Solution 9.1

(a) V_m = **50 V**.

(b) Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{30} = \underline{0.2094s}$ = **209.4ms**

(c) Frequency f = ω/(2π) = 30/(2π) = **4.775 Hz**.

(d) At t=1ms, v(0.01) = 50cos(30x0.01rad + 10˚) = 50cos(1.72˚ + 10˚) = **44.48 V** and ωt = **0.3 rad**.

Solution 9.2

(a) amplitude = **15 A**

(b) $\omega = 25\pi =$ **78.54 rad/s**

(c) $f = \dfrac{\omega}{2\pi} =$ **12.5Hz**

(d) $I_s = 15\angle 25°$ A

$I_s(2\text{ ms}) = 15\cos((500\pi)(2\times 10^{-3}) + 25°) =$

$15\cos(\pi + 25°) = 15\cos(205°) =$ **–13.595 A**

Solution 9.3

(a) $10 \sin(\omega t + 30°) = 10 \cos(\omega t + 30° - 90°) = \mathbf{10\cos(\omega t - 60°)}$

(b) $-9 \sin(8t) = \mathbf{9\cos(8t + 90°)}$

(c) $-20 \sin(\omega t + 45°) = 20 \cos(\omega t + 45° + 90°) = \mathbf{20\cos(\omega t + 135°)}$

(a) $10\cos(\omega t - 60°)$, (b) $9\cos(8t + 90°)$, (c) $20\cos(\omega t + 135°)$

Solution 9.4

Design a problem to help other students to better understand sinusoids.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

(a) Express $v = 8\cos(7t + 15°)$ in sine form.

(b) Convert $i = -10\sin(3t - 85°)$ to cosine form.

Solution

(a) $v = 8\cos(7t + 15°) = 8\sin(7t + 15° + 90°) = \mathbf{8\sin(7t + 105°)}$

(b) $i = -10\sin(3t - 85°) = 10\cos(3t - 85° + 90°) = \mathbf{10\cos(3t + 5°)}$

Solution 9.5

$v_1 = 45 \sin(\omega t + 30°)$ V $= 45 \cos(\omega t + 30° - 90°) = 45 \cos(\omega t - 60°)$ V

$v_2 = 50 \cos(\omega t - 30°)$ V

This indicates that the phase angle between the two signals is **30°** and that $\boldsymbol{v_1}$ **lags** $\boldsymbol{v_2}$.

Solution 9.6

(a) $v(t) = 10 \cos(4t - 60°)$

$i(t) = 4 \sin(4t + 50°) = 4 \cos(4t + 50° - 90°) = 4 \cos(4t - 40°)$

Thus, **i(t) leads v(t) by 20°**.

(b) $v_1(t) = 4 \cos(377t + 10°)$

$v_2(t) = -20 \cos(377t) = 20 \cos(377t + 180°)$

Thus, **v_2(t) leads v_1(t) by 170°**.

(c) $x(t) = 13 \cos(2t) + 5 \sin(2t) = 13 \cos(2t) + 5 \cos(2t - 90°)$

$\mathbf{X} = 13\angle 0° + 5\angle -90° = 13 - j5 = 13.928\angle -21.04°$

$x(t) = 13.928 \cos(2t - 21.04°)$

$y(t) = 15 \cos(2t - 11.8°)$

phase difference $= -11.8° + 21.04° = 9.24°$

Thus, **y(t) leads x(t) by 9.24°**.

Solution 9.7

If $f(\phi) = \cos\phi + j\sin\phi$,

$$\frac{df}{d\phi} = -\sin\phi + j\cos\phi = j(\cos\phi + j\sin\phi) = jf(\phi)$$

$$\frac{df}{f} = j\,d\phi$$

Integrating both sides

$\ln f = j\phi + \ln A$

$f = Ae^{j\phi} = \cos\phi + j\sin\phi$

$f(0) = A = 1$

i.e. $\mathbf{f(\phi) = e^{j\phi} = \cos\phi + j\sin\phi}$

Solution 9.8

(a) $\dfrac{60\angle 45^\circ}{7.5 - j10} + j2 = \dfrac{60\angle 45^\circ}{12.5\angle -53.13^\circ} + j2$

$= 4.8\angle 98.13^\circ + j2 = -0.6788 + j4.752 + j2$

$=$ **$-0.6788 + j6.752$**

(b) $(6-j8)(4+j2) = 24 - j32 + j12 + 16 = 40 - j20 = 44.72\angle -26.57^\circ$

$$\frac{32\angle -20^\circ}{(6-j8)(4+j2)} + \frac{20}{-10+j24} = \frac{32\angle -20^\circ}{44.72\angle -26.57^\circ} + \frac{20}{26\angle 112.62^\circ}$$

$= 0.7156\angle 6.57^\circ + 0.7692\angle -112.62^\circ = 0.7109 + j0.08188 - 0.2958 - j0.71$

$=$ **$0.4151 - j0.6281$**

(c) $20 + (16\angle -50^\circ)(13\angle 67.38^\circ) = 20 + 208\angle 17.38^\circ = 20 + 198.5 + j62.13$

$=$ **$218.5 + j62.13$**

Solution 9.9

(a) $$(5\angle 30°)(6 - j8 + 1.1197 + j0.7392) = (5\angle 30°)(7.13 - j7.261)$$
$$= (5\angle 30°)(10.176\angle -45.52°) =$$

50.88∠–15.52°.

(b) $$\frac{(10\angle 60°)(35\angle -50°)}{(-3 + j5) = (5.83\angle 120.96°)} =$$ **60.02∠–110.96°.**

Solution 9.10

Design a problem to help other students to better understand phasors.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Given that $z_1 = 6 - j8$, $z_2 = 10\angle{-30^\circ}$, and $z_3 = 8e^{-j120^\circ}$, find:
(a) $z_1 + z_2 + z_3$
(b) $z_1 z_2 / z_3$

Solution

(a) $z_1 = 6 - j8, \;\; z_2 = 8.66 - j5, \text{ and } \; z_3 = -4 - j6.9282$
$z_1 + z_2 + z_3 =$ **(10.66 – j19.928)Ω**

(b) $\frac{z_1 z_2}{z_3} = [(10\angle{-53.13^\circ})(10\angle{-30^\circ})/(8\angle{-120^\circ})] = 12.5\angle{36.87^\circ}\Omega =$ **(10 + j7.5) Ω**

Solution 9.11

(a) $V = \underline{21 < -15^o \text{ V}}$

(b) $i(t) = 8\sin(10t + 70^o + 180^o) = 8\cos(10t + 70^o + 180^o - 90^o) = 8\cos(10t + 160^o)$

I = 8∠160° mA

(c) $v(t) = 120\sin(10^3 t - 50^o) = 120\cos(10^3 t - 50^o - 90^o)$

V = 120∠–140° V

(d) $i(t) = -60\cos(30t + 10^o) = 60\cos(30t + 10^o + 180^o)$

I = 60∠–170° mA

Solution 9.11

Let **X** = 4∠40° and **Y** = 20∠–30°. Evaluate the following quantities and express your results in polar form.

(X + **Y**)/**X***
(X - **Y**)*
(**X** + **Y**)/**X**

X = 3.064+j2.571; **Y** = 17.321–j10

(a) **(X** + **Y**)**X*** = $(20.38 - j7.429)(4\angle -40^\circ)$
$= (21.69\angle -20.03^\circ)(4\angle -40^\circ) = 86.76\angle -60.03^\circ$
= **86.76∠–60.03˚**

(b) **(X – Y)*** = (–14.257+j12.571)* = **19.41∠–139.63˚**

(c) (**X** + **Y**)/**X** **=** (21.69∠–20.03˚)/(4∠40˚) = **5.422∠–60.03˚**

Solution 9.13

(a) $\mathbf{(-0.4324 + \mathit{j}0.4054) + (-0.8425 - \mathit{j}0.2534) = \underline{-1.2749 + \mathit{j}0.1520}}$

(b) $\dfrac{50\angle -30^o}{24\angle 150^o} = \underline{-2.0833}$ = **–2.083**

(c) (2+j3)(8-j5) –(-4) = **35 +j14**

Solution 9.14

(a) $\dfrac{3-j14}{-7+j17}=\dfrac{14.318\angle -77.91°}{18.385\angle 112.38°}=0.7788\angle 169.71°=\underline{-0.7663+j0.13912}$

(b) $\dfrac{(62.116+j231.82+138.56-j80)(60-j80)}{(67+j84)(16.96+j10.5983)}=\dfrac{24186-6944.9}{246.06+j2134.7}=\underline{-1.922-j11.55}$

(c) $\left[\dfrac{10+j20}{3+j4}\right]^2\sqrt{(10+j5)(16-j20)}$

$= [(22.36\angle 63.43°)/(5\angle 53.13°)]^2[(11.18\angle 26.57°)(25.61\angle -51.34°)]^{0.5}$
$= [4.472\angle 10.3°]^2[286.3\angle -24.77°]^{0.5} = (19.999\angle 20.6°)(16.921\angle -12.38°) = 338.4\angle 8.22°$

or **334.9+j48.38**

Solution 9.15

(a) $\begin{vmatrix} 10+j6 & 2-j3 \\ -5 & -1+j \end{vmatrix}$ = -10 – j6 + j10 – 6 + 10 – j15

= **–6 – j11**

(b) $\begin{vmatrix} 20\angle -30^\circ & -4\angle -10^\circ \\ 16\angle 0^\circ & 3\angle 45^\circ \end{vmatrix}$ = $60\angle 15^\circ + 64\angle -10^\circ$

= 57.96 + j15.529 + 63.03 – j11.114

= **120.99 + j4.415**

(c)
$$\left|\begin{array}{ccc} 1-j & -j & 0 \\ j & 1 & -j \\ 1 & j & 1+j \\ 1-j & -j & 0 \\ j & 1 & -j \end{array}\right|$$

= (1–j)1(1+j) +$j^2$0+1(–j)(–j) – 0(1) – (1–j)(–j)j – j(–j)(1+j)

= 2–1 – 1 + j – 1 – j = **–1**

Solution 9.16

(a) $-20\cos(4t + 135°) = 20\cos(4t + 135° - 180°)$

$= 20\cos(4t - 45°)$

The phasor form is $\mathbf{20\angle{-45°}}$

(b) $8\sin(20t + 30°) = 8\cos(20t + 30° - 90°)$

$= 8\cos(20t - 60°)$

The phasor form is $\mathbf{8\angle{-60°}}$

(c) $20\cos(2t) + 15\sin(2t) = 20\cos(2t) + 15\cos(2t - 90°)$

The phasor form is $20\angle 0° + 15\angle{-90°} = 20 - j15 = \mathbf{25\angle{-36.87°}}$

Solution 9.17

$$V = V_1 + V_2 = 10 < -60^o + 12 < 30^o = 5 - j8.66 + 10.392 + j6 = 15.62 < -9.805^o$$

v(t) = **15.62cos(50t–9.8°) V**

Solution 9.18

(a) $v_1(t) = \mathbf{60\ cos(t + 15°)}$

(b) $\mathbf{V}_2 = 6 + j8 = 10\angle 53.13°$

$v_2(t) = \mathbf{10\ cos(40t + 53.13°)}$

(c) $i_1(t) = \mathbf{2.8\ cos(377t - \pi/3)}$

(d) $\mathbf{I}_2 = -0.5 - j1.2 = 1.3\angle 247.4°$

$i_2(t) = \mathbf{1.3\ cos(10^3 t + 247.4°)}$

Solution 9.19

(a) $3\angle 10° - 5\angle -30° = 2.954 + j0.5209 - 4.33 + j2.5$

$= -1.376 + j3.021$

$= 3.32\angle 114.49°$

Therefore, $3\cos(20t + 10°) - 5\cos(20t - 30°)$

$=$ $\mathbf{3.32\cos(20t + 114.49°)}$

(b) $40\angle -90° + 30\angle -45° = -j40 + 21.21 - j21.21$

$= 21.21 - j61.21$

$= 64.78\angle -70.89°$

Therefore, $40\sin(50t) + 30\cos(50t - 45°) =$ $\mathbf{64.78\cos(50t - 70.89°)}$

(c) Using $\sin\alpha = \cos(\alpha - 90°)$,

$20\angle -90° + 10\angle 60° - 5\angle -110° = -j20 + 5 + j8.66 + 1.7101 + j4.699$

$= 6.7101 - j6.641$

$= 9.44\angle -44.7°$

Therefore, $20\sin(400t) + 10\cos(400t + 60°) - 5\sin(400t - 20°)$

$=$ $\mathbf{9.44\cos(400t - 44.7°)}$

Solution 9.20

$7.5\cos(10t+30^\circ)$ A can be represented by $7.5\angle 30^\circ$ and $120\cos(10t+75^\circ)$ V can be represented by $120\angle 75^\circ$. Thus,

$\mathbf{Z} = \mathbf{V}/\mathbf{I} = (120\angle 75^\circ)/(7.5\angle 30^\circ) = 16\angle 45^\circ$ or $\mathbf{(11.314+j11.314)\ \Omega}$.

