Solution 11.1

$v(t) = 160\cos(50t)$
$i(t) = -33\sin(50t-30^\circ) = 33\cos(50t-30^\circ+180^\circ-90^\circ) = 33\cos(50t+60^\circ)$

$p(t) = v(t)i(t) = 160x33\cos(50t)\cos(50t+60^\circ)$
$= 5280(1/2)[\cos(100t+60^\circ)+\cos(60^\circ)] =$ **$[1.320+2.640\cos(100t+60^\circ)]$ kW**.

$P = [V_mI_m/2]\cos(0-60^\circ) = 0.5x160x33x0.5 =$ **1.320 kW**.

Solution 11.2

Given the circuit in Fig. 11.35, find the average power supplied or absorbed by each element.

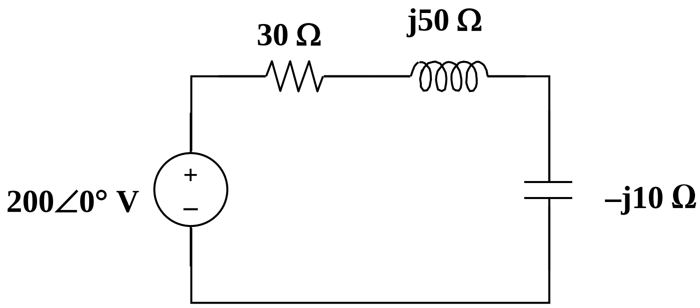
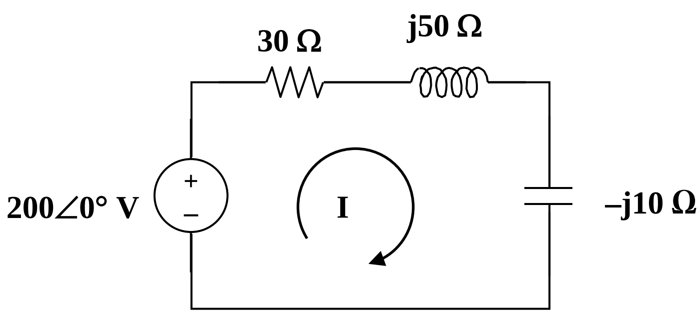


Figure 11.35
For Prob. 11.2.

Solution

Step 1. First we can write one mesh equation and solve for I. Once we have I, we can then find the average power absorbed by each element. Obviously the source will have a negative power absorbed meaning it is supplying power. One last comment, since we still have not covered rms values, we will treat the 200 V as a peak value.

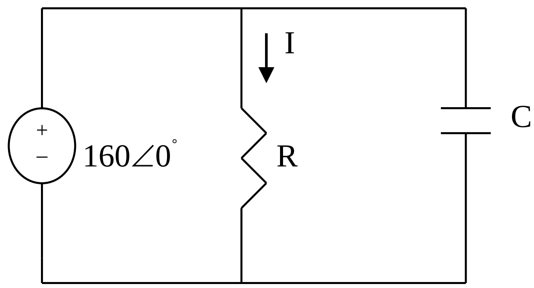


$-200 + 30\mathbf{I} + j50\mathbf{I} + (-j10)\mathbf{I} = 0$ or $\mathbf{I} = 200/(30+j40)$. Finally, $P_{30} = \mathbf{I}(\mathbf{I})^*30$, $P_{j50} = \mathbf{0}$, $P_{-j10} = \mathbf{0}$, and $P_{200} = -|\mathbf{V}|\,|\mathbf{I}|\cos(\theta)$

Step 2. $\mathbf{I} = 200/50\angle 53.13° = 4\angle{-53.13°}$ A. Thus,

P_{30} = **480 W** and P_{200} = **–480 W**.

Solution 11.3



$$90\ \mu\text{F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{j90x10^{-6}x2x10^{3}} = -j5.5556$$

I = 160/60 = 2.667A

The average power delivered to the load is the same as the average power absorbed by the resistor which is

$$P_{avg} = 0.5|I|^2 60 = \mathbf{213.4\ W}.$$

Solution 11.4

Using Fig. 11.36, design a problem to help other students better understand instantaneous and average power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the average power dissipated by the resistances in the circuit of Fig. 11.36. Additionally, verify the conservation of power. Note, we do not talk about rms values of voltages and currents until Section 11.4, all voltages and currents are peak values.

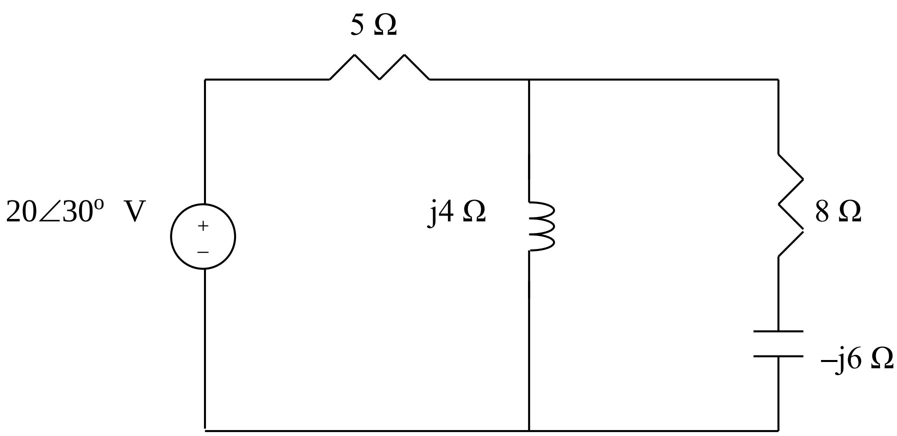
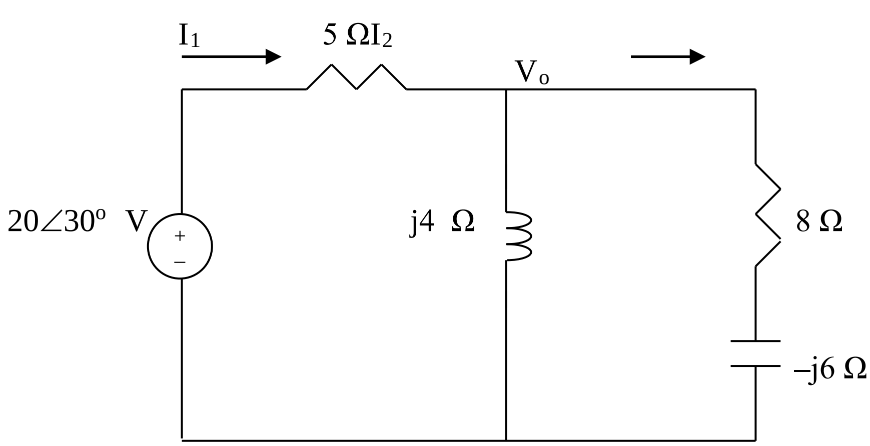


Figure 11.36 For Prob. 11.4.

Solution

We apply nodal analysis. At the main node,



$$\frac{20<30^o - V_o}{5} = \frac{V_o}{j4} + \frac{V_o}{8-j6} \quad \longrightarrow \quad V_o = 5.152 + j10.639 = 11.821\angle 64.16^\circ$$

For the 5-Ω resistor,

$$I_1 = \frac{20<30^o - V_o}{5} = 2.438 < -3.0661^o \quad \text{A}$$

The average power dissipated by the resistor is

$$P_1 = \frac{1}{2}|I_1|^2 R_1 = \frac{1}{2}x2.438^2 x5 = \underline{14.86 \text{ W}}$$

For the 8-Ω resistor,

$$\mathbf{I_2} = V_o/(8–j6) = (11.812/10)\angle(64.16+36.87)^\circ = 1.1812\angle 101.03^\circ \text{ A}$$

The average power dissipated by the resistor is

$$P_2 = 0.5|I_2|^2R_2 = 0.5(1.1812)^2 8 = \mathbf{5.581\ W}$$

The complex power supplied is

$$\mathbf{S} = 0.5(V_s)(I_1)^* = 0.5(20\angle 30^\circ)(2.438\angle 3.07^\circ) = 24.38\angle 33.07^\circ$$

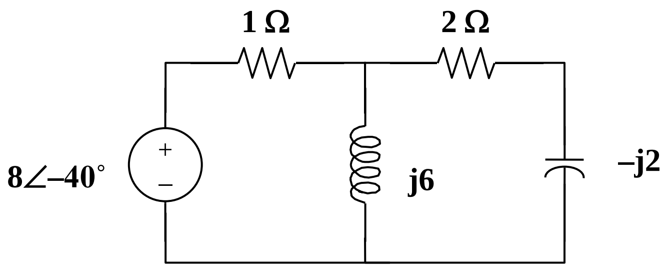
$$= \mathbf{(20.43+13.303)\ VA}$$

Adding P_1 and P_2 gives the real part of **S**, showing the conservation of power.

P = 14.86+5.581 = **20.44 W** which checks nicely.

Solution 11.5

Converting the circuit into the frequency domain, we get:



$$I_{1\Omega} = \frac{8\angle -40^\circ}{1 + \dfrac{j6(2 - j2)}{j6 + 2 - j2}} = 1.6828\angle -25.38^\circ$$

$$P_{1\Omega} = \frac{1.6828^2}{2} 1 = \underline{1.4159\,\text{W}}$$

$P_{1\Omega}$ = **1.4159 W**

$P_{3H} = P_{0.25F}$ = **0 W**

$$|I_{2\Omega}| = \left| \frac{j6}{j6 + 2 - j2} 1.6828\angle -25.38^\circ \right| = 2.258$$

$$P_{2\Omega} = \frac{2.258^2}{2} 2 = \underline{5.097\,\text{W}}$$

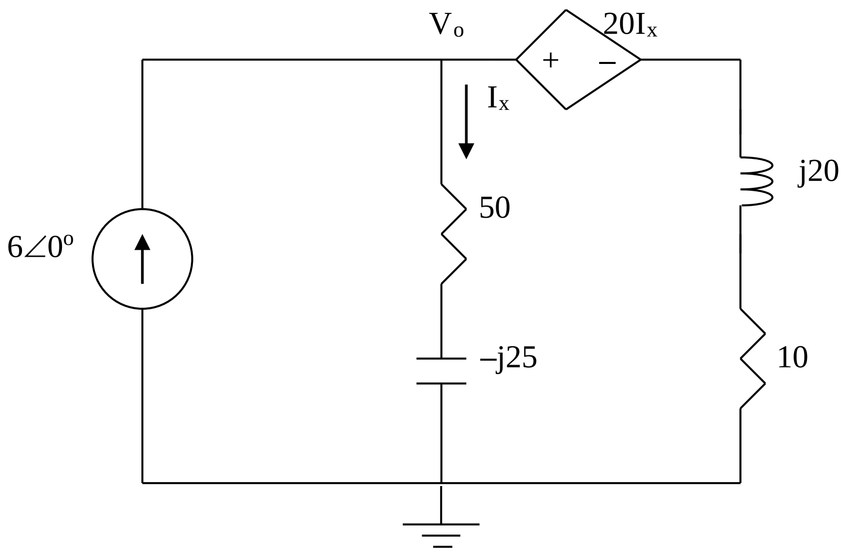
$P_{2\Omega}$ = **5.097 W**

Solution 11.6

$$20\text{ mH} \longrightarrow j\omega L = j10^3 x20x10^{-3} = j20$$

$$40\mu F \rightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x40x10^{-6}} = -j25$$

We apply nodal analysis to the circuit below.



$$-6 + \frac{V_o - 20I_x}{10 + j20} + \frac{V_o - 0}{50 - j25} = 0$$

But $I_x = \dfrac{V_o}{50 - j25}$. Substituting this and solving for V_o leads

$$\left(\frac{1}{10 + j20} - \frac{20}{(10 + j20)}\frac{1}{(50 - j25)} + \frac{1}{50 - j25}\right)V_o = 6$$

$$\left(\frac{1}{22.36\angle 63.43^\circ} - \frac{20}{(22.36\angle 63.43^\circ)(55.9\angle -26.57^\circ)} + \frac{1}{55.9\angle -26.57^\circ}\right)V_o = 6$$

$$(0.02 - j0.04 - 0.012802 + j0.009598 + 0.016 + j0.008)V_o = 6$$

$(0.0232 - j0.0224)V_o = 6$ or $V_o = 6/(0.03225\angle -43.99^\circ) = 186.05\angle 43.99^\circ$ volts.

$|I_x| = 186.05/55.9 = 3.328$

We can now calculate the average power absorbed by the 50-Ω resistor.

$P_{avg} = [(3.328)^2/2]x50 =$ **276.8 W**.

Solution 11.7

Given the circuit of Fig. 11.40, find the average power absorbed by the 10-Ω resistor.

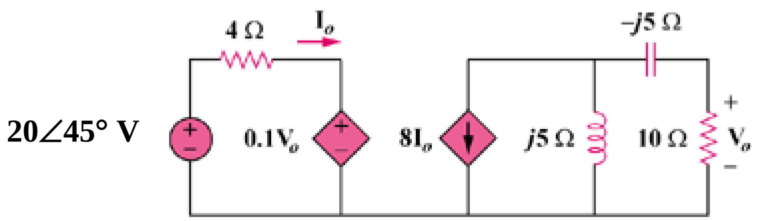


Figure 11.40
For Prob. 11.7.

Solution

Applying KVL to the left-hand side of the circuit,

$$20\angle 45^\circ = 4\mathbf{I}_o + 0.1\mathbf{V}_o \qquad (1)$$

Applying KCL to the right side of the circuit,

$$8\mathbf{I}_o + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1}{10 - j5} = 0$$

But,
$$\mathbf{V}_o = \frac{10}{10 - j5}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \frac{10 - j5}{10}\mathbf{V}_o$$

Hence,
$$8\mathbf{I}_o + \frac{10 - j5}{j50}\mathbf{V}_o + \frac{\mathbf{V}_o}{10} = 0$$

$$\mathbf{I}_o = j0.025\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1),

$$20\angle 45^\circ = 0.1\mathbf{V}_o(1 + j)$$

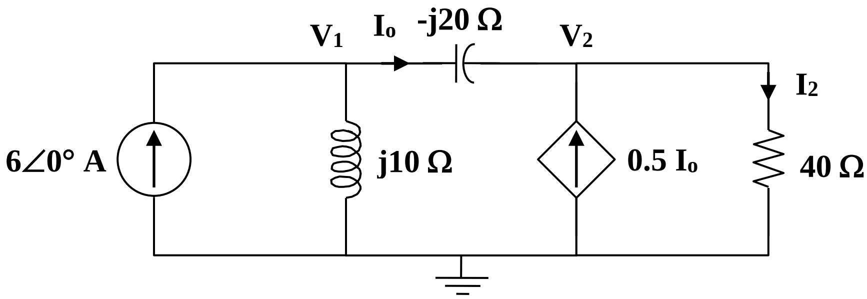
$$\mathbf{V}_o = \frac{200\angle 45^\circ}{1 + j}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{10} = \frac{20}{\sqrt{2}}\angle 0^\circ$$

$$P = \frac{1}{2}|\mathbf{I}_1|^2 R = \left(\frac{1}{2}\right)\left(\frac{400}{2}\right)(10) = \mathbf{1\ kW}$$

Solution 11.8

We apply nodal analysis to the following circuit.



At node 1,

$$6 = \frac{\mathbf{V}_1}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} \quad \mathbf{V}_1 = j120 - \mathbf{V}_2 \qquad (1)$$

At node 2,

$$0.5\mathbf{I}_o + \mathbf{I}_o = \frac{\mathbf{V}_2}{40}$$

But, $$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$

Hence, $$\frac{1.5(\mathbf{V}_1 - \mathbf{V}_2)}{-j20} = \frac{\mathbf{V}_2}{40}$$

$$3\mathbf{V}_1 = (3 - j)\mathbf{V}_2 \qquad (2)$$

Substituting (1) into (2),

$$j360 - 3\mathbf{V}_2 - 3\mathbf{V}_2 + j\mathbf{V}_2 = 0$$

$$\mathbf{V}_2 = \frac{j360}{6 - j} = \frac{360}{37}(-1 + j6)$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{40} = \frac{9}{37}(-1 + j6)$$

$$P = \frac{1}{2}\left|\mathbf{I}_2\right|^2 R = \frac{1}{2}\left(\frac{9}{\sqrt{37}}\right)^2 (40) = \mathbf{43.78\ W}$$

Solution 11.9

For the op amp circuit in Fig. 11.41, $\mathbf{V}_s = 2\angle 30^o$ V. Find the average power absorbed by the 20-kΩ resistor.

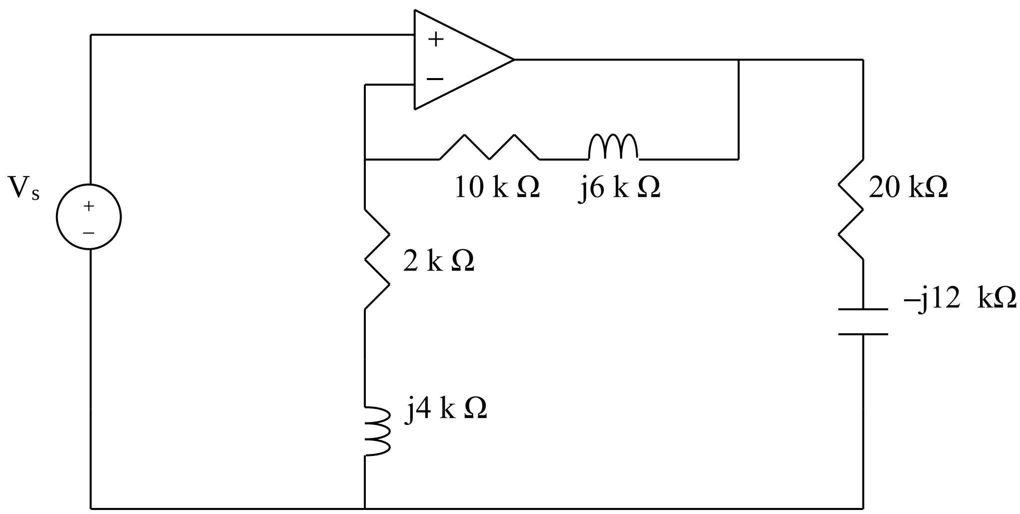


Figure 11.41
For Prob. 11.9.

Solution

This is a non-inverting op amp circuit. At the output of the op amp,

$$\mathbf{V_o} = \left(1+\frac{\mathbf{Z}_2}{\mathbf{Z}_1}\right)\mathbf{V_s} = \left(1+\frac{(10+j6)10^3}{(2+j4)10^3}\right)\mathbf{V_s} = \left(1+\frac{11.6619\angle 30.964^o}{4.4721\angle 63.435^o}\right)2\angle 30^o$$

$= (1+2.6077\angle{-32.471^\circ})2\angle 30^\circ = (1+2.2-j1.4)2\angle 30^\circ$
$= (3.4928\angle{-23.629^\circ})(2\angle 30^\circ) = 6.9856\angle 6.371^\circ = (6.9425+j0.77516)$ V.

The current through the 20-kΩ resistor is

$$\mathbf{I_o} = \frac{\mathbf{V_o}}{20k - j12k} = (6.9856\angle 6.371^\circ)/(23.3238k\angle{-30.964^\circ}) = 0.29951\angle 37.335^\circ$$
mA

or $|\mathbf{I_o}| = 0.2995$ mA.

$P = [|\mathbf{I_o}|^2/2]R = [0.2995^2/2]10^{-6}x20x10^3$

$= \mathbf{897\ \mu W}$

Solution 11.10

No current flows through each of the resistors. Hence, for each resistor, $P = \mathbf{0\ W}$. It should be noted that the input voltage will appear at the output of each of the op amps.

Solution 11.11

$\omega = 377, \qquad R = 10^4, \qquad C = 200 \times 10^{-9}$

$\omega RC = (377)(10^4)(200 \times 10^{-9}) = 0.754$

$\tan^{-1}(\omega RC) = 37.02°$

$$Z_{ab} = \frac{10k}{\sqrt{1 + (0.754)^2}} \angle -37.02° = 7.985 \angle -37.02° \text{ k}\Omega$$

$i(t) = 33\sin(377t + 22°) = 33\cos(377t - 68°)\ mA$

$\mathbf{I} = 33\angle -68°$ mA

$$S = \frac{I^2 Z_{ab}}{2} = \frac{\left(33x10^{-3}\right)^2 (7.985\angle -37.02°) \times 10^3}{2}$$

$\mathbf{S} = 4.348\angle -37.02°$ VA

$$P = |S|\cos(37.02) = \mathbf{3.472\ W}$$

Solution 11.2

For the circuit shown in Fig. 11.44, determine the load impedance $\boldsymbol{Z_L}$ for maximum power transfer (to $\boldsymbol{Z_L}$). Calculate the maximum power absorbed by the load.

