

$$[1] P(\emptyset) = 0$$

$$[2] P(S) = 1$$

$$[3] 0 \leq P(A) \leq 1$$

$$[4] P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$[5] P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$[6] P(A - B) = P(A \cap B^c) = P(A) - P(A \cap B)$$

$$[7] P(A^c) = 1 - P(A)$$

$$[8] P(A^c \cap B^c) = P(A \cup B)^c$$

$$[9] P(A^c \cup B^c) = P(A \cap B)^c$$

23 P

4

$$P(A \cup B) = 0.76$$

$$P(A) = ?$$

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$, P(A \cup B^c) = 0.87$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.76$$

$$P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c) = 0.87$$

$$P(A) + 1 - P(B) - P(A) + P(A \cap B) = 0.87$$

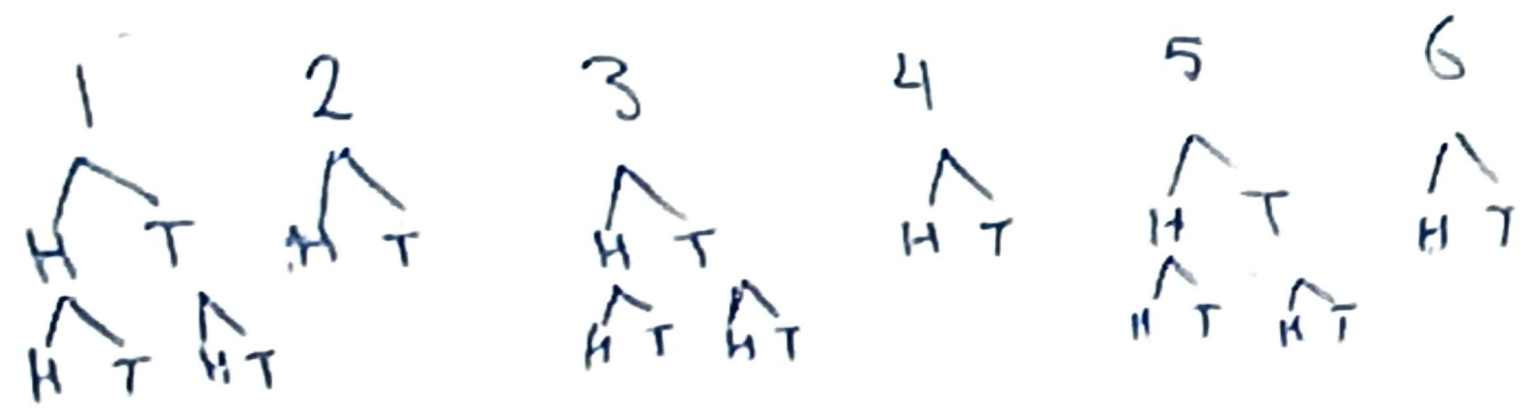
$$1 - P(B) + P(A \cap B) = 0.87$$

$$1 + P(A) = 0.76 + 0.87 \Rightarrow P(A) = 0.63$$

23 P  
 [5]

dice : 2, 4, 6  $\Rightarrow$  Gin once

dice : 1, 3, 5  $\Rightarrow$  Gin twice



$S = \{1HH, 1HT, 1TH, 1TT, 2H, 2T, \dots, 6T\}$

$$\frac{2^3}{2^3} P$$

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Let A be the event of having at least one head

$$A = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$$

$$P(A) = \frac{7}{8}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = P(A)$$

$$A, B \text{ independent} \Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

II

$$S = \{(1,1), (1,2), (1,3), \dots, (6,6)\}$$

Let A be the event that the sum is 9

Let B be the event that at least one die showed 6

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{2}{36}}{\frac{4}{36}} = \frac{1}{2}$$

$$A = \{(3,6), (4,5), (5,4), (6,3)\} \quad P(A) = \frac{4}{36}$$

$$B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,1), \dots, (6,6)\}$$

$$A \cap B = \{(3,6), (6,3)\} \quad P(A \cap B) = \frac{2}{36}$$

Q6] A, B independent

$$P(A \cap B^c)$$

$$= P(A) - P(A \cap B)$$

$$= P(A) - P(A) \cdot P(B)$$

$$= P(A) [1 - P(B)]$$

$$= P(A) - P(B^c)$$

A, B<sup>c</sup> are independent

??

