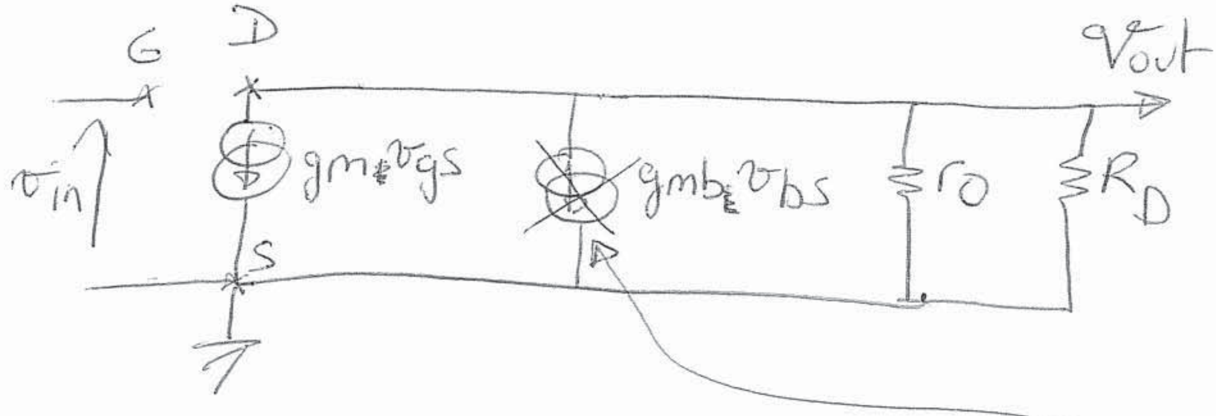
Problem 1:

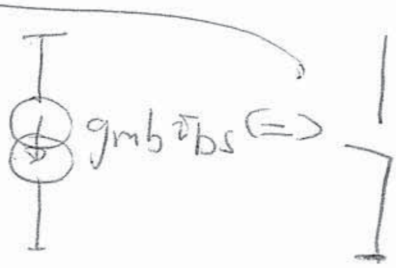
1) small-signal gain $A_v = \dfrac{v_{out}}{v_{in}}$



$v_b = v_s = 0 \qquad g_{mb} v_{bs} = 0 \; (1)$

$v_g = v_{in} \Rightarrow v_{gs} = v_{in} \quad (2)$

$v_{out} = -g_m v_{gs} (r_o // R_D) \quad (3)$

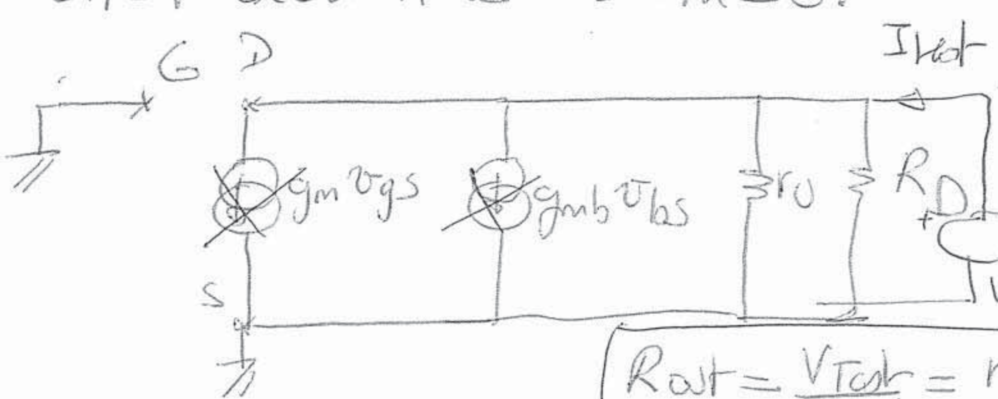


$(2) \Rightarrow (3)$: $v_{out} = -g_m v_{in} (r_o // R_D)$

$$\Rightarrow \boxed{A_v = \frac{v_{out}}{v_{in}} = -g_m \frac{r_o R_D}{r_o + R_D}}$$

2) input impedance looking in the gate is $\boxed{R_{in} = \infty}$ since no current can flow into the gate.

