Solution 10.1

We first determine the input impedance.

$$1H \quad \longrightarrow \quad j\omega L = j1x10 = j10$$

$$1F \quad \longrightarrow \quad 1/j\omega C = \frac{1}{j10x1} = -j0.1$$

$$Z_{in} = 1 + \left(\frac{1}{j10} + \frac{1}{-j0.1} + \frac{1}{1}\right)^{-1} = 1.0101 - j0.1 = 1.015 < -5.653^{o}$$

$$I = \frac{2 < 0^{o}}{1.015 < -5.653^{o}} = 1.9704 < 5.653^{o}$$

i(t) = **1.9704cos(10t+5.65°) A**

Solution 10.2

Using Fig. 10.51, design a problem to help other students better understand nodal analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for V_o in Fig. 10.51, using nodal analysis.

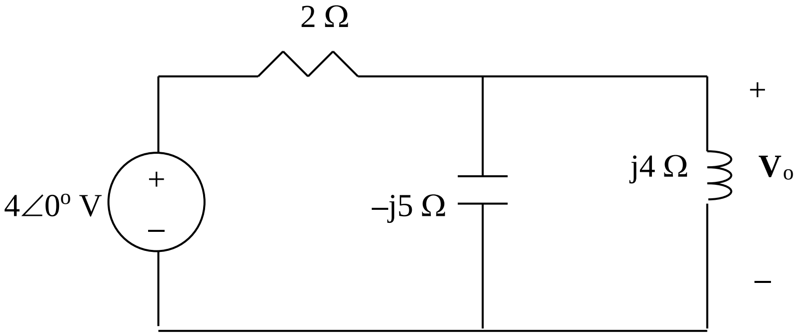
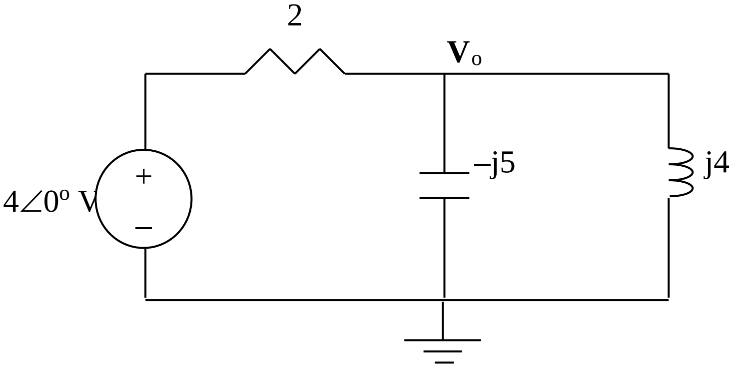


Figure 10.51 For Prob. 10.2.

Solution

Consider the circuit shown below.



At the main node,

$$\frac{4-V_o}{2}=\frac{V_o}{-j5}+\frac{V_o}{j4} \quad \longrightarrow \quad 40=V_o(10+j)$$

$\mathbf{V_o}$ = 40/(10–j) = (40/10.05)∠5.71° = **3.98∠5.71° V**

Solution 10.3

$$\omega = 4$$

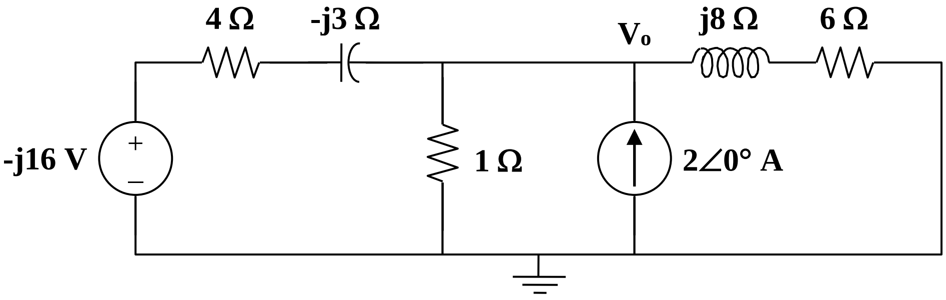
$$2\cos(4t) \longrightarrow 2\angle 0^\circ$$

$$16\sin(4t) \longrightarrow 16\angle -90^\circ = -j16$$

$$2\text{ H} \longrightarrow j\omega L = j8$$

$$1/12\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4)(1/12)} = -j3$$

The circuit is shown below.



Applying nodal analysis,

$$\frac{-j16 - \mathbf{V}_o}{4 - j3} + 2 = \frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{6 + j8}$$

$$\frac{-j16}{4 - j3} + 2 = \left(1 + \frac{1}{4 - j3} + \frac{1}{6 + j8}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{3.92 - j2.56}{1.22 + j0.04} = \frac{4.682\angle -33.15^\circ}{1.2207\angle 1.88^\circ} = 3.835\angle -35.02^\circ$$

Therefore,

$v_o(t) =$ **3.835cos(4t – 35.02°) V**

Solution 10.4

Compute $v_o(t)$ in the circuit of Fig. 10.53.

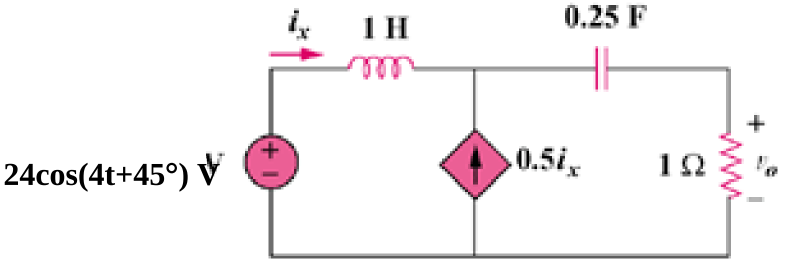
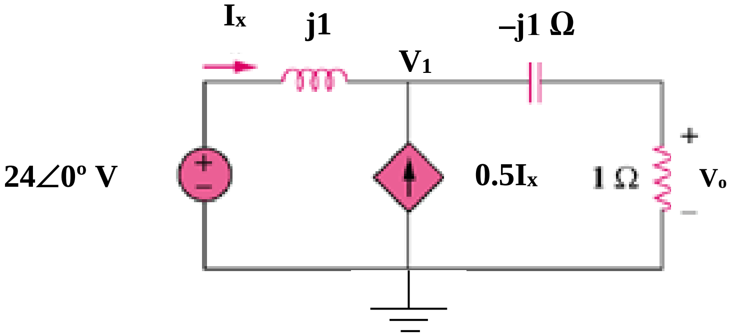


Figure 10.53
For Prob. 10.4.

Solution

Step 1. Convert the circuit into the frequency domain and solve for the node voltage, V_1, using nodal analysis. The find the current $I_C = V_1/[1+(1/(j4x0.25)]$ which then produces $V_o = 1xI_C$. Finally, convert the capacitor voltage back into the time domain.



Note that we represented $24\cos(4t+45^\circ)$ volts by $24\angle 0^\circ$ V. That will make our calculations easier and all we have to do is to offset our answer by a 45°.

Our node equation is $[(V_1-24)/j] - (0.5I_x) + [(V_1-0)/(1-j)] = 0$. We have two unknowns, therefore we need a constraint equation. $I_x = [(24-V_1)/j] = j(V_1-24)$. Once we have V_1, we can find $I_o = V_1\,/(1-j)$ and $V_o = 1xI_o$.

Step 2. Now all we need to do is to solve our equations.

$[(V_1-24)/j] - [0.5j(V_1-24] + [(V_1-0)/(1-j)] = [-j-j0.5+0.5+j0.5]V_1 + j24+j12 = 0$
or

$[0.5–j]V_1 = –j36$ or $V_1 = j36/(–0.5+j) = (36\angle 90^o)/(1.118\angle 116.57^o)$
$= 32.2\angle –26.57^o$ V.

Finally, $I_x = V_1/(1–j) = (32.2\angle –26.57^o) (0.7071\angle 45^o) = 22.769\angle 18.43^o$ A and $V_o = 1xI_o = 22.77\angle 18.43^o$ V or

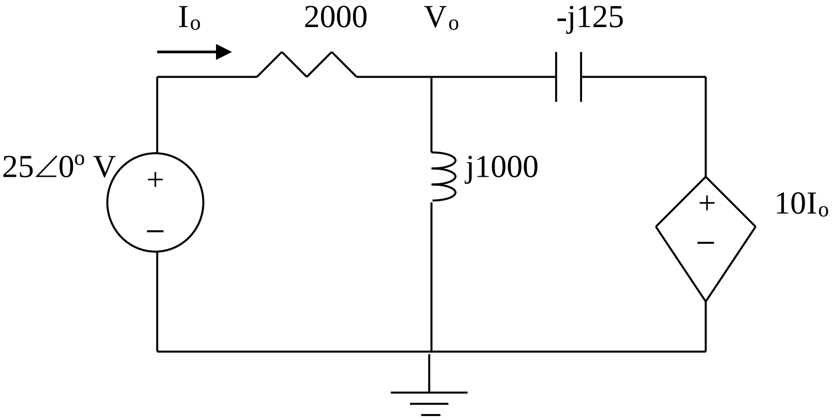
$$v_o(t) = 22.77\cos(4t+45^o+18.43^o) = \mathbf{22.77\cos(4t+63.43^o)\ volts}.$$

Solution 10.5

$$0.25H \quad \longrightarrow \quad j\omega L = j0.25x4x10^3 = j1000$$

$$2\mu F \quad \longrightarrow \quad {}^{1}\!/_{j\omega C} = \frac{1}{j4x10^3x2x10^{-6}} = -j125$$

Consider the circuit as shown below.



At node V_o,

$$\frac{V_o - 25}{2000} + \frac{V_o - 0}{j1000} + \frac{V_o - 10I_o}{-j125} = 0$$

$$V_o - 25 - j2V_o + j16V_o - j160I_o = 0$$

$$(1 + j14)V_o - j160I_o = 25$$

But $I_o = (25–V_o)/2000$

$$(1 + j14)V_o - j2 + j0.08V_o = 25$$

$$V_o = \frac{25 + j2}{1 + j14.08} = \frac{25.08\angle 4.57°}{14.115\angle 58.94°}1.7768\angle -81.37°$$

Now to solve for i_o,

$$I_o = \frac{25 - V_o}{2000} = \frac{25 - 0.2666 + j1.7567}{2000} = 12.367 + j0.8784\,\text{mA}$$

$$= 12.398\angle 4.06°$$

i_o = **$12.398\cos(4x10^3t + 4.06^\circ)$ mA**.

Problem 10.6

Determine $\mathbf{V}_x$ shown in Fig. 10.55

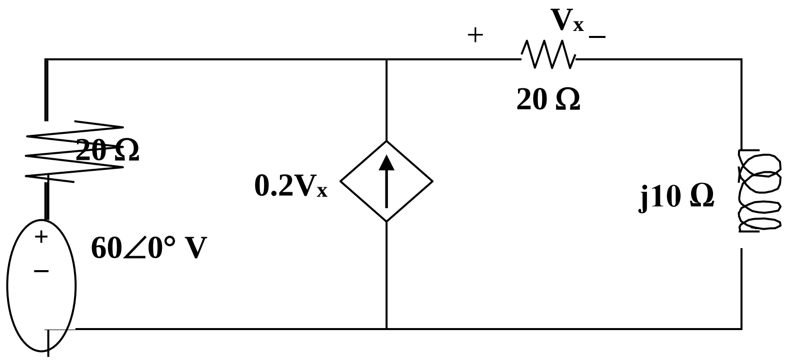


Figure 10.55
For Prob. 10.6.

Solution

Let V_o be the voltage across the dependent current source. Using nodal analysis we get:

$$[(V_o–60)/20] – 0.2V_x + [(V_o–0)/(20+j10)] = 0 \text{ where } V_x = V_o[20/(20+j10)]$$

This leads this to $\{0.05 – [4/(20+j10)] + [1/(20+j10)]\}V_o = 3$ or

$$(1+j0.5–3)V_o = (–2+J0.5)V_o = 3(20+j10) \text{ or } V_o = 3(20+j10)/(–2+j0.5) \text{ or}$$

$$V_x = 3(20)/(–2+j0.5) = 60/(2.06155\angle 165.96°) = \mathbf{29.1\angle –165.96° \ V}.$$

Solution 10.7

At the main node,

$$\frac{120\angle -15^{o} - V}{40 + j20} = 6\angle 30^{o} + \frac{V}{-j30} + \frac{V}{50} \quad \longrightarrow \quad \frac{115.91 - j31.058}{40 + j20} - 5.196 - j3 = V\left(\frac{1}{40 + j20} + \frac{j}{30} + \frac{1}{50}\right)$$

$$V = \frac{-3.1885 - j4.7805}{0.04 + j0.0233} = \underline{124.08\angle -154^{o}\ \ V}$$

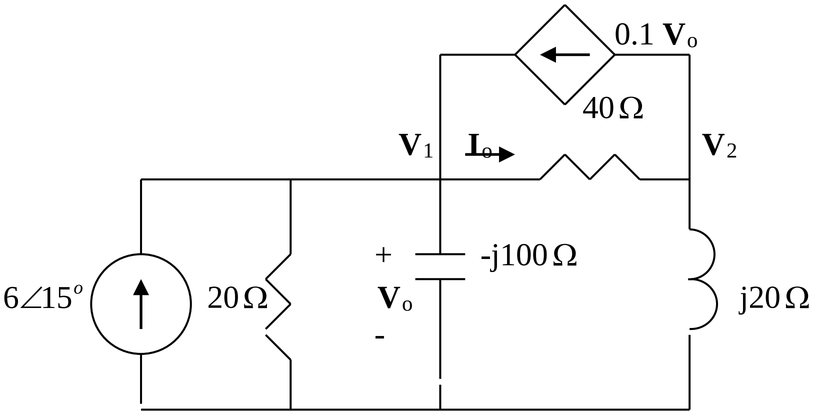
Solution 10.8

$\omega = 200,$

$$100\text{mH} \quad \longrightarrow \quad j\omega L = j200x0.1 = j20$$

$$50\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j200x50x10^{-6}} = -j100$$

The frequency-domain version of the circuit is shown below.



At node 1,

$$6\angle 15^o + 0.1V_1 = \frac{V_1}{20} + \frac{V_1}{-j100} + \frac{V_1 - V_2}{40}$$

or $\quad 5.7955 + j1.5529 = (-0.025 + j0.01)V_1 - 0.025V_2 \qquad (1)$

At node 2,

$$\frac{V_1 - V_2}{40} = 0.1V_1 + \frac{V_2}{j20} \quad \longrightarrow \quad 0 = 3V_1 + (1 - j2)V_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} (-0.025 + j0.01) & -0.025 \\ 3 & (1 - j2) \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} (5.7955 + j1.5529) \\ 0 \end{pmatrix} \quad \text{or} \quad AV = B$$

Using MATLAB,

V = inv(A)*B

leads to $V_1 = -70.63 - j127.23$, $V_2 = -110.3 + j161.09$

$$I_o = \frac{V_1 - V_2}{40} = 7.276\angle -82.17^o$$

Thus,

$$\underline{\boldsymbol{i_o(t) = 7.276\cos(200t - 82.17^o)\ \text{A}}}$$

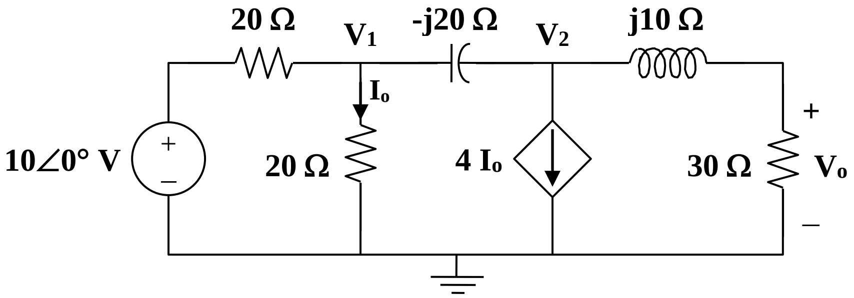
Solution 10.9

$$10\cos(10^3 t) \longrightarrow 10\angle 0°, \quad \omega = 10^3$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

Consider the circuit shown below.



At node 1,

$$\frac{10-\mathbf{V}_1}{20} = \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20}$$

$$10 = (2+j)\mathbf{V}_1 - j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-j20} = (4)\frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_2}{30+j10}, \text{ where } \mathbf{I}_o = \frac{\mathbf{V}_1}{20} \text{ has been substituted.}$$

$$(-4+j)\mathbf{V}_1 = (0.6+j0.8)\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{0.6+j0.8}{-4+j}\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1)

$$10 = \frac{(2+j)(0.6+j0.8)}{-4+j}\mathbf{V}_2 - j\mathbf{V}_2$$

or

$$\mathbf{V}_2 = \frac{170}{0.6-j26.2}$$

$$\mathbf{V}_o = \frac{30}{30+j10}\mathbf{V}_2 = \frac{3}{3+j}\cdot\frac{170}{0.6-j26.2} = 6.154\angle 70.26°$$

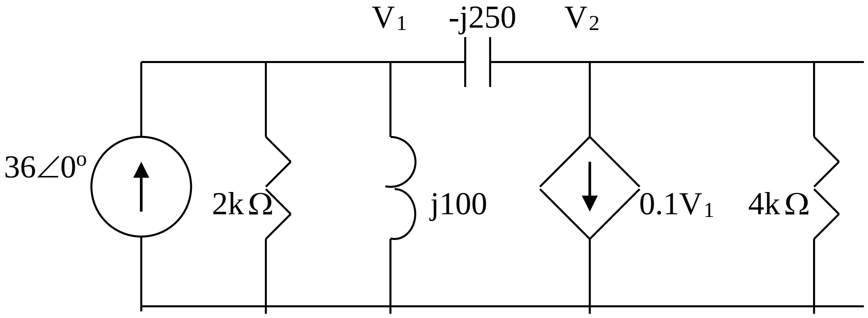
Therefore, $v_o(t) =$ **$6.154\cos(10^3 t + 70.26°)$ V**

Solution 10.10

$$50\,\text{mH} \longrightarrow j\omega L = j2000x50x10^{-3} = j100, \quad \omega = 2000$$

$$2\mu F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j2000x2x10^{-6}} = -j250$$

Consider the frequency-domain equivalent circuit below.



At node 1,

$$36 = \frac{V_1}{2000} + \frac{V_1}{j100} + \frac{V_1 - V_2}{-j250} \longrightarrow 36 = (0.0005 - j0.006)V_1 - j0.004V_2$$

(1)

At node 2,

$$\frac{V_1 - V_2}{-j250} = 0.1V_1 + \frac{V_2}{4000} \longrightarrow 0 = (0.1 - j0.004)V_1 + (0.00025 + j0.004)V_2$$

(2)

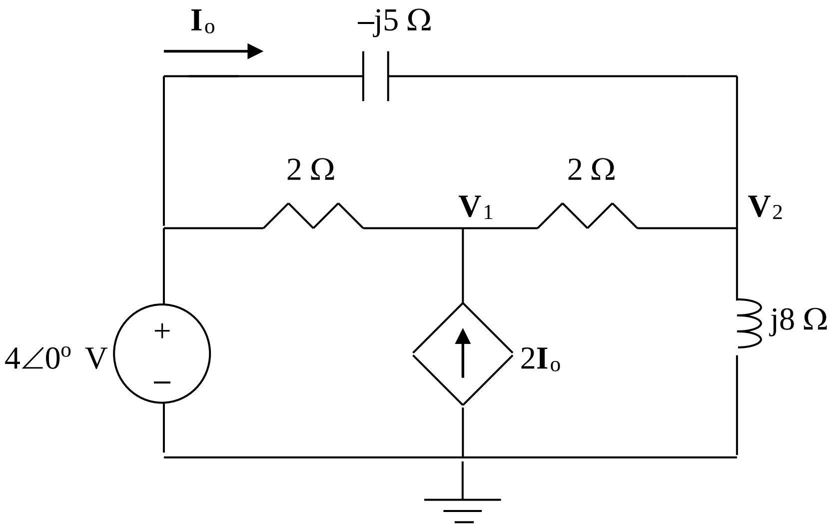
Solving (1) and (2) gives

$$V_o = V_2 = -535.6 + j893.5 = 8951.1\angle 93.43^o$$

v_o (t) = **8.951 sin(2000t +93.43°) kV**

Solution 10.11

Consider the circuit as shown below.



