24	91,7	83,9	85	43	90	100	0,311957	A24	100	90	83,9	91,7	85	43	0,311957
A27	79,2	83,9	65	54,2	81,5	91,7	0,21721	A25	91,7	95,1	80,4	87,5	80	41,7	0,208333
A25	87,5	80,4	80	41,7	95,1	91,7	0,208333	A26	100	85,9	85,7	87,5	70	42,3	0,126812
A30	66,7	85,7	80	51,5	72,2	83,3	0,132609	A27	91,7	81,5	83,9	79,2	65	54,2	0,21721
A26	87,5	85,7	70	42,3	85,9	100	0,126812	A28	83,3	81,5	80,4	87,5	65	48,2	0,102355
A28	87,5	80,4	65	48,2	81,5	83,3	0,102355	A29	75	83,1	75	83,3	75	47,3	0,074094
A29	83,3	75	75	47,3	83,1	75	0,074094	A30	83,3	72,2	85,7	66,7	80	51,5	0,132609
A34	62,5	82,1	85	48,5	63,7	66,7	0,036957	A31	83,3	89,1	85,7	70,8	75	35,1	-0,02083
A36	70,8	66,1	60	52,4	80,3	66,7	-0,005958	A32	75	80,3	82,1	70,8	80	39	-0,01866
A35	66,7	75	60	47,3	81,7	91,7	-0,00906	A33	75	75	75	62,5	80	46,1	-0,04783
A32	70,8	82,1	80	39	80,3	75	-0,01866	A34	66,7	63,7	82,1	62,5	85	48,5	0,036957
A31	70,8	85,7	75	35,1	89,1	83,3	-0,02083	A35	91,7	81,7	75	66,7	60	47,3	-0,00906
A41	66,7	46,4	45	65,8	82,4	75	-0,04094	A36	66,7	80,3	66,1	70,8	60	52,4	-0,00598
A33	62,5	75	80	46,1	75	75	-0,04783	A37	91,7	67,8	76,8	62,5	80	36,6	-0,13696
A38	66,7	82,1	90	27,7	65,3	66,7	-0,12373	A42	83,3	77,5	71,4	66,7	55	44	-0,14855
A37	62,5	76,8	80	36,6	67,8	91,7	-0,13696	A38	66,7	65,3	82,1	66,7	90	27,7	-0,12373
A42	66,7	71,4	55	44	77,5	83,3	-0,14855	A39	66,7	61,1	76,8	62,5	80	42,9	-0,17283
A40	58,3	69,6	50	44,9	90,5	83,3	-0,17174	A40	83,3	90,5	69,6	58,3	50	44,9	-0,17174
A39	62,5	76,8	80	42,9	61,1	66,7	-0,17283	A41	75	82,4	46,4	66,7	45	65,8	-0,04094
A46	58,3	58,9	55	58,6	55,6	75	-0,19529	A43	66,7	74,3	66,1	66,7	80	34,7	-0,22572
A43	66,7	66,1	80	34,7	74,3	66,7	-0,22572	A44	83,3	73,4	50	75	70	33,3	-0,30707
A50	62,5	64,3	35	50,6	75,2	66,7	-0,22681	A45	75	73,1	57,1	75	60	36	-0,28696
A54	41,7	60,7	30	52,1	76,6	75	-0,28043	A49	100	64,4	69,6	62,5	50	36,3	-0,34167
A45	75	57,1	60	36	73,1	75	-0,28696	A47	66,7	62	75	62,5	85	21	-0,29275
A47	62,5	75	85	21	62	66,7	-0,29275	A46	75	55,6	58,9	58,3	55	58,6	-0,19529
A52	54,2	51,8	60	52,1	56,3	66,7	-0,29891	A48	66,7	63,2	64,3	58,3	60	46,1	-0,3
A48	58,3	64,3	60	46,1	63,2	66,7	-0,3	A50	66,7	75,2	64,3	62,5	35	50,6	-0,22681
A51	50	67,9	55	47,5	68,8	58,3	-0,30036	A51	58,3	68,8	67,9	50	55	47,5	-0,30036
A44	75	50	70	33,3	73,4	83,3	-0,30707	A52	66,7	56,3	51,8	54,2	60	52,1	-0,29891
A49	62,5	69,6	50	36,3	64,4	100	-0,34167	A53	58,3	60,9	60,7	45,8	65	43,8	-0,38732
A53	45,8	60,7	65	43,8	60,9	58,3	-0,38732	A54	75	76,6	60,7	41,7	30	52,1	-0,28043
A61	62,5	33,9	50	53,6	35,9	50	-0,43732	A55	58,3	54,9	53,6	45,8	55	48,2	-0,4471
A55	45,8	53,6	55	48,2	54,9	58,3	-0,4471	A56	66,7	59,3	57,1	45,8	50	42,3	-0,53188
A56	45,8	57,1	50	42,3	59,3	66,7	-0,53188	A57	58,3	53,7	51,8	54,2	55	38,4	-0,58949
A65	45,8	51,8	20	48,5	38,7	66,7	-0,57391	A58	75	55,3	51,8	58,3	50	26,8	-0,64239
A57	54,2	51,8	55	38,4	53,7	58,3	-0,58949	A59	41,7	54,2	55,4	50	55	36,5	-0,59583
A59	50	55,4	55	36,5	54,2	41,7	-0,59583	A60	66,7	49,5	48,2	50	55	35,1	-0,66051
A58	58,3	51,8	50	26,8	55,3	75	-0,64239	A65	66,7	38,7	51,8	45,8	20	48,5	-0,57391
A62	45,8	42,9	40	33,9	69,9	66,7	-0,65471	A62	66,7	69,9	42,9	45,8	40	33,9	-0,65471
A60	50	48,2	55	35,1	49,5	66,7	-0,66051	A67	50	54,2	53,6	45,8	25	30,1	-0,77464
A63	33,3	55,4	60	23,2	59,7	41,7	-0,66395	A64	58,3	49,3	53,6	37,5	60	24,1	-0,71377
A64	37,5	53,6	60	24,1	49,3	58,3	-0,71377	A63	41,7	59,7	55,4	33,3	60	23,2	-0,66395
A67	45,8	53,6	25	30,1	54,2	50	-0,77464	A66	50	59,7	37,5	41,7	50	22,6	-0,78641
A66	41,7	37,5	50	22,6	59,7	50	-0,78641	A61	50	35,9	33,9	62,5	50	53,6	-0,43732
A68	29,2	26,8	50	35,7	43,3	41,7	-0,8192	A68	41,7	43,3	26,8	29,2	50	35,7	-0,8192
A70	20,8	35,7	30	17,3	53	66,7	-0,89496	A69	33,3	52,3	48,2	33,3	25	22,3	-0,91105
	33,3	48,2	25	22,3	52,3	33,3	-0,91105	A70	66,7	53	35,7	20,8	30	17,3	-0,89946

 (a) Netflow-Angle ($\nabla = 13149$)

 (b) Best found table ($\nabla = 13535$)

Figure 11: Best cities ranking - Evaluations

4.3 Environmental Performance Index (G20)

For our second case study, we use the Environmental Performance Index (EPI), a joint project between the Yale Center for Environmental Law & Policy (YCELP) and the Center

	Crit2	Crit5	Crit1	Crit6	Crit3	Crit4	NetFlows		Crit5	Crit2	Crit3	Crit6	Crit4	Crit1	Netflows
A6	0.768116	0.84058	0.942029	0.681159	0.42029	0.710145	0.72029	A1	0.84058	0.898551	0.927536	0.797103	0.710145	0.623188	0.716848
A15	0.623188	0.637681	0.623188	0.42029	0.521739	0.710145	0.562138	A3	0.985507	0.898551	0.724638	0.594203	0.710145	0.84058	0.795109
A5	0.898551	0.985507	0.84058	0.594203	0.724638	0.710145	0.777899	A5	0.84058	0.898551	0.927536	0.942029	0.304348	0.275362	0.777899
A10	0.898551	0.637681	0.84058	0.478261	0.724638	0.710145	0.696739	A7	0.84058	0.376812	0.289855	1	0.710145	0.942029	0.759239
A8	0.898551	0.463768		1	0.217391	0.927536	0.710145	A2	0.84058	0.898551	0.623188	0.826087	0.710145	0.84058	0.744746
A19	0.623188	0.84058	0.275362	-0.02899	0.521739	0.710145	0.415036	A6	0.84058	0.768116	0.42029	0.681159	0.710145	0.942029	0.72029
A26	0.376812	0.318841	-0.02899	-0.31884	0.289855	0.710145	0.126812	A4	0.84058	0.898551	0.927536	0.73913	0.304348	0.623188	0.804167
A21	0.623188	0.637681	0.623188	-0.26087	0.521739	0.710145	0.391848	A9	0.637681	0.898551	0.724638	0.478261	0.710145	0.84058	0.709964
A24	0.623188	0.231884	0.623188	-0.21739	0.362319	0.710145	0.311957	A10	0.463768	0.898551	0.927536	0.217391	0.710145	1	0.696739
A25	0.376812	0.086957	0.275362	-0.36232	0.826087	0.304348	0.208333	A8	0.463768	0.898551	0.927536	0.768116	0.304348	0.623188	0.673337
A23	0.623188	0.637681	0.623188	-0.3913	0.521739	0.710145	0.359239	A13	0.463768	0.376812	0.927536	0.971014	0.710145	-0.02899	0.356568
A22	0.376812	0.885505	0.942029	-0.02899	0.086957	0.374819		A14	0.637681	0.376812	0.521739	0.913043	0.710145	0.275362	0.60163
A31	0.072464	0.318841	0.057971	-0.637681	0.333333	0.086957	0.02089	A12	0.84058	0.623188	0.724638	0.565217	0.710145	0.275362	0.59058
A32	0.072464	0.15942	0.275362	-0.42029	0.057971	-0.14493	-0.01866	A11	0.637681	0.376812	0.724638	0.710145	0.623188	0.583152	
A33	-0.05797	0.15942	0.84058	-0.7971	-0.33333	-0.46377	-0.12373	A15	0.637681	0.623188	0.521739	0.42029	0.710145	0.623188	0.562138
A37	-0.26087	0.028986	0.275362	-0.47826	-0.30435	0.304348	-0.13666	A16	0.463768	0.623188	0.521739	0.855072	0.710145	-0.02899	0.522464
A34	-0.26087	0.15942	0.623188	0.173913	-0.3913	-0.46377	0.036957	A18	0.463768	0.623188	0.724638	0.246377	0.710145	0.275362	0.512862
A47	-0.26087	-0.05797	0.623188	-0.97101	-0.44928	-0.46377	-0.29275	A17	0.318841	0.623188	0.724638	0.449275	0.086957	0.623188	0.465339
A43	-0.05797	-0.26087	0.275362	-0.68116	-0.13043	-0.46377	-0.25275	A20	0.84058	0.623188	0.521739	-0.02899	0.710145	0.275362	0.184748
A33	-0.26087	-0.05797	0.275362	-0.08698	-0.10145	-0.14493	-0.04783	A23	0.637681	0.623188	0.521739	-0.3919	0.710145	0.623188	0.359239
A44	0.144928	-0.76812	-0.02899	-0.73913	-0.15946	0.086957	-0.30707	A21	0.637681	0.623188	0.521739	-0.26087	0.710145	0.623188	0.391848
A39	-0.26087	0.028986	0.275362	-0.26087	-0.47826	-0.46377	-0.17283	A22	0.985507	0.376812	-0.02899	0.086957	0.0942029	0.374819	
A45	0.144928	-0.46377	-0.27536	-0.56522	-0.18841	-0.14493	-0.28696	A19	0.463768	0.231884	0.246377	0.652174	0.710145	0.275362	0.150306
A63	-0.92754	-0.52174	-0.27536	-0.88406	-0.55072	-0.94203	-0.66395	A24	0.231884	0.623188	0.362319	-0.21739	0.710145	0.623188	0.311957
A64	-0.88406	-0.5942	-0.27536	-0.85507	-0.91304	-0.73913	-0.71377	A28	0.318841	0.376812	0.289855	-0.31884	0.710145	-0.02899	0.102355
A58	-0.43478	-0.69568	-0.68116	-0.82606	-0.68116	-0.14493	-0.64238	A25	0.231884	0.188406	0.15942	0.536232	0.304348	-0.13043	0.208333
A66	-0.84058	-0.91304	-0.68116	-0.91304	-0.55072	-0.85507	-0.78644	A26	0.086957	0.376812	0.826087	-0.36232	0.304348	0.275362	0.126812
A49	-0.26087	-0.17391	-0.68116	-0.53623	-0.36232	0.710145	-0.34167	A35	0.318841	0.072464	0.333333	-0.63768	0.086957	0.057971	-0.00906
A60	-0.5942	-0.81159	-0.47826	-0.63768	-0.88406	-0.46377	-0.66051	A27	0.318841	-0.05797	-0.21739	0.304348	0.086957	0.275362	0.217271
A70	-1	-0.94203	-0.889855	-1	-0.82609	-0.46377	-0.89846	A29	0.086957	0.376812	0.115942	0.115942	0.086957	-0.13043	0.074094
A69	-0.92754	-0.81159	-0.95652	-0.94203	-0.85507	-1	-0.91105	A34	0.15942	0.072464	0.057971	-0.42029	-0.14493	0.275362	0.036957
A59	-0.5942	-0.52174	-0.47826	-0.50725	-0.75362	-0.94203	-0.595583	A33	-0.05797	-0.05797	0.15942	0.028986	0.304348	-0.27536	-0.04783
A67	-0.72464	-0.5942	-0.95652	-0.76812	-0.75362	-0.85507	-0.77464	A30	-0.05797	0.231884	0.217391	0.028986	-0.14493	0.057971	0.132609
A57	-0.52174	-0.69568	-0.47826	-0.44928	-0.7971	-0.73913	-0.58949	A32	-0.26087	0.072464	0.057971	0.391304	-0.46377	-0.27536	-0.01866
A53	-0.72464	-0.37681	-0.13043	-0.18841	-0.50725	-0.73913	-0.38732	A31	0.15942	-0.26087	-0.39131	0.173913	-0.46377	0.623188	-0.02083
A68	-0.97101	-1	-0.68116	-0.5942	-0.94203	-0.94203	-0.8192	A40	-0.13043	-0.05797	0.014493	-0.15942	0.086957	-0.47826	-0.17174
A62	-0.72464	-0.88406	-0.82606	-0.71014	-0.24638	-0.46377	-0.65471	A36	0.85507	-0.05797	0.188406	0.884058	-0.14493	-0.7971	-0.00598
A56	-0.72464	-0.46377	-0.68116	-0.31884	-0.5942	-0.46377	-0.53188	A38	0.15942	-0.05797	-0.33333	-0.7971	-0.46377	0.84058	-0.12373
A48	-0.43478	-0.318841	-0.27536	-0.08696	-0.42029	-0.46377	-0.3	A37	-0.05797	-0.26087	-0.10145	-0.08696	-0.14493	0.275362	-0.13696
A55	-0.72464	-0.5942	-0.47826	0.115942	-0.71014	-0.73913	-0.4471	A42	0.028986	-0.26087	-0.47826	-0.26087	-0.46377	0.275362	-0.148855
A51	-0.5942	-0.21739	-0.47826	0.072464	-0.27536	-0.73913	-0.30036	A41	-0.17391	-0.43478	0.391304	-0.13043	0.086957	-0.68116	-0.04094
A65	-0.72464	-0.69568	-1	0.173913	-0.97101	-0.46377	-0.57399	A39	0.028986	-0.26087	-0.30435	-0.47826	0.304348	0.275362	-0.17283
A42	-0.05797	-0.13043	-0.47826	-0.15942	0.014493	0.086957	-0.14855	A53	-0.17391	-0.26087	-0.36232	-0.53623	0.710145	-0.68116	-0.38732
A52	-0.52174	-0.69568	-0.27536	0.347826	-0.62319	-0.46377	-0.29891	A52	-0.76812	0.144928	-0.15942	-0.73913	0.086957	-0.02899	-0.29891
A61	-0.26087	-0.97101	-0.68116	0.507246	-1	-0.85507	-0.43732	A48	-0.05797	-0.26087	-0.44928	-0.97101	-0.46377	0.623188	-0.3
A40	-0.43478	-0.17391	-0.68116	-0.13043	0.391304	0.086957	-0.17174	A44	-0.26087	-0.10343	-0.05797	0.15942	-0.46377	0.275362	-0.30707
A50	-0.26087	-0.318841	-0.85507	0.275362	-0.724638	-0.02746	-0.22688	A45	-0.318841	-0.26087	0.072464	0.536232	-0.46377	-0.85507	-0.28696
A54	-0.84058	-0.318841	-0.89855	0.347826	-0.376812	-0.02899	-0.14493	A47	-0.46377	0.144928	-0.18841	-0.56522	-0.14493	-0.27536	-0.29275
A46	-0.43478	-0.42029	-0.47826	0.623188	-0.652174	-0.652174	-0.19529	A43	-0.42029	-0.43478	-0.47826	-0.027409	-0.08696	-0.46377	-0.275362
A36	0.072464	-0.26087	-0.27536	0.391304	0.521739	0.710145	0.583152	A50	-0.318841	-0.43478	-0.47826	-0.527362	-0.027409	-0.46377	-0.22681
A41	-0.05797	-0.85507	-0.7971	0.884058	0.388406	0.14493	-0.04094	A51	-0.21739	-0.5942	-0.27536	0.072464	-0.73913	-0.47826	-0.203036
A35	-0.05797	-0.05797	-0.27536	0.028986	0.15942	0.304348	-0.09096	A46	-0.37681	-0.84058	-0.02899	0.347826	-0.14493	-0.89855	-0.19529
A27	0.188406	0.231884	-0.13043	0.536232	0.115942	0.304348	0.21721	A54	-0.37681	-0.72464	-0.18841	-0.73913	-0.13043	-0.28043	
A12	0.376812	0.463768	-0.02899	0.971014	0.927536	0.710145	0.590598	A49	-0.69565	-0.52174	-0.62319	0.347826	-0.46377	-0.27536	-0.34167
A16	0.623188	0.463768	-0.2899	0.855074	0.521739	0.710145	0.522468	A61	-0.69565	-0.43478	-0.68116	-0.82609	-0.14493	-0.68116	-0.43732
A28	0.376812	0.086957	-0.13043	0.115942	0.115942	0.086957	0.20355	A57	-0.46377	-0.72464	-0.5492	-0.31884	-0.46377	-0.68116	-0.58949
A11	0.376812	0.637681													

