Solution 7.1

(a) τ=RC = 1/200

For the resistor, V=iR= $56e^{-200t} = 8Re^{-200t}x10^{-3} \longrightarrow R = \frac{56}{8} = \underline{7\ k\Omega}$

$$C = \frac{1}{200R} = \frac{1}{200X7X10^3} = \underline{0.7143\mu F}$$

(b) τ =1/200= **5 ms**

(c) If value of the voltage at = 0 is 56 .

$$\frac{1}{2}x56 = 56e^{-200t} \longrightarrow e^{200t} = 2$$

$$200t_o = \ln 2 \longrightarrow t_o = \frac{1}{200}\ln 2 = \underline{3.466\ ms}$$

Solution 7.2

$\tau = R_{th}C$
where R_{th} is the Thevenin equivalent at the capacitor terminals.

$R_{th} = 120 \,||\, 80 + 12 = 60\ \Omega$
$\tau = 60x0.05 =$ **3 s**.

Solution 7.3

$R = 6k\ \ +40k\text{x}(25k+35k)/(40k+25k+35k) = 6k+2400k/100 = 30\ k\Omega$.

$\tau = RC = 30k\text{x}50\text{x}10^{-12} = \mathbf{1.5\ \mu s}$.

Solution 7.4

For t<0, $v(0^-)$=40 V.

For t >0. we have a source-free RC circuit.

$$\tau = RC = 2x10^3 x10x10^{-6} = 0.02$$

$$v(t) = v(0)e^{-t/\tau} = \underline{40e^{-50t} \text{ V}}$$

Solution 7.5

Using Fig. 7.85, design a problem to help other students to better understand source-free RC circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit shown in Fig. 7.85, find i(t), t>0.

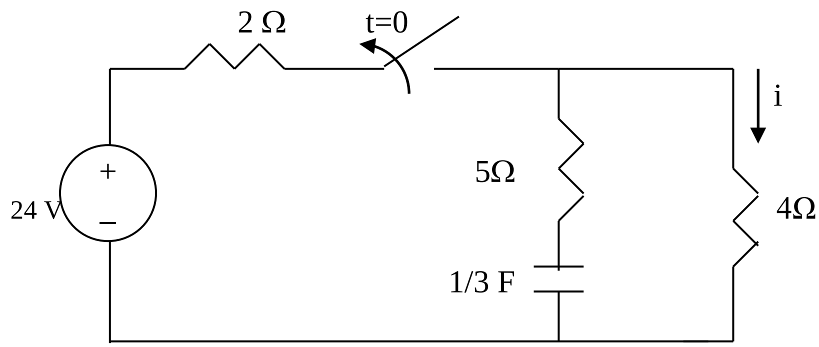


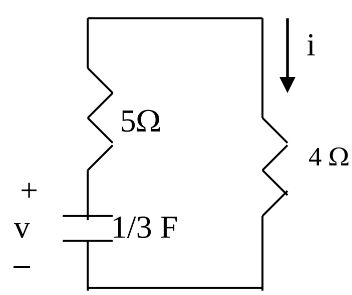
Figure 7.85 For Prob. 7.5.

Solution

Let v be the voltage across the capacitor. For t <0,

$$v(0^-) = \frac{4}{2+4}(24) = 16 \text{ V}$$

For t >0, we have a source-free RC circuit as shown below.



$$\tau = RC = (4+5)\frac{1}{3} = 3s$$

$$v(t) = v(0)e^{-t/\tau} = 16e^{-t/3}\ \text{V}$$

$$i(t) = -C\frac{dv}{dt} = -\frac{1}{3}\left(-\frac{1}{3}\right)16e^{-t/3} = \underline{1.778e^{-t/3}\ \text{A}}$$

Solution 7.6

The switch in Fig. 7.85 has been closed for a long time, and it opens at $t = 0$. Find $v(t)$ for $t \geq 0$.

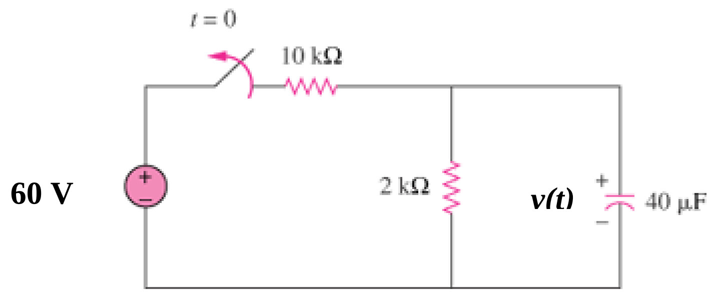


Figure 7.85
For Prob. 7.6.

Solution

$v(0) = [2x60/(10+2)] = 10$ V and $\tau = RC = 2{,}000x40x10^{-6} = 0.08$.

Note, $1/0.08 = 12.5/s$

This then leads to $v(t) = \mathbf{[10e^{-12.5t}]\ V}$ for all $0 \leq t$.

Solution 7.7

Assuming that the switch in Fig. 7.87 has been in position A for a long time and is moved to position B at t=0. Then at t = 1second, the switch moves from B to C. Find $v_C(t)$ for $t \geq 0$.

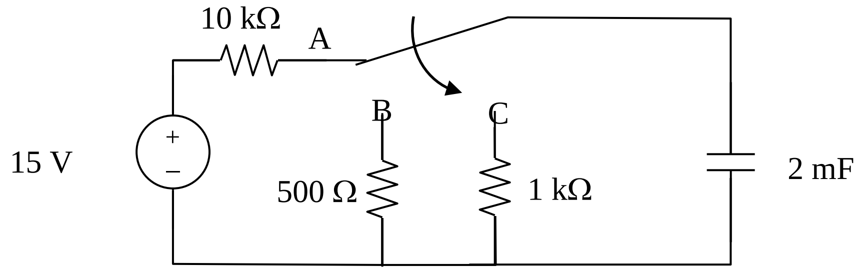


Figure 7.87
For Prob. 7.7

Solution

Step 1. Determine the initial voltage on the capacitor. Clearly it charges to 15 volts when the switch is at position A because the circuit has reached steady state.

This then leaves us with two simple circuits, the first a 500 **Ω** resistor in series with a 2 mF capacitor and an initial charge on the capacitor of 15 volts. The second circuit which exists from t = 1 sec to infinity. The initial condition for the second circuit will be $v_C(1)$ from the first circuit. The time constant for the first circuit is (500)(0.002) = 1 sec and the time constant for the second circuit is (1,000)(0.002) = 2 sec. $v_C(\infty) = 0$ for both circuits.

Step 2.

$v_C(t)$ = [**$15e^{-t}$]u(t) volts** for $0 < t < 1$ sec and = $15e^{-1}e^{-2(t-1)}$ at t = 1 sec, and

= **[$5.518e^{-2(t-1)}$]u(t–1) volts** for 1 sec $< t < \infty$.

[$15e^{-t}$] volts for $0 < t < 1$ sec, **[$5.518e^{-2(t-1)}$] volts** for 1 sec $< t < \infty$.

Solution 7.8

(a) $\tau = RC = \dfrac{1}{4}$

$$-i = C\frac{dv}{dt}$$

$$-0.2e^{-4t} = C(10)(-4)e^{-4t} \longrightarrow C = \mathbf{5\ mF}$$

$$R = \frac{1}{4C} = \mathbf{50\ \Omega}$$

(b) $\tau = RC = \dfrac{1}{4} = \mathbf{0.25\ s}$

(c) $w_C(0) = \dfrac{1}{2}CV_0^2 = \dfrac{1}{2}(5\times 10^{-3})(100) = \mathbf{250\ mJ}$

(d) $w_R = \dfrac{1}{2}\times\dfrac{1}{2}CV_0^2 = \dfrac{1}{2}CV_0^2\left(1-e^{-2t_0/\tau}\right)$

$$0.5 = 1 - e^{-8t_0} \longrightarrow e^{-8t_0} = \frac{1}{2}$$

or $e^{8t_0} = 2$

$$t_0 = \frac{1}{8}\ln(2) = \mathbf{86.6\ ms}$$

Solution 7.9

The switch in Fig. 7.89 opens at t=0. Find v_o for $t > 0$.

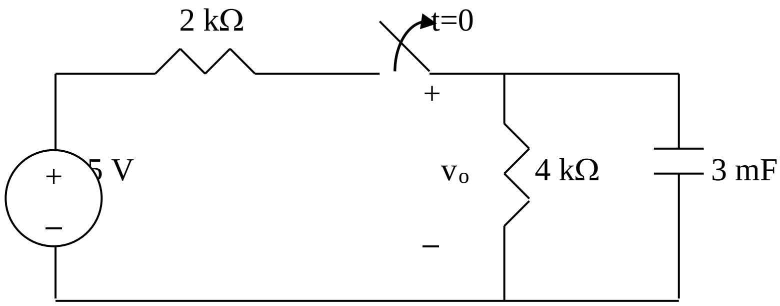


Figure 7.89
For Prob. 7.9.

Solution

For $t < 0$, the switch is closed so that

$v_o(0) = [4/(2+4)]15 = 10$ V.

For $t > 0$, we have a source-free RC circuit where $\tau = 4{,}000\text{x}0.003 = 12$ s.

Thus,
$v_o(t) =$ **[$10e^{-t/12}$] V** for all $t \geq 0$.

Solution 7.10

For t<0, $v(0^-) = \frac{3}{3+9}(36V) = \underline{9\ \text{V}}$

For t>0, we have a source-free RC circuit

$$\tau = RC = 3x10^3 x20x10^{-6} = 0.06s$$

$v_o(t)$ = $\mathbf{9e^{-16.667t}}$ **V**

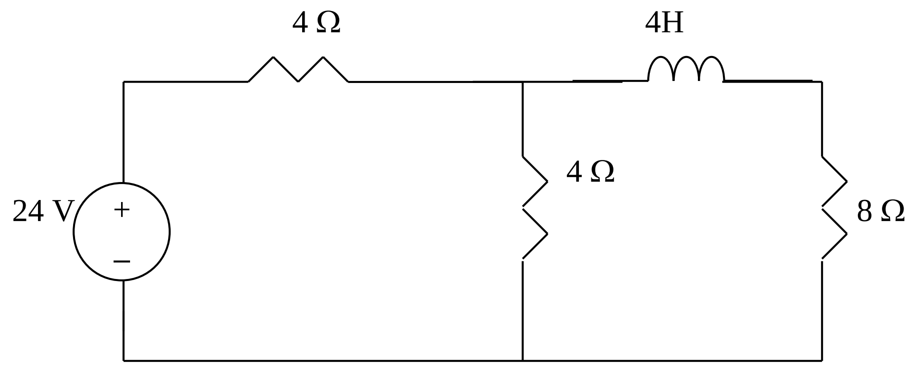
Let the time be t_o.

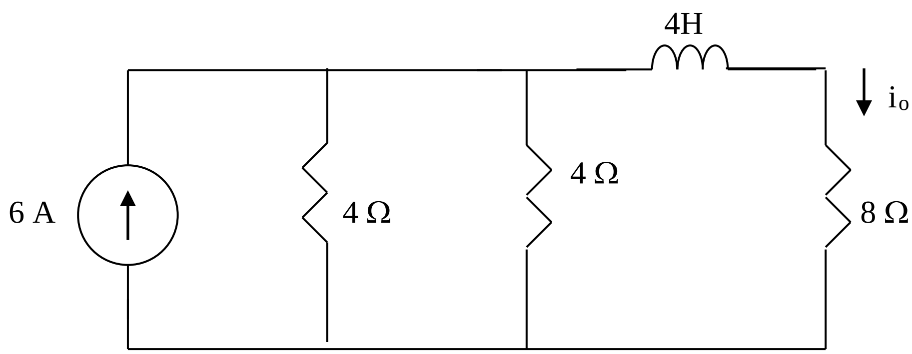
$3 = 9e^{-16.667t_o}$ or $e^{16.667t_o} = 9/3 = 3$

$t_o = \ln(3)/16.667 =$ **65.92 ms**.

Solution 7.11

For $t<0$, we have the circuit shown below.





$4||4 = 4x4/8=2$
$i_o(0^-) = [2/(2+8)]6 = 1.2$ A

For $t>0$, we have a source-free RL circuit.

$\tau = \dfrac{L}{R} = \dfrac{4}{4+8} = 1/3$ thus,

$i_o(t) = \mathbf{1.2e^{-3t}}$ **A**.

Solution 7.12

Using Fig. 7.92, design a problem to help other students better understand source-free RL circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The switch in the circuit in Fig. 7.90 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.

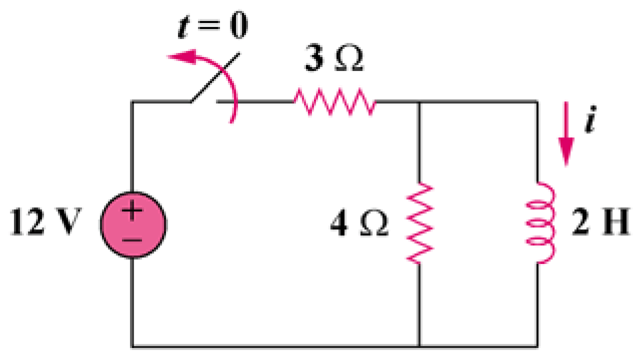
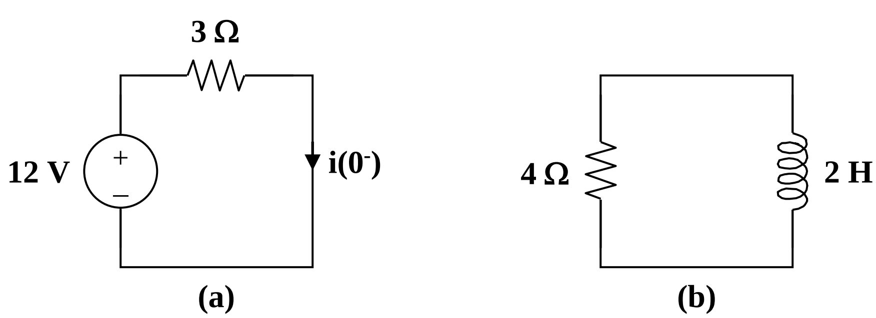


Figure 7.90

Solution

When t < 0, the switch is closed and the inductor acts like a short circuit to dc. The 4 Ω resistor is short-circuited so that the resulting circuit is as shown in Fig. (a).



$$i(0^-) = \frac{12}{3} = 4\text{ A}$$

Since the current through an inductor cannot change abruptly,

$$i(0) = i(0^-) = i(0^+) = 4\text{ A}$$

When t > 0, the voltage source is cut off and we have the RL circuit in Fig. (b).

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5$$

Hence,

$$i(t) = i(0)\,e^{-t/\tau} = \mathbf{4e^{-2t}\ A}$$

Solution 7.13

(a) $\tau = \dfrac{1}{10^3} = \underline{1ms}$ = **1 ms**.

v(t) = i(t)R = $80e^{-1000t}$ V = $R5e^{-1000t}x10^{-3}$ or R = 80,000/5 = **16 kΩ**.

But τ = L/R = $1/10^3$ or L = $16x10^3/10^3$ = **16 H**.

(b) The energy dissipated in the resistor is

$$w = \int_0^{0.0005} pdt = \int_0^{0.0005} 0.4e^{-2000t}dt = -\frac{0.4}{2000}e^{-2000t}\Big|_0^{0.0005}$$

= $200(1-e^{-1})x10^{-6}$ = **126.42 μJ**.

