12.5 For a Y-connected load, the time-domain expressions for three line-to-neutral voltages at the terminals are:

$$v_{AN} = 120 \cos(\omega t + 32°) \text{ V}$$
$$v_{BN} = 120 \cos(\omega t - 88°) \text{ V}$$
$$v_{CN} = 120 \cos(\omega t + 152°) \text{ V}$$

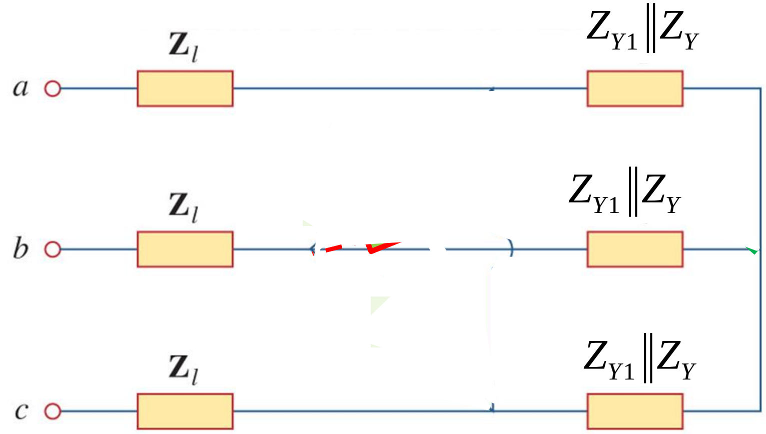
Write the time-domain expressions for the line-to-line voltages v_{AB}, v_{BC}, and v_{CA}.

Solution:

$$v_{AB} = 120\sqrt{3}\cos(wt + 32° + 30°) = 207.8\cos(wt + 62°)$$
$$v_{BC} = 207.8\cos(wt - 58°)$$
$$v_{CA} = 207.8\cos(wt + 182°) = 207.8\cos(wt - 178°)$$

Problem 12.15 P546

The circuit in Fig. 12.48 is excited by a balanced three-phase source with a line voltage of 210 V. If $\mathbf{Z}_l = 1 + j1\ \Omega$, $\mathbf{Z}_\Delta = 24 - j30\ \Omega$, and $\mathbf{Z}_Y = 12 + j5\ \Omega$, determine the magnitude of the line current of the combined loads.



Solution: $Z_\Delta \Rightarrow Z_{Y1}$

$$Z_{Y1} = \frac{1}{3} Z_\Delta = 8 - j10\ \Omega$$

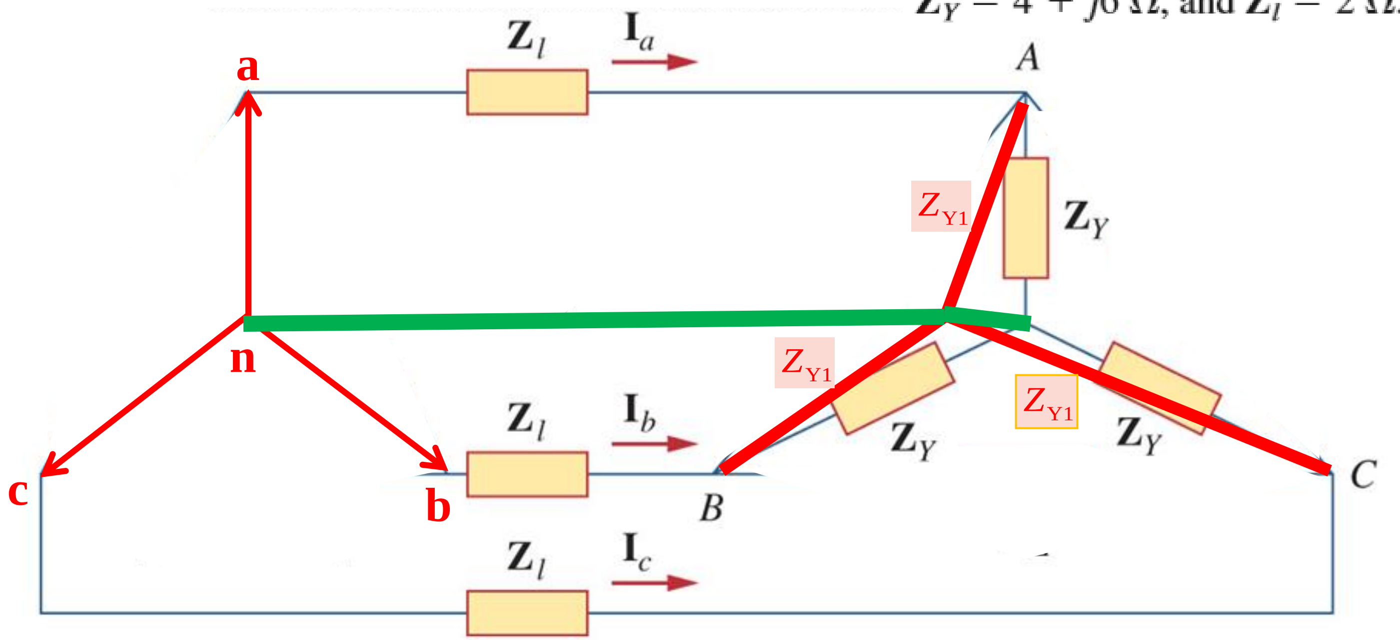
$$\text{So}: Z_{Ytotal} = Z_l + Z_{Y1} \| Z_Y = 1 + j1 + (8 - j10) \| (12 + j15)$$