3) To determine output impedance, Apply V_{Test} at the output, measure I_{test}. Remove all other excitations $\rightarrow v_{in} = 0$.



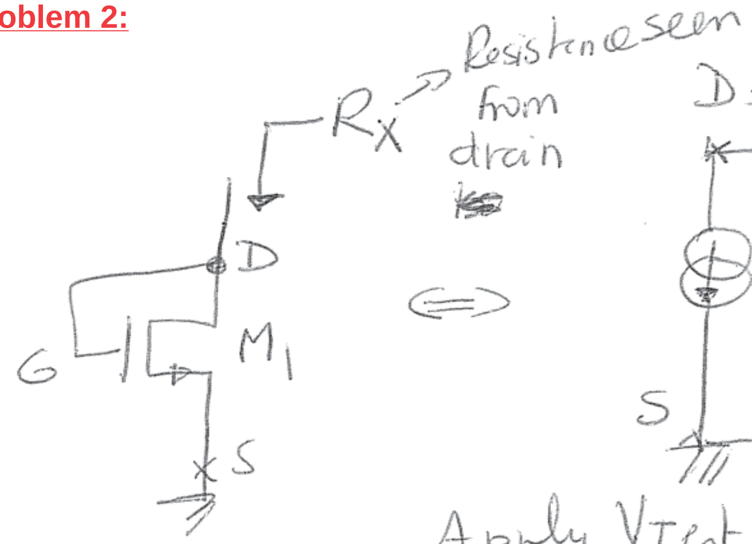
$v_g = v_s = 0$

$\Rightarrow v_{gs} = 0 \Rightarrow$ $g_m v_{gs} \Rightarrow$ turned OFF

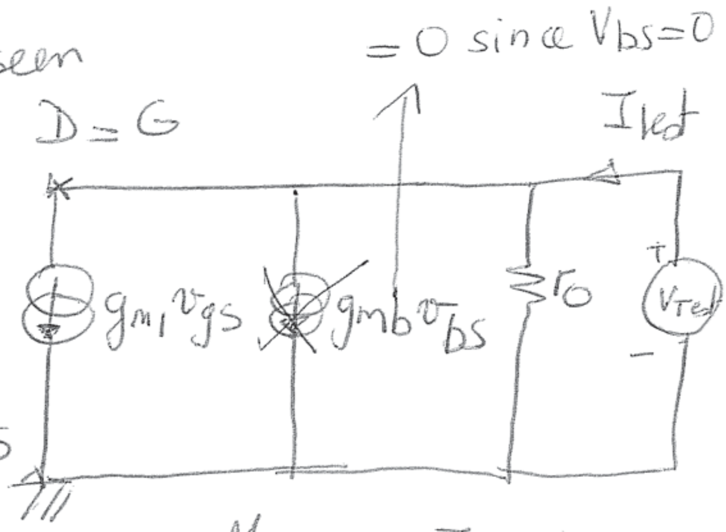
$v_{bs} = 0 \Rightarrow$ V_{Test} turned OFF

$$\boxed{R_{out} = \frac{V_{Test}}{I_{Test}} = r_o // R_D}$$

Problem 2:



$\Longleftrightarrow$



Apply V_{Test} ⟹ Measure I_{test}

↳ between drain and ground

$R_X = ?$ $\quad R_X = \dfrac{V_{Test}}{I_{test}}$

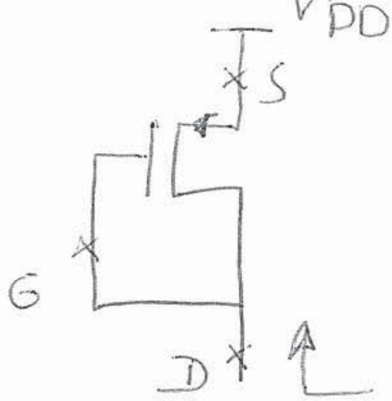
$\cancel{I_{test} = \frac{V_{test}}{r_o} + \frac{V_{test}}{g_{m1} v_{gs}} = \frac{V_{test}}{r_o} + \frac{V_{test}}{g_{m1} V_{Test}}}$