(a) **16 kΩ, 16 H, 1 ms** **(b) 126.42 μJ**

Solution 7.14

$$R_{Th} = (40+20)//(10+30) = \frac{60x40}{100} = 24k\Omega$$

$$\tau = L/R = \frac{5x10^{-3}}{24x10^{3}} = \underline{0.2083\mu s}$$

Solution 7.15

(a) $R_{Th} = 2 + 10 // 40 = 10\Omega, \qquad \tau = \dfrac{L}{R_{Th}} = 5/10 = \underline{0.5s}$

(b) $R_{Th} = 40 // 160 + 48 = 40\Omega, \qquad \tau = \dfrac{L}{R_{Th}} = (20x10^{-3})/80 = \underline{0.25\,ms}$

(a) **10 Ω, 500 ms** **(b) 40 Ω, 250 µs**

Solution 7.16

$$\tau = \frac{L_{eq}}{R_{eq}}$$

(a) $L_{eq} = L$ and $R_{eq} = R_2 + \frac{R_1 R_3}{R_1 + R_3} = \frac{R_2(R_1 + R_3) + R_1 R_3}{R_1 + R_3}$

$$\tau = \frac{\mathbf{L(R_1 + R_3)}}{\mathbf{R_2(R_1 + R_3) + R_1 R_3}}$$

(b) where $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$ and $R_{eq} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = \frac{R_3(R_1 + R_2) + R_1 R_2}{R_1 + R_2}$

$$\tau = \frac{\mathbf{L_1 L_2 (R_1 + R_2)}}{\mathbf{(L_1 + L_2)(R_3(R_1 + R_2) + R_1 R_2)}}$$

Solution 7.17

Consider the circuit of Fig. 7.97. Find $v_0(t)$ if $i(0) = 15$ A and $v(t) = 0$.

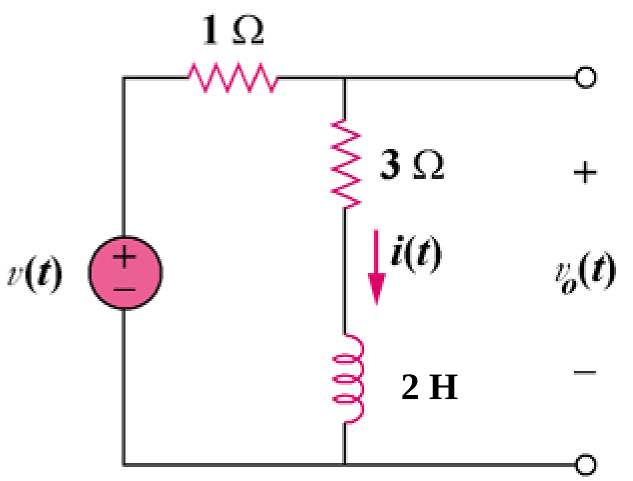


Figure 7.97
For Prob. 7.17.

Solution

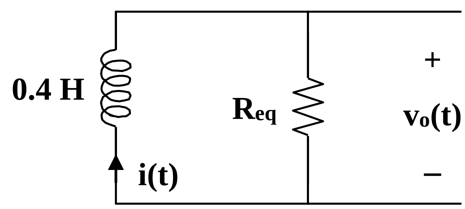
$i(t) = i(0)e^{-t/\tau}$ where $\tau = L/R_{eq} = 2/4 = (½)$ s. Additionally $v_o(t) = 3i(t) + 2di(t)/dt$.

Thus, $i(t) = [15e^{-2t}]u(t)$ A and $v_o(t) = [45e^{-2t}]u(t) - [(2)(2)15e^{-2t}]u(t)$

$=$ **$[-15e^{-2t}]$ V** for all $t \geq 0$.

Solution 7.18

If $v(t) = 0$, the circuit can be redrawn as shown below.



$$R_{eq} = 2 \,\|\, 3 = \frac{6}{5}, \qquad \tau = \frac{L}{R} = \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$$

$$i(t) = i(0)\,e^{-t/\tau} = 5e^{-3t}$$

$$v_o(t) = -L\frac{di}{dt} = \frac{-2}{5}(-3)5e^{-3t} = \mathbf{6\,e^{-3t}\ V}$$

Solution 7.19

In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 5$ A.

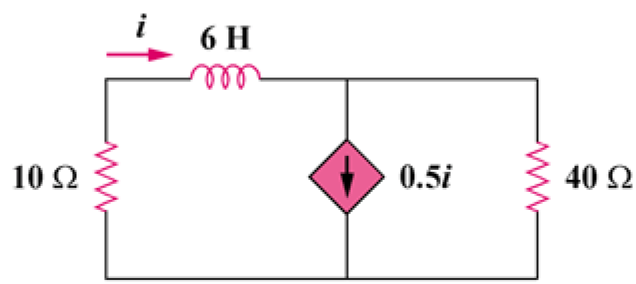
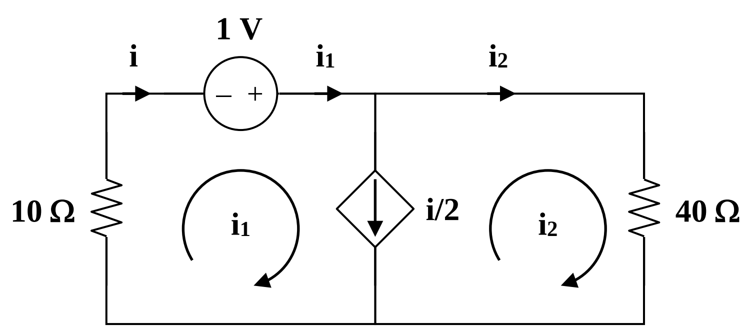


Figure 7.99
For Prob. 7.19.

Solution



To find R_{th} we replace the inductor by a 1-V voltage source as shown above.

$10i_1 - 1 + 40i_2 = 0$

But $\quad i = i_2 + i/2 \quad$ and $\quad i = i_1$

i.e. $\quad i_1 = 2i_2 = i$

$$10i - 1 + 20i = 0 \longrightarrow i = \frac{1}{30}$$

$$R_{th} = \frac{1}{i} = 30\,\Omega$$

$$\tau = \frac{L}{R_{th}} = \frac{6}{30} = 0.2\text{ s}$$

$i(t) = \mathbf{5e^{-5t}u(t)\ A}$.

Solution 7.20

(a) $\tau = \dfrac{L}{R} = \dfrac{1}{50} \longrightarrow R = 50L$

$v = -L\dfrac{di}{dt}$

$90e^{-50t} = -L(30)(-50)e^{-50t} \longrightarrow L = \mathbf{60\ mH}$

$R = 50L = \mathbf{3\ \Omega}$

(b) $\tau = \dfrac{L}{R} = \dfrac{1}{50} = \mathbf{20\ ms}$

(c) $w = \dfrac{1}{2}Li^2(0) = \dfrac{1}{2}(0.06)(30)^2 = \mathbf{27\ J}$

The value of the energy remaining at 10 ms is given by:

$w_{10} = 0.03(30e^{-0.5})^2 = 0.03(18.196)^2 = 9.933$ J.

So, the fraction of the energy dissipated in the first 10 ms is given by:

(27–9.933)/27 = 0.6321 or **63.21%**.

Solution 7.21

In the circuit in Fig. 7.101, find the value of *R* for which the steady-state energy stored in the inductor will be 2 J.

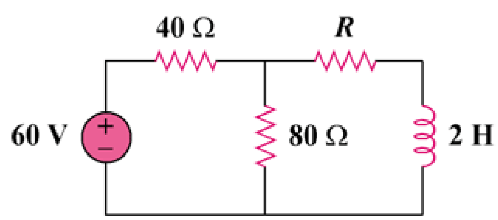
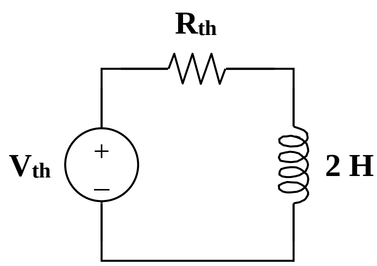


Figure 7.101
For Prob. 7.21.

Solution

The circuit can be replaced by its Thevenin equivalent shown below.



$$V_{th} = \frac{80}{80+40}(60) = 40 \text{ V}$$

$$R_{th} = 40 \,||\, 80 + R = \frac{80}{3} + R$$

$$I = i(0) = i(\infty) = \frac{V_{th}}{R_{th}} = \frac{40}{80/3 + R}$$

w = (1/2)LI^2 = 0.5(2)[40/(R+80/3)]^2 = 2 or 40/(R+80/3) = 1.4142, thus,

R+80/3 = 40/1.4142 or R = 28.285–26.667 = **1.618 Ω**.

Solution 7.23

Since the 2 Ω resistor, 1/3 H inductor, and the (3+1) Ω resistor are in parallel, they always have the same voltage.

$$-\mathrm{i} = \frac{10}{2} + \frac{10}{3+1} = 7.5 \quad \longrightarrow \quad i(0) = -7.5$$

The Thevenin resistance $\mathrm{R_{th}}$ at the inductor's terminals is

$$\mathrm{R_{th}} = 2 \,\|\, (3+1) = \frac{4}{3}, \qquad \tau = \frac{\mathrm{L}}{\mathrm{R_{th}}} = \frac{1/3}{4/3} = \frac{1}{4}$$

$$i(t) = i(0)e^{-t/\tau} = -7.5\mathrm{e}^{-4t}, \quad t > 0$$

$$v_L = v_o = L\frac{di}{dt} = -7.5(-4)(1/3)\,\mathrm{e}^{-4t}$$

$$\mathrm{v_o} = \mathbf{10e^{-4t}}\ \boldsymbol{V}, \quad \boldsymbol{t > 0}$$

$$\mathrm{v_x} = \frac{1}{3+1}\mathrm{v_L} = \mathbf{2.5\,e^{-4t}\ V}, \quad \boldsymbol{t > 0}$$

Solution 7.24

(a) $v(t) = \mathbf{-5u(t)}$

(b) $i(t) = -10\left[u(t) - u(t-3)\right] + 10\left[u(t-3) - u(t-5)\right]$

$= \mathbf{-10u(t) + 20u(t-3) - 10u(t-5)}$

(c) $x(t) = (t-1)\left[u(t-1) - u(t-2)\right] + \left[u(t-2) - u(t-3)\right]$

$+ (4-t)\left[u(t-3) - u(t-4)\right]$

$= (t-1)u(t-1) - (t-2)u(t-2) - (t-3)u(t-3) + (t-4)u(t-4)$

$= \mathbf{r(t-1) - r(t-2) - r(t-3) + r(t-4)}$

(d) $y(t) = 2u(-t) - 5\left[u(t) - u(t-1)\right]$

$= \mathbf{2u(-t) - 5u(t) + 5u(t-1)}$

Solution 7.25

Design a problem to help other students to better understand singularity functions.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

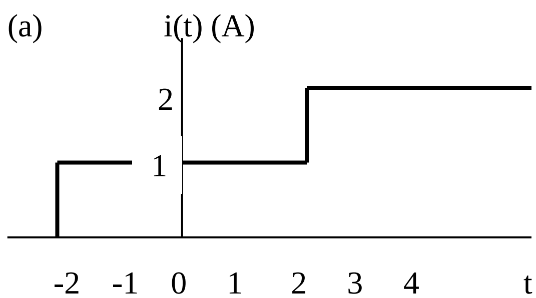
Problem

Sketch each of the following waveforms.
(a) i(t) = [u(t–2)+u(t+2)] A
(b) v(t) = [r(t) – r(t–3) + 4u(t–5) – 8u(t–8)] V

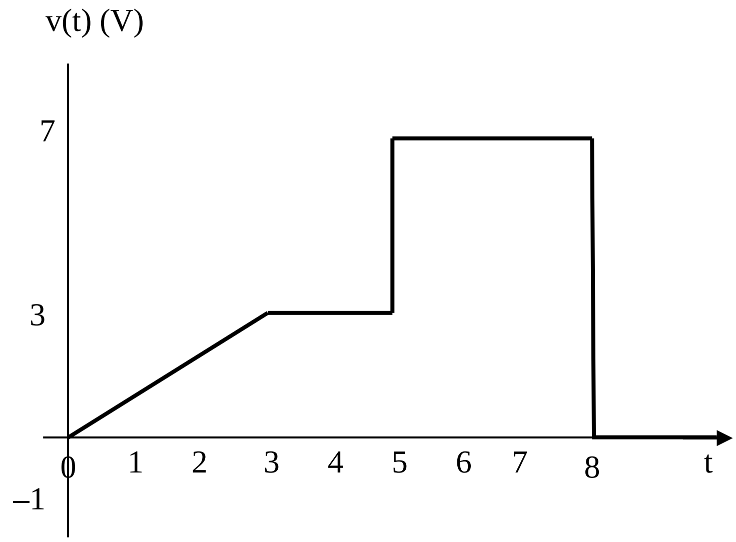
Solution

The waveforms are sketched below.

(a)



(b)



Solution 7.26

(a) $\mathrm{v_1(t) = u(t+1) - u(t) + \left[u(t-1) - u(t) \right]}$

$\mathrm{v_1(t) =}$ $\mathbf{u(t+1) - 2u(t) + u(t-1)}$

(b) $\mathrm{v_2(t) = (4-t)\left[u(t-2) - u(t-4) \right]}$

$\mathrm{v_2(t) = \text{-}(t-4)u(t-2) + (t-4)u(t-4)}$

$\mathrm{v_2(t) =}$ $\mathbf{2u(t-2) - r(t-2) + r(t-4)}$

(c) $\mathrm{v_3(t) = 2\left[u(t-2) - u(t-4) \right] + 4\left[u(t-4) - u(t-6) \right]}$

$\mathrm{v_3(t) =}$ $\mathbf{2u(t-2) + 2u(t-4) - 4u(t-6)}$

(d) $\mathrm{v_4(t) = \text{-}t\left[u(t-1) - u(t-2) \right] = \text{-}t\,u(t-1) + t\,u(t-2)}$

$\mathrm{v_4(t) = (\text{-}t+1-1)u(t-1) + (t-2+2)u(t-2)}$

$\mathrm{v_4(t) =}$ $\mathbf{\text{-}\,r(t-1) - u(t-1) + r(t-2) + 2u(t-2)}$

Solution 7.27

v(t)= **[5u(t+1)+10u(t)–25u(t–1)+15u(t-2)] V**

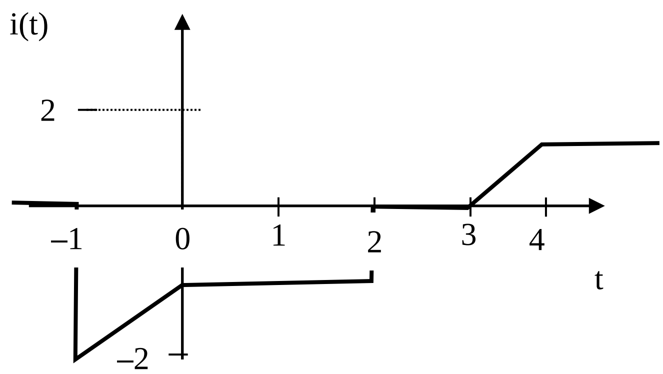
Solution 7.28

Sketch the waveform represented by

$i(t)$ = [r(t+1) – r(t) – 2u(t+1) +u(t–2) + r(t–3) – r(t–4)] A.

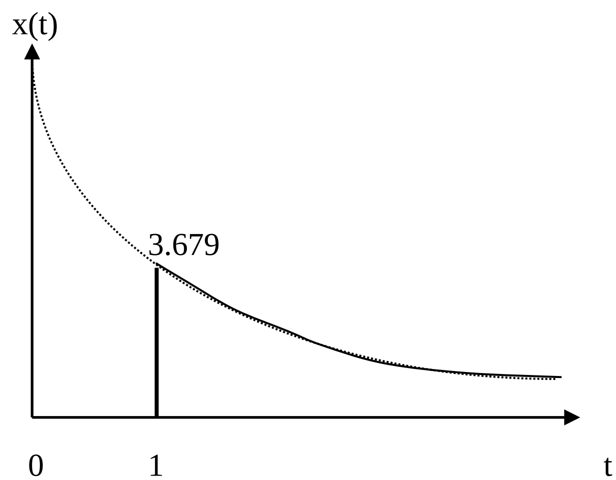
Solution

$i(t)$ is sketched below.

