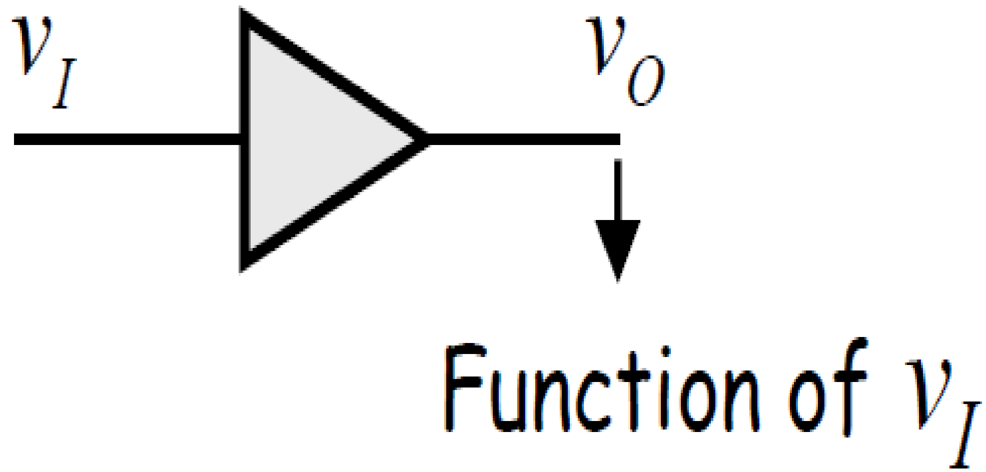
Lecture 9

The Operational Amplifier

Abstraction

Operational Amplifier (Op amp) Abstraction



Op-Amp

- The operational amplifier ("op amp") is a fundamental circuit building block of analog circuit design. An op amp is a very versatile integrated circuit IC that can be used to do accomplish many functions (more on this later).

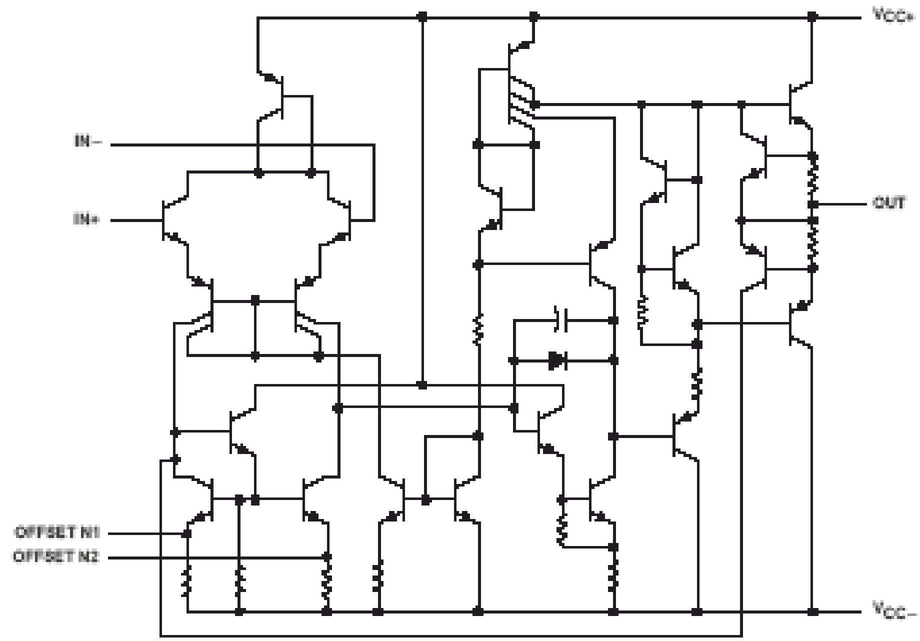
- The name "operational amplifier" originates from the fact that this type of amplifier can be used to perform mathematical operations such as the scaling, summation and integration of analog signals.

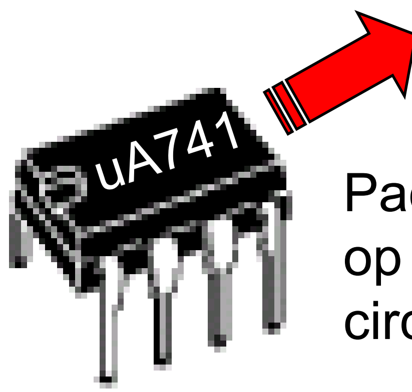
- The μA 709, introduced by Fairchild Semiconductors in 1965, was one of the first widely used general-purpose IC operational amplifiers.

A look inside the uA741 Op amp

- An op amp comprises a large number of components such as transistors, resistors, diodes, and capacitors, all of which are integrated on a single chip.

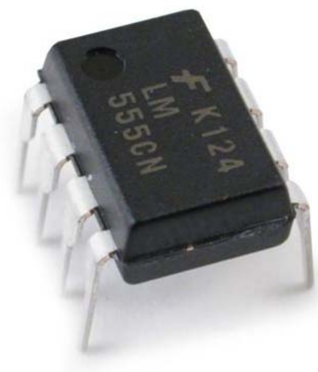
- In the following, we will not discuss what is inside the op amp. We will treat the op amp as a circuit building block and study its terminal characteristics and applications.

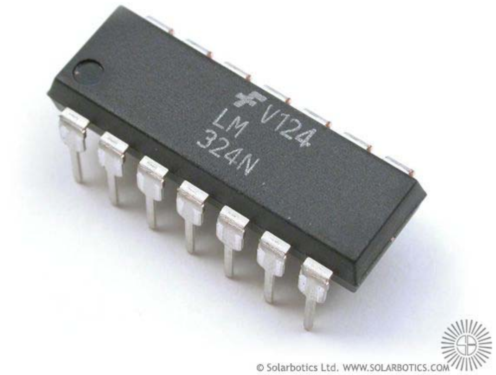


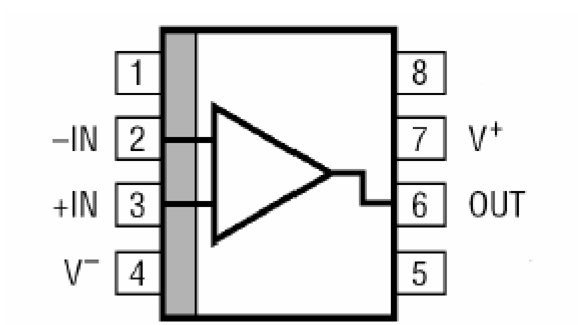


Packaged op amp circuit chip

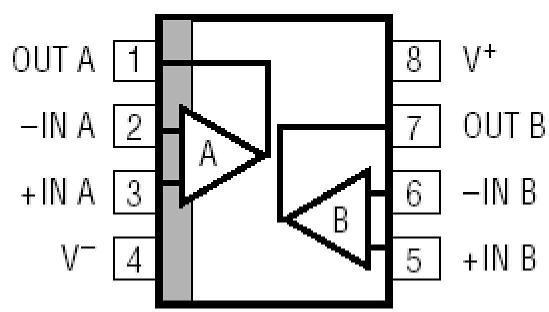
Features of Modern Op-Amps



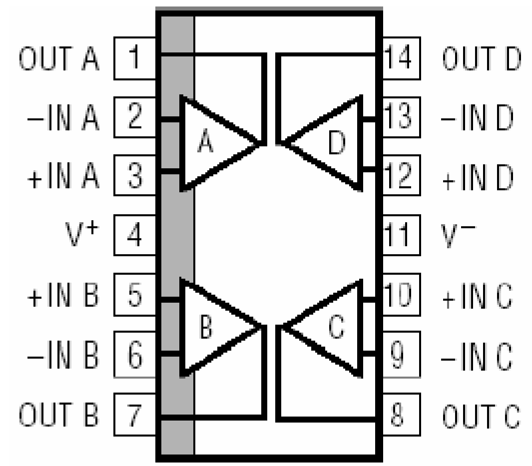




Single



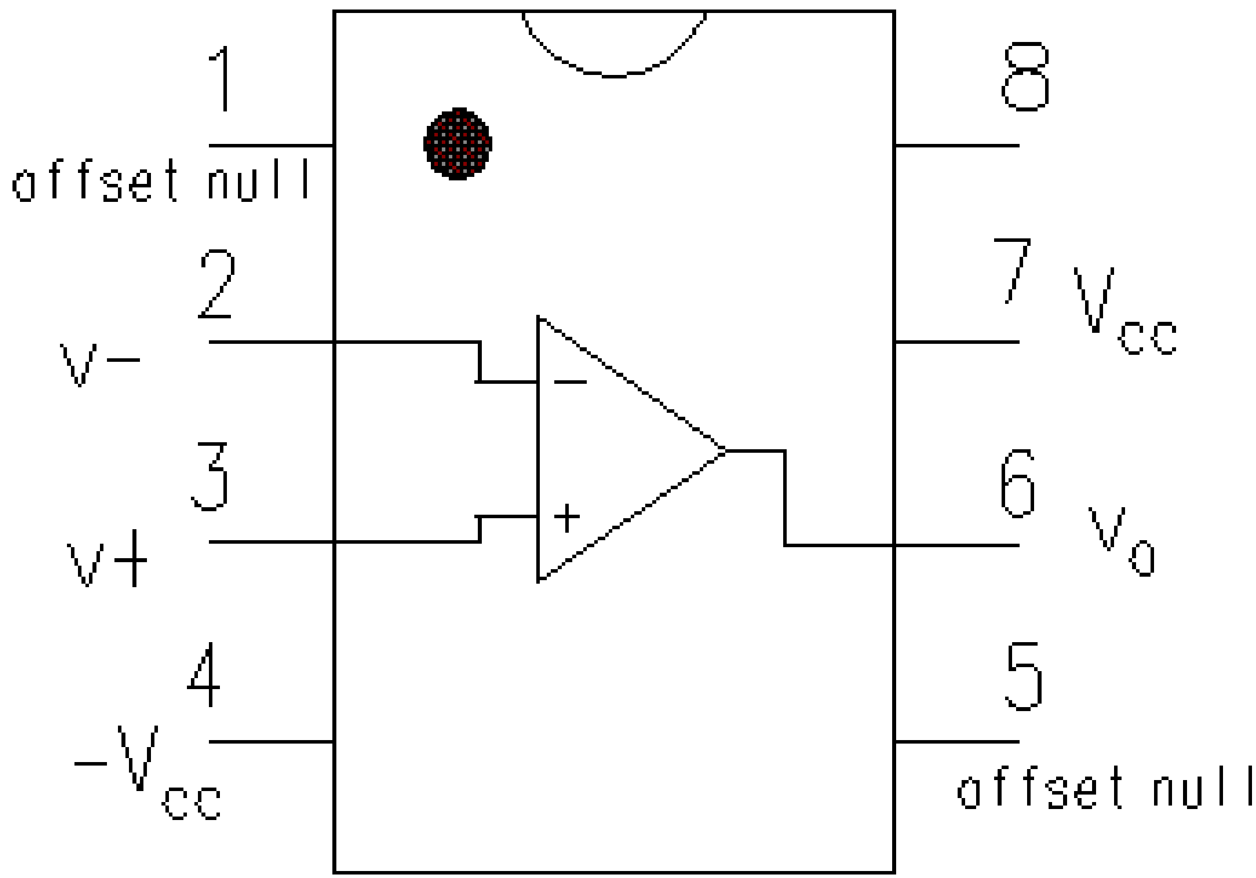
Dual



Quad

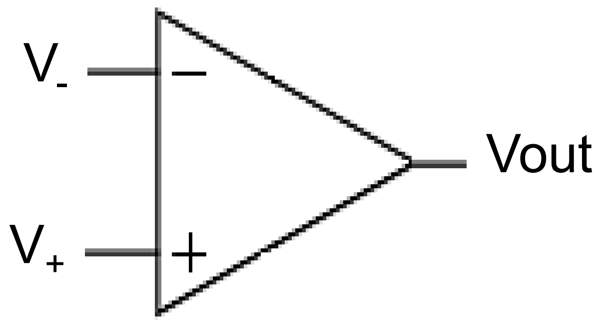
- Integrated Circuit — Standard pinouts
- Multiple op-amps on a single chip — inexpensive

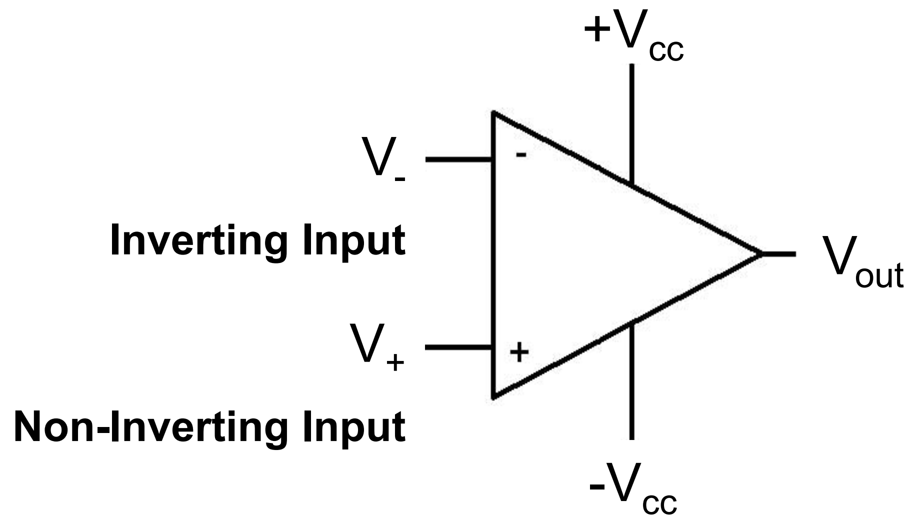
Typical 8 Pin Op-Amp



Op-Amp Terminals

Op amp symbol most often used





Op amp symbol with power supplies

- The op amp requires dc power to operate. Typically, the op amp requires both positive and negative power supplies ($-V_{CC}$ and $+V_{CC}$).
- From a signal point of view, there are 3 critical connections:

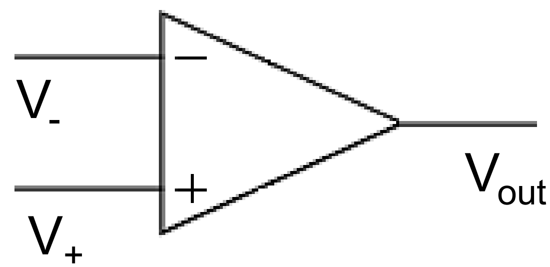
  the inverting input terminal denoted by '–'

  the non-inverting input terminal denoted by '+'

  the output terminal

The Op amp is a differential amplifier

- The op amp is designed to amplify the difference between the voltage signals applied to the two input terminals and then multiply it by some gain factor A such that the voltage at the output terminal is $V_{out}=A(V_+ - V_-)$



Op-Amp

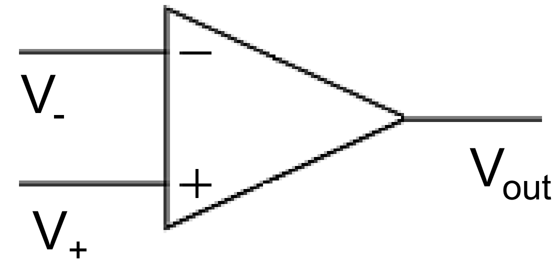
- The op amp is designed to respond only to the difference signal and hence should ignore any signal common to both inputs. That is if $V_+=V_-=1V$, then the output will ideally be zero.

- The gain factor A is referred to as the op amp differential gain or open-loop gain. It represents the maximum voltage gain available from the device.

Amplifier Gain

- All op-amps can be represented by the formula:

$$V_{out} = A\,(V_{+} - V_{-})$$

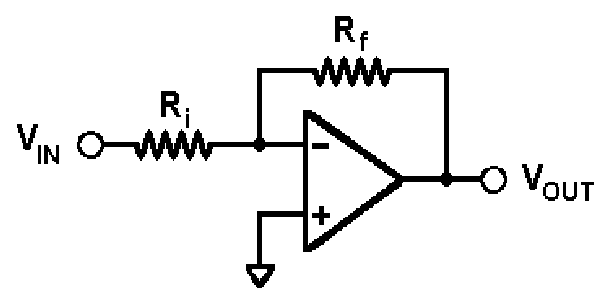


Op-Amp

where A is the gain, and is a property of the individual op-amp

- This gain should be distinguished from the gain of the op-amp circuit which is generally denoted by A_v

$$A_v = V_{out} / V_{in}$$



Op-Amp Circuit

- A potential source of confusion comes from failing to properly distinguish between the op-amp and the op-amp circuit

Characteristics of an Ideal Op-Amp

- An ideal op amp is an ideal differential amplifier with:

  Infinite gain A for the differential input signal.

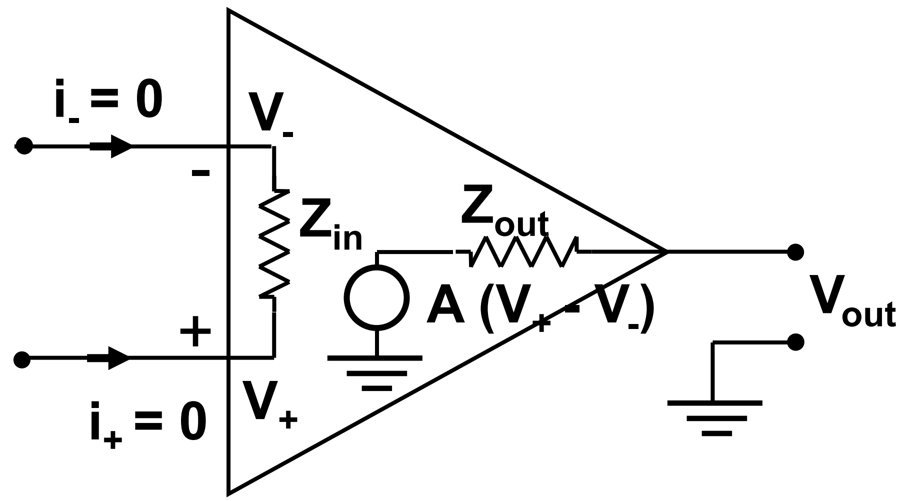
  Zero gain for the common-mode input signal.

