Solution 12.1

(a) If $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle -30° = \mathbf{231\angle -30°\ V}$$

$$\mathbf{V}_{bn} = \mathbf{231\angle -150°\ V}$$

$$\mathbf{V}_{cn} = \mathbf{231\angle -270°\ V}$$

(b) For the acb sequence,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0° - V_p\angle 120°$$

$$\mathbf{V}_{ab} = V_p\left(1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = V_p\sqrt{3}\angle -30°$$

i.e. in the acb sequence, $\mathbf{V}_{ab}$ lags $\mathbf{V}_{an}$ by 30°.

Hence, if $\mathbf{V}_{ab} = 400$, then

$$\mathbf{V}_{an} = \frac{400}{\sqrt{3}}\angle 30° = \mathbf{231\angle 30°\ V}$$

$$\mathbf{V}_{bn} = \mathbf{231\angle 150°\ V}$$

$$\mathbf{V}_{cn} = \mathbf{231\angle -90°\ V}$$

Solution 12.2

Since phase c lags phase a by 120°, this is an **acb sequence**.

$$\mathbf{V}_{bn} = 120\angle(30° + 120°) = \mathbf{120\angle 150°}\ \boldsymbol{V}$$

Solution 12.3

Given a balanced Y-connected three-phase generator with a line-to-line voltage of $\mathbf{V_{ab}} = 100\angle 45°$ V and $\mathbf{V_{bc}} = 100\angle 165°$ V, determine the phase sequence and the value of $\mathbf{V_{ca}}$.

Solution

Since $\mathbf{V_{bc}}$ leads $\mathbf{V_{ab}}$ by 120° we have a **acb** sequence and $\mathbf{V_{ca}} = \mathbf{100\angle{-75°}\ V}$.

Solution 12.4

Knowing the line-to-line voltages we can calculate the wye voltages and can let the value of V_a be a reference with a phase shift of zero degrees.

$V_L = 440 = \sqrt{3}\,V_p$ or $V_p = 440/1.7321 = 254$ V or $\mathbf{V_{an}} = 254\angle 0°$ V which determines, using abc rotation, both $\mathbf{V_{bn}} = 254\angle{-120°}$ and $\mathbf{V_{cn}} = 254\angle 120°$.

$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y = 254/(40\angle 30°) = \mathbf{6.35\angle{-30°}\ A}$

$\mathbf{I}_b = \mathbf{I}_a\angle{-120°} = \mathbf{6.35\angle{-150°}\ A}$

$\mathbf{I}_c = \mathbf{I}_a\angle{+120°} = \mathbf{6.35\angle 90°\ A}$

Solution 12.5

$\mathbf{V}_{AB}$ = 1.7321x$\mathbf{V}_{AN}\angle$+30˚ = 207.8$\angle$(32˚+30˚) = 207.8$\angle$62˚ V or

v_{AB} = **207.8cos(ωt+62˚) V**
which also leads to,
v_{BC} = **207.8cos(ωt–58˚) V**

and
v_{CA} = **207.8cos(ωt+182˚) V**

207.8cos(ωt+62˚) V, 207.8cos(ωt–58˚) V, 207.8cos(ωt+182˚) V

Solution 12.6

Using Fig. 12.41, design a problem to help other students to better understand balanced wye-wye connected circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the Y-Y circuit of Fig. 12.41, find the line currents, the line\ voltages, and the load voltages.

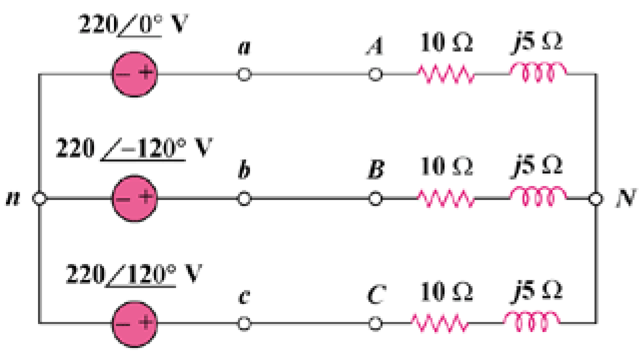


Figure 12.41

Solution

$$\mathbf{Z}_Y = 10 + j5 = 11.18\angle 26.56^\circ$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{220\angle 0^\circ}{11.18\angle 26.56^\circ} = \mathbf{19.68\angle -26.56^\circ\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = \mathbf{19.68\angle -146.56^\circ\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = \mathbf{19.68\angle 93.44^\circ\ A}$$

The line voltages are

$$\mathbf{V}_{ab} = 220\sqrt{3}\angle 30^\circ = \mathbf{381\angle 30^\circ\ V}$$

$$\mathbf{V}_{bc} = \mathbf{381\angle -90^\circ\ V}$$

$$\mathbf{V}_{ca} = \mathbf{381\angle -210^\circ\ V}$$

The load voltages are

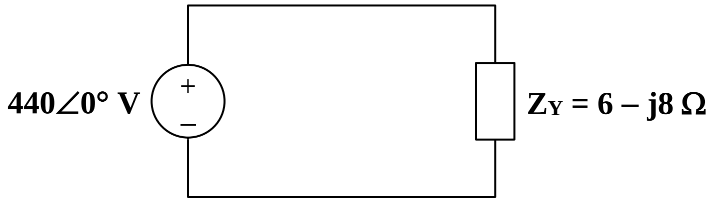
$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \mathbf{V}_{an} = \mathbf{220\angle 0^\circ\ V}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{bn} = \mathbf{220\angle -120^\circ\ V}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{cn} = \mathbf{220\angle 120^\circ\ V}$$

Solution 12.7

This is a balanced Y-Y system.



Using the per-phase circuit shown above,

$$\mathbf{I}_a = \frac{440\angle 0^\circ}{6 - j8} = \mathbf{44\angle 53.13^\circ\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = \mathbf{44\angle -66.87^\circ\ A}$$

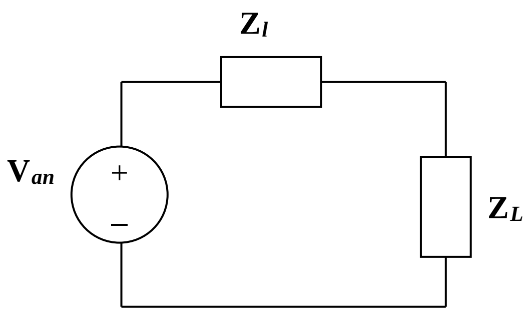
$$\mathbf{I}_c = \mathbf{I}_a \angle 120^\circ = \mathbf{44\angle 173.13^\circ\ A}$$

Solution 12.8

In a balanced three-phase wye-wye system, the source is an acb-sequence of voltages and $\mathbf{V}_{cn} = 120\angle 35^\circ$ V rms. The line impedance per phase is (1+j2)Ω, while the per phase impedance of the load is (11+j14) Ω. Calculate the line currents and the load voltages.

Solution

Consider the per phase equivalent circuit shown below.



Since the sequence is acb and $\mathbf{V}_{cn} = 120\angle 35^\circ$ V, then $\mathbf{V}_{an} = 120\angle 155^\circ$ V, and $\mathbf{V}_{bn} = 120\angle -85^\circ$ V.

$\mathbf{I}_a = \mathbf{V}_{an}/(\mathbf{Z}_l + \mathbf{Z}_L) = (120\angle 155^\circ)/(12+j16) = (120\angle 155^\circ)/(20\angle 53.13^\circ)$

$= \mathbf{6\angle 101.87^\circ}$ **amps**.

$\mathbf{I}_b = \mathbf{I}_a \angle 120^\circ = \mathbf{6\angle 221.87^\circ}$ **amps**.

$\mathbf{I}_c = \mathbf{I}_a \angle -120^\circ = \mathbf{6\angle -18.13^\circ}$ **amps**.

$\mathbf{V}_{La} = \mathbf{I}_a\mathbf{Z}_L = (6\angle 101.87^\circ)(11+j14) = (6\angle 101.87^\circ)(17.8045\angle 51.843^\circ)$

$= \mathbf{106.83\angle 153.71^\circ}$ **volts**.

$\mathbf{V}_{Lb} = \mathbf{V}_{La} \angle 120^\circ = \mathbf{106.83\angle -86.29^\circ}$ **volts**.

$\mathbf{V}_{Lc} = \mathbf{V}_{La} \angle -120^\circ = \mathbf{106.83\angle 33.71^\circ}$ **volts**.

Solution 12.9

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_L + \mathbf{Z}_Y} = \frac{120\angle 0°}{20 + j15} = \mathbf{4.8\angle -36.87°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a \angle -120° = \mathbf{4.8\angle -156.87°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{4.8\angle 83.13°\ A}$$

As a balanced system, $\mathbf{I}_n = \mathbf{0\ A}$

Solution 12.10

For the circuit in Fig. 12.43, determine the current in the neutral line.

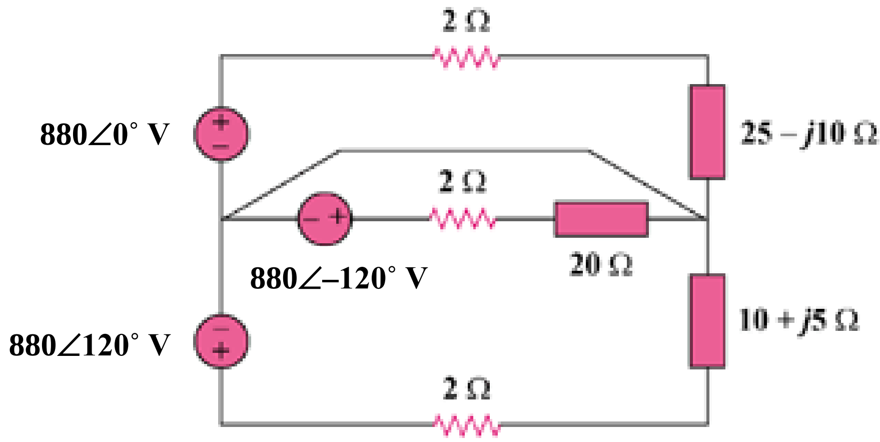


Figure 12.43
For Probs. 12.10 and 12.58.

Solution

Since the neutral line is present, we can solve this problem on a per-phase basis.

For phase a,

$$\mathbf{I_a} = \frac{\mathbf{V_{an}}}{\mathbf{Z_A} + 2} = \frac{880\angle\mathbf{0}°}{27 - \mathbf{j}10} = \frac{880}{28.7924\angle -20.323°} = 30.564\angle\mathbf{20.323}°$$

For phase b,

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_B + 2} = \frac{880\angle\text{-}120°}{22} = 40\angle\text{-}120°$$

For phase c,

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_C + 2} = \frac{880\angle 120°}{12 + j5} = \frac{880\angle 120°}{13\angle 22.62°} = 67.69\angle 97.38°$$

The current in the neutral line is

$$\mathbf{I}_\text{n} = \text{-}(\mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c}) \text{ or } \text{-}\mathbf{I}_\text{n} = \mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c}$$

$$\text{-}\mathbf{I_n} = (28.661 + \mathbf{j}10.6152) + (-20 - \mathbf{j}34.641) + (-8.6947 + \mathbf{j}67.129)$$

$\mathbf{I_n}$ = 0.0337 – j43.103 = **43.1∠–89.96° A**

Solution 12.11

In the wye-delta system shown in Fig. 12.44, the source is a positive sequence with $\mathbf{V}_{an}$ = 440∠0˚ V and phase impedance $\mathbf{Z}_P$ = (2 – j3) Ω. Calculate the line voltage V_L and the line current I_L.

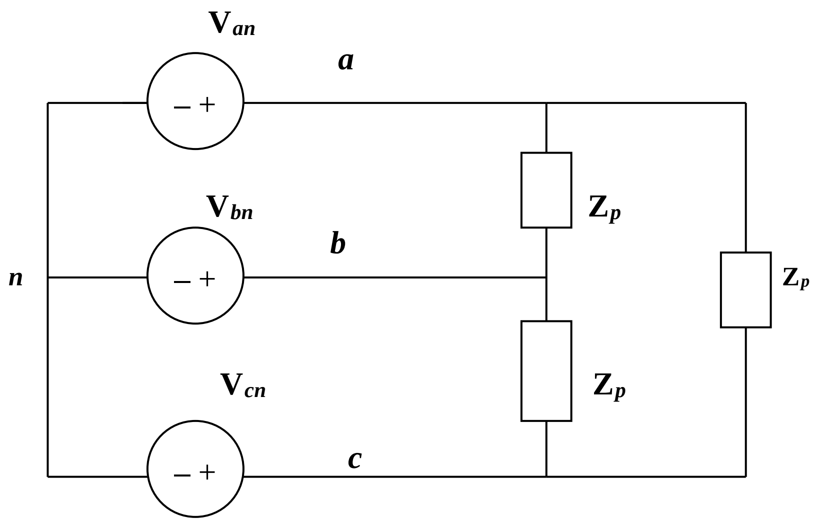


Figure 12.44
For Prob. 12.11.

Solution

Given that V_p = 440 and that the system is balanced, $V_L = 1.7321V_p$ = **762.1 V**.

$I_p = V_L/|2–j3|$ = 762.12/3.6056 = 211.37 A and

I_L = 1.7321x211.37 = **366.1 A**.

Solution 12.12

Using Fig. 12.45, design a problem to help other students to better understand wye-delta connected circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for the line currents in the **Y-Δ** circuit of Fig. 12.45. Take $\mathbf{Z}_\Delta = 60\angle 45°\Omega$.

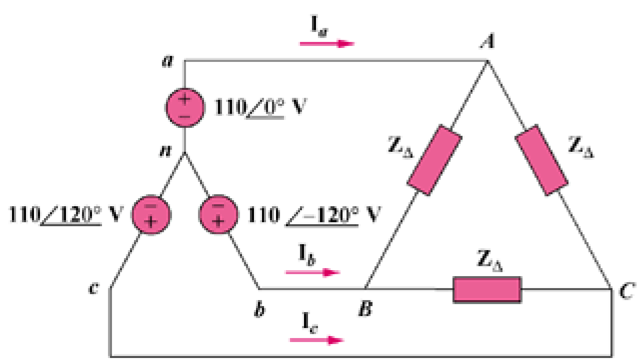
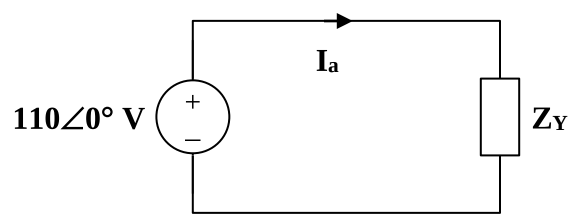


Figure 12.45

Solution

Convert the delta-load to a wye-load and apply per-phase analysis.

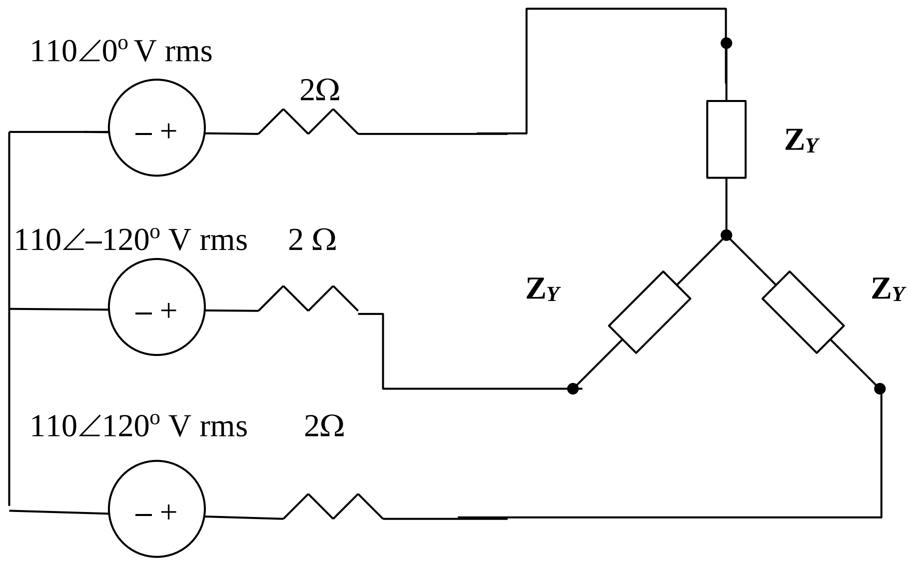


$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 45°\,\Omega$$

$$\mathbf{I}_a = \frac{110\angle 0°}{20\angle 45°} = \mathbf{5.5\angle -45°\ A}$$
$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{5.5\angle -165°\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{5.5\angle 75°\ A}$$

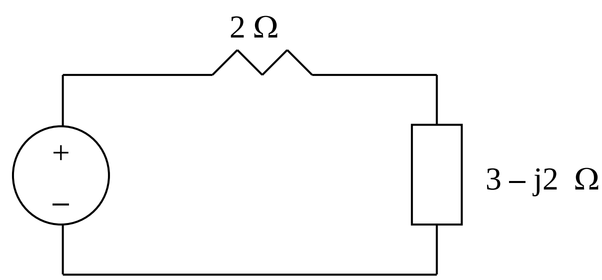
Solution 12.13

Convert the delta load to wye as shown below.



$$Z_Y = \frac{1}{3} Z_\Delta = 3 - j2\ \Omega$$

We consider the single phase equivalent shown below.



$\mathbf{I_a} = 110/(2 + 3 - j2) = 20.43\angle 21.8°$ A

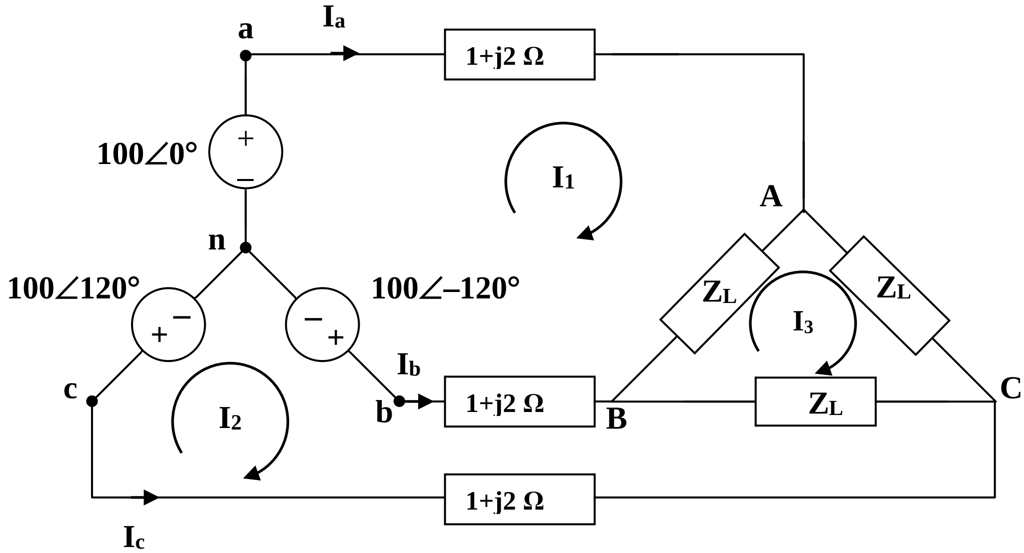
$I_L = |\mathbf{I_a}| = \mathbf{20.43\ A}$

$S = 3|\mathbf{I_a}|^2 Z_Y = 3(20.43)^2(3–j2) = 4514\angle –33.96˚ = 3744 – j2522$

P = Re(S) = **3.744 kW**.

Solution 12.14

We apply mesh analysis with $Z_L = (12+j12)\ \Omega$.



For mesh 1,

$$-100 + 100\angle -120^o + I_1(14 + j16) - (1 + j2)I_2 - (12 + j12)I_3 = 0 \text{ or}$$

$$(14 + j16)I_1 - (1 + j2)I_2 - (12 + j12)I_3 = 100 + 50 - j86.6 = 150 + j86.6 \qquad (1)$$

For mesh 2,

$$100\angle 120^o - 100\angle -120^o - I_1(1 + j2) - (12 + j12)I_3 + (14 + j16)I_2 = 0 \text{ or}$$

$$-(1 + j2)I_1 + (14 + j16)I_2 - (12 + j12)I_3 = -50 - j86.6 + 50 - j86.6 = -j173.2 \qquad (2)$$

For mesh 3,

$$-(12 + j12)I_1 - (12 + j12)I_2 + (36 + j36)I_3 = 0 \text{ or } \mathbf{I_3 = I_1 + I_2} \qquad (3)$$

Solving for $\mathbf{I_1}$ and $\mathbf{I_2}$ using (1) to (3) gives

$\mathbf{I_1} = 12.804\angle -50.19°$ A = (8.198 – j9.836) A and
$\mathbf{I_2} = 12.804\angle -110.19°$ A = (–4.419 – j12.018) A

$$\mathbf{I_a = I_1 = 12.804\angle -50.19°\ A}$$

$$\mathbf{I_b = I_2 - I_1 = 12.804\angle -170.19°\ A}$$

$$\mathbf{I_c = -I_2 = 12.804\angle 69.81^\circ\ A}$$

As a check we can convert the delta into a wye circuit. Thus,

$\mathbf{Z_Y}$ = (12+j12)/3 = 4+j4 and $\mathbf{I_a}$ = 100/(1+j2+4+j4) = 100/(5+j6)
= 100/(7.8102∠50.19°) =

12.804 ∠–50.19° A.