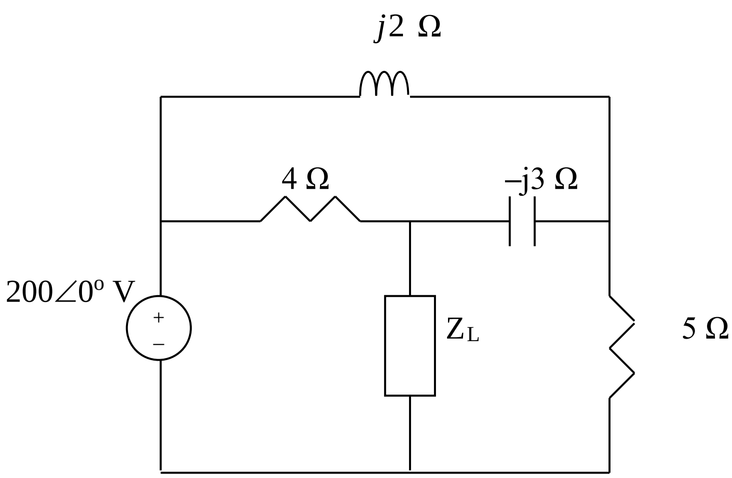
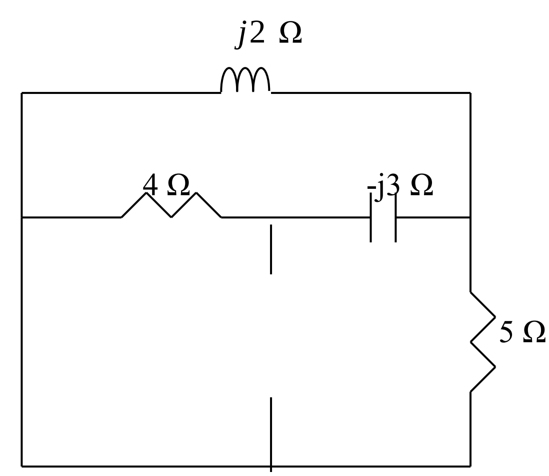


Figure 11.44
For Prob. 11.12.

Solution

We find the Thevenin impedance using the circuit below.

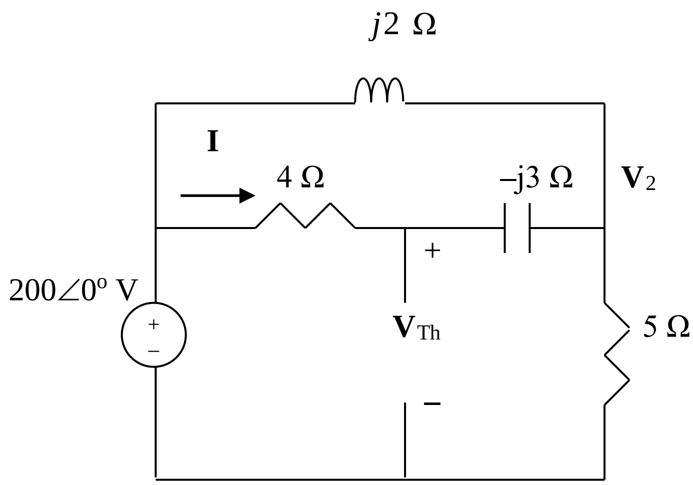


We note that the inductor is in parallel with the 5-Ω resistor and the combination is in series with the capacitor. That whole combination is in parallel with the 4-Ω resistor. Thus,

$$Z_{Thev} = \frac{4\left(-j3+\frac{5xj2}{5+j2}\right)}{4-j3+\frac{5xj2}{5+j2}} = \frac{4(0.6896-j1.2758)}{4.69-j1.2758} = \frac{4(1.4502\angle-61.61^\circ)}{4.86\angle-15.22^\circ}$$
$$= 1.1936\angle-46.39^\circ$$

$\mathbf{Z_{Thev}}$ = 0.8233 – j0.8642 or $\mathbf{Z_L}$ = **[823.3 + j864.2] mΩ**.

We obtain V_{Th} using the circuit below. We apply nodal analysis.



$$\frac{\mathbf{V}_2-200}{4-\mathbf{j}3}+\frac{\mathbf{V}_2-200}{\mathbf{j}2}+\frac{\mathbf{V}_2-0}{5}=0$$
$$(0.16+\mathbf{j}0.12-\mathbf{j}0.5+0.2)\mathbf{V}_2 = (0.16+\mathbf{j}0.12-\mathbf{j}0.5)200$$
$$(0.5235\angle-46.55^\circ)\mathbf{V}_2 = (0.4123\angle-67.17^\circ)200 = 82.46\angle-67.17^\circ$$

Thus, $\mathbf{V_2}$ = 157.52∠–20.62˚V = 147.43–j55.473

I = (200 – V_2)/(4 – j3) = (200 – 147.43 + j55.473)/(4 – j3)
= (52.57+j55.473)/(4–j3) = (76.426∠46.54°)/(5∠–36.87°
= 15.285∠83.41° = 1.7542+j15.184

$\mathbf{V_{Thev}}$ = 200 – 4**I** = 200 – 7.0168 – j60.736 = [192.983 – j60.736] V
= 202.31∠–17.47˚V

To calculate the maximum power to the load, we can use eq. 11.14,
$I_L = V_{THEV}/(2r_l)$, to obtain,

|$\mathbf{I_L}$| = (202.31/(2x0.8233)) = 122.865 A

P_{avg} = [(|$\mathbf{I_L}$|)^{2}0.8233]/2 = **6.214 kW**.

Solution 11.13

For maximum power transfer to the load, $\mathbf{Z_L}$ = **[120 – j60] Ω**.

$I_L = 165/(240) = 0.6875$ A

$$P_{avg} = [|I_L|^2 120]/2 = \mathbf{28.36\ W}.$$

Solution 11.14

Using Fig. 11.45, design a problem to help other students to better understand maximum average power transfer to a load ***Z***.

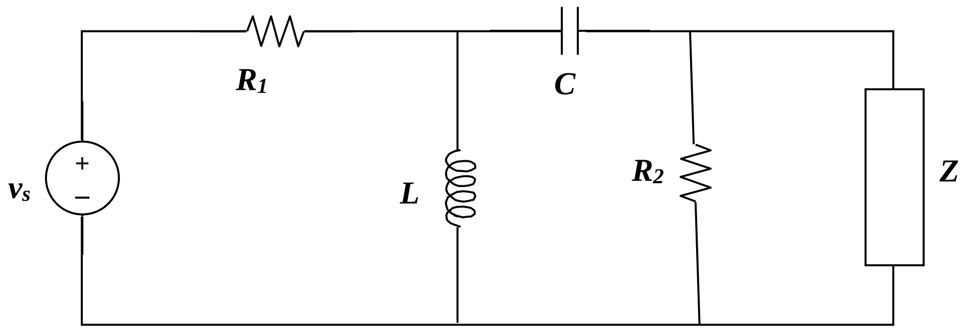
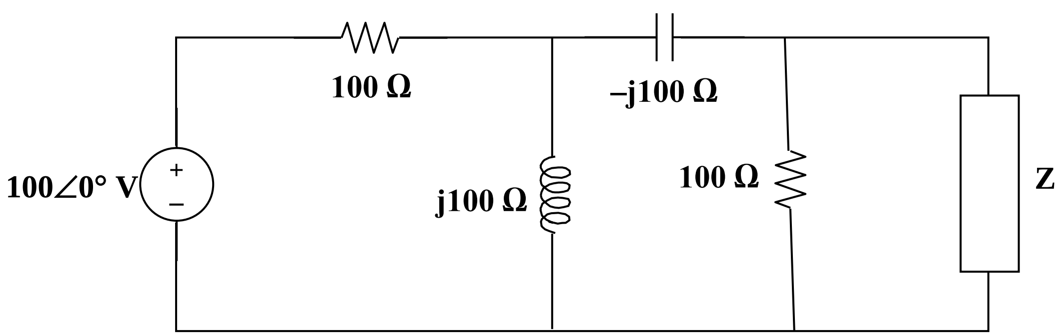


Figure 11.45
For Prob. 11.14.

Although there are many ways to work this problem, this is an example of how a student might pose and solve the problem.

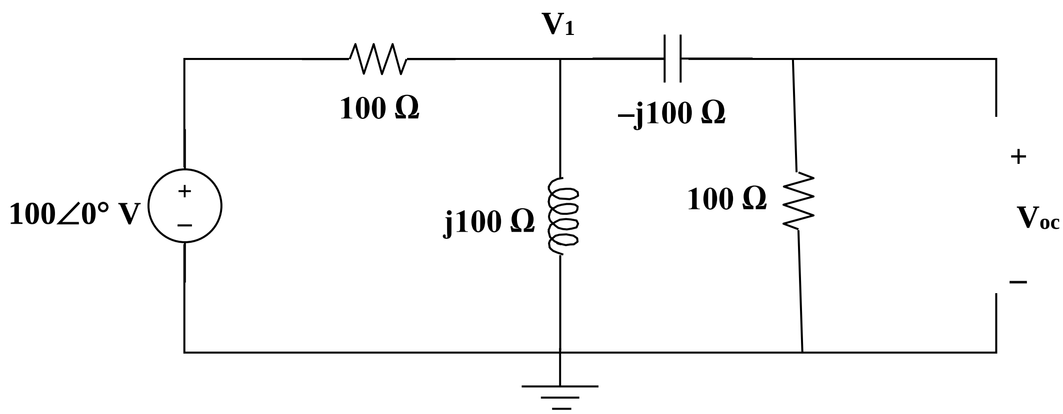
Problem Statement

It is desired to transfer maximum power to the load ***Z*** in the circuit shown below. Find ***Z*** and the maximum average power. Let $v_s = 100 \sin(100t)$ V.



Solution

Step 1. We need to find the Thevenin equivalent at the terminals of **Z**. In order to do this we need to find $\mathbf{V}_{oc}$ and $\mathbf{Z}_{eq}$. Finding the open circuit voltage, $\mathbf{V}_{oc}$, we use the following circuit to help us write the nodal equation,
$[(\mathbf{V}_1–100)/100] + [(\mathbf{V}_1–0)/(j100)] + [(\mathbf{V}_1–\mathbf{V}_{oc})/(–j100)] = 0$ and
$[(\mathbf{V}_{oc}–\mathbf{V}_1)/(–j100)] + [(\mathbf{V}_{oc}–0)/100] = 0$. Solving this will give us $\mathbf{V}_{oc} = \mathbf{V}_{Thev}$.



To find $\mathbf{Z_{eq}}$ all we need to do is set the voltage source to zero and then find the impedance looking in from the right. $\mathbf{Z_{eq}} = 100||[-j100+j100||100]$.
Once we have Z_{eq} we now have the load that will give us maximum power transfer, $\mathbf{Z} = (\mathbf{Z_{eq}})^*$.

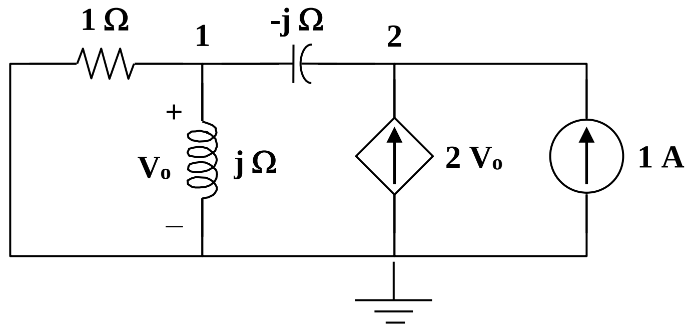
Step 2. $[(\mathbf{V_{oc}}-\mathbf{V_1})/(-j100)] + [(\mathbf{V_{oc}}-0)/100] = 0$ or $j0.01\mathbf{V_1} = (0.01+j0.01)\mathbf{V_{oc}}$ or $\mathbf{V_1} = [(0.01+j0.01)/(j0.01)]\mathbf{V_{oc}} = (1-j)\mathbf{V_{oc}}$. Now we substitute this into the first equation. $[(\mathbf{V_1}-100)/100] + [(\mathbf{V_1}-0)/(j100)] + [(\mathbf{V_1}-\mathbf{V_{oc}})/(-j100)] = 0$ or $(0.01-j0.01+j0.01)\mathbf{V_1} - (j0.01)\mathbf{V_{oc}} = 1$ or $(0.01)\mathbf{V_1} - j0.01\mathbf{V_{oc}} = 1$ which leads to $(0.01)(1-j)\mathbf{V_{oc}} - j0.01\mathbf{V_{oc}} = (0.01-j0.02)\mathbf{V_{oc}} = 1$ or $\mathbf{V_{oc}} = 1/(0.022361\angle -63.43°$ V $= 44.72\angle 63.43°$ V $= \mathbf{V_{Thev}}$.

$100||j100 = 100(j100)/(100+j100) = 100(0.5+j0.5) = 50+j50$.
$100||(-j100+50+j50) = 100(50-j50)/(150-j50)$
$= 100(70.71\angle -45°)/(158.11\angle -18.43°) = 44.72\angle -26.57°\ \Omega$ or
$\mathbf{Zeq} = 44.72\angle -26.57°\ \Omega = (40 - j20)\ \Omega$ or

$$\mathbf{Z} = (\mathbf{Z_{eq}})^* = \mathbf{(40+j20)\ \Omega} \text{ and}$$
$$P_{avg} = [|\mathbf{V_{Thev}}|]^2/[4(\mathbf{Zeq} - \mathbf{Z})] = \mathbf{6.25\ W}.$$

Solution 11.15

To find $\mathbf{Z}_{eq}$, insert a 1-A current source at the load terminals as shown in Fig. (a).



(a)

At node 1,

$$\frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{j} = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow \mathbf{V}_o = j\mathbf{V}_2 \tag{1}$$

At node 2,

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow 1 = j\mathbf{V}_2 - (2+j)\mathbf{V}_o \tag{2}$$

Substituting (1) into (2),

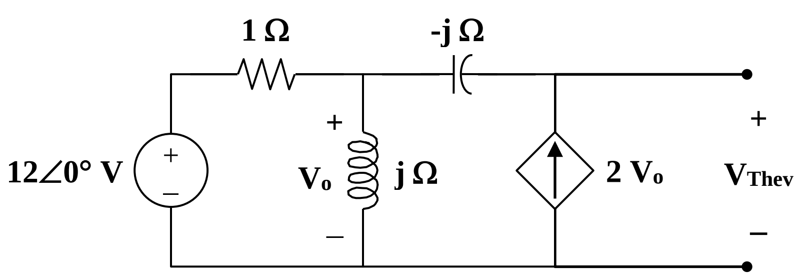
$$1 = j\mathbf{V}_2 - (2+j)(j)\mathbf{V}_2 = (1-j)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{1}{1-j}$$

$$\mathbf{Z}_{eq} = \frac{\mathbf{V}_2}{1} = \frac{1+j}{2} = 0.5 + j0.5$$

$$\mathbf{Z}_L = \mathbf{Z}_{eq}^* = \mathbf{[0.5 - \mathit{j}0.5]\,\Omega}$$

We now obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

$$-2\mathbf{V}_o + \frac{\mathbf{V}_o - 12}{1} + \frac{\mathbf{V}_o}{j} = 0$$

$$\mathbf{V}_o = \frac{-12}{1+j}$$

$$-\mathbf{V}_o - (-j \times 2\,\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Thev} = (1-j2)\mathbf{V}_o = \frac{(-12)(1-j2)}{1+j}$$

$$P_{max} = \frac{\left[\left[\dfrac{V_{Thev}}{0.5+j0.5+0.5-j0.5}\right]\right]^2}{2}0.5 = \frac{\left(\dfrac{12\sqrt{5}}{\sqrt{2}}\right)^2}{2(2x0.5)^2}0.5$$

$$= \mathbf{90\ W}$$

Solution 11.16

For the circuit in Fig. 11.47, find the maximum power that can be delivered to the load $\mathbf{Z}_L$.

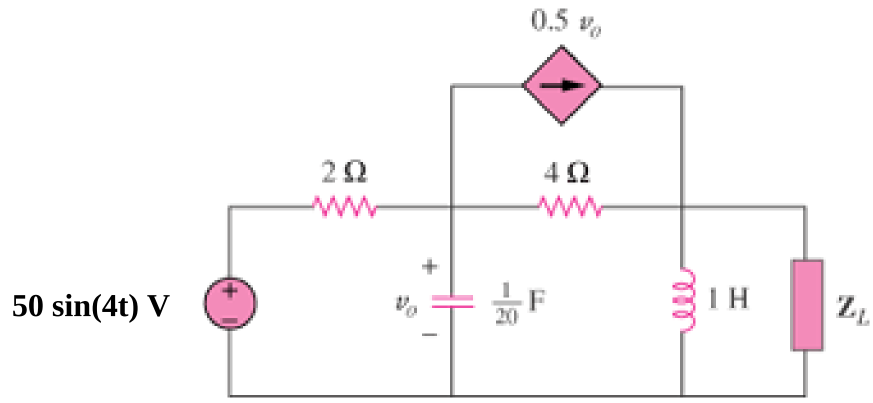
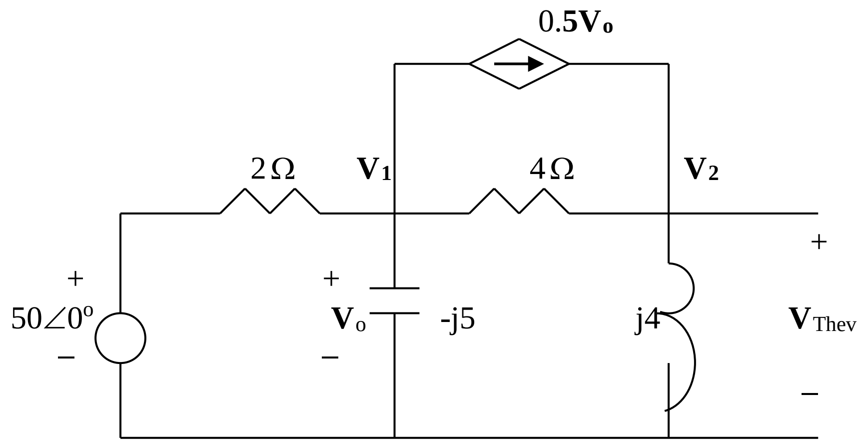


Figure 11.47
For Prob. 11.16.

Solution

$$\omega = 4, \quad 1\text{H} \longrightarrow j\omega L = j4, \quad 1/20\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/20} = -j5$$

We find the Thevenin equivalent at the terminals of Z_L. To find V_{Thev}, we use the circuit shown below.



At node 1,

$$\frac{50-\mathbf{V}_1}{2} = \frac{\mathbf{V}_1}{-\mathbf{j}5} + 0.5\mathbf{V}_1 + \frac{\mathbf{V}_1-\mathbf{V}_2}{4} \longrightarrow 25 = \mathbf{V}_1(1.25+\mathbf{j}0.2) - 0.25\mathbf{V}_2$$

(1)

At node 2,

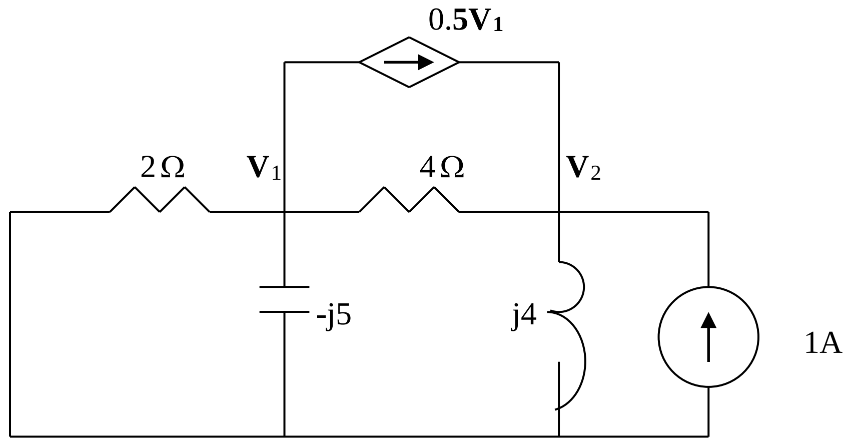
$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{4} + 0.5\mathbf{V}_1 = \frac{\mathbf{V}_2}{\mathbf{j}4} \quad\longrightarrow\quad 0 = 0.75\mathbf{V}_1 + \mathbf{V}_2(-0.25 + \mathbf{j}0.25) \quad \text{or}$$

$\mathbf{V_1} = (0.33333\text{–j}0.33333)\mathbf{V_2} = (0.4714\angle\text{–}45°)\mathbf{V_2}$ (2)

Substituting (2) into (1) leads to

$(1.25\text{+j}0.2)(\ 0.33333\text{–j}0.33333)\mathbf{V_2} - 0.25\mathbf{V_2} = 25$
$= [0.41666\text{+}0.066666 - 0.25 + \text{j}(0.066666\text{–}0.41666)]\mathbf{V_2} = (0.23332\text{–j}0.35)\mathbf{V_2}$
$= (0.42064\angle\text{–}56.311°)\mathbf{V_2}$ or $\mathbf{V_2} = 25/(0.42064\angle\text{–}56.311°) = 59.433\angle 56.311°$ V
$= (32.967 + \text{j}49.452)$ V $= \mathbf{V_{Thev}}$.

