Gaussian Hill - FlowDirection and FlowAccumulation

February 17, 2020

1 Calculating FlowDirection and Flowaccumulation on a Gaussian Hill

Bodo Bookhagen and Aljoscha Rheinwalt

Note: richDEM will not work on Windows OS - this notebook will only run on Linux/Ubuntu or MacOS-X

First, define Gaussian Function and analytical (first and second order) derivatives. Define the slope function (see 'Gaussian Hill and DEM analysis.ipynb') for more details. For analytical details see Rheinwalt et al., 2019 (JGR-Earth Surface: https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JF004827) and Smith et al., 2019 (Earth Surface Dynamics: https://www.earth-surf-dynam.net/7/475/2019/).

[7]:

```
import numpy as np
import richdem as rd
from matplotlib import pyplot as pl
from matplotlib.colors import LogNorm
pl.rcParams['figure.figsize'] = [14, 7]

def gaussian_hill_elevation(n, b = 2.5):
    x, y = np.meshgrid(np.linspace(-b,b,n),
                       np.linspace(-b,b,n))
    z = np.exp(-x*x-y*y)
    return (x, y, z)

def gaussian_hill_slope(n, b = 2.5):
    x, y = np.meshgrid(np.linspace(-b,b,n),
                       np.linspace(-b,b,n))
    r = np.sqrt(x*x+y*y)
    return 2*r*np.exp(-r*r)

def gaussian_hill_curvature(n, b = 2.5):
    x, y = np.meshgrid(np.linspace(-b,b,n),
                       np.linspace(-b,b,n))
    r = np.sqrt(x*x+y*y)
    return (1 - 2*r*r)*2*np.exp(-r*r)
```

```
def gaussian_hill_sca(n, b = 2.5):
    x, y = np.meshgrid(np.linspace(-b,b,n),
                       np.linspace(-b,b,n))
    r = np.sqrt(x*x+y*y)
    return r/2.0

def np_slope(x, y, z):
    d = y[1,0] - y[0,0]
    dy, dx = np.gradient(z, d)
    return np.sqrt(dx*dx+dy*dy)

def np_curvature(x, y, z):
    d = y[1,0] - y[0,0]
    dy, dx = np.gradient(z, d)
    dz = np.sqrt(dx*dx+dy*dy)
    dy, dx = np.gradient(dz, d)
    return np.sqrt(dx*dx+dy*dy)
```

Next, calculate Flowdirection using richDEM (rd.FlowAccumulation). Here, we use the Multiple-Flow-Direction (MFD) after Freeman, 1991).