So, the answer does check.

Solution 12.15

Convert the delta load, $\mathbf{Z}_{\Delta}$, to its equivalent wye load.

$$\mathbf{Z}_{Ye} = \frac{\mathbf{Z}_{\Delta}}{3} = 8 - j10$$

$$\mathbf{Z}_{p} = \mathbf{Z}_{Y} \parallel \mathbf{Z}_{Ye} = \frac{(12 + j5)(8 - j10)}{20 - j5} = 8.076\angle\text{-}14.68^\circ$$

$$\mathbf{Z}_{p} = 7.812 - j2.047$$

$$\mathbf{Z}_{T} = \mathbf{Z}_{p} + \mathbf{Z}_{L} = 8.812 - j1.047$$

$$\mathbf{Z}_{T} = 8.874\angle\text{-}6.78^\circ$$

We now use the per-phase equivalent circuit.

$$\mathbf{I}_{a} = \frac{V_p}{\mathbf{Z}_{p} + \mathbf{Z}_{L}}, \qquad \text{where} \quad V_p = \frac{210}{\sqrt{3}}$$

$$\mathbf{I}_{a} = \frac{210}{\sqrt{3}\,(8.874\angle\text{-}6.78^\circ)} = 13.66\angle 6.78^\circ$$

$$\mathbf{I}_{L} = \left|\mathbf{I}_{a}\right| = \mathbf{13.66\ A}$$

Solution 12.16

A balanced delta-connected load has a phase current $\mathbf{I}_{AC} = 5\angle{-30°}$ A.

(a) Determine the three line currents assuming that the circuit operates in the positive phase sequence.

(b) Calculate the load impedance if the line voltage is $\mathbf{V}_{AB} = 440\angle 0°$ V.

Solution

(a) $\mathbf{I}_{CA} = -\mathbf{I}_{AC} = 5\angle(-30° + 180°) = 5\angle 150°$

This implies that

$$\mathbf{I}_{AB} = 5\angle 30°$$
$$\mathbf{I}_{BC} = 5\angle -90°$$

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \mathbf{8.66\angle 0°\ A}$$
$$\mathbf{I}_b = \mathbf{8.66\angle{-120°}\ A}$$
$$\mathbf{I}_c = \mathbf{8.66\angle 120°\ A}$$

(b) $\mathbf{Z}_\Delta = \dfrac{\mathbf{V}_{AB}}{\mathbf{I}_{AB}} = \dfrac{440\angle 0°}{5\angle 30°} = \mathbf{88\angle{-30°}\ \Omega}.$

Solution 12.17

A positive sequence wye connected source where $\mathbf{V_{an}} = 120\angle 90°$ V, is connected to a delta connected load where $\mathbf{Z_L} = (60+j45)\ \Omega$. Determine the line currents.

Solution

First the voltages are $\mathbf{V_{an}} = 120\angle 90°$ V, $\mathbf{V_{bn}} = 120\angle -30°$ V, and $\mathbf{V_{cn}} = 120\angle -150°$ V. The phase load is $\mathbf{Z_\Delta} = 75\angle 36.87°\ \Omega$.

$$\mathbf{Z_Y} = \mathbf{Z_\Delta}/3 = 25\angle 36.87°\ \Omega$$

Thus,

$$\mathbf{I_a} = \mathbf{V_{an}}/25\angle 36.87° = 120\angle 90°/25\angle 36.87° = \mathbf{4.8\angle 53.13°\ A}.$$
$$\mathbf{I_b} = 120\angle -30°/25\angle 36.87° = \mathbf{4.8\angle -66.87°\ A}.$$
$$\mathbf{I_c} = 120\angle -150°/25\angle 36.87° = \mathbf{4.8\angle 173.13°\ A}.$$

Solution 12.18

$$\mathbf{V}_{AB} = \mathbf{V}_{an}\sqrt{3}\angle 30° = (220\angle 60°)(\sqrt{3}\angle 30°) = 381.1\angle 90°$$

$$\mathbf{Z}_{\Delta} = 12 + j9 = 15\angle 36.87°$$

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{381.1\angle 90°}{15\angle 36.87°} = \mathbf{25.4\angle 53.13^\circ\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{25.4\angle{-66.87}^\circ\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{25.4\angle 173.13^\circ\ A}$$

Solution 12.19

For the Δ-Δ circuit of Fig. 12.50, calculate the phase and line currents.

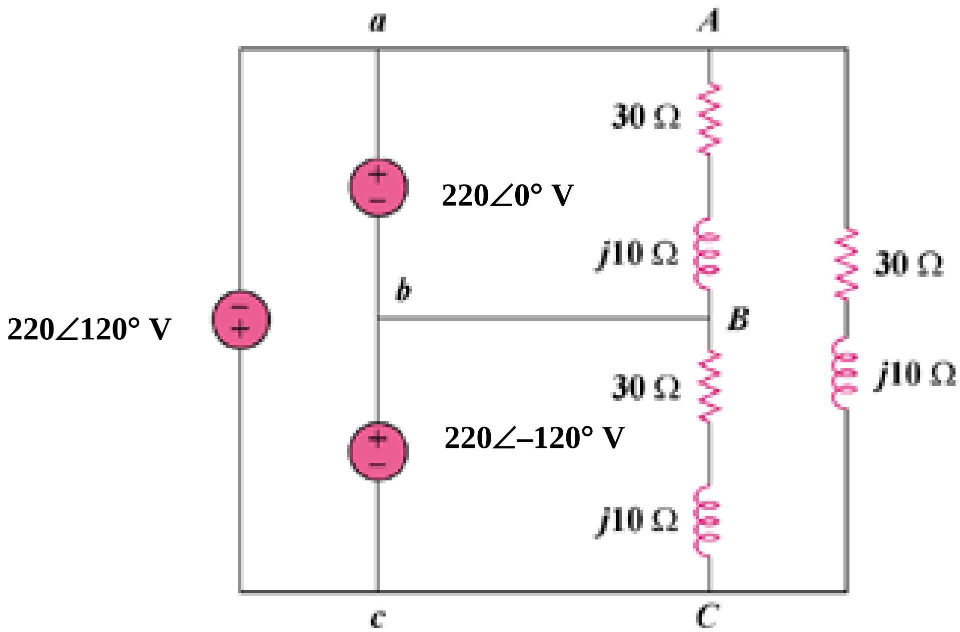


Figure 12.50
For Prob. 12.19.

Solution

$$\mathbf{Z}_{\Delta} = 30 + j10 = 31.62\angle 18.43°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{\Delta}} = \frac{440\angle 0°}{31.62\angle 18.43°} = \mathbf{13.915\angle{-18.43}°\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120°} = \mathbf{13.915\angle{-138.43}°\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{13.915\angle 101.57°\ A}$$

The line currents are

$$\mathbf{I}_{a} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}\sqrt{3}\angle{-30°}$$

$$\mathbf{I}_{a} = 13.915\sqrt{3}\angle{-48.43°} = \mathbf{24.1\angle{-48.43}°\ A}$$

$$\mathbf{I}_{b} = \mathbf{I}_{a}\angle{-120°} = \mathbf{24.1\angle{-168.43}°\ A}$$

$$\mathbf{I}_{c} = \mathbf{I}_{a}\angle 120° = \mathbf{24.1\angle 71.57°\ A}$$

Solution 12.20

Using Fig. 12.51, design a problem to help other students to better understand balanced delta-delta connected circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Refer to the Δ-Δ circuit in Fig. 12.51. Find the line and phase currents. Assume that the load impedance is 12 + j9Ω per phase.

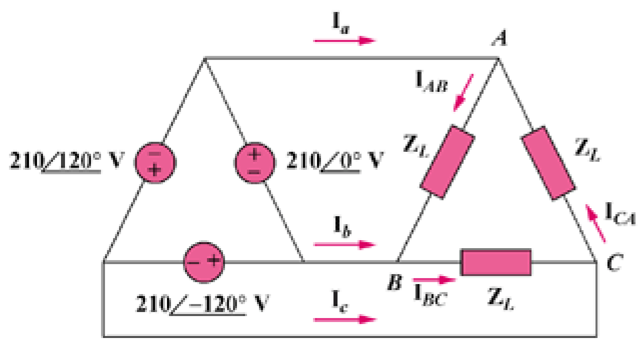


Figure 12.51

Solution

$$\mathbf{Z}_\Delta = 12 + j9 = 15\angle 36.87°$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{210\angle 0°}{15\angle 36.87°} = \mathbf{14\angle -36.87° \ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle -120° = \mathbf{14\angle -156.87° \ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{14\angle 83.13° \ A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30° = \mathbf{24.25\angle -66.87° \ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120° = \mathbf{24.25\angle -186.87° \ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{24.25\angle 53.13° \ A}$$

Solution 12.21

Three 440-volt generators, form a delta connected source which is connected to a balanced delta connected load of $\mathbf{Z}_L$ = (8.66 + j5) Ω per phase as shown in Fig. 12.52. Determine the value of $\mathbf{I}_{BC}$ and $\mathbf{I}_{aA}$. What is the pf of the load?

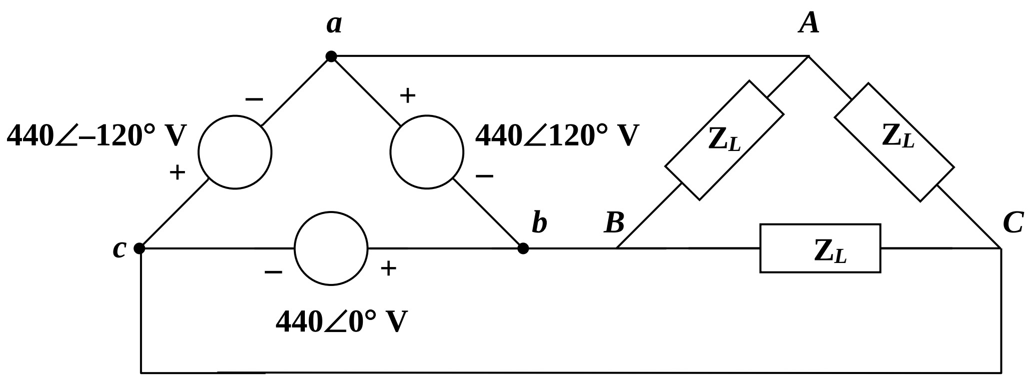


Figure 12.52
For Prob. 12.21.

Solution

$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_L = 440/(10\angle 30°) = \mathbf{44\angle{-30}°\ A}$.

$\mathbf{I}_{aA} = \mathbf{I}_{AC} + \mathbf{I}_{AB} = [440\angle 60°/(10\angle 30°)] + [440\angle 120°/(10\angle 30°)]$
$= [44\angle 30°] + [44\angle 90°] = 38.105 + j22 + j44 = 38.105 + j66 = \mathbf{76.21\angle 60°\ A}$.

pf = 8.66/10 = **0.866**.

Solution 12.22

Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_L = (114 + j87)\ \Omega$ and $\mathbf{Z}_l = (2 + j)\ \Omega$.

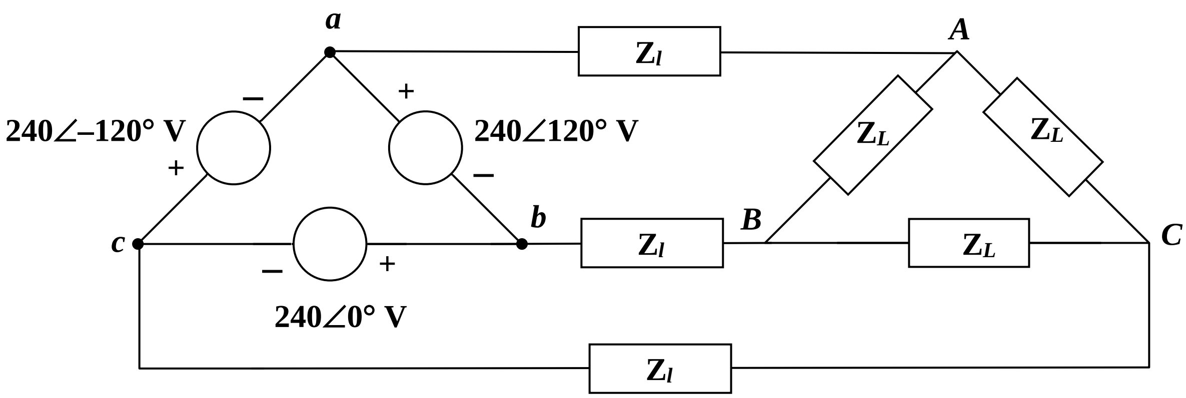
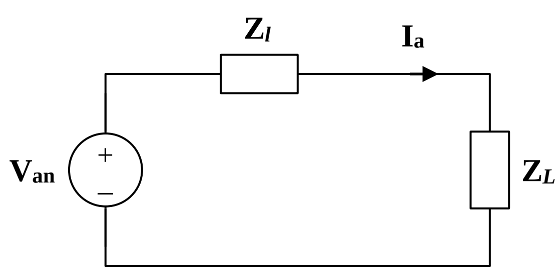


Figure 12.53
For Prob. 12.22.

Solution

Due to the line impedances, converting the Δ-connected source to a Y-connected source will make solving this problem easier.



Therefore,

$$\mathbf{V}_{an} = \frac{240}{\sqrt{3}}\angle 90^\circ = 138.564\angle 90^\circ\ \text{V}\ ,\ \mathbf{V}_{bn} = 138.564\angle -30^\circ\ \text{V, and}$$

$\mathbf{V}_{cn} = 138.564\angle -150^\circ$ V. The angles for the wye connected sources can be seen graphically by noting that the above circuit accurately shows the angles associated with the delta connected source and that the corresponding wye connected sources connect at the center, labeled n, of the delta connected sources. Also, $\mathbf{Z}_p = (114+j87)/3 = (38 + j29)\ \Omega$.

Finally, $\mathbf{I}_{aA} = 138.564\angle 90^\circ/[38+2+j(29+1)] = 138.564\angle 90^\circ/(50\angle 36.87^\circ)$ or

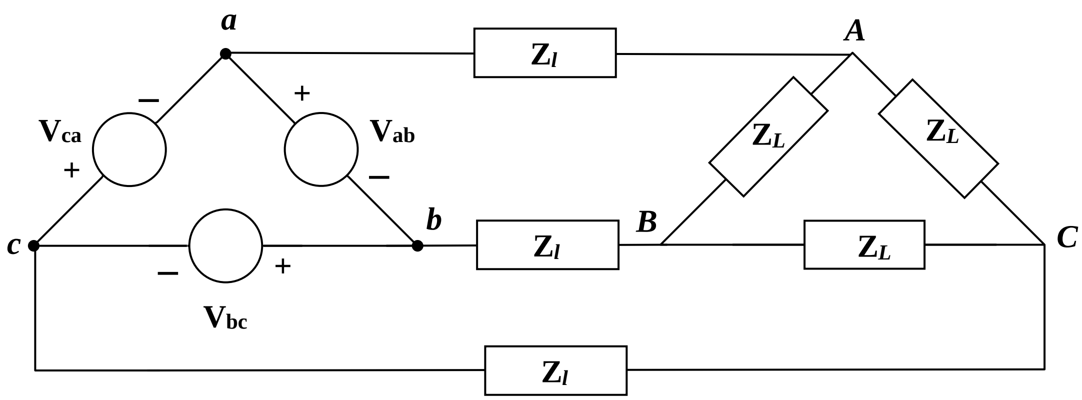
$\mathbf{I}_{aA} = \mathbf{2.772\angle 53.13^\circ\ A}$
$\mathbf{I}_{bB} = \mathbf{2.772\angle -66.87^\circ\ A}$
$\mathbf{I}_{cC} = \mathbf{2.772\angle 173.13^\circ\ A}$

Solution 12.23*

A balanced delta connected source is connected to a balanced delta connected load where $\mathbf{Z}_L = (80 + j60)\ \Omega$ and $\mathbf{Z}_l = (2 + j)\ \Omega$. Given that the load voltages are $\mathbf{V}_{AB} = 100\angle 0°$ V, $\mathbf{V}_{BC} = 100\angle 120°$ V, and $\mathbf{V}_{CA} = 100\angle{-120°}$ V. Calculate the source voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$.

Solution

We know that $\mathbf{I}_{aA} = \mathbf{I}_{AB} + \mathbf{I}_{AC} = \mathbf{V}_{AB}/\mathbf{Z}_L + \mathbf{V}_{AC}/\mathbf{Z}_L = [100/(100\angle 37.87°)] + [100\angle 60°/(100\angle 36.87°)] = 1\angle{-36.87°} + 1\angle 23.13° = 0.8–j0.6 + 0.91962 +j0.39282 = 1.71962 – j0.20718 = 1.7321\angle{-6.87°}$ A, $\mathbf{I}_{bB} = \mathbf{I}_{BA} + \mathbf{I}_{BC} = \mathbf{V}_{BA}/\mathbf{Z}_L + \mathbf{V}_{BC}/\mathbf{Z}_L = [100\angle 180°/(100\angle 37.87°)] + [100\angle 120°/(100\angle 36.87°)] = 1\angle 143.13° + 1\angle 83.13° = –0.8+j0.6 + 0.119617 + j0.99282 = –0.68038 + j1.59282 = 1.73205\angle 113.13°$ A, and $\mathbf{I}_{cC} = \mathbf{I}_{CA} + \mathbf{I}_{CB} = \mathbf{V}_{CA}/\mathbf{Z}_L + \mathbf{V}_{CB}/\mathbf{Z}_L = [100\angle{-120°}/(100\angle 37.87°)] + [100\angle{-60°}/(100\angle 36.87°)] = 1\angle{-157.87°} + 1\angle{-96.87°} = –0.9263315–j0.376709 – 0.119617–j0.99282$
$= –1.0459485–j1.369529 = 1.7233\angle{-127.37°}$ A. Finally we need $\mathbf{Z}_l = 2.23607\angle 26.565°$.



Clearly $\mathbf{V}_{ab} = \mathbf{I}_{aA}\mathbf{Z}_l + \mathbf{V}_{AB} – \mathbf{I}_{bB}\mathbf{Z}_l = (1.7321\angle{-6.87°})(2.23607\angle 26.56°) + 100 – (1.73205\angle 113.13°)(2.23607\angle 26.565°) = 3.8731\angle 19.69° + 100 – (3.873\angle 139.69°) = 3.6466+j1.30497 + 100 – (–2.95338+j2.50553) = 106.6 –j1.20056 =$ **$106.61\angle{-0.65°}$ V**, $\mathbf{V}_{bc} = \mathbf{I}_{bB}\mathbf{Z}_l + \mathbf{V}_{BC} – \mathbf{I}_{cC}\mathbf{Z}_l = (1.73205\angle 113.13°)(2.23607\angle 26.56°) + 100\angle 120°$
$– (1.7233\angle{-127.37°})(2.23607\angle 26.56°) = 3.8534\angle 139.69° + 100\angle 120°$
$– (3.8534\angle{-100.81°}) = –2.93843+j2.4929 – 50 + j86.6 – (–0.72272–j3.785)$
$= –52.216+j92.878 =$ **$106.55\angle 119.34°$ V**, and $\mathbf{V}_{ca} = \mathbf{I}_{cC}\mathbf{Z}_l + \mathbf{V}_{CA} – \mathbf{I}_{aA}\mathbf{Z}_l$
$= (1.7233\angle{-127.37°})(2.23607\angle 26.56°) + 100\angle{-120°}$
$– (1.7321\angle{-6.87°})(2.23607\angle 26.56°) = 3.8534\angle{-100.81°} – 50–j86.6 – (3.8731\angle 19.69°)$
$= –0.72272–j3.785 – 50–j86.6 – (3.6466+j1.305)$
$= –54.369 – j91.69 =$ **$106.6\angle{-120.67°}$ V**.

$\mathbf{V}_{ab}$ = **$106.61\angle{-0.65°}$ V** , $\mathbf{V}_{bc}$ = **$106.55\angle 119.34°$ V**, $\mathbf{V}_{ca}$ = **$106.6\angle{-120.67°}$ V**.

Solution 12.24

A balanced delta-connected source has phase voltage $\mathbf{V}_{ab}$ = 880∠30° V and a positive phase sequence. If this is connected to a balanced delta-connected load, find the line and phase currents. Take the load impedance per phase as 60 ∠30°**Ω** and line impedance per phase as 1 + j1**Ω**.

