

Ch③ Special Probability Distributions

discrete $P(x)$

نحوه دلائل احکمین

① Binomial Dist.

• تجربة زنگر، مفرد مجموعات n

• النتائج $\{0, 1, 2, \dots, n\}$ يتحققون بحسب قانون الاحتمالات P والمتغير q

• مجموع النتائج متساوية

$$(P+q=1)$$

$$q=1-P$$

• $P(x) = {}^n_C_x P^x q^{n-x}$, $x = 0, 1, 2, 3, \dots, n$

• $E(x) = np$

• $V(x) = npq$

• $\sigma = \sqrt{npq}$

• $M_x(t) = [q + p e^t]^n$

- ٣٥% -

- ١٥ -

الدوار، نسبة ٤٠ رسم صندوق

Ch③ Special Probability Distributions

- ① Suppose the experiment of tossing a coin 5 times;
 ② write the probability function describes this expm
 ③ find the following probabilities

(a) The Head appears exactly 3 times?
 (b) " " more than 2 times?
 (c) " = less than or equal 4 times?
 (d) " = will not appear

$n=5$ $P = \frac{1}{2}$, $q = \frac{1}{2}$

$$① P(X) = {}^n C_x P^x q^{n-x}, x=0, 1, 2, \dots, n$$

$$P(X) = {}^5 C_x (0.5)^x (0.5)^{5-x}, x=0, 1, 2, 3, 4, 5$$

$$② a P(X=3) = {}^5 C_3 (0.5)^3 (0.5)^{5-3} = 0.3125$$

$$\begin{aligned}
 b P(X > 2) &= P(3) + P(4) + P(5) \\
 &= {}^5 C_3 (0.5)^3 (0.5)^{5-3} + {}^5 C_4 (0.5)^4 (0.5)^{5-4} + {}^5 C_5 (0.5)^5 (0.5)^{5-5} \\
 c P(X \leq 4) &= 1 - P(X > 4) \\
 &= 1 - P(5) = 1 - {}^5 C_5 (0.5)^5 (0.5)^{5-5} \\
 d P(X=0) &= {}^5 C_0 (0.5)^0 (0.5)^5
 \end{aligned}$$

Ch(3) Special Probability Distributions

Q Suppose the experiment of tossing a coin 5 times;
 ① write the probability function describes this expm

- find ① $E(x)$ ② $V(x)$
 ③ σ ④ $M_x(t)$

② use $M_x(t)$ to find

- ⑤ $E(x)$, ⑥ $V(x)$

(sol)

$$P(x) = \binom{5}{x} (0.5)^x (0.5)^{5-x}, \quad x=0, 1, 2, 3, 4, 5$$

$$\textcircled{1} \quad E(x) = n\bar{P} = 5(0.5) = \underline{\underline{2.5}}$$

$$\textcircled{2} \quad V(x) = n\bar{P}q = 5(0.5)(0.5) = 1.25$$

$$\textcircled{3} \quad \sigma = \sqrt{n\bar{P}q} = \sqrt{1.25} = \underline{\underline{1.118}}$$

$$\textcircled{4} \quad \boxed{M_x(t)} = [9 + Pe^t]^5 = \underline{\underline{[0.5 + 0.5e^t]^5}}$$

$$\begin{aligned} \textcircled{5} \quad & \left. \left\{ \begin{aligned} & E(x) = \frac{d M_x(t)}{dt} \Big|_{t=0} \\ & V(x) = \frac{d^2 M_x(t)}{dt^2} \Big|_{t=0} \end{aligned} \right\} \right. \\ & = 5 \left[\underbrace{0.5 + 0.5e^t}_{+e^t} \right]^4 (0.5e^t) \Big|_{t=0} \\ & = 5 \left[0.5 + 0.5e^0 \right]^4 (0.5e^0) \\ & = \underline{\underline{2.5}} \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad & E(x^2) = \left. \frac{d^2 M_x(t)}{dt^2} \right|_{t=0} \\ & = 5 \cdot \frac{4(0.5 + 0.5e^t)^3 (0.5e^t)^2 + (0.5e^t) 5 \cdot (0.5 + 0.5e^t)}{t=0} \\ & = 20(0.5)^2 + (0.5)(5) = \underline{\underline{7.5}} \end{aligned}$$

$$\begin{aligned} \bullet \quad & V(x) = E(x^2) - (E(x))^2 \\ & = 7.5 - (2.5)^2 = \underline{\underline{1.25}} \end{aligned}$$

