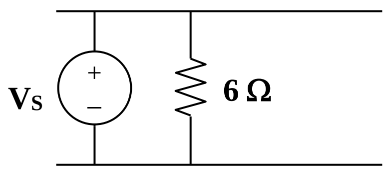
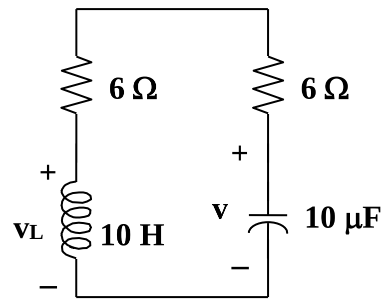
Solution 8.1

(a) At $t = 0-$, the circuit has reached steady state so that the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0-) = 12/6 = 2A,\ v(0-) = 12V$$

$$\text{At } t = 0+,\ i(0+) = i(0-) = \mathbf{2A},\ v(0+) = v(0-) = \mathbf{12V}$$

(b) For $t > 0$, we have the equivalent circuit shown in Figure (b).

$$v_L = Ldi/dt \text{ or } di/dt = v_L/L$$

Applying KVL at $t = 0+$, we obtain,

$$v_L(0+) - v(0+) + 10i(0+) = 0$$

$$v_L(0+) - 12 + 20 = 0, \text{ or } v_L(0+) = -8$$

Hence, $di(0+)/dt = -8/2 = \mathbf{-4\ A/s}$

Similarly, $i_C = Cdv/dt$, or $dv/dt = i_C/C$

$$i_C(0+) = -i(0+) = -2$$

$$dv(0+)/dt = -2/0.4 = \mathbf{-5\ V/s}$$

(c) As t approaches infinity, the circuit reaches steady state.

$$i(\infty) = \mathbf{0\ A},\ v(\infty) = \mathbf{0\ V}$$

Solution 8.2

Using Fig. 8.63, design a problem to help other students better understand finding initial and final values.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit of Fig. 8.63, determine:

(a) $i_R(0^+)$, $i_L(0^+)$, and $i_C(0^+)$,

(b) $di_R(0^+)/dt$, $di_L(0^+)/dt$, and $di_C(0^+)/dt$,

(c) $i_R(\infty)$, $i_L(\infty)$, and $i_C(\infty)$.

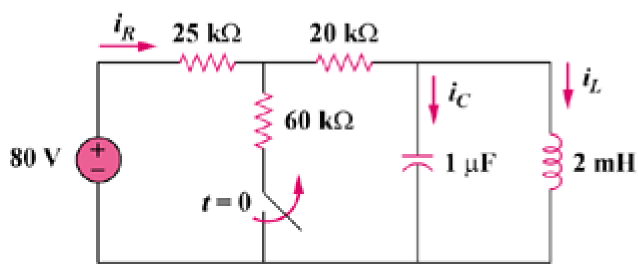
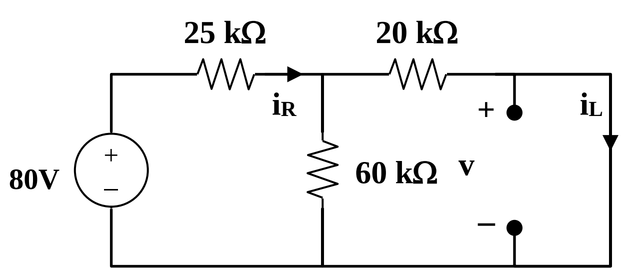


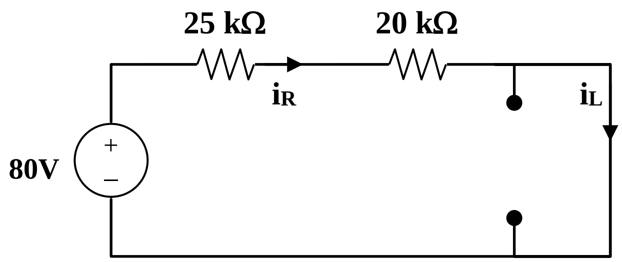
Figure 8.63

Solution

(a) At t = 0-, the equivalent circuit is shown in Figure (a).



(a)



(b)

$60||20 = 15$ kohms, $i_R(0\text{-}) = 80/(25 + 15) = 2$mA.

By the current division principle,

$$i_L(0\text{-}) = 60(2\text{mA})/(60 + 20) = 1.5 \text{ mA}$$

$$v_C(0\text{-}) = 0$$

At $t = 0+$,

$$v_C(0+) = v_C(0\text{-}) = 0$$

$$i_L(0+) = i_L(0\text{-}) = \mathbf{1.5\ mA}$$

$$80 = i_R(0+)(25 + 20) + v_C(0\text{-})$$

$$i_R(0+) = 80/45\text{k} = \mathbf{1.778\ mA}$$

But, $i_R = i_C + i_L$

$$1.778 = i_C(0+) + 1.5 \text{ or } i_C(0+) = \mathbf{0.278\ mA}$$

(b) $v_L(0+) = v_C(0+) = 0$

But, $v_L = Ldi_L/dt$ and $di_L(0+)/dt = v_L(0+)/L = 0$

$$di_L(0+)/dt = \mathbf{0}$$

Again, $80 = 45i_R + v_C$

$$0 = 45di_R/dt + dv_C/dt$$

But, $dv_C(0+)/dt = i_C(0+)/C = 0.278 \text{ mamps}/1\ \mu\text{F} = 278 \text{ V/s}$

Hence, $di_R(0+)/dt = (-1/45)dv_C(0+)/dt = -278/45$

$$di_R(0+)/dt = \mathbf{-6.1778\ A/s}$$

$$\text{Also,} \quad i_R = i_C + i_L$$

$$di_R(0+)/dt = di_C(0+)/dt + di_L(0+)/dt$$

$$-6.1788 = di_C(0+)/dt + 0, \text{ or } di_C(0+)/dt = \mathbf{-6.1788\ A/s}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (b).

$$i_R(\infty) = i_L(\infty) = 80/45k = \mathbf{1.778\ mA}$$

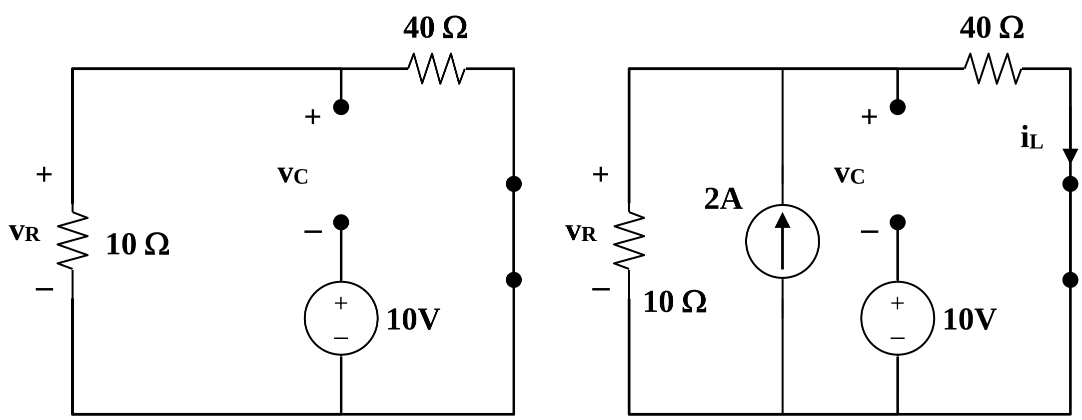
$$i_C(\infty) = Cdv(\infty)/dt = \mathbf{0}.$$

Solution 8.3

At $t = 0^-$, $u(t) = 0$. Consider the circuit shown in Figure (a). $i_L(0^-) = 0$, and $v_R(0^-) = 0$. But, $-v_R(0^-) + v_C(0^-) + 10 = 0$, or $v_C(0^-) = -10V$.

(a) At $t = 0^+$, since the inductor current and capacitor voltage cannot change abruptly, the inductor current must still be equal to **0A**, the capacitor has a voltage equal to **–10V**. Since it is in series with the +10V source, together they represent a direct short at $t = 0^+$. This means that the entire 2A from the current source flows through the capacitor and not the resistor. Therefore, $v_R(0^+) = \mathbf{0\ V}$.

(b) At $t = 0^+$, $v_L(0+) = 0$, therefore $Ldi_L(0+)/dt = v_L(0^+) = 0$, thus, $di_L/dt = \mathbf{0A/s}$, $i_C(0^+) = 2$ A, this means that $dv_C(0^+)/dt = 2/C = \mathbf{8\ V/s}$. Now for the value of $dv_R(0^+)/dt$. Since $v_R = v_C + 10$, then $dv_R(0^+)/dt = dv_C(0^+)/dt + 0 = \mathbf{8\ V/s}$.



(a) (b)

(c) As t approaches infinity, we end up with the equivalent circuit shown in Figure (b).

$$i_L(\infty) = 10(2)/(40 + 10) = \mathbf{400\ mA}$$

$$v_C(\infty) = 2[10\|40] - 10 = 16 - 10 = \mathbf{6V}$$

$$v_R(\infty) = 2[10\|40] = \mathbf{16\ V}$$

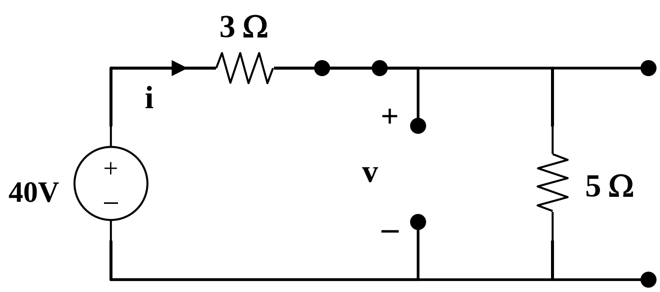
Solution 8.4

(a) At $t = 0^-$, $u(-t) = 1$ and $u(t) = 0$ so that the equivalent circuit is shown in Figure (a).

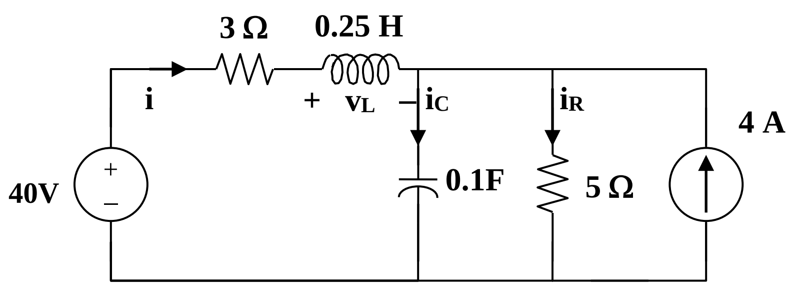
$$i(0^-) = 40/(3 + 5) = 5\text{A}, \text{ and } v(0^-) = 5i(0^-) = 25\text{V}.$$

Hence, $i(0^+) = i(0^-) = \mathbf{5A}$

$v(0^+) = v(0^-) = \mathbf{25V}$



(a)



(b)

(b) $i_C = Cdv/dt$ or $dv(0^+)/dt = i_C(0^+)/C$

For $t = 0^+$, $4u(t) = 4$ and $4u(-t) = 0$. The equivalent circuit is shown in Figure (b). Since i and v cannot change abruptly,

$$i_R = v/5 = 25/5 = 5\text{A}, \; i(0^+) + 4 = i_C(0^+) + i_R(0^+)$$

$$5 + 4 = i_C(0^+) + 5 \text{ which leads to } i_C(0^+) = 4$$

$$dv(0^+)/dt = 4/0.1 = \mathbf{40\ V/s}$$

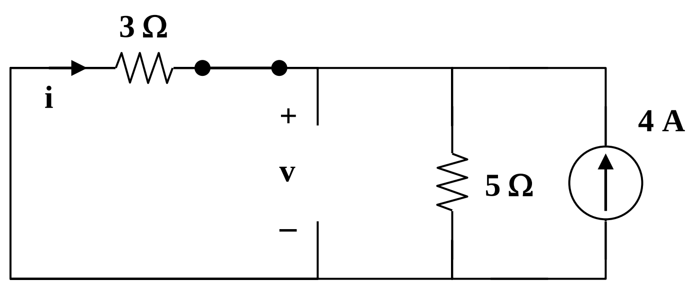
Similarly, $v_L = Ldi/dt$ which leads to $di(0^+)/dt = v_L(0^+)/L$

$$3i(0^+) + v_L(0^+) + v(0^+) = 0$$

$$15 + v_L(0^+) + 25 = 0 \text{ or } v_L(0^+) = -40$$

$$di(0^+)/dt = -40/0.25 = \mathbf{-160\ A/s}$$

(c) As t approaches infinity, we have the equivalent circuit in Figure (c).



(c)

$$i(\infty) = -5(4)/(3 + 5) = \mathbf{-2.5\ A}$$

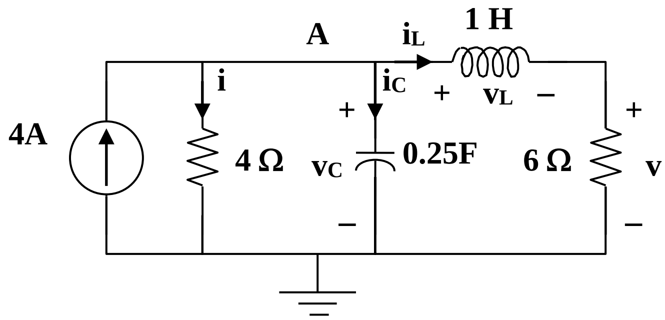
$$v(\infty) = 5(4 - 2.5) = \mathbf{7.5\ V}$$

Solution 8.5

(a) For $t < 0$, $4u(t) = 0$ so that the circuit is not active (all initial conditions = 0).

$$i_L(0-) = 0 \text{ and } v_C(0-) = 0.$$

For $t = 0+$, $4u(t) = 4$. Consider the circuit below.



Since the 4-ohm resistor is in parallel with the capacitor,

$$i(0+) = v_C(0+)/4 = 0/4 = \mathbf{0\ A}$$

Also, since the 6-ohm resistor is in series with the inductor,
$v(0+) = 6i_L(0+) =$ **0V**.

(b) $di(0+)/dt = d(v_R(0+)/R)/dt = (1/R)dv_R(0+)/dt = (1/R)dv_C(0+)/dt$

$$= (1/4)4/0.25 \text{ A/s} = \mathbf{4\ A/s}$$

$v = 6i_L$ or $dv/dt = 6di_L/dt$ and $dv(0+)/dt = 6di_L(0+)/dt = 6v_L(0+)/L = 0$

$$\text{Therefore } dv(0+)/dt = \mathbf{0\ V/s}$$

(c) As t approaches infinity, the circuit is in steady-state.

$$i(\infty) = 6(4)/10 = \mathbf{2.4\ A}$$

$$v(\infty) = 6(4 - 2.4) = \mathbf{9.6\ V}$$

Solution 8.6

(a) Let i = the inductor current. For $t < 0$, $u(t) = 0$ so that

$$i(0) = 0 \text{ and } v(0) = 0.$$

For $t > 0$, $u(t) = 1$. Since, $v(0+) = v(0-) = 0$, and $i(0+) = i(0-) = 0$.

$$v_R(0+) = Ri(0+) = \mathbf{0\ V}$$

Also, since $v(0+) = v_R(0+) + v_L(0+) = 0 = 0 + v_L(0+)$ or $v_L(0+) = \mathbf{0\ V}$.
(1)

(b) Since $i(0+) = 0$, $i_C(0+) = V_S/R_S$

But, $i_C = Cdv/dt$ which leads to $dv(0+)/dt = V_S/(CR_S)$
(2)

From (1), $dv(0+)/dt = dv_R(0+)/dt + dv_L(0+)/dt$
(3)

$v_R = iR$ or $dv_R/dt = Rdi/dt$
(4)

But, $v_L = Ldi/dt$, $v_L(0+) = 0 = Ldi(0+)/dt$ and $di(0+)/dt = 0$ (5)

From (4) and (5), $dv_R(0+)/dt = \mathbf{0\ V/s}$

From (2) and (3), $dv_L(0+)/dt = dv(0+)/dt = \mathbf{V_s/(CR_s)}$

(c) As t approaches infinity, the capacitor acts like an open circuit, while the inductor acts like a short circuit.

$$v_R(\infty) = \mathbf{[R/(R + R_s)]V_s}$$

$$v_L(\infty) = \mathbf{0\ V}$$

Solution 8.7

$\alpha = [R/(2L)] = 20x10^3/(2x0.2x10^{-3}) = 50x10^6$

$\omega_o = [1/(LC)^{0.5}] = 1/(0.2x10^{-3}x5x10^{-6})^{0.5} = 3.162\ x10^4$

$\alpha > \omega_o \longrightarrow \underline{\text{overdamped}}$

overdamped

Solution 8.8

Design a problem to help other students better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The branch current in an *RLC* circuit is described by the differential equation

$$\frac{d^2 i}{dt^2} + 6\frac{di}{dt} + 9i = 0$$

and the initial conditions are $i(0) = 0$, $di(0)/dt = 4$. Obtain the characteristic equation and determine $i(t)$ for $t > 0$.

Solution

$$\mathbf{s^2 + 6s + 9 = 0}, \text{ thus } s_{1,2} = \frac{-6 \pm \sqrt{6^2 - 36}}{2} = -3, \text{ repeated roots.}$$

$$i(t) = [(A + Bt)e^{-3t}],\ i(0) = 0 = A$$

$$di/dt = [Be^{-3t}] + [-3(Bt)e^{-3t}]$$

$$di(0)/dt = 4 = B.$$

$$\text{Therefore, } i(t) = \mathbf{[4te^{-3t}]\ A}$$

Solution 8.9

$s^2 + 10s + 25 = 0$, thus $s_{1,2} = \dfrac{-10 \pm \sqrt{10-10}}{2} = \mathbf{-}5$, repeated roots.

$$i(t) = [(A + Bt)e^{-5t}],\ \ i(0) = 10 = A$$

$$di/dt = [Be^{-5t}] + [-5(A + Bt)e^{-5t}]$$

$$di(0)/dt = 0 = B - 5A = B - 50 \text{ or } B = 50.$$

Therefore, $i(t) = \mathbf{[(10 + 50t)e^{-5t}]\ A}$

Solution 8.10

The differential equation that describes the current in an *RLC* network is

$$3\frac{di^2}{dt^2}+15\frac{di}{dt}+12i=0$$

Given that $i(0) = 0$, $di(0)/dt = 6$ mA/s, obtain $i(t)$.

Solution

$$s^2 + 5s + 4 = 0, \text{ thus } s_{1,2} = \frac{-5\pm\sqrt{25-16}}{2} = -4, -1.$$

$$i(t) = (Ae^{-4t} + Be^{-t}), \; i(0) = 0 = A + B, \text{ or } B = -A$$

$$di/dt = (-4Ae^{-4t} - Be^{-t})$$

$$di(0)/dt = 0.006 = -4A - B = -3A \text{ or } A = -0.006/3 = -0.002 \text{ and } B = 0.002.$$

Therefore, $i(t) = \mathbf{(-2e^{-4t} + 2e^{-t})\ mA.}$

Solution 8.11

$$s^2 + 2s + 1 \;=\; 0,\;\; \text{thus}\;\; s_{1,2} \;=\; \frac{-2 \pm \sqrt{4-4}}{2} \;=\; -1,\;\; \text{repeated roots.}$$

$$v(t) \;=\; [(A + Bt)e^{-t}],\;\; v(0) \;=\; 10 \;=\; A$$

$$dv/dt \;=\; [Be^{-t}] + [-(A + Bt)e^{-t}]$$

$$dv(0)/dt \;=\; 0 \;=\; B - A \;=\; B - 10 \;\; \text{or} \;\; B \;=\; 10.$$

Therefore, $v(t) \;=\; \mathbf{[(10 + 10t)e^{-t}]\ V}$

Solution 8.12

(a) Overdamped when $C > 4L/(R^2) = 4x1.5/2500 = 2.4x10^{-3}$, or

$C >$ **2.4 mF**

(b) Critically damped when $C =$ **2.4 mF**

(c) Underdamped when $C <$ **2.4 mF**

Solution 8.13

Let R||60 = R_o. For a series RLC circuit,

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.01x4}} = 5$$

For critical damping, $\omega_o = \alpha = R_o/(2L) = 5$

or $R_o = 10L = 40 = 60R/(60 + R)$

which leads to R = **120 ohms**

Solution 8.14

When the switch is in position A, $v(0^-)= 0$ and $i_L(0) = 80/40 = 2$ A. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

When the switch is in position A, $v(0^-)= 0$. When the switch is in position B, we have a source-free series RCL circuit.

$$\alpha = \frac{R}{2L} = \frac{10}{2x4} = 1.25$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x4}} = 1$$

Since $\alpha > \omega_o$, we have overdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.25 \pm \sqrt{1.562}$$ –0.5 and –2 0.9336

$$v(t) = Ae^{-2t} + Be^{-0.5t} \qquad (1)$$

$$v(0) = 0 = A + B \qquad (2)$$

$i_C(0) = C(dv(0)/dt) = -2$ or $dv(0)/dt = -2/C = -8$.

But $$\frac{dv(t)}{dt} = -2Ae^{-2t} - 0.5Be^{-0.5t}$$

$$\frac{dv(0)}{dt} = -2A - 0.5B = -8 \qquad (3)$$

Solving (2) and (3) gives A= 1.3333 and B = –1.3333

$v(t) =$ $\mathbf{5.333e^{-2t} - 5.333e^{-0.5t}}$ **V**.