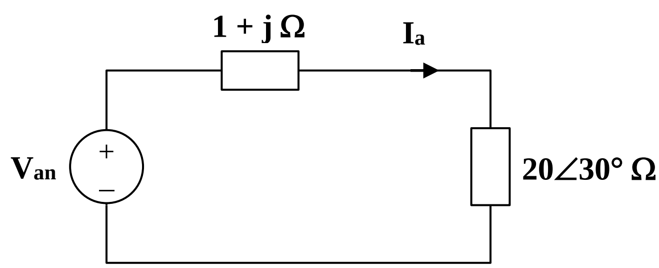
Solution

Convert both the source and the load to their wye equivalents.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 20\angle 30° = 17.32 + j10$$

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle\text{-}30° = 508.07\angle 0°$$

We now use per-phase analysis.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{(1+j)+(17.32+j10)} = \frac{508.07}{21.37\angle 31°} = \mathbf{23.77\angle{-31}°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle\text{-}120° = \mathbf{23.77\angle{-151}°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120° = \mathbf{23.77\angle 89°\ A}$$

But $\quad \mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle\text{-}30°$

$$\mathbf{I}_{AB} = \frac{23.77\angle\text{-}31°}{\sqrt{3}\angle\text{-}30°} = \mathbf{13.724\angle{-1}°\ A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle\text{-}120° = \mathbf{13.724\angle{-121}°\ A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle 120° = \mathbf{13.724\angle 119°\ A}$$

Solution 12.25

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$\mathbf{I}_a = \frac{440\angle(10° - 30°)}{\sqrt{3}\,\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.318\angle -24.78°$

$$\mathbf{I}_a = \frac{440\angle -20°}{\sqrt{3}(14.318\angle -24.78°)} = \mathbf{17.742\angle 4.78°\ amps}.$$

$\mathbf{I_b = I_a\angle -120° = 17.742\angle -115.22°\ amps}$.

$\mathbf{I_c = I_a\angle +120° = 17.742\angle 124.78°\ amps}$.

Solution 12.26

Using Fig. 12.55, design a problem to help other students to better understand balanced delta connected sources delivering power to balanced wye connected loads.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the balanced circuit in Fig. 12.55, $\mathbf{V}_{ab} = 125\angle 0°$ V. Find the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

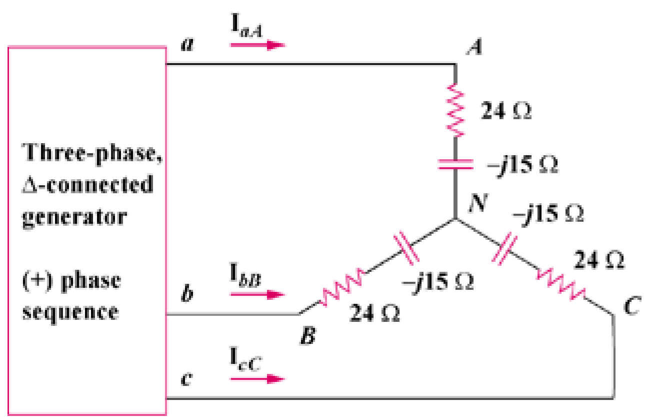


Figure 12.55

Solution

Transform the source to its wye equivalent.

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30° = 72.17\angle -30°$$

Now, use the per-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}}, \qquad \mathbf{Z} = 24 - j15 = 28.3\angle -32°$$

$$\mathbf{I}_{aA} = \frac{72.17\angle -30°}{28.3\angle -32°} = \mathbf{2.55\angle 2°\ A}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{aA}\angle -120° = \mathbf{2.55\angle -118°\ A}$$

$$\mathbf{I}_{cC} = \mathbf{I}_{aA}\angle 120° = \mathbf{2.55\angle 122°\ A}$$

Solution 12.27

Since Z_L and Z_ℓ are in series, we can lump them together so that

$$Z_Y = 2 + j + 6 + j4 = 8 + j5$$

$$I_a = \frac{\frac{V_P}{\sqrt{3}} < -30^o}{Z_Y} = \frac{208 < -30^o}{\sqrt{3}(8 + j5)}$$

$$V_L = (6 + j4)I_a = \frac{208(0.866 - j0.5)(6 + j4)}{\sqrt{3}(8 + j5)} = 80.81 - j43.54$$

$|V_L|$ = **91.79 V**

Solution 12.28

The line-to-line voltages in a wye-load have a magnitude of 880 V and are in the positive sequence at 60 Hz. If the loads are balanced with $Z_1 = Z_2 = Z_3 = 25\angle30^\circ$, find all line currents and phase voltages.

Solution

$V_L = |V_{ab}| = 880 = \sqrt{3}V_P$ or $V_P = 880/1.7321 = 508.05$

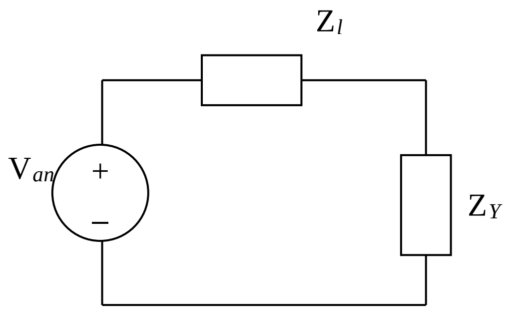
For reference, let V_{AN} = $\mathbf{508.05\angle0^\circ}$ **V** which leads to
V_{BN} = $\mathbf{508.05\angle{-120}^\circ}$ **V** and V_{CN} = $\mathbf{508.05\angle120^\circ}$ **V**.

The line currents are found as follows,

$I_a = V_{AN}/Z_Y = 508.05/25\angle30^\circ$ = $\mathbf{20.32\angle{-30}^\circ}$ **A**.
This leads to, I_b = $\mathbf{20.32\angle{-150}^\circ}$ **A** and I_c = $\mathbf{20.32\angle90^\circ}$ **A**.

Solution 12.29

We can replace the delta load with a wye load, $Z_Y = Z_\Delta/3 = 17+j15\Omega$. The per-phase equivalent circuit is shown below.



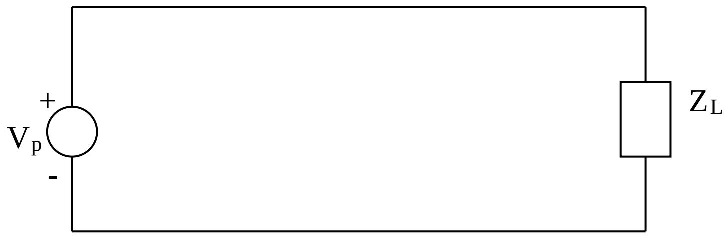
$\mathbf{I_a} = \mathbf{V_{an}}/|\mathbf{Z}_Y + \mathbf{Z}_l| = 240/|17+j15+0.4+j1.2| = 240/|17.4+j16.2| = 240/23.77 = 10.095$

$\mathbf{S} = 3[(I_a)^2(17+j15)] = 3x101.91(17+j15)$

$= \mathbf{[5.197+j4.586]\ kVA}$.

Solution 12.30

Since this a balanced system, we can replace it by a per-phase equivalent, as shown below.



$$\overline{S} = 3\overline{S}_p = \frac{3V^2_p}{Z^*_p}, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$\overline{S} = \frac{V^2_L}{Z^*_p} = \frac{(208)^2}{30\angle -45^o} = 1.4421\angle 45^o \text{ kVA}$$

$$\boldsymbol{P = S\cos\theta = \underline{1.02\,\text{kW}}}$$

Solution 12.31

A balanced delta-connected load is supplied by a 60-Hz three-phase source with a line voltage of 480V. Each load phase draws 24 kW at a lagging power factor of 0.8. Find:

(a) the load impedance per phase
(b) the line current
(c) the value of capacitance needed to be connected in parallel with each load phase to minimize the current from the source.

Solution

(a)

$$P_p = 24{,}000, \quad \cos\theta = 0.8, \quad S_p = \frac{P_P}{\cos\theta} = 24/0.8 = 30\text{kVA} \text{ and } \theta = 36.87°$$

$$Q_p = S_P \sin\theta = 18\,\text{kVAR}$$

$$\overline{S} = 3\overline{S}_p = 3(24 + j18) = 72 + j54\,\text{kVA}$$

For delta-connected load, $V_p = V_L = 480$ (rms). But

$$\overline{S} = \frac{3V^2{}_p}{Z^*{}_p} \longrightarrow Z^*{}_p = \frac{3V^2{}_p}{S} = \frac{3(480)^2}{(72 + j54)x10^3}, \quad \underline{Z_P = [6.144 + j4.608]\Omega}$$

(b) $$P_p = \sqrt{3}V_L I_L \cos\theta \longrightarrow I_L = \frac{24{,}000}{\sqrt{3}x480x0.8} = \mathbf{36.08\ A}$$

(c) We find C to bring the power factor to unity

$$Q_c = Q_p = 18\,\text{kVA} \longrightarrow C = \frac{Q_c}{\omega V^2{}_{rms}} = \frac{18{,}000}{2\pi x60x480^2} = \mathbf{207.2\ \mu F}.$$

Solution 12.32

Design a problem to help other students to better understand power in a balanced three-phase system.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A balanced wye load is connected to a 60-Hz three-phase source with $V_{ab} = 240\angle 0^\circ$V. The load has lagging pf =0.5 and each phase draws 5 kW. (a) Determine the load impedance Z_Y. (b) Find I_a, I_b, and I_c.

Solution

(a) $|V_{ab}| = \sqrt{3}V_p = 240 \quad \longrightarrow \quad V_p = \dfrac{240}{\sqrt{3}} = 138.56$

$$V_{an} = V_p < -30^o$$

$$pf = 0.5 = \cos\theta \quad \longrightarrow \quad \theta = 60^o$$

$$P = S\cos\theta \quad \longrightarrow \quad S = \frac{P}{\cos\theta} = \frac{5}{0.5} = 10 \text{ kVA}$$

$$Q = S\sin\theta = 10\sin 60 = 8.66$$

$$S_p = 5 + j8.66 \text{ kVA}$$

But

$$S_P = \frac{V_p^2}{Z_p^*} \quad \longrightarrow \quad Z_p^* = \frac{V_p^2}{S_p} = \frac{138.56^2}{(5 + j8.66)x10^3} = 0.96 - j1.663$$

$\mathbf{Z_P = [0.96 + j1.663]\ \Omega}$

(b) $I_a = \dfrac{V_{an}}{Z_Y} = \dfrac{138.56 < -30^o}{0.96 + j1.6627} = \underline{72.17 < -90^o \text{ A}} =$ **72.17∠–90° A**

$I_b = I_a < -120^o = \underline{72.17 < -210^o \text{ A}} =$ **72.17∠150° A**

$I_c = I_a < +120^o = \underline{72.17 < 30^o \text{ A}} =$ **72.17∠30° A**

Solution 12.33

$$\mathbf{S} = \sqrt{3}\,V_L I_L \angle\theta$$

$$S = \left|\mathbf{S}\right| = \sqrt{3}\,V_L I_L$$

For a Y-connected load,

$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p$

$$S = 3\,V_p I_p$$

$$I_L = I_p = \frac{S}{3V_p} = \frac{4800}{(3)(208)} = \mathbf{7.69\ A}$$

$$V_L = \sqrt{3}\,V_p = \sqrt{3} \times 208 = \mathbf{360.3\ V}$$

Solution 12.34

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{220}{\sqrt{3}}$$

$$\mathbf{I}_a = \frac{V_p}{\mathbf{Z}_Y} = \frac{220}{\sqrt{3}\,(10 - j16)} = \frac{127.02}{18.868\angle -58^\circ} = 6.732\angle 58^\circ$$

$I_L = I_p =$ **6.732A**

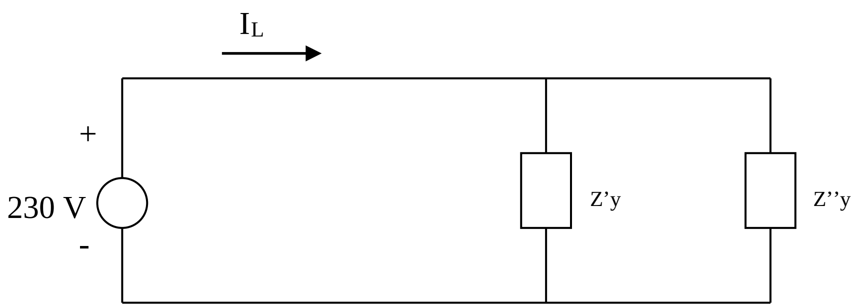
$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta = \sqrt{3} \times 220 \times 6.732\angle -58^\circ = 2565\angle -58^\circ$$

S = [1.3592–j2.175] kVA

Solution 12.35

(a) This is a balanced three-phase system and we can use per phase equivalent circuit. The delta-connected load is converted to its wye-connected equivalent

$$Z''_y = \frac{1}{3}Z_\Delta = (60 + j30)/3 = 20 + j10$$



$$Z_y = Z'_y // Z''_y = (40 + j10)//(20 + j10) = 13.5 + j5.5$$

$$\mathbf{I_L = \frac{230}{13.5 + j5.5} = [14.61 - j5.953]\ A}$$

(b) $\mathbf{S = 3V_s I^*_L = [10.081 + j4.108]\ kVA}$

(c) pf = P/S = **0.9261**

Solution 12.36

(a) S = 1 [0.75 + sin(cos^{-1}0.75)] = **0.75 + j0.6614 MVA**

(b) $\overline{S} = 3V_p I^*{}_p \quad \longrightarrow \quad I^*{}_p = \dfrac{S}{3V_p} = \dfrac{(0.75 + j0.6614)x10^6}{3x4200} = 59.52 + j52.49$

$$\boldsymbol{P_L = |\, I_p\,|^2\ R_l = (79.36)^2(4) = \underline{25.19\,\text{kW}}}$$

(c) $\boldsymbol{V_s = V_L + I_p(4 + j) = 4.4381 - j0.21\,\text{kV} = \underline{4.443\angle\text{-}2.709^{\circ}\ \text{kV}}}$

Solution 12.37

The total power measured in a three-phase system feeding a balanced wye-connected load is 12 kW at a power factor of 0.6 leading. If the line voltage is 440 V, calculate the line current $\boldsymbol{I_L}$ and the load impedance $\mathbf{Z}_{\boldsymbol{Y}}$.

Solution

$$S = \frac{P}{pf} = \frac{12}{0.6} = 20\,\text{kVA} \quad \text{also } \theta = -53.13°$$

$$\mathbf{S} = S\angle\theta = 20{,}000\angle{-53.13°} = [12 - j16]\ \text{kVA}.$$

But $$\mathbf{S} = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L\angle\theta = \sqrt{3}V_L I_L\angle\theta$$

$$I_L = \frac{20\times10^3}{\sqrt{3}\times440} = \mathbf{26.24\ A}$$

$$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2\mathbf{Z}_p$$

For a Y-connected load, $I_L = I_p$.

$$\mathbf{Z}_p = \frac{\mathbf{S}}{3|I_L|^2} = \frac{(12-j16)\times10^3}{(3)(26.2432)^2} = \frac{(12-j16)x10^3}{2066.117}$$

$$\mathbf{Z}_p = \mathbf{(5.808 - j7.744)\ \Omega}$$

Solution 12.38

As a balanced three-phase system, we can use the per-phase equivalent shown below.

$$\mathbf{I}_a = \frac{110\angle 0°}{(1+j2)+(9+j12)} = \frac{110\angle 0°}{10+j14}$$

$$\mathbf{S}_p = |\mathbf{I}_a|^2 \mathbf{Z}_Y = \frac{(110)^2}{(10^2+14^2)} \cdot (9+j12)$$

The complex power is

$$\mathbf{S} = 3\mathbf{S}_p = 3\frac{(110)^2}{296} \cdot (9+j12)$$

$$\mathbf{S} = \mathbf{(1.1037+j1.4716)\ kVA}$$

Solution 12.39

Find the real power absorbed by the load in Fig. 12.58.

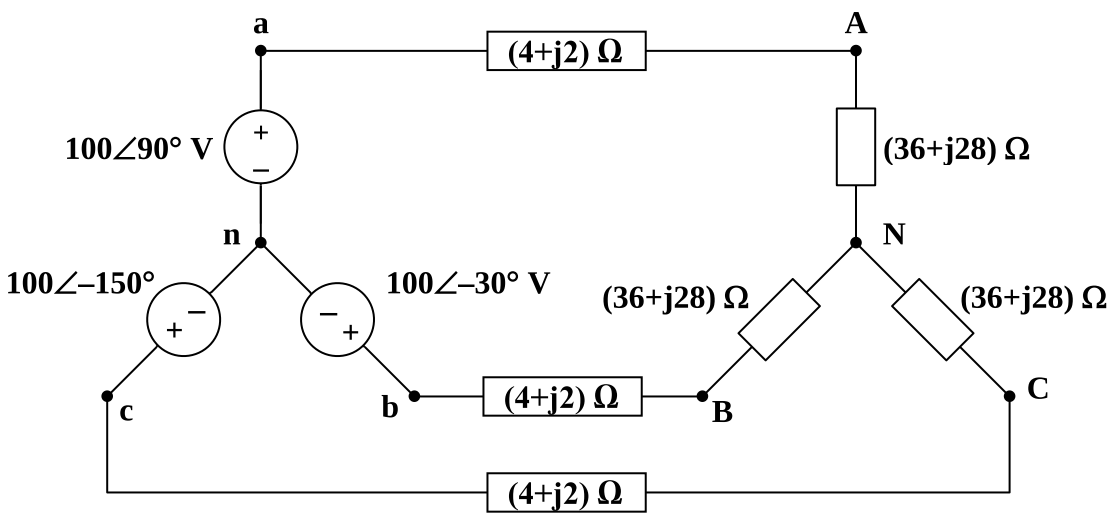


Figure 12.58
For Prob. 12.39.

Solution

To find power delivered to the load, we need to determine the current through the load. Since the load is balanced, the current through the load is equal to

$\mathbf{I}_{aA} = \mathbf{V}_{an}/(\mathbf{Z}_l+\mathbf{Z}_L) = j100/(4+j2 + 36+j28) = j100/(40+j30) = j100/(50\angle 36.87°) = 2\angle 53.13°$ A.

$P = (I_{aA})(36)(I_{aA})^* = (2)^2(36) = 144$ W for a total power absorbed equal to

$P_{Tot} = 3\text{x}144 = \mathbf{432\ W}$.

Solution 12.40

Transform the delta-connected load to its wye equivalent.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 7 + j8$$

Using the per-phase equivalent circuit above,

$$\mathbf{I}_a = \frac{100\angle 0°}{(1 + j0.5) + (7 + j8)} = 8.567\angle -46.75°$$

For a wye-connected load,

$$I_p = I_a = |\mathbf{I}_a| = 8.567$$

$$\mathbf{S} = 3|\mathbf{I}_p|^2 \mathbf{Z}_p = (3)(8.567)^2(7 + j8)$$

$$P = \text{Re}(\mathbf{S}) = (3)(8.567)^2(7) = \mathbf{1.541\ kW}$$

Solution 12.41

$$S = \frac{P}{pf} = \frac{5\ \text{kW}}{0.8} = 6.25\ \text{kVA}$$

But $\quad S = \sqrt{3}\, V_L I_L$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{6.25 \times 10^3}{\sqrt{3} \times 400} = \mathbf{9.021\ A}$$

Solution 12.42

The load determines the power factor.

$$\tan\theta = \frac{40}{30} = 1.333 \quad \longrightarrow \quad \theta = -53.13°$$

$$\text{pf} = \cos\theta = 0.6 \quad \text{(leading)}$$

$$\mathbf{S} = 7.2 - j\left(\frac{7.2}{0.6}\right)(0.8) = 7.2 - j9.6 \text{ kVA}$$

But $\quad \mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$

$$\left|\mathbf{I}_p\right|^2 = \frac{\mathbf{S}}{3\mathbf{Z}_p} = \frac{(7.2 - j9.6)\times 10^3}{(3)(30 - j40)} = 80$$

$$I_p = 8.944 \text{ A}$$

$$I_L = I_p = \mathbf{8.944\ A}$$

$$V_L = \frac{S}{\sqrt{3}\, I_L} = \frac{12\times 10^3}{\sqrt{3}\,(8.944)} = \mathbf{774.6\ V}$$

Solution 12.43

$\mathbf{S} = 3\left|\mathbf{I}_p\right|^2 \mathbf{Z}_p$, $\qquad I_p = I_L$ for Y-connected loads

$\mathbf{S} = (3)(13.66)^2(7.812 - j2.047)$

$$\mathbf{S} = \mathbf{[4.373 - \mathit{j}1.145]}\ \boldsymbol{kVA}$$

Solution 12.44

For a Δ-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\,I_p$$

$$S = \sqrt{3}\,V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{\sqrt{(12^2+5^2)}\times 10^3}{\sqrt{3}\,(240)} = 31.273$$

At the source,

$$\mathbf{V}_L^{'} = \mathbf{V}_L + \mathbf{I}_L\,\mathbf{Z}_l + \mathbf{I}_L\mathbf{Z}_l$$

$$\mathbf{V}_L^{'} = 240\angle 0° + 2(31.273)(1+j3) = 240+62.546+j187.638$$

$$\mathbf{V}_L^{'} = 302.546+j187.638 = 356\angle 31.81°$$

$$\left|\mathbf{V}_L^{'}\right| = \mathbf{356\ V}$$

Also, at the source,

S' = $3(31.273)^2(1+j3) + (12{,}000+j5{,}000) = 2{,}934+12{,}000+j(8{,}802+5{,}000)$
= $14{,}934+j13{,}802 = 20{,}335\angle 42.744°$ thus, $\theta = 42.744°$.

pf = cos(42.744°) = **0.7344**

Checking, $V_Y = 240/1.73205 = 138.564$, $\mathbf{S} = 3(138.564)^2/(Z_Y)^* = 12{,}000+l5{,}000$, and $Z_Y = 57{,}600/(12{,}000–j5{,}000) = 57.6/(13\angle –22.62°) = 4.4308\angle 22.62° = 4.09+j1.70416$. The total load seen by the source is $1+j3+4.09+j1.70416 = 5.09+j4.70416 = 6.9309\angle 42.74°$ per phase. This leads to $\theta = \mathrm{Tan}^{-1}(4.70416/5.09) = \mathrm{Tan}^{-1}(0.9242) = 42.744°$. Clearly, the answer checks. $I_l = 138.564/4.4308 = 31.273$ A. Again the answer checks. Finally, $3(31.273)^2(5.09+j4.70416) = 2{,}934(6.9309\angle 42.74°) = 20{,}335\angle 42.74°$, the same as we calculated above.

