LapPlace Transforms

$$\int_0^{\pi/6} \sin 3x \, dx = \frac{\cos 3x}{-3}\Big|_0^{\pi/6} = -\frac{1}{3}\left[\cos 3\frac{\pi}{6} - \cos 0\right]$$

$$= -\frac{1}{3}\left[0 - 1\right] = \frac{1}{3}$$

$$\int_0^1 e^{-2x} \, dx = \frac{e^{-2x}}{-2}\Big|_0^1 = -\frac{1}{2}\left[e^{-2} - 1\right]$$

$$\int_{-2}^1 f(x) \, dx \qquad f(x) = \begin{cases} 2x - 1 & x \geq 0 \\ 2x + 1 & x < 0 \end{cases}$$

$$= \int_{-2}^0 (2x+1)\,dx + \int_0^1 (2x-1)\,dx$$

$$= 2 \cdot \frac{x^2}{2}\Big|_{-2}^0 + x\Big|_{-2}^0 + 2\frac{x^2}{2}\Big|_0^1 - x\Big|_0^1$$

$$= -4 + 2 + 1 - 1 =$$

Laplace Transforms

Integration by parts

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$\int_0^1 x\,e^{-3x}\,dx = -\frac{1}{3}x e^{-3x}\Big|_0^1 + \frac{1}{3}\int_0^1 e^{-3x}\,dx$$

$u = x$, $dv = e^{-3x}\,dx$ $\quad = -\frac{1}{3}x e^{-3x}\Big|_0^1 - \frac{1}{9}e^{-3x}\Big|_0^1$

$du = dx$, $v = -\frac{1}{3}e^{-3x}$ $\quad = -\frac{1}{3}e^{-3} - \frac{1}{9}(e^{-3} - 1)$

Laplace Transforms

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$.

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin ax \sin bx$

$= \frac{1}{2}\left(\cos(a-b)x - \cos(a+b)x\right)$

$e^{\infty} = \infty$

$e^{-\infty} = 0$

Laplace Transform

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

Improper Integral

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$$

Laplace Transforms

$$\mathcal{L}(f(t)) = \int_0^\infty f(t)\, e^{-st}\, dt, \quad s \in \mathbb{R},\ t > 0$$

$$\mathcal{L}(1) = \int_0^\infty 1 \cdot e^{-st}\, dt = \lim_{b \to \infty} \int_0^b e^{-st}\, dt$$

$$= \lim_{b \to \infty} -\frac{1}{s} e^{-st} \Big|_0^b$$

$$\boxed{\mathcal{L}(1) = \frac{1}{s}, \quad s > 0}$$

$$= -\frac{1}{s} \lim_{b \to \infty} \left(e^{-sb} - 1 \right)$$

$$= -\frac{1}{s}(0 - 1) = \frac{1}{s}, \quad s > 0$$

CamScanner

Laplace Transforms

$$\mathcal{L}(t) = \lim_{b \to \infty} \int_0^b t\, e^{-st}\, dt$$

$$\boxed{\mathcal{L}(t) = \frac{1}{s^2}, \quad s > 0}$$

$$u = t, \quad dv = e^{-st}\, dt$$

$$du = dt, \quad v = -\frac{1}{s} e^{-st}$$

$$= \lim_{b \to \infty} \left[-\frac{1}{s} e^{-st}\, t \Big|_0^b + \frac{1}{s} \int_0^b e^{-st}\, dt \right]$$

$$= \lim_{b \to \infty} \left[-\frac{1}{s} \left(e^{-sb}\, b \right) - \frac{1}{s^2} \left(e^{-sb} - 1 \right) \right]$$

$$= \frac{1}{s^2} \quad s > 0$$

LapPlace Transforms

$$\mathcal{L}(e^{at}) = \lim_{b \to \infty} \int_0^b e^{at} \cdot e^{-st}\, dt$$

$$= \lim_{b \to \infty} \int_0^b e^{-(s-a)t}\, dt$$

$$= \lim_{b \to \infty} -\frac{1}{s-a}\left[e^{-(s-a)b} - 1\right]$$

$$= \frac{1}{s-a}, \quad s > a$$

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LaPlace Transforms

$\mathcal{L}(1) = \frac{1}{s}, \quad s > 0$

$\mathcal{L}(t) = \frac{1}{s^2}, \quad s > 0$

$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}, \quad s > 0$

$\mathcal{L}(e^{at}) = \frac{1}{s-a}, \quad s > a$

$\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}, \quad s > 0$

$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}, \quad s > 0$

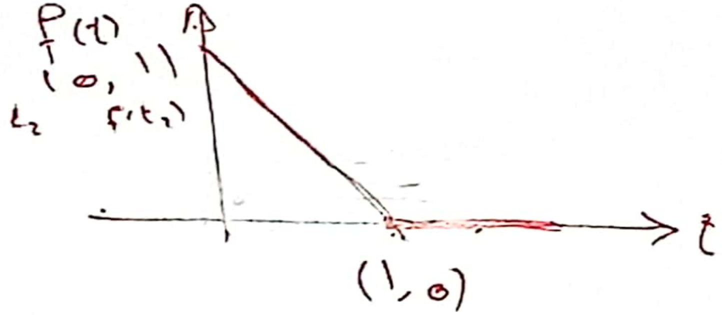
$\mathcal{L}(\sinh at) = \frac{a}{s^2-a^2}, \quad s > |a|$