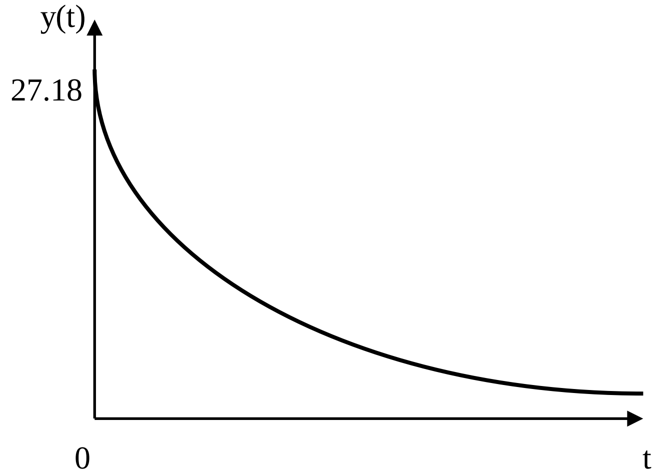


Solution 7.29

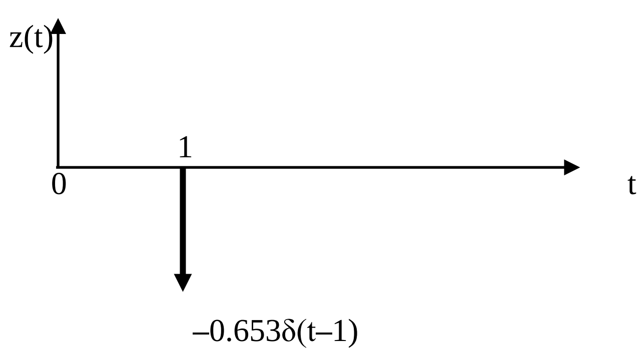
(a)



(b)



(c) $z(t) = \cos 4t\delta(t-1) = \cos 4\delta(t-1) = -0.6536\delta(t-1)$, which is sketched below.



Solution 7.30

(a) $\int_{-\infty}^{\infty} 4t^2\,\delta(t-1)\,dt = 4t^2\big|_{t=1} = \mathbf{4}$

(b) $\int_{-\infty}^{\infty} 4t^2\cos(2\pi t)\,\delta(t-0.5)\,dt = 4t^2\cos(2\pi t)\big|_{t=0.5} = \cos\pi = \mathbf{-1}$

Solution 7.31

(a) $\int_{-\infty}^{\infty}\left[e^{-4t^2}\,\delta(t-2)\right]dt = e^{-4t^2}\Big|_{t=2} = e^{-16} = \mathbf{112\times 10^{-9}}$

(b) $\int_{-\infty}^{\infty}\left[5\delta(t) + e^{-t}\,\delta(t) + \cos 2\pi t\,\delta(t)\right]dt = \left(5 + e^{-t} + \cos(2\pi t)\right)\Big|_{t=0} = 5+1+1 = \mathbf{7}$

Solution 7.32

(a) $\int_1^t u(\lambda)d\lambda = \int_1^t 1 d\lambda = \lambda \Big|_1^t = \underline{t-1}$

(b) $\int_0^4 r(t-1)dt = \int_0^1 0dt + \int_1^4 (t-1)dt = \frac{t^2}{2} - t\Big|_1^4 = \underline{4.5}$

(c) $\int_1^5 (t-6)^2 \delta(t-2)dt = (t-6)^2\Big|_{t=2} = \underline{16}$

Solution 7.33

The voltage across a 10-mH inductor is 45δ (*t* – 2) mV. Find the inductor current, assuming that the inductor is initially uncharged.

Solution

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\, d\tau + i(0)$$

$$i(t) = \frac{10^{-3}}{10\times 10^{-3}}\int_0^t 45\delta(\tau - 2)\, d\tau + 0 = 4.5\text{u(t–2) A.}$$

It should be noted that the integration of the impulse function, $\delta(t-t_0)$, produces the unit step, $u(t-t_0)$. Whatever the multiplier (f(t)) of the impulse function at $t = t_0$ ends up multiplying the unit step by the same amount ($f(t_0)$) in this case f(2) = 4.5.

i(t) = **4.5u(t–2) A**.

Solution 7.34

(a)
$$\frac{d}{dt}\left[u(t-1)\,u(t+1)\right] = \delta(t-1)u(t+1) +$$
$$u(t-1)\delta(t+1) = \delta(t-1)1 + 0\delta(t+1) = \underline{\delta(t-1)}$$

(b)
$$\frac{d}{dt}\left[r(t-6)\,u(t-2)\right] = u(t-6)u(t-2) +$$
$$r(t-6)\delta(t-2) = u(t-6)1 + 0\delta(t-2) = \underline{u(t-6)}$$

(c)
$$\frac{d}{dt}\left[\sin 4t\; u(t-3)\right] = 4\cos 4t\; u(t-3) + \sin 4t\delta(t-3)$$
$$= 4\cos 4t\; u(t-3) + \sin 4x3\delta(t-3)$$
$$= \underline{4\cos 4t\; u(t-3) - 0.5366\delta(t-3)}$$

Solution 7.35

(a)

$$v = Ae^{-2t}, \qquad v(0) = A = -1$$

v(t) = $\mathbf{-e^{-2t}u(t)}$ **V**

(b)

$$i = Ae^{3t/2}, \qquad i(0) = A = 2$$

i(t) = $\mathbf{2e^{-1.5t}u(t)}$ **A**

Solution 7.36

(a) $v(t) = A + Be^{-t}, \quad t > 0$

$A = 1, \; v(0) = 0 = 1 + B \quad \text{or} \qquad B = -1$

$v(t) = \mathbf{1 - e^{-t}\ V, \quad t > 0}$

(b) $v(t) = A + Be^{t/2}, \quad t > 0$

$A = -3, \quad v(0) = -6 = -3 + B \qquad \text{or} \qquad B = -3$

$v(t) = \mathbf{-3\left(1 + e^{t/2}\right) V, \quad t > 0}$

Solution 7.37

Let $v = v_h + v_p$, $v_p = 10$.

$$\dot{v}_h + \frac{1}{4} v_h = 0 \longrightarrow v_h = Ae^{-t/4}$$

$$v = 10 + Ae^{-0.25t}$$

$$v(0) = 2 = 10 + A \longrightarrow A = -8$$
$$v = 10 - 8e^{-0.25t}$$

(a) $\boldsymbol{\tau = \underline{4s}}$

(b) $\boldsymbol{v(\infty) = 10\ \mathrm{V}}$

(c) $\boldsymbol{\underline{v = \left(10 - 8e^{-0.25t}\right)u(t)\ V}}$

Solution 7.38

Let $i = i_p + i_h$

$$\dot{i}_h + 3i_h = 0 \longrightarrow i_h = Ae^{-3t}u(t)$$

Let $i_p = ku(t)$, $\dot{i}_p = 0$, $3ku(t) = 2u(t) \longrightarrow k = \frac{2}{3}$

$$i_p = \frac{2}{3}u(t)$$

$$i = (Ae^{-3t} + \frac{2}{3})u(t)$$

If i(0) =0, then A + 2/3 = 0, i.e. A=-2/3. Thus,

$$\underline{\boldsymbol{i = \frac{2}{3}(1 - e^{-3t})u(t)}}$$

Solution 7.39

(a) Before t = 0,

$$v(t) = \frac{1}{4+1}(20) = \mathbf{4\ V}$$

After t = 0,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$\tau = RC = (4)(2) = 8, \quad v(0) = 4, \qquad v(\infty) = 20$$

$$v(t) = 20 + (4 - 20)\, e^{-t/8}$$

$$v(t) = 20 - 16\, e^{-t/8} \text{ V}$$

(b) Before t = 0, $v = v_1 + v_2$, where v_1 is due to the 12-V source and v_2 is due to the 2-A source.

$v_1 = 12$ V

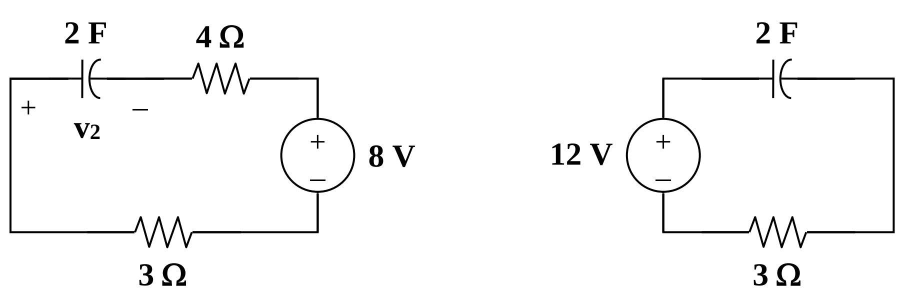
To get v_2, transform the current source as shown in Fig. (a).

$v_2 =$ -8 V

Thus,

$$v = 12 - 8 = \mathbf{4\ V}$$

After t = 0, the circuit becomes that shown in Fig. (b).



(a) (b)

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 4, \qquad \tau = RC = (2)(3) = 6$$

$$v(t) = 12 + (4 - 12)\, e^{-t/6}$$

$$v(t) = \mathbf{12 - 8\, e^{-t/6}\ \ V}$$

Solution 7.40

(a) Before t = 0, v = **12 V**.

After t = 0, $v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$

$v(\infty) = 4, \qquad v(0) = 12, \qquad \tau = RC = (2)(3) = 6$

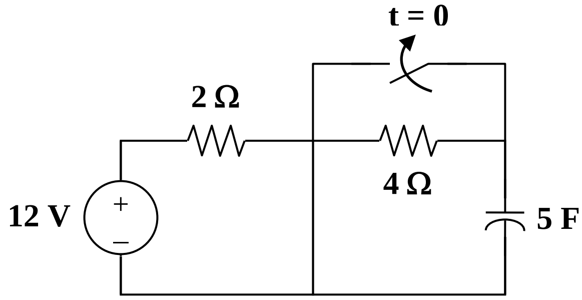
$v(t) = 4 + (12 - 4)\, e^{-t/6}$

$v(t) = \qquad \mathbf{4 + 8e^{-t/6}\ V}$

(b) Before t = 0, v = **12 V**.

After t = 0, $v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$

After transforming the current source, the circuit is shown below.



$v(0) = 12, \qquad v(\infty) = 12, \qquad \tau = RC = (2)(5) = 10$

v = **12 V**

Solution 7.41

Using Fig. 7.108, design a problem to help other students to better understand the step response of an RC circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 7.108, find $v(t)$ for $t > 0$.

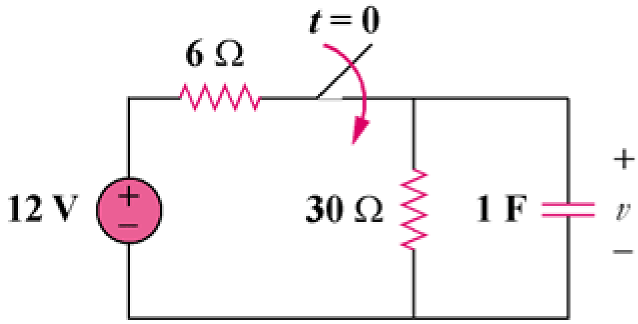


Figure 7.108

Solution

$$v(0) = 0, \qquad v(\infty) = \frac{30}{36}(12) = 10$$

$$R_{eq}C = (6 \parallel 30)(1) = \frac{(6)(30)}{36} = 5$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

$$v(t) = 10 + (0 - 10)\, e^{-t/5}$$

$$v(t) = \underline{10\,(1 - e^{-0.2t})\; u(t)\text{V}}$$

Solution 7.42

(a) If the switch in Fig. 7.109 has been open for a long time and is closed at $t = 0$, find $v_o(t)$.

(b) Suppose that the switch has been closed for a long time and is opened at $t = 0$. Find $v_o(t)$.

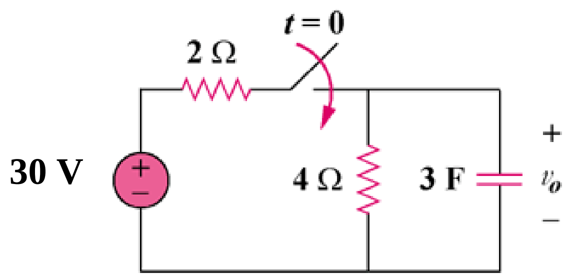


Figure 7.109
For Prob. 7.42.

Solution

(a) $v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right] e^{-t/\tau}$

$v_o(0) = 0, \qquad v_o(\infty) = \dfrac{4}{4+2}(30) = 20V$

$\tau = R_{eq}C_{eq}, \qquad R_{eq} = 2 \,||\, 4 = \dfrac{4}{3}$

$\tau = \dfrac{4}{3}(3) = 4$

$v_o(t) = 20 - 20e^{-t/4}$

$v_o(t)$ = $\mathbf{20(1-e^{-0.25t})u(t)}$ **V**.

(b) For this case, $v_o(\infty) = 0$ so that

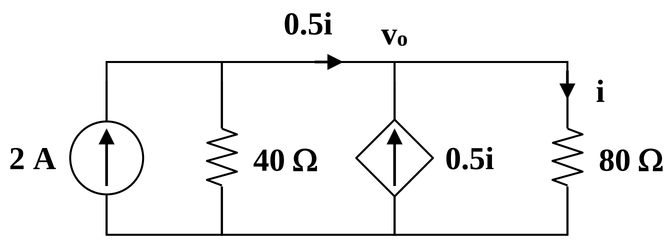
$v_o(t) = v_o(0)e^{-t/\tau}$

$v_o(0) = \dfrac{4}{4+2}(30) = 20, \qquad \tau = RC = (4)(3) = 12$

v_o = $\mathbf{[20\{1-u(t)\} + (20e^{-t/12})u(t)]}$ **V**.

Solution 7.43

Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



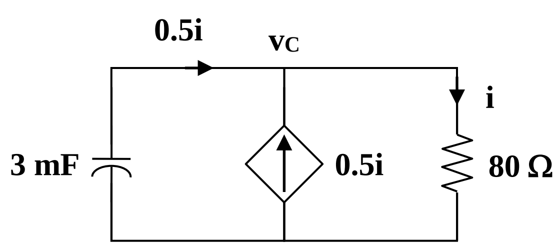
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

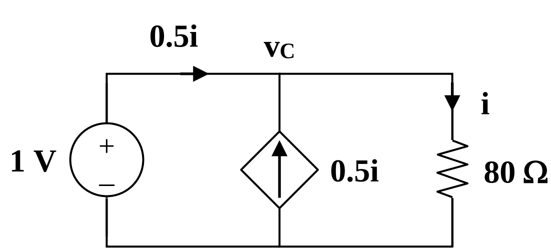
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64 \text{ V}$$

$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \boldsymbol{800e^{-t/480}u(t)\ mA}$$

Solution 7.44

The switch in Fig. 7.111 has been in position *a* for a long time. At $t = 0$, it moves to position *b*. Calculate $i(t)$ for all $t > 0$.

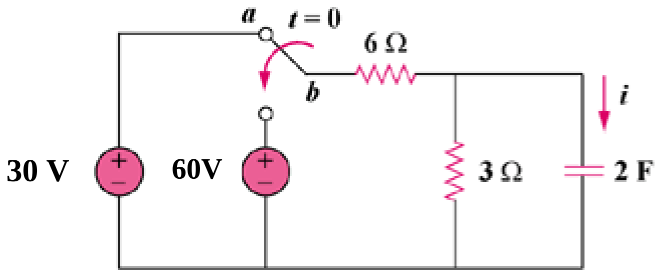


Figure 7.111
For Prob. 7.44.

Solution

Let v(t) be the voltage across the capacitor and $R_{eq} = 6 \| 3 = 2\,\Omega$ and $\tau = RC = 4$, therefore, $v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$

Using voltage division,

$$v(0) = \frac{3}{3+6}(30) = 10\,V\,, \qquad v(\infty) = \frac{3}{3+6}(60) = 20\,V$$

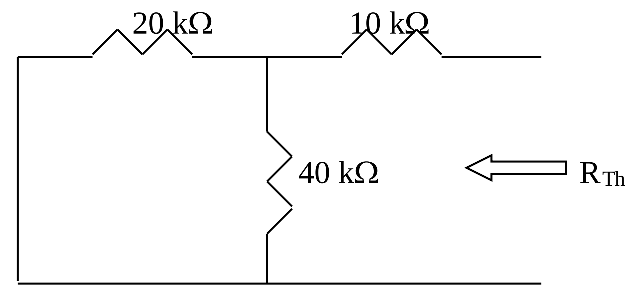
Thus,

$$v(t) = 20 + (10-20)\ e^{-t/4} = 20 - 10\ e^{-t/4}$$

$$i(t) = C\frac{dv}{dt} = (2)(-10)\left(-1/4\right)e^{-t/4} = \mathbf{(5e^{-0.25t})u(t)\ A}.$$

Solution 7.45

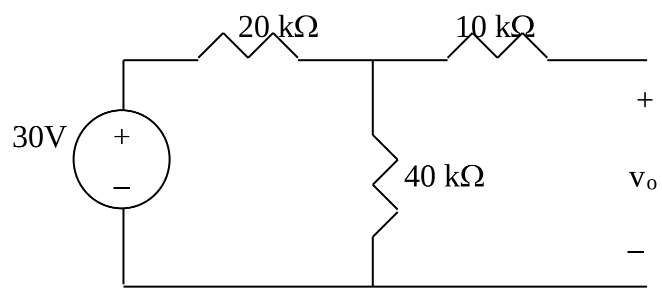
To find R_{Th}, consider the circuit shown below.



$$R_{Th} = 10 + 20//40 = 10 + \frac{20x40}{60} = \frac{70}{3} k\Omega$$

$$\tau = R_{Th}C = \frac{70}{3} x10^3 x3x10^{-6} = 0.07$$

To find $v_o(\infty)$, consider the circuit below.