Solution 12.45

$$\mathbf{S} = \sqrt{3}\, V_L I_L \angle\theta$$

$$I_L = \frac{|\mathbf{S}|\angle -\theta}{\sqrt{3}\, V_L}, \qquad |\mathbf{S}| = \frac{P}{pf} = \frac{450\times 10^3}{0.708} = 635.6 \text{ kVA}$$

$$\mathbf{I}_L = \frac{(635.6)\angle -\theta}{\sqrt{3}\times 440} = 834\angle -45^\circ \text{ A}$$

At the source,

$$\mathbf{V}_L = 440\angle 0^\circ + \mathbf{I}_L\,(0.5 + j2)$$
$$\mathbf{V}_L = 440 + (834\angle -45^\circ)(2.062\angle 76^\circ)$$
$$\mathbf{V}_L = 440 + 1719.7\angle 31^\circ$$
$$\mathbf{V}_L = 1914.1 + j885.7$$

$$\mathbf{V}_L = \mathbf{2.109\angle 24.83^\circ\ V}$$

Solution 12.46

For the wye-connected load,

$$I_L = I_p, \qquad V_L = \sqrt{3}\,V_p \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3|\mathbf{V}_p|^2}{\mathbf{Z}^*} = \frac{3|\mathbf{V}_L/\sqrt{3}|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{|\mathbf{V}_L|^2}{\mathbf{Z}^*} = \frac{(110)^2}{100} = 121\text{ W}$$

For the delta-connected load,

$$V_p = V_L, \qquad I_L = \sqrt{3}\,I_p, \qquad I_p = V_p / \mathbf{Z}$$

$$\mathbf{S} = 3\mathbf{V}_p\mathbf{I}_p^* = \frac{3|\mathbf{V}_p|^2}{\mathbf{Z}^*} = \frac{3|\mathbf{V}_L|^2}{\mathbf{Z}^*}$$

$$\mathbf{S} = \frac{(3)(110)^2}{100} = 363\text{ W}$$

This shows that the **delta-connected load** will absorb three times more average power than the wye-connected load using the same elements.. This is also evident from $\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3}$.

Solution 12.47

$\text{pf} = 0.8 \quad \text{(lagging)} \quad \longrightarrow \quad \theta = \cos^{-1}(0.8) = 36.87°$
$\mathbf{S}_1 = 250\angle 36.87° = 200 + \text{j}150 \text{ kVA}$

$\text{pf} = 0.95 \quad \text{(leading)} \quad \longrightarrow \quad \theta = \cos^{-1}(0.95) = -18.19°$
$\mathbf{S}_2 = 300\angle -18.19° = 285 - \text{j}93.65 \text{ kVA}$

$\text{pf} = 1.0 \quad \longrightarrow \quad \theta = \cos^{-1}(1) = 0°$
$\mathbf{S}_3 = 450 \text{ kVA}$

$\mathbf{S}_T = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 935 + \text{j}56.35 = 936.7\angle 3.45° \text{ kVA}$

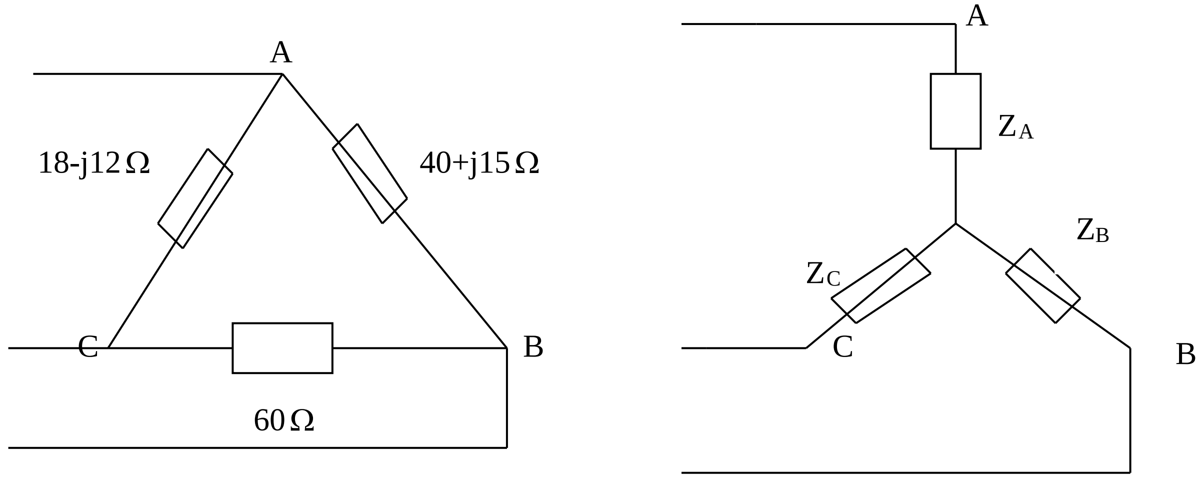
$\left| \mathbf{S}_T \right| = \sqrt{3}\, V_L I_L$

$I_L = \dfrac{936.7 \times 10^3}{\sqrt{3}\,(13.8 \times 10^3)} = \mathbf{39.19 \text{ A rms}}$

$\text{pf} = \cos\theta = \cos(3.45°) = \mathbf{0.9982 \quad (lagging)}$

Solution 12.48

(a) We first convert the delta load to its equivalent wye load, as shown below.

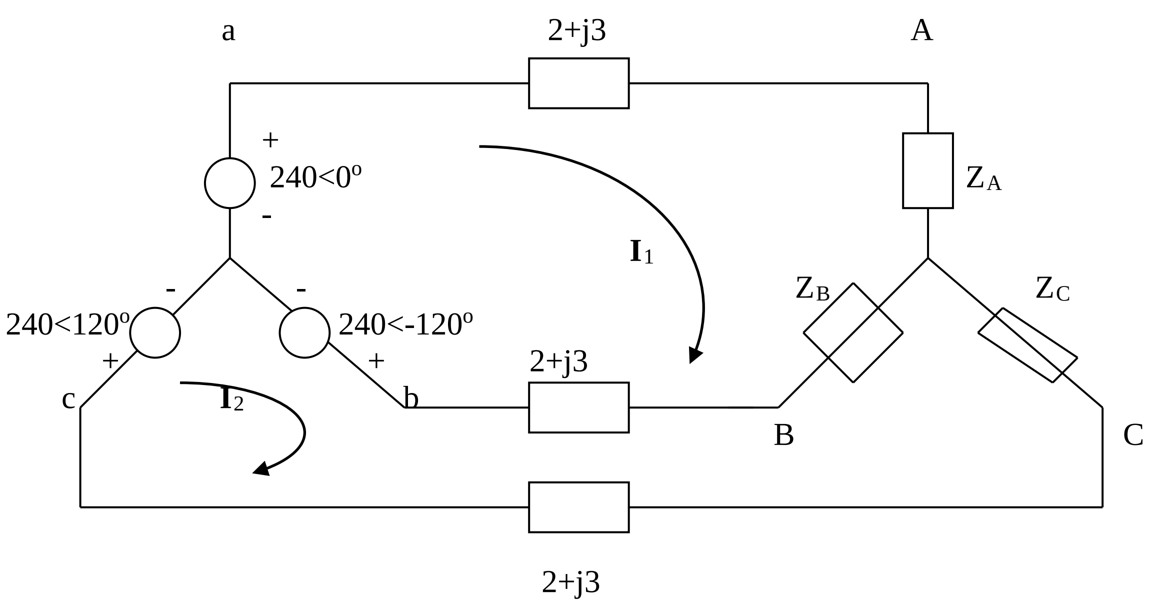


$$Z_A = \frac{(40+j15)(18-j12)}{118+j3} = 7.577 - j1.923$$

$$Z_B = \frac{60(40+j15).}{118+j3} = 20.52 + j7.105$$

$$Z_C = \frac{60(18-j12)}{118+j3} = 8.992 - j6.3303$$

The system becomes that shown below.



We apply KVL to the loops. For mesh 1,

$$-240+240\angle -120^o + I_1(2Z_l + Z_A + Z_B) - I_2(Z_B + Z_l) = 0$$

or

$$(32.097 + j11.13)I_1 - (22.52 + j10.105)I_2 = 360 + j207.85 \qquad (1)$$

For mesh 2,

$$240\angle 120^o - 240\angle -120^o - I_1(Z_B + Z_l) + I_2(2Z_l + Z_B + Z_C) = 0$$

or

$$-(22.52 + j10.105)I_1 + (33.51 + j6.775)I_2 = -j415.69 \qquad (2)$$

Solving (1) and (2) gives

$$I_1 = 23.75 - j5.328, \qquad I_2 = 15.165 - j11.89$$

$$\boldsymbol{I_{aA} = I_1 = \underline{24.34\angle -12.64^o \text{ A}}, \qquad I_{bB} = I_2 - I_1 = \underline{10.81\angle -142.6^o \text{ A}}}$$

$$\boldsymbol{I_{cC} = -I_2 = \underline{19.27\angle 141.9^o \text{ A}}}$$

(b) $\mathbf{S_a = (240\angle 0^o)(24.34\angle 12.64^o) = 5841.6\angle 12.64^o}$

$\mathbf{S_b = (240\angle -120^o)(10.81\angle 142.6^o) = 2594.4\angle 22.6^o}$

$\mathbf{S_c = (240\angle 120^o)(19.27\angle -141.9^o) = 4624.8\angle -21.9^o}$

$$\mathbf{S = S_a + S_b + S_c = 12.386 + j0.55 \ \ kVA = \underline{12.4\angle 2.54^o \ kVA}}$$

Solution 12.49

Each phase load consists of a 20-ohm resistor and a 10-ohm inductive reactance. With a line voltage of 480 V rms, calculate the average power taken by the load if:

(a) the three phase loads are delta-connected,
(b) the loads are wye-connected.

Solution

(a) For the delta-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L = 480 \text{ (rms)},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x480^2}{(20 - j10)} = \frac{(13{,}824 + j6{,}912)k}{500} = (27.648 + j13.824)k$$

P = **27.65 kW**

(b) For the wye-connected load, $Z_p = 20 + j10\Omega, \quad V_p = V_L / \sqrt{3},$

$$S = \frac{3V^2{}_p}{Z^*{}_p} = \frac{3x480^2}{3(20 - j10)} = (9.216 + j4.608)kVA$$

P = **9.216 kW**

Solution 12.50

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 8(0.6 + j0.8) = 4.8 + j6.4\,\text{kVA}, \qquad \overline{S}_1 = 3\,\text{kVA}$$

Hence,

$$\overline{S}_2 = \overline{S} - \overline{S}_1 = 1.8 + j6.4\,\text{kVA}$$

But $\overline{S}_2 = \dfrac{3V^2{}_p}{Z^*{}_p}, \quad V_p = \dfrac{V_L}{\sqrt{3}} \longrightarrow \overline{S}_2 = \dfrac{.V^2{}_L}{Z^*{}_p}$

$$\boldsymbol{Z^*{}_p = \frac{V^*{}_L}{\overline{S}_2} = \frac{240^2}{(1.8 + j6.4)x10^3} \longrightarrow \underline{Z_p = 2.346 + j8.34\Omega}}$$

Solution 12.51

Consider the wye-delta system shown in Fig. 12.60. Let $\mathbf{Z}_1 = 100\ \Omega$, $\mathbf{Z}_2 = j100\ \Omega$, and $\mathbf{Z}_3 = -j100\ \Omega$. Determine the phase currents, $\mathbf{I}_{AB}$, $\mathbf{I}_{BC}$, and $\mathbf{I}_{CA}$, and the line currents, $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

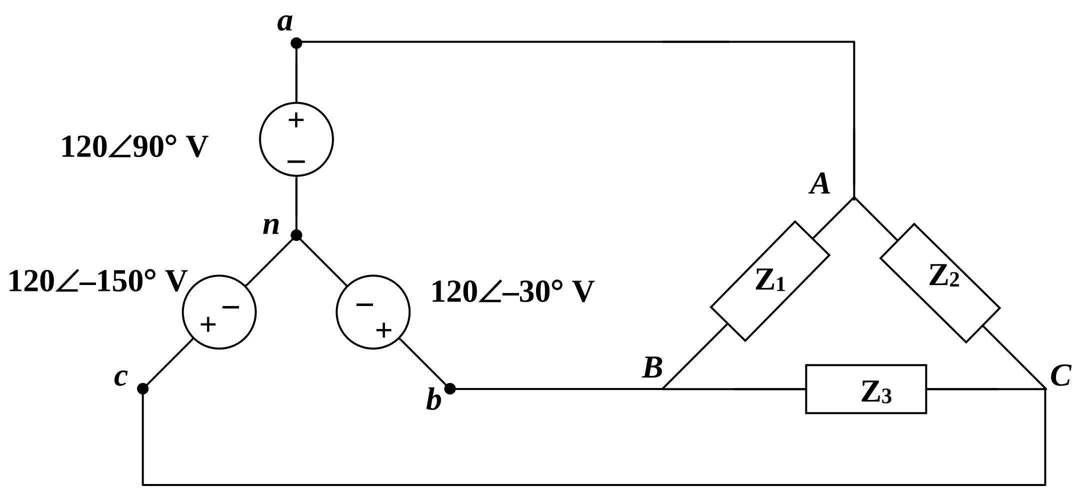


Figure 12.60
For Prob. 12.51.

Solution

Step 1. First we need to determine the Phase voltages, $\mathbf{V}_{AB} = \mathbf{V}_{an} - \mathbf{V}_{bn}$, $\mathbf{V}_{BC} = \mathbf{V}_{bn} - \mathbf{V}_{cn}$, and $\mathbf{V}_{CA} = \mathbf{V}_{cn} - \mathbf{V}_{an}$. Then we can calculate phase currents, $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_1$, $\mathbf{I}_{BC}$, $= \mathbf{V}_{BC}/\mathbf{Z}_3$, and $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_2$. Finally, we can now calculate the line currents, $\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$, $\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB}$, and $\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC}$.

Step 2. $\mathbf{V}_{AB} = \mathbf{V}_{ab} - \mathbf{V}_{bn} = 120\angle 90° - 120\angle -30° = j120 - 103.923 + j60$
$= -103.923 + j180 = 207.846\angle 120°$ V, $\mathbf{V}_{BC} = \mathbf{V}_{bn} - \mathbf{V}_{cn}$
$= 120\angle -30° - 120\angle -150° = 103.923 - j60 + 103.923 + j60 = 207.846$ V, and
$\mathbf{V}_{CA} = \mathbf{V}_{cn} - \mathbf{V}_{an} = 120\angle -150° - j120 = -103.923 - j60 - j120 = -103.923 - j180$
$= 207.846\angle -120°$ V.

$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_1 = 207.846\angle 120°/100 = \mathbf{2.078\angle 120°\ A}$,
$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_3 = 207.846\angle 0°/(-j100) = \mathbf{2.078\angle 90°\ A}$,
and $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_2 = 207.846\angle -120°/(j100) = \mathbf{2.078\angle 150°\ A}$.

Finally, $\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 2.07846\angle 120° - 2.07846\angle 30°$
$= -1.03923 + j1.8 - 1.8 - j1.03923 = -2.83923 + j0.76077 = \mathbf{2.939\angle 165°\ A}$,
$\mathbf{I}_{bB} = \mathbf{I}_{BC} - \mathbf{I}_{AB} = 2.07846\angle 90° - 2.07846\angle 120° = j2.07846 + 1.03923 - j1.8$
$= 1.03923 + j0.27846 = \mathbf{1.07589\angle 15°\ A}$, and $\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 2.07846\angle 150°$
$-\ 2.07846\angle 90° = -1.8 + j1.03923 - j2.07846 = -1.8 - j1.03923 = \mathbf{2.078\angle -150°\ A}$.

Solution 12.52

A four-wire wye-wye circuit has

$$\mathbf{V}_{an} = 220\ \angle 120°, \qquad \mathbf{V}_{bn} = 220\ \angle 0°$$
$$\mathbf{V}_{cn} = 220\ \angle{-120°}\ \text{V}$$

If the impedances are

$$\mathbf{Z}_{AN} = 20\ \angle 60°, \qquad \mathbf{Z}_{BN} = 30\ \angle 0°$$
$$\mathbf{Z}_{cn} = 40\ \angle 30°\ \Omega$$

find the current in the neutral line.

Solution

Since the neutral line is present, we can solve this problem on a per-phase basis.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{AN}} = \frac{220\angle 120°}{20\angle 60°} = 11\angle 60°$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_{BN}} = \frac{220\angle 0°}{30\angle 0°} = 7.3333\angle 0°$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_{CN}} = \frac{220\angle\text{-}120°}{40\angle 30°} = 5.5\angle\text{-}150°$$

Thus,

$$-\mathbf{I}_\text{n} = \mathbf{I}_\text{a} + \mathbf{I}_\text{b} + \mathbf{I}_\text{c}$$
$$-\mathbf{I}_\text{n} = 11\angle 60° + 7.3333\angle 0° + 5.5\angle\text{-}150°$$
$$-\mathbf{I}_\text{n} = (5.5 + j9.5263) + (7.3333) + (-4.7631 - j2.75)$$
$$-\mathbf{I}_\text{n} = 8.0702 + j6.7763 = 10.538\angle 40.02°$$

$$\mathbf{I}_\text{n} = \mathbf{10.538\angle{-139.98°}\ A}$$

Solution 12.53

Using Fig. 12.61, design a problem that will help other students to better understand unbalanced three-phase systems.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the wye-wye system shown in Fig. 12.61, loads connected to the source are unbalanced. (a) Calculate I_a, I_b, and I_c. (b) Find the total power delivered to the load. Take V_P = 240 V rms.