G20, excluding the European Union member. We have thus an evaluation table (see Table 8) composed of 19 countries evaluated on 9 criteria: air quality, health impacts, water sanitation, agriculture, biodiversity habitat, climate energy, fisheries, forests, water resources.

Table 8 Environmental Performance Index (G20) - Evaluation table

Perm	Country	1 Air Quality	2 Health Impacts	3 Water Sanitation	4 Agriculture	5 Habitat	6 Energy	7 Climate	8 Fisheries	9 Forests	Water Resources	EPI Ranking
1	Australia	98.33	100	100	66.46	83.08	47.67	19.37	100	92.33	3	
2	Germany	78.5	100	100	65.31	100	62.77	13.4	31.35	95.18	6	
3	United Kingdom	95.82	100	100	66.03	70.11	54.24	0	43.06	97.93	12	
4	Italy	80.85	100	63.51	58.87	79.77	63.41	24.93	55.41	91.44	22	
5	Canada	97.85	100	95.9	62.52	58.4	59.85	21.54	16.64	80.42	24	
6	Japan	84.79	99.2	100	46.48	73.53	43.54	25.34	55.41	71.26	26	
7	France	89.44	100	100	65.55	54.45	49.83	0	37.94	83.8	27	
8	USA	96.41	95.33	86.48	61.53	63.35	56.45	3.34	14.35	63.66	33	
9	Saudi Arabia	84.45	94.68	83.48	92	93.7	46.63	6.43	0	28.54	35	
10	South Korea	62.24	96.93	85.92	46.98	50.4	41.55	22.24	33.76	83.68	43	
11	Mexico	87.09	76.67	46.2	55.21	62.32	51.35	25.34	19.87	37.45	65	
12	Turkey	84.07	66.06	71.43	56.67	32.62	46.52	21.9	52.35	48.93	66	
13	South Africa	94.4	47.51	36.08	79.2	63.96	49.87	2.52	100	27.86	72	
14	Russia	94.36	83.12	45.17	16.93	53.39	61.02	12.73	35.07	21.5	73	
15	Brazil	97.64	68.59	50.44	74.51	66.74	53.82	24.68	10.81	10.87	77	
16	Argentina	99.64	85.07	75.7	96	44.88	16.79	15.68	0	11.75	93	
17	Indonesia	75.31	67.55	24.29	51.85	78.08	45.25	25.8	7.75	0.02	112	
18	China	18.81	76.23	33.15	33.85	66.63	65.16	14.68	25.34	18.18	118	
19	India	23.24	50.04	26.28	58.4	39.18	35.24	22.64	35.07	10.49	155	

Source: [24].

The GAIA plane is represented in Figure 13. We can see that the criteria are separated in three groups. Two of them are clearly opposed: fisheries vs. air quality and agriculture. For instance, Indonesia, China and India have some of the best evaluations regarding fisheries while their scores on air quality and agriculture are amongst the worst of our set. The opposite features can be observed for countries such as UK, Canada, France and the USA.



Figure 13: GAIA plane for the Environmental Performance Index (G20)

For this case, we will analyse the two other combinations: the Netflow-Weight combination for the evaluation table and the Proximity-Proximity combination for the unicriterion netflow table, alongside the two best tables found with our genetic algorithm, as shown respectively in Figures 14 and 15.

The first combination, in Figure 14a does not display any particular structure in the organisation of the evaluations. Indeed, in this case, we have enriched the table by displaying the criteria with the highest weight first. The ∇ -indicator reaches a high value (1556) compared to the best found (1619, Figure 14b). However this is a coincidence given that the weights are not linked to the evaluations.

In the second combination (Figure 15a), the alternatives and criteria are ordered depending on their proximity to the decision axis. We can therefore observe that the best countries are located on the top of the table which contributes to the high value of the obtained ∇ -indicator (1417).

The best found table (Figure 15b) with a ∇ -indicator of 1465 does not seem to hold better ordinal properties. This is explained by the heterogeneous nature of the profiles, as observed in the GAIA plane. When comparing this table with the best one for the evaluations, this heterogeneity is emphasized. A particular example of that statement can be observed for criterion 7 (fisheries) where the evaluations range only between 0 and 25.8 whereas the corresponding unicriterion netflows will range between -0.944 and 1.

	Crit1	Crit2	Crit3	Crit5	Crit6	Crit9	Crit7	Crit8	Crit4	NetFlows
A1	98,33	100	100	83,08	47,67	92,33	19,37	100	66,46	0,625926
A3	95,82	100	100	70,11	54,24	97,93	0	43,06	66,03	0,518519
A2	78,5	100	100	100	62,77	95,18	13,4	31,35	65,31	0,474074
A5	97,85	100	95,9	58,4	59,85	80,42	21,54	16,64	62,52	0,37963
A4	80,85	100	63,51	79,77	63,41	91,44	24,93	55,41	58,87	0,353703
A7	89,44	100	100	54,45	49,83	83,8	0	37,94	65,55	0,246297
A6	84,79	99,2	100	73,53	43,54	71,26	25,34	55,41	46,48	0,225
A8	96,41	95,33	86,48	63,35	56,45	63,66	3,34	14,35	61,53	0,161111
A9	84,45	94,68	83,48	93,7	46,63	28,54	6,43	0	92	-0,02963
A15	97,64	68,59	50,44	66,74	53,82	10,87	24,68	10,81	74,51	-0,06759
A11	87,09	76,67	46,2	62,32	51,35	37,45	25,34	19,87	55,21	-0,14352
A14	94,36	83,12	45,17	53,39	61,02	21,5	12,73	35,07	16,93	-0,17315
A10	62,24	96,93	85,92	50,4	41,55	83,68	22,24	33,76	46,98	-0,18056
A16	99,64	85,07	75,7	44,88	16,79	11,75	15,68	0	96	-0,18518
A13	94,4	47,51	36,08	63,96	49,87	27,86	2,52	100	79,2	-0,23611
A18	18,81	76,23	33,15	66,63	65,16	18,18	14,68	25,34	33,85	-0,34537
A12	84,07	66,06	71,43	32,62	46,52	48,93	21,9	52,35	56,67	-0,3537
A17	75,31	67,55	24,29	78,08	45,25	0,02	25,8	7,75	51,85	-0,51667
A19	23,24	50,04	26,28	39,18	35,24	10,49	22,64	35,07	58,4	-0,75278

(a) Netflow-Weight ($\nabla = 1556$)

	Crit2	Crit1	Crit5	Crit3	Crit9	Crit4	Crit6	Crit8	Crit7	Netflows
A1	100	98,33	83,08	100	92,33	66,46	47,67	100	19,37	0,625926
A3	100	95,82	70,11	100	97,93	66,03	54,24	43,06	0	0,518519
A2	100	78,5	100	100	95,18	65,31	62,77	31,35	13,4	0,474074
A4	100	80,85	79,77	63,51	91,44	58,87	63,41	55,41	24,93	0,353703
A5	100	97,85	58,4	95,9	80,42	62,52	59,85	16,64	21,54	0,37963
A7	100	89,44	54,45	100	83,8	65,55	49,83	37,94	0	0,246297
A6	99,2	84,79	73,53	100	71,26	46,48	43,54	55,41	25,34	0,225
A8	95,33	96,41	63,35	86,48	63,66	61,53	56,45	14,35	3,34	0,161111
A9	94,68	84,45	93,7	83,48	28,54	92	46,63	0	6,43	-0,02963
A15	68,59	97,64	66,74	50,44	10,87	74,51	53,82	10,81	24,68	-0,06759
A11	76,67	87,09	62,32	46,2	37,45	55,21	51,35	19,87	25,34	-0,14352
A10	96,93	62,24	50,4	85,92	83,68	46,98	41,55	33,76	22,24	-0,18056
A16	85,07	99,64	44,88	75,7	11,75	96	16,79	0	15,68	-0,18518
A13	47,51	94,4	63,96	36,08	27,86	79,2	49,87	100	2,52	-0,23611
A12	66,06	84,07	32,62	71,43	48,93	56,67	46,52	52,35	21,9	-0,3537
A14	83,12	94,36	53,39	45,17	21,5	16,93	61,02	35,07	12,73	-0,17315
A18	76,23	18,81	66,63	33,15	18,18	33,85	65,16	25,34	14,68	-0,34537
A17	67,55	75,31	78,08	24,29	0,02	51,85	45,25	7,75	25,8	-0,51667
A19	50,04	23,24	39,18	26,28	10,49	58,4	35,24	35,07	22,64	-0,75278

(b) Best found table ($\nabla = 1619$)**Figure 14:** Environmental Performance Index - Evaluations

5 Conclusion

Adequately reordered tables can give us interesting insights into the problems we tackle. This contribution shows how to achieve such results based on the PROMETHEE methodology. Several possibilities of tables have been proposed, taking into account information such as the netflows, the weight or the position of the alternatives and criteria on the GAIA plane.