Solution 9.21

(a) $F = 5\angle 15^o - 4\angle\underline{-30^o - 90^o} = 6.8296 + j4.758 = 8.3236\angle 34.86^o$

$$\underline{\boldsymbol{f(t) = 8.324\cos(30t + 34.86^o)}}$$

(b) $G = 8\angle -90^o + 4\angle 50^o = 2.571 - j4.9358 = 5.565\angle -62.49^o$

$$\underline{\boldsymbol{g(t) = 5.565\cos(t - 62.49^o)}}$$

(c) $\mathrm{H} = \dfrac{1}{\mathrm{j}\omega}\left(10\angle 0^{\mathrm{o}} + 50\angle -90^{\mathrm{o}}\right), \qquad \omega = 40$

i.e. $\mathrm{H} = 0.25\angle -90^{\mathrm{o}} + 1.25\angle -180^{\mathrm{o}} = -\mathrm{j}0.25 - 1.25 = 1.2748\angle -168.69^{\mathrm{o}}$

$$\mathrm{h(t)} \boldsymbol{= 1.2748\cos(40t - 168.69^\circ)}$$

Solution 9.22

Let $\quad$ f(t) $= 10v(t) + 4\dfrac{dv}{dt} - 2\int_{-\infty}^{t} v(t)dt$

$$F = 10V + j\omega 4V - \frac{2V}{j\omega}, \qquad \omega = 5, \qquad V = 55\angle 45^o$$

$$F = 10V + j20V + j0.4V = (10 + j20.4)V = 22.72\angle 63.89°(55\angle 45°) = 1249.6\angle 108.89^o$$

f(t) = **1249.6cos(5t+108.89°)**

Solution 9.23

(a) v = [110sin(20t+30˚) + 220cos(20t–90˚)] V leads to $\mathbf{V} = 110\angle(30^\circ-90^\circ) + 220\angle-90^\circ$ $= 55-j95.26 - j220 = 55-j315.3 = 320.1\angle-80.11^\circ$ or

v = **320.1cos(20t–80.11˚) A**.

(b) i = [30cos(5t+60˚)–20sin(5t+60˚)] A leads to $\mathbf{I} = 30\angle60^\circ - 20\angle(60^\circ-90^\circ) =$ $15+j25.98 - (17.321-j10) = -2.321+j35.98 = 36.05\angle93.69^\circ$ or

i = **36.05cos(5t+93.69˚) A**.

(a) 320.1cos(20t–80.11˚) A, (b) 36.05cos(5t+93.69˚) A

Solution 9.24

(a)

$$\mathbf{V} + \frac{\mathbf{V}}{j\omega} = 10\angle 0°, \quad \omega = 1$$

$$\mathbf{V}(1 - j) = 10$$

$$\mathbf{V} = \frac{10}{1 - j} = 5 + j5 = 7.071\angle 45°$$

Therefore,

v(t) = **7.071cos(t + 45°) V**

(b)

$$j\omega\mathbf{V} + 5\mathbf{V} + \frac{4\mathbf{V}}{j\omega} = 20\angle(10° - 90°), \quad \omega = 4$$

$$\mathbf{V}\left(j4 + 5 + \frac{4}{j4}\right) = 20\angle -80°$$

$$\mathbf{V} = \frac{20\angle -80°}{5 + j3} = 3.43\angle -110.96°$$

Therefore,

v(t) = **3.43cos(4t – 110.96°) V**

Solution 9.25

(a)

$$2j\omega\mathbf{I} + 3\mathbf{I} = 4\angle 45°, \quad \omega = 2$$

$$\mathbf{I}(3 + j4) = 4\angle 45°$$

$$I = \frac{4\angle 45°}{3 + j4} = \frac{4\angle 45°}{5\angle 53.13°} = 0.8\angle -8.13°$$

Therefore, i(t) = **800cos(2t – 8.13°) mA**

(b)

$$10\frac{\mathbf{I}}{j\omega} + j\omega\mathbf{I} + 6\mathbf{I} = 5\angle 22°, \quad \omega = 5$$

$$(-j2 + j5 + 6)\mathbf{I} = 5\angle 22°$$

$$\mathbf{I} = \frac{5\angle 22°}{6 + j3} = \frac{5\angle 22°}{6.708\angle 26.56°} = 0.745\angle -4.56°$$

Therefore, i(t) = **745 cos(5t – 4.56°) mA**

Solution 9.26

$$j\omega\mathbf{I} + 2\mathbf{I} + \frac{\mathbf{I}}{j\omega} = 1\angle 0°, \quad \omega = 2$$

$$\mathbf{I}\left(j2 + 2 + \frac{1}{j2}\right) = 1$$

$$\mathbf{I} = \frac{1}{2 + j1.5} = 0.4\angle -36.87°$$

Therefore, i(t) = **0.4 cos(2t – 36.87°)**

Solution 9.27

$$j\omega\mathbf{V} + 50\mathbf{V} + 100\frac{\mathbf{V}}{j\omega} = 110\angle\text{-}10°, \quad \omega = 377$$

$$\mathbf{V}\left(j377 + 50 - \frac{j100}{377}\right) = 110\angle\text{-}10°$$

$$\mathbf{V}\,(380.6\angle 82.45°) = 110\angle\text{-}10°$$

$$\mathbf{V} = 0.289\angle\text{-}92.45°$$

Therefore, v(t) = **289 cos(377t – 92.45°) mV**.

Solution 9.28

Determine the current that flows through a 20-Ω resistor connected in parallel with a voltage source v_s = 120 cos(377t+37˚) V.

Solution

$$i(t) = \frac{v_s(t)}{R} = \frac{120\cos(377t + 37°)}{20} = \mathbf{6cos(377t+37°)\ A}.$$

Solution 9.29

Given that $v_C(0)$ = 2cos(155°) V, what is the instantaneous voltage across a 2-μF capacitor when the current through it is I = 4 sin ($10^6\ t$ + 25°) A?

Solution

$$\mathbf{Z} = \frac{1}{j\omega C} = \frac{1}{j(10^6)(2\times 10^{-6})} = -j0.5$$

$$\mathbf{V} = \mathbf{IZ} = (4\angle 25°)(0.5\angle -90°) = 2\angle -65°$$

Therefore v(t) = **2 sin(10^6t – 65°) V**.

Solution 9.30

Since R and C are in parallel, they have the same voltage across them. For the resistor,

$$V = I_R R \quad \longrightarrow \quad I_R = V / R = \frac{100 < 20^o}{40k} = 2.5 < 20^o \text{ mA}$$

$$i_R = \underline{2.5\cos(60t + 20^o) \text{ mA}}$$

For the capacitor,

$$i_C = C\frac{dv}{dt} = 50x10^{-6}(-60)x100\sin(60t + 20^o) = \underline{-300\sin(60t + 20^o) \text{ mA}}$$

Solution 9.31

A series RLC circuit has R= 80 Ω, L=240 mH, and C = 5mF. If the input voltage is v(t)= 115cos(2t), find the current flowing through the circuit.

Solution

$$L = 240mH \quad \longrightarrow \quad j\omega L = j2x240x10^{-3} = j0.48$$

$$C = 5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2x5x10^{-3}} = -j100$$

$$Z = 80 + j0.48 - j100 = 80 - j99.52 = 127.688\angle -51.21^\circ\ \Omega.$$

$I = V/Z = 115\angle 0^\circ/(80-j99.52) = 115/(127.688\angle -51.21^\circ) = 0.9006\angle 51.21^\circ$

Thus,

i(t) = **900.6cos(2t+51.21°) mA**

Solution 9.32

Using Fig. 9.40, design a problem to help other students to better understand phasor relationships for circuit elements.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Two elements are connected in series as shown in Fig. 9.40.
If i = 12 cos (2t - 30°) A, find the element values.

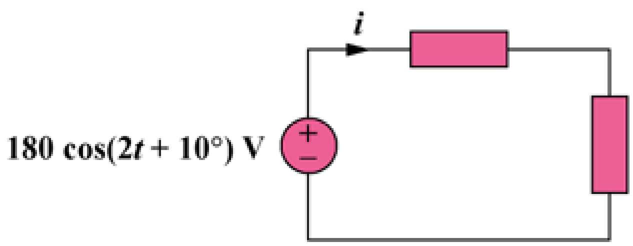


Figure 9.40

Solution

$\mathbf{V} = 180\angle 10°$, $\mathbf{I} = 12\angle -30°$, $\omega = 2$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{180\angle 10°}{12\angle -30°} = 15\angle 40° = 11.49 + j9.642\ \Omega$$

One element is a resistor with R = **11.49 Ω**.
The other element is an inductor with $\omega L = 9.642$ or L = **4.821 H**.

Solution 9.33

A series *RL* circuit is connected to a 220-V ac source. If the voltage across the resistor is 170 V, find the voltage across the inductor.

Solution

$220\angle\theta = v_R + jv_L$ where $220 = \sqrt{v_R^2 + v_L^2}$

$v_L = \sqrt{220^2 - v_R^2}$

$v_L = \sqrt{220^2 - 170^2} =$ **139.64 V**

Solution 9.34

$v_o = 0$ when $jX_L - jX_C = 0$ so $X_L = X_C$ or $\omega L = \dfrac{1}{\omega C} \longrightarrow \omega = \dfrac{1}{\sqrt{LC}}$.

$$\omega = \frac{1}{\sqrt{(5\times10^{-3})(20\times10^{-3})}} = \mathbf{100\ rad/s}$$

Solution 9.35

Find current i in the circuit of Fig. 9.42, when $v_s(t)$ = 115cos(200t) V.

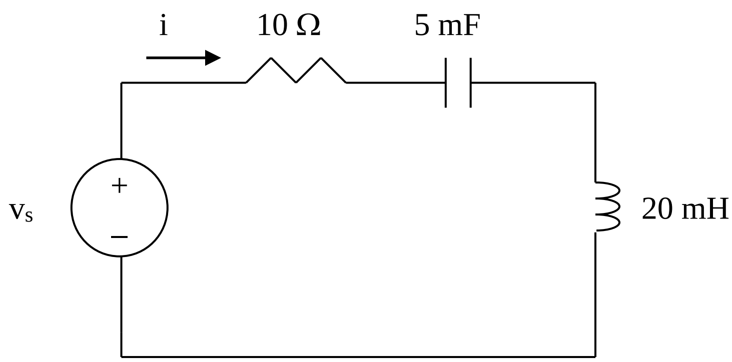


Figure 9.42
For Prob. 9.35.

Solution

$$5mF \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200x5x10^{-3}} = -j$$

$$20mH \quad \longrightarrow \quad j\omega L = j20x10^{-3}x200 = j4$$

$$Z_{in} = 10 - j + j4 = 10 + j3 = 10.44\angle 16.699°$$

Thus, I = V_s/Z_{in} = $115\angle 0°/10.44\angle 16.699°$ = $11.015\angle{-16.7°}$

i(t) = **11.015cos(200t–16.7°) A**

Solution 9.36

Using Fig. 9.43, design a problem to help other students to better understand impedance.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit in Fig. 9.43, determine *i*. Let v_s = 60 cos(200*t* - 10°) V.

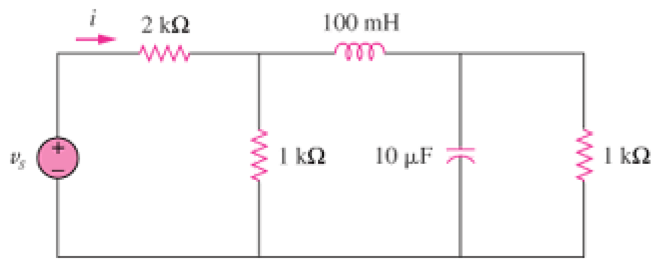


Figure 9.43

Solution

Let Z be the input impedance at the source.

$$100\text{ mH} \longrightarrow j\omega L = j200x100x10^{-3} = j20$$

$$10\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x10^{-6}x200} = -j500$$

1000//-j500 = 200 –j400
1000//(j20 + 200 –j400) = 242.62 –j239.84

$$Z = 2242.62 - j239.84 = 2255\angle -6.104^o$$

$$I = \frac{60\angle -10^o}{2255\angle -6.104^o} = 26.61\angle -3.896^o \text{ mA}$$

$$\mathbf{i = \underline{266.1\cos(200t - 3.896^o)}\,mA}$$

Solution 9.37

Determine the admittance **Y** for the circuit in Fig. 9.44.

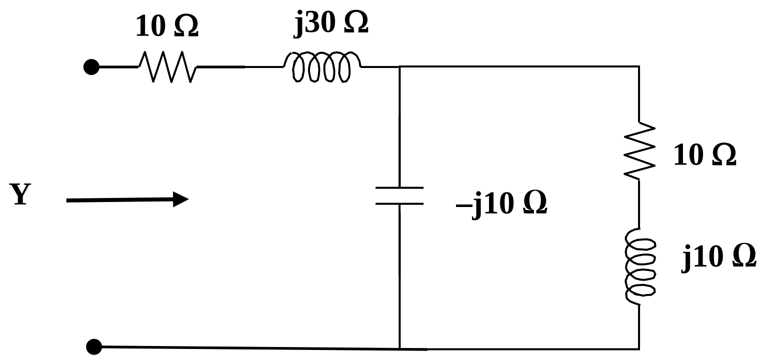


Figure 9.44
For Prob. 9.37.

Solution

Let us start with **Z** = 1/**Y** = 10+j30 + (–j10)(10+j10)/(–j10+10+j10)
= 10 + j30 + (100–j100)/10 = 20+j20 and **Y** = 1/**Z** = 1/28.284∠45°
= 0.035355∠–45° or

Y = 0.025–j0.025 = **(25–j25) mS**.

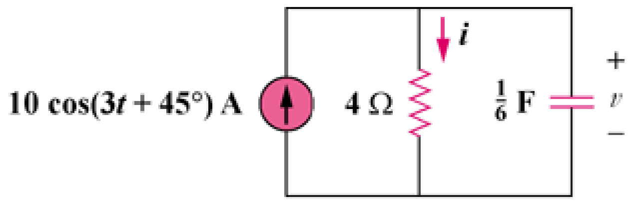
Solution 9.38

Using Fig. 9.45, design a problem to help other students to better understand admittance.

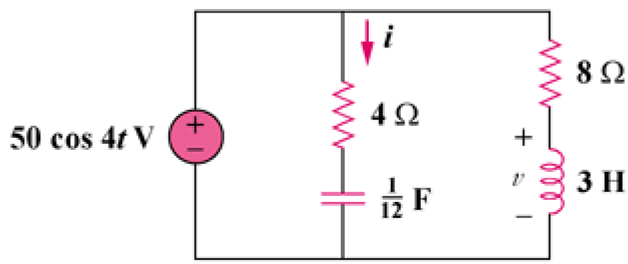
Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find *i*(*t*) and *v*(*t*) in each of the circuits of Fig. 9.45.