To obtain R_{eq}, consider the circuit shown below. We replace $\mathbf{Z_L}$ by a 1-A current source.



At node 1,

$$\frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1}{-\mathbf{j}5} + 0.5\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{4} = 0 \quad\longrightarrow\quad 0 = \mathbf{V}_1(1.25 + \mathbf{j}0.2) - 0.25\mathbf{V}_2 \text{ or}$$

$\mathbf{V_1} = [0.25/(1.2659\angle 9.09°)]\mathbf{V_2} = (0.197488\angle\text{–}9.09°)\mathbf{V_2}$
(3)

At node 2,

$$1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{4} + 0.5\mathbf{V}_1 = \frac{\mathbf{V}_2}{\mathbf{j}4} \quad\longrightarrow\quad -1 = 0.75\mathbf{V}_1 + \mathbf{V}_2(-0.25 + \mathbf{j}0.25)$$

(4)

Substituting (3) into (4) gives,

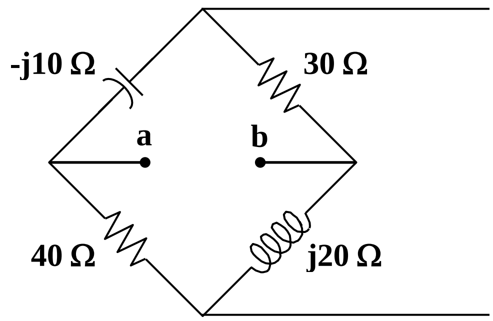
$0.75(0.197488\angle\text{–}9.09°)\mathbf{V_2} + (\text{–}0.25\text{+j}0.25)\mathbf{V_2} = \text{–}1$
$= (0.146256\text{–j}0.0234 - 0.25 + \text{j}0.25)\mathbf{V_2} = (\text{–}0.103744\text{+j}0.2266)\mathbf{V_2}$
$= (0.24922\angle 114.6°)\mathbf{V_2}$ or $\mathbf{V_2} = 4.0125\angle 65.4°$ V $= (1.67033\text{+j}3.6483)$ V.

$$\mathbf{Z_{eq}} = \frac{\mathbf{V}_2}{1} = 4.0125\angle 65.4^\circ\ \Omega \text{ and } \mathbf{Z_L} = 4.0125\angle -65.4^\circ\ \Omega$$

$$\mathbf{P}_{\text{max}} = \frac{|\mathbf{V_{Th}}|^2}{4[\mathbf{Z_{eq}} + \mathbf{Z_L}]} = \frac{59.433^2}{8\mathbf{x}1.67033} = \mathbf{264.34\ W}.$$

Solution 11.17

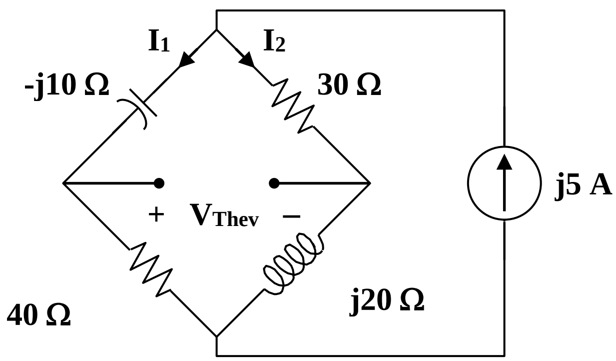
We find Z_{eq} at terminals a-b following Fig. (a).



(a)

$$\mathbf{Z}_{eq} = \left(-j10+30\right) \| \left(j20+40\right) = \frac{(30-j10)(40+j20)}{70+j10} = \mathbf{20\ \Omega} = \mathbf{Z_L}$$

We obtain $\mathbf{V}_{Thev}$ from Fig. (b).



(b)

Using current division,

$$\mathbf{I}_1 = \frac{30+\text{j}20}{70+\text{j}10}(\text{j}5) = \text{-}1.1+\text{j}2.3$$

$$\mathbf{I}_2 = \frac{40-\text{j}10}{70+\text{j}10}(\text{j}5) = 1.1+\text{j}2.7$$

$$\mathbf{V}_{\text{Th}} = 30\mathbf{I}_2 + \text{j}10\mathbf{I}_1 = 10+\text{j}70$$

$$P_{\max} = \frac{\left|\mathbf{V}_{Th}\right|^2}{2\left(Z_{eq}+Z_L\right)^2} Z_L = \frac{5000}{(2)(2x20)^2} 20 = \mathbf{31.25\ W}$$

Solution 11.18

Find the value of $\mathbf{Z}_L$ in the circuit of Fig. 11.49 for maximum power transfer.

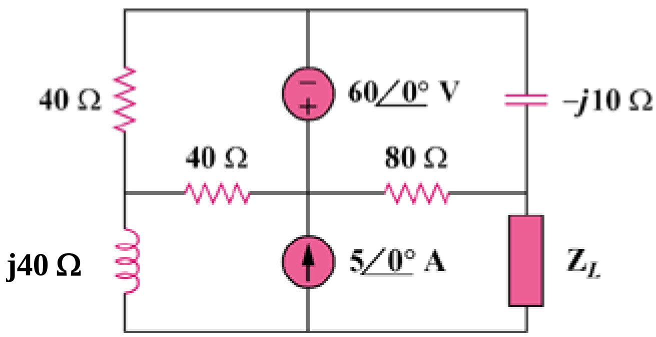
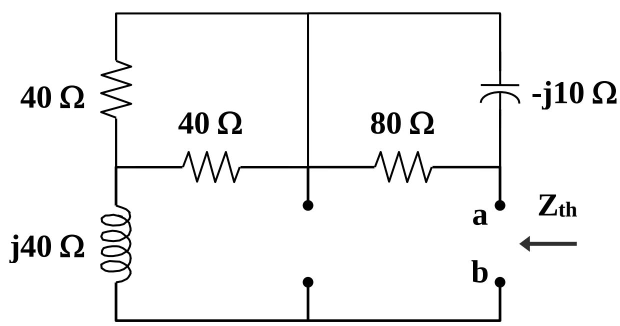


Figure 11.49
For Prob. 11.18.

Solution

We find $\mathbf{Z}_{\text{Th}}$ at terminals a-b as shown in the figure below.



$$\mathbf{Z}_{Th} = j40 + 40 \,||\, 40 + 80 \,||\, (\text{-j}10) = \text{j}40 + 20 + \frac{(80)(\text{-j}10)}{80 - \text{j}10}$$

$$\mathbf{Z}_{Th} = 21.23 + j30.154$$

$$\mathbf{Z}_{\text{L}} = \mathbf{Z}^*_{\text{Th}} = \mathbf{[21.23{-}j30.15]\ \Omega}$$

Solution 11.19

The variable resistor R in the circuit of Fig. 11.50 is adjusted until it absorbs the maximum average power. Find R and the maximum average power absorbed.

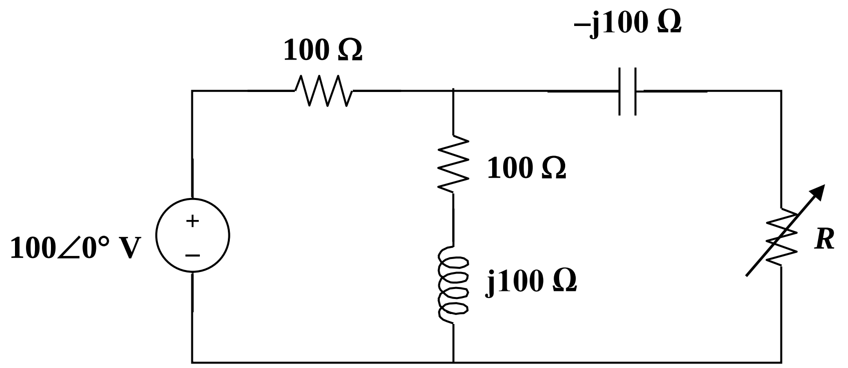
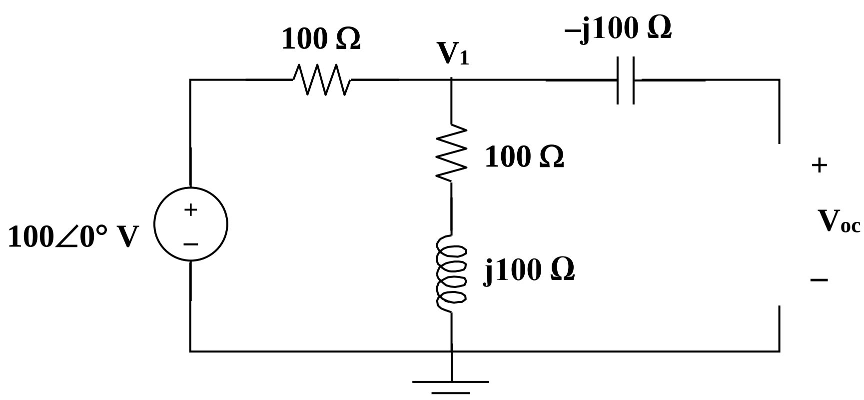


Figure 11.50
For Prob. 11.19.

Solution

Step 1. We first remove R from the circuit and then find the Thevenin equivalent circuit. Once we have $\mathbf{V}_{\mathbf{Thev}}$ and $\mathbf{Z}_{\mathbf{eq}}$ we then know that for maximum power transfer to the load, R must be equal to $|\mathbf{Z}_{\mathbf{eq}}|$ and $P_{avg} = |\mathbf{V}_{\mathbf{Thev}}|^2/(8R)$. We now find $\mathbf{V}_{\mathbf{Thev}}$ by writing and solving a nodal equation for the circuit shown below. To find $\mathbf{Z}_{\mathbf{eq}}$, we just set the source to zero (a short) and determine the impedance looking in from the right. $\mathbf{Z}_{\mathbf{eq}} = –j100 + 100(100+j100)/(100+100+j100)$.



$[(V_1–100)/100] + [(V_1–0)/(100+j100)] + 0 = 0$ and $\mathbf{V}_{\mathbf{oc}} = \mathbf{V}_{\mathbf{Thev}} = \mathbf{V}_{\mathbf{1}}$.

Step 2. $\mathbf{Z}_{\mathbf{eq}} = –j100+100(1.4142\angle 45°)/(2.2361\angle 26.57°) = –j100+63.244\angle 18.43° = –j100 + 60 + j20 = (60 – j80)\ \Omega = 100\angle –53.13°\ \Omega$.

The node equation becomes, $(0.01+0.005–j0.005)\mathbf{V}_{\mathbf{1}} = 1 = 0.0158114\angle –18.43°\mathbf{V}_{\mathbf{1}}$ or $\mathbf{V}_{\mathbf{1}} = 63.246\angle 18.43°$. Thus,

$$R = \mathbf{100\ \Omega}$$

and $|\mathbf{I}| = 63.246/|60–j80+100| = 63.246/178.885 = 0.353557$ A and

the maximum $P_{avg} = [(0.353557)^2/2]100 = \mathbf{6.25\ W}$.

Solution 11.20

The load resistance R_L in Fig. 11.51 is adjusted until it absorbs the maximum average power. Calculate the value of R_L and the maximum average power.

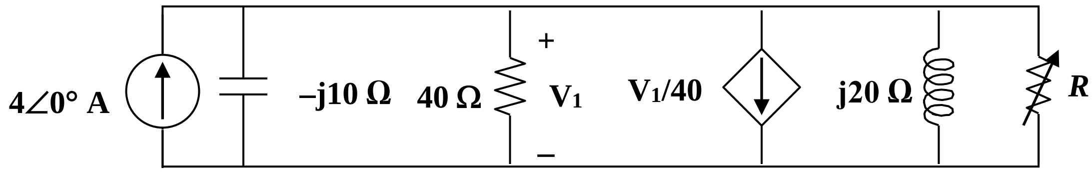
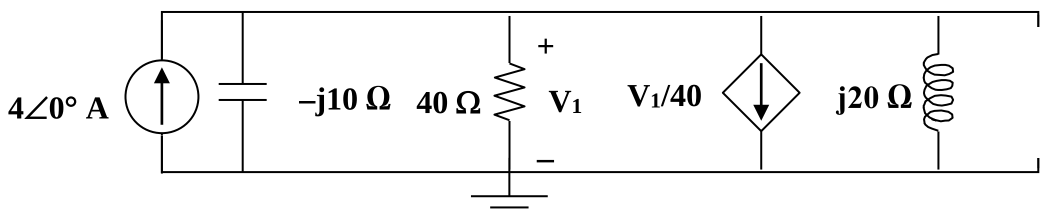


Figure 11.51
For Prob. 11.20.

Solution

Step 1. The easiest way to solve this problem is to find the Thevenin equivalent circuit and then we can now solve for R = $|\mathbf{Z_{eq}}|$ and $P_{avg} = |\mathbf{I}|^2R/2$ where $\mathbf{I} = \mathbf{V_{Thev}}/(\mathbf{Z_{eq}}+R)$. To find the Thevenin equivalent circuit we take R out of the circuit and then find $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$ which gives $\mathbf{V_{Thev}} = \mathbf{V_{oc}}$ and $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$.



We note that $\mathbf{V_1} = \mathbf{V_{oc}} = \mathbf{V_{Thev}}$ and the open circuit nodal equation becomes,
$-4 + [(\mathbf{V_1}-0)/(-j10)] + [(\mathbf{V_1}-0)/(40)] + [\mathbf{V_1}/40] + [(\mathbf{V_1}-0)/(j20)] = 0$.

For I_{sc} we note that $\mathbf{V_1} = 0$ because of the short which means that $\mathbf{I_{sc}} = 4$ A.

Step 2. $(j0.1 + 0.025 - j0.05 + 0.025)\mathbf{V_1} = 4$ or $\mathbf{V_1} = 4/(0.05+j0.05)$ or $\mathbf{V_1} = 4/(0.07071\angle 45°) = 56.569\angle -45°$ V. Therefore, $\mathbf{Z_{eq}} = 56.569\angle -45°/4 = 14.142\angle -45°$ Ω.

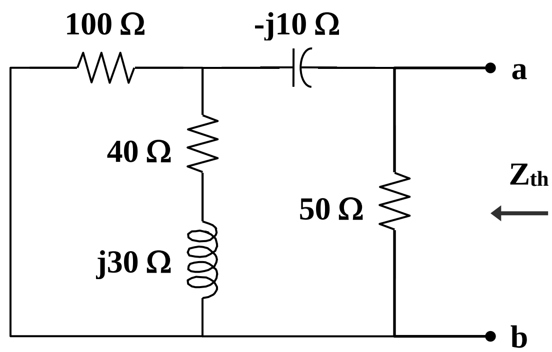
R = **14.142 Ω**

and $\mathbf{I} = (56.569\angle -45°)/(7.071-j7.071+14.142)$
$= (56.569\angle -45°)/(21.213-j7.071) = (56.569\angle -45°)/(22.36\angle -18.435°)$
$= 2.5299\angle -26.565°$ A. Therefore,

$P_{avg} = (2.5299)^2R/2 =$ **45.26 W**.

Solution 11.21

We find $\mathbf{Z}_{Th}$ at terminals a-b, as shown in the figure below.



$$\mathbf{Z}_{Th} = 50 \,||\, [\,-j10 + 100 \,||\, (40 + j30)\,]$$

where $100 \,||\, (40 + j30) = \dfrac{(100)(40 + j30)}{140 + j30} = 31.707 + j14.634$

$$\mathbf{Z}_{Th} = 50 \,||\, (31.707 + j4.634) = \frac{(50)(31.707 + j4.634)}{81.707 + j4.634}$$

$$\mathbf{Z}_{Th} = 19.5 + j1.73$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \mathbf{19.58\ \Omega}$$

Solution 11.22

i(t) = [2–2cos(2t)] amps

$$I_{rms}^2 = \frac{1}{\pi}\left[\int_0^{\pi}\left[2-2\cos\left(2t\right)\right]^2 dt\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} 4dt + \int_0^{\pi}\left[-4\cos(2t)\right]dt + \int_0^{\pi} 4\cos^2(2t)dt\right]$$

$$= \frac{1}{\pi}\left[4\pi + 0 + 4\int_0^{\pi}\left[\frac{1+\cos(4t)}{2}\right]dt\right] = \frac{1}{\pi}\left[4\pi + 4\left(\frac{\pi}{2}\right)\right] = 6$$

$$\mathbf{I_{rms}} = \sqrt{6} = 2.449\,\mathbf{amps}$$

Solution 11.23

Using Fig. 11.54, design a problem to help other students to better understand how to find the rms value of a waveshape.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the rms value of the voltage shown in Fig. 11.54.

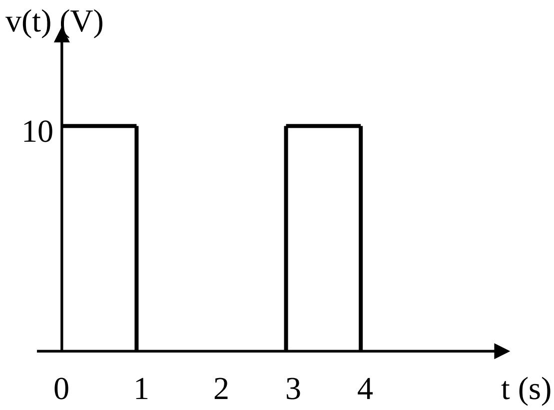


Figure 11.54 For Prob. 11.23.

Solution

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{3}\int_0^1 10^2 dt = \frac{100}{3}$$

V_{rms} = **5.7735 V**

Solution 11.24

$$T = 2, \qquad v(t) = \begin{cases} 5, & 0 < t < 1 \\ -5, & 1 < t < 2 \end{cases}$$

$$V_{rms}^2 = \frac{1}{2}\left[\int_0^1 5^2 \, dt + \int_1^2 (-5)^2 \, dt\right] = \frac{25}{2}[1+1] = 25$$

$$V_{rms} = \mathbf{5\ V}$$

Solution 11.25

Find the rms value of the signal shown in Fig. 11.56.

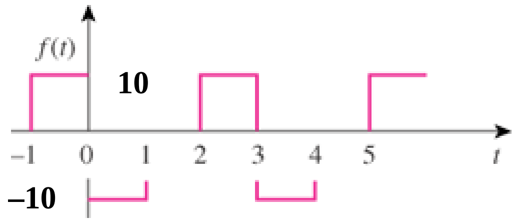


Figure 11.56
For Prob. 11.25.