At node 1,

$$\frac{V_1 - 4}{2} - 2I_o + \frac{V_1 - V_2}{2} = 0$$
$$V_1 - 0.5V_2 - 2I_o = 2$$

But, $I_o = (4–V_2)/(–j5) = –j0.2V_2 + j0.8$

Now the first node equation becomes,

$V_1 – 0.5V_2 + j0.4V_2 – j1.6 = 2$ or
$V_1 + (–0.5+j0.4)V_2 = 2 + j1.6$

At node 2,

$$\frac{V_2 - V_1}{2} + \frac{V_2 - 4}{-j5} + \frac{V_2 - 0}{j8} = 0$$
$$–0.5V_1 + (0.5 + j0.075)V_2 = j0.8$$

Using MATLAB to solve this, we get,

```
>> Y=[1,(-0.5+0.4i);-0.5,(0.5+0.075i)]

Y =
```

```
  1.0000          -0.5000 + 0.4000i
 -0.5000           0.5000 + 0.0750i

>> I=[(2+1.6i);0.8i]

I =

  2.0000 + 1.6000i
       0 + 0.8000i

>> V=inv(Y)*I

V =

  4.8597 + 0.0543i
  4.9955 + 0.9050i
```

$I_o = -j0.2V_2 + j0.8 = -j0.9992 + 0.01086 + j0.8 = 0.01086 - j0.1992$

$= \mathbf{199.5\angle 86.89^\circ\ mA}$.

Solution 10.12

Using Fig. 10.61, design a problem to help other students to better understand Nodal analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

By nodal analysis, find i_o in the circuit in Fig. 10.61.

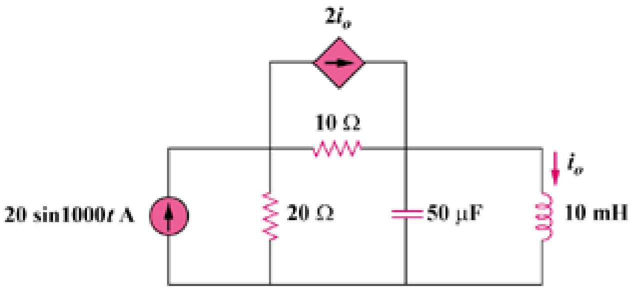


Figure 10.61

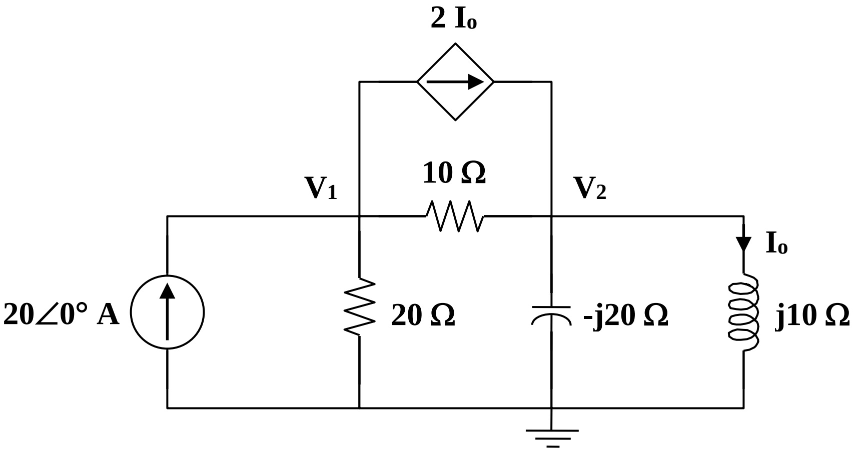
Solution

$$20\sin(1000t) \longrightarrow 20\angle 0°, \quad \omega = 1000$$

$$10 \text{ mH} \longrightarrow j\omega L = j10$$

$$50\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^3)(50\times 10^{-6})} = -j20$$

The frequency-domain equivalent circuit is shown below.



At node 1,

$$20 = 2\mathbf{I}_o + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10},$$ where

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10}$$

$$20 = \frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1}{20} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10}$$

$$400 = 3\mathbf{V}_1 - (2 + j4)\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{2\mathbf{V}_2}{j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{\mathbf{V}_2}{-j20} + \frac{\mathbf{V}_2}{j10}$$

$$j2\mathbf{V}_1 = (-3 + j2)\mathbf{V}_2$$

or $$\mathbf{V}_1 = (1 + j1.5)\mathbf{V}_2 \qquad (2)$$

Substituting (2) into (1),

$$400 = (3 + j4.5)\mathbf{V}_2 - (2 + j4)\mathbf{V}_2 = (1 + j0.5)\mathbf{V}_2$$

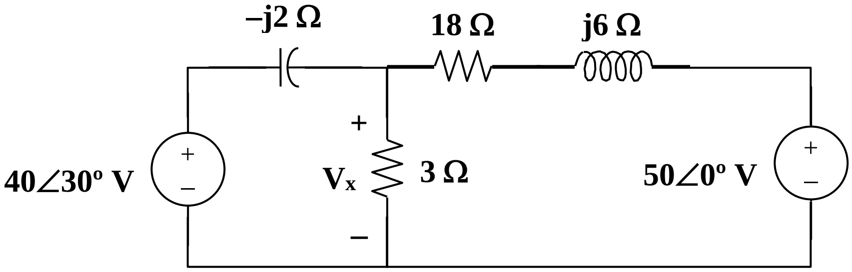
$$\mathbf{V}_2 = \frac{400}{1 + j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_2}{j10} = \frac{40}{j(1 + j0.5)} = 35.74\angle -116.6^\circ$$

Therefore, $i_o(t) =$ **35.74 sin(1000t – 116.6°) A**

Solution 10.13

Nodal analysis is the best approach to use on this problem. We can make our work easier by doing a source transformation on the right hand side of the circuit.



$$\frac{V_x - 40\angle 30^\circ}{-j2} + \frac{V_x}{3} + \frac{V_x - 50}{18 + j6} = 0$$

which leads to V_x = **29.36∠62.88° A**.

Solution 10.14

At node 1,

$$\frac{0-\mathbf{V}_1}{-j2}+\frac{0-\mathbf{V}_1}{10}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30^\circ$$

$$-(1+j2.5)\mathbf{V}_1 - j2.5\mathbf{V}_2 = 173.2 + j100 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_2}{j2}+\frac{\mathbf{V}_2}{-j5}+\frac{\mathbf{V}_2-\mathbf{V}_1}{j4}=20\angle 30^\circ$$

$$-j5.5\mathbf{V}_2 + j2.5\mathbf{V}_1 = 173.2 + j100 \qquad (2)$$

Equations (1) and (2) can be cast into matrix form as

$$\begin{bmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{bmatrix}\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}=\begin{bmatrix} -200\angle 30^\circ \\ 200\angle 30^\circ \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1+j2.5 & j2.5 \\ j2.5 & -j5.5 \end{vmatrix} = 20 - j5.5 = 20.74\angle -15.38^\circ$$

$$\Delta_1 = \begin{vmatrix} -200\angle 30^\circ & j2.5 \\ 200\angle 30^\circ & -j5.5 \end{vmatrix} = j3(200\angle 30^\circ) = 600\angle 120^\circ$$

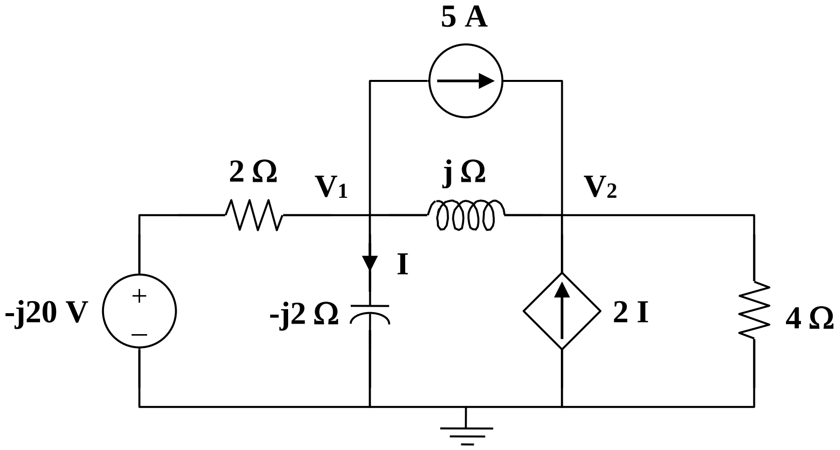
$$\Delta_2 = \begin{vmatrix} 1+j2.5 & -200\angle 30^\circ \\ j2.5 & 200\angle 30^\circ \end{vmatrix} = (200\angle 30^\circ)(1+j5) = 1020\angle 108.7^\circ$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 28.93\angle 135.38^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 49.18\angle 124.08^\circ \text{ V}$$

Solution 10.15

We apply nodal analysis to the circuit shown below.



At node 1,

$$\frac{-j20 - \mathbf{V}_1}{2} = 5 + \frac{\mathbf{V}_1}{-j2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j}$$

$$-5 - j10 = (0.5 - j0.5)\,\mathbf{V}_1 + j\,\mathbf{V}_2 \qquad (1)$$

At node 2,

$$5 + 2\,\mathbf{I} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j} = \frac{\mathbf{V}_2}{4},$$

where $\mathbf{I} = \frac{\mathbf{V}_1}{-j2}$

$$\mathbf{V}_2 = \frac{5}{0.25 - j}\,\mathbf{V}_1 \qquad (2)$$

Substituting (2) into (1),

$$-5 - j10 - \frac{j5}{0.25 - j} = 0.5\,(1 - j)\,\mathbf{V}_1$$

$$(1 - j)\,\mathbf{V}_1 = -10 - j20 - \frac{j40}{1 - j4}$$

$$(\sqrt{2}\angle -45°)\,\mathbf{V}_1 = -10 - j20 + \frac{160}{17} - \frac{j40}{17}$$

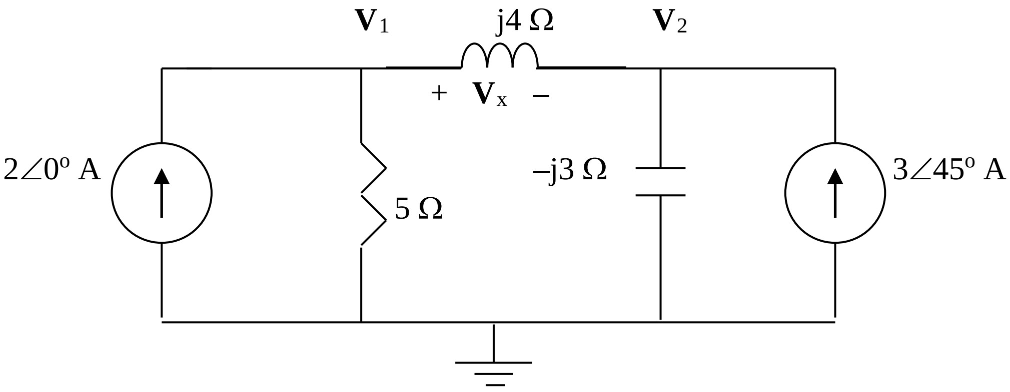
$$\mathbf{V}_1 = 15.81\angle 313.5°$$

$$\mathbf{I} = \frac{\mathbf{V}_1}{-j2} = (0.5\angle 90°)(15.81\angle 313.5°)$$

$$\mathbf{I} = \mathbf{7.906\angle 43.49°\ A}$$

Solution 10.16

Consider the circuit as shown in the figure below.



At node 1,

$$-2+\frac{V_1-0}{5}+\frac{V_1-V_2}{j4}=0 \qquad (1)$$

$$(0.2-j0.25)V_1+j0.25V_2=2$$

At node 2,

$$\frac{V_2-V_1}{j4}+\frac{V_2-0}{-j3}-3\angle 45^\circ=0 \qquad (2)$$

$$j0.25V_1+j0.08333V_2=2.121+j2.121$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 0.2-j0.25 & j0.25 \\ j0.25 & j0.08333 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} 2 \\ 2.121+j2.121 \end{bmatrix}$$

Solving this using MATLAB, we get,

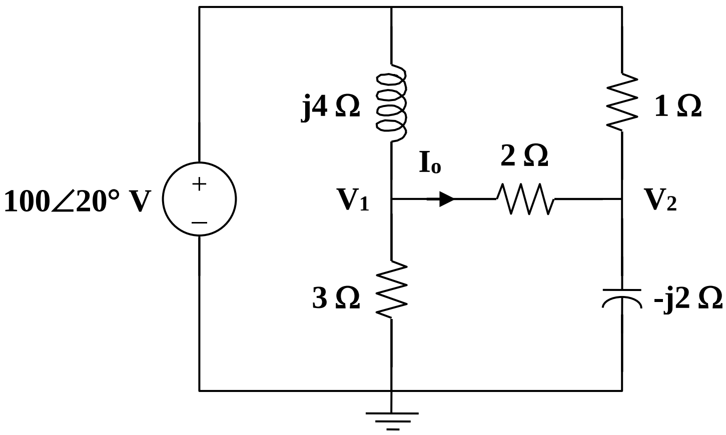
```
>> Y=[(0.2-0.25i),0.25i;0.25i,0.08333i]

Y =

  0.2000 - 0.2500i       0 + 0.2500i
       0 + 0.2500i       0 + 0.0833i

>> I=[2;(2.121+2.121i)]
```

```
I =

   2.0000
   2.1210 + 2.1210i

>> V=inv(Y)*I

V =

   5.2793 - 5.4190i
   9.6145 - 9.1955i
```

$$V_s = V_1 - V_2 = -4.335 + j3.776 = \mathbf{5.749\angle 138.94^\circ\ V}.$$

Solution 10.17

Consider the circuit below.



At node 1,

$$\frac{100\angle 20° - \mathbf{V}_1}{j4} = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2}$$

$$100\angle 20° = \frac{\mathbf{V}_1}{3}(3 + j10) - j2\,\mathbf{V}_2$$

(1)

At node 2,

$$\frac{100\angle 20° - \mathbf{V}_2}{1} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\mathbf{V}_2}{-j2}$$

$$100\angle 20° = -0.5\,\mathbf{V}_1 + (1.5 + j0.5)\,\mathbf{V}_2$$

(2)

From (1) and (2),

$$\begin{bmatrix} 100\angle 20° \\ 100\angle 20° \end{bmatrix} = \begin{bmatrix} -0.5 & 0.5(3 + j) \\ 1 + j10/3 & -j2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} -0.5 & 1.5 + j0.5 \\ 1 + j10/3 & -j2 \end{vmatrix} = 0.1667 - j4.5$$

$$\Delta_1 = \begin{vmatrix} 100\angle 20° & 1.5 + j0.5 \\ 100\angle 20° & -j2 \end{vmatrix} = -55.45 - j286.2$$

$$\Delta_2 = \begin{vmatrix} -0.5 & 100\angle 20° \\ 1 + j10/3 & 100\angle 20° \end{vmatrix} = -26.95 - j364.5$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = 64.74\angle\text{-}13.08^\circ$$

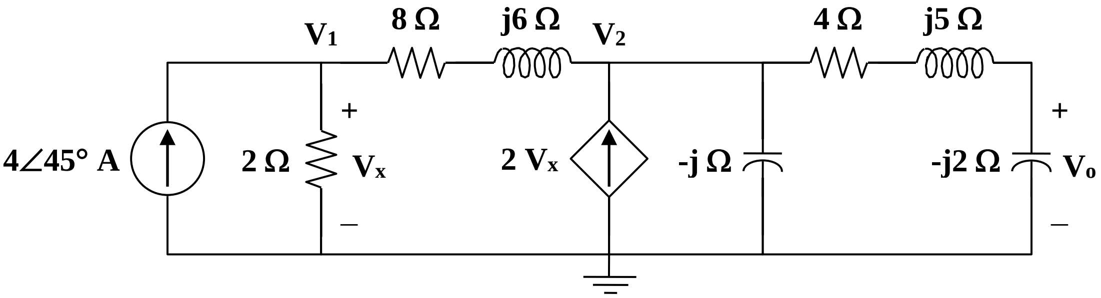
$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 81.17\angle\text{-}6.35^\circ$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - \mathbf{V}_2}{2} = \frac{\Delta_1 - \Delta_2}{2\Delta} = \frac{\text{-}28.5 + j78.31}{0.3333 - j9}$$

$$\mathbf{I}_o = \mathbf{9.25\angle\text{-}162.12^\circ\ A}$$

Solution 10.18

Consider the circuit shown below.



At node 1,

$$4\angle 45° = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8+j6}$$

$$200\angle 45° = (29 - j3)\,\mathbf{V}_1 - (4 - j3)\,\mathbf{V}_2$$

(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{8+j6} + 2\mathbf{V}_x = \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{4 + j5 - j2},$$

where $\mathbf{V}_x = \mathbf{V}_1$

$$(104 - j3)\,\mathbf{V}_1 = (12 + j41)\,\mathbf{V}_2$$

$$\mathbf{V}_1 = \frac{12 + j41}{104 - j3}\mathbf{V}_2$$

(2)

Substituting (2) into (1),

$$200\angle 45° = (29 - j3)\frac{(12 + j41)}{104 - j3}\mathbf{V}_2 - (4 - j3)\,\mathbf{V}_2$$

$$200\angle 45° = (14.21\angle 89.17°)\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{200\angle 45°}{14.21\angle 89.17°}$$

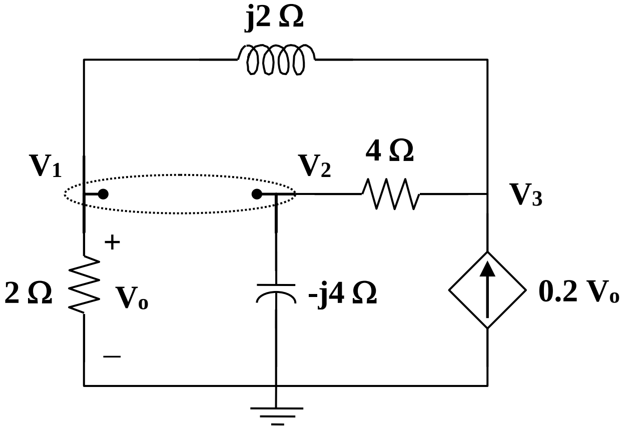
$$\mathbf{V}_o = \frac{-j2}{4 + j5 - j2}\mathbf{V}_2 = \frac{-j2}{4 + j3}\mathbf{V}_2 = \frac{-6 - j8}{25}\mathbf{V}_2$$

$$\mathbf{V}_o = \frac{10\angle 233.13°}{25} \cdot \frac{200\angle 45°}{14.21\angle 89.17°}$$

$$\mathbf{V}_o = \mathbf{5.63\angle 189°\ V}$$

Solution 10.19

We have a supernode as shown in the circuit below.



Notice that $\mathbf{V}_o = \mathbf{V}_1$.