(a)



(b)

Figure 9.45

Solution

(a) $\frac{1}{6}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = \text{-j2}$

$$\mathbf{I} = \frac{\text{- j2}}{4 - \text{j2}}(10\angle 45°) = 4.472\angle\text{-}18.43°$$

Hence, i(t) = **4.472 cos(3t – 18.43°) A**

$$\mathbf{V} = 4\mathbf{I} = (4)(4.472\angle\text{-}18.43°) = 17.89\angle\text{-}18.43°$$

Hence, v(t) = **17.89 cos(3t – 18.43°) V**

(b) $\frac{1}{12}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = \text{-j3}$

$3\text{ H} \longrightarrow \quad j\omega L = j(4)(3) = j12$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0°}{4 - j3} = 10\angle 36.87°$$

Hence, i(t) = **10 cos(4t + 36.87°) A**

$$\mathbf{V} = \frac{j12}{8 + j12}(50\angle 0°) = 41.6\angle 33.69°$$

Hence, v(t) = **41.6 cos(4t + 33.69°) V**

Solution 9.39

For the circuit shown in Fig. 9.46, find Z_{eq} and use that to find current I. Let ω=10 rad/s.

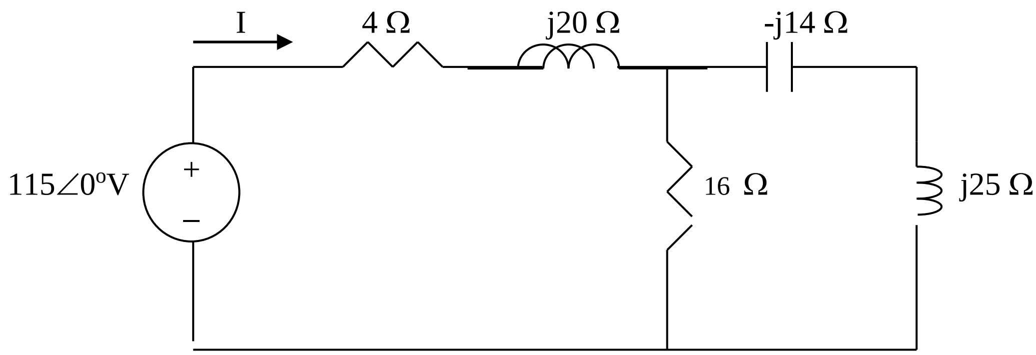


Figure 9.46
For Prob. 9.39.

Solution

$Z_{eq} = 4 + j20 + [16(–j14+j25)/(16–j14+j25)] = 4 + j20 + j176(16–j11)/(256+121)$
$= 4 + j20 + (1,936+j2,816)/377 = (9.135 + j27.47)\ \Omega.$

= **(9.135+j27.47) Ω = 28.95∠71.61° Ω**.

$I = V/Z_{eq} = 115/28.95\angle 71.61° = 3.972\angle –71.61°$

i(t) = **3.972cos(10t–71.61°) A**

Solution 9.40

In the circuit of Fig. 9.47, find i_o(t) when:

(a) ω = 1 rad/s
(b) ω = 5 rad/s
(c) ω = 10 rad/s

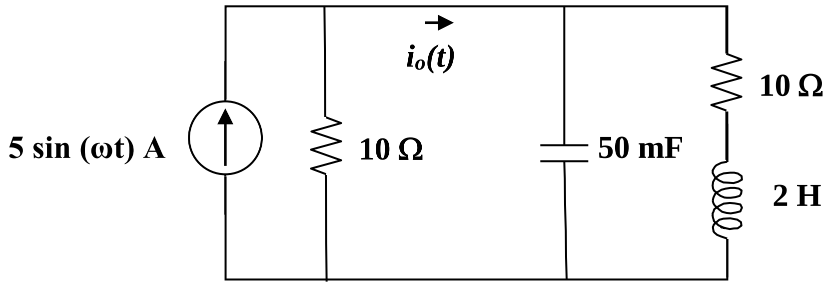


Figure 9.47
For Prob. 9.40.

Solution

It will help to convert the current source resistor into the Thevenin equivalent or 50 V in series with 10 Ω.

(a) For ω = 1 rad/s, the inductor becomes j2 and the capacitor becomes –j20 which leads to Z = 10 + (–j20)(10+j2)/(–j20+10+j2) = 10 + (40–j200)/(10–j18)
= 10 + 203.96∠–78.69°/(20.591∠–60.945°) = 10 + 9.9053∠–17.745°
= 10 + 9.434 – j3.0189 = 19.667∠–8.83°
$I_o = 50/Z_{in}$ = 2.542∠8.83° A or i_o(t) = **2.542sin(t+8.83°) A**.

(b) For ω = 5 rad/s, the inductor becomes j10 and the capacitor becomes –j4 which leads to Z = 10 + (–j4)(10+j10)/(–j4+10+j10) = 10 + (40–j40)/(10+j6)
= 10 + 56.569∠–45°/(11.6619∠30.9638°) = 10 + 4.8508∠–75.964°
= 10 + 1.17647 – j4.706 = 12.12683∠–22.834°
$I_o = 50/Z_{in}$ = 4.1231∠22.83° A or i_o(t) = **4.123sin(5t+22.83°) A**.

(c) For ω = 10 rad/s, the inductor becomes j20 and the capacitor becomes –j2 which leads to Z = 10 + (–j2)(10+j20)/(–j2+10+j20) = 10 + (40–j20)/(10+j18)
= 10 + 44.721∠–26.565°/(20.591∠60.945°) = 10 + 2.1719∠–87.51°
= 10 + 0.0944 – j2.1698 = 10.325∠–12.13°
$\mathbf{I_o} = 50/\mathbf{Z_{in}}$ = 4.843∠12.13° A or i_o(t) = **4.843sin(10t+12.13°) A**.

Solution 9.41

Find $v(t)$ in the *RLC* circuit of Fig. 9.48.

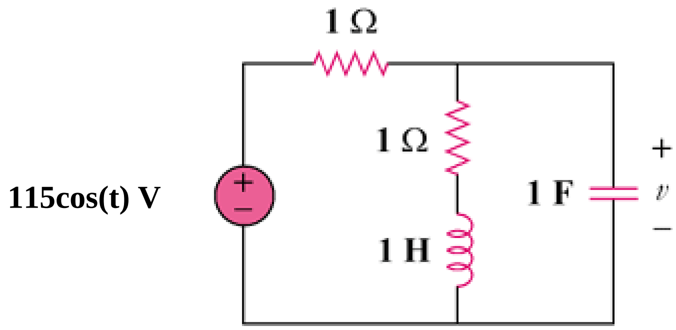


Figure 9.48
For Prob. 9.41.

Solution

$$\omega = 1,$$

$$1\text{ H} \longrightarrow j\omega L = j(1)(1) = j$$

$$1\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1)(1)} = -j$$

$$\mathbf{Z} = 1 + (1+j) \,||\, (-j) = 1 + \frac{-j+1}{1} = 2 - j$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{115}{2-j}, \qquad \mathbf{I}_c = (1+j)\mathbf{I}$$

$$\mathbf{V} = (-j)(1+j)\mathbf{I} = (1-j)\mathbf{I} = \frac{(1-j)(115)}{2-j} = 72.74\angle -18.43°$$

Thus,

v(t) = **72.74cos(t – 18.43°) V**

Solution 9.42

$$\omega = 200$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(50 \times 10^{-6})} = -j100$$

$$0.1\ \text{H} \longrightarrow j\omega L = j(200)(0.1) = j20$$

$$50 \parallel -j100 = \frac{(50)(-j100)}{50 - j100} = \frac{-j100}{1 - j2} = 40 - j20$$

$$\mathbf{V}_o = \frac{j20}{j20 + 30 + 40 - j20}(60\angle 0°) = \frac{j20}{70}(60\angle 0°) = 17.14\angle 90°$$

Thus,

$v_o(t) =$ **17.14 sin(200t + 90°) V**

or

$v_o(t) =$ **17.14 cos(200t) V**

Solution 9.43

Find current $\mathbf{I_o}$ in the circuit shown in Fig. 9.50.

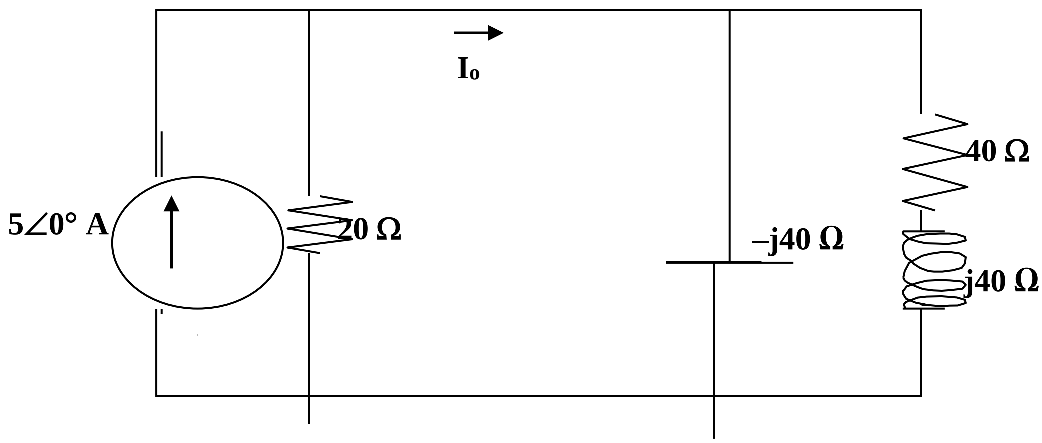


Figure 9.50
For Prob. 9.43.

Solution

First we convert the current source into a voltage source ($100\angle0°$ V) in series with 20 Ω. This then gives us an input impedance equal to,

$\mathbf{Z_{in}} = 20 + (-j40)(40+j40)/(-j40+40+j40) = 20 + (1{,}600-j1{,}600)/40 = 20+40-j40$
$= 60-j40 = 72.111\angle-33.69°$.

$\mathbf{I_o} = 100/(72.111\angle-33.69° = \mathbf{1.3868\angle33.69°\ A}$.

Solution 9.44

Calculate $i(t)$ in the circuit of Fig. 9.51.

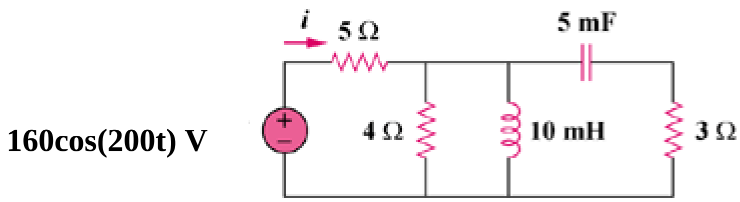


Figure 9.51
For Prob. 9.44.

Solution

$\omega = 200$

$$10\text{ mH} \longrightarrow j\omega L = j(200)(10\times 10^{-3}) = j2$$

$$5\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(200)(5\times 10^{-3})} = -j$$

$$\mathbf{Y} = \frac{1}{4} + \frac{1}{j2} + \frac{1}{3-j} = 0.25 - j0.5 + \frac{3+j}{10} = 0.55 - j0.4$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{0.55 - j0.4} = 1.1892 + j0.865$$

$$\mathbf{I} = \frac{160\angle 0°}{5+\mathbf{Z}} = \frac{160\angle 0°}{6.1892 + j0.865} = \frac{160}{6.2494\angle 7.956°} = 25.6\angle -7.956°$$

Thus,
i(t) = **25.6cos(200t – 7.96°) A**

Solution 9.45

Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

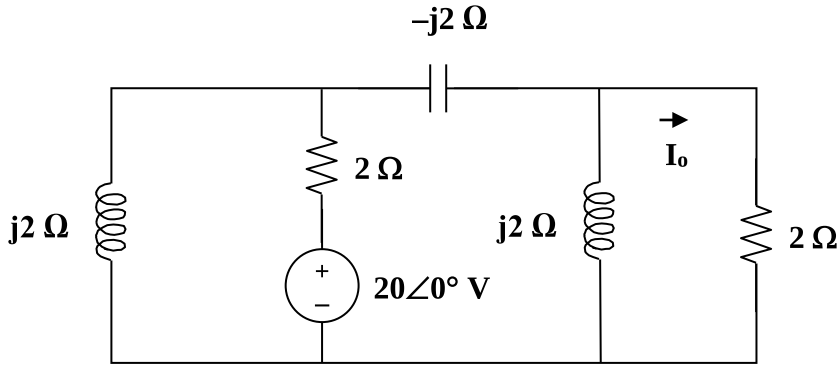


Figure 9.52
For Prob. 9.45.

Solution

There are different ways to solve this problem. We could identify two unknown node voltages and solve for the one that can give us $\mathbf{I_o}$. So, set the bottom as the reference node and let $\mathbf{V_1}$ be the node at the top of the left hand node and $\mathbf{V_2}$ be the node voltage on the right. Thus, $\mathbf{I_o} = \mathbf{V_2}/2$.

The nodal equations are $[(V_1–0)/j2]+[(V_1–20)/2]+[(V_1–V_2)/(–j2)] = 0$ and $[(V_2–V_1)/(–j2)]+[(V_2–0)/j2]+[(V_2–0)/2] = 0$. Simplifying them we get,

$(–j0.5+0.5+j0.5)V_1 – (j0.5)V_2 = 10$ and $–(j0.5)V_1 + (j0.5–j0.5+0.5)V_2 = 0$ or $0.5V_1 – j0.5V_2 = 10$ and $–j0.5V_1 + 0.5V_2 = 0$ or $V_1 = –jV_2$.