  Infinite input impedance (does not load a circuit).

  Zero output impedance (allows the op amp to provide a constant output voltage, independent of the current drawn from the source).

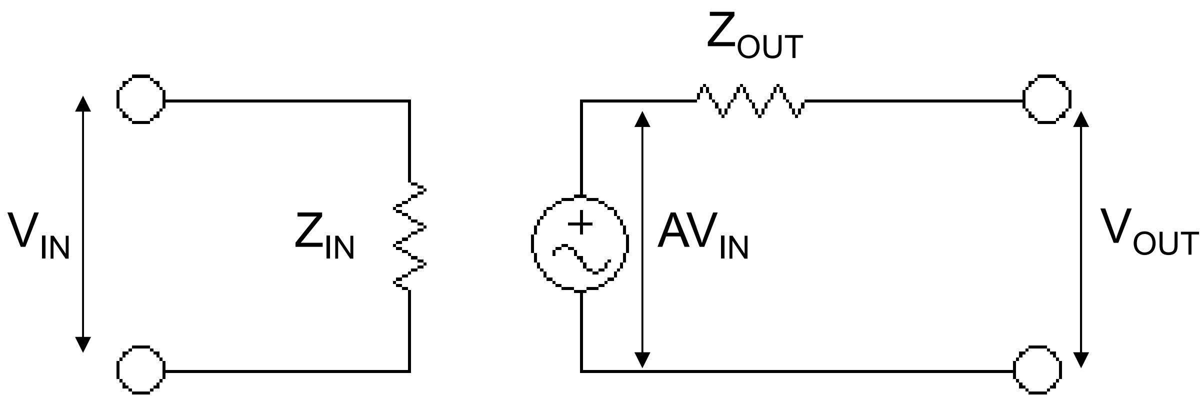
  Infinite bandwidth (i.e. will amplify signals of any frequency with equal gain).

- Amplification gain $A = \infty$
- Input impedance $Z_{in} = \infty$
- Input currents $I_+ = I_- = 0$
- Output impedance $Z_{out} = 0$



Impedances

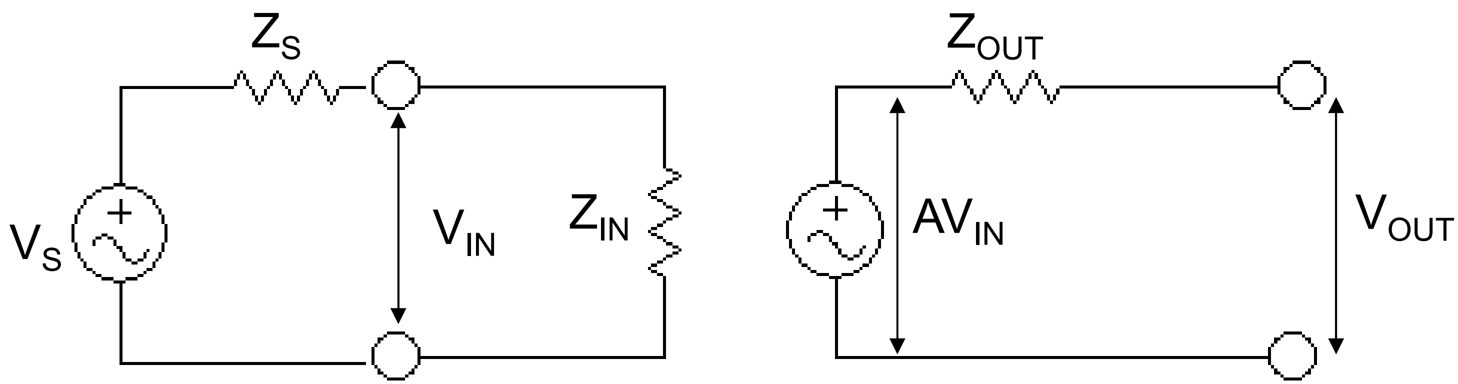
- Why do we care about the input and output impedance?
- Simplest "black box" amplifier model:



- The amplifier measures voltage across Z_{IN}, then generates a voltage which is larger by a factor A
- This voltage generator, in series with the output resistance Z_{OUT}, is connected to the output port.
- A should be a constant (i.e. gain is linear)

Input impedance

- Attach an input - a source voltage V_S plus source impedance Z_S



- Note the voltage divider $Z_S + Z_{IN}$.

  $V_{IN}=V_S(Z_{IN}/(Z_{IN}+Z_S))$

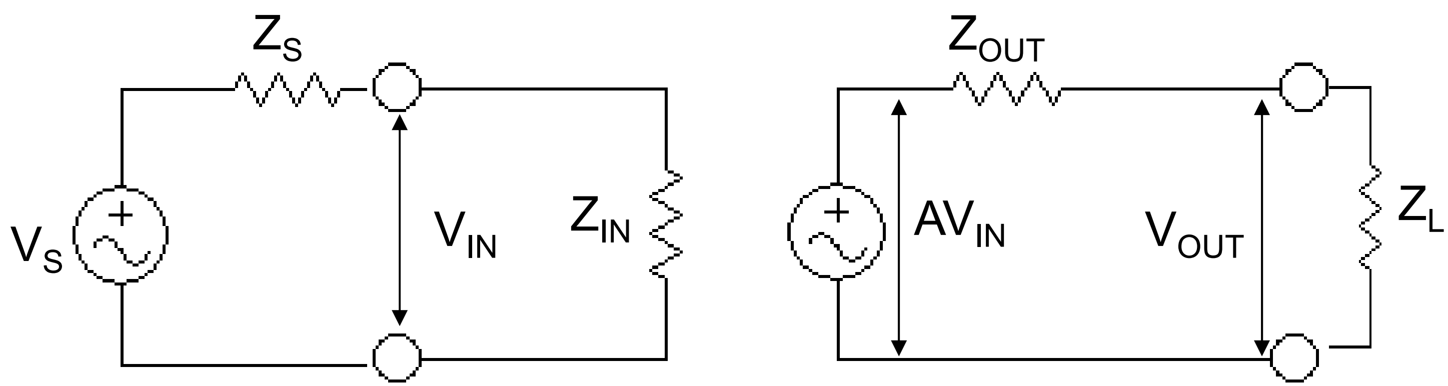
  We want $V_{IN} = V_S$ regardless of source impedance

  So want Z_{IN} to be large.

- The ideal amplifier has an infinite input impedance

Output impedance

- Attach a load - an output circuit with a resistance RL



- Note the voltage divider $Z_{OUT} + Z_L$.

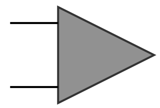
  $V_{OUT}=AV_{IN}(Z_L/(Z_L+Z_{OUT})$

  Want $V_{OUT}=AV_{IN}$ regardless of load

  We want Z_{OUT} to be small.

- The ideal amplifier has zero output impedance

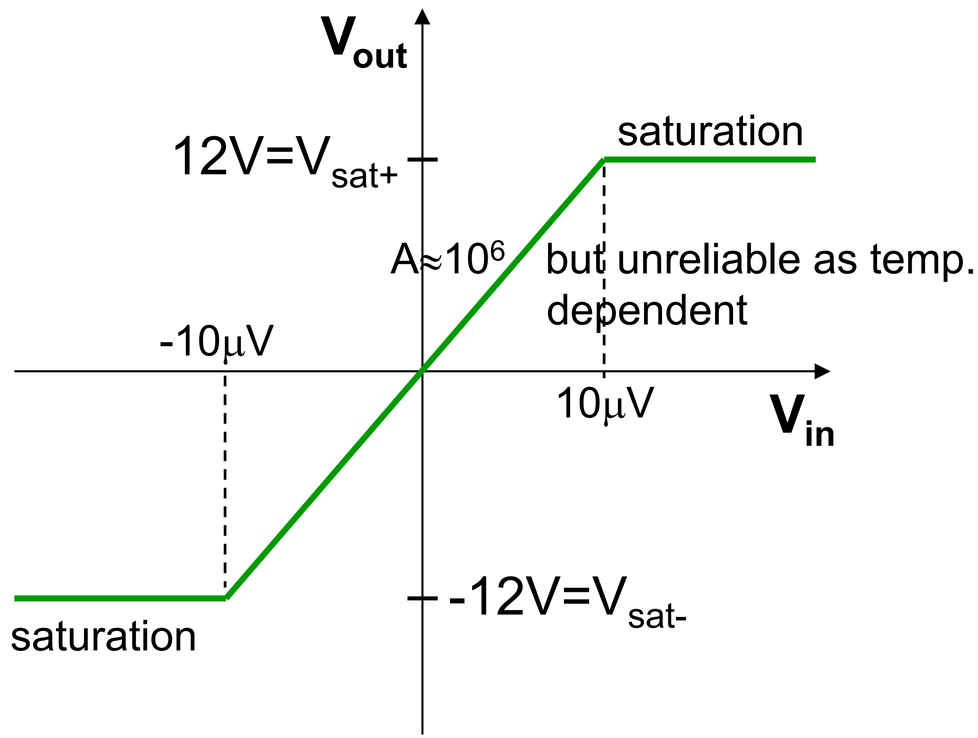
Ideal v. Real Op-Amps

	Ideal Op-Amp 	Typical Op-Amp 
Operational Gain	infinity	10^5 - 10^9
Input Resistance	infinity	$10^6\ \Omega$ (BJT) $10^9\ \Omega$ - $10^{12}\ \Omega$ (FET)
Input Current	0	10^{-12} – 10^{-8} A
Output Resistance	0	0 – 1000 Ω
Bandwidth	unlimited	Attenuates and phases at high frequencies (depends on slew rate) => 1-20 MHz
Temperature	independent	Influence on Bandwidth and gain

Saturation

- The ideal op amp circuit model assumes that output can be infinitely large.

- In practice, the power supplies define the upper and lower limits.



\+ Saturation:

$V_{out} = V_{sat+} \approx V_{vcc+}$

Linear Mode:

$V_{out} = A\,(V_{+} - V_{-})$

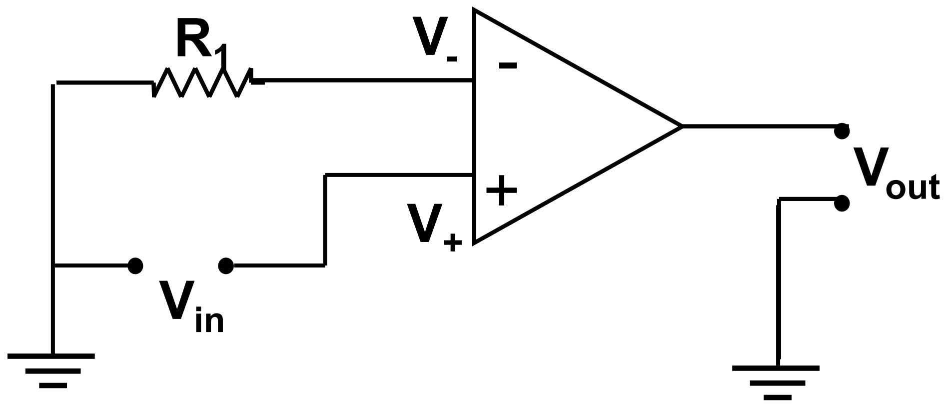
\- Saturation:

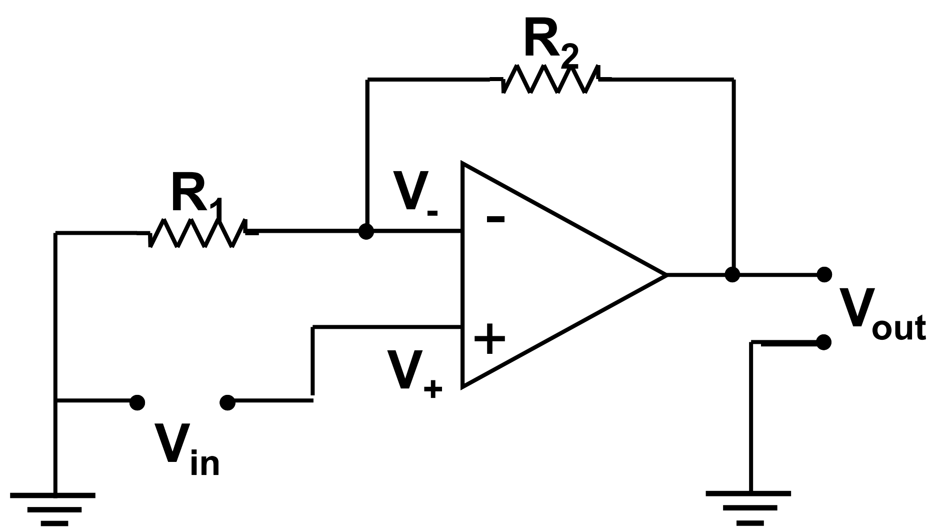
$V_{out} = V_{sat-} \approx V_{vcc-}$

- Op amp saturation has nothing to do with MOS saturation.

Open-Loop vs. Closed-Loop

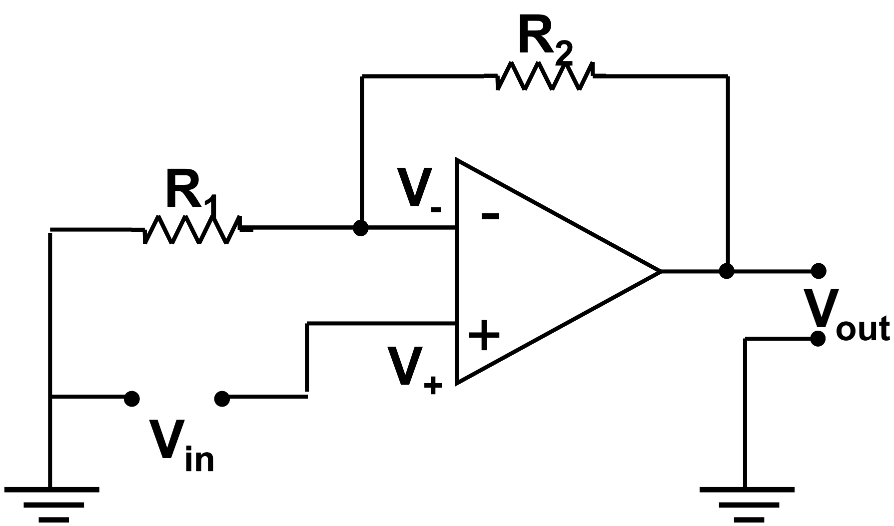
- In contrast to open-loops, closed-loop op-amps have feedback

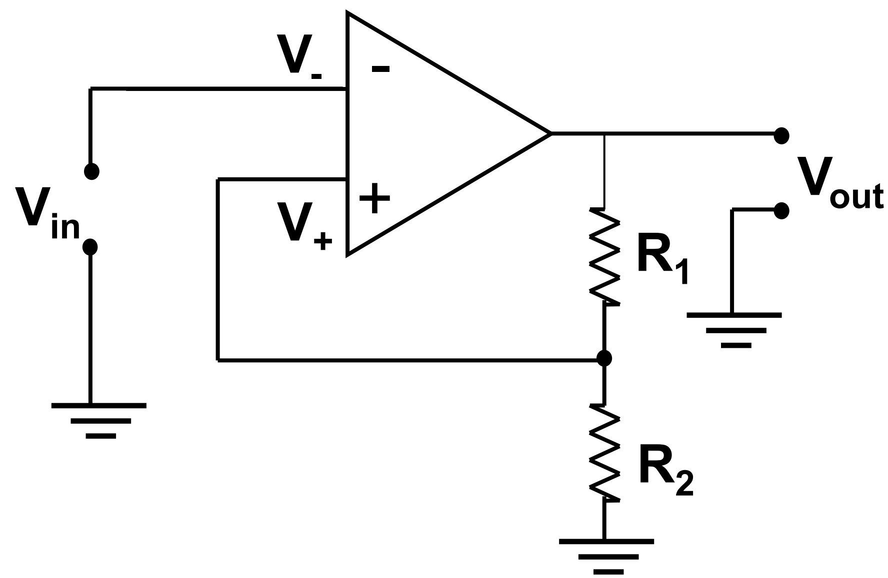




Negative vs. Positive Feedback

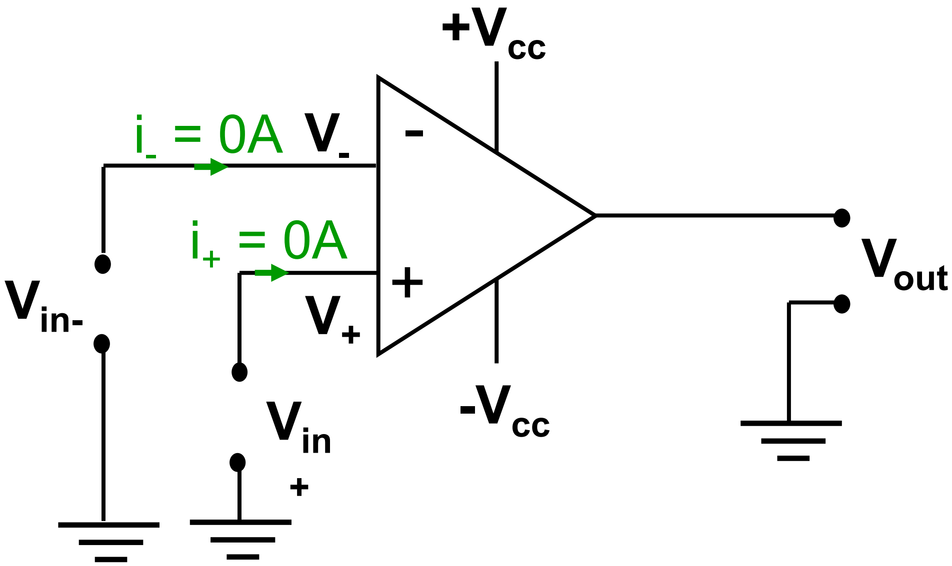
- Closed loops either have negative or positive feedback
- Negative feedback leads to the inverting input, positive to the non-inverting input

