Game Playing

Outline

- Game playing
- Game trees
  - Minimax
  - Alpha-beta pruning
  - Adding randomness

Why study games?

- Clear criteria for success
- It's a good reasoning problem, formal and nontrivial.
- Direct comparison with humans and other computer
- Fun
- Games often define very large search spaces
  - chess 35^{100} nodes in search tree, 10^{40} legal states

Games vs. Search Problems

- Unpredictable opponent → specifying a move for every possible opponent reply

- Time limits → unlikely to find goal, must approximate

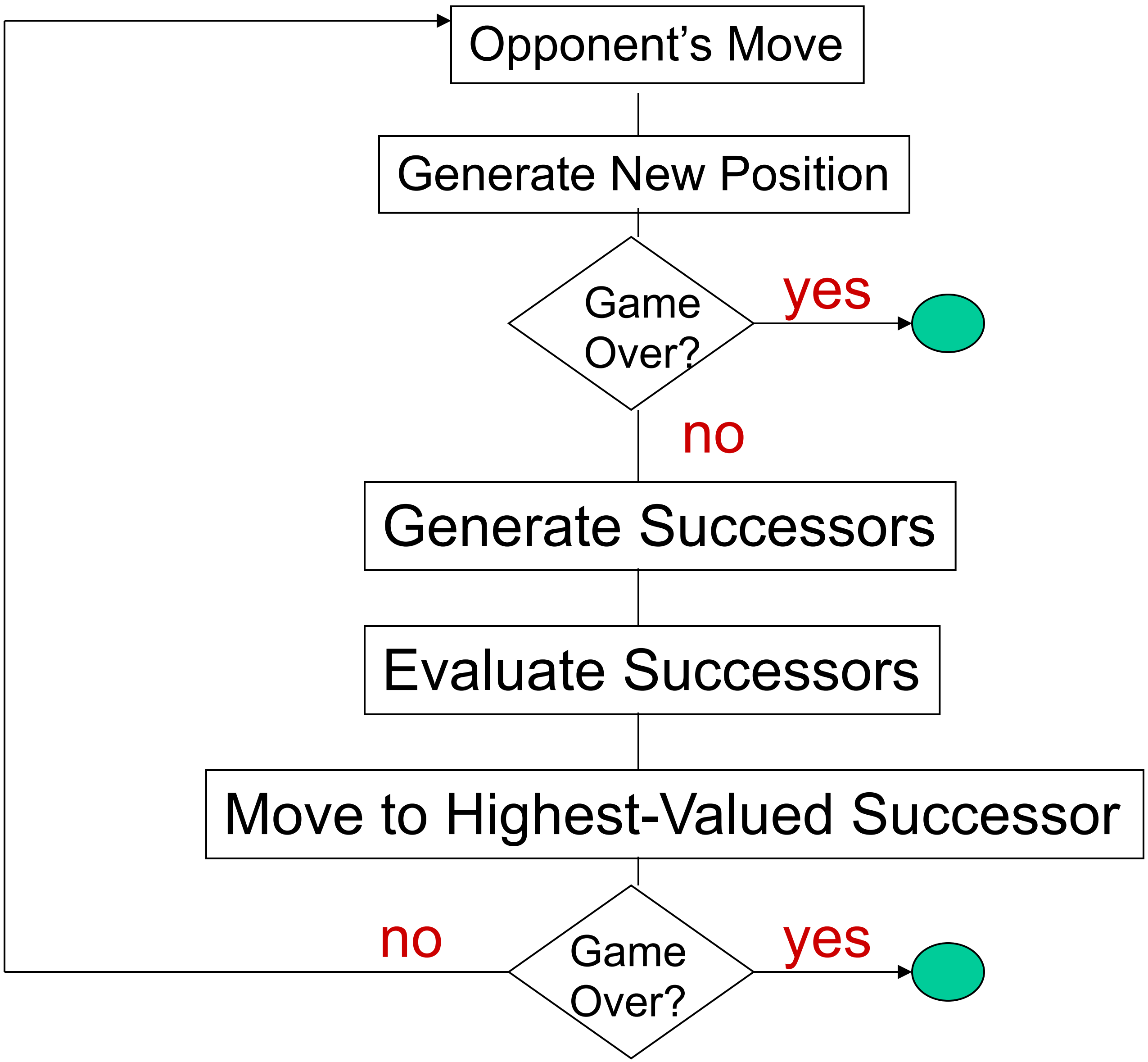
Typical case

- 2-person game
- Players alternate moves
- **Zero-sum**: one player's loss is the other's gain
- **Perfect information**: both players have access to complete information about the state of the game. No information is hidden from either player.
- No chance (e.g., using dice) involved
- Examples: Tic-Tac-Toe, Checkers, Chess, Go, Nim, Othello
- Not: Bridge, Solitaire, Backgammon, ...

How to play a game

- A way to play such a game is to:
  - Consider all the legal moves you can make
  - Compute the new position resulting from each move
  - Evaluate each resulting position and determine which is best
  - Make that move
  - Wait for your opponent to move and repeat
- Key problems are:
  - Representing the “board”
  - Generating all legal next boards
  - Evaluating a position

Two-Player Game



Evaluation function

- **Evaluation function** or **static evaluator** is used to evaluate the "goodness" of a game position.
  - Contrast with heuristic search where the evaluation function was a non-negative estimate of the cost from the start node to a goal and passing through the given node
- The zero-sum assumption allows us to use a single evaluation function to describe the goodness of a board with respect to both players.
  - **f(n) >> 0**: position n good for me and bad for you
  - **f(n) << 0**: position n bad for me and good for you
  - **f(n) near 0**: position n is a neutral position
  - **f(n) = +infinity**: win for me
  - **f(n) = -infinity**: win for you

Evaluation function examples

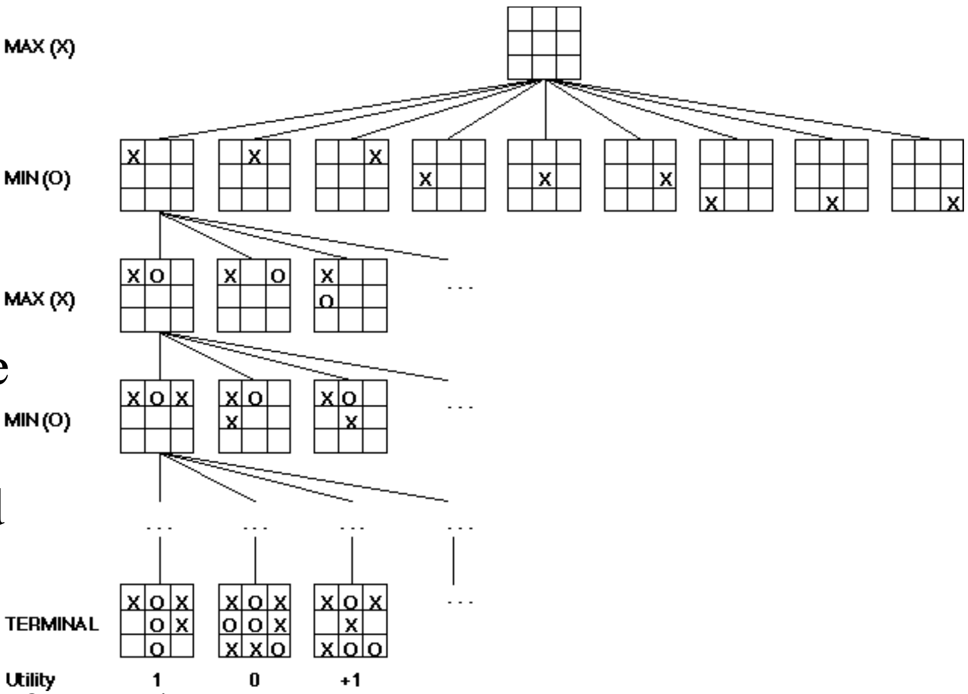
- Example of an evaluation function for Tic-Tac-Toe:

  f(n) = [# of 3-lengths open for me] - [# of 3-lengths open for you]

  where a 3-length is a complete row, column, or diagonal
- Alan Turing's function for chess
  - **f(n) = w(n)/b(n)** where w(n) = sum of the point value of white's pieces and b(n) = sum of black's
- Most evaluation functions are specified as a weighted sum of position features:

