MT1003 Applied Help Sheet

Daniel Laing

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1 Difference Equations

1.1 2nd Order $(L4)$

Form: $A_{n+2} + bA_{n+1} + cA_n = F(n)$ with $A_0 = \alpha,\ A_1 = \beta,\ b, c, \alpha, \beta \in \mathbb{R}$
$(A_n)_{CF}$ has sol'n determined by roots of $p(x) = x^2 + bx + c$:

Distinct, Real Roots: $x_{1,2} = \lambda_1, \lambda_2 \to A_n = k_1\lambda_1^n + k_2\lambda_2^n$

Repeated Roots: $x_{1,2} = \lambda \to A_n = \lambda^n(k_1 + k_2 n)$

Complex Roots: $x_{1,2} = p \pm qi = re^{\pm i\vartheta} \to A_n = r^n\left(k_1 \cos(n\vartheta) + k_2 \sin(n\vartheta)\right)$