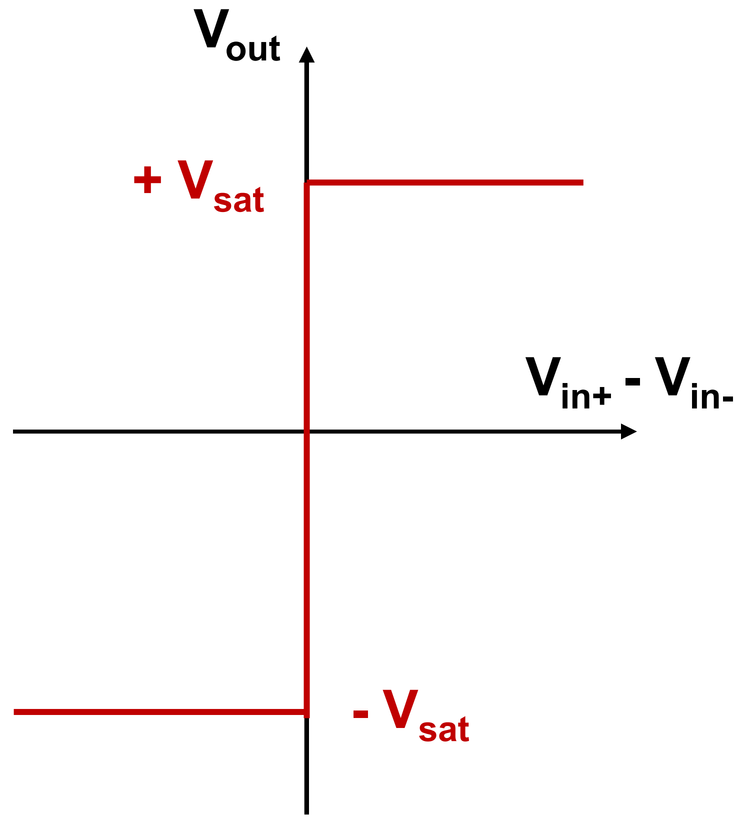




Comparator

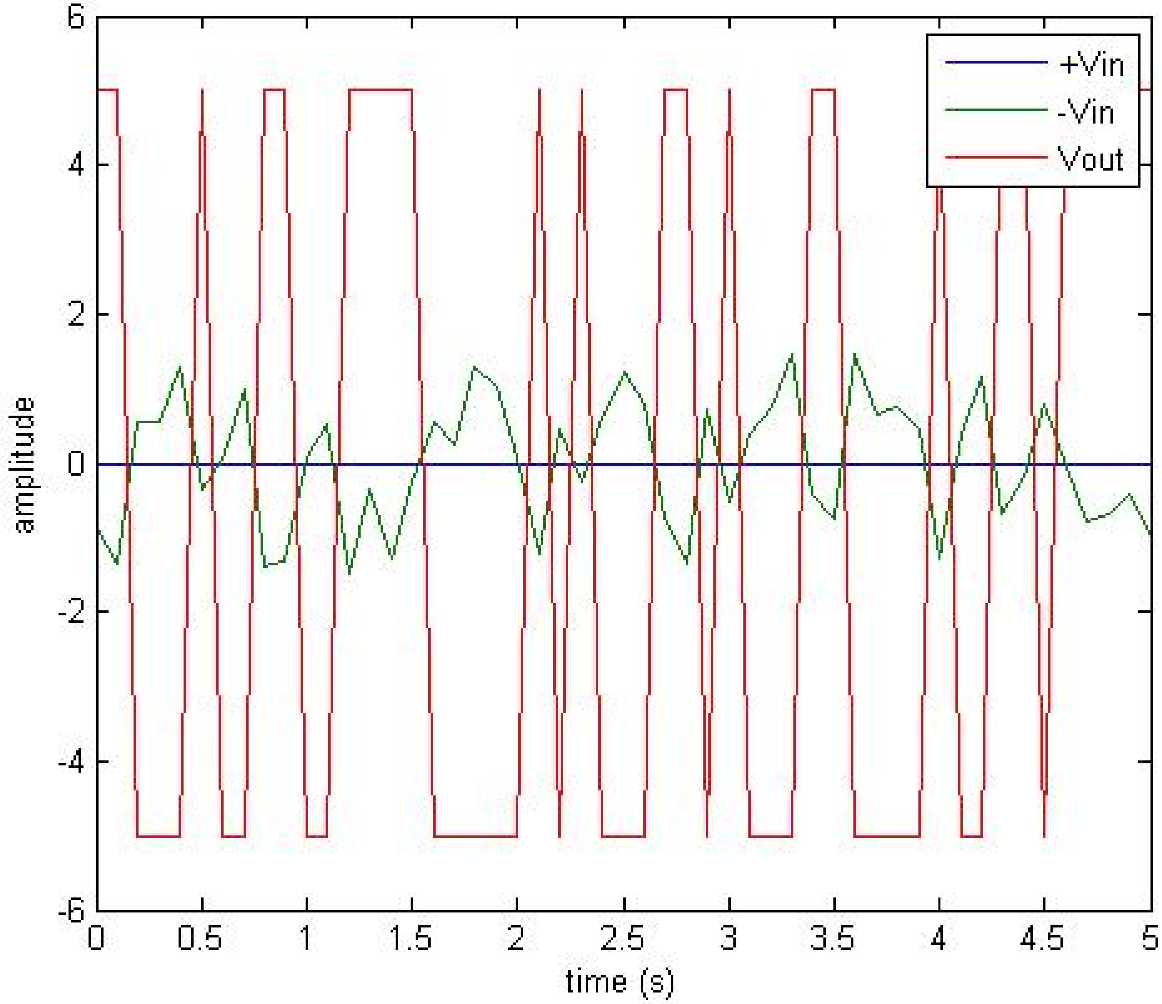
- A comparator is an example of an open-loop op-amp





If $V_+ > V_-$	$V_{out} = V_{sat} \approx V_{cc}$
If $V_+ < V_-$	$V_{out} = -V_{sat} \approx -V_{cc}$

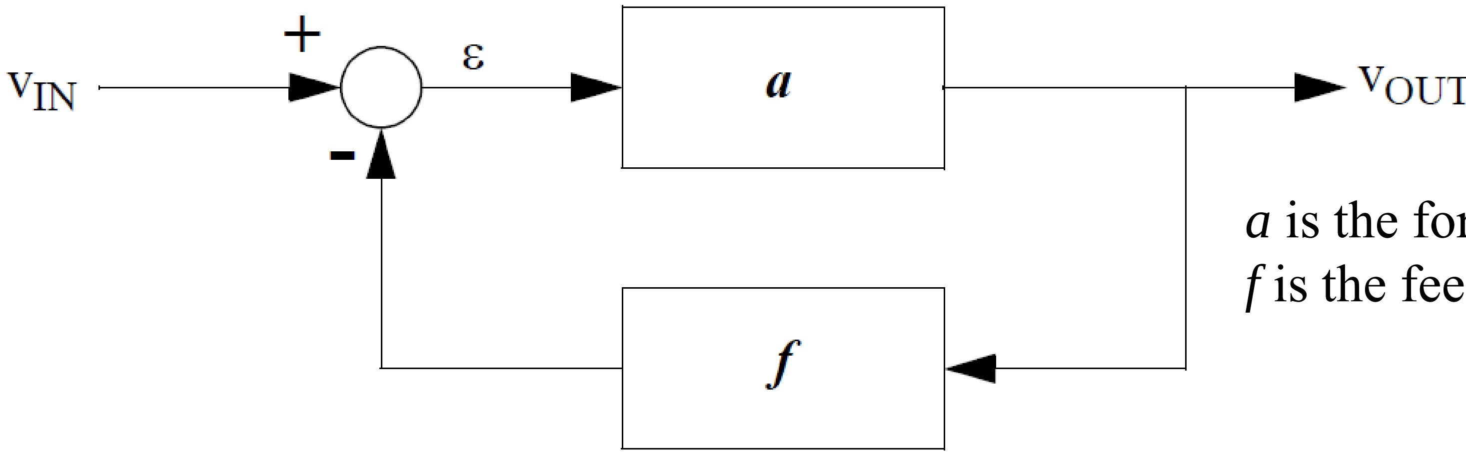
Comparator



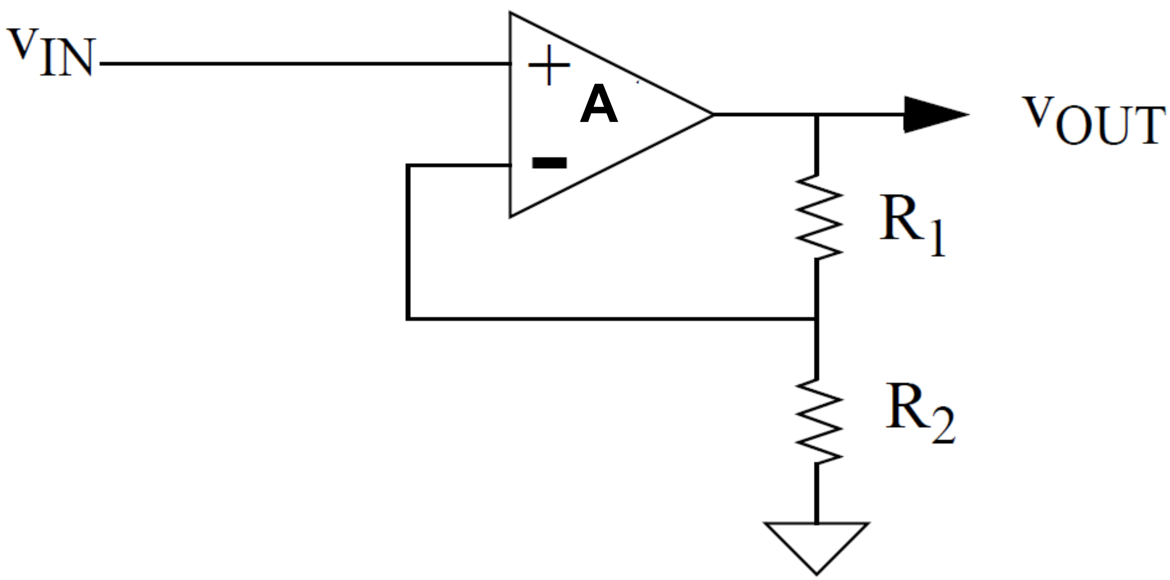
Distortion

- With op amps, the problem is not one of sufficient amplification gain. Trivial to achieve very large gains.
- Rather, the problem is distortion as op amp is implemented using non-linear elements (the transistors)
- The gain of the op amp can exhibit large variations over time. For example, its gain is very sensitive to temperature changes.
- The gain of the op amp is thus not constant.
- To address the issue of signal distortion, Harold Black proposed the negative feedback amplifier, in 1927.

Negative Feedback Amplifier



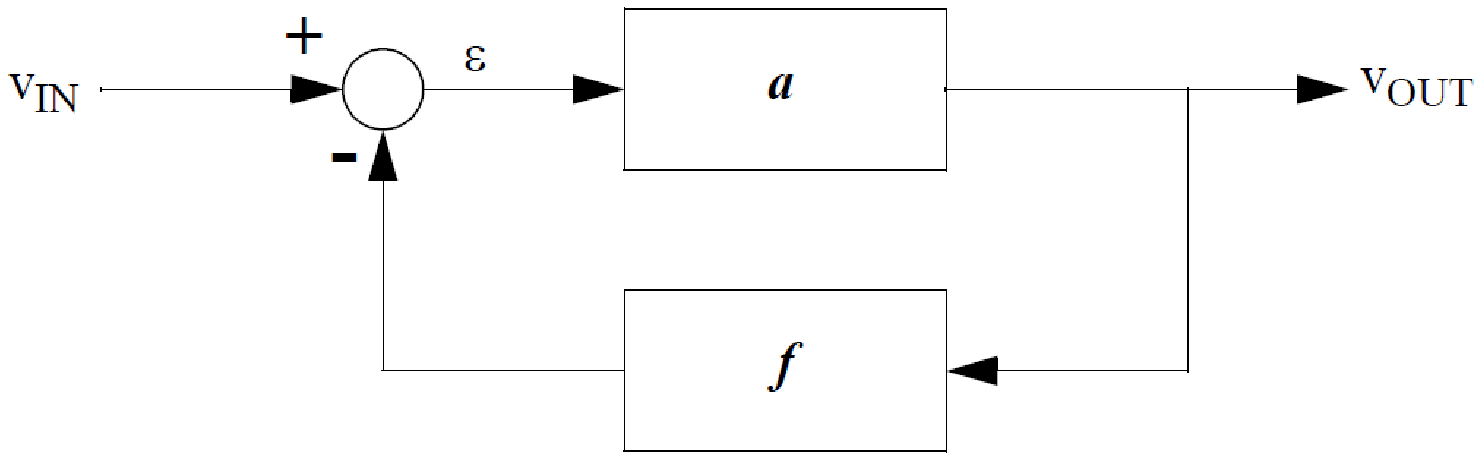
a is the forward gain
f is the feedback factor



$$a = A$$

$$f = \frac{R_2}{R_1 + R_2}$$

Negative Feedback Amplifier

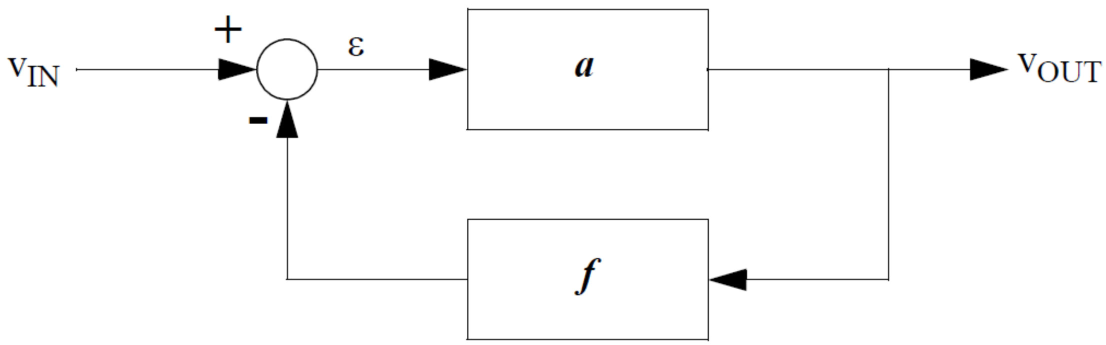


- The overall closed-loop gain Av of this system is:

$$A_v = \frac{a}{1+af} \approx \frac{1}{f}$$

when you make $af >> 1$

Negative Feedback Amplifier

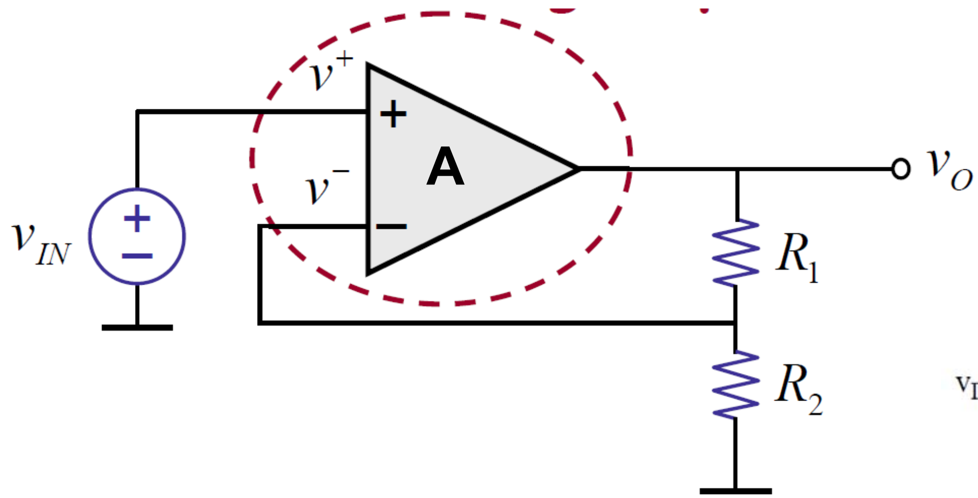


$$A_v = \frac{a}{1+af} \approx \frac{1}{f}$$

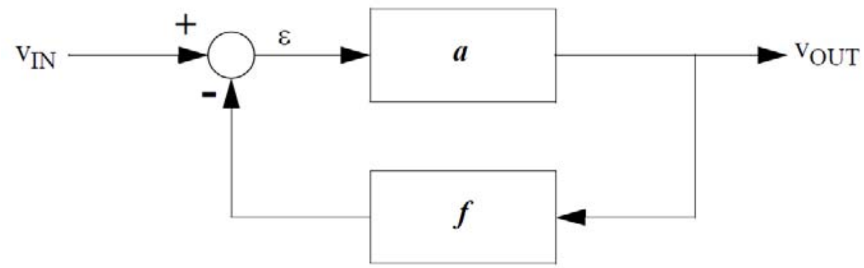
- The feedback factor f can be implemented with perfectly linear elements, such as resistive voltage dividers, so that the overall closed-loop behavior is linear even though the amplifier in the block a is not.

- That is, it doesn't matter that a exhibits all sorts of nonlinear behavior as long as af >>1 under all conditions of interest. The only trade-off is that the overall, closed-loop gain A_v is much smaller than the forward gain a. However, if gain is cheap, but low distortion isn't, then negative feedback is a marvelous solution to a very difficult problem.

