MT1003 Pure Help Sheet

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1 Graphs

1.1 Directed Graphs

Consists of set V of *vertices* and set E of ordered pairs from V making *edges*. Written: $\Gamma = (V, E)$.
Let Γ be a directed graph:

Walk – Sequence of verticies and edges on Γ.

Path – Walk on Γ $s.t.$ no vertex occurs twice.

Circuit – Closed walk on Γ.

Simple Circuit – Closed path on Γ.

Loop – Edge of form (v, v) with $v \in V$.

1.2 Types & Properties of Graphs

Undirected Graph – A graph $s.t.$ $\forall\ (v_i, v_j) \in \Gamma\ \exists\ (v_j, v_i)$. Usually just called *graph*.

Multigraph – Graph with multiple edges between verticies.

Simple Graph – Undirected, finite, no loops, no multiple edges.

Connected Graph – $\exists$ path between any two distinct verticies a, b.

Vertex Degree – # of edges incedent on vertex.

Regular Graph – All verticies have same degree.

Complete Graph – Graph, K_n, is simple, connected with n verticies.

Null Graph – Graph, N_n, with n verticies and no edges. Also a regular graph.

Cycle Length – Graph C_n $s.t.$ it forms a circle.

1.3 Trees

Tree – Connected, no circuits.

Spanning Tree – Subgraph of Γ which contains all verticies and is a tree. Minimal connectivity.

Theorem 8.22: $\exists$ a unique path between any two verticies on a tree.
Proof: Since T is connected, $\exists \geqslant 1$ path from $a \to b$. I 1 then $\exists$ a circuit in T $\therefore$ not a tree.
Theorem 8.23: Let Γ be *undirected graph*. Γ connected if and only if it has spanning tree.
Theorem 8.24: $\exists$ 3 non-iosmprphic trees with 5 verticies.
Theorem 8.25: Let $T = (V, E)$ be a tree.
Then $|E| = |V| - 1$.
Proof: [THINGS]

1.4 Eulerian & Hamiltonian Graphs

Eulerian Graph – $\exists$ circuit in Γ passing though every vertex and edge exactly once.

Semi-Eulerian Graph – $\exists$ walk in Γ passing though every vertex and edge exactly once.