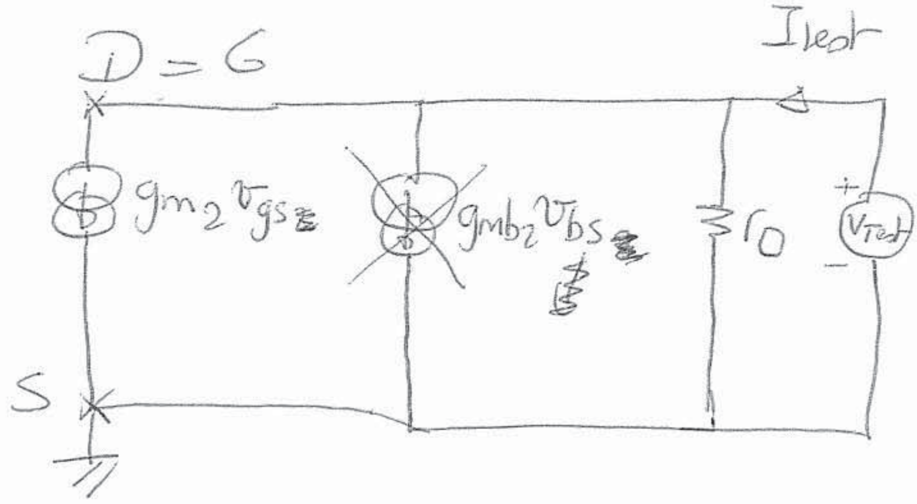
KCL:

$$I_{test} = \frac{V_{test}}{r_o} + g_{m1} v_{gs}$$

$$= \frac{V_{Test}}{r_o} + g_{m1} V_{Test}$$

$$\boxed{R_X = \frac{V_{test}}{I_{test}} = \frac{r_o}{1 + g_{m1} r_o} = r_o \,//\, \frac{1}{g_{m1}}}$$





R_y → resistance seen from drain

$v_{bs2} = 0$ since $V_{bulk} = V_{DD}$ and $V_S = V_{DD}$ } dc

KCL:

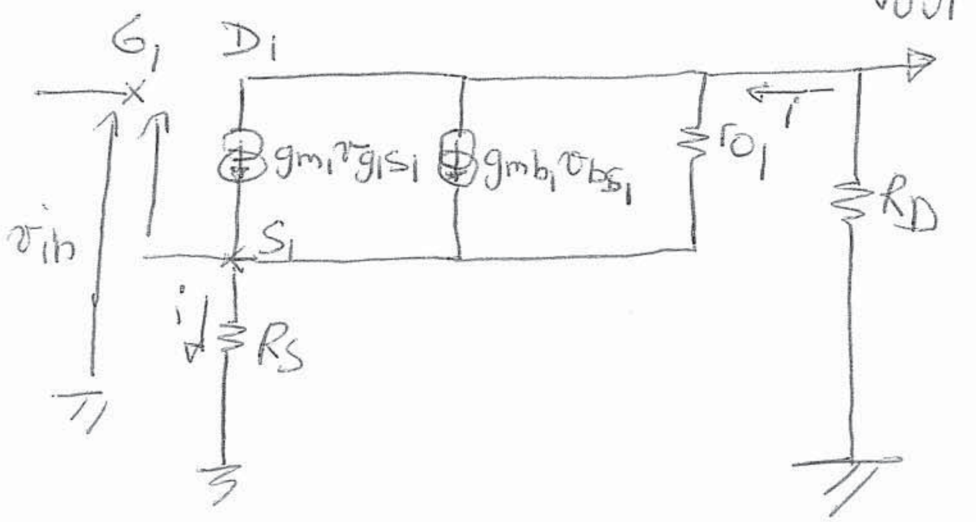
$$I_{test} = g_{m2} v_{gs2} + \frac{V_{Test}}{r_o}$$