$v_o(\infty) = [40/(40+20)]30 = 20$ V

$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]e^{-t/0.07}$ = $\mathbf{[20 - 15e^{-14.286t}]u(t)\ V}$.

Solution 7.46

$$\tau = R_{Th}C = (2+6)x0.25 = 2s, \qquad v(0) = 0, \qquad v(\infty) = 6i_s = 6x5 = 30$$

$$\boldsymbol{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} = \underline{30(1 - e^{-t/2})}\, u(t)\ V}$$

Solution 7.47

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 7.114 if $v(0) = 0$.

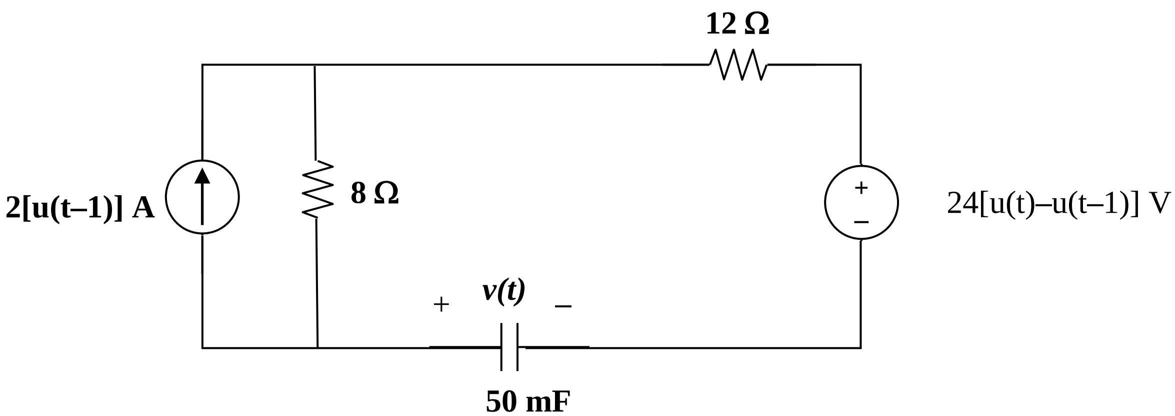


Figure 7.114
For Prob. 7.47.

Solution

For $t < 0$, $u(t) = 0$, $u(t-1) = 0$, $v(0) = 0$

For $0 < t < 1$, $\tau = (8+12)0.05 = 1$ s.

$v(0) = 0$, $v(\infty) = 24$ V.

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(t) = 24\left(1 - e^{-t}\right)$$

For $t > 1$, $v(1) = 24(1 - e^{-1}) = 15.171$ V.

At $t = \infty$ the voltage source is now a short circuit and the capacitor is an open circuit thus, $v(\infty) = -(2)(8) = -16$ V.

Now $v(t) = -16 + [15.171 - (-16)]e^{-(t-1)} = -16 + 31.17\, e^{-(t-1)}$

Thus,

$$v(t) = \begin{cases} 24\left(1 - e^{-t}\right)V, & 0 < t < 1 \\ [-16 + 31.17\, e^{-(t-1)}]\, V, & 1 < t \end{cases}$$

Solution 7.48

For $t < 0$, $u(-t) = 1$,

For $t > 0$, $u(-t) = 0$, $v(\infty) = 0$

$R_{th} = 20 + 10 = 30$, $\tau = R_{th}C = (30)(0.1) = 3$

$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$

$v(t) = \mathbf{10\,e^{-t/3}\ V}$

$$i(t) = C\frac{dv}{dt} = (0.1)\left(\frac{-1}{3}\right)10\,e^{-t/3}$$

$$i(t) = \mathbf{\frac{-1}{3}e^{-t/3}\ A}$$

Solution 7.49

For $0 < t < 1$, $\quad v(0) = 0$, $\qquad v(\infty) = (2)(4) = 8$

$$R_{eq} = 4 + 6 = 10, \qquad \tau = R_{eq}C = (10)(0.5) = 5$$

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(t) = 8\left(1 - e^{-t/5}\right) \text{ V}$$

For $t > 1$, $\qquad v(1) = 8\left(1 - e^{-0.2}\right) = 1.45$, $\qquad v(\infty) = 0$

$$v(t) = v(\infty) + \left[v(1) - v(\infty) \right] e^{-(t-1)/\tau}$$

$$v(t) = 1.45 e^{-(t-1)/5} \text{ V}$$

Thus,

$$v(t) = \begin{cases} \mathbf{8\left(1 - e^{-t/5}\right) V,} & \mathbf{0 < t < 1} \\ \mathbf{1.45 e^{-(t-1)/5} \ V,} & \mathbf{t > 1} \end{cases}$$

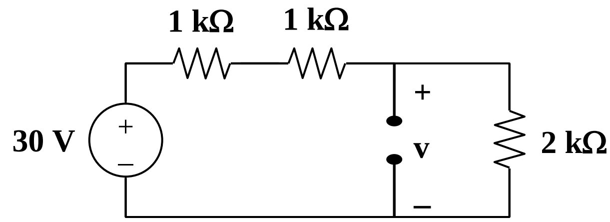
Solution 7.50

For the capacitor voltage,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(0) = 0$$

For t > 0, we transform the current source to a voltage source as shown in Fig. (a).



(a)

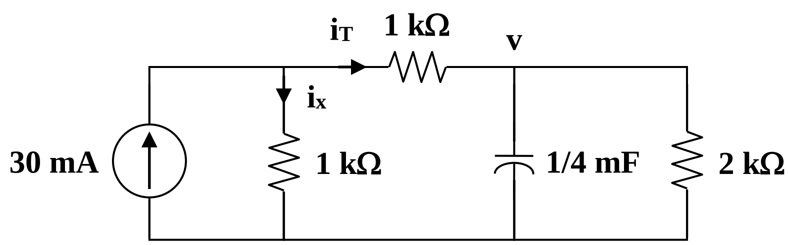
$$v(\infty) = \frac{2}{2+1+1}(30) = 15 \text{ V}$$

$$R_{th} = (1+1) \parallel 2 = 1 \text{ k}\Omega$$

$$\tau = R_{th}C = 10^3 \times \frac{1}{4} \times 10^{-3} = \frac{1}{4}$$

$$v(t) = 15\left(1 - e^{-4t}\right), \quad t > 0$$

We now obtain i_x from v(t). Consider Fig. (b).



(b)

$$i_x = 30 \text{ mA} - i_T$$

But $\quad i_T = \dfrac{v}{R_3} + C\dfrac{dv}{dt}$

$$i_T(t) = 7.5\left(1 - e^{-4t}\right) \text{ mA} + \frac{1}{4} \times 10^{-3} (-15)(-4)e^{-4t} \text{ A}$$

$$i_T(t) = 7.5\left(1 + e^{-4t}\right) \text{ mA}$$

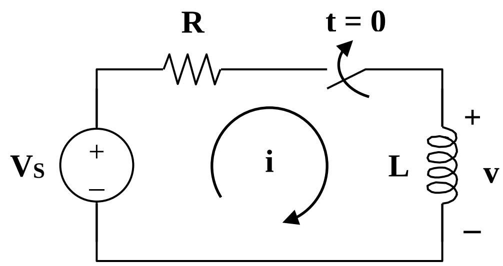
Thus,

$$i_x(t) = 30 - 7.5 - 7.5e^{-4t} \text{ mA}$$

$$i_x(t) = \mathbf{7.5\left(3 - e^{-4t}\right) mA, \quad t > 0}$$

Solution 7.51

Consider the circuit below.



After the switch is closed, applying KVL gives

$$V_S = Ri + L\frac{di}{dt}$$

or $$L\frac{di}{dt} = -R\left(i - \frac{V_S}{R}\right)$$

$$\frac{di}{i - V_S/R} = \frac{-R}{L}dt$$

Integrating both sides,

$$\ln\left(i - \frac{V_S}{R}\right)\Bigg|_{I_0}^{i(t)} = \frac{-R}{L}t$$

$$\ln\left(\frac{i - V_S/R}{I_0 - V_S/R}\right) = \frac{-t}{\tau}$$

or $$\frac{i - V_S/R}{I_0 - V_S/R} = e^{-t/\tau}$$

$$\mathbf{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}}$$

which is the same as Eq. (7.60).

Solution 7.52

Using Fig. 7.118, design a problem to help other students to better understand the step response of an RL circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 7.118, find $i(t)$ for $t > 0$.

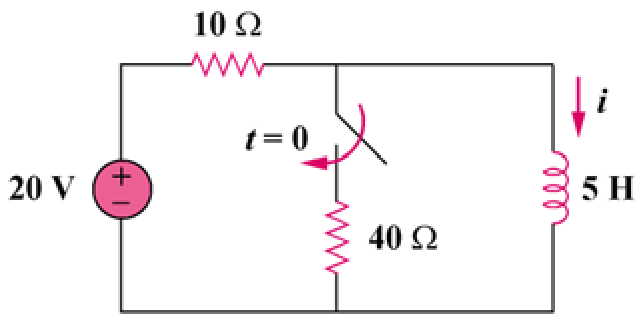


Figure 7.118

Solution

$$i(0) = \frac{20}{10} = 2\text{ A}, \qquad i(\infty) = 2\text{ A}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \mathbf{2\ A}$$

Solution 7.53

(a) Before t = 0, $i = \dfrac{25}{3+2} = \mathbf{5\ A}$

After t = 0, $i(t) = i(0)e^{-t/\tau}$

$$\tau = \frac{L}{R} = \frac{4}{2} = 2, \qquad i(0) = 5$$

$$i(t) = \boldsymbol{5e^{-t/2}\ u(t)A}$$

(b) Before t = 0, the inductor acts as a short circuit so that the 2 Ω and 4 Ω resistors are short-circuited.

$i(t) = \mathbf{6\ A}$

After t = 0, we have an RL circuit.

$$i(t) = i(0)e^{-t/\tau}, \qquad \tau = \frac{L}{R} = \frac{3}{2}$$

$$i(t) = \boldsymbol{6e^{-2t/3}\ u(t)A}$$

Solution 7.54

(a) Before t = 0, i is obtained by current division or

$$i(t) = \frac{4}{4+4}(2) = \mathbf{1\ A}$$

After t = 0,

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$\tau = \frac{L}{R_{eq}}, \qquad R_{eq} = 4 + (4 \,\|\, 12) = 7\ \Omega$$

$$\tau = \frac{3.5}{7} = \frac{1}{2}$$

$$i(0) = 1, \qquad i(\infty) = \frac{(4 \,\|\, 12)}{4 + (4 \,\|\, 12)}(2) = \frac{3}{4+3}(2) = \frac{6}{7}$$

$$i(t) = \frac{6}{7} + \left(1 - \frac{6}{7}\right) e^{-2t}$$

$$i(t) = \mathbf{\frac{1}{7}\left(6 - e^{-2t}\right)\ A}$$

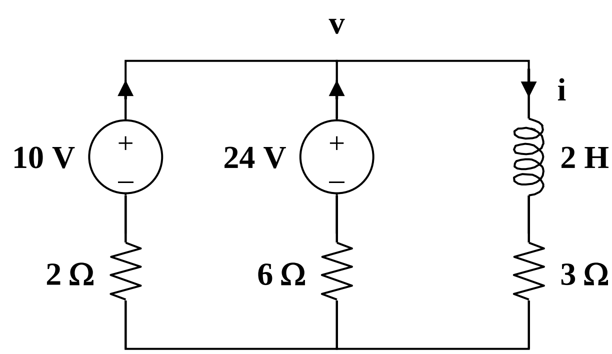
(b) Before t = 0, $i(t) = \frac{10}{2+3} = \mathbf{2\ A}$

After t = 0, $R_{eq} = 3 + (6 \,\|\, 2) = 4.5$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{4.5} = \frac{4}{9}$$

$$i(0) = 2$$

To find $i(\infty)$, consider the circuit below, at t = when the inductor becomes a short circuit,



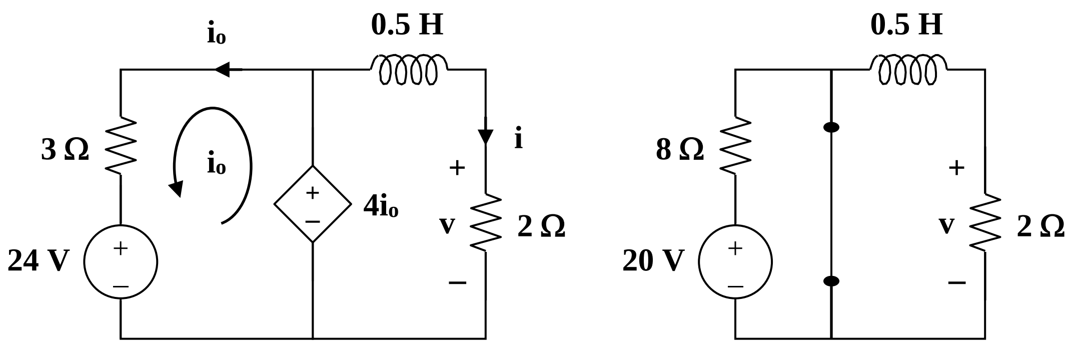
$$\frac{10 - v}{2} + \frac{24 - v}{6} = \frac{v}{3} \longrightarrow v = 9 \quad i(\infty) = \frac{v}{3} = 3\text{ A and}$$

$$i(t) = 3 + (2 - 3)e^{-9t/4}$$

$$i(t) = \mathbf{3 - e^{-9t/4}\ A}$$

Solution 7.55

For t < 0, consider the circuit shown in Fig. (a).



(a) (b)

$$3i_o + 24 - 4i_o = 0 \longrightarrow i_o = 24$$

$$v(t) = 4i_o = \mathbf{96\ V} \qquad i = \frac{v}{2} = 48\ \text{A}$$

For t > 0, consider the circuit in Fig. (b).

