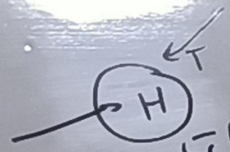
Ch① Probabilities

- **Random Experiment**
- **Sample Space** فضاء العينة
  The set of all outcomes from random experiments, denoted by S.

ex① toss Coin one time.

$S = \{ H, T \}$



عدد الحالات = $2^1 = 2$

ex② toss Coin two time.

$S = \{ HH, HT, TH, TT \}$

$2^{②} = 4$

Ch(1) Probabilities

ex(3) toss coin 3 times

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$2^3 = 8$

ex(4) toss die / dice (نرد) one time

$S = \{1, 2, 3, 4, 5, 6\}$

$6^1 = 6$

ex(5) toss dice two times

$6^2 = 36$

$S = \{(1,1), (1,2), (1,3) \dots (1,6), (2,1), (2,2), (2,3) \dots (2,6), \dots (6,1), (6,1) \dots (6,6)\}$

Events الحوادث :

Ch(1) Probabilities

(1)

A subset of sample space

we denote it by Capital letters

A , B , . . .

$S = \{HH, HT, TH, TT\}$

(ex)

(ex) toss Coin two time the event of showing

(H: Head) only on time

$A = \{HT, TH\}$

ظهور الصورة مرة واحدة فقط

Events الحوادث

Probabilities

A subset of Sample Space

we denote it by Capital letters $= \{HH, HT, TH, TT\}$

$A, B, \ldots$

ex dice two time: The event that The Sum of the two faces $= 9$

$B = \{(3,6), (6,3), (4,5), (5,4)\}$

ex The 1st face > 2nd face

$S = \{(1,1), (1,2) \ldots (1,6)$
$(2,1) (2,2) \ldots (2,6)$
$(3,1) \ldots (3,6)$
$\vdots$
$(6,1) \ldots (6,6)\}$

$C = \{(2,1),$
$(3,1),(3,2),$
$(4,1)(4,2)(4,3),$
$(5,1),(5,2)(5,3)(5,4),$
$(6,1),(6,2)(6,3),(6,4),(6,5)\}$

Classic Def. of Probabilities Cnt'd Probabilities

$$P(A) = \frac{N(A)}{N(S)}$$

(ex) $S = \{HH, HT, TH, TT\}$

$A = \{HT, TH\}$

$B = \{HH, TH, HT\}$

$C = \{TT\}$

$P(A) = \frac{2}{4} = 0.5$

$P(B) = \frac{3}{4} = 0.75$

$P(C) = \frac{1}{4} = 0.25$

Modern Def. of Probabilities

Axioms

① $P(\phi) = 0$ (value)

ϕ ← set

impossible event

② $P(S) = 1$

S ← Certain event

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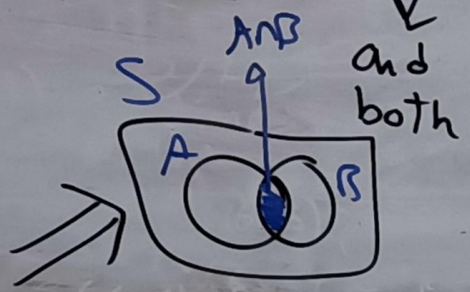
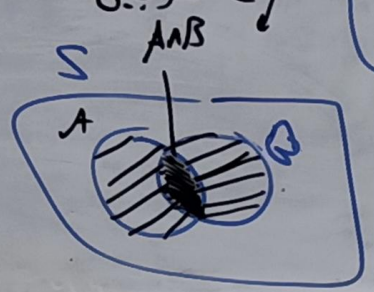
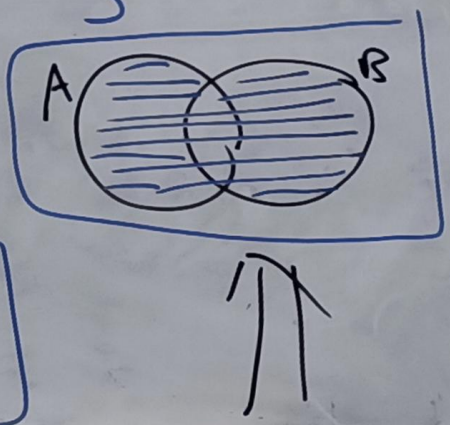
③ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\cup$ ← or, either of