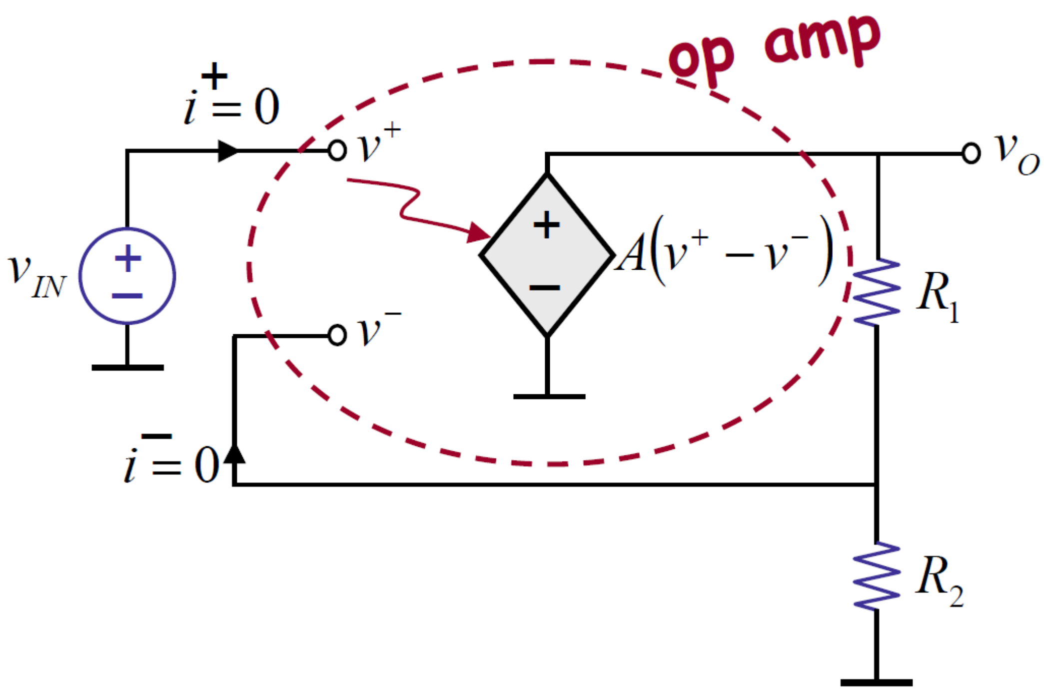
Non-inverting amplifier



$$a = A$$



Equivalent circuit model



$$f = \frac{R_2}{R_1 + R_2}$$

Negative Feedback Amplifier

Let us analyze the circuit:

Find v_O in terms of v_{IN}, etc.

$$v_O = A\left(v^+ - v^-\right)$$

$$= A\left(v_{IN} - v_O \frac{R_2}{R_1 + R_2}\right)$$

$$v_O\left(1 + \frac{AR_2}{R_1 + R_2}\right) = Av_{IN}$$

$$v_O = \frac{Av_{IN}}{1 + \dfrac{AR_2}{R_1 + R_2}}$$

What happens when "A" is very large?

Non-inverting amplifier

Let's see... When A is large

$$v_O = \frac{Av_{IN}}{\cancel{1} + \frac{AR_2}{R_1 + R_2}} \approx \frac{\cancel{A}v_{IN}}{\frac{\cancel{A}R_2}{R_1 + R_2}}$$

$$\approx v_{IN} \underbrace{\frac{(R_1 + R_2)}{R_2}}_{\text{gain}}$$

Suppose

$$A = 10^6$$

$$R_1 = 9R$$

$$R_2 = R$$

Non-inverting amplifier

$$v_O = \frac{10^6 \cdot v_{IN}}{1 + \frac{10^6 R}{9R + R}}$$

$$= \frac{\cancel{10^6} \cdot v_{IN}}{\cancel{1+} \cancel{10^6} \cdot \frac{1}{10}}$$

$$v_O \approx v_{IN} \cdot 10$$

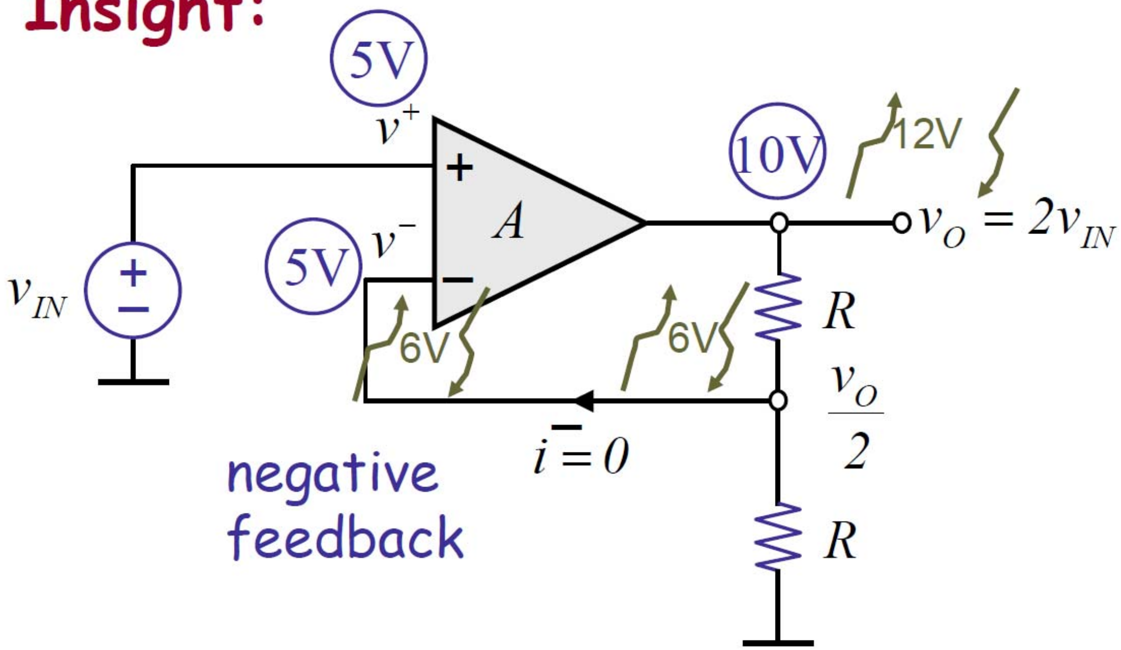
Gain:

- determined by resistor ratio
- insensitive to A, temperature, fab variations

Non-inverting amplifier

Why did this happen?

Insight:



e.g. $v_{IN} = 5\text{V}$

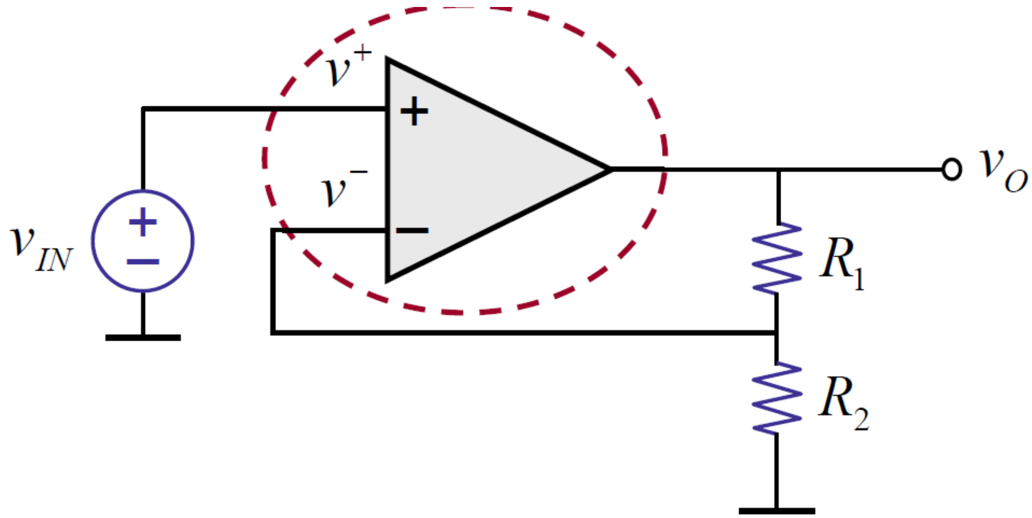
Suppose I perturb the circuit…

(e.g., force v_O momentarily to 12V somehow).

Stable point is when $v^+ \approx v^-$.

Key: negative feedback → portion of output fed to $-ve$ input.

More op amp insights

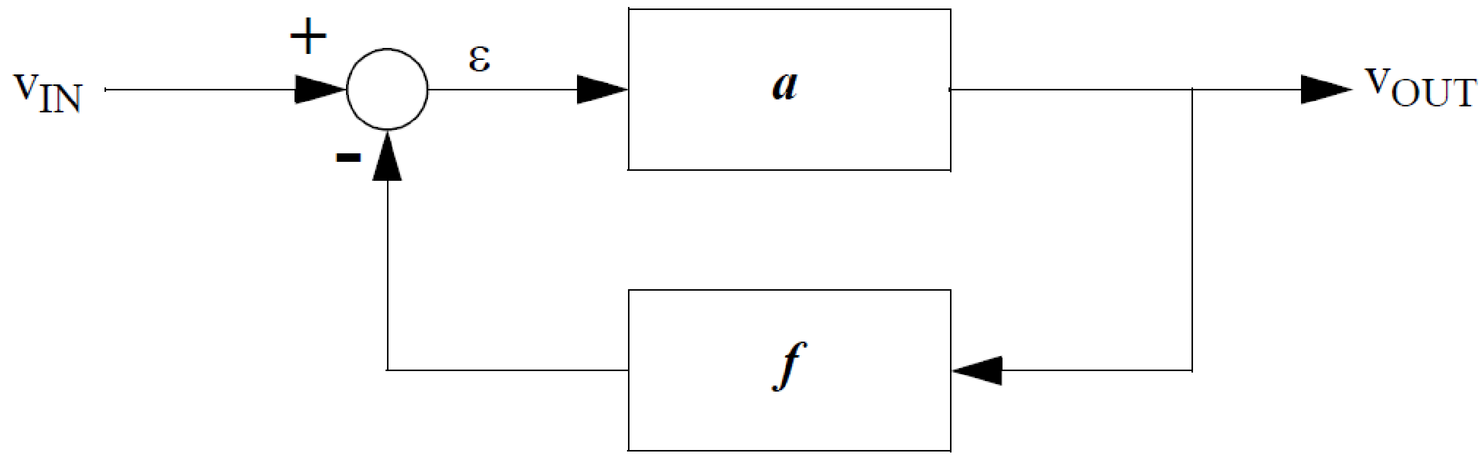


Observe, under negative feedback,

$$v^+ - v^- = \frac{v_O}{A} = \frac{\left(\frac{R_1 + R_2}{R_1}\right) v_{IN}}{A} \rightarrow 0$$

$\boxed{v^+ \approx v^-}$ →yields an easier analysis method (under negative feedback).

Negative Feedback Amplifier

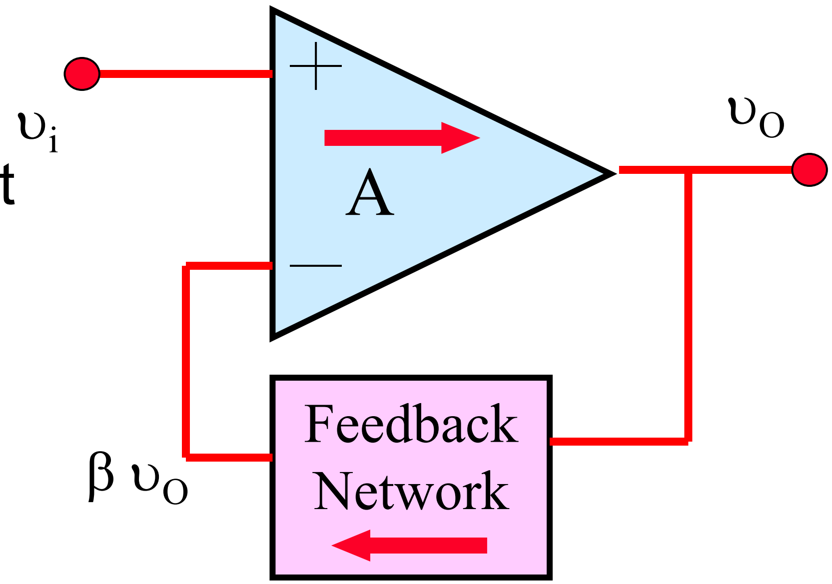


- Negative feedback drives the output voltage into a direction that tends to "zero" the input differential ε voltage when you make $af >> 1$.

$$A_v = \frac{v_{OUT}}{v_{IN}} = \frac{a\varepsilon}{v_{IN}} = \frac{a}{1+af} \quad \Rightarrow \quad \frac{\varepsilon}{v_{IN}} = \frac{1}{1+af}$$

Negative feedback: a corrective mechanism

- In an op-amp with no feedback (comparator), there is no corrective mechanism, and the output voltage will go to full saturation with the tiniest amount of input differential voltage.



Negative feedback

- Negative feedback drives the op amp output voltage into a direction that tends to “zero” the input differential voltage.

- As a result, negative feedback enables an op-amp to work in its linear region, as opposed to merely being either positively or negatively fully saturated. Negative feedback opposes any change at the output. If the output tries to rise (or fall), the loop will respond to force it back to the nominal value.

Calculation Rules for Op-Amps

- Assumptions:

  Calculation based on the models of an ideal op-amp ($Z_{in} = \infty$, $Z_{out} = 0$, $A = \infty$)

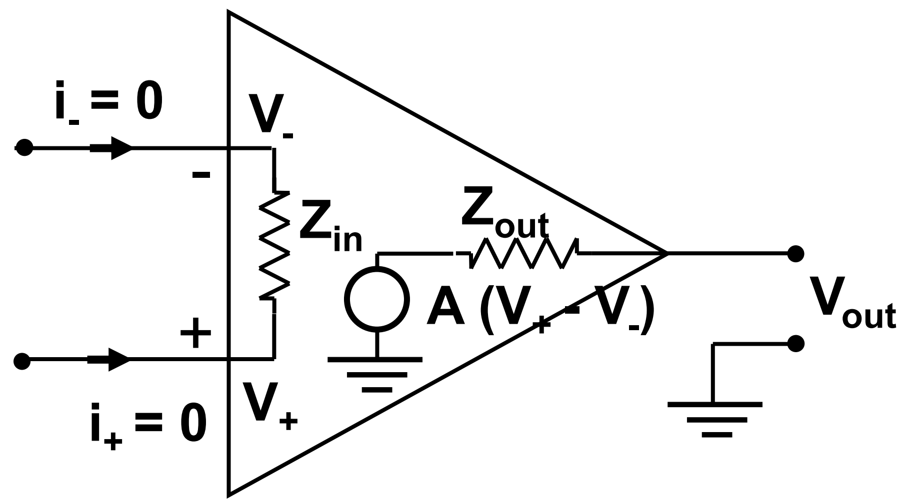
  Op-Amp operates in its linear amplifying mode (V_{out} between saturation borders)

