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Learning and Competition in the Differentiated Products Market

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Contents

1 Introduction

Firms on real markets need to make pricing and product design decisions in diverse consumer environments with limited information. Consumers have different preferences and willingness to pay for product attributes, and firms tend not to directly see them and cannot accurately forecast demand. Rather, firms use these data over time to direct their decisions. Knowing how companies set prices and attribute products in such contexts is a central question in industrial economics and microeconomics.

Conventional economic models commonly concern firm behavior with static equilibrium models while using a strong information assumption to establish firm performance. Traditional price competition classic models assume that firms know what is needed and are solving problems analytically in order to reach the equilibrium approach. Although the theoretical contribution of these models is crucial, they lack the insights from two characteristic areas central to many real-world markets: the heterogeneity of consumer preferences and the fact that firms typically learn about demand through experimentation rather than necessarily perfect information.

This paper constructs some of the components of the structural model of consumer demand in which individual consumers vary in their willingness to pay and prefer product attributes. To compete in a market for distinctively priced goods, companies decide not just prices but also features of the products. Demand is the outcome of individual consumer choices, probabilistically dependent on price and the level of conformity of all features of the product with consumer preferences. Consequently, aggregate demand and firm profits are endogenous results of heterogeneous economic decisions at the micro level.

Because those profit functions usually do not support analytical solutions, the firms optimization problems are solved numerically. Rather than simply assuming that a firm immediately calculates equilibrium strategies, it employs a simulation-based model in which firms investigate market effects and adapt their alternatives. In both cases, the method provides an opportunity to study market behavior under a limited amount of data, whilst also serving as a bridge between theoretical demand models and algorithmic decision making. To clearly demonstrate how consumer heterogeneity affects pricing, product design and overall, the model is first devised around one firm. The model may be replicated with several firms engaged in a given market in which each firm's customer demand is uncertain in terms of the degree of uncertainty on consumers' demand and the behavior of its competitors. Under such circumstances, equilibrium results are reached by repeated interactions and learning, rather than through explicit decision making.

The thesis adds to knowledge on price and product competition in differentiated markets by applying a solid demand structure with mathematical and simulation-based solution methods. This approach is firmly rooted in economic theory, although it recognises the reality of features in real-world scenarios like learning, experimenting and incomplete information that cannot be accurately modeled with an analytical approach. For this reason, it offers a flexible basis for studying firm behavior in modern markets characterized by a range of product-based distinctions and consumer heterogeneity, hence product differentiation as a key element of study.

2 Literature review

The place for literature review.

3 Mathematical models

Here we give mathematical formulation of all models, but also for every model we numerically solve the model for some selected values of initial and exogenous parameters. We need to make those variables exactly the same as the one we take for simulations so that we can compare the results.

3.1 A model with a single firm

Herein, we describe a mathematical model of a market with a single firm. The model's aim is to explain how heterogeneity among consumers creates demand and how a firm will choose prices and product attributes in response to that demand. Since the resulting profit function does not admit a closed-form solution, the decision problem of the firm is analyzed using a numerical, algorithmic approach reflecting learning under incomplete information.

3.1.1 A model of an agent population

We consider a market populated by a large number N of agents. Agents are heterogeneous along two dimensions: Reservation price, where each i consumer has a reservation price $R_i > 0$ showing the maximum price at which the consumer would buy the product, and Taste (the ideal product characteristic), where each consumer i has a preferred product characteristic $S_i \in [0, 1]$. We assume that the random variables R and S are independent.

The pair of these random variables is taken from continuous distributions. In the numerical implementation used throughout the thesis, the Gamma distribution was used for the variable R to reflect that there is a certain common reservation price in the population, but that there are some deviations as well. The type of distribution for random variable S is about the particular style preference of an agent. Because we want the values of this distribution to fall in the interval [0,1], we used the Beta distribution. The choice of the Beta distribution was made because it contains the Uniform distribution, but also it may reflect the fact that there is a common value for the style choice with some variation. Therefore, we are ensured that reservation prices are positive and tastes are bounded on [0,1]. These distributions provide a flexible and economically convincing description of consumer heterogeneity.

According to the model, agents make a binary decision: either purchase one unit of the product or not buys at all. In that way, the first step for the agent is to look at the price of the good. The utility of the agent from facing the good with price p and style o is the following

$$u(r, s, p, o) = -\alpha\frac{p}{r} - \beta(s-o)^2$$

where r is a realization of the random variable R, s is a realization of the random variable S, p is the offered price of a good (assumed non-negative) with the style o. We can view U as the random variable:

$$U = -\alpha\frac{p}{R} - \beta(S-o)^2$$

In the above formula, the parameters r and s describe the consumer (the agent) and the parameters p and o describe the good. Both these parameters are controlled by a firm producing the good.