Solution 8.15

Given that s_1 = -10 and s_2 = -20, we recall that

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -20$$

Clearly, $s_1 + s_2 = -2\alpha = -30$ or $\alpha = 15 = R/(2L)$ or $R = 60L$ (1)

$$s_1 = -15 + \sqrt{15^2 - \omega_o^2} = -10 \text{ which leads to } 15^2 - \omega_o^2 = 25$$

$$\text{or } \omega_o = \sqrt{225 - 25} = \sqrt{200} = 1/\sqrt{LC}, \text{ thus } LC = 1/200 \quad (2)$$

Since we have a series RLC circuit, $i_L = i_C = Cdv_C/dt$ which gives,

$$i_L/C = dv_C/dt = [200e^{-20t} - 300e^{-30t}] \text{ or } i_L = 100C[2e^{-20t} - 3e^{-30t}]$$

But, i is also $= 20\{[2e^{-20t} - 3e^{-30t}]x10^{-3}\} = 100C[2e^{-20t} - 3e^{-30t}]$

Therefore, $C = (0.02/10^2) =$ **200 μF**

$L = 1/(200C) =$ **25 H**

$R = 30L =$ **750 ohms**

Solution 8.16

At $t = 0$, $i(0) = 0$, $v_C(0) = 40x30/50 = 24V$

For $t > 0$, we have a source-free RLC circuit.

$$\alpha = R/(2L) = (40 + 60)/5 = 20 \text{ and } \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x2.5}} = 20$$

$$\omega_o = \alpha \text{ leads to critical damping}$$

$$i(t) = [(A + Bt)e^{-20t}], \; i(0) = 0 = A$$

$$di/dt = \{[Be^{-20t}] + [-20(Bt)e^{-20t}]\},$$

$$\text{but } di(0)/dt = -(1/L)[Ri(0) + v_C(0)] = -(1/2.5)[0 + 24]$$

Hence, $B = -9.6$ or $i(t) = $ **$[-9.6te^{-20t}]$ A**

Solution 8.17

$$i(0) = I_0 = 0,\ v(0) = V_0 = 4x5 = 20$$
$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0) = -4(0+20) = -80$$
$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}\frac{1}{25}}} = 10$$
$$\alpha = \frac{R}{2L} = \frac{10}{2\frac{1}{4}} = 20,\ which\,is > \omega_o.$$
$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -20 \pm \sqrt{300} = -20 \pm 10\sqrt{3} = -2.679, -37.32$$
$$i(t) = A_1e^{-2.679t} + A_2e^{-37.32t}$$
$$i(0) = 0 = A_1 + A_2,\ \frac{di(0)}{dt} = -2.679A_1 - 37.32A_2 = -80$$
$$This\,leads\,to\,A_1 = -2.309 = -A_2$$
$$i(t) = 2.309\left(e^{-37.32t} - e^{-2.679t}\right)$$

$$Since, v(t) = \frac{1}{C}\int_0^t i(t)dt + 20,\ we\,get$$

v(t) = **[$21.55e^{-2.679t} - 1.55e^{-37.32t}$] V**

Solution 8.18

When the switch is off, we have a source-free parallel RLC circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2, \qquad \alpha = \frac{1}{2RC} = 0.5$$

$$\alpha < \omega_o \qquad \longrightarrow \qquad \text{underdamped case} \quad \omega_\mathrm{d} = \sqrt{\omega_o^{\ 2} - \alpha^2} = \sqrt{4 - 0.25} = 1.936$$

$I_o(0)$ = i(0) = initial inductor current = 100/5 = 20 A

$V_o(0)$ = v(0) = initial capacitor voltage = 0 V

$$v(t) = e^{-\alpha t}(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)) = e^{-0.5\alpha t}(A_1 \cos(1.936t) + A_2 \sin(1.936t))$$

v(0) =0 = A_1

$$\frac{dv}{dt} = e^{-0.5\alpha t}(-0.5)(A_1 \cos(1.936t) + A_2 \sin(1.936t))$$
$$+ e^{-0.5\alpha t}(-1.936A_1 \sin(1.936t) + 1.936A_2 \cos(1.936t))$$

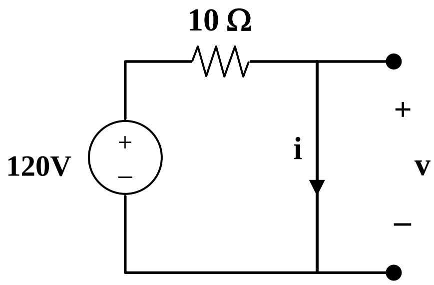
$$\frac{dv(0)}{dt} = -\frac{(V_o + RI_o)}{RC} = -\frac{(0+20)}{1} = -20 = -0.5A_1 + 1.936A_2 \qquad \longrightarrow \qquad A_2 = -10.333$$

Thus,

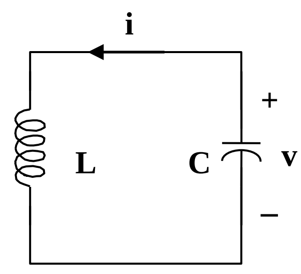
$$\underline{\boldsymbol{v(t) = [-10.333e^{-0.5t} \sin(1.936t)]volts}}$$

Solution 8.19

For $t < 0$, the equivalent circuit is shown in Figure (a).



(a)



(b)

$$i(0) = 120/10 = 12, \ v(0) = 0$$

For $t > 0$, we have a series RLC circuit as shown in Figure (b) with $R = 0 = \alpha$.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4}} = 0.5 = \omega_d$$

$$i(t) = [A\cos 0.5t + B\sin 0.5t], \ i(0) = 12 = A$$

$$v = -Ldi/dt, \text{ and } -v/L = di/dt = 0.5[-12\sin 0.5t + B\cos 0.5t],$$

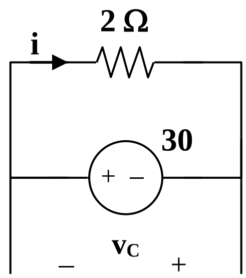
$$\text{which leads to } -v(0)/L = 0 = B$$

Hence, $i(t) = 12\cos 0.5t$ A and $v = 0.5$

However, $v = -Ldi/dt = -4(0.5)[-12\sin 0.5t] =$ **24sin(0.5t) V**

Solution 8.20

For $t < 0$, the equivalent circuit is as shown below.



$$v(0) = -30 \text{ V} \quad \text{and} \quad i(0) = 30/2 = 15 \text{ A}$$

For $t > 0$, we have a series RLC circuit.

$$\alpha = R/(2L) = 2/(2x0.5) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{0.5x1/4} = 2\sqrt{2}$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{8-4} = 2$$

$$i(t) = (A\cos(2t) + B\sin(2t))e^{-2t}$$

$$i(0) = 15 = A$$

$$di/dt = -2(15\cos(2t) + B\sin(2t))e^{-2t} + (-2x15\sin(2t) + 2B\cos(2t))e^{-\alpha t}$$

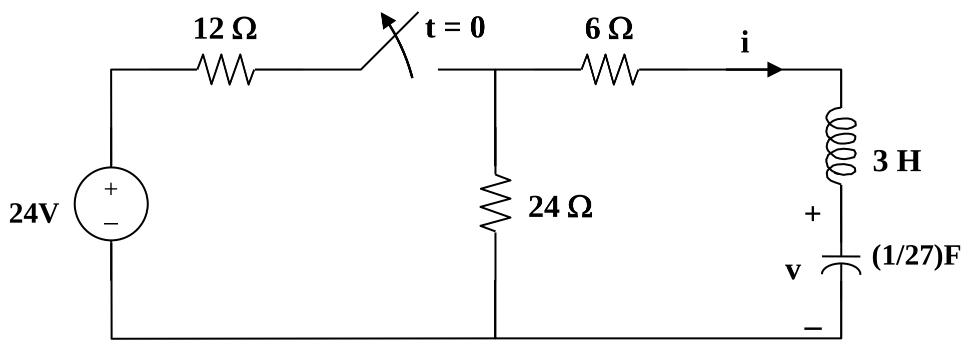
$$di(0)/dt = -30 + 2B = -(1/L)[Ri(0) + v_C(0)] = -2[30 - 30] = 0$$

Thus, $B = 15$ and $i(t) =$ $\mathbf{(15\cos(2t) + 15\sin(2t))e^{-2t}}$ **A**

Solution 8.21

By combining some resistors, the circuit is equivalent to that shown below.

$60\|(15 + 25) = 24$ ohms.



At $t = 0-$, $i(0) = 0$, $v(0) = 24x24/36 = 16V$

For $t > 0$, we have a series RLC circuit. $R = 30$ ohms, $L = 3$ H, $C = (1/27)$ F

$$\alpha = R/(2L) = 30/6 = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{3x1/27} = 3, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm \sqrt{5^2 - 3^2} = -9, -1$$

$$v(t) = [Ae^{-t} + Be^{-9t}],\ v(0) = 16 = A + B \qquad (1)$$

$$i = Cdv/dt = C[-Ae^{-t} - 9Be^{-9t}]$$

$$i(0) = 0 = C[-A - 9B] \text{ or } A = -9B \qquad (2)$$

From (1) and (2), $B = -2$ and $A = 18$.

Hence, $v(t) = \mathbf{(18e^{-t} - 2e^{-9t})\ V}$

Solution 8.22

Compare the characteristic equation with eq. (8.8), i.e.

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

we obtain

$$\frac{R}{L} = 100 \quad \longrightarrow \quad L = \frac{R}{100} = \frac{2000}{100} = \underline{20H}$$

$$\frac{1}{LC} = 10^6 \quad \rightarrow \quad C = \frac{1}{10^6 L} = \frac{10^{-6}}{20} = \underline{50 \text{ nF}}$$

Solution 8.23

Let $C_o = C + 0.01$. For a parallel RLC circuit,

$\alpha = 1/(2RC_o)$, $\omega_o = 1/\sqrt{LC_o}$

$\alpha = 1 = 1/(2RC_o)$, we then have $C_o = 1/(2R) = 1/20 = 50$ mF

$\omega_o = 1/\sqrt{0.02x0.05} = 141.42 > \alpha$ (underdamped)

$C_o = C + 10$ mF $= 50$ mF or $C =$ **40 mF**

Solution 8.24

When the switch is in position A, the inductor acts like a short circuit so

$$i(0^-) = 4$$

When the switch is in position B, we have a source-free parallel RCL circuit

$$\alpha = \frac{1}{2RC} = \frac{1}{2x10x10x10^{-3}} = 5$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{4}x10x10^{-3}}} = 20$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -5 + \sqrt{25-400} = -5 + j19.365$$

$$i(t) = e^{-5t}\left(A_1 \cos 19.365t + A_2 \sin 19.365t\right)$$

$$i(0) = 4 = A_1$$

$$v = L\frac{di}{dt} \longrightarrow \frac{di(0)}{dt} = \frac{v(0)}{L} = 0$$

$$\frac{di}{dt} = eA^{-5t}\left(-5A_1 \cos 19.365t - 5A_2 \sin 19.365t - 19.365A_1 \sin 19.365t + 19.365A_2 \cos 19.365t\right)$$

0 = [di(0)/dt] = –5A$_1$ + 19.365A$_2$ or A$_2$ = 20/19.365 = 1.0328

i(t) = **e^{-5t}[4cos(19.365t) + 1.0328sin(19.365t)] A**

Solution 8.25

Using Fig. 8.78, design a problem to help other students to better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In the circuit in Fig. 8.78, calculate $i_o(t)$ and $v_o(t)$ for $t > 0$.

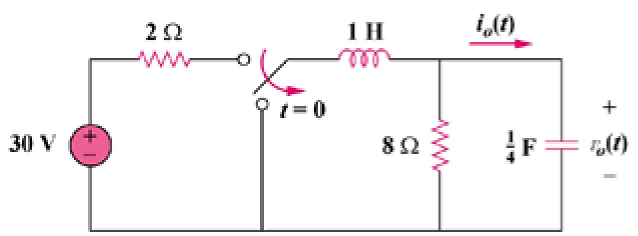


Figure 8.78

Solution

In the circuit in Fig. 8.76, calculate $i_o(t)$ and $v_o(t)$ for t>0.

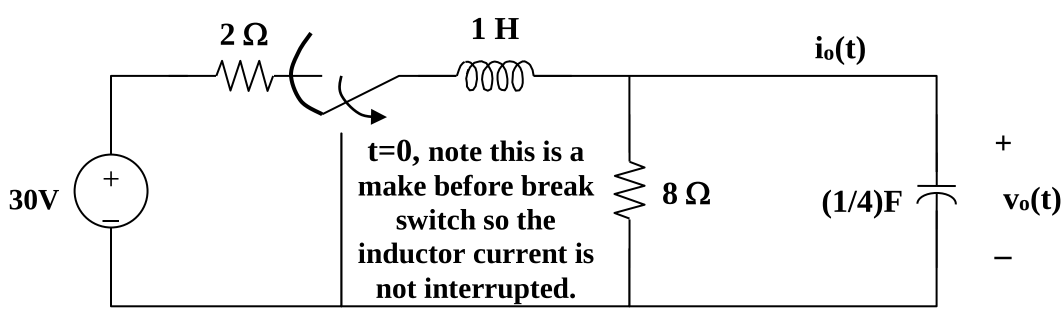


Figure 8.78 For Problem 8.25.

At t = 0^-, $v_o(0) = (8/(2 + 8)(30) = 24$

For t > 0, we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = ¼$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/4} = 2$$

Since α is less than ω_o, we have an under-damped response.

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4 - (1/16)} = 1.9843$$

$$v_o(t) = (A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t}$$

$$v_o(0) = 30(8/(2+8)) = 24 = A_1 \text{ and } i_o(t) = C(dv_o/dt) = 0 \text{ when } t = 0.$$

$$dv_o/dt = -\alpha(A_1\cos\omega_d t + A_2\sin\omega_d t)e^{-\alpha t} + (-\omega_d A_1\sin\omega_d t + \omega_d A_2\cos\omega_d t)e^{-\alpha t}$$

at $t = 0$, we get $dv_o(0)/dt = 0 = -\alpha A_1 + \omega_d A_2$

Thus, $A_2 = (\alpha/\omega_d)A_1 = (1/4)(24)/1.9843 = 3.024$

$v_o(t) =$ **$(24\cos1.9843t + 3.024\sin1.9843t)e^{-t/4}$ volts**.

$i_0(t) = Cdv/dt = 0.25[-24(1.9843)\sin1.9843t + 3.024(1.9843)\cos1.9843t - 0.25(24\cos1.9843t) - 0.25(3.024\sin1.9843t)]e^{-t/4}$
$=$ **$[-12.095\sin1.9843t]e^{-t/4}$ A**.

Solution 8.26

$$s^2 + 2s + 5 = 0, \text{ which leads to } s_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j4$$

These roots indicate an underdamped circuit which has the generalized solution given as:

$$i(t) = I_s + [(A_1\cos(4t) + A_2\sin(4t))e^{-t}],$$

At $t = \infty$, $(di(t)/dt) = 0$ and $(d^2i(t)/dt^2) = 0$ so that
$I_s = 10/5 = 2$ (from $(d^2i(t)/dt^2)+2(di(t)/dt)+5 = 10$)

$$i(0) = 2 = 2 + A_1, \text{ or } A_1 = 0$$

$$di/dt = [(4A_2\cos(4t))e^{-t}] + [(-A_2\sin(4t))e^{-t}] = 4 = 4A_2, \text{ or } A_2 = 1$$

$i(t) =$ **[2 + sin(4te^{-t})] amps**

Solution 8.27

$$s^2 + 4s + 8 \;=\; 0 \text{ leads to } s \;=\; \frac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$$

$$v(t) \;=\; V_s + (A_1\cos 2t + A_2\sin 2t)e^{-2t}$$

$$8V_s = 24 \text{ means that } V_s = 3$$

$$v(0) = 0 = 3 + A_1 \text{ leads to } A_1 = -3$$

$$dv/dt = -2(A_1\cos 2t + A_2\sin 2t)e^{-2t} + (-2A_1\sin 2t + 2A_2\cos 2t)e^{-2t}$$

$$0 = dv(0)/dt = -2A_1 + 2A_2 \text{ or } A_2 = A_1 = -3$$

v(t) = **[3 – 3(cos(2t) + sin(2t))e^{-2t}] volts**.

Solution 8.28

The characteristic equation is

$$Ls^2 + Rs + \frac{1}{C} = 0 \quad \longrightarrow \quad \frac{1}{2}s^2 + 4s + \frac{1}{0.2} = 0 \quad \longrightarrow \quad s^2 + 8s + 10 = 0$$

$$s_{1,2} = \frac{-8 \pm \sqrt{64 - 40}}{2} = \quad -6.45 \text{ and } -1.5505$$

$$\text{i(t)} = \text{i}_\text{s} + \text{Ae}^{-6.45\text{t}} + \text{Be}^{-1.5505\text{t}}$$

But $[i_s/C] = 10$ or $i_s = 0.2x10 = 2$

$$i(t) = 2 + Ae^{-6.45t} + Be^{-1.5505t}$$

$$\text{i(0)} = 1 = 2 + \text{A} + \text{B} \ \text{ or } \ \text{A} + \text{B} = -1 \ \text{ or } \text{A} = -1-\text{B} \qquad (1)$$

$$\frac{di(t)}{dt} = -6.45Ae^{-6.45t} - 1.5505Be^{-1.5505t}$$

$$but \ \frac{di(0)}{dt} = 0 = -6.45A - 1.5505B \qquad (2)$$

Solving (1) and (2) gives –6.45(–1–B) – 1.5505B = 0 or (6.45–1.5505)B = –6.45
B = –6.45/(4.9) = –1.3163 and A = –1–1.3163 = –2.3163

A= –2.3163, B= –1.3163

Hence,

i(t) = **[2–2.3163e$^{-6.45t}$ –1.3163e$^{-1.5505t}$] A**.

Solution 8.29

(a) $s^2 + 4 = 0$ which leads to $s_{1,2} = \pm j2$ (an undamped circuit)

$$v(t) = V_s + A\cos 2t + B\sin 2t$$

$$4V_s = 12 \text{ or } V_s = 3$$

$$v(0) = 0 = 3 + A \text{ or } A = -3$$

$$dv/dt = -2A\sin 2t + 2B\cos 2t$$

$$dv(0)/dt = 2 = 2B \text{ or } B = 1, \text{ therefore } v(t) = \mathbf{(3 - 3\cos 2t + \sin 2t)\ V}$$

(b) $s^2 + 5s + 4 = 0$ which leads to $s_{1,2} = -1, -4$

$$i(t) = (I_s + Ae^{-t} + Be^{-4t})$$

$$4I_s = 8 \text{ or } I_s = 2$$

$$i(0) = -1 = 2 + A + B, \text{ or } A + B = -3 \qquad (1)$$

$$di/dt = -Ae^{-t} - 4Be^{-4t}$$

$$di(0)/dt = 0 = -A - 4B, \text{ or } B = -A/4 \qquad (2)$$

From (1) and (2) we get $A = -4$ and $B = 1$

$$i(t) = \mathbf{(2 - 4e^{-t} + e^{-4t})\ A}$$

(c) $s^2 + 2s + 1 = 0$, $s_{1,2} = -1, -1$

$$v(t) = [V_s + (A + Bt)e^{-t}], \quad V_s = 3.$$

$$v(0) = 5 = 3 + A \text{ or } A = 2$$

$$dv/dt = [-(A + Bt)e^{-t}] + [Be^{-t}]$$

$$dv(0)/dt = -A + B = 1 \text{ or } B = 2 + 1 = 3$$

$$v(t) = \mathbf{[3 + (2 + 3t)e^{-t}]\ V}$$

(d) $s^2 + 2s + 5 = 0$, $s_{1,2} = -1 + j2, -1 - j2$

$$i(t) = [I_s + (A\cos 2t + B\sin 2t)e^{-t}], \text{ where } 5I_s = 10 \text{ or } I_s = 2$$

$$i(0) = 4 = 2 + A \text{ or } A = 2$$

$$di/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [(-2A\sin 2t + 2B\cos 2t)e^{-t}]$$

$$di(0)/dt = -2 = -A + 2B \text{ or } B = 0$$

$$i(t) = \mathbf{[2 + (2\cos 2t)e^{-t}]\ A}$$

Solution 8.30

The step responses of a series RLC circuit are

$v_C(t) = [40–10e^{–2000t}–10e^{–4000t}]$ volts, $t > 0$ and
$i_L(t) = [3e^{–2000t}+6e^{–4000t}]$ m A, $t > 0$.

(a) Find C. (b) Determine what type of damping exhibited by the circuit.

Solution

Step 1. For a series RLC circuit, $i_R(t) = i_L(t) = i_C(t)$.

We can determine C from $i_C(t) = i_L(t) = C(dv_C/dt)$ and we can determine that the circuit is **overdamped** since the exponent value are real and negative.

Step 2. $C(dv_C/dt) = C[20{,}000e^{–2000t}+40{,}000e^{–4000t}] = 0.003e^{–2000t}+0.006e^{–4000t}$ or

$$C = 0.003/20{,}000 = \mathbf{150\ \eta F}.$$

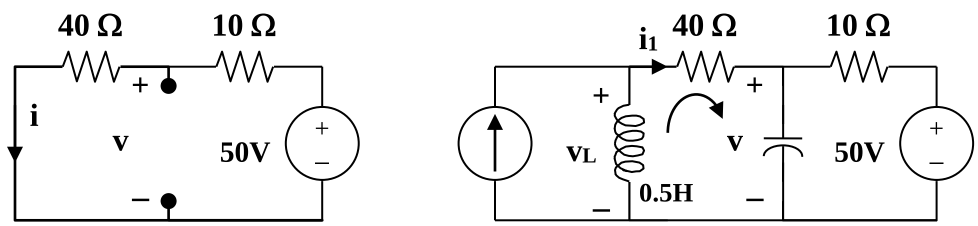
Solution 8.31

For t = 0-, we have the equivalent circuit in Figure (a). For t = 0+, the equivalent circuit is shown in Figure (b). By KVL,

$$v(0+) = v(0-) = 40, \quad i(0+) = i(0-) = 1$$

By KCL, $2 = i(0+) + i_1 = 1 + i_1$ which leads to $i_1 = 1$.
By KVL, $-v_L + 40i_1 + v(0+) = 0$ which leads to $v_L(0+) = 40x1 + 40 = 80$

$$v_L(0+) = \mathbf{80\ V}, \quad v_C(0+) = \mathbf{40\ V}$$



(a) (b)

Solution 8.32

For the circuit in Fig. 8.80, find $v(t)$ for $t > 0$.

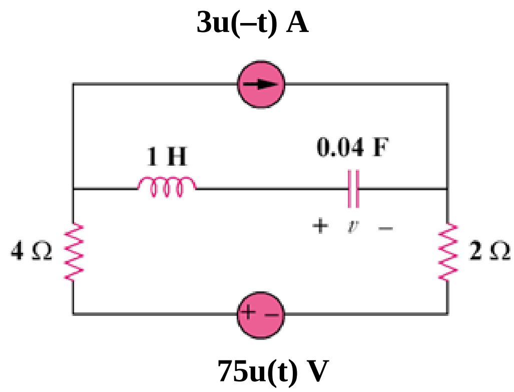
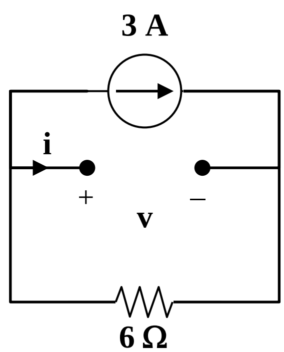


Figure 8.80
For Prob. 8.32.