Finally, $–j0.5V_2 – j0.5V_2 = 10$ or $V_2 = 10/(–j) = j10$. Thus,

$\mathbf{I_o} = j10/2 = j5 = \mathbf{5\angle 90°\ A}$.

Solution 9.46

If v_s = 100sin(10t+18°) V in the circuit in Fig. 9.53, find i_o.

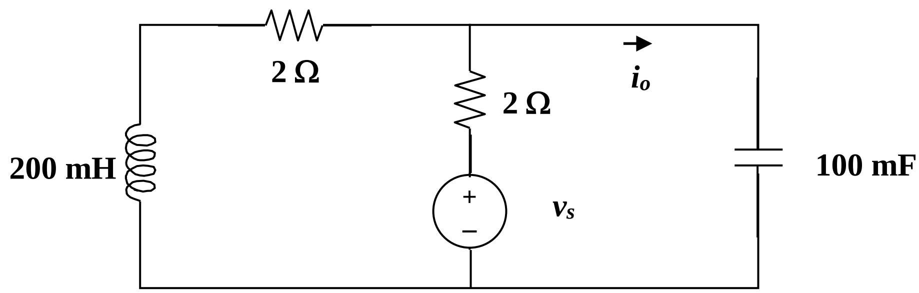


Figure 9.53
For Prob. 9.46.

Solution

Let $V_s = 100\angle 0°$ V (we will account for the 18° when we convert I_o back into the time domain). The inductor becomes j2 Ω and the capacitor becomes –j Ω. We can now write a nodal equation and $I_o = V_C/(-j)$. The nodal equation will give us V_C.

$$[(V_C-0)/(2+j2)] + [(V_C-100)/2] + [(V_C-0)/(-j)] = 0 \text{ or}$$
$$(0.25-j0.25+0.5+j)V_C = (0.75+j0.75)V_C$$
$$= (1.06066\angle 45°)V_C = 50 \text{ or } V_C = 47.14\angle -45° \text{ which leads to}$$

$I_o = 47.14\angle -45°/(-j) = 47.14\angle 45°$. Compensating for the 18° we get,

i_o = **47.14sin(10t+63°) A**.

Solution 9.47

In the circuit shown in Fig. 9.54, determine the value of $i_s(t)$.

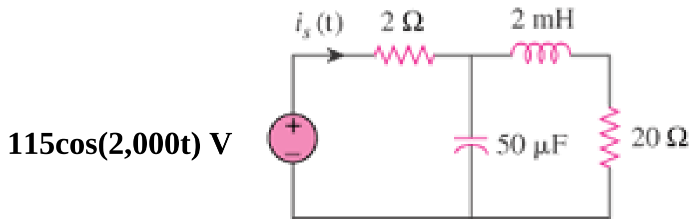
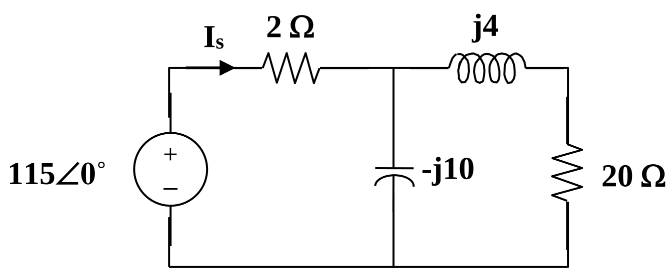


Figure 9.54
For Prob. 9.47.

Solution

First, we convert the circuit into the frequency domain.



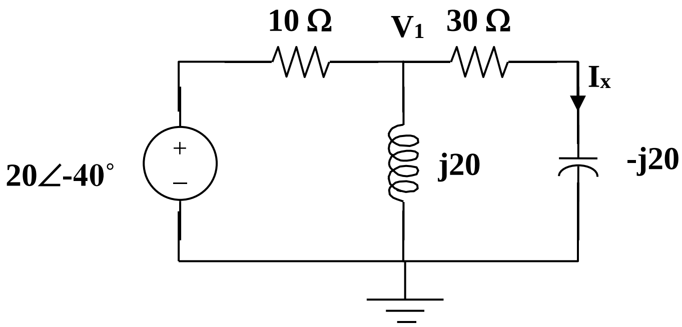
$$I_s = \frac{115}{2 + \dfrac{-j10(20+j4)}{-j10+20+j4}} = \frac{115}{2 + \dfrac{203.961\angle -78.69^\circ}{20.8806\angle -16.699^\circ}} = \frac{115}{2 + 9.76773\angle -61.991^\circ}$$

$$= \frac{115}{2 + 4.58703 - j8.623673} = \frac{115}{10.8516\angle -52.63^\circ} = 10.598\angle 52.63^\circ$$

$i_s(t)$ = **10.598cos(2000t +52.63°) A**

Solution 9.48

Converting the circuit to the frequency domain, we get:



We can solve this using nodal analysis.

$$\frac{V_1 - 20\angle -40°}{10} + \frac{V_1 - 0}{j20} + \frac{V_1 - 0}{30 - j20} = 0$$

$$V_1(0.1 - j0.05 + 0.02307 + j0.01538) = 2\angle -40°$$

$$V_1 = \frac{2\angle 40°}{0.12307 - j0.03462} = 15.643\angle -24.29°$$

$$I_x = \frac{15.643\angle -24.29°}{30 - j20} = 0.4338\angle 9.4°$$

$$i_x = \underline{0.4338\sin(100t + 9.4°)\,A}$$

Solution 9.49

Find $v_s(t)$ in the circuit of Fig. 9.56 if the current i_x through the 1-Ω resistor is 8 sin 200t A.

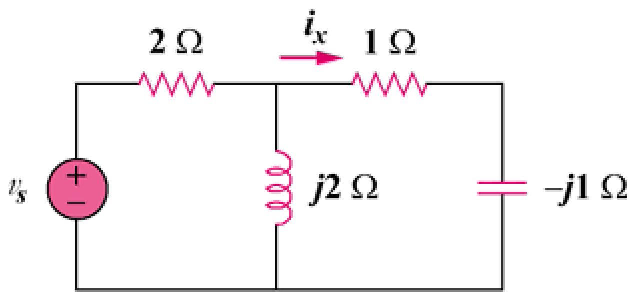
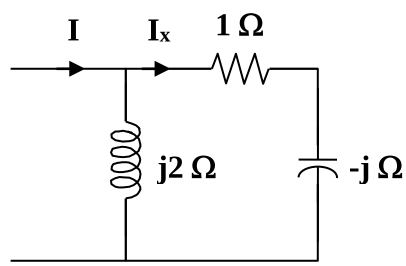


Figure 9.56
For Prob. 9.49.

Solution

$$\mathbf{Z}_T = 2 + j2 \,||\, (1-j) = 2 + \frac{(j2)(1-j)}{1+j} = 4$$



$$\mathbf{I}_x = \frac{j2}{j2+1-j}\mathbf{I} = \frac{j2}{1+j}\mathbf{I}, \qquad \text{where } \mathbf{I}_x = 8\angle 0° = 8$$

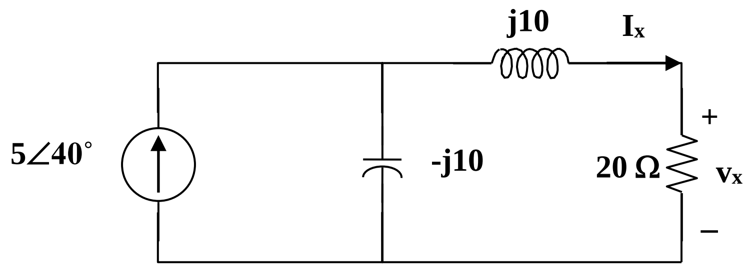
$$\mathbf{I} = \frac{1+j}{j2}\mathbf{I}_x = \frac{8+j8}{j2}$$

$$\mathbf{V}_s = \mathbf{I}\,\mathbf{Z}_T = \frac{8+j8}{j2}(4) = \frac{16(1+j)}{j} = 16(1-j) = 22.627\angle{-45°}$$

$v_s(t) =$ **22.63 sin(200t – 45°) V**

Solution 9.50

Since ω = 100, the inductor = j100x0.1 = j10 Ω and the capacitor = $1/(j100x10^{-3})$ = -j10Ω.



Using the current dividing rule:

$$I_x = \frac{-j10}{-j10+20+j10}5\angle 40° = -j2.5\angle 40° = 2.5\angle -50°$$
$$V_x = 20I_x = 50\angle -50°$$

$v_x(t)$ = **50cos(100t–50°) V**

Solution 9.51

If the voltage v_o across the 2-Ω resistor in the circuit of Fig. 9.58 is 90cos(2t) V, obtain i_s.

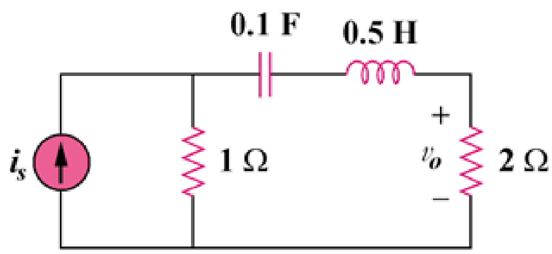


Figure 9.58
For Prob. 9.51.

Solution

$$0.1\,\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(0.1)} = -j5$$

$$0.5\,\text{H} \longrightarrow j\omega L = j(2)(0.5) = j$$

The current **I** through the 2-Ω resistor is

$$\mathbf{I} = \frac{1}{1 - j5 + j + 2}\mathbf{I}_s = \frac{\mathbf{I}_s}{3 - j4}, \qquad \text{where} \quad \mathbf{I} = \frac{90}{2}\angle 0° = 45$$

$$\mathbf{I}_s = (45)(3 - j4) = 225\angle -53.13°$$

Therefore,

$$i_s(t) = \mathbf{225cos(2t – 53.13°) A}$$

Solution 9.52

If $\mathbf{V}_o = 8\angle 30°$ V in the circuit of Fig. 9.59, find $\mathbf{V}_s$.

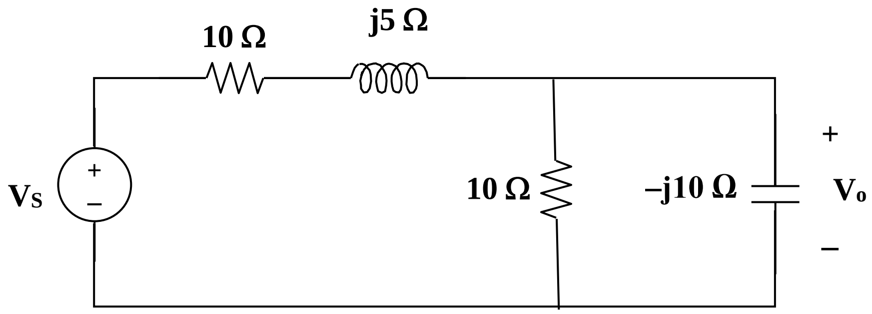


Figure 9.59
For Prob. 9.52

Solution

We can treat $\mathbf{V}_o$ as the node voltage for the circuit and then write the node equations. We get,

$$[(\mathbf{V}_o-\mathbf{V}_s)/(10+j5)] + [(\mathbf{V}_o-0)/10] + [(\mathbf{V}_o-0)/(-j10)] = 0 \text{ this leads to}$$
$$\mathbf{V}_s/(10+j5) = (0.08-j0.04+0.1+j0.1)\mathbf{V}_o = (0.18+j0.06)8\angle 30° \text{ or}$$
$$\mathbf{V}_s = [(0.189737\angle 18.435°)(8\angle 30°)(11.18034\angle 26.565° \text{ thus,}$$

$$\mathbf{V}_s = \mathbf{16.971\angle 75°\ V}.$$

Solution 9.53

Find $\mathbf{I}_0$ in the circuit in Fig. 9.60.

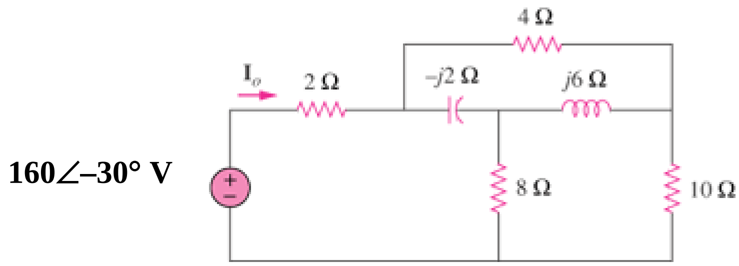
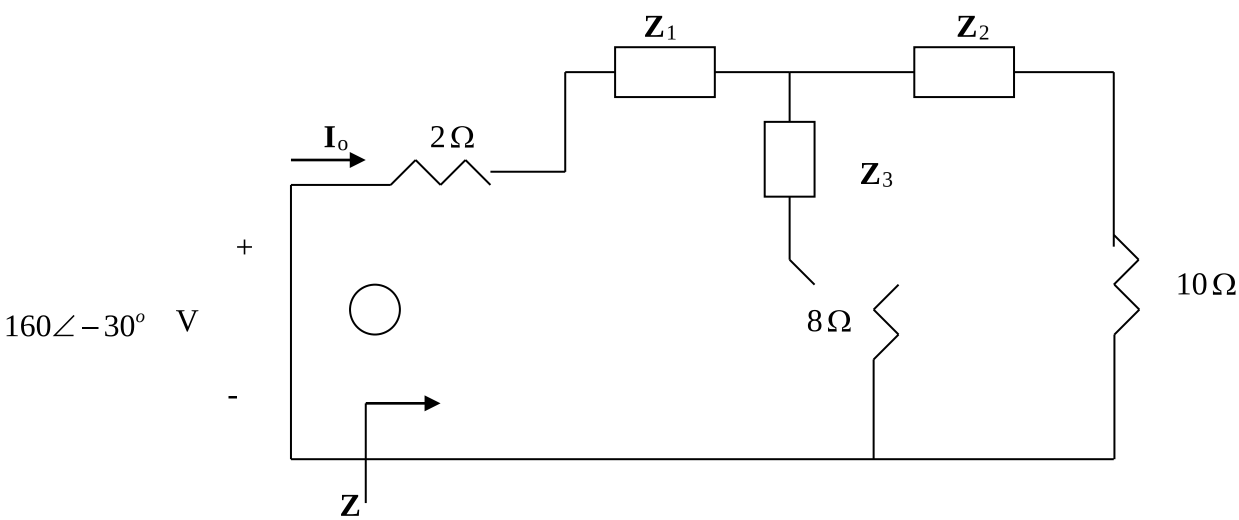


Figure 9.60
For Prob. 9.53.