$\mathcal{L}(\cosh at) = \frac{s}{s^2-a^2}, \quad s > |a|$

Ex:

Laplace Transforms

$\mathcal{L}(f(t))$:



$$f(t) = \begin{cases} -t+1, & 0 \le t \le 1 \\ 0 & t \ge 1 \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^\infty f(t) e^{-st} dt = \int_0^1 (-t+1) dt + \cancel{\int_1^\infty 0 \, e^{-st} dt}$$

$$= -\frac{t^2}{2}\Big|_0^1 + t\Big|_0^1 = -\left(\frac{1}{2}\right) + 1 = \frac{1}{2}$$

Ex:

LaPlace Transforms

$\mathcal{L}(f(t))$:

$$f(t) = \begin{cases} 0 & , 0 \le t < 1 \\ t & , 1 < t < 2 \\ 1 & , t > 2 \end{cases}$$

$$\mathcal{L}(f(t)) = \int_0^1 0 \, e^{-st} dt + \int_1^2 t e^{-st} dt + \int_2^\infty e^{-st} dt$$

$= 0$

$\boxed{s > 0}$

Ex)

Laplace Transforms

$\mathcal{L}(f(t))$: $f(t) = \begin{cases} -t+1, & 0 \le t \le 1 \\ 0, & t \ge 1 \end{cases}$

$$\mathcal{L}(f(t)) = \int_0^1 (-t+1)\, e^{-st}\, dt + \int_1^\infty \underbrace{0\, e^{-st}}_{=0}\, dt$$

$$= -\underbrace{\int_0^1 t\, e^{-st}\, dt}_{= I_1} + \underbrace{\int_0^1 e^{-st}\, dt}_{= I_2}$$

$u = t$, $dv = e^{-st}\, dt$

$du = dt$, $v = -\frac{1}{s} e^{-st}$

$I_1 = -\frac{1}{s} e^{-st}\, t \Big|_0^1 + \frac{1}{s}\int_0^1 e^{-st}\, dt$

$= -\frac{1}{s}(e^{-s}) - \frac{1}{s^2}(e^{-s} - 1)$

$I_2 = -\frac{1}{s} e^{-st} \Big|_0^1 = -\frac{1}{s}(e^{-s} - 1)$

Linearity

LapPlace Transforms

$$\mathcal{L}(a\,f(t) + b\,g(t)) = a\,\mathcal{L}(f(t)) + b\,\mathcal{L}(g(t))$$

Ex: $\mathcal{L}(6t+9) = 6\,\mathcal{L}(t) + 9\,\mathcal{L}(1)$

$$= \frac{6}{s^2} + \frac{9}{s} \qquad s > 0$$

Ex: $\mathcal{L}(\sin^2 t) = \mathcal{L}\left(\frac{1}{2}(1-\cos 2t)\right)$

$$= \frac{1}{2}\mathcal{L}(1) - \frac{1}{2}\mathcal{L}(\cos 2t)$$

$$= \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\,\frac{s}{s^2+4}$$

$s > 0$

Linearity

LapPace Transforms

Ex $\mathcal{L}(\sin 2x \sin 3x)$

$\sin 2x \sin 3x = \frac{1}{2}\left(\cos(2-3)x - \cos(2+3)x\right)$

$\mathcal{L}\left(\frac{1}{2}\left(\cos(-x) - \cos 5x\right)\right)$

$= \frac{1}{2}\mathcal{L}(\cos -x) - \frac{1}{2}\mathcal{L}(\cos 5x)$

$= \frac{1}{2}\frac{s}{s^2+1} - \frac{1}{2}\frac{s}{s^2+25}$

$s > 0$

Linearity

Laplace Transforms

Ex $\mathcal{L}(\sin(at+b))$

$= \mathcal{L}(\sin(at)\cos b + \cos at \sin b)$

$= \cos b \cdot \mathcal{L}(\sin at) + \sin b \, \mathcal{L}(\cos at)$

$= \cos b \cdot \dfrac{a}{s^2+a^2} + \sin b \cdot \dfrac{s}{s^2+a^2}$

Linearity

Laplace Transforms

$\mathcal{L}(\sin^2 t) = \mathcal{L}\left(\frac{1}{2}(1-\cos 2t)\right)$

$= \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\cdot\frac{s}{s^2+4}$

Ex) $\mathcal{L}\left(e^{4t} + e^{2t} + t^3 + \sin^2 t\right)$.

$= \mathcal{L}(e^{4t}) + \mathcal{L}(e^{2t}) + \mathcal{L}(t^3) + \mathcal{L}(\sin^2 t)$

$= \frac{1}{s-4} + \frac{1}{s-2} + \frac{3!}{s^4} + \frac{1}{2}\cdot\frac{1}{s} - \frac{1}{2}\cdot\frac{s}{s^2+4}$