$\cap$ ← and, both





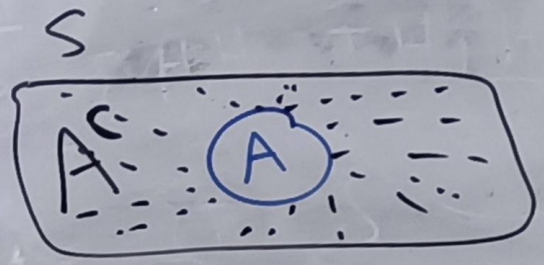


Modern Def. of Probabilities — Ch① Probabilities

↳ • Axioms

④ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

⑤ $P(A^c) = 1 - P(A)$

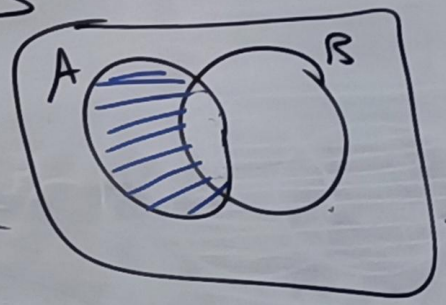


$P(S) = 1$

$P(A) + P(A^c) = 1$

⑥ • $P(A \cap B^c) = P(A) - P(A \cap B)$

• $P(B \cap A^c) = P(B) - P(A \cap B)$



⑦ • $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$

• $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B)$

Conditional Probability

الاحتمال المشروط

Cond Probabilities

(1)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

$P(A \mid B)$

A: الحدث الأساسي

B: الحدث المعطى

- Condition that
- if we know that
- Given that
- it happen to be
- if

- what is the Probability
- find ...

- $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$

Example (1) $P(A)=\frac{1}{2}$ $P(B)=\frac{1}{3}$ Ch(1) Probabilities

$P(A\cap B)=\frac{1}{4}$ Find The following Probabilities

$0 \le P(A) \le 1$

① $P(A^c) = 1-P(A) = 1-\frac{1}{2} = 0.5$

$P(B^c) = 1-P(B) = 1-\frac{1}{3} = \frac{2}{3} = 0.66$

② $P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{1}{2}+\frac{1}{3}-\frac{1}{4} = 7/12$

③ $P(A\cap B^c) = P(A)-P(A\cap B) = \frac{1}{2}-\frac{1}{4} = 0.25$

④ $P(A^c\cap B) = P(B)-P(A\cap B) = \frac{1}{3}-\frac{1}{4} = 1/12$

⑤ $P(A^c\cup B^c) = P(A\cap B)^c = 1-P(A\cap B) = 1-\frac{1}{4} = 0.75$

⑥ $P(A^c\cap B^c) = P(A\cup B)^c = 1-P(A\cup B) = 1-7/12 = 5/12$

⑦ $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/4}{1/3} = 3/4 = 0.75$

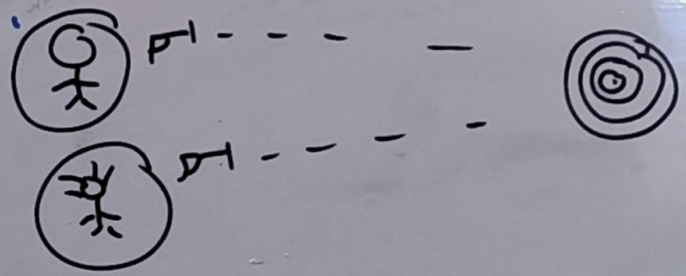
⑧ $P(A^c|B) = \frac{P(A^c\cap B)}{P(B)} = \frac{1/12}{1/3} = 3/12 = 0.25$