Furthermore, an indicator has been proposed to evaluate the ordinal properties of these tables and compare them. It also serves as a fitness function for a genetic algorithm that

	Crit3	Crit2	Crit9	Crit5	Crit6	Crit8	Crit1	Crit4	Crit7	NetFlows
A3	0,777778	0,722222	1	0,333333	0,333333	0,444444	0,444444	0,444444	-0,944444	0,518519
A1	0,777778	0,722222	0,777778	0,777778	-0,22222	0,944444	0,888889	0,555556	0	0,625926
A2	0,777778	0,722222	0,888889	1	0,777778	-0,11111	-0,555556	0,222222	-0,333333	0,474074
A7	0,777778	0,722222	0,555556	-0,44444	-0,11111	0,333333	0,111111	0,333333	-0,944444	0,246297
A5	0,444444	0,722222	0,333333	-0,33333	0,555556	-0,444444	0,777778	0,111111	0,111111	0,37963
A8	0,333333	0,111111	0,111111	-0,11111	0,444444	-0,555556	0,555556	0	-0,666667	0,161111
A4	-0,22222	0,722222	0,666667	0,666667	0,888889	0,722222	-0,44444	-0,11111	0,666667	0,353703
A6	0,777778	0,333333	0,222222	0,444444	-0,666667	0,722222	-0,11111	-0,777778	0,833333	0,225
A9	0,111111	0	-0,22222	0,888889	-0,33333	-0,944444	-0,22222	0,888889	-0,555556	-0,02963
A10	0,222222	0,222222	0,444444	-0,666667	-0,777778	0	-0,777778	-0,666667	0,333333	-0,18056
A16	0	-0,11111	-0,666667	-0,777778	-1	-0,944444	1	1	-0,11111	-0,18518
A13	-0,666667	-1	-0,333333	0	0,944444	0,333333	0,777778	-0,777778	-0,23611	
A18	-0,777778	-0,444444	-0,555556	0,111111	1	-0,22222	-1	-0,888889	-0,22222	-0,34537
A15	-0,33333	-0,555556	-0,777778	0,222222	0,222222	-0,666667	0,666667	0,666667	0,555556	-0,06759
A14	-0,555556	-0,22222	-0,44444	-0,555556	0,666667	0,166667	0,222222	-1	-0,44444	-0,17315
A11	-0,44444	-0,33333	-0,11111	-0,22222	0,111111	-0,33333	0	-0,44444	0,833333	-0,14352
A12	-0,11111	-0,777778	0	-1	-0,444444	0,555556	-0,333333	-0,333333	0,222222	-0,3537
A17	-1	-0,666667	-1	0,555556	-0,555556	-0,777778	-0,666667	-0,555556	1	-0,51667
A19	-0,88889	-0,88889	-0,88889	-0,88889	-0,88889	0,166667	-0,88889	-0,22222	0,444444	-0,75278

(a) Proximity-Proximity ($\nabla = 1417$)

	Crit3	Crit2	Crit8	Crit9	Crit4	Crit1	Crit6	Crit7	Crit5	Netflows
A1	0,777778	0,722222	0,944444	0,777778	0,555556	0,888889	-0,22222	0	0,777778	0,625926
A3	0,777778	0,722222	0,444444	1	0,444444	0,444444	0,333333	-0,944444	0,333333	0,518519
A2	0,777778	0,722222	-0,11111	0,888889	0,222222	-0,555556	0,777778	-0,33333	1	0,474074
A4	-0,22222	0,722222	0,722222	0,666667	-0,11111	-0,44444	0,888889	0,666667	0,666667	0,353703
A5	0,444444	0,722222	-0,44444	0,333333	0,111111	0,777778	0,555556	0,111111	-0,33333	0,37963
A7	0,777778	0,722222	0,333333	0,555556	0,333333	0,111111	-0,11111	-0,944444	-0,44444	0,246297
A6	0,777778	0,333333	0,722222	0,222222	-0,777778	-0,11111	-0,666667	0,833333	0,444444	0,225
A8	0,333333	0,111111	-0,555556	0,111111	0	0,555556	0,444444	-0,666667	-0,11111	0,161111
A9	0,111111	0	-0,94444	-0,22222	0,888889	-0,22222	-0,33333	-0,555556	0,888889	-0,02963
A15	-0,33333	-0,555556	-0,666667	-0,777778	0,666667	0,666667	0,222222	0,555556	0,222222	-0,06759
A11	-0,44444	-0,33333	-0,33333	-0,11111	-0,44444	0	0,111111	0,833333	-0,22222	-0,14352
A13	-0,666667	-1	0,944444	-0,33333	0,777778	0,333333	0	-0,777778	0	-0,23611
A10	0,222222	0,222222	0	0,444444	-0,666667	-0,777778	-0,777778	0,333333	-0,666667	-0,18056
A16	0	-0,11111	-0,94444	-0,666667	1	1	-1	-0,11111	-0,777778	-0,18518
A12	-0,11111	-0,777778	0,555556	0	-0,33333	-0,33333	-0,44444	0,222222	-1	-0,3537
A14	-0,555556	-0,22222	0,166667	-0,44444	-1	0,222222	0,666667	-0,44444	-0,555556	-0,17315
A18	-0,777778	-0,44444	-0,22222	-0,555556	-0,88889	-1	0,222222	0,111111	-0,34537	
A17	-1	-0,666667	-0,777778	-1	-0,555556	-0,666667	-0,555556	1	0,555556	-0,51667
A19	-0,88889	-0,88889	0,166667	-0,88889	-0,22222	-0,88889	-0,88889	0,444444	-0,88889	-0,75278

(b) Best found table ($\nabla = 1465$)**Figure 15:** Environmental Performance Index - Unicriterion netflows

will search for better representations. When comparing the obtained tables, we can note that the results with the ∇ -indicators and the ratio R seem consistent.

Indeed, as shown in Figure 16, the combination Netflow-Angle has a ratio R of approximately 95% in most cases. This is mostly due to the size of the used dataset. For the best cities ranking, the high number of alternatives will have a greater impact on the ∇ -indicator whereas in small sets, such as the EPI, the position of the criteria can have more influence, especially when they are conflicting with the decision axis. The same statement applies for the Netflow-Weight and Proximity-Proximity combinations.

	Best cities ranking subset				Best cities ranking				Environmental Performance Index			
	Evaluations		Unicriterion Flows		Evaluations		Unicriterion Flows		Evaluations		Unicriterion flows	
	Nabla	R	Nabla	R	Nabla	R	Nabla	R	Nabla	R	Nabla	R
GA Best	648	100,00%	603	100,00%	13535	100,00%	13289	100,00%	1619	100,00%	1465	100,00%
GA Worst	149	0,00%	205	0,00%	2962	0,00%	3199	0,00%	643	0,00%	838	0,00%
Netflow-Angle	625	95,39%	589	96,48%	13149	96,35%	13209	99,21%	1392	76,74%	1430	94,42%
Netflow-Weight	533	76,95%	597	98,49%	12959	94,55%	13186	98,98%	1556	93,55%	1439	95,85%
Angle-Angle	424	55,11%	388	45,98%	7662	44,45%	7722	44,83%	1176	54,61%	1214	59,97%
Proxi-Proxi	611	92,59%	567	90,95%	12695	92,06%	12475	91,93%	1503	88,11%	1417	92,34%

Figure 16: Comparison of the obtained results based on the ∇ -indicator and the ratio R

For the Angle-Angle combination, it is interesting to note the values are all close to 50%. This is in accordance with the way these tables are built. Scanning the GAIA plane and going from the best to the worst to the best alternatives ensures that only half of the table will respect the ordinal properties that we defined for the ∇ -indicator. All the tables used for these computations are available in the appendices (see Sections A, B, and C).

To the best of our knowledge, this is the first attempt at reordering tables using multicriteria information. This study has shown that multicriteria-enriched tables can still hold interesting ordinal properties. This could be easily generalised to other methodologies. Instead of netflows, other aggregators could be considered.

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Appendices

A Best cities ranking subset

	Crit4	Crit5	Crit3	Crit1	Crit6	Crit2	NetFlows		Crit6	Crit1	Crit3	Crit2	Crit5	Crit4	NetFlows
A2	100	96,4	91,2	95	58,9	95,8	0,704808	A2	58,9	95	91,2	95,8	96,4	100	0,704808
A1	100	96,4	85,9	95	75	87,5	0,694231	A1	75	95	85,9	87,5	96,4	100	0,694231
A3	100	90,7	91,7	80	67,3	75,9	0,694231	A3	87,3	80	91,7	87,5	92,9	100	0,694231
A4	100	89,3	91,7	70	65,2	81,9	0,541346	A4	65,2	70	91,7	91,7	89,3	100	0,541346
A5	100	92,9	91,7	85	42,0	91,7	0,446154	A5	42,9	85	91,7	91,7	92,9	100	0,446154
A6	100	85,7	85,9	70	42,3	87,5	0,030769	A6	42,3	70	85,9	87,5	85,7	100	0,030769
A7	83,3	85,7	89,1	75	35,1	70,8	-0,03846	A7	35,1	75	89,1	70,8	85,7	83,3	-0,03846
A8	66,7	66,1	80,3	60	52,4	70,8	-0,14615	A8	52,4	60	80,3	70,8	66,1	66,7	-0,14615
A9	75	46,4	82,4	45	65,8	66,7	-0,17885	A9	65,8	45	82,4	66,7	46,4	75	-0,17885
A10	75	58,9	55,6	55	58,6	58,3	-0,30865	A10	58,6	55	55,6	58,3	58,9	75	-0,30865
A11	58,3	67,9	68,8	55	47,5	50	-0,35481	A11	47,5	55	68,8	50	67,9	58,3	-0,35481
A13	33,9	35,9	50	53,6	62,5	-0,575	A13	53,6	50	35,9	62,5	33,9	50	-0,575	
A12	66,7	57,1	59,3	50	42,3	45,8	-0,65577	A12	42,3	50	59,3	45,8	57,1	66,7	-0,65577
A14	50	37,5	59,7	50	22,6	41,7	0,82692	A14	22,6	50	59,7	41,7	37,5	50	0,82692

(a) Netflow-angle

(b) Netflow-Weight

	Crit4	Crit5	Crit3	Crit1	Crit6	Crit2	NetFlows		Crit2	Crit4	Crit5	Crit3	Crit1	Crit6	NetFlows
A2	100	96,4	91,2	95	58,9	95,8	0,704808	A4	91,7	100	89,3	91,7	70	65,2	0,541346
A5	100	92,9	91,7	85	42,9	91,7	0,446154	A3	87,5	100	92,9	91,7	80	67,3	0,667308
A6	100	85,7	85,9	70	42,3	87,5	0,030769	A1	87,5	100	96,4	85,9	95	75	0,694231
A7	83,3	85,7	89,1	75	35,1	70,8	-0,03846	A2	95,8	100	96,4	91,2	95	59,3	0,667308
A14	37,5	59,7	50	32,6	42,3	65,8	-0,03846	A5	91,7	100	92,9	91,7	85	42,9	0,446154
A12	66,7	57,1	59,3	50	42,3	45,8	-0,65577	A6	87,5	100	85,7	85,9	70	42,3	0,030769
A11	58,3	67,9	68,8	55	47,5	50	-0,35481	A7	70,8	83,3	85,7	89,1	75	35,1	-0,03846
A8	66,7	66,1	80,3	60	52,4	70,8	-0,14615	A9	66,7	75	46,4	82,4	45	65,8	-0,17885
A13	50	33,9	35,9	50	53,6	62,5	-0,575	A10	58,3	75	58,9	55,6	55	58,6	-0,30865
A10	75	58,9	55,6	55	58,6	58,3	-0,30865	A13	62,5	50	33,9	35,9	50	53,6	-0,575
A9	75	46,4	82,4	45	65,8	66,7	-0,17885	A8	70,8	66,7	66,1	80,3	60	52,4	-0,14615
A1	100	96,4	85,9	95	75	87,5	0,694231	A11	50	58,3	67,9	68,8	55	47,5	-0,35481
A3	100	92,9	91,7	80	67,3	87,5	0,667308	A12	45,8	66,7	57,1	59,3	50	42,3	-0,65577
A4	100	89,3	91,7	70	65,2	91,7	0,541346	A14	41,7	50	37,5	59,7	50	22,6	0,82692