At the supernode,

$$\frac{\mathbf{V}_3 - \mathbf{V}_2}{4} = \frac{\mathbf{V}_2}{\text{- j4}} + \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{\text{j2}}$$

$$0 = (2 - \text{j2})\,\mathbf{V}_1 + (1 + \text{j})\,\mathbf{V}_2 + (\text{-1} + \text{j2})\,\mathbf{V}_3 \qquad (1)$$

At node 3,

$$0.2\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_3}{\text{j2}} = \frac{\mathbf{V}_3 - \mathbf{V}_2}{4}$$

$$(0.8 - \text{j2})\,\mathbf{V}_1 + \mathbf{V}_2 + (\text{-1} + \text{j2})\,\mathbf{V}_3 = 0 \qquad (2)$$

Subtracting (2) from (1),

$$0 = 1.2\mathbf{V}_1 + \text{j}\,\mathbf{V}_2 \qquad (3)$$

But at the supernode,

$$\mathbf{V}_1 = 12\angle 0° + \mathbf{V}_2$$

or $\mathbf{V}_2 = \mathbf{V}_1 - 12 \qquad (4)$

Substituting (4) into (3),

$$0 = 1.2\mathbf{V}_1 + \text{j}\,(\mathbf{V}_1 - 12)$$

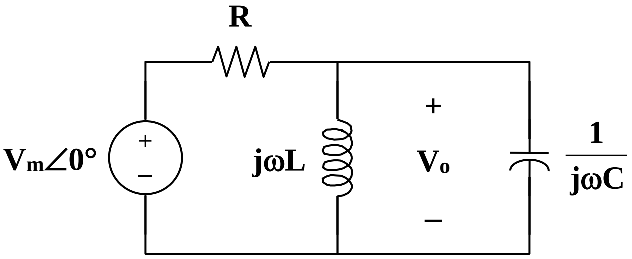
$$\mathbf{V}_1 = \frac{\text{j12}}{1.2 + \text{j}} = \mathbf{V}_o$$

$$\mathbf{V}_o = \frac{12\angle 90°}{1.562\angle 39.81°}$$

$$\mathbf{V}_o = \mathbf{7.682\angle 50.19°\ V}$$

Solution 10.20

The circuit is converted to its frequency-domain equivalent circuit as shown below.



Let $$\mathbf{Z} = j\omega L \parallel \frac{1}{j\omega C} = \frac{\frac{L}{C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$$

$$\mathbf{V}_o = \frac{\mathbf{Z}}{R+\mathbf{Z}} V_m = \frac{\frac{j\omega L}{1-\omega^2 LC}}{R + \frac{j\omega L}{1-\omega^2 LC}} V_m = \frac{j\omega L}{R\,(1-\omega^2 LC) + j\omega L} V_m$$

$$\mathbf{V}_o = \frac{\omega L\, V_m}{\sqrt{R^2\,(1-\omega^2 LC)^2 + \omega^2 L^2}} \angle\left(90° - \tan^{-1}\frac{\omega L}{R\,(1-\omega^2 LC)}\right)$$

If $\mathbf{V}_o = A\angle\phi$, then

$$A = \frac{\boldsymbol{\omega} \mathbf{L}\, \mathbf{V_m}}{\sqrt{\mathbf{R}^2\,(\mathbf{1}-\boldsymbol{\omega}^2\mathbf{LC})^2 + \boldsymbol{\omega}^2\mathbf{L}^2}}$$

and $$\phi = \mathbf{90° - tan^{-1}} \frac{\boldsymbol{\omega}\mathbf{L}}{\mathbf{R\,(1-}\boldsymbol{\omega}^2\mathbf{LC)}}$$

Solution 10.21

(a) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{1}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{1}{1 - \omega^2 LC + j\omega RC}$$

At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \mathbf{1}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \frac{\mathbf{-j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}$$

(b) $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

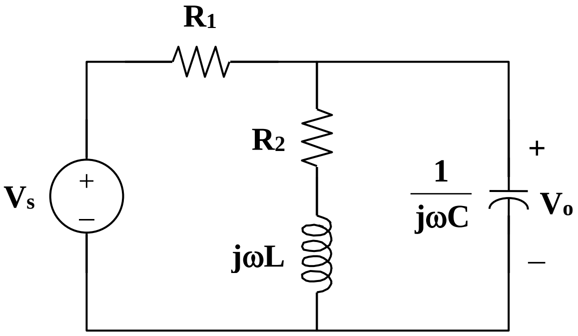
At $\omega = 0$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0}$$

As $\omega \to \infty$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1} = \mathbf{1}$$

At $\omega = \dfrac{1}{\sqrt{LC}}$, $$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-1}{jRC \cdot \dfrac{1}{\sqrt{LC}}} = \frac{\mathbf{j}}{\mathbf{R}}\sqrt{\frac{\mathbf{L}}{\mathbf{C}}}$$

Solution 10.22

Consider the circuit in the frequency domain as shown below.



Let $\mathbf{Z} = (R_2 + j\omega L) \,||\, \dfrac{1}{j\omega C}$

$$\mathbf{Z} = \frac{\dfrac{1}{j\omega C}(R_2 + j\omega L)}{R_2 + j\omega L + \dfrac{1}{j\omega C}} = \frac{R_2 + j\omega L}{1 + j\omega R_2 - \omega^2 LC}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{Z}}{\mathbf{Z} + R_1} = \frac{\dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}{R_1 + \dfrac{R_2 + j\omega L}{1 - \omega^2 LC + j\omega R_2 C}}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \boldsymbol{\frac{R_2 + j\omega L}{R_1 + R_2 - \omega^2 LCR_1 + j\omega(L + R_1 R_2 C)}}$$

Solution 10.23

$$\frac{V - V_s}{R} + \frac{V}{j\omega L + \frac{1}{j\omega C}} + j\omega CV = 0$$

$$V + \frac{j\omega RCV}{-\omega^2 LC + 1} + j\omega RCV = V_s$$

$$\left(\frac{1 - \omega^2 LC + j\omega RC + j\omega RC - j\omega^3 RLC^2}{1 - \omega^2 LC} \right) V = V_s$$

$$\boldsymbol{V = \underline{\frac{(1 - \omega^2 LC)V_s}{1 - \omega^2 LC + j\omega RC(2 - \omega^2 LC)}}}$$

Solution 10.24

Design a problem to help other students to better understand mesh analysis.

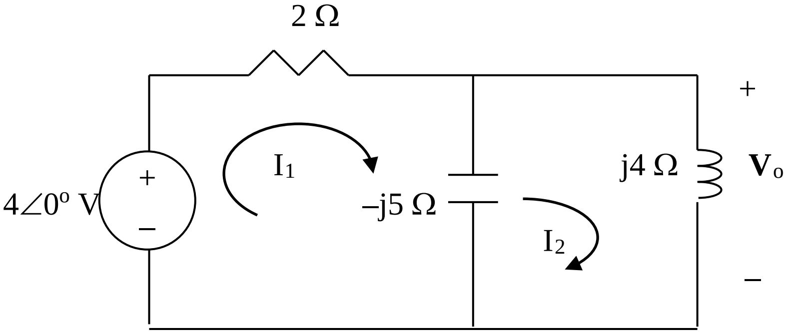
Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use mesh analysis to find $\mathbf{V}_o$ in the circuit in Prob. 10.2.

Solution

Consider the circuit as shown below.



For mesh 1,

$$4 = (2 - j5)I_1 + j5I_1 \qquad (1)$$

For mesh 2,

$$0 = j5I_1 + (j4 - j5)I_2 \quad \longrightarrow \quad I_1 = \frac{1}{5}I_2 \qquad (2)$$

Substituting (2) into (1),

$$4 = (2 - j5)\frac{1}{5}I_2 + j5I_2 \quad \longrightarrow \quad I_2 = \frac{1}{0.1 + j}$$

$\mathbf{V}_o$ = j4$\mathbf{I}_2$ = j4/(0.1+j) = j4/(1.00499∠84.29°) = **3.98∠5.71° V**

Solution 10.25

$$\omega = 2$$

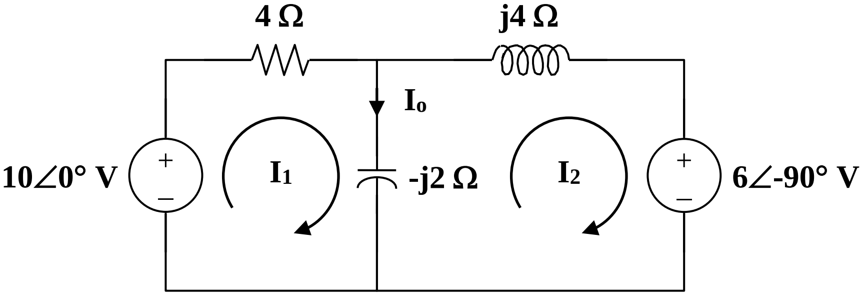
$$10\cos(2t) \longrightarrow 10\angle 0°$$

$$6\sin(2t) \longrightarrow 6\angle -90° = -j6$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.25\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

The circuit is shown below.



For loop 1,

$$-10 + (4 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$
$$5 = (2 - j)\mathbf{I}_1 + j\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j2\mathbf{I}_1 + (j4 - j2)\mathbf{I}_2 + (-j6) = 0$$
$$\mathbf{I}_1 + \mathbf{I}_2 = 3 \qquad (2)$$

In matrix form (1) and (2) become

$$\begin{bmatrix} 2-j & j \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$\Delta = 2(1 - j), \qquad \Delta_1 = 5 - j3, \qquad \Delta_2 = 1 - j3$$

$$\mathbf{I}_o = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{4}{2(1-j)} = 1 + j = 1.4142\angle 45°$$

Therefore,

$$i_o(t) = \mathbf{1.4142cos(2t + 45°)\ A}$$

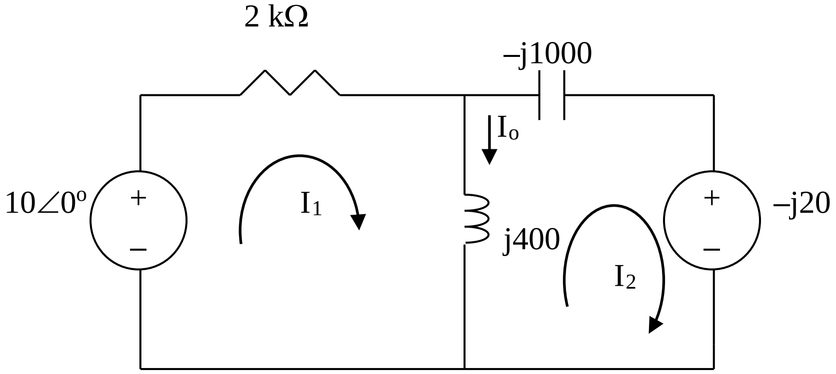
Solution 10.26

$$0.4H \quad \longrightarrow \quad j\omega L = j10^3 x0.4 = j400$$

$$1\mu F \quad \longrightarrow \quad 1/j\omega C = \frac{1}{j10^3 x10^{-6}} = -j1000$$

$$20\sin\left(10^3 t\right) = 20\cos\left(10^3 t - 90°\right) which\, leads\, to\, 20\angle -90° = -j20$$

The circuit becomes that shown below.



For loop 1,

$$-10 + (12000 + j400)I_1 - j400I_2 = 0 \quad \longrightarrow \quad 1 = (200 + j40)I_1 - j40I_2 \text{ (1)}$$

For loop 2,

$$-j20 + (j400 - j1000)I_2 - j400I_1 = 0 \quad \longrightarrow \quad -12 = 40I_1 + 60I_2$$

(2)

In matrix form, (1) and (2) become

$$\begin{bmatrix} 1 \\ -12 \end{bmatrix} = \begin{bmatrix} 200 + j40 & -j40 \\ 40 & 60 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this leads to

I_1 =0.0025-j0.0075, I_2 = -0.035+j0.005

$I_o = I_1 - I_2$ = 0.0375 – j0.0125 = 39.5∠–18.43° mA

$i_o(t)$ = **39.5cos(10^3t–18.43°) mA**

Solution 10.27

Using mesh analysis, find **I**1 and **I**2 in the circuit of Fig. 10.75 as shown in the text.

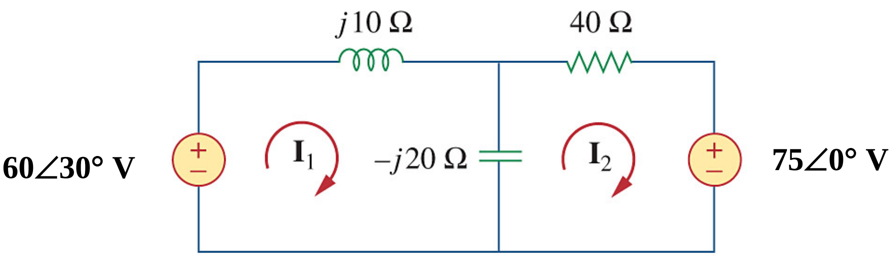


Figure 10.75
For Prob. 10.27.

Solution

For mesh 1,

$$-60\angle 30° + (j10 - j20)\mathbf{I}_1 + j20\mathbf{I}_2 = 0$$

$$6\angle 30° = \text{-}j\mathbf{I}_1 + j2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$75\angle 0° + (40 - j20)\mathbf{I}_2 + j20\mathbf{I}_1 = 0$$

$$7.5 = \text{-}j2\mathbf{I}_1 - (4 - j2)\mathbf{I}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 6\angle 30° \\ 7.5 \end{bmatrix} = \begin{bmatrix} \text{-}j & j2 \\ \text{-}j2 & \text{-}(4 - j2) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = \text{-}2 + 4\text{j} = 4.472\angle 116.56°$$

$$\Delta_1 = \text{-}(6\angle 30°)(4 - j2) - j15 = 31.515\angle 211.8°$$

$$\Delta_2 = \text{-}j7.5 + 12\angle 120° = 6.66\angle 154.27°$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \mathbf{7.047\angle 95.24°\ A}$$

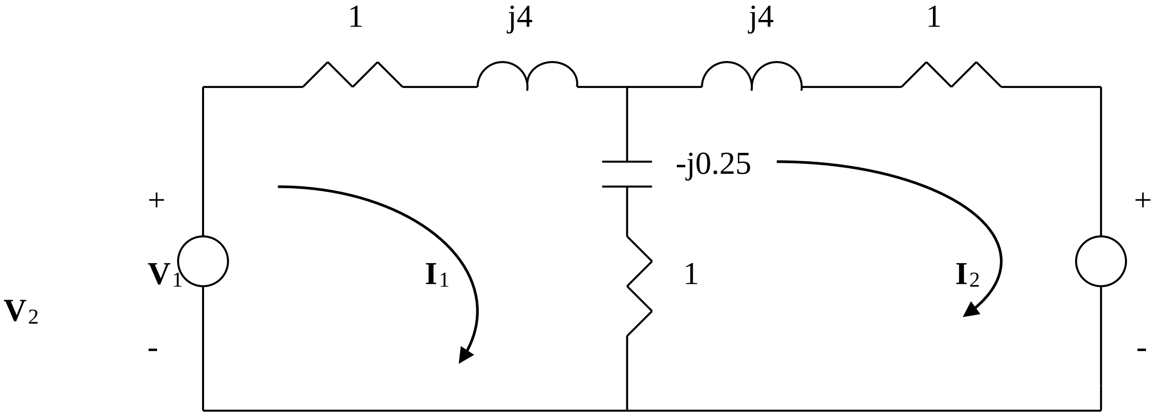
$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.4892\angle 37.71°\ A}$$

Solution 10.28

$$1\text{H} \longrightarrow j\omega L = j4, \qquad 1\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j1x4} = -j0.25$$

The frequency-domain version of the circuit is shown below, where

$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o \ .$$



$$V_1 = 10\angle 0^o, \qquad V_2 = 20\angle -30^o$$

Applying mesh analysis,

$$10 = (2 + j3.75)I_1 - (1 - j0.25)I_2 \qquad (1)$$

$$-20\angle -30^o = -(1 - j0.25)I_1 + (2 + j3.75)I_2 \qquad (2)$$

From (1) and (2), we obtain

$$\begin{pmatrix} 10 \\ -17.32 + j10 \end{pmatrix} = \begin{pmatrix} 2 + j3.75 & -1 + j0.25 \\ -1 + j0.25 & 2 + j3.75 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

Solving this leads to

$$I_1 = 2.741\angle -41.07^o, \qquad I_2 = 4.114\angle 92^o$$

Hence,

$i_1(t)$ = **2.741cos(4t–41.07°)A**, $i_2(t)$ = **4.114cos(4t+92°)A**.

Solution 10.29

Using Fig. 10.77, design a problem to help other students better understand mesh analysis.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

By using mesh analysis, find **I**₁ and **I**₂ in the circuit depicted in Fig. 10.77.

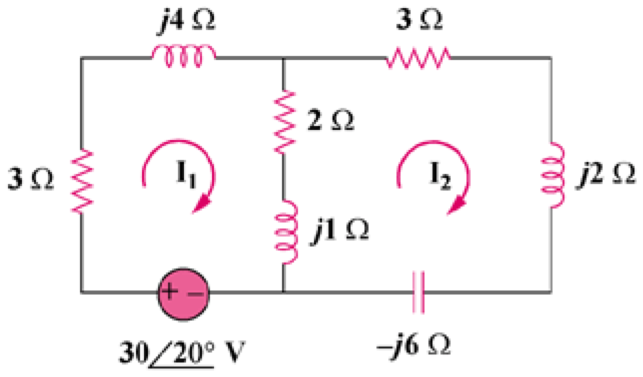


Figure 10.77

Solution

For mesh 1,

$$(5+j5)\mathbf{I}_1-(2+j)\mathbf{I}_2-30\angle 20^\circ=0$$
$$30\angle 20^\circ=(5+j5)\mathbf{I}_1-(2+j)\mathbf{I}_2$$
(1)

For mesh 2,

$$(5+j3-j6)\mathbf{I}_2-(2+j)\mathbf{I}_1=0$$
$$0=-(2+j)\mathbf{I}_1+(5-j3)\mathbf{I}_2$$
(2)

From (1) and (2),

$$\begin{bmatrix}30\angle 20^\circ\\0\end{bmatrix}=\begin{bmatrix}5+j5 & -(2+j)\\-(2+j) & 5-j3\end{bmatrix}\begin{bmatrix}\mathbf{I}_1\\\mathbf{I}_2\end{bmatrix}$$

$$\Delta=37+j6=37.48\angle 9.21^\circ$$
$$\Delta_1=(30\angle 20^\circ)(5.831\angle -30.96^\circ)=175\angle -10.96^\circ$$

$$\Delta_2 = (30\angle 20°)(2.356\angle 26.56°) = 67.08\angle 46.56°$$

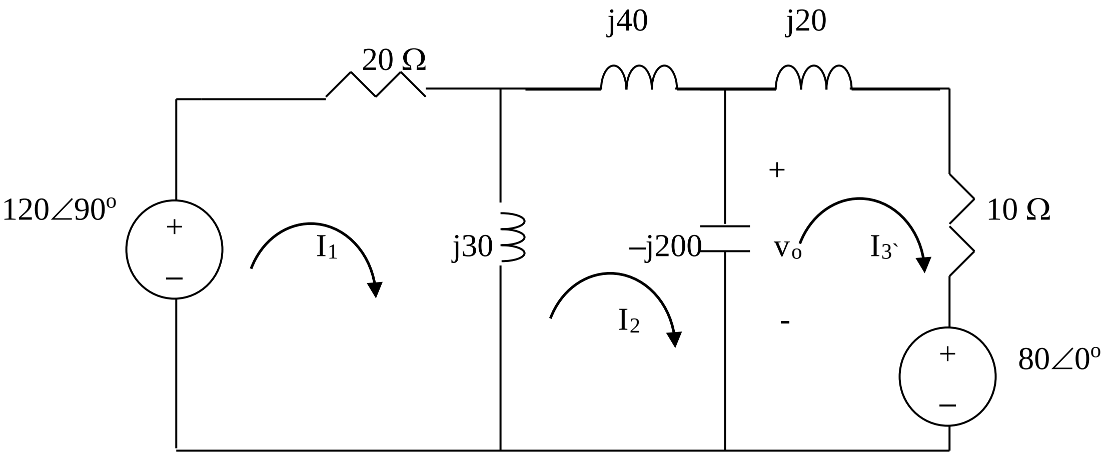
$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \mathbf{4.67\angle{-20.17}°\ A}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.79\angle 37.35°\ A}$$

Solution 10.30

$$300mH \longrightarrow j\omega L = j100x300x10^{-3} = j30$$
$$200mH \longrightarrow j\omega L = j100x200x10^{-3} = j20$$
$$400mH \longrightarrow j\omega L = j100x400x10^{-3} = j40$$
$$50\mu F \longrightarrow {}^{1}\!/_{j\omega C} = \frac{1}{j100x50x10^{-6}} = -j200$$

The circuit becomes that shown below.