This specification takes into account such key economic factors, as decreasing of the utility as price increases relative to the agent's wish to pay r and with the squared distance between the consumer's ideal taste s and the product characteristic o.

Agents are assumed to choose with some probability. The probability that an agent with characteristics (r, s) buys the product is given by a logit rule:

$$P(buy) = \frac{\exp\big(u\big)}{1 + \exp\big(u\big)}$$

And the probability of not buy is given by the following formula:

$$P(\text{not buy}) = 1 - \frac{\exp\big(u\big)}{1 + \exp\big(u\big)}$$

The view of the probability of buying as a random variable P is the following:

$$\mathcal{P} = P(\text{buy}) = \frac{\exp(U)}{1 + \exp(U)} = \frac{\exp\big(-\alpha\frac{p}{R} - \beta(S-o)^2\big)}{1 + \exp\big(-\alpha\frac{p}{R} - \beta(S-o)^2\big)}.$$

This formulation suggests that buyers who have a greater extent of utility will be more inclined to purchase products, as long as one may still randomize individual behavior. Specifically, the probabilistic choice interpretation serves to model markets with incomplete information and repeated interaction.

3.1.2 A model of a firm

A single firm supplies one product to the market. The firm chooses two decision variables: a price $p \geq 0$ and a product characteristic (style) $s \in [0,1]$. For convenience, to indicate the expected profit of a single firm offering a good at price p and with the style o, we assume that cost function $c(x) = c * x$, and $c > /0$. Therefore, the cost function is linear. The expected profit of a firm is then the following:

$$\begin{aligned}
\mathbb{E}(\text{profit}) &= \mathbb{E}\big(p \cdot \text{demand} - c(\text{demand})\big) \\
&= \mathbb{E}\big(p \cdot \text{demand} - c \cdot \text{demand}\big) \\
&= \mathbb{E}\big(p \cdot N \cdot \mathcal{P} - c \cdot N \cdot \mathcal{P}\big) \\
&= \mathbb{E}\big((p-c) \cdot N \cdot \mathcal{P}\big) \\
&= (p-c) \cdot N \cdot \mathbb{E}(\mathcal{P}).
\end{aligned}$$

$where$:

$$\begin{aligned}
\mathbb{E}(\mathcal{P}) &= \mathbb{E}\left(\frac{\exp\big(-\alpha\frac{p}{r} - \beta(s-o)^2\big)}{1+\exp\big(-\alpha\frac{p}{r} - \beta(s-o)^2\big)}\right) \\
&= \int_0^\infty \int_0^\infty \frac{\exp\big(-\alpha\frac{p}{r} - \beta(s-o)^2\big)}{1+\exp\big(-\alpha\frac{p}{r} - \beta(s-o)^2\big)} f_R(r)\, f_S(s)\, ds\, dr.
\end{aligned}$$

Thus, the expected profit function generalizes the dilemma faced by a company: raising prices increases profit per unit of output, but reduces demand by decreasing the probability of buying. Since the expected buying probability is determined by a multivariate integral, the optimization problem cannot be solved analytically. Instead, it is approached numerically using a learning algorithm that simulates the firm's behavior under conditions of incomplete information:

$$\max_{\substack{p \geq 0 \\ o \in [0,1]}} \ \Pi(p,o) = (p-c)\, N\, \mathbb{E}[P(p,o)]\,.$$

3.1.3 Algorithm for a firm

Rather than assuming that the firm has full knowledge of the demand function, the model adopts an algorithmic perspective in which the firm learns from market outcomes. The firm follows a repeated five-step procedure:

1. A population of consumers is generated by drawing (R_i, S_i) from their respective distributions. This population remains fixed throughout the analysis.

2. The firm selects initial values for price p and product characteristic o.

3. Given (p, o), each consumer makes a purchase decision according to the probabilistic rule defined by the utility function.

4. The firm observes aggregate demand and computes realized (or expected) profit.

5. The firm adjusts either price or product characteristic by comparing current profit with profits obtained under small alternative changes. The firm adopts the change that improves profit and repeats the process.

This algorithm takes into account the idea that firms do not solve their optimization problems instantaneously but instead experiment, observe outcomes, and gradually improve their decisions. In the deterministic implementation used in the simulations, the firm evaluates expected demand (the sum of individual purchase probabilities), which corresponds to averaging outcomes over many repeated sales periods

3.2 A model with two firms

4 Implementation of mathematical models

In this chapter we describe how we implemented the above mathematical models. So we do not give here any results, we just describe how we implement a consumer, how we implement a firm, what is an algorithm of the behavior of the firms, and so on and so forth.

5 Simulation results

We show the results of simulations for various initial and exogenous parameters.

5.1 Simulations for a single firm

5.2 Simulations for two firms

6 Conclusions

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Streszczenie

Tutaj zamieszczają Państwo streszczenie pracy. Streszczenie powinno być długości około pół strony.