Solution

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^1 (-10)^2 dt + \int_1^2 0dt + \int_2^3 10^2 dt\right]$$

$$= \frac{1}{3}[100 + 0 + 100] = \frac{200}{3}$$

$$f_{rms} = \sqrt{\frac{200}{3}} = 8.165$$

f_{rms} = **8.165**

Solution 11.26

$$\mathrm{T} = 4, \qquad v(t) = \begin{cases} 5 & 0 < t < 2 \\ 20 & 2 < \mathrm{t} < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 10^2 \ dt + \int_2^4 (20)^2 \ dt \ \right] = \frac{1}{4}[200 + 800] = 250$$

$$\mathrm{V_{rms}} = \mathbf{15.811\ V}.$$

Solution 11.27

$T = 5, \qquad i(t) = t, \quad 0 < t < 5$

$$I_{rms}^2 = \frac{1}{5}\int_0^5 t^2\,dt = \frac{1}{5}\cdot\frac{t^3}{3}\Big|_0^5 = \frac{125}{15} = 8.333$$

$I_{rms} =$ **2.887 A**

Solution 11.28

$$V_{rms}^2 = \frac{1}{5}\left[\int_0^2 (4t)^2\ dt + \int_2^5 0^2\ dt\right]$$

$$V_{rms}^2 = \frac{1}{5}\cdot\frac{16t^3}{3}\Big|_0^2 = \frac{16}{15}(8) = 8.533$$

$$V_{rms} = \mathbf{2.92\ V}$$

$$P = \frac{V_{rms}^2}{R} = \frac{8.533}{2} = \mathbf{4.267\ W}$$

Solution 11.29

$$\mathrm{T} = 20, \qquad i(t) = \begin{cases} 60 - 6t & 5 < t < 15 \\ \text{-}120 + 6\text{t} & 15 < \text{t} < 25 \end{cases}$$

$$I_{eff}^2 = \frac{1}{20}\left[\int_5^{15} (60 - 6t)^2 \, dt + \int_{15}^{25} (\text{-}120 + 6\text{t})^2 \, dt\right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\int_5^{15} (900 - 180t + 9t^2) \, dt + \int_{15}^{25} (9\text{t}^2 - 360\text{t} + 3600) \, dt\right]$$

$$I_{eff}^2 = \frac{1}{5}\left[\left(900t - 90t^2 + 3t^3\right)\Big|_5^{15} + \left(3t^3 - 180t^2 + 3600t\right)\Big|_{15}^{25}\right]$$

$$I_{eff}^2 = \frac{1}{5}[750 + 750] = 300$$

$$\mathrm{I}_{\mathrm{eff}} = \mathbf{17.321\ A}$$

$$\mathrm{P} = \mathrm{I}_{\mathrm{eff}}^2\mathrm{R} = (17.321)^2 \mathrm{x} 12 = \mathbf{3.6\ kW}.$$

Solution 11.30

$$v(t) = \begin{cases} t & 0 < t < 2 \\ -1 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 t^2\ dt + \int_2^4 (-1)^2\ dt\right] = \frac{1}{4}\left[\frac{8}{3} + 2\right] = 1.1667$$

V_{rms} = **1.08 V**

Solution 11.31

$$V^2{}_{rms} = \frac{1}{2}\int_0^2 v(t)dt = \frac{1}{2}\left[\int_0^1 (2t)^2\, dt + \int_1^2 (-4)^2\, dt\right] = \frac{1}{2}\left[\frac{4}{3} + 16\right] = 8.6667$$

$\boldsymbol{V_{rms}}$ **=** **<u>2.944 V</u>**

Solution 11.32

$$I_{rms}^2 = \frac{1}{2}\left[\int_0^1 (10t^2)^2\ dt + \int_1^2 0\ dt\right]$$

$$I_{rms}^2 = 50\int_0^1 t^4\ dt = 50 \cdot \frac{t^5}{5}\Big|_0^1 = 10$$

$$I_{rms} = \mathbf{3.162\ A}$$

Solution 11.33

Determine the rms value for the waveform in Fig. 11.64.

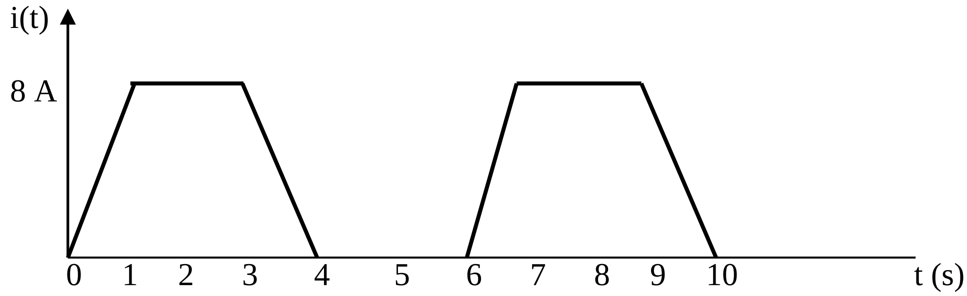


Figure 11.64
For Prob. 11.33.

Solution

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)\,dt = \frac{1}{6}\left[\int_0^1 64t^2 dt + \int_1^3 64 dt + \int_3^4 (-8t+32)^2 dt\right]$$

$$I_{rms}^2 = \frac{1}{6}\left[64\frac{t^3}{3}\bigg|_0^1 + 64(3-1) + \left(64\frac{t^3}{3} - 256t^2 + 1024t\right)\bigg|_3^4\right]$$

$= 28.43$

I_{rms} = **5.332 A**

Solution 11.34

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^2 (3t)^2\, dt + \int_2^3 6^2\, dt\right]$$

$$= \frac{1}{3}\left[\left.\frac{9t^3}{3}\right|_0^2 + 36\right] = 20$$

$$f_{rms} = \sqrt{20} = 4.472$$

f_{rms} = **4.472**

Solution 11.35

$$V_{rms}^2 = \frac{1}{6}\left[\int_0^1 10^2\ dt + \int_1^2 20^2\ dt + \int_2^4 30^2\ dt + \int_4^5 20^2\ dt + \int_5^6 10^2\ dt\right]$$

$$V_{rms}^2 = \frac{1}{6}[100 + 400 + 1800 + 400 + 100] = 466.67$$

$$V_{rms} = \mathbf{21.6\ V}$$

Solution 11.36

(a) I_{rms} = $\underline{10\ \text{A}}$

(b) $V^2_{rms} = 4^2 + \left(\dfrac{3}{\sqrt{2}}\right)^2 \longrightarrow V_{rms} = \sqrt{16 + \dfrac{9}{2}} = \underline{\mathbf{4.528\ V}}$ (checked)

(c) $I_{rms} = \sqrt{64 + \dfrac{36}{2}} = \underline{\mathbf{9.055\ A}}$

(d) $V_{rms} = \sqrt{\dfrac{25}{2} + \dfrac{16}{2}} = \underline{\mathbf{4.528\ V}}$

Solution 11.37

Design a problem to help other students to better understand how to determine the rms value of the sum of multiple currents.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Calculate the rms value of the sum of these three currents:

$i_1 = 8$, $i_2 = 4\sin(t + 10°)$, $i_3 = 6\cos(2t + 30°)$ A

Solution

$$i = i_1 + i_2 + i_3 = 8 + 4\sin(t + 10^o) + 6\cos(2t + 30^o)$$

$$\boldsymbol{I_{rms} = \sqrt{I^2_{1rms} + I^2_{2rms} + I^2_{3rms}} = \sqrt{64 + \frac{16}{2} + \frac{36}{2}} = \sqrt{90} = \underline{9.487\ \mathrm{A}}}$$

Solution 11.38

For the power system in Fig. 11.67, find: (a) the average power, (b) the reactive power, (c) the power factor. Note that 440 V is an rms value.

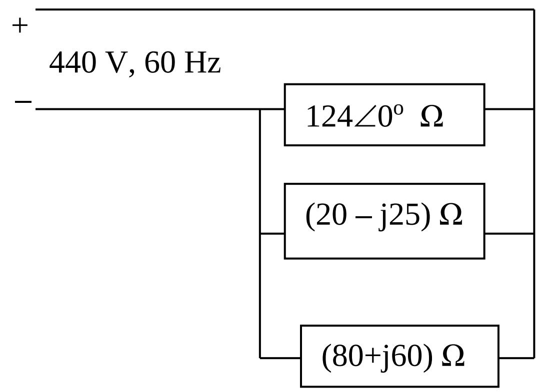


Figure 11.67
For Prob. 11.38.

Solution

$\mathbf{S_1} = V^2/(\mathbf{Z_1})^* = 1.56129$ kW.

$\mathbf{S_2} = V^2/(\mathbf{Z_2})^* = 193{,}600/(32.0156\angle 51.34° = 6{,}047.05\angle{-51.34°}$ VA
$= 3.7776$ kW – j4.7219 kVAR

$\mathbf{S_3} = V^2/(\mathbf{Z_3})^* = 193{,}600/(100\angle{-36.87°} = 1{,}936\angle 36.87°$ VA
$= 1.5488$ kW + j1.1616 lVAR.

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3} = (1.56129+3.7776+1.5488)$ kW + j(0–4.7219+1.1616) kVAR
= 6.888 kW – j3.56 kVAR.
Therefore,

(a) P = Re(**S**) = **6.888 kW**

(b) Q = Im (**S**) = **–3.56 kVAR (leading)**

(c) pf = $\cos [\tan^{-1}([-3.56/6.888])] \cos \{\tan^{-1}[-0.51684]\}$
$= \cos(-27.332°) =$ **0.89**

Solution 11.39

An ac motor with impedance $\mathbf{Z_L} = (2 + j1.2)\ \Omega$ is supplied by a 220-V, 60-Hz source. (a) Find pf, P, and Q. (b) Determine the capacitor required to be connected in parallel with the motor so that the power factor is corrected to unity.

Solution

(a) $\mathbf{Z_L} = 2 + j1.2 = 2.3324\angle 30.964^\circ$

pf = cos(30.964) = **0.8575**

$\mathbf{S} = V^2/(\mathbf{Z_L})^* = 48{,}400/(2.3324\angle{-30.964^\circ}) = 20{,}751\angle 30.964^\circ$
= 17.794 kW + j 10.676 kVAR.

P = **17.794 kW**

Q = **10.676 kVAR (lagging)**

(b) $X_C = V^2/Q_C = 48{,}400/10{,}676 = 4.5335 = 1/(377C)$ or
C = **585.1 μF**.

{It is important to note that this capacitor will see a peak voltage of 220$\sqrt{2}$ = 311.08V, this means that the specifications on the capacitor must be at least this or greater!}

Solution 11.40

Design a problem to help other students to better understand apparent power and power factor.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A load consisting of induction motors is drawing 80 kW from a 220-V, 60 Hz power line at a pf of 0.72 lagging. Find the capacitance of a capacitor required to raise the pf to 0.92.

Solution

$$pf1 = 0.72 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 43.94^0$$
$$pf2 = 0.92 = \cos\theta_2 \quad \longrightarrow \quad \theta_2 = 23.07^0$$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{80x10^3(0.9637 - 0.4259)}{2\pi x 60 x (220)^2} = \underline{2.4\ \text{mF}},$$

{Again, we need to note that this capacitor will be exposed to a peak voltage of 311.08V and must be rated to at least this level, preferably higher!}

Solution 11.41

(a) $$- j2 \,\|\, (j5 - j2) = -j2 \,\|\, -j3 = \frac{(-j2)(-j3)}{j} = -j6$$

$$\mathbf{Z}_T = 4 - j6 = 7.211\angle -56.31°$$

pf $= \cos(-56.31°) =$ **0.5547 (leading)**

(b) $$j2 \,\|\, (4 + j) = \frac{(j2)(4 + j)}{4 + j3} = 0.64 + j1.52$$

$$\mathbf{Z} = 1 \,\|\, (0.64 + j1.52 - j) = \frac{0.64 + j0.44}{1.64 + j0.44} = 0.4793\angle 21.5°$$

pf $= \cos(21.5°) =$ **0.9304 (lagging)**

Solution 11.42

(a) S=120, $pf = 0.707 = \cos\theta \quad \longrightarrow \quad \theta = 45^o$

$$S = S\cos\theta + jS\sin\theta = \underline{84.84 + j84.84 \text{ VA}}$$

(b) $S = V_{rms} I_{rms} \quad \longrightarrow \quad I_{rms} = \frac{S}{V_{rms}} = \frac{120}{110} = \underline{1.091 \text{ A rms}}$

(c) $S = I_{rms}^2 Z \quad \longrightarrow \quad Z = \frac{S}{I_{rms}^2} = \underline{71.278 + j71.278\ \Omega}$

(d) If Z = R + jωL, then R = **71.278 Ω**

$$\omega L = 2\pi fL = 71.278 \quad \longrightarrow \quad L = \frac{71.278}{2\pi x 60} = \underline{0.1891 \text{ H}} = \mathbf{189.1\ mH}.$$

Solution 11.43

Design a problem to help other students to better understand complex power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The voltage applied to a 10-ohm resistor is

$v(t) = 5 + 3\cos(t + 10°) + \cos(2t + 30°)$ V

(a) Calculate the rms value of the voltage.

(b) Determine the average power dissipated in the resistor.

Solution

(a) $$\boldsymbol{V_{rms} = \sqrt{V^2_{1rms} + V^2_{2rms} + V^2_{3rms}} = \sqrt{25 + \frac{9}{2} + \frac{1}{2}} = \sqrt{30} = \underline{5.477\ \text{V}}}$$

(b) $$\boldsymbol{P = \frac{V^2_{rms}}{R} = 30/10 = \underline{3\ \text{W}}}$$

Solution 11.44

$$40\mu F \longrightarrow \frac{1}{j\omega C}=\frac{1}{j2000x40x10^{-6}}=-j12.5$$

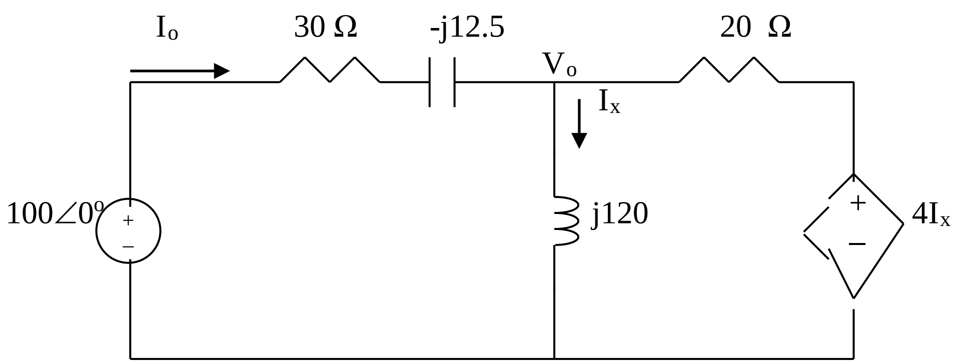
$$60mH \longrightarrow j\omega L = j2000x60x10^{-3} = j120$$

We apply nodal analysis to the circuit shown below.

$$\frac{100-V_o}{30-j12.5}+\frac{4I_x-V_o}{20}=\frac{V_o}{j120}$$

But $I_x=\dfrac{V_o}{j120}$. Solving for V_o leads to

$$V_o = 2.9563+j1.126$$



$$I_o=\frac{100-V_o}{30-j12.5}=2.7696+j1.1165$$

$$S=\frac{1}{2}V_sI_o^*=\frac{1}{2}(100)(2.7696-j.1165)=\underline{138.48-j55.825\ \text{VA}}$$

S = (138.48 – j55.82) VA

Solution 11.45

(a) $\boldsymbol{V^2_{rms} = 20^2 + \frac{60^2}{2} = 2200} \quad \longrightarrow \quad \boldsymbol{V_{rms} = \underline{46.9\ \mathrm{V}}}$

$$\boldsymbol{I_{rms} = \sqrt{1^2 + \frac{0.5^2}{2}} = \sqrt{1.125} = \underline{1.061A}}$$

(b) p(t) = v(t)i(t) = 20 + 60cos100t – 10sin100t – 30(sin100t)(cos100t); clearly the average power = **20W**.

Solution 11.46

(a) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (220\angle 30°)(0.5\angle -60°) = 110\angle -30°$

$\mathbf{S} = \mathbf{[95.26 - \mathit{j}55]}\,\boldsymbol{VA}$

Apparent power = **110 VA**
Real power = **95.26 W**
Reactive power = **55 VAR**
pf is **leading** because current leads voltage

(b) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (250\angle -10°)(6.2\angle 25°) = 1550\angle 15°$

$\mathbf{S} = \mathbf{[497.2 + \mathit{j}401.2]}\,\boldsymbol{VA}$

Apparent power = **1550 VA**
Real power = **1497.2 W**
Reactive power = **401.2 VAR**
pf is **lagging** because current lags voltage

(c) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (120\angle 0°)(2.4\angle 15°) = 288\angle 15°$

$\mathbf{S} = \mathbf{[278.2 + \mathit{j}74.54]}\,\boldsymbol{VA}$

Apparent power = **288 VA**
Real power = **278.2 W**
Reactive power = **74.54 VAR**
pf is **lagging** because current lags voltage

(d) $\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (160\angle 45°)(8.5\angle -90°) = 1360\angle -45°$

$\mathbf{S} = \mathbf{[961.7 - j961.7]\ VA}$

Apparent power = **1360 VA**
Real power = **961.7 W**
Reactive power = **- 961.7 VAR**
pf is **leading** because current leads voltage

Solution 11.47

For each of the following cases, find the complex power, the average power, and the reactive power:

(a) $v(t) = 169.7 \sin(377t + 45°)$ V, $i(t) = 5.657 \sin(377t)$ A

(b) $v(t) = 339.4 \sin(377t + 90°)$ V, $i(t) = 5.657 \sin(377t + 45°)$ A

(c) $\mathbf{V} = 900\ \angle 90°$ V rms, $\mathbf{Z} = 75\ \angle 45°\ \Omega$

(d) $\mathbf{I} = 100\ \angle 60°$ A rms, $\mathbf{Z} = 50\ \angle 60°\ \Omega$

Solution

Step 1. In the first two cases we need to convert the time varying voltages and currents into complex rms (rms magnitudes are equal to peak values divided by 1.4142) values. $\mathbf{S} = \mathbf{VI}^* = \mathbf{VV}^*/\mathbf{Z}^* = \mathbf{IZI}^* = P + jQ$, where **S** is the complex power, P is the average power, and Q is the reactive power.

Step 2.

(a) $\mathbf{V} = (169.7/1.4142)\angle 45° = 120\angle 45°$ V and
$\mathbf{I} = (5.657/1.4142)\angle 0° = 4\angle 0°$ A.
Now $\mathbf{S} = \mathbf{480\angle 45°\ VA} = \mathbf{339.4\ W} + j\mathbf{339.4\ VAR}$

(b) $\mathbf{V} = 240\angle 90°$ V and $\mathbf{I} = 4\angle 45°$ A which leads to
$\mathbf{S} = (240\angle 90°)(4\angle -45°) = \mathbf{960\angle 45°\ VA} = \mathbf{678.8\ W + j678.8\ VAR}$.

(c) $\mathbf{S} = 900^2/(75\angle -45°) = \mathbf{10.8\angle 45°\ kVA} = \mathbf{7.637\ kW} + j\mathbf{7.637\ kVAR}$.

(d) $\mathbf{S} = 100^2(50\angle 60°) = \mathbf{500\angle 60°\ kVA} = \mathbf{250\ kW} + j\mathbf{433\ kVAR}$.