For mesh 1,

$$-120 < 90^o + (20 + j30)I_1 - j30I_2 = 0 \longrightarrow j120 = (20 + j30)I_1 - j30I_2 \qquad (1)$$

For mesh 2,

$$-j30I_1 + (j30 + j40 - j200)I_2 + j200I_3 = 0 \longrightarrow 0 = -3I_1 - 13I_2 + 20I_3 \qquad (2)$$

For mesh 3,

$$80 + j200I_2 + (10 - j180)I_3 = 0 \rightarrow -8 = j20I_2 + (1 - j18)I_3 \qquad (3)$$

We put (1) to (3) in matrix form.

$$\begin{bmatrix} 2 + j3 & -j3 & 0 \\ -3 & -13 & 20 \\ 0 & j20 & 1 - j18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} j12 \\ 0 \\ -8 \end{bmatrix}$$

This is an excellent candidate for MATLAB.

```
>> Z=[(2+3i),-3i,0;-3,-13,20;0,20i,(1-18i)]

Z =

   2.0000 + 3.0000i        0 - 3.0000i        0
  -3.0000           -13.0000           20.0000
        0                  0 +20.0000i   1.0000 -18.0000i

>> V=[12i;0;-8]

V =

        0 +12.0000i
        0
  -8.0000

>> I=inv(Z)*V

I =

   2.0557 + 3.5651i
   0.4324 + 2.1946i
   0.5894 + 1.9612i
```

$V_o = –j200(I_2 – I_3) = –j200(–0.157+j0.2334) = 46.68 + j31.4 = 56.26\angle 33.93^\circ$

$v_o = \mathbf{56.26cos(100t + 33.93^\circ)\ V}$.

Solution 10.31

Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

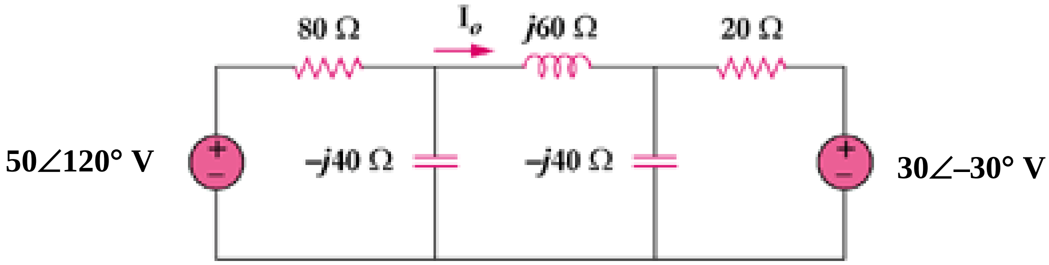
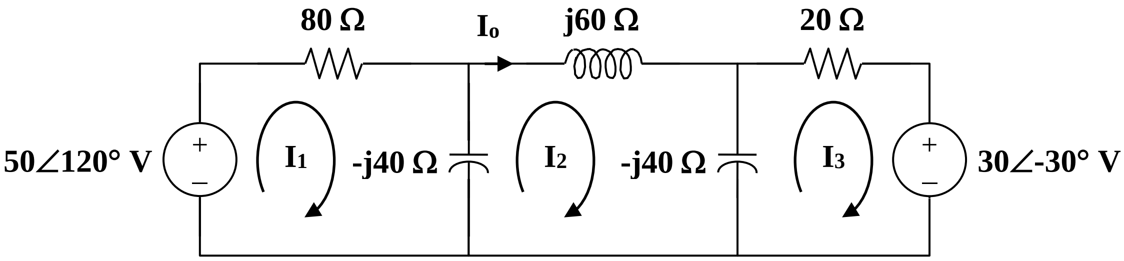


Figure 10.79
For Prob. 10.31.

Solution

Consider the network shown below.



For loop 1,

$$-50\angle 120° + (80 - j40)\mathbf{I}_1 + j40\mathbf{I}_2 = 0$$

$$5\angle 20° = 4(2 - j)\mathbf{I}_1 + j4\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$\text{j}40\mathbf{I}_1 + (\text{j}60 - \text{j}80)\mathbf{I}_2 + \text{j}40\mathbf{I}_3 = 0$$

$$0 = 2\mathbf{I}_1 - \mathbf{I}_2 + 2\mathbf{I}_3 \qquad (2)$$

For loop 3,

$$30\angle -30° + (20 - j40)\mathbf{I}_3 + j40\mathbf{I}_2 = 0$$

$$-3\angle -30° = j4\mathbf{I}_2 + 2(1 - j2)\mathbf{I}_3 \qquad (3)$$

From (2),

$$2\mathbf{I}_3 = \mathbf{I}_2 - 2\mathbf{I}_1$$

Substituting this equation into (3),

$$-3\angle -30° = -2(1 - j2)\mathbf{I}_1 + (1 + j2)\mathbf{I}_2 \qquad (4)$$

From (1) and (4),

$$\begin{bmatrix} 5\angle 120° \\ -3\angle -30° \end{bmatrix} = \begin{bmatrix} 4(2-j) & j4 \\ -2(1-j2) & 1+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8-\mathrm{j}4 & -\mathrm{j}4 \\ -2+\mathrm{j}4 & 1+\mathrm{j}2 \end{vmatrix} = 32+\mathrm{j}20 = 37.74\angle 32°$$

$$\Delta_2 = \begin{vmatrix} 8-j4 & 5\angle 120° \\ -2+j4 & -3\angle -30° \end{vmatrix} = -2.464 + j41.06 = 41.125\angle 93.44°$$

$$\mathbf{I}_o = \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{1.0897\angle 61.44°\ A}$$

Solution 10.32

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit of Fig. 10.80 using mesh analysis.

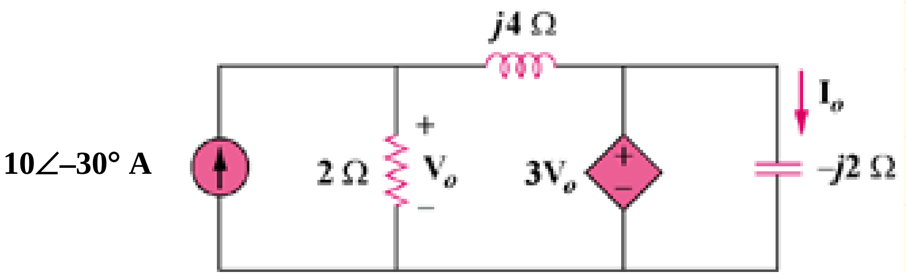
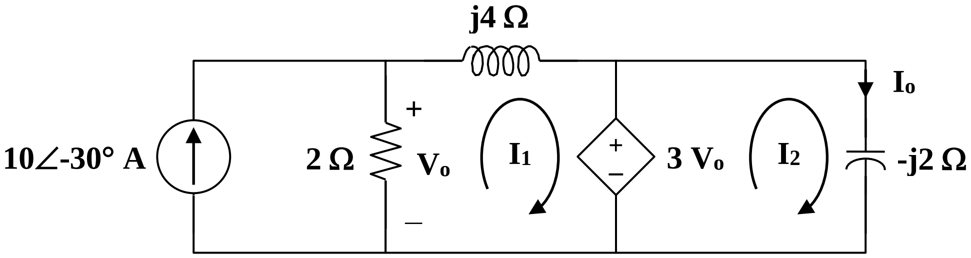


Figure 10.80
For Prob. 10.32.

Solution

Consider the circuit below.



For mesh 1,

$$(2+j4)\mathbf{I}_1 - 2(10\angle{-30°}) + 3\mathbf{V}_o = 0$$

where $$\mathbf{V}_o = 2(10\angle{-30°} - \mathbf{I}_1)$$

Hence,

$$(2+j4)\mathbf{I}_1 - 20\angle{-30°} + 6(10\angle{-30°} - \mathbf{I}_1) = 0$$

$$10\angle{-30°} = (1-j)\mathbf{I}_1$$

or $$\mathbf{I}_1 = 25\sqrt{2}\angle 15°$$

$$\mathbf{I}_o = \frac{3\mathbf{V}_o}{-j2} = \frac{3}{-j2}(2)(10\angle{-30°} - \mathbf{I}_1)$$

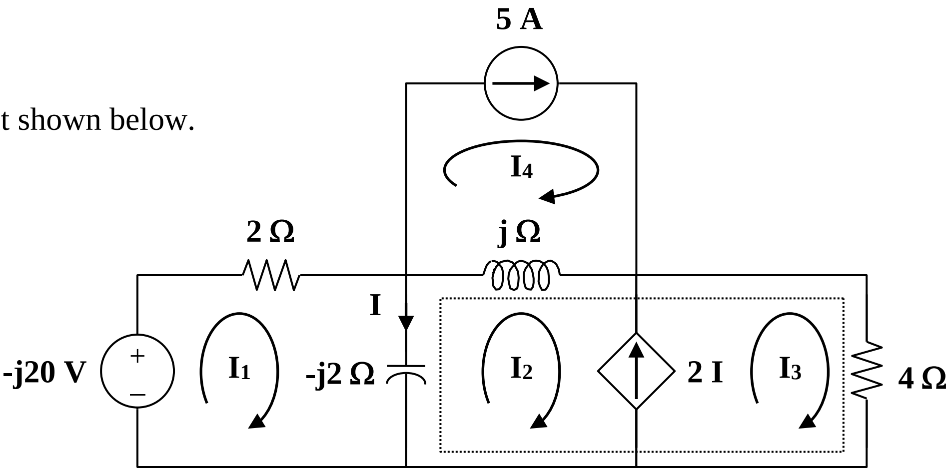
$$\mathbf{I}_o = j3(10\angle{-30°} - 5\sqrt{2}\angle 15°)$$

$$\mathbf{I}_o = \mathbf{21.21\angle 15°\ A}$$

$$\mathbf{V}_o = \frac{-j2\mathbf{I}_o}{3} = \mathbf{5.657\angle{-75°}\ V}$$

Solution 10.33

Consider the circuit shown below.



For mesh 1,

$$j20 + (2 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

$$(1 - j)\mathbf{I}_1 + j\mathbf{I}_2 = -j10 \qquad (1)$$

For the supermesh,

$$(j - j2)\mathbf{I}_2 + j2\mathbf{I}_1 + 4\mathbf{I}_3 - j\mathbf{I}_4 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_3 - \mathbf{I}_2 = 2\mathbf{I} = 2(\mathbf{I}_1 - \mathbf{I}_2)$$

$$\mathbf{I}_3 = 2\mathbf{I}_1 - \mathbf{I}_2 \qquad (3)$$

For mesh 4,

$$\mathbf{I}_4 = 5 \qquad (4)$$

Substituting (3) and (4) into (2),

$$(8 + j2)\mathbf{I}_1 - (-4 + j)\mathbf{I}_2 = j5 \qquad (5)$$

Putting (1) and (5) in matrix form,

$$\begin{bmatrix} 1 - j & j \\ 8 + j2 & 4 - j \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} -j10 \\ j5 \end{bmatrix}$$

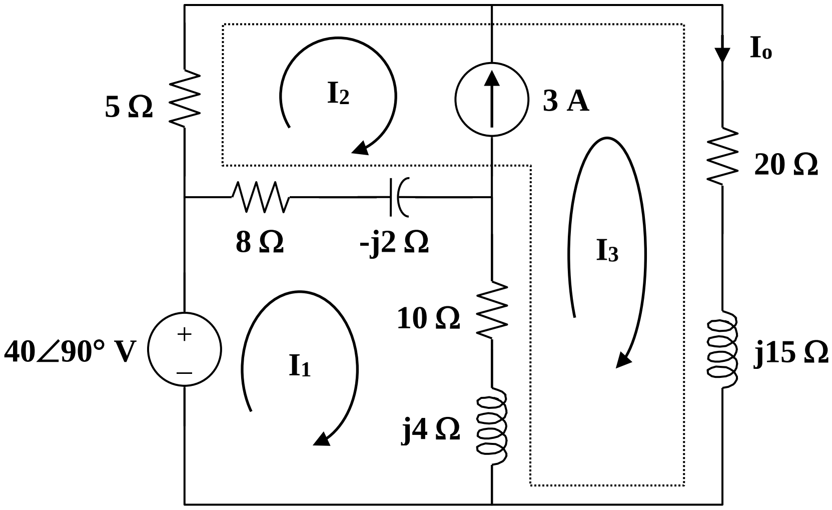
$$\Delta = -3 - j5, \qquad \Delta_1 = -5 + j40, \qquad \Delta_2 = -15 + j85$$

$$\mathbf{I} = \mathbf{I}_1 - \mathbf{I}_2 = \frac{\Delta_1 - \Delta_2}{\Delta} = \frac{10 - j45}{-3 - j5} =$$

7.906∠43.49° A

Solution 10.34

The circuit is shown below.



For mesh 1,

$$- j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (30 + j19)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (2)$$

Also,

$$\mathbf{I}_2 = \mathbf{I}_3 - 3 \qquad (3)$$

Adding (1) and (2) and incorporating (3),

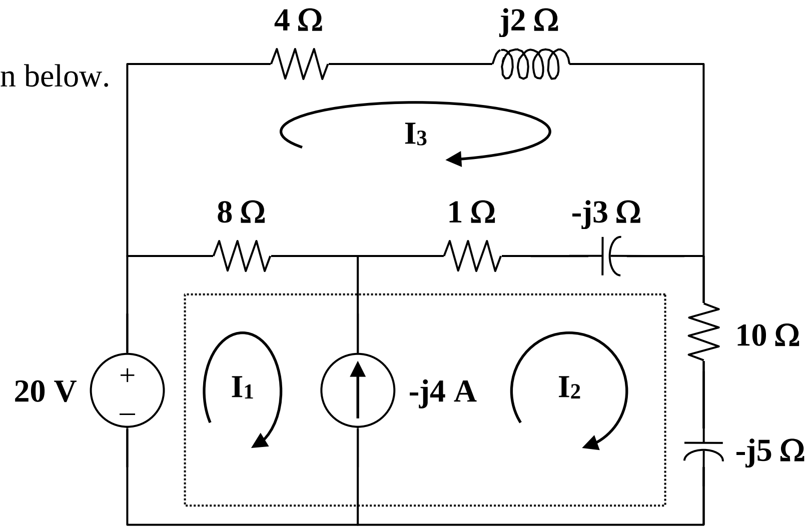
$$- j40 + 5(\mathbf{I}_3 - 3) + (20 + j15)\mathbf{I}_3 = 0$$

$$\mathbf{I}_3 = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48°$$

$$\mathbf{I}_o = \mathbf{I}_3 = \mathbf{1.465\angle 38.48°\ A}$$

Solution 10.35

Consider the circuit shown below.



For the supermesh,

$$-20 + 8\mathbf{I}_1 + (11 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$$

Also,

$$\mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$$

For mesh 3,

$$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$$

Substituting (2) into (1),

$$(19 - j8)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 20 - j32 \qquad (4)$$

Substituting (2) into (3),

$$-(9 - j3)\mathbf{I}_2 + (13 - j)\mathbf{I}_3 = j32 \qquad (5)$$

From (4) and (5),

$$\begin{bmatrix} 19 - j8 & -(9 - j3) \\ -(9 - j3) & 13 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 20 - j32 \\ j32 \end{bmatrix}$$

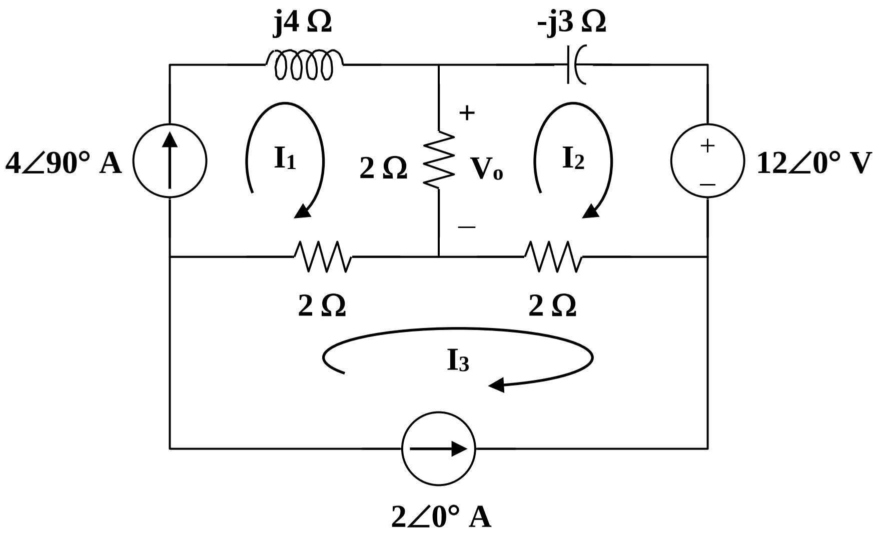
$$\Delta = 167 - j69, \qquad \Delta_2 = 324 - j148$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{324 - j148}{167 - j69} = \frac{356.2\angle -24.55°}{180.69\angle -22.45°}$$

$$\mathbf{I}_2 = \mathbf{1.971\angle{-2.1}° \ A}$$

Solution 10.36

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 4\angle 90° = j4 \quad \text{and} \quad \mathbf{I}_3 = \text{-}2$$

For mesh 2,

$$(4 - j3)\mathbf{I}_2 - 2\mathbf{I}_1 - 2\mathbf{I}_3 + 12 = 0$$

$$(4 - j3)\mathbf{I}_2 - j8 + 4 + 12 = 0$$

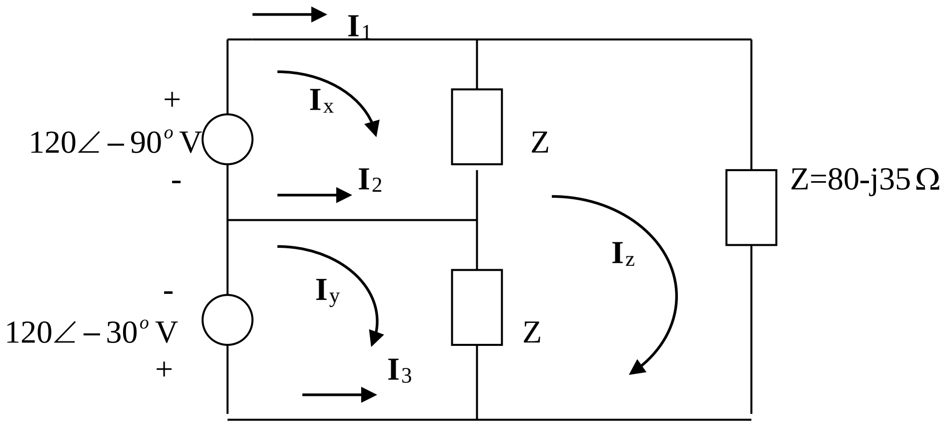
$$\mathbf{I}_2 = \frac{\text{-}16 + j8}{4 - j3} = \text{-}3.52 - j0.64$$

Thus,

$$\mathbf{V}_o = 2(\mathbf{I}_1 - \mathbf{I}_2) = (2)(3.52 + j4.64) = 7.04 + j9.28$$

$$\mathbf{V}_o = \mathbf{11.648\angle 52.82°\ V}$$

Solution 10.37



For mesh x,

$$ZI_x - ZI_z = -j120 \qquad (1)$$

For mesh y,

$$ZI_y - ZI_z = -120\angle 30^{o} = -103.92 + j60 \qquad (2)$$

For mesh z,

$$-ZI_x - ZI_y + 3ZI_z = 0 \qquad (3)$$

Putting (1) to (3) together leads to the following matrix equation:

$$\begin{pmatrix} (80-j35) & 0 & (-80+j35) \\ 0 & (80-j35) & (-80+j35) \\ (-80+j35) & (-80+j35) & (240-j105) \end{pmatrix}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} -j120 \\ -103.92+j60 \\ 0 \end{pmatrix} \longrightarrow \quad AI = B$$

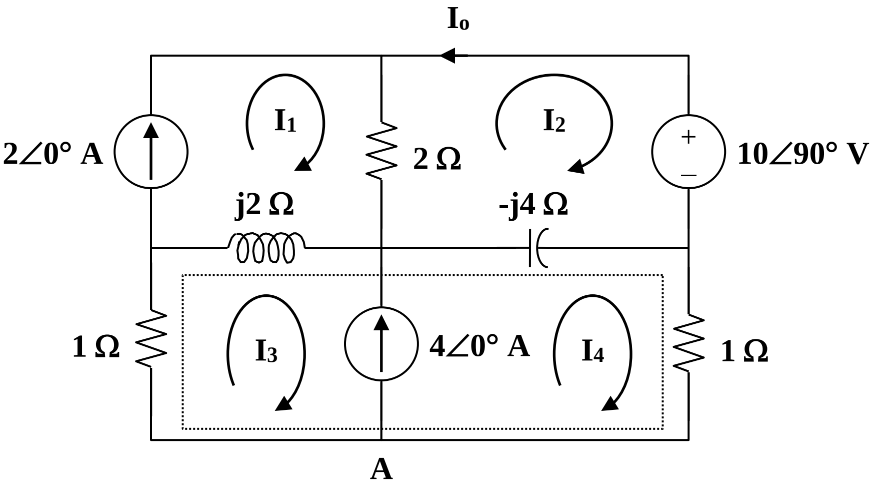
Using MATLAB, we obtain

$$I = inv(A)*B = \begin{pmatrix} -0.2641 - j2.366 \\ -2.181 - j0.954 \\ -0.815 - j1.1066 \end{pmatrix}$$

$$\boldsymbol{I_1 = I_x = -0.2641 - j2.366 = \underline{2.38\angle -96.37^{o}}\ A}$$
$$\boldsymbol{I_2 = I_y - I_x = -1.9167 + j1.4116 = \underline{2.38\angle 143.63^{o}}\ A}$$
$$\boldsymbol{I_3 = -I_y = 2.181 + j0.954 = \underline{2.38\angle 23.63^{o}\ A}}$$

Solution 10.38

Consider the circuit below.