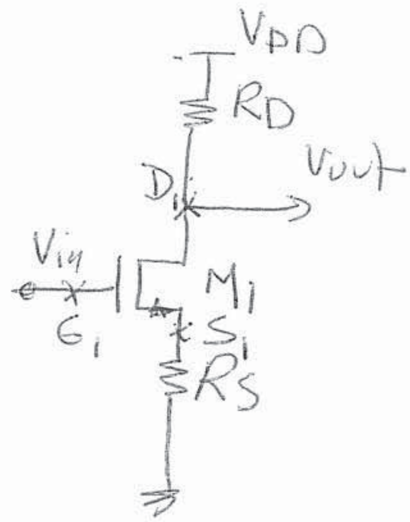
since $v_{gs} = V_{Test}$

$$\rightarrow \boxed{R_y = \frac{V_{test}}{I_{test}} = \frac{r_o}{1 + g_{m2} r_o} = r_o \,/\!/\, \frac{1}{g_{m2}}}$$

Problem 3:

small-signal gain $A_v = \dfrac{v_{out}}{v_{in}}$?





$$v_{bs_1} = 0 - v_{s_1} = -v_{s_1} .$$

$$v_{g_1 s_1} = v_{in} - v_{s_1} .$$

$$v_{s_1} = R_S \cdot i = -R_S \frac{V_{out}}{R_D} \qquad (1)$$

KCL in S_1 ⇒ $g_{m1} v_{g_1 s_1} + g_{mb_1} v_{bs_1} + \dfrac{V_{out} - V_{s_1}}{r_{o_1}} = \dfrac{V_{s_1}}{R_S}$

$$\Rightarrow g_{m1}(v_{in} - v_{s_1}) + g_{mb_1}(-v_{s_1}) + \frac{v_{out} - v_{s_1}}{r_{o_1}} = \frac{v_{s_1}}{R_S}$$

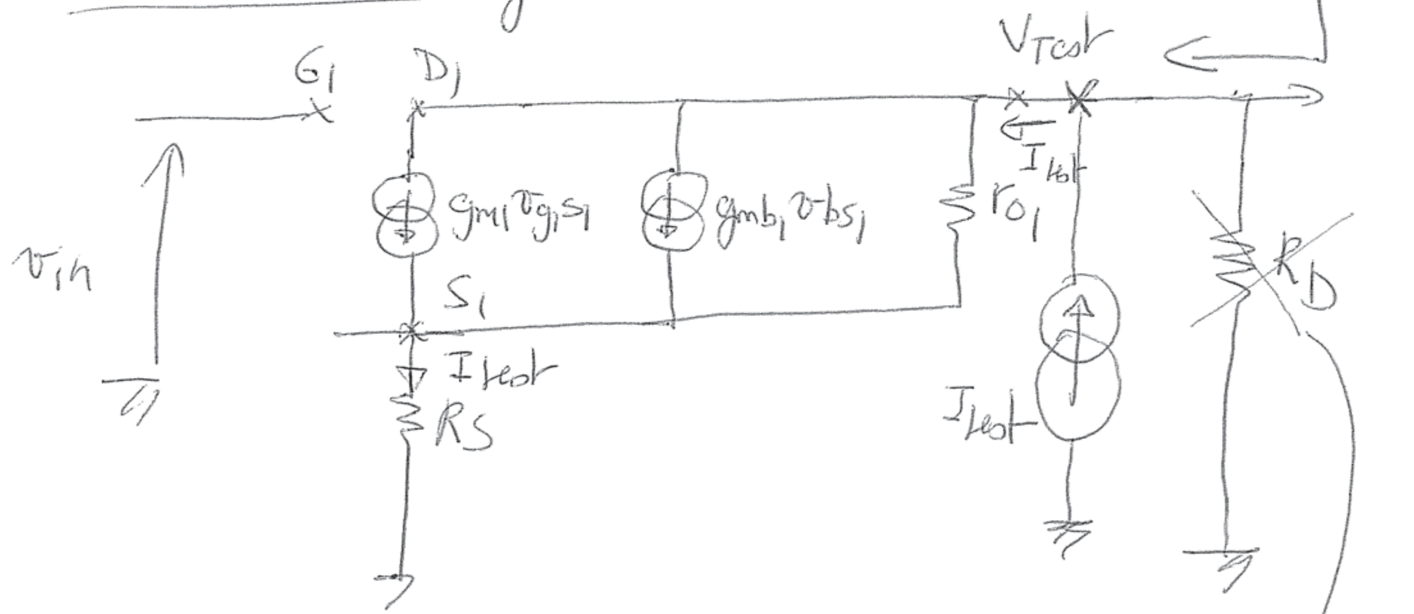
$$\Rightarrow g_{m1} v_{in} = -\frac{v_{out}}{r_{o_1}} + v_{s_1}\left(\frac{1}{R_S} + \frac{1}{r_{o_1}} + g_{mb_1} + g_{m1}\right) \qquad (2)$$