Solution

Convert the delta to wye subnetwork as shown below.



$$Z_1 = \frac{-j2x4}{4+j4} = \frac{8\angle -90^\circ}{5.6569\angle 45^\circ} = -1 - j1, \qquad Z_2 = \frac{j6x4}{4+j4} = 3 + j3,$$

$$Z_3 = \frac{12}{4+j4} = 1.5 - j1.5$$

$$(Z_3 + 8)//(Z_2 + 10) = (9.5 - j1.5)//(13 + j3) = 5.691\angle 0.21^\circ = 5.691 + j0.02086$$

$$Z = 2 + Z_1 + 5.691 + j0.02086 = 6.691 - j0.9791$$

$$\mathbf{I_o} = \frac{160\angle -30^o}{Z} = \frac{160\angle -30^o}{6.7623\angle -8.33^o} = \mathbf{23.66\angle{-21.67^o}\ A}.$$

Solution 9.54

In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 30\angle 0°$ A.

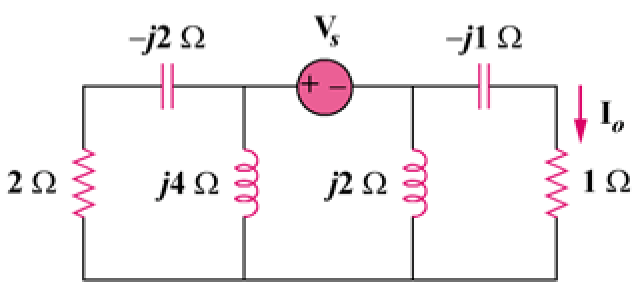
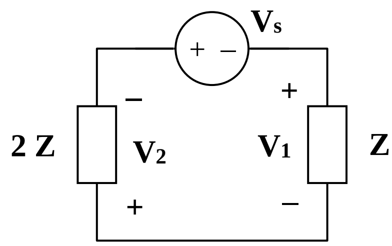


Figure 9.61
For Prob. 9.54.

Solution

Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1-j) = 30(1-j)$$

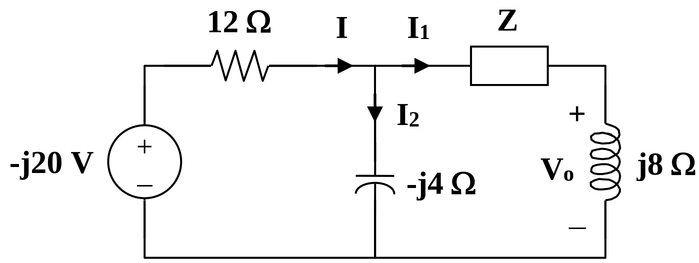
$$\mathbf{V}_2 = 2\mathbf{V}_1 = 60(1-j)$$

$\mathbf{V_2} + \mathbf{V_s} + \mathbf{V_1} = 0$ or

$$\mathbf{V}_s = -\mathbf{V}_1 - \mathbf{V}_2 = -90(1-j) = (90\angle 180°)(1.4142\angle -45°)$$

$$\mathbf{V}_s = \mathbf{127.28\angle 135°\ V}$$

Solution 9.55



$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{j8} = \frac{4}{j8} = \text{-j}0.5$$

$$\mathbf{I}_2 = \frac{\mathbf{I}_1(\mathbf{Z} + j8)}{-j4} = \frac{(\text{-j}0.5)(\mathbf{Z} + j8)}{-j4} = \frac{\mathbf{Z}}{8} + j$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = \text{-j}0.5 + \frac{\mathbf{Z}}{8} + j = \frac{\mathbf{Z}}{8} + j0.5$$

$$-j20 = 12\mathbf{I} + \mathbf{I}_1(\mathbf{Z} + j8)$$

$$-j20 = 12\left(\frac{\mathbf{Z}}{8} + \frac{j}{2}\right) + \frac{-j}{2}(\mathbf{Z} + j8)$$

$$-4 - j26 = \mathbf{Z}\left(\frac{3}{2} - j\frac{1}{2}\right)$$

$$\mathbf{Z} = \frac{-4 - j26}{\frac{3}{2} - j\frac{1}{2}} = \frac{26.31\angle 261.25°}{1.5811\angle -18.43°} = 16.64\angle 279.68°$$

Z = (2.798 – j16.403) Ω

Solution 9.56

$$50\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j377x50x10^{-6}} = -j53.05$$

$$60mH \quad \longrightarrow \quad j\omega L = j377x60x10^{-3} = j22.62$$

$$Z_{in} = 12 - j53.05 + j22.62 // 40 = \underline{21.692 - j35.91\ \Omega}$$

Solution 9.57

$$2\text{H} \longrightarrow j\omega L = j2$$

$$1\text{F} \longrightarrow \frac{1}{j\omega C} = -j$$

$$\text{Z} = 1 + \text{j}2 // (2 - \text{j}) = 1 + \frac{\text{j}2(2-\text{j})}{\text{j}2 + 2 - \text{j}} = 2.6 + \text{j}1.2$$

$$\boldsymbol{Y = {}^{1}\!/\!_{Z} = \underline{0.3171 - j0.1463\ \text{S}}}$$

Solution 9.58

Using Fig. 9.65, design a problem to help other students to better understand impedance combinations.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

At $\omega = 50$ rad/s, determine $\mathbf{Z}_{in}$ for each of the circuits in Fig. 9.65.

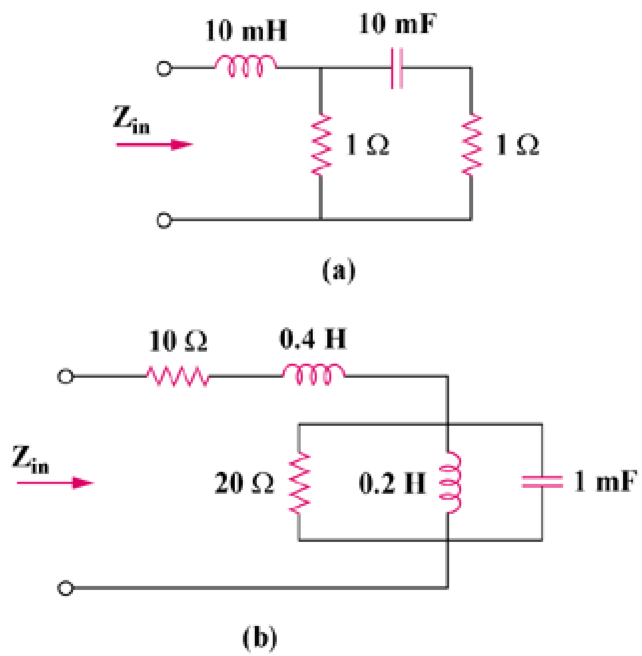


Figure 9.65

Solution

(a) 10 mF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(50)(10\times10^{-3})} = \text{-}j2$

$$10\text{ mH} \longrightarrow j\omega L = j(50)(10\times10^{-3}) = j0.5$$

$$\mathbf{Z}_{in} = j0.5 + 1\,||\,(1 - j2)$$

$$\mathbf{Z}_{in} = j0.5 + \frac{1 - j2}{2 - j2}$$

$$\mathbf{Z}_{in} = j0.5 + 0.25(3 - j)$$

$$\mathbf{Z}_{in} = \mathbf{0.75 + j0.25\ \Omega}$$

(b) $0.4\ \text{H} \longrightarrow j\omega L = j(50)(0.4) = j20$

$0.2\ \text{H} \longrightarrow j\omega L = j(50)(0.2) = j10$

$$1\ \text{mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(50)(1\times 10^{-3})} = -j20$$

For the parallel elements,

$$\frac{1}{\mathbf{Z}_p} = \frac{1}{20} + \frac{1}{j10} + \frac{1}{-j20}$$

$$\mathbf{Z}_p = 10 + j10$$

Then,

$$\mathbf{Z}_{in} = 10 + j20 + \mathbf{Z}_p = \mathbf{20 + j30\ \Omega}$$

Solution 9.59

For the network in Fig. 9.66, find $\mathbf{Z}_{in}$. Let $\omega = 100$ rad/s.

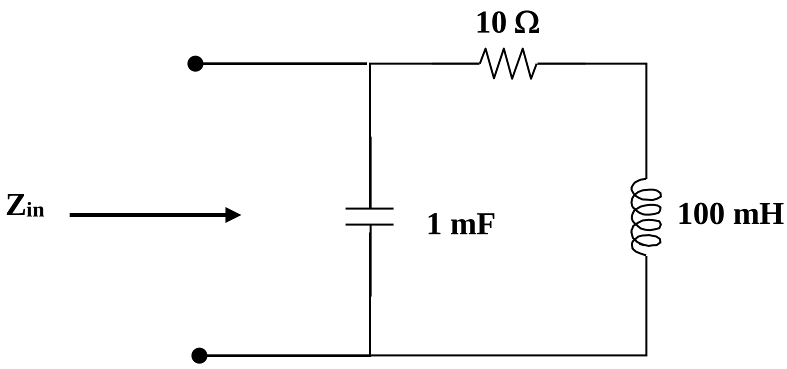


Figure 9.66
For Prob. 9.59.

Solution

At $\omega = 100$ rad/s the capacitor becomes $-j10\ \Omega$ and the inductor becomes $j10\ \Omega$. This then leads to $Z_{in} = (-j10)(10+j10)/(-j10+10+j10) = (100-j100)/10$ or

$$Z_{in} = \mathbf{(10 - j10)\ \Omega = 14.142\angle{-45^\circ}\ \Omega}.$$

Solution 9.60

$$Z = (25 + j15) + (20 - j50)//(30 + j10) = 25 + j15 + 26.097 - j5.122$$

Z = **(51.1+j9.878) Ω**

Solution 9.61

All of the impedances are in parallel.

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{1-j} + \frac{1}{1+j2} + \frac{1}{j5} + \frac{1}{1+j3}$$

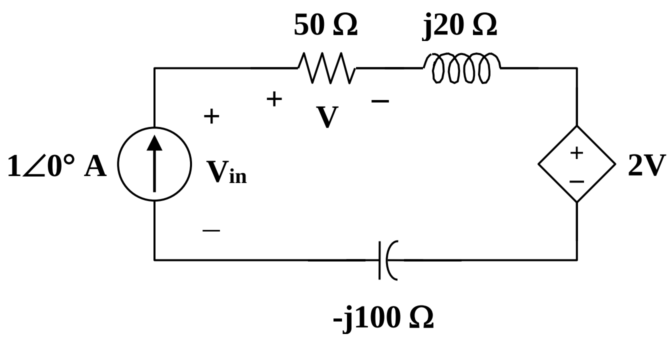
$$\frac{1}{\mathbf{Z}_{eq}} = (0.5 + j0.5) + (0.2 - j0.4) + (-j0.2) + (0.1 - j0.3) = 0.8 - j0.4$$

$$\mathbf{Z}_{eq} = \frac{1}{0.8 - j0.4} = \mathbf{(1 + j0.5)\ \Omega}$$

Solution 9.62

$$2\text{ mH} \longrightarrow j\omega L = j(10\times10^{3})(2\times10^{-3}) = j20$$

$$1\,\mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10\times10^{3})(1\times10^{-6})} = -j100$$



$$\mathbf{V} = (1\angle 0°)(50) = 50$$

$$\mathbf{V}_{in} = (1\angle 0°)(50 + j20 - j100) + (2)(50)$$
$$\mathbf{V}_{in} = 50 - j80 + 100 = 150 - j80$$

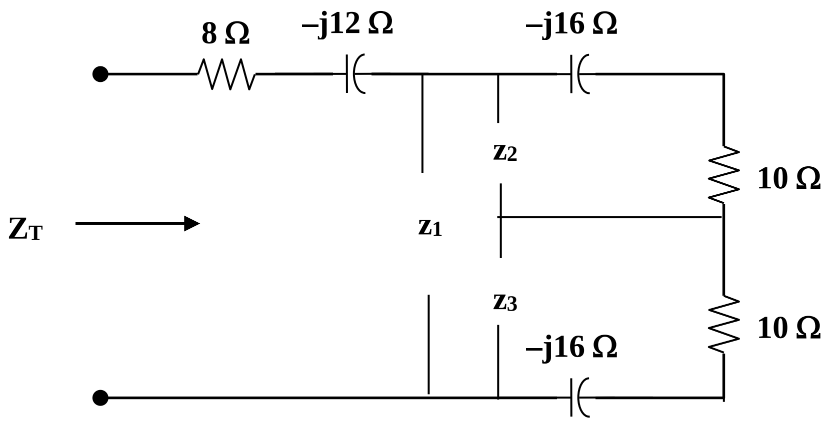
$$\mathbf{Z}_{in} = \frac{\mathbf{V}_{in}}{1\angle 0°} = \mathbf{150 - j80\ \Omega}$$

Solution 9.63

First, replace the wye composed of the 20-ohm, 10-ohm, and j15-ohm impedances with the corresponding delta.

$$z_1 = \frac{200 + j150 + j300}{10} = 20 + j45$$

$$z_2 = \frac{200 + j450}{j15} = 30 - j13.333, \; z_3 = \frac{200 + j450}{20} = 10 + j22.5$$



Now all we need to do is to combine impedances.

$$z_2 \| (10 - j16) = \frac{(30 - j13.333)(10 - j16)}{40 - j29.33} = 8.721 - j8.938$$

$$z_3 \| (10 - j16) = 21.70 - j3.821$$

$$Z_T = 8 - j12 + z_1 \| (8.721 - j8.938 + 21.7 - j3.821) = \underline{34.69 - j6.93\Omega}$$

Solution 9.64

Find $\mathbf{Z_T}$ and $\mathbf{V}$ in the circuit shown in Fig. 9.71. Let the value of the inductance be j20 Ω.