⑨ $P(B|A^c) = \frac{P(B\cap A^c)}{P(A^c)} = \frac{1/12}{1/2} = 1/6$

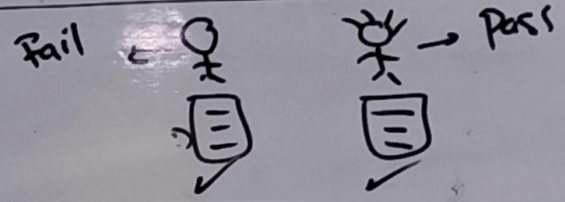
الاستقلال
Independence

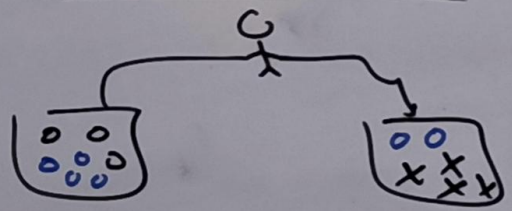
وقوع أحدهم لا يؤثر على وقوع الآخر

ex



ex

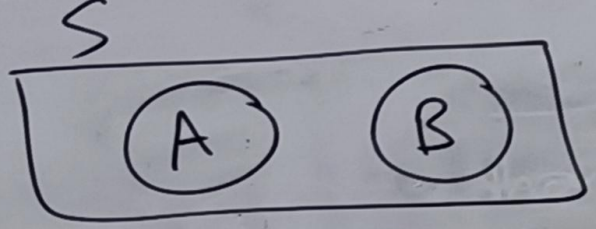




$$P(A \cap B) = P(A) \cdot P(B)$$

التناافي
Disjoint Event (Mutually Exclusive)

وقوع أحدهم يمنع حدوث الآخر



$$A \cap B = \phi$$

ex ⟶ أشارة المرور

$$P(A \cap B) = 0$$

Example Let $P(A) = 1/4$, $P(B) = 2/5$

for (A) and (B) Independent Events; Find

① $P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{2}{5} = 0.1$

② $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B) = \frac{1}{4} + \frac{2}{5} - 0.1 = 0.55$

③ $P(A \cap B^c) = P(A) - P(A \cap B) = \frac{1}{4} - 0.1$

④ $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B) = 1 - 0.1 = 0.9$

⑤ $P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B) = 1 - \left[\frac{1}{4} + \frac{2}{5} - 0.1\right] = 0.45$

⑥ $P(A \mid B) = \frac{P(A) \cdot P(B)}{P(B)} = P(A) = \frac{1}{4}$

⑦ $P(B \mid A) = \frac{P(A) \cdot P(B)}{P(A)} = P(B) = \frac{2}{5}$

⑧ $P(A^c \mid B) = P(A^c) = 1 - P(A) = 0.75$

⑨ $P(A^c \mid B^c) = P(A^c) = \quad = 0.75$

Example Let $P(A) = 1/4$, $P(B) = 2/5$

for (A) and (B) disjoint Events; Find

① $P(A \cap B) = P(\phi) = 0$

② $P(A \cup B) = P(A) + P(B) - 0 = \frac{1}{4} + \frac{2}{5} - 0 = 0.65$

③ $P(A \cap B^c) = P(A) - P(A \cap B) = P(A) - 0 = \frac{1}{4}$

④ $P(A^c \cup B^c) = 1 - P(A \cap B) =$ (1)

⑤ $P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.65 = 0.35$

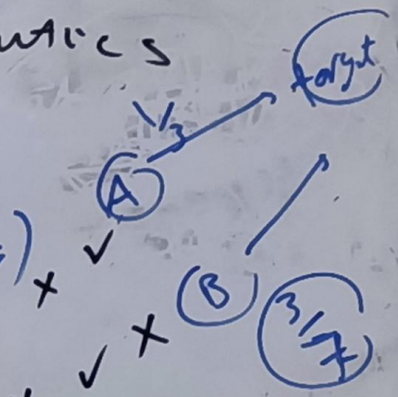
⑥ $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$

⑦ $P(B|A) = \frac{P(B \cap A)}{P(A)} = 0$

⑧ $P(A^c|B) = \frac{P(A^c \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = 1$ ($P(A \cap B) \to 0$)

⑨ $P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{0.35}{1 - 2/5} = 7/12$

Example ② let (A) and (B) shoot at the same time a target, $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{7}$, find the following probabilities



① The target will be hit.

$= P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{3} + \frac{3}{7} - \left(\frac{1}{3}\right)\left(\frac{3}{7}\right)$

② (A) hit (and) (B) not

$= P(A \cap B^c) = P(A) - P(A \cap B) = P(A) - P(A)P(B) =$

③ at least one of Them will hit.