(1) in (2) ⇒ $g_{m_1} v_{in} = -\dfrac{v_{out}}{r_{o_1}} - \dfrac{R_S}{R_D} V_{out}\left(\dfrac{1}{R_S} + \dfrac{1}{r_{o_1}} + g_{mb_1} + g_{m_1}\right)$

$$A_v = \frac{v_{out}}{v_{in}} = -g_{m1} \cdot \frac{1}{\dfrac{1}{r_{o_1}} + \dfrac{1}{R_D} + \dfrac{R_S}{r_{o_1} R_D} + \dfrac{R_S}{R_D}(g_{m_1} + g_{mb1})}$$

$$\boxed{A_v = -g_{m1} \cdot \frac{r_{o_1} R_D}{r_{o_1} + R_D + R_S + r_{o_1} R_S (g_{m_1} + g_{mb1})}}$$

Resistance looking into the drain :



$R_{drain} = R_D // R_{Test}$

so we Apply here I_{Test} and determine V_{Test}.

$v_{in} = 0$ during this experiment.

removed to compute R_{Test}

$v_{G1} = 0 \Rightarrow v_{g_1 s_1} = -V_{s_1}$ (1)

$v_{bs_1} = -V_{s_1}$ (2)

with $V_{s_1} = R_s I_{Test}$ (3)

KCL in S_1 : $g_{m_1} v_{g_1 s_1} + g_{mb_1} v_{bs_1} + \frac{v_{Test} - v_{s_1}}{r_{o_1}} = \frac{v_{s_1}}{R_S}$ (4)

(1) (2) (3) into (4) gives : $-g_{m_1} v_{s_1} - g_{mb_1} v_{s_1} + \frac{v_{Test}}{r_{o_1}} - \frac{v_{s_1}}{r_{o_1}} = \frac{v_{s_1}}{R_S}$