  Negative feedback configuration

Calculation Rules

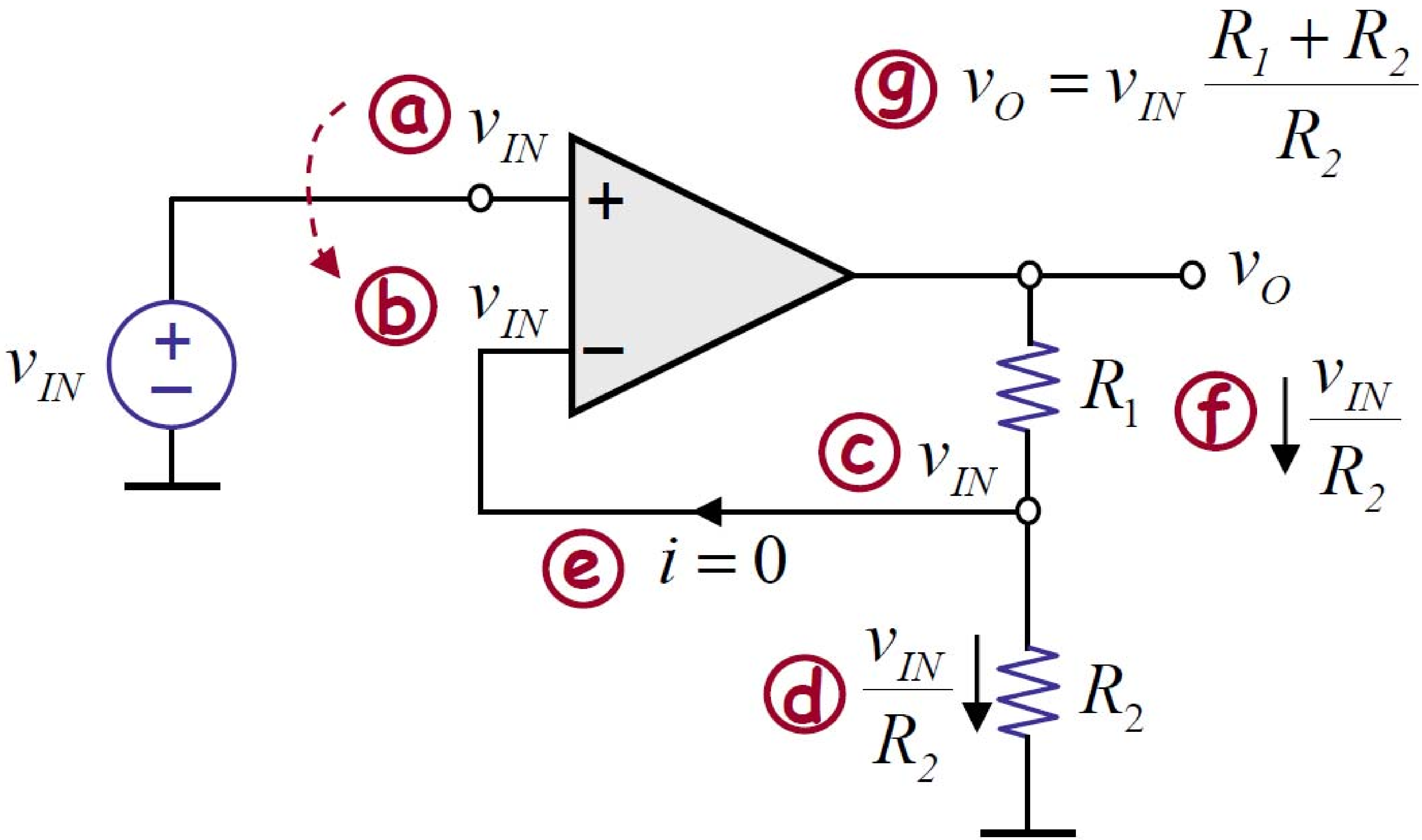
(1) $i_+ = i_- = 0$

(2) $V_+ = V_-$

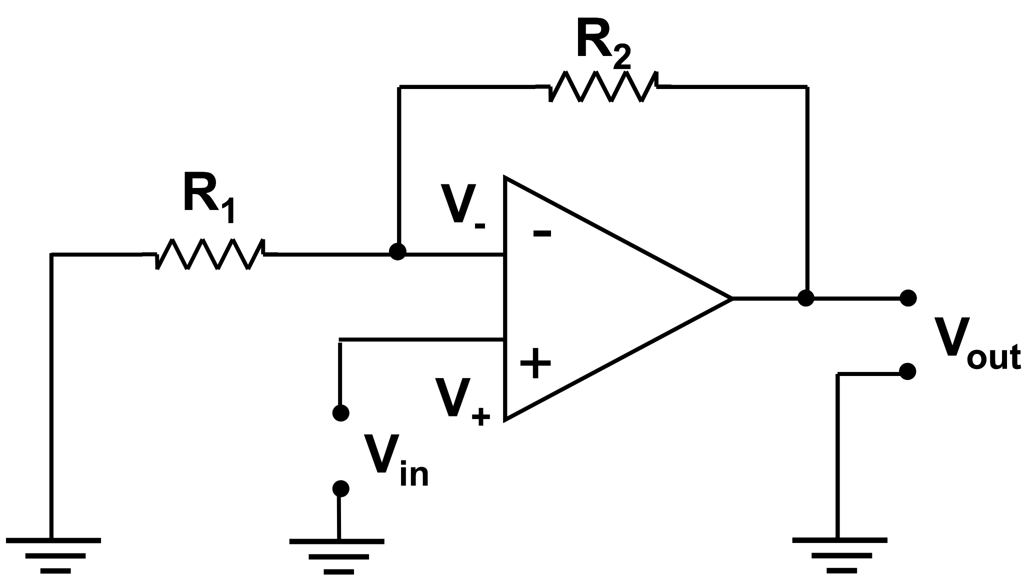


An easier op amp analysis method

$$v^+ \approx v^- \qquad i^+ \approx 0 \qquad i^- \approx 0$$



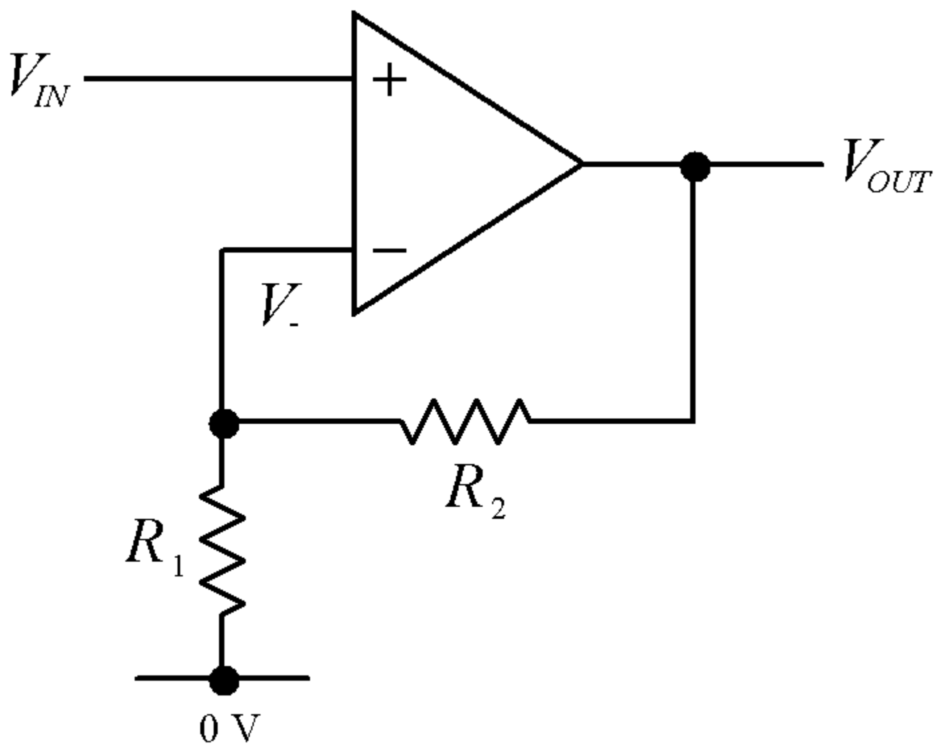
Non-inverting Op-Amp



Circuit Characteristics

- Input voltage is amplified with a **positive gain**
- Output connected to inverting input (V_-)
- Inverting input leading to ground
- Input voltage connected to non-inverting input (V_+)

Non-inverting Op-Amp



Remember the rules for negative feedback:

$$I_+ = 0 \;\; and \;\; I_- = 0$$

$$V_+ = V_-$$

$$V_{IN} = V_+ = V_- = V_{OUT} \frac{R_1}{R_1 + R_2}$$

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

Non-inverting Op-Amp - Example

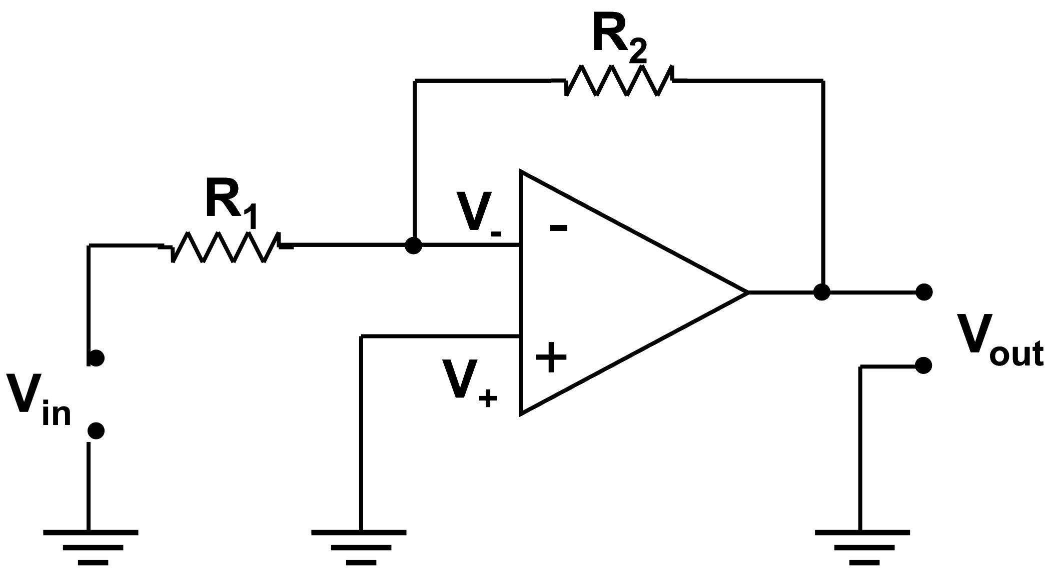
- A non-inverting op-amp has an input voltage of 2 V. R_1 = 6 kΩ, R_2 = 30 kΩ. What is the output voltage?

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in} = \left(1 + \frac{30\text{k}\Omega}{6\text{k}\Omega}\right) 2\text{V} = 12\text{V}$$

- The saturation output voltage of an non-inverting op-amp is V_{sat}=13 V. R_1 = 10 kΩ, R_2 = 10 kΩ. Determine the maximum input voltage so that the output voltage does not saturate.

$$V_{in} = \frac{V_{out}}{\left(1 + \frac{R_2}{R_1}\right)} = \frac{13\text{V}}{\left(1 + \frac{10\text{k}\Omega}{10\text{k}\Omega}\right)} = 6.5\text{V}$$

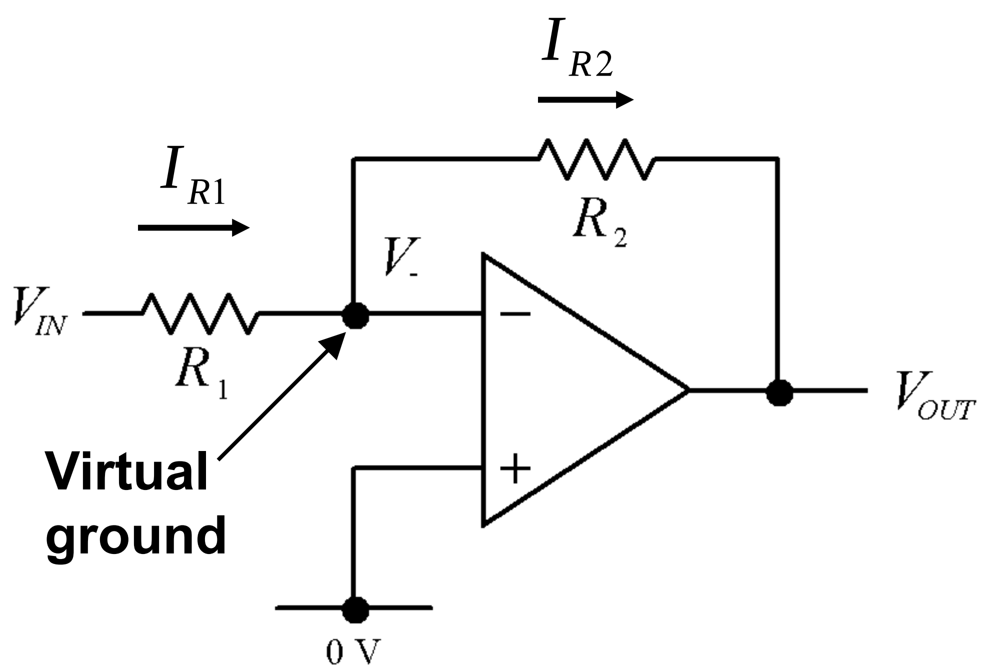
Inverting Op-Amp



Circuit Characteristics

- Output connected to inverting input (V_-)
- Non-inverting input leading to ground
- Input voltage connected to inverting input (V_-)
- Input voltage is amplified with a negative gain

Inverting Op-Amp



Use the following rules:

$$I_+ = 0 \;\; and \;\; I_- = 0$$

$$V_+ = V_-$$

$$V_- = V_+ = 0$$

$$I_- = 0 \Rightarrow I_{R1} = I_{R2}$$

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{V_{IN} - 0}{R_1}$$

$$I_{R2} = \frac{V_{R2}}{R_2} = \frac{0 - V_{OUT}}{R_2}$$

$$\therefore \frac{V_{IN} - 0}{R_1} = \frac{0 - V_{OUT}}{R_2}$$

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1}$$

Inverting Op-Amp - Example

- Let's assume we need to create an output signal of 10 V.

  V_{cc+} = 30 V, R_1 = 10 kΩ, V_{in} = - 5 V.

  How do we have to choose R_2?

  $V_{out} = -(R_2/R_1) \times V_{in}$

  $\rightarrow R_2 = -V_{out} \times R_1/V_{in} = (-10\text{ V} \times 10\text{ k}\Omega) / (-5\text{ V}) = 20\text{ k}\Omega$

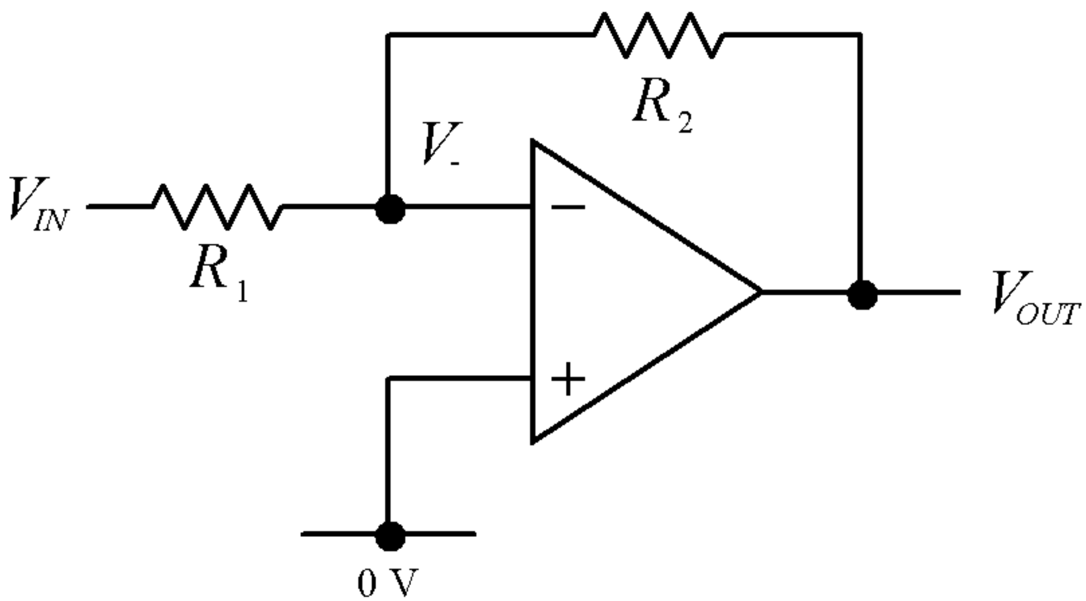
- What would be Vout , if Vcc+ = 30 V, R_1 = 5 kΩ, R_2 = 20 kΩ and Vin = -10 V?

  $V_{out} = -(R_2/R_1) \times V_{in} = (-20\text{ k}\Omega / 5\text{ k}\Omega) \times (-10\text{V}) = 40\text{ V}$???

  No! Since $V_{out} > V_{cc+}$ $\rightarrow V_{out} = V_{cc+}$

Input impedance of Inverting Op-Amp

- Unlike the non-inverting amplifier, the input current is non-zero therefore the input impedance is finite:



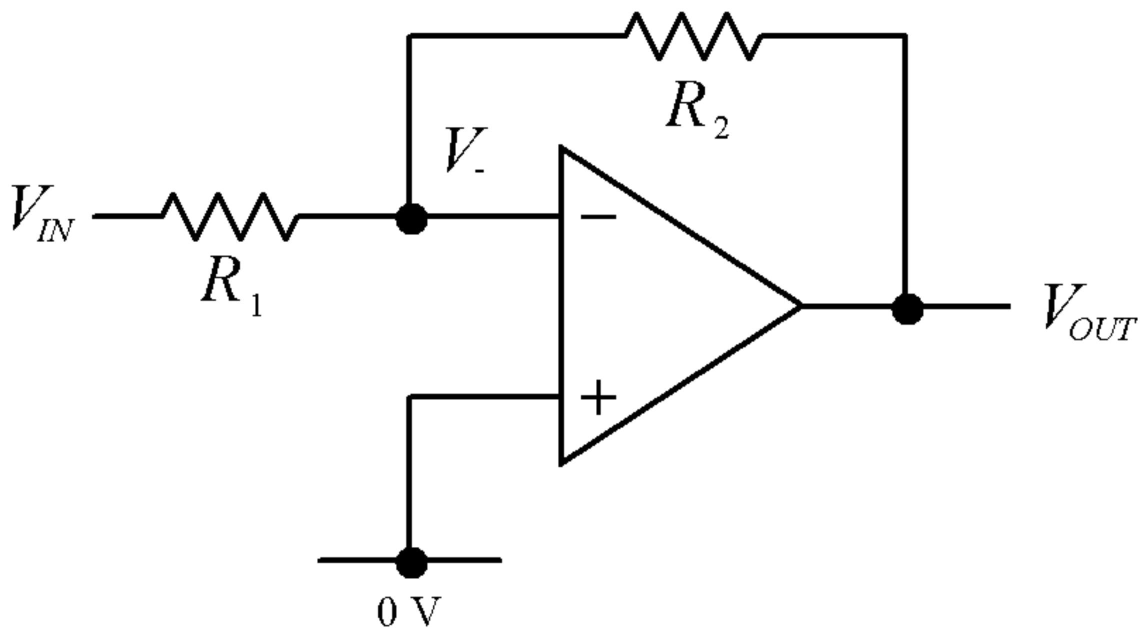
$$R_{IN} = \frac{V_{IN}}{I_{IN}}$$

$$I_{IN} = \frac{V_{IN} - 0}{R_1}$$

$$\therefore R_{IN} = R_1$$

Example of Inverting Op-Amp

- To design an inverting amplifier with a voltage gain of –10 and an input impedance of 1 kΩ:



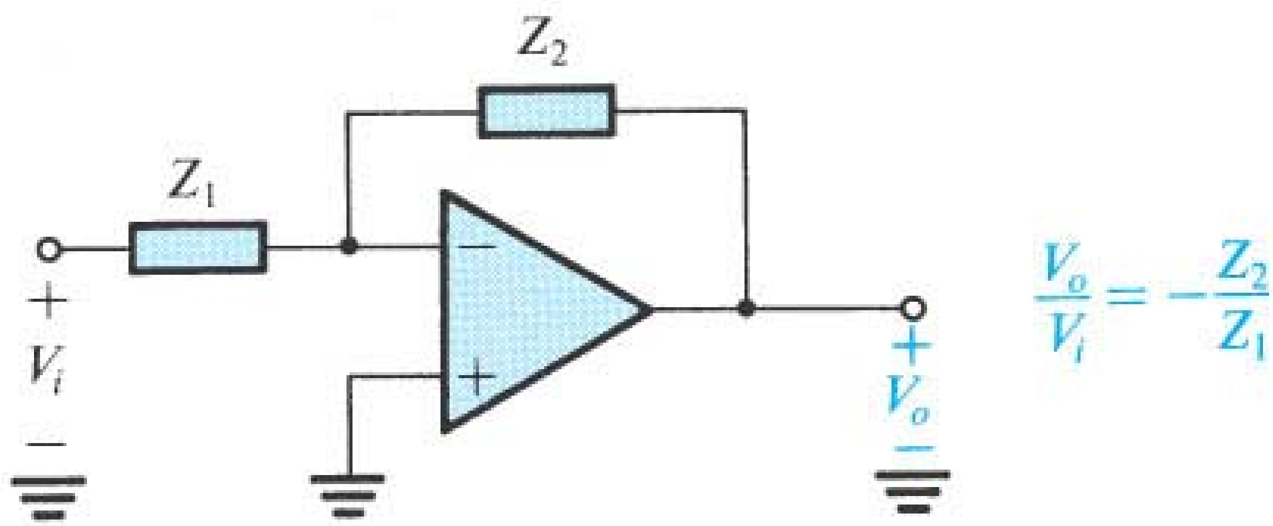
$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1} = -10$$

$$\Rightarrow R_2 = 10 R_1$$

$$R_{IN} = R_1 = 1\,k\Omega$$

$$\therefore R_2 = 10\,k\Omega$$

Inverting configuration with general impedances



- By placing different circuit elements into Z_1 and Z_2, we can get interesting operations. Some examples …

