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Learning and Competition in the Differentiated Products Market

Field of study: Advanced Analytics – Big Data
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Introduction

Why the topic matters?

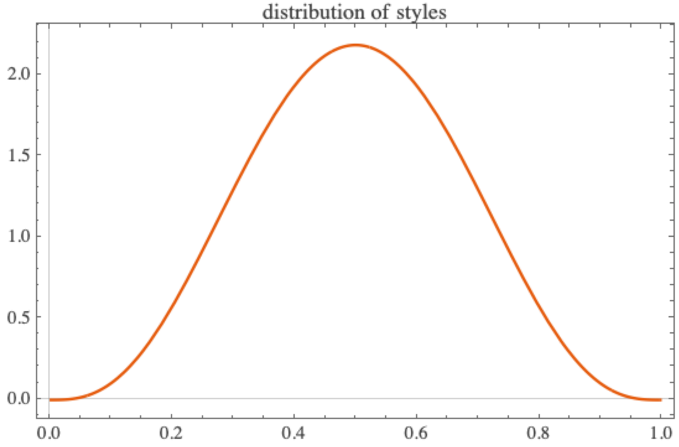
- Firms choose price and product design;
- Consumers differ in income and tastes;
- Competition creates strategic interaction.

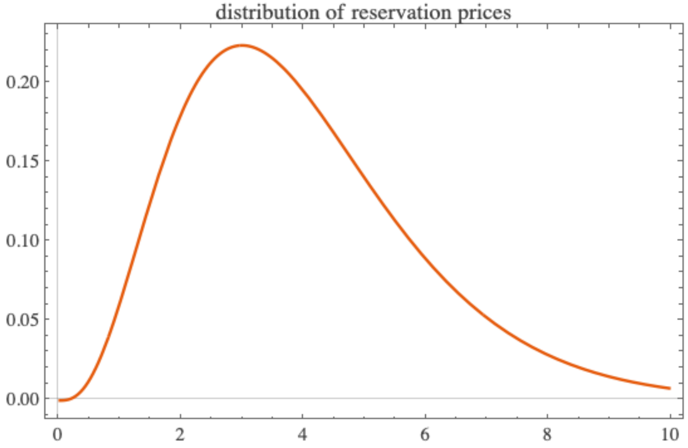
Research objectives:

- How does a single firm learn optimal price and style;
- What changes under competition;
- How do structural parameters affect equilibrium.

Model Overview (Consumers and Utility):

- Market populated with large number N of consumers.
- Two dimensions of consumer: reservation price $r_i > 0$ and taste $s_i \in [0, 1]$.
- The good is characterized by the price $\mathbf{p} \geq \mathbf{0}$ and style $\mathbf{o} \in [\mathbf{0}, \mathbf{1}]$.
- The distribution of tastes (Gamma(4,1)) and reservation prices (Beta(4,4)):





Model Overview (Consumers and Utility):

- The utility of consumer:

$$u(r, s, p, o) = -\alpha \frac{p}{r} - \beta (s - o)^2$$

- Becomes random variable U if taken from u:

$$U = -\alpha \frac{p}{R} - \beta (S - o)^2$$

- With the probability of buying:

$$\mathcal{B} = \mathbb{P}(\text{buy}) = \frac{\exp(U)}{1+\exp(U)} = \frac{\exp\left(-\alpha \frac{p}{R} - \beta (S - o)^2\right)}{1+\exp\left(-\alpha \frac{p}{R} - \beta (S - o)^2\right)}$$

Model Overview (Firm):

- The expected profit of a firm:

$$\Pi(o,p) = \mathbb{E}(\text{profit}) = (p \ - c) \cdot N \cdot \mathbb{E}(\mathcal{B})$$

Where:

$$\mathbb{E}(\mathcal{B}) = \left(\frac{\exp\left(-\alpha \frac{p}{R} - \beta (S \ - o)^2\right)}{1 + \exp\left(-\alpha \frac{p}{R} - \beta (S \ - o)^2\right)} \right) = \int_0^1 \int_0^\infty \frac{\exp\left(-\alpha \frac{p}{R} - \beta (S \ - o)^2\right)}{1 + \exp\left(-\alpha \frac{p}{R} - \beta (S \ - o)^2\right)} f_R(r)\, f_S(s)\, ds\, dr$$

- Therefore, the firm chooses:

$$max \Pi(p,o)$$

- Because there is no closed-form solution, the numerical optimization methodology is implemented.

Algorithm of a single firm

- We set the maximum number of steps T and simulation follows $t = 1, 2, \ldots, T$.
- We fix the number of agents N and keep it fixed (each agent is assigned realizations of the random variables R and S.
- We define two values of incremental changes to prices (δ_p) and styles (δ_o).
- The firm selects initial central values for price p_o and product characteristic o_0.
- For each of the elements in the local search grid, consumers make purchase decisions.
- The firm observes the realized aggregate demands and computes realized profits.
- The firm creates a new local search grid around the new center (t inscreases by 1).
- If $t < T$, we go to step 5. If $t = T$, then stop.

Single Firm Results

Main findings:

- Profit is low at very low prices, increases as price increases, reaches a maximum and decreases when price becomes too high.
- Rapid convergence of learning dynamics.
- Higher α (price sensitivity) leads to lower optimal price and profit.
- Optimal style is $o^* \approx \mathbb{E}[\mathrm{s}]$.

When taste distribution becomes skewed:

- Equilibrium positioning shifts.
- Price remains relatively stable.

Duopoly extension

Profit in duopoly:

$$\Pi_j = (p - c) \cdot N \cdot \mathbb{E}(\mathcal{B}_j)$$

Sequential best-response dynamics:

- Firm 1 updates given Firm 2.
- Firm 2 updates given Firm 1.
- Convergence to stationary outcome.

The two-firm model constitutes a normal-form game in which each firm chooses $\boldsymbol{p_j}$ and $\boldsymbol{o_j}$ to maximize expected profit given the competitor's strategy. A Nash equilibrium is a strategy profile $(p_1^*, o_1^*, p_2^*, o_2^*)$ such that each firm's strategy is a best response to others

Duopoly results

Competition effects:

- Lower prices than monopoly
- Lower per-firm profits
- Convergence to symmetric positioning
- No strong endogenous differentiation

Symmetric equilibrium:

$$p_1^* = p_2^* \, , o_1^* = o_2^*$$

Contributions and Conclusions

- Contributions:
  - Unified model of price and positioning
  - Numerical equilibrium computation
  - Comparative statics under monopoly and duopoly
- Key conclusions:
  - Price sensitivity determines markup
  - Taste distribution determines positioning
  - Competition lowers prices but does not induce differentiation under symmetry

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