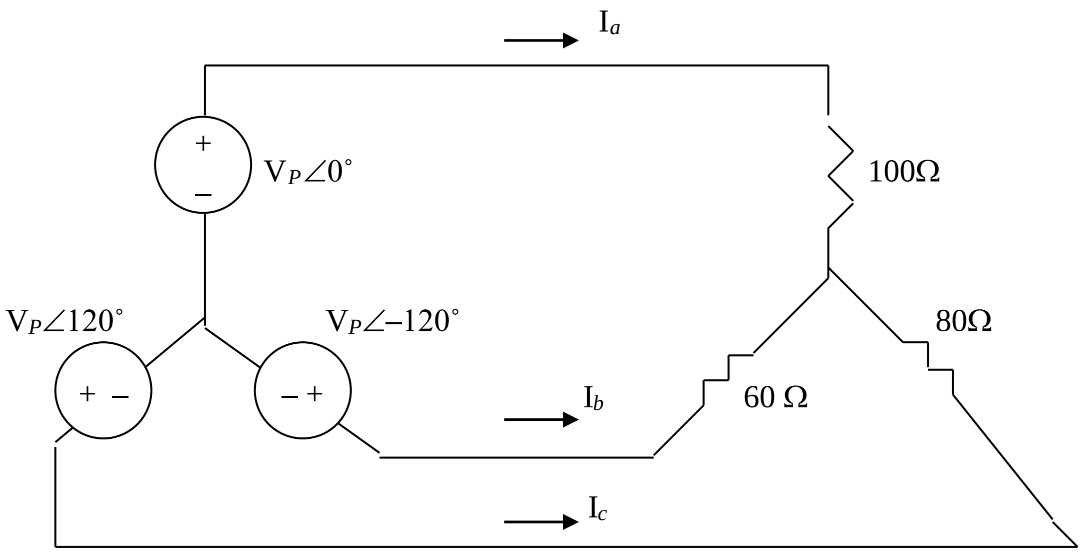
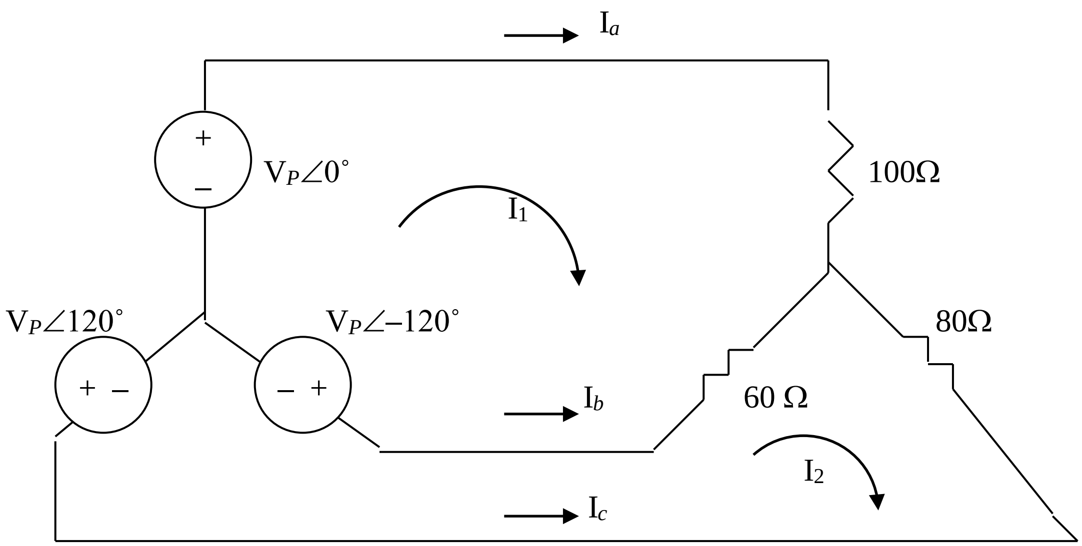


Figure 12.61 For Prob. 12.53.

Solution

Applying mesh analysis as shown below, we get.



$240\angle -120^\circ - 240 + 160\mathbf{I_1} - 60\mathbf{I_2} = 0$ or $160\mathbf{I_1} - 60\mathbf{I_2} = 360 + j207.84$ (1)

$240\angle 120^\circ - 240\angle -120^\circ - 60\mathbf{I_1} + 140\mathbf{I_2} = 0$ or $-60\mathbf{I_1} + 140\mathbf{I_2} = -j415.7$ (2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 160 & -60 \\ -60 & 140 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 360 + j207.84 \\ -j415.7 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Z=[160,-60;-60,140]
Z =
   160   -60
   -60   140
>> V=[(360+207.8i);-415.7i]
V =
  1.0e+002 *
   3.6000 + 2.0780i
        0 - 4.1570i
>> I=inv(Z)*V
I =
   2.6809 + 0.2207i
   1.1489 - 2.8747i
```

$I_1 = 2.681+j0.2207$ and $I_2 = 1.1489–j2.875$

$$I_a = I_1 = \mathbf{2.69\angle 4.71^\circ\ A}$$

$$I_b = I_2 - I_1 = -1.5321-j3.096 = \mathbf{3.454\angle -116.33^\circ\ A}$$

$$I_c = -I_2 = \mathbf{3.096\angle 111.78^\circ\ A}$$

$$S_a = |I_a|^2\ Z_a = (2.69)^2 x100 = 723.61$$
$$S_b = |I_b|^2\ Z_b = (3.454)^2 x60 = 715.81$$
$$S_c = |I_c|^2\ Z_c = (3.0957)^2 x80 = 766.67$$

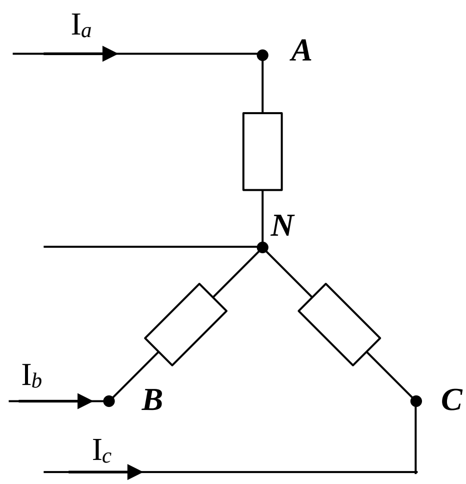
$$S = S_a + S_b + S_c = \underline{2.205\ \text{kVA}}$$

Solution 12.54

A balanced three-phase Y-source with V_P = 880 V rms drives a wye-connected three-phase load with phase impedance Z_{AN} = 80 Ω, Z_{BN} = 60+j90 Ω, and Z_{CN} = j80 Ω. Calculate the line currents and total complex power delivered to the load. Assume that the neutrals are connected.

Solution

Consider the load as shown below.



Assume $\mathbf{V_{AN}} = 880\angle 0°$ V, $\mathbf{V_{BN}} = 880\angle 120°$ V, and $\mathbf{V_{CN}} = 880\angle -120°$ V.
$\mathbf{I_a}$ = 880/80 = $\mathbf{11\angle 0°\ A}$, $\mathbf{I_b} = 880\angle 120°/(60+j90) = 880\angle 120°/(108.17\angle 56.13°)$
= $\mathbf{8.135\angle 63.87°\ A}$, and $\mathbf{I_c} = 880\angle -120°/(j80)$ = $\mathbf{11\angle 150°\ A}$.

$\mathbf{S_a} = V_{AN}(I_a)^* = 880\text{x}11 = 9.68$ kW, $\mathbf{S_b} = 880\angle 120°(8.135\angle -63.87°)$
= $7.159\angle 56.13°$ kVA = 3.99 kW + j5.944 kVAR, and
$\mathbf{S_c} = (880\angle -120°)(11\angle -150°) = 9.68\angle 90°$ kVA = j9.68 kVAR.

$\mathbf{S} = \mathbf{S_a} + \mathbf{S_b} + \mathbf{S_c}$ = (9.68+3.99)kW + j(5.944+9.68)kVAR or

$$\mathbf{S = 13.67\ kW + j15.624\ kVAR = 20.76\angle 48.82°\ kVA}.$$

Chapter 12, Solution 55.

A three-phase supply, with the line-to-line voltage of 240 V rms, has the unbalanced load as shown in Fig. 12.62. Find the line currents and the total complex power delivered to the load.

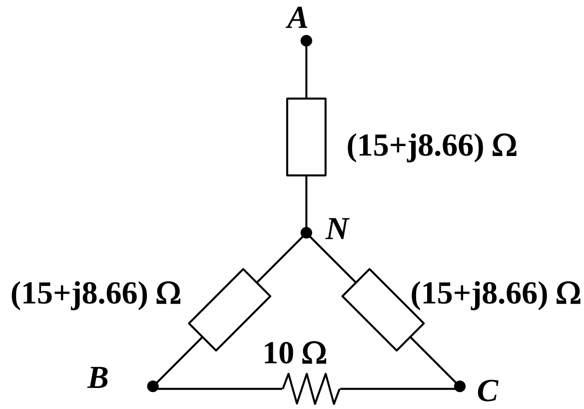


Figure 12.62
For Prob. 12.55.

Solution
To solve this problem we need to arbitrarily select phase angles for the sources which then enables us to find line currents as well as complex power delivered to the load.

Step 1. Let $\mathbf{V_{AB}} = 240\angle 0°$ V, $\mathbf{V_{BC}} = 240\angle 120°$ V, and $\mathbf{V_{CA}} = 240\angle -120°$ V. We can treat this as two different circuits and then use superposition to find the line currents and total complex power.

The first circuit consists of a balanced wye with the phase voltages (see Fig. 12.19) of $\mathbf{V_{an}} = 138.564\angle -30°$, $\mathbf{V_{bn}} = 138.564\angle -150°$, and $\mathbf{V_{cn}} = 138.564\angle 90°$, Therefore, the line currents for this are equal to, $\mathbf{I_{aA}} = \mathbf{V_{an}}/(17.32\angle 30°)$, $\mathbf{I_{bB}} = \mathbf{V_{bn}}/(17.32\angle 30°)$, and $\mathbf{I_{cC}} = \mathbf{V_{cn}}/(17.32\angle 30°)$.

Finally, we note that the current that flows through the 10-Ω resistor impacts the line currents, $\mathbf{I_{bB}}$ and $\mathbf{I_{cC}}$. Let us call the current through the resistor as $\mathbf{I_{BC}}$. $\mathbf{I_{BC}} = \mathbf{V_{BC}}/10$. Thus, $(\mathbf{I_{bB}})' = \mathbf{I_{bB}} + \mathbf{I_{BC}}$ and $(\mathbf{I_{cC}})' = \mathbf{I_{cC}} - \mathbf{I_{BC}}$.

The last thing we need to do is calculate $\mathbf{S_{Tot}} = 3|\mathbf{I_{line}}|^2(15+j8.66) + |\mathbf{I_{AB}}|^2(10)$.

Step 2. $\mathbf{I_{aA}} = (138.564\angle -30°)/(17.32\angle 30°) = \mathbf{8\angle -60°\ A}$,
$\mathbf{I_{bB}} = (138.564\angle -150°)/(17.32\angle 30°) = 8\angle 180° = -8$, and
$\mathbf{I_{cC}} = (138.564\angle 90°)/(17.32\angle 30°) = 8\angle 60° = 4 + j6.9282$. $\mathbf{I_{BC}} = (240\angle 120°)/10$
$= 24\angle 120° = -12 + j20.785$. Thus, $(\mathbf{I_{bB}})' = -8 - 12 + j20.785 = -20+j20.785$
$= \mathbf{28.84\angle 133.9°\ A}$ and $(\mathbf{I_{cC}})' = \mathbf{I_{cC}} - \mathbf{I_{BC}} = = 4 + j6.9282 +12 - j20.785$
$= 16 - j13.8568 = \mathbf{21.17\angle -40.89°\ A}$.
$\mathbf{S_{Tot}} = 3|\mathbf{I_{line}}|^2(15+j86.6) + |\mathbf{I_{BC}}|^2(10) = 3(8)^2(15+j8.66) + (24)^2(10)$
$= 2{,}880 + j1{,}662.72 + 5{,}760 = $ **8.64 kW + j1.6627 kVAR**.

Solution 12.56

Using Fig. 12.63, design a problem to help other students to better understand unbalanced three-phase systems.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Refer to the unbalanced circuit of Fig. 12.63. Calculate:

(a) the line currents

(b) the real power absorbed by the load

(c) the total complex power supplied by the source

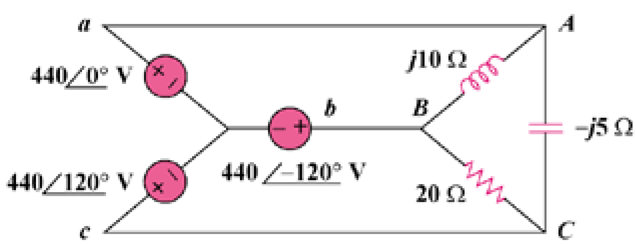
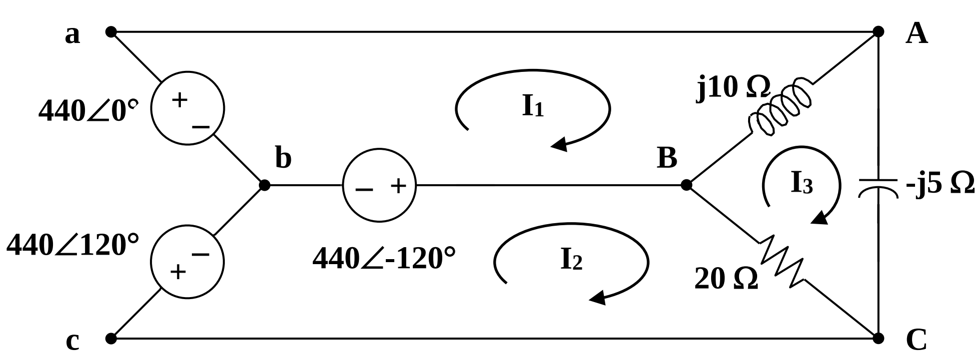


Figure 12.63

Solution

(a) Consider the circuit below.



For mesh 1,

$$440\angle -120° - 440\angle 0° + j10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_1 - \mathbf{I}_3 = \frac{(440)(1.5 + j0.866)}{j10} = 76.21\angle -60° \qquad (1)$$

For mesh 2,

$$440\angle 120° - 440\angle -120° + 20(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \frac{(440)(j1.732)}{20} = j38.1 \qquad (2)$$

For mesh 3,

$$j10(\mathbf{I}_3 - \mathbf{I}_1) + 20(\mathbf{I}_3 - \mathbf{I}_2) - j5\mathbf{I}_3 = 0$$

Substituting (1) and (2) into the equation for mesh 3 gives,

$$\mathbf{I}_3 = \frac{(440)(-1.5 + j0.866)}{j5} = 152.42\angle 60° \qquad (3)$$

From (1),

$$\mathbf{I}_1 = \mathbf{I}_3 + 76.21\angle -60° = 114.315 + j66 = 132\angle 30°$$

From (2),

$$\mathbf{I}_2 = \mathbf{I}_3 - j38.1 = 76.21 + j93.9 = 120.93\angle 50.94°$$

$$\mathbf{I}_a = \mathbf{I}_1 = \mathbf{132\angle 30°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = -38.105 + j27.9 = \mathbf{47.23\angle 143.8°\ A}$$

$$\mathbf{I}_c = -\mathbf{I}_2 = \mathbf{120.9\angle 230.9°\ A}$$

(b) $$\mathbf{S}_{AB} = \left|\mathbf{I}_1 - \mathbf{I}_3\right|^2 (j10) = j58.08 \text{ kVA}$$

$$\mathbf{S}_{BC} = \left|\mathbf{I}_2 - \mathbf{I}_3\right|^2 (20) = 29.04 \text{ kVA}$$

$$\mathbf{S}_{CA} = \left|\mathbf{I}_3\right|^2 (-j5) = (152.42)^2(-j5) = -j116.16 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = 29.04 - j58.08 \text{ kVA}$$

Real power absorbed = **29.04 kW**

(c) Total complex supplied by the source is

$$\mathbf{S} = \mathbf{29.04 - j58.08\ kVA}$$

Solution 12.57

Determine the line currents for the three-phase circuit in Fig. 12.64.
Let $\mathbf{V}_a = 220\angle 0°$, $\mathbf{V}_b = 220\angle{-120°}$, $\mathbf{V}_c = 220\angle 120°$ V.

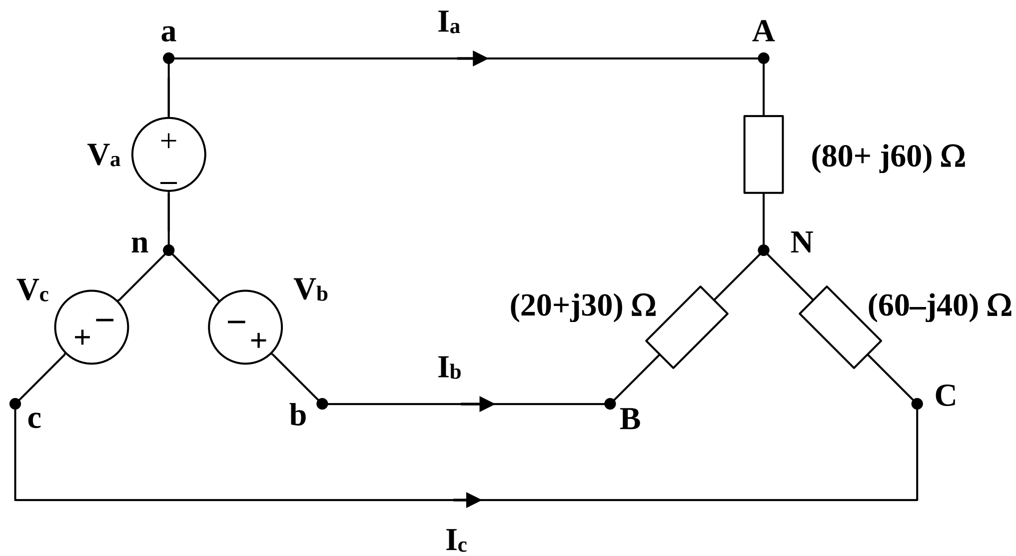
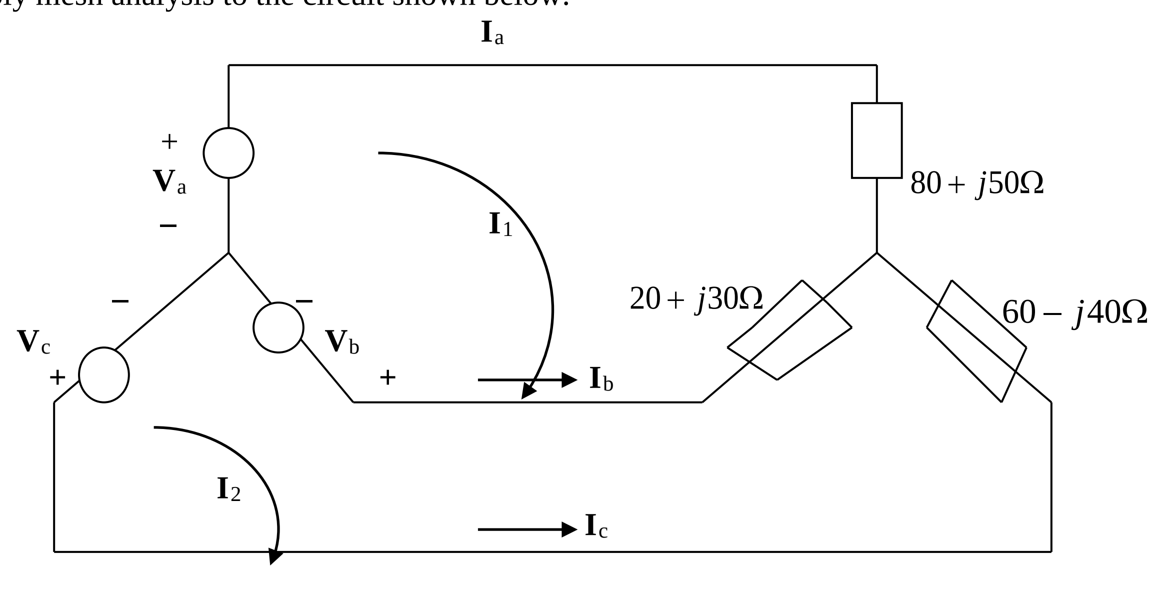


Figure 12.64
For Prob. 12.57.

Solution

We apply mesh analysis to the circuit shown below.



$\mathbf{V_a}$ = 220 V, $\mathbf{V_b}$ = (–110 – j190.53) V, $\mathbf{V_c}$ = (–110 + j190.53) V

$$(100+j80)I_1-(20+j30)I_2=V_a-V_b=330+j190.53 \qquad (1)$$
$$-(20+j30)I_1+(80-j10)I_2=V_b-V_c=-j381.1 \qquad (2)$$

Solving (1) and (2) using MATLAB gives,

```
>> Z=[100+80j,-20-30j;-20-30j,80-10j]

Z =

   1.0e+02 *

   1.0000 + 0.8000i  -0.2000 - 0.3000i
  -0.2000 - 0.3000i   0.8000 - 0.1000i

>> V=[330+190.53j;-381.1j]