  $f(n) = w_1 * feat_1(n) + w_2 * feat_2(n) + ... + w_n * feat_k(n)$
- Example features for chess are piece count, piece placement, squares controlled, etc.
- Deep Blue had over 8000 features in its evaluation function

Game trees



- Problem spaces for typical games are represented as trees
- Root node represents the current board configuration; player must decide the best single move to make next
- **Static evaluator function** rates a board position. f(board) = real number with f>0 “white” (me), f<0 for black (you)
- Arcs represent the possible legal moves for a player
- If it is **my turn** to move, then the root is labeled a "**MAX**" node; otherwise it is labeled a "**MIN**" node, indicating **my opponent's turn**.
- Each level of the tree has nodes that are all MAX or all MIN; nodes at level i are of the opposite kind from those at level i+1

Game Tree (2-player, Deterministic, Turns)

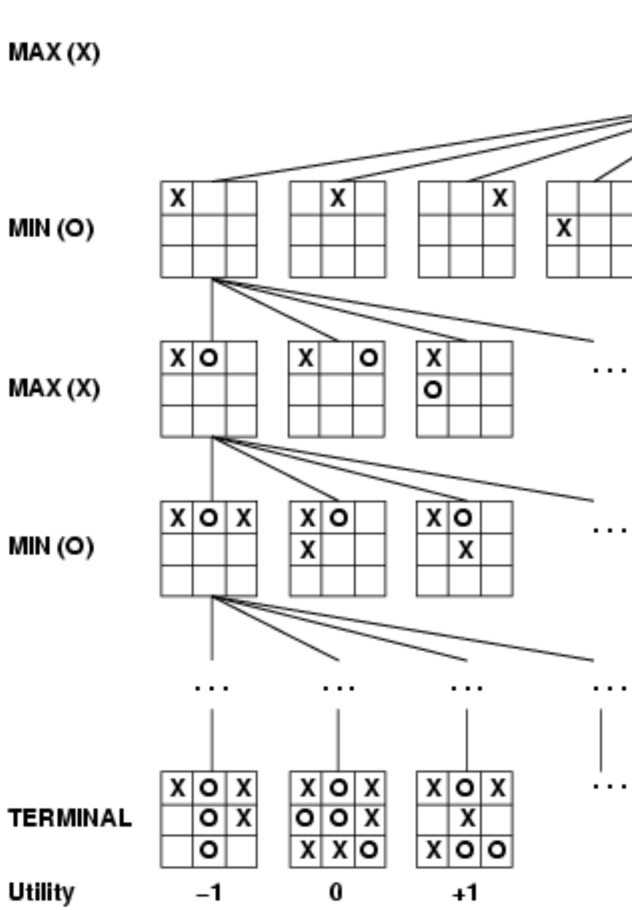
computer's turn

opponent's turn

computer's turn

opponent's turn

leaf nodes are evaluated



The computer is **Max**.
The opponent is **Min**.

At the leaf nodes, the **utility function** is employed. Big value means good, small is bad.

Mini-Max Terminology

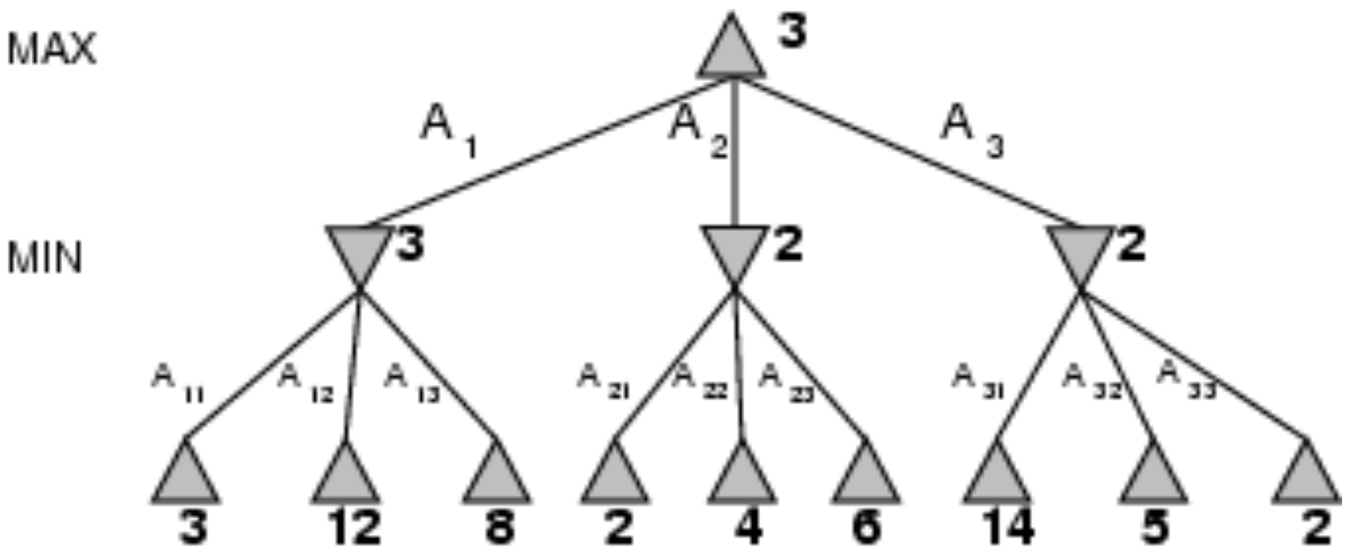
- utility function: the function applied to leaf nodes
- backed-up value
  - of a max-position: the value of its largest successor
  - of a min-position: the value of its smallest successor
- minimax procedure: search down several levels; at the bottom level apply the utility function, back-up values all the way up to the root node, and that node selects the move.

Minimax procedure

- Create start node as a MAX node with current board configuration
- Expand nodes down to some **depth** (a.k.a. **ply**) of lookahead in the game
- Apply the evaluation function at each of the leaf nodes
- “Back up” values for each of the non-leaf nodes until a value is computed for the root node
  - At MIN nodes, the backed-up value is the **minimum** of the values associated with its children.
  - At MAX nodes, the backed-up value is the **maximum** of the values associated with its children.
- Pick the operator associated with the child node whose backed-up value determined the value at the root

