PH1012 Help Sheet

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Quantum Phenomena

Quantisation ($n \in \mathbb{N}$)

Energy of photon (γ): $E_\gamma = nh\nu$

Angular momentum: $l = n\hbar$

Particle Nature of Light

1. Photoelectric Effect:
   → light is absorbed in "chunks".
2. X-ray Production:
   → light is produced in "chunks".
3. Compton Scattering:
   → light has momentum.

Photoelectric Effect ($L2$)

Classical: → electron receives oscillating force, magnitude increases until ejected.

Quantum: → electron receives kick from γ, energy transferred. Energy breaks attraction (work function, W):

$$E_k = h\nu - W, \quad eV_0 = E_{kmax}$$

$$\implies eV_0 = h\nu - W$$

X-ray Production ($L3$)

Produced by "breaking energy" of electron passing nucleus.
Inner shell interactions give characteristic radiation (K_α and K_β).
e^- completely stopped gives λ_{min}:

$$\rightarrow \lambda_{min} = \frac{hc}{eV}$$

Compton Scattering ($L4$)

γ incident on electron scatters:
→ assumes γ has momentum.

But see 2 peaks, unchanged & shifted, shift depends on angle:

$$\rightarrow \Delta\lambda = \frac{h}{mc}(1-\cos\vartheta) \quad \left(= \frac{2h}{mc}\sin^2\frac{\vartheta}{2}\right)$$

λ shift from scattering e^-.

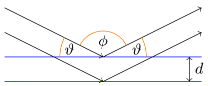
Unchanged λ from strongly bound e^-, effectively interacting with atom:
→ ≈∞ mass $\implies \Delta\lambda \rightarrow 0$

Matter Waves ($L5$)

Double slit with single γ:
→ γ interferes with itself.
Can't predict landing place, only probability, $P \propto |E|^2$ Position not defined until measurement.
All matter has wave-like properties:

$$\rightarrow \lambda = \frac{h}{p}$$

Bragg Scattering



Needs:

1. $\phi \approx 2\vartheta$
2. $\lambda \approx$ atomic size (d).
3. Moderate acceleration ($V \approx 50V$).

$$2d\sin\vartheta = n\lambda \quad n \in \mathbb{N}$$

Larger particles also interfere
e.g. small viruses, vitamins.
Accelerated e^- have energy:

$$\rightarrow E_{e^-} = \frac{h}{\sqrt{2m_e eV}}$$

Uncertainty Principle ($L6$)

$$\Delta x \Delta p_x \geqslant \frac{\hbar}{2\pi}$$

$$E = \frac{h}{\sqrt{2m_e eV}}$$

$$\frac{d^2\Psi(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]\Psi(x) = 0$$

Reduced mass: $\mu = \dfrac{m_1 + m_2}{m_1 m_2}$

$$E = \frac{h^2 n^2}{8mL^2} \quad n \in \mathbb{N}$$

$$\frac{1}{\lambda} = R_\infty Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \quad R_\infty = \frac{1}{(4\pi\varepsilon_0)^2} \cdot \frac{me^4}{4\pi\hbar^3 c}$$

$$E_n = -\frac{Z^2}{n^2}R_y, \quad R_y = hcR_\infty$$

$$r_n = a_0\frac{n^2}{Z}, \quad a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$$

Mechanics

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\vartheta}{dt^2}$$

$$v = r\omega \left(= r\frac{d\vartheta}{dt}\right), \qquad a = r\alpha \left(= r\frac{d\omega}{dt}\right)$$

$$F_{central} = \frac{mv^2}{r} = mr\omega^2$$

$$|\vec{F}| = G\frac{m_1 m_2}{r^2}$$

$$g(r) = \frac{GM(r)}{r^2}$$

$$P^2 = \frac{4\pi^2}{GM}a^3$$

$$U(r) = -G\frac{Mm}{r}$$

Lasers

$$\Delta\nu = \frac{c\Delta\lambda}{\lambda^2}$$

$$R = \frac{A_{21}}{\rho(\nu)B_{21}} = \exp\left[\frac{h\nu}{k_B T}\right] - 1$$

$$I_2 = I_1 e^{2\alpha L} R_1 R_2$$

Amp. threshold: $e^{2\alpha L} R_1 R_2 = 1$

Max. efficiency: $\eta = \frac{\nu_{emission}}{\nu_{pump}} = \frac{\lambda_{pump}}{\lambda_{lasing}}$

$$2L_{optical} = m\lambda_m \quad m \in \mathbb{N}$$

Reflectivity, $R = \left(\frac{n_1 - n_0}{n_1 + n_0}\right)^2$

$$\Delta\lambda_{FSR} = \frac{\lambda^2}{2L_{opt}}$$

$$P_{peak} = \frac{E_{pulse}}{t_{pulse}}$$

$$P_{average} = PRF \cdot E_{pulse}$$

$$t_{pulse} \approx \frac{2L_{opt}}{c(1 - Ref)}$$

$$t_{interval} = \frac{2L_{opt}}{c}$$