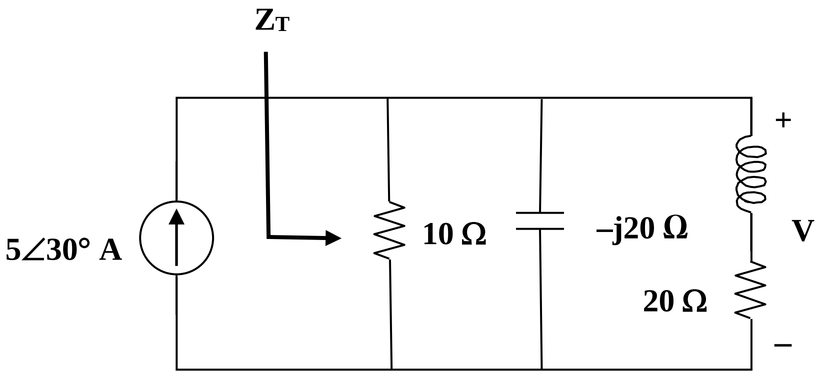


Figure 9.71
For Prob. 9.64.

Solution

$[1/\mathbf{Z_T}] = 0.1 + j0.05 + 0.025 - j0.025 = 0.125 + j0.025 = 0.127475\angle 11.31°$ or
$\mathbf{Z_T} = 7.8447\angle{-11.31°} = (7.6924 - j1.53848)$.

$\mathbf{Z_T} = \mathbf{(7.692 - j1.5385)\ \Omega}$

$\mathbf{V} = \mathbf{I}x\mathbf{Z_T} = (5\angle 30°)(7.8447\angle{-11.31°}) = \mathbf{39.22\angle 18.69°\ V}$.

Solution 9.65

$$\mathbf{Z}_T = 2 + (4 - j6) \,||\, (3 + j4)$$

$$\mathbf{Z}_T = 2 + \frac{(4 - j6)(3 + j4)}{7 - j2}$$

$$\mathbf{Z}_T = \mathbf{6.83 + j1.094\ \Omega} = 6.917\angle 9.1^\circ\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{120\angle 10^\circ}{6.917\angle 9.1^\circ} = \mathbf{17.35\angle 0.9^\circ\ A}$$

Solution 9.66

For the circuit in Fig. 9.73, calculate $\mathbf{Z_T}$ and $\mathbf{V_{ab}}$.

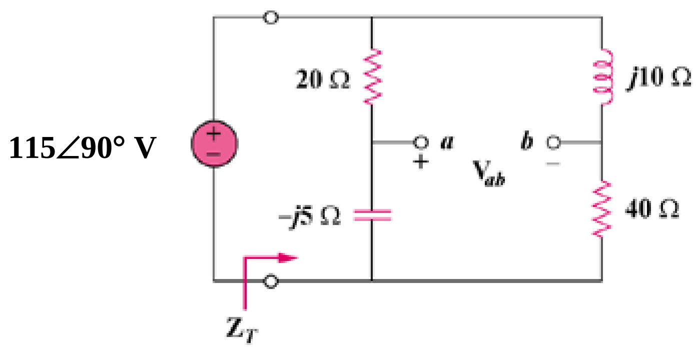


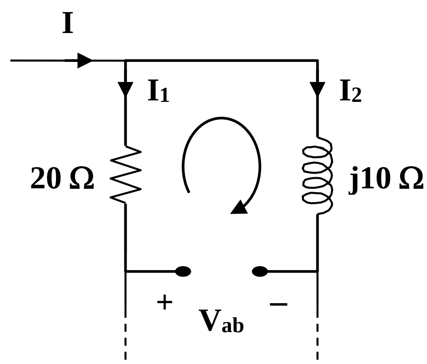
Figure 9.73
For Prob. 9.66.

Solution

$$\mathbf{Z}_T = (20 - j5) \,||\, (40 + j10) = \frac{(20 - j5)(40 + j10)}{60 + j5} = \frac{170}{145}(12 - j)$$

$$\mathbf{Z}_T = \mathbf{14.069 - j1.172\ \Omega} = \mathbf{14.118\angle\text{-}4.76°\ \Omega}$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{115\angle 90°}{14.118\angle\text{-}4.76°} = 8.1456\angle 94.76°$$



$$\mathbf{I}_1 = \frac{40 + j10}{60 + j5}\mathbf{I} = \frac{8 + j2}{12 + j}\mathbf{I}$$

$$\mathbf{I}_2 = \frac{20 - j5}{60 + j5}\mathbf{I} = \frac{4 - j}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = -20\mathbf{I}_1 + j10\mathbf{I}_2$$

$$\mathbf{V}_{ab} = \frac{-(160 + j40)}{12 + j}\mathbf{I} + \frac{10 + j40}{12 + j}\mathbf{I}$$

$$\mathbf{V}_{ab} = \frac{-150}{12 + j}\mathbf{I} = \frac{(-12 + j)(150)}{145}\mathbf{I}$$

$$\mathbf{V}_{ab} = (12.457\angle 175.24°)(8.1456\angle 97.76°)$$

$$\mathbf{V}_{ab} = \mathbf{101.47\angle 273°\ V}$$

Solution 9.67

(a) 20 mH $\longrightarrow$ $j\omega L = j(10^3)(20\times10^{-3}) = j20$

12.5 μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(12.5\times10^{-6})} = -j80$

$$\mathbf{Z}_{in} = 60 + j20 \,||\, (60 - j80)$$

$$\mathbf{Z}_{in} = 60 + \frac{(j20)(60 - j80)}{60 - j60}$$

$$\mathbf{Z}_{in} = 63.33 + j23.33 = 67.494\angle 20.22°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \mathbf{14.8\angle{-20.22}°\ mS}$$

(b) 10 mH $\longrightarrow$ $j\omega L = j(10^3)(10\times10^{-3}) = j10$

20 μF $\longrightarrow$ $\dfrac{1}{j\omega C} = \dfrac{1}{j(10^3)(20\times10^{-6})} = -j50$

$30 \,||\, 60 = 20$

$$\mathbf{Z}_{in} = -j50 + 20 \,||\, (40 + j10)$$

$$\mathbf{Z}_{in} = -j50 + \frac{(20)(40 + j10)}{60 + j10} = -j50 + 20(41.231\angle 14.036°)/(60.828\angle 9.462°)$$

$= -j50 + (13.5566\angle 4.574° = -j50 + 13.51342 + j1.08109$

$= 13.51342 - j48.9189 = 50.751\angle -74.56°$

$$\mathbf{Z}_{in} = 13.5 - j48.92 = 50.75\angle -74.56°$$

$$\mathbf{Y}_{in} = \frac{1}{\mathbf{Z}_{in}} = \mathbf{19.704\angle 74.56°\ mS} = 5.246 + j18.993\ \text{mS}$$

Solution 9.68

$$\mathbf{Y}_{eq} = \frac{1}{5-j2} + \frac{1}{3+j} + \frac{1}{-j4}$$

$$\mathbf{Y}_{eq} = (0.1724 + j0.069) + (0.3 - j0.1) + (j0.25)$$

$$\mathbf{Y}_{eq} = \mathbf{(472.4 + j219)\ mS}$$

Solution 9.69

$$\frac{1}{\mathbf{Y}_o} = \frac{1}{4} + \frac{1}{-j2} = \frac{1}{4}(1 + j2)$$

$$\mathbf{Y}_o = \frac{4}{1 + j2} = \frac{(4)(1 - j2)}{5} = 0.8 - j1.6$$

$$\mathbf{Y}_o + j = 0.8 - j0.6$$

$$\frac{1}{\mathbf{Y}_o'} = \frac{1}{1} + \frac{1}{-j3} + \frac{1}{0.8 - j0.6} = (1) + (j0.333) + (0.8 + j0.6)$$

$$\frac{1}{\mathbf{Y}_o'} = 1.8 + j0.933 = 2.028\angle 27.41^\circ$$

$$\mathbf{Y}_o' = 0.4932\angle -27.41^\circ = 0.4378 - j0.2271$$

$$\mathbf{Y}_o' + j5 = 0.4378 + j4.773$$

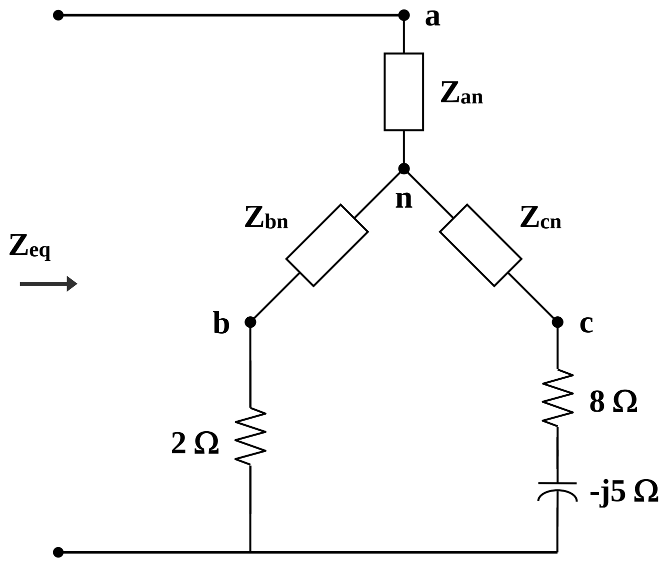
$$\frac{1}{\mathbf{Y}_{eq}} = \frac{1}{2} + \frac{1}{0.4378 + j4.773} = 0.5 + \frac{0.4378 - j4.773}{22.97}$$

$$\frac{1}{\mathbf{Y}_{eq}} = 0.5191 - j0.2078$$

$$\mathbf{Y}_{eq} = \frac{0.5191 - j0.2078}{0.3126} = \mathbf{(1.661 + j0.6647)\ S}$$

Solution 9.70

Make a delta-to-wye transformation as shown in the figure below.



$$\mathbf{Z}_{an} = \frac{(\text{-}j10)(10+j15)}{5-j10+10+j15} = \frac{(10)(15-j10)}{15+j5} = 7-j9$$

$$\mathbf{Z}_{bn} = \frac{(5)(10+j15)}{15+j5} = 4.5+j3.5$$

$$\mathbf{Z}_{cn} = \frac{(5)(\text{-}j10)}{15+j5} = \text{-}1-j3$$

$$\mathbf{Z}_{eq} = \mathbf{Z}_{an} + (\mathbf{Z}_{bn}+2) \,||\, (\mathbf{Z}_{cn}+8-j5)$$

$$\mathbf{Z}_{eq} = 7-j9+(6.5+j3.5) \,||\, (7-j8)$$

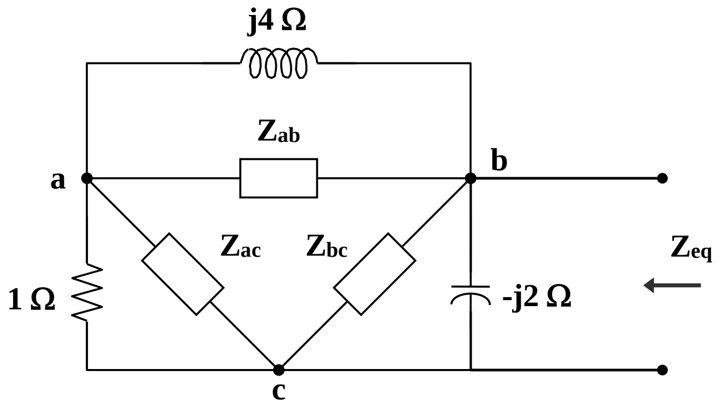
$$\mathbf{Z}_{eq} = 7-j9+\frac{(6.5+j3.5)(7-j8)}{13.5-j4.5}$$

$$\mathbf{Z}_{eq} = 7-j9+5.511-j0.2$$

$$\mathbf{Z}_{eq} = 12.51-j9.2 = \mathbf{15.53\angle\text{-}36.33^\circ\ \Omega}$$

Solution 9.71

We apply a wye-to-delta transformation.



$$\mathbf{Z}_{ab} = \frac{2 - j2 + j4}{j2} = \frac{2 + j2}{j2} = 1 - j$$

$$\mathbf{Z}_{ac} = \frac{2 + j2}{2} = 1 + j$$

$$\mathbf{Z}_{bc} = \frac{2 + j2}{-j} = -2 + j2$$

$$j4 \,||\, \mathbf{Z}_{ab} = j4 \,||\, (1 - j) = \frac{(j4)(1 - j)}{1 + j3} = 1.6 - j0.8$$

$$1 \,||\, \mathbf{Z}_{ac} = 1 \,||\, (1 + j) = \frac{(1)(1 + j)}{2 + j} = 0.6 + j0.2$$

$$j4 \,||\, \mathbf{Z}_{ab} + 1 \,||\, \mathbf{Z}_{ac} = 2.2 - j0.6$$

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{1}{-j2} + \frac{1}{-2 + j2} + \frac{1}{2.2 - j0.6}$$