Minimax

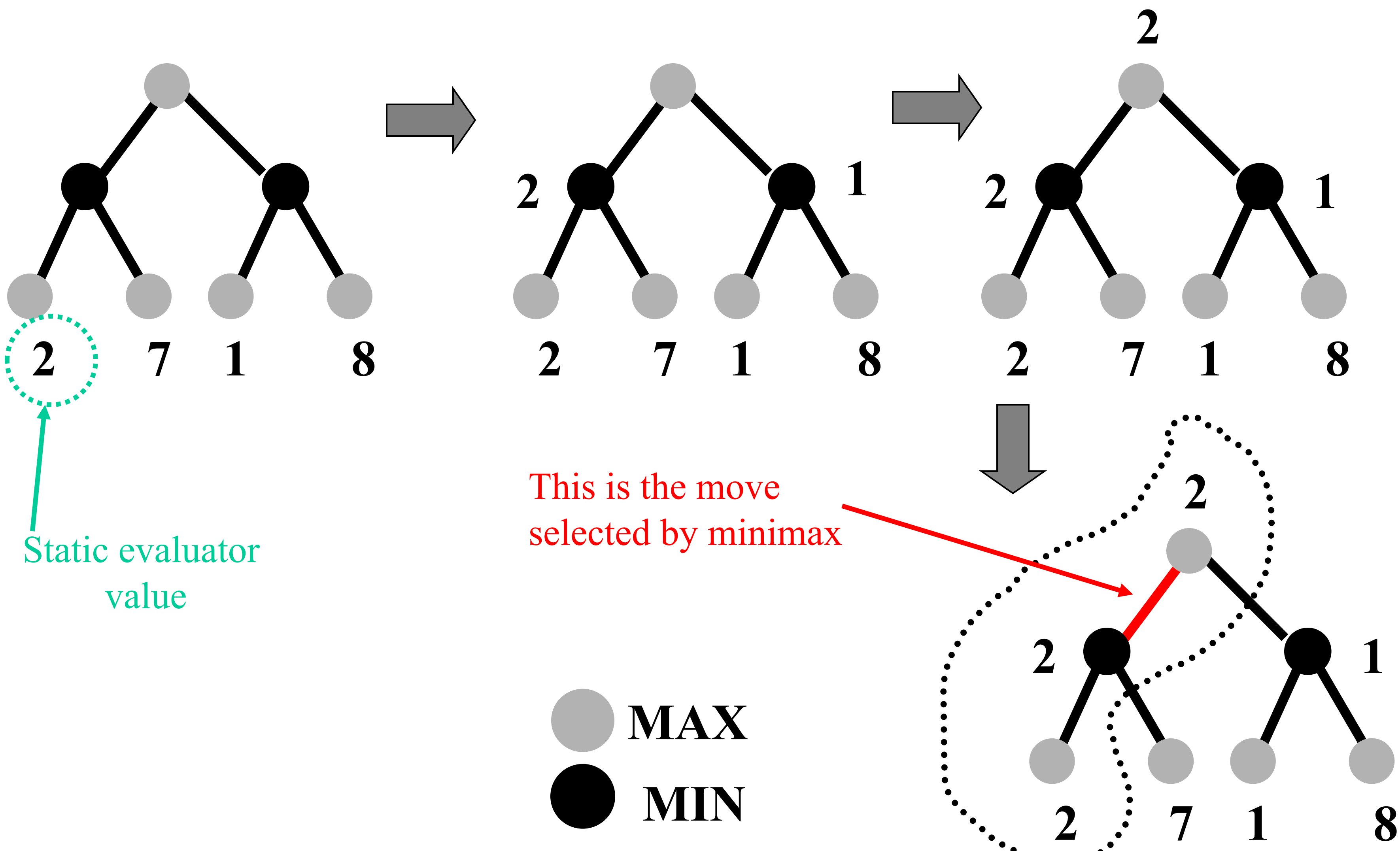
- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:



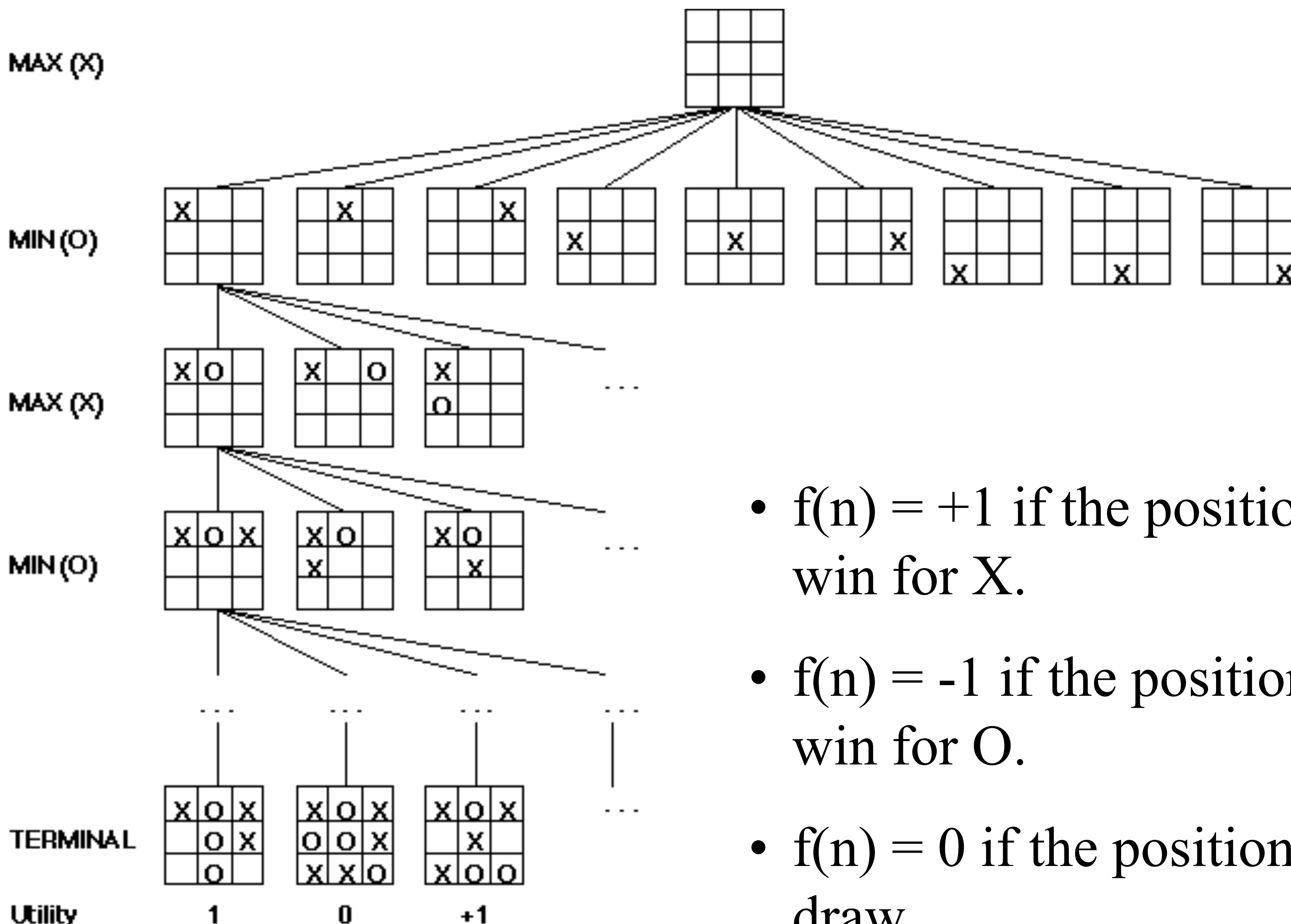
Minimax Strategy

- Why do we take the min value every other level of the tree?
- These nodes represent the opponent's choice of move.
- The computer assumes that the human will choose that move that is of least value to the computer.

Minimax Algorithm



Partial Game Tree for Tic-Tac-Toe



- f(n) = +1 if the position is a win for X.
- f(n) = -1 if the position is a win for O.
- f(n) = 0 if the position is a draw.

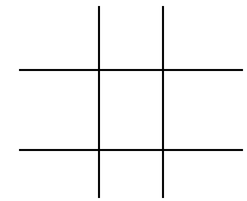
Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
   v ← MAX-VALUE(state)
   return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do
      v ← MAX(v, MIN-VALUE(s))
   return v
```