(c) Angle-Angle

(d) Proximity-Proximity

Figure 17: Visualisation - Best cities ranking subset - Evaluations

	Crit4	Crit5	Crit3	Crit1	Crit6	Crit2	NetFlows		Crit2	Crit4	Crit5	Crit3	Crit1	Crit6	NetFlows
A2	0,615385	0,923077	0,538462	0,923077	0,384615	1	0,704808	A2	0,384615	0,923077	0,538462	1	0,230797	0,153845	0,704808
A1	0,615385	0,923077	0,538462	0,923077	0,384615	1	0,694231	A1	0,694231	0,923077	0,538462	0,384615	0,230797	0,153845	0,694231
A3	0,615385	0,615385	0,846154	0,538462	0,846154	0,384615	0,667308	A3	0,846154	0,538462	0,846154	0,384615	0,153845	0,667308	0,694231
A4	0,615385	0,846154	0,846154	0,538462	0,153845	0,384615	0,541346	A4	0,538462	0,153845	0,846154	0,384615	0,230797	0,153845	0,694231
A5	0,615385	0,615385	0,846154	0,692308	0,384612	0,769231	0,446154	A5	0,384612	0,692308	0,846154	0,384615	0,153845	0,446154	0,667308
A6	0,615385	0,153845	0,846154	0,538462	0,153845	0,384615	0,030769	A6	0,846153	0,153845	0,384615	0,153845	0,230797	0,153845	0,694231
A7	0,615385	0,153845	0,846154	0,384615	0,153845	0,384615	0,-03846	A7	0,153845	0,384615	0,153845	0,384615	0,230797	0,153845	0,694231
A8	0,615385	0,230797	0,384615	0,30769	0,384615	0,384615	0,-03846	A8	0,-03846	0,384615	0,384615	0,384615	0,230797	0,153845	0,-03846
A10	0,615385	0,384615	0,384615	0,30769	0,384615	0,384615	0,-03846	A10	0,384615	0,384615	0,384615	0,384615	0,230797	0,153845	0,-03846
A11	0,69231	0,07692	0,384615	0,30769	0,384615	0,384615	0,-03846	A11	0,30769	0,07692	0,384615	0,384615	0,230797	0,153845	0,-03846
A13	0,92308	-1	-1	0,69231	0,076923	0,384615	-0,575	A13	0,076923	0,69231	-1	0,384615	0,230797	0,153845	-0,575
A12	-0,46154	-0,53846	-0,69231	-0,69231	-0,61538	-0,84615	-0,65577	A12	-0,846158	-0,69231	-0,69231	-0,84615	-0,46154	-0,65577	0,82692
A14	-0,92308	-0,84615	-0,53846	-0,69231	-1	-1	-0,82692	A14	-1	-0,69231	-0,53846	-1	-0,84615	-0,92308	0,82692

(a) Netflow-angle

(b) Netflow-Weight

	Crit4	Crit5	Crit3	Crit1	Crit6	Crit2	NetFlows		Crit2	Crit4	Crit5	Crit3	Crit1	Crit6	NetFlows
A2	0,615385	0,923077	0,538462	0,923077	0,384615	1	0,704808	A4	0,769231	0,615385	0,846154	0,846154	0,538462	0,541346	0,694231
A5	0,615385	0,615385	0,846154	0,538462	0,846154	0,384615	0,667308	A1	0,384615	0,615385	0,923077	0,153845	0,230797	0,153845	0,694231
A6	0,615385	0,153845	0,846154	0,538462	0,153845	0,384615	0,030769	A2	0,769231	0,615385	0,615385	0,384615	0,230797	0,541346	0,694231
A7	0,615385	0,153845	0,846154	0,384615	0,153845	0,384615	0,03846	A5	0,384615	0,615385	0,615385	0,384615	0,230797	0,541346	0,694231
A14	0,92308	-0,84615	-0,53846	-0,69231	-1	-1	-0,82692	A7	0,076923	0,153845	0,384615	0,384615	0,230797	0,153845	0,694231
A12	-0,46154	-0,53846	-0,69231	-0,69231	-0,61538	-0,84615	-0,65577	A9	-0,230797	-0,153845	-0,69231	-0,69231	-0,692308	-0,17885	
A11	-0,69231	-0,07692	-0,384615	-0,30769	-0,384615	-0,384615	-0,38462	A10	-0,384615	-0,384615	-0,384615	-0,384615	-0,30769	-0,30865	
A8	-0,46154	-0,230797	-0,384615	-0,30769	-0,230769	-0,384615	-0,14615	A13	-0,384615	-0,230797	-0,153845	-0,153845	-0,692308	-0,575	
A13	-0,92308	-1	-1	0,69231	0,076923	0,384615	-0,575	A12	-0,230797	-0,153845	-0,69231	-0,69231	-0,692308	-0,575	
A10	-0,153845	-0,384615	-0,69231	-0,69231	-0,61538	-0,84615	-0,30865	A14	-0,230797	-0,153845	-0,69231	-0,69231	-0,692308	-0,575	
A5	0,615385	0,923077	0,538462	0,923077	0,384615	1	0,694231	A8	-0,230797	-0,230797	-0,230797	-0,230797	-0,230797	-0,230797	
A1	0,615385	0,615385	0,846154	0,538462	0,846154	0,384615	0,667308	A11	-0,92308	-0,69231	-0,384615	-0,384615	-0,384615	-0,384615	
A3	0,615385	0,153845	0,846154	0,538462	0,846154	0,384615	0,667308	A12	-0,84615	-0,615385	-0,53846	-0,53846	-0,69231	-0,69231	
A4	0,615385	0,384615	0,846154	0,538462	0,153845	0,									

B Best cities ranking

	Crit2	Crit5	Crit1	Crit6	Crit3	Crit4	NetFlows		Crit6	Crit1	Crit3	Crit2	Crit5	Crit4	NetFlows
A4	100	96,4	85	63,7	97,2	100	0,804167	A4	63,7	85	97,2	100	96,4	100	0,804167
A3	100	96,4	90	64	93,5	100	0,795109	A3	64	90	93,5	100	96,4	100	0,795109
A5	100	100	90	55,7	94,4	100	0,777899	A5	55,7	90	94,4	100	100	100	0,777899
A7	100	96,4	85	61,7	97,2	91,7	0,759239	A7	61,7	85	97,2	100	96,4	91,7	0,759239
A2	100	96,4	80	71,3	97,2	91,7	0,744746	A2	71,3	80	97,2	100	96,4	91,7	0,744746
A6	95,8	96,4	95	58,9	91,2	100	0,72029	A6	58,9	95	91,2	95,8	96,4	100	0,72029
A1	87,5	96,4	95	75	85,9	100	0,716848	A1	75	95	85,9	87,5	96,4	100	0,716848
A9	100	89,3	85	62,5	97,2	91,7	0,709964	A9	62,5	85	97,2	100	89,3	91,7	0,709964
A10	100	92,9	90	53,3	94,4	100	0,696739	A10	53,3	90	94,4	100	92,9	100	0,696739
A8	100	89,3	100	50	97,2	100	0,67337	A8	50	100	97,2	100	89,3	100	0,67337
A13	87,5	92,9	85	61,3	94,4	100	0,635688	A13	61,3	85	94,4	87,5	92,9	100	0,635688
A14	91,7	96,4	80	55,1	94,4	100	0,60163	A14	55,1	80	94,4	91,7	96,4	100	0,60163
A12	87,5	89,3	70	72,6	97,2	100	0,59058	A12	72,6	80	97,2	87,5	89,3	100	0,59058
A11	87,5	92,9	80	67,3	91,7	100	0,583152	A11	67,3	80	91,7	87,5	92,9	100	0,583152
A15	91,7	92,9	85	52,7	91,7	100	0,562138	A15	52,7	85	91,7	91,7	92,9	100	0,562138
A16	91,7	89,3	70	65,2	91,7	100	0,522464	A16	65,2	70	91,7	91,7	89,3	100	0,522464
A18	91,7	85,7	85	53	94,4	83,3	0,512862	A18	53	85	94,4	91,7	85,7	83,3	0,512862
A17	91,7	89,3	80	50,3	94,4	100	0,465399	A17	50,3	80	94,4	91,7	89,3	100	0,465399
A20	83,3	89,3	80	58,8	85,6	100	0,418478	A20	58,8	80	85,6	83,3	89,3	100	0,418478
A19	91,7	96,4	80	46,7	91,7	100	0,415036	A19	46,7	80	91,7	91,7	96,4	100	0,415036
A21	91,7	92,9	85	42,9	91,7	100	0,391848	A21	42,9	85	91,7	91,7	92,9	100	0,391848
A22	87,5	100	95	46,7	76,6	83,3	0,374819	A22	46,7	95	76,6	87,5	100	83,3	0,374819
A23	91,7	92,9	85	39,3	91,7	100	0,359239	A23	39,3	85	91,7	91,7	92,9	100	0,359239
A24	91,7	83,9	85	43	99	100	0,311957	A24	43	85	90	91,7	83,9	100	0,311957
A27	79,2	83,9	65	54,2	81,5	91,7	0,21721	A27	54,2	65	81,5	79,2	83,9	91,7	0,21721
A25	87,5	80,4	80	41,7	95,1	91,7	0,208333	A25	41,7	80	95,1	87,5	80,4	91,7	0,208333
A30	66,7	85,7	80	51,5	72,2	83,3	0,132609	A30	51,5	80	72,2	66,7	85,7	83,3	0,132609
A26	87,5	85,7	70	42,3	85,9	100	0,126812	A26	42,3	70	85,9	87,5	85,7	100	0,126812
A28	87,5	80,4	65	48,2	81,5	83,3	0,102355	A28	48,2	65	81,5	87,5	80,4	83,3	0,102355
A29	83,3	75	75	47,3	83,1	75	0,074094	A29	47,3	75	83,1	83,3	75	75	0,074094
A34	62,5	82,1	85	48,5	63,7	66,7	0,036957	A34	48,5	85	63,7	62,5	82,1	66,7	0,036957
A36	70,8	66,1	60	52,4	80,3	66,7	-0,00598	A36	52,4	60	80,3	70,8	66,1	66,7	-0,00598
A35	66,7	75	60	47,3	81,7	91,7	-0,00906	A35	47,3	60	81,7	66,7	75	91,7	-0,00906
A32	70,8	82,1	80	39	80,3	75	-0,01866	A32	39	80	80,3	70,8	82,1	75	-0,01866
A31	70,8	85,7	75	35,1	89,1	83,3	-0,02083	A31	35,1	75	89,1	70,8	85,7	83,3	-0,02083
A41	66,7	46,4	45	65,8	82,4	75	-0,04094	A41	65,8	45	82,4	66,7	46,4	75	-0,04094
A33	62,5	75	80	46,1	75	75	-0,04783	A33	46,1	80	75	62,5	75	75	-0,04783
A38	66,7	82,1	90	27,7	65,3	66,7	-0,12373	A38	27,7	90	65,3	66,7	82,1	66,7	-0,12373
A37	62,5	76,8	80	36,6	67,8	91,7	-0,13696	A37	36,6	80	67,8	62,5	76,8	91,7	-0,13696
A42	66,7	71,4	55	44	77,5	83,3	-0,14855	A42	44	55	77,5	66,7	71,4	83,3	-0,14855
A40	58,3	69,6	50	44,9	90,5	83,3	-0,17174	A40	44,9	50	90,5	58,3	69,6	83,3	-0,17174
A39	62,5	76,8	80	42,9	61,1	66,7	-0,17283	A39	42,9	80	61,1	62,5	76,8	66,7	-0,17283
A46	58,3	58,9	55	58,6	55,6	75	-0,19529	A46	58,6	55	55,6	58,3	58,9	75	-0,19529
A43	66,7	66,1	80	34,7	74,3	66,7	-0,22572	A43	34,7	80	74,3	66,7	66,1	66,7	-0,22572
A50	62,5	64,3	35	50,6	75,2	66,7	-0,22681	A50	50,6	35	75,2	62,5	64,3	66,7	-0,22681
A54	41,7	60,7	30	52,1	76,6	75	-0,28043	A54	52,1	30	76,6	41,7	60,7	75	-0,28043
A45	75	57,1	60	36	73,1	75	-0,28696	A45	36	60	73,1	75	57,1	75	-0,28696
A47	62,5	75	85	21	62	66,7	-0,29275	A47	21	85	62	62,5	75	66,7	-0,29275
A52	54,2	51,8	60	52,1	56,3	66,7	-0,29891	A52	52,1	60	56,3	54,2	51,8	66,7	-0,29891
A48	58,3	64,3	60	46,1	63,2	66,7	-0,3	A48	46,1	60	63,2	58,3	64,3	66,7	-0,3
A51	50	67,9	55	47,5	68,8	58,3	-0,30036	A51	47,5	55	68,8	50	67,9	58,3	-0,30036
A44	75	50	70	33,3	73,4	83,3	-0,30707	A44	33,3	70	73,4	75	50	83,3	-0,30707
A49	62,5	69,6	50	36,3	64,4	100	-0,34167	A49	36,3	50	64,4	62,5	69,6	100	-0,34167
A53	45,8	60,7	65	43,8	60,9	58,3	-0,38732	A53	43,8	65	60,9	45,8	60,7	58,3	-0,38732
A61	62,5	33,9	50	53,6	35,9	50	-0,43732	A61	53,6	50	35,9	62,5	33,9	50	-0,43732
A55	45,8	53,6	55	48,2	54,9	58,3	-0,4471	A55	48,2	55	54,9	45,8	53,6	58,3	-0,4471
A56	45,8	57,1	50	42,3	59,3	66,7	-0,53188	A56	42,3	50	59,3	45,8	57,1	66,7	-0,53188
A65	45,8	51,8	20	48,5	38,7	66,7	-0,57391	A65	48,5	20	38,7	45,8	51,8	66,7	-0,57391
A57	54,2	51,8	55	38,4	53,7	58,3	-0,58949	A57	38,4	55	53,7	54,2	51,8	58,3	-0,58949
A59	50	55,4	55	36,5	54,2	41,7	-0,59583	A59	36,5	55	54,2	50	55,4	41,7	-0,59583
A58	58,3	51,8	50	26,8	55,3	75	-0,64239	A58	26,8	50	55,3	58,3	51,8	75	-0,64239
A62	45,8	42,9	40	33,9	69,9	66,7	-0,65471	A62	33,9	40	69,9	45,8	42,9	66,7	-0,65471
A60	50	48,2	55	35,1	49,5	66,7	-0,66051	A60	35,1	55	49,5	50	48,2	66,7	-0,66051
A63	33,3	55,4	60	23,2	59,7	41,7	-0,66395	A63	23,2	60	59,7	33,3	55,4	41,7	-0,66395
A64	37,5	53,6	60	24,1	49,3	58,3	-0,71377	A64	24,1	60	49,3	37,5	53,6	58,3	-0,71377
A67	45,8	53,6	25	30,1	54,2	50	-0,77464	A67	30,1	25	54,2	45,8	53,6	50	-0,77464
A66	41,7	37,5	50	22,6	59,7	50	-0,78641	A66	22,6	50	59,7	41,7	37,5	50	-0,78641
A68	29,2	26,8	50	35,7	43,3	41,7	-0,8192	A68	35,7	50	43,3	29,2	26,8	41,7	-0,8192
A70	20,8	35,7	30	17,3	53	66,7	-0,89946	A70	17,3	30	53	20,8	35,7	66,7	-0,89946
A69	33,3	48,2	25	22,3	52,3	33,3	-0,91105	A69	22,3	25	52,3	33,3	48,2	33,3	-0,91105

