Exercise 123

a) LS estimator

$$\hat{\beta} = \underset{x}{argmin}||Y - \mathbf{X}\beta|| = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T Y$$

Some matrix multiplication:

$$(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T = \begin{pmatrix} 1.88 & -1.82 & 1.26 & -1.41 & -2.85 & 3.95 \\ -1.04 & 1.21 & -0.67 & 0.96 & 1.83 & -2.29 \end{pmatrix}$$

Result:

$$\hat{\beta} = (-35.95, 59.44)^T$$

b) estimated residuals

$$\hat{U} = Y - \mathbf{X}\hat{\beta} = \begin{pmatrix} 57 \\ 68 \\ 59 \\ 66 \\ 72 \\ 52 \end{pmatrix} - \begin{pmatrix} 57 \\ 68 \\ 59.16 \\ 66.8 \\ 71.05 \\ 51 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -0.16 \\ -0.8 \\ 0.95 \\ 1 \end{pmatrix}$$

c) alternative model

New design matrix:

$$\mathbf{X} = \begin{pmatrix} 1.5700 & 1.7500 & 1.60 & 1.7300 & 1.80 & 1.4700 \\ 2.4649 & 3.0625 & 2.56 & 2.9929 & 3.24 & 2.1609 \end{pmatrix}$$