Ch(3) Special Probability Distributions

(Ex)

$$n = 20$$

$$P = 0.7$$

$$q = 0.3$$

سوچی دلخواه
جذب کننده
فکر کننده
فکر کننده
فکر کننده

Find the following

① $E(X)$

② $V(X)$

③ $M_X(t)$

④ Find the following problem

⑤ $P(X=0)$

⑥ $P(X > 17)$

⑦ $P(X \leq 3)$

⑧ $P(X \geq 2)$

$$P(X) = \frac{20}{x} \binom{20}{x} (0.7)^x (0.3)^{20-x}, x=0, 1, 2, \dots, 20$$

① $E(X) = nP = 20(0.7) =$

② $V(X) = nPq = 20(0.7)(0.3) =$

③ $M_X(t) = (0.3 + 0.7e^t)^{20}$

④ ⑤ $P(X=0) = \binom{20}{0} (0.7)^0 (0.3)^{20}$
 $= 3.48 \times 10^{-11}$

(b) $P(X > 17) = P(X=18) + P(X=19) + P(X=20)$

$$= \binom{20}{18} (0.7)^2 (0.3)^2 + \binom{20}{19} (0.7)^1 (0.3)^1 + \binom{20}{20} (0.7)^0 (0.3)^0$$

(c) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$$= \binom{20}{0} 0.7^0 0.3^{20} + \binom{20}{1} 0.7^1 (0.3)^1 + \binom{20}{2} 0.7^2 (0.3)^2 + \binom{20}{3} 0.7^3 (0.3)^3$$

(d) $P(X \geq 2) = 1 - P(X \leq 1)$

$$= 1 - (P(0) + P(1))$$

$$= 1 - \left[\binom{20}{0} 0.7^0 (0.3)^{20} + \binom{20}{1} 0.7^1 (0.3)^1 \right]$$

Ch(3) Discrete Probability Distributions

② Poisson Distribution

توزيع بواسون

* تجربة تكرار حثيرة ملحوظة تحدث في كل دورة ملحوظة *

$$f_x(x)$$

أرجوكم ملحوظة

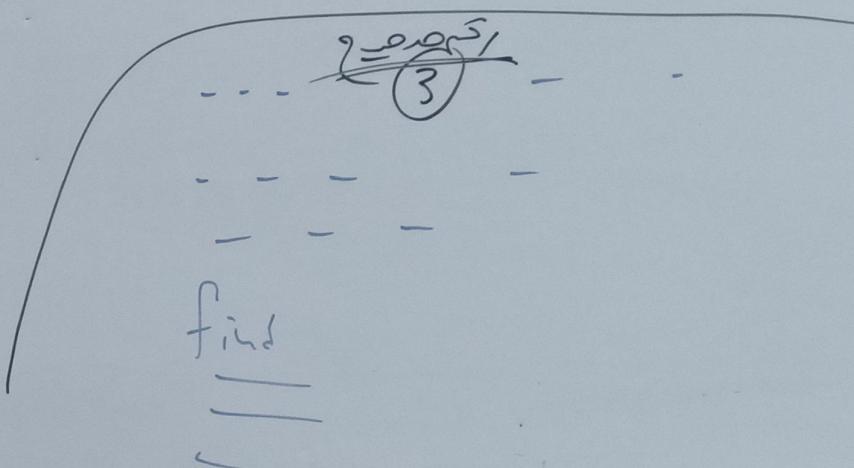
$$P(X) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad X = 0, 1, 2, \dots$$