Solution

For t = 0^-, the equivalent circuit is shown below.



$$i(0^-) = 0, \quad v(0^-) = -3x6 = -18 \text{ V}$$

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 6/2 = 3, \quad \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.04}$$

$$s_{1,2} = -3 \pm \sqrt{9 - 25} = -3 \pm j4$$

Thus, $v(t) = V_s + [(A\cos 4t + B\sin 4t)e^{-3t}]$

where V_s = final capacitor voltage = 75 V
$v(t) = 75 + [(A\cos 4t + B\sin 4t)e^{-3t}]$

$v(0) = -18 = 75 + A$ which gives $A = -93$.

$i(0) = 0 = Cdv(0)/dt$

$dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$

$0 = dv(0)/dt = -3A + 4B$ or $B = (3/4)A = -69.75$

$v(t) =$ $\mathbf{\{75 + [(-93\cos 4t - 69.75\sin 4t)e^{-3t}]\}}$ **V for all t > 0.**

Solution 8.33

Find $v(t)$ for $t > 0$ in the circuit in Fig. 8.81.

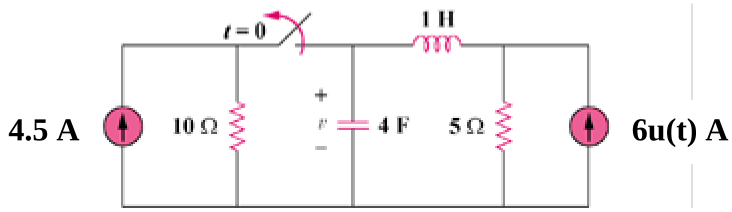
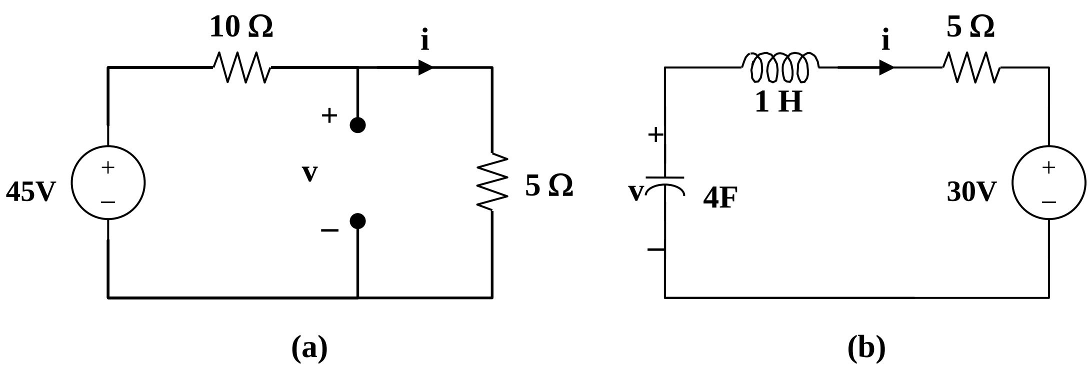


Figure 8.81
For Prob. 8.33.

Solution

We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).



$$i(0) = 45/15 = 3 \text{ A}, \quad v(0) = 5x45/15 = 15 \text{ V}$$

For $t > 0$, we have a series RLC circuit, shown in (b).

$$\alpha = R/(2L) = 5/2 = 2.5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5, \text{ clearly } \alpha > \omega_o \text{ (overdamped response)}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.949, -0.0505$$

$$v(t) = V_s + [A_1e^{-4.949t} + A_2e^{-0.0505t}], \quad V_s = 30 \text{ V}.$$

$$v(0) = 15 = 30 + A_1 + A_2 \quad \text{or}$$
$$A_2 = -15 - A_1$$
(1)

$$i(0) = -Cdv(0)/dt \;\text{ or }\; dv(0)/dt = -3/4 = -0.75$$

Hence, $-0.75 = -4.949A_1 - 0.0505A_2$ (2)

From (1) and (2), $-0.75 = -4.949A_1 + 0.0505(15 + A_1)$ or
$-4.898A_1 = -0.75 - 0.7575 = -1.5075$

$$A_1 = 0.3078, \; A_2 = -15.308$$

$$v(t) = \mathbf{[30 + 0.3078e^{-4.949t} - 15.308e^{-0.05t}]\ V\ for\ all\ t > 0}.$$

Solution 8.34

Calculate $i(t)$ for $t > 0$ in the circuit in Fig. 8.82.

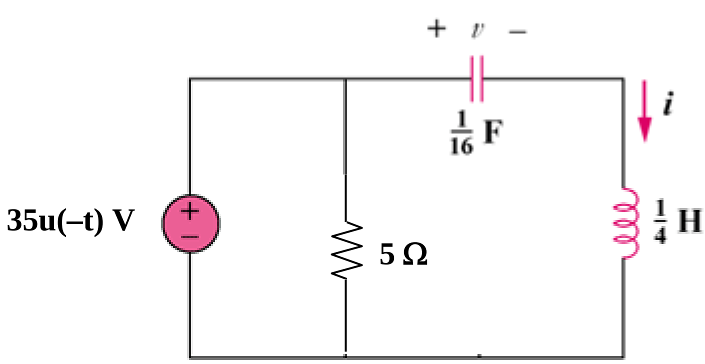


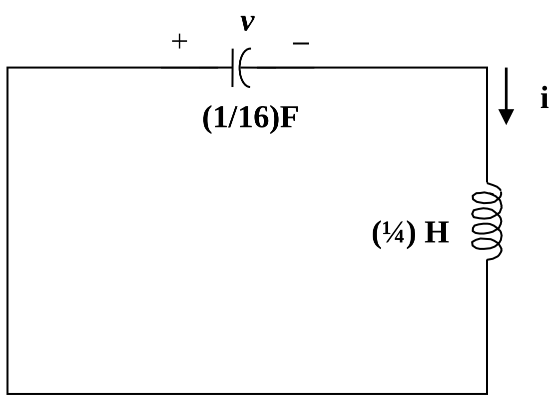
Figure 8.82
For Prob. 8.34.

Solution

Before t = 0, the capacitor acts like an open circuit while the inductor behaves like a short circuit.

$$i(0) = 0, \; v(0) = 35 \text{ V}$$

For t > 0, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free, series RLC circuit.

$$\alpha = R/(2L) = 0, \; \omega_o = 1/\sqrt{LC} = 1/\sqrt{\frac{1}{16} x \frac{1}{4}} = 8, \; s = \pm j8$$

Since α is equal to zero, we have an undamped response. Therefore,

$$i(t) = A_1\cos(8t) + A_2\sin(8t) \quad \text{where } i(0) = 0 = A_1$$

$$di(0)/dt = (1/L)v_L(0) = -(1/L)v(0) = -4x35 = -140$$

However, $di/dt = 8A_2\cos(8t)$, thus, $di(0)/dt = -140 = 8A_2$ which leads to
$A_2 = -17.5$

Now we have $\quad i(t) = \mathbf{-17.5\sin(8t)\ A}$.

Solution 8.35

Using Fig. 8.83, design a problem to help other students to better understand the step response of series *RLC* circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 8.83.

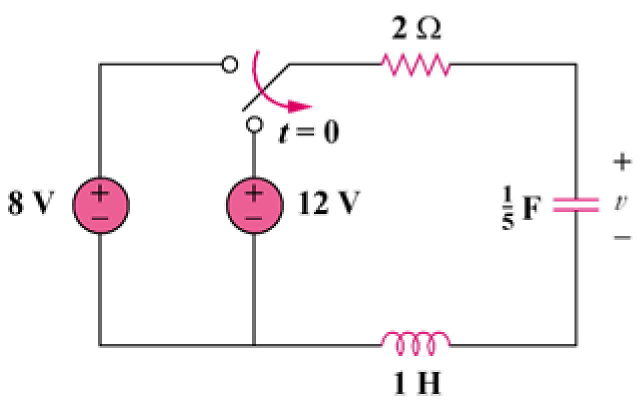


Figure 8.83

Solution

At t = 0-, $i_L(0) = 0$, $v(0) = v_C(0) = 8$ V

For t > 0, we have a series RLC circuit with a step input.

$$\alpha = R/(2L) = 2/2 = 1, \ \omega_o = 1/\sqrt{LC} = 1/\sqrt{1/5} = \sqrt{5}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm j2$$

$$v(t) = V_s + [(A\cos 2t + B\sin 2t)e^{-t}], \ V_s = 12.$$

$$v(0) = 8 = 12 + A \text{ or } A = -4, \ i(0) = Cdv(0)/dt = 0.$$

$$\text{But } dv/dt = [-(A\cos 2t + B\sin 2t)e^{-t}] + [2(-A\sin 2t + B\cos 2t)e^{-t}]$$

$$0 = dv(0)/dt = -A + 2B \text{ or } 2B = A = -4 \text{ and } B = -2$$

$$v(t) = \mathbf{\{12 - (4\cos 2t + 2\sin 2t)e^{-t}\ V}.$$

Solution 8.36

Obtain $v(t)$ and $i(t)$ for $t > 0$ in the circuit in Fig. 8.84.

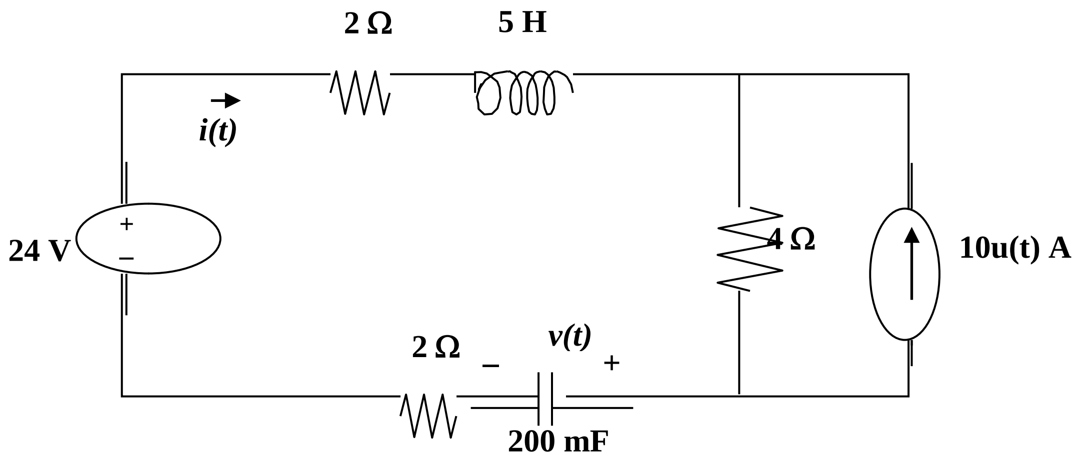
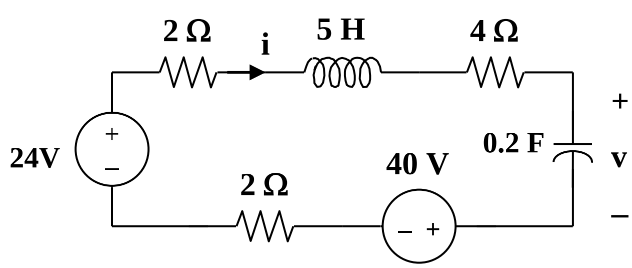


Figure 8.84
For Prob. 8.36.

Solution

For t = 0^-, 10u(t) A = 0. Thus, i(0) = 0, and v(0) = 24 V.

For t > 0, we have the series RLC circuit shown below.



$$\alpha = R/(2L) = (2 + 2 + 4)/(2x5) = 0.8$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.2} = 1$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -0.8 \pm j0.6$$

$$v(t) = V_s + [(A\cos(0.6t) + B\sin(0.6t))e^{-0.8t}]$$

$$V_s = v_{ss} = 24 - 40 = -16 \text{ V and } v(0) = 24 = -16 + A \text{ or } A = +40$$

$$i(0) = Cdv(0)/dt = 0$$

But $dv/dt = [-0.8(A\cos(0.6t) + B\sin(0.6t))e^{-0.8t}] + [0.6(-A\sin(0.6t) + B\cos(0.6t))e^{-0.8t}]$

$0 = dv(0)/dt = -0.8A + 0.6B$ which leads to $B = 0.8x(40)/0.6 = 53.33$

$$v(t) = \mathbf{\{-16 + [(40\cos(0.6t) + 53.33\sin(0.6t))e^{-0.8t}]\}u(t)\ \ V}$$

$$i = Cdv/dt$$

$$= 0.2\{[-0.8(40\cos(0.6t) + 53.33\sin(0.6)t)e^{-0.8t}] + [0.6(-40\sin(0.6t) + 53.33\cos(0.6t))e^{-0.8t}]\}$$

$$i(t) = \mathbf{[-13.333\sin(0.6t)e^{-0.8t}]u(t)\ A}.$$

Solution 8.37

For the network in Fig. 8.85, solve for $i(t)$ for $t > 0$.

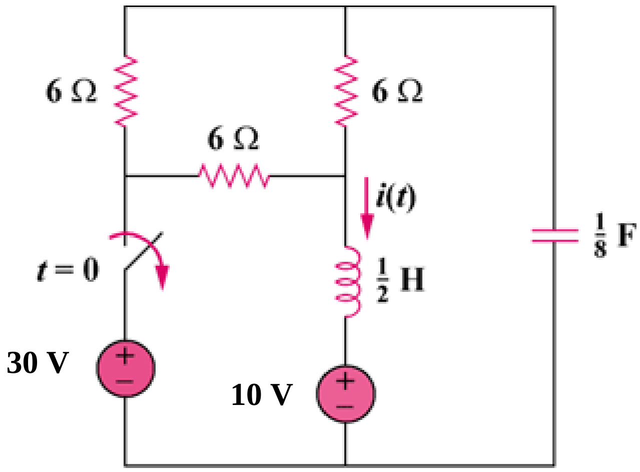
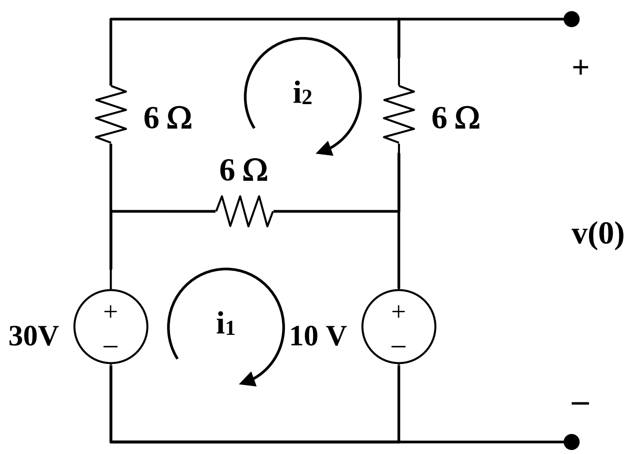


Figure 8.85
For Prob. 8.37.

Solution

For $t = 0^-$, the equivalent circuit is shown below.



$$18i_2 - 6i_1 = 0 \text{ or } i_1 = 3i_2 \qquad (1)$$

$$-30 + 6(i_1 - i_2) + 10 = 0 \text{ or } i_1 - i_2 = 20/6 = 10/3 \qquad (2)$$

From (1) and (2), $(2/3)i_1 = 10/3$ or $i_1 = 5$ and $i_2 = i_1 - 10/3 = 5/3$

$$i(0) = i_1 = 5A$$

$$-10 - 6i_2 + v(0) = 0$$

$$v(0) = 10 + 6x5/3 = 20$$

For $t > 0$, we have a series RLC circuit.

$$R = 6||12 = 4$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/8)} = 4$$

$$\alpha = R/(2L) = (4)/(2x(1/2)) = 4$$

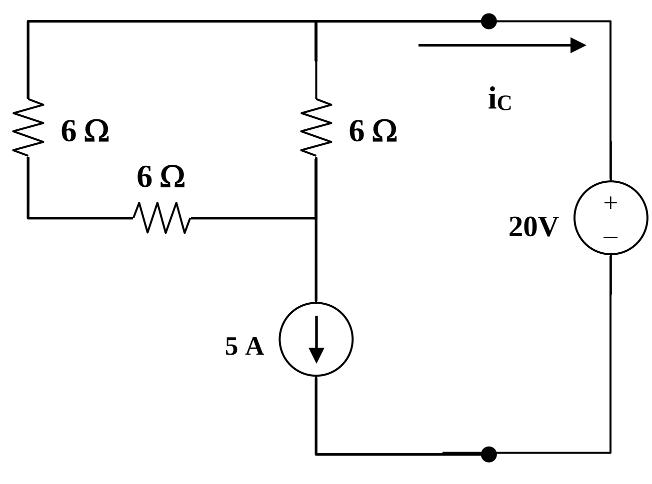
$\alpha = \omega_o$, therefore the circuit is critically damped

$$v(t) = V_s + [(A + Bt)e^{-4t}], \text{ and } V_s = v_{ss} = 10$$

$$v(0) = 20 = 10 + A, \text{ or } A = 10$$

$$i_C = Cdv/dt = C[-4(10 + Bt)e^{-4t}] + C[(B)e^{-4t}]$$

To find $i_C(0)$ we need to look at the circuit right after the switch is opened. At this time, the current through the inductor forces that part of the circuit to act like a current source and the capacitor acts like a voltage source. This produces the circuit shown below. Clearly, $i_C(0+)$ must equal $-i_L(0) = -5A$.



$$i_C(0) = -5 = C(-40 + B) \text{ which leads to } -40 = -40 + B \text{ or } B = 0$$

$$i_C = Cdv/dt = (1/8)[-4(10 + 0t)e^{-4t}] + (1/8)[(0)e^{-4t}]$$

$$i_C(t) = [-(1/2)(10)e^{-4t}]$$

$$i(t) = -i_C(t) = \mathbf{5e^{-4t}\ A\ for\ all\ t > 0}.$$

Solution 8.38

Refer to the circuit in Fig. 8.86. Calculate $i(t)$ for $t > 0$.

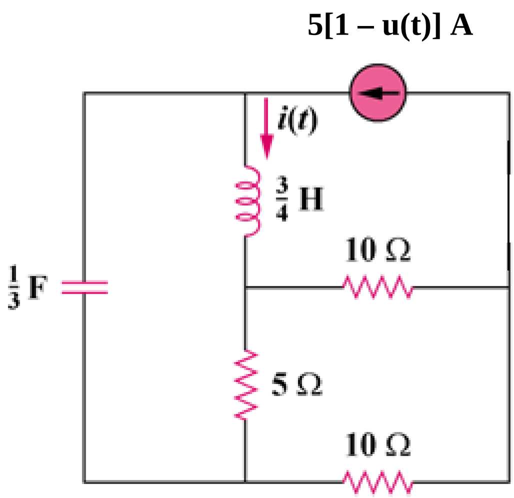
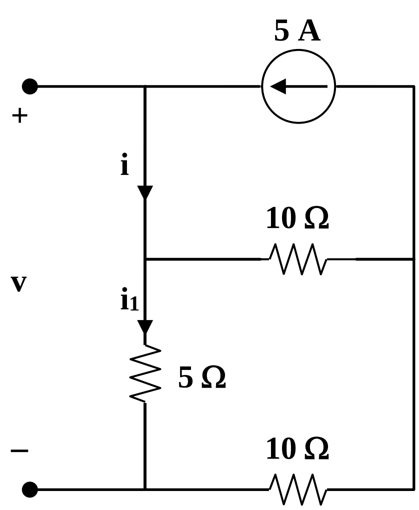


Figure 8.86
For Prob. 8.38.

Solution

At $t = 0^-$, the equivalent circuit is as shown.



$i(0) = 5$ A, $i_1(0) = 10(5)/(10 + 15) = 2$ A

$v(0) = 5i_1(0) = 10$ V

For $t > 0$, we have a source-free series RLC circuit.

$R = 5||(10 + 10) = 4$ ohms

$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/3)(3/4)} = 2$

$\alpha = R/(2L) = (4)/(2x(3/4)) = 8/3$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.66667 \pm 1.763834 = -4.43\ 05, -0.90283$

$i(t) = [Ae^{-4.431t} + Be^{-0.9028t}]$ and $i(0) = A + B = 5$ or $A = 5 - B$

$di(0)/dt = (1/L)[-Ri(0) + v(0)] = (4/3)(-4x5 + 10) = -40/3 = -13.33333$

Hence, $-13.3333 = -4.4305A - 0.90283B = -22.1525 + 4.4305B - 0.90283B$

$3.52767B = 8.8192$ or $B = 2.5$ and $A = 5 - 2.5 = 2.5$. Thus,

$$i(t) = \mathbf{[2.5e^{-4.431t} + 2.5e^{-0.9028t}]\ A}.$$

Solution 8.39

Determine $v(t)$ for $t > 0$ in the circuit in Fig. 8.87.

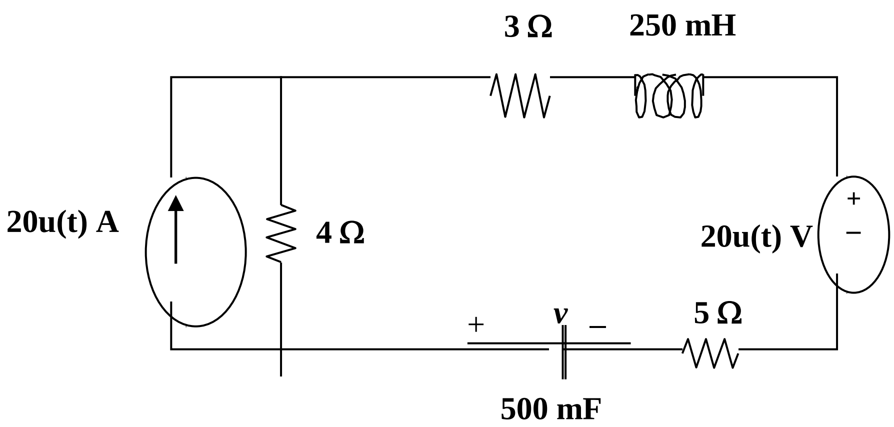


Figure 8.87
For Prob. 8.39.

Solution

For $t = 0^-$, the source voltages are equal to zero thus, the initial conditions are $v(0) = 0$ and $i_L(0) = 0$.