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(0) = 48, \qquad i(\infty) = 0$$

$$R_{th} = 2\ \Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{0.5}{2} = \frac{1}{4}$$

$$i(t) = (48)\,e^{-4t}$$

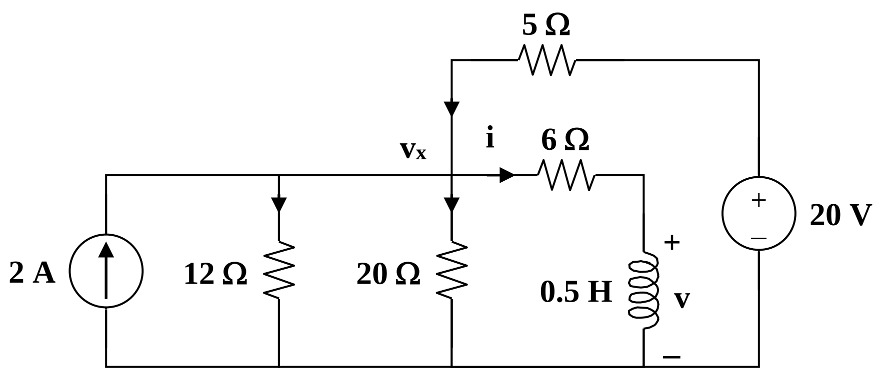
$$v(t) = 2\,i(t) = \mathbf{96\,\boldsymbol{e}^{-4t}\ \boldsymbol{u(t)V}}$$

Solution 7.56

$$R_{eq} = 6 + 20 \parallel 5 = 10\ \Omega, \qquad \tau = \frac{L}{R} = 0.05$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

i(0) is found by applying nodal analysis to the following circuit.



$$2 + \frac{20 - v_x}{5} = \frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} \longrightarrow v_x = 12$$

$$i(0) = \frac{v_x}{6} = 2\ \text{A}$$

Since $20 \parallel 5 = 4$,

$$i(\infty) = \frac{4}{4+6}(4) = 1.6$$

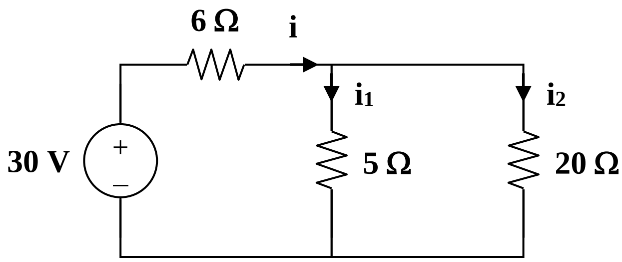
$$i(t) = 1.6 + (2 - 1.6)e^{-t/0.05} = 1.6 + 0.4e^{-20t}$$

$$v(t) = L\frac{di}{dt} = \frac{1}{2}(0.4)(-20)e^{-20t}$$

$$v(t) = \mathbf{-4e^{-20t}\ V}$$

Solution 7.57

At $t = 0^-$, the circuit has reached steady state so that the inductors act like short circuits.



$$i = \frac{30}{6+(5\,\|\,20)} = \frac{30}{10} = 3, \qquad i_1 = \frac{20}{25}(3) = 2.4, \qquad i_2 = 0.6$$

$$i_1(0) = 2.4 \text{ A}, \qquad i_2(0) = 0.6 \text{ A}$$

For t > 0, the switch is closed so that the energies in L_1 and L_2 flow through the closed switch and become dissipated in the 5 Ω and 20 Ω resistors.

$$i_1(t) = i_1(0)\,e^{-t/\tau_1}, \qquad \tau_1 = \frac{L_1}{R_1} = \frac{2.5}{5} = \frac{1}{2}$$

$$i_1(t) = \mathbf{2.4e^{-2t}u(t)\ A}$$

$$i_2(t) = i_2(0)\,e^{-t/\tau_2}, \qquad \tau_2 = \frac{L_2}{R_2} = \frac{4}{20} = \frac{1}{5}$$

$$i_2(t) = \mathbf{600e^{-5t}u(t)\ mA}$$

Solution 7.58

For t < 0, $v_o(t) = 0$

For t > 0, $i(0) = 10$, $i(\infty) = \dfrac{20}{1+3} = 5$

$$R_{th} = 1 + 3 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{1/4}{4} = \frac{1}{16}$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 5\left(1 + e^{-16t}\right) \text{A}$$

$$v_o(t) = 3i + L\frac{di}{dt} = 15\left(1 + e^{-16t}\right) + \frac{1}{4}(-16)(5)e^{-16t}$$

$$v_o(t) = \mathbf{15 - 5e^{-16t}\ \ V}$$

Solution 7.59

Determine the step response $v_o(t)$ to $i_s = 6u(t)$ A in the circuit of Fig. 7.124.

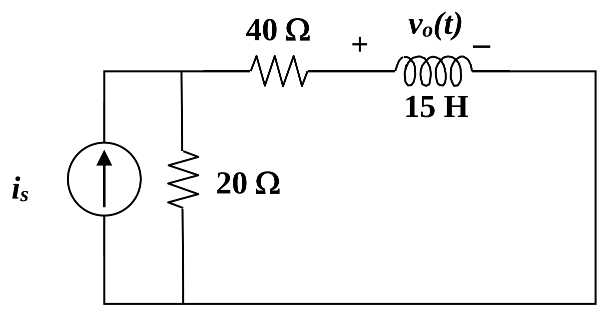


Figure 7.124
For Prob. 7.59.

Solution

Let i(t) be the current through the inductor.
For t < 0, $i_s = 0$ A and i(0) = 0 A.
For t > 0, $R_{eq} = 20+40 = 60\ \Omega$ and $\tau = (L/R_{eq}) = 15/60 = 0.25$ s.

At t = ∞, the inductor becomes a short and the current through the 40 Ω can be found by using current division, i(∞) = 6x20/(20+40) = 2 A.

Thus, $i(t) = [2–2e^{–4t}]$ A and $v_o(t) = Ldi/dt = 15[(–4)(–2e^{–4t})]$ or

$v_o(t) =$ $\mathbf{120e^{–4t}u(t)\ V}$.

Solution 7.60

Let I be the inductor current.

For t < 0, $\quad u(t) = 0 \longrightarrow i(0) = 0$

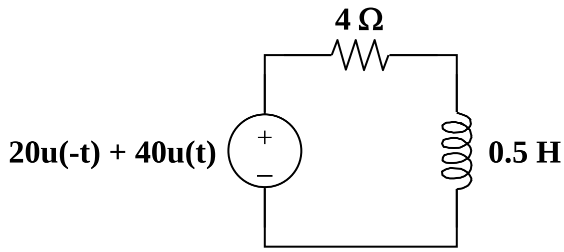
For t > 0, $\quad R_{eq} = 5 \| 20 = 4\,\Omega, \quad \tau = \dfrac{L}{R_{eq}} = \dfrac{8}{4} = 2$

$$i(\infty) = 4$$
$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$
$$i(t) = 4\left(1 - e^{-t/2}\right)$$

$$v(t) = L\frac{di}{dt} = (8)(-4)\left(\frac{-1}{2}\right)e^{-t/2}$$
$$v(t) = \mathbf{16\,e^{-0.5t}\ \ V}$$

Solution 7.61

The current source is transformed as shown below.



$$\tau = \frac{L}{R} = \frac{1/2}{4} = \frac{1}{8}, \qquad i(0) = 5, \qquad i(\infty) = 10$$

$$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = \mathbf{(10 - 5e^{-8t})u(t)\ A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-5)(-8)\, e^{-8t}$$

$$v(t) = \mathbf{20e^{-8t}u(t)\ V}$$

Solution 7.62

$$\tau = \frac{L}{R_{eq}} = \frac{2}{3 \| 6} = 1$$

For 0 < t < 1, $u(t-1) = 0$ so that

$$i(0) = 0, \qquad i(\infty) = \frac{1}{6}$$

$$i(t) = \frac{1}{6}\left(1 - e^{-t}\right)$$

For t > 1, $i(1) = \frac{1}{6}\left(1 - e^{-1}\right) = 0.1054$

$$i(\infty) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$i(t) = 0.5 + (0.1054 - 0.5)\,e^{-(t-1)}$$

$$i(t) = 0.5 - 0.3946\,e^{-(t-1)}$$

Thus,

$$i(t) = \begin{cases} \mathbf{\frac{1}{6}\left(1 - e^{-t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{0.5 - 0.3946\,e^{-(t-1)}\ A} & \mathbf{t > 1} \end{cases}$$

Solution 7.63

For t < 0, $u(-t) = 1$, $i(0) = \frac{10}{5} = 2$

For t > 0, $u(-t) = 0$, $i(\infty) = 0$

$R_{th} = 5 \| 20 = 4\,\Omega$, $\tau = \frac{L}{R_{th}} = \frac{0.5}{4} = \frac{1}{8}$

$i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$

$$i(t) = \mathbf{2e^{-8t}u(t)\ A}$$

$$v(t) = L\frac{di}{dt} = \left(\frac{1}{2}\right)(-8)(2)\,e^{-8t}$$

$$v(t) = \mathbf{-8e^{-8t}u(t)\ V}$$

$2e^{-8t}u(t)$ A, $-8e^{-8t}u(t)$ V

Solution 7.64

Determine the value of $i_L(t)$ and the total energy dissipated by the circuit from t = 0 sec to t = ∞ sec. The value of $i_{in}(t)$ is equal to [6 – 6u(t)] A.

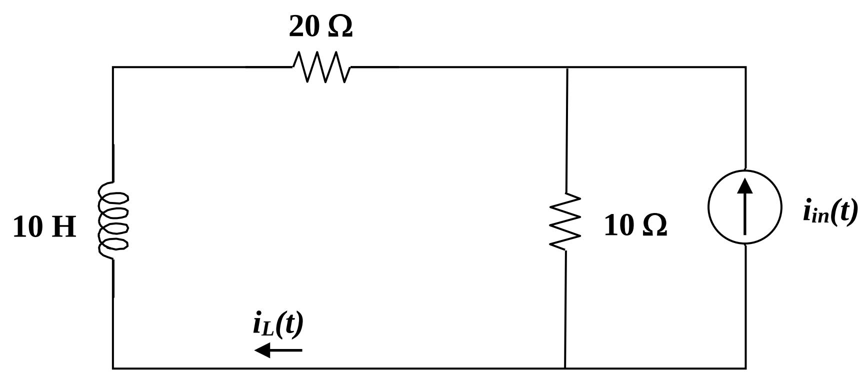


Figure 7.129
For Prob. 7.64.

Solution

For t < 0, the value of i_{in} = 6 A. The value of i_L can be found by using current division, $i_L(0) = –6x10/(20+10) = –2$ A.

For 0 < t i_{in} = 0 A and $i_L(\infty) = –(0)(1/3) = 0$ A and $\tau = L/R = 10/30 = 1/3$. Thus,

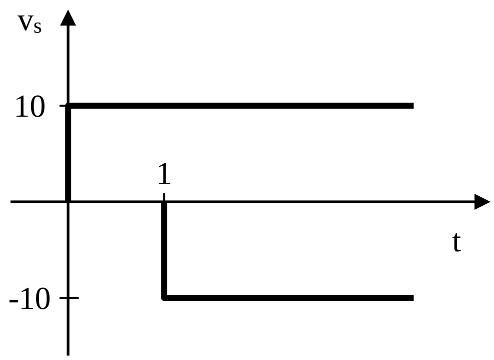
$$i_L(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$
$$= \mathbf{-2e^{-3t}\ A}.$$

To find the total energy that will be dissapaited in the circuit from t = 0 to ∞ we only need to recognize that the inductor is the only divice supplying power to the circuit after t = 0. Thus, the total energy dissapatied by the circuit is equal to the energy stored in the inductor at t = 0.

$$w = (1/2)Li_L(0)^2 = 0.5x10x(-2)^2 = \mathbf{20\ J}.$$

Solution 7.65

Since $v_s = 10\left[u(t) - u(t-1)\right]$, this is the same as saying that a 10 V source is turned on at t = 0 and a -10 V source is turned on later at t = 1. This is shown in the figure below.



For 0 < t < 1, $i(0) = 0$, $\quad i(\infty) = \frac{10}{5} = 2$

$$R_{th} = 5 \,||\, 20 = 4, \qquad \tau = \frac{L}{R_{th}} = \frac{2}{4} = \frac{1}{2}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

$$i(t) = 2\left(1 - e^{-2t}\right) \text{ A}$$

$$i(1) = 2\left(1 - e^{-2}\right) = 1.729$$

For t > 1, $i(\infty) = 0$ $\quad$ since $v_s = 0$

$$i(t) = i(1)e^{-(t-1)/\tau}$$

$$i(t) = 1.729e^{-2(t-1)} \text{ A}$$

Thus,

$$i(t) = \begin{cases} \mathbf{2\left(1 - e^{-2t}\right) A} & \mathbf{0 < t < 1} \\ \mathbf{1.729e^{-2(t-1)} \; A} & \mathbf{t > 1} \end{cases}$$

Solution 7.66

Using Fig. 7.131, design a problem to help other students to better understand first-order op amp circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the op-amp circuit of Fig. 7.131, find v_o. Assume that v_s changes abruptly from 0 to 1 V at t=0. Find v_o.

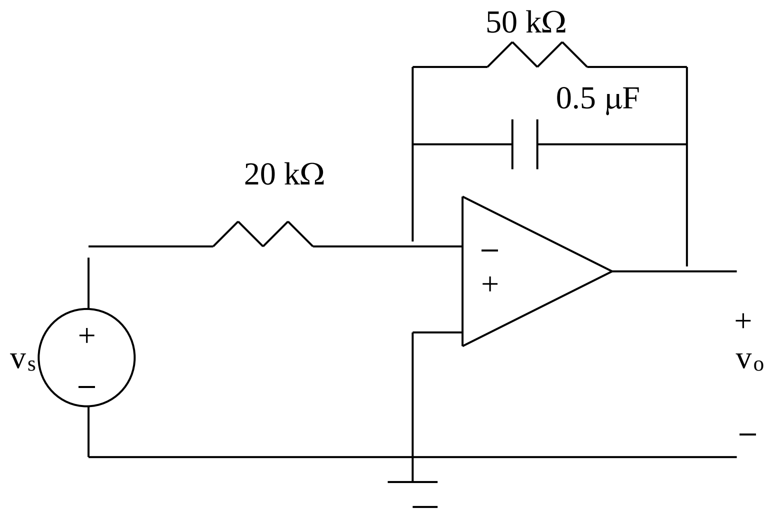


Figure 7.131 For Prob. 7.66.

Solution

For t<0-, v_s =0 so that $v_o(0)=0$
:Let v be the capacitor voltage
For t>0, v_s =1. At steady state, the capacitor acts like an open circuit so that we have an inverting amplifier

$$v_o(\infty) = -(50k/20k)(1V) = -2.5 \text{ V}$$

$$\tau = RC = 50\text{x}10^3\text{x}0.5\text{x}10^{-6} = 25 \text{ ms}$$

$$v_o(t) = v_o(\infty) + (v_o(0) - v_o(\infty))e^{-t/0.025} = \underline{\mathbf{2.5(e^{-40t} - 1)\ V}}.$$

Solution 7.67

If $v(0) = 10$ V, find $v_o(t)$ for $t > 0$ in the op amp circuit in Fig. 7.132.
Let $R = 100$ kΩ and $C = 20$ μF.

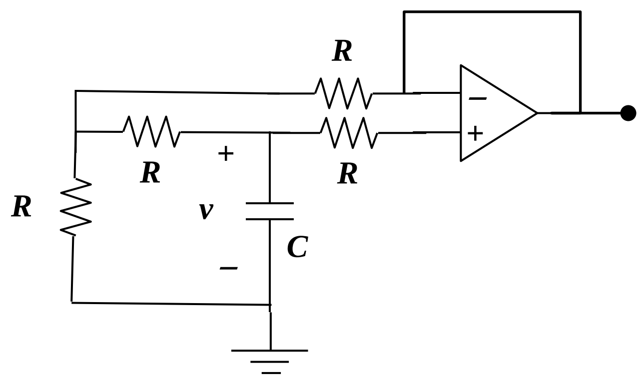
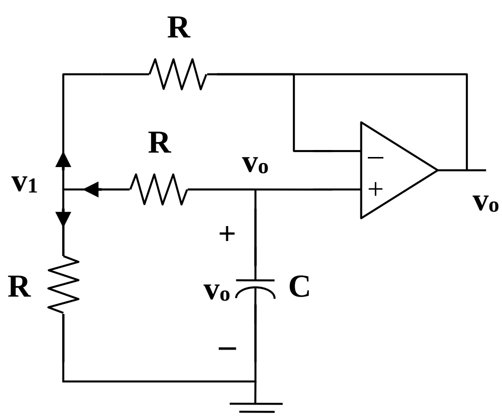


Figure 7.132
For Prob. 7.67.

Solution

In this circuit, the resistor between the capacitor and the positive input terminal of the op amp can be neglected since the current through it has to be equal to zero. This then results in the circuit shown below. Clearly this is a voltage follower circuit with $v_o = v$.



At node 1, $[(v_1–0)/R] + [(v_1–v_o)/R] + [(v_1–v_o)/R] = 0$ or $(3/R)v_1 = (2/R)v_o$ or $v_1 = (2/3)v_o$.

At the noninverting terminal,

$$C\frac{dv_o}{dt} + \frac{v_o - v_1}{R} = 0$$

$$-RC\frac{dv_o}{dt} = v_o - v_1 = v_o - \frac{2}{3}v_o = \frac{1}{3}v_o$$

$$\frac{dv_o}{dt} = -\frac{v_o}{3RC} \text{ or } v_o(t) = V_T e^{-t/3RC}$$

$V_T = v_o(0) = 10$ V and $\tau = 3RC = 3x10^5x2x10^{-5} = 6$ s.