- $E(X) = \lambda$

- $V(X) = \lambda$

- $\sigma = \sqrt{\lambda}$

- $M_X(t) = e^{\lambda [e^t - 1]}$



<p>⑤ $X \sim \text{Poisson}(\lambda=0.4)$. Find</p> <p>① $P(X=0)$</p> <p>② $P(X=4)$</p> <p>③ $P(X \geq 3)$</p> <p>④ $P(X < 5)$</p>	<p>⑥ Fns</p> <p>a) $E(X)$</p> <p>b) $V(X)$</p> <p>c) σ</p> <p>d) $M_X(t)$</p>	<p>⑦ $P(X=4) = \frac{0.4^4 e^{-0.4}}{4!}$</p>
		<p>⑧ $P(X \geq 3) = 1 - P(X < 3)$</p>
		<p>= $1 - \left(\frac{0.4^3 e^{-0.4}}{3!} \right)$</p>
		<p>⑨ $P(X < 5) = P(0) + P(1) + P(2) + P(3) + P(4)$</p>
		<p>⑩ a) $E(X) = \lambda = 0.4$</p>
		<p>b) $V(X) = \lambda = 0.4$</p>
		<p>c) $\sigma = \sqrt{V(X)} = \sqrt{0.4}$</p>
		<p>d) $M_X(t) = e^{\lambda(e^t - 1)}$</p>
		<p>= $e^{0.4(e^t - 1)}$</p>
		<p>⑪ $P(X=0) = \frac{0.4^0 e^{-0.4}}{0!} =$</p>

Continuous Probability Function

① Normal Distribution

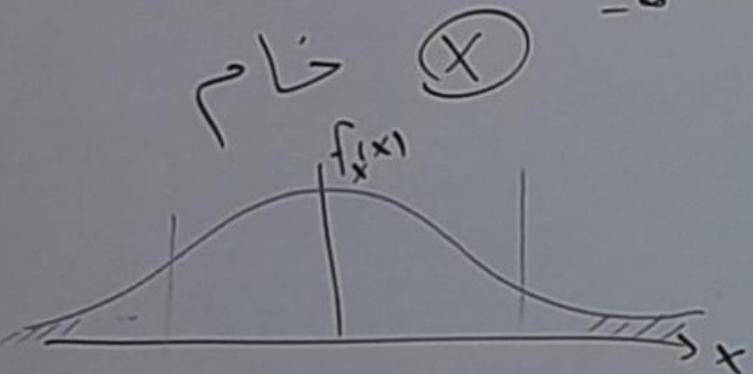
الدوري الطبيعي

لعمارات ذات طبقات

متر

$$-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2$$

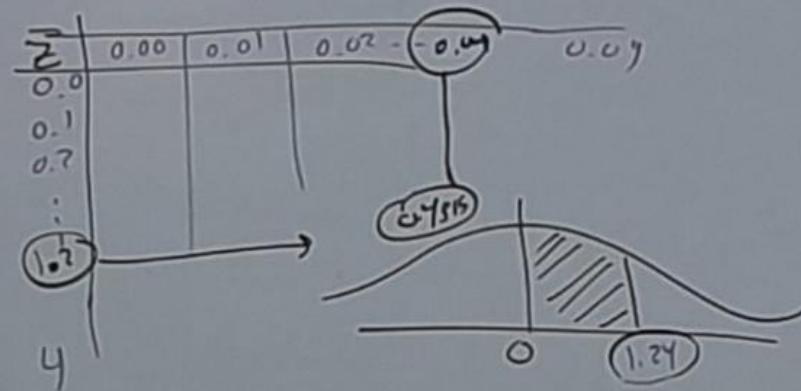
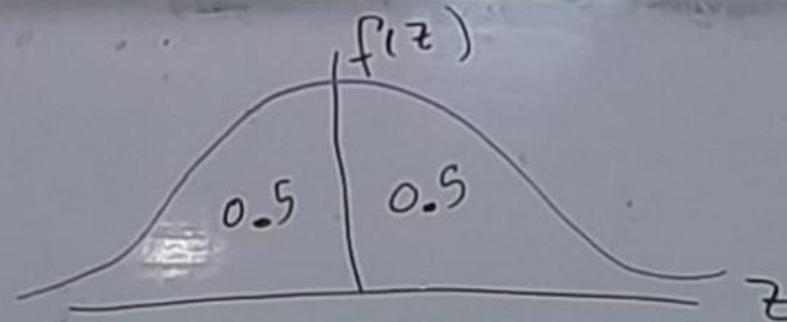
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2} \quad -\infty < x < \infty$$