V =

   1.0e+02 *

   3.3000 + 1.9053i
   0.0000 - 3.8110i

>> I = inv(Z)*V

I =

   3.7233 - 1.2170i
   1.8178 - 3.4445i or
```

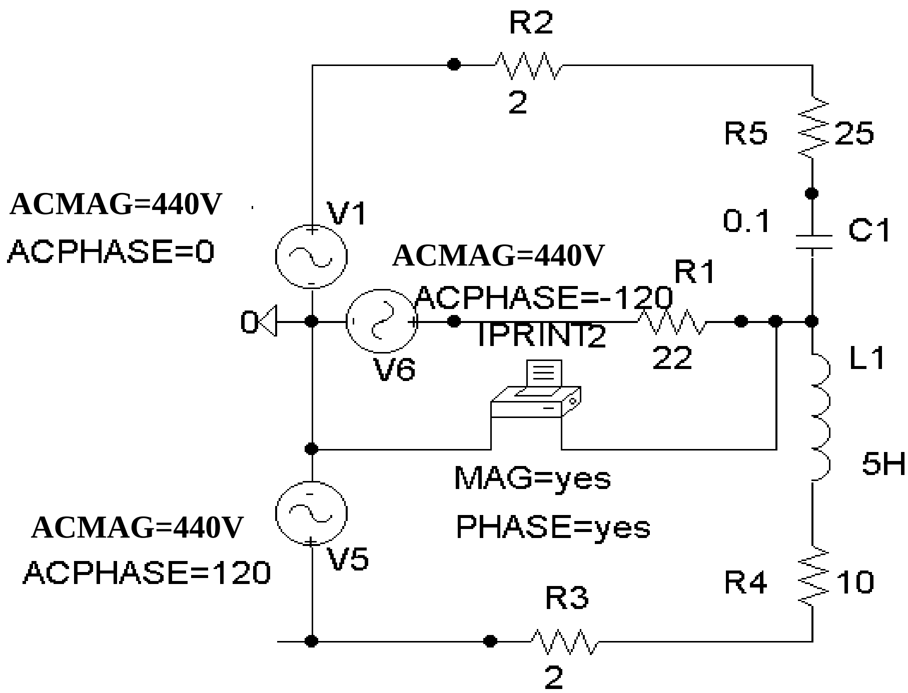
$\mathbf{I_a} = \mathbf{I_1}$ = 3.7233–j1.217 = $\mathbf{3.917\angle{-18.1^\circ}}$ **A**,
$\mathbf{I_b} = -\mathbf{I_1} + \mathbf{I_2}$ = –1.9055–j2.2275 = $\mathbf{2.931\angle{-130.55}}$ **A**,
and $\mathbf{I_c} = -\mathbf{I_2}$ = –1.8178+j3.4445 = $\mathbf{3.895\angle 117.82^\circ}$ **A**.

Solution 12.58

The schematic is shown below. IPRINT is inserted in the neutral line to measure the current through the line. In the AC Sweep box, we select Total Ptss = 1, Start Freq. = 0.1592, and End Freq. = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	2.156 E+01	–8.997 E+01

i.e. I_n = **$21.56\angle{-89.97°}$ A**

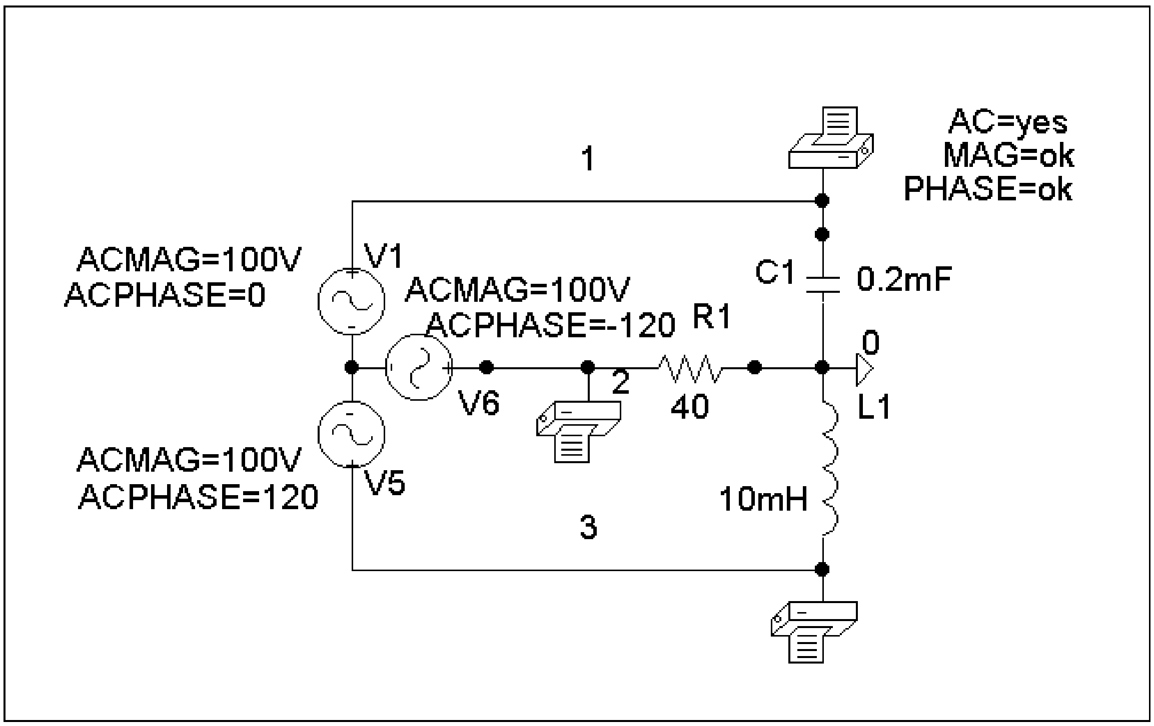


Solution 12.59

The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, we obtain an output file which includes

FREQ	VM(1)	VP(1)
6.000 E+01	2.206 E+02	–3.456 E+01
FREQ	VM(2)	VP(2)
6.000 E+01	2.141 E+02	–8.149 E+01
FREQ	VM(3)	VP(3)
6.000 E+01	4.991 E+01	–5.059 E+01

i.e. V_{AN} = **220.6∠–34.56°**, V_{BN} = **214.1∠–81.49°**, V_{CN} = **49.91∠–50.59° V**

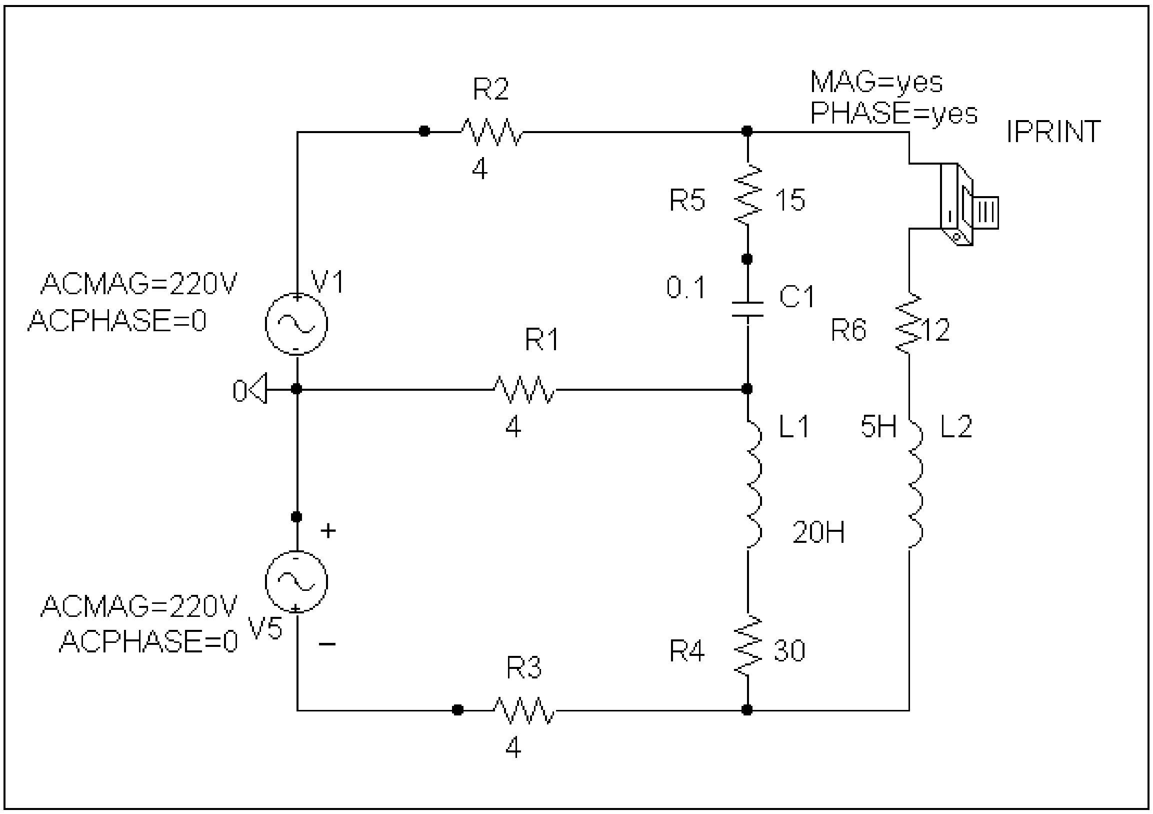


Solution 12.60

The schematic is shown below. IPRINT is inserted to give I_o. We select Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. Upon simulation, the output file includes

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	1.953 E+01	–1.517 E+01

from which, I_o = **19.53∠–15.17° A**



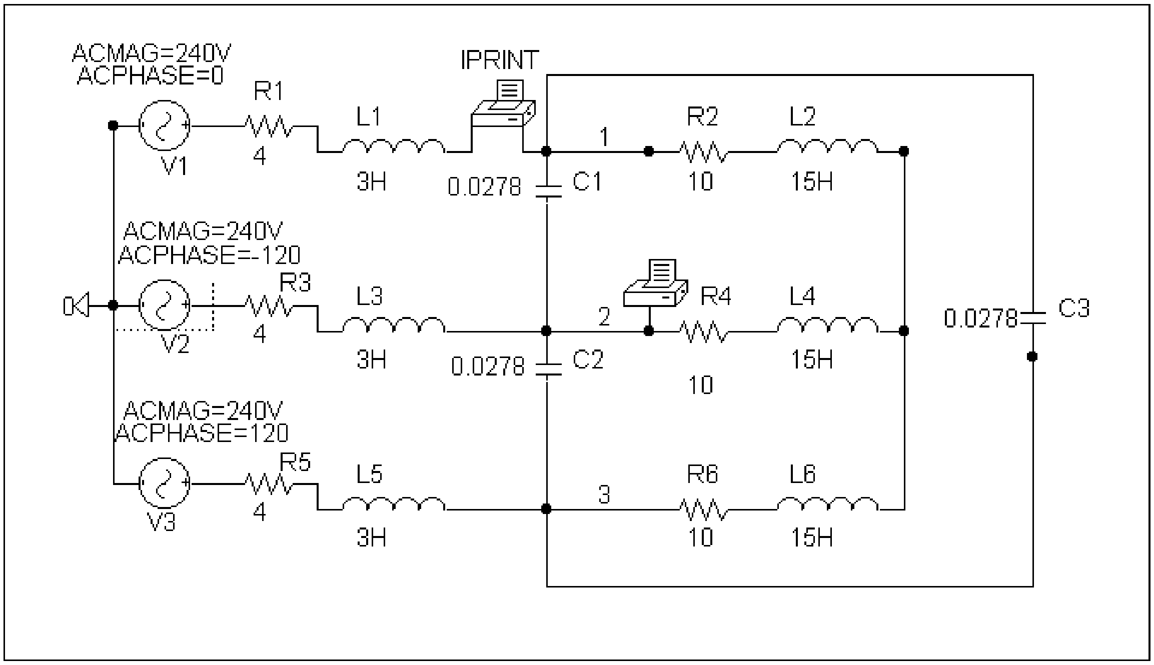
Solution 12.61

The schematic is shown below. Pseudo-components IPRINT and PRINT are inserted to measure I_{aA} and V_{BN}. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. Once the circuit is simulated, we get an output file which includes

FREQ	VM(2)	VP(2)
1.592 E–01	2.308 E+02	–1.334 E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.115 E+01	3.699 E+01

from which

$$I_{aA} = \mathbf{11.15\angle 37° \ A}, \quad V_{BN} = \mathbf{230.8\angle -133.4° \ V}$$



Solution 12.62

Using Fig. 12.68, design a problem to help other students to better understand how to use *PSpice* to analyze three-phase circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The circuit in Fig. 12.68 operates at 60 Hz. Use *PSpice* to find the source current $\mathbf{I}_{ab}$ and the line current $\mathbf{I}_{bB}$.

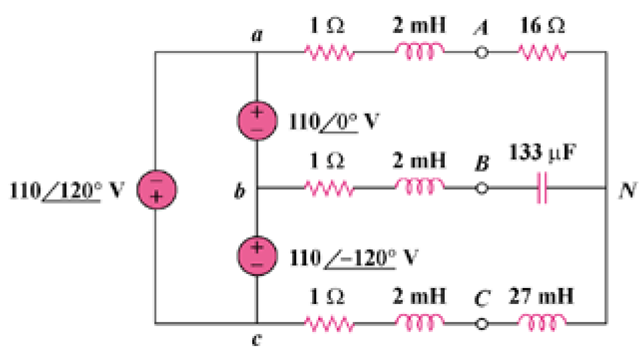


Figure 12.68

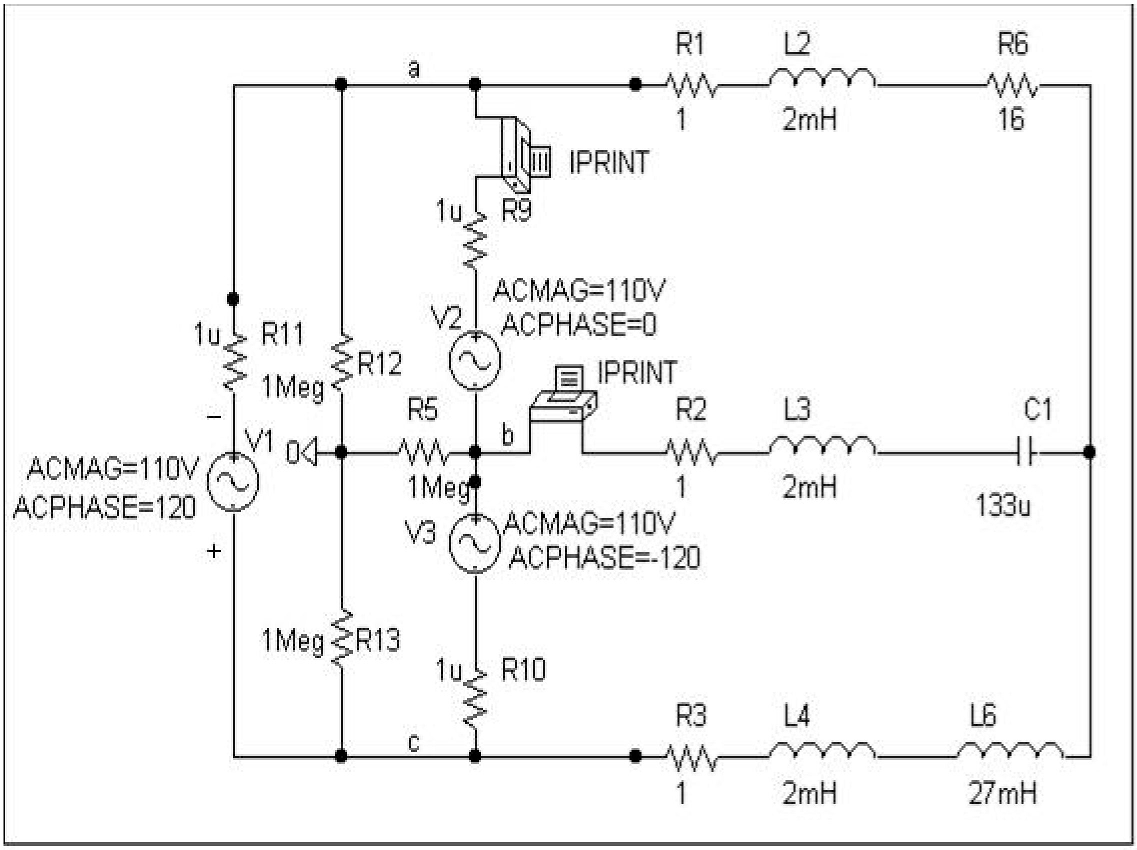
Solution

Because of the delta-connected source involved, we follow Example 12.12. In the AC Sweep box, we type Total Pts = 1, Start Freq = 60, and End Freq = 60. After simulation, the output file includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.000 E+01	5.960 E+00	–9.141 E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000 E+01	7.333 E+07	1.200 E+02

From which

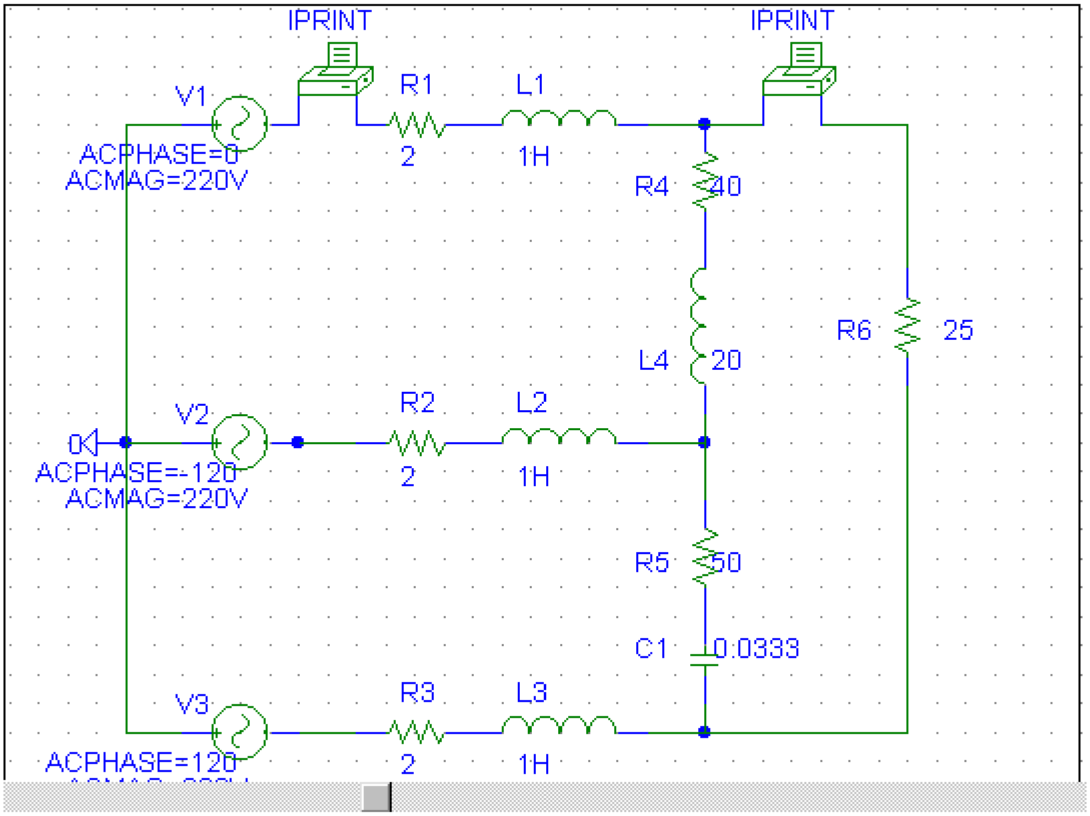
$$I_{ab} = \mathbf{3.432\angle{-46.31}^\circ\ A},\quad I_{bB} = \mathbf{10.39\angle{-78.4}^\circ\ A}$$


R1
L2
R6
a
1
2mH
16
IPRINT
1u
R9
ACMAG=110V
ACPHASE=0
V2
1u
R11
R12
1Meg
IPRINT
R5
R2
L3
C1
V1
0
b
ACMAG=110V
ACPHASE=120
1Meg
1
2mH
133u
V3
ACMAG=110V
ACPHASE=-120
1Meg
R13
1u
R10
R3
L4
L6
c
1
2mH
27mH

Solution 12.63

Let $\omega = 1$ so that $L = X/\omega = 20\,\text{H}$, and $C = \dfrac{1}{\omega X} = 0.0333\,\text{F}$

The schematic is shown below..



When the file is saved and run, we obtain an output file which includes the following:

FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E-01 1.867E+01 1.589E+02

FREQ IM(V_PRINT2)IP(V_PRINT2)

1.592E-01 1.238E+01 1.441E+02

From the output file, the required currents are:

$$\underline{\boldsymbol{I_{aA} = 18.67\angle 158.9^o \text{ A}, \quad I_{AC} = 12.38\angle 144.1^o \text{ A}}}$$

Solution 12.64

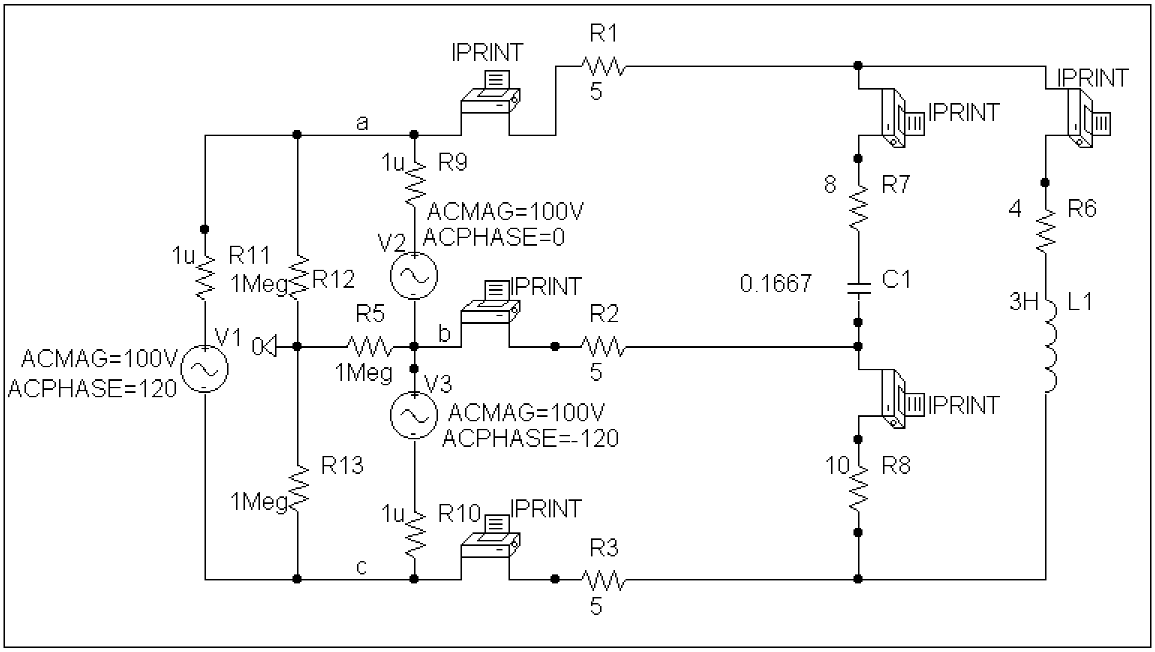
We follow Example 12.12. In the AC Sweep box we type Total Pts = 1,
Start Freq = 0.1592, and End Freq = 0.1592. After simulation the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.710 E+00	7.138 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.781 E+07	–1.426 E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	3.898 E+00	–5.076 E+00
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
1.592 E–01	3.547 E+00	6.157 E+01
FREQ	IM(V_PRINT5)	IP(V_PRINT5)
1.592 E–01	1.357 E+00	9.781 E+01
FREQ	IM(V_PRINT6)	IP(V_PRINT6)
1.592 E–01	3.831 E+00	–1.649 E+02

from this we obtain

$$I_{aA} = \mathbf{4.71\angle 71.38^\circ\ A},\ I_{bB} = \mathbf{6.781\angle{-142.6^\circ}\ A},\ I_{cC} = \mathbf{3.898\angle{-5.08^\circ}\ A}$$

$$I_{AB} = \mathbf{3.547\angle 61.57^\circ\ A},\ I_{AC} = \mathbf{1.357\angle 97.81^\circ\ A},\ I_{BC} = \mathbf{3.831\angle{-164.9^\circ}\ A}$$

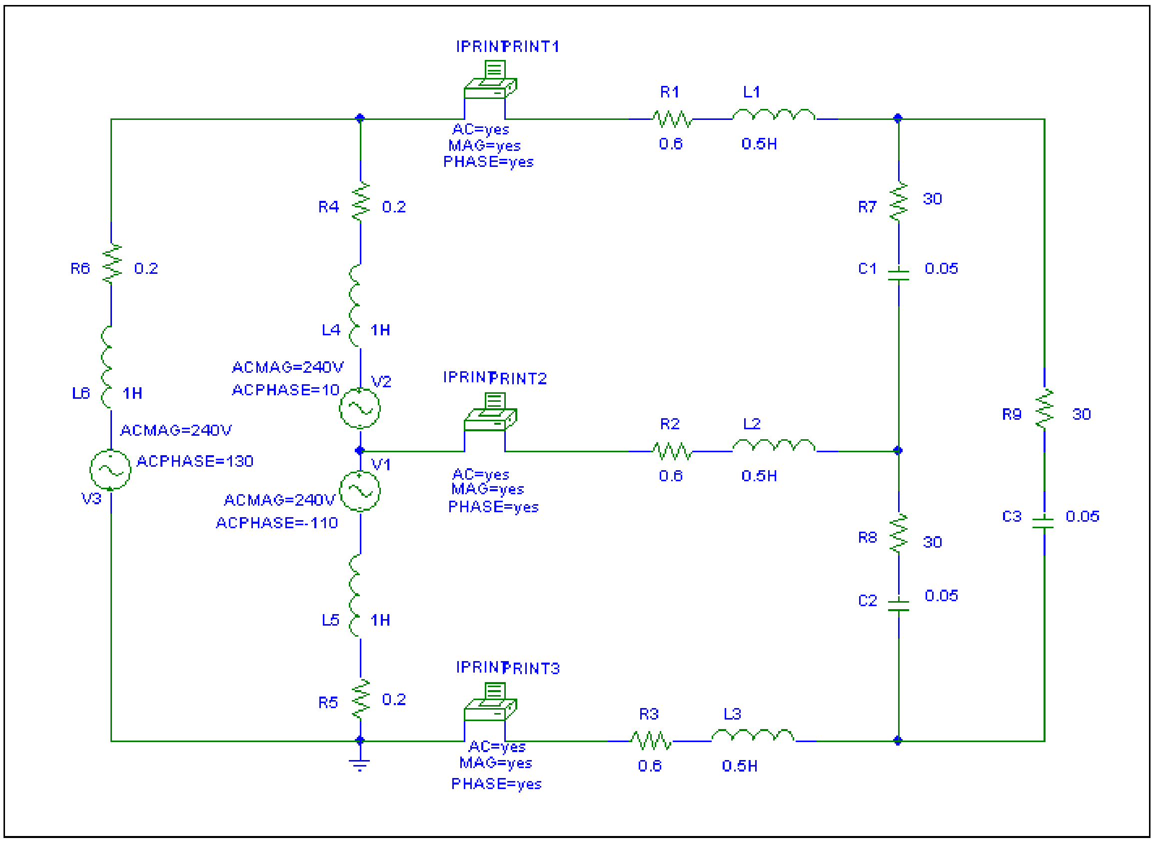

IPRINT
R1
5
a
1u
R9
ACMAG=100V
ACPHASE=0
V2
IPRINT
IPRINT
IPRINT
8
R7
4
R6
1u
R11
1Meg
R12
R5
R2
0.1667
C1
3H
L1
V1
0
b
ACMAG=100V
ACPHASE=120
1Meg
5
V3
ACMAG=100V
ACPHASE=-120
IPRINT
R13
1Meg
10
R8
1u
R10
IPRINT
R3
c
5

Solution 12.65

Due to the delta-connected source, we follow Example 12.12. We type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. The schematic is shown below. After it is saved and simulated, we obtain an output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	1.140E+01	8.664E+00
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.140E+01	-1.113E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	1.140E+01	1.287E+02

Thus, I_{aA} = **11.02∠12° A**, I_{bB} = **11.02∠–108° A**, I_{cC} = **11.02∠132° A**



Since this is a balanced circuit, we can perform a quick check. The load resistance is large compared to the line and source impedances so we will ignore them (although it would not be difficult to include them).

Converting the sources to a Y configuration we get:

$$V_{an} = 138.56 \angle{-20^\circ} \text{ Vrms}$$

and

$$Z_Y = 10 - j6.667 = 12.019\angle{-33.69^\circ}$$

Now we can calculate,

$$I_{aA} = (138.56 \angle{-20^\circ})/(12.019\angle{-33.69^\circ}) = 11.528\angle 13.69^\circ$$

Clearly, we have a good approximation which is very close to what we really have.

Solution 12.66

A three-phase, four-wire system operating with a 480-V line voltage is shown in Fig. 12.71. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

(a) the voltage to neutral
(b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$
(c) the readings of the wattmeters
(d) the total power absorbed by the load

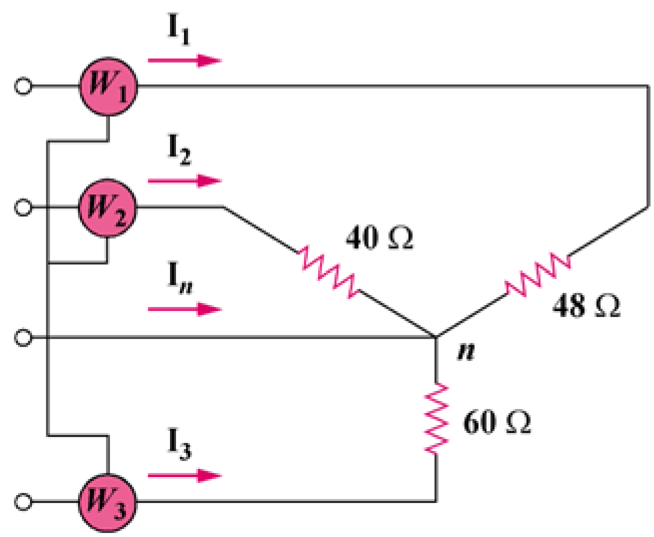


Figure 12.71
For Prob. 12.66.

Solution

(a) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{480}{\sqrt{3}} = \mathbf{277.1\ V}$

(b) Because the load is unbalanced, we have an unbalanced three-phase system. Assuming an abc sequence,

$$\mathbf{I}_1 = \frac{277.128\angle 0°}{48} = \mathbf{5.774\angle 0°\ A}$$

$$\mathbf{I}_2 = \frac{277.128\angle\text{-}120°}{40} = \mathbf{6.928\angle{-120}°\ A}$$

$$\mathbf{I}_3 = \frac{277.128\angle 120°}{60} = \mathbf{4.619\angle 120°\ A}$$

$$\mathbf{I}_n = -\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3 = -5.774 - (-3.464 - j6) - (-2.31 + j4)$$

$= (0 + j2)\ \text{A} = \mathbf{2\angle 90°\ A}$.

Hence,

$|\mathbf{I_1}| = 5.774$ A, $|\mathbf{I_2}| = 6.928$ A, $|\mathbf{I_3}| = 4.619$ A.

(c) $P_1 = I_1^2 R_1 = (5.774)^2(48) = \mathbf{1.6003\ kW}$

$P_2 = I_2^2 R_2 = (6.928)^2(40) = \mathbf{1.9199\ kW}$

$P_3 = I_3^2 R_3 = (4.619)^2(60) = \mathbf{1.2801\ kW}$

(d) $\mathrm{P_T} = \mathrm{P_1} + \mathrm{P_2} + \mathrm{P_3} = \mathbf{4.8\ kW}$

Solution 12.67

(a) The power to the motor is

$$P_T = S\cos\theta = (260)(0.85) = 221 \text{ kW}$$

The motor power per phase is

$$P_p = \frac{1}{3}P_T = 73.67 \text{ kW}$$

Hence, the wattmeter readings are as follows:

$$W_a = 73.67 + 24 = \mathbf{97.67\ kW}$$

$$W_b = 73.67 + 15 = \mathbf{88.67\ kW}$$

$$W_c = 73.67 + 9 = \mathbf{82.67\ kW}$$

(b) The motor load is balanced so that $I_N = 0$.

For the lighting loads,

$$I_a = \frac{24{,}000}{120} = 200 \text{ A}$$

$$I_b = \frac{15{,}000}{120} = 125 \text{ A}$$

$$I_c = \frac{9{,}000}{120} = 75 \text{ A}$$

If we let

