常微与偏微课程作业

贾博方

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题目1.

$$x' = x + y \quad x(0) = 2 \quad y' = 4x + y \quad y(0) = 3$$

解答.

$$x' = x + y$$

$$y = x' - x$$

$$y' = x'' - x'$$

$$y' = 4x + y$$

$$x'' - x' = 4x + x' - x$$

$$x'' - 2x' - 3x = 0$$

$$m^2 - 2m - 3 = 0$$

$$(m - 3)(m + 1) = 0$$

$$m1 = 3 \quad m2 = -1$$

$$x(t) = c1e^{3t} + c2e^{-t}$$

$$x' = 3c1e^{3t} - c2e^{-t}$$

$$y = x' - x$$

$$y(t) = 3c1e^{3t} - c2e^{-t} - (c1e^{3t} + c2e^{-t})$$

$$y(t) = 2c1e^{3t} - 2c2e^{-t}$$

$$x(0)=2 \quad c1+c2=2$$

$$y(0)=3 \quad 2c1-2c2=3$$

$$c1=\frac{7}{4} \quad c2=\frac{1}{4}$$

$$x(t)=\frac{7}{4}e^{3t}+\frac{1}{4}e^{-t}$$

$$y(t)=\frac{7}{2}e^{3t}-\frac{1}{2}e^{-t}$$

题目2.

$$\frac{d^3y}{dt^3}-2\frac{d^2y}{dt^2}-\frac{dy}{dt}+2y=0$$

解答.

$$m^3-2m^2-m+2=0$$

$$(m-1)(m^2-m-2)=0$$

$$(m-1)(m-2)(m+1)=0$$

$$m1=1 \quad m2=2 \quad m3=-1$$

$$y(t)=c1e^{t}+c2e^{2t}+c3e^{-t}$$

题目3.

$$\frac{d^3y}{dt^3}-6\frac{d^2y}{dt^2}+5\frac{dy}{dt}+12y=0$$

解答.

$$m^3-6m^2+5m+12=0$$

$$(m-3)(m^2-3m-4)=0$$

$$(m-3)(m-4)(m+1)=0$$

$$m1=3 \quad m2=2 \quad m3=-1$$

$$y(t)=c1e^{3t}+c2e^{4t}+c3e^{-t}$$