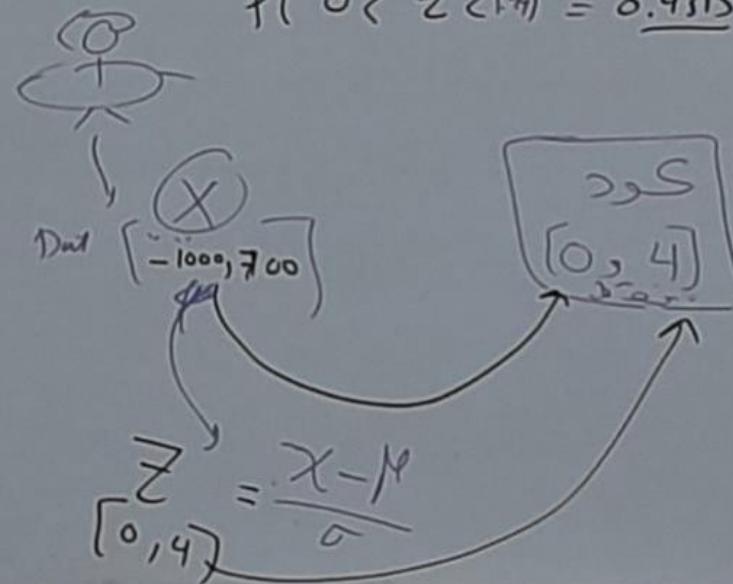
* Standard Normal Dist--

$$Z = \frac{X - \mu}{\sigma}$$

حوال
[0, 4)



$$P(0 < Z < 1.96) = 0.4755$$



Continuous Probability Function

① Normal Distribution التوزيع الطبيعي

$$X \sim N(\mu, \sigma^2)$$

Ex: let $X \sim N(16, 25)$

find the following probabilities

$$\textcircled{1} \quad P(16 < X < 18)$$

$$\textcircled{2} \quad P(-17 < X < 16)$$

$$\textcircled{3} \quad P(17 \leq X \leq 18)$$

$$\textcircled{4} \quad P(14 < X < 15)$$

$$\textcircled{5} \quad P(15 < X < 18)$$

$$\textcircled{6} \quad P(X > 18)$$

$$\textcircled{7} \quad P(X < 18)$$

$$\textcircled{8} \quad P(X < 14)$$

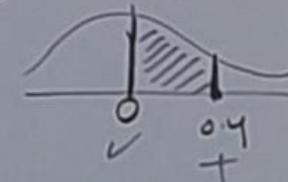
$$\textcircled{9} \quad P(X > 14)$$

$$\textcircled{10} \quad P(X = 13)$$

(sol)