$P(A \cap B^c) + P(A^c \cap B) + P(A \cap B)$

④ at most " " = = =

$P(A \cap B^c) + P(A^c \cap B) + P(A^c \cap B^c)$

⑤ only one of Them will hit The target

$P(A \cap B^c) + P(A^c \cap B)$

⑥ احتمال انه يصيب واحد على الاقل $= 1 - P(\text{الاحتمال المضاد}) = 1 - P(A \cup B)$

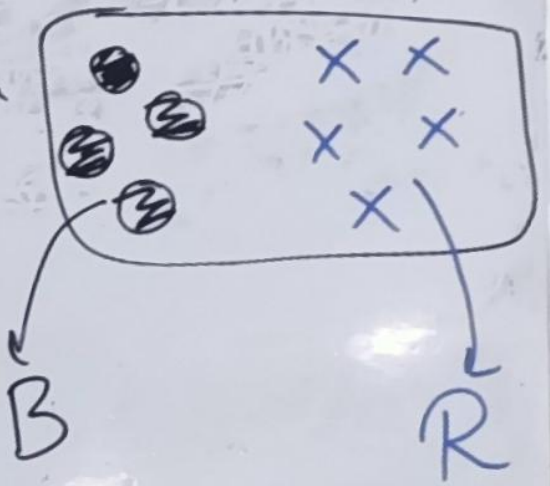
- Multiplication Rule

قاعدة الضرب

بشكل عشوائي . at random

بشكل متتابع . succession for

بدون ارجاع . without replacement



في حالة سحب كرتين على التوالي دون ارجاع وبشكل عشوائي

- $P(B_1 \cap R_2) = P(B_1) \cdot P(R_2 / B_1)$

التتابع

$$= \frac{4}{9} \cdot \frac{5}{8} = \frac{20}{72}$$

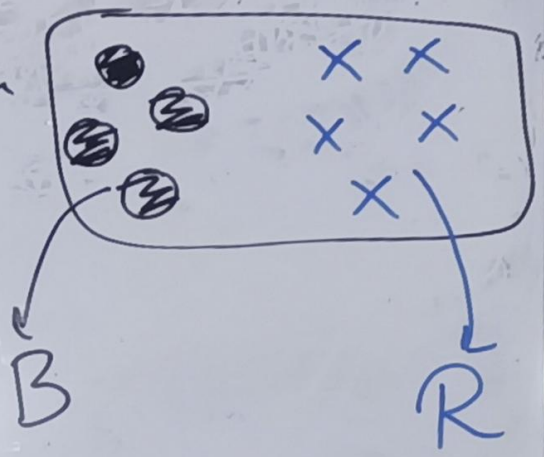
- $P[B_1 \cap B_2] = P(B_1) \cdot P(B_2 | B_1) = \frac{4}{9} \cdot \frac{3}{8} = \frac{12}{72}$

• Multiplication Rule قاعدة الضرب

بشكل عشوائي . at random

بشكل متتابع . succession form

بدون ارجاع . without replacement
بدون اعادة replacement



Generally

- $P(A \cap B) = P(A) \cdot P[B|A]$

- $P[A \cap B \cap C] = P(A) \cdot P[B|A] \cdot P[C|A \cap B]$

- $P[A \cap B \cap C \cap D] = P(A) \cdot P(B|A) \cdot P[C|(A \cap B)] \cdot P[D|A \cap B \cap C]$

• Total Probability

الاحتمال الكلي

Find

$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \cdots + P(A_n \cap B)$

$P(B)$ اكرن الغلس

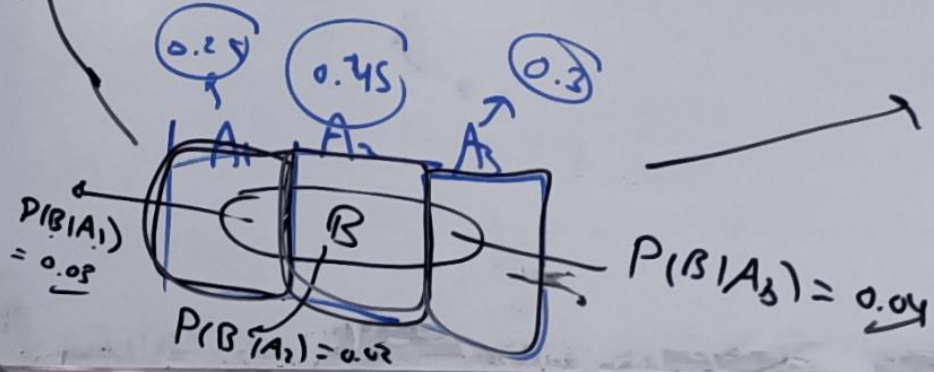
$P(A_1 \cap B)$ فاحص العزب

$P(B) = P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + \cdots P(A_n) \cdot P(B|A_n)$

