Ex find $\mathcal{L}(e^{at}\, t)$

Sol

$$\mathcal{L}(t) = \frac{1}{s^2}, \quad s>0$$

$$\mathcal{L}(e^{at}\, t) = \frac{1}{(s-a)^2}, \quad s>a$$

Ex

$$\mathcal{L}(t^2 \cosh \pi t) = \mathcal{L}\left(t^2\, \frac{e^{\pi t} + e^{-\pi t}}{2}\right)$$

$$= \frac{1}{2}\mathcal{L}(t^2 e^{\pi t}) + \frac{1}{2}\mathcal{L}(t^2 e^{-\pi t})$$

$$= \frac{1}{2}\cdot\frac{2}{(s-\pi)^3} + \frac{1}{2}\cdot\frac{2}{(s+\pi)^3} = \frac{1}{(s-\pi)^3} + \frac{1}{(s+\pi)^3} = \frac{(s+\pi)^3 + (s-\pi)^3}{(s-\pi)^3 (s+\pi)^3}$$

$$= \frac{s^3 + 3s^2\pi + 3s\pi^2 + \pi^3 + s^3 - 3s^2\pi + 3s\pi^2 - \pi^3}{(s^2-\pi^2)^3}$$

Multiplication by t^n
(Differentiation of Laplace Transforms)

If $\mathcal{L}(f(t)) = F(s)$, then

$$\mathcal{L}(t^n f(t)) = (-1)^n \frac{d^n}{ds^n} F(s)$$

Ex

$$\mathcal{L}(t \sin 3t) = (-1)^1 \frac{d}{ds}\left(\frac{3}{s^2+9}\right) = -3 \frac{-1}{(s^2+9)^2} 2s = \frac{6s}{(s^2+9)^2}$$

$$\mathcal{L}(\sin 3t) = \frac{3}{s^2+9}, \quad s > 0$$

$$\frac{d}{dt}\frac{1}{f(t)}$$

$$= -\frac{1}{f(t)^2}\cdot f'(t)$$

$$\frac{d}{dt}\frac{f(t)}{g(t)} = \frac{g(t)\,f'(t) - f(t)\,g'(t)}{g^2(t)}$$

$$= -3\,\frac{d}{ds}\left(\frac{1}{s^2+9}\right)$$

$$= -3\cdot\frac{-1}{(s^2+9)^2}\,2s$$

Ex $\mathcal{L}(t^2 \sin 7t) = (-1)^2 \frac{d^2}{ds^2}\left(\frac{7}{s^2+49}\right)$

$\mathcal{L}(\sin 7t) = \frac{7}{s^2+49}$

$= 7 \frac{d^2}{ds^2}\left(\frac{1}{s^2+49}\right)$

$\frac{d}{ds}\left(\frac{1}{s^2+49}\right) = \frac{-1}{(s^2+49)^2} \cdot 2s = \frac{-2s}{(s^2+49)^2}$

$\frac{d^2}{ds^2}\left(\frac{1}{s^2+49}\right) = \frac{d}{ds}\left(\frac{-2s}{(s^2+49)^2}\right)$

$= -2 \frac{(s^2+49)^2 \, 1 - s \, 2(s^2+49) 2s}{(s^2+49)^4}$

ﺑﺴﻂ

النمط الإفتراضي

Ex $\mathcal{L}(3te^{5t}\sin 7t)$

$$= 3\,\mathcal{L}(t\underbrace{e^{5t}\sin 7t}_{f(t)}) = 3(-1)^1\frac{d}{ds}\frac{7}{(s-5)^2+49}$$

$$\mathcal{L}(\sin 7t) = \frac{7}{s^2+49}$$

$$= -21\frac{d}{ds}\frac{1}{(s-5)^2+49}$$

$$\mathcal{L}(e^{5t}\sin 7t) = \frac{7}{(s-5)^2+49}$$

$$= -21\cdot\frac{-1}{((s-5)^2+49)^2}\,2(s-5)$$

النمط الإفتراضي

Ex $\mathcal{L}\left(\frac{1}{2a^3}(\sin at - at\cos at)\right)$

$$= \frac{1}{2a^3}\mathcal{L}(\sin at) - \frac{1}{2a^2}\mathcal{L}(t\cos at)$$

$$= \frac{1}{2a^3}\cdot\frac{a}{s^2+a^2} - \frac{1}{2a^2}\cdot\frac{s^2-a^2}{(s^2+a^2)^2}$$

$$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$$

$$\mathcal{L}(t\cos at) = -\frac{d}{ds}\frac{s}{s^2+a^2} = -\frac{s^2+a^2-2s^2}{(s^2+a^2)^2} = \frac{s^2-a^2}{(s^2+a^2)^2}$$