[8]:

```
# Gaussian hill
n = 234
x, y, z = gaussian_hill_elevation(n)
d = y[1,0] - y[0,0]
sca = rd.FlowAccumulation(rd.rdarray(z, no_data = -9999), method = 'Freeman',␣
 ↪exponent = 1.1)
sca *= d
```

```
Warning! No geotransform defined. Choosing a standard one! (Top left cell's top
let corner at <0,0>; cells are 1x1.)
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```
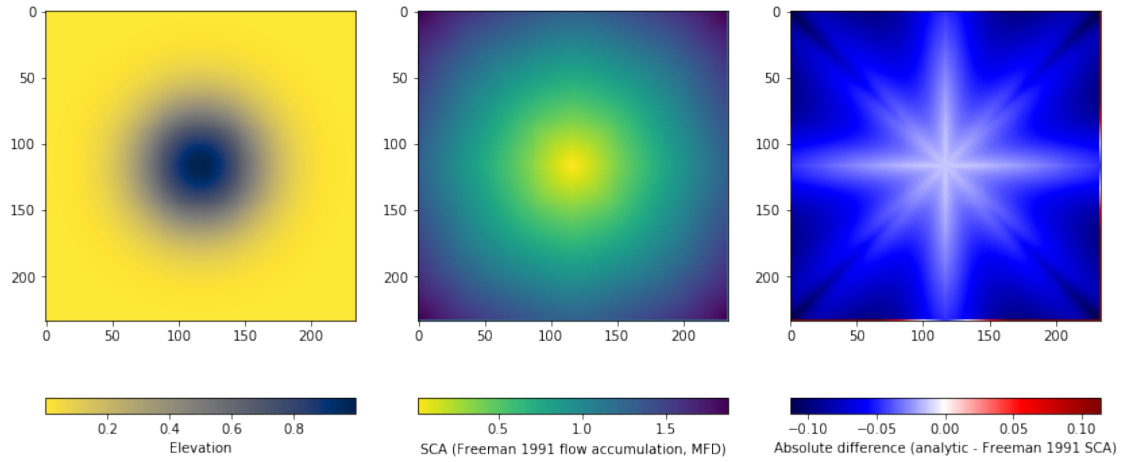
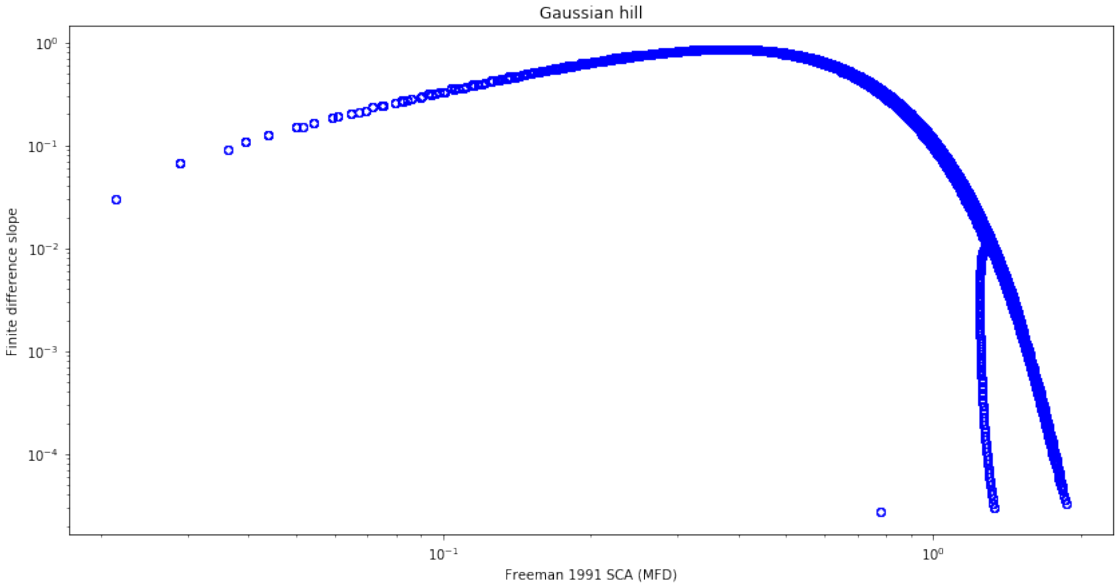

Then visualize the gaussian Hill and the flow accumulation (drainage area). Because

[9]:

```
fg, ax = pl.subplots(1, 3)
im = ax[0].imshow(z, cmap = pl.cm.cividis_r)
cb = fg.colorbar(im, ax = ax[0], orientation = 'horizontal')
cb.set_label('Elevation')

im = ax[1].imshow(sca, cmap = pl.cm.viridis_r)
cb = fg.colorbar(im, ax = ax[1], orientation = 'horizontal')
cb.set_label('SCA (Freeman 1991 flow accumulation, MFD)')

v = gaussian_hill_sca(n) - sca
vmin = v.min()
im = ax[2].imshow(v, cmap = pl.cm.seismic, vmin = vmin, vmax = -vmin)
```

```
cb = fg.colorbar(im, ax = ax[2], orientation = 'horizontal')
cb.set_label('Absolute difference (analytic - Freeman 1991 SCA)')
pl.show()

pl.title('Gaussian hill')
pl.loglog(sca, np_slope(x, y, z), 'bo', mfc = 'none')
pl.xlabel('Freeman 1991 SCA (MFD)')
pl.ylabel('Finite difference slope')
pl.show()
```





2 Repeat Analysis for Gaussian Channel Head

[10]:

```
# Gaussian valley head
n = 234
x, y, z = gaussian_hill_elevation(n)
d = y[1,0] - y[0,0]
z = 1 - z

#x, y, z = x[:,n//2:], y[:,n//2:], z[:,n//2:]

sca = rd.FlowAccumulation(rd.rdarray(z, no_data = -9999), method = 'Freeman',␣
 ↪exponent = 1.1)
sca *= d

fg, ax = pl.subplots(1, 2)
im = ax[0].imshow(z, cmap = pl.cm.cividis_r)
cb = fg.colorbar(im, ax = ax[0], orientation = 'horizontal')
cb.set_label('Elevation')

im = ax[1].imshow(sca, cmap = pl.cm.viridis_r, norm = LogNorm())
cb = fg.colorbar(im, ax = ax[1], orientation = 'horizontal')
cb.set_label('Freeman 1991 flow accumulation (MFD SCA)')
pl.show()

pl.title('Gaussian pit')
pl.loglog(sca, np_slope(x, y, z), 'bo', mfc = 'none')
pl.xlabel('Freeman 1991 SCA (MFD)')
pl.ylabel('Finite difference slope')
pl.show()
```