For $t > 0$,

$$R = 3 + 5 + 4 = 12 \text{ ohms}$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{(1/2)(1/4)} = \sqrt{8}$$

$$\alpha = R/(2L) = (12)/(0.5) = 24$$

Since $\alpha > \omega_o$, we have an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -24 \pm 23.833 = -47.83, \ -0.167$$

Thus, $$v(t) = V_s + [Ae^{-47.83t} + Be^{-0.167t}], \text{ where}$$

$V_s = v_{ss} = -80 + 20 = -60$ volts.

$$v(0) = 0 = -60 + A + B \text{ or } 60 = A + B \qquad (1)$$

$$i(0) = Cdv(0)/dt = 0$$

But, $$dv(0)/dt = -47.83A - 0.167B = 0 \text{ or}$$

$$B = -286.4A \qquad (2)$$

From (1) and (2), $A + (–286.4)A = 60$ or $A = 60/(–285.4) = –0.21023$ and
$B = –286.4x(–0.21023) = 60.21$

$$v(t) = \mathbf{[–60 + (–0.2102e^{-47.83t}+60.21e^{-0.167t})]u(t)\ volts}.$$

Solution 8.40

The switch in the circuit of Fig. 8.88 is moved from position *a* to *b* at $t = 0$. Assume that the voltage across the capacitor is equal to zero at t = 0 and that the switch is a make before break switch. Determine $i(t)$ for $t > 0$.

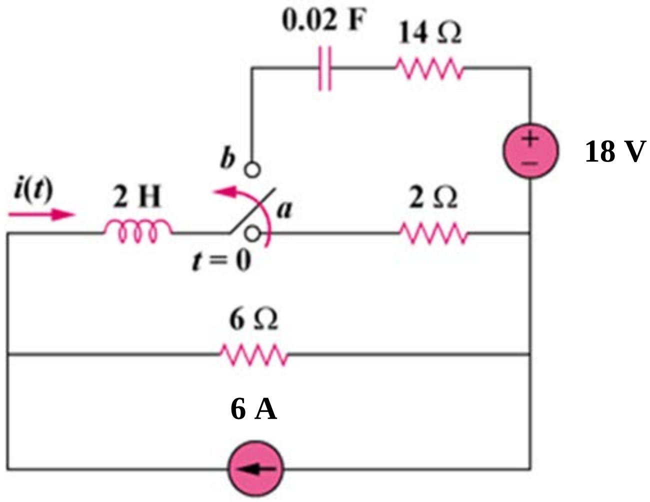
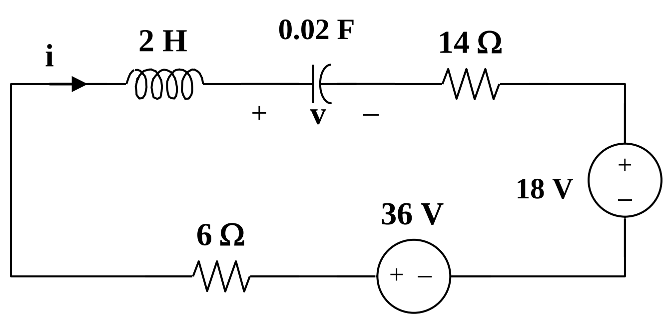


Figure 8.88
For Prob. 8.40.

Solution

At $t = 0^-$, $v_C(0) = 0$ and $i_L(0) = i(0) = (6/(6 + 2))6 = 4.5$ A.

For $t > 0$, we have a series RLC circuit with a step input as shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x0.02} = 5$$

$$\alpha = R/(2L) = (6 + 14)/(2x2) = 5$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$v(t) = V_s + [(A + Bt)e^{-5t}], \quad V_s = v_{ss} = 36 - 18 = 18 \text{ V}.$$

$v(0) = 0 = 18 + A$ or $A = -18$.

$i = Cdv/dt = C\{[Be^{-5t}] + [-5(A + Bt)e^{-5t}]\}$

$i(0) = 4.5 = C[-5A + B] = 0.02[90 + B]$ or $B = 135$.

Thus, $i(t) = 0.02\{[135e^{-5t}] + [-5(-18 + 135t)e^{-5t}]\}$

$$i(t) = \mathbf{[(4.5 - 13.5t)e^{-5t}]\ A}.$$

Solution 8.41

For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

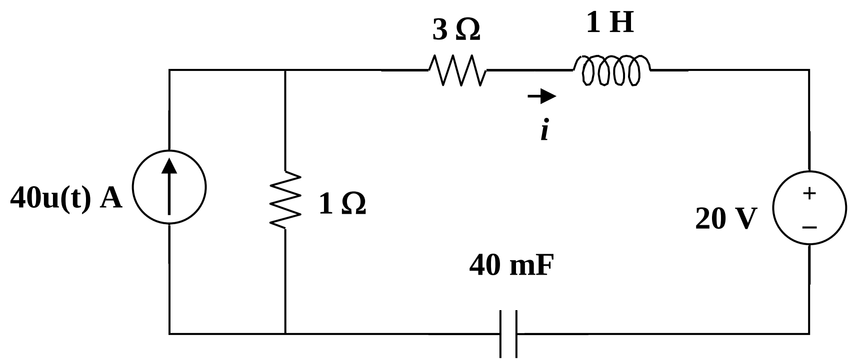


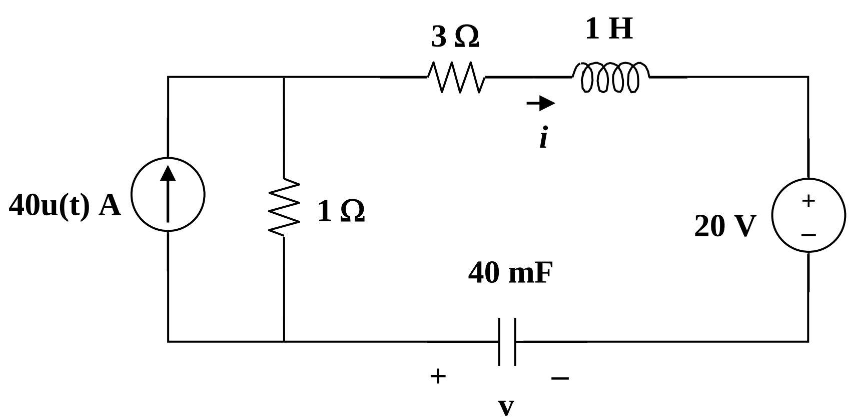
Figure 8.89
For Prob. 8.41.

Solution

At $t = 0^-$, $i(0) = 0$, and

$$v(0) = 20 \text{ V}$$

For $t > 0$, we have a series RLC circuit shown below.



$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5 \text{ rad/sec}$$

$$\alpha = R/(2L) = (4)/(2x1) = 2$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2 \pm j4.583$$

Thus, $v(t) = V_{ss} + [(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}]$,

where $\omega_d = 4.583$ and $V_{ss} = -20$ V

$v(0) = 20 = -20 + A$ or $A = 40$

$i(t) = -Cdv/dt$
$= C(2)\,[(A\cos(\omega_d t) + B\sin(\omega_d t))e^{-2t}] - C\omega_d[(-A\sin(\omega_d t) + B\cos(\omega_d t))e^{-2t}]$

$i(0) = 0 = 2A - \omega_d B$

$B = 2A/\omega_d = 80/(4.583) = 17.456$

$i(t) = C\{[(0\cos(\omega_d t) + (2B + \omega_d A)\sin(\omega_d t))]e^{-2t}\}$

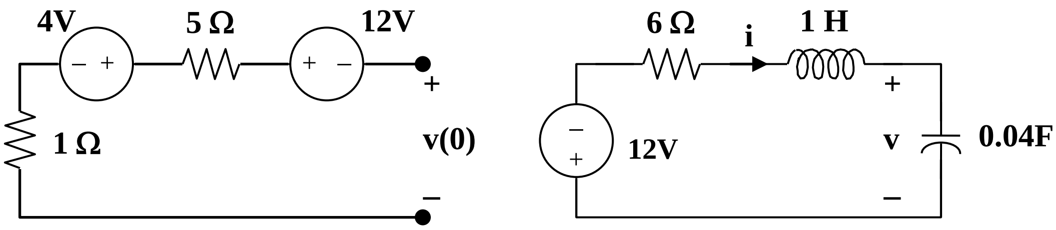
$= (0.04)\{[(34.192 + 183.32)\ \sin(\omega_d t))]e^{-2t}\}$

$i(t) =$ $\mathbf{[8.7\sin(4.583t)e^{-2t}]u(t)\ A}$.

Solution 8.42

For $t = 0$-, we have the equivalent circuit as shown in Figure (a).

$$i(0) = i(0) = 0, \text{ and } v(0) = 4 - 12 = -8V$$



(a) (b)

For $t > 0$, the circuit becomes that shown in Figure (b) after source transformation.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x1/25} = 5$$

$$\alpha = R/(2L) = (6)/(2) = 3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -3 \pm j4$$

Thus, $$v(t) = V_s + [(A\cos 4t + B\sin 4t)e^{-3t}], \quad V_s = -12$$

$$v(0) = -8 = -12 + A \text{ or } A = 4$$

$$i = Cdv/dt, \text{ or } i/C = dv/dt = [-3(A\cos 4t + B\sin 4t)e^{-3t}] + [4(-A\sin 4t + B\cos 4t)e^{-3t}]$$

$$i(0) = -3A + 4B \text{ or } B = 3$$

$$v(t) = \mathbf{\{-12 + [(4\cos 4t + 3\sin 4t)e^{-3t}]\}\ A}$$

Solution 8.43

The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8$ Np/s and $\omega_d = 30$ rad/s.

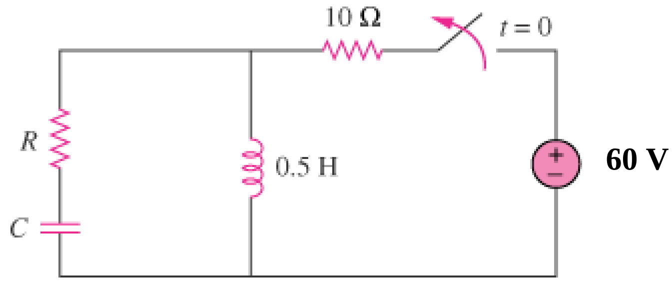


Figure 8.91
For Prob. 8.43.

Solution

For t>0, we have a source-free series RLC circuit.

$$\alpha = \frac{R}{2L} \longrightarrow R = 2\alpha L = 2x8x0.5 = \mathbf{8\Omega}$$

$$\omega_d = \sqrt{\omega_o^{\,2} - \alpha^2} = 30 \longrightarrow \omega_o = \sqrt{900 + 64} = \sqrt{964}$$

$$\boldsymbol{\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_o^2 L} = \frac{1}{964x0.5} = 2.075\ \mathrm{mF}}$$

Solution 8.44

$$\alpha = \frac{R}{2L} = \frac{1000}{2x1} = 500, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-9}}} = 10^4$$

$\omega_o > \alpha \quad \longrightarrow \quad$ **underdamped.**

Solution 8.45

In the circuit of Fig. 8.92, find $v(t)$ and $i(t)$ for $t > 0$.

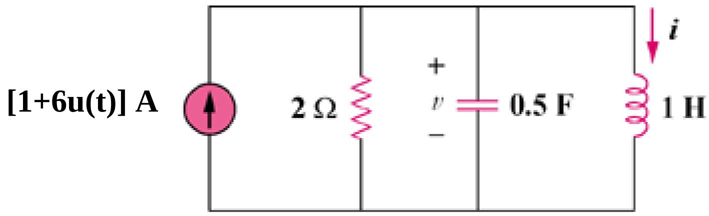


Figure 8.92
For Prob. 8.45.

Solution

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.5} = \sqrt{2}$$

$$\alpha = 1/(2RC) = (1)/(2x2x0.5) = 0.5$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\omega_o^2 - \alpha^2} = -0.5 \pm j1.3229$$

Thus, $$i(t) = I_s + [(A\cos 1.3229t + B\sin 1.3229t)e^{-0.5t}], \quad I_s = 6$$

$$i(0) = 1 = 6 + A \text{ or } A = -5$$

$$v = v_C(0) = v_L(0) = Ldi(0)/dt = 0$$

$$di/dt = [1.3229(-A\sin 1.3229t + B\cos 1.3229t)e^{-0.5t}] + [-0.5(A\cos 1.3229t + B\sin 1.3229t)e^{-0.5t}]$$

$$di(0)/dt = 0 = 1.3229B - 0.5A \text{ or } B = 0.5(-5)/1.3229 = -1.8898$$

Thus, $$i(t) = \mathbf{\{6 - [(5\cos 1.3229t + 1.8898\sin 1.3229t)e^{-t/2}]\}\ A}$$

To find v(t) we use $v(t) = v_L(t) = Ldi(t)/dt$.

From above,
$$di/dt = [1.3229(-A\sin 1.3229t + B\cos 1.3229t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

Thus,

$$v(t) = Ldi/dt = [1.323(-A\sin 1.323t + B\cos 1.323t)e^{-0.5t}] + [-0.5(A\cos 1.323t + B\sin 1.323t)e^{-0.5t}]$$

$$= [1.3229(5\sin 1.3229t - 1.8898\cos 1.3229t)e^{-0.5t}] + [(2.5\cos 1.3229t + 0.9449\sin 1.3229t)e^{-0.5t}]$$

$$v(t) = [(-0\cos 1.323t + 4.536\sin 1.323t)e^{-0.5t}]\ \text{V}$$

$$= \mathbf{[(7.559\sin 1.3229t)e^{-t/2}]\ V}.$$

Solution 8.46

Using Fig. 8.93, design a problem to help other students to better understand the step response of a parallel *RLC* circuit.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $i(t)$ for $t > 0$ in the circuit in Fig. 8.93.

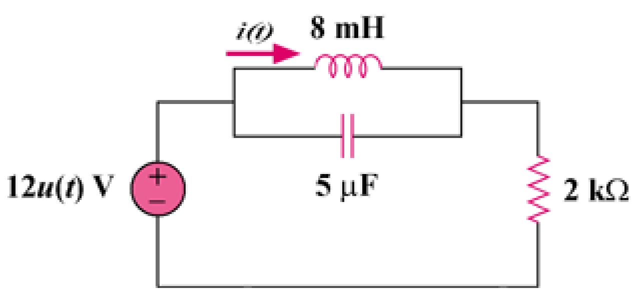
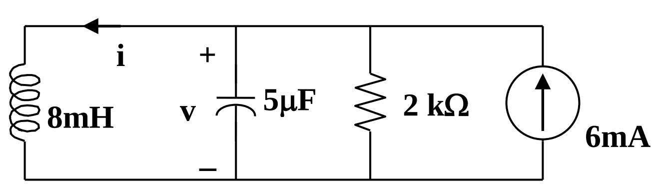


Figure 8.93

Solution

For $t = 0-$, $u(t) = 0$, so that $v(0) = 0$ and $i(0) = 0$.

For $t > 0$, we have a parallel RLC circuit with a step input, as shown below.



$$\alpha = 1/(2RC) = (1)/(2x2x10^3\ x5x10^{-6}) = 50$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{8x10^{-3}x5x10^{-6}} = 5{,}000$$

Since $\alpha < \omega_o$, we have an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} \cong -50 \pm j5{,}000$$

Thus, $$i(t) = I_s + [(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}], \quad I_s = 6\text{mA}$$

$$i(0) = 0 = 6 + A \text{ or } A = -6\text{mA}$$

$$v(0) = 0 = Ldi(0)/dt$$

$$di/dt = [5{,}000(-A\sin 5{,}000t + B\cos 5{,}000t)e^{-50t}] + [-50(A\cos 5{,}000t + B\sin 5{,}000t)e^{-50t}]$$

$$di(0)/dt = 0 = 5{,}000B - 50A \text{ or } B = 0.01(-6) = -0.06\text{mA}$$

Thus, $$i(t) = \mathbf{\{6 - [(6\cos 5{,}000t + 0.06\sin 5{,}000t)e^{-50t}]\}\ mA}$$

Solution 8.47

Find the output voltage $v_o(t)$ in the circuit of Fig. 8.94.

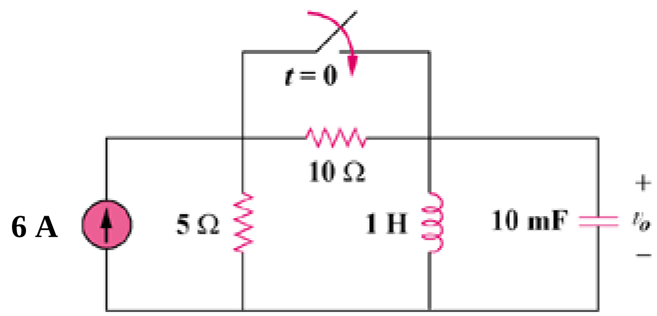


Figure 8.94
For Prob. 8.47.

Solution

At $t = 0^-$, we obtain, $i_L(0) = 6x5/(10 + 5) = 2$ A, and $v_o(0) = 0$.

For $t > 0$, the 10-ohm resistor is short-circuited and we have a parallel RLC circuit with a step input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.01) = 10$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.01} = 10$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -10$$

Thus, $i(t) = I_{ss} + [(A + Bt)e^{-10t}]$, $I_{ss} = 6$, $i(0) = 2 = 6 + A$ or $A = -4$

$$v_o = Ldi/dt = [Be^{-10t}] + [-10(A + Bt)e^{-10t}]$$

$$v_o(0) = 0 = B - 10A \text{ or } B = -40$$

Thus, $v_o(t) = \mathbf{(400te^{-10t})\ V}$.

Solution 8.48

For $t = 0^-$, we obtain $i(0) = -6/(1 + 2) = -2$ and $v(0) = 2x1 = 2$.

For $t > 0$, the voltage is short-circuited and we have a source-free parallel RLC circuit.

$$\alpha = 1/(2RC) = (1)/(2x1x0.25) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x0.25} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $i(t) = [(A + Bt)e^{-2t}]$, $i(0) = -2 = A$

$$v = Ldi/dt = [Be^{-2t}] + [-2(-2 + Bt)e^{-2t}]$$

$$v_o(0) = 2 = B + 4 \text{ or } B = -2$$

Thus,

$$i(t) = \mathbf{[(-2 - 2t)e^{-2t}]\ A}$$

and $v(t) = \mathbf{[(2 + 4t)e^{-2t}]\ V}$

Solution 8.49

Determine $i(t)$ for $t > 0$ in the circuit of Fig. 8.96.

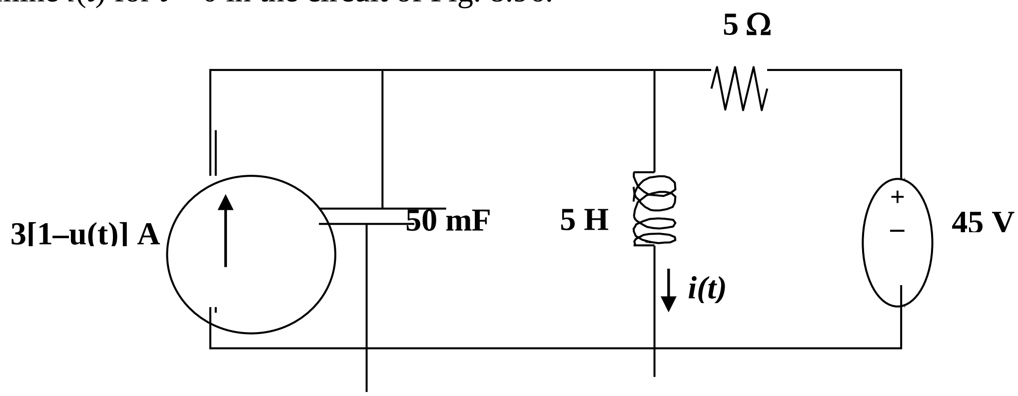


Figure 8.96
For Prob. 8.49.

Solution

For $t = 0^-$, $i(0) = 3 + 45/5 = 12$ A and $v(0) = 0$.

For $t > 0$, we have a parallel *RLC* circuit with a step change in the input.

$$\alpha = 1/(2RC) = (1)/(2x5x0.05) = 2$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x0.05} = 2$$

Since $\alpha = \omega_o$, we have a critically damped response.

$$s_{1,2} = -2$$

Thus, $$i(t) = I_{ss} + [(A + Bt)e^{-2t}], \quad I_{ss} = 9$$

$$i(0) = 12 = 9 + A \text{ or } A = 3$$

$$v = Ldi/dt \text{ or } v/L = di/dt = [Be^{-2t}] + [-2(A + Bt)e^{-2t}]$$

$$v(0)/L = 0 = di(0)/dt = B - 2x3 \text{ or } B = 6$$

Thus, $i(t) =$ $\mathbf{\{9 + [(3 + 6t)e^{-2t}]\}u(t)\ A}$

Solution 8.50

For the circuit in Fig. 8.97, find $i(t)$ for $t > 0$.

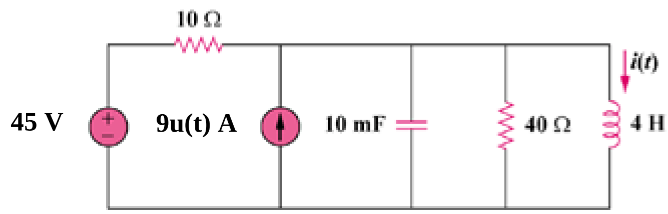
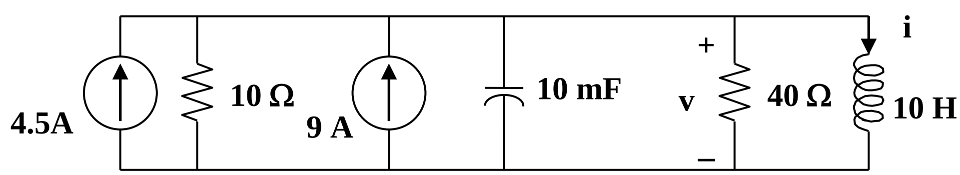


Figure 8.97
For Prob. 8.50.

Solution

For t = 0^-, 9u(t) = 0, v(0) = 0, and i(0) = 45/10 = 4.5 A.

For t > 0, we have a parallel RLC circuit.



$$I_{ss} = 4.5 + 9 = 13.5 \text{ A and } R = 10||40 = 8 \text{ ohms}$$

$$\alpha = 1/(2RC) = (1)/(2x8x0.01) = 25/4 = 6.25$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{4x0.01} = 5$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -10, -2.5$$

Thus, $$i(t) = I_{ss} + [Ae^{-10t}] + [Be^{-2.5t}], \quad I_{ss} = 13.5$$

$$i(0) = 4.5 = 13.5 + A + B \text{ or } A + B = -9$$

$$di/dt = [-10Ae^{-10t}] + [-2.5Be^{-2.5t}],$$

$$v(0) = 0 = Ldi(0)/dt \text{ or } di(0)/dt = 0 = -10A - 2.5B \text{ or } B = -4A$$

Thus, $A - 4A = -9$ or $A = 3$ and $B = -12$.

