常微与偏微课程作业

贾博方

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题目1.

$$\frac{d^3y}{dt^3} - 2\frac{d^2y}{dt^2} - \frac{dy}{dt} + 2y = 0$$

解答.

$$m^3 - 2m^2 - m + 2 = 0$$

$$(m-1)(m^2 - m - 2) = 0$$

$$(m-1)(m-2)(m+1) = 0$$

$$m1 = 1 \quad m2 = 2 \quad m3 = -1$$

$$y(t) = c1e^t + c2e^{2t} + c3e^{-t}$$

题目2.

$$\frac{d^3y}{dt^3} - 6\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 12y = 0$$

解答.

$$m^3 - 6m^2 + 5m + 12 = 0$$

$$(m-3)(m^2 - 3m - 4) = 0$$

$$(m-3)(m-4)(m+1) = 0$$

$$m1 = 3 \quad m2 = 2 \quad m3 = -1$$

$$y(t) = c1e^{3t} + c2e^{4t} + c3e^{-t}$$

题目3.

$$\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=t+e^{-t}$$

解答.

$$r^3+r^2+r+1=0$$

$$(r+1)(r^2+1)=0$$

$$r1=-1 \quad r2=i \quad r3=-i$$

$$\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=t$$

$$\phi 1(t)=At+B$$

$$\phi 1'(t)=A$$

$$\phi 1''(t)=0$$

$$\phi 1'''(t)=0$$

$$0+0+A+At+B=t$$

$$A=1 \quad B=-1$$

$$\phi 1(t)=t-1$$

$$\frac{d^3y}{dt^3}+\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=e^{-t}$$

$$\phi 2(t)=Cte^{-t}$$

$$\phi 2'(t)=Ce^{-t}(1-t)$$

$$\phi 2''(t)=Ce^{-t}(t-2)$$

$$\phi 2'''(t)=Ce^{-t}(3-t)$$

$$e^{-t}C(3-t+t-2+1-t+t)=e^{-t}$$

$$C=\frac{1}{2}$$

$$\phi 2(t)=\frac{1}{2}te^{-t}$$

$$\phi(t)=\phi 1(t)+\phi 2(t)$$

$$\phi(t)=t-1+\frac{1}{2}te^{-t}$$