Backward interpolation = الاستكمال خلفي

X	x_0	x_1	...	x_{n-2}	x_{n-1}	x_n
y	y_0	y_1	...	y_{n-2}	y_{n-1}	y_n

x_s x_s x_{n+1} x

find $y(x_s)$

Step ① $h = x_n - x_{n-1}$

$$S = \frac{x_s - x_n}{h}$$

Step ②

X	y	∇y	$\nabla^2 y$	$\nabla^3 y$	...
x_0	y_0				
	⋮	⋮	⋮		
x_{n-1}	y_{n-1}	∇y_{n-1}	$\nabla^2 y_{n-1}$		
x_n	y_n	∇y_n	$\nabla^2 y_n$		

$x^2 + 2$

Step ③

$$y(x_s) = y_n + \frac{S}{1!}\nabla y_n + \frac{S(S+1)}{2!}\nabla^2 y_n + \frac{S(S+1)(S+2)}{3!}\nabla^3 y_n + \dots$$

ex

X	1	3	5	7
y	3	11	27	51

find $y(6)$

Step ① $h = 2$ $\quad S = \dfrac{x - 7}{2}$

Step ②

X	y	∇y	$\nabla^2 y$	$\nabla^3 y$
1	3			
		8		
3	11		8	
		16		0
5	27		8	
		24		
7	51			

$y(6) = 6^2 + 2$
$= 38$

Step ③

$$y(x) = 51 + \frac{\left(\frac{x-7}{2}\right)}{1!}(24) + \frac{\left(\frac{x-7}{2}\right)\left(\frac{x-7}{2} + \frac{2}{2}\right)}{2!}(8) + 0$$

$$y(x) = 51 + 12(x-7) + \frac{(x-7)(x-5)}{1} + 0$$

$$= x^2 - 12x + 35 + 51 + 12x - 84 = \boxed{x^2 + 2}$$

$y' = 2x$

$y'(6) = 12$

x	x_0	x_1	...	x_n
y	y_0	y_1	...	y_n

على فترات غير متساوية

find $y(x)$

طريقة لاجرانج

Lagrange.

نفرض

$$y(x) = a_0(x-x_1)(x-x_2)(x-x_3)\cdots(x-x_n)$$
$$+\,a_1(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)$$
$$\vdots$$
$$+\,a_n(x-x_0)(x-x_1)(x-x_2)\cdots(x-x_{n-1})$$

اذا نعوض $\boxed{x=x_0}$

$$y_0 = a_0(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)$$

$$a_0 = \frac{1}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}\,y_0$$

نعوض $x=x_1$

$$y_1 = a_1(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)$$

$$a_1 = \frac{1}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}\,y_1$$

بالمثل

$$a_n = \frac{1}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}\,y_n$$

نعوض في * لنحصل قيم $a_n \dots a_1, a_0$

$$y(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}\,y_0$$
$$+\,\frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}\,y_1$$
$$\vdots$$
$$+\,\frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}\,y_n$$

X	x_0	x_1	…	x_{n-1}	x_n
y	y_0	y_1			y_n

على فترات غير متساوية

find $y(x)$

طريقة لاجرانج

$$y(x) = l_0 y_0 + l_1 y_1 + \dots l_n y_n$$

الصيغة العامة

$$l_i(x) = \prod_{\substack{i=0 \\ j=0 \\ i \neq j}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

$$y(x) = \sum_{i=0}^{n} l_i y_i$$

Step 1 نكون معاملات لاجرانج l_i

Step 2 $y(x) = \sum_{i=0}^{n} l_i y_i$

نعوض $x = x_1$

$$y_1 = a_1 (x_1 - x_0)(x_1 - x_2) \dots (x_1 - x_n)$$

$$a_1 = \frac{1}{(x_1 - x_0)(x_1 - x_2) \dots (x_1 - x_n)} y_1$$

بالمثل

$$a_n = \frac{1}{(x_n - x_0)(x_n - x_1) \dots (x_n - x_{n-1})} y_n$$

نعوض في * كل من قيم $a_n \dots a_1, a_0$

$$y(x) = \frac{(x - x_1)(x - x_2) \dots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \dots (x_0 - x_n)} y_0 + \frac{(x - x_0)(x - x_2) \dots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \dots (x_1 - x_n)} y_1 + \vdots + \frac{(x - x_0)(x - x_1) \dots (x - x_{n-1})}{(x_n - x_0)(x_n - x_1) \dots (x_n - x_{n-1})} y_n$$

على فترات غير متساوية

ex) Use Lagrange M

x	1	2	4
y	5	8	20

اك

Find
① $y(x)$
② $y(3)$
③ $y'(3)$

step①

x_0 1 $\quad x_1$ 2 $\quad x_2$ 4

$$l_0(x) = \frac{(x-2)(x-4)}{(1-2)(1-4)} = \frac{1}{3}(x^2-6x+8)$$

$$l_1(x) = \frac{(x-1)(x-4)}{(2-1)(2-4)} = -\frac{1}{2}(x^2-5x+4)$$

$$l_2(x) = \frac{(x-1)(x-2)}{(4-1)(4-2)} = \frac{1}{6}(x^2-3x+2)$$

step②
$$y(x) = l_0 y_0 + l_1 y_1 + l_2 y_2$$

①
$$y(x) = \frac{5}{3}(x^2-6x+8) - \frac{8}{2}(x^2-5x+4) + \frac{20}{6}(x^2-3x+2)$$

$$y(x) = x^2 + 0x + 4 = \boxed{x^2+4}$$

② $y(3) = 3^2 + 4 = 13$

③ $y'(x) = 2x$

$$y'(3) = 2(3) = \boxed{6}$$

على فترات غير متساوية

ex) Use Lagrange M

Find $y(7)$

x	2	3	5	8
y	29	105	497	2045

الحل

1) $x_0 = 2$, $x_1 = 3$, $x_2 = 5$, $x_3 = 8$

$$l_0 = \frac{(7-3)(7-5)(7-8)}{(2-3)(2-5)(2-8)} = 4/9$$

$$l_1 = \frac{(7-2)(7-5)(7-8)}{(3-2)(3-5)(3-8)} = \boxed{-1}$$

$$l_2 = \frac{(7-2)(7-3)(7-8)}{(5-2)(5-3)(5-8)} = \boxed{\frac{10}{9}}$$

$$l_3 = \frac{(7-2)(7-3)(7-5)}{(8-2)(8-3)(8-5)} = \frac{4}{9}$$

$$y(7) = l_0 y_0 + l_1 y_1 + l_2 y_2 + l_3 y_3$$

$$= \frac{4}{9}(29) + (-1)(105) + \left(\frac{10}{9}\right)(497) + \left(\frac{4}{9}\right)(2045)$$

$$= 1369$$