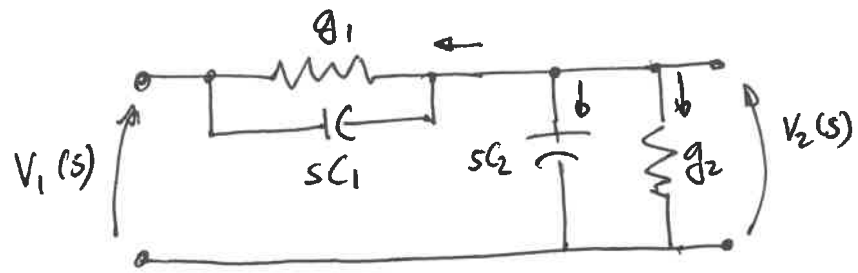
ENSC 3021 TUTORIAL 4A

Q1.



$I = GV$

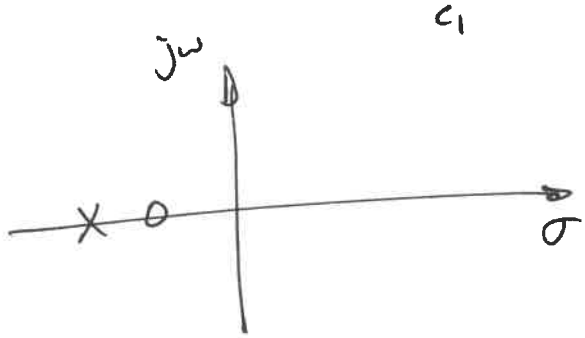
Require $\dfrac{V_2(s)}{V_1(s)}$

$$(sC_2+g_2)V_2(s) + (V_2(s) - V_1(s))(sC_1+g_1) = 0$$

$$(s(C_2+C_1) + (g_2+g_1))V_2(s) = (sC_1+g_1)V_1(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{sC_1+g_1}{s(C_2+C_1)+g_1+g_2} = \frac{C_1}{C_2+C_1}\,\frac{s+\frac{g_1}{C_1}}{s+\frac{g_1+g_2}{C_1+C_2}}$$

zero @ $s = -\dfrac{g_1}{C_1}$ pole @ $s = -\dfrac{g_1+g_2}{C_1+C_2}$



if $\dfrac{g_1}{C_1} = \dfrac{g_1+g_2}{C_1+C_2}$

then $\dfrac{V_2(s)}{V_1(s)} = k = \dfrac{C_1}{C_2+C_1}$

$\Rightarrow g_1C_1 + g_1C_2 = g_1C_1 + g_2C_1$

$\Rightarrow g_1C_2 = g_2C_1$

$\Rightarrow \dfrac{C_2}{R_1} = \dfrac{C_1}{R_2}$

$\Rightarrow R_2C_2 = C_1R_1$

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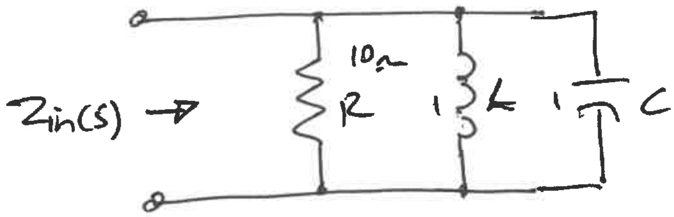
Q2.



$\omega_0 = 1 r/s$.

$$Z_{in}(s) \quad \frac{1}{\frac{1}{R} + \frac{1}{s} + s}$$

$$= \frac{Rs}{Rs^2 + s + R}$$

$$z_{in}(s=j\omega)\Big|_{\omega=1r/s} = \frac{jR}{-R + j + R} = \frac{R}{1} = 10\Omega.$$

if want impedance to be 1000 Ω then impedance scaling $\underline{b = 100}$

if want frequency to be scaled to 10^6 r/s then require $a = 10^6$.

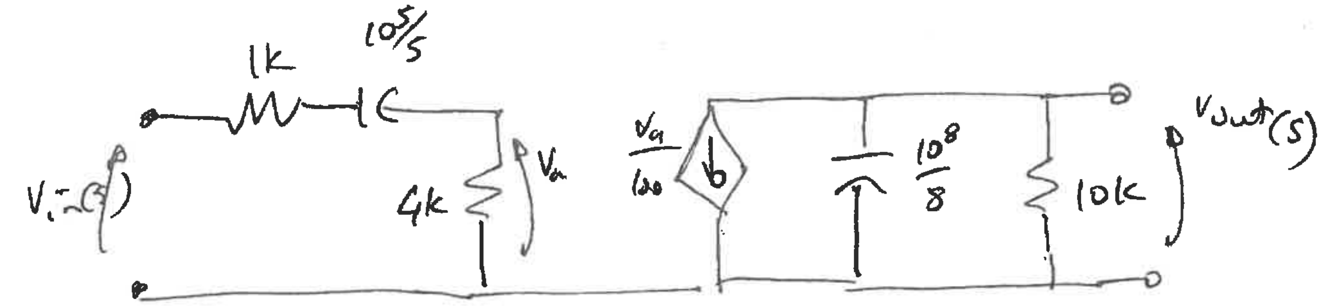
so $R^* = bR = 100 \times 10 = \underline{\underline{1000\Omega}}$