Clearly, $i(t) = \mathbf{\{ 13.5 + [3e^{-10t}] + [-12e^{-2.5t}]\}\ A}$.

Solution 8.51

Let i = inductor current and v = capacitor voltage.

$$\text{At } t = 0,\ v(0) = 0 \text{ and } i(0) = i_o.$$

For t > 0, we have a parallel, source-free LC circuit (R = ∞).

$$\alpha = 1/(2RC) = 0 \text{ and } \omega_o = 1/\sqrt{LC} \text{ which leads to } s_{1,2} = \pm j\omega_o$$

$$v = A\cos\omega_o t + B\sin\omega_o t,\ v(0) = 0\ A$$

$$i_C = Cdv/dt = -i$$

$$dv/dt = \omega_o B\sin\omega_o t = -i/C$$

$$dv(0)/dt = \omega_o B = -i_o/C \text{ therefore } B = i_o/(\omega_o C)$$

$$v(t) = \mathbf{-(i_o/(\omega_o C))\sin\omega_o t\ V \text{ where } \omega_o = 1/\sqrt{\mathit{LC}}}$$

Solution 8.52

The step response of a parallel *RLC* circuit is

$$v = 10 + 20e^{-300t} (\cos 400t - 2 \sin 400t) \text{ V}, \; t \geq 0$$

when the inductor is 25 mH. Find *R* and *C*.

Solution

$$\alpha = 300 = \frac{1}{2RC} \qquad (1)$$

$$\omega_d = \sqrt{\omega_o{}^2 - \alpha^2} = 400 \quad \longrightarrow \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 160{,}000 + 90{,}000 = \frac{1}{LC} \qquad (2)$$

From (2),

$$C = \frac{1}{250{,}000 x 25 x 10^{-3}} = \mathbf{160\ \mu F}$$

From (1),

$$R = \frac{1}{2\alpha C} = \frac{1}{2 x 300 x 160 x 10^{-6}} = \mathbf{10.417\ \Omega}.$$

Solution 8.53

After being open for a day, the switch in the circuit of Fig. 8.99 is closed at t=0. Find the differential equation describing i(t), t >0.

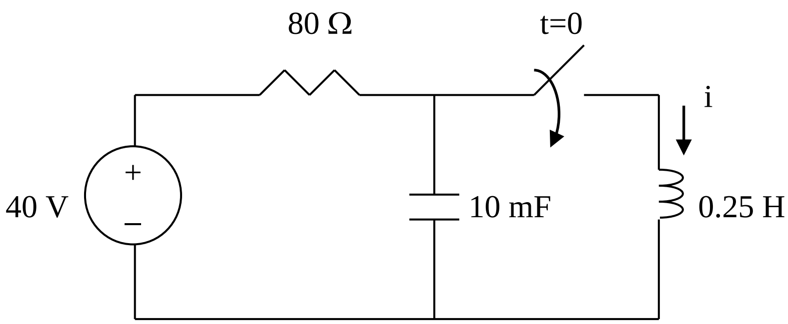
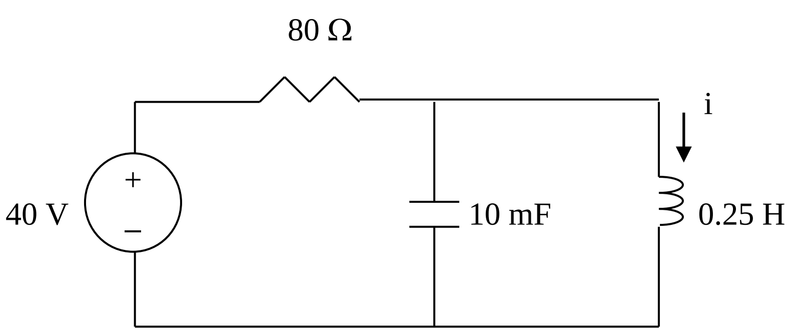


Figure 8.99
For Prob. 8.53.

Solution

For $t < 0$, $i(0) = 0$ and $v_C(0) = 40$.

For $t > 0$, we have the circuit as shown below.



$[(40–v_C)/80] = 0.01[dv_C/dt] + i$ or $40 = v_C + 0.8[dv_C/dt] + 80i$. But v_C is also $= 0.25di/dt$ which leads to $40 = 0.25[di/dt] + (0.8)(0.25)[d^2i/dt^2] + 80i$. Simplifying we get,

$$\mathbf{(d^2i/dt^2) + 1.25(di/dt) + 400i = 200}.$$

Solution 8.54

Using Fig. 8.100, design a problem to help other students better understand general second-order circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 8.100, let I = 9A, R_1 = 40 Ω, R_2 = 20 Ω, C = 10 mF, R_3 = 50 Ω, and L = 20 mH. Determine: (a) $i(0^+)$ and $v(0^+)$, (b) $di(0^+)/dt$ and $dv(0^+)/dt$, (c) $i(\infty)$ and $v(\infty)$.

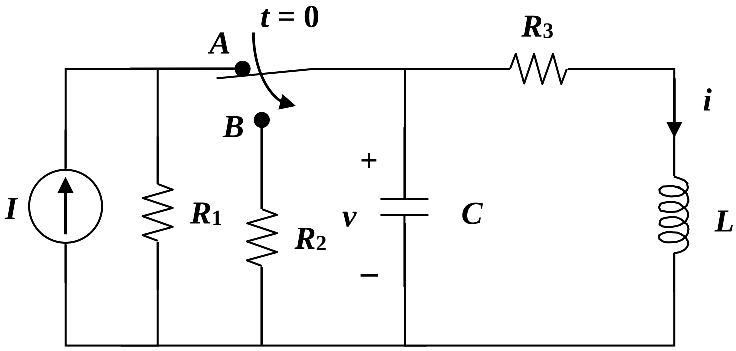
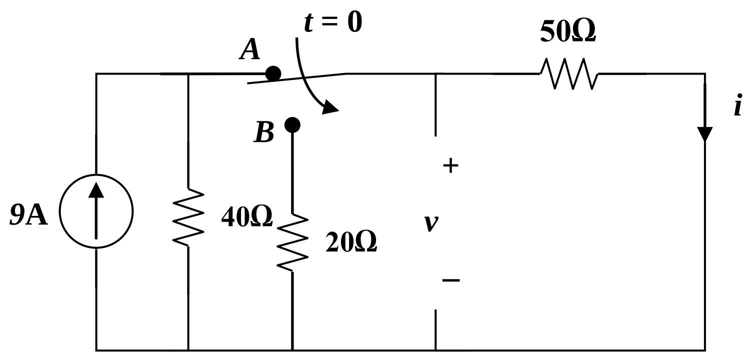


Figure 8.100
For Prob. 8.54.

Solution

(a) When the switch is at A, the circuit has reached steady state. Under this condition, the circuit is as shown below.



(a) When the switch is at A, $i(0^-) = 9[(40\text{x}50)/(40+50)]/50 = 4$ A and $v(0^-) = 50i(0^-) = 200$ V. Since the current flowing through the inductor cannot change in zero time, $i(0^+) = i(0^-) =$ **4 A**. Since the voltage across the capacitor cannot change in zero time, $v(0^-) = v(0^-) =$ **200 V**.

(b) For the inductor, $v_L = L(di/dt)$ or $di(0^+)/dt = v_L(0^+)/0.02$.

At $t = 0^+$, the right hand loop becomes,

$$-200 + 50\text{x}4 + v_L(0^+) = 0 \text{ or } v_L(0^+) = 0 \text{ and } (di(0^+)/dt) = \underline{\mathbf{0}}.$$

For the capacitor, $i_C = C(dv/dt)$ or $dv(0^+)/dt = i_C(0^+)/0.01$.

At $t = 0^+$, and looking at the current flowing out of the node at the top of the circuit,

$$((200–0)/20) + i_C + 4 = 0 \text{ or } i_C = –14 \text{ A}.$$

Therefore,

$$dv(0^+)/dt = –14/0.01 = \mathbf{–1.4\ kV/s.}$$

(c) When the switch is in position B, the circuit reaches steady state. Since it is source-free, i and v decay to zero with time.

Thus,

$$i(\infty) = \mathbf{0\ A} \text{ and } v(\infty) = \mathbf{0\ V}.$$

Solution 8.55

For the circuit in Fig. 8.101, find $v(t)$ for $t > 0$. Assume that $i(0^+) = 2$ A.

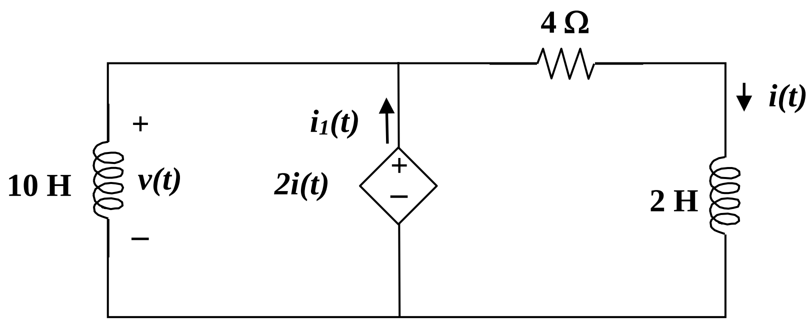
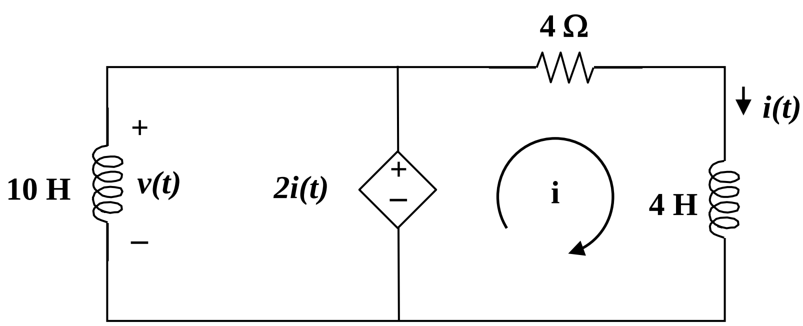


Figure 8.101
For Prob. 8.55.

Solution



We find that $i_1 = v(t) = 2i(t)$.

The inductor on the left does not affect the voltage so it can be neglected. Writing a mesh equation we get $-2i + 4i + 4di/dt = 0$ or $(di/dt) + 0.5i = 0$. This is a first order linear differential equation which has a solution equal to $i(t) =$
$[A + Be^{-t/\tau}]u(t)$ A. We have $\tau = 4/2 = 2$, $A = i(\infty) = 0$ and $A+B = i(0) = 2$ A $= B$. This leads to, $i(t) = 2e^{-t/2}$ A for all $t > 0$. Thus,

$v(t) = 2i(t) =$ **$[4e^{-t/2}]$ V** for all $t > 0$.

Solution 8.56

In the circuit of Fig. 8.102, find $i(t)$ for $t > 0$.

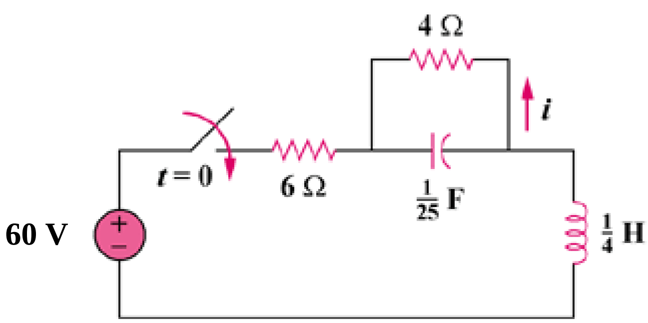
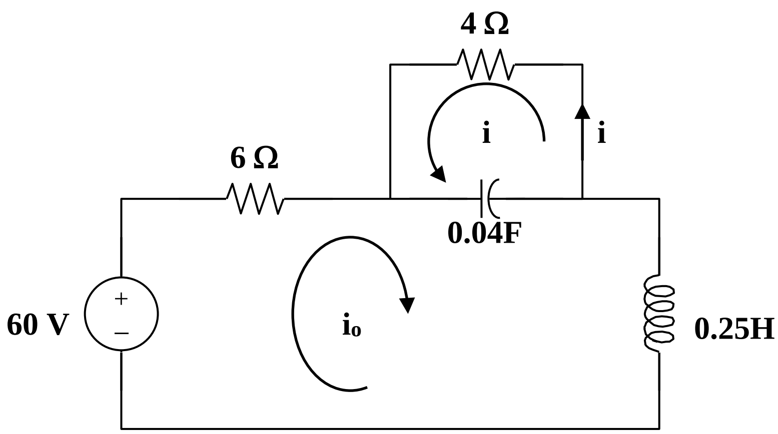


Figure 8.102
For Prob. 8.56.

Solution

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop and letting v = the capacitor voltage positive on the left,

$-60 + 6i_o + 0.25di_o/dt + v = 0$ where $i_o = [v/4] + 0.04dv/dt$ which leads to

$-60 + 1.5v + 0.24[dv/dt] + 0.0625[dv/dt] + 0.01[d^2v/dt^2] + v = 0$ or

$[d^2v/dt^2] + 30.25[dv/dt] + 250v = 6{,}000$ which leads to the characteristic equation,

$s^2 + 30.25s + 250 = 0$. $s_{1,2} = [–30.25±(915.0625–1{,}000)^{0.5}]/2 = [–30.25±j9.21615]/2 = –15.125 ±j4.608$. Since we have an underdamped system the response v can be expressed as,

$v = V_{ss} + [A_1\cos(4.608t) + A_2\sin(4.608t)]e^{–15.125t}$ where $V_{ss} = 24$ V and $v(0) = 0 = V_{ss} + A_1$ or $A_1 = –24$ V. Since the initial current through the capacitor = 0, then $dv(0)/dt = 0$. $dv(0)/dt = [–(–24)(4.608)\sin(0) + A_2(4.608)\cos(0)]e^{–0} – 15.125[–24\cos(0) + A_2\sin(0)]e^{–0}$ or $0 = 4.608A_2 + 363$ or $A_2 = –78.78$.

Thus, $v = [24 + [–24\cos(4.608t) – 78.78\sin(4.608t)]e^{–15.125t}$ V for all $t > 0$.

Since $i = –v/4 =$ **$[–6 + [6\cos(4.608t) + 19.695\sin(4.608t)]e^{–15.125t}$ A**.

Solution 8.57

Given the circuit shown in Fig. 8.103, determine the characteristic equation of the circuit and the values for *i(t)* and *v(t)* for all t > 0.

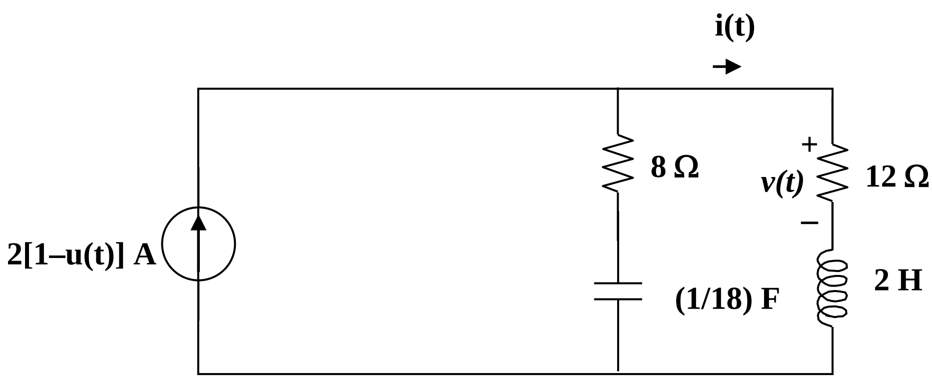


Figure 8.103
For Prob. 8.57.

Solution

Let v_C = capacitor voltage (plus on top and negative on the bottom) and i = inductor current. At t = 0^+, the circuit has reached steady-state and the current source goes to zero.

$$v_C(0^+) = 24 \text{ V and } i(0^+) = 2\text{A}$$

We now have a source-free RLC circuit.

$$R = 8 + 12 = 20 \text{ ohms, } L = 2 \text{ H, } C = (1/18) \text{ F.}$$

$$\alpha = R/(2L) = (20)/(2x2) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{2x(1/18)} = 3$$

Since $\alpha > \omega_o$, we have a overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \text{ and } -9$$

Thus, the characteristic equation is (s + 1)(s + 9) = 0 or $\mathbf{s^2 + 10s + 9 = 0}$.

$$i(t) = [Ae^{-t} + Be^{-9t}] \text{ and } i(0) = 2 = A + B \text{ or } B = 2 - A.$$

To determine A and B will need a second equation or to determine di(0)/dt.

We only need to evaluate the loop equation at $t = 0^+$ or

$-v_C(0) + 20i(0) + Ldi(0^+)/dt = 0$ or $di(0^+)/dt = [24–40]/2 = –8$
$= –A–9B$ or $= –A–9(2–A) = –18+8A = –8$ or $A = 10/8 = 1.25$ and
$B = 2 – (1.25) = 0.75$. Thus,

$$i(t) = \mathbf{[1.25e^{-t} + 0.75e^{-9t}]u(t)\ A}.$$

Finally,

$$v(t) = 12i(t) = \mathbf{[15e^{-t} + 9e^{-9t}]u(t)\ V}.$$

Solution 8.58

In the circuit of Fig. 8.104, the switch has been in position 1 for a long time but moved to position 2 at $t = 0$. Find:

(a) $v(0^+)$, $dv(0^+)/dt$
(b) $v(t)$ for $t \geq 0$.

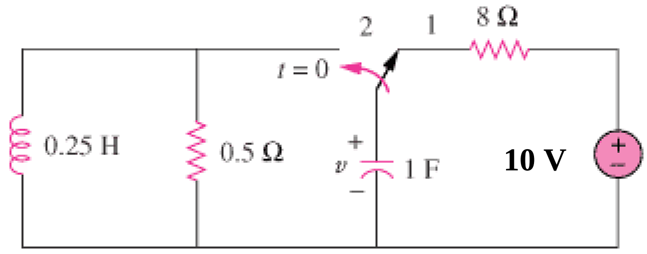


Figure 8.104
For Prob. 8.58.

Solution

(a) Let i =inductor current, v = capacitor voltage i(0) =0, $v(0^+)$ = **10 V**.

$$\frac{dv(0)}{dt} = -\frac{[v(0) + Ri(0)]}{RC} = -\frac{(10+0)}{0.5} = \mathbf{-20\ V/s}.$$

(b) For $t \geq 0$, the circuit is a source-free RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2x0.5x1} = 1, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25x1}} = 2$$

$$\omega_0 = \sqrt{\omega^2{}_o - \alpha^2} = \sqrt{4-1} = 1.732$$

Thus,

$$v(t) = e^{-t}(A_1 \cos 1.732t + A_2 \sin 1.732t)$$

v(0) = 10 V = A_1

$$\frac{dv}{dt} = -e^{-t}A_1 \cos 1.732t - 1.732e^{-t}A_1 \sin 1.732t - e^{-t}A_2 \sin 1.732t + 1.732e^{-t}A_2 \cos 1.732t$$

$$\frac{dv(0)}{dt} = -20 = -A_1 + 1.732A_2 \quad \longrightarrow \quad A_2 = -5.774$$

v(t) = **[10cos(1.732t) – 5.774sin(1.732t)]e^{-t} V for all t > 0**.

Solution 8.59

The switch in Fig. 8.105 has been in position 1 for $t < 0$. At $t = 0$, it is moved from position 1 to the top of the capacitor at t = 0. Please note that the switch is a make before break switch, it stays in contact with position 1 until it makes contact with the top of the capacitor and then breaks the contact at position 1. Given that the initial voltage across the capacitor is equal to zero, determine $v(t)$.

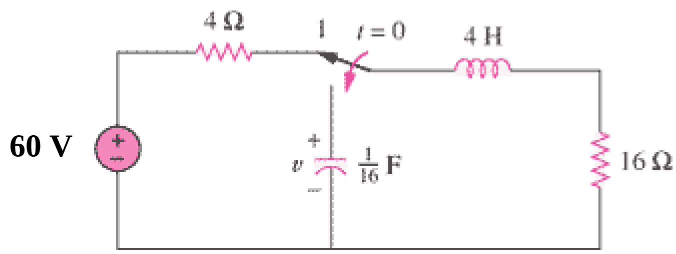


Figure 8.105
For Prob. 8.59.

Solution

Let i = inductor current and v = capacitor voltage
v(0) = 0, i(0) = 60/(4+16) = 3 A
For t>0, the circuit becomes a source-free series RLC with

$$\alpha = \frac{R}{2L} = \frac{16}{2x4} = 2, \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4x1/16}} = 2, \qquad \longrightarrow \qquad \alpha = \omega_o = 2$$

$$i(t) = Ae^{-2t} + Bte^{-2t}$$

i(0) = 3 = A

$$\frac{di}{dt} = -2Ae^{-2t} + Be^{-2t} - 2Bte^{-2t}$$

$$\frac{di(0)}{dt} = -2A + B = -\frac{1}{L}[Ri(0) - v(0)] \qquad \longrightarrow \qquad -2A + B = -\frac{1}{4}(48-0), \qquad B = -6$$

i(t) = [3e^{-2t} – 6te^{-2t}] and

$$v = \frac{1}{C}\int_0^t -id\tau + v(0) = -48\int_0^t e^{-2\tau}d\tau + 96\int_0^t \tau e^{-2\tau}d\tau = +24e^{-2\tau}\Big|_0^t + \frac{96}{4}e^{-2\tau}(-2\tau - 1)\Big|_0^t$$

v = **–48te^{-2t} V**.

Solution 8.60

Obtain i_1 and i_2 for $t > 0$ in the circuit of Fig. 8.106.