(a) Netflow-Angle

(b) Netflow-Weight

Figure 19: Visualisation - Best cities ranking - Evaluations

	Crit2	Crit5	Crit1	Crit6	Crit3	Crit4	NetFlows		Crit4	Crit3	Crit2	Crit5	Crit6	Crit1	NetFlows
A6	95,8	96,4	95	58,9	91,2	100	0,72029	A17	100	94,4	91,7	89,3	50,3	80	0,465399
A15	91,7	92,9	85	52,7	91,7	100	0,562138	A18	83,3	94,4	91,7	85,7	53	85	0,512862
A5	100	100	90	55,7	94,4	100	0,777899	A29	75	83,1	83,3	75	47,3	75	0,074094
A10	100	92,9	90	53,3	94,4	100	0,696739	A3	100	93,5	100	96,4	64	90	0,795109
A8	100	89,3	100	50	97,2	100	0,67337	A7	91,7	97,2	100	96,4	61,7	85	0,759239
A19	91,7	96,4	80	46,7	91,7	100	0,415036	A4	100	97,2	100	96,4	63,7	85	0,804167
A26	87,5	85,7	70	42,3	85,9	100	0,126812	A1	100	85,9	87,5	96,4	75	95	0,716848
A21	91,7	92,9	85	42,9	91,7	100	0,391848	A13	100	94,4	87,5	92,9	61,3	85	0,635688
A24	91,7	83,9	85	43	90	100	0,311957	A14	100	94,4	91,7	96,4	55,1	80	0,60163
A25	87,5	80,4	80	41,7	95,1	91,7	0,208333	A9	91,7	97,2	100	89,3	62,5	85	0,709964
A23	91,7	92,9	85	39,3	91,7	100	0,359239	A30	83,3	72,2	66,7	85,7	51,5	80	0,132609
A22	87,5	100	95	46,7	76,6	83,3	0,374819	A2	91,7	97,2	100	96,4	71,3	80	0,744746
A31	70,8	85,7	75	35,1	89,1	83,3	-0,02083	A20	100	85,6	83,3	89,3	58,8	80	0,418478
A32	70,8	82,1	80	39	80,3	75	0,01866	A11	100	91,7	87,5	92,9	67,3	80	0,583152
A38	66,7	82,1	90	27,7	65,3	66,7	-0,12373	A28	83,3	81,5	87,5	80,4	48,2	65	0,102355
A37	62,5	76,8	80	36,6	67,8	91,7	-0,13696	A16	100	91,7	91,7	89,3	65,2	70	0,522464
A34	62,5	82,1	85	48,5	63,7	66,7	0,036957	A6	100	91,2	95,8	96,4	58,9	95	0,72029
A47	62,5	75	85	21	62	66,7	-0,29275	A15	100	91,7	91,7	92,9	52,7	85	0,562138
A43	66,7	66,1	80	34,7	74,3	66,7	-0,22572	A5	100	94,4	100	100	55,7	90	0,777899
A33	62,5	75	80	46,1	75	75	-0,04783	A10	100	94,4	100	92,9	53,3	90	0,696739
A44	75	50	70	33,3	73,4	83,3	0,30707	A12	100	97,2	87,5	89,3	72,6	70	0,59058
A39	62,5	76,8	80	42,9	61,1	66,7	-0,17283	A8	100	97,2	100	89,3	50	100	0,67337
A45	75	51,7	60	36	73,1	75	-0,28696	A19	100	91,7	91,7	96,4	46,7	80	0,415036
A63	33,3	55,4	60	23,2	59,7	41,7	-0,66395	A27	91,7	81,5	79,2	83,9	54,2	65	0,21721
A64	37,5	53,6	60	24,1	49,3	58,3	-0,71377	A26	100	85,9	87,5	85,7	42,3	70	0,126812
A58	58,3	51,8	50	26,8	55,3	75	-0,64239	A21	100	91,7	91,7	92,9	42,9	85	0,391848
A66	41,7	37,5	50	22,6	59,7	50	-0,78641	A24	100	90	91,7	83,9	43	85	0,311957
A49	62,5	69,6	50	36,3	64,4	100	-0,34167	A25	91,7	95,1	87,5	80,4	41,7	80	0,208333
A60	50	48,2	55	35,1	49,5	66,7	-0,66051	A23	100	91,7	91,7	92,9	39,3	85	0,359239
A70	20,8	35,7	30	17,3	53	66,7	-0,89946	A22	83,3	76,6	87,5	100	46,7	95	0,374819
A69	33,3	48,2	25	22,3	52,3	33,3	-0,91108	A35	91,7	81,7	66,7	75	47,3	60	-0,00906
A59	50	55,4	55	36,5	54,2	41,7	-0,59583	A31	83,3	89,1	70,8	85,7	35,1	75	-0,02083
A67	45,8	53,6	25	30,1	54,2	50	-0,77464	A41	75	82,4	66,7	46,4	65,8	45	-0,04094
A57	54,2	51,8	55	38,4	53,7	58,3	-0,58949	A32	75	80,3	70,8	82,1	39	80	-0,01866
A53	45,8	60,7	65	43,8	60,9	58,3	-0,38732	A36	66,7	80,3	70,8	66,1	52,4	60	-0,00598
A68	29,2	26,8	50	35,7	43,3	41,7	-0,18192	A38	66,7	65,3	66,7	82,1	27,7	90	-0,12373
A62	45,8	42,9	40	33,9	69,9	66,7	-0,65471	A46	75	55,6	58,3	58,9	58,6	55	-0,19529
A56	45,8	57,1	50	42,3	59,3	66,7	-0,53188	A37	91,7	67,8	62,5	76,8	36,6	80	-0,13696
A48	58,3	64,3	60	46,1	63,2	66,7	-0,3	A54	75	76,6	41,7	60,7	52,1	30	-0,280403
A55	45,8	53,6	55	48,2	54,9	58,3	-0,44711	A50	66,7	75,2	62,5	64,3	50,6	35	-0,22681
A51	50	67,9	55	47,5	68,8	58,3	-0,30036	A40	83,3	90,5	58,3	69,6	44,9	50	-0,17174
A65	45,8	51,8	20	48,5	38,7	66,7	-0,57391	A34	66,7	63,7	62,5	82,1	48,5	85	0,036957
A42	66,7	71,4	55	44	77,5	83,3	-0,14855	A47	66,7	62	62,5	75	21	85	-0,29275
A52	54,2	51,8	60	52,1	56,3	66,7	-0,29891	A61	50	35,9	62,5	33,9	53,6	50	-0,43732
A61	62,5	33,9	50	53,6	35,9	50	-0,43732	A43	66,7	74,3	66,7	66,1	34,7	80	-0,22572
A40	58,3	69,6	50	44,9	90,5	83,3	-0,17174	A52	66,7	56,3	54,2	51,8	52,1	60	-0,29891
A50	62,5	64,3	35	50,6	75,2	66,7	-0,22681	A42	83,3	77,5	66,7	71,4	44	55	-0,14855
A54	41,7	60,7	30	52,1	76,6	75	-0,28043	A65	66,7	38,7	45,8	51,8	48,5	20	-0,57391
A46	58,3	58,9	55	58,6	55,6	75	-0,19529	A33	75	75	62,5	75	46,1	80	-0,04783
A36	70,8	66,1	60	52,4	80,3	66,7	-0,00598	A51	58,3	68,8	50	67,9	47,5	55	-0,30036
A41	66,7	46,4	45	65,8	82,4	75	-0,04094	A55	58,3	54,9	45,8	53,6	48,2	55	-0,44711
A35	66,7	75	60	47,3	81,7	91,7	-0,00906	A44	83,3	73,4	75	50	33,3	70	-0,30707
A27	79,2	83,9	65	54,2	81,5	91,7	0,21721	A39	66,7	61,1	62,5	76,8	42,9	80	-0,17283
A12	87,5	89,3	70	72,6	97,2	100	0,509058	A48	66,7	63,2	58,3	64,3	46,1	60	-0,3
A16	91,7	89,3	70	65,2	91,7	100	0,522464	A56	66,7	59,3	45,8	57,1	42,3	50	-0,53188
A28	87,5	80,4	65	48,2	81,5	83,3	0,102355	A45	75	73,1	75	57,1	36	60	-0,28696
A11	87,5	92,9	80	67,3	91,7	100	0,583152	A62	66,7	69,9	45,8	42,9	33,9	40	-0,65471
A20	83,3	89,3	80	58,8	85,6	100	0,418478	A68	41,7	43,3	29,2	26,8	35,7	50	-0,8192
A2	100	96,4	80	71,3	97,2	91,7	0,744746	A53	58,3	60,9	45,8	60,7	43,8	65	-0,38732
A30	66,7	85,7	80	51,5	72,2	83,3	0,132609	A57	58,3	53,7	54,2	51,8	38,4	55	-0,58949
A9	100	89,3	85	62,5	97,2	91,7	0,709964	A67	50	54,2	45,8	53,6	30,1	25	-0,77464
A14	91,7	96,4	80	55,1	94,4	100	0,60163	A59	41,7	54,2	50	55,4	36,5	55	-0,59583
A13	87,5	92,9	85	61,3	94,4	100	0,635688	A69	33,3	52,3	33,3	48,2	22,3	25	-0,91105
A1	87,5	96,4	95	75	85,9	100	0,716848	A70	66,7	53	20,8	35,7	17,3	30	-0,89946
A4	100	96,4	85	63,7	97,2	100	0,804167	A60	66,7	49,5	50	48,2	35,1	55	-0,66051
A7	100	96,4	85	61,7	97,2	91,7	0,759239	A49	100	64,4	62,5	69,6	36,3	50	-0,34167
A3	100	96,4	90	64	93,5	100	0,795109	A66	50	59,7	41,7	37,5	22,6	50	-0,78641
A29	83,3	75	75	47,3	83,1	75	0,074094	A63	41,7	59,7	33,3	55,4	23,2	60	-0,66395
A18	91,7	85,7	85	53	94,4	83,3	0,512862	A64	58,3	49,3	37,5	53,6	24,1	60	-0,71377
A17	91,7	89,3	80	50,3	94,4	100	0,465399	A58	75	55,3	58,3	51,8	26,8	50	-0,64239

(c) Angle-Angle

(d) Proximity-Proximity

Figure 19: Visualisation - Best cities ranking - Evaluations (cont.)