Thus,

$v_o(t) =$ **$10e^{-t/6}u(t)$ A**.

Solution 7.68

This is a very interesting problem which has both an ideal solution as well as a realistic solution. Let us look at the ideal solution first. Just before the switch closes, the value of the voltage across the capacitor is zero which means that the voltage at both terminals input of the op amp are each zero. As soon as the switch closes, the output tries to go to a voltage such that both inputs to the op amp go to 4 volts. The ideal op amp puts out whatever current is necessary to reach this condition. An infinite (impulse) current is necessary if the voltage across the capacitor is to go to 8 volts in zero time (8 volts across the capacitor will result in 4 volts appearing at the negative terminal of the op amp). So v_o will be equal to **8 volts** for all t > 0.

What happens in a real circuit? Essentially, the output of the amplifier portion of the op amp goes to whatever its maximum value can be. Then this maximum voltage appears across the output resistance of the op amp and the capacitor that is in series with it. This results in an exponential rise in the capacitor voltage to the steady-state value of 8 volts.

$$vC(t) = V_{\text{op amp max}}(1 - e^{-t/(R_{out}C)}) \text{ volts, for all values of vC less than 8 V,}$$

$$= \mathbf{8\ V} \text{ when t is large enough so that the 8 V is reached.}$$

Solution 7.69

Let v_x be the capacitor voltage.
For t < 0, $v_x(0) = 0$

For t > 0, the 20 kΩ and 100 kΩ resistors are in series and together, they are in parallel with the capacitor since no current enters the op amp terminals. As $t \rightarrow \infty$, the capacitor acts like an open circuit so that

$$v_o(\infty) = \frac{-4}{10}(20+100) = -48$$

$$R_{th} = 20+100 = 120\ \text{k}\Omega, \qquad \tau = R_{th}C = (120\times10^3)(25\times10^{-3}) = 3000$$

$$v_o(t) = v_o(\infty) + [v_o(0) - v_o(\infty)]\, e^{-t/\tau}$$

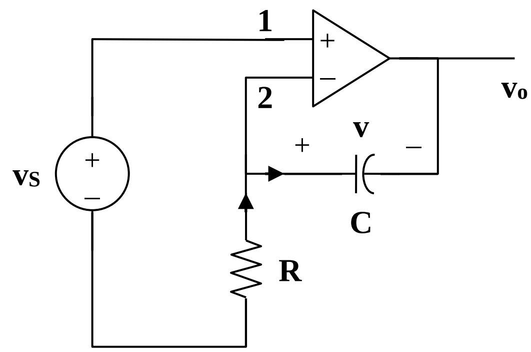
$$v_o(t) = -48\left(1 - e^{-t/3000}\right)\text{V} = \mathbf{48(e^{-t/3000} - 1)u(t)V}$$

Solution 7.70

Let v = capacitor voltage.

For t < 0, the switch is open and $v(0) = 0$.

For t > 0, the switch is closed and the circuit becomes as shown below.



$$v_1 = v_2 = v_s \tag{1}$$

$$\frac{0 - v_s}{R} = C\frac{dv}{dt} \tag{2}$$

where $v = v_s - v_o \quad \longrightarrow \quad v_o = v_s - v$ (3)

From (1),

$$\frac{dv}{dt} = \frac{v_s}{RC} = 0$$

$$v = \frac{-1}{RC}\int v_s \; dt + v(0) = \frac{-t\,v_s}{RC}$$

Since v is constant,

$$RC = (20\times10^{3})(5\times10^{-6}) = 0.1$$

$$v = \frac{-20\,t}{0.1}\text{ mV} = -200t\text{ mV}$$

From (3),

$$v_o = v_s - v = 20 + 200\,t$$

$$v_o = \mathbf{20(1+10t)\ mV}$$

Solution 7.71

For the op amp circuit in Fig. 7.136, suppose v_s = 10u(t) V. Find *v(t)* for t > 0.

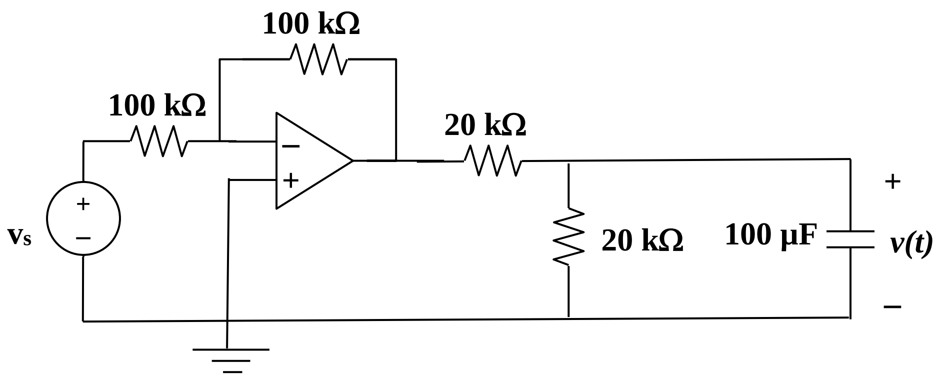


Figure 7.136
For Prob. 7.71.

Solution

We recognize that the op amp operates as an inverting op amp whose output is equal to $-v_s$ = –10u(t) V. Since the output of the op amp acts like an ideal voltage source, we can determine the Thevenin equivalent circuit, as seen by the capacitor, with $V_{Thev} = V_{oc}$ = –10u(t)[20k/(20k+20k)] = –5u(t) V and R_{eq} = 20kx20k/(20k+20k) = 10 kΩ. Next we get $\tau = R_{eq}C = 10^4 x 10^{-4} = 1$ s.

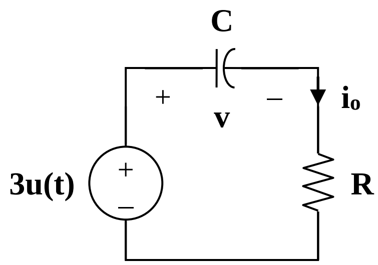
Additionally since the input voltage is equal to zero until t = 0, the value of v(0) = 0.

Finally v(∞) = –5 V which leads to,

$$v(t) = \mathbf{[-5 + 5e^{-t}]u(t)\ V}.$$

Solution 7.72

The op amp acts as an emitter follower so that the Thevenin equivalent circuit is shown below.



Hence,

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(0) = -2\text{ V}, \quad v(\infty) = 3\text{ V}, \quad \tau = RC = (10\times10^{3})(10\times10^{-6}) = 0.1$$

$$v(t) = 3 + (-2 - 3)\,e^{-10t} = 3 - 5\,e^{-10t}$$

$$i_o = C\frac{dv}{dt} = (10\times10^{-6})(-5)(-10)\,e^{-10t}$$

$$i_o = \mathbf{0.5\,e^{-10t}\ mA, \quad t > 0}$$

Solution 7.73

For the op amp circuit of Fig. 7.138, let R_1 = 10 kΩ, R_f = 30 kΩ, $C = 20\ \mu F$, and $v(0)$ = 1 V. Find v_o.

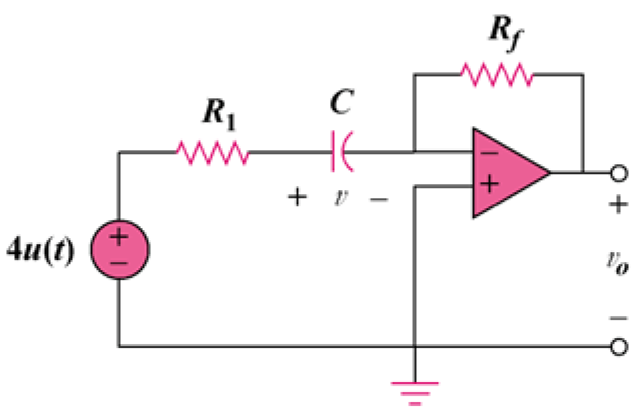
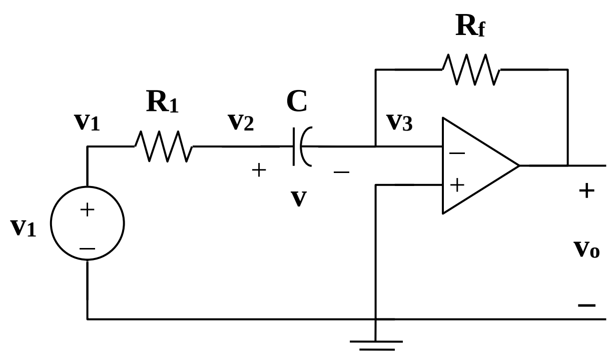


Figure 7.138
For Prob. 7.73.

Solution

Consider the circuit below.



At node 2,

$$\frac{v_1 - v_2}{R_1} = C\frac{dv}{dt} \tag{1}$$

At node 3,

$$C\frac{dv}{dt} = \frac{v_3 - v_o}{R_f} \tag{2}$$

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$. Hence, (1) becomes

$$\frac{v_1 - v}{R_1} = C\frac{dv}{dt}$$

$$v_1 - v = R_1 C\frac{dv}{dt}$$

or $$\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$$

which is similar to Eq. (7.42). Hence,

$$v(t) = \begin{cases} v_T & t < 0 \\ v_1 + \left(v_T - v_1\right)e^{-t/\tau} & t > 0 \end{cases}$$

where $v_T = v(0) = 1$ and $v_1 = 4$

$$\tau = R_1 C = (10\times10^3)(20\times10^{-6}) = 0.2$$

$$v(t) = \begin{cases} 1 & t < 0 \\ 4 - 3e^{-5t} & t > 0 \end{cases}$$

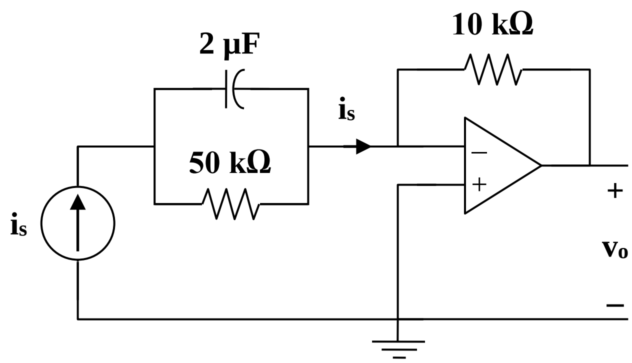
From (2),

$$v_o = -R_f C\frac{dv}{dt} = -(30\times10^3)(20\times10^{-6})(15e^{-5t})$$

$v_o = -9e^{-5t}, \quad t > 0$ or v_o = **$(-9e^{-5t})u(t)$ V**.

Solution 7.74

Let v = capacitor voltage. For t < 0, v(0) = 0



For t > 0, $i_s = 10\ \mu A$.

Since the current through the feedback resistor is i_s, then

$$v_o = -i_s x10^4 \text{ volts} = -10^{-5}x10^4 = \mathbf{-100\ mV}.$$

It is interesting to look at the capacitor voltage.

$$i_s = C\frac{dv}{dt} + \frac{v}{R}$$
$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$$

It is evident that

$$\tau = RC = (2\times10^{-6})(50\times10^{3}) = 0.1$$

At steady state, the capacitor acts like an open circuit so that i_s passes through R. Hence,

$$v(\infty) = i_s R = (10\times10^{-6})(50\times10^{3}) = 0.5\text{ V}$$

Then the voltage across the capacitor is,

$$v(t) = \mathbf{500(1-e^{-10t})\ mV}.$$

Solution 7.75

In the circuit of Fig. 7.140, find v_o and i_o, given that $v_s = 10[1 - e^{-t}]u(t)$ V.

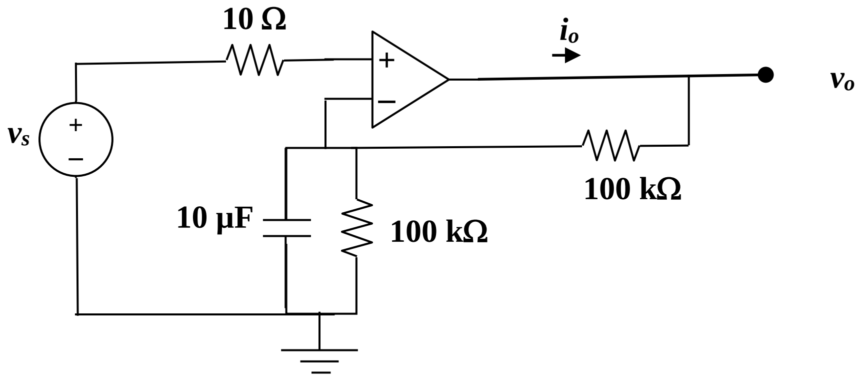


Figure 7.140
For Prob. 7.75.

Solution

Let v_a = voltage at the noninverting terminal and let v_b = voltage at the inverting terminal.

Since $v_s = 0$ for all $t < 0$, all the initial voltages are equal to 0.

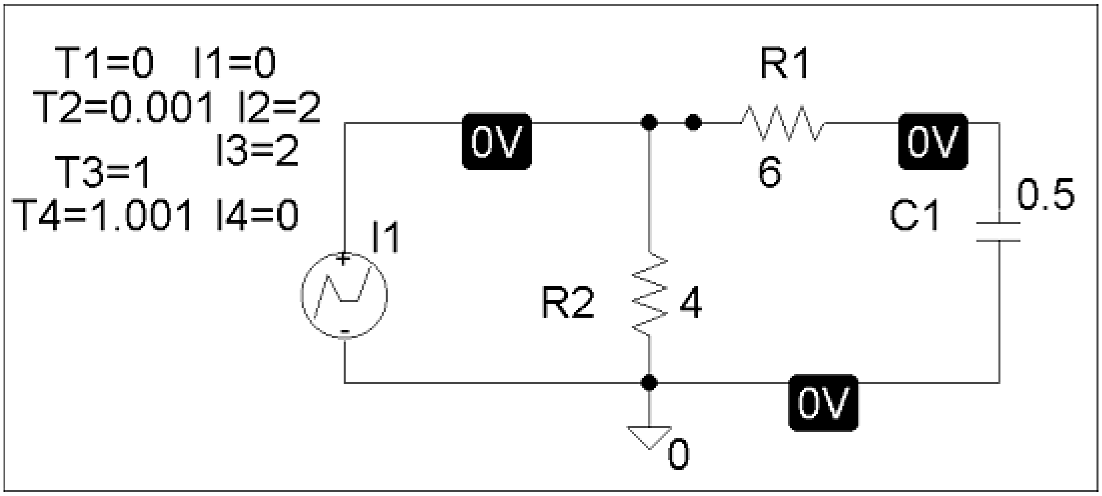
For $t > 0$, $v_a = v_b = v_s = 10[1 - e^{-t}]$.

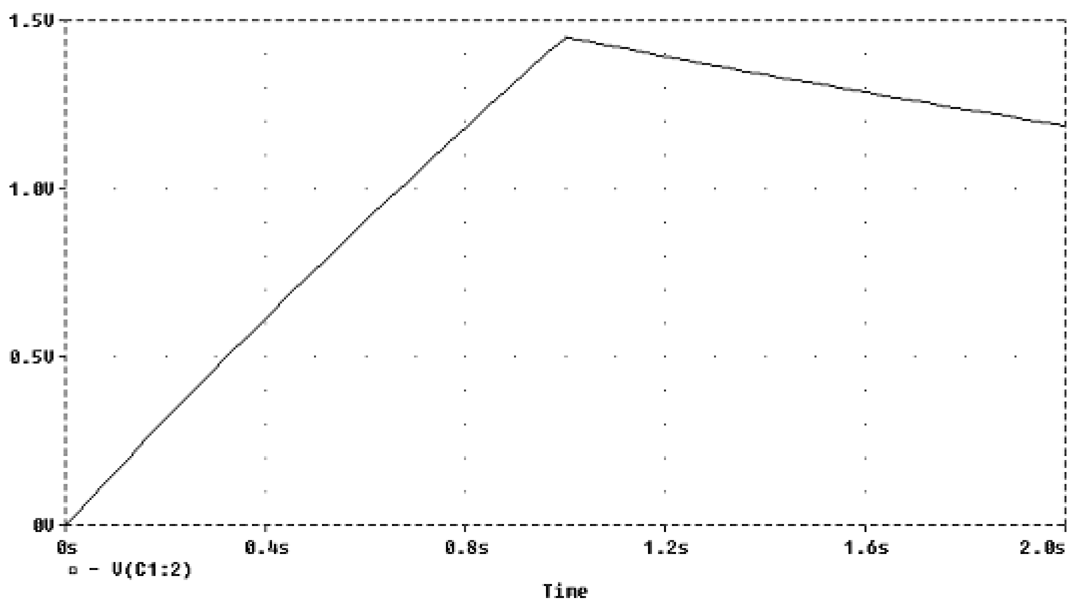
At v_b, $10^{-5}(dv_b/dt) + [(v_b-0)/10^5] + [(v_b-v_o)/10^5] + 0 = 0$. Since $dv_s/dt = 10e^{-t}$ we then get $v_o = 10e^{-t} + 2v_s = 10e^{-t} + 20 - 20e^{-t} =$ **$[20 - 10e^{-t}]u(t)$ V**.

