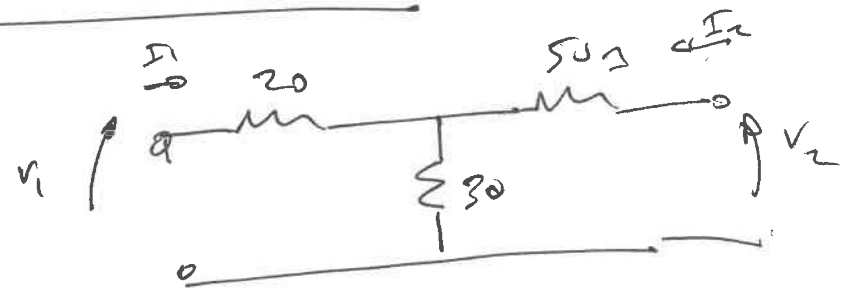
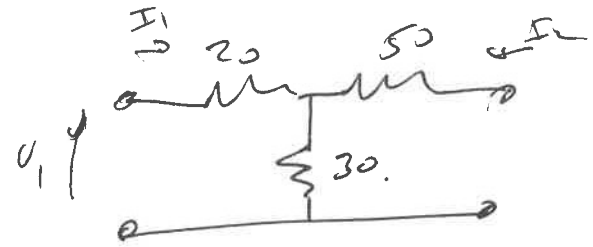
Tute 5 Q1 worked Solutions



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$



$$I_1 = V_1/50$$

$$Z_{11} = 50$$

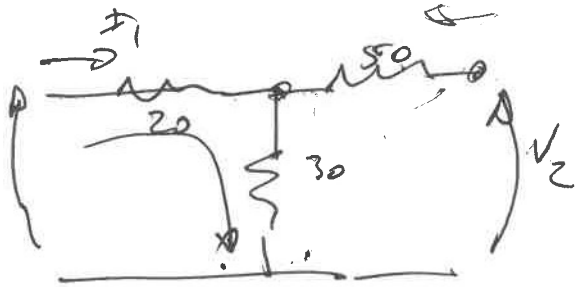
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 80$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$V_1 = \frac{30}{80} V_2 \qquad I_2 = \frac{V_2}{80}$$

$$\frac{V_1}{I_2} = \frac{\frac{30}{80} V_2}{\frac{V_2}{80}} = 30$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$



$$I_1 = \frac{V_1}{50}$$

$$V_2 = \frac{30}{50} V_1 \qquad \frac{V_2}{I_1} = \frac{V_1 \frac{30}{50}}{\frac{V_1}{50}} = 30$$