```
function MIN-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← ∞
   for a, s in SUCCESSORS(state) do
      v ← MIN(v, MAX-VALUE(s))
   return v
```

Tic Tac Toe



- Let p be a position in the game
- Define the utility function f(p) by
  - f(p) =
    - largest positive number if p is a win for computer
    - smallest negative number if p is a win for opponent
    - RCDC – RCDO
  - where RCDC is number of rows, columns and diagonals in which computer could still win
  - and RCDO is number of rows, columns and diagonals in which opponent could still win.

Sample Evaluations

- X = Computer; O = Opponent

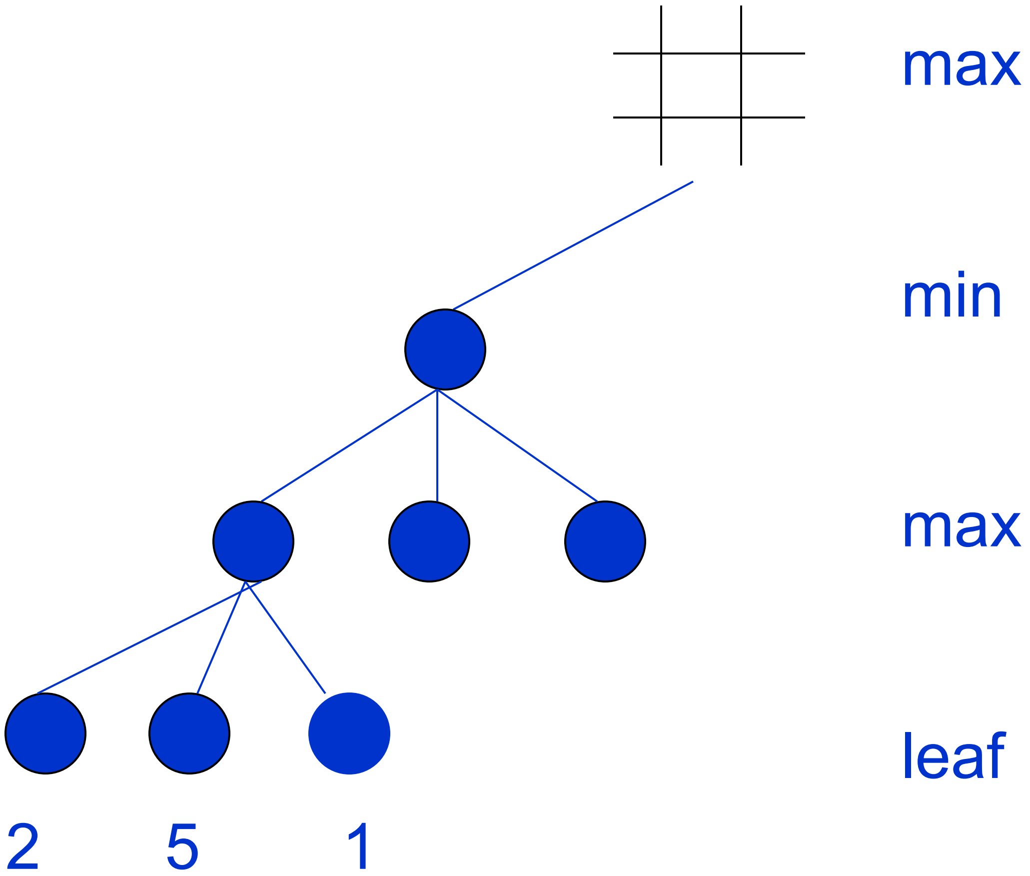
	O	
	X	

O	O	X
X	X	

	X	O
rows		
cols		
diags		

	X	O
rows		
cols		
diags		

Minimax is done depth-first



Properties of Minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

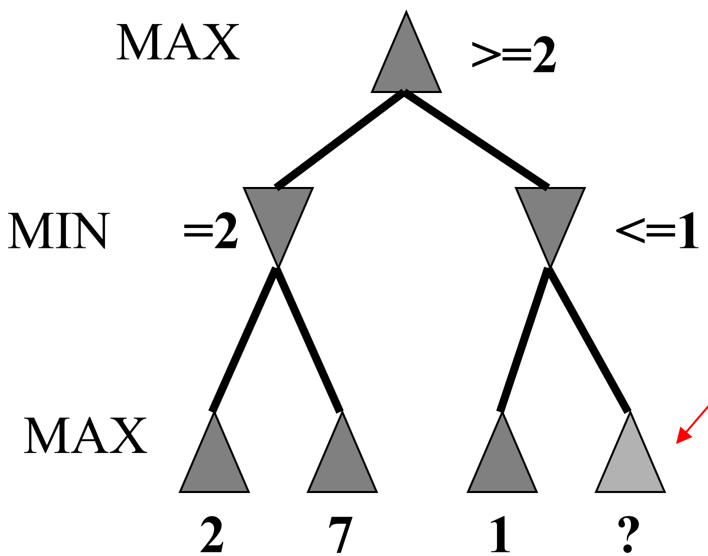
- For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  → exact solution completely infeasible

Need to speed it up.

Alpha-beta pruning

Alpha-beta pruning

