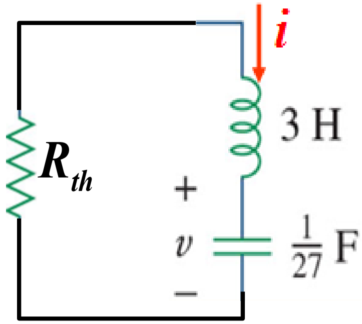
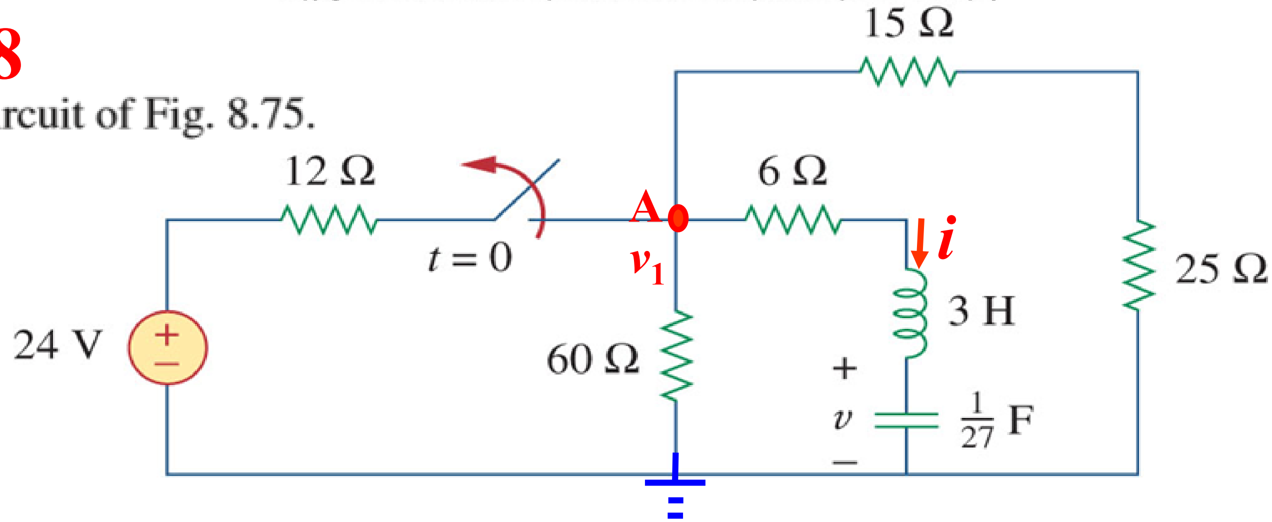
Problem 8.21 P358

Calculate $v(t)$ for $t > 0$ in the circuit of Fig. 8.75.





Solution:

When $t > 0$, the circuit is :

$$R_{th} = 6 + 60 \| (15 + 25) = 30\Omega$$

Using KVL: $iR_{th} + 3\frac{di}{dt} + v = 0$

But $i = C\frac{dv}{dt}$, Hence we find :

$$\frac{d^2v}{dt^2} + 10\frac{dv}{dt} + 9v = 0$$

The characteristic equation:

$$s^2 + 10s + 9 = 0 \qquad s_1 = -9, s_2 = -1$$

$$\text{So : } v(t) = A_1 e^{-9t} + A_2 e^{-t}$$

That is: $v(0^+) = A_1 + A_2 \cdots\cdots (1)$

$$\frac{dv(0)}{dt} = -9A_1 - 9A_2 \cdots\cdots (2)$$

$t < 0$, node A: $\dfrac{24 - v_1}{12} = \dfrac{v_1}{60} + \dfrac{v_1}{15 + 25}$

$$v_1 = 16\text{ V}$$

So: $v(0^+) = v_1 = 16\text{ V} \cdots\cdots (3)$

And $i(0^+) = 0\text{ A}$

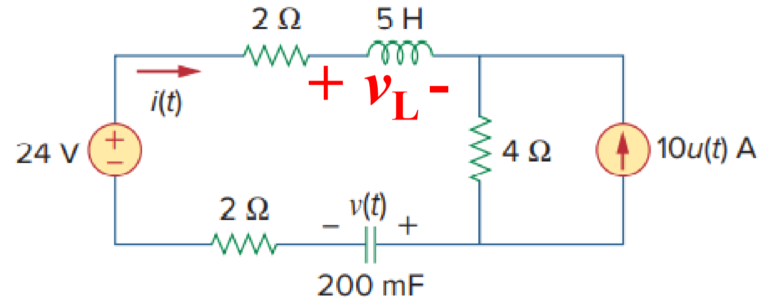
As: $\dfrac{dv(0)}{dt} = \dfrac{1}{C} i(0) = 0 \cdots\cdots (4)$