Now, $i_o = [(v_o-v_s)/10^5] = \{[20 - 10e^{-t}] - [10-10e^{-t}]\}/10^5 =$ **100 μA**.

Solution 7.76

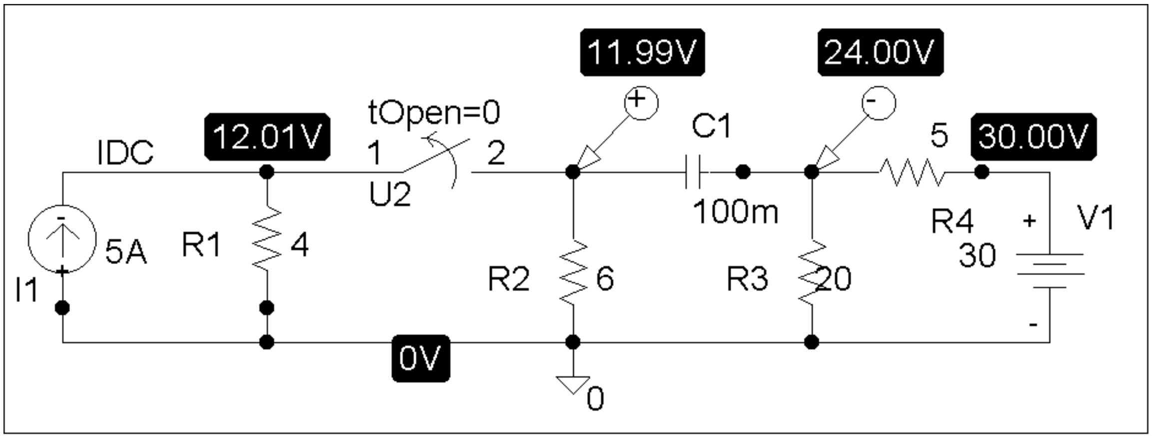
The schematic is shown below. For the pulse, we use IPWL and enter the corresponding values as attributes as shown. By selecting Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s since the width of the input pulse is 1 s. After saving and simulating the circuit, we select Trace/Add and display –V(C1:2). The plot of V(t) is shown below.

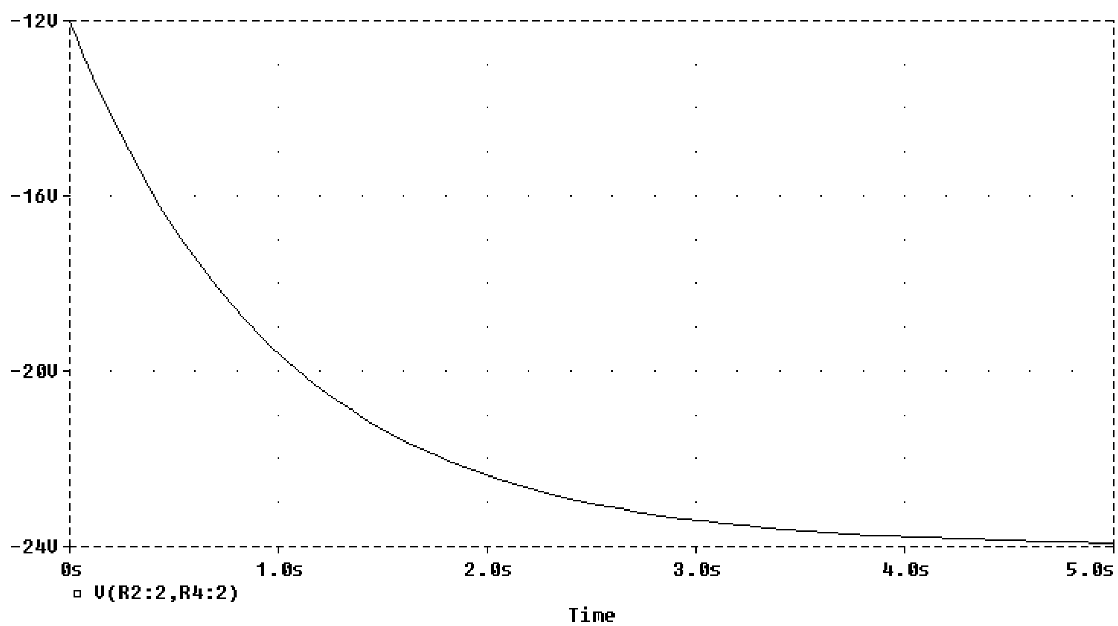




Solution 7.77

The schematic is shown below. We click Marker and insert Mark Voltage Differential at the terminals of the capacitor to display V after simulation. The plot of V is shown below. Note from the plot that V(0) = 12 V and V(∞) = -24 V which are correct.



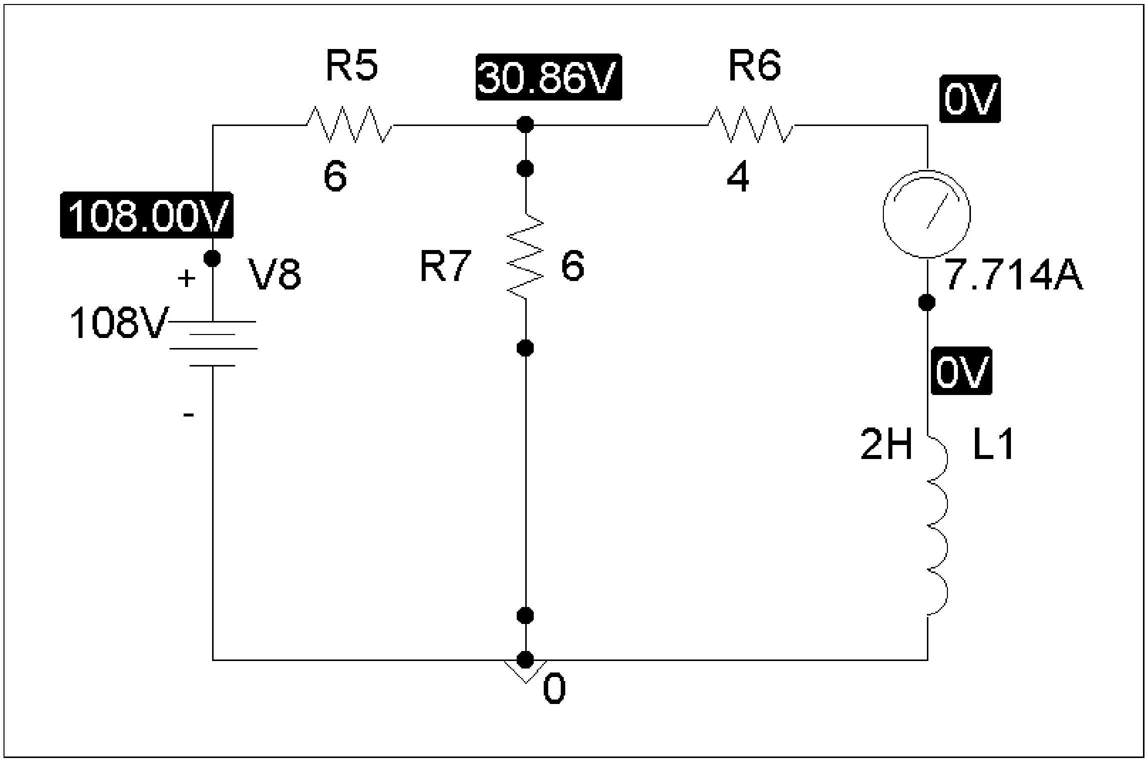


Solution 7.78

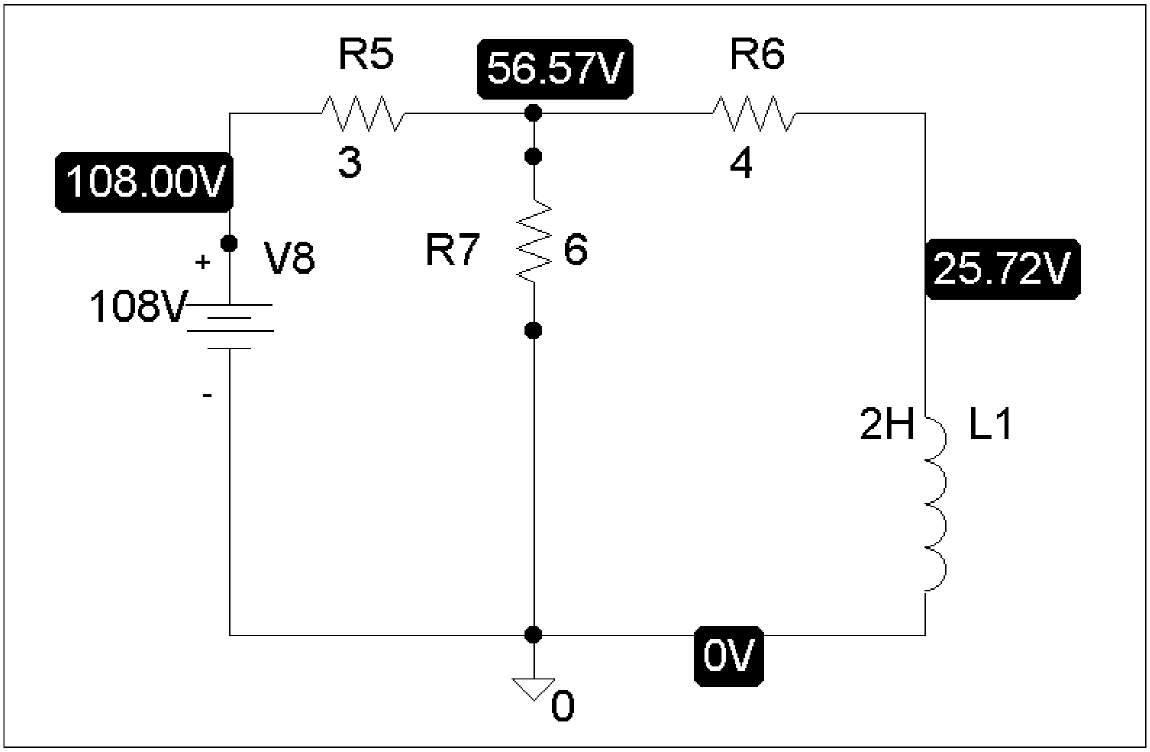
(a) When the switch is in position (a), the schematic is shown below. We insert IPROBE to display i. After simulation, we obtain,

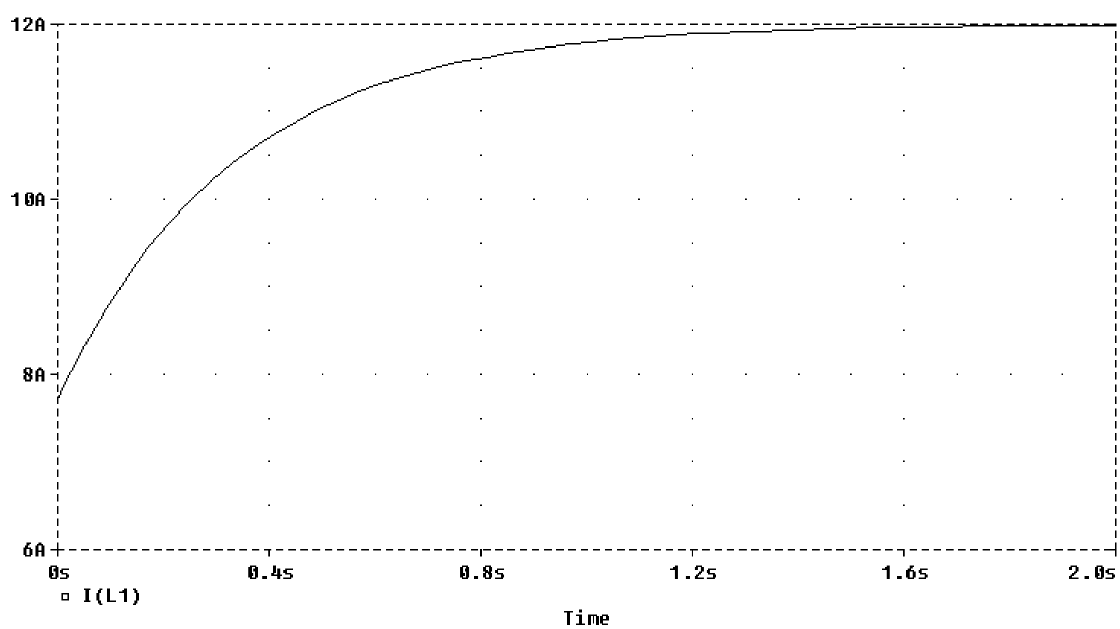
$$i(0) = 7.714 \text{ A}$$

from the display of IPROBE.



(b) When the switch is in position (b), the schematic is as shown below. For inductor I1, we let IC = 7.714. By clicking <u>Analysis/Setup/Transient</u>, we let Print Step = 25 ms and Final Step = 2 s. After Simulation, we click <u>Trace/Add</u> in the probe menu and display I(L1) as shown below. Note that $i(\infty) = 12$A, which is correct.


R5
56.57V
R6
3
4
108.00V
V8
+
108V
-
R7
6
25.72V
2H
L1
0V
0


12A
10A
8A
6A
0s
0.4s
0.8s
1.2s
1.6s
2.0s
I(L1)
Time

Solution 7.79

In the circuit in Fig. 7.143, determine i_o.

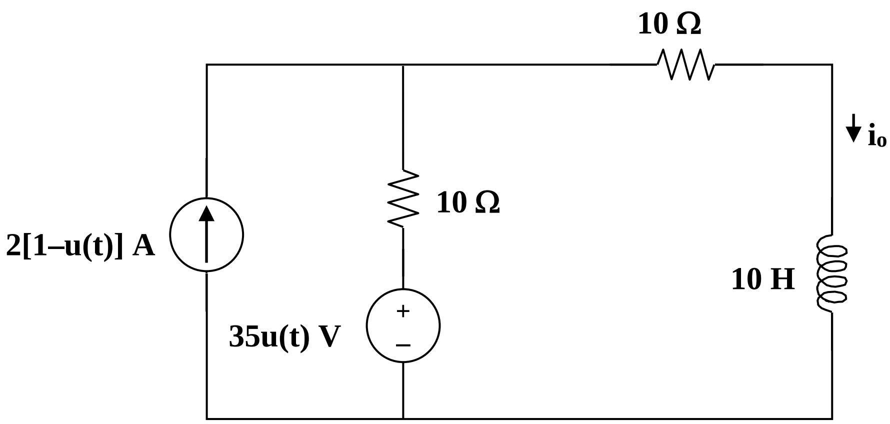


Figure 7.143
For Prob. 7.79.

Solution

For all t < 0, the voltage source is equal to zero (a short) and $i_o = 2x10/(10+10) = 1$ A.

For all 0 < t, the voltage source is equal to 35 V and the current source is equal to zero (an open circuit). At $t = \infty$, $i_o(\infty) = 35/20 = 1.75$ A. Additionally, $R_{eq} = 20\ \Omega$ and $\tau = L/R_{eq} = 10/20 = 0.5$ sec.

Finally,

$i_o(t) = 1.75 + [1–1.75]e^{-2t} =$ $\mathbf{[1.75 - 0.75e^{-2t}]u(t)\ A}$.

Solution 7.80

In the circuit of Fig. 7.144, find the value of i_o for all values of $0 < t$.