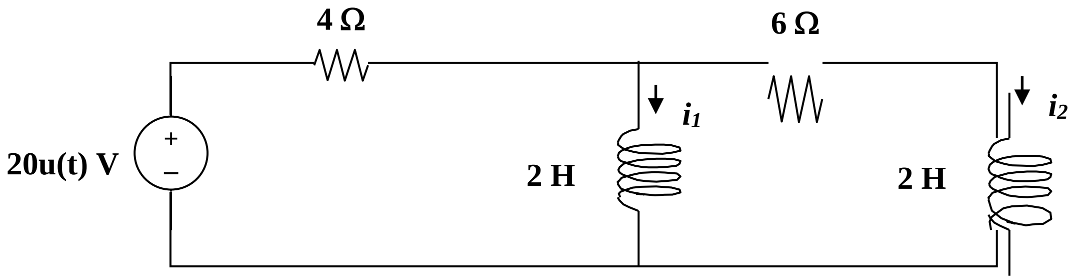


Figure 8.106
For Prob. 8.60.

Solution

Since the independent source is equal to zero until t = 0, $i_1(0) = i_2(0) = 0$.

Applying nodal analysis and letting the voltage at node 1 be v_1 (the voltage across the first inductor) and at node 2 be v_2 (the voltage across the second inductor) we get,

$[(v_1–20)/4] + i_1 + i_2 = 0$ and $[(v_2–v_1)/6] + i_2 = 0$. Also, $v_1 = 2di_1/dt$ and $v_2 = 2di_2/dt$.

The first equation gives us, $[(2di_1/dt)/4] + i_1 + i_2 = 5$ or $i_2 = 5 – 0.5(di_1/dt) – i_1$. We can take the derivative of this and get $(di_2/dt) = –0.5(d^2i_1/dt^2) – di_1/dt$.

The second equation gives us $(di_2/dt) – (di_1/dt) + 3\, i_2 = 0$ or
$[–0.5(d^2i_1/dt^2) – di_1/dt] – (di_1/dt) + 3[5 – 0.5(di_1/dt) – i_1] = 0$ or
$(d^2i_1/dt^2) + 7(di_1/dt) + 6i_1 = 30$.

We have the following $s^2 + 7s + 6 = 0 = (s+1)(s+6)$ and overdamped circuit.

Thus, $i_1(t) = I_s + [Ae^{–t} + Be^{–6t}]$, $I_s = 20/4 = 5$ A. Now let t = 0 we get,
$i_1(0) = 0 = 5 + A + B$ or $A = –5 – B$. Since the two inductors are open at t = 0 we get
$v_1(0^+) = 20\text{ V} = 2di_1(0^+)$ or $(di_1(0^+)/dt) = 10 = –A – 6B = –(–5–B) – 6B = 5–5B$ or $B = –1$ and $A = –4$ which gives us,

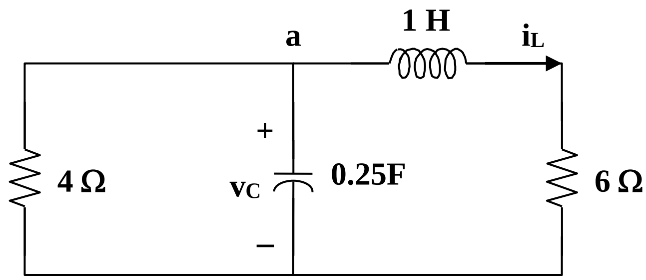
$i_1(t) =$ **$[5 – 4e^{–t} – e^{–6t}]u(t)$ A**.

We can use $i_2 = 5 – 0.5(di_1/dt) – i_1$. to find i_2. $= 5 – 2e^{–t} – 3e^{–6t} – 5 + 4e^{–t} + e^{–6t}$ or

$i_2(t) =$ **$[2e^{–t} – 2e^{–6t}]u(t)$ A**.

Solution 8.61

For $t > 0$, we obtain the natural response by considering the circuit below.



At node a, $v_C/4 + 0.25dv_C/dt + i_L = 0$ (1)

But, $v_C = 1di_L/dt + 6i_L$ (2)

Combining (1) and (2),

$$(1/4)di_L/dt + (6/4)i_L + 0.25d^2i_L/dt^2 + (6/4)di_L/dt + i_L = 0$$

$$d^2i_L/dt^2 + 7di_L/dt + 10i_L = 0$$

$$s^2 + 7s + 10 = 0 = (s + 2)(s + 5) \text{ or } s_{1,2} = -2, -5$$

Thus, $i_L(t) = i_L(\infty) + [Ae^{-2t} + Be^{-5t}]$,

where $i_L(\infty)$ represents the final inductor current $= 4(4)/(4 + 6) = 1.6$

$i_L(t) = 1.6 + [Ae^{-2t} + Be^{-5t}]$ and $i_L(0) = 1.6 + [A+B]$ or $-1.6 = A+B$ (3)

$$di_L/dt = [-2Ae^{-2t} - 5Be^{-5t}]$$

and $di_L(0)/dt = 0 = -2A – 5B$ or $A = -2.5B$ (4)

From (3) and (4), $A = -8/3$ and $B = 16/15$

$$i_L(t) = 1.6 + [-(8/3)e^{-2t} + (16/15)e^{-5t}]$$

$$v(t) = 6i_L(t) = \mathbf{\{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}\ V}$$

$$v_C = 1di_L/dt + 6i_L = [\ (16/3)e^{-2t} - (16/3)e^{-5t}] + \{9.6 + [-16e^{-2t} + 6.4e^{-5t}]\}$$

$$v_C = \{9.6 + [-(32/3)e^{-2t} + 1.0667e^{-5t}]\}$$

$$i(t) = v_C/4 = \mathbf{\{2.4 + [-2.667e^{-2t} + 0.2667e^{-5t}]\}\ A}$$

Solution 8.62

Find the response $v(t)$ for $t > 0$ in the circuit in Fig. 8.107. Let $R = 8\ \Omega$, $L = 2$ H, and $C = 125$ mF.

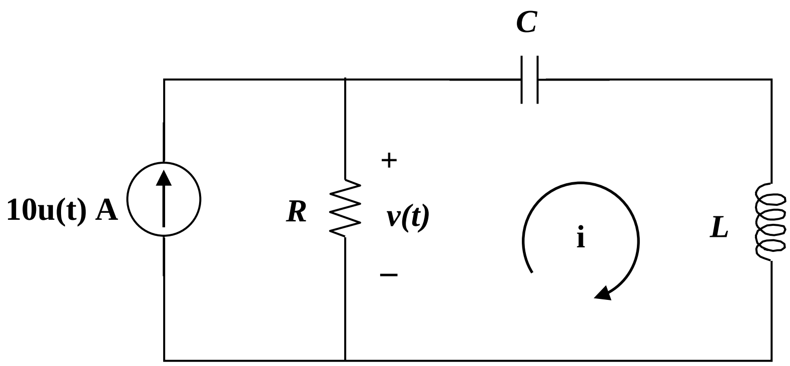


Figure 8.107
For Prob. 8.62.

Solution

This is actually a series RLC circuit where $\alpha = R/(2L) = 2$ and $\omega_0 = 1/\sqrt{LC} = 2$ and $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2$. Clearly we have a critically damped circuit.

A straight forward way to solve this is to solve for i and then solve for $v(t) = 8[10 - i]u(t)$ V.

$i = I_{ss} + [(A + Bt)e^{-2t}]$ where $I_{ss} = 0$. Additionally $i(0) = 0 = 0 + A$ or $A = 0$. Next $di/dt = B[e^{-2t} - 2te^{-2t}]$ or $di(0)/dt = B$. We note that $v_L = L\,di/dt$ or $di/dt = v_L/L$. This leads to $di(0)/dt = 80/2 = 40 = B$. Thus, $i = [40te^{-2t}]u(t)$ A.

Finally,

$$v(t) = \mathbf{[80 - 320te^{-2t}]u(t)\ V}.$$

Solution 8.63

$$\frac{v_s - 0}{R} = C\frac{d(0 - v_o)}{dt} \quad \longrightarrow \quad \frac{v_s}{R} = -C\frac{dv_o}{dt}$$

$$v_o = L\frac{di}{dt} \quad \longrightarrow \quad \frac{dv_o}{dt} = L\frac{d^2 i}{dt^2} = -\frac{v_s}{RC}$$

Thus,

$$\boldsymbol{\frac{d^2 i(t)}{dt^2} = -\frac{v_s}{RCL}}$$

Solution 8.64

Using Fig. 8.109, design a problem to help other students to better understand second-order op amp circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the differential equation for $v_o(t)$ in the network of Fig. 8.109.

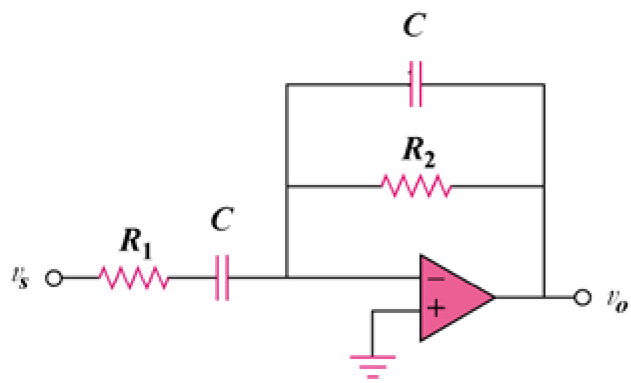
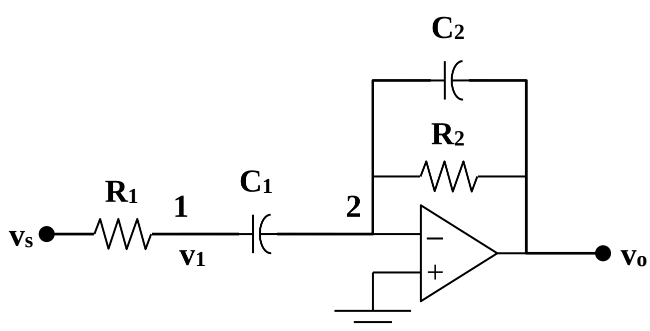


Figure 8.109

Solution



At node 1, $(v_s - v_1)/R_1 = C_1\, d(v_1 - 0)/dt$ or $v_s = v_1 + R_1C_1dv_1/dt$ (1)

At node 2, $C_1dv_1/dt = (0 - v_o)/R_2 + C_2d(0 - v_o)/dt$

or $-R_2C_1dv_1/dt = v_o + R_2C_2dv_o/dt$ (2)

From (1) and (2), $(v_s - v_1)/R_1 = C_1\, dv_1/dt = -(1/R_2)(v_o + R_2C_2dv_o/dt)$

or $v_1 = v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt)$ (3)

Substituting (3) into (1) produces,

$$v_s = v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt) + R_1C_1d\{v_s + (R_1/R_2)(v_o + R_2C_2dv_o/dt)\}/dt$$

$$= v_s + (R_1/R_2)(v_o) + (R_1C_2)dv_o/dt + R_1C_1dv_s/dt + (R_1R_1C_1/R_2)dv_o/dt + ((R_1)^2 C_1C_2)[d^2v_o/dt^2]$$

$$((R_1)^2 C_1C_2)[d^2v_o/dt^2] + [(R_1C_2) + (R_1R_1C_1/R_2)]dv_o/dt + (R_1/R_2)(v_o) = -R_1C_1dv_s/dt$$

Simplifying we get,

$$[d^2v_o/dt^2] + \{[(R_1C_2) + (R_1R_1C_1/R_2)]/((R_1)^2 C_1C_2)\}dv_o/dt + \{(R_1/R_2)(v_o)/((R_1)^2 C_1C_2)\} = -\{R_1C_1/((R_1)^2 C_1C_2)\}dv_s/dt$$

$$\mathbf{d^2v_o/dt^2 + [(1/R_1C_1) + (1/(R_2C_2))]dv_o/dt + [1/(R_1R_2C_1C_2)](v_o) = -[1/(R_1C_2)]dv_s/dt}$$

Another way to successfully work this problem is to give actual values of the resistors and capacitors and determine the actual differential equation. Alternatively, one could give a differential equations and ask the other students to choose actual value of the differential equation.

Solution 8.65

At the input of the first op amp,

$$(v_o - 0)/R \;=\; Cd(v_1 - 0) \qquad (1)$$

At the input of the second op amp,

$$(-v_1 - 0)/R \;=\; Cdv_2/dt \qquad (2)$$

Let us now examine our constraints. Since the input terminals are essentially at ground, then we have the following,

$$v_o \;=\; -v_2 \;\text{ or }\; v_2 \;=\; -v_o \qquad (3)$$

Combining (1), (2), and (3), eliminating v_1 and v_2 we get,

$$\frac{d^2v_o}{dt^2} - \left(\frac{1}{R^2C^2}\right)v_o = \frac{d^2v_o}{dt^2} - 100v_o = 0$$

Which leads to $s^2 - 100 \;=\; 0$

Clearly this produces roots of –10 and +10.

And, we obtain,

$$v_o(t) \;=\; (Ae^{+10t} + Be^{-10t})V$$

$$\text{At } t = 0,\; v_o(0+) \;=\; -v_2(0+) \;=\; 0 \;=\; A + B,\; \text{thus } B \;=\; -A$$

This leads to $v_o(t) \;=\; (Ae^{+10t} - Ae^{-10t})V$. Now we can use $v_1(0+) \;=\; 2V$.

From (2), $v_1 \;=\; -RCdv_2/dt \;=\; 0.1dv_o/dt \;=\; 0.1(10Ae^{+10t} + 10Ae^{-10t})$

$$v_1(0+) \;=\; 2 \;=\; 0.1(20A) \;=\; 2A \;\text{ or }\; A \;=\; 1$$

Thus, $v_o(t) \;=\; \mathbf{(e^{+10t} - e^{-10t})\ V}$

It should be noted that this circuit is unstable (clearly one of the poles lies in the right-half-plane).

Solution 8.66

Obtain the differential equations for $v_o(t)$ in the op amp circuit in Fig. 8.111.

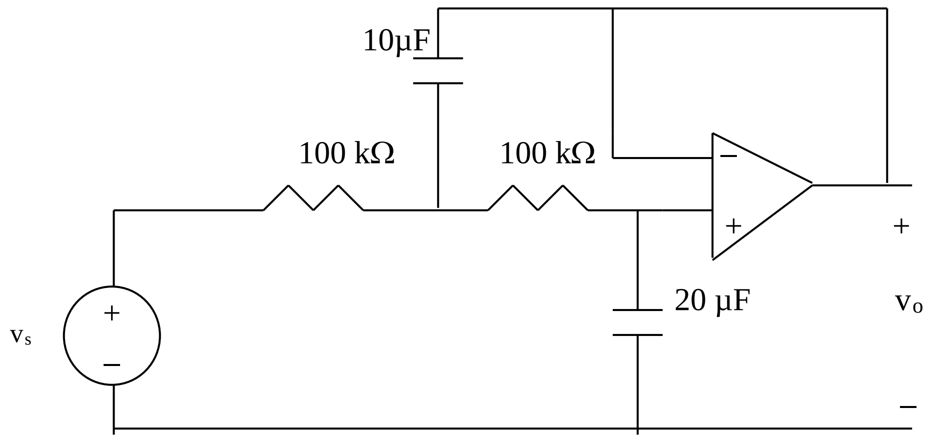
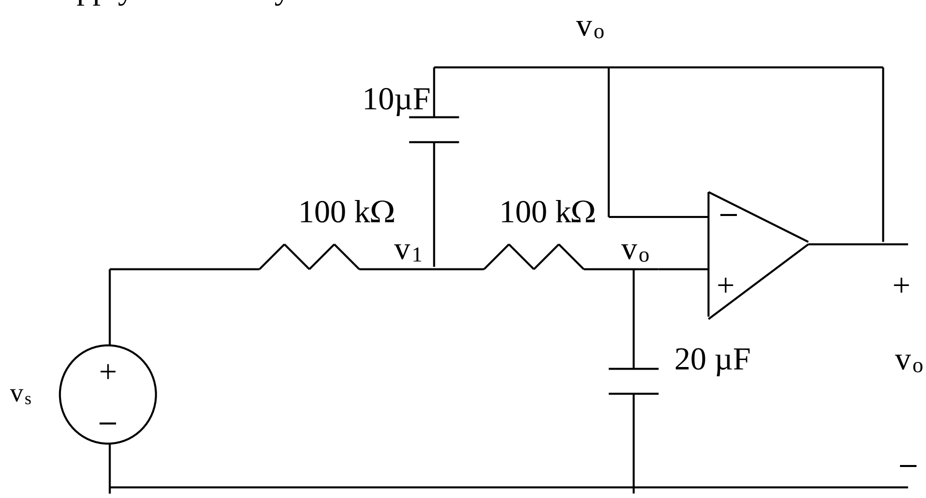


Figure 8.111
For Prob. 8.66.

Solution

We apply nodal analysis to the circuit as shown below.



At node 1,

$$\frac{v_1 - v_s}{10^5} + 10^{-5}\frac{d(v_1 - v_o)}{dt} + \frac{v_1 - v_o}{10^5} = 0 \; or \; v_s = 2v_1 - v_o + \frac{dv_1}{dt} - \frac{dv_o}{dt}$$

At node 2,

$$\frac{v_o - v_1}{10^5} + 2x10^{-5}\frac{d(v_o - 0)}{dt} + 0 = 0\,or\,v_1 = v_o + 2\frac{dv_o}{dt}$$

This leads to $v_s = 2\left(v_o + 2\frac{dv_o}{dt}\right) - v_o + \frac{d\left(v_o + 2\frac{dv_o}{dt}\right)}{dt} - \frac{dv_o}{dt}$ or

$$\mathbf{v_s = [2(d^2v_o/dt^2) + 4(dv_o/dt) + v_o]}$$

Solution 8.67

At node 1,

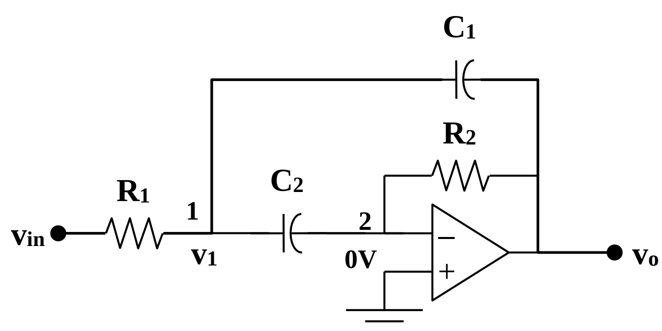
$$\frac{v_{in} - v_1}{R_1} = C_1 \frac{d(v_1 - v_o)}{dt} + C_2 \frac{d(v_1 - 0)}{dt} \qquad (1)$$

At node 2, $$C_2 \frac{d(v_1 - 0)}{dt} = \frac{0 - v_o}{R_2}, \text{ or } \frac{dv_1}{dt} = \frac{-v_o}{C_2 R_2} \qquad (2)$$

From (1) and (2),

$$v_{in} - v_1 = -\frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} - R_1 C_1 \frac{dv_o}{dt} - R_1 \frac{v_o}{R_2}$$

$$v_1 = v_{in} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{dv_o}{dt} + R_1 \frac{v_o}{R_2} \qquad (3)$$



From (2) and (3),

$$-\frac{v_o}{C_2 R_2} = \frac{dv_1}{dt} = \frac{dv_{in}}{dt} + \frac{R_1 C_1}{C_2 R_2}\frac{dv_o}{dt} + R_1 C_1 \frac{d^2 v_o}{dt^2} + \frac{R_1}{R_2}\frac{dv_o}{dt}$$

$$\frac{d^2 v_o}{dt^2} + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{dv_o}{dt} + \frac{v_o}{C_1 C_2 R_2 R_1} = -\frac{1}{R_1 C_1}\frac{dv_{in}}{dt}$$

But $C_1 C_2 R_1 R_2 = 10^{-4} \text{x} 10^{-4} \text{x} 10^{4} \text{x} 10^{4} = 1$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{2}{R_2 C_1} = \frac{2}{10^4 \text{x} 10^{-4}} = 2$$

$$\frac{d^2v_o}{dt^2} + 2\frac{dv_o}{dt} + v_o = -\frac{dv_{in}}{dt}$$

Which leads to $s^2 + 2s + 1 = 0$ or $(s + 1)^2 = 0$ and $s = -1, -1$

Therefore, $v_o(t) = [(A + Bt)e^{-t}] + V_f$

As t approaches infinity, the capacitor acts like an open circuit so that

$$V_f = v_o(\infty) = 0$$

$v_{in} = 10u(t)$ mV and the fact that the initial voltages across each capacitor is 0

means that $v_o(0) = 0$ which leads to $A = 0$.

$$v_o(t) = [Bte^{-t}]$$

$$\frac{dv_o}{dt} = [(B - Bt)e^{-t}] \qquad (4)$$

From (2),
$$\frac{dv_o(0+)}{dt} = -\frac{v_o(0+)}{C_2R_2} = 0$$

From (1) at $t = 0+$,

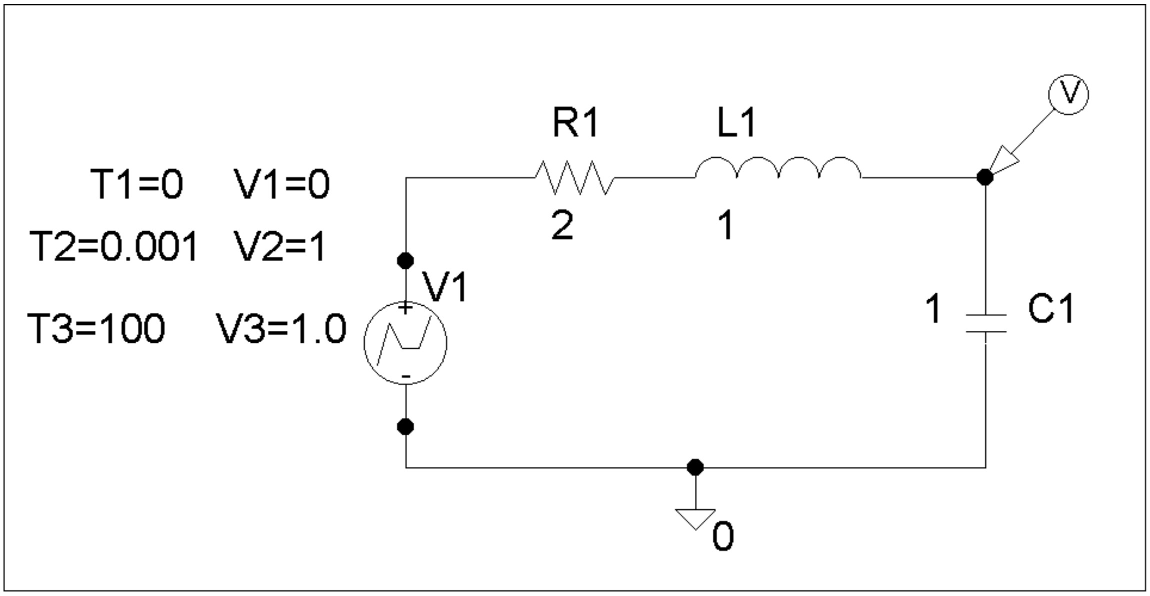
$$\frac{1-0}{R_1} = -C_1\frac{dv_o(0+)}{dt} \text{ which leads to } \frac{dv_o(0+)}{dt} = -\frac{1}{C_1R_1} = -1$$