Clearly,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(2 - j4)\mathbf{I}_2 - 2\mathbf{I}_1 + j4\mathbf{I}_4 + 10\angle 90° = 0 \qquad (2)$$

Substitute (1) into (2) to get

$$(1 - j2)\mathbf{I}_2 + j2\mathbf{I}_4 = 2 - j5$$

For the supermesh,

$$(1 + j2)\mathbf{I}_3 - j2\mathbf{I}_1 + (1 - j4)\mathbf{I}_4 + j4\mathbf{I}_2 = 0$$

$$j4\mathbf{I}_2 + (1 + j2)\mathbf{I}_3 + (1 - j4)\mathbf{I}_4 = j4 \qquad (3)$$

At node A,

$$\mathbf{I}_3 = \mathbf{I}_4 - 4 \qquad (4)$$

Substituting (4) into (3) gives

$$j2\mathbf{I}_2 + (1 - j)\mathbf{I}_4 = 2(1 + j3) \qquad (5)$$

From (2) and (5),

$$\begin{bmatrix} 1 - j2 & j2 \\ j2 & 1 - j \end{bmatrix} \begin{bmatrix} \mathbf{I}_2 \\ \mathbf{I}_4 \end{bmatrix} = \begin{bmatrix} 2 - j5 \\ 2 + j6 \end{bmatrix}$$

$$\Delta = 3 - j3, \qquad \Delta_1 = 9 - j11$$

$$\mathbf{I}_o = \mathbf{-I}_2 = \frac{-\Delta_1}{\Delta} = \frac{-(9 - j11)}{3 - j3} = \frac{1}{3}(-10 + j)$$

$$\mathbf{I}_o = \mathbf{3.35\angle 174.3° \ A}$$

Solution 10.39

For mesh 1,

$$(28 - j15)I_1 - 8I_2 + j15I_3 = 12\angle 64^o \qquad (1)$$

For mesh 2,

$$-8I_1 + (8 - j9)I_2 - j16I_3 = 0 \qquad (2)$$

For mesh 3,

$$j15I_1 - j16I_2 + (10 + j)I_3 = 0 \qquad (3)$$

In matrix form, (1) to (3) can be cast as

$$\begin{pmatrix} (28 - j15) & -8 & j15 \\ -8 & (8 - j9) & -j16 \\ j15 & -j16 & (10 + j) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} 12\angle 64^o \\ 0 \\ 0 \end{pmatrix} \quad \text{or} \quad AI = B$$

Using MATLAB,

I = inv(A)*B

$$I_1 = -0.128 + j0.3593 = \mathbf{381.4\angle 109.6° \ mA}$$
$$I_2 = -0.1946 + j0.2841 = \mathbf{344.3\angle 124.4° \ mA}$$
$$I_3 = 0.0718 - j0.1265 = \mathbf{145.5\angle{-60.42}° \ mA}$$
$$I_x = I_1 - I_2 = 0.0666 + j0.0752 = \mathbf{100.5\angle 48.5° \ mA}$$

381.4∠109.6° mA, 344.3∠124.4° mA, 145.5∠–60.42° mA, 100.5∠48.5° mA

Solution 10.40

Find i_o in the circuit shown in Fig. 10.85 using superposition.

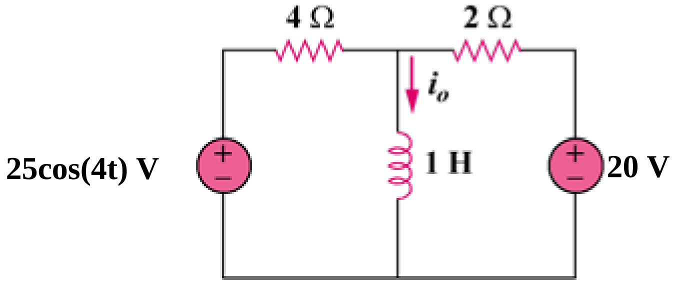
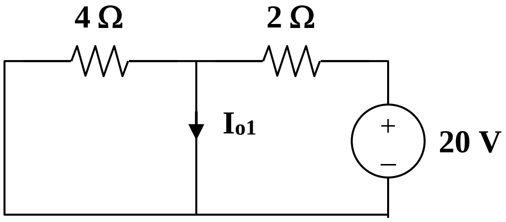


Figure 10.85
For Prob. 10.40.

Solution

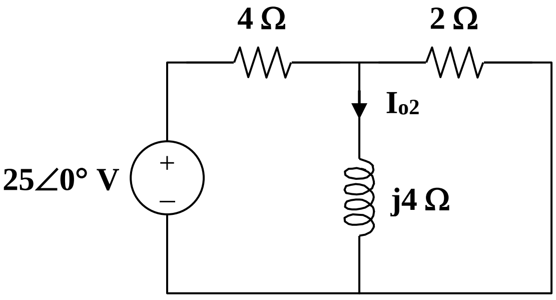
Let $\mathbf{I_o} = \mathbf{I_{o1}} + \mathbf{I_{o2}}$, where I_{o1} is due to the dc source and I_{o2} is due to the ac source. For I_{o1}, consider the circuit in Fig. (a).
Clearly,



(a)

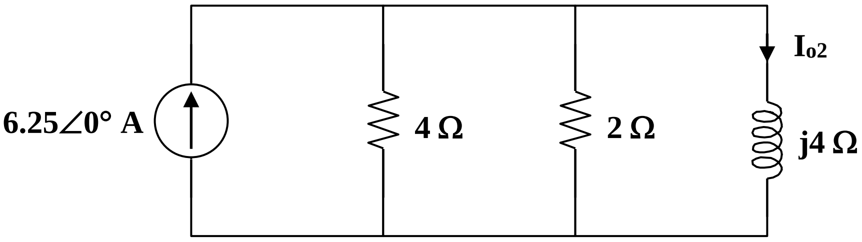
$\mathbf{I_{o1}} = 20/2 = 10$ A

For $\mathbf{I_{o2}}$, consider the circuit in Fig. (b).



(b)

If we transform the voltage source, we have the circuit in Fig. (c), where $4 \| 2 = 4/3\,\Omega$.



(c)

By the current division principle,

$$\mathbf{I}_{o2} = \frac{4/3}{4/3 + j4}(6.25\angle 0°)$$

$$\mathbf{I}_{o2} = 0.625 - j1.875 = 1.9764\angle\text{-}71.56°$$

Thus, $$I_{o2} = 1.9764\cos(4t - 71.56°)\ A$$

Therefore,

$i_o = i_{o1} + i_{o2} =$ **[10 + 1.9764cos(4t–71.56°)] A**

Solution 10.41

Find v_o for the circuit in Fig. 10.86 assuming that $i_s(t)$ = 2sin(2t) + 3cos(4t) A.

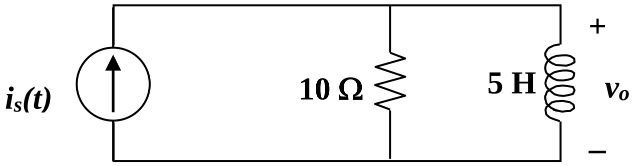


Figure 10.86
For Prob. 10.41.

Solution

This problem is easily solved using superposition. $V_o = I_s[10(j5\omega)]/(10+j5\omega)$.

For $\omega = 2$ rad/s we get $V_o' = 2(j100)/(10+j10) = 14.142\angle 45^o$ A and for
$\omega = 4$ rad/s we get $V_o'' = 3(j200)/(10+j20) = j600/(22.361\angle 63.43^\circ$
$= 26.83\angle 26.57^\circ$ or

v_o = **[14.142sin(2t+45º) + 26.83cos(4t+26.57º)] V**.

Solution 10.42

Using Fig. 10.87, design a problem to help other students to better understand the superposition theorem.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Solve for I_o in the circuit of Fig. 10.87.

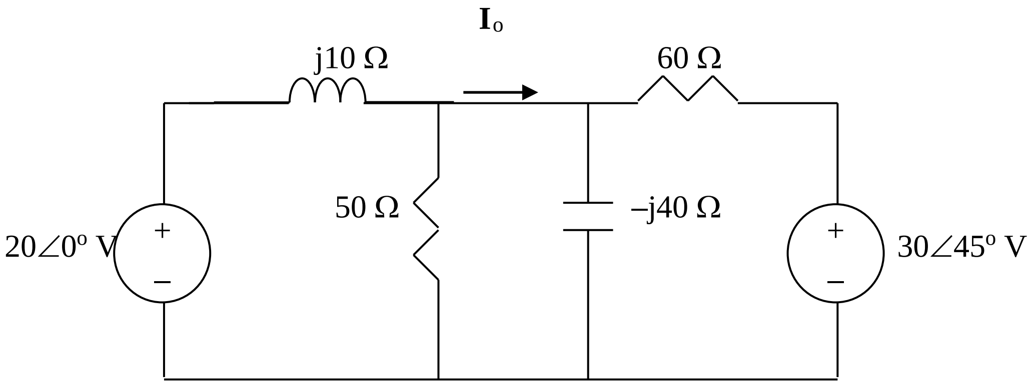
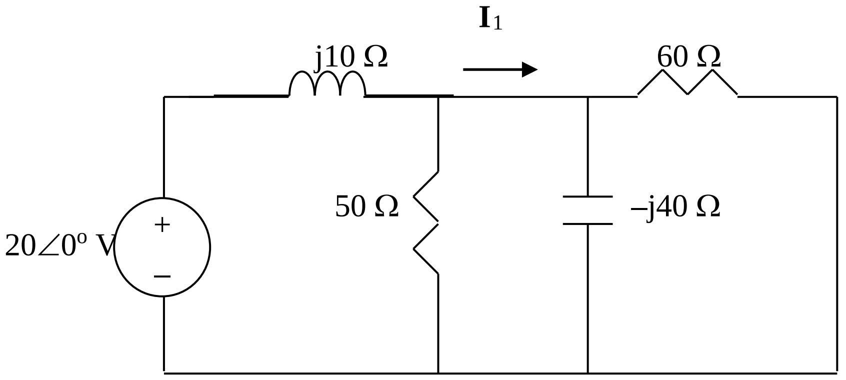


Figure 10.87 For Prob. 10.42.

Solution

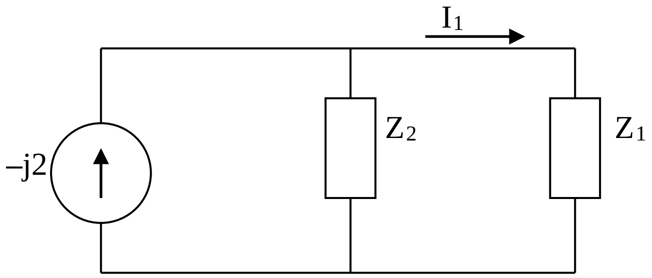
Let $I_o = I_1 + I_2$

where I_1 and I_2 are due to 20<0° and 30<45° sources respectively. To get I_1, we use the circuit below.



Let Z_1 = -j40//60 = 18.4615 –j27.6927, Z_2 = j10//50=1.9231 + j9.615

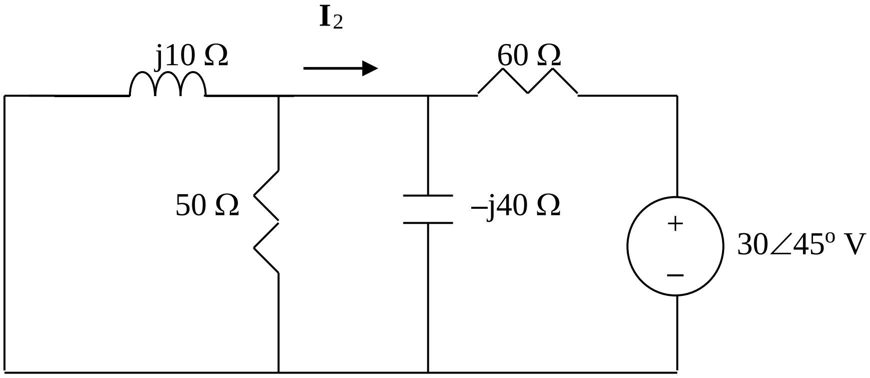
Transforming the voltage source to a current source leads to the circuit below.



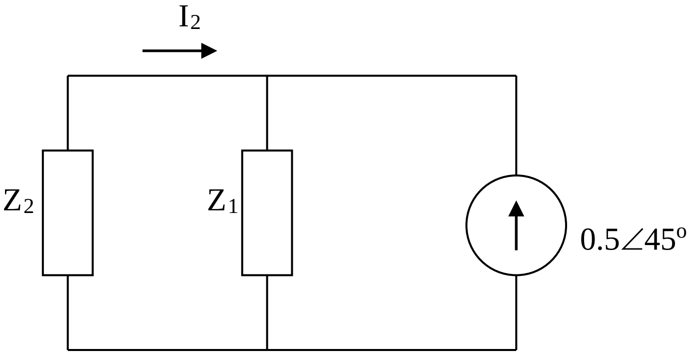
Using current division,

$$I_1 = \frac{Z_2}{Z_1 + Z_2}(-j2) = 0.6217 + j0.3626$$

To get I2, we use the circuit below.



After transforming the voltage source, we obtain the circuit below.



Using current division,

$$I_2 = \frac{-Z_1}{Z_1 + Z_2}(0.5 < 45^o) = -0.5275 - j0.3077$$

Hence, **I₀** = **I₁ + I₂ =** 0.0942+j0.0509 = **109∠30° mA**.

Solution 10.43

Using the superposition principle, find i_x in the circuit of Fig. 10.88.

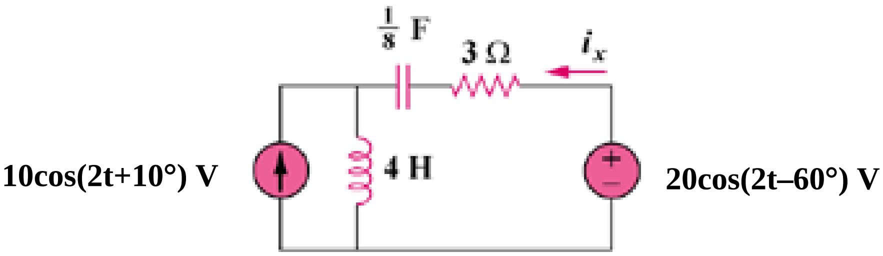


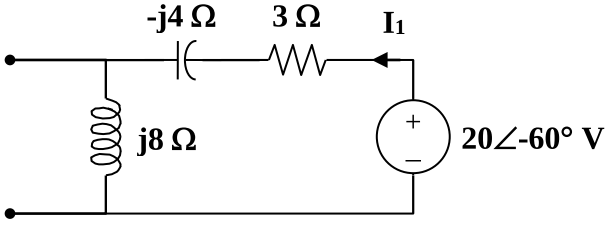
Figure 10.88
For Prob. 10.43.

Solution

Let $\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2$, where $\mathbf{I}_1$ is due to the voltage source and $\mathbf{I}_2$ is due to the current source.

$$\begin{aligned}
&\omega = 2 \\
&10\cos(2t+10°) \longrightarrow 10\angle 10° \\
&20\cos(2t-60°) \longrightarrow 20\angle\text{-}60° \\
&4\text{ H} \longrightarrow j\omega\text{L} = \text{j}8 \\
&\frac{1}{8}\text{ F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(2)(1/8)} = \text{-j}4
\end{aligned}$$

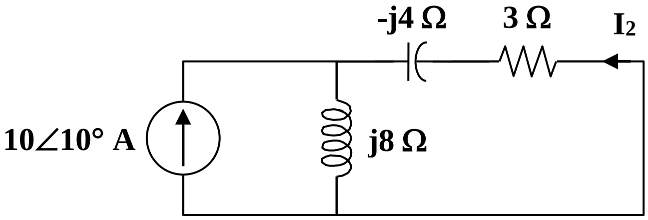
For $\mathbf{I}_1$, consider the circuit in Fig. (a).



(a)

$$\mathbf{I}_1 = \frac{20\angle\text{-}60°}{3 + j8 - j4} = \frac{20\angle\text{-}60°}{3 + j4}$$

For $\mathbf{I}_2$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{-\,\text{j}8}{3+\text{j}8-\text{j}4}(10\angle 10°) = \frac{-\,\text{j}80\angle 10°}{3+j4}$$

$$\mathbf{I}_x = \mathbf{I}_1 + \mathbf{I}_2 = \frac{1}{3+j4}(20\angle -60° - j80\angle 10°)$$

$$\mathbf{I}_x = \frac{99.02\angle -76.04°}{5\angle 53.13°} = 19.804\angle -129.17°$$

Therefore, i_x = **19.804cos(2t – 129.17°) A**

Solution 10.44

Use superposition principle to obtain v_x in the circuit of Fig. 10.89. Let v_s = 50 sin2*t* V and i_s = 12 cos(6*t* + 10°) A.

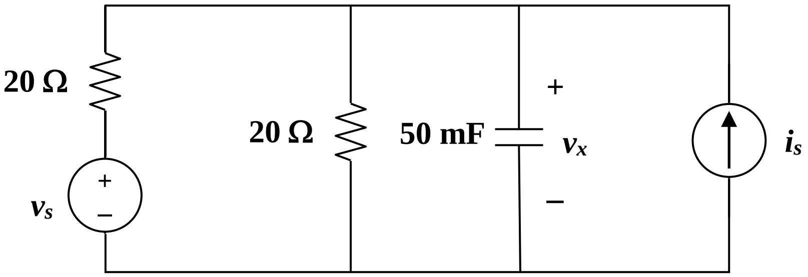


Figure 10.89
For Prob. 10.44.

Solution

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the voltage source and current source respectively.

For v_1, ω = 2 rad/s and the capacitive reactance is equal to –j10 Ω and V_s = 50 V. The resulting nodal equation becomes, $[(V_1–50)/20] + [(V_1–0)/20] + [(V_1–0)/(–j10)] + 0 = 0$.