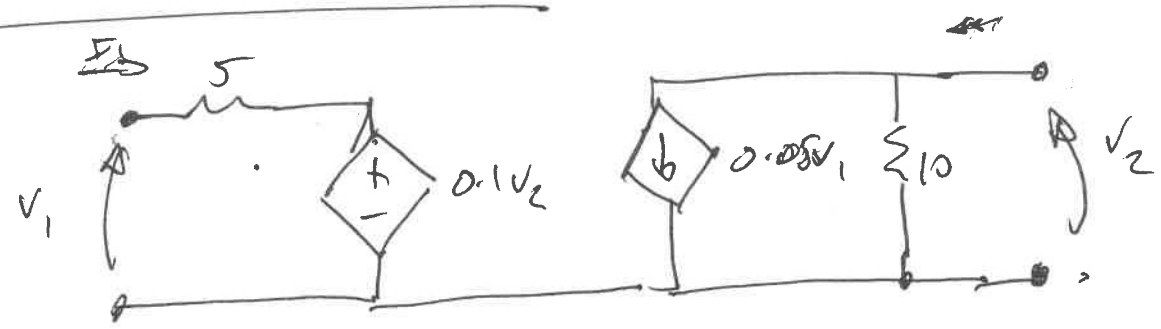
$$Z_{21} = 30$$

$$Z_{21} = Z_{12}.$$

$$Z_{11} = 50$$
$$Z_{22} = 80$$
$$Z_{21} = Z_{12} = 30$$

TUTE 5 worked solutions

Q2



$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$V_2 = -0.5V_1$$

$$I_1 = \frac{V_1 - 0.1V_2}{5}$$

$$I_1 = \frac{V_1 + 0.05V_1}{5} = \frac{1.05}{5}V_1$$

$$z_{11} = \frac{5}{1.05} = 4.76$$

$$z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \quad \frac{V_1}{\frac{1.05V_1}{5}} = \frac{5}{1.05} = 4.76$$

$$z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$V_1 = 0.1V_2$$

$$I_2 = \frac{V_2}{10} + 0.05V_1$$

$$I_2 = 0.1V_2 + 0.005V_2$$

$$I_2 = 0.105V_2$$

$$z_{22} = \frac{V_2}{I_2} = \frac{V_2}{0.105V_2} = \frac{1}{0.105} = 9.52$$

$$z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$V_1 = 0.1V_2$$

$$I_2 = \frac{V_2}{10} + 0.05V_1$$

$$I_2 = \frac{10V_1}{10} + 0.05V_1 = 1.05V_1$$

$$z_{12} = \frac{V_1}{I_2} = \frac{V_1}{1.05V_1} = 0.952$$

Tute 5 worked solutions

Q2

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$I_1 = \frac{V_1 - 0.4V_2}{5}$$

$$V_2 = -0.5V_1$$

$$I_1 = \frac{-2V_2 - 0.1V_2}{5}$$

$$= \frac{-2.1V_2}{5}$$

$$z_{21} = \frac{V_2}{\frac{-2.1V_2}{5}} = \frac{-5}{2.1} = -2.38$$

$z_{11} = 4.76$

$z_{22} = 9.52$

$z_{12} = 0.952$

$z_{21} = -2.38$

Tute 5

Q3

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = 10^3 I_1 + 10 I_2 \quad (1)$$

$$V_2 = -10^6 I_1 + 10^4 I_2 \quad (2)$$

$$I_1 = \frac{V_S - V_1}{1000} \qquad \boxed{10^3 I_1 = V_S - V_1} \quad (3)$$

$$\frac{V_2}{10^4} = = -I_2 \quad (4)$$

(3) in (1) $\quad V_1 = V_S - V_1 + 10 I_2$

$$2V_1 = V_S + 10 I_2 \qquad I_2 = \frac{2V_1 - V_S}{10} \quad (6)$$

from (2) + (4) $\quad -10^4 I_2 = -10^6 I_1 + 10^4 I_2$

$$-I_2 = -100 I_1 + I_2$$

$$2 I_2 = 100 I_1 \qquad I_2 = 50 I_1 \quad (5)$$

(1) + (5) $\quad V_1 = \frac{10^3}{50} I_2 + 10 I_2 = 30 I_2$

$$\frac{V_1}{30} = \frac{2V_1 - V_S}{10} \qquad V_1 \frac{5}{6} = V_S$$

$$V_1 = \frac{6}{5} V_S$$

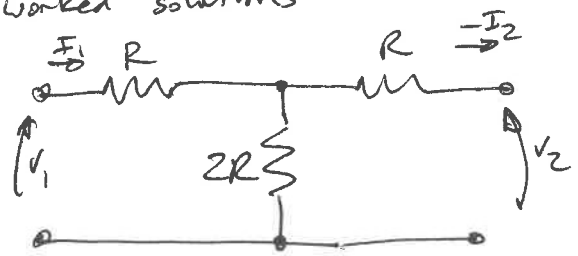
$$V_2 = -10^4 I_2 = \frac{-10^4 V_1}{30}$$