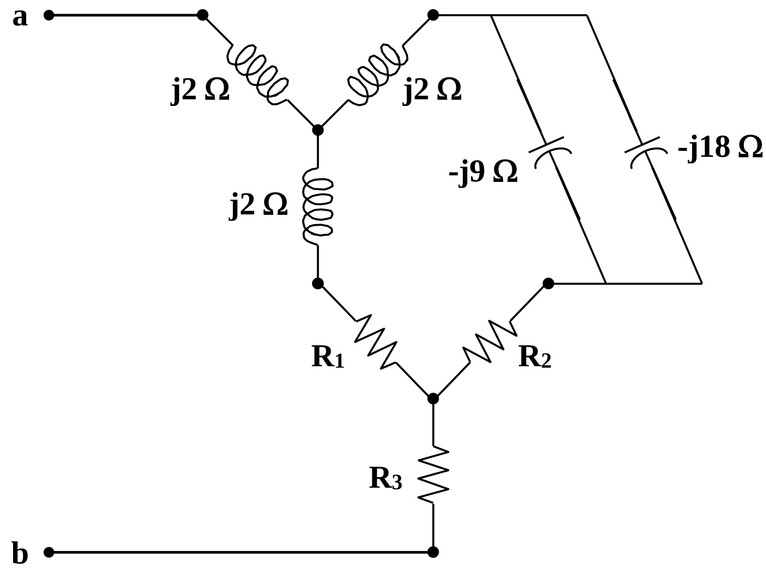
$$= j0.5 - 0.25 - j0.25 + 0.4231 + j0.1154$$

$$= 0.173 + j0.3654 = 0.4043\angle 64.66^\circ$$

$$\mathbf{Z}_{eq} = 2.473\angle{-64.66^\circ}\ \Omega = \mathbf{(1.058 - j2.235)\ \Omega}$$

Solution 9.72

Transform the delta connections to wye connections as shown below.



$$-\,j9 \parallel -j18 = -j6,$$

$$R_1 = \frac{(20)(20)}{20+20+10} = 8\,\Omega, \qquad R_2 = \frac{(20)(10)}{50} = 4\,\Omega, \qquad R_3 = \frac{(20)(10)}{50} = 4\,\Omega$$

$$\mathbf{Z}_{ab} = j2 + (j2+8) \parallel (j2 - j6 + 4) + 4$$

$$\mathbf{Z}_{ab} = 4 + j2 + (8 + j2) \parallel (4 - j4)$$

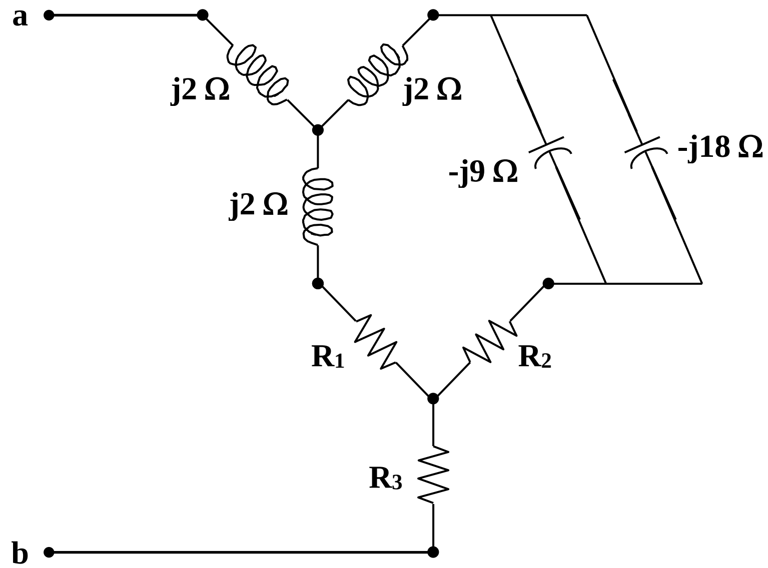
$$\mathbf{Z}_{ab} = 4 + j2 + \frac{(8+j2)(4-j4)}{12 - j2}$$

$$\mathbf{Z}_{ab} = 4 + j2 + 3.567 - j1.4054$$

$$\mathbf{Z}_{ab} = \mathbf{(7.567 + j0.5946)\ \Omega}$$

Solution 9.73

Transform the delta connection to a wye connection as in Fig. (a) and then transform the wye connection to a delta connection as in Fig. (b).



$$\mathbf{Z}_1 = \frac{(j8)(-j6)}{j8 + j8 - j6} = \frac{48}{j10} = -j4.8$$

$$\mathbf{Z}_2 = \mathbf{Z}_1 = -j4.8$$

$$\mathbf{Z}_3 = \frac{(j8)(j8)}{j10} = \frac{-64}{j10} = j6.4$$

$$(2 + \mathbf{Z}_1)(4 + \mathbf{Z}_2) + (4 + \mathbf{Z}_2)(\mathbf{Z}_3) + (2 + \mathbf{Z}_1)(\mathbf{Z}_3) =$$
$$(2 - j4.8)(4 - j4.8) + (4 - j4.8)(j6.4) + (2 - j4.8)(j6.4) = 46.4 + j9.6$$

$$\mathbf{Z}_a = \frac{46.4 + j9.6}{j6.4} = 1.5 - j7.25$$

$$\mathbf{Z}_b = \frac{46.4 + j9.6}{4 - j4.8} = 3.574 + j6.688$$

$$\mathbf{Z}_c = \frac{46.4 + j9.6}{2 - j4.8} = 1.727 + j8.945$$

$$j6 \parallel \mathbf{Z}_b = \frac{(6\angle 90^\circ)(7.583\angle 61.88^\circ)}{3.574 + j12.688} = 07407 + j3.3716$$

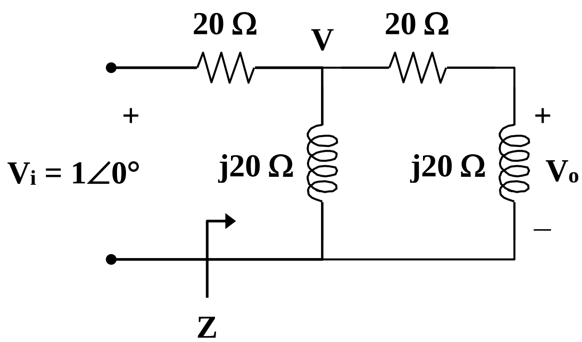
$$-j4 \parallel \mathbf{Z}_a = \frac{(-j4)(1.5 - j7.25)}{1.5 - j11.25} = 0.186 - j2.602$$

$$\text{j}12 \parallel \mathbf{Z}_c = \frac{(12\angle 90°)(9.11\angle 79.07°)}{1.727 + \text{j}20.945} = 0.5634 + \text{j}5.1693$$

$$\mathbf{Z}_{eq} = (\text{j}6 \parallel \mathbf{Z}_b) \parallel (\text{-j}4 \parallel \mathbf{Z}_a + \text{j}12 \parallel \mathbf{Z}_c)$$
$$\mathbf{Z}_{eq} = (0.7407 + \text{j}3.3716) \parallel (0.7494 + \text{j}2.5673)$$
$$\mathbf{Z}_{eq} = 1.508\angle 75.42°\ \Omega = \mathbf{(0.3796 + j1.46)\ \Omega}$$

Solution 9.74

One such RL circuit is shown below.



We now want to show that this circuit will produce a 90° phase shift.

$$\mathbf{Z} = \text{j}20 \,||\, (20 + \text{j}20) = \frac{(\text{j}20)(20+\text{j}20)}{20+\text{j}40} = \frac{-20+\text{j}20}{1+\text{j}2} = 4(1+\text{j}3)$$

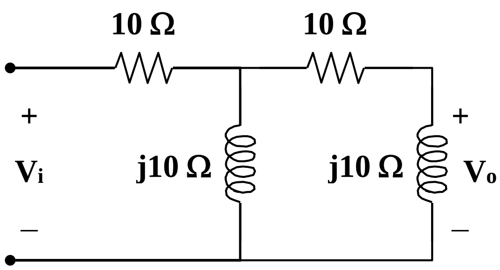
$$\mathbf{V} = \frac{\mathbf{Z}}{\mathbf{Z}+20}\mathbf{V}_\text{i} = \frac{4+\text{j}12}{24+\text{j}12}(1\angle 0°) = \frac{1+\text{j}3}{6+\text{j}3} = \frac{1}{3}(1+\text{j})$$

$$\mathbf{V}_\text{o} = \frac{\text{j}20}{20+\text{j}20}\mathbf{V} = \left(\frac{\text{j}}{1+\text{j}}\right)\left(\frac{1}{3}(1+\text{j})\right) = \frac{\text{j}}{3} = 0.3333\angle 90°$$

This shows that the output leads the input by 90°.

Solution 9.75

Since $\cos(\omega t) = \sin(\omega t + 90°)$, we need a phase shift circuit that will cause the output to lead the input by 90°. **This is achieved by the RL circuit shown below, as explained in the previous problem**.



This can also be obtained by an RC circuit.

Solution 9.76

(a) $v_2 = 8\sin 5t = 8\cos(5t - 90^o)$

v$_1$ leads v$_2$ by 70^o.

(b) $v_2 = 6\sin 2t = 6\cos(2t - 90^o)$

v$_1$ leads v$_2$ by 180^o.

(c) $v_1 = -4\cos 10t = 4\cos(10t + 180^o)$

$v_2 = 15\sin 10t = 15\cos(10t - 90^o)$

v$_1$ leads v$_2$ by 270^o.

Solution 9.77

Refer to the *RC* circuit in Fig. 9.81.
(a) Calculate the phase shift at 2 MHz.
(b) Find the frequency where the phase shift is 45°.

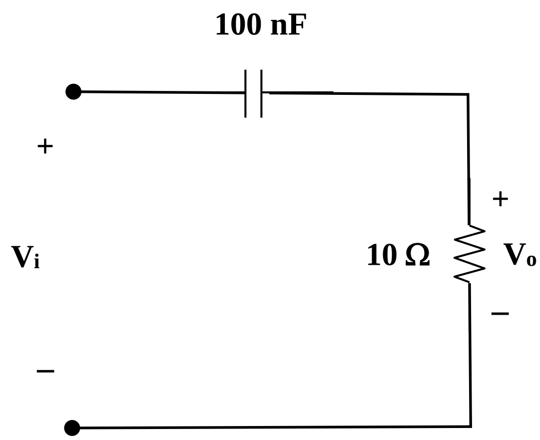


Figure 9.81
For Prob. 9.77.

Solution

In the frequency domain, the capacitance is equal to $-j10^7/\omega$ which leads to,

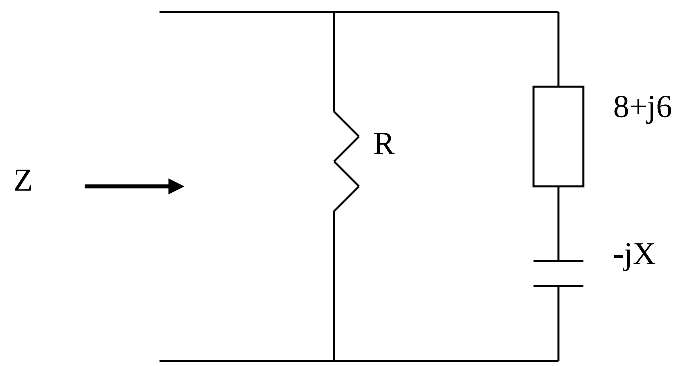
$V_o = V_i(10)/(10-j10^7/\omega)$.

For $\omega = 2\text{x}10^6$ $V_o = V_i(10)/(10-j5) = V_i(10)/(11.18034\angle-26.565°) = 0.8944V_i\angle 26.57°$ which produces a phase shift = **26.57º (lagging)**.

For a 45° phase shift we need to have $(10^7/\omega) = 10$ or

ω = **1 MHz**.

Solution 9.78



$$Z = R//[8 + j(6 - X)] = \frac{R[8 + j(6 - X)]}{R + 8 + j(6 - X)} = 5$$

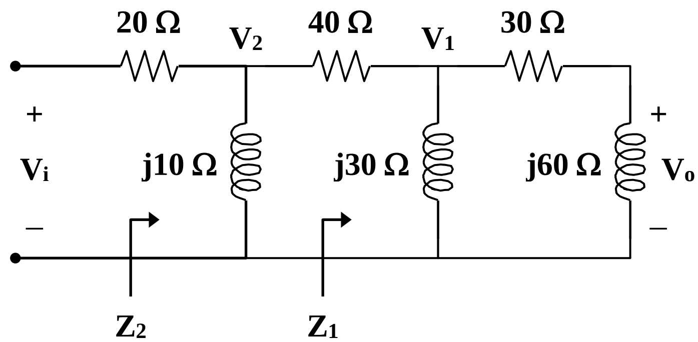
i.e. 8R + j6R – jXR = 5R + 40 + j30 –j5X

Equating real and imaginary parts:

8R = 5R + 40 which leads to **R=13.333Ω**
6R-XR =30-5X which leads to **X= 6 Ω**.

Solution 9.79

(a) Consider the circuit as shown.



$$\mathbf{Z}_1 = j30 \parallel (30 + j60) = \frac{(j30)(30 + j60)}{30 + j90} = 3 + j21$$

$$\mathbf{Z}_2 = j10 \parallel (40 + \mathbf{Z}_1) = \frac{(j10)(43 + j21)}{43 + j31} = 1.535 + j8.896 = 9.028\angle 80.21°$$

Let $\mathbf{V}_i = 1\angle 0°$.

$$\mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2 + 20}\mathbf{V}_i = \frac{(9.028\angle 80.21°)(1\angle 0°)}{21.535 + j8.896}$$

$$\mathbf{V}_2 = 0.3875\angle 57.77°$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + 40}\mathbf{V}_2 = \frac{3 + j21}{43 + j21}\mathbf{V}_2 = \frac{(21.213\angle 81.87°)(0.3875\angle 57.77°)}{47.85\angle 26.03°}$$

$$\mathbf{V}_1 = 0.1718\angle 113.61°$$

$$\mathbf{V}_o = \frac{j60}{30 + j60}\mathbf{V}_1 = \frac{j2}{1 + j2}\mathbf{V}_1 = \frac{2}{5}(2 + j)\mathbf{V}_1$$

$$\mathbf{V}_o = (0.8944\angle 26.56°)(0.1718\angle 113.6°)$$

$$\mathbf{V}_o = 0.1536\angle 140.2°$$

Therefore, the phase shift is **140.2°**

(b) The phase shift is **leading**.