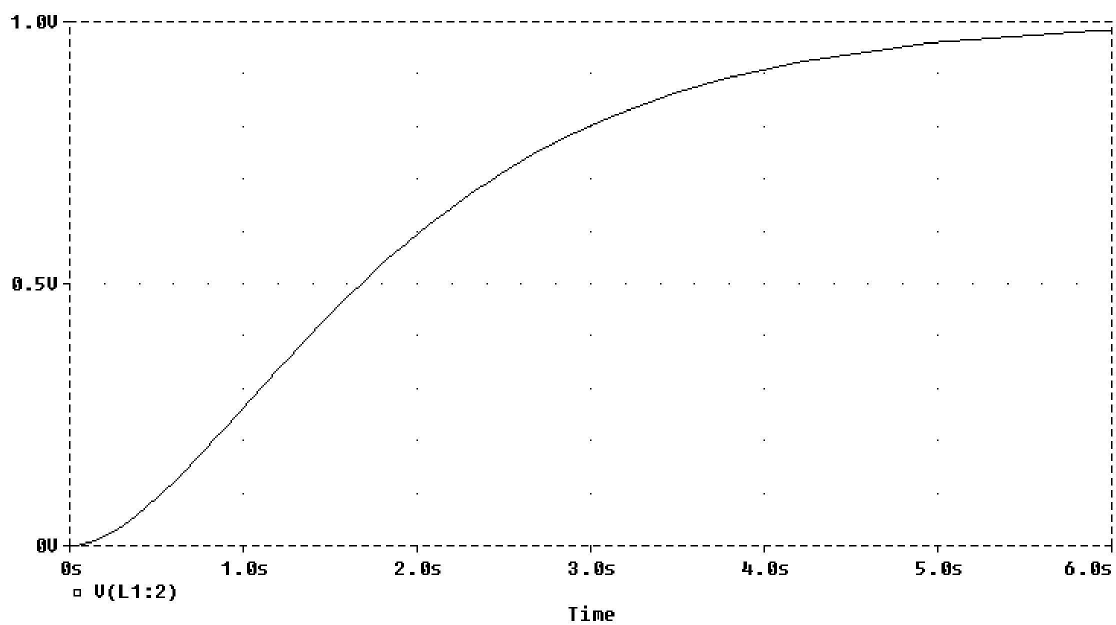
Substituting this into (4) gives $B = -1$

Thus, $v(t) = \mathbf{-te^{-t}u(t)\ V}$

Solution 6.68

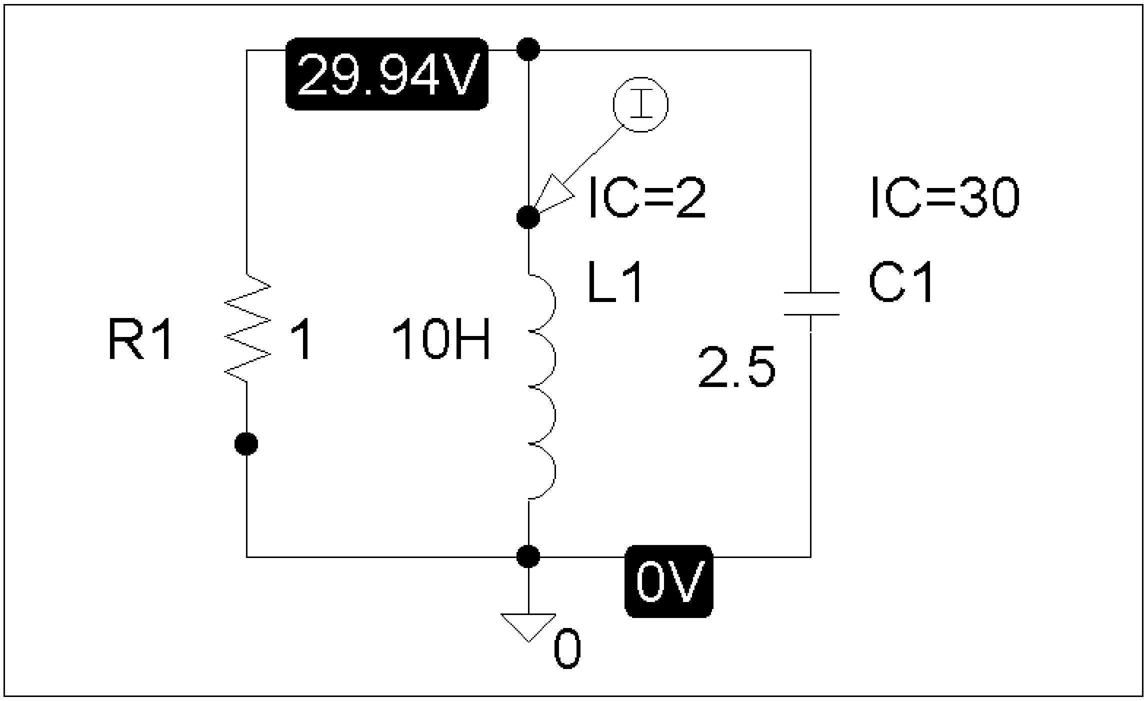
The schematic is as shown below. The unit step is modeled by VPWL as shown. We insert a voltage marker to display V after simulation. We set Print Step = 25 ms and final step = 6s in the transient box. The output plot is shown below.

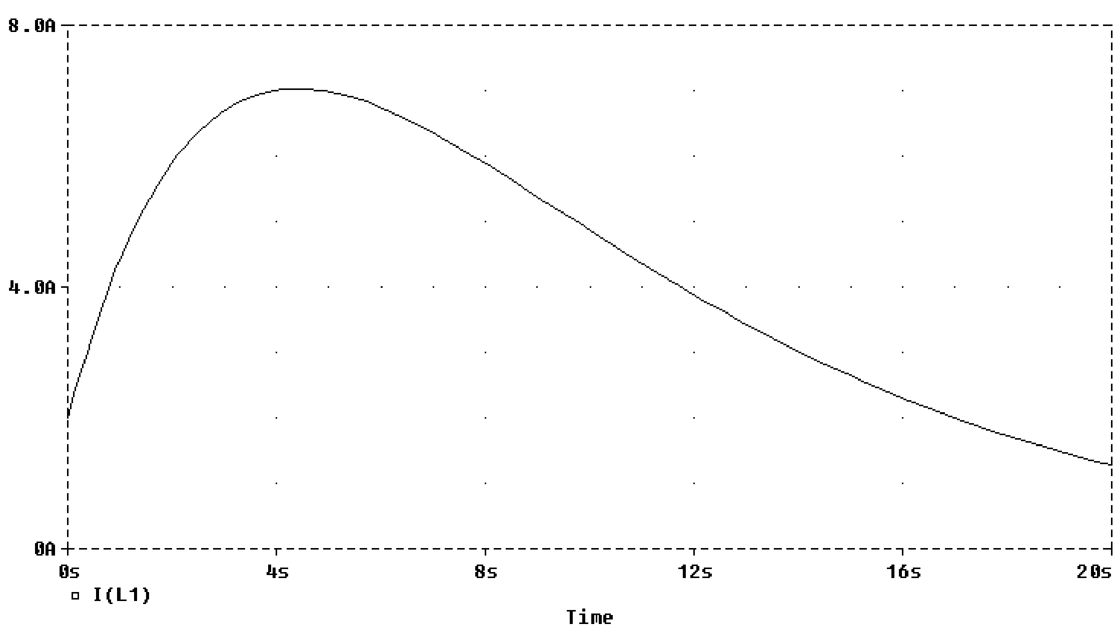




Solution 8.69

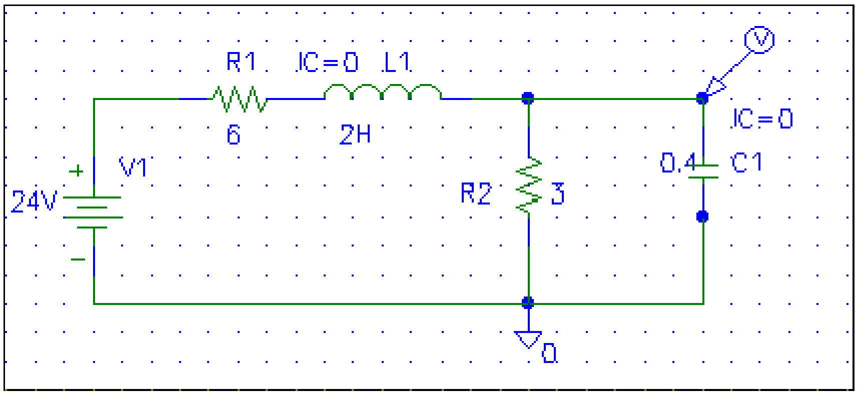
The schematic is shown below. The initial values are set as attributes of L1 and C1. We set Print Step to 25 ms and the Final Time to 20s in the transient box. A current marker is inserted at the terminal of L1 to automatically display i(t) after simulation. The result is shown below.



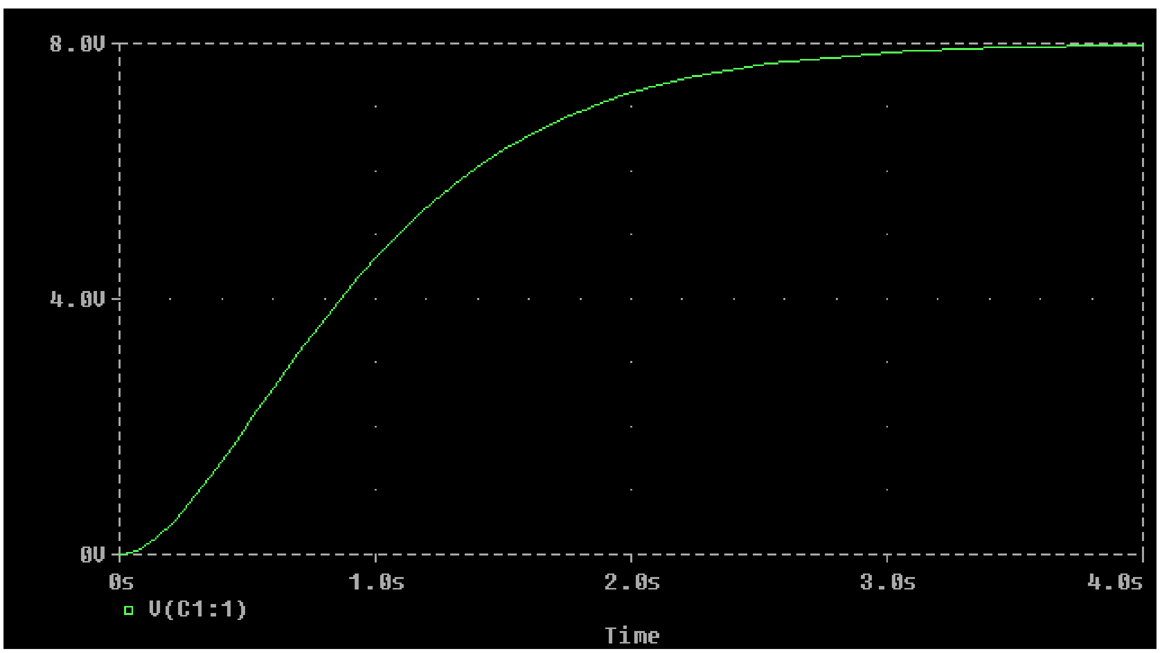


Solution 8.70

The schematic is shown below.

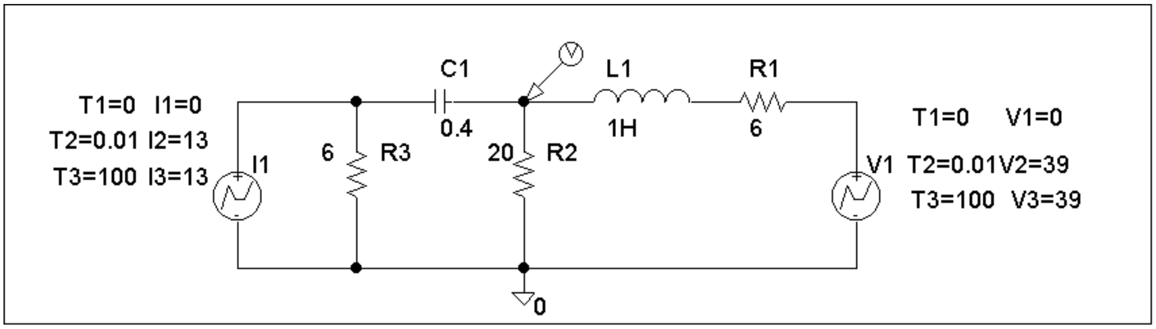


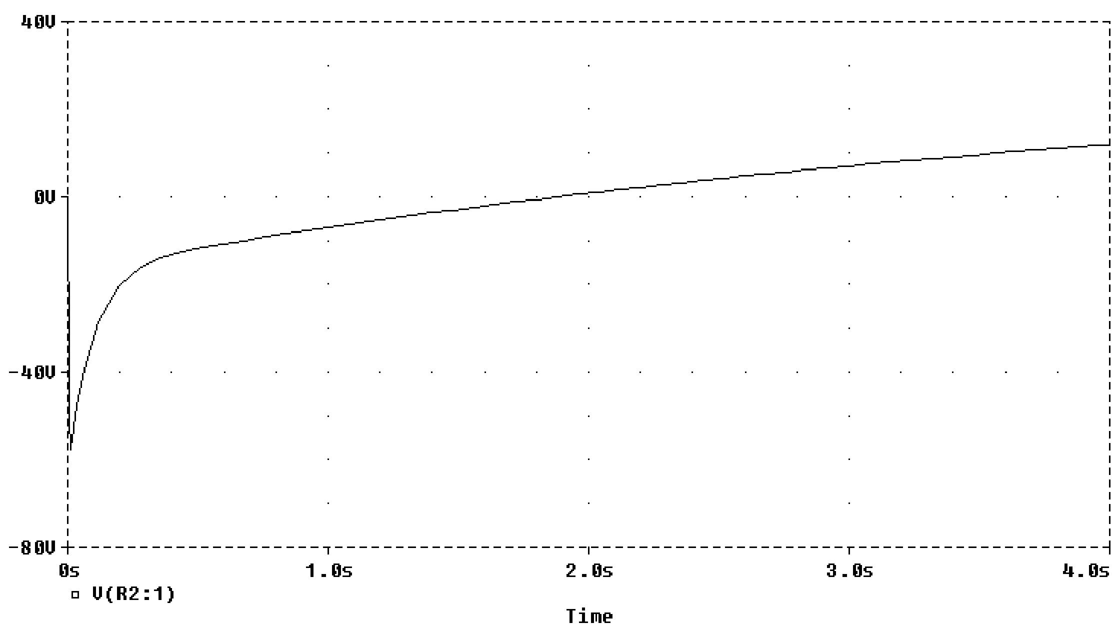
After the circuit is saved and simulated, we obtain the capacitor voltage v(t) as shown below.



Solution 8.71

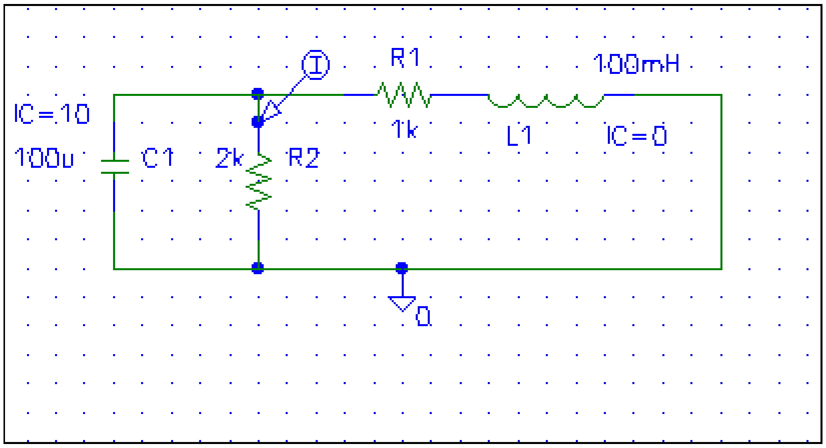
The schematic is shown below. We use VPWL and IPWL to model the 39 u(t) V and 13 u(t) A respectively. We set Print Step to 25 ms and Final Step to 4s in the Transient box. A voltage marker is inserted at the terminal of R2 to automatically produce the plot of v(t) after simulation. The result is shown below.



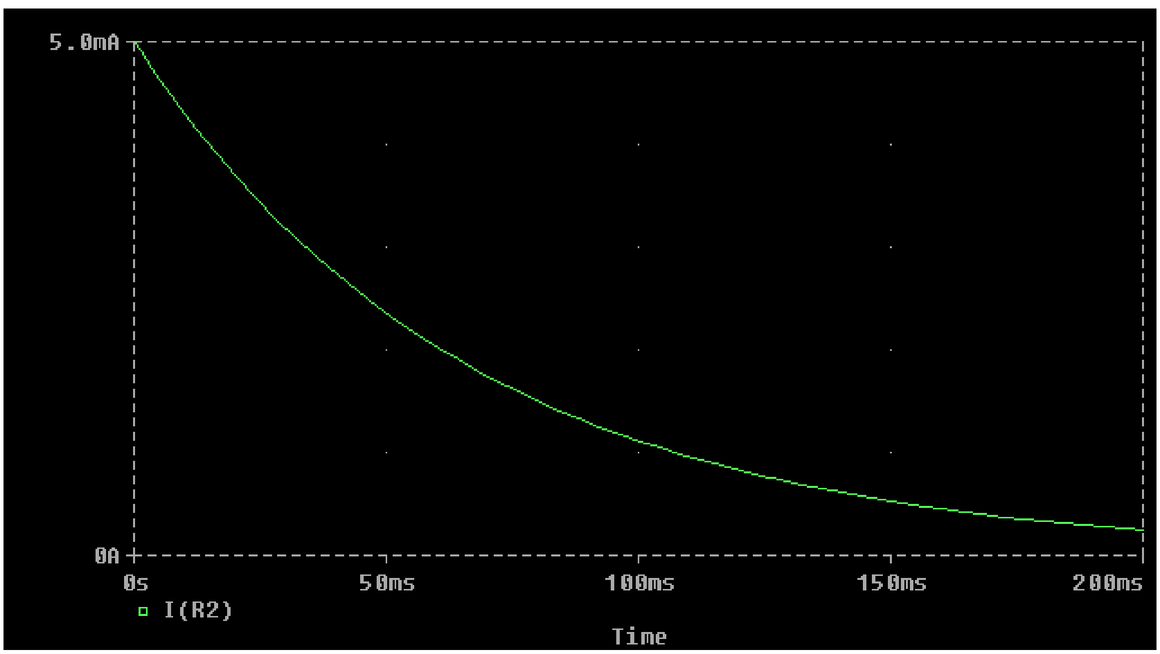


Solution 8.72

When the switch is in position 1, we obtain IC=10 for the capacitor and IC=0 for the inductor. When the switch is in position 2, the schematic of the circuit is shown below.



When the circuit is simulated, we obtain i(t) as shown below.



Solution 8.73

Design a problem, using PSpice, to help other students to better understand source-free *RLC* circuits.

Although there are many ways to work this problem, this is an example based on a somewhat similar problem worked in the third edition.

Problem

The step response of an *RLC* circuit is given by

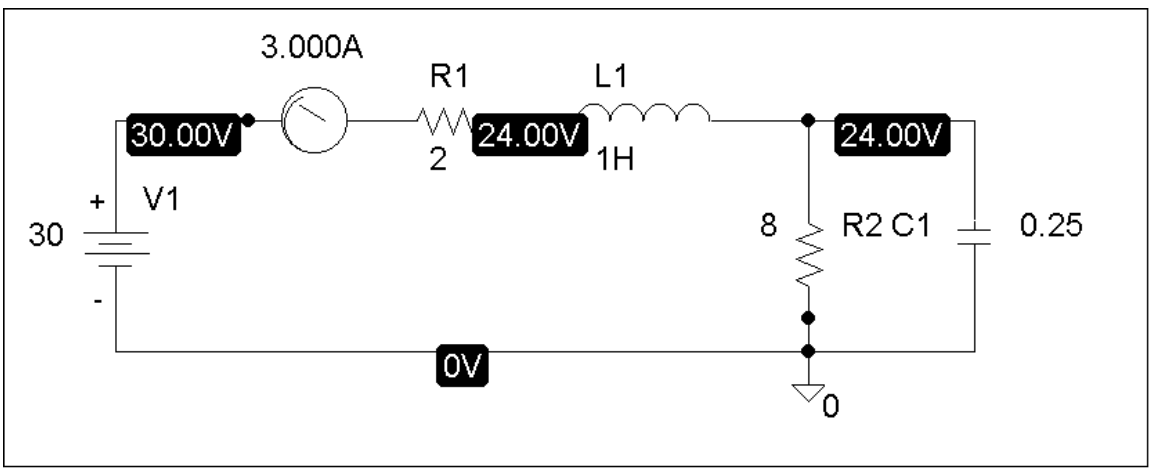
$$\frac{d^2 i_L}{dt^2} + 0.5\frac{di_L}{dt} + 4i_L = 0$$

Given that $i_L(0) = 3$ A and $v_C(0) = 24$ V, solve for v_C(t) and $I_C(t)$.