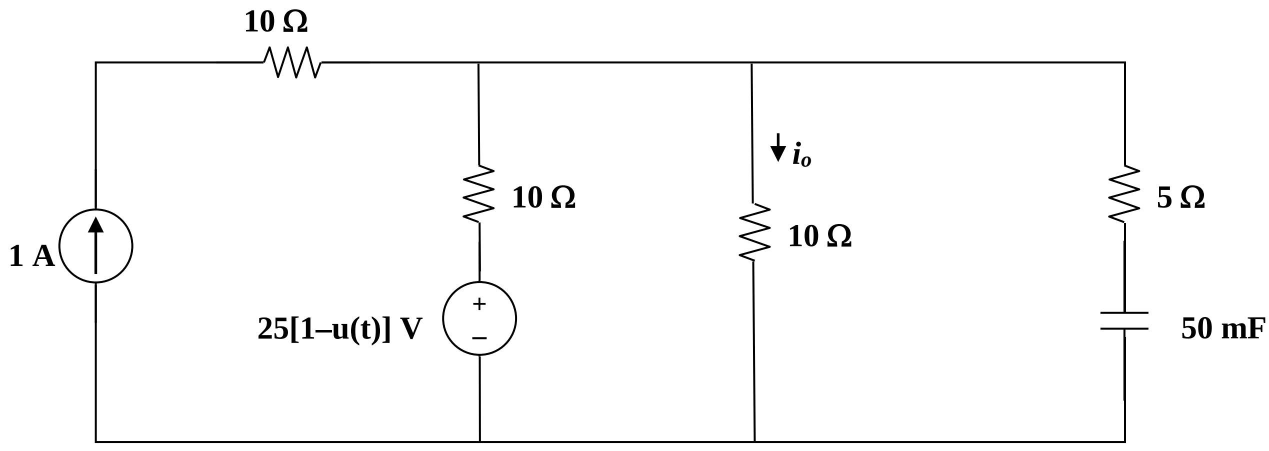


Figure 7.144
For Prob. 7.80.

Solution

For all values of $t < 0$, the current source is equal to 1 A and the voltage source is equal to 25 V. In addition the capacitor is equal to an open circuit. Thus, if we let v_o be the voltage at the top node and taking the bottom node as reference we get,

$-1 + [(v_o-25)/10] + [(v_o-0)/10] = 0$ and $v_o = 35/2 = 17.5$ V and $v_C(0) = 17.5$ V. Note, we can neglect the resistor in series with the current source.

For all values of $0 < t$, the current source is still equal to 1 A but the voltage source is now equal to zero (a short). We can now use the following equations to find i_o.

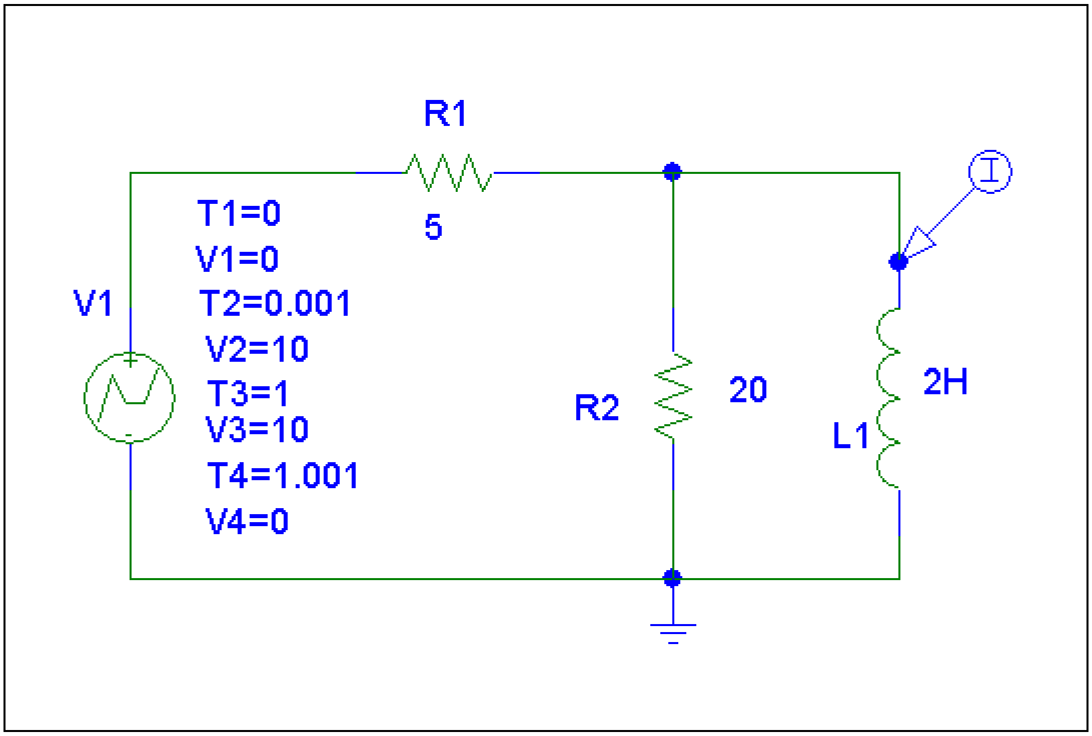
At $t = \infty$, the capacitor is an open circuit and the value of $v_C(\infty) = 5$ V, $R_{eq} = [10x10/(10+10)] + 5 = 10\ \Omega$, and $\tau = 10x0.05 = 0.5$ sec.

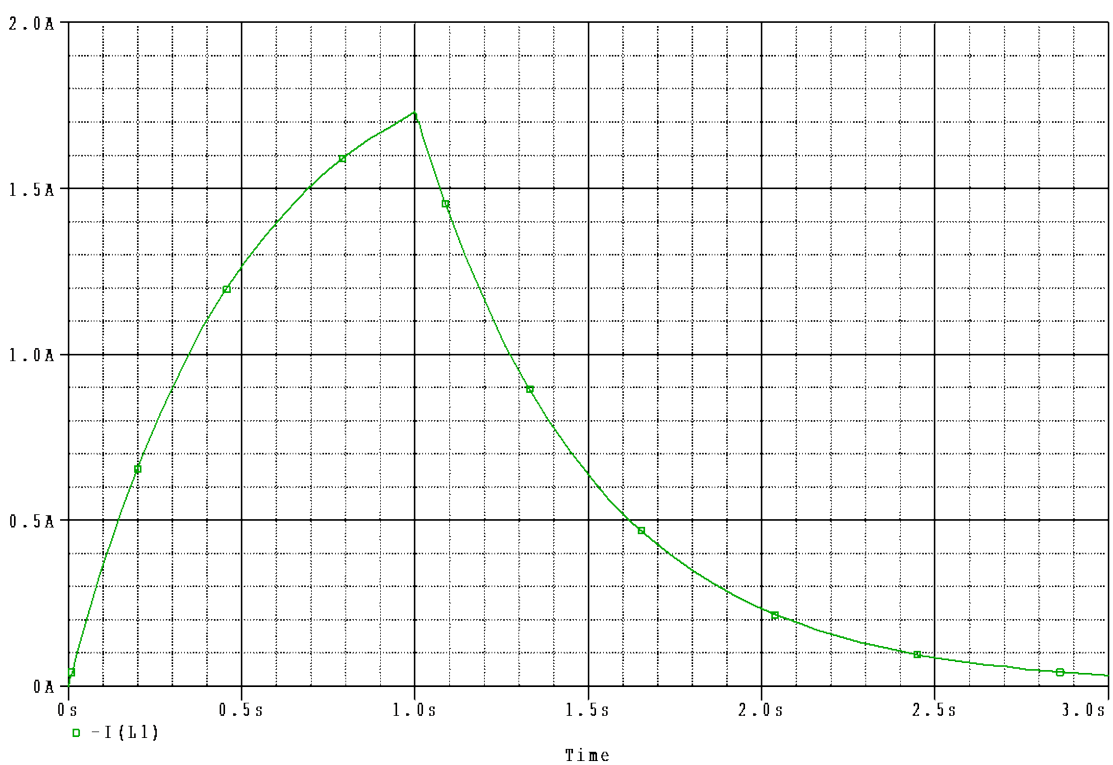
$v_C = [5 + 12.5e^{-2t}]u(t)$ V and $-1 + [(v_o-0)/10] + [(v_o-0)/10] + [(v_o-v_C)/5] = 0$ or $(0.1+0.1+0.2)v_o = 1 + 0.2v_C = 1 + 1 + 2.5e^{-2t} = 2 + 2.5e^{-2t}$ or $v_o = 5 + 6.25e^{-2t}$. Thus,

$i_o = v_o/10 =$ $\mathbf{[500+625e^{-2t}]u(t)}$ **mA**.

Solution 7.81

The schematic is shown below. We use VPWL for the pulse and specify the attributes as shown. In the Analysis/Setup/Transient menu, we select Print Step = 25 ms and final Step = 3 S. By inserting a current marker at one terminal of LI, we automatically obtain the plot of i after simulation as shown below.





Solution 7.82

$$\tau = RC \quad \longrightarrow \quad R = \frac{\tau}{C} = \frac{3\times 10^{-3}}{100\times 10^{-6}} = \mathbf{30\,\Omega}$$

Solution 7.83

$$v(\infty) = 120, \qquad v(0) = 0, \qquad \tau = RC = 34x10^6\,x15x10^{-6} = 510s$$

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \quad \longrightarrow \quad 85.6 = 120(1 - e^{-t/510})$$

Solving for t gives

$$t = 510\ln 3.488 = 637.16s$$

speed = 4000m/637.16s = **6.278m/s**

Solution 7.84

A capacitor with a value of 10 mF has a leakage resistance of 2 MΩ. How long does it take the voltage across the capacitor to decay to 40% of the initial voltage to which the capacitor is charged? Assume that the capacitor is charged and then set aside by itself.

Solution

The voltage across a charged capacitor is equal to $v_C(t) = v_C(0)e^{-t/\tau}$ where $\tau = R_{leak}C = (2x10^6)(0.01) = 2x10^4$. Thus,

$0.4v_C(0) = v_C(0)e^{-t/20,000}$ or $-t/20,000 = \ln(0.4) = -0.91629$ or t = 18.326 ks or

t = **5.091 hours**.

Solution 7.85

(a) The light is on from 75 volts until 30 volts. During that time we essentially have a 120-ohm resistor in parallel with a 6-μF capacitor.

$v(0) = 75$, $v(\infty) = 0$, $\tau = 120x6x10^{-6} = 0.72$ ms

$v(t_1) = 75e^{-t_1/\tau} = 30$ which leads to $t_1 = -0.72\ln(0.4)$ ms = **659.7 μs** of lamp on time.

(b) $\tau = RC = (4\times10^6)(6\times10^{-6}) = 24$ s

Since $v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$

$$v(t_1) - v(\infty) = [v(0) - v(\infty)]e^{-t_1/\tau} \qquad (1)$$

$$v(t_2) - v(\infty) = [v(0) - v(\infty)]e^{-t_2/\tau} \qquad (2)$$

Dividing (1) by (2),

$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau\ln\left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)}\right)$$

$$t_0 = 24\ln\left(\frac{75-120}{30-120}\right) = 24\ln(2) = \mathbf{16.636\ s}$$

Solution 7.86

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(\infty) = 12, \qquad v(0) = 0$$

$$v(t) = 12\left(1 - e^{-t/\tau}\right)$$

$$v(t_0) = 8 = 12\left(1 - e^{-t_0/\tau}\right)$$

$$\frac{8}{12} = 1 - e^{-t_0/\tau} \quad \longrightarrow \quad e^{-t_0/\tau} = \frac{1}{3}$$

$$t_0 = \tau \ln(3)$$

For $R = 100\ k\Omega$,

$$\tau = RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2\ s$$

$$t_0 = 0.2 \ln(3) = 0.2197\ s$$

For $R = 1\ M\Omega$,

$$\tau = RC = (1 \times 10^6)(2 \times 10^{-6}) = 2\ s$$

$$t_0 = 2 \ln(3) = 2.197\ s$$

Thus,

$$\mathbf{0.2197\ s < t_0 < 2.197\ s}$$

Solution 7.87

Let i be the inductor current.

For t < 0, $i(0^-) = \frac{120}{100} = 1.2 \text{ A}$

For t > 0, we have an RL circuit

$$\tau = \frac{L}{R} = \frac{50}{100+400} = 0.1, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 1.2 e^{-10t}$$

At t = 100 ms = 0.1 s,

$i(0.1) = 1.2 e^{-1} =$ **441mA**

which is the same as the current through the resistor.

Solution 7.88

(a) $\tau = RC = (300 \times 10^{3})(200 \times 10^{-12}) = 60\ \mu s$

As a differentiator,

$T > 10\tau = 600\ \mu s = 0.6\ ms$

i.e. $T_{min} = \mathbf{0.6\ ms}$

(b) $\tau = RC = 60\ \mu s$

As an integrator,

$T < 0.1\tau = 6\ \mu s$

i.e. $T_{max} = \mathbf{6\ \mu s}$

Solution 7.89

Since $\tau < 0.1T = 1\ \mu s$

$$\frac{L}{R} < 1\ \mu s$$

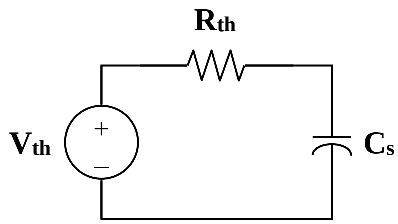
$$L < R \times 10^{-6} = (200 \times 10^{3})(1 \times 10^{-6})$$

$$\mathbf{L < 200\ mH}$$

Solution 7.90

We determine the Thevenin equivalent circuit for the capacitor C_s.

$$v_{th} = \frac{R_s}{R_s + R_p} v_i, \qquad R_{th} = R_s \parallel R_p$$



The Thevenin equivalent is an RC circuit. Since

$$v_{th} = \frac{1}{10} v_i \quad \longrightarrow \quad \frac{1}{10} = \frac{R_s}{R_s + R_p}$$

$$R_s = \frac{1}{9} R_p = \frac{6}{9} = \mathbf{\frac{2}{3}\ M\Omega}$$

Also,

$$\tau = R_{th} C_s = 15\ \mu s$$

where $R_{th} = R_p \parallel R_s = \dfrac{6(2/3)}{6+2/3} = 0.6\ M\Omega$

$$C_s = \frac{\tau}{R_{th}} = \frac{15\times10^{-6}}{0.6\times10^{6}} = \mathbf{25\ pF}$$

Solution 7.91

$$i_o(0) = \frac{12}{50} = 240\text{ mA}, \qquad i(\infty) = 0$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 240\, e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{2}{R}$$

$$i(t_0) = 10 = 240\, e^{-t_0/\tau}$$

$$e^{t_0/\tau} = 24 \quad \longrightarrow \quad t_0 = \tau \ln(24)$$

$$\tau = \frac{t_0}{\ln(24)} = \frac{5}{\ln(24)} = 1.573 = \frac{2}{R}$$

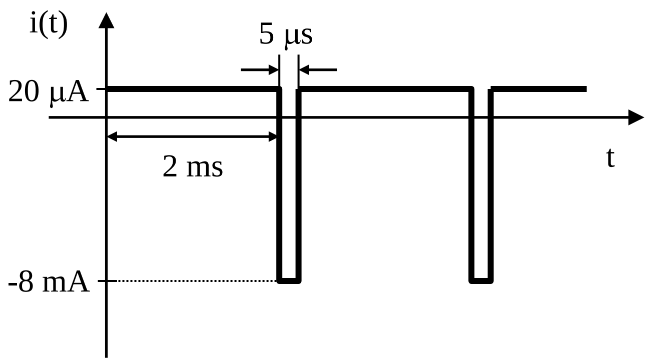
$$R = \frac{2}{1.573} = \mathbf{1.271\,\Omega}$$

Solution 7.92

$$i = C\frac{dv}{dt} = 4\times10^{-9}\cdot\begin{cases}\dfrac{10}{2\times10^{-3}} & 0 < t < t_R \\ \dfrac{-10}{5\times10^{-6}} & t_R < t < t_D\end{cases}$$

$$i(t) = \begin{cases}20\ \mu A & 0 < t < 2\text{ ms} \\ -8\text{ mA} & 2\text{ ms} < t < 2\text{ ms} + 5\ \mu s\end{cases}$$

which is sketched below.



(not to scale)