- We can improve on the performance of the minimax algorithm through **alpha-beta pruning**
- Basic idea: *"If you have an idea that is surely bad, don't take the time to see how truly awful it is."* -- Pat Winston



- We don't need to compute the value at this node.
- No matter what it is, it can't affect the value of the root node.

Alpha-Beta Procedure

- The alpha-beta procedure can speed up a depth-first minimax search.
- Alpha: a lower bound on the value that a max node may ultimately be assigned

$v \geq \alpha$

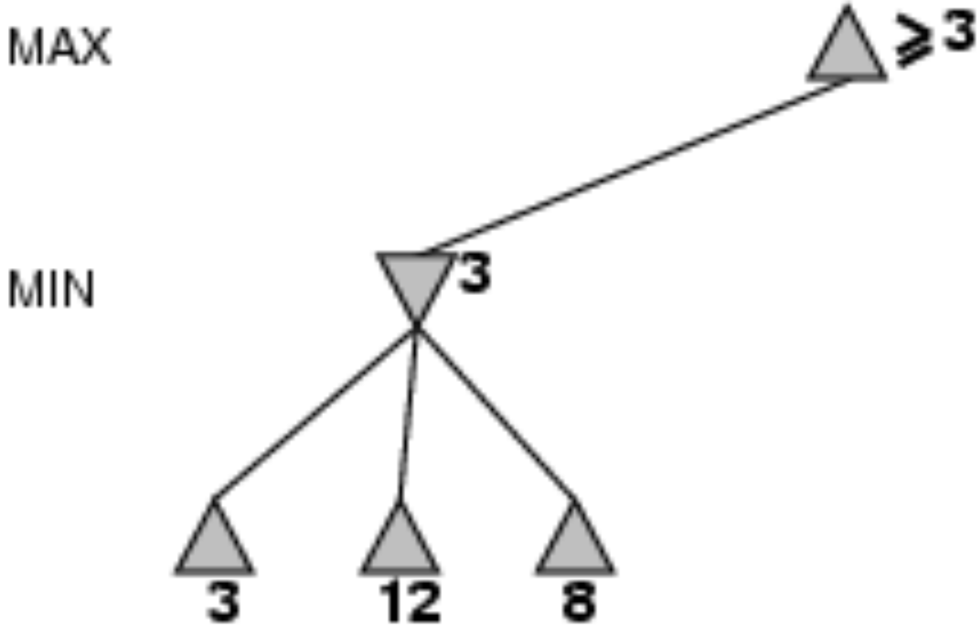
- Beta: an upper bound on the value that a minimizing node may ultimately be assigned

$v \leq \beta$

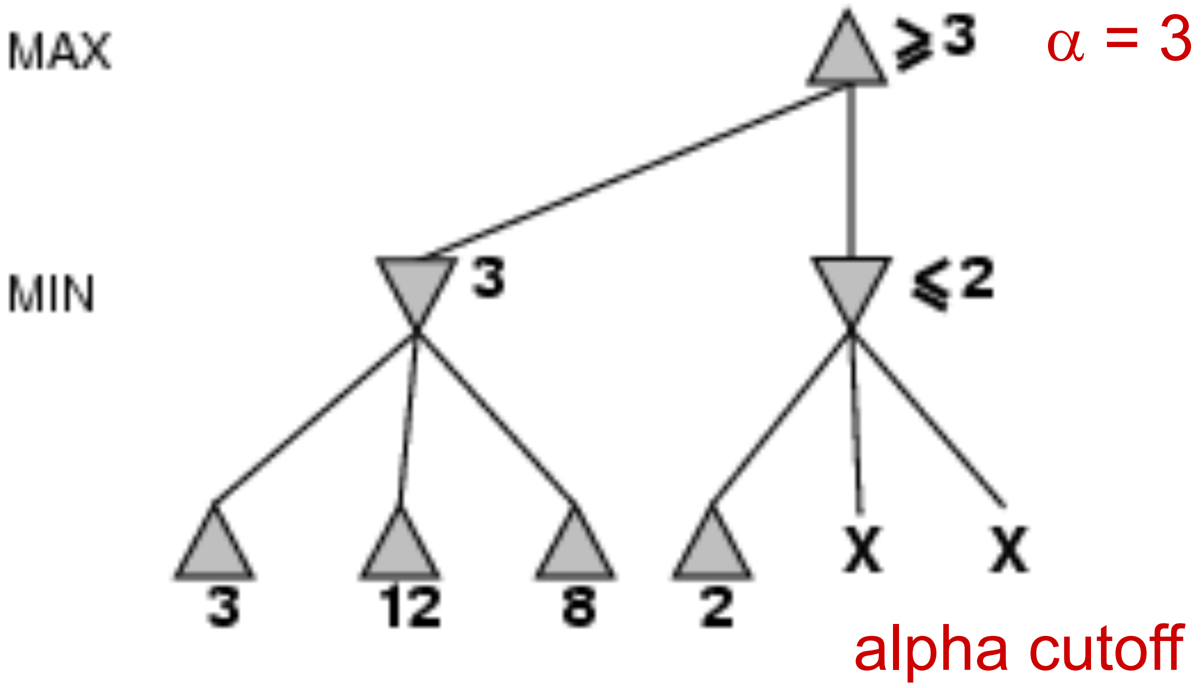
Alpha-beta pruning

- Traverse the search tree in depth-first order
- At each **MAX** node n, **alpha(n)** = maximum value found so far
- At each **MIN** node n, **beta(n)** = minimum value found so far
  - Note: The alpha values start at -infinity and only increase, while beta values start at +infinity and only decrease.