```
Warning! No geotransform defined. Choosing a standard one! (Top left cell's top
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```
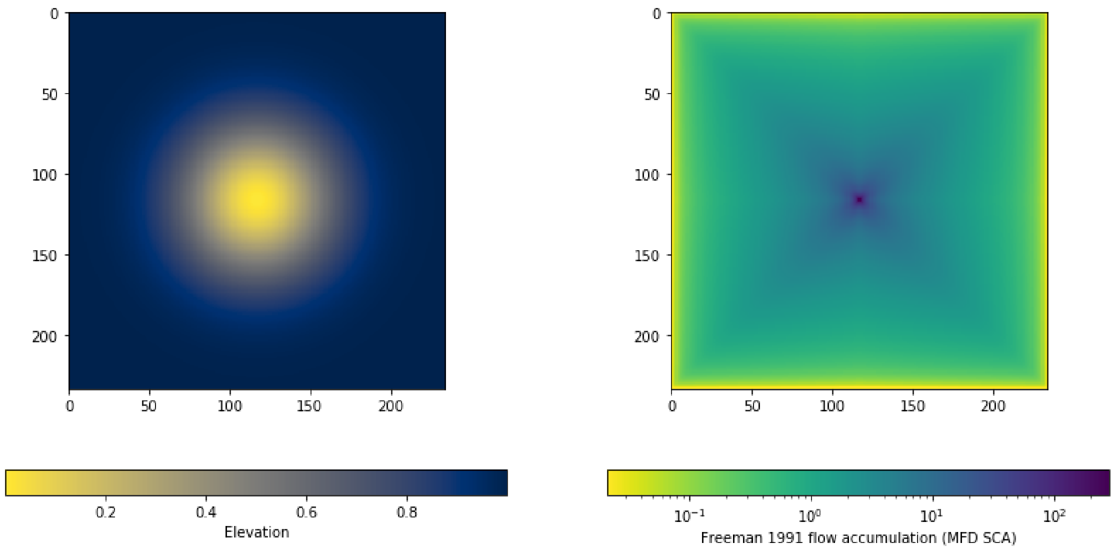
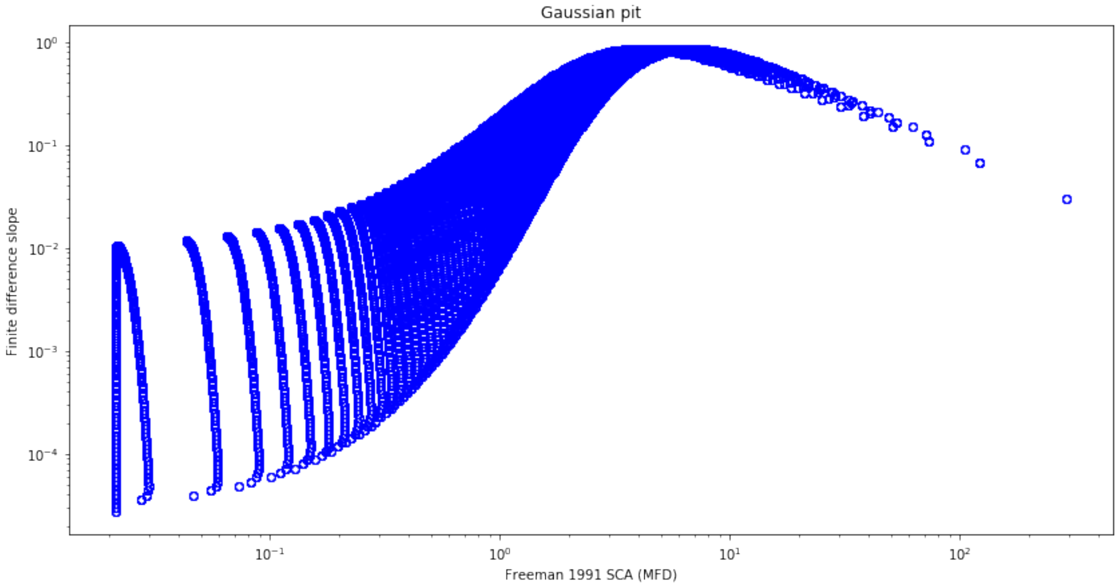


0
50
100
150
200
0
50
100
150
200
0.2
0.4
0.6
0.8
Elevation
10^{-1}
10^{0}
10^{1}
10^{2}
Freeman 1991 flow accumulation (MFD SCA)


Gaussian pit
Finite difference slope
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^{0}
10^{1}
10^{2}
Freeman 1991 SCA (MFD)

[ ]: