Q.1 Reduce the following Boolean expressions to the required number of literals.

(a) $ABC+\overline{A}\,\overline{B}C+\overline{A}BC+AB\overline{C}+\overline{A}\,\overline{B}\,\overline{C}$ to five literals

(a) $AB C + \overline{A}\,\overline{B}C + \overline{A}BC + AB\overline{C} + \overline{A}\,\overline{B}\,\overline{C}$

$= BC\cancel{(A+\overline{A})} + \overline{A}\,\overline{B}\cancel{(C+\overline{C})} + AB\overline{C}$

$= BC + \overline{A}\,\overline{B} + AB\overline{C}$

$= B(C + A\overline{C}) + \overline{A}\,\overline{B}$

$= B(C+A)\cancel{(C+\overline{C})} + \overline{A}\,\overline{B}$

$= B(A+C) + \overline{A}\,\overline{B}$

(b) $\overline{\left[(\overline{CD}) + A\right]} + A + CD + AB$ to three literals

(b) $\overline{\left((\overline{CD}) + A\right)} + A + CD + AB$

$= CD\overline{A} + A + CD + AB$

$= CD\cancel{(\overline{A}+1)} + A\cancel{(1+B)}$

$= CD + A$

(c) $(A + C + D)\,(A + C + \overline{D})\,(A + \overline{C} + D)\,(A + \overline{B})$ to four literals.

(c) $(A+C+D)(A+C+\overline{D})(A+\overline{C}+D)(A+\overline{B})$

$$= \left[(A+C)(A+C) + (A+C)\overline{D} + (A+C)D + \cancel{D\overline{D}}\right](A+\overline{C}+D)(A+\overline{B})$$

$$= (A+C)\underbrace{\left[(A+C) + \underbrace{\overline{D} + D}_{1}\right]}_{1}(A+\overline{C}+D)(A+\overline{B})$$

$$= (A+C)(A+\overline{C}+D)(A+\overline{B})$$

$$= (AA + A\overline{C} + AD + CA + \cancel{C\overline{C}} + CD)(A+\overline{B})$$

$$= \left[A\underbrace{\left[A + D + \underbrace{\overline{C} + C}_{1}\right]}_{1} + CD\right](A+\overline{B})$$

$$= (A + CD)(A + \overline{B}) \quad = \quad A + \overline{B}CD$$

(since $A + ACD + A\overline{B} = A$)

Q.2 For each of the problems in Q.1, draw up a truth table and show that the simplified expressions are equivalent to the original Boolean expressions.

(a)

A	B	C	ABC	$\bar{A}\bar{B}C$	$\bar{A}BC$	$AB\bar{C}$	$\bar{A}\bar{B}\bar{C}$	OUTPUT
0	0	0	0	0	0	0	1	1
0	0	1	0	1	0	0	0	1
0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	0	0	1
1	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	1	0	0	0	0	1	0	1
1	1	1	1	0	0	0	0	1

A	B	C	$B(A+C)$	$\bar{A}\bar{B}$	OUTPUT
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	1	0	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	1	0	1
1	1	1	1	0	1

Thus $B(A+C) + \bar{A}\bar{B}$

$= ABC + \bar{A}\bar{B}C + \bar{A}BC + AB\bar{C} + \bar{A}\bar{B}\bar{C}$

(b)

ABCD	$\overline{\overline{CD}+A}$	A	CD	AB	Z	CD	A	Z^*
0000	0	0	0	0	0	0	0	0
0001	0	0	0	0	0	0	0	0
0010	0	0	0	0	0	0	0	0
0011	1	0	1	0	1	1	0	1
0100	0	0	0	0	0	0	0	0
0101	0	0	0	0	0	0	0	0
0110	0	0	0	0	0	0	0	0
0111	1	0	1	0	1	1	0	1
1000	0	1	0	0	1	0	1	1
1001	0	1	0	0	1	0	1	1
1010	0	1	0	0	1	0	1	1
1011	0	1	1	0	1	1	1	1
1100	0	1	0	1	1	0	1	1
1101	0	1	0	1	1	0	1	1
1110	0	1	0	1	1	0	1	1
1111	0	1	1	1	1	1	1	1

