# Operators عوامل

## I Forward Differences

$\Delta$ Delta دلتا

$$\Delta y_i = y_{i+1} - y_i$$

$$\Delta f(x) = f(x+h) - f(x)$$

$$\Delta y_0 = y_1 - y_0$$

ex) $\Delta y_5 = y_6 - y_5$

$\Delta y_{3/2} = y_{5/2} - y_{3/2}$

$\Delta \sin(x) = \sin(x+h) - \sin x$

## Table of forward Differences

Given $y = f(x) \Rightarrow$

| $x$ | $x_0$ | $x_1$ ... | $x_n$ |
|---|---|---|---|
| $y=f(x)$ | $y_0$ | $y_1$ ... | $y_n$ |

| $X$ | $Y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| $x_0$ | $y_0$ | $\Delta y_0 = y_1 - y_0$ | | |
| $x_1$ | $y_1$ | $\Delta y_1 = y_2 - y_1$ | $\Delta^2 y_0$ | $\Delta^3 y_0$ |
| $x_2$ | $y_2$ | | $\Delta^2 y_1$ | |
| ⋮ | ⋮ | | ⋮ | |
| $x_n$ | $y_n$ | | | |

# Operators عوامل

## (I) Forward Differences

(ex) $y = f(x) = x^2 + 1$ , $x = 0, 1, 2, 3, 4$

| x | y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| 0 | 1 | | | | |
| | | 1 | | | |
| 1 | 2 | | 2 | | |
| | | 3 | | 0 | |
| 2 | 5 | | 2 | | 0 |
| | | 5 | | 0 | |
| 3 | 10 | | 2 | | |
| | | 7 | | | |
| 4 | 17 | | | | |

$2(64) - 1$

## (II) Backward operator ∇ (Nabla نابلا)

$$\nabla y_i = y_i - y_{i-1}$$

$$\nabla f(x) = f(x) - f(x-h)$$

(ex) $\nabla y_4 = y_4 - y_3$

$$\nabla y_{\frac{5}{2}} = y_{\frac{5}{2}} - y_{\frac{3}{2}}$$

(ex) $f(x) = 2x^3 - 1$ , $x = 1, 2, 3, 4$

| x | y | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ |
|---|---|---|---|---|
| 1 | 1 | | | |
| | | 14 | | |
| 2 | 15 | | 24 | |
| | | 38 | | 12 |
| 3 | 53 | | 36 | |
| | | 74 | | |
| 4 | 127 | | | |

# Operators

## (III) Shift Operator

$$E\,y_{(i)} = y_{i+1}$$

$$E\,f(x) = f(x+h)$$

(ex) $E\,y_7 = y_8$

$E\,y_{\frac{1}{2}} = y_{\frac{3}{2}}$

$$E^{(k)} y_i = y_{i+k}$$

(ex) $E^5 y_3 = y_8$

## العوامل E

Relation $\Delta$ , $E$

$$\Delta y_i = y_{i+1} - y_i$$

$$\Delta y_i = E\,y_i - y_i$$

$$\Delta (y_i) = [E - 1]\,y_i$$

$$\Delta \equiv E - 1$$

$$1 + \Delta = E$$

$\nabla$ , $E$

$$\nabla y_i = y_i - y_{i-1}$$

$$\nabla y_i = y_i - E^{-1} y_i$$

$$\nabla y_i = (1 - E^{-1})\,y_i$$

$$\nabla \equiv 1 - E^{-1}$$

$$E^{-1} = 1 - \nabla$$

$$E = \frac{1}{1-\nabla}$$

# Inter Polation

# الاستكمال

| السنة | 1973 | 1975 | 1977 | .... | 2019 |
|---|---|---|---|---|---|
| السكان | 66M | 68M | 71M | .... | 104M |

$x_s$

*

| x | $x_0$ | $x_1$ | $x_2$ | ... | $x_n$ |
|---|---|---|---|---|---|
| y | $y_0$ | $y_1$ | $y_2$ | ... | $y_n$ |

Find $y(x_s)$ استكمال امامي

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$y(x_s) = y_s = E^s y_0 = (1+\Delta)^s y_0$$

$$= \left(1 + s\Delta + \frac{s(s-1)}{2!}\Delta^2 + \frac{s(s-1)(s-2)}{3!}\Delta^3 + \cdots\right) y_0$$

$$= y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \cdots$$

Step ① $h = x_{i+1} - x_i$ $\qquad s = \frac{x_s - x_0}{h}$

Step ② Constru table $\Delta$

| x | y | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| $x_0$ | $y_0$ | $\Delta y_0$ | |
| $x_1$ | $y_1$ | | $\Delta^2 y_0$ |
| ... | ... | | |
| $x_n$ | $y_n$ | | |

Step ③ $y_s = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \cdots$

ex

| x | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| y | 3 | 11 | 27 | 51 |

find $y(2)$

sol

Step ① $h = 2$ $\quad S = \frac{x_s - x_0}{h} = \frac{2-1}{2} = \frac{1}{2}$

Step ②

| x | y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 1 | 3 | 8 | | |
| 3 | 11 | 16 | 8 | 0 |
| 5 | 27 | 24 | 8 | |
| 7 | 51 | | | |

Step ③ $y(x_s) = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0$

$y(2) = 3 + \left(\frac{1}{2}\right)(8) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(8) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(0)$

$= 6$

Ex. 1

$= 6$

6

$y(x_s) = y_s = E^s y_0 = (1+\Delta)^s y_0$

$= \left[1 + s\Delta + \frac{s(s-1)}{2!}\Delta^2 + \frac{s(s-1)(s-2)}{3!}\Delta^3 + \cdots\right] y_0$

$= y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \cdots$

Step ① $h = x_{i+1} - x_i$ $\quad S = \frac{x_s - x_0}{h}$

Step ② Constn table $\Delta$

| x | y | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| $x_0$ | $y_0$ | $\Delta y_0$ | $\Delta^2 y_0$ |
| $x_1$ | $y_1$ | | |
| $x_2$ | $y_2$ | | |

Step ③ $y_s = y_0 + s\Delta y_0 + \frac{s(s-1)}{2!}\Delta^2 y_0 + \frac{s(s-1)(s-2)}{3!}\Delta^3 y_0 + \cdots$

ex

| x | 2 | 4 | 6 |
|---|---|---|---|
| y | 12 | 6 | 24 |

find $y(3)$ $h = 2$ $s = \frac{1}{2}$

| x | y | $\Delta y$ | $\Delta^2 y$ |
|---|---|---|---|
| 2 | 12 | -6 | 24 |
| 4 | 6 | 18 | |
| 6 | 24 | | |

$y(x_s) =$

$$y(3) = 12 + \left(\frac{1}{2}\right)(-6) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(24)$$

$$= 12 - 3 - 3 = \boxed{6}$$