Solution 11.48

(a) $\mathbf{S} = \mathrm{P} - \mathrm{jQ} = \mathbf{[269 - \mathit{j}150]}\,\boldsymbol{VA}$

(b) $\mathrm{pf} = \cos\theta = 0.9 \quad \longrightarrow \quad \theta = 25.84°$

$$\mathrm{Q} = \mathrm{S}\sin\theta \quad \longrightarrow \quad \mathrm{S} = \frac{\mathrm{Q}}{\sin\theta} = \frac{2000}{\sin(25.84°)} = 4588.31$$

$$\mathrm{P} = \mathrm{S}\cos\theta = 4129.48$$

$\mathbf{S} = \mathbf{[4.129 - \mathit{j}2]}\,\boldsymbol{kVA}$

(c) $\mathrm{Q} = \mathrm{S}\sin\theta \quad \longrightarrow \quad \sin\theta = \dfrac{\mathrm{Q}}{\mathrm{S}} = \dfrac{450}{600} = 0.75$

$\theta = 48.59$, $\qquad \mathrm{pf} = 0.6614$

$$\mathrm{P} = \mathrm{S}\cos\theta = (600)(0.6614) = 396.86$$

$\mathbf{S} = \mathbf{[396.9 + \mathit{j}450]}\,\boldsymbol{VA}$

(d) $\mathrm{S} = \dfrac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \dfrac{(220)^2}{40} = 1210$

$$\mathrm{P} = \mathrm{S}\cos\theta \quad \longrightarrow \quad \cos\theta = \frac{\mathrm{P}}{\mathrm{S}} = \frac{1000}{1210} = 0.8264$$

$\theta = 34.26°$

$$\mathrm{Q} = \mathrm{S}\sin\theta = 681.25$$

$\mathbf{S} = \mathbf{[1 + \mathit{j}0.6812]}\,\boldsymbol{kVA}$

Solution 11.49

(a) $\mathbf{S} = 4 + j\dfrac{4}{0.86}\sin(\cos^{-1}(0.86))$ kVA

$\mathbf{S} = \mathbf{[4 + \mathit{j}2.373]}\ \boldsymbol{kVA}$

(b) $\text{pf} = \dfrac{\text{P}}{\text{S}} = \dfrac{1.6}{2}0.8 = \cos\theta \longrightarrow \sin\theta = 0.6$

$\mathbf{S} = 1.6 - j2\sin\theta = \mathbf{[1.6 - \mathit{j}1.2]}\ \boldsymbol{kVA}$

(c) $\mathbf{S} = \mathbf{V}_{\text{rms}}\,\mathbf{I}^{*}_{\text{rms}} = (208\angle 20°)(6.5\angle 50°)$ VA

$\mathbf{S} = 1.352\angle 70° = \mathbf{[0.4624 + \mathit{j}1.2705]}\ \boldsymbol{kVA}$

(d) $\mathbf{S} = \dfrac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \dfrac{(120)^2}{40 - j60} = \dfrac{14400}{72.11\angle -56.31°}$

$\mathbf{S} = 199.7\angle 56.31° = \mathbf{[110.77 + \mathit{j}166.16]}\ \boldsymbol{VA}$

Solution 11.50

(a) $\mathbf{S} = \mathrm{P} - \mathrm{jQ} = 1000 - \mathrm{j}\dfrac{1000}{0.8}\sin(\cos^{-1}(0.8))$

$\mathbf{S} = 1000 - \mathrm{j}750$

But, $\mathbf{S} = \dfrac{|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{Z}^*}$

$$\mathbf{Z}^* = \frac{|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{S}} = \frac{(220)^2}{1000 - \mathrm{j}750} = 30.98 + \mathrm{j}23.23$$

$\mathbf{Z} = \mathbf{[30.98 - \mathit{j}23.23]\,\Omega}$

(b) $\mathbf{S} = |\mathbf{I}_{\mathrm{rms}}|^2\,\mathbf{Z}$

$$\mathbf{Z} = \frac{\mathbf{S}}{|\mathbf{I}_{\mathrm{rms}}|^2} = \frac{1500 + \mathrm{j}2000}{(12)^2} = \mathbf{[10.42 + \mathit{j}13.89]\,\Omega}$$

(c) $\mathbf{Z}^* = \dfrac{|\mathbf{V}_{\mathrm{rms}}|^2}{\mathbf{S}} = \dfrac{|\mathbf{V}|^2}{2\mathbf{S}} = \dfrac{(120)^2}{(2)(4500\angle 60°)} = 1.6\angle -60°$

$\mathbf{Z} = 1.6\angle 60° = \mathbf{[0.8 + \mathit{j}1.386]\,\Omega}$

Solution 11.51

For the entire circuit in Fig. 11.70, calculate:

(a) the power factor
(b) the average power delivered by the source
(c) the reactive power
(d) the apparent power
(e) the complex power

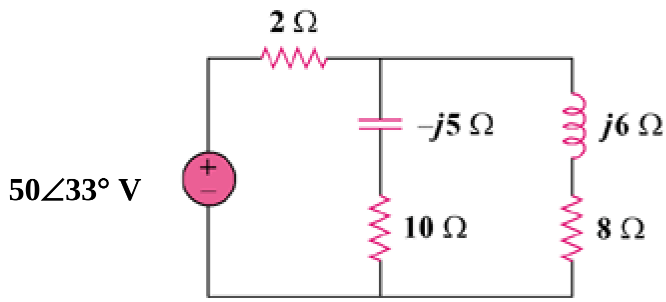


Figure 11.70
For Prob. 11.51.

Solution

(a) $\mathbf{Z}_T = 2 + (10 - j5) \parallel (8 + j6)$

$$\mathbf{Z}_T = 2 + \frac{(10 - j5)(8 + j6)}{18 + j} = 2 + \frac{110 + j20}{18 + j}$$

$$\mathbf{Z}_T = 8.152 + j0.768 = 8.188\angle 5.382^\circ$$

pf $= \cos(5.382^\circ) =$ **0.9956 (lagging)**

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = \dfrac{|\mathbf{V}|^2}{(\mathbf{Z}_T)^*} = \dfrac{(50)^2}{(8.188\angle \mathbf{-5.382}^\circ)}$

$\mathbf{S} = 305.325\angle 5.382^\circ$

$P = S\cos\theta =$ **304 W**

(c) $Q = S\sin\theta =$ **28.64 VAR**

(d) $S = |\mathbf{S}| =$ **305.3 VA**

(e) $\mathbf{S} = 305.325\angle 5.382^\circ =$ **(304+j28.64) VA**

Solution 11.52

$$S_A = 2000 + j\frac{2000}{0.8}0.6 = 2000 + j1500$$
$$S_B = 3000x0.4 - j3000x0.9165 = 1200 - j2749$$
$$S_C = 1000 + j500$$
$$S = S_A + S_B + S_C = 4200 - j749$$

(a) $pf = \dfrac{4200}{\sqrt{4200^2 + 749^2}} = \mathbf{0.9845\ leading}$

(b) $S = V_{rms}I^*_{rms} \longrightarrow I^*_{rms} = \dfrac{4200 - j749}{120\angle 45^\circ} = 35.55\angle -55.11^\circ$

I_{rms} = $\mathbf{35.55\angle 55.11^\circ\ A}$.

Solution 11.53

$S = S_A + S_B + S_C = 4000(0.8–j0.6) + 2400(0.6+j0.8) + 1000 + j500$

$= 5640 + j20 = 5640\angle 0.2^\circ$

(a) $$\boldsymbol{I_{rms}^{*} = \frac{S_B}{V_{rms}} + \frac{S_A + S_C}{V_{rms}} = \frac{S}{V_{rms}} = \frac{5640\angle 0.2^\circ}{120\angle 30^\circ} = 47\angle -29.8^\circ}$$

$$\boldsymbol{I = 47\angle 29.8^\circ = \underline{47\angle 29.8^\circ\ A}}$$

(b) pf = $\cos(0.2^\circ)$ **≈ <u>1.0 lagging</u>**.

Solution 11.54

For the network in Fig. 11.73, find the complex power absorbed by each element.

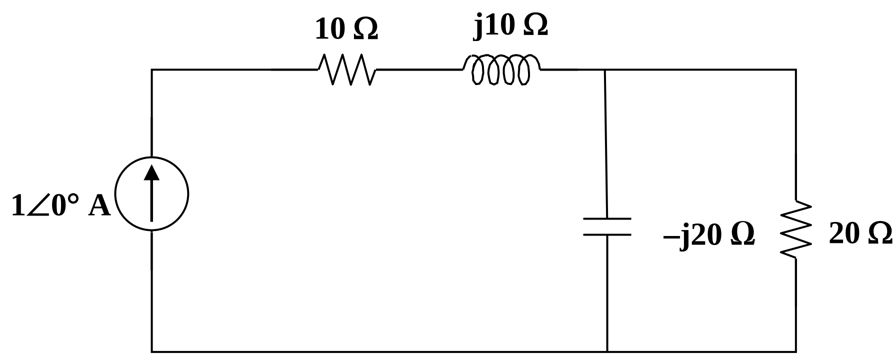


Figure 11.73
For Prob. 11.54.

Solution

Step 1. $P_{10} = (1)^2 10$, $Q_{j10} = (1)^2(10)$, and we need to determine the current division to find the current through the –j20 Ω, $\mathbf{I}_{-j20} = 1(20)/(20–j20)$, and the 20Ω, $\mathbf{I}_{20} = 1(–j20)/(20–j20)$. Finally $Q_{-j20} = |\mathbf{I}_{-j20}|^2(20)$ and $P_{20} = |\mathbf{I}_{20}|^2 20$. Lastly, the power absorbed by the current source can be expressed as
$\mathbf{S}_{\mathbf{absorbed}} = –(P_{10} + P_{20} + jQ_{j10} – jQ_{-j20})$.

Step 2. $P_{10} =$ **10 W**, $Q_{j10} =$ **10 VAR**, $\mathbf{I}_{-j20} = 20/(28.282\angle–45°) = 0.7071\angle45°$ A, and
$\mathbf{I}_{20} = (20\angle–90°)/(28.282\angle–45°) = 0.70711\angle–45°$ A. Thus,

$P_{10} =$ **10 W**, $P_{20} =$ **10 W,**
$Q_{j10} =$ **10 VAR,** $Q_{-j20} =$ **10 VAR** and
S = –(10+10+j10–j10) = **–20 W**.

Solution 11.55

Using Fig. 11.74, design a problem to help other students to better understand the conservation of AC power.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the complex power absorbed by each of the five elements in the circuit of Fig. 11.74.

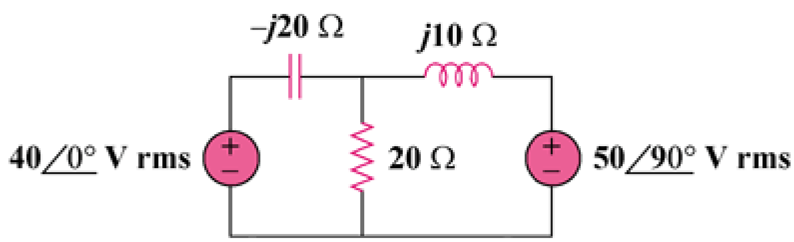
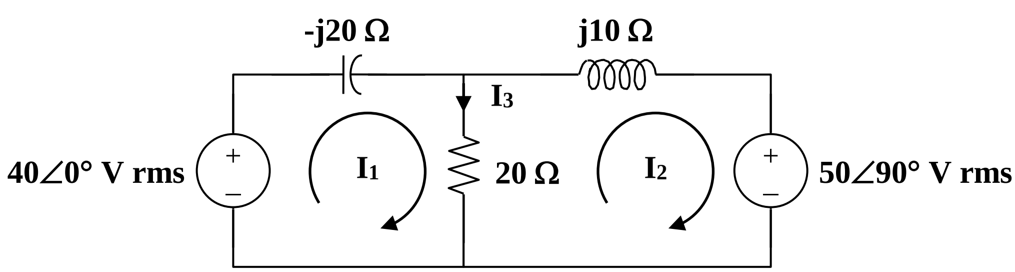


Figure 11.74

Solution

We apply mesh analysis to the following circuit.



For mesh 1,

$$40 = (20 - j20)\,I_1 - 20\,I_2$$

$$2 = (1 - j)\,I_1 - I_2 \qquad (1)$$

For mesh 2,

$$-j50 = (20 + j10)\,I_2 - 20\,I_1$$

$$-j5 = -2\,I_1 + (2 + j)\,I_2 \qquad (2)$$

Putting (1) and (2) in matrix form,

$$\begin{bmatrix} 2 \\ -j5 \end{bmatrix} = \begin{bmatrix} 1-j & -1 \\ -2 & 2+j \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 1 - j, \qquad \Delta_1 = 4 - j3, \qquad \Delta_2 = -1 - j5$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{4 - j3}{1 - j} = \frac{1}{2}(7 + j) = 3.535\angle 8.13°$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-1 - j5}{1 - j} = 2 - j3 = 3.605\angle -56.31°$$

$$I_3 = I_1 - I_2 = (3.5 + j0.5) - (2 - j3) = 1.5 + j3.5 = 3.808\angle 66.8°$$

For the 40-V source,

$$\mathbf{S} = \mathbf{-V\,I_1^*} = -(40)\left(\frac{1}{2}\cdot(7 - j)\right) = \mathbf{[-140 + j20]\ VA}$$

For the capacitor,

$$\mathbf{S} = \left|\mathbf{I}_1\right|^2 \mathbf{Z}_c = \mathbf{-\ j250\ VA}$$

For the resistor,

$$\mathbf{S} = \left|\mathbf{I}_3\right|^2 R = \mathbf{290\ VA}$$

For the inductor,

$$\mathbf{S} = \left|\mathbf{I}_2\right|^2 \mathbf{Z}_L = \mathbf{j130\ VA}$$

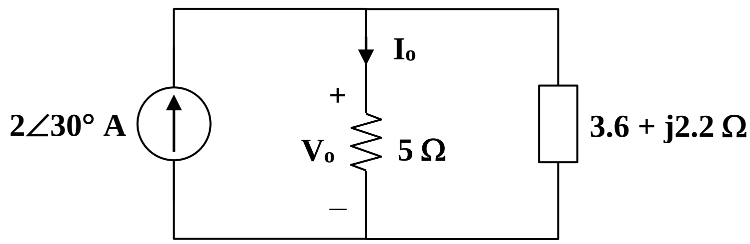
For the j50-V source,

$$\mathbf{S} = \mathbf{V\,I_2^*} = (j50)(2 + j3) = \mathbf{[-150 + j100]\ VA}$$

Solution 11.56

$$-j2 \parallel 6 = \frac{(6)(-j2)}{6-j2} = \frac{12\angle -90^\circ}{6.32456\angle -18.435^\circ} = 1.897365\angle -71.565^\circ = 0.6 - j1.8$$
$$3 + j4 + [(-j2) \parallel 6] = 3.6 + j2.2$$

The circuit is reduced to that shown below.



$$\mathbf{I}_o = \frac{3.6 + j2.2}{8.6 + j2.2}(2\angle 30^\circ) = \frac{4.219\angle 31.4296^\circ}{8.87694\angle 14.3493^\circ}(2\angle 30^\circ) = 0.95055\angle 47.08^\circ$$

$$\mathbf{V}_o = 5\mathbf{I}_o = 4.75275\angle 47.08^\circ$$

$$\mathbf{S} = \mathbf{V}_o \mathbf{I}_s^* = (4.75275\angle 47.08^\circ)(2\angle -30^\circ)$$
$$\mathbf{S} = 9.5055\angle 17.08^\circ = \mathbf{(9.086+j2.792)\ VA}$$

Solution 11.57

For the circuit in Fig. 11.76, find the average, reactive, and complex power delivered by the dependent voltage source.

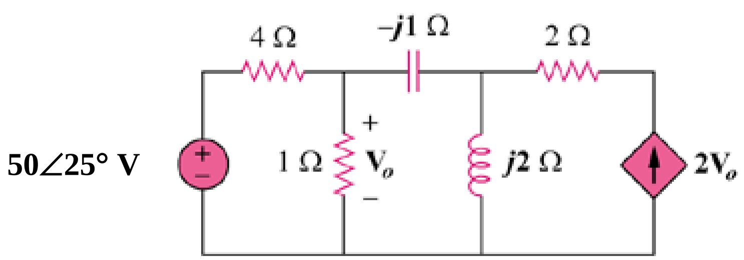
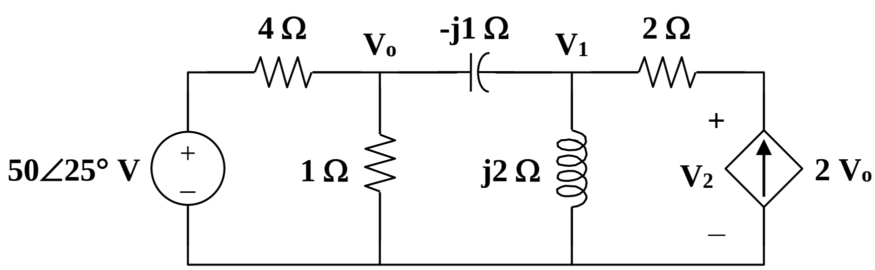


Figure 11.76
For Prob. 11.57.

Solution

Consider the circuit as shown below.



At node o,

$$\frac{50-\mathbf{V}_o}{4}=\frac{\mathbf{V}_o}{1}+\frac{\mathbf{V}_o-\mathbf{V}_1}{-j}$$ (Note, we are neglecting the angle on the source since we are only interested in power from the dependent source.)

$$50=(5+j4)\mathbf{V}_o-j4\mathbf{V}_1 \qquad (1)$$

At node 1,

$$\frac{\mathbf{V}_o-\mathbf{V}_1}{-j}+2\mathbf{V}_o=\frac{\mathbf{V}_1}{j2}$$

$$\mathbf{V}_1=(2-j4)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1), $50=(5+j4-j8-16)\mathbf{V}_o$

$$\mathbf{V}_o=\frac{-50}{11+j4}, \qquad \mathbf{V}_1=\frac{(-50)(2-j4)}{11+j4}$$

The voltage across the dependent source is

$$\mathbf{V}_2 = \mathbf{V}_1 + (2)(2\,\mathbf{V}_o) = \mathbf{V}_1 + 4\,\mathbf{V}_o$$

$$\mathbf{V}_2 = \frac{-50}{11+\text{j}4}\cdot(2-j4+4) = \frac{(-50)(6-j4)}{11+j4}$$

$$\mathbf{S} = \mathbf{V}_2\,\mathbf{I}^* = \mathbf{V}_2\,(2\,\mathbf{V}_o^*)$$

$$\mathbf{S} = \frac{(-50)(6-\mathbf{j}4)}{11+\mathbf{j}4}\cdot\frac{\mathbf{-100}}{\mathbf{11-j4}} = \left(\frac{5{,}000}{137}\right)(6-\mathbf{j}4)$$

$$\mathbf{S} = \mathbf{(219 - j145.99)\ VA}$$

Solution 11.58

Obtain the complex power delivered to the 10-kΩ resistor in Fig. 11.77 below.

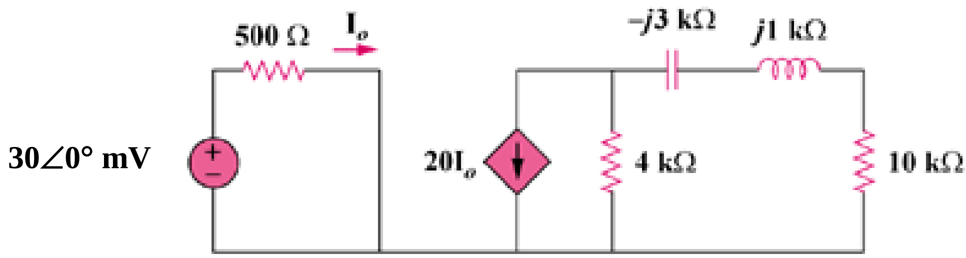


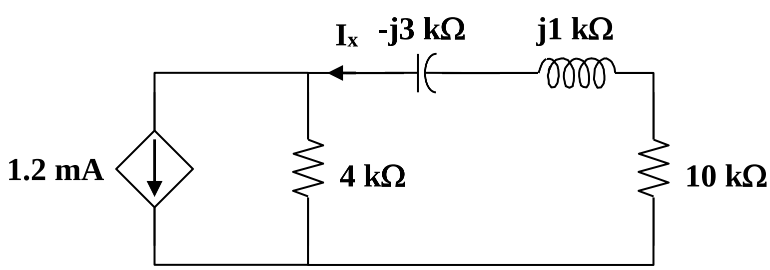
Figure 11.77
For Prob. 11.58.

Solution

From the left portion of the circuit,

$$\mathbf{I}_o = \frac{0.03}{500} = 60\ \mu A$$

$20\mathbf{I}_o = 1.2\ mA$ which then leads to the following circuit,



From the right portion of the circuit,

$$\mathbf{I}_x = \frac{4}{4+10+j-j3}(1.2\ mA) = \frac{2.4}{7-j}\ mA$$

$$\mathbf{S} = |\mathbf{I}_x|^2 R = \frac{(2.4\times 10^{-3})^2}{50}\cdot(10\times 10^3)\ 1.152$$

$$\mathbf{S} = \mathbf{1.152\ mVA}$$

It should be noted that even though we give the answer in VA, the complex power delivered to a resistor is always in watts so, a value of **S = 1.152 mW** would also be correct.

Solution 11.59

Calculate the reactive power in the inductor and capacitor in the circuit of Fig. 11.78.