النمط الإفتراضي

Ex Evaluate $\int_0^\infty t e^{-2t} \sin t \, dt$

$s = 2$

$$\mathcal{L}(t \sin t) = \int_0^\infty t \sin t \, e^{-st} \, dt$$

$$\boxed{s = 2}$$

$$\frac{4}{25}$$

$$\mathcal{L}(\sin t) = \frac{1}{s^2+1}$$

$$\mathcal{L}(t \sin t) = -\frac{d}{ds} \frac{1}{s^2+1} = -\frac{-1}{(s^2+1)^2} \cdot 2s = \frac{2s}{(s^2+1)^2}$$

$$\int_a^b \frac{f'(t)}{f(t)}\,dt$$

$$= \ln|f(t)|\Big|_a^b$$

$$\frac{1}{2}\int_c^d \frac{2t\,dt}{t^2+a^2} =$$

$$= \frac{1}{2}\ln|t^2+a^2|\Big|_c^d$$

$$\int_c^d \frac{du}{u^2+a^2}$$

$$= \frac{1}{a}\tan^{-1}\frac{u}{a}$$

Division by t

Integration of Laplace transforms

If $\mathcal{L}\{f(t)\} = F(s)$, then $\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_s^\infty F(u)\,du$

Ex: $\mathcal{L}\left(\frac{\sin at}{t}\right) = \int_s^\infty \frac{a\,du}{u^2+a^2} = a\int_s^\infty \frac{du}{u^2+a^2}$

$\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$

$= a \cdot \frac{1}{a}\tan^{-1}\frac{u}{a}\Big|_s^\infty$

$= \tan^{-1}\infty - \tan^{-1}\frac{s}{a} = \frac{\pi}{2} - \tan^{-1}\frac{s}{a} = \cot^{-1}\frac{s}{a}$

$= \tan^{-1}\frac{a}{s}$

النمط الافتراضي

Ex $\mathcal{L}\left(\frac{2(1-\cos at)}{t}\right)$

$$= 2\mathcal{L}\left(\frac{1}{t}\right) - 2\mathcal{L}\left(\frac{\cos at}{t}\right) = \left(2\ln u - 2\,\frac{1}{2}\ln(u^2+1)\right)\Big|_s^\infty$$

$$\mathcal{L}\left(\frac{1}{t}\right) = \int_s^\infty \frac{du}{u} = \ln u \Big|_s^\infty$$

$$= \ln u^2 - \ln(u^2+1)\Big|_s^\infty$$

$$= \ln \frac{u^2}{u^2+1}\Big|_s^\infty$$

$$\mathcal{L}(1) = \frac{1}{s}$$

$$\mathcal{L}\left(\frac{\cos at}{t}\right) = \int_s^\infty \frac{u}{u^2+1}\,du = \frac{1}{2}\ln(u^2+1)\Big|_s^\infty$$

$$= \lim_{b\to\infty} \ln\frac{u^2}{u^2+1}\Big|_s^b$$

$$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$$

$$= \ln 1 - \ln\frac{u^2}{u^2+1}$$

الق‍ائمة أدوات رمز ال... تبديل القلم الممحاة شكل تخطيط ال... تراجع تكرار تحديد مشترك مسح 11/11

$$\lim_{u \to \infty} \ln \frac{u^2}{u^2+1}$$

$$\ln \frac{\infty}{\infty}$$

$$= \lim_{u \to \infty} \ln \frac{1}{1 + \frac{1}{u^2}}$$

$$= \ln 1 = 0$$

0

$$\int_c^d \frac{du}{u^2+a^2}$$

$$= \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$\tan^{-1} \frac{s}{a} + \cot^{-1} \frac{s}{a} = \frac{\pi}{2}$$

$$\cot^{-1} \frac{s}{a} = \tan^{-1} \frac{a}{s}$$

$$\ln a^n = n \ln a$$

$$\ln a - \ln b = \ln \frac{a}{b}$$

النمط الإفتراضي

Ex $\mathcal{L}\left(\frac{2(e^t - \cos t)}{t}\right)$

$$= 2\,\mathcal{L}\left(\frac{e^t}{t}\right) - 2\,\mathcal{L}\left(\frac{\cos t}{t}\right) = 2\int_s^\infty \frac{du}{u-1} - 2\int_s^\infty \frac{u\,du}{u^2+1}$$

$\mathcal{L}(e^t) = \frac{1}{s-1}$

$\mathcal{L}(\cos t) = \frac{s}{s^2+1}$

$$= 2\ln(u-1) - \ln(u^2+1)\Big|_s^\infty$$

$$= \ln\frac{(u-1)^2}{u^2+1}\Big|_s^\infty$$

$$= 0 - \ln\frac{(s-1)^2}{s^2+1}$$

12/12

Ex $$\int_0^\infty \frac{e^{-t}-e^{-3t}}{t}\,dt$$

$$\mathcal{L}\left(\frac{e^{-t}-e^{-3t}}{t}\right)=\int_0^\infty \underbrace{\frac{e^{-t}-e^{-3t}}{t}}_{f(t)}\,e^{-st}\,dt$$

$$\parallel \quad \boxed{s=0}$$

$$\mathcal{L}\left(\frac{e^{-t}}{t}\right)-\mathcal{L}\left(\frac{e^{-3t}}{t}\right)=\int_s^\infty \frac{du}{u+1}-\int_s^\infty \frac{du}{u+3}=\ln(u+1)-\ln(u+3)\Big|_s^\infty$$

$$\mathcal{L}(e^{-t})=\frac{1}{s+1},\quad \mathcal{L}(e^{-3t})=\frac{1}{s+3}$$

$$=\ln\frac{u+1}{u+3}\Big|_s^\infty$$

$$=0-\ln\frac{s+1}{s+3}=-\ln\frac{1}{3}=\ln 3$$