  Summer

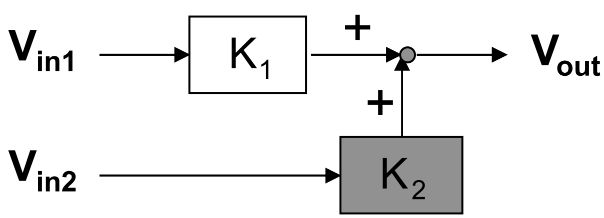
  Differentiator

  Integrator

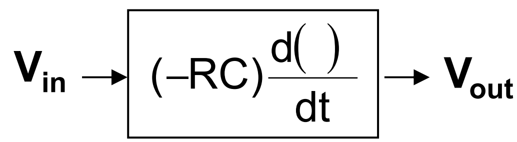
Op-Amps for Maths

- Closed-loop operational amplifiers with negative feedback can be used to implement various mathematic operations:

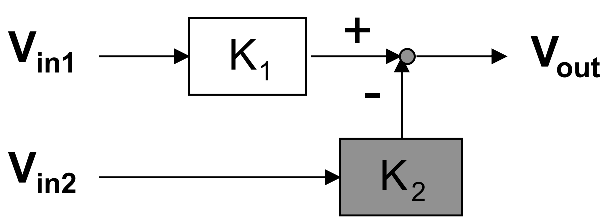
  - Summing



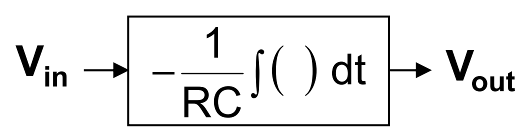
  - Derivative



  - Subtracting

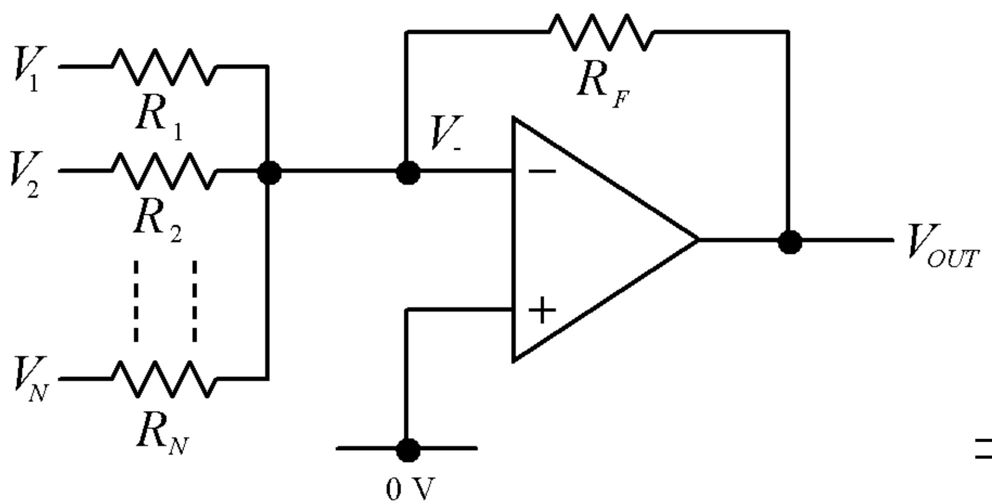


  - Integrating



Inverting Weighted Summer

- This circuit is an important application of the inverting configuration. It performs is a weighted sum of the input signals $v_1, v_2, \ldots, v_n$. The only constraint is that all the summing coefficients are of the same sign.



Apply Kirchoff's current law to the V_- node:

$$I_{R1} + I_{R2} + \ldots + I_{RN} + I_{RF} = 0$$

Like the inverting amp, $V_- = 0$

$$\Rightarrow \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_N}{R_N} + \frac{V_{OUT}}{R_F} = 0$$

$$\Rightarrow V_{OUT} = -R_F \left[\frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_N}{R_N} \right]$$

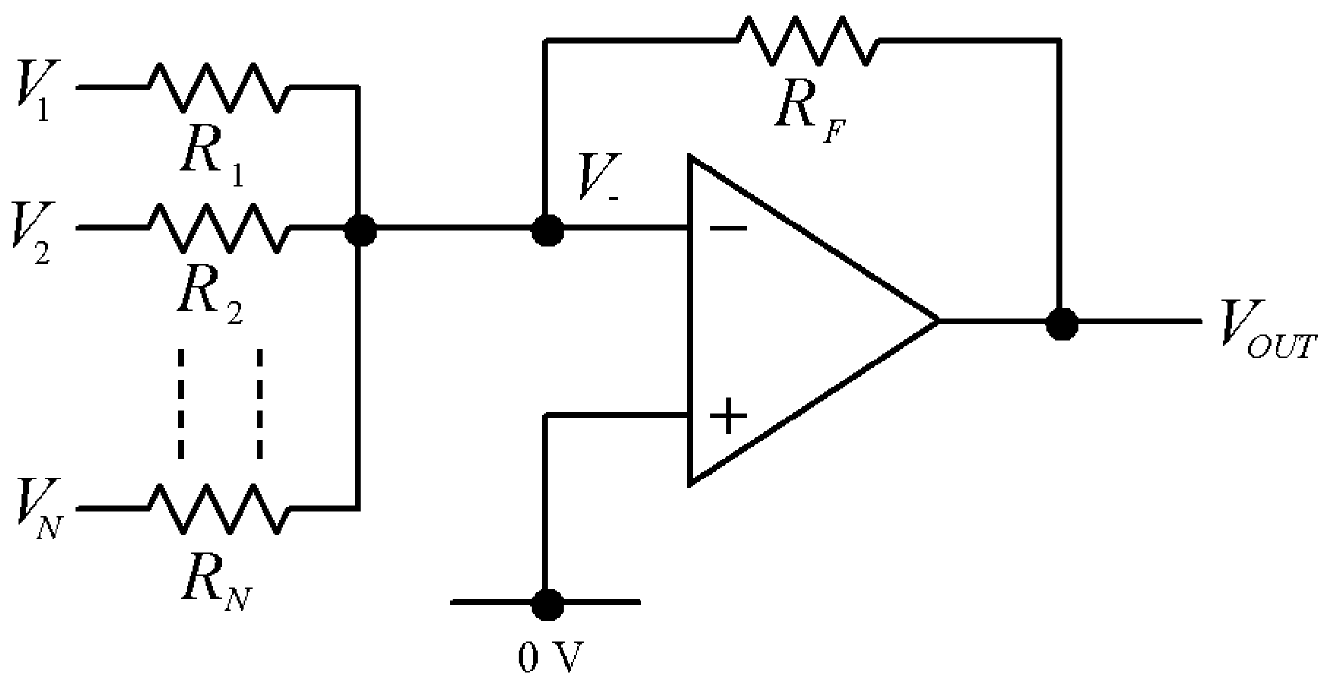
Input impedances

- Input impedance seen by:

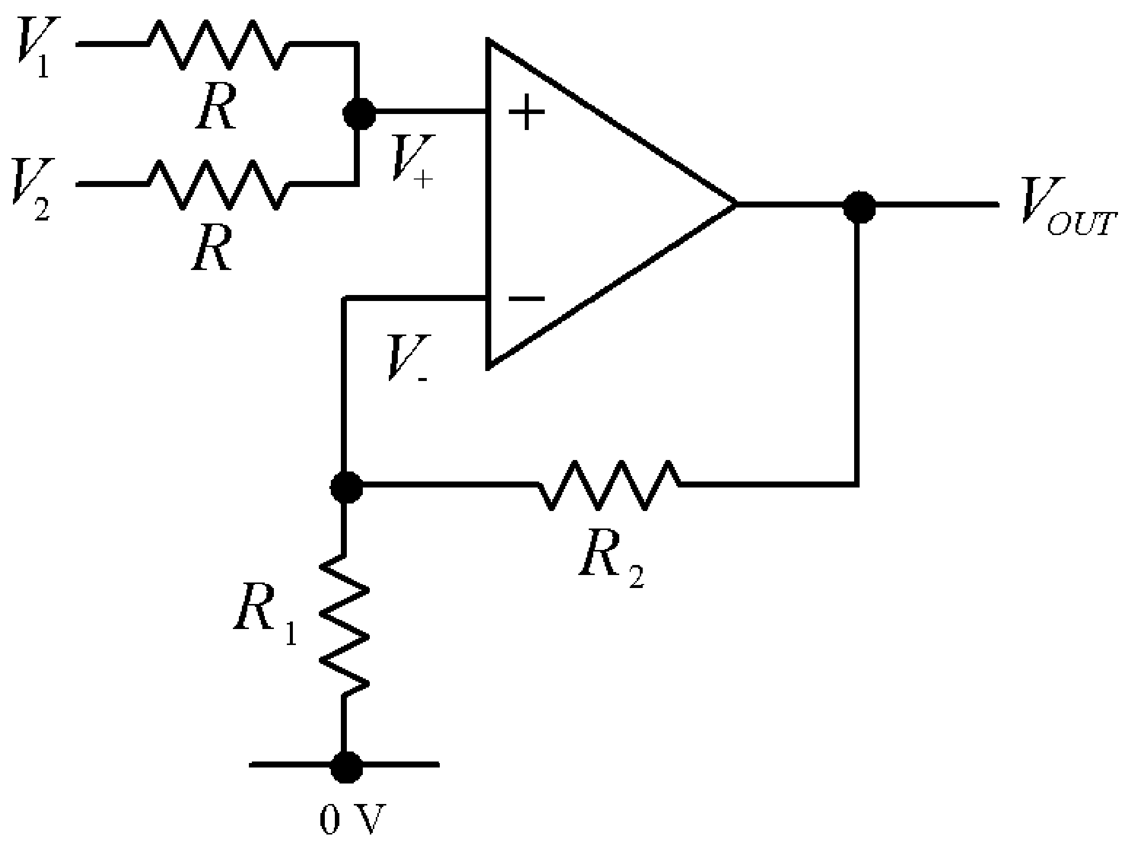
  Input 1 = R_1

  Input 2 = R_2

  Input N = R_N



Non-inverting Weighted Summer



$$\frac{V_{OUT}}{V_+} = 1 + \frac{R_2}{R_1}$$

$$\frac{V_1 - V_+}{R} + \frac{V_2 - V_+}{R} = 0$$

$$\Rightarrow V_+ = \frac{V_1 + V_2}{2}$$

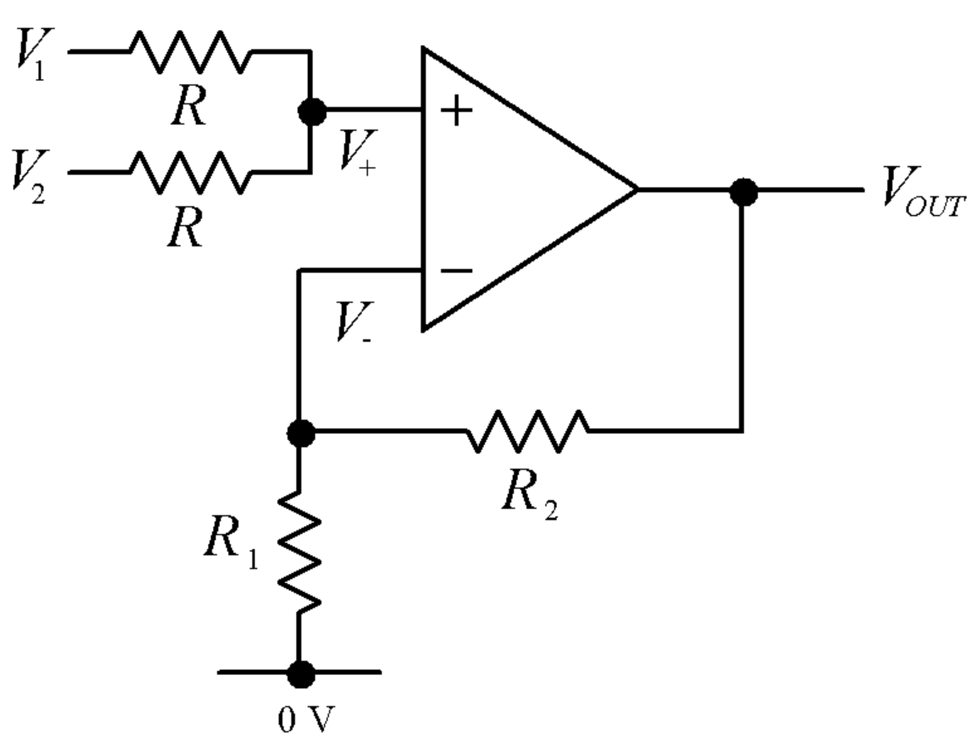
$$\therefore V_{OUT} = \left[1 + \frac{R_2}{R_1}\right]\frac{V_1 + V_2}{2}$$

Limitation of Non-inverting Weighted Summer

- Input current depends on the level of all the other inputs.

$$e.g.\quad I_{IN1} = \frac{(V_1 - V_2)}{2R}$$

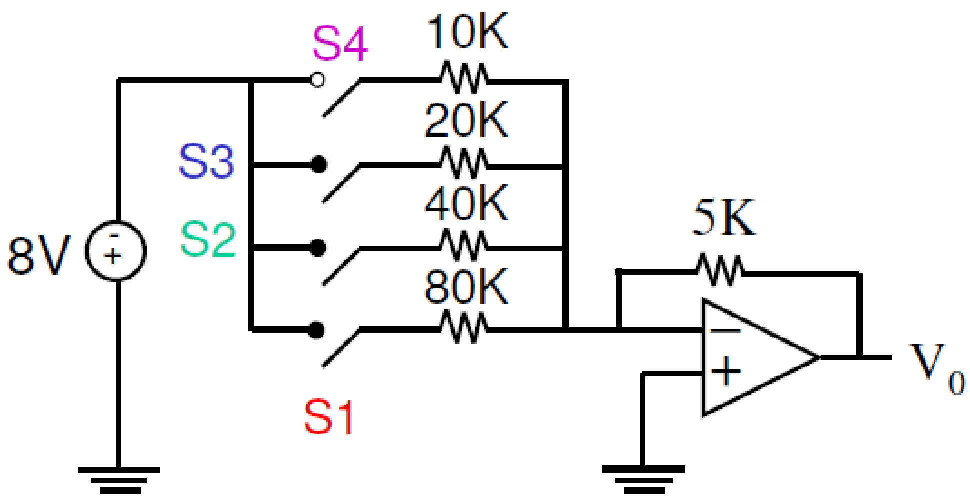
- The effective input impedance is not constant.
- This can be undesirable.
- Inverting configuration is usually preferred.



Application: Digital-to-Analog Conversion

A DAC can be used to convert the digital representation of an audio signal into an analog voltage that is then used to drive speakers -- so that you can hear it!