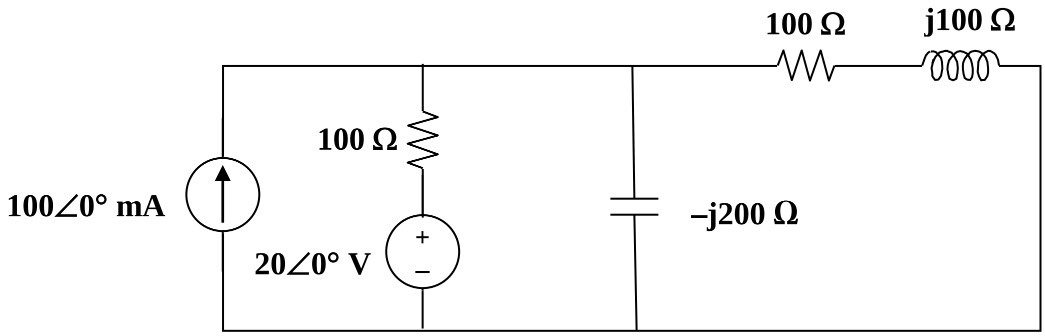
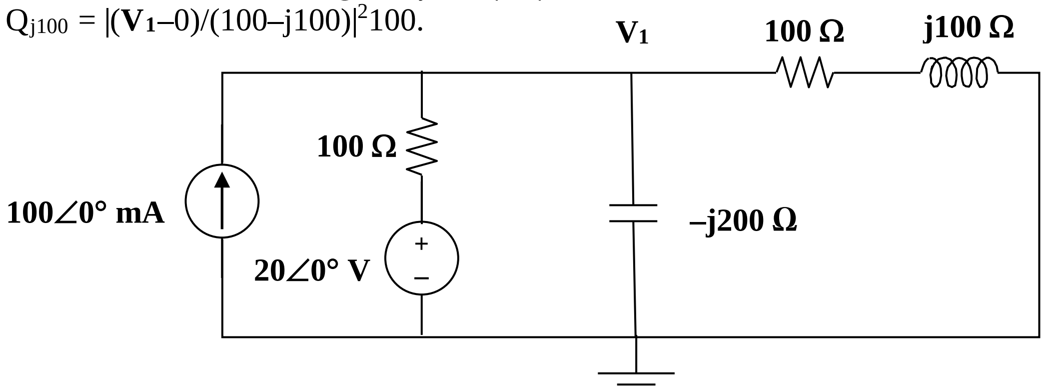


Figure 11.78
For Prob. 11.59.

Solution

Step 1. We write a nodal equation and solve for $\mathbf{V_1}$ in the following circuit. Once we have $\mathbf{V_1}$ we can get $Q_{-j200} = |\mathbf{V_1}|^2/200$ and $Q_{j100} = |(\mathbf{V_1}-0)/(100-j100)|^2 100$.



$$-0.1 + [(\mathbf{V_1}-20)/100] + [(\mathbf{V_1}-0)/(-j200)] + [(\mathbf{V_1}-0)/(100+j100)] = 0.$$

Step 2. $(0.01+j0.005+0.005-j0.005)\mathbf{V_1} = 0.3$ or $\mathbf{V_1} = 0.3/(0.015) = 20$ V.

$Q_{-j200} = 20^2/200 = 2$ VAR and $Q_{j100} = (20/141.42)^2 100 = 2$ VAR or

$$\mathbf{S_{-j200} = -j2\ VAR} \text{ and } \mathbf{S_{j100} = j2\ VAR}.$$

Solution 11.60

$$S_1 = 20 + j\frac{20}{0.8}\sin(\cos^{-1}(0.8)) = 20 + j15$$

$$S_2 = 16 + j\frac{16}{0.9}\sin(\cos^{-1}(0.9)) = 16 + j7.749$$

$$S = S_1 + S_2 = 36 + j22.749 = 42.585\angle 32.29°$$

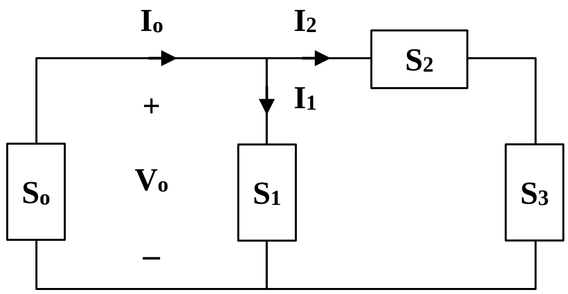
But $\quad S = V_o I^* = 6V_o$

$$V_o = \frac{S}{6} = \mathbf{7.098\angle 32.29°}$$

$$pf = \cos(32.29°) = \mathbf{0.8454 \quad (lagging)}$$

Solution 11.61

Consider the network shown below.



$$\mathbf{S}_2 = 1.2 - \text{j}0.8 \text{ kVA}$$

$$\mathbf{S}_3 = 4 + \text{j}\frac{4}{0.9}\sin(\cos^{-1}(0.9)) = 4 + \text{j}1.937 \text{ kVA}$$

Let $\mathbf{S}_4 = \mathbf{S}_2 + \mathbf{S}_3 = 5.2 + \text{j}1.137 \text{ kVA}$

But $\mathbf{S}_4 = \mathbf{V}_o \mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_4}{\mathbf{V}_o} = \frac{(5.2 + j1.137) \times 10^3}{100\angle 90°} = 11.37 - j52$$

$$\mathbf{I}_2 = 11.37 + j52$$

Similarly, $\mathbf{S}_1 = \sqrt{2} - \text{j}\frac{\sqrt{2}}{0.707}\sin(\cos^{-1}(0.707)) = \sqrt{2}(1 - \text{j}) \text{ kVA}$

But $\mathbf{S}_1 = \mathbf{V}_o \mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{V_o} = \frac{(1.4142 - j1.4142) \times 10^3}{j100} = -14.142 - j14.142$$

$$\mathbf{I}_1 = -14.142 + \text{j}14.142$$

$$\mathbf{I}_o = \mathbf{I}_1 + \mathbf{I}_2 = -2.772 + \text{j}66.14 = \mathbf{66.2\angle 92.4° \text{ A}}$$

$$\mathbf{S}_o = V_o I_o^*$$

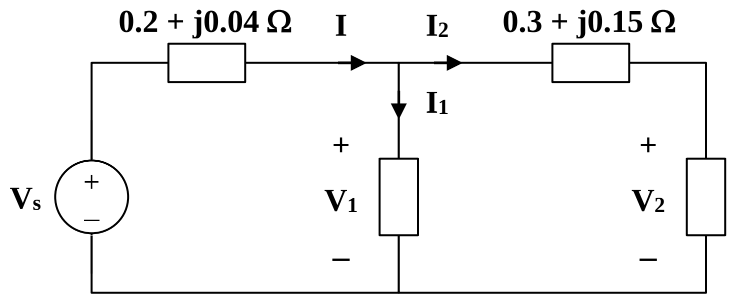
$$\mathbf{S}_o = (100\angle 90°)(66.2\angle -92.4°)\ VA$$

$$\mathbf{S}_\text{o} = \mathbf{6.62\angle{-2.4}° \text{ kVA}}$$

66.2∠92.4° A, 6.62∠–2.4° kVA

Solution 11.62

Consider the circuit below.



$$\mathbf{S}_2 = 15 - j\frac{15}{0.8}\sin(\cos^{-1}(0.8)) = 15 - j11.25$$

But $\quad \mathbf{S}_2 = \mathbf{V}_2\,\mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}_2} = \frac{15 - j11.25}{120}$$

$$\mathbf{I}_2 = 0.125 + j0.09375$$

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{I}_2\,(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120 + (0.125 + j0.09375)(0.3 + j0.15)$$

$$\mathbf{V}_1 = 120.02 + j0.0469$$

$$\mathbf{S}_1 = 10 + j\frac{10}{0.9}\sin(\cos^{-1}(0.9)) = 10 + j4.843$$

But $\quad \mathbf{S}_1 = \mathbf{V}_1\,\mathbf{I}_1^*$

$$\mathbf{I}_1^* = \frac{\mathbf{S}_1}{\mathbf{V}_1} = \frac{11.111\angle 25.84°}{120.02\angle 0.02°}$$

$$\mathbf{I}_1 = 0.093\angle -25.82° = 0.0837 - j0.0405$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 0.2087 + j0.053$$

$$\mathbf{V}_s = \mathbf{V}_1 + \mathbf{I}\,(0.2 + j0.04)$$

$$\mathbf{V}_s = (120.02 + j0.0469) + (0.2087 + j0.053)(0.2 + j0.04)$$

$$\mathbf{V}_s = 120.06 + j0.0658$$

$$\mathbf{V}_s = \mathbf{120.06\angle 0.03°\ V}$$

Solution 11.63

Find $\mathbf{I}_o$ in the circuit of Fig. 11.82.

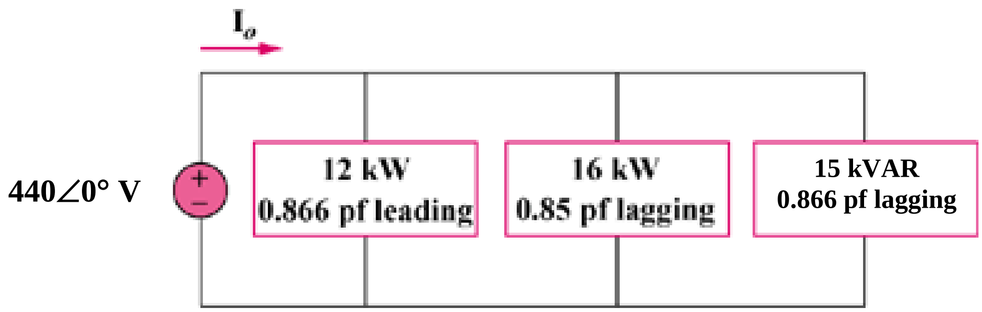


Figure 11.82
For Prob. 11.63.

Solution

Let $\quad \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$.

$\mathbf{S}_1 = \mathbf{P} + \mathbf{jQ} = 12\mathbf{k} - \mathbf{jQ_C}$ where $\tan\theta = Q_C/12k$ and $\theta = \cos^{-1}(0.866) = 30°$ and $Q_C = 12k(\tan(30°)) = 6.9282k$ so $\mathbf{S_1} = 12k - j6.9282$.

$$\mathbf{S}_2 = 16 + j\frac{16}{0.85}\sin(\cos^{-1}(0.85)) = 16 + j9.916$$

$$\mathbf{S}_3 = \frac{(15)(0.866)}{\sin(\cos^{-1}(0.866))} + j15 = 25.98 + j15$$

$$\mathbf{S} = 53.98 + j17.987 = \mathbf{V}\,\mathbf{I}_o^* = 56.898\angle 18.43°\text{ kVA}$$

$$\mathbf{I}_o^* = \frac{\mathbf{S}}{\mathbf{V}} = \frac{(53.98 + j17.987)x10^3}{440} = (56.898\angle 18.43°\text{ kVA})/(440\text{ V})$$

$$\mathbf{I}_o = \mathbf{129.31\angle 18.43^\circ\ A}$$

Solution 1l.64

Determine I_s in the circuit shown in Fig. 11.83, if the voltage source supplies 6 kW and 1.2 kVAR (leading).

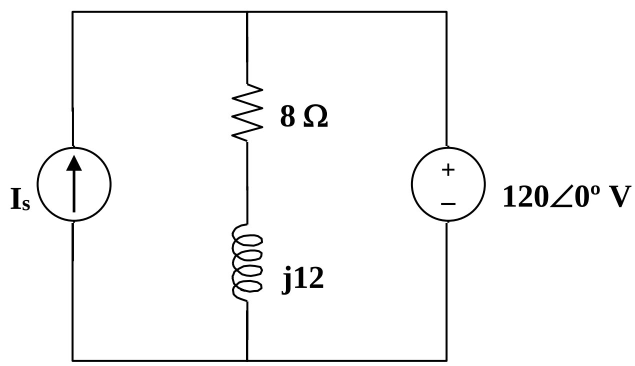
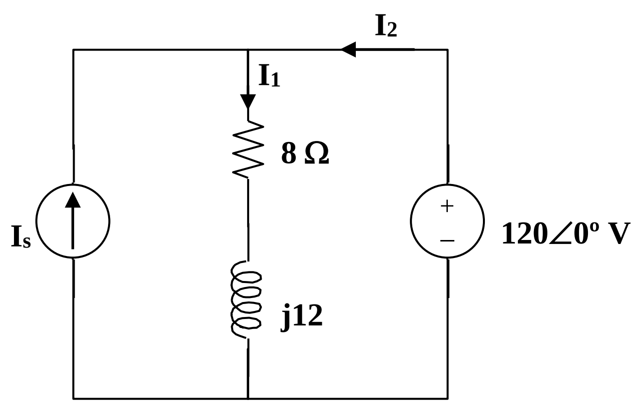


Figure 11.83
For Prob. 11.64.

Solution



$\mathbf{I_s} + \mathbf{I_2} = \mathbf{I_1}$ or $\mathbf{I_s} = \mathbf{I_1} - \mathbf{I_2}$

$$I_1 = \frac{120}{8 + j12} = 4.615 - j6.923$$

But, $$S = VI_2^* \longrightarrow I_2^* = \frac{S}{V} = \frac{6{,}000 - j1{,}200}{120} = 50 - j10$$

$$or\ I_2 = 50 + j10$$

$$\mathbf{I_s} = \mathbf{I_1} - \mathbf{I_2} = -45.385 - j16.923 = \mathbf{48.44\angle{-159.55^\circ}\ A}.$$

Solution 11.65

$$C = 1\,\text{nF} \;\longrightarrow\; \frac{1}{j\omega C} = \frac{-j}{10^4 \times 10^{-9}} = -j100\ \text{k}\Omega$$

At the noninverting terminal,

$$\frac{4\angle 0° - \mathbf{V}_o}{100} = \frac{\mathbf{V}_o}{-j100} \;\longrightarrow\; \mathbf{V}_o = \frac{4}{1+j}$$

$$\mathbf{V}_o = \frac{4}{\sqrt{2}}\angle -45°$$

$$v_o(t) = \frac{4}{\sqrt{2}}\cos(10^4 t - 45°)$$

$$P = \frac{V_{rms}^2}{R} = \left(\frac{4}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\right)^2\left(\frac{1}{50\times 10^3}\right)\text{W}$$

$$P = \mathbf{80\ \mu W}$$

Solution 11.66

Obtain the average power absorbed by the 10-Ω resistor in the op amp circuit in Fig. 11.85.

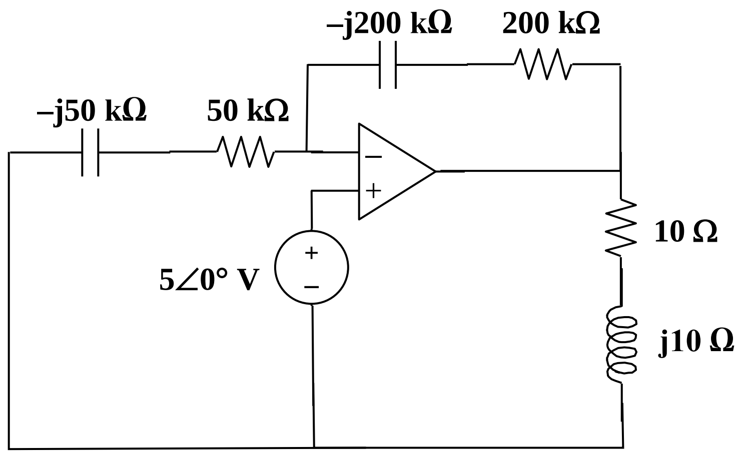
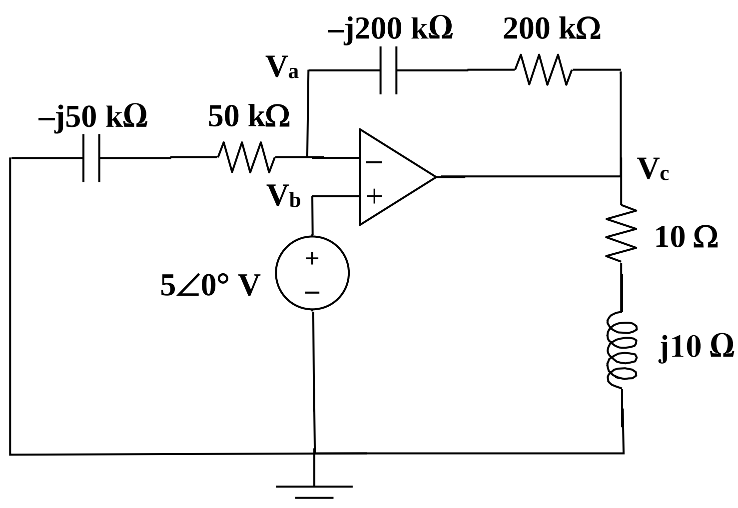


Figure 11.85
For Prob. 11.66.

Solution

Step 1. First we identify a reference node and then label the unknown nodes. Finally we write the node equations and use the constraint equation to solve for the unknown nodes.



$[(\mathbf{V_a}-0)/(50\text{k}-j50\text{k})] + [(\mathbf{V_a}-\mathbf{V_c})/(200\text{k}-j200\text{k})] + 0 = 0$ and $\mathbf{V_a} = \mathbf{V_b} = 5$ V. This allows us to solve for $\mathbf{V_c}$. The current through the 10 Ω resistor = $\mathbf{V_c}/(10+j10)$.

Step 2. $\mathbf{V_c}/(200k–j200k) = [5/(50k–j50k)] + [5/(200k–j200k)] = 25/(200k–j200k)$
or

$\mathbf{V_c} = 25$ V and $|\mathbf{I_{10}}| = |25/(14.142\angle 45°)| = 1.76778$ and

$$P_{10} = (1.76778)^2 10 = \mathbf{31.25\ W}.$$

Solution 11.67

For the op amp shown in Fig. 11.86, calculate:

(a) the complex power delivered by the voltage source
(b) the average power dissipated by the 10-Ω resistor

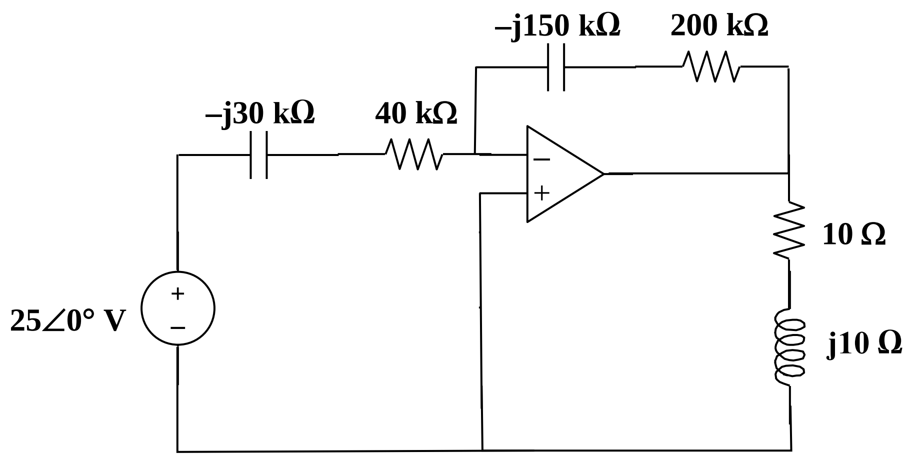
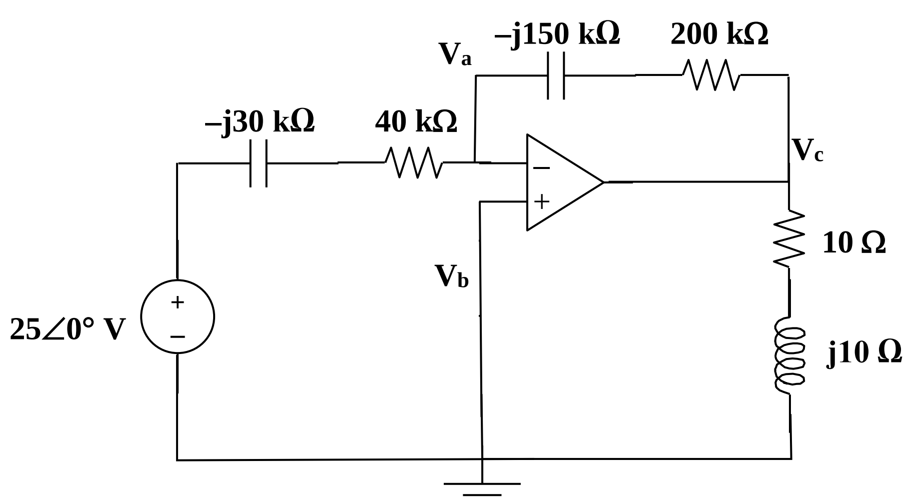


Figure 11.86
For Prob. 11.67.

Solution

Step 1. First we establish a reference node and then identify the unknown nodes. Next we write node equations and apply the constraint equation.



$[(\mathbf{V_a}–25)/(40k–j30k)] + [(\mathbf{V_a}–\mathbf{V_c})/(200k–j150k)] + 0 = 0$ and $\mathbf{V_a} = \mathbf{V_b} = 0$.
Finally,
$\mathbf{I_{10}} = (\mathbf{V_c}–0)/(10+j10)$. For the complex power delivered by the source,

$\mathbf{I_s}$ = 25/(40k–j30k) and $\mathbf{S_{delivered}}$ = 25$(\mathbf{I_s})^*$.

Step 2. $\mathbf{V_c}$/(200k–j150k) = –25/(40k–j30k) or $\mathbf{V_c}$ = –125 V and $\mathbf{I_{10}}$ = –125/(14.142∠45°)
= 8.8389∠135° A and $\mathbf{I_s}$ = 25/(50k∠–36.87°) = 0.5∠36.87° mA.