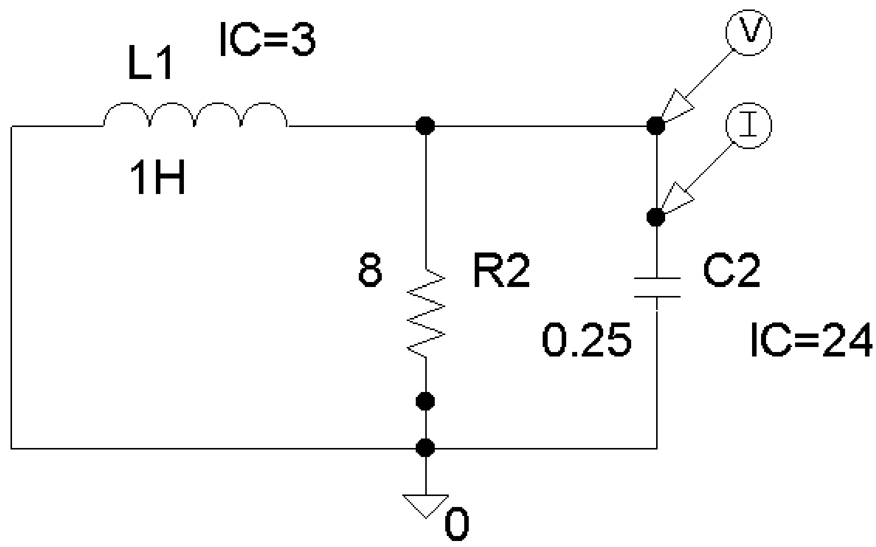
Solution

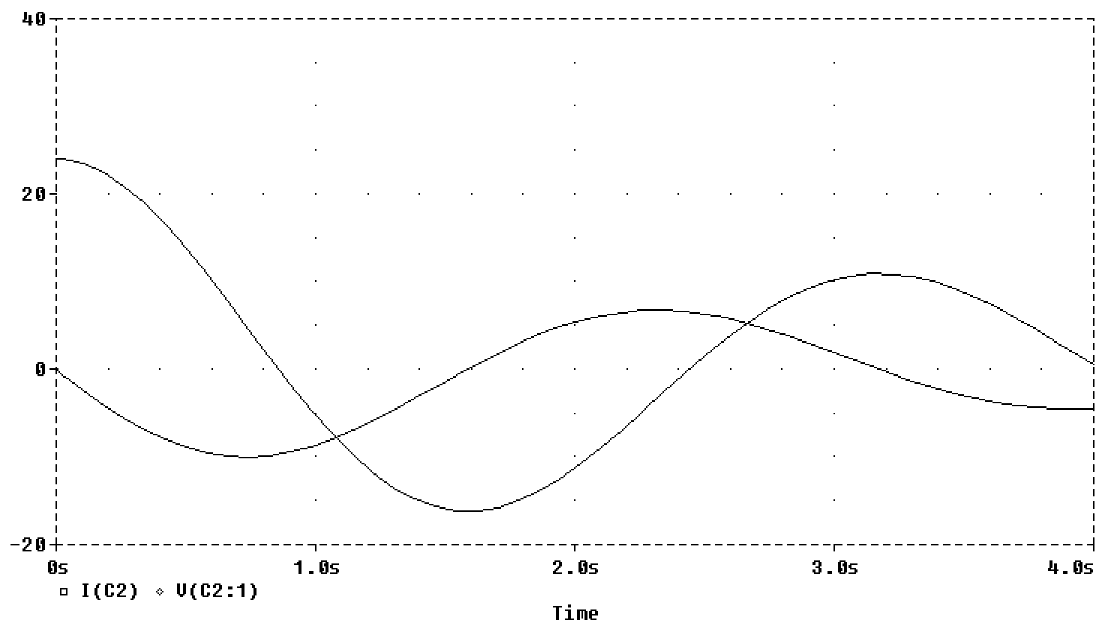
(a) For t < 0, we have the schematic below. When this is saved and simulated, we obtain the initial inductor current and capacitor voltage as

$$i_L(0) = 3 \text{ A} \quad \text{and } v_c(0) = 24 \text{ V}.$$



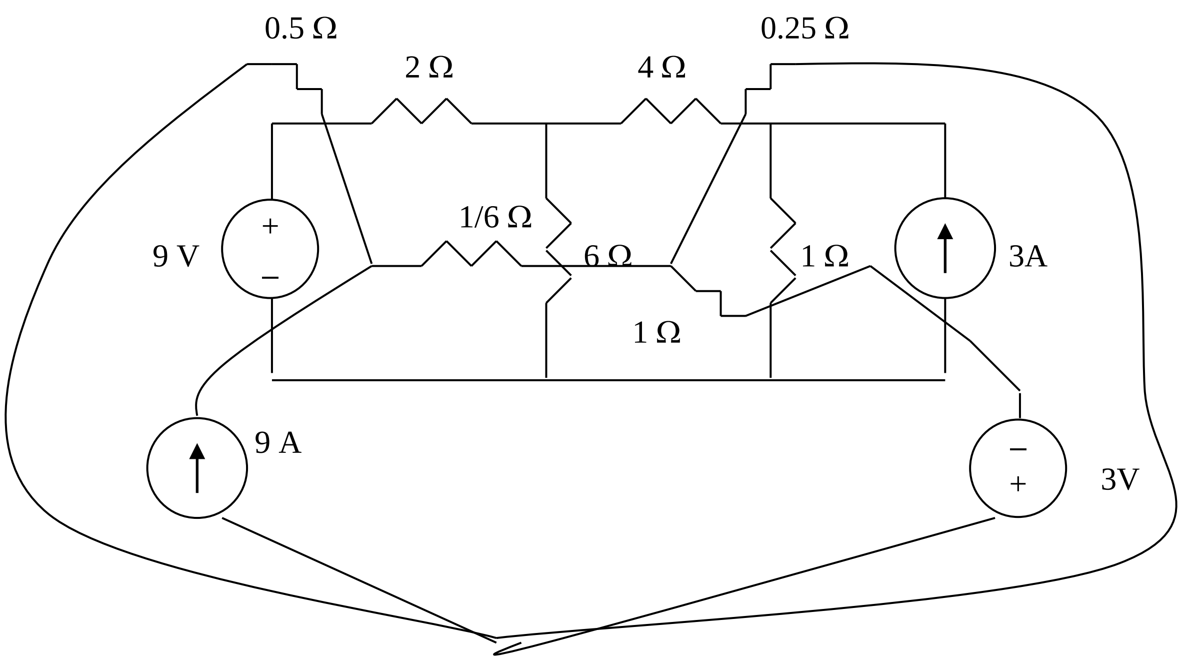
(b) For $t > 0$, we have the schematic shown below. To display i(t) and v(t), we insert current and voltage markers as shown. The initial inductor current and capacitor voltage are also incorporated. In the Transient box, we set Print Step = 25 ms and the Final Time to 4s. After simulation, we automatically have $i_o(t)$ and $v_o(t)$ displayed as shown below.



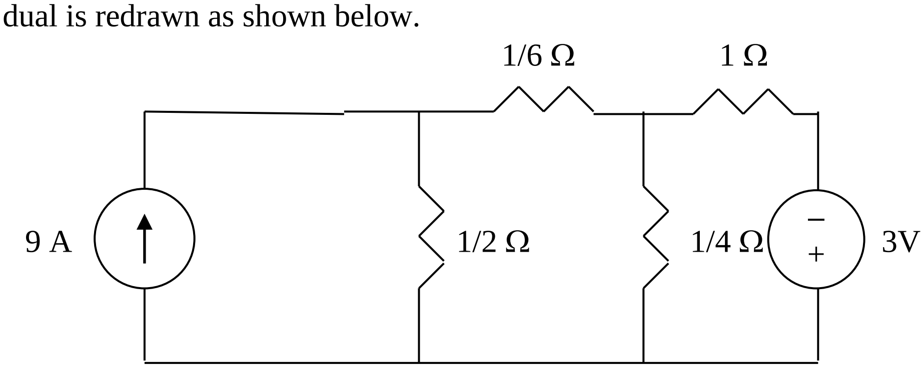


Solution 8.74

The dual is constructed as shown below.

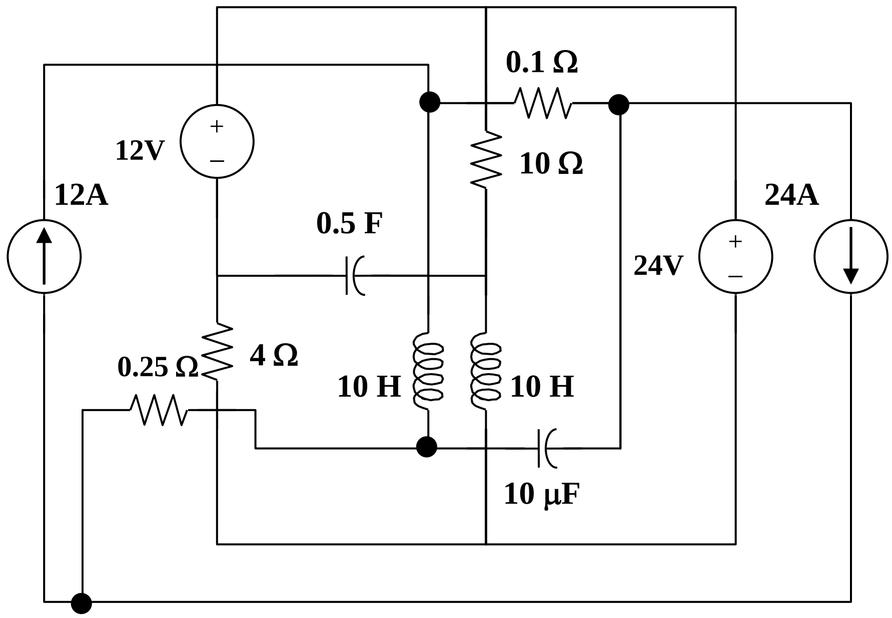


The dual is redrawn as shown below.

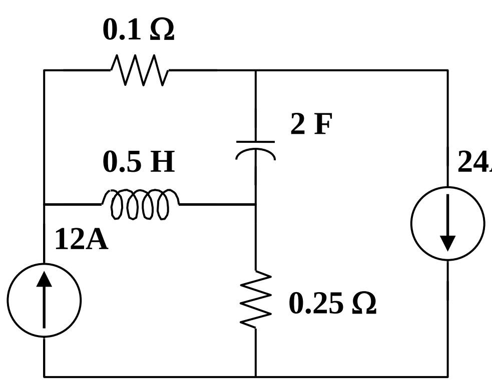


Solution 8.75

The dual circuit is connected as shown in Figure (a). It is redrawn in Figure (b).



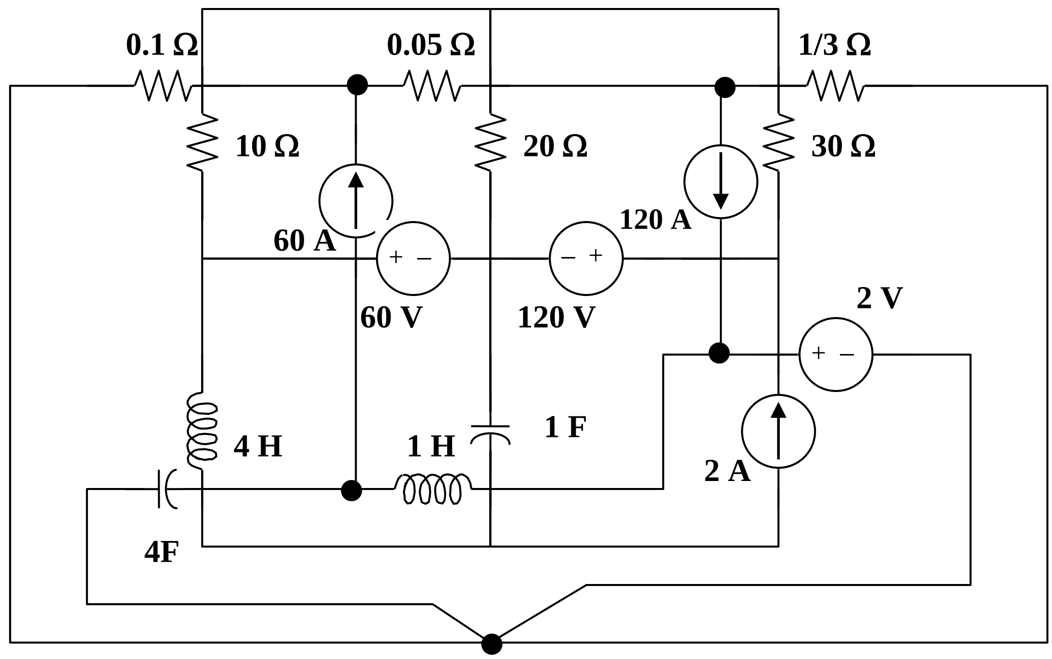
(a)



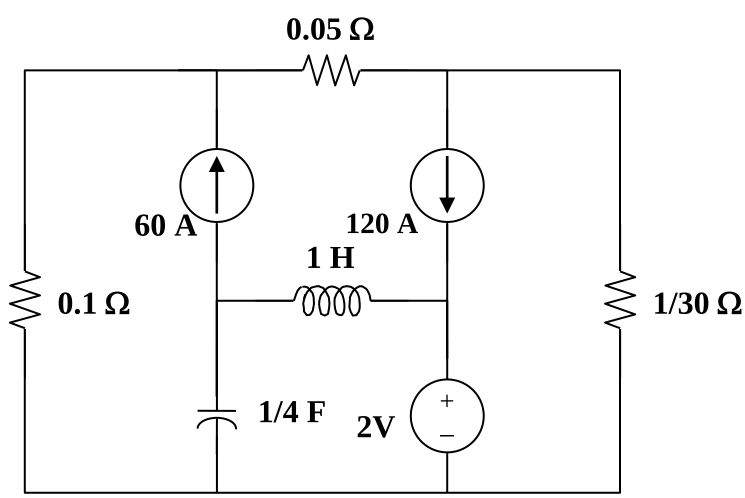
(b)

Solution 8.76

The dual is obtained from the original circuit as shown in Figure (a). It is redrawn in Figure (b).



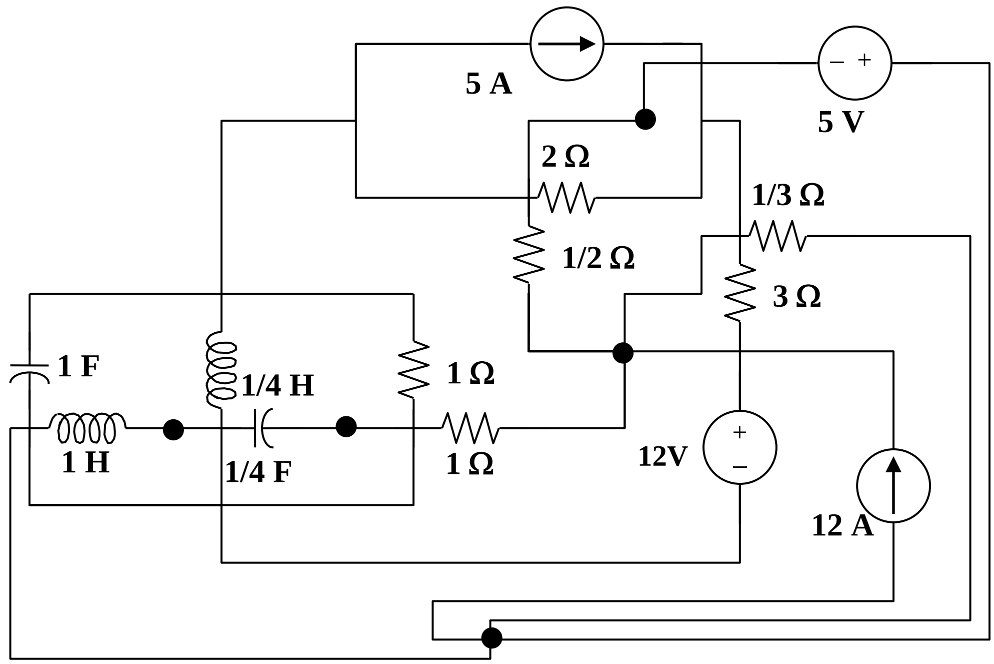
(a)



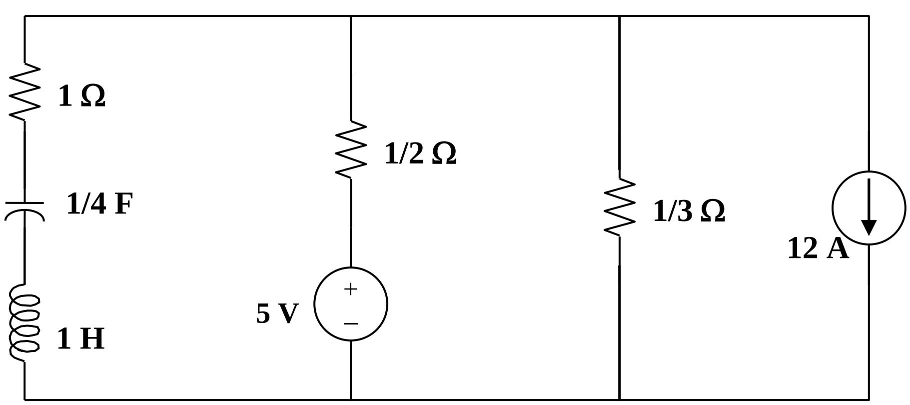
(b)

Solution 8.77

The dual is constructed in Figure (a) and redrawn in Figure (b).



(a)



(b)

Solution 8.78

The voltage across the igniter is $v_R = v_C$ since the circuit is a parallel RLC type.

$$v_C(0) = 12, \text{ and } i_L(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x3x1/30) = 5$$

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{60x10^{-3}x1/30} = 22.36$$

$\alpha < \omega_o$ produces an underdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm j21.794$$

$$v_C(t) = e^{-5t}(A\cos 21.794t + B\sin 21.794t) \qquad (1)$$

$$v_C(0) = 12 = A$$

$$dv_C/dt = -5[(A\cos 21.794t + B\sin 21.794t)e^{-5t}]$$

$$+ 21.794[(-A\sin 21.794t + B\cos 21.794t)e^{-5t}] \qquad (2)$$

$$dv_C(0)/dt = -5A + 21.794B$$

But, $\quad dv_C(0)/dt = -[v_C(0) + Ri_L(0)]/(RC) = -(12 + 0)/(1/10) = -120$

Hence, $\quad -120 = -5A + 21.794B$, leads to $B \ (5x12 - 120)/21.794 = -2.753$

At the peak value, $\quad dv_C(t_o)/dt = 0$, i.e.,

$$0 = A + B\tan 21.794t_o + (A21.794/5)\tan 21.794t_o - 21.794B/5$$

$$(B + A21.794/5)\tan 21.794t_o = (21.794B/5) - A$$

$$\tan 21.794t_o = [(21.794B/5) - A]/(B + A21.794/5) = -24/49.55 = -0.484$$

Therefore, $\quad 21.7945t_o = |-0.451|$

$$t_o = |-0.451|/21.794 = \mathbf{20.68\ ms}$$

Solution 8.79

A load is modeled as a 100-mH inductor in parallel with a 12-Ω resistor. A capacitor is needed to be connected to the load so that the network is critically damped at 60 Hz. Calculate the size of the capacitor.

Solution

For critical damping of a parallel RLC circuit,

$$\alpha = \omega_o \quad \longrightarrow \quad \frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

Hence,

$$C = \frac{L}{4R^2} = \frac{0.1}{4x144} = \mathbf{173.61\mu F}.$$

Solution 8.80

$$t_1 = 1/|s_1| = 0.1\text{x}10^{-3} \text{ leads to } s_1 = -1000/0.1 = -10{,}000$$

$$t_2 = 1/|s_2| = 0.5\text{x}10^{-3} \text{ leads to } s_1 = -2{,}000$$

$$s_1 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$$

$$s_1 + s_2 = -2\alpha = -12{,}000, \text{ therefore } \alpha = 6{,}000 = R/(2L)$$

$$L = R/12{,}000 = 50{,}000/12{,}000 = \mathbf{4.167H}$$

$$s_2 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -2{,}000$$

$$\alpha - \sqrt{\alpha^2 - \omega_o^2} = 2{,}000$$

$$6{,}000 - \sqrt{\alpha^2 - \omega_o^2} = 2{,}000$$

$$\sqrt{\alpha^2 - \omega_o^2} = 4{,}000$$

$$\alpha^2 - \omega_o^2 = 16\text{x}10^6$$

$$\omega_o^2 = \alpha^2 - 16\text{x}10^6 = 36\text{x}10^6 - 16\text{x}10^6$$

$$\omega_o = 10^3\sqrt{20} = 1/\sqrt{LC}$$

$$C = 1/(20\text{x}10^6\text{x}4.167) = \mathbf{12\ nF}$$

Solution 8.81

$t = 1/\alpha = 0.25$ leads to $\alpha = 4$

But, α 1/(2RC) or, $C = 1/(2\alpha R) = 1/(2x4x200) =$ **625 μF**

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

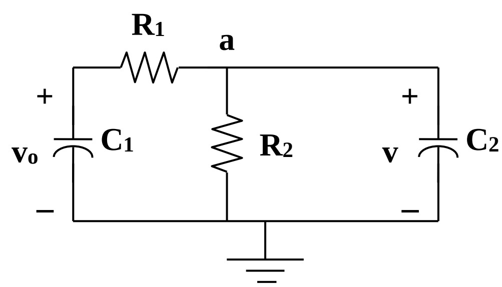
$$\omega_o^2 = \omega_d^2 + \alpha^2 = (2\pi 4x10^3)^2 + 16 \cong (2\pi 4x10^3 0^2 = 1/(LC)$$

This results in $L = 1/(64\pi^2 x10^6 x625x10^{-6}) =$ **2.533 μH**

Solution 8.82

For t = 0-, v(0) = 0.

For t > 0, the circuit is as shown below.



At node a,

$$(v_o - v/R_1 = (v/R_2) + C_2 dv/dt$$

$$v_o = v(1 + R_1/R_2) + R_1C_2\ dv/dt$$

$$60 = (1 + 5/2.5) + (5x10^6\ x5x10^{-6})dv/dt$$

$$60 = 3v + 25dv/dt$$

$$v(t) = V_s + [Ae^{-3t/25}]$$

where $\quad 3V_s = 60$ yields $V_s = 20$

$$v(0) = 0 = 20 + A \text{ or } A = -20$$

$$v(t) = \mathbf{20(1 - e^{-3t/25})V}$$

Solution 8.83

$$i = i_D + C dv/dt \qquad (1)$$

$$-v_s + iR + L di/dt + v = 0 \qquad (2)$$

Substituting (1) into (2),

$$v_s = Ri_D + RC dv/dt + L di_D/dt + LC d^2v/dt^2 + v = 0$$

$$LC d^2v/dt^2 + RC dv/dt + Ri_D + L di_D/dt = v_s$$

$$\mathbf{d^2v/dt^2 + (R/L)dv/dt + (R/LC)i_D + (1/C)di_D/dt = v_s/LC}$$