- **Beta cutoff**: Given a MAX node n, cut off the search below n (i.e., don't generate or examine any more of n's children) if alpha(n) >= beta(i) for some MIN node ancestor i of n.
- **Alpha cutoff:** stop searching below MIN node n if beta(n) <= alpha(i) for some MAX node ancestor i of n.

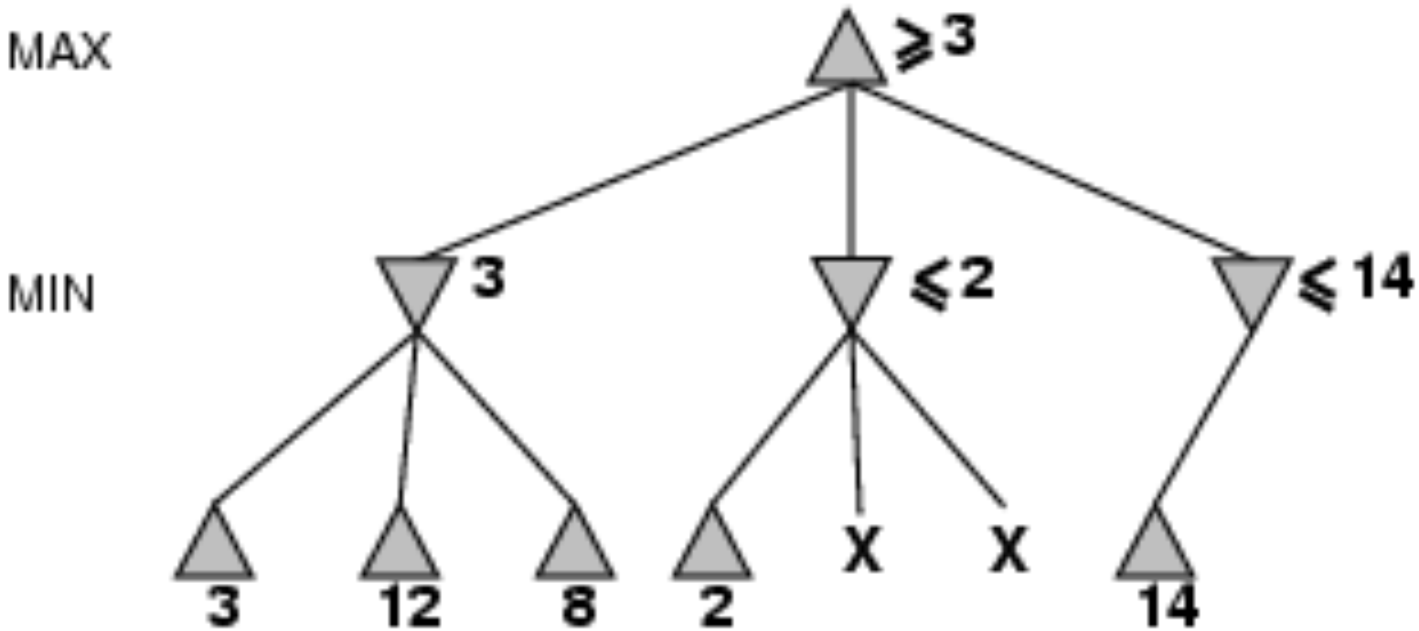
α-β pruning example



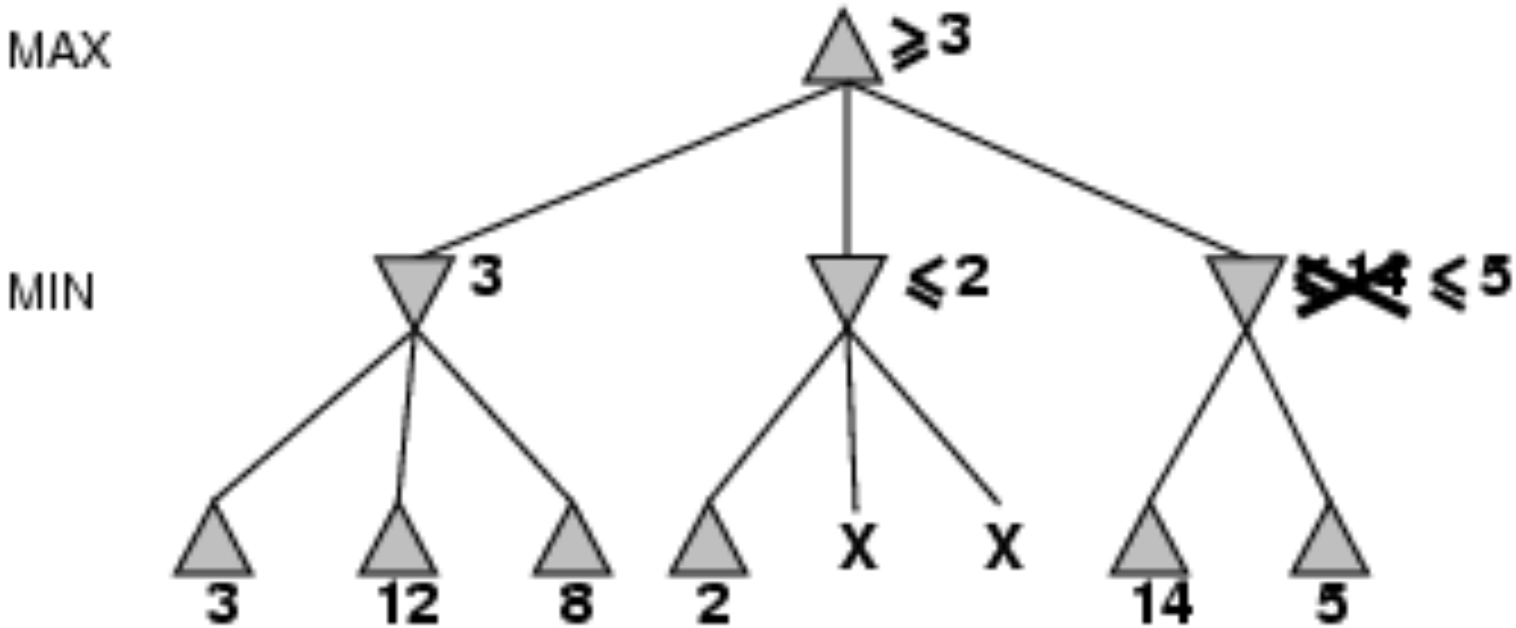
α-β pruning example



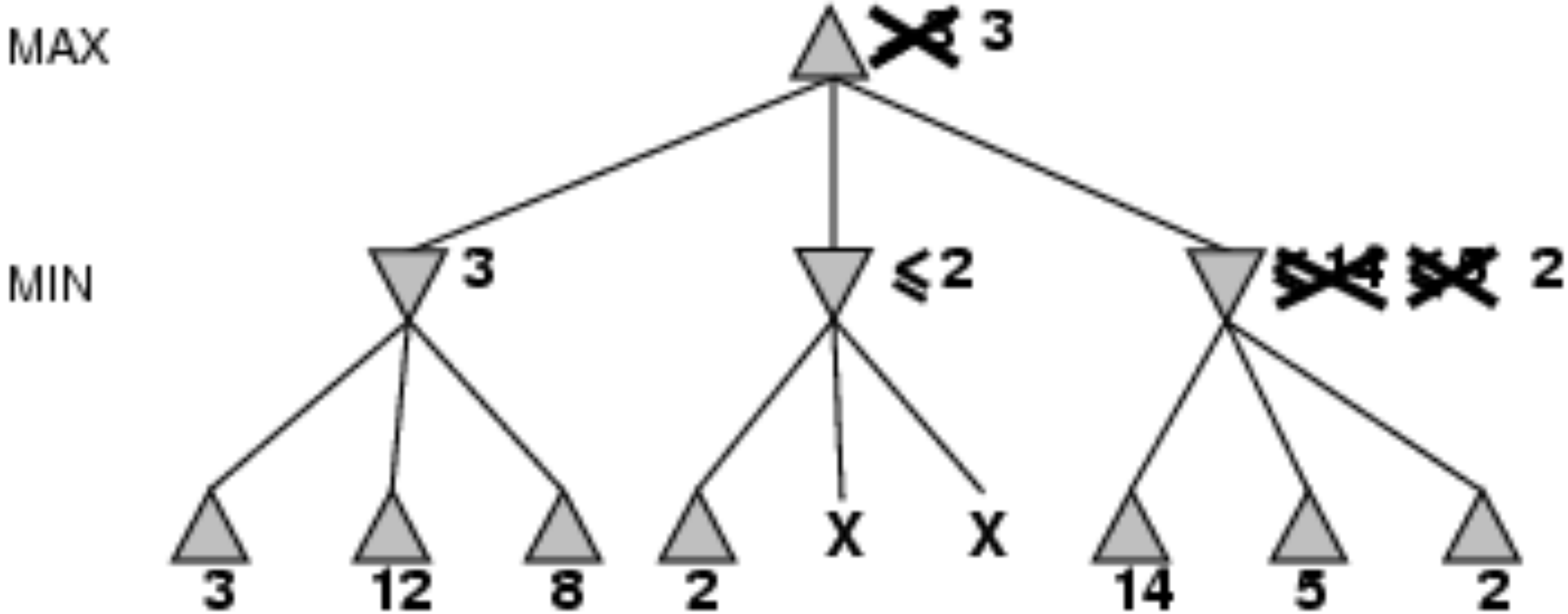
α-β pruning example



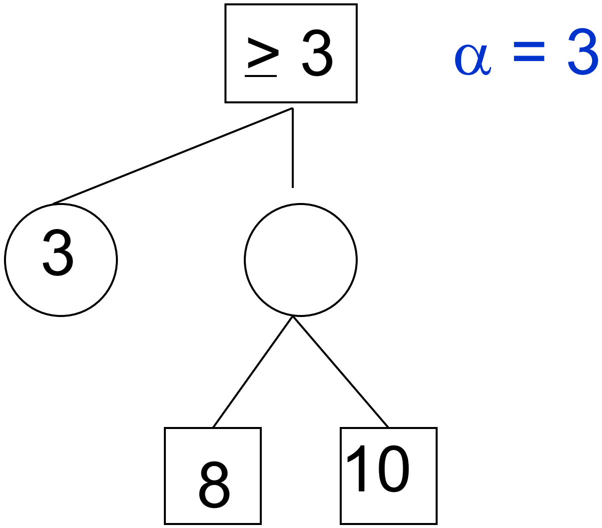
α-β pruning example



α-β pruning example

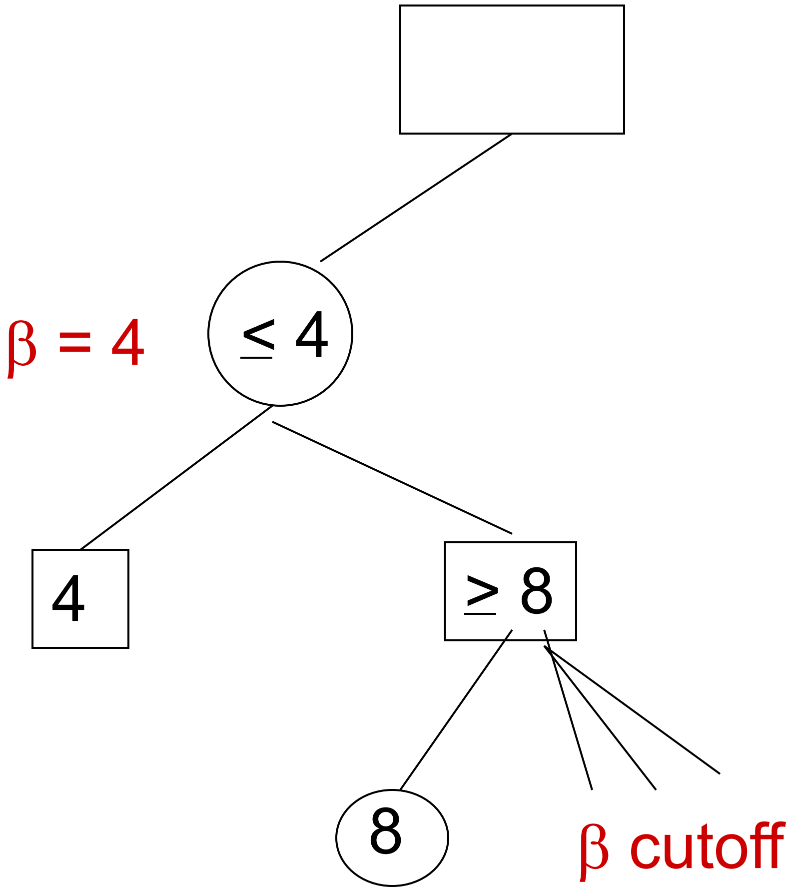


Alpha Cutoff

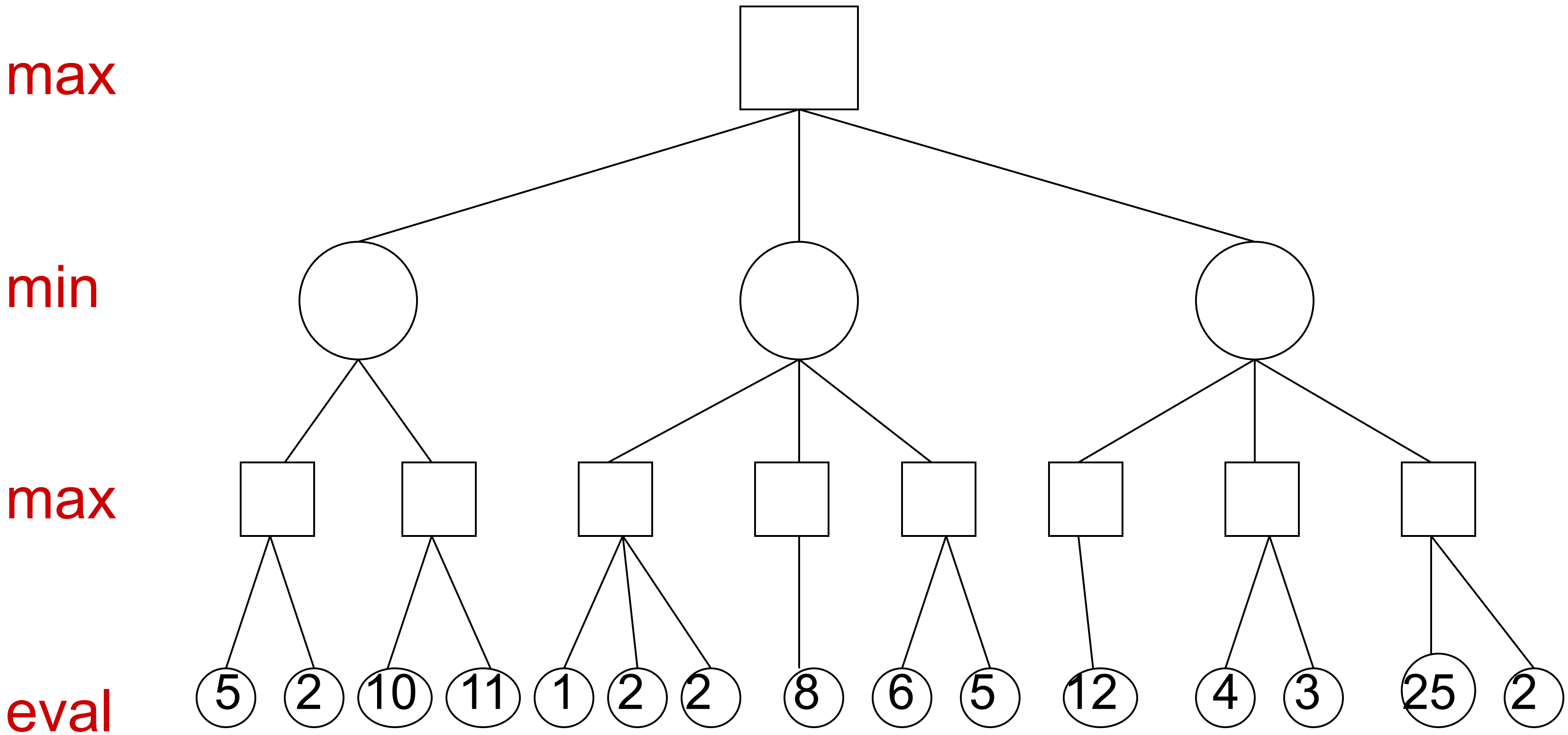


What happens here? Is there an alpha cutoff?

Beta Cutoff



Alpha-Beta Pruning



Properties of α-β

- Pruning does not affect final result. This means that it gets the exact same result as does full minimax.

- Good move ordering improves effectiveness of pruning

- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search

- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

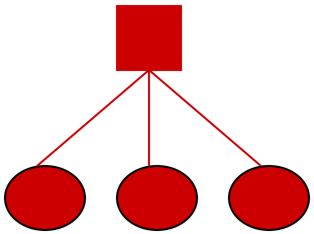
The α-β algorithm