	Crit2	Crit5	Crit1	Crit6	Crit3	Crit4	NetFlows		Crit6	Crit1	Crit3	Crit2	Crit5	Crit4	NetFlows
A4	0.898551	0.84058	0.623188	0.797101	0.927536	0.710145	0.804167	A4	0.797101	0.623188	0.927536	0.898551	0.84058	0.710145	0.804167
A3	0.898551	0.84058	0.84058	0.826087	0.623188	0.710145	0.795109	A3	0.826087	0.84058	0.623188	0.898551	0.84058	0.710145	0.795109
A5	0.898551	0.985507	0.84058	0.594203	0.724638	0.710145	0.777899	A5	0.594203	0.84058	0.724638	0.898551	0.985507	0.710145	0.777899
A7	0.898551	0.84058	0.623188	0.793193	0.927536	0.304348	0.759239	A7	0.73913	0.623188	0.927536	0.898551	0.84058	0.304348	0.759239
A2	0.898551	0.84058	0.275362	0.942029	0.927536	0.304348	0.744746	A2	0.942029	0.275362	0.927536	0.898551	0.84058	0.304348	0.744746
A6	0.768116	0.84058	0.942029	0.681159	0.42029	0.710145	0.72029	A6	0.681159	0.942029	0.42029	0.768116	0.84058	0.710145	0.72029
A1	0.376812	0.84058	0.942029	1	0.289855	0.710145	0.716848	A1	1	0.942029	0.289855	0.376812	0.84058	0.710145	0.16848
A9	0.898551	0.463768	0.623188	0.768116	0.927536	0.304348	0.709964	A9	0.768116	0.623188	0.927536	0.898551	0.463768	0.304348	0.709964
A10	0.898551	0.637681	0.84058	0.478261	0.724638	0.710145	0.696739	A10	0.478261	0.84058	0.724638	0.898551	0.376811	0.710145	0.696739
A8	0.898551	0.463768	1	0.217391	0.927536	0.710145	0.67337	A8	0.217391	1	0.927536	0.898551	0.463768	0.710145	0.67337
A13	0.376812	0.637681	0.623188	0.710145	0.724638	0.710145	0.635688	A13	0.710145	0.623188	0.927536	0.376812	0.376811	0.710145	0.635688
A14	0.623188	0.84058	0.275362	0.565217	0.724638	0.710145	0.60163	A14	0.565217	0.275362	0.724638	0.623188	0.84058	0.710145	0.60163
A12	0.376812	0.463768	-0.02899	0.971014	0.927536	0.710145	0.59058	A12	0.971014	-0.02899	0.927536	0.376812	0.463768	0.710145	0.59058
A11	0.376812	0.637681	0.275362	0.913043	0.521739	0.710145	0.583152	A11	0.913043	0.275362	0.521739	0.376812	0.376811	0.710145	0.583152
A15	0.623188	0.637681	0.623188	0.42029	0.521739	0.710145	0.562138	A15	0.42029	0.623188	0.521739	0.623188	0.463768	0.710145	0.562138
A16	0.623188	0.463768	-0.02899	0.855078	0.521739	0.710145	0.522464	A16	0.855072	-0.02899	0.521739	0.623188	0.376811	0.710145	0.522464
A18	0.623188	0.318841	0.623188	0.449275	0.724638	0.086957	0.512862	A18	0.449275	0.623188	0.724638	0.623188	0.318841	0.086957	0.512862
A17	0.623188	0.463768	0.275362	0.246377	0.724638	0.710145	0.465399	A17	0.246377	0.275362	0.724638	0.623188	0.463768	0.710145	0.465399
A20	0.231884	0.463768	0.275362	0.652174	0.246377	0.710145	0.418478	A20	0.652174	0.275362	0.246377	0.231884	0.463768	0.710145	0.418478
A19	0.623188	0.84058	0.275362	-0.02899	0.521739	0.710145	0.415036	A19	-0.02899	0.275362	0.521739	0.623188	0.84058	0.710145	0.415036
A21	0.623188	0.637681	0.623188	0.26087	0.521739	0.710145	0.391848	A21	0.26087	0.623188	0.521739	0.623188	0.376811	0.710145	0.391848
A22	0.376812	0.8985507	0.942029	-0.02899	0.086957	0.710145	0.747819	A22	-0.02899	0.942029	-0.02899	0.376812	0.985507	0.086957	0.747819
A23	0.623188	0.637681	0.623188	-0.02899	0.521739	0.710145	0.359239	A23	-0.3913	0.623188	0.521739	0.623188	0.376811	0.710145	0.359239
A24	0.623188	0.231884	0.623188	-0.21739	0.362319	0.710145	0.311957	A24	-0.21739	0.623188	0.362319	0.623188	0.231884	0.710145	0.311957
A27	0.188404	0.231884	-0.13043	0.536239	0.115942	0.304348	0.21721	A27	0.536232	-0.13043	0.115942	0.188406	0.231884	0.304348	0.21721
A25	0.376812	0.086957	0.275362	0.36232	0.826087	0.304348	0.208333	A25	-0.36232	0.275362	0.826087	0.376812	0.086957	0.304348	0.208333
A30	-0.05797	0.318841	0.275362	0.304348	-0.21739	0.086957	0.132609	A30	0.304348	0.275362	-0.21739	0.05797	0.188406	0.086957	0.132609
A26	0.376812	0.318841	-0.02899	-0.31884	0.289855	0.710145	0.126812	A26	-0.31884	-0.02899	0.289855	0.376812	0.318841	0.710145	0.126812
A28	0.376812	0.086957	-0.13043	0.115942	0.115942	0.086957	0.102355	A28	0.115942	-0.13043	0.115942	0.376812	0.086957	0.102355	0.102355
A29	0.231884	-0.05797	0.057971	0.028986	0.217391	-0.14493	0.074094	A29	0.028986	-0.057971	0.217391	0.231884	-0.05797	-0.14493	0.074094
A34	-0.26087	0.15942	0.623188	0.173913	-0.3913	-0.46377	0.036957	A34	0.173913	0.623188	-0.3913	-0.26087	0.15942	-0.46377	0.036957
A36	0.072464	-0.26087	-0.27536	0.391304	0.057971	-0.46377	-0.00598	A36	0.391304	-0.26087	-0.27536	0.057971	0.072464	-0.26087	-0.00598
A35	-0.05797	-0.05797	-0.27536	0.028986	0.15942	0.304348	0.00906	A35	0.028986	-0.27536	0.15942	-0.05797	0.05797	0.304348	0.00906
A32	0.072464	0.15942	0.275362	-0.42029	0.057971	-0.14493	-0.01866	A32	-0.42029	0.275362	0.057971	0.072464	0.15942	-0.14493	-0.01866
A31	0.072464	0.318841	0.057971	-0.63768	0.333333	0.086957	-0.02083	A31	-0.63768	0.057971	0.333333	0.072464	0.318841	0.086957	-0.02083
A41	-0.05797	-0.085507	-0.7971	0.84058	0.188406	-0.14493	-0.04094	A41	0.884058	-0.7971	0.188406	-0.05797	-0.85507	-0.14493	-0.04094
A33	-0.26087	-0.05797	0.275362	-0.086959	-0.10145	-0.14493	0.047813	A33	-0.086956	0.275362	-0.10145	-0.26087	-0.05797	-0.14493	0.047813
A38	-0.05797	0.15942	0.84058	-0.7971	-0.333333	-0.46377	0.12373	A38	-0.7971	0.84058	-0.333333	-0.05797	0.15942	-0.46377	0.12373
A37	-0.26087	0.028986	0.275362	-0.47826	-0.304348	0.304348	0.13696	A37	-0.47826	0.275362	-0.304343	-0.26087	0.028986	0.304348	0.13696
A42	-0.05797	-0.13043	-0.47826	-0.15942	0.014493	0.086957	-0.14855	A42	-0.15942	-0.47826	0.014493	-0.05797	-0.13043	0.086957	-0.14855
A40	-0.43478	-0.17391	-0.68116	-0.13043	0.391304	0.086957	-0.17174	A40	-0.13043	-0.68116	0.391304	-0.43478	-0.17391	0.086957	-0.17174
A39	-0.26087	0.028986	0.275362	-0.26087	-0.47826	-0.46377	-0.17283	A39	-0.26087	0.275362	-0.47826	-0.26087	0.028986	-0.46377	-0.17283
A46	-0.43478	-0.42029	-0.47826	0.623188	-0.65217	-0.14493	-0.19529	A46	0.623188	-0.47826	-0.65217	-0.43478	-0.42029	-0.14493	-0.19529
A43	-0.05797	0.173913	-0.47826	-0.724638	-0.13043	-0.46377	0.22572	A43	-0.68116	0.275362	-0.13043	-0.05797	-0.26087	-0.46377	0.22572
A50	-0.26087	-0.31884	-0.85507	0.275362	-0.07246	-0.46377	0.22681	A50	0.275362	-0.85507	-0.07246	-0.26087	-0.31884	-0.46377	0.22681
A54	-0.84058	-0.7971	-0.68116	0.347826	-0.02899	-0.14493	0.280403	A54	0.347826	-0.89855	-0.347826	-0.02899	-0.84058	-0.37681	-0.280403
A45	0.144928	-0.46377	-0.27536	-0.56522	-0.18841	-0.14493	0.28696	A45	-0.56522	-0.27536	-0.18841	0.144928	-0.46377	-0.14493	-0.28696
A47	-0.26087	-0.05797	0.623188	-0.97101	-0.44928	-0.46377	0.29275	A47	-0.97101	0.623188	-0.44928	-0.26087	-0.05797	-0.46377	0.29275
A52	-0.52174	-0.69565	-0.27536	0.347826	-0.62319	-0.46377	0.29891	A52	0.347826	-0.27536	-0.62319	-0.52174	-0.69565	-0.46377	-0.29891
A48	-0.43478	-0.31884	-0.27536	-0.086956	-0.42029	-0.46377	-0.3	A48	-0.086956	-0.27536	-0.42029	-0.43478	-0.31884	-0.46377	-0.3
A51	-0.5942	-0.21739	-0.47826	0.072463	-0.27536	-0.73913	-0.30036	A51	0.072464	-0.47826	-0.27536	-0.5942	-0.21739	-0.73913	-0.30036
A44	0.144928	-0.76812	-0.02899	-0.73913	-0.15942	0.086957	-0.30707	A44	-0.73913	-0.02899	-0.15942	0.144928	-0.76812	0.086957	-0.30707
A49	-0.26087	-0.17391	-0.68116	-0.53623	-0.36232	-0.710145	-0.34167	A49	-0.53623	-0.68116	-0.36232	-0.5942	-0.52174	-0.94203	-0.59583
A53	-0.72464	-0.37681	-0.13043	-0.18841	-0.50725	-0.73913	-0.38732	A53	-0.18841	-0.13043	-0.50725	-0.72464	-0.37681	-0.73913	-0.38732
A61	-0.26087	-0.97101	-0.68116	0.507246	-1	-0.85507	-0.43732	A61	0.507246	-0.68116	-1	-0.26087	-0.97101	-0.85507	-0.43732
A55	-0.72464	-0.5942	-0.47826	0.115942	-0.71014	-0.73913	-0								