$\mathbf{S_{delivered}}$ = 25(0.5∠–36.87°) = **12.5∠–36.87° mVA** and
$P_{avg} = (8.8389)^2 10$ = **78.13 W**.

Solution 11.68

Let $\mathbf{S} = \mathbf{S}_R + \mathbf{S}_L + \mathbf{S}_c$

where

$$\mathbf{S}_R = P_R + jQ_R = \frac{1}{2} I_o^2 R + j0$$

$$\mathbf{S}_L = P_L + jQ_L = 0 + j\frac{1}{2} I_o^2 \omega L$$

$$\mathbf{S}_c = P_c + jQ_c = 0 - j\frac{1}{2} I_o^2 \cdot \frac{1}{\omega C}$$

Hence,

$$\mathbf{S} = \frac{\mathbf{1}}{\mathbf{2}} \mathbf{I}_o^2 \left[\mathbf{R} + \mathbf{j}\left(\boldsymbol{\omega} \mathbf{L} - \frac{\mathbf{1}}{\boldsymbol{\omega} \mathbf{C}} \right) \right]$$

Solution 11.69

Refer to the circuit shown in Fig. 11.88.

(a) What is the power factor?
(b) What is the average power dissipated?
(c) What is the value of the capacitance that will give a unity power factor when connected to the load?

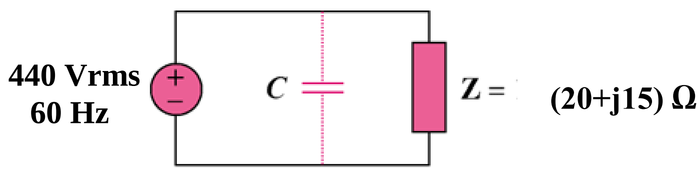


Figure 11.88
For Prob. 11.69.

Solution

(a) Given that $\mathbf{Z} = 20 + j15$

$$\tan\theta = \frac{15}{20} \longrightarrow \theta = 36.87°$$

$$\text{pf} = \cos\theta = \mathbf{0.8}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(440)^2}{25\angle -36.87°} = 6{,}195.2 + j4{,}646.4$$

The average power absorbed $= \text{P} = \text{Re}(\mathbf{S}) = \mathbf{6.195\ kW}$

(c) For unity power factor, $\theta_1 = 0°$, which implies that the reactive power due to the capacitor is $Q_C = 4.6464$ kVAR

But $$Q_c = \frac{V^2}{X_c} = \omega C V^2$$

$$C = \frac{Q_c}{\omega V^2} = \frac{(4{,}646.4)}{(2\pi)(60)(440)^2} = \mathbf{63.66\ \mu F}$$

Solution 11.70

Design a problem to help other students to better understand power factor correction.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

An 880-VA, 220-V, 50-Hz load has a power factor of 0.8 lagging. What value of parallel capacitance will correct the load power factor to unity?

Solution

$$\text{pf} = \cos\theta = 0.8 \longrightarrow \quad \sin\theta = 0.6$$

$$Q = S\sin\theta = (880)(0.6) = 528$$

If the power factor is to be unity, the reactive power due to the capacitor is

$$Q_c = Q = 528 \text{ VAR}$$

$$\text{But} \quad Q = \frac{V_{rms}^2}{X_c} = \omega C V^2 \longrightarrow C = \frac{Q_c}{\omega V^2}$$

$$C = \frac{(528)}{(2\pi)(50)(220)^2} = \mathbf{34.72\ \mu F}$$

Solution 11.71

(a) For load 1,

Q_1 = 60 kVAR, pf = 0.85 or $\theta_1 = 31.79^\circ$

$Q_1 = S_1 \sin\theta_1$ = 60k or S_1 = 113.89k and P_1 = 113.89cos(31.79) = 96.8kW

$\mathbf{S}_1$ = 96.8 + j60 kVA

For load 2, $\mathbf{S}_2$ = 90 – j50 kVA

For load 3, $\mathbf{S}_3$ =100 kVA

Hence,

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} + \mathbf{S_3} = 286.8 + j10\text{kVA} = 287\angle 2^\circ\text{kVA}$

But $\mathbf{S} = (V_{rms})^2/Z^*$ or $Z^* = 120^2/287\angle 2^\circ\text{k} = 0.05017\angle{-2^\circ}$

Thus, $Z = \mathbf{0.05017\angle 2^\circ\Omega}$ or **[50.14 + j1.7509] mΩ**.

(b) From above, pf = cos2° = **0.9994**.

(c) $I_{rms} = V_{rms}/Z = 120/0.05017\angle 2^\circ = \mathbf{2.392\angle{-2^\circ}\ kA}$ or **[2.391 – j0.08348] kA**.

Solution 11.72

(a) $P = S\cos\theta_1 \quad \longrightarrow \quad S = \dfrac{P}{\cos\theta_1} = \dfrac{2.4}{0.8} = 3.0\text{ kVA}$

$pf = 0.8 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 36.87^o$

$Q = S\sin\theta_1 = 3.0\sin 36.87^o = 1.8\text{ kVAR}$

Hence, S = 2.4 + j1.8 kVA

$S_1 = \dfrac{P_1}{\cos\theta} = \dfrac{1.5}{0.707} = 2.122\text{ kVA}$

$pf = 0.707 = \cos\theta \quad \longrightarrow \quad \theta = 45^o$

$Q_1 = P_1 = 1.5\text{ kVAR} \quad \longrightarrow \quad \mathrm{S}_1 = 1.5 + j1.5\text{ kVA}$

Since, $S = S_1 + S_2 \quad \longrightarrow \quad S_2 = S - S_1 = (2.4 + j1.8) - (1.5 + j1.5) = 0.9 + j0.3\text{ kVA}$

$S_2 = 0.9497 < 18.43^o$

pf = cos 18.43^o = <u>0.9487</u>

(b) $pf = 0.9 = \cos\theta_2 \quad \longrightarrow \quad \theta_2 = 25.84^o$

$$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{2400(\tan 36.87 - \tan 25.84)}{2\pi x 60 x (120)^2} = \underline{117.5\ \mu\text{F}}$$

Solution 11.73

(a) $\mathbf{S} = 10 - j15 + j22 = 10 + j7 \text{ kVA}$

$S = |\mathbf{S}| = \sqrt{10^2 + 7^2} = \mathbf{12.21\ kVA}$

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* \longrightarrow \mathbf{I}^* = \dfrac{\mathbf{S}}{\mathbf{V}} = \dfrac{10{,}000 + j7{,}000}{240}$

$\mathbf{I} = 41.667 - j29.167 = \mathbf{50.86\angle -35^\circ\ A}$

(c) $\theta_1 = \tan^{-1}\left(\dfrac{7}{10}\right) = 35^\circ,\qquad \theta_2 = \cos^{-1}(0.96) = 16.26^\circ$

$Q_c = P_1[\tan\theta_1 - \tan\theta_2] = 10[\tan(35^\circ) - \tan(16.26^\circ)]$
$Q_c = \mathbf{4.083\ kVAR}$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{4083}{(2\pi)(60)(240)^2} = \mathbf{188.03\ \mu F}$$

(d) $\mathbf{S}_2 = P_2 + jQ_2,\ P_2 = P_1 = 10 \text{ kW}$

$Q_2 = Q_1 - Q_c = 7 - 4.083 = 2.917 \text{ kVAR}$

$\mathbf{S}_2 = 10 + j2.917 \text{ kVA}$
But $\mathbf{S}_2 = \mathbf{V}\mathbf{I}_2^*$

$$\mathbf{I}_2^* = \frac{\mathbf{S}_2}{\mathbf{V}} = \frac{10{,}000 + j2917}{240}$$
$\mathbf{I}_2 = 41.667 - j12.154 = \mathbf{43.4\angle -16.26^\circ\ A}$

Solution 11.74

(a) $\theta_1 = \cos^{-1}(0.8) = 36.87°$

$$S_1 = \frac{P_1}{\cos\theta_1} = \frac{24}{0.8} = 30 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = (30)(0.6) = 18 \text{ kVAR}$$

$$\mathbf{S}_1 = 24 + j18 \text{ kVA}$$

$$\theta_2 = \cos^{-1}(0.95) = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{40}{0.95} = 42.105 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 13.144 \text{ kVAR}$$

$$\mathbf{S}_2 = 40 + j13.144 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144 \text{ kVA}$$

$$\theta = \tan^{-1}\left(\frac{31.144}{64}\right) = 25.95°$$

$$\text{pf} = \cos\theta = \mathbf{0.8992}$$

(b) $\theta_2 = 25.95°$, $\theta_1 = 0°$

$$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95°) - 0] = 31.144 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{31{,}144}{(2\pi)(60)(120)^2} = \mathbf{5.74 \text{ mF}}$$

Solution 11.75

Consider the power system shown in Fig. 11.90. Calculate:

(a) the total complex power
(b) the power factor
(c) the parallel capacitance necessary to establish a unity power factor

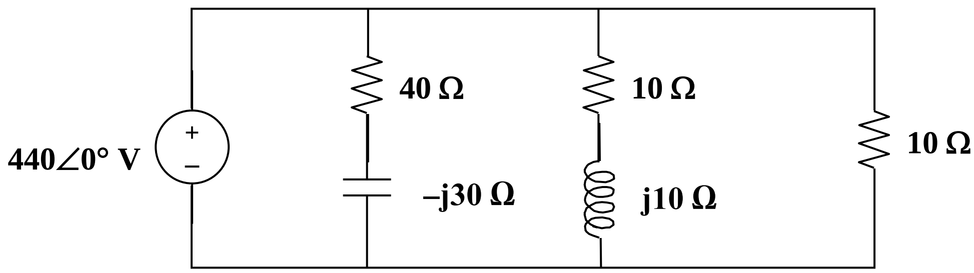


Figure 11.90
For Prob. 11.75.

Solution

Step 1. $S_1 = (440)^2/(40–j30)^*$, $S_2 = (440)^2/(10+j10)^*$, and $S_3 = (440)^2/10$.
$S_{Tot} = S_1 + S_2 + S_3 = P_{Tot} + jQ_{Tot}$. Now pf = $\cos[\text{Tan}^{-1}(Q_{Tot}/P_{Tot})]$.
Finally $Q_{Tot} = Q_C = (440)^2/X_C$ and $X_C = 1/(377C)$ assuming 60 Hz.

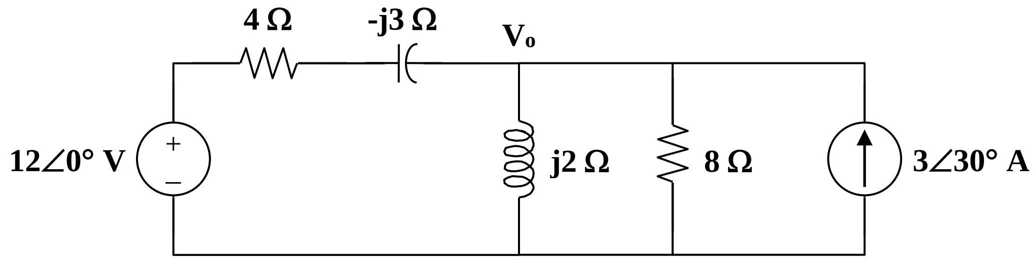
(a) $S_1 = 193{,}600/(50\angle 36.87°) = 3{,}872\angle –36.87° = (3{,}097.6–j2{,}323.2)$ VA, $S_2 = 193{,}600/(14.142\angle –45° = 13{,}689.7\angle 45° = (9{,}680+j9{,}680)$ VA, and $S_3 = 193{,}600/10 = 19{,}3600$. $S_{Tot} = [(3.0976+9.68+19.36)+j(–2.3232+9.68)]$ kVA = **32.14 kW + j7.357 kVAR**.

(b) pf = $\cos[\text{Tan}^{-1}(7.357/32.14)] = \cos(12.893°) =$ **0.9748**

(c) $Q_C = 7{,}357 = 193{,}600/X_C$ and $X_C = 193{,}600/7{,}357 = 26.315 = 1/(377C)$ or $C =$ **100.8 μF**.

Solution 11.76

The wattmeter reads the real power supplied by the current source. Consider the circuit below.



$$3\angle 30^\circ + \frac{12 - \mathbf{V}_o}{4 - j3} = \frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o}{8}$$

$$\mathbf{V}_o = \frac{36.14 + j23.52}{2.28 - j3.04} = 0.7547 + j11.322 = 11.347\angle 86.19^\circ$$

$$\mathbf{S} = \mathbf{V}_o\,\mathbf{I}_o^* = (11.347\angle 86.19^\circ)(3\angle -30^\circ)$$
$$\mathbf{S} = 34.04\angle 56.19^\circ \text{ VA}$$

$$\text{P} = \text{Re}(\mathbf{S}) = \mathbf{18.942\ W}$$

Solution 11.77

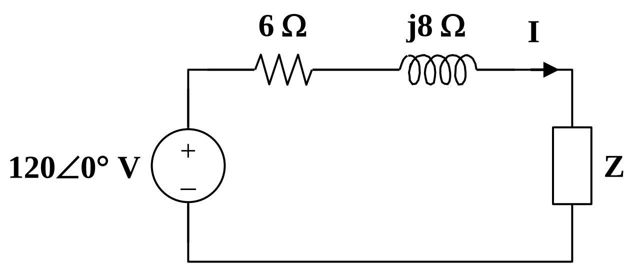
The wattmeter measures the power absorbed by the parallel combination of 0.1 F and 150 Ω.

$$120\cos(2t) \longrightarrow 120\angle 0°, \qquad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j8$$

$$0.1\text{ F} \longrightarrow \frac{1}{j\omega C} = \text{-}j5$$

Consider the following circuit.



$$\mathbf{Z} = 15 \,\|\, (\text{-}j5) = \frac{(15)(\text{-}j5)}{15 - j5} = 1.5 - j4.5$$

$$\mathbf{I} = \frac{120}{(6 + j8) + (1.5 - j4.5)} = 14.5\angle\text{-}25.02°$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}\left|\mathbf{I}\right|^2 \mathbf{Z} = \frac{1}{2}\cdot(14.5)^2(1.5 - j4.5)$$
$$\mathbf{S} = 157.69 - j473.06 \text{ VA}$$

The wattmeter reads

$$P = \text{Re}(\mathbf{S}) = \mathbf{157.69\ W}$$

Solution 11.78

Find the wattmeter reading of the circuit shown in Fig. 11.93 below.

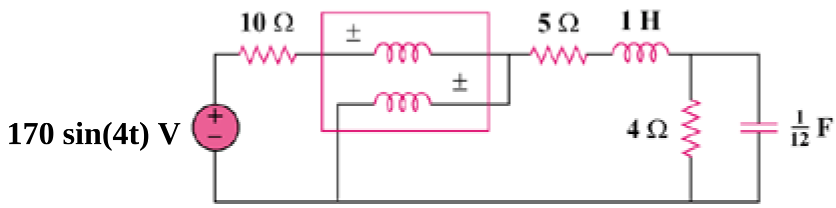


Figure 11.93
For Prob. 11.78.

Solution

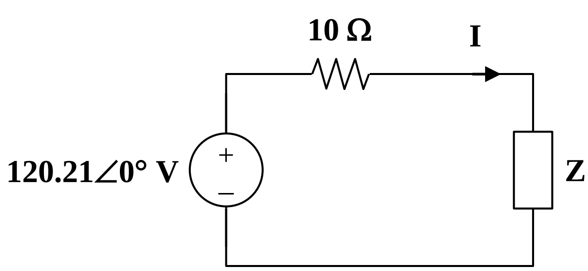
The wattmeter reads the power absorbed by the element to its right side.

$$170\sin(4t) \longrightarrow 120.21\angle\mathbf{0}°, \qquad \omega = 4$$

$$1\text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = -j3$$

Consider the following circuit.



$$\mathbf{Z} = 5 + j4 + 4 \,||\, -j3 = 5 + j4 + \frac{(4)(-j3)}{4 - j3}$$

$$\mathbf{Z} = 6.44 + j2.08$$

$$\mathbf{I} = \frac{120.21}{6.7676\angle 17.9°} = 17.7626\angle -17.9°$$

$$\mathbf{S} = |\mathbf{I}|^2\,\mathbf{Z} = (17.7626)^2(6.44 + j2.08) = (2{,}032 + j656.3)\text{VA}$$

$$P = \text{Re}(\mathbf{S}) = \mathbf{2.032\ kW}$$

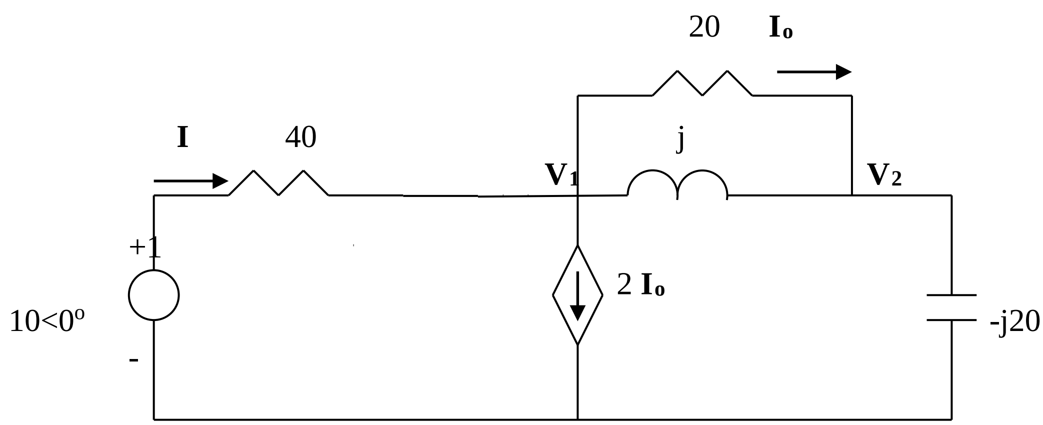
Solution 11.79

The wattmeter reads the power supplied by the source and partly absorbed by the 40-Ω resistor.

$\omega = 100,$

$$10\,\text{mH} \longrightarrow j100x10x10^{-3} = j, \quad 500\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j100x500x10^{-6}} = -j20$$

The frequency-domain circuit is shown below.



At node 1,

$$\frac{10 - V_1}{40} = 2I_o + \frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{3(V_1 - V_2)}{20} + \frac{V_1 - V_2}{j} \longrightarrow \qquad (1)$$

$$10 = (7 - j40)V_1 + (-6 + j40)V_2$$

At node 2,

$$\frac{V_1 - V_2}{j} + \frac{V_1 - V_2}{20} = \frac{V_2}{-j20} \longrightarrow 0 = (20 + j)V_1 - (19 + j)V_2 \qquad (2)$$

Solving (1) and (2) yields $V_1 = 1.5568 - j4.1405$

$$I = \frac{10 - V_1}{40} = 0.2111 + j0.1035, \quad S = \frac{1}{2}V_1 I^* = -0.04993 - j0.5176$$

$$P = \text{Re}(S) = \mathbf{50\ mW}.$$

Solution 11.80

The circuit of Fig. 11.95 portrays a wattmeter connected into an ac network.

(a) Find the load current.

(b) Calculate the wattmeter reading.

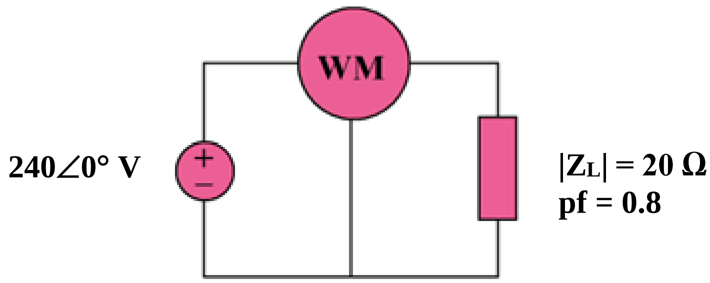


Figure 11.95
For Prob. 11.80.

Solution

(a) $$|\mathbf{I}| = \frac{|\mathbf{V}|}{|\mathbf{Z}|} = \frac{240}{20} = \mathbf{12\ A}$$

(b) $$|\mathbf{S}| = \frac{|\mathbf{V}|^2}{|\mathbf{Z}|} = \frac{(240)^2}{20} = 2{,}880\text{ VA}$$

P = S cos(θ) = 2,880(0.8) = **2.304 kW**.

