

Multiple decrements tables with lifecontingencies package

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Abstract

This paper introduces the `mdt` class within **lifecontingencies** R package, that handles multiple decrements models for actuarial applications. Applied examples will show how to perform demographic and actuarial calculation with the package.

Keywords: actuarial mathematics, multiple decrement models, **lifecontingencies**.

1. Introduction

1.1. Multiple Decrement Models within R

As of 2014 no R package provides easy tools to manage multiple decrement tables for actuarial applications. Multiple decrement tables are the basis of many applications, as demography, medicine and actuarial science.

Until now no R package provides a good tool to manage multiple decrement tables, even if [Deshmukh \(2012\)](#) provides an R based focus on multiple decrement tables with applications in R. The topic is deeply related to multistate analysis of life histories on which [Willekens \(2014\)](#) provide a very good introduction.

This paper introduces the `mdt` class that has been specifically engineered to manage multiple decrements models with R. Applied examples will follows.

1.2. A review of Discrete Multiple Decrement Mathematics

Following notation in [Finan \(2014\)](#), we provide definitions of the key quantities that allow to understand the main concepts regarding Multiple Decrement (MD) theory. Be $l_x^{(\tau)} = \sum_{j=1 \dots m} l_x^{(j)}$ survivors to age x that will, at future ages, be fully depleted by m causes of decrement. $d_x^{(j)} = l_x^{(j)} - l_{x+1}^{(j)}$ represents the expected number of lives exiting from the population between ages x and $x + 1$ due to decrement j . Therefore it follows that ${}_n d_x^{(j)} = \sum_{t=0 \dots n-1} d_{x+t}^{(j)}$. The probability that a life x will leave the group within one year as a result of decrement j is ${}_n q_x^{(j)} = \frac{{}_n d_x^{(j)}}{l_x^{(\tau)}}$. It follows that $q_x^{(\tau)} = \sum_{j=1}^m q_x^{(j)}$ and that ${}_t q_x^{(\tau)} = 1 - {}_t p_x^{(\tau)} = \sum_j {}_t q_x^{(\tau)}$.

2. Multiple Decrement Models within lifecontingencies Package

2.1. The mdt class

Examples in this paper are worked on slides provided in [Valdez \(2011\)](#). First of all, we load the R package.

```
R> library(lifecontingencies)
```

Then we create a `mdt` class object. We can use the first example found on ([?](#), p. 4).

```
R> valdezDf<-data.frame(
+           x=c(50:54),
+           lx=c(4832555,4821937,4810206,4797185,4782737),
+           hearth=c(5168, 5363, 5618, 5929, 6277),
+           accidents=c(1157, 1206, 1443, 1679,2152),
+           other=c(4293,5162,5960,6840,7631)
+ )
R> valdezMdt<-new("mdt",name="ValdezExample",table=valdezDf)
```

Added fictional decrement below last `x` and completed `x` and `lx` until zero....
Completed the table at top, all decrements on first cause

The `mdt` class is an S4 class object ([Chambers 2008](#)) comprised by a character slot `name` and a `data.frame` slot `table` that is composed by following columns:

1. `x`: the age, from 0 to ω .
2. `lx`: the subject living (at risk) at the beginning of age.
3. one or more columns for different causes of decrements.

Values within `table` item represents absolute number of subjects at risk at the beginning of age x and dying for cause j during period $x - x + 1$.

Within the various methods defined within the `mdt` class, `setValidity` performs consistency checks to properly create the `mdt` object. In particular, it verifies whether:

1. `x` and `lx` exist and that they are consistent. `x` should start from 0 and flows by increments of one. The first `lx` value should be equal to the sum of all decrements and that $l_x = l_{x-1} - (d_{x-1,1} + d_{x-1,2} + \dots + d_{x-1,k})$ for any x .
2. If the decrements (or `x` and `lx`) have been provided only for partial ages, the table is completed below (from 0 to l_{x-1}) assuming a decrement rate of 0.01 for the first cause of death.
3. if the decrements at last provided age, ω , do not sum to l_ω , the table is incremented by one row such as $l_{\omega+1} = l_\omega - (d_{\omega,1} + d_{\omega,2} + \dots + d_{\omega,j})$.

As shown, when the table is sanitized the operations performed are reported on logs.

An internal function, `.tableSanitizer` tries to fix the limitations on the input table in order it to meet the class definition requirements.

Table can be viewed thanks to a `print` and `show` method (output omitted for simplicity). Similarly, it is possible to export a `mdt` to a `data.frame` or to a `markovchainList` object (from `markovchain` package).

```
R> print(valdezMdt)
```

```
R> valdezDf<-as(valdezMdt,"data.frame")
```

```
R> require(markovchain)
```

```
R> valdezMarkovChainList<-as(valdezMdt,"markovchainList")
```

Two specific methods have been defined for `mdt` class objects: `getOmega`, that returns the maximum attainable age (similar to the one of `lifetable` class), and `getDecrements`, that returns the decrements (by means of the names within table slot different from `x` and `lx`).

```
R> getOmega(valdezMdt)
```

```
[1] 55
```

```
R> getDecrements(valdezMdt)
```

```
[1] "hearth"      "accidents" "other"
```

A summary method is available as well.

```
R> summary(valdezMdt)
```

```
This is Multiple Decrements Table: ValdezExample
```

```
Omega age is: 55
```

```
Stored decrements are: hearth accidents other
```

2.2. Decrement probabilities calculation

The `lifecontingencies` package makes easy to compute $d_x^{(j)}$, ${}_nd_x^{(j)}$ as well as ${}_nd_x^{(\tau)}$ quantities thanks to `dxt` function.

```
R> dxt(valdezMdt,x=51,decrement="other")
```

```
[1] 5162
```

```
R> dxt(valdezMdt,x=51,t=2, decrement="other")
```

```
[1] 11122
```

```
R> dxt(valdezMdt,x=51)
```

```
[1] 11731
```

Probabilities could be computed as well.

```
R> dxt(valdezMdt,x=51,t=2, decrement="other")
```

```
[1] 11122
```

```
R> pxt(valdezMdt,x=50,t=3)
```

```
[1] 0.9926809
```

```
R> qxt(valdezMdt,x=53,t=2,decrement=1)
```

```
[1] 0.002544409
```

It is possible to generate random trajectories of a life subject to multiple cause of decrements as the following code shows.

```
R> rmdt(n = 2,object = valdezMdt,x = 50,t = 2)
```

```

  1      2
50 "alive" "alive"
51 "alive" "alive"
52 "alive" "alive"
```

3. Actuarial Applications

The package now offers limited capabilities to fit multiple decrement insurances, e.g. $A_{x:\overline{n}|}^1$ ⁽¹⁾

The example in (Finan 2014, p. 674), cites: A 3-year term issued to (16) pays 20,000 at the end of year of death if death results from an accident. The mdt table is below created.

```

R> myTable<-data.frame(x=c(16,17,18),
+   lx=c(20000,17600,14520),
+   da=c(1300,1870,2380),
+   doc=c(1100,1210,1331)
+ )
R> myMdt<-new("mdt",table=myTable,name="Sample")
```

Added fictional decrement below last x and completed x and lx until zero....
Completed the table at top, all decrements on first cause

The value of $A_{16:\overline{3}|}^1{}^{(a)}$ is below calculated

```
R> Axn.mdt(object=myMdt,x=16,i=.1,decrement="da")
```

```
[1] 0.1363636
```

References

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