$L^* = \frac{b}{a} L = \frac{100}{10^6} \times 1 = \underline{\underline{1 \times 10^{-4} H}}$

$C^* = \frac{1}{ab} C = \frac{1}{100 \times 10^{6}} = \underline{\underline{1 \times 10^{-8} F}}$

$$\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10^{-12}}} = \underline{1 \times 10^6 r/s} \checkmark$$

Q3.



$$V_a(s) = \frac{4\times10^3}{5\times10^3 + \frac{10^5}{s}} V_{in}(s)$$

$$V_{out}(s) = -\frac{V_a(s)}{100} \times \frac{1}{\frac{1}{10^4} + s10^{-8}}$$

$$= -\frac{V_a(s)}{100} \frac{10^4}{1\times10^{-4}s + 1}$$

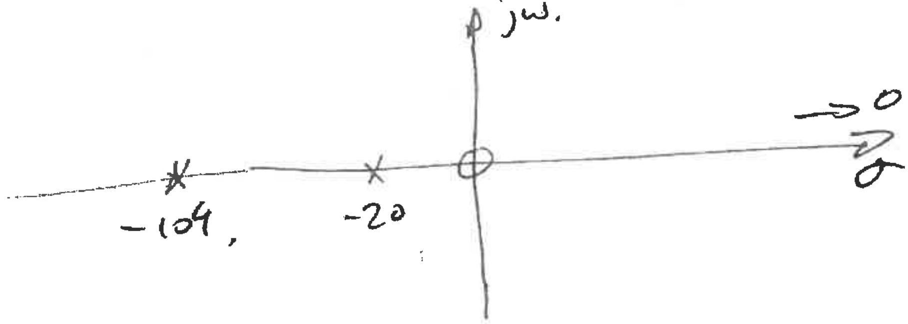
$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{4\times10^3 s}{10^5 + 5\times10^3 s} \times \frac{10^4}{1\times10^{-4}s + 1} \times \frac{1}{100}$$

a) $$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{4s}{(1+5\times10^{-2}s)(1+1\times10^{-4}s)}$$

b) Zero @ $s=0$, $s=\infty$

poles @ $s = -\frac{1}{5\times10^{-2}} = -20$

$s = -\frac{1}{1\times10^{-4}} = -10^4$



Q3.

c) $$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-4s}{(1+s/20)(1+s/10^4)}$$

$s = j\omega$ $\quad j\omega = j0.1$

$$\frac{-4j0.1}{\left(1+\frac{j0.1}{20}\right)\left(1+\frac{j0.1}{10^4}\right)} \leftarrow 1$$

$$\sim \underline{-j0.4}$$

$\omega = 200\,r/s$

$$\frac{-4j200}{\left(1+\frac{j200}{20}\right)\left(1+\frac{j200}{10^4}\right)}$$

$$\sim \frac{-4j200}{j10\,(1+\sim)}$$

$$\sim -4\frac{j200}{j10} = 4\times 20 = 80$$

$$\boxed{\frac{V_{out}}{V_{in}} \sim -\underline{80}}$$

$j\omega = 10^6\,r/s$

$$\frac{V_{out}}{V_{in}} = \frac{-4j10^6}{\left(1+j\frac{10^6}{20}\right)\left(1+j\frac{10^6}{10^4}\right)} \quad = j\frac{4}{5}$$

$$\sim \frac{-4j10^6}{(-1)\frac{10^6\times 10^2}{20}} = \frac{4j10^6\times 20}{10^8}$$

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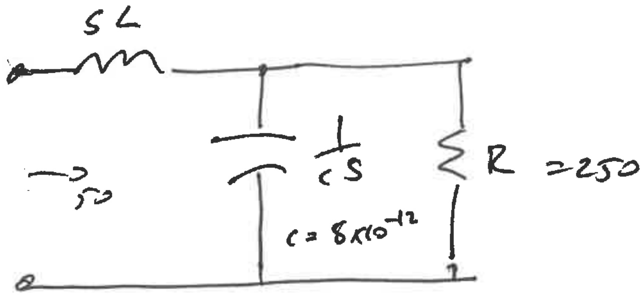
Q4