	Crit2	Crit5	Crit1	Crit6	Crit3	Crit4	NetFlows		Crit4	Crit3	Crit2	Crit5	Crit6	Crit1	NetFlows
A6	0,768116	0,84058	0,942029	0,681159	0,42029	0,710145	0,72029	A17	0,710145	0,724638	0,623188	0,463768	0,246377	0,275362	0,465399
A15	0,623188	0,637681	0,623188	0,42029	0,521739	0,710145	0,562138	A18	0,086957	0,724638	0,623188	0,318841	0,449275	0,623188	0,512862
A5	0,898551	0,985507	0,84058	0,594203	0,724638	0,710145	0,777899	A29	-0,14493	0,217391	0,231884	-0,05797	0,028986	0,057973	0,074094
A10	0,898551	0,637681	0,84058	0,478261	0,724638	0,710145	0,696739	A3	0,710145	0,623188	0,988551	0,84058	0,826087	0,84058	0,795109
A8	0,898551	0,463768	1	0,217391	0,927536	0,710145	0,67337	A7	0,304348	0,927536	0,988551	0,84058	0,739103	0,623188	0,759239
A19	0,623188	0,84058	0,275362	-0,02899	0,521739	0,710145	0,415036	A4	0,710145	0,927536	0,988551	0,84058	0,797101	0,623188	0,804167
A26	0,376812	0,318841	-0,02899	0,31884	0,289855	0,710145	0,126812	A1	0,710145	0,289855	0,376812	0,84058	1	0,942029	0,716848
A21	0,623188	0,637681	0,623188	-0,26087	0,521739	0,710145	0,391848	A13	0,710145	0,724638	0,376812	0,637681	0,710145	0,623188	0,355688
A24	0,623188	0,231884	0,623188	0,21739	0,362319	0,710145	0,311957	A14	0,710145	0,724638	0,623188	0,84058	0,655217	0,275362	0,60163
A25	0,376812	0,086957	0,275362	-0,36232	0,826087	0,304348	0,208333	A9	0,304348	0,927536	0,988551	0,463768	0,768116	0,623188	0,709964
A23	0,623188	0,637681	0,623188	-0,3913	0,521739	0,710145	0,359239	A30	0,086957	-0,21739	-0,05797	0,318841	0,304348	0,275362	0,132609
A22	0,376812	0,985507	0,942029	-0,02899	0,02899	0,086957	0,374819	A2	0,304348	0,927536	0,988551	0,84058	0,942029	0,275362	0,744746
A31	0,072464	0,318841	0,971	0,637681	0,333333	0,086957	-0,02083	A20	0,710145	0,426377	0,231884	0,463768	0,652174	0,275362	0,184478
A32	0,072464	0,15942	0,275362	0,42029	0,057971	0,14493	0,01866	A11	0,710145	0,521739	0,376812	0,637681	0,913043	0,275362	0,583152
A38	-0,05797	0,15942	0,84058	-0,7971	-0,33333	-0,46377	-0,12373	A28	0,086957	0,115942	0,376812	0,086957	0,115942	-0,13043	0,102355
A37	-0,26087	0,028986	0,275362	-0,47826	0,304348	0,313699	A16	0,710145	0,521739	0,623188	0,463768	0,855072	-0,02899	0,522464	
A34	-0,26087	0,15942	0,623188	0,173913	-0,3913	-0,46377	0,036957	A6	0,710145	0,42029	0,768116	0,84058	0,681159	0,942029	0,72029
A47	-0,26087	-0,05797	0,623188	-0,9710	-0,44922	-0,46377	-0,29275	A15	0,710145	0,521739	0,623188	0,637681	0,42029	0,623188	0,562138
A43	-0,05797	-0,26087	0,275362	-0,68116	-0,13043	-0,46377	-0,22572	A5	0,710145	0,724638	0,988551	0,985507	0,594203	0,84058	0,777899
A33	-0,26087	-0,05797	0,275362	-0,08699	-0,10145	-0,14493	-0,04783	A10	0,710145	0,724638	0,988551	0,637681	0,478261	0,84058	0,696739
A44	0,144928	-0,76812	-0,02899	0,73913	0,15942	0,086957	-0,30707	A12	0,710145	0,927536	0,376812	0,463768	0,971014	-0,02899	0,59058
A39	-0,26087	0,028986	0,275362	-0,26087	-0,47826	-0,46377	-0,17283	A8	0,710145	0,927536	0,988551	0,463768	0,217391	1	0,67337
A45	0,144928	-0,46377	-0,27536	-0,56522	-0,18841	-0,14493	-0,28699	A19	0,710145	0,521739	0,623188	0,84058	-0,02899	0,275362	0,151036
A63	-0,92754	-0,52174	-0,27536	-0,88404	-0,55072	-0,94203	-0,66399	A27	0,304348	0,115942	0,188406	0,231884	0,536232	-0,13043	0,21721
A64	-0,88406	-0,5942	-0,27536	-0,85507	-0,91304	-0,73913	-0,71377	A26	0,710145	0,289855	0,376812	0,318841	-0,31884	-0,02899	0,126812
A58	-0,43478	-0,69565	-0,68116	0,82609	0,68116	-0,14493	0,64239	A21	0,710145	0,521739	0,623188	0,637681	-0,26087	0,623188	0,391848
A66	-0,84058	-0,91304	-0,68116	0,91304	-0,55072	-0,85507	-0,78641	A24	0,710145	0,362319	0,623188	0,231884	-0,21739	0,623188	0,311957
A49	-0,26087	-0,17391	-0,68116	-0,53623	-0,36232	0,710145	-0,34167	A25	0,304348	0,826087	0,376812	0,086957	-0,36232	0,275362	0,208333
A60	-0,5942	-0,81159	-0,47826	-0,63768	-0,88406	-0,46377	-0,660501	A23	0,710145	0,521739	0,623188	0,637681	-0,3913	0,359239	
A70	-1	-0,94203	-0,89855	-1	-0,82609	-0,46377	-0,89946	A22	0,086957	-0,02899	0,376812	0,985507	-0,02899	0,942029	0,374819
A69	-0,92754	-0,81159	-0,95652	-0,94203	-0,85507	-1	-0,91106	A35	0,304348	0,15942	0,188406	0,231884	-0,27536	-0,00906	
A59	-0,5942	-0,52174	-0,47826	-0,50725	-0,75362	-0,94203	-0,595893	A31	0,086957	0,333333	0,072464	0,318841	-0,63768	0,057971	-0,02083
A67	-0,72464	-0,5942	-0,95652	-0,76812	-0,75362	-0,85507	-0,77464	A41	-0,14493	0,188406	-0,05797	-0,85507	0,84058	-0,7971	-0,04094
A57	-0,52174	-0,69565	-0,47826	-0,44928	-0,7971	-0,73913	-0,58949	A32	-0,14493	0,057971	0,72464	0,15942	-0,42029	0,275362	-0,01866
A53	-0,72464	-0,37681	-0,13043	-0,18841	-0,50725	-0,73913	-0,38732	A36	-0,63377	0,057971	0,72464	-0,72464	-0,26087	0,391304	-0,27536
A68	-0,97101	-1	-0,68116	-0,5942	-0,94203	-0,94203	-0,8192	A38	-0,63377	-0,33333	-0,05797	0,15942	-0,7971	0,84058	-0,12373
A62	-0,72464	-0,88406	-0,82609	-0,71014	-0,24638	-0,65477	-0,65471	A46	-0,14493	-0,65217	-0,43478	-0,42029	0,623188	-0,47826	-0,19529
A56	-0,72464	-0,46377	-0,68116	-0,31884	-0,5942	-0,46377	-0,53181	A37	0,304348	-0,304348	-0,26087	0,289898	-0,47826	0,275362	-0,13696
A48	-0,43478	-0,31884	-0,27536	-0,08699	-0,42029	-0,46377	-0,3	A54	-0,14493	-0,02899	-0,84058	-0,37681	0,347826	-0,89855	-0,28043
A55	-0,72464	-0,5942	-0,47826	0,115942	-0,71014	-0,73913	-0,4471	A50	-0,63377	-0,07246	-0,26087	-0,31884	0,275362	-0,85507	-0,22681
A51	-0,5942	-0,21739	-0,47826	0,072464	-0,27536	-0,73913	-0,30036	A40	0,086957	0,391304	-0,43478	-0,17391	-0,13043	-0,68116	-0,17174
A65	-0,72464	-0,69565	-1	0,173913	-0,97101	-0,46377	-0,57391	A34	-0,63377	-0,3913	-0,26087	0,15942	0,173913	0,623188	0,036957
A42	-0,05797	-0,13043	-0,47826	-0,15942	0,014493	0,086957	-0,14855	A47	-0,63377	-0,44928	-0,26087	-0,05797	-0,97101	0,623188	-0,29275
A52	-0,52174	-0,69565	-0,27536	0,347826	0,62319	-0,62319	-0,46377	A61	-0,85507	-1	-0,26087	0,97101	0,507246	-0,68116	-0,43732
A61	-0,26087	-0,97101	-0,68116	0,507246	-1	-0,85507	-0,43732	A43	-0,63377	-0,13043	-0,05797	-0,26087	0,68116	0,275362	-0,22572
A40	-0,43478	-0,17391	-0,68116	-0,13043	0,391304	0,086957	-0,17174	A52	-0,63377	-0,62319	-0,52174	-0,69565	0,347826	-0,27536	-0,29891
A50	-0,26087	-0,31884	-0,85507	0,275362	-0,724638	-0,07246	-0,22681	A42	0,086957	0,014493	-0,05797	-0,13043	-0,15942	-0,47826	-0,148855
A54	-0,84058	-0,37681	-0,89855	0,347826	-0,28989	-0,14493	0,280403	A65	-0,63377	-0,97101	-0,72464	-0,69565	0,173913	-3	-0,57391
A46	-0,43478	-0,42029	-0,47826	0,623188	-0,65217	-0,14493	-0,19529	A33	-0,14493	-0,10445	-0,26087	-0,05797	-0,08696	0,275362	-0,04783
A36	0,072464	-0,26087	-0,27536	0,391304	0,057971	-0,46377	0,00598	A51	-0,73913	-0,27536	-0,5942	-0,21739	0,072464	-0,47826	-0,30036
A41	-0,05797	-0,85507	-0,7971	0,884058	0,188406	-0,14493	-0,04094	A55	-0,73913	-0,71014	-0,72464	-0,5942	0,15942	-0,47826	-0,4471
A35	-0,05797	-0,27536	0,028986	0,15942	0,304348	0,00909	A44	0,086957	-0,15942	0,144928	-0,76812	-0,73913	-0,02899	-0,30707	
A27	0,188406	0,231884	-0,13043	0,536232	0,15942	0,304348	0,21721	A39	-0,63377	-0,47826	-0,26087	0,028986	-0,26087	0,275362	-0,17283
A12	0,376812	0,463768	-0,02899	0,971014	0,927536	0,710145	0,59058	A48	-0,63377	-0,42029	-0,43478	-0,31884	-0,08696	-0,27536	-0,3
A16	0,623188	0,463768	-0,21739	0,855072	0,521739	0,710145	0,522464	A56	-0,63377	-0,5942	-0,72464	-0,46377	-0,31884	-0,68116	-0,53188
A28	0,376812	0,086957	-0,13043	0,115942	0,115942	0,086957	0,102355	A45	-0,14493	-0,18841	0,144928	-0,46377	-0,56522	-0,27536	-0,28696
A11	0,376812	0,637681	0,275362	0,913043	0,52										

C Environmental Performance Index (G20)

	Crit3	Crit1	Crit4	Crit7	Crit8	Crit6	Crit5	Crit9	Crit2	NetFlows
A1	100	98,33	66,46	19,37	100	47,67	83,08	92,33	100	0,625926
A3	100	98,82	66,03	0	43,06	54,24	70,11	97,93	100	0,518519
A2	100	98,33	66,46	19,37	100	47,67	83,08	92,33	100	0,625926
A5	95,9	97,89	62,52	21,54	38,65	59,05	58,4	85,42	100	0,379851
A4	63,51	82,85	58,87	24,93	55,41	63,41	79,77	91,44	100	0,353703
A7	100	89,44	65,55	37,94	49,91	54,47	83,8	100	0,246297	
A6	100	84,45	66,48	25,26	18,71	56,67	73,53	71,26	99,1	0,222
A8	86,49	84,45	61,29	3,24	0	46,63	93,7	28,54	94,68	-0,2963
A9	83,48	84,45	62,24	6,43	0	46,63	93,7	28,54	94,68	-0,2963
A15	93,44	97,64	74,51	24,06	10,81	53,82	66,74	10,87	88,59	-0,6759
A11	46,2	87,09	55,21	25,34	19,87	51,35	62,32	37,45	76,67	-0,14352
A13	51,51	97,64	74,51	24,06	10,81	53,82	66,74	10,87	88,59	-0,6759
A10	85,82	62,24	46,98	22,24	33,76	41,55	50,4	83,68	96,93	-0,19756
A16	75,7	99,64	96	15,68	0	16,79	44,88	11,75	85,07	-0,18518
A13	36,08	94,4	79,2	2,52	55,100	49,87	63,96	27,86	47,51	-0,23611
A13	31,81	93,83	33,86	18,86	29,81	65,84	78,81	18,18	76,23	-0,45357
A12	71,43	84,07	54,67	21,8	5,23	44,52	82,62	0	66,06	-0,3527
A17	24,29	75,31	51,85	25,8	3,75	45,25	78,08	0,02	67,55	-0,15667
A19	26,28	23,24	58,4	22,64	35,07	45,24	39,18	10,49	50,04	0,57527

