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UWA – ENSC3015 Signals and Systems

Please complete your details below:

Surname: ______________________________ Number: ____________________

Signature: _____________________________ Date: __________________

10:58am, Monday, September 18, 2017 in **Tattersall LT**

Class Test 2:
Laplace and z-Transform Methods

Time allowed: 45 minutes
Max mark: **50**, Assessment: **5%**[1]

This paper contains:
9 pages, 6 questions

Candidates should attempt **all** questions and show **all** working with numerical answers to **3** decimal places in the spaces provided after each question, show as much working as possible to gain maximum marks. You can use the blank page on the reverse side for rough working, but these pages will not be marked

FOR THE ATTACHMENTS PLEASE REFER TO THE SEPARATED PAGES

[1] If you do better in the exam this test will not contribute to your unit marks and the 5% will come from the final exam performance. However if you do better in this test compared to the final exam then this test will be included in the unit marks.

Question 1 (10 marks)

Determine the Laplace transform (using the method indicated), sketch the pole-zero plot (showing ALL zeroes and poles) and then shade in the associated region of convergence for each of the following continuous-time functions:

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$, do this by direct integration using the defining equation (do NOT use the tables)

(b) $x(t) = te^{-2|t|}$, do this using the table of transform pairs and properties.

Question 2 (8 marks)

Consider a continuous-time LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

(a) Derive the Laplace transforms of $x(t)$ and $h(t)$ (you can use the transform pair table for this).

(b) Use the convolution property to determine the Laplace transform $Y(s)$ of the output $y(t)$.

(c) Determine the output $y(t)$ by taking the inverse Laplace transform of $Y(s)$.

(d) Verify your result from (c) by directly calculating the convolution $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$ (show your working)

Question 3 (6 marks)

Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation:

$$\frac{d^2}{dt^2}y(t) - \frac{d}{dt}y(t) - 2y(t) = x(t)$$

(a) By using the Laplace transform determine the transfer function, $H(s)$, expressed as a ratio of two polynomials in s

(b) Sketch the pole-zero pattern of $H(s)$ on the s-plane.

(c) From the pole-zero diagram can a physical implementation of this system be stable for any input $x(t)$?

Question 4 (6 marks)

Determine the z-transform and the associated region of convergence depicted on a pole-zero plot (showing ALL zeros and poles) for the discrete-time function: $x[n] = \left(\frac{1}{2}\right)^{n+1} u[n+3]$. You can do this by using the table of pairs and properties or direct summation (where you can use: $\sum_{n=k}^{\infty} \beta^n = \beta^k/(1-\beta)$ for $|\beta| < 1$)

Question 5 (10 marks)

Consider a discrete-time LTI system for which the input $x[n]$ and output $y[n]$ are related by the difference equation:

$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n]$$

where it is evident that this system is anti-causal due to the $y[n+1]$.

(a) By using the z-transform determine the transfer function, $H(z)$, expressed as a ratio of two polynomials in z

(b) Sketch the pole-zero pattern of $H(z)$ on the z-plane.

(c) Now by first expressing $H(z)$ as a ratio of two polynomials in z^{-1} carry out a partial fraction expansion (to allow taking the inverse z-transform in the next step).

(d) Find the impulse response sequence $h[n]$ which ensures the system is stable.

Question 6 (10 marks)

Consider a system with transfer function:

$$H(z) = \frac{6(5z-1)}{(2z-1)(3z+1)}$$

(a) Assume the system is stable, can it be causal?

(b) What is the difference equation which implements a stable system with this transfer function?

(c) Use adder, multiplier and unit delay modules to implement your system of (b) in hardware given $x[n]$ is the input and $y[n]$ is the output.

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Laplace Transform and Transfer Function, $H(s)$ ($x(t)$ is input, $y(t)$ is output)

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt, \qquad Y(s) = H(s)X(s)$$

Laplace Transform Pairs

$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} > -a$
$te^{-at}u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} > -a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} < -a$
$-te^{-at}u(-t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} < -a$

Laplace Transform Properties

Signal	*Bilateral Transform* $x(t) \xleftrightarrow{\mathcal{L}} X(s)$ $y(t) \xleftrightarrow{\mathcal{L}} Y(s)$	*ROC* $s \in R_x$ $s \in R_y$
$ax(t) + by(t)$	$aX(s) + bY(s)$	At least $R_x \cap R_y$
$x(t-\tau)$	$e^{-s\tau}X(s)$	R_x
$e^{s_o t}x(t)$	$X(s - s_o)$	$R_x + \mathrm{Re}\{s_o\}$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	$\frac{R_x}{\lvert a\rvert}$
$x(t) * y(t)$	$X(s)Y(s)$	At least $R_x \cap R_y$
$-tx(t)$	$\frac{d}{ds}X(s)$	R_x
$\frac{d}{dt}x(t)$	$sX(s)$	At least R_x
$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{X(s)}{s}$	At least $R_x \cap \{\mathrm{Re}\{s\} > 0\}$

Partial Fraction Expansion Algorithm for $X(s)$ ($M < N$)

Simple, distinct poles, $d_i \neq d_j$

$$X(s) = \sum_{k=1}^{N} \frac{A_k}{(s-d_k)}, \quad \text{where } A_k = (s-d_k)X(s)|_{s=d_k}$$

Causality and Stability of $H(s)$

For a system to be causal the ROC of $H(s)$ must be to the right of all poles in the s-plane.

For a system to be stable the ROC if $H(s)$ must include the $j\omega$-axis in the s-plane.

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z-Transform and Transfer Function, $H(z)$ ($x[n]$ is input, $y[n]$ is output)

$$X(z) = \sum_{k=-\infty}^{\infty} x[k]z^{-k}, \qquad Y(z) = H(z)X(z)$$

z-Transform Pairs

$\alpha^n u[n]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$n\alpha^n u[n]$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-\alpha^n u[-n-1]$	$\dfrac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$-n\alpha^n u[-n-1]$	$\dfrac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$

z-Transform Properties

Signal	*Bilateral Transform* $x[n] \xleftrightarrow{z} X(z)$ $y[n] \xleftrightarrow{z} Y(z)$	*ROC* $z \in R_x$ $z \in R_y$
$ax[n] + by[n]$	$aX(z) + bY(z)$	At least $R_x \cap R_y$
$x[n-k]$	$z^{-k}X(z)$	R_x, except possibly $\lvert z\rvert = 0, \infty$
$\alpha^n x[n]$	$X\left(\dfrac{z}{\alpha}\right)$	$\lvert\alpha\rvert R_x$
$x[-n]$	$X\left(\dfrac{1}{z}\right)$	$\dfrac{1}{R_x}$
$x[n] * y[n]$	$X(z)Y(z)$	At least $R_x \cap R_y$
$nx[n]$	$-z\dfrac{d}{dz}X(z)$	R_x, except possibly addition or deletion of $z = 0$

Partial Fraction Expansion Algorithm for $X(z)$ ($M < N$)
Simple, distinct poles, $d_i \neq d_j$

$$X(z) = \sum_{k=1}^{N} \frac{A_k}{(1-d_k z^{-1})}, \quad \text{where } A_k = (1-d_k z^{-1})X(z)\big|_{z=d_k}$$

Causality and Stability of $H(z)$
For a system to be causal the ROC of $H(z)$ must be to the exterior of the circle centred at the origin in the z-plane which contains all poles.
For a system to be stable the ROC if $H(z)$ must include the unit circle in the z-plane.

Computational structures for discrete-time LTI systems require three elements:

gain or multiplier element	p_1
summer or accumulator element	Σ
unit delay element	z^{-1}