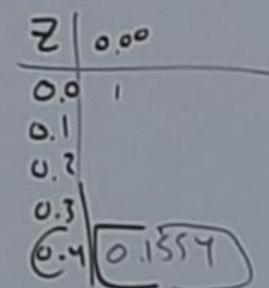
$$\textcircled{1} \quad P(16 < X < 18) = P\left[\frac{16-16}{5} < \frac{X-16}{5} < \frac{18-16}{5}\right]$$

$$= P[0 < Z < 0.4]$$



$$= \Phi(0.4)$$

$$= 0.1554$$



$$\overline{P(0 < Z < 0.4) = 0.1554}$$

Continuous Probability Function

① Normal Distribution التوزيع الطبيعي

$$X \sim N(\mu, \sigma^2)$$

Ex: Let $X \sim N(16, 25)$

Find the following probabilities

① $P(16 < X < 18)$

② $P(15.2 < X < 16)$

③ $P(17 \leq X \leq 18)$

④ $P(14 < X < 15)$

⑤ $P(15 < X < 18)$

⑥ $P(X > 18)$

⑦ $P(X < 18)$

⑧ $P(X < 14)$

⑨ $P(X > 14)$

⑩ $P(X = 13)$

(Sol)

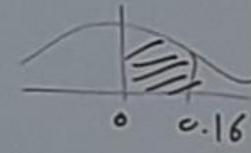
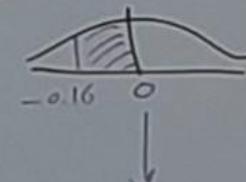
⑪ $P(15.2 < X < 16)$

$$= P\left(\frac{15.2 - 16}{5} < \frac{X - \mu}{\sigma} < \frac{16 - 16}{5}\right)$$

$$= P[-0.16 < Z < 0]$$

$$= P[0 < Z < 0.16]$$

$$= 0.0636$$



$$\underline{P(0 < Z < 0.16) = 0.0636}$$

continuous probability function

① Normal Distribution
النرمالية

$$X \sim N(\mu, \sigma^2)$$

Ex) let $X \sim N(16, 25)$

find the following probabilities

$$\textcircled{1} \quad P(16 < X < 18)$$

$$\textcircled{2} \quad P(15.2 < X < 16)$$

$$\textcircled{3} \quad P(17 \leq X \leq 18)$$

$$\textcircled{4} \quad P(14 < X < 15)$$

$$\textcircled{5} \quad P(15 < X < 18)$$

$$\textcircled{6} \quad P(X > 18)$$

$$\textcircled{7} \quad P(X < 18)$$

$$\textcircled{8} \quad P(X < 14)$$

$$\textcircled{9} \quad P(X > 14)$$

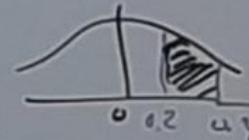
$$\textcircled{10} \quad P(X = 13)$$

(Sol)