"Weighted-adder D/A converter"



4-Bit D/A

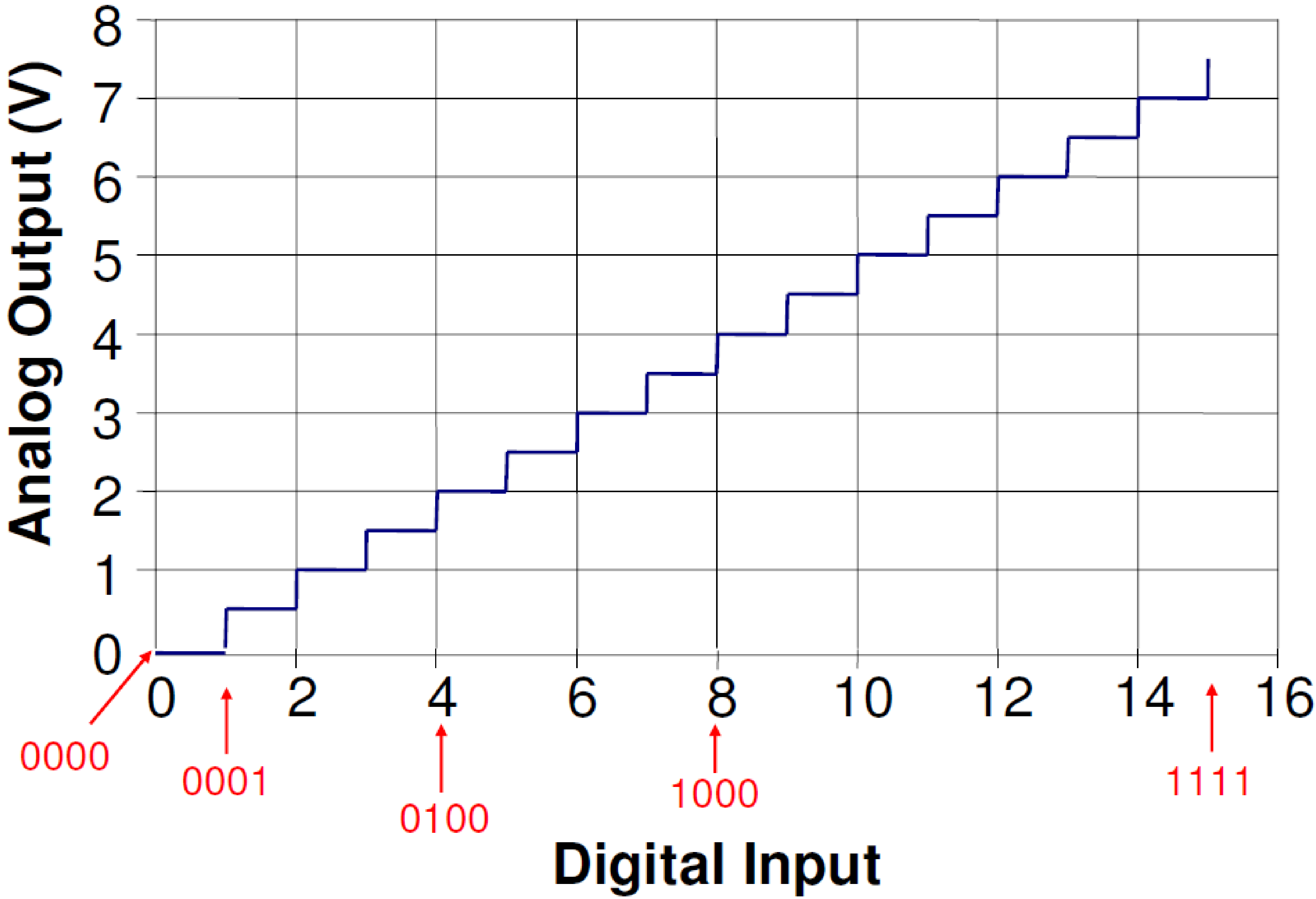
(Transistors are used as electronic switches)

Binary number	Analog output (volts)
0 0 0 0	0
0 0 0 1	.5
0 0 1 0	1
0 0 1 1	1.5
0 1 0 0	2
0 1 0 1	2.5
0 1 1 0	3
0 1 1 1	3.5
1 0 0 0	4
1 0 0 1	4.5
1 0 1 0	5
1 0 1 1	5.5
1 1 0 0	6
1 1 0 1	6.5
1 1 1 0	7
1 1 1 1	7.5

MSB LSB

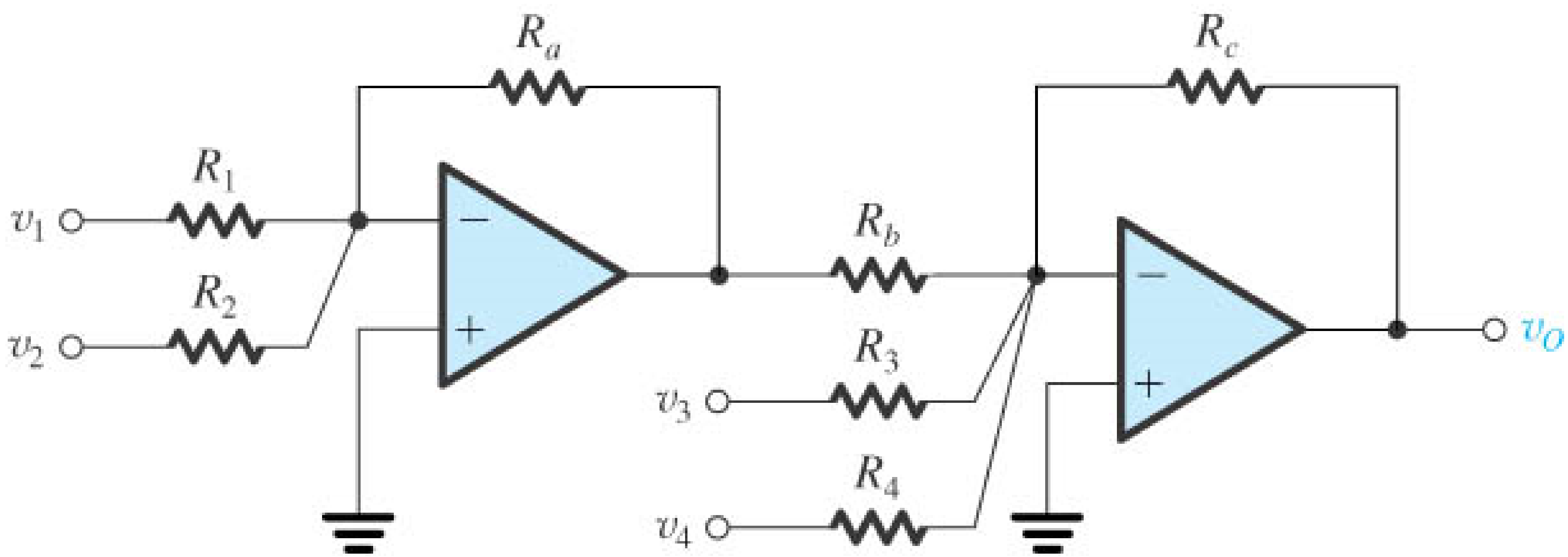
S1 closed if LSB =1
S2 " if next bit = 1
S3 " if " " = 1
S4 " if MSB = 1

Characteristic of 4-Bit DAC



Summing signals with opposite signs

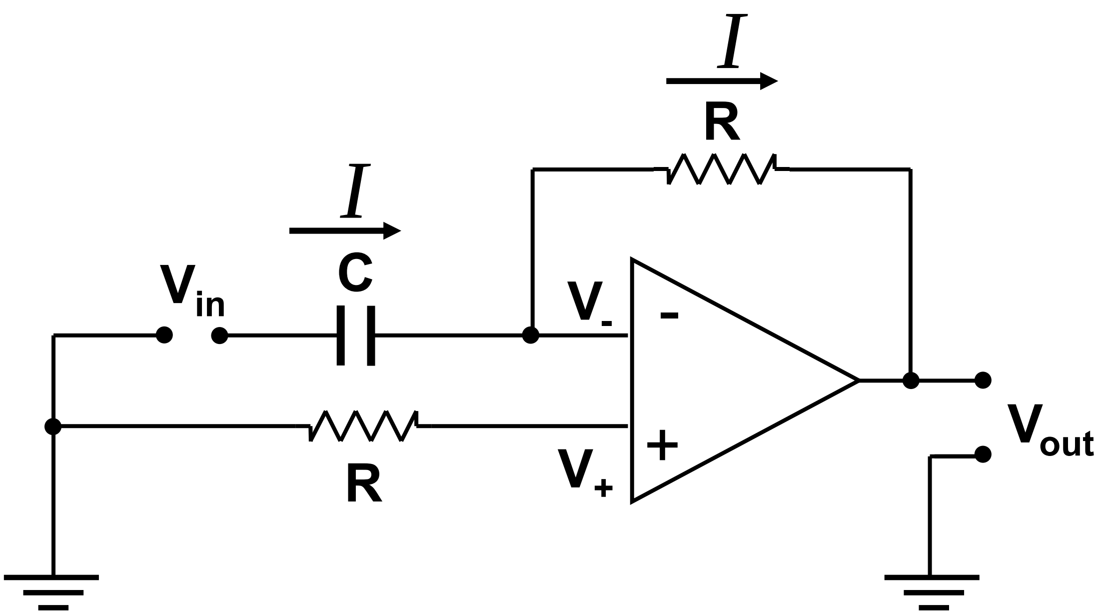
- Such a function can be implemented using 2 op amps:

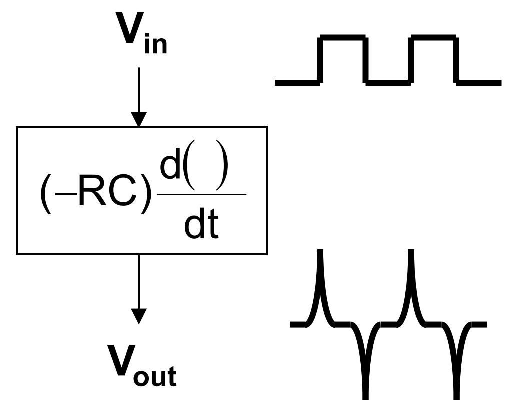


$$v_O = v_1\left(\frac{R_a}{R_1}\right)\left(\frac{R_c}{R_b}\right) + v_2\left(\frac{R_a}{R_2}\right)\left(\frac{R_c}{R_b}\right) - v_3\left(\frac{R_c}{R_3}\right) - v_4\left(\frac{R_c}{R_4}\right)$$

Derivative Op-Amp

- Circuit in closed-loop inverting configuration. It performs the mathematical function of differentiation of the input. The output is large when the rate of change of the input is large.





$$I_+ = 0 \;\; and \;\; I_- = 0$$

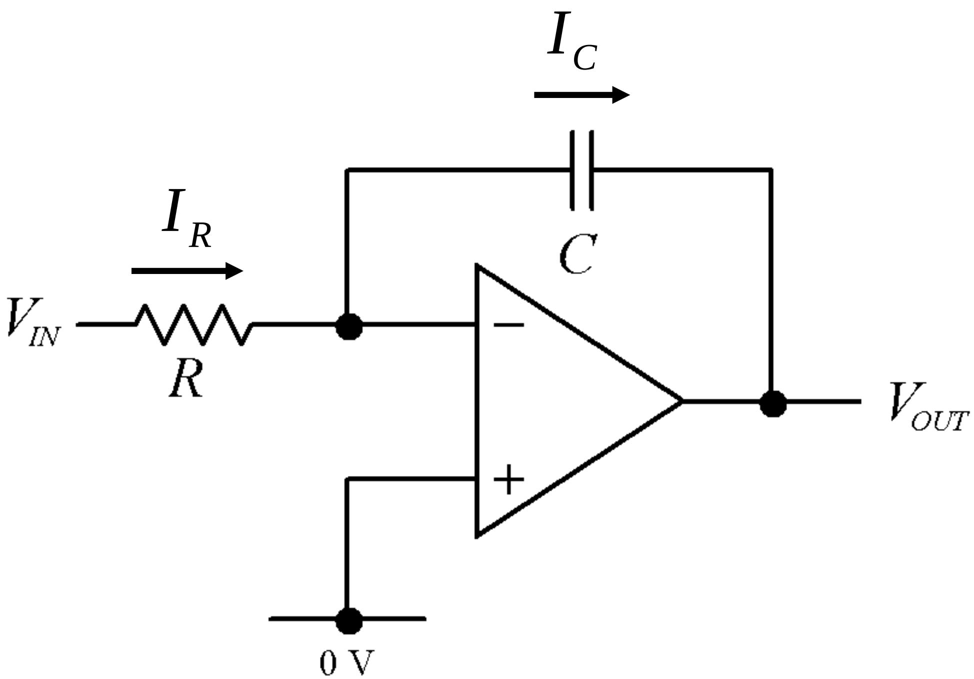
$$V_- = V_+ = 0$$

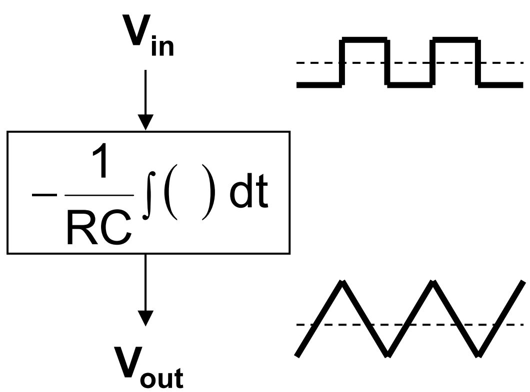
$$I(t) = C\frac{dV_{in}(t)}{dt}$$

$$V_{OUT} = -CR\frac{dV_{in}(t)}{dt}$$

Integrator Op-Amp

- This circuit is another application of the inverting configuration. It performs the mathematical function of integration of the input.





$$I_+ = 0 \ \ and \ \ I_- = 0$$

$$V_- = V_+ = 0$$

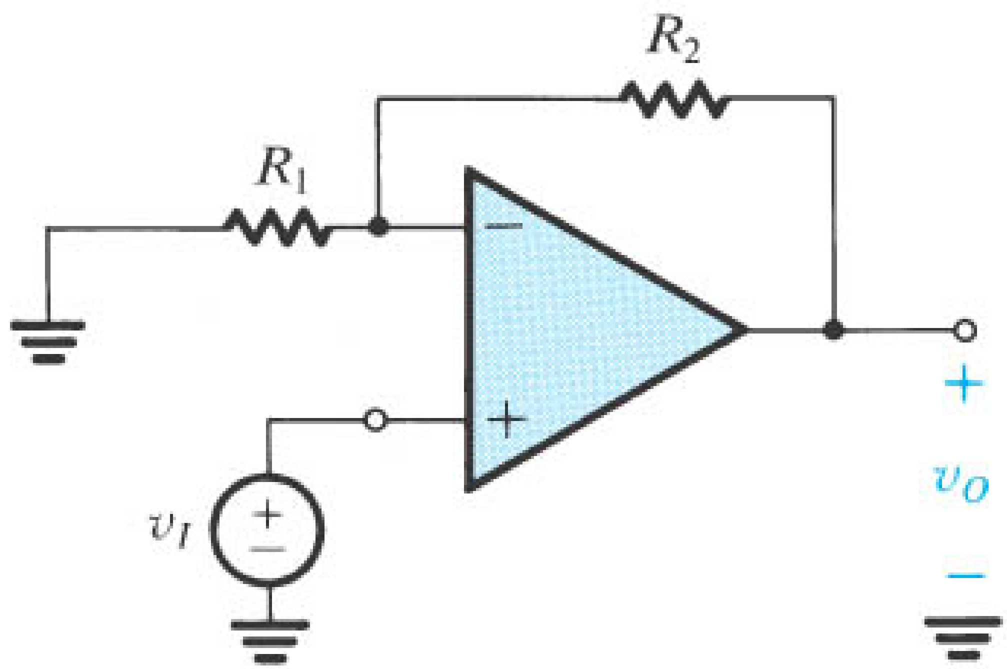
$$I_R = I_C$$

$$\therefore \frac{V_{IN} - 0}{R} = C\frac{dV_C}{dt} = C\frac{d}{dt}(0 - V_{OUT})$$

$$\Rightarrow V_{OUT} = -\frac{1}{RC}\int V_{IN}\,dt$$

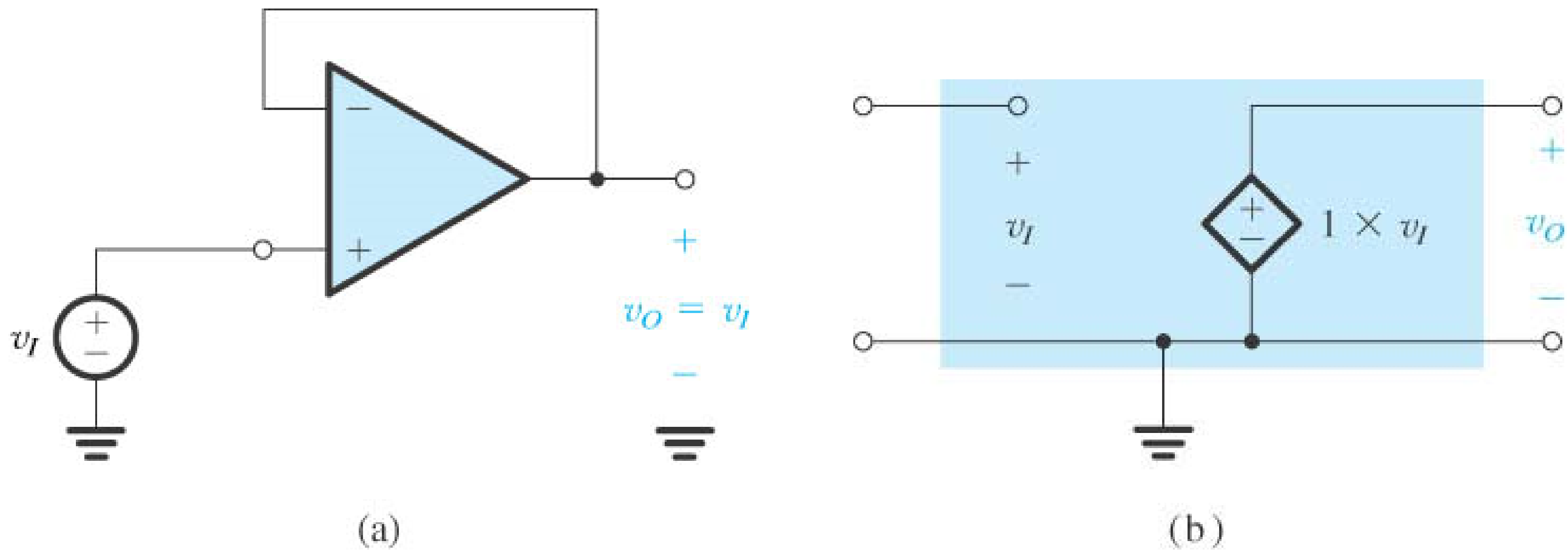
Op amps in the non-inverting configuration

- Another important characteristic of this circuit, which is not provided by the inverting configuration, is that the non-inverting op amp configuration draws no current from the source, meaning the closed-loop non-inverting configuration has an infinite input impedance.



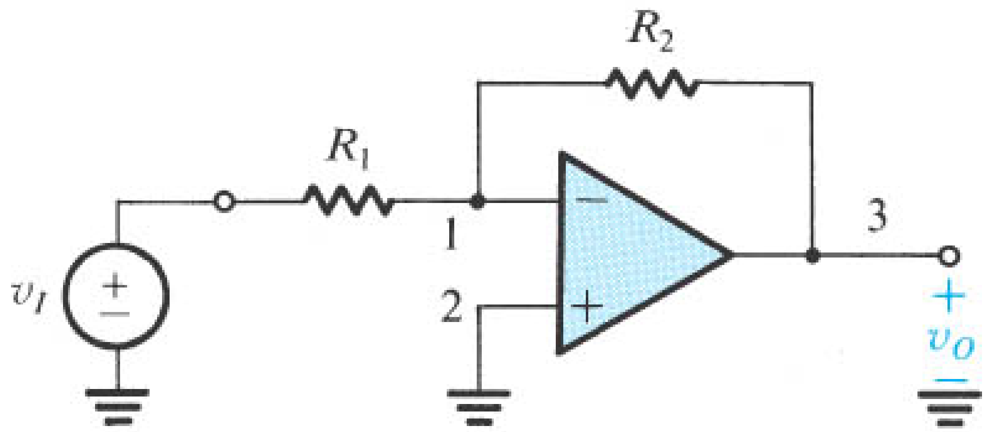
Follower – Special case of the Non-inverting

- In the non-inverting amplifier, we can let R_2=0 and R_1=∞. This circuit is referred to as voltage follower, since the output "follows" the input. In the ideal case, we have R_{in}=∞, R_{out}=0, and the follower has the equivalent circuit shown below.
- Used as a buffer to give effective isolation of the output from the signal source. Its draws very little power from the signal source, avoiding "loading" effects.