$$= 8.874 \angle -7^\circ$$

$$\text{Then}: I_l = \frac{210/\sqrt{3}}{|Z_{Ytotal}|} = 13.66\ \text{A}$$

Problem 12.22 P547

Find the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ in the three-phase network of Fig. 12.53 below. Take $\mathbf{Z}_\Delta = 12 - j15\ \Omega$, $\mathbf{Z}_Y = 4 + j6\ \Omega$, and $\mathbf{Z}_l = 2\ \Omega$.



Solution:

$$Z_\Delta \Rightarrow Z_{Y1}\text{:}\quad Z_{Y1} = \frac{1}{3} Z_\Delta = 4 - j5\ \Omega$$

$$Z_{Ytotal} = Z_l + Z_{Y1} \| Z_Y = 2 + (4 - j5) \| (4 + j6) = 7.723\text{-}j0.215$$

$$V_{an} = \frac{440\angle 0^\circ}{\sqrt{3}\angle 30^\circ} = 254\angle -30^\circ \qquad I_a = \frac{V_{an}}{Z_{Ytotal}} = 32.88\angle -28^\circ$$

$$I_b = I_a \angle -120^\circ = 32.88\angle -148^\circ$$

$$I_c = I_a \angle 120^\circ = 32.88\angle 92^\circ$$

12.47 The following three parallel-connected three-phase loads are fed by a balanced three-phase source:

Load 1: 250 kVA, 0.8 pf lagging

Load 2: 300 kVA, 0.95 pf leading

Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

Solution:

$$S_1 = 250 \text{ kVA}$$

$$\text{pf}_1 = 0.8 \text{ lagging}$$

$$\text{So:}\theta_1 = \cos^{-} 0.8 = 37°$$

$$P_1 = S_1 \times \cos\theta_1 = 200 \text{ kW}$$

$$Q_1 = S_1 \times \sin\theta_1 = 150 \text{ kVAR}$$

$$S_2 = 300\text{kVA}$$

$$\text{pf}_2 = 0.95 \text{ leading}$$

$$\text{So:}\theta_2 = -\cos^{-} 0.95 = \text{-}18°$$

$$P_2 = S_2 \times \cos\theta_2 = 285 \text{ kW}$$

$$Q_2 = S_2 \times \sin\theta_2 = -93 \text{ kVAR}$$

$$S_3 = 450\text{kVA}$$

$$\text{pf}_3 = 1$$

$$\text{So:}P_3 = 450 \text{ kW}$$

$$Q_3 = 0 \text{ kVAR}$$

$$\text{So}: \text{S} = P_1 + P_2 + P_3 + j(Q_1 + Q_2 + Q_3)$$

$$= 935 + j57 = 936.7\angle 3.5° \text{ kVA}$$

$$\text{As}: |\text{S}| = \sqrt{3} V_l I_l$$

$$\Rightarrow I_l = \frac{936.7 \times 10^3}{\sqrt{3} \times 13.8 \times 10^3} = 39.2 \text{ A rms}$$

$$\text{pf} = \cos 3.5° = 0.998 \text{ lagging}$$

$$\text{or: pf} = \frac{P}{S} = \frac{935}{936.7} = 0.998 \text{ lagging}$$

12.50 A balanced three-phase source with $\mathbf{V}_L = 240$ V rms is supplying 8 kVA at 0.6 power factor lagging to two wye-connected parallel loads. If one load draws 3 kW at unity power factor, calculate the impedance per phase of the second load.

Solution:

$$S = 8 \text{ kVA}. \ \ \text{pf} = 0.6 \text{ lagging}$$

So: $P = S \times \text{pf} = 4.8 \text{ kW}$

$$Q = \sqrt{S^2 - P^2} = 6.4 \text{ kVAR}$$

$$P_1 = 3 \text{ kW} . \ \ \text{pf}_1 = 1$$

So: $Q_1 = 0 \text{ kVAR}$

So: $P_2 = P\text{-}P_1 = 1.8 \text{ kW}$

$$Q_2 = Q\text{-}Q_1 = 6.4 \text{ kVAR}$$

So: $S_2 = \sqrt{P_2^2 + Q_2^2} = 6.65 \text{ kVA}$

As: $V_p = \dfrac{V_l}{\sqrt{3}} = \dfrac{240}{\sqrt{3}}$ V rms

$$S_2 = \frac{3V_p^2}{|Z_2|}$$

So: $|Z_2| = \dfrac{240^2}{6.65 \times 10^3} = 8.66\Omega$

$$\theta_{Z_2} = \tan^{-}\left(\frac{Q_2}{P_2}\right) = \tan^{-}\left(\frac{6.4}{1.8}\right) = 74°$$

So: $Z_2 = |Z_2| \angle \theta_{Z_2} = 8.66\angle 74°$

$$= 2.39 + j8.32 \ \Omega$$