$$\textcircled{3} \quad P[17 < X < 18]$$

$$= P\left(\frac{17-16}{5} < \frac{X-\mu}{\sigma} < \frac{18-16}{5}\right)$$

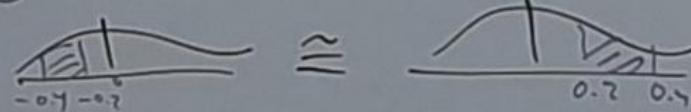
$$= P(0.2 < Z < 0.4)$$



$$= P(0 < Z < 0.4) - P(0 < Z < 0.2)$$

$$= \alpha - \beta$$

$$\textcircled{4} \quad P[14 < X < 15] = P(-0.4 < Z < -0.2)$$



$$= \alpha - \beta$$

$$\textcircled{P(0 < Z < 0.4)} = \alpha$$

$$\textcircled{P(0 < Z < 0.2)} = \beta$$

continuous probability function

① Normal Distribution التوزيع الطبيعي

$$X \sim N(\mu, \sigma^2)$$

(Ex) let $X \sim N(16, 25)$

find the following probabilities

① $P(16 < X < 18)$

② $P(15.2 < X < 16)$

③ $P(17 \leq X \leq 18)$

④ $P(14 < X < 15)$

⑤ $P(15 < X < 18)$

⑥ $P(X > 18)$

⑦ $P(X < 18)$

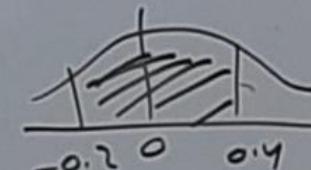
⑧ $P(X < 14)$

⑨ $P(X > 14)$

⑩ $P(X = 13)$

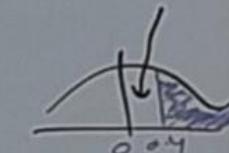
⑪ $P(15 < X < 18)$

$$= P(-0.2 < Z < 0.4)$$



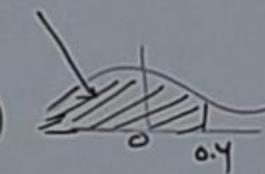
$$= P(0 < Z < 0.4) + P(0 < Z < 0.2)$$

⑫ $P(X > 18) = P(Z > 0.4)$



$$= 0.5 - P[0 < Z < 0.4]$$

⑬ $P(X < 18) = P(Z < 0.4)$



$$= 0.5 + P[0 < Z < 0.4]$$

⑭ $P(0 < Z < 0.4) = d$

⑮ $P(0 < Z < 0.2) = r$

Continuous Probability Function

Normal Distribution النرمالية

$$X \sim N(\mu, \sigma^2)$$

Ex) Let $X \sim N(16, 25)$

Find the following

$$\textcircled{1} \quad P(16 < X < 18)$$

$$\textcircled{2} \quad P(15.7 < X < 16)$$

$$\textcircled{3} \quad P(17 \leq X \leq 18)$$

$$\textcircled{4} \quad P(14 < X < 15)$$

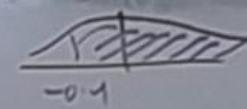
$$\textcircled{5} \quad P(15 < X < 18)$$

$$\textcircled{6} \quad P(X > 18)$$

$$\textcircled{7} \quad P(X < 18)$$

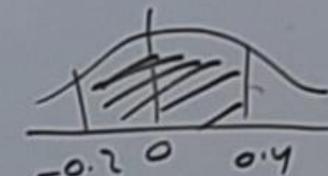
$$\textcircled{8} \quad P(X < 14) = P(X < -0.4)$$

$$\textcircled{9} \quad P(X > 14) \\ \textcircled{10} \quad P(X = 13) = 0$$



$$\textcircled{5} \quad P(15 < X < 18)$$

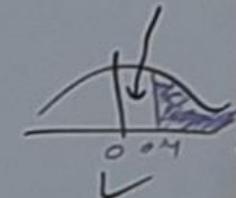
$$= P(-0.2 < Z < 0.4)$$



$$= P(0 < Z < 0.4) + P(0 < Z < 0.2)$$

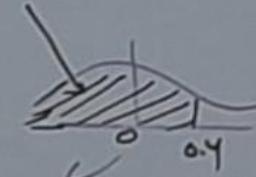
$$\textcircled{6} \quad P(X > 18) = P(Z > 0.4)$$

$$= 0.5 - P[0 < Z < 0.4]$$



$$\textcircled{7} \quad P(X < 18) = P(Z < 0.4)$$

$$= 0.5 + P[0 < Z < -0.4]$$



$$P(0 < Z < 0.4) = \alpha$$

$$P(0 < Z < 0.2) = \beta$$

④ for $X \sim \text{Normal}(75, 625)$
 find ① and ② that
 satr firs

$$\textcircled{1} P[X \geq a] = 0.2743$$

$$\begin{aligned} & (\sigma)^2 \\ & \sigma = 25 \end{aligned}$$

$$\therefore P[0 < z < \frac{a-75}{25}] = 0.2257$$

$$0.8, \quad \frac{a-75}{25} = 0.6$$

$$a = 75 + (25)(0.6) = 90$$

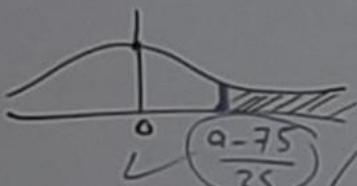
$$\textcircled{2} P[X \leq b] = 0.3446$$