From(1)(2)(3)(4): $A_1 = -2; A_2 = 18$

$$\text{So : } v(t) = -2e^{-9t} + 18e^{-t}$$

Problem 8.36 P360

8.36 Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit of Fig. 8.84



Solution:

When $t > 0$, using KVL with all independent sources are turned off :

$$(2+2+4)i + v + v_L = 0 \cdots\cdots (1)$$

As: $i = C\frac{dv}{dt} = 200\times 10^{-3}\frac{dv}{dt} = \frac{1}{5}\frac{dv}{dt} \cdots (2)$

$$v_L = L\frac{di}{dt} = 5\times\frac{1}{5}\frac{d^2v}{dt^2} = \frac{d^2v}{dt^2} \cdots\cdots (3)$$

From(1)(2)(3): $5\frac{d^2v}{dt^2} + 8\frac{dv}{dt} + 5v = 0$

$$5S^2 + 8S + 5 = 0$$

$$S_{1、2} = -0.8 \pm j0.6$$

$t \to \infty$: $v(\infty) = 24 - 4\times 10 = -16\text{V}$

So: $v(t) = -16 + (A_1\sin 0.6t + A_2\cos 0.6t)e^{-0.8t} \cdots (3)$

Hence: $v(0^+) = -16 + A_2 \cdots\cdots (4)$

$$\frac{dv(0^+)}{dt} = 0.6A_1 - 0.8A_2 \cdots\cdots (5)$$

As: $v(0^+) = v(0^-) = 24\text{V} \cdots\cdots (6)$

$i(0^+) = i(0^-) = 0\text{ A}$

And: $\frac{dv(0^+)}{dt} = \frac{1}{C}i(0^+) = 0 \cdots\cdots (7)$

From(4)(5)(6)(7):

$$A_1 = \frac{160}{3}; A_2 = 40$$

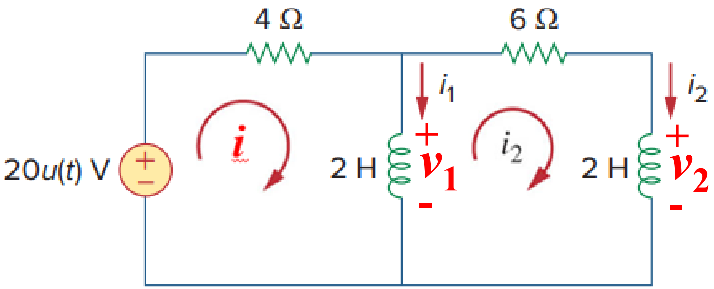
So:

$$v(t) = -16 + (\frac{160}{3}\sin 0.6t + 40\cos 0.6t)e^{-0.8t}$$

From(2): $i == \frac{1}{5}\frac{dv}{dt} = -\frac{40}{3}\sin 0.6te^{-0.8t}$

Problem 8.60 P363

Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106



Solution:

When $t > 0$, using KCL with 20V − sources turned off :

$$i:\quad 4i + 2\frac{d(i-i_2)}{dt} = 0 \cdots\cdots (1)$$

$$i_2:\ 6i_2 + 2\frac{di_2}{dt} - 2\frac{d(i-i_2)}{dt} = 0 \cdots (2)$$

$$(1)(2) \Longrightarrow \quad \frac{d^2 i_2}{dt^2} + 7\frac{di_2}{dt} + 6i_2 = 0$$