Simplifying we get $(0.05+0.05+j0.1)V_1 = (0.1+j0.1)V_1 = 2.5$ or $V_1 = 17.678\angle–45°$ or $v_1(t) = 17.678\sin(2t–45°)$ V.

For v_2, ω = 6 rad/s and the capacitive reactance is equal to –j(10/3) Ω and I_s = 12 A. Note we will adjust the angle after we calculate the value of V_2 during the conversion back into the time domain. The resulting nodal equation becomes, $[(V_2–0)/20] + [(V_2–0)/20] +$
$[(V_2–0)/(–j10/3)] –12 = 0$.

Simplifying we get $(0.05+0.5+j0.3)V_2 = 12$ or $V_2 = 12/(0.1+j0.3) = 12/(0.31623\angle 71.57°)$ or $V_2 = 37.95\angle–71.57°$ V or $v_2(t) = 37.95\cos(6t–61.57°)$ V.

v_x = **[17.678sin(2t–45°)+37.95cos(6t–61.57°)] V**.

Solution 10.45

Use superposition to find *i(t)* in the circuit of Fig. 10.90.

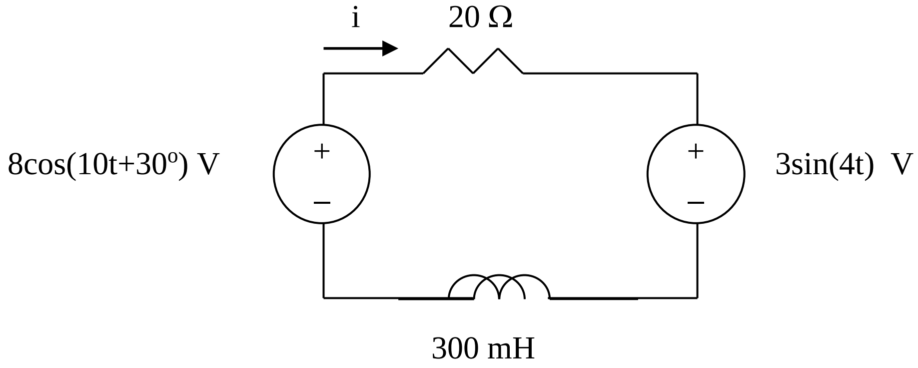
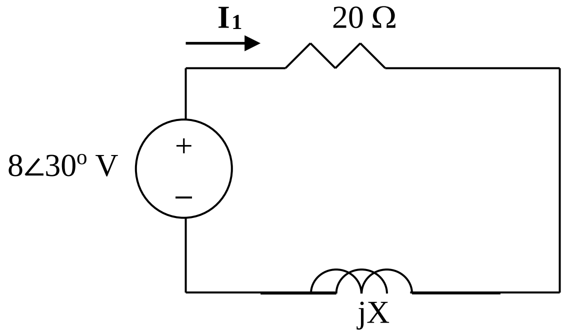


Figure 10.90
For Prob. 10.45.

Solution

Let $i = i_1 + i_2$, where i_1 and i_2 are due to 8cos(10t +30°) and 3sin4t sources respectively. To find i_1, consider the circuit below.



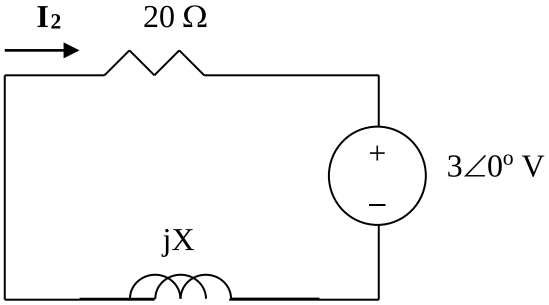
$$X = \omega L = 10x300x10^{-3} = 3$$

Type equation here.

$$\mathbf{I}_1 = \frac{8\angle 30°}{20 + j3} = \frac{8\angle 30°}{20.22\angle 8.53°} = 0.3956\angle 21.47°$$

$i_1(t)$ = 395.6cos(10t+21.47°) mA.

To find $i_2(t)$, consider the circuit below,



$$X = \omega L = 4x300x10^{-3} = 1.2$$

$$\mathbf{I}_2 = -\frac{3\angle 0^\circ}{20 + j1.2} = \frac{3\angle 180^\circ}{20.036\angle 3.43^\circ} = 0.14975\angle 176.57^\circ \text{ or}$$

$i_2(t) = 149.75\sin(4t+176.57°)$ mA.

Thus,

$i(t) = i_1(t) + i_2(t) =$ **[395.6cos(10t+21.47°) + 149.75sin(4t+176.57°)] mA**.

Solution 10.46

Solve for $v_o(t)$ in the circuit of Fig. 10.91 using the superposition principle.

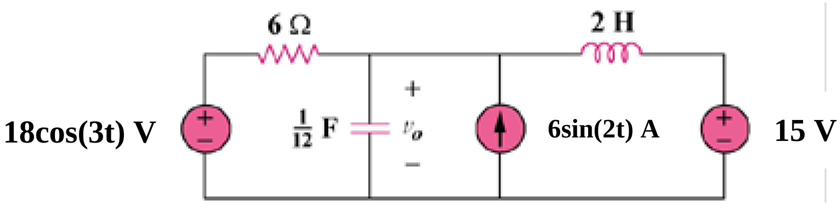
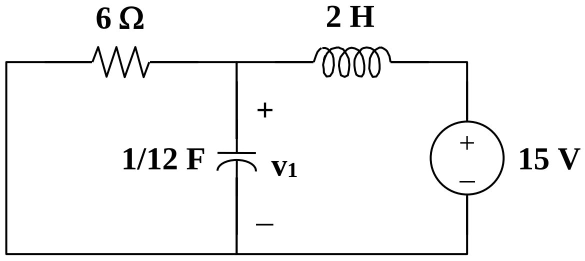


Figure 10.91
For Prob. 10.46.

Solution

Let $v_o = v_1 + v_2 + v_3$, where $\mathbf{V_1}$, $\mathbf{V_2}$, and $\mathbf{V_3}$ are respectively due to the 15-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



(a)

The capacitor is open to dc, while the inductor is a short circuit. Hence,

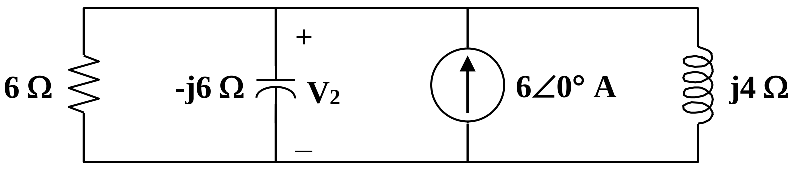
$$v_1 = 15\,V$$

For v_2, consider the circuit in Fig. (b).

$$\omega = 2$$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = -j6$$



(b)

Applying nodal analysis,

$$6 = \frac{\mathbf{V}_2}{6} + \frac{\mathbf{V}_2}{-j6} + \frac{\mathbf{V}_2}{j4} = \left(\frac{1}{6} + \frac{j}{6} - \frac{j}{4}\right)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{36}{1 - j0.5} = 32.18\angle 26.57°$$

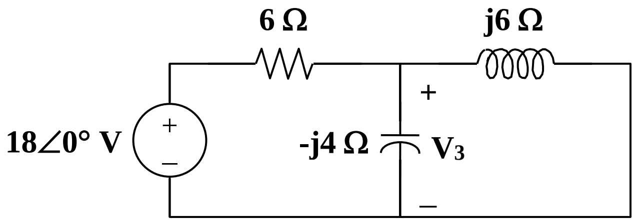
Hence, $v_2 = 32.18\sin(2t + 26.57°)\,V$

For v_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow \text{j}\omega\text{L} = \text{j6}$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(3)(1/12)} = \text{-j4}$$



(c)

At the non-reference node,

$$\frac{18 - \mathbf{V}_3}{6} = \frac{\mathbf{V}_3}{-j4} + \frac{\mathbf{V}_3}{j6}$$

$$\mathbf{V}_3 = \frac{18}{1 + j0.5} = 16.1\angle -26.57°$$

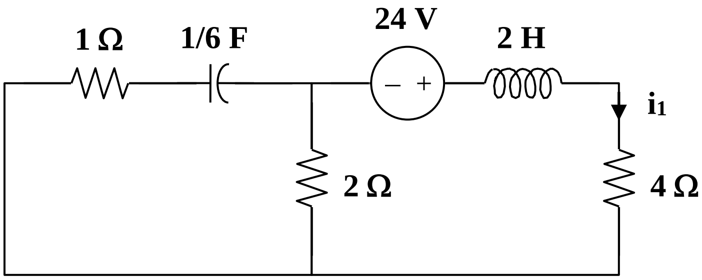
Hence, $v_3 = 16.1\cos(3t - 26.57°)\,V$

Therefore,

$v_o(t)$ = **[15+32.18sin(2t+26.57°)+16.1cos(3t–26.57°)] V**

Solution 10.47

Let $i_o = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are respectively due to the 24-V dc source, the ac voltage source, and the ac current source. For i_1, consider the circuit in Fig. (a).



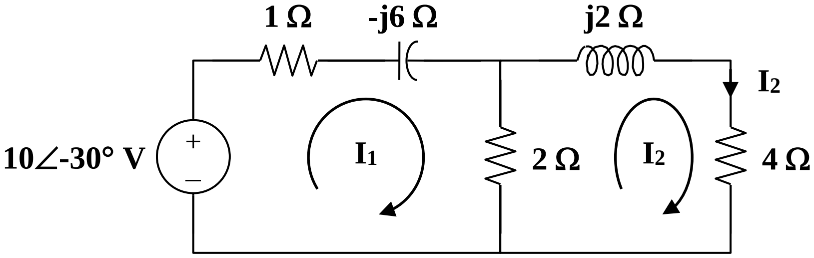
(a)

Since the capacitor is an open circuit to dc,

$$i_1 = \frac{24}{4+2} = 4 \text{ A}$$

For i_2, consider the circuit in Fig. (b).

$$\omega = 1$$
$$2 \text{ H} \longrightarrow j\omega L = j2$$
$$\frac{1}{6} \text{F} \longrightarrow \frac{1}{j\omega C} = \text{-}j6$$



(b)

For mesh 1,

$$\text{-}10\angle\text{-}30° + (3 - j6)\mathbf{I}_1 - 2\mathbf{I}_2 = 0$$
$$10\angle\text{-}30° = 3(1 - 2j)\mathbf{I}_1 - 2\mathbf{I}_2 \qquad (1)$$

For mesh 2,

$$0 = \text{-}2\mathbf{I}_1 + (6 + j2)\mathbf{I}_2$$
$$\mathbf{I}_1 = (3 + j)\mathbf{I}_2 \qquad (2)$$

Substituting (2) into (1)

$$10\angle -30° = 13 - j15\mathbf{I}_2$$

$$\mathbf{I}_2 = 0.504\angle 19.1°$$

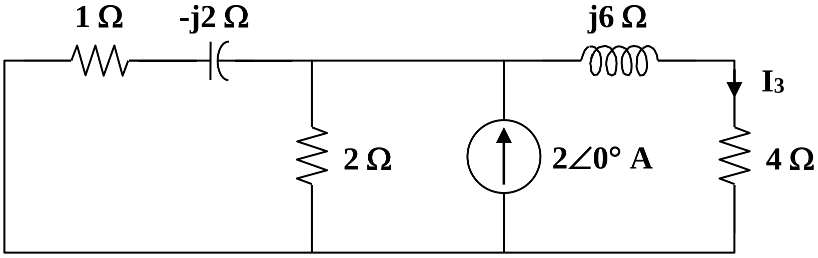
Hence, $i_2 = 0.504\sin(t + 19.1°)$ A

For i_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{6}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/6)} = -j2$$



(c)

$$2\,\|\,(1 - j2) = \frac{2(1 - j2)}{3 - j2}$$

Using current division,

$$\mathbf{I}_3 = \frac{\frac{2(1-j2)}{3-j2}\cdot(2\angle 0°)}{4 + j6 + \frac{2(1-j2)}{3-j2}} = \frac{2(1-j2)}{13 + j3}$$

$$\mathbf{I}_3 = 0.3352\angle -76.43°$$

Hence $i_3 = 0.3352\cos(3t - 76.43°)$ A

Therefore, $i_o =$ **[4 + 0.504 sin(t + 19.1°) + 0.3352 cos(3t – 76.43°)] A**

Solution 10.48

Find i_o in the circuit in Fig. 10.93 using superposition.

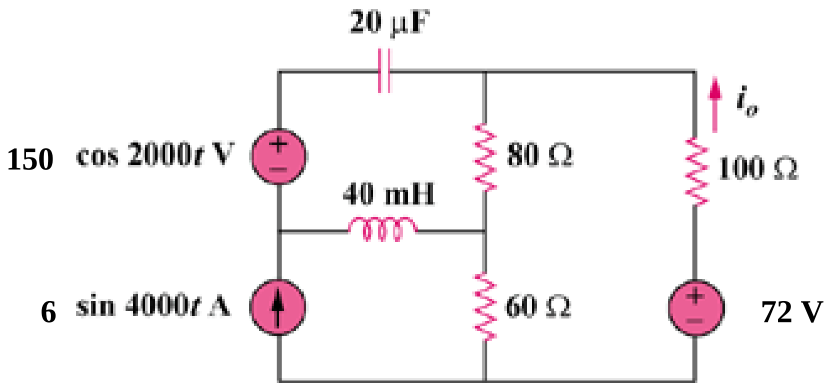


Figure 10.93
For Prob. 10.48.

Solution

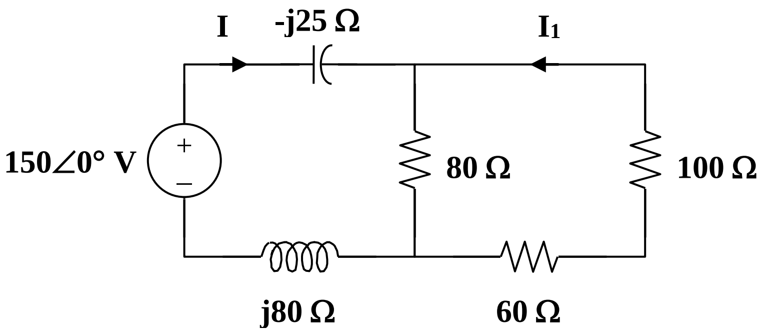
Let $i_o = i_1 + i_2 + i_3$, where I_1 is due to the ac voltage source, I_2 is due to the dc voltage source, and I_3 is due to the ac current source. For I_1, consider the circuit in Fig. (a).

$$\omega = 2000$$

$$50\cos(2000t) \longrightarrow 50\angle 0°$$

$$40\text{ mH} \longrightarrow j\omega L = j(2000)(40\times10^{-3}) = j80$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times10^{-6})} = -j25$$



(a)

$$80\,\|\,(60+100) = 160/3$$

$$\mathbf{I} = \frac{150}{160/3 + j80 - j25} = \frac{90}{32 + j33}$$

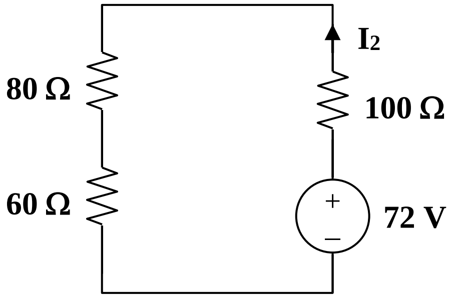
Using current division,

$$\mathbf{I}_1 = \frac{-80\mathbf{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{30\angle 180°}{46\angle 45.9°}$$

$$\mathbf{I}_1 = 0.6522\angle 134.1°$$

Hence, $i_1 = 0.6522\cos(2000t + 134.1°)\ A$

For $\mathbf{I}_2$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{72}{80+60+100} = 0.3\ A$$

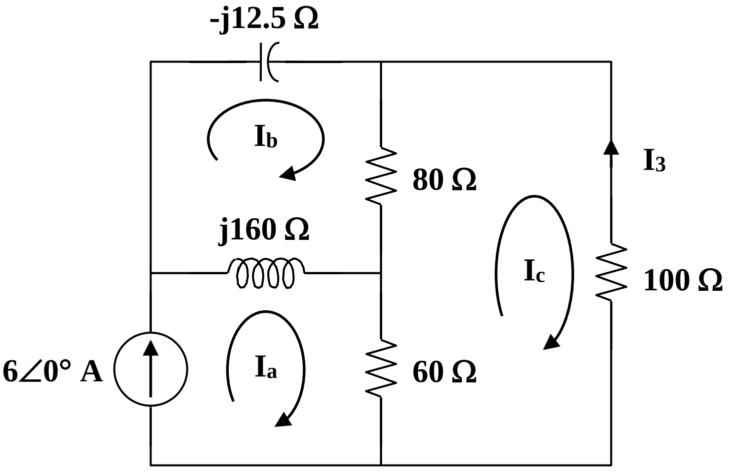
For $\mathbf{I}_3$, consider the circuit in Fig. (c).

$$\omega = 4000$$

$$2\cos(4000\text{t}) \longrightarrow 2\angle 0°$$

$$40\text{ mH} \longrightarrow \text{j}\omega\text{L} = \text{j}(4000)(40\times 10^{-3}) = \text{j}160$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{\text{j}\omega\text{C}} = \frac{1}{\text{j}(4000)(20\times 10^{-6})} = \text{-j}12.5$$



(c)

`

For mesh 1,

$$\mathbf{I}_a = 6 \text{ A} \qquad (1)$$

For mesh 2,

$$(80 + j160 - j12.5)\mathbf{I_b} - j160\mathbf{I}_a - 80\mathbf{I}_c = 0$$

Simplifying and substituting (1) into this equation yields

$$(8 + j14.75)\mathbf{I}_b - 8\mathbf{I}_c = j96 \qquad (2)$$

For mesh 3,

$$240\mathbf{I}_c - 60\mathbf{I}_a - 80\mathbf{I}_b = 0$$

Simplifying and substituting (1) into this equation yields

$$\mathbf{I_b} = 3\mathbf{I_c} - 4.5 \qquad (3)$$

Substituting (3) into (2) yields

$$(16 + j44.25)\mathbf{I_c} = 36 + j162.375$$

$$\mathbf{I}_c = \frac{36 + j162.375}{16 + j44.25} = 3.5346\angle 7.38°$$

$$\mathbf{I}_3 = \text{-}\mathbf{I_c} = -3.535\angle 7.38°$$

Hence, $$\mathbf{i_{O}}_3 = 3.535\sin(4000\mathbf{t} - 172.62°) \ \mathbf{A}$$

Therefore,

i_O = **{0.3 + 0.6522cos(2000t + 134.1°) + 3.535sin(4000t – 172.62°)} A**

Solution 10.49

Using source transformation, find *i* in the circuit of Fig. 10.94.

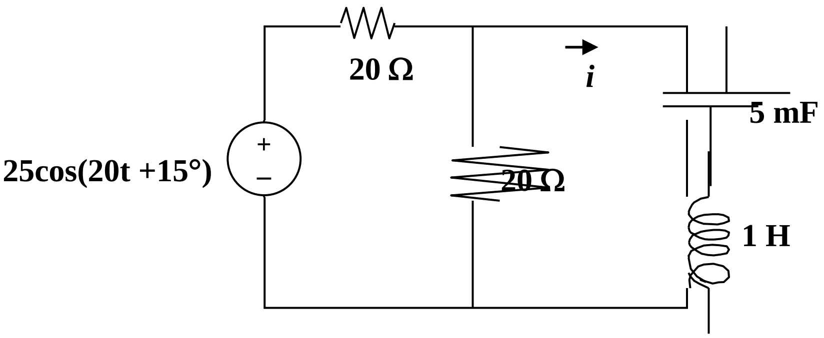


Figure 10.94
For Prob. 10.49.

