常微与偏微课程作业

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题目1.

$$\frac{dy}{dt} + y\cos t = 0$$

解答.

$$a(t) = \cos t$$

$$\mu(t) = e^{\int a(t)dt}$$

$$= e^{\int \cos t dt}$$

$$= e^{\sin t}$$

$$e^{\sin t}(\frac{dy}{dt} + y\cos t) = 0$$

$$\frac{dy}{dt}e^{\sin t}y = 0$$

$$e^{\sin t}y = C$$

$$y = Ce^{-\sin t}$$

题目2.

$$\frac{dy}{dt} + y\sqrt{t}\sin t = 0$$

解答.

$$a(t) = \sqrt{t}\sin t$$

$$\mu(t) = e^{\int a(t)dt}$$

$$
\begin{aligned}
&= e^{\int \sqrt{t}\sin t dt} \\
&= e^{\int \sqrt{t}\sin t dt} \\
e^{\int \sqrt{t}\sin t dt}(\frac{dy}{dt} + y\sqrt{t}\sin t) &= 0 \\
\frac{dy}{dt}e^{\int \sqrt{t}\sin t dt} &= 0 \\
e^{\int \sqrt{t}\sin t dt}y &= C \\
y &= Ce^{-\int \sqrt{t}\sin t dt}
\end{aligned}
$$

题目3.

$$\frac{dy}{dt} + \frac{2t}{1+t^2}y = \frac{1}{1+t^2}$$

解答.

$$
\begin{aligned}
a(t) &= \frac{2t}{1+t^2} \\
\mu(t) &= e^{\int a(t)dt} \\
&= e^{\int \frac{2t}{1+t^2}dt} \\
&= e^{ln(1+t^2)} \\
&= 1+t^2 \\
(\frac{dy}{dt} + \frac{2t}{1+t^2}y)(1+t^2) &= \frac{1}{1+t^2} \\
\frac{dy}{dt}(1+t^2)y &= \frac{1}{1+t^2} \\
(1+t^2)y &= \frac{1}{1+t^2} \\
(1+t^2)y &= \int \frac{1}{1+t^2}(1+t^2)dt \\
(1+t^2)y &= t + C \\
y &= \frac{t}{1+t^2} + \frac{C}{1+t^2}
\end{aligned}
$$

题目4.

$$\frac{dy}{dt} + y = te^t$$

解答.

$$a(t) = 1$$

$$\mu(t) = e^{\int a(t)dt}$$

$$= e^{\int dt}$$

$$= e^t$$

$$(\frac{dy}{dt} + y)e^t = te^t \cdot e^t$$

$$\frac{de^t}{dt}y = te^t \cdot e^t$$

$$e^t y = \int te^t dt + C$$

$$y = \frac{e^2 t}{2} + C$$

$$= \frac{e^t}{2} + \frac{C}{e^t}$$

题目5.

$$\frac{dy}{dt} + \sqrt{1+t^2}y = 0 \quad y(0) = \sqrt{5}$$

解答.

$$a(t) = \sqrt{1+t^2}$$

$$\mu(t) = e^{\int a(t)dt}$$

$$= e^{\int \sqrt{1+t^2}dt}$$

$$= e^{\frac{ln(\sqrt{1+t^2}+t)}{2} + \frac{t\sqrt{1+t^2}}{2}}$$

$$\frac{dy}{dt}(e^{\frac{ln(\sqrt{1+t^2}+t)}{2} + \frac{t\sqrt{1+t^2}}{2}}) = 0$$

$$\int_0^t \frac{d}{ds}e^{\frac{ln(\sqrt{1+s^2}+s)}{2} + \frac{s\sqrt{1+s^2}}{2}}y(s)ds = C$$

$$[e^{\frac{ln(\sqrt{1+t^2}+t)}{2} + \frac{t\sqrt{1+t^2}}{2}}]y - \sqrt{5}$$

$$y = \frac{\sqrt{5}}{e^{\frac{ln(\sqrt{1+t^2}+t)}{2} + \frac{t\sqrt{1+t^2}}{2}}}$$

题目6.

$$\frac{dy}{dt} + \sqrt{1+t^2}e^{-t}y = 0 \quad y(0) = 1$$

解答.

$$\begin{aligned}
a(t) &= \sqrt{1+t^2}e^{-t} \\
\mu(t) &= e^{\int a(t)dt} \\
&= e^{\int \sqrt{1+t^2}e^{-t}dt} \\
e^{\int \sqrt{1+t^2}e^{-t}dt}(\frac{dy}{dt} &+ y\sqrt{1+t^2}e^{-t}) = 0 \\
\frac{d}{dt}(e^{\int \sqrt{1+t^2}e^{-t}dt}) &= 0 \\
\int_0^t \frac{d}{ds}e^{\int \sqrt{1+s^2}e^{-s}ds}y(s)ds &= 0 \\
e^{\int \sqrt{1+t^2}e^{-t}dt}y - 1 &= 0 \\
y &= \frac{1}{e^{\int \sqrt{1+t^2}e^{-t}dt}}
\end{aligned}$$

题目7.

$$\frac{dy}{dt} + \sqrt{1+t^2}e^{-t}y = 0$$

$$\begin{aligned}
a(t) &= \sqrt{1+t^2}e^{-t} \\
\mu(t) &= e^{\int a(t)dt} \\
&= e^{\int \sqrt{1+t^2}e^{-t}dt} \\
e^{\int \sqrt{1+t^2}e^{-t}dt}(\frac{dy}{dt} &+ y\sqrt{1+t^2}e^{-t}) = 0 \\
\frac{d}{dt}(e^{\int \sqrt{1+t^2}e^{-t}dt}) &= 0 \\
\int_0^t \frac{d}{ds}e^{\int \sqrt{1+s^2}e^{-s}ds}y(s)ds &= 0 \\
e^{\int \sqrt{1+t^2}e^{-t}dt}y &= 0
\end{aligned}$$

$$y = 0$$

解答.

题目8.

$$\frac{dy}{dt} - 2ty = t$$

解答.

$$a(t) = -2t$$

$$\mu(t) = e^{\int a(t)dt}$$

$$= e^{-t^2}$$

$$e^{-t^2}(\frac{dy}{dt} - 2ty) = t$$

$$\frac{d}{dt}e^{-t^2}y = te^{-t^2}$$

$$\int_0^t \frac{d}{ds}e^{-s^2}y(s)ds = \int_0^t se^{-s^2}ds$$

$$e^{-t^2}y - 1 = -\frac{e^{-t^2}}{2} + \frac{1}{2}$$

$$e^{-t^2}y = \frac{3 - e^{-t^2}}{2}$$

$$y = \frac{3 - e^{-t^2}}{2e^{-t^2}}$$

$$= \frac{3}{2e^{-t^2}} - \frac{1}{2}$$

$$= \frac{3e^{t^2} - 1}{2}$$