The unity-gain buffer or follower amplifier and its equivalent circuit model.

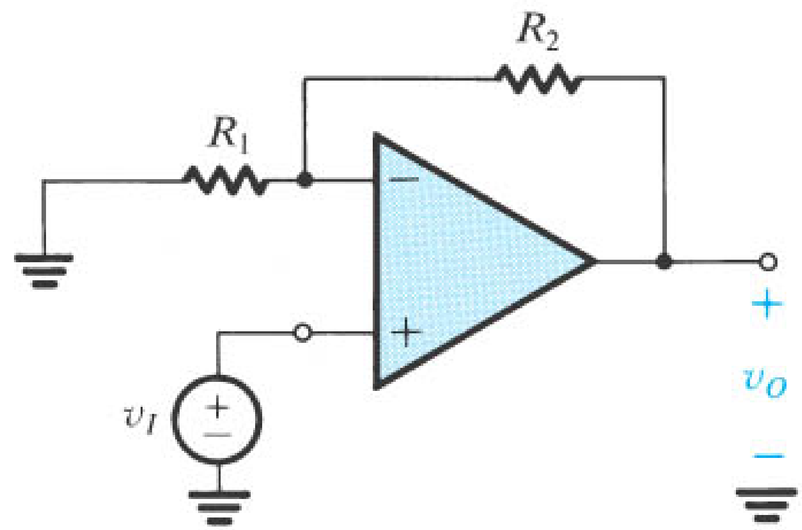
Inverting and Non-Inverting compared



Inverting configuration

$v_O / v_I = - R_2/R_1$

$R_{in} = R_1$



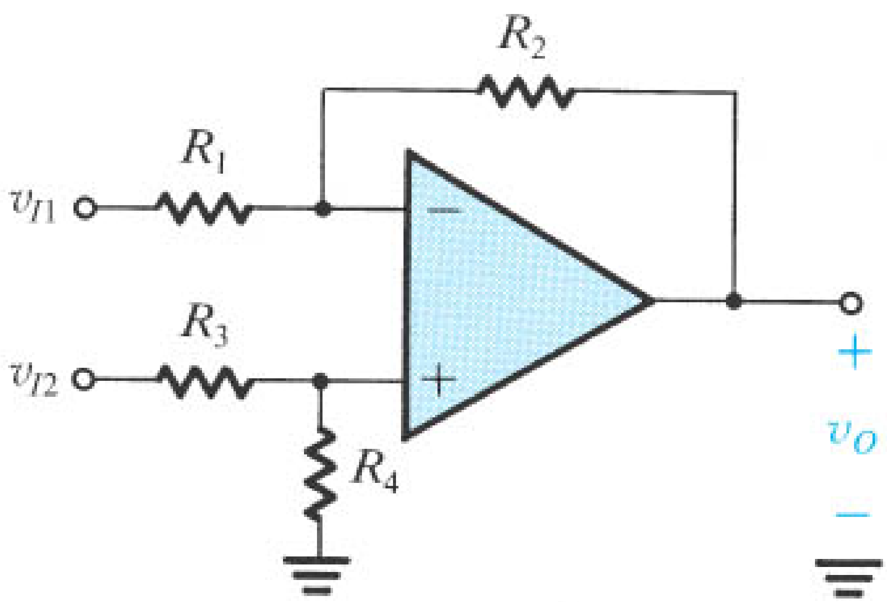
Non-inverting configuration

$v_O/v_I = 1+(R_2/R_1)$

R_{in} infinite

Difference amplifier

- Combining the non-inverting and inverting configurations allows us to take the difference between 2 input signals vI1 and vI2.

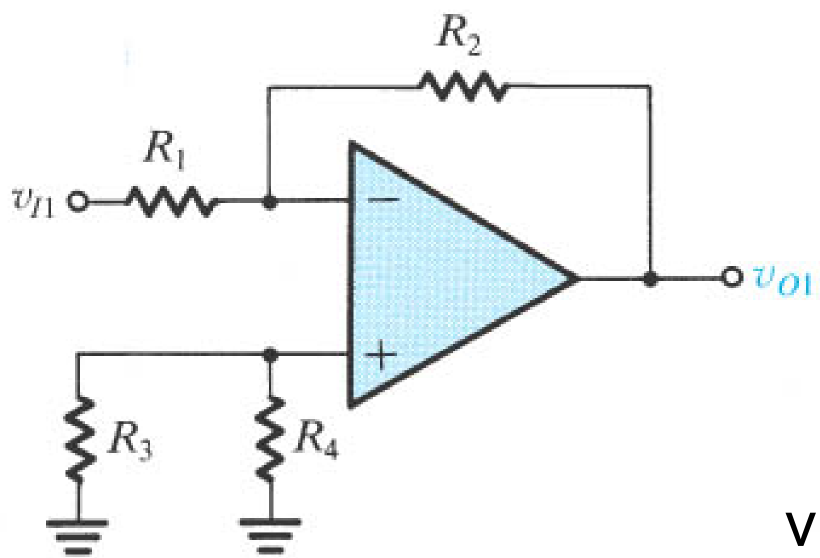


- Use superposition:

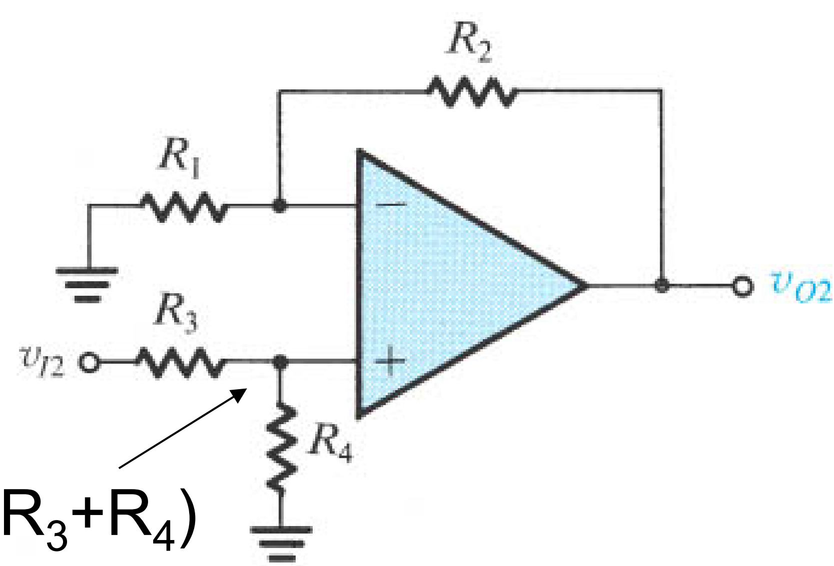
set v_{I1}= 0, solve for v_O (non-inverting config.)

set v_{I2}= 0, solve for v_O (inverting config.)

Difference amplifier



(a)



$v_{I2}\ R_4/(R_3+R_4)$

(b)

$v_{O1} = -(R_2/R_1)v_{I1}$

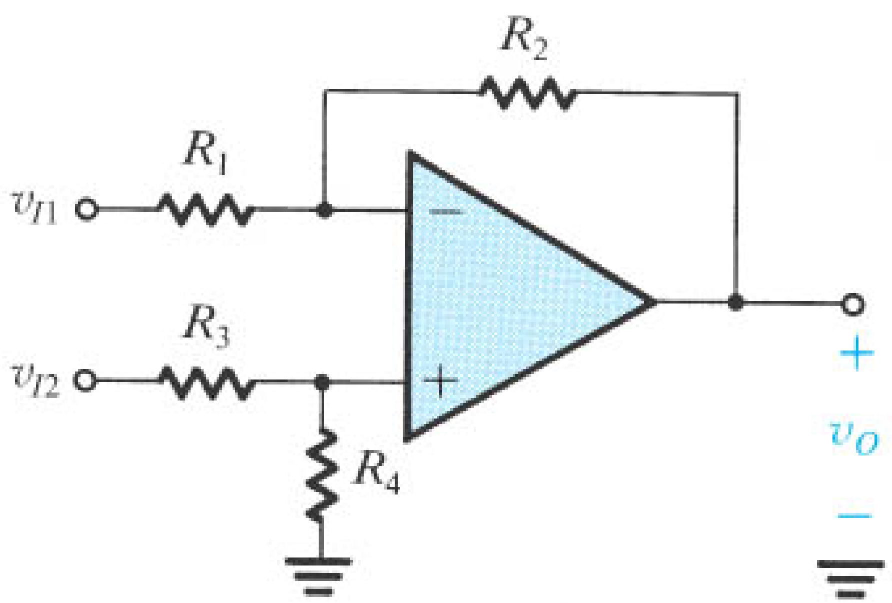
$v_{O2} = (1 + R_2/R_1)\ [R_4/(R_3+R_4)]v_{I2}$

Add the two results and rearrange:

$$\boxed{v_O = (1 + R_2/R_1) \,/\, (1+R_3/R_4)\ v_{I2} - (R_2/R_1)v_{I1}}$$

Design of difference amplifiers

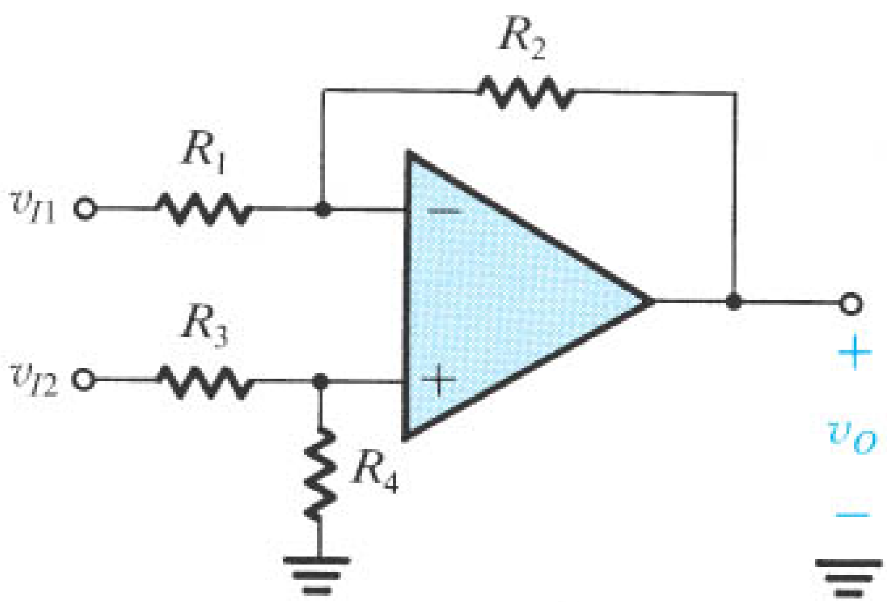
$$v_O = (1 + R_2/R_1) / (1+R_3/R_4)\, v_{I2} - (R_2/R_1)v_{I1}$$



- For $v_O = v_{I2} - v_{I1}$
  - set $R_2=R_1=R$ and $R_3=R_4=R$.
- For $v_O = 3v_{I2} - 2v_{I1}$
  - set $R_1=R$, $R_2=2R$ and $R_3=0$.

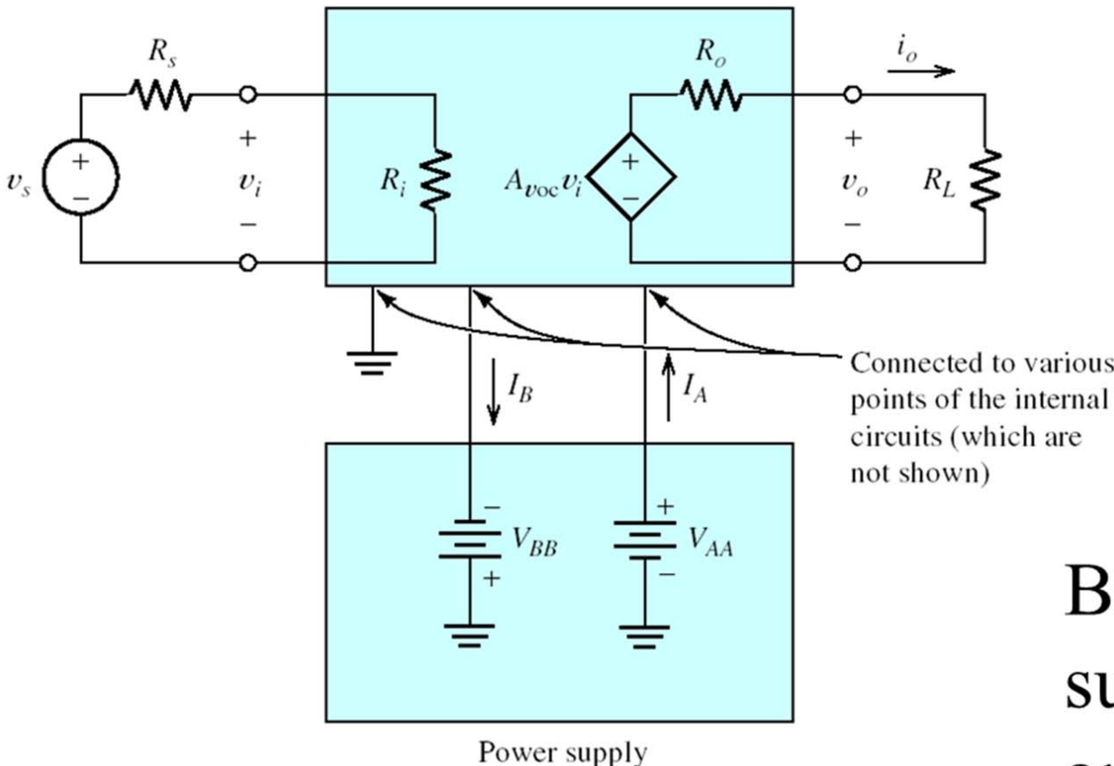
- Note that an op amp is a difference amplifier by itself. However, its very high (ideally infinite) gain makes it impossible to use it by itself. Instead feedback is used to create a circuit whose closed-loop gain is finite, predictable, and stable (more on this later).

Input resistance of difference amplifiers



- When measuring R_{in} at one input, ground all other inputs:
- $R_{in1}=R_1$ seen from v_{I1}, same as inverting config.
- $R_{in2}=R_3 + R_4$ seen from v_{I2}.

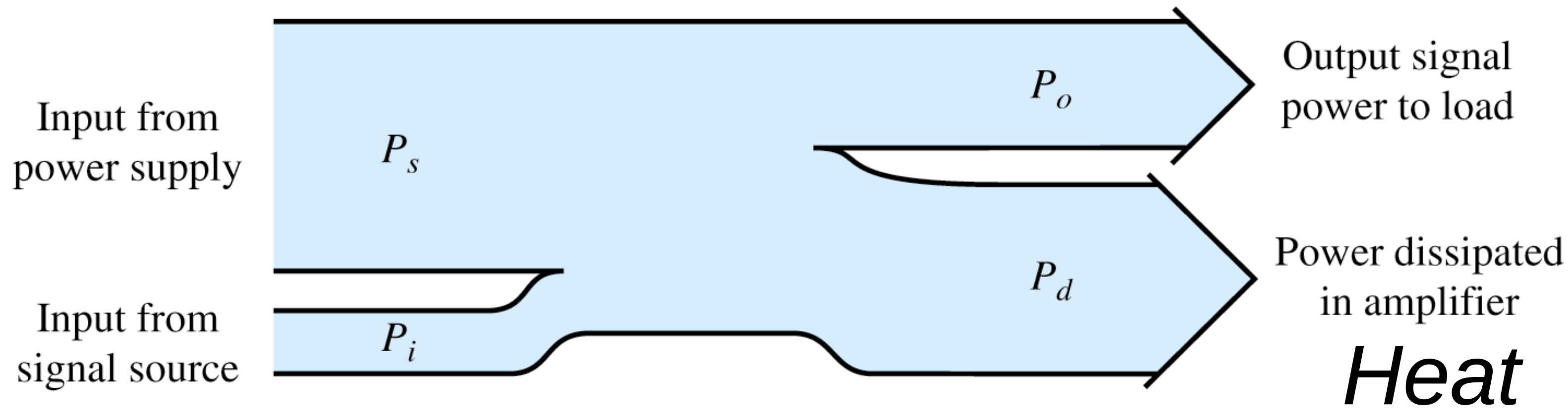
POWER SUPPLIES AND EFFICIENCY



$$P_S = V_{AA} I_A + V_{BB} I_B$$

Both power supplies are supplying power to the amplifier

POWER SUPPLIES AND EFFICIENCY



$$P_o > P_i$$

The additional power comes from the power supply. The efficiency is given by:

$$\eta = \frac{P_o}{P_s} x 100$$

Ideal Op-Amp Linear Circuit Analysis

$$1.\quad I_+ = 0 \text{ and } I_- = 0$$

$$2.\quad V_- = V_+$$

$$3.\quad V_{OUT} = \text{anything}!$$

- When negative feedback is applied, the use of these assumptions makes design/analysis much easier (although not necessarily simple!).

- Of course, they aren't really true…

Acknowledgments

- Lecture slides are mainly based on the lecture materials from F. Bousaid (UWA).

- Credit is acknowledged where credit is due. Please refer to the full list of references.