```
function ALPHA-BETA-SEARCH(state) returns an action
   inputs: state, current state in game

   v ← MAX-VALUE(state, −∞, +∞)
   return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
   inputs: state, current state in game
           α, the value of the best alternative for MAX along the path to state
           β, the value of the best alternative for MIN along the path to state

   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do
      v ← MAX(v, MIN-VALUE(s, α, β))
      if v ≥ β then return v      cutoff
      α ← MAX(α, v)
   return v
```



The α-β algorithm

function MIN-VALUE(*state*, α, β) **returns** *a utility value*

  inputs: *state*, current state in game

    α, the value of the best alternative for MAX along the path to *state*

    β, the value of the best alternative for MIN along the path to *state*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

  $v \leftarrow +\infty$

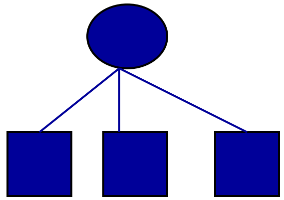
  for a, s in SUCCESSORS(*state*) **do**

    $v \leftarrow$ MIN(v, MAX-VALUE(s, α, β))

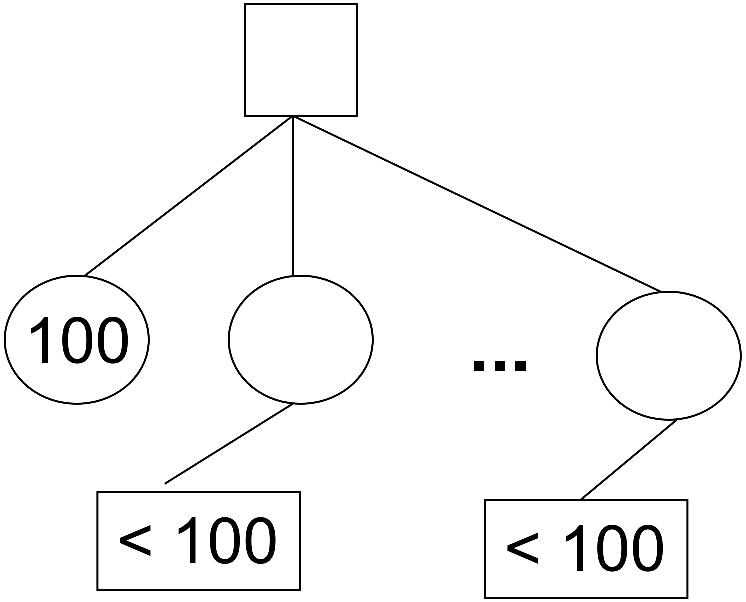
    if $v \leq \alpha$ **then return** v cutoff

    $\beta \leftarrow$ MIN(β, v)

  return v



When do we get alpha cutoffs?

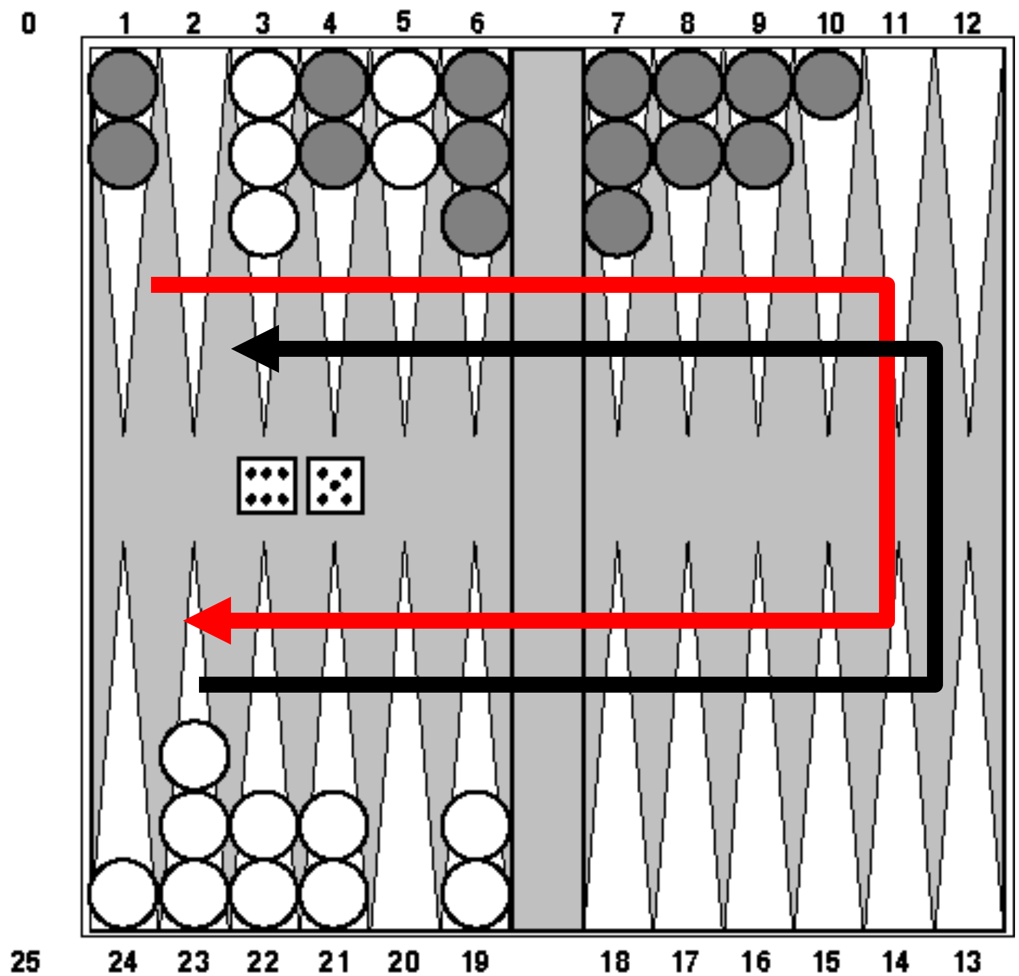


Effectiveness of alpha-beta

- Alpha-beta is guaranteed to compute the same value for the root node as computed by minimax, with less or equal computation
- **Worst case:** no pruning, examining b^d leaf nodes, where each node has b children and a d-ply search is performed
- **Best case:** examine only $(2b)^{d/2}$ leaf nodes.