$$\mathbf{I}_a = I_a\angle 0° = 200\angle 0° \text{ A}$$

$$\mathbf{I}_b = 125\angle -120° \text{ A}$$

$$\mathbf{I}_c = 75\angle 120° \text{ A}$$

Then,

$$-\mathbf{I}_N = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c$$

$$-\mathbf{I}_N = 200 + (125)\left(-0.5 - j\frac{\sqrt{3}}{2}\right) + (75)\left(-0.5 + j\frac{\sqrt{3}}{2}\right)$$

$$-\mathbf{I}_N = 100 - j43.3 \text{ A}$$

$$\left|\mathbf{I}_N\right| = \mathbf{108.97\ A}$$

Solution 12.68

(a) $S = \sqrt{3}\,V_L I_L = \sqrt{3}(330)(8.4) = \mathbf{4801\ VA}$

(b) $P = S\cos\theta \longrightarrow \text{pf} = \cos\theta = \dfrac{P}{S}$

$$\text{pf} = \frac{4500}{4801.24} = \mathbf{0.9372}$$

(c) For a wye-connected load,
$I_p = I_L = \mathbf{8.4\ A}$

(d) $V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{330}{\sqrt{3}} = \mathbf{190.53\ V}$

Solution 12.69

For load 1,

$$\overline{S}_1 = S_1\cos\theta_1 + jS_1\sin\theta_1$$

$$pf = 0.85 = \cos\theta_1 \quad \longrightarrow \quad \theta_1 = 31.79^o$$

$$\overline{S}_1 = 13.6 + j8.43 \quad \text{kVA}$$

For load 2,

$$\overline{S_2} = 12x0.6 + j12x0.8 = 7.2 + j9.6 \ \ \text{kVA}$$

For load 3,

$$\overline{S}_3 = 8 + j0 \quad \text{kVA}$$

Therefore,
S = S_1 + S_2 + S_3 = [28.8+j18.03] kVA

Although we can solve this using a delta load, it will be easier to assume our load is wye connected. We also need the wye voltages and will assume that the phase angle on V_{an} = 208/1.73205 = 120.089 is –30 degrees.

Since
S = 3**VI*** or **I*** = **S**/(3**V**) = (33,978∠32.048°)/[3(120.089)∠–30°] = 94.31∠62.05° A.

I_a = 94.31∠–62.05° A, **I_b = 94.31∠177.95° A**, **I_c = 94.31∠57.95° A**

I = 138.46 – j86.68 = **163.35∠-32° A**.

Solution 12.70

$$P_T = P_1 + P_2 = 1200 - 400 = 800$$

$$Q_T = P_2 - P_1 = -400 - 1200 = -1600$$

$$\tan\theta = \frac{Q_T}{P_T} = \frac{-1600}{800} = -2 \quad \longrightarrow \quad \theta = -63.43°$$

$$\text{pf} = \cos\theta = \mathbf{0.4472} \quad \textbf{(leading)}$$

$$Z_p = \frac{V_L}{I_L} = \frac{240}{6} = 40$$

$$\mathbf{Z_p = 40\angle -63.43°\ \Omega}$$

Solution 12.71

(a) If $\mathbf{V}_{ab} = 208\angle 0°$, $\mathbf{V}_{bc} = 208\angle -120°$, $\mathbf{V}_{ca} = 208\angle 120°$,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{Ab}} = \frac{208\angle 0°}{20} = 10.4\angle 0°$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_{BC}} = \frac{208\angle -120°}{10\sqrt{2}\angle -45°} = 14.708\angle -75°$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_{CA}} = \frac{208\angle 120°}{13\angle 22.62°} = 16\angle 97.38°$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA} = 10.4\angle 0° - 16\angle 97.38°$$
$$\mathbf{I}_{aA} = 10.4 + 2.055 - j15.867$$
$$\mathbf{I}_{aA} = 20.171\angle -51.87°$$

$$\mathbf{I}_{cC} = \mathbf{I}_{CA} - \mathbf{I}_{BC} = 16\angle 97.83° - 14.708\angle -75°$$
$$\mathbf{I}_{cC} = 30.64\angle 101.03°$$

$$P_1 = |\mathbf{V}_{ab}||\mathbf{I}_{aA}|\cos(\theta_{V_{ab}} - \theta_{I_{aA}})$$
$$P_1 = (208)(20.171)\cos(0° + 51.87°) = \mathbf{2.590\ \textit{kW}}$$

$$P_2 = |\mathbf{V}_{cb}||\mathbf{I}_{cC}|\cos(\theta_{V_{cb}} - \theta_{I_{cC}})$$

But $\mathbf{V}_{cb} = -\mathbf{V}_{bc} = 208\angle 60°$

$$P_2 = (208)(30.64)\cos(60° - 101.03°) = \mathbf{4.808\ \textit{kW}}$$

(b) $P_T = P_1 + P_2 = 7398.17 \text{ W}$

$$Q_T = \sqrt{3}(P_2 - P_1) = 3840.25 \text{ VAR}$$
$$\mathbf{S}_T = P_T + jQ_T = 7398.17 + j3840.25 \text{ VA}$$
$$S_T = |\mathbf{S}_T| = \mathbf{8.335\ \textit{kVA}}$$

Solution 12.72

From ***Problem 12.11,***

$\mathbf{V}_{AB} = 220\angle 130° \text{ V}$ and $\mathbf{I}_{aA} = 30\angle 180° \text{ A}$

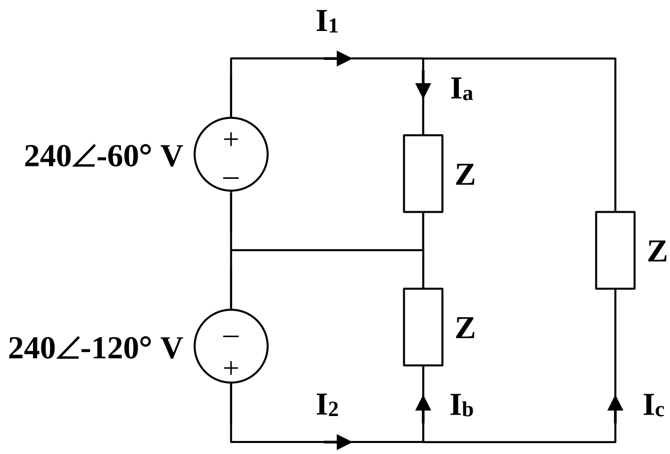
$P_1 = (220)(30)\cos(130° - 180°) = \mathbf{4.242} \; \boldsymbol{kW}$

$\mathbf{V}_{CB} = \mathbf{-V}_{BC} = 220\angle 190°$
$\mathbf{I}_{cC} = 30\angle -60°$

$P_2 = (220)(30)\cos(190° + 60°) = \mathbf{-\ 2.257} \; \boldsymbol{k}\mathbf{W}$

Solution 12.73

Consider the circuit as shown below.



$$\mathbf{Z} = 10 + j30 = 31.62\angle 71.57°$$

$$\mathbf{I}_a = \frac{240\angle -60°}{31.62\angle 71.57°} = 7.59\angle -131.57°$$

$$\mathbf{I}_b = \frac{240\angle -120°}{31.62\angle 71.57°} = 7.59\angle -191.57°$$

$$\mathbf{I}_c\,\mathbf{Z} + 240\angle -60° - 240\angle -120° = 0$$

$$\mathbf{I}_c = \frac{-240}{31.62\angle 71.57°} = 7.59\angle 108.43°$$

$$\mathbf{I}_1 = \mathbf{I}_a - \mathbf{I}_c = 13.146\angle -101.57°$$

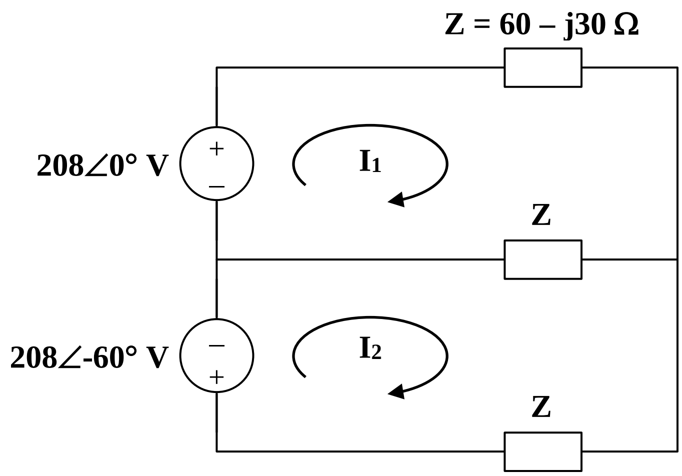
$$\mathbf{I}_2 = \mathbf{I}_b + \mathbf{I}_c = 13.146\angle 138.43°$$

$$P_1 = \mathrm{Re}\left[\mathbf{V}_1\,\mathbf{I}_1^*\right] = \mathrm{Re}\left[(240\angle -60°)(13.146\angle 101.57°)\right] = \mathbf{2.360}\ \boldsymbol{kW}$$

$$P_2 = \mathrm{Re}\left[\mathbf{V}_2\,\mathbf{I}_2^*\right] = \mathrm{Re}\left[(240\angle -120°)(13.146\angle -138.43°)\right] = \mathbf{-632.8\ W}$$

Solution 12.74

Consider the circuit shown below.



For mesh 1,

$$208 = 2\mathbf{Z}\mathbf{I}_1 - \mathbf{Z}\mathbf{I}_2$$

For mesh 2,

$$-208\angle -60° = -\mathbf{Z}\mathbf{I}_1 + 2\mathbf{Z}\mathbf{I}_2$$

In matrix form,

$$\begin{bmatrix} 208 \\ -208\angle -60° \end{bmatrix} = \begin{bmatrix} 2\mathbf{Z} & -\mathbf{Z} \\ -\mathbf{Z} & 2\mathbf{Z} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 3\mathbf{Z}^2, \quad \Delta_1 = (208)(1.5 + j0.866)\mathbf{Z}, \quad \Delta_2 = (208)(j1.732)\mathbf{Z}$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{(208)(1.5 + j0.866)}{(3)(60 - j30)} = 1.789\angle 56.56°$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{(208)(j1.732)}{(3)(60 - j30)} = 1.79\angle 116.56°$$

$$P_1 = \text{Re}\left[\mathbf{V}_1 \mathbf{I}_1^*\right] = \text{Re}\left[(208)(1.789\angle -56.56°)\right] = \mathbf{208.98\ W}$$

$$P_2 = \text{Re}\left[\mathbf{V}_2 (-\mathbf{I}_2)^*\right] = \text{Re}\left[(208\angle -60°))(1.79\angle 63.44°)\right] = \mathbf{371.65\ W}$$

Solution 12.75

(a) $$I = \frac{V}{R} = \frac{12}{600} = \mathbf{20\ mA}$$

(b) $$I = \frac{V}{R} = \frac{120}{600} = \mathbf{200\ mA}$$

Solution 12.76

If both appliances have the same power rating, P,

$$I = \frac{P}{V_s}$$

For the 120-V appliance, $I_1 = \frac{P}{120}$.

For the 240-V appliance, $I_2 = \frac{P}{240}$.

$$\text{Power loss} = I^2 R = \begin{cases} \dfrac{P^2 R}{120^2} & \text{for the 120-V appliance} \\ \dfrac{P^2 R}{240^2} & \text{for the 240-V appliance} \end{cases}$$

Since $\frac{1}{120^2} > \frac{1}{240^2}$, **the losses in the 120-V appliance are higher**.

Solution 12.77

A three-phase generator supplied 10 kVA at a power factor of 0.85 lagging. If 7.5 kW are delivered to the load and line losses are 160 W per phase, what are the losses in the generator?

Solution

$P_g = P_T - P_{load} - P_{line}$, $\qquad$ pf $= 0.85$

But $\quad P_T = 10k \cos(\theta) = 10k(0.85) = 8.5$ kW
$P_g = 8.5$ kW – 7.5 kW – (3)(160) W = **520 W**

Solution 12.78

$$\cos\theta_1 = \frac{51}{60} = 0.85 \quad \longrightarrow \quad \theta_1 = 31.79^\circ$$
$$Q_1 = S_1 \sin\theta_1 = (60)(0.5268) = 31.61\,\text{kVAR}$$

$$P_2 = P_1 = 51\,\text{kW}$$
$$\cos\theta_2 = 0.95 \quad \longrightarrow \quad \theta_2 = 18.19^\circ$$
$$S_2 = \frac{P_2}{\cos\theta_2} = 53.68\text{ kVA}$$
$$Q_2 = S_2 \sin\theta_2 = 16.759\text{ kVAR}$$

$$Q_c = Q_1 - Q_2 = 3.61 - 16.759 = 14.851\,\text{kVAR}$$

For each load,

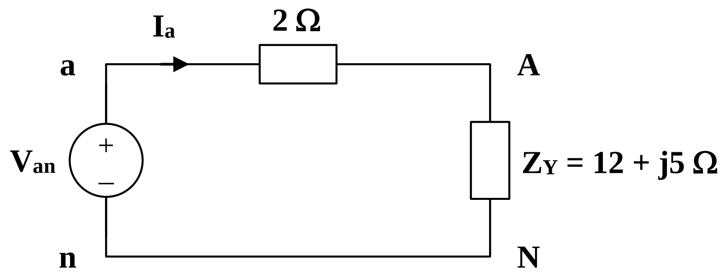
$$Q_{c1} = \frac{Q_c}{3} = 4.95\text{ kVAR}$$
$$C = \frac{Q_{c1}}{\omega V^2} = \frac{4950}{(2\pi)(60)(440)^2} = \mathbf{67.82\ \mu F}$$

Solution 12.79

A balanced three-phase generator has an *abc* phase sequence with phase voltage $\mathbf{V}_{an}$ = 554.3∠0° V. The generator feeds an induction motor which may be represented by a balanced Y-connected load with an impedance of 12 + j5 Ω per phase. Find the line currents and the load voltages. Assume a line impedance of 2 Ω per phase.

Solution

Consider the per-phase equivalent circuit below.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y + 2} = \frac{554.3\angle 0°}{14 + j5} = \frac{554.3}{14.866\angle 19.65°} = \mathbf{37.29\angle{-19.65°}\ A}$$

Thus,

$$\mathbf{I}_b = \mathbf{I}_a \angle{-120°} = \mathbf{37.29\angle{-139.65°}\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a \angle 120° = \mathbf{37.29\angle 100.35°\ A}$$

