$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$e = 2.7$

$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}(1)\, dt = \left.\frac{e^{-st}}{-s}\right|_0^\infty$$

$e^{\infty} = (2.7)^{\infty} = \infty$

$e^{-\infty} = \frac{1}{\infty} = \frac{1}{\frac{1}{0}} = 0$

$$= -\frac{1}{s}\left[e^{-\infty} - e^{0}\right]$$

$$= -\frac{1}{s}\left[0 - 1\right] = \frac{1}{s}$$

$$\mathcal{L}\{\sin(\underline{\omega t} + \delta)\} = \mathcal{L}\{\sin\omega t \cos\delta + \cos\omega t \sin\delta\}$$

$$= \cos\delta\, \mathcal{L}\{\sin\omega t\} + \sin\delta\, \mathcal{L}\{\cos\omega t\}$$

$$= \cos\delta\left(\frac{\omega}{s^2+\omega^2}\right) + \sin\delta\left(\frac{s}{s^2+\omega^2}\right)$$

$$= \frac{\omega\cos\delta + s\sin\delta}{s^2+\omega^2}$$

\#

① $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$

$\mathcal{L}\{1\} = \frac{1}{s}$

$\mathcal{L}\{e^{at}(1)\} = \frac{1}{s-a}$

② $\mathcal{L}\{e^{at} t^{b}\} = \frac{\Gamma(b+1)}{(s-a)^{b+1}}$

③ $\mathcal{L}\{e^{at} t^{n}\} = \frac{n!}{(s-a)^{n+1}}$

④ $\mathcal{L}\{e^{at} e^{bt}\} = \frac{1}{(s-a)-b}$

⑤ $\mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2+b^2}$

⑥ $\mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2+b^2}$

⑦ $\mathcal{L}\{e^{at}\sinh bt\} = \frac{b}{(s-a)^2-b^2}$

⑧ $\mathcal{L}\{e^{at}\cosh bt\} = \frac{s-a}{(s-a)^2-b^2}$

(i) $\mathcal{L}\{t^3 e^{-3t}\} = \mathcal{L}\{e^{-3t}(t^3)\}$, $\mathcal{L}\{t^3\} = \frac{3!}{s^4} = \frac{6}{s^4}$

$= \frac{6}{(s-(-3))^4} = \frac{6}{(s+3)^4}$ #

(ii) $\mathcal{L}\{e^{-3t}(2\cos 5t - 3\sin 5t)\}$

$= 2\mathcal{L}\{e^{-3t}\cos 5t\} - 3\mathcal{L}\{e^{-3t}\sin 5t\} = 2\,\frac{s-(-3)}{(s-(-3))^2+25} - 3\,\frac{5}{(s-(-3))^2+25}$

$= \frac{2(s+3)-15}{(s+3)^2+25} = \frac{2s+6-15}{s^2+6s+9+25} = \boxed{\frac{2s-9}{s^2+6s+34}}$

(iii) $\mathcal{L}\left\{e^{t}(3\sinh 2t - 5\cosh 2t)\right\}$

H.W $\mathcal{L}\left\{e^{-t}(3\sinh 2t - 5\cosh 2t)\right\}$

$$= 3\mathcal{L}\left\{e^{t}\sinh 2t\right\} - 5\mathcal{L}\left\{e^{t}\cosh 2t\right\}$$

$$= 3\,\frac{2}{(s-1)^2-4} - 5\,\frac{s-1}{(s-1)^2-4}$$

$$= \frac{6-5(s-1)}{(s-1)^2-4} = \frac{6-5s+5}{s^2-2s+1-4} = \boxed{\frac{11-5s}{s^2-2s-3}}$$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-st}(g(t))\,dt = \int e^{-st}(0) + \int_{\frac{\pi}{2}}^{\infty} e^{-st}\cos\left(t-\frac{\pi}{2}\right)dt$$

let $t-\frac{\pi}{2} = u \Rightarrow dt = du \qquad t = u + \frac{\pi}{2}$

at $t = \frac{\pi}{2} \rightarrow u = 0$

at $t = \infty \rightarrow u = \infty$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-st}\cos u\,du = \int_0^\infty e^{-s\left(u+\frac{\pi}{2}\right)}\cos u\,du$$