(c) If $\mathbf{V}_i = 120\text{ V}$, then

$$\mathbf{V}_o = (120)(0.1536\angle 140.2°) = 18.43\angle 140.2°\text{ V}$$

and the magnitude is **18.43 V**.

Solution 9.80

$$200\text{ mH} \longrightarrow \quad j\omega L = j(2\pi)(60)(200\times 10^{-3}) = j75.4\,\Omega$$

$$\mathbf{V}_o = \frac{j75.4}{R+50+j75.4}\mathbf{V}_i = \frac{j75.4}{R+50+j75.4}(120\angle 0°)$$

(a) When $R = 100\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{150+j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{167.88\angle 26.69°}$$

$\mathbf{V}_o =$ **$53.89\angle 63.31°$ V**

(b) When $R = 0\,\Omega$,

$$\mathbf{V}_o = \frac{j75.4}{50+j75.4}(120\angle 0°) = \frac{(75.4\angle 90°)(120\angle 0°)}{90.47\angle 56.45°}$$

$\mathbf{V}_o =$ **$100\angle 33.55°$ V**

(c) To produce a phase shift of 45°, the phase of $\mathbf{V}_o = 90° + 0° - \alpha = 45°$.

Hence, α = phase of $(R + 50 + j75.4) = 45°$.

For α to be 45°, $R + 50 = 75.4$

Therefore, $R =$ **$25.4\ \Omega$**

Solution 9.81

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + \dfrac{1}{j\omega C_x}$.

$$\mathbf{Z}_x = \frac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$$

$$R_x + \frac{1}{j\omega C_x} = \frac{R_3}{R_1}\left(R_2 + \frac{1}{j\omega C_2}\right)$$

$$R_x = \frac{R_3}{R_1}R_2 = \frac{1200}{400}(600) = \mathbf{1.8\ k\Omega}$$

$$\frac{1}{C_x} = \left(\frac{R_3}{R_1}\right)\left(\frac{1}{C_2}\right) \longrightarrow C_x = \frac{R_1}{R_3}C_2 = \left(\frac{400}{1200}\right)(0.3\times 10^{-6}) = \mathbf{0.1\ \mu F}$$

Solution 9.82

$$C_x = \frac{R_1}{R_2} C_s = \left(\frac{100}{2000}\right)(40 \times 10^{-6}) = \mathbf{2\ \mu F}$$

Solution 9.83

$$L_x = \frac{R_2}{R_1} L_s = \left(\frac{500}{1200}\right)(250 \times 10^{-3}) = \mathbf{104.17\ mH}$$

Solution 9.84

Let $\mathbf{Z}_1 = R_1 \parallel \dfrac{1}{j\omega C_s}$, $\mathbf{Z}_2 = R_2$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_x = R_x + j\omega L_x$.

$$\mathbf{Z}_1 = \frac{\dfrac{R_1}{j\omega C_s}}{R_1 + \dfrac{1}{j\omega C_s}} = \frac{R_1}{j\omega R_1 C_s + 1}$$

Since $\mathbf{Z}_x = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2$,

$$R_x + j\omega L_x = R_2 R_3 \frac{j\omega R_1 C_s + 1}{R_1} = \frac{R_2 R_3}{R_1}(1 + j\omega R_1 C_s)$$

Equating the real and imaginary components,

$$\mathbf{R_x = \frac{R_2 R_3}{R_1}}$$

$$\omega L_x = \frac{R_2 R_3}{R_1}(\omega R_1 C_s) \text{ implies that}$$

$$\mathbf{L_x = R_2 R_3 C_s}$$

Given that $R_1 = 40\ k\Omega$, $R_2 = 1.6\ k\Omega$, $R_3 = 4\ k\Omega$, and $C_s = 0.45\ \mu F$

$$R_x = \frac{R_2 R_3}{R_1} = \frac{(1.6)(4)}{40}\ k\Omega = 0.16\ k\Omega = \mathbf{160\ \Omega}$$

$$L_x = R_2 R_3 C_s = (1.6)(4)(0.45) = \mathbf{2.88\ H}$$

Solution 9.85

Let $\mathbf{Z}_1 = R_1$, $\mathbf{Z}_2 = R_2 + \dfrac{1}{j\omega C_2}$, $\mathbf{Z}_3 = R_3$, and $\mathbf{Z}_4 = R_4 \parallel \dfrac{1}{j\omega C_4}$.

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

Since $\mathbf{Z}_4 = \dfrac{\mathbf{Z}_3}{\mathbf{Z}_1}\mathbf{Z}_2 \longrightarrow \mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$,

$$\frac{-jR_4R_1}{\omega R_4 C_4 - j} = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

$$\frac{-jR_4R_1(\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} = R_3R_2 - \frac{jR_3}{\omega C_2}$$

Equating the real and imaginary components,

$$\frac{R_1R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2R_3 \qquad (1)$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \qquad (2)$$

Dividing (1) by (2),

$$\frac{1}{\omega R_4 C_4} = \omega R_2 C_2$$

$$\omega^2 = \frac{1}{R_2 C_2 R_4 C_4}$$

$$\omega = 2\pi f = \frac{1}{\sqrt{R_2 C_2 R_4 C_4}}$$

$$\mathbf{f = \frac{1}{2\pi\sqrt{R_2 R_4 C_2 C_4}}}$$

Solution 9.86

$$\mathbf{Y} = \frac{1}{240} + \frac{1}{j95} + \frac{1}{-j84}$$

$$\mathbf{Y} = 4.1667 \times 10^{-3} - j0.01053 + j0.0119$$

$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1000}{4.1667 + j1.37} = \frac{1000}{4.3861\angle 18.2°}$$

$\mathbf{Z} = \mathbf{228\angle\text{-}18.2° \ \Omega}$

Solution 9.87

The network in Fig. 9.87 is part of the schematic describing an industrial electronic sensing device. What is the total impedance of the circuit at 4 kHz?

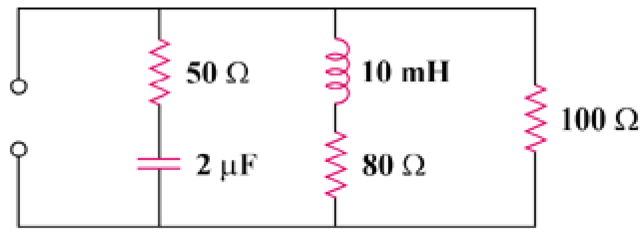


Figure 9.87
For Prob. 9.87.

Solution

$$\mathbf{Z}_1 = 50 + \frac{1}{j\omega C} = 50 + \frac{-j}{(2\pi)(4\times10^3)(2\times10^{-6})}$$
$$\mathbf{Z}_1 = 50 - j19.8944 = 53.813\angle{-21.697^\circ}$$

$$\mathbf{Z}_2 = 80 + j\omega L = 80 + j(2\pi)(4\times10^3)(10\times10^{-3})$$
$$\mathbf{Z}_2 = 80 + j251.327 = 263.752\angle 72.343^\circ$$

$$\mathbf{Z}_3 = 100$$

$$\frac{1}{\mathbf{Z}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3}$$
$$\frac{1}{\mathbf{Z}} = (0.0185829)\angle 21.697^\circ + (0.0037914)\angle{-72.343^\circ} + 0.01$$
$$= 0.0172663 + j0.0068701 + 0.00115 - j0.0036128 + 0.01 = 0.0284163 + j0.0032573$$
$$= 0.028602\angle 6.539^\circ \text{ or}$$

Z = 1/0.028602∠6.539° = **34.96∠–6.54° Ω**
= **(34.73 – j3.982) Ω**.

Solution 9.88

(a) $\mathbf{Z} = -j20 + j30 + 120 - j20$

$\mathbf{Z} = \mathbf{(120 - j10)\ \Omega}$

(b) If the frequency were halved, $\frac{1}{\omega C} = \frac{1}{2\pi f\ C}$ would cause the capacitive impedance to double, while $\omega L = 2\pi f\ L$ would cause the inductive impedance to halve.

Thus,

$\mathbf{Z} = -j40 + j15 + 120 - j40$

$\mathbf{Z} = \mathbf{(120 - j65)\ \Omega}$

Solution 9.89

An industrial load is modeled as a series combination of an inductor and a resistance as shown in Fig. 9.89. Calculate the value of a capacitor C across the series combination so that the net impedance is resistive at a frequency of 2 kHz.

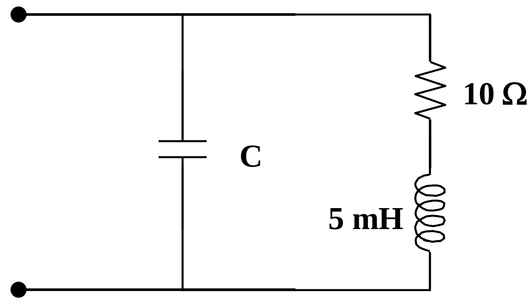


Figure 9.89
For Prob. 9.89.

Solution

Step 1.

There are different ways to solve this problem but perhaps the easiest way is to convert the series R L elements into their parallel equivalents. Then all you need to do is to make the inductance and capacitance cancel each other out to result in a purely resistive circuit.

$X_L = 2x10^3x5x10^{-3} = 10$ which leads to $\mathbf{Y} = 1/(10+j10) = 0.05–j0.05$ or a 20**Ω** resistor in parallel with a j20**Ω** inductor. $X_c = 1/(2x10^3C)$ and the parallel combination of the capacitor and inductor is equal to,

$[(–jX_C)(j20)/(–jX_C+j20)]$.

Step 2.

Now we just need to set $X_C = 20 = 1/(2x10^3C)$ which will create an open circuit.

$$C = 1/(20x2x10^3) = \mathbf{25\ \mu F}.$$

Solution 9.90

Let $\mathbf{V}_s = 145\angle 0°$, $X = \omega L = (2\pi)(60)\,L = 377\,L$

$$\mathbf{I} = \frac{\mathbf{V}_s}{80 + R + jX} = \frac{145\angle 0°}{80 + R + jX}$$

$$\mathbf{V}_1 = 80\,\mathbf{I} = \frac{(80)(145)}{80 + R + jX}$$

$$50 = \left|\frac{(80)(145)}{80 + R + jX}\right| \qquad (1)$$

$$\mathbf{V}_o = (R + jX)\,\mathbf{I} = \frac{(R + jX)(145\angle 0°)}{80 + R + jX}$$

$$110 = \left|\frac{(R + jX)(145)}{80 + R + jX}\right| \qquad (2)$$

From (1) and (2),

$$\frac{50}{110} = \frac{80}{|R + jX|}$$

$$|R + jX| = (80)\left(\frac{11}{5}\right)$$

$$R^2 + X^2 = 30976 \qquad (3)$$

From (1),

$$|80 + R + jX| = \frac{(80)(145)}{50} = 232$$

$$6400 + 160R + R^2 + X^2 = 53824$$

$$160R + R^2 + X^2 = 47424 \qquad (4)$$

Subtracting (3) from (4),

$$160R = 16448 \longrightarrow R = \mathbf{102.8\ \Omega}$$

From (3),

$$X^2 = 30976 - 10568 = 20408$$

$$X = 142.86 = 377L \longrightarrow L = \mathbf{378.9\ mH}$$

Solution 9.91

Figure 9.91 shows a series combination of an inductance and a resistance. If it is desired to connect a capacitor in parallel with the series combination such that the net impedance is resistive at 10 kHz, what is the required value of *C*?

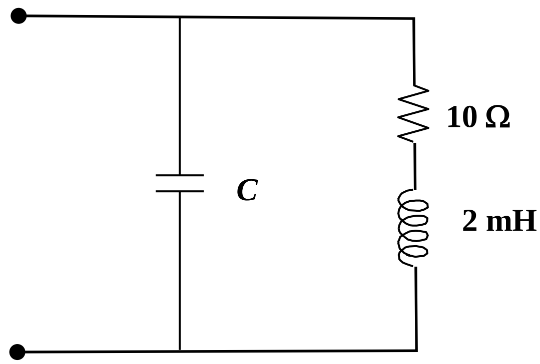


Figure 9.91
For Prob. 9.91.

Solution

At 10 kHz the inductive reactance is equal to j20 Ω and the capacitive reactance is equal to –j/(10^4C). The easiest way to eliminate the effect of the inductor with the capacitor is to convert the resistor and inductor to admittance. Thus,

1/(10+j20) = (10–j20)/(100+400) = 0.02–j0.04. This leads to 10^4C = 0.04 or

C = 0.04x10^{-4} = **4 μF**.

Note the resultant resistance is equal to 1/0.02 = 50 Ω.

Solution 9.92

(a) $\boldsymbol{Z_o = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{100\angle 75^o}{450\angle 48^o\, x10^{-6}}} = \underline{471.4\angle 13.5^o\,\Omega}}$

(b) $\boldsymbol{\gamma = \sqrt{ZY} = \sqrt{100\angle 75^o\, x450\angle 48^o\, x10^{-6}} = \underline{212.1\angle 61.5^o}\ mS}$

Solution 9.93

$$\mathbf{Z} = \mathbf{Z}_s + 2\,\mathbf{Z}_\ell + \mathbf{Z}_L$$
$$\mathbf{Z} = (1 + 0.8 + 23.2) + j(0.5 + 0.6 + 18.9)$$
$$\mathbf{Z} = 25 + j20$$

$$\mathbf{I}_L = \frac{\mathbf{V}_S}{\mathbf{Z}} = \frac{115\angle 0^\circ}{32.02\angle 38.66^\circ}$$

$$\mathbf{I}_L = \mathbf{3.592\angle{-38.66^\circ}\ A}$$