$$\boxed{\frac{V_2}{V_1} = \frac{-10^4}{30} = -333}$$

$$\frac{V_1}{I_1} = 50 \times 30 \, I_1$$

$$\boxed{\frac{V_1}{I_1} = 1500}$$

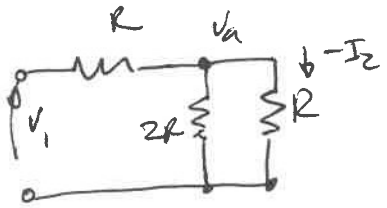
TUTR 5 Q4 worked solutions



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad \frac{V_1}{\frac{2}{3}V_1} = \frac{3}{2} = \underline{\underline{1.5}}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = \frac{V_1}{\frac{2}{5R}V_1} = \frac{5R}{2} = \underline{\underline{2.5R}}$$



$$-I_2 = \frac{V_a}{R}$$

$$V_a = V_1 \frac{2R\|R}{R + 2R\|R}$$

$$2R\|R = \frac{1}{\frac{1}{2R}+\frac{1}{R}} = \frac{2R^2}{3R} = \frac{2}{3}R$$

$$V_a = \frac{\frac{2}{3}R}{R+\frac{2}{3}R}V_1 = \frac{\frac{2}{3}}{\frac{3+2}{3}}V_1 = \frac{2}{5}V_1$$

$$-I_2 = \frac{2}{5R}V_1$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}, \qquad I_1 = \frac{V_1}{3R}$$

$$V_2 = \frac{2R}{3R}V_1 = \frac{2}{3}V_1$$

$$= \frac{\frac{V_1}{3R}}{\frac{2}{3}V_1} = \frac{1}{2R} = \underline{\underline{0.5/R}}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=0}$$

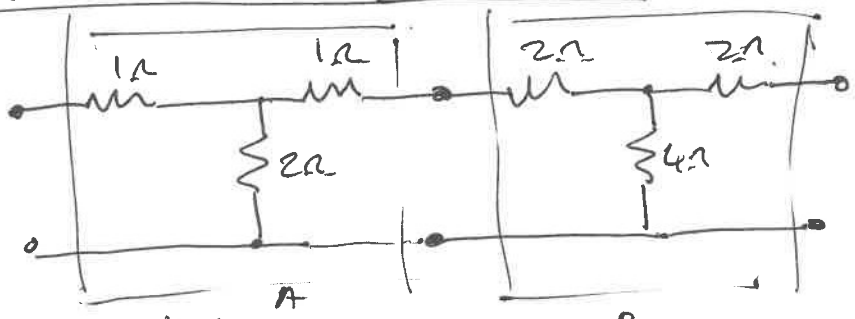
from before $-I_2 = \frac{2}{5R}V_1$

$$I_1 = \frac{V_1}{R+\frac{2}{3}R} = \frac{3V_1}{5R}$$

$$= \frac{\frac{3}{5R}V_1}{\frac{2}{5R}V_1} = 1.5$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.5 & 2.5R \\ \frac{0.5}{R} & 1.5 \end{bmatrix}$$

TUTE 5 Q4 worked solutions continued



Network A

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.5 & 2.5 \\ 0.5 & 1.5 \end{bmatrix}$$

Network B

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.5 & 5 \\ 0.25 & 1.5 \end{bmatrix}$$

TOTAL $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \text{Network} \\ A \end{bmatrix} \begin{bmatrix} \text{network} \\ B \end{bmatrix}$

$$= \begin{bmatrix} 1.5 & 2.5 \\ 0.5 & 1.5 \end{bmatrix} \begin{bmatrix} 1.5 & 5 \\ 0.25 & 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} 1.5 \times 1.5 + 2.5 \times 0.25, & 1.5 \times 5 + 2.5 \times 1.5 \\ 0.5 \times 1.5 + 1.5 \times 0.25, & 0.5 \times 5 + 1.5 \times 1.5 \end{bmatrix}$$

$$= \begin{bmatrix} 2.875, & 11.25 \\ 1.125, & 4.75 \end{bmatrix}$$