$$= e^{-\frac{s\pi}{2}}\int_0^\infty e^{-su}\cos u\,du = e^{-\frac{s\pi}{2}}\mathcal{L}\{\cos u\}$$

$$= e^{-\frac{s\pi}{2}}\left(\frac{s}{s^2+1}\right)$$

Ex: Find $\mathcal{L}\{g(t)\}$; $g(t) = \begin{cases} (t-2)^3 & , t > 2 \\ 0 & , t < 2 \end{cases}$

$$\mathcal{L}\{g(t)\} = \int_0^2 e^{-st}(0)\,dt + \int_2^\infty e^{-st}(t-2)^3\,dt$$

let $t-2 = u \Rightarrow dt = du$, at $t = 2 \to u = 0$

$t = u+2$ at $t = \infty \to u = \infty$

$$= \int_0^\infty e^{-s(u+2)} u^3\,du = e^{-2s}\int_0^\infty e^{-su} u^3\,du$$

$$= e^{-2s}\mathcal{L}\{u^3\} = e^{-2s}\left(\frac{3!}{s^4}\right)$$

Find $\mathcal{L}\{f(\lambda t)\}$

(i) $f(t) = t^n \to \mathcal{L}\{(\lambda t)^n\}$, $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$

$$= \frac{1}{\lambda}\,\frac{n!}{\left(\frac{s}{\lambda}\right)^{n+1}} = \frac{n!}{\lambda\,\frac{s^{n+1}}{\lambda^{n+1}}} = \boxed{\frac{\lambda^n n!}{s^{n+1}}} \#$$

(ii) $f(t) = e^t \to \mathcal{L}\{e^{\lambda t}\} = \dfrac{1}{\lambda}\,\dfrac{1}{\frac{s}{\lambda} - 1} = \dfrac{1}{\cancel{\lambda}}\,\dfrac{1}{\frac{s-\lambda}{\cancel{\lambda}}}$

$$= \boxed{\frac{1}{s-\lambda}} \#$$

Ex If $\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$

Find $\boxed{\mathcal{L}\left\{\frac{\sin at}{t}\right\}}$

$$\mathcal{L}\left\{\frac{\sin at}{at}\right\} = \cancel{\frac{1}{a}}\,\mathcal{L}\left\{\frac{\sin at}{t}\right\} = \cancel{\frac{1}{a}}\tan^{-1}\frac{1}{\left(\frac{s}{a}\right)} = \boxed{\tan^{-1}\frac{a}{s}}$$

$$\mathcal{L}\left\{\frac{\sin at}{t}\right\}$$

$$(n+1)\left(\frac{n!}{s\, s^{n+1}}\right) = \cancel{s}\left(\ldots\right)$$

$$L\left(t^{n+1}\right) = \frac{(n+1)n!}{s^{n+2}} = \frac{(n+1)!}{s^{n+2}}$$

Ex Given $L(t^3) = \frac{6}{s^4}$ $\Rightarrow$ Find $L\left(t^6\right)$

$f(t) = t^6$

$f'(t) = 6t^5$

$f''(t) = 30t^4$

$f'''(t) = 120t^3$

$L(f''') = s^3L(f) - s^2f(0) - sf'(0) - f''(0)$

$120L(t^3) = s^3L(t^6) - s^2(0) - 0 - 0$

$120L(t^3) = s^3\left[\frac{6!}{s^7}\right] = \frac{720\cancel{s^3}}{\cancel{s^7}}$

$L(t^3) = \frac{720}{120s^4} = \frac{72}{12s^4} = \boxed{\frac{6}{s^4}}$ #

$$-b^2\mathcal{L}\{t\sin bt\} + 2b\,\mathcal{L}\{\cos bt\} = s^2\mathcal{L}\{t\sin bt\} - s(0) - 0$$

$$2b\,\mathcal{L}\{\cos bt\} = (b^2+s^2)\,\mathcal{L}\{t\sin bt\}$$

$$2b\left(\frac{s}{s^2+a^2}\right) = (b^2+s^2)\,\mathcal{L}\{\;y\;\}$$

$$\boxed{\mathcal{L}\{t\sin bt\} = \frac{2bs}{(b^2+s^2)(s^2+a^2)}}$$