sol

$$\textcircled{1} P[X \geq a] = 0.2743$$

$$P\left[\frac{X-75}{25} \geq \frac{a-75}{25}\right] = 0.2743$$

$$P[z \geq \frac{a-75}{25}] = 0.2743$$



$$0.5 - P[0 < z < \frac{a-75}{25}] = 0.2743$$

$$\overbrace{\hspace{10em}}^{\text{P}(0 < z < 0.6) = 0.2257}$$

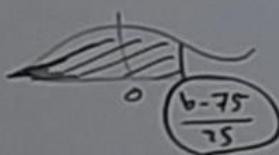
④ For $X \sim \text{Normal}(75, 625)$
 find a) and b) that
 satr firs

$$\textcircled{1} P[X \geq a] = 0.2743$$

$$\textcircled{2} P[X < b] = 0.3446$$

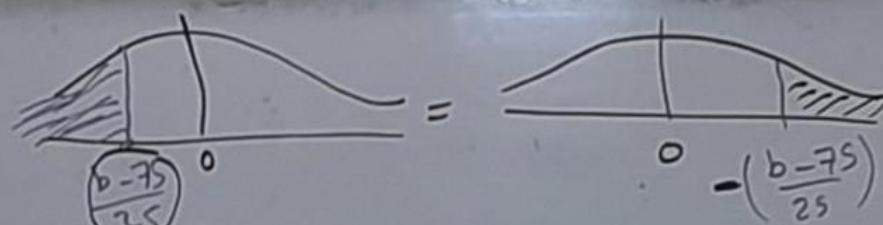
$$\textcircled{3} P[X < b] = 0.3446$$

$$P[Z < \frac{b-75}{25}] = 0.3446$$



$$0.5 + P[0 < Z < \frac{b-75}{25}] = 0.3446$$

$$P[0 < Z < \frac{b-75}{25}] \quad \text{خاص} \times$$



$$\Rightarrow 0.5 - P[0 < Z < -\frac{(b-75)}{25}] = 0.3446$$

$$P[0 < Z < -\frac{(b-75)}{25}] = 0.1554$$

$$-\frac{(b-75)}{25} = 0.4$$

$$\frac{b-75}{25} = -0.4$$

$$b = 75 - 25(0.4) \approx 65$$

$$P(0 < Z < 0.4) = 0.1554$$

④ Gamma Distribution

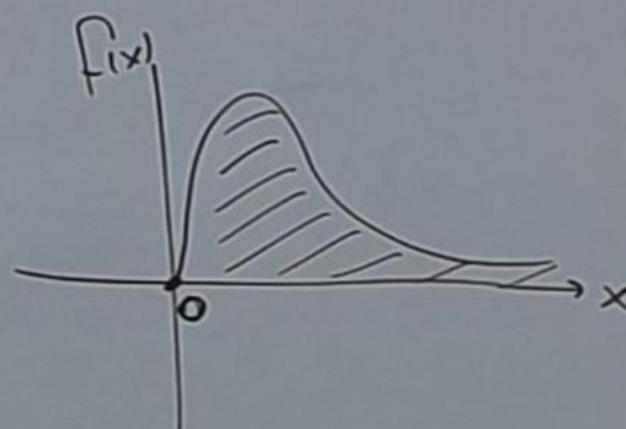
لولج ٤٠١٩

$$f_x(x) = \frac{1}{\Gamma(\alpha)} \frac{\alpha^{\alpha}}{\theta^{\alpha}} X^{\alpha-1} e^{-\frac{x}{\theta}}, \quad X > 0$$