  - Result is you can search twice as deep as minimax!
- **Best case** is when each player's best move is the first alternative generated
- In Deep Blue, they found empirically that alpha-beta pruning meant that the average branching factor at each node was about 6 instead of about 35!

Games of chance

- Backgammon is a two-player game with **uncertainty**.
- Players roll dice to determine what moves to make.
- White has just rolled *5 and 6* and has four legal moves:
  - 5-10, 5-11
  - 5-11, 19-24
  - 5-10, 10-16
  - 5-11, 11-16
- Such games are good for exploring decision making in adversarial problems involving skill and luck.



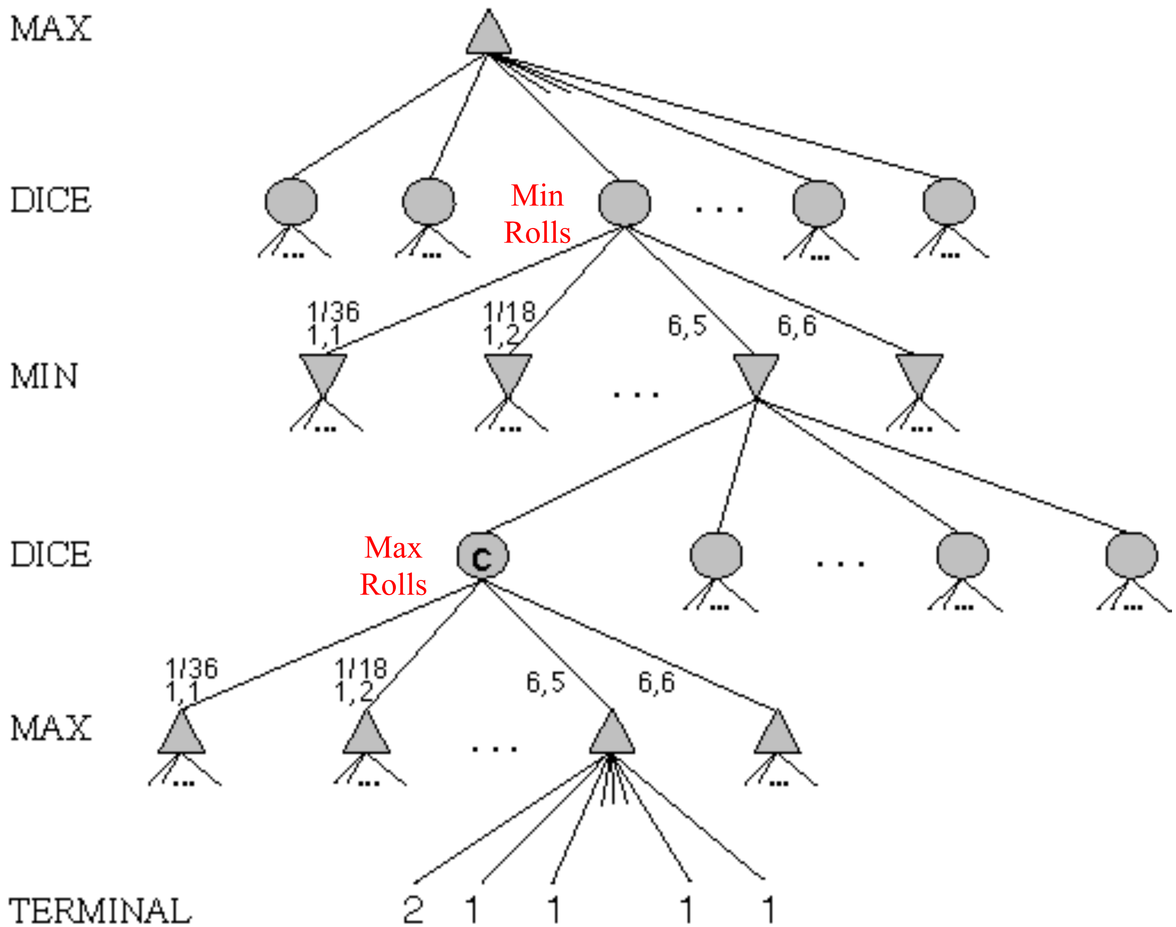
Game trees with chance nodes

- **Chance nodes** (shown as circles) represent random events
- For a random event with N outcomes, each chance node has N distinct children; a probability is associated with each
- (For 2 dice, there are 21 distinct outcomes)
- Use minimax to compute values for MAX and MIN nodes
- Use **expected values** for chance nodes
- For chance nodes over a max node, as in C:

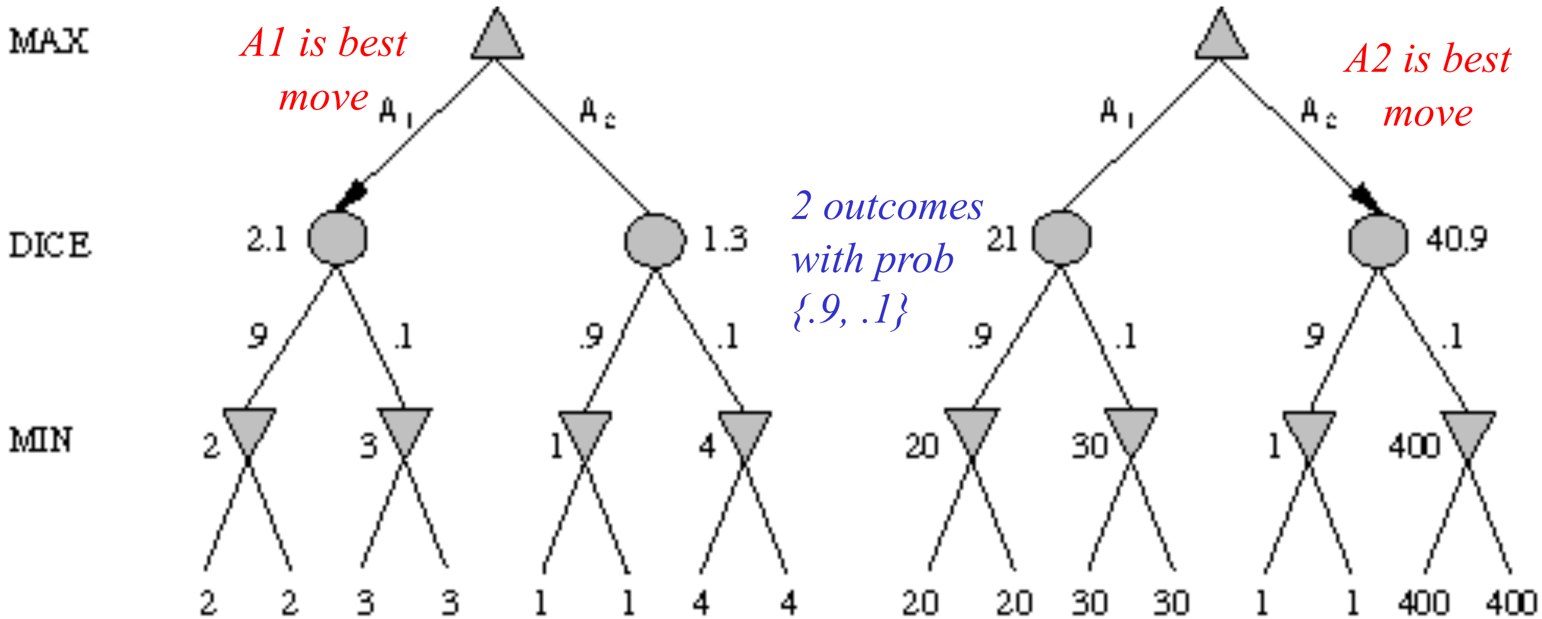
expectimax(C) = $\sum_i (P(d_i) * \text{maxvalue}(i))$

- For chance nodes over a min node:

expectimin(C) = $\sum_i (P(d_i) * \text{minvalue}(i))$



Meaning of the evaluation function



- Dealing with probabilities and expected values means we have to be careful about the "meaning" of values returned by the static evaluator.
- Note that a "relative-order preserving" change of the values would not change the decision of minimax, but could change the decision with chance nodes.
- Linear transformations are OK