(a) Netflow-angle

	Crit1	Crit2	Crit3	Crit5	Crit6	Crit9	Crit7	Crit8	Crit4	NetFlows
A1	98,33	100	100	83,08	47,67	92,33	19,37	100	66,44	0,625926
A3	95,82	100	100	70,11	54,24	97,93	0	43,06	66,03	0,518519
A2	98,33	100	100	62,77	47,67	83,08	13,4	38,8	66,03	0,625926
A5	95,9	97,89	62,52	21,54	38,65	59,05	58,4	85,42	100	0,379851
A4	80,85	100	63,51	28,77	63,41	91,44	24,93	55,41	88,59	0,353703
A7	89,44	100	63,51	54,47	49,83	83,8	24,0	37,94	65,55	0,246297
A6	84,79	99,2	100	73,53	43,54	71,26	25,34	55,41	46,44	0,222
A8	80,85	98,44	63,51	38,53	0	43,06	83,8	14,09	63,83	0,151111
A9	84,45	94,68	83,48	93,7	46,63	28,54	6,43	0	55,21	-0,2963
A15	97,64	65,99	66,74	50,4	50,4	10,87	24,06	10,81	74,51	-0,6759
A11	87,09	76,67	46,2	62,32	51,35	37,45	25,34	19,87	55,21	-0,14352
A14	81,45	83,12	45,17	53,2	61,39	21,5	12,73	35,07	16,89	-0,17913
A10	62,24	85,89	55,82	50,1	50,1	16,59	28,83	21,76	46,29	-0,28529
A16	99,62	85,07	75,7	44,88	16,79	11,75	15,68	0	76	-0,18518
A13	94,4	47,51	36,08	63,96	49,87	27,86	2,52	79,2	0	-0,23611
A18	18,81	76,23	33,15	36,66	65,16	18,18	14,08	25,34	33,86	-0,45357
A12	84,07	75,31	51,85	25,8	3,75	48,25	82,62	52	56,67	-0,3527
A17	73,31	61,55	24,29	78,08	49,25	0,02	25,8	17,75	51,85	-0,15667
A19	23,24	50,04	26,28	39,18	35,24	10,49	22,64	35,07	58,4	0,57527

(b) Netflow-Weight

	Crit1	Crit2	Crit3	Crit5	Crit6	Crit9	Crit7	Crit8	Crit4	NetFlows	
A3	100	98,33	66,46	19,37	100	43,06	54,24	70,11	97,93	100	0,518519
A1	100	98,82	66,03	0	43,06	47,67	83,08	92,33	100	0,625926	
A2	100	98,33	66,46	19,37	100	43,06	54,24	70,11	97,93	100	0,625926
A5	95,9	97,89	62,52	21,54	38,65	59,05	58,4	85,42	100	0,379851	
A8	86,48	94,65	61,53	3,34	14,35	56,45	63,35	61,66	95,33	0,151111	
A9	83,48	84,45	62,24	6,43	0	46,63	93,7	28,54	94,68	-0,2963	
A16	75,7	99,64	96	15,68	0	16,79	44,88	11,75	85,07	-0,18518	
A13	80,85	98,44	79,2	2,52	55,2	49,83	49,83	27,86	47,51	-0,23611	
A15	50,44	97,64	74,51	24,06	10,81	53,82	66,74	88,59	0,07579		
A26	26,28	21,24	58,4	22,64	35,07	35,24	39,18	10,49	50,04	0,57527	
A17	24,29	75,31	51,85	25,8	3,75	45,25	78,08	0,02	67,55	-0,15667	
A11	71,43	84,07	56,67	21,9	5,23	46,52	92,62	48,93	66,06	-0,3527	
A14	45,17	94,36	61,93	12,73	35,07	61,02	53,39	21,5	83,12	-0,17311	
A18	33,15	18,81	33,85	14,68	25,34	65,16	66,63	18,18	76,23	-0,45357	
A10	85,92	62,24	46,98	22,24	33,76	41,55	50,4	81,68	96,93	-0,18056	
A6	84,45	94,36	46,2	62,32	41,55	43,06	71,26	35,07	50,04	0,151111	
A4	63,51	80,85	58,87	24,93	55,41	63,41	79,77	91,44	100	0,353703	
A2	100	78,5	65,31	13,4	31,35	62,77	100	95,18	100	0,474074	

(c) Angle-Angle

	Crit3	Crit2	Crit1	Crit5	Crit6	Crit9	Crit7	Crit8	Crit4	NetFlows
A3	100	100	92,28	70,11	54,24	43,05	95,86	64,03	93,7	0,518519
A1	100	92,23	83,08	47,67	100	98,33	66,46	19,37	100	0,625926
A2	100	100	95,18	34,05	62,77	31,35	78,5	65,31	13,4	0,474074
A7	100	100	83,12	34,05	37,94	30,83	89,3	65,33	13,4	0,474074
A5	95,9	100	80,82	58,4	58,85	16,64	97,84	65,31	13,4	0,474074
A8	86,48	95,33	63,35	63,35	56,45	14,35	96,41	61,53	3,34	0,161111
A4	63,51	100	91,44	79,77	63,41	55,41	80,85	58,87	46,48	0,353703
A6	100	99,2	71,26	73,53	43,54	55,41	84,79	46,48	25,34	0,222
A9	85,92	94,65	28,54	34,05	0	43,06	95,18	65,31	13,4	0,474074
A16	75,7	85,07	11,75	44,88	16,79	0	99,64	96	15,68	-0,18518
A13	36,08	47,51	27,86	63,96	49,87	100	94,4	79,2	2,52	-0,23611
A15	50,44	68,59	10,87	66,74	51,82	10,81	97,64	74,51	24,68	-0,06759
A14	45,17	81,12	21,5	53,39	61,02	35,07	94,36	16,93	12,73	-0,17311
A11	46,2	76,67	37,45	62,32	51,35	19,87	87,09	55,21	25,34	-0,14352
A17	71,43	84,07	48,25	62,32	52,32	52,32	52,32	52,32	52,32	0,161111
A12	24,29	63,55	0,02	78,08	49,25	7,78	75,31	51,85	25,8	-0,15667
A19	26,28	50,04	10,49	39,18	35,24	35,07	23,24	58,4	22,64	0,57527

(d) Proximity-Proximity

Figure 21: Visualisation - EPI - Evaluations

	Cn1	Cn2	Cn3	Cn4	Cn5	Cn6	Cn7	Cn8	Cn9	Cn10	Cn11	NetFlow
A1	0.777778	0.888889	0.555556	0	0.944444	-0.222222	0.777778	0.777778	1	0.722222	0.518519	
A3	0.777778	0.444444	0.444444	-0.944444	0.444444	0.333333	0.333333	0.333333	1	0.722222	0.518519	
A2	0.777778	-0.555556	0.222222	-0.333333	-0.111111	0.777778	1	0.888889	0.777778	0.474074		
A5	0.777778	0.777778	0.111111	0.111111	0.444444	0.555556	-0.333333	0.333333	0.777778	0.777778	0.373079	
A4	-0.222222	0.888889	0.444444	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.373079
A7	0.777778	0.111111	0.333333	-0.333333	0.333333	0.111111	-0.444444	0.555556	0.777778	0.777778	0.246297	
A6	0.777778	-0.111111	0.777778	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.246297
A8	0.333333	0.555556	0	-0.666667	-0.555556	0.444444	-0.111111	0.111111	0.111111	0.111111	0.111111	0.111111
A9	0.111111	0.222222	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.076759
A15	-0.3333	0.222222	0.666667	0.555556	0.666667	0.222222	0.222222	0.222222	0.377778	0.555556	0.076759	
A10	-0.444444	0	0.444444	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.14952
A11	-0.555556	0.222222	-0.444444	-0.444444	0.166667	0.666667	-0.555556	-0.444444	-0.222222	0.17315		
A14	-0.111111	-0.333333	0.333333	-0.222222	0.555556	0.444444	0	0.777778	-0.333333	0.333333	0.17315	
A12	-0.111111	-0.666667	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.518519
A17	-0.888889	0.888889	0.222222	0.444444	0.166667	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.475278
A19												

(a) Netflow-angle

	Cn1	Cn3	Cn5	Cn7	Cn9	Cn11	Cn13	Cn15	Cn17	Cn19	Cn21	NetFlow
A1	0.888889	0.722222	0.777778	0.777778	-0.222222	0.777778	0.777778	0.777778	0.777778	0.722222	0.518519	
A3	0.444444	0.722222	0.777778	0.333333	0.333333	-0.944444	0.444444	0.444444	0.444444	0.444444	0.444444	0.518519
A2	-0.555556	0.722222	0.777778	0.777778	0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.474074
A5	0.777778	0.722222	0.444444	-0.333333	0.555556	0.333333	0.111111	0.444444	0.111111	0.111111	0.111111	0.37963
A4	-0.444444	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.362979
A7	0.111111	0.722222	0.777778	-0.444444	0.111111	0.555556	-0.944444	0.333333	0.333333	0.333333	0.333333	0.246297
A6	-0.111111	0.333333	0.777778	0.444444	-0.666667	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.246297
A9	0.555556	0.111111	0.333333	0.111111	0.444444	0.111111	-0.666667	-0.555556	0.111111	0.111111	0.111111	0.161111
A8	-0.222222	0.111111	0.111111	0.111111	0.333333	0.333333	0.333333	0.333333	0.333333	0.333333	0.333333	0.296129
A15	0.666667	0.555556	0.222222	0.222222	0.111111	0.111111	0.444444	0.444444	0.444444	0.444444	0.444444	0.444444
A16	0	-0.333333	0.444444	0.444444	-0.555556	-0.666667	0.444444	-0.444444	-0.166667	-0.166667	-0.166667	-0.17315
A10	-0.777778	0.222222	0.222222	0.666667	0.777778	0.444444	0.444444	0.444444	0.444444	0.444444	0.444444	0.444444
A13	0.333333	-1	-0.666667	0	0	0.333333	-0.777778	0.944444	0.777778	0.777778	0.777778	-0.23611
A18	-0.444444	-0.777778	0.111111	0	1	0.555556	-0.222222	-0.222222	0.888889	0.888889	0.888889	-0.45357
A12	-0.333333	-0.777778	-0.111111	-0.444444	0	0.222222	0.555556	0.555556	-0.333333	-0.333333	-0.333333	-0.3537
A17	-0.666667	-0.666667	0	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.555556	0.518519
A19	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.475278

(b) Netflow-Weight

	Cn1	Cn3	Cn5	Cn7	Cn9	Cn11	Cn13	Cn15	Cn17	Cn19	Cn21	NetFlow
A1	0.888889	0.722222	0.777778	0.777778	-0.222222	0.777778	0.777778	0.777778	0.777778	0.722222	0.518519	
A3	0.444444	0.722222	0.777778	0.333333	0.333333	-0.944444	0.444444	0.444444	0.444444	0.444444	0.444444	0.518519
A2	-0.555556	0.722222	0.777778	0.777778	0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.474074
A5	0.444444	0.777778	0.111111	0.111111	0.444444	0.555556	-0.333333	0.333333	0.722222	0.722222	0.37963	
A4	-0.444444	0.333333	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.362979
A7	-0.666667	0.333333	0.777778	-0.777778	0.944444	0	0	0.333333	-0.777778	0.944444	0.777778	-0.23611
A16	-0.333333	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.362979
A15	-0.333333	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.666667	0.362979
A18	-0.111111	0	1	0	1	0	0	0	0	0	0	0
A13	-0.111111	0	1	0	1	0	0	0	0	0	0	0
A10	-0.222222	0.777778	0.333333	0.777778	0.333333	0.777778	0.333333	0.777778	0.333333	0.777778	0.333333	0.296129
A6	-0.777778	-0.111111	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.474074
A12	-0.444444	-0.111111	0.666667	0.722222	0.722222	0.666667	0.444444	0.444444	0.222222	0.333333	0.333333	0.222222
A4	-0.222222	0.222222	0.222222	0.222222	0.111111	0.333333	-0.333333	-0.333333	-0.111111	0.111111	0.111111	0.111111
A16	0.666667	0	1	0.333333	0	0	0	0.944444	0.333333	0.777778	0.777778	-0.23611
A18	-0.777778	-0.444444	0.555556	0.111111	0	1	0	0.222222	0.888889	0.888889	0.888889	-0.45357
A14	0.333333	0.222222	0.444444	-0.555556	0.666667	0.111111	0.333333	0.333333	0	0.444444	0.444444	-0.17315
A11	-0.444444	-0.333333	-0.111111	-0.222222	0.111111	0.333333	-0.944444	-0.444444	0	0.444444	0.444444	-0.14952
A12	-0.111111	-0.777778	0	-1	-0.444444	0.555556	-0.777778	-0.666667	-0.555556	-0.333333	-0.222222	-0.3537
A17	-0.666667	-0.666667	0	0.555556	-0.555556	0.111111	0.777778	-0.666667	-0.555556	-0.333333	-0.222222	-0.3537
A19	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.888889	0.475278

(c) Angle-Angle

	Cn1	Cn3	Cn5	Cn7	Cn9	Cn11	Cn13	Cn15	Cn17	Cn19	Cn21	NetFlow
A3	0.777778	0.722222	0.777778	0.777778	-0.222222	0.777778	0.777778	0.777778	0.777778	0.722222	0.518519	
A1	0.777778	0.888889	0.555556	0	0.944444	0.944444	0.944444	0.944444	0.944444	0.944444	0.944444	0.518519
A2	0.777778	0.111111	0.333333	0.333333	-0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.474074
A7	0.777778	0.111111	0.333333	0.333333	-0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.111111	0.474074
A5	0.444444	0.722222	0.555556	-0.444444	-0.111111	0.333333	0.555556	-0.444444	0.111111	0.111111	0.111111	0.37963
A4	-0.222222	0.222222	0.222222	0.222222	0.111111	0.333333	-0.222222	-0.222				

Résumé