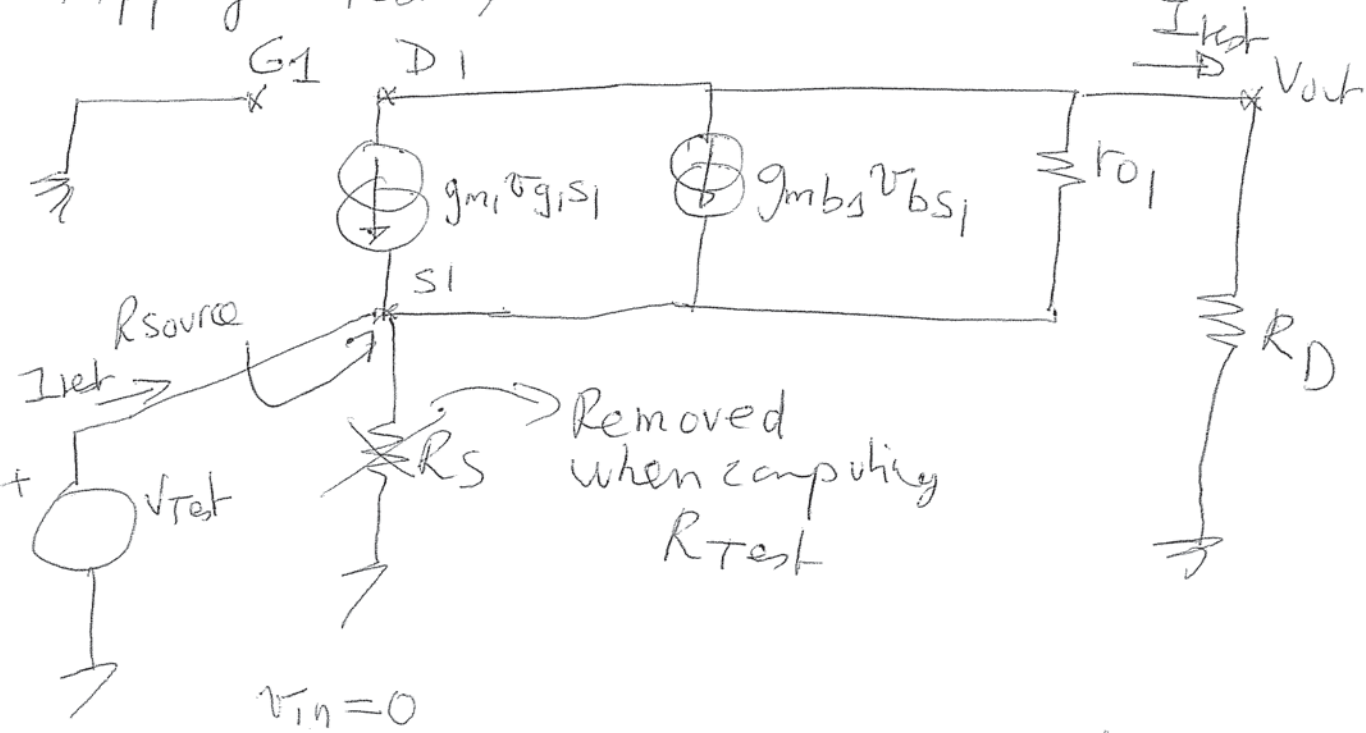
$$\frac{v_{Test}}{r_{o_1}} = R_s I_{Test} \left(\frac{1}{R_S} + (g_{m_1} + g_{mb_1}) + \frac{1}{r_{o_1}} \right)$$

$$\boxed{\text{so } R_{Test} = \frac{v_{Test}}{I_{Test}} = r_{o_1} R_S (g_{m_1} + g_{mb_1}) + r_{o_1} + R_S}$$

Resistance looking into the source:

$$\boxed{R_{source} = R_S // R_{Test}}$$

with R_{Test} resistance measured at the source of the transistor, with $V_{in}=0$.
Apply V_{Test}, measure I_{test}.



$v_{in} = 0$

$v_{g_1 s_1} = -v_{s_1}$ and $v_{bs_1} = -v_{s_1}$ (1)

$v_{s_1} = V_{Test}$ (2)