Solution 11.81

Design a problem to help other students to better understand how to correct power factor to values other than unity.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A 120-V rms, 60-Hz electric hair dryer consumes 600 W at a lagging pf of 0.92. Calculate the rms-valued current drawn by the dryer.

How would you power factor correct this to a value of 0.95?

Solution

P = 600 W, $pf = 0.92 \quad \longrightarrow \quad \theta = 23.074^o$

$$P = S\cos\theta \quad \longrightarrow \quad S = \frac{P}{0.92} = 652.17 \text{ VA}$$

S = P+jQ = 600 + j652.17sin23.09° = 600 +j255.6

But $S = V_{rms} I_{rms}^*$.

$$I_{rms}^* = \frac{S}{V_{rms}} = \frac{600 + j255.6}{120}$$

$$I_{rms} = 5 - j2.13 = \mathbf{5.435\angle{-23.07}^\circ\ A}.$$

To correct this to a pf = 0.95, I would add a capacitor in parallel with the hair dryer (remember, series compensation will increase the power delivered to the load and probably burn out the hair dryer.

pf = 0.95 = 600/S or S = 631.6 VA and θ = 18.19˚ and VARs = 197.17

Thus,

$VARs_{cap}$ = 255.6 – 197.17 = 58.43 = 120xI_C or I_C = 58.43/120 = 0.4869A

Next,

X_C = 120/0.4869 = 246.46 = 1/(377xC) or C = **10.762 μF**

Solution 11.82

(a) $P_1 = 5{,}000, \qquad Q_1 = 0$

$$P_2 = 30{,}000x0.82 = 24{,}600, \qquad Q_2 = 30{,}000\sin(\cos^{-1} 0.82) = 17{,}171$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = 29{,}600 + j17{,}171$$

$$\boldsymbol{S = |\overline{S}| = \underline{34.22\,\text{kV}}A}$$

(b) $\qquad$ Q = **17.171 kVAR**

(c) $pf = \dfrac{P}{S} = \dfrac{29{,}600}{34{,}220} = 0.865$

$$\boldsymbol{Q_c = P(\tan\theta_1 - \tan\theta_2)}$$
$$\boldsymbol{= 29{,}600\left[\tan(\cos^{-1} 0.865) - \tan(\cos^{-1} 0.9)\right] = \underline{2833\ \text{VAR}}}$$

(c) $$\boldsymbol{C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{2833}{2\pi x 60 x 240^2} = \underline{130.46\mu\,\text{F}}}$$

Solution 11.83

(a) $\overline{S} = \frac{1}{2}VI^* = \frac{1}{2}(210\angle 60^o)(8\angle -25^o) = 840\angle 35^o$

$$\boldsymbol{P = S\cos\theta = 840\cos 35^o = \underline{688.1\ \text{W}}}$$

(b) S = **<u>840 VA</u>**

(c) $\boldsymbol{Q = S\sin\theta = 840\sin 35^o = \underline{481.8\ \text{VAR}}}$

(d) $\boldsymbol{pf = P/S = \cos 35^o = \underline{0.8191\ \text{(lagging)}}}$

Solution 11.84

(a) Maximum demand charge $= 2{,}400 \times 30 = \$72{,}000$

Energy cost $= \$0.04 \times 1{,}200 \times 10^3 = \$48{,}000$

Total charge = **\$120,000**

(b) To obtain \$120,000 from 1,200 MWh will require a flat rate of

$\frac{\$120{,}000}{1{,}200 \times 10^3}$ per kWh = **\$0.10 per kWh**

Solution 11.85

A regular household system of a single-phase three-wire allows the operation of both 120-V and 240-V, 60-Hz appliances. The household circuit is modeled as shown in Fig. 11.96. Calculate: (a) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_n$, (b) the total complex power supplied, (c) the overall power factor of the circuit.

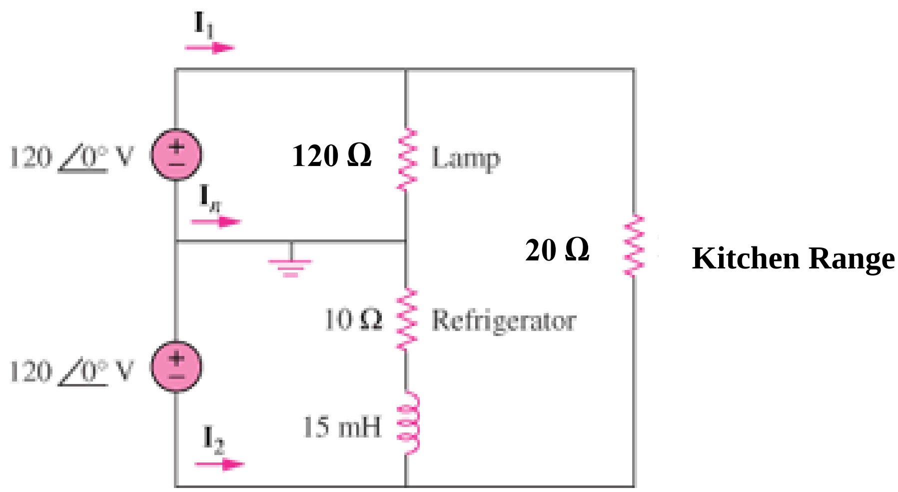


Figure 11.96
For Prob. 11.85.

Solution

(a) 15 mH $\longrightarrow$ $j2\pi x60x15x10^{-3} = j5.655$

Since we know the voltage across each device we can calculate the current through each one. $\mathbf{I}_{\mathbf{Lamp}}$ = 120/120 = 1 A, $\mathbf{I}_{\mathbf{Ref}}$ = 120/(10+j5.655), and $\mathbf{I}_{\mathbf{range}}$ = 240/20 = 12 A. KCL gives us $\mathbf{I}_1 = \mathbf{I}_{\mathbf{Lamp}} + \mathbf{I}_{\mathbf{range}}$ = **13 A**, $\mathbf{I}_n = \mathbf{I}_{\mathbf{Ref}} - \mathbf{I}_{\mathbf{Lamp}}$, and $\mathbf{I}_2 = -\mathbf{I}_{\mathbf{Ref}} - \mathbf{I}_{\mathbf{range}}$. $\mathbf{I}_{\mathbf{Ref}}$ = 120/(11.4882∠29.488°)
= (10.4455∠–29.488°) A = (9.0924 – j5.1417) A. Thus,
$\mathbf{I}_n$ = 9.0924 – 1 – j5.1417 = **(8.092 – j5.142) A** = **9.588∠–32.43° A** and
$\mathbf{I}_2$ = –9.0924 – 12 + j5.1417 = **(–21.09 + j5,142) A** = **21.71∠166.3° A**.

(b) The complex power delivered by each source is given by $\mathbf{S}_1 = 120(I_1)^*$ and $\mathbf{S}_2 = 120(-I_2)^*$ and $\mathbf{S}_{\mathbf{Tot}} = \mathbf{S}_1 + \mathbf{S}_2$. $\mathbf{S}_1$ = 120(13) = 1.56 kW and
$\mathbf{S}_2 = 120(21.71\angle{-13.7°})^*$ = 2.6052∠13.7° kVA = 2.531 kW + j617 VAR, thus, **S** = **4.091 kW + j0.617 kVAR**.

(c) Finally, pf = P/|S| = 4.091/4.1373 = **0.9888 (lagging)**.

Solution 11.86

For maximum power transfer

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* \longrightarrow \mathbf{Z}_i = \mathbf{Z}_{Th} = \mathbf{Z}_L^*$$

$$\mathbf{Z}_L = R + j\omega L = 75 + j(2\pi)(4.12\times10^6)(4\times10^{-6})$$
$$\mathbf{Z}_L = 75 + j103.55\,\Omega$$

$$\mathbf{Z}_i = \mathbf{[75 - \mathit{j}103.55]\,\Omega}$$

Solution 11.87

$$\mathbf{Z} = R \pm jX$$

$$\mathbf{V}_R = \mathbf{I} R \quad \longrightarrow \quad R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{80}{50 \times 10^{-3}} = 1.6 \text{ k}\Omega$$

$$|\mathbf{Z}|^2 = R^2 + X^2 \quad \longrightarrow \quad X^2 = |\mathbf{Z}|^2 - R^2 = (3)^2 - (1.6)^2$$

$$X = 2.5377 \text{ k}\Omega$$

$$\theta = \tan^{-1}\left(\frac{X}{R}\right) = \tan^{-1}\left(\frac{2.5377}{1.6}\right) = 57.77°$$

$$\text{pf} = \cos\theta = \mathbf{0.5333}$$

Solution 11.88

(a) $\mathbf{S} = (110)(2\angle 55°) = 220\angle 55°$

$P = S\cos\theta = 220\cos(55°) = \mathbf{126.2\ W}$

(b) $S = \left|\mathbf{S}\right| = \mathbf{220\ VA}$

Solution 11.89

(a) Apparent power $= S =$ **12 kVA**

$$P = S\cos\theta = (12)(0.78) = 9.36 \text{ kW}$$
$$Q = S\sin\theta = 12\sin(\cos^{-1}(0.78)) = 7.51 \text{ kVAR}$$

$$\mathbf{S} = P + jQ = \mathbf{[9.36 + \mathit{j}7.51]\ \mathit{kVA}}$$

(b) $$\mathbf{S} = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} \longrightarrow \mathbf{Z}^* = \frac{|\mathbf{V}|^2}{\mathbf{S}} = \frac{(210)^2}{(9.36 + j7.51)\times 10^3} = 2.866 - j2.3$$

$$\mathbf{Z} = \mathbf{[2.866 + j2.3]\ \Omega}$$

Solution 11.90

Original load :

$$P_1 = 2000 \text{ kW}, \qquad \cos\theta_1 = 0.85 \longrightarrow \theta_1 = 31.79°$$

$$S_1 = \frac{P_1}{\cos\theta_1} = 2352.94 \text{ kVA}$$

$$Q_1 = S_1 \sin\theta_1 = 1239.5 \text{ kVAR}$$

Additional load :

$$P_2 = 300 \text{ kW}, \qquad \cos\theta_2 = 0.8 \longrightarrow \theta_2 = 36.87°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 375 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = 225 \text{ kVAR}$$

Total load :

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = (P_1 + P_2) + j(Q_1 + Q_2) = P + jQ$$

$$P = 2000 + 300 = 2300 \text{ kW}$$

$$Q = 1239.5 + 225 = 1464.5 \text{ kVAR}$$

The minimum operating pf for a 2300 kW load and not exceeding the kVA rating of the generator is

$$\cos\theta = \frac{P}{S_1} = \frac{2300}{2352.94} = 0.9775$$

or $\theta = 12.177°$

The maximum load kVAR for this condition is

$$Q_m = S_1 \sin\theta = 2352.94 \sin(12.177°)$$

$$Q_m = 496.313 \text{ kVAR}$$

The capacitor must supply the difference between the total load kVAR (i.e. Q) and the permissible generator kVAR (i.e. Q_m). Thus,

$$Q_c = Q - Q_m = \mathbf{968.2 \text{ kVAR}}$$

Solution 11.91

The nameplate of an electric motor has the following information:

Line voltage: 220 V rms
Line current: 15 A rms
Line frequency: 60 Hz
Power: 2700 W

Determine the power factor (lagging) of the motor. Find the value of the capacitance *C* that must be connected across the motor to raise the pf to unity.

Solution

I = V/Z which leads to $Z = [220/15]\angle\theta = 14.6667\angle\theta$, $S = (220)(15)\angle\theta = 3.3\angle\theta$ kVA, where $\cos^{-1}(2700/3300) = \cos^{-1}(0.818182) = 35.097°$, and $X_L = 3300\sin(35.097°) = 1897.38 = X_C$. This leads to $C = 1/[377(1897.38)] =$ **1.398 μF**.

pf = **0.8182** (lagging)

C = **1.398 μF**

0.8182 (lagging), **1.398 μF**

Solution 11.92

As shown in Fig. 11.97, a 550-V feeder line supplies an industrial plant consisting of a motor drawing 90 kW at 0.8 pf (inductive), a capacitor with a rating of 20 kVAR, and lighting drawing 10 kW.

(a) Calculate the total reactive power and apparent power absorbed by the plant.

(b) Determine the overall pf.

(c) Find the magnitude of the current in the feeder line.

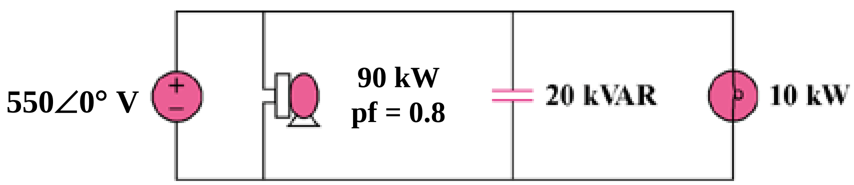


Figure 11.97
For Prob. 11.92.

Solution

(a) Apparent power drawn by the motor is

$$S_m = \frac{P}{\cos\theta} = \frac{90}{0.8} = 112.5\ kVA$$

$$Q_m = \sqrt{S^2 - P^2} = \sqrt{(112.5)^2 - (90)^2} = 67.5\ kVAR$$

Total real power

$$P = P_m + P_c + P_L = 90 + 0 + 10 = 100\ kW$$

Total reactive power

$$Q = Q_m + Q_c + Q_L = 67.5 - 20 + 0 = \mathbf{47.5\ kVAR}$$

Total apparent power

$$\mathrm{S} = \sqrt{\mathrm{P}^2 + \mathrm{Q}^2} = \mathbf{110.71\ kVA}$$

(b) $$pf = \frac{P}{S} = \frac{100}{110.71} = \mathbf{0.9033}$$

(c) |**I**| = S/V = 110,710/550 = **201.3 A**.

Solution 11.93

(a) $P_1 = (5)(0.7457) = 3.7285 \text{ kW}$

$$S_1 = \frac{P_1}{pf} = \frac{3.7285}{0.8} = 4.661 \text{ kVA}$$

$$Q_1 = S_1 \sin(\cos^{-1}(0.8)) = 2.796 \text{ kVAR}$$

$$\mathbf{S}_1 = 3.7285 + j2.796 \text{ kVA}$$

$$P_2 = 1.2 \text{ kW}, \qquad Q_2 = 0 \text{ VAR}$$

$$\mathbf{S}_2 = 1.2 + j0 \text{ kVA}$$

$$P_3 = (10)(120) = 1.2 \text{ kW}, \qquad Q_3 = 0 \text{ VAR}$$

$$\mathbf{S}_3 = 1.2 + j0 \text{ kVA}$$

$$Q_4 = 1.6 \text{ kVAR}, \qquad \cos\theta_4 = 0.6 \longrightarrow \sin\theta_4 = 0.8$$

$$S_4 = \frac{Q_4}{\sin\theta_4} = 2 \text{ kVA}$$

$$P_4 = S_4 \cos\theta_4 = (2)(0.6) = 1.2 \text{ kW}$$

$$\mathbf{S}_4 = 1.2 - j1.6 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$

$$\mathbf{S} = 7.3285 + j1.196 \text{ kVA}$$

Total real power = **7.3285 kW**

Total reactive power = **1.196 kVAR**

(b) $$\theta = \tan^{-1}\left(\frac{1.196}{7.3285}\right) = 9.27^\circ$$

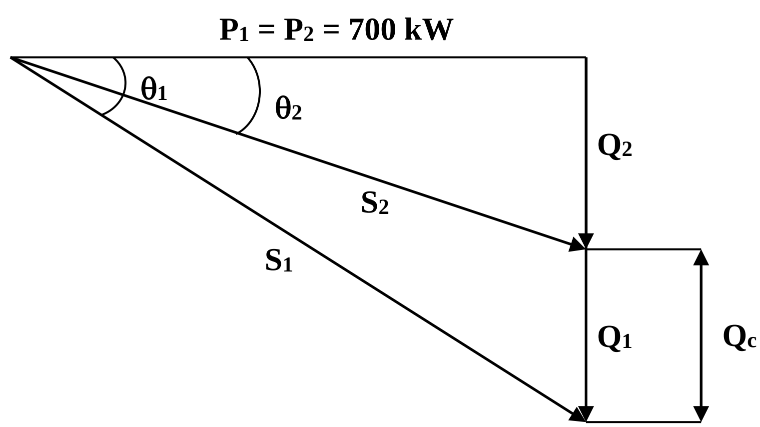
$$pf = \cos\theta = \mathbf{0.987}$$

Solution 11.94

$$\cos\theta_1 = 0.7 \longrightarrow \theta_1 = 45.57°$$
$$S_1 = 1\text{ MVA} = 1000\text{ kVA}$$
$$P_1 = S_1\cos\theta_1 = 700\text{ kW}$$
$$Q_1 = S_1\sin\theta_1 = 714.14\text{ kVAR}$$

For improved pf,

$$\cos\theta_2 = 0.95 \longrightarrow \theta_2 = 18.19°$$
$$P_2 = P_1 = 700\text{ kW}$$
$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{700}{0.95} = 736.84\text{ kVA}$$
$$Q_2 = S_2\sin\theta_2 = 230.08\text{ kVAR}$$



(a) Reactive power across the capacitor

$$Q_c = Q_1 - Q_2 = 714.14 - 230.08 = 484.06\text{ kVAR}$$

Cost of installing capacitors $= \$30 \times 484.06 =$ **$14,521.80**

(b) Substation capacity released $= S_1 - S_2 = 1000 - 736.84 = 263.16$ kVA

Saving in cost of substation and distribution facilities $= \$120 \times 263.16 =$ **$31,579.20**

(c) **Yes**, because (a) is greater than (b). Additional system capacity obtained by using capacitors costs only 46% as much as new substation and distribution facilities.

Solution 11.95

(a) Source impedance $\mathbf{Z}_s = R_s - jX_c$

Load impedance $\mathbf{Z}_L = R_L + jX_2$

For maximum load transfer

$$\mathbf{Z}_L = \mathbf{Z}_s^* \longrightarrow R_s = R_L, \quad X_c = X_L$$

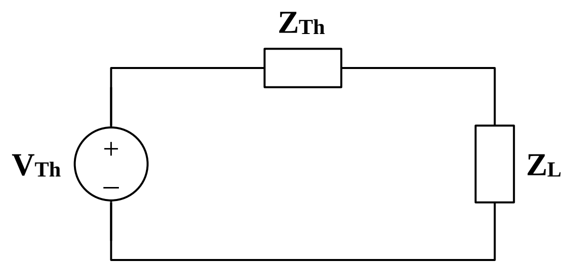
$$X_c = X_L \longrightarrow \frac{1}{\omega C} = \omega L$$

or $\quad \omega = \dfrac{1}{\sqrt{LC}} = 2\pi f$

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(80\times 10^{-3})(40\times 10^{-9})}} = \mathbf{2.814\ kHz}$$

(b) $$P = \left(\frac{V_S}{(10+4)}\right)^2 4 = \left(\frac{4.6}{14}\right)^2 4 = \mathbf{431.8\ mW} \quad \text{(since } V_s \text{ is in rms)}$$

Solution 11.96



(a) $V_{Th} = 146\text{ V}, \quad 300\text{ Hz}$

$Z_{Th} = 40 + j8\,\Omega$

$Z_L = Z_{Th}^* = \mathbf{[40 - \mathit{j}8]\,\Omega}$

(b) $$P = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(146)^2}{(8)(40)} = \mathbf{66.61\ W}$$

Solution 11.97

A power transmission system is modeled as shown in Fig. 11.99. If $\mathbf{V}_s = 440\angle 0°$ rms, find the average power absorbed by the load.

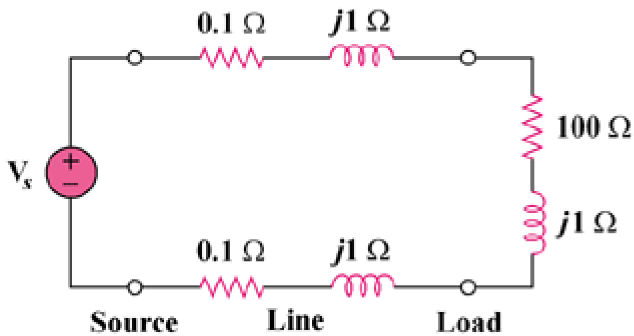


Figure 11.99
For Problem 11.97.

Solution

$$Z_T = (2)(0.1 + j) + (100 + j20) = 100.2 + j22\,\Omega$$

$$I = \frac{V_s}{Z_T} = \frac{440}{100.2 + j22}$$

$$P = |I|^2 R_L = 100|I|^2 = \frac{(100)(440)^2}{(100.2)^2 + (22)^2} = \frac{19{,}360{,}000}{10{,}040 + 484}$$

= **1.8396 kW**.