$$Z = \overline{\overline{CD}+A} + A + CD + AB$$
$$Z^* = CD + A$$

By the truth table, $Z = Z^*$

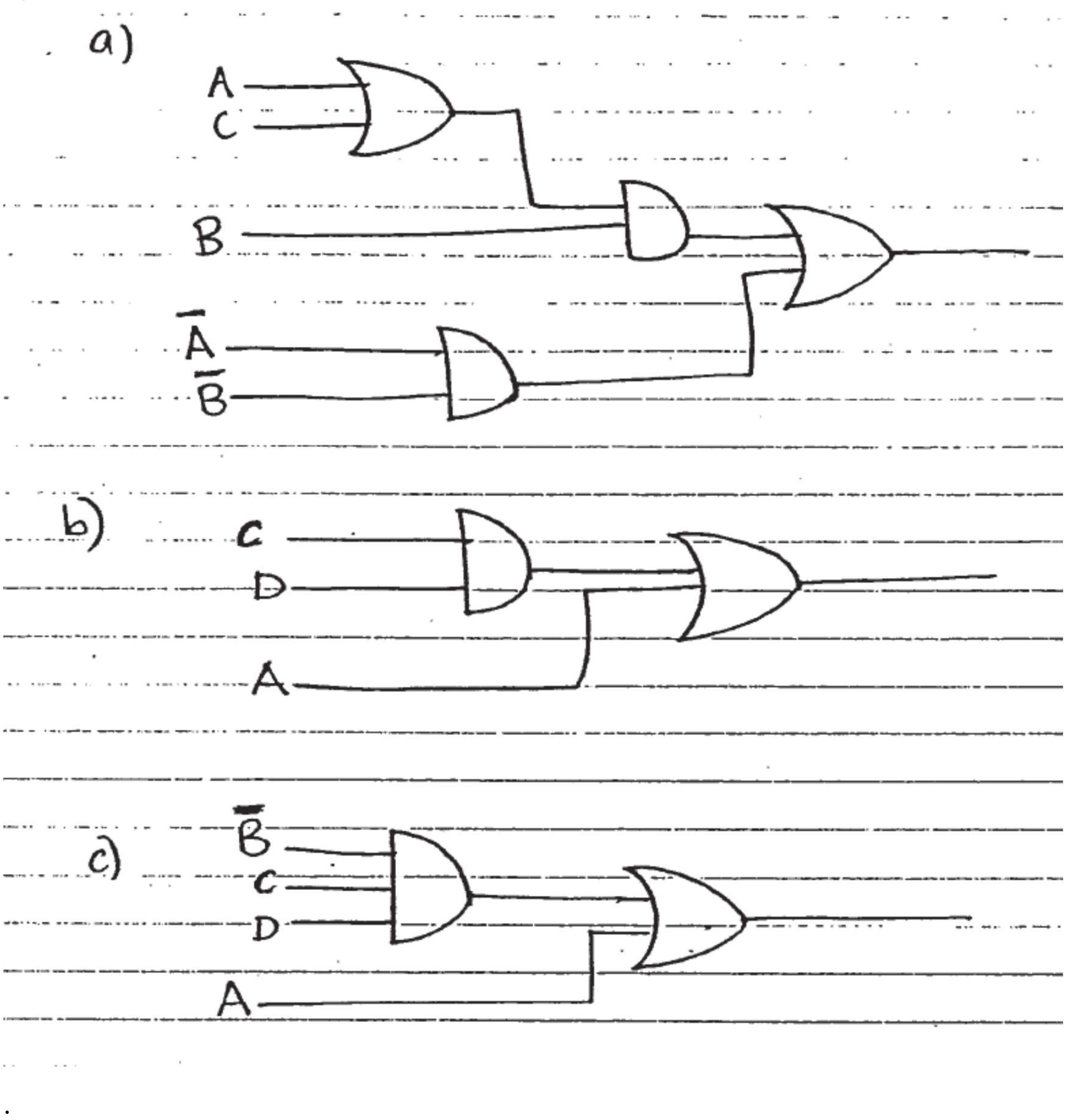
(c)

ABCD	$A+C+D$	$A+C+\overline{D}$	$A+\overline{C}+D$	$A+\overline{B}$	Z	A	$\overline{B}CD$	Z^*
0000	0	1	1	1	0	0	0	0
0001	1	0	1	1	0	0	0	0
0010	1	1	0	1	0	0	0	0
0011	1	1	1	1	1	0	1	1
0100	0	1	1	1	0	0	0	0
0101	1	0	1	0	0	0	0	0
0110	1	1	0	0	0	0	0	0
0111	1	1	1	0	0	0	0	0
1000	1	1	1	1	1	1	0	1
1001	1	1	1	1	1	1	0	1
1010	1	1	1	1	1	1	0	1
1011	1	1	1	1	1	1	1	1
1100	1	1	1	1	1	1	0	1
1101	1	1	1	1	1	1	0	1
1110	1	1	1	1	1	1	0	1
1111	1	1	1	1	1	1	1	1

$$Z = (A+C+D)(A+C+\overline{D})(A+\overline{C}+D)(A+\overline{B}), \quad Z^* = A + \overline{B}CD$$

Again $Z = Z^*$

Q.3 For each of the problems in Q.1, draw a logic circuit implementation of the simplified expressions using AND, OR and NOT gates.



Q4. Use Karnaugh maps to simplify the following Boolean function in:

(a) Sum-of-products, and
(b) Product-of-sums form.

$$F = \overline{W}\,(\overline{X}Y + \overline{X}\,\overline{Y} + XYZ) + \overline{X}\,\overline{Z}\,(Y + W)$$

$$\text{"don't care"} = \overline{W}X\,(\overline{Y}Z + Y\overline{Z}) + WYZ$$

$$F = \bar{W}(\bar{X}Y + \bar{X}\bar{Y} + XYZ) + \bar{X}\bar{Z}(Y + W)$$

$$d = \bar{W}X(\bar{Y}Z + Y\bar{Z}) + WYZ$$

(a)

WX \ YZ	00	01	11	10
00	1	1	1	1
01		X	1	X
11			X	
10	1		X	1

$\bar{W}Z$

$\bar{X}\bar{Z}$

$$F = \bar{X}\bar{Z} + \bar{W}Z$$

(b)

WX \ YZ	00	01	11	10
00				
01	0	X		X
11	0	0	X	0
10		0	X	

$(\bar{W} + \bar{Z})$

$(\bar{X} + Z)$

$$F = (\bar{W} + \bar{Z})(\bar{X} + Z)$$