$$P(B) = \sum_{i=1}^{n} P(A_i)\, P(B|A_i)$$

$A_1, A_2 \ldots A_n$ are Partion of S

- $A_i \cap A_j = \phi$ ، $i \neq j$
- $A_1 \cup A_2 \cup \cdots \cup A_n = S$



مثال

Q Find

① $P(B)$

② $P(A_3 \cap B)$

③ $P(A_1 \cap B)$

i	$P(A)$	$P(B \mid A)$
1	0.25	0.03
2	0.45	0.02
3	0.3	0.04

Sol

① total

$$P(B) = \sum_{i=1}^{3} P(A_i)P(B \mid A_i) = P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + P(A_3) \cdot P(B \mid A_3)$$

$$= (0.25)(0.03) + (0.45)(0.02) + (0.3)(0.04)$$

$$=$$

② $P(A_3 \cap B) = P(A_3) \cdot P(B \mid A_3) = (0.3)(0.04) =$

③ $P(A_1 \cap B) = P(A_1) \cdot P(B \mid A_1) = (0.25)(0.03) =$

Q Find

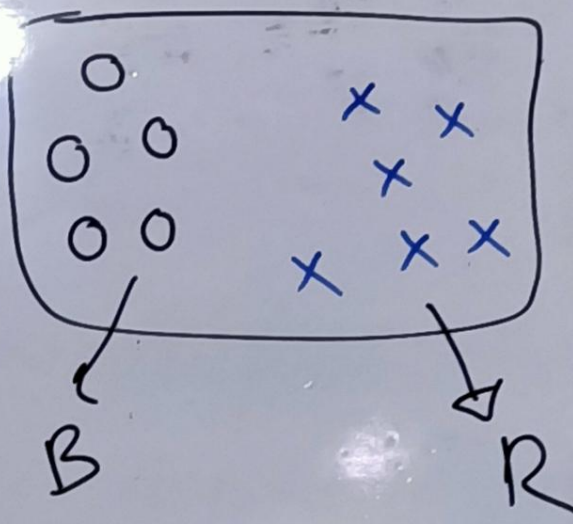
with replacement

مثال لحجر بإرجاع

① $P(B_1 \cap B_2 \cap R_3)$ ←

$= P(B_1) \cdot P(B_2) \cdot P(R_3)$

$= \frac{5}{11} \cdot \frac{5}{11} \cdot \frac{6}{11}$



Example

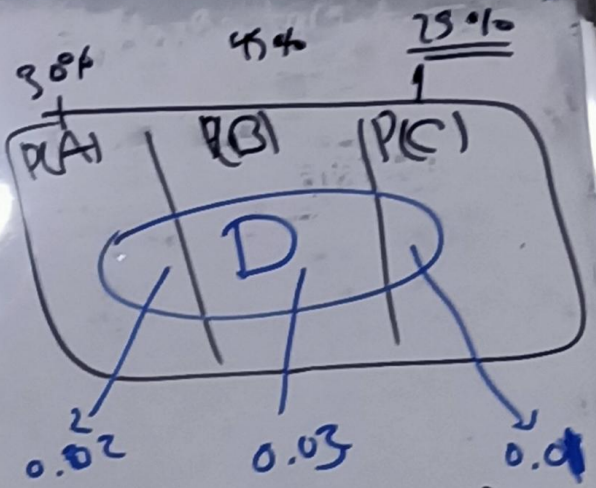
three machine

Find

① P(D)

② P(A ∩ D)

Sol



① $P(D) = P(A)\cdot P(D|A) + P(B)\cdot P(D|B) + P(C)\cdot P(D|C)$

$= (30\%)(0.02) + (45\%)(0.03) + (25\%)(0.01)$

② $P(A \cap D) = P(A)\cdot P(D|A)$

$= (0.30)(0.02)$