$$\mathbf{V}_{AN} = \mathbf{I}_a \mathbf{Z}_Y = (37.286\angle{-19.65°})(13\angle 22.62°) = \mathbf{484.7\angle 2.97°\ V}$$

Thus,

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle{-120°} = \mathbf{484.7\angle{-117.03°}\ V}$$

$$\mathbf{V}_{CN} = \mathbf{V}_{AN}\angle 120° = \mathbf{484.7\angle 122.97°\ V}$$

Solution 12.80

$$S = S_1 + S_2 + S_3 = 6[0.83 + j\sin(\cos^{-1} 0.83)] + S_2 + 8(0.7071 - j0.7071)$$
$$S = 10.6368 - j2.31 + S_2 \text{ kVA} \qquad (1)$$

But
$$S = \sqrt{3} V_L I_L \angle\theta = \sqrt{3}(208)(84.6)(0.8 + j0.6) \text{ VA} = 24.383 + \text{j}18.287 \text{ kVA} \qquad (2)$$

From (1) and (2),
$$S_2 = 13.746 + j20.6 = 24.76\angle 56.28 \text{ kVA}$$
Thus, the unknown load is **24.76 kVA at 0.5551 pf lagging**.

Solution 12.81

$$\text{pf} = 0.8 \quad (\text{leading}) \quad \longrightarrow \quad \theta_1 = -36.87°$$
$$\mathbf{S}_1 = 150\angle -36.87° \text{ kVA}$$

$$\text{pf} = 1.0 \quad \longrightarrow \quad \theta_2 = 0°$$
$$\mathbf{S}_2 = 100\angle 0° \text{ kVA}$$

$$\text{pf} = 0.6 \quad (\text{lagging}) \quad \longrightarrow \quad \theta_3 = 53.13°$$
$$\mathbf{S}_3 = 200\angle 53.13° \text{ kVA}$$

$$\mathbf{S}_4 = 80 + j95 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$
$$\mathbf{S} = 420 + j165 = 451.2\angle 21.45° \text{ kVA}$$

$$S = \sqrt{3}\, V_L I_L$$

$$I_L = \frac{S}{\sqrt{3}\, V_L} = \frac{451.2 \times 10^3}{\sqrt{3} \times 480} = 542.7 \text{ A}$$

For the line,

$$\mathbf{S}_L = 3 I_L^2 \mathbf{Z}_L = (3)(542.7)^2 (0.02 + j0.05)$$
$$\mathbf{S}_L = 17.67 + j44.18 \text{ kVA}$$

At the source,

$$\mathbf{S}_T = \mathbf{S} + \mathbf{S}_L = 437.7 + j209.2$$
$$\mathbf{S}_T = 485.1\angle 25.55° \text{ kVA}$$

$$V_T = \frac{S_T}{\sqrt{3}\, I_L} = \frac{485.1 \times 10^3}{\sqrt{3} \times 542.7} = \mathbf{516 \text{ V}}$$

Solution 12.82

$$\overline{S}_1 = 400(0.8 + j0.6) = 320 + j240\,\text{kVA}, \qquad \overline{S}_2 = 3\frac{V^2{}_p}{Z^*{}_p}$$

For the delta-connected load, $V_L = V_p$

$$\overline{S}_2 = 3x\frac{(2400)^2}{10 - j8} = 1053.7 + j842.93\,\text{kVA}$$

$$\overline{S} = \overline{S}_1 + \overline{S}_2 = 1.3737 + j1.0829\,\text{MVA}$$

Let I = $I_1 + I_2$ be the total line current. For I_1,

$$S_1 = 3V_p I^*{}_1, \qquad V_p = \frac{V_L}{\sqrt{3}}$$

$$I^*{}_1 = \frac{S_1}{\sqrt{3}V_L} = \frac{(320 + j240)x10^3}{\sqrt{3}(2400)}, \qquad I_1 = 76.98 - j57.735$$

For I_2, convert the load to wye.

$$I_2 = I_p\sqrt{3}\angle -30^o = \frac{2400}{10 + j8}\sqrt{3}\angle -30^o = 273.1 - j289.76$$

$$I = I_1 + I_2 = 350 - j347.5$$

$$\boldsymbol{V_s = V_L + V_{line} = 2400 + I(3 + j6) = 5.185 + j1.405\,\text{kV} \quad \longrightarrow \quad |V_s| = \underline{5.372\,\text{kV}}}$$

Solution 12.83

$$S_1 = 120x746x0.95(0.707 + j0.707) = 60.135 + j60.135\,\text{kVA}, \qquad \text{S}_2 = 80\,\text{kVA}$$

$$S = S_1 + S_2 = 140.135 + j60.135\,\text{kVA}$$

$$\textbf{But } |\boldsymbol{S}| = \sqrt{3}\boldsymbol{V}_L\boldsymbol{I}_L \qquad \longrightarrow \qquad \boldsymbol{I}_L = \frac{|\boldsymbol{S}|}{\sqrt{3}\boldsymbol{V}_L} = \frac{\mathbf{152.49}x\mathbf{10^3}}{\sqrt{3}x\mathbf{480}} = \underline{\mathbf{183.42\ A}}$$

Solution 12.84

We first find the magnitude of the various currents.

For the motor,

$$I_L = \frac{S}{\sqrt{3}\,V_L} = \frac{4000}{440\sqrt{3}} = 5.248 \text{ A}$$

For the capacitor,

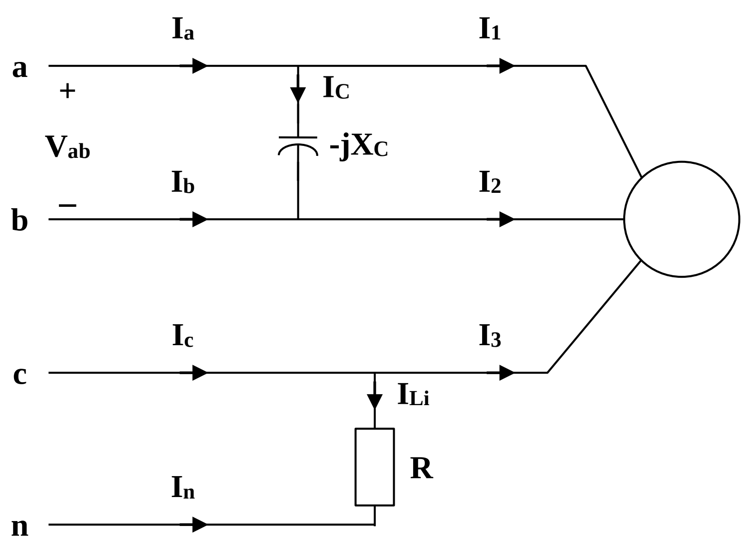
$$I_C = \frac{Q_c}{V_L} = \frac{1800}{440} = 4.091 \text{ A}$$

For the lighting,

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_{Li} = \frac{P_{Li}}{V_p} = \frac{800}{254} = 3.15 \text{ A}$$

Consider the figure below.



If $\mathbf{V}_{an} = V_p\angle 0°$, $\quad \mathbf{V}_{ab} = \sqrt{3}\,V_p\angle 30°$

$$\mathbf{V}_{cn} = V_p\angle 120°$$

$$\mathbf{I}_C = \frac{\mathbf{V}_{ab}}{-jX_C} = 4.091\angle 120°$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = 4.091\angle(\theta + 30°)$$

where $\theta = \cos^{-1}(0.72) = 43.95°$

$$\mathbf{I}_1 = 5.249\angle 73.95°$$

$$\mathbf{I}_2 = 5.249\angle -46.05°$$

$$\mathbf{I}_3 = 5.249\angle 193.95°$$

$$\mathbf{I}_{Li} = \frac{\mathbf{V}_{cn}}{R} = 3.15\angle 120°$$

Thus,

$$\mathbf{I}_a = \mathbf{I}_1 + \mathbf{I}_C = 5.249\angle 73.95° + 4.091\angle 120°$$
$$\mathbf{I}_a = \mathbf{8.608\angle 93.96°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_C = 5.249\angle -46.05° - 4.091\angle 120°$$
$$\mathbf{I}_b = \mathbf{9.271\angle -52.16°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_3 + \mathbf{I}_{Li} = 5.249\angle 193.95° + 3.15\angle 120°$$
$$\mathbf{I}_c = \mathbf{6.827\angle 167.6°\ A}$$

$$\mathbf{I}_n = -\mathbf{I}_{Li} = \mathbf{3.15\angle -60°\ A}$$

Solution 12.85

Let $\quad Z_Y = R$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138.56 \text{ V}$$

$$P = V_p I_p = \frac{27}{2} = 9 \text{ kW} = \frac{V_p^2}{R}$$

$$R = \frac{V_p^2}{P} = \frac{(138.56)^2}{9000} = 2.133\,\Omega$$

Thus, $\quad Z_Y = \mathbf{2.133\,\Omega}$

Solution 12.86

For the single-phase three-wire system in Fig. 12.77, find currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{nN}$.

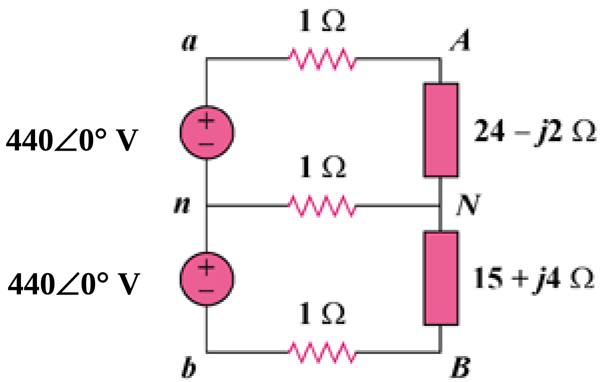
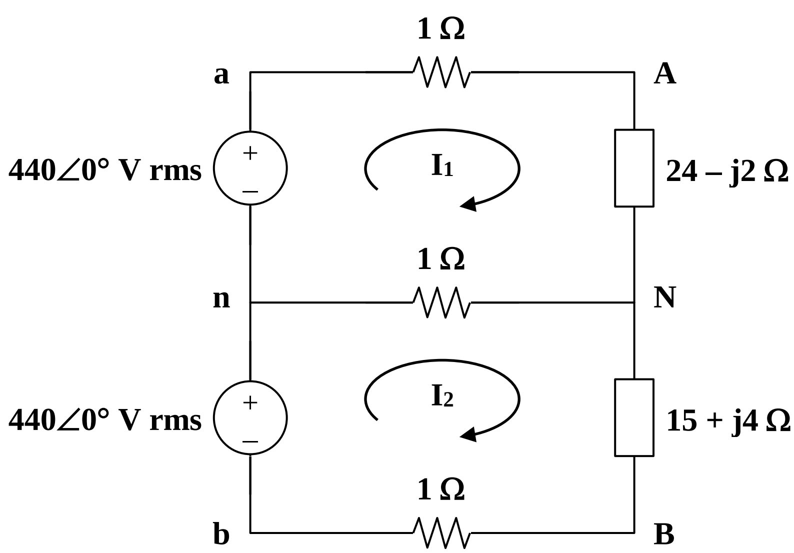


Figure 12.77
For Prob. 12.86.

Solution

Consider the circuit shown below.



For the two meshes,

$$440 = (26 - j2)\mathbf{I}_1 - \mathbf{I}_2 \qquad (1)$$

$$440 = (17 + j4)\mathbf{I}_2 - \mathbf{I}_1 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 440 \\ 440 \end{bmatrix} = \begin{bmatrix} 26 - j2 & \text{-}1 \\ \text{-}1 & 17 + j4 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = 449 + \text{j}70\,, \quad \Delta_1 = (440)(18 + j4)\,, \quad \Delta_2 = (440)(27 - j2)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{440 \times 18.44\angle 12.53^\circ}{454.42\angle 8.86^\circ} = \mathbf{17.8548\angle 3.67^\circ\ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{440 \times 27.07\angle\text{-}4.24^\circ}{454.42\angle 8.86^\circ} = \mathbf{26.211\angle{-13.1}^\circ\ A}$$

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_1 = \mathbf{17.8548\angle 3.67^\circ\ A}$$

$$\mathbf{I}_{\text{bB}} = \text{-}\mathbf{I}_2 = \mathbf{26.211\angle 166.9^\circ\ A}$$

$$\mathbf{I}_{nN} = \mathbf{I}_2 - \mathbf{I}_1 = 25.529 - \text{j}5.9408 - 17.8182 - \text{j}1.14288$$

$= 7.711 - \text{j}7.084 =$ $\mathbf{(10.471\angle{-42.57}^\circ)\ A}$

Solution 12.87

Consider the single-phase three-wire system shown in Fig. 12.78. Find the current in the neutral wire and the complex power supplied by each source. Take $\mathbf{V}_s$ as a $220\angle 0°$–V, 60-Hz source.

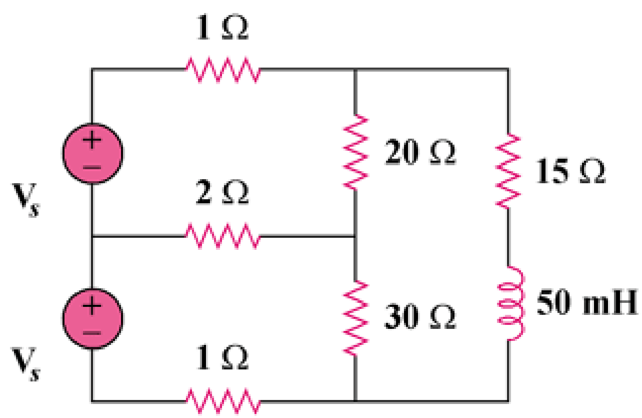
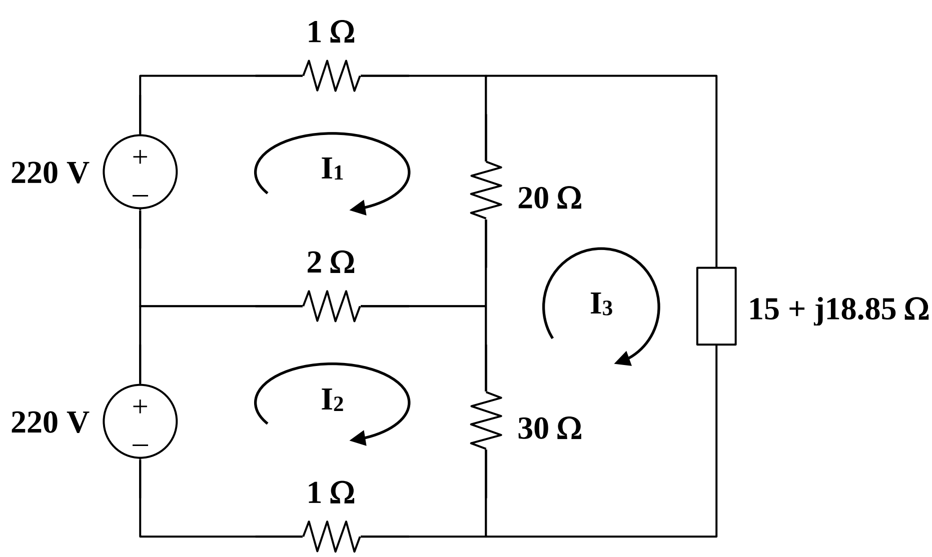


Figure 12.78
For Prob. 12.87.

Solution

$L = 50 \text{ mH} \quad \longrightarrow \quad j\omega L = j(2\pi)(60)(50\text{x}10^{-3}) = j18.85$

Consider the circuit below.



Applying KVl to the three meshes, we obtain

$$23\mathbf{I}_1 - 2\mathbf{I}_2 - 20\mathbf{I}_3 = 220 \qquad (1)$$

$$-2\mathbf{I}_1 + 33\mathbf{I}_2 - 30\mathbf{I}_3 = 220 \qquad (2)$$

$$-20\mathbf{I}_1 - 30\mathbf{I}_2 + (65 + j18.85)\mathbf{I}_3 = 0 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 23 & -2 & -20 \\ -2 & 33 & -30 \\ -20 & -30 & 65 + j18.85 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 220 \\ 220 \\ 0 \end{bmatrix}$$

$$\Delta = 12{,}775 + j14{,}232, \qquad \Delta_1 = (220)(1975 + j659.8)$$

$$\Delta_2 = (220)(1825 + j471.3), \qquad \Delta_3 = (220)(1450)$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{220 \times 2082\angle 18.47°}{19214\angle 48.09°} = 23.951\angle{-29.62°}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{220 \times 1884.9\angle 14.48°}{19124\angle 48.09°} = 21.675\angle{-33.61°}$$

$$\mathbf{I}_n = \mathbf{I}_2 - \mathbf{I}_1 = \frac{\Delta_2 - \Delta_1}{\Delta} = \frac{(220)(-150 - j188.5)}{12{,}775 + j14{,}231.75} = \mathbf{2.77\angle{-176.6°}\ A}$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = (220)(23.951\angle 29.62°) = \mathbf{(4.581 + j2.604)\ kVA}$$

$$\mathbf{S}_2 = \mathbf{V}_2\mathbf{I}_2^* = (220)(21.675\angle 33.61°) = \mathbf{(3.971 + j2.64)\ kVA}$$