$$S^2 + 7S + 6 = 0$$

$$S_1 = -6; S_2 = -1$$

$$t \to \infty: i_1(\infty) = \frac{20}{4} = 5\text{ A}$$

$$\text{So: } i_1(t) = 5 + A_1 e^{-6t} + A_2 e^{-t} \cdots (3)$$

$$\text{Hence: } i_1(0^+) = A_1 + A_2 \cdots\cdots\cdots (4)$$

$$\frac{di_1(0^+)}{dt} = -6A_1 - A_2 \cdots\cdots (5)$$

$$As: i_1(0^+) = i_1(0^-) = 0\text{ A} \cdots\cdots (6)$$

$$i_2(0^+) = i_2(0^-) = 0\text{ A}$$

$$\text{And:} \frac{di_1(0^+)}{dt} = \frac{1}{2}v_1(0^+)$$

$$v_1(0^+) = 20 - 4[i_1(0^+) + i_2(0^+)]$$
$$=20$$

$$\text{So: } \frac{di_1(0^+)}{dt} = \frac{1}{2}v_1(0^+) = 10 \cdots (7)$$

$$\text{From(4)(5)(6)(7): } A_1 = -1; A_2 = -4 \cdots (8)$$

$$(8) \Longrightarrow (3): \quad i_1(t) = 5 - e^{-6t} - 4e^{-t} \cdots (9)$$

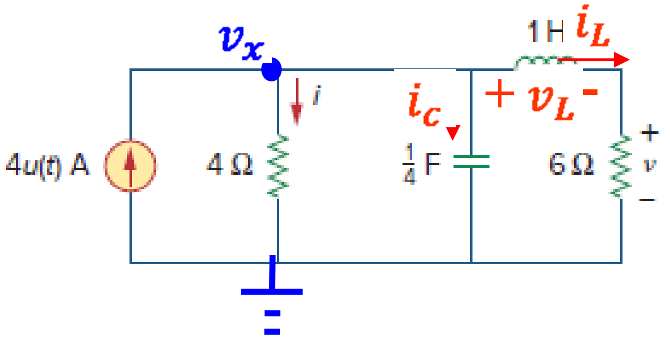
$$i_2(t) = \frac{20 - v_1}{4} - i_1(t) \cdots (10)$$

$$v_1 = 2\frac{di_1(t)}{dt} = 12e^{-6t} - 8e^{-t} \cdots (11)$$

$$(9)(11) \Longrightarrow (10): \ i_2(t) = -2e^{-6t} + 2e^{-t}$$

Problem 8.61 P363

For the circuit in Prob. 8.5, find i and v for $t > 0$.



Solution:

When t >0,using KCL with 4u(t)A-source turned off:

$$\frac{v_x}{4}+\frac{1}{4}\frac{dv_x}{dt}+i_L=0 \cdots\cdots(1)$$

$$v_x=\frac{di_L}{dt}+6i_L \cdots\cdots\cdots\cdots(2)$$

(2)→(1) : $$\frac{d^2i_L}{dt^2}+7\frac{di_L}{dt}+10i_L=0$$

$$S^2+7S+10=0 \longrightarrow S_1=-5、S_2=-2$$

$t\to\infty$: $$4=\frac{v_x}{4}+\frac{v_x}{6} \longrightarrow v_x(\infty)=\frac{48}{5}$$

So: $$v_x=\frac{48}{5}+A_1e^{-5t}+A_2e^{-2t}\cdots\cdots(3)$$

Hence: $$v_x(0^+)=\frac{48}{5}+A_1+A_2\cdots\cdots(4)$$

$$\frac{dv_x(0^+)}{dt}=-5A_1-2A_2\cdots\cdots(5)$$

As: $$v_x(0^+)=v_x(0^-)=0\cdots\cdots(6)$$

$$i_L(0^+)=i_L(0^-)=0$$

And: $$\frac{dv_x(0^+)}{dt}=4i_C(0^+)$$

$$t=0^+:4=\frac{v_x(0^+)}{4}+i_C(0^+)+i_L(0^+)\longrightarrow i_C(0^+)=4$$

So: $$\frac{dv_x(0^+)}{dt}=4i_C(0^+)=16\cdots\cdots(7)$$

$$(6)(7)\to(4)(5):\ A_1=\frac{16}{15}、A_2=-\frac{32}{3}$$

So: $$v_x=\frac{48}{5}+\frac{16}{15}e^{-5t}-\frac{32}{3}e^{-2t}$$

$$i=\frac{v_x}{4}=\frac{12}{5}+\frac{4}{15}e^{-5t}-\frac{8}{3}e^{-2t}$$

$$v=6i_L=6\left(4-i-\frac{1}{4}\frac{dv_x}{dt}\right)$$

$$=\frac{48}{5}+\frac{32}{5}e^{-5t}-16e^{-2t}$$