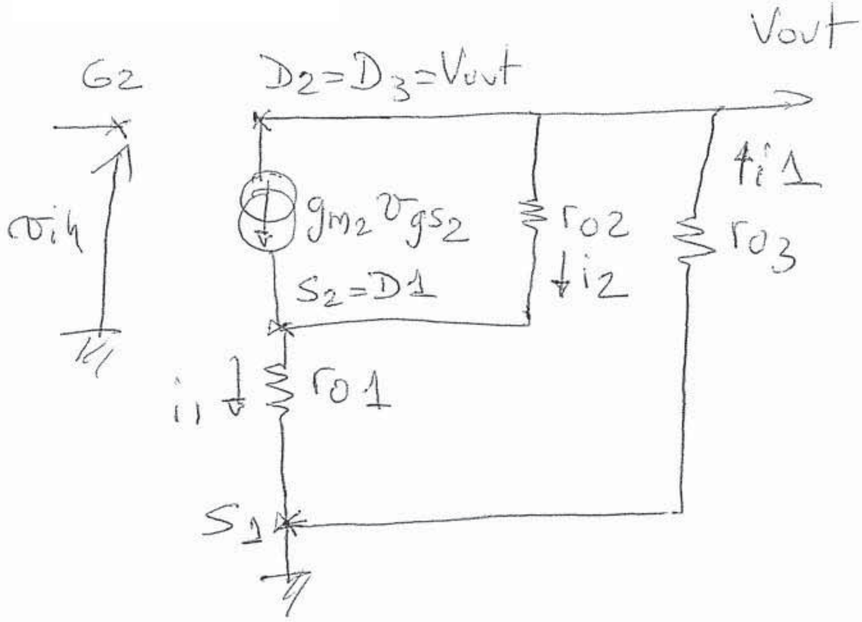
KCL in S_1 $\Rightarrow$ $I_{test} + g_{m1} v_{g_1 s_1} + g_{mb_1} v_{bs_1} + \frac{V_{out} - V_{test}}{r_{o1}} = 0$

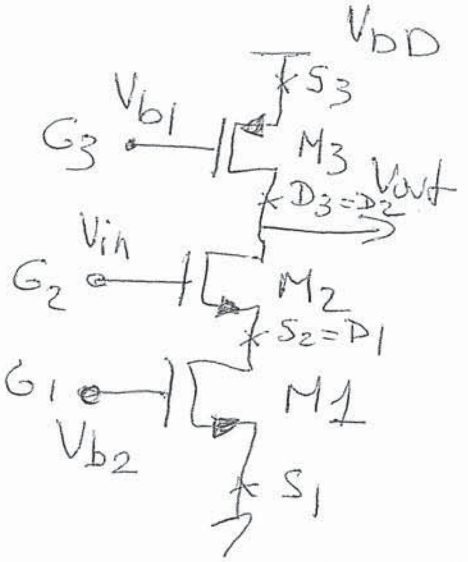
given that $V_{out} = R_D I_{test}$. and $v_{s_1} = V_{Test}$

$\Rightarrow$ $I_{test}\left(1 + \frac{R_D}{r_{o1}}\right) = V_{Test}\left(g_{m1} + g_{mb_1} + \frac{1}{r_{o1}}\right)$

$$\boxed{R_{Test} = \frac{V_{Test}}{I_{test}} = \frac{r_{o1} + R_D}{1 + r_{o1}(g_{m1} + g_{mb_1})}}$$

Problem 4:





V_{DD}, V_{b1} and V_{b2} dc values $\Rightarrow v_{S3} = v_{G1} = v_{G3} = 0$ in small-signal analysis

$$v_{g3s3} = v_{g1s1} = 0$$
$$v_{g2s2} = v_{in} - v_{s2}$$

KCL in S_2: $g_{m2}(v_{in} - v_{s2}) + \dfrac{V_{out} - v_{s2}}{r_{o2}} = \dfrac{v_{s2}}{r_{o1}}$ (1)

Ohm's law: $V_{out} = -r_{o3}\, i_1 = -r_{o3}\dfrac{v_{s2}}{r_{o1}}$

$$\rightarrow \boxed{v_{s2} = -\frac{r_{o1}}{r_{o3}} v_{out}} \quad (2)$$

(2) in (1) $\Rightarrow g_{m2}\left(v_{in} + \dfrac{r_{o1}}{r_{o3}} v_{out}\right) + \dfrac{v_{out}}{r_{o2}} + \dfrac{r_{o1}}{r_{o3} r_{o2}} v_{out} = -\dfrac{r_{o1} v_{out}}{r_{o3} r_{o1}}$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = -g_{m2} \cdot \frac{r_{o3} r_{o2}}{r_{o3} + r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}}$$

$$\boxed{A_v = -g_{m2} \cdot \frac{r_{o3} r_{o2}}{r_{o3} + r_{o1} + r_{o2}(1 + g_{m2} r_{o1})}}$$