$Z_{in}(s)$ →



$$Z_{in}(s) = sL + \frac{1}{\frac{1}{R} + cs}$$

$$= sL + \frac{R}{1 + Rcs}$$

$$= \frac{sL + s^2 LRC + R}{1 + Rcs}$$

$$\boxed{Z_{in}(s) = sL + \frac{250}{1 + 2\times10^{-9} s}}$$

$s = j\omega \quad \omega = 10^9$

$$= j\omega L + \frac{250}{1 + j\omega\, 2\times10^{-9}} = j10^9 L + \frac{250}{1+2j}$$

$$= j10^9 L + \frac{250\,(1-2j)}{(1+2j)(1-2j)} = j10^9 L + \left(\frac{250 - 500j}{5}\right)$$

$$= j10^9 L + 50 - 100j$$

$$\Rightarrow 10^9 L = 100$$

$$L = \frac{10^2}{10^9} = 10^{-7}\,H$$

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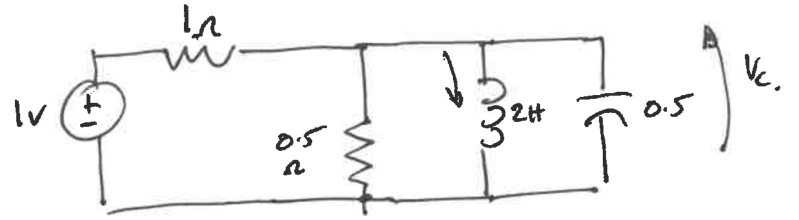
Q5

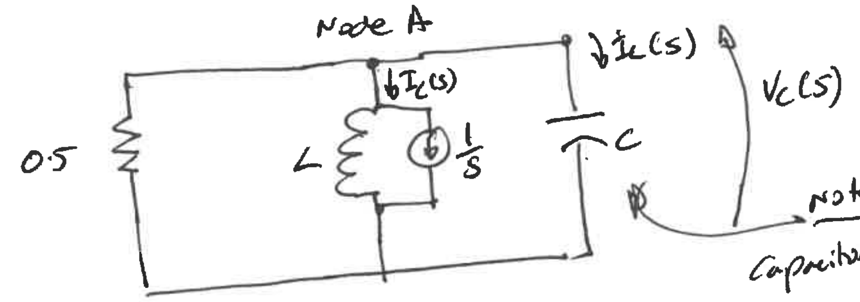
First initial conditions

a)



$i_L(0^+) = 1V / 1\Omega = \underline{1A}$ $V_C(0^+) = \underline{0}$.

Laplace transform circuit after switch opens.



Note for capacitor no initial condition voltage source as $V_C(0^+) = 0$.

KCL at the node A:

$$\frac{V_c(s)}{0.5} + I_L(s) + I_c(s) = 0$$

$$\frac{V_c(s)}{0.5} + \frac{1}{sL}V_c(s) + \frac{1}{s} + sCV_c(s) = 0$$

$$2sL\,V_c(s) + V_c(s) + L + s^2LC\,V_c(s) = 0$$

$$V_c(s)\left(s^2LC + 2sL + 1\right) = -L$$

$$V_c(s) = \frac{-L}{LCs^2 + 2Ls + 1}$$ $L=2$ $C=0.5$

$$V_c(s) = \frac{-2}{s^2 + 4s + 1}$$

$V_c(t=0)$: $\lim_{s\to\infty} sV_c(s) = \frac{-2\times\infty}{(\infty+3.73)(\infty+0.268)} = \underline{\underline{0}}$ (ie $\frac{1}{\infty}$)

$V_c(t=\infty)$: $\lim_{s\to 0} sV_c(s) = \frac{-2\times 0}{(0+3.73)(0+0.268)} = \underline{\underline{0}}$

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Q5 b) $V_c(s) = \frac{-2}{s^2+4s+1} = \frac{-2}{(s+3.73)(s+0.268)}$

So poles @ $s = -3.73$, $s = -0.268$

zeros @ $s = \pm\infty$

c) $\frac{C_1}{s+3.73} + \frac{C_2}{s+0.268}$

$C_i = (s-p_i)F(s)\big|_{s=p_i}$

$C_1 = \frac{-2}{-3.73+0.268} = \underline{\underline{0.578}}$

$C_2 = \frac{-2}{(s+3.73)}\bigg|_{s=-0.268} = \frac{-2}{-0.268+3.73} = \underline{\underline{-0.578}}$

$V_c(s) = \frac{0.578}{s+3.73} - \frac{0.578}{s+0.268}$

$V_c(t) = \underline{\left(0.578e^{-3.73t} - 0.578e^{-0.268t}\right)u(t)}$