Solution

First we convert the circuit into the frequency domain and use source transformation to change the voltage source in series with the 20 Ω resistor into a (25∠15°)/20 = 1.25∠15° A current source in parallel with a 20 Ω resistor. Now the two parallel 20 Ω resistors can be turned into a single (20)(20)/(20+20) = 10 Ω resistor. Now we convert the current source in parallel with the 10 Ω resistor into a (1.25∠15°)(10) = 12.5∠15° V voltage source in series with a 10Ω resistor.

Now we get I = (12.5∠15°)/[10–j(1/((20)(0.005))+j(20)(1)] = (12.5∠15°)/(10–j10+j20) = (12.5∠15°)/(14.142∠45°) = 0.8839∠–30°. Thus,

i = **883.9cos(20t–30°) mA**.

Solution 10.50

Using Fig. 10.95, design a problem to help other students to better understand source transformation.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use source transformation to find v_o in the circuit in Fig. 10.95.

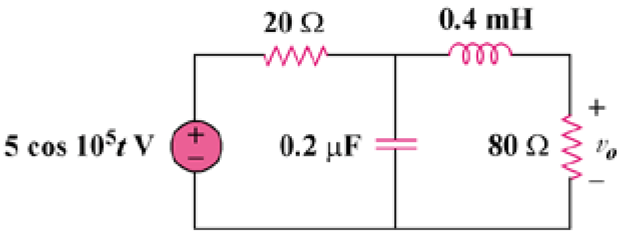


Figure 10.95

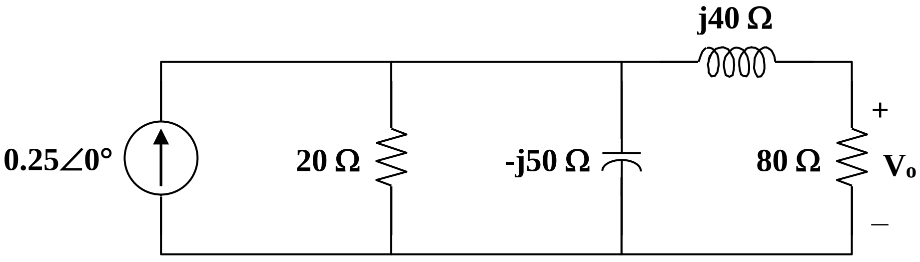
Solution

$$5\cos(10^5 t) \longrightarrow 5\angle 0°, \quad \omega = 10^5$$

$$0.4\text{ mH} \longrightarrow j\omega L = j(10^5)(0.4\times 10^{-3}) = j40$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^5)(0.2\times 10^{-6})} = -j50$$

After transforming the voltage source, we get the circuit in Fig. (a).

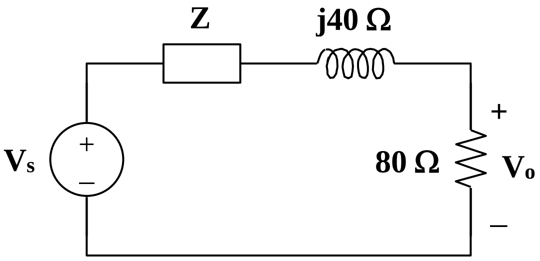


(a)

Let $\quad \mathbf{Z} = 20 \,||\, -j50 = \dfrac{-j100}{2 - j5}$

and $\quad \mathbf{V}_s = (0.25\angle 0°)\,\mathbf{Z} = \dfrac{-j25}{2 - j5}$

With these, the current source is transformed to obtain the circuit in Fig.(b).



(b)

By voltage division,

$$\mathbf{V}_o = \frac{80}{\mathbf{Z}+80+j40}\mathbf{V}_s = \frac{80}{\frac{-j100}{2-j5}+80+j40}\cdot\frac{-j25}{2-j5}$$

$$\mathbf{V}_o = \frac{8(-j25)}{36-j42} = 3.615\angle -40.6°$$

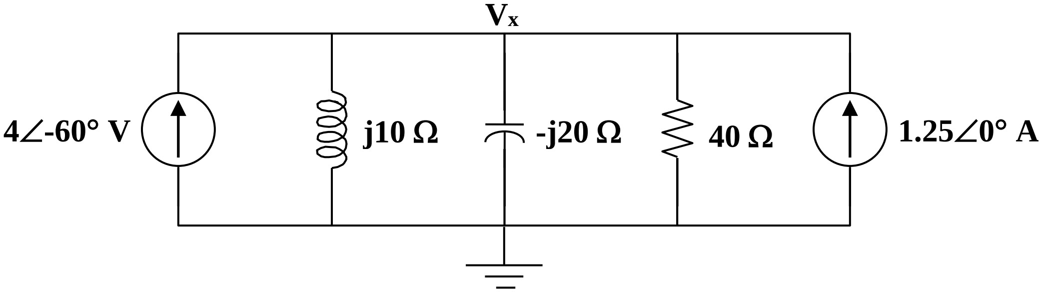
Therefore, $v_o =$ **$3.615\cos(10^5 t - 40.6°)$ V**

Solution 10.51

There are many ways to create this problem, here is one possible solution. Let $V_1 = 40\angle 30°$ V, $X_L = 10\ \Omega$, $X_C = 20\ \Omega$, $R_1 = R_2 = 80\ \Omega$, and $V_2 = 50$ V.

If we let the voltage across the capacitor be equal to V_x, then
$\mathbf{I_o} = [\mathbf{V_x}/(–j20)] + [(\mathbf{V_x}–50)/80] = (0.0125+j0.05)\mathbf{V_x} – 0.625 = (0.051539\angle 75.96°)\mathbf{V_x} – 0.625$.

The following circuit is obtained by transforming the voltage sources.



$\mathbf{V_x} = (4\angle\text{-}60°+1.25)/(–j0.1+j0.05+0.025) = (2–j3.4641+1.25)/(0.025–j0.05)$
$= (3.25–j3.4641)/(0.025–j0.05) = (4.75\angle–46.826°)/(0.055902\angle–63.435°)$
$= 84.97\angle 16.609°$ V.

Therefore,

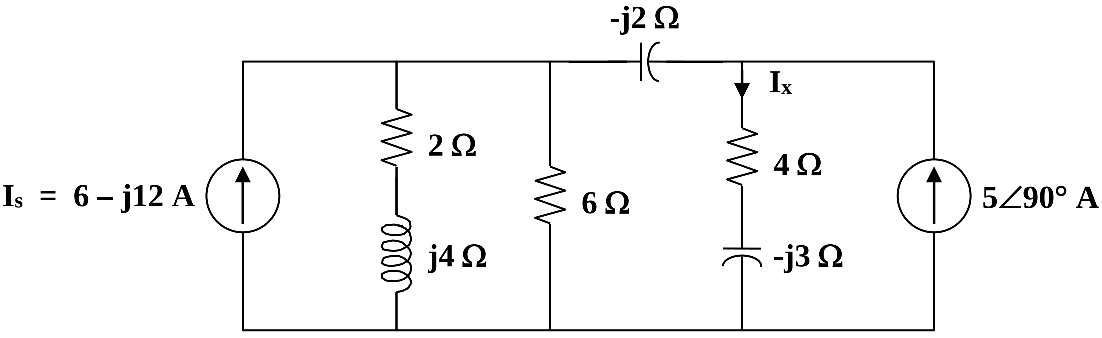
$\mathbf{I_o} = (0.051539\angle 75.96°)(84.97\angle 16.609°) – 0.625 = 4.3793\angle 92.569° – 0.625$
$= –0.196291+j4.3749 – 0.625 = –0.821291+j4.3749 =$ **$4.451\angle 100.63°$ A**.

Solution 10.52

We transform the voltage source to a current source.

$$\mathbf{I}_s = \frac{60\angle 0°}{2 + j4} = 6 - j12$$

The new circuit is shown in Fig. (a).

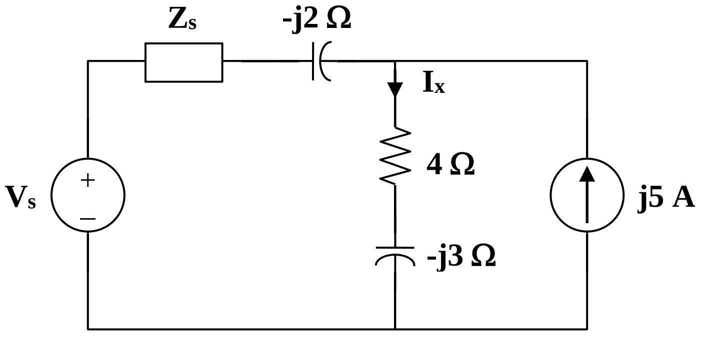


(a)

Let $$\mathbf{Z}_s = 6 \,||\, (2 + j4) = \frac{6(2 + j4)}{8 + j4} = 2.4 + j1.8$$

$$\mathbf{V}_s = \mathbf{I}_s\,\mathbf{Z}_s = (6 - j12)(2.4 + j1.8) = 36 - j18 = 18(2 - j)$$

With these, we transform the current source on the left hand side of the circuit to a voltage source. We obtain the circuit in Fig. (b).

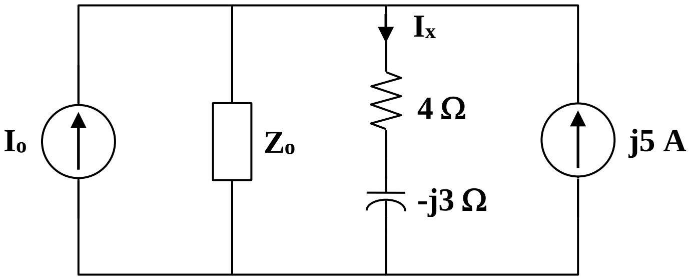


(b)

Let $$\mathbf{Z}_o = \mathbf{Z}_s - j2 = 2.4 - j0.2 = 0.2(12 - j)$$

$$\mathbf{I}_o = \frac{\mathbf{V}_s}{\mathbf{Z}_o} = \frac{18(2 - j)}{0.2(12 - j)} = 15.517 - j6.207$$

With these, we transform the voltage source in Fig. (b) to a current source. We obtain the circuit in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_x = \frac{\mathbf{Z}_o}{\mathbf{Z}_o + 4 - j3}(\mathbf{I}_o + j5) = \frac{2.4 - j0.2}{6.4 - j3.2}(15.517 - j1.207)$$

$$\mathbf{I}_x = 5 + j1.5625 = \mathbf{5.238\angle 17.35° \ A}$$

Solution 10.53

Use the concept of source transformation to find $\mathbf{V}_o$ in the circuit of Fig. 10.97.

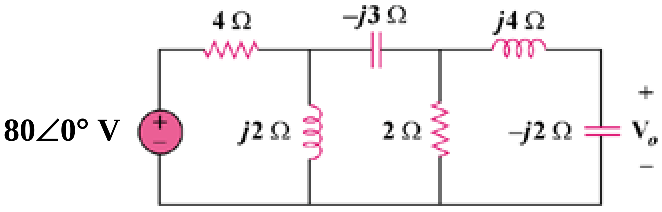
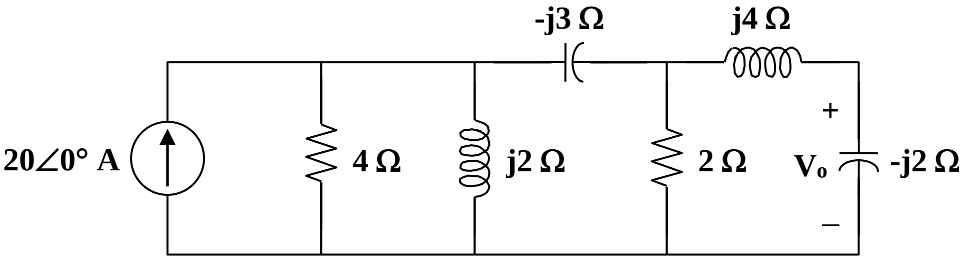


Figure 10.97
For Prob. 10.53.

Solution

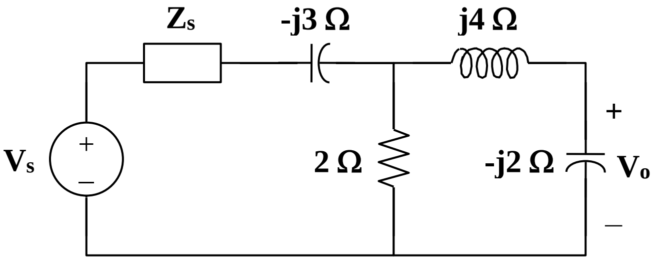
We transform the voltage source to a current source to obtain the circuit in Fig. (a).



(a)

Let
$$\mathbf{Z}_s = 4 \,||\, j2 = \frac{j8}{4+j2} = 0.8 + j1.6$$
$$\mathbf{V}_s = (20\angle 0°)\,\mathbf{Z}_s = (20)(0.8 + j1.6) = 16 + j32$$

With these, the current source is transformed so that the circuit becomes that shown in Fig. (b).

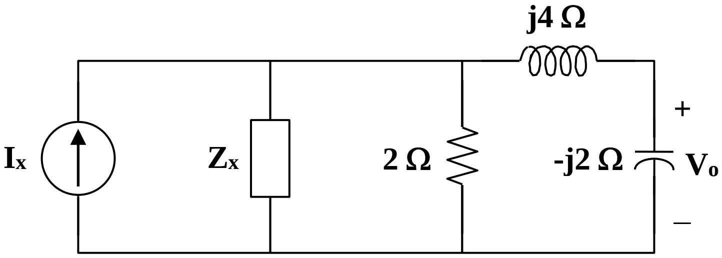


(b)

Let $$\mathbf{Z}_x = \mathbf{Z}_s - j3 = 0.8 - j1.4$$

$$\mathbf{I}_x = \frac{\mathbf{V}_s}{\mathbf{Zx}} = \frac{16 + \mathbf{j}32}{0.8 - \mathbf{j}1.4} = -12.3076 + \mathbf{j}18.4616$$

With these, we transform the voltage source in Fig. (b) to obtain the circuit in Fig. (c).

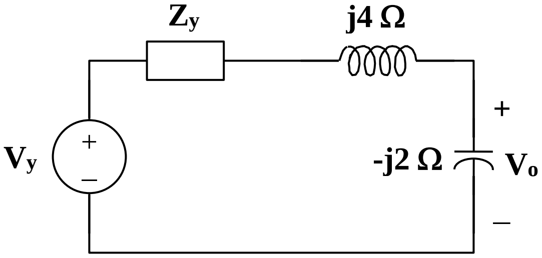


(c)

Let $$\mathbf{Z}_y = 2 \,\|\, \mathbf{Z}_x = \frac{1.6 - j2.8}{2.8 - j1.4} = 0.8571 - j0.5714$$

$$\mathbf{V}_y = \mathbf{I}_x\,\mathbf{Z}_y = (-12.3076 + j18.4616)\cdot(0.8571 - j0.5714) = j22.8572 \text{ V}.$$

With these, we transform the current source to obtain the circuit in Fig. (d).



(d)

Using current division,

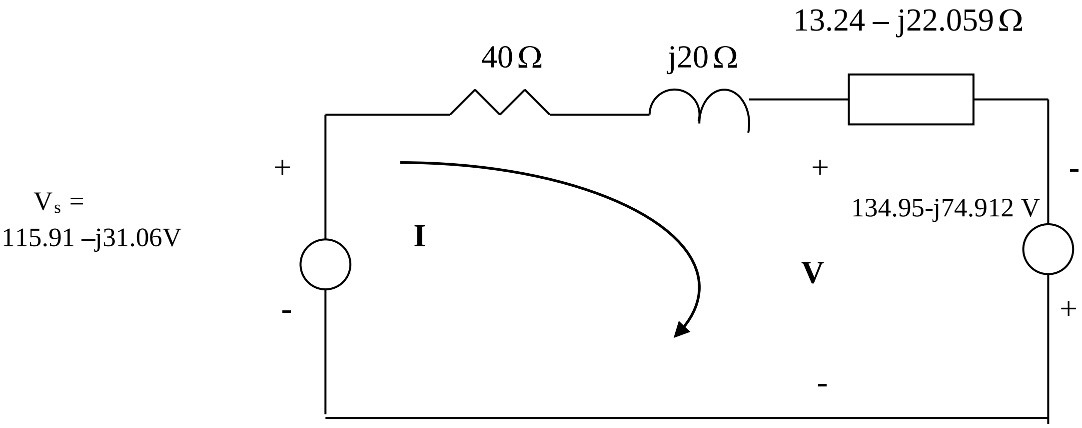
$$\mathbf{V}_o = \frac{-j2}{\mathbf{Z}_y + j4 - j2}\mathbf{V}_y = \frac{-j2(j22.8572)}{0.8571 - j0.5714 + j4 - j2} = \mathbf{(14.116 - j23.532)\ V}.$$

$$\mathbf{V_o = 27.44\angle{-59.04^\circ}\ V}.$$

Solution 10.54

$$50\,//(-j30) = \frac{50x(-j30)}{50-j30} = 13.24 - j22.059$$

We convert the current source to voltage source and obtain the circuit below.



Applying KVL gives

-115.91 + j31.058 + (53.24-j2.059)I -134.95 + j74.912 = 0

or $I = \frac{-250.86 + j105.97}{53.24 - j2.059} = -4.7817 + j1.8055$

But $-V_S + (40 + j20)I + V = 0 \longrightarrow V = V_S - (40 + j20)I$

$$\boldsymbol{V = 115.91 - j31.05 - (40 + j20)(-4.7817 + j1.8055) = \underline{124.06\angle -154^o\ \text{V}}}$$

which agrees with the result in Prob. 10.7.

Solution 10.55

Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* for each of the circuits in Fig. 10.98.

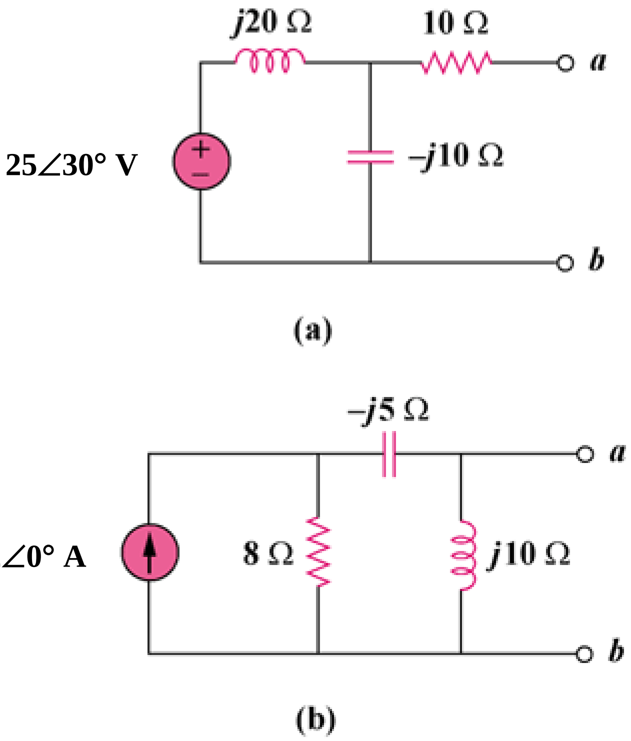
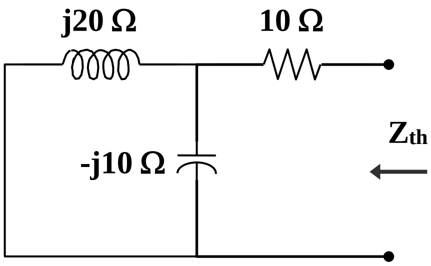


Figure 10.98
For Prob. 10.55.

Solution

(a) To find $\mathbf{Z}_{\text{th}}$, consider the circuit in Fig. (a).