$$P(A \cap B^c) = P(A) \cdot P(B^c)$$



[6] A, B independent

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= P(A \cup B)^c \\ &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A) \cdot P(B) \\ &= P(A^c) - P(B) [1 - P(A)] \\ &= P(A^c) - P(B) \cdot P(A^c) \end{aligned}$$

??

$$\begin{aligned} P(\bar{A} \cap \bar{B}) &= P(\bar{A}) \cdot P(\bar{B}) \\ &= P(A^c) [1 - P(B)] \\ &= P(A^c) \cdot P(B^c) \end{aligned}$$

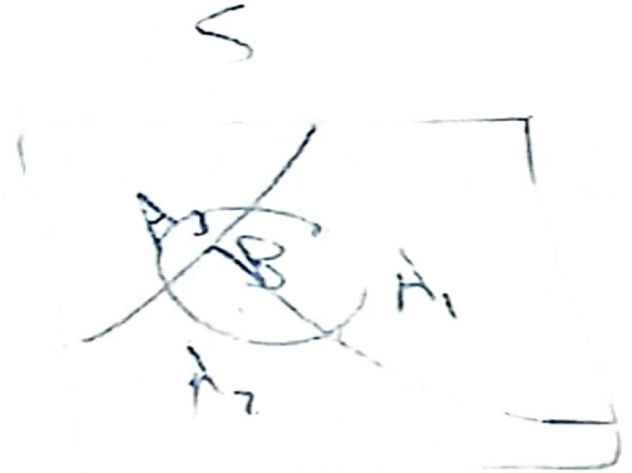


$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(B) = P(A \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$$

$$P(B) = P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + P(A_3) \cdot P(B|A_3)$$

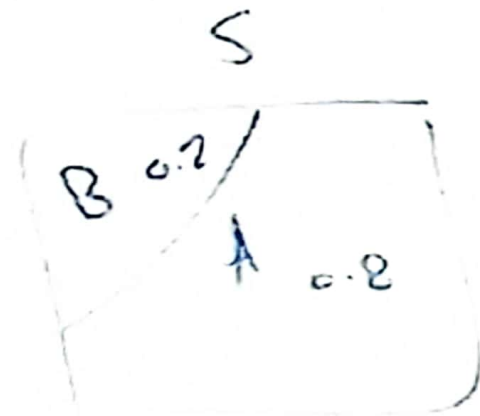
$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{P(A_1) \cdot P(B|A_1)}{P(B)}$$



$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$P(A) = 0.8, P(B) = 0.2$$

Let D be the event the product is defective



$$P(D|A) = 0.05$$

$$P(D|B) = 0.01$$

$$\begin{aligned} \text{a) } P(A \cap D^c) &= P(A) - P(A \cap D) \\ &= P(A) - P(A) \cdot P(D|A) \\ &= 0.8 - (0.8)(0.05) = \end{aligned}$$

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$P(A) = 0.8, \quad P(B) = 0.2, \quad P(D|A) = 0.05$$

$$P(D|B) = 0.01$$

$$\begin{aligned} \text{(b)} \quad P(D) &= P(D \cap A) + P(D \cap B) \\ &= P(A) \cdot P(D|A) + P(B) \cdot P(D|B) \\ &= (0.8)(0.05) + (0.2)(0.01) = \end{aligned}$$

$$P(A) = 0.3, \quad P(B) = 0.45, \quad P(C) = 0.25$$

Let  $D$  be event that the product is defective

$$P(D|A) = 0.02$$

$$P(D|B) = 0.03$$

$$P(D|C) = 0.01$$

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{P(A) \cdot P(D|A)}{P(D)}$$

$$\begin{aligned} P(D) &= P(D \cap A) + P(D \cap B) + P(D \cap C) \\ &= P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C) \end{aligned}$$