- $E(X) = \alpha \theta$

- $V(X) = \alpha \theta^2$

- $M_X(t) = (1 - \theta t)^{-\alpha}$



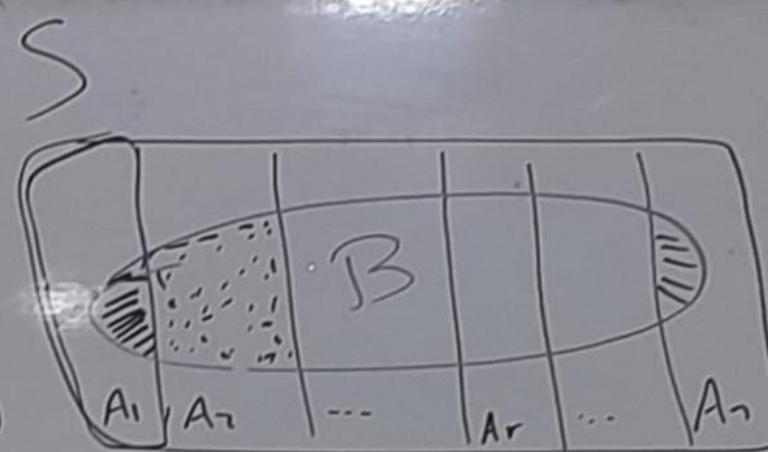
$$\alpha, \theta > 0$$

Total probability —

$$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + \dots + P(A_n \cap B)$$

$$= P(A_1) P(B|A_1) + P(A_2) P(B|A_2) + \dots + P(A_n) P(B|A_n)$$

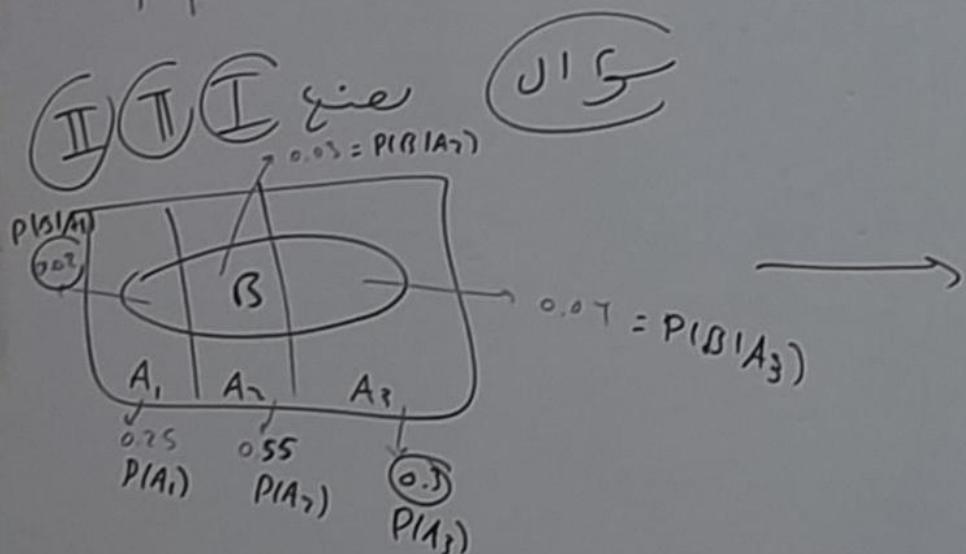
$$= \sum_{i=1}^n P(A_i) P(B|A_i)$$



$$\textcircled{G} A_i \cap A_j = \emptyset$$

$$\textcircled{Q} A_1 \cup A_2 \cup \dots \cup A_n = S$$

II T I e re g ie



$$\begin{aligned}
 P(B) &= \sum_{i=1}^3 P(A_i) P(B|A_i) \\
 &= P(A_1) P(B|A_1) + P(A_2) P(B|A_2) + P(A_3) P(B|A_3) \\
 &= (0.25)(0.03) + (0.55)(0.02) + (0.3)(0.04)
 \end{aligned}$$