

# MT1003 PURE HELP SHEET

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**Theorem 8.24:**  $\exists$  3 non-isomorphic trees with 5 vertices.

**Theorem 8.25:** Let  $T = (V, E)$  be a tree.

Then  $|E| = |V| - 1$ .

PROOF: [THINGS]

## 1 Graphs

### 1.1 Directed Graphs

Consists of set  $V$  of *vertices* and set  $E$  of ordered pairs from  $V$  making *edges*.  
Written:  $\Gamma = (V, E)$ .

Let  $\Gamma$  be a directed graph:

**Walk** – Sequence of vertices and edges on  $\Gamma$ .

**Path** – Walk on  $\Gamma$  s.t. no vertex occurs twice.

**Circuit** – Closed walk on  $\Gamma$ .

**Simple Circuit** – Closed path on  $\Gamma$ .

**Loop** – Edge of form  $(v, v)$  with  $v \in V$ .

### 1.2 Types & Properties of Graphs

**Undirected Graph** – A graph s.t.  
 $\forall (v_i, v_j) \in \Gamma \exists (v_j, v_i)$ . Usually just called *graph*.

**Multigraph** – Graph with multiple edges between vertices.

**Simple Graph** – Undirected, finite, no loops, no multiple edges.

**Connected Graph** –  $\exists$  path between any two distinct vertices  $a, b$ .

**Vertex Degree** – # of edges incident on vertex.

**Regular Graph** – All vertices have same degree.

**Complete Graph** – Graph,  $K_n$ , is simple, connected with  $n$  vertices.

**Null Graph** – Graph,  $N_n$ , with  $n$  vertices and no edges. Also a regular graph.

**Cycle Length** – Graph  $C_n$  s.t. it forms a circle.

### 1.3 Trees

**Tree** – Connected, no circuits.

**Spanning Tree** – Subgraph of  $\Gamma$  which contains all vertices and is a tree.  
Minimal connectivity.

**Theorem 8.22:**  $\exists$  a unique path between any two vertices on a tree.

PROOF: Since  $T$  is connected,  $\exists \geq 1$  path from  $a \rightarrow b$ . If 1 then  $\exists$  a circuit in  $T$   $\therefore$  not a tree.

**Theorem 8.23:** Let  $\Gamma$  be *undirected graph*.  $\Gamma$  connected if and only if it has spanning tree.

### 1.4 Eulerian & Hamiltonian Graphs

**Eulerian Graph** –  $\exists$  circuit in  $\Gamma$  passing through every vertex and edge exactly once.

**Semi-Eulerian Graph** –  $\exists$  walk in  $\Gamma$  passing through every vertex and edge exactly once.