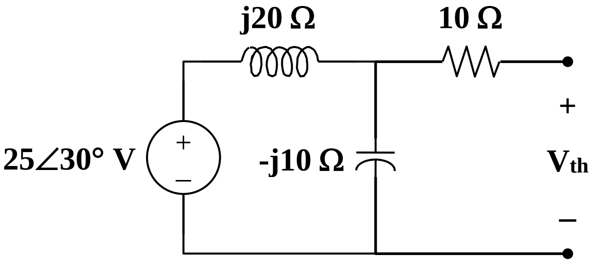


(a)

$$\mathbf{Z}_{\text{N}} = \mathbf{Z}_{\text{th}} = 10 + \text{j}20 \,||\, (\text{-j}10) = 10 + \frac{(\text{j}20)(\text{-j}10)}{\text{j}20 - \text{j}10}$$

$$= 10 - \text{j}20 = \mathbf{22.36\angle{-63.43}°\ \Omega}$$

To find $\mathbf{V}_{\text{th}}$, consider the circuit in Fig. (b).

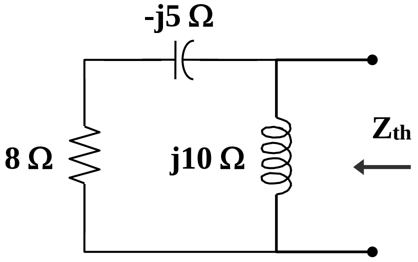


(b)

$$\mathbf{V}_{th} = \frac{-j10}{j20 - j10}(25\angle 30°) = \mathbf{25\angle{-150}°\ V}$$

$$\mathbf{I}_{\mathbf{N}} = \frac{\mathbf{V}_{\mathbf{th}}}{\mathbf{Z}_{\mathbf{th}}} = \frac{25\angle -150°}{22.36\angle \text{-}63.43°} = \mathbf{1.1181\angle{-86.57}°\ A}$$

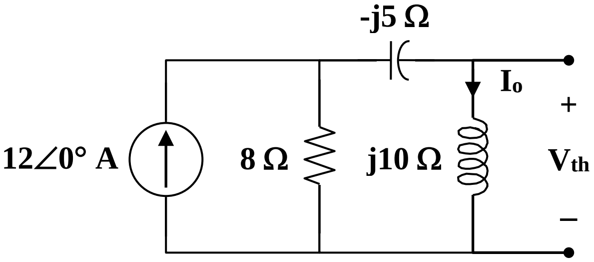
(b) To find $\mathbf{Z}_{\text{th}}$, consider the circuit in Fig. (c).



(c)

$$\mathbf{Z}_{\text{N}} = \mathbf{Z}_{\text{th}} = \text{j}10 \,||\, (8 - \text{j}5) = \frac{(\text{j}10)(8 - \text{j}5)}{\text{j}10 + 8 - \text{j}5} = \mathbf{10\angle 26°\ \Omega}$$

To obtain $\mathbf{V}_{\text{th}}$, consider the circuit in Fig. (d).



(d)

By current division,

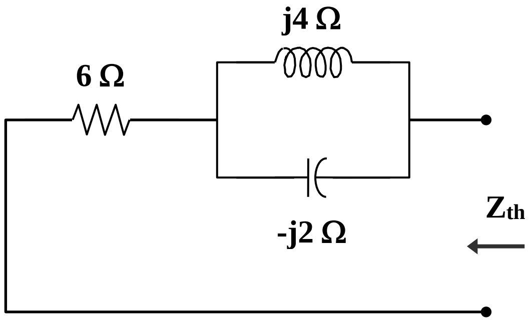
$$\mathbf{I}_o = \frac{8}{8 + j10 - j5}(12\angle 0°) = \frac{96}{8 + j5}$$

$$\mathbf{V}_{th} = j10\mathbf{I}_o = \frac{j960}{8 + j5} = \mathbf{101.76\angle 58° \ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{101.76\angle 58°}{10\angle 26°} = \mathbf{10.176\angle 32° \ A}$$

Solution 10.56

(a) To find $\mathbf{Z}_{\text{th}}$, consider the circuit in Fig. (a).



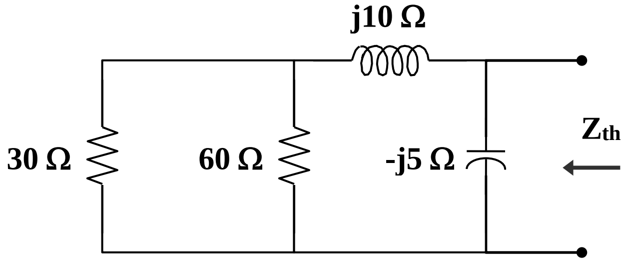
(a)

$$\mathbf{Z}_{\text{N}} = \mathbf{Z}_{\text{th}} = 6 + \text{j}4 \,||\, (\text{-j}2) = 6 + \frac{(\text{j}4)(\text{-j}2)}{\text{j}4 - \text{j}2} = 6 - \text{j}4$$
$$= \mathbf{7.211\angle\text{-}33.69°\ \Omega}$$

By placing short circuit at terminals a-b, we obtain,
$\mathbf{I}_{\text{N}} = \mathbf{2\angle 0°\ A}$

$$\mathbf{V}_{\text{th}} = \mathbf{Z}_{\text{th}}\,\mathbf{I}_{\text{th}} = (7.211\angle -33.69°)(2\angle 0°) = \mathbf{14.422\angle\text{-}33.69°\ V}$$

(b) To find $\mathbf{Z}_{\text{th}}$, consider the circuit in Fig. (b).

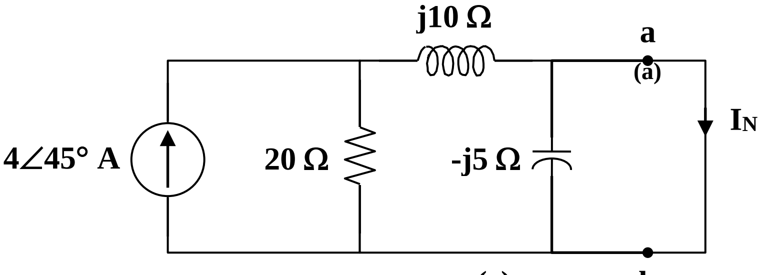


(b)

$$30 \,||\, 60 = 20$$

$$\mathbf{Z}_{\text{N}} = \mathbf{Z}_{\text{th}} = \text{-j}5 \,||\, (20 + \text{j}10) = \frac{(\text{-j}5)(20 + \text{j}10)}{20 + \text{j}5}$$
$$= \mathbf{5.423\angle\text{-}77.47°\ \Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, we transform the voltage source and combine the 30 Ω and 60 Ω resistors. The result is shown in Fig. (c).



(c)

$$\mathbf{I}_N = \frac{20}{20 + j10}(4\angle 45°) = \frac{2}{5}(2 - j)(4\angle 45°)$$

$$= \mathbf{3.578\angle 18.43°\ A}$$

$$\mathbf{V}_{th} = \mathbf{Z}_{th}\,\mathbf{I}_N = (5.423\angle -77.47°)\,(3.578\angle 18.43°)$$

$$= \mathbf{19.4\angle\text{-}59°\ V}$$

Solution 10.57

Using Fig. 10.100, design a problem to help other students to better understand Thevenin and Norton equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Thevenin and Norton equivalent circuits for the circuit shown in Fig. 10.100.

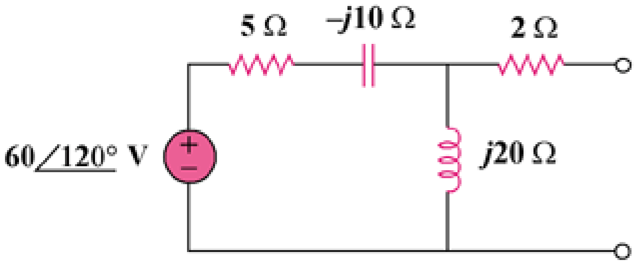
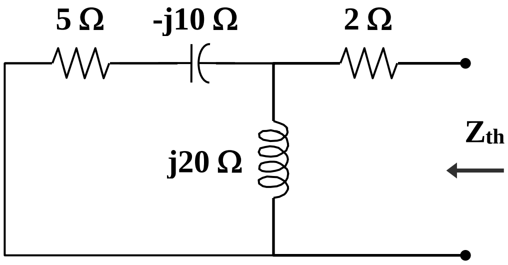


Figure 10.100

Solution

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).

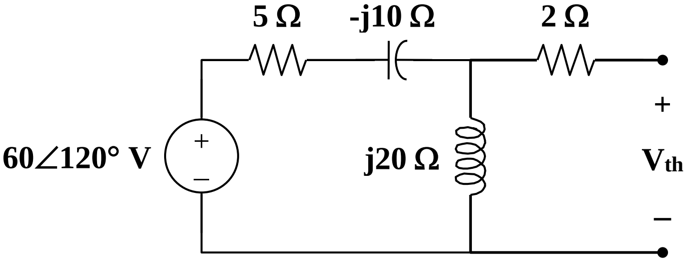


(a)

$$\mathbf{Z}_N = \mathbf{Z}_{th} = 2 + j20 \,\|\, (5 - j10) = 2 + \frac{(j20)(5-j10)}{5+j10}$$

$$= 18 - j12 = \mathbf{21.633\angle\text{-}33.7°\ \Omega}$$

To find $\mathbf{V}_{\text{th}}$, consider the circuit in Fig. (b).



(b)

$$\mathbf{V}_{\text{th}} = \frac{\text{j}20}{5-\text{j}10+\text{j}20}(60\angle 120°) = \frac{\text{j}4}{1+\text{j}2}(60\angle 120°)$$

$$= \mathbf{107.3\angle 146.56°\ V}$$

$$\mathbf{I}_{\text{N}} = \frac{\mathbf{V}_{\text{th}}}{\mathbf{Z}_{\text{th}}} = \frac{107.3\angle 146.56°}{21.633\angle -33.7°} = \mathbf{4.961\angle\text{-}179.7°\ A}$$

Solution 10.58

For the circuit depicted in Fig. 10.101, find the Thevenin equivalent circuit at terminals *a*-*b*.

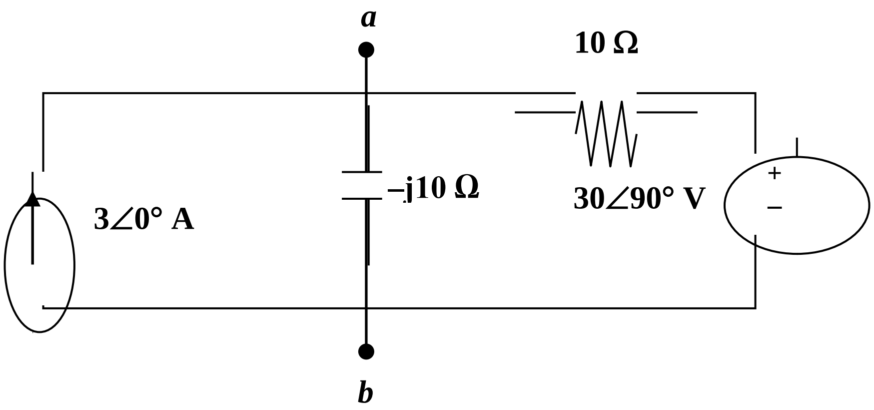


Figure 10.101
For Prob. 10.58.

Solution

The easiest way to do this is to find $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$. Writing a nodal equation at $\mathbf{V_{ab}}$ will give us $\mathbf{V_{oc}} = \mathbf{V_{ab}}$. $-3 + [(\mathbf{V_{ab}}-0)/(-j10)] + [(V_{ab}-j30)/10] = 0$ or $(0.1+j0.1)\mathbf{V_{ab}} = 3+j3$ or

$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = 3(1+j)/[0.1(1+j)] = \mathbf{30\ V}$.

I_{sc} is fairly easy in that shorting a to b shorts out the capacitor. Therefore,
$I_{sc} = 3 + [(j30)/10] = 3+j3$. Thus,

$\mathbf{Z_{eq}} = \mathbf{V_{Thev}}/\mathbf{I_{sc}} = 30/[3(1+j)] = \mathbf{(5-j5)\ \Omega}$.

Solution 10.59

Calculate the output impedance of the circuit shown in Fig. 10.102.

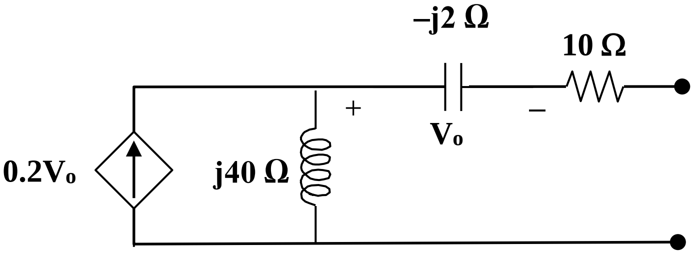
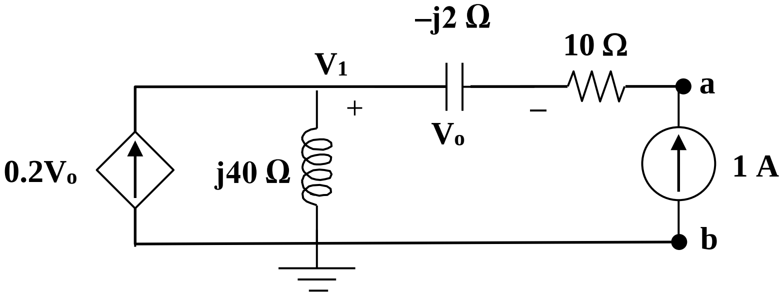


Figure 10.102
For Prob. 10.59.

Solution

Since there are no independent sources, we need to inject a current, best value is to make it 1 amp, into the terminals on the right and then to determine the voltage at the terminals.



Clearly $\mathbf{V_o} = –(–j2) = j2$ and $V_1 = (0.2V_o + 1)j40 = (1+j0.4)j40 = –16+j40$ V.
Next, $\mathbf{V_{ab}} = 10 – j2 – 16 + j40 = –6+j38 = 38.47\angle 98.97°$ V or

$$\mathbf{Z_{eq} = (–6+j38)\ \Omega}.$$

Solution 10.60

Find the Thevenin equivalent of the circuit in Fig. 10.103 as seen from:

(a) terminals *a-b*
(b) terminals *c-d*

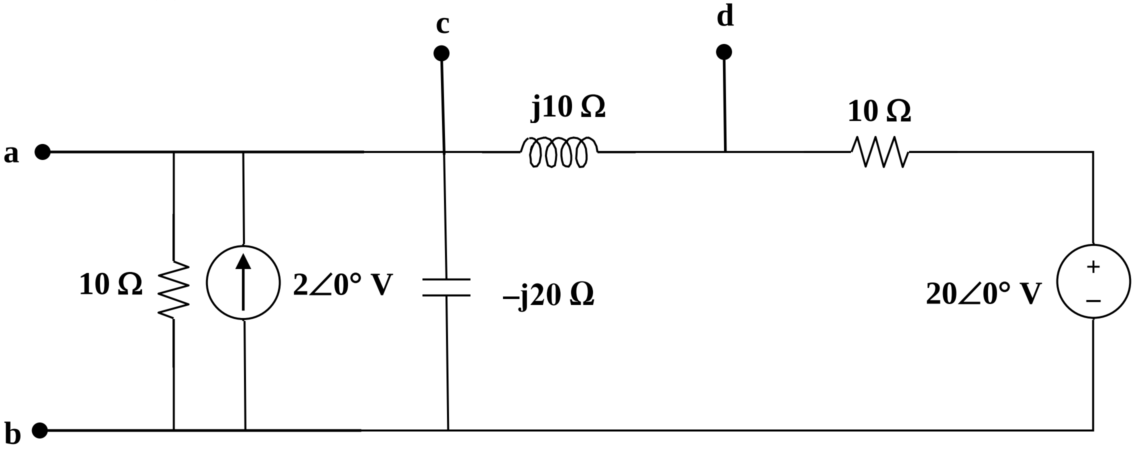


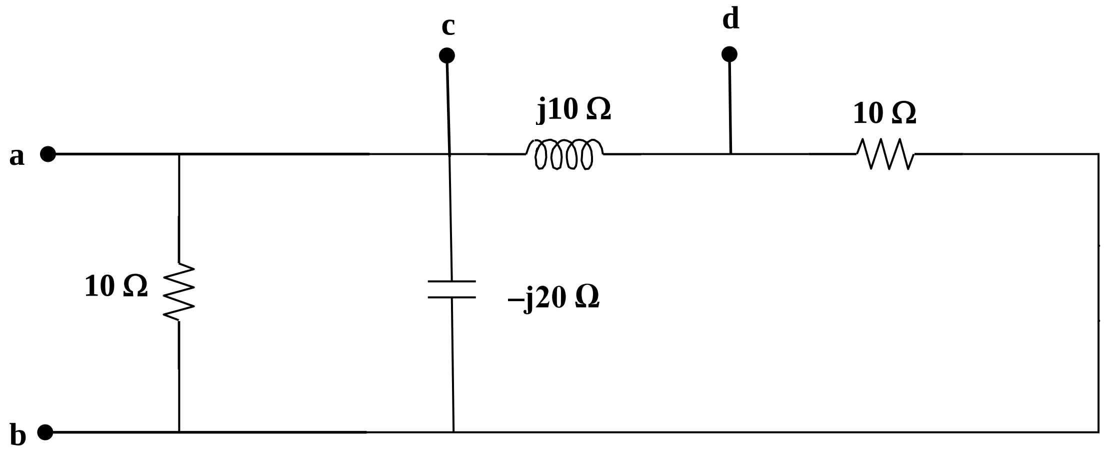
Figure 10.103
For Prob. 10.60.

Solution

Let us find the Thevenin equivalent circuits by finding $\mathbf{V_{ab}}$ and $\mathbf{V_{cd}}$ in the above circuit which gives us the Thevenin voltages. Next we set the independent sources to zero and find $\mathbf{Z_{ab}}$ and $\mathbf{Z_{cd}}$ which are the Thevenin impedances.

We start by observing we only have one unknown node voltage, $\mathbf{V_c}$ which leads to $[(\mathbf{V_c}–0)/10] – 2 + [(\mathbf{V_c}–0)/(–j20)] + [(\mathbf{V_c}–20)/(10+j10)] = 0$. Now we get,

$[0.1+j0.05+(10–j10)/(100+100)]\mathbf{V_c} = [0.1+j0.05+0.05–j0.05]\mathbf{V_c} = 0.15\mathbf{V_c}$
$= 2 + 20(10–j10)/200 = 2 + 1 – j = 3–j$ or $\mathbf{V_c} = 20–j6.6667$. Clearly $\mathbf{V_{ab}} = \mathbf{V_c} = \mathbf{V_{Thevab}} =$ **21.08∠18.44° V**. Let I = the current flowing left to right through the inductor. Thus, $\mathbf{I} = [(\mathbf{V_c}–20)/(10+j10)] = –j6.6667(0.05–j0.05) = –0.33333–j0.33333$ which gives us $\mathbf{V_{cd}} = j10(–0.33333–j0.33333) = 3.3333–j3.3333 =$ **4.714∠–45° V**.



For ab, $1/Z_{eq} = 0.1 + j0.05 + 0.05 - j0.05 = 0.15$ or $Z_{eq} = (20/3)\ \Omega$.

For cd, $1/Z_{eq} = (j10)\{[(10)(-j20)/(10-j20)]+10\}/(j10+\{[(10)(-j20)/(10-j20)]+10\})$.

$[(10)(-j20)/(10-j20)]+10 = [-j200(10+j20)/(100+400)]+10 = 8 - j4 + 10 = 18 - j4$
$= (j10)\{18-j4\}/(j10+\{18-j4\}) = (40+j180)/(18+j6)$
$= 184.391\angle 77.471°/(18.9737\angle 18.435°) = (9.7183\angle 59.036°)\ \Omega = (5 + j25/3)\ \Omega$.

(a) $\mathbf{V_{Thev} = 21.08\angle 18.44°\ V}$ and $\mathbf{Z_{eq}} = \mathbf{(20/3)\ \Omega}$,
(b) $\mathbf{V_{Thev} = 4.714\angle -45°\ V}$ and $\mathbf{Z_{eq}} = \mathbf{(9.7183\angle 59.04°)\ \Omega}$ or = $\mathbf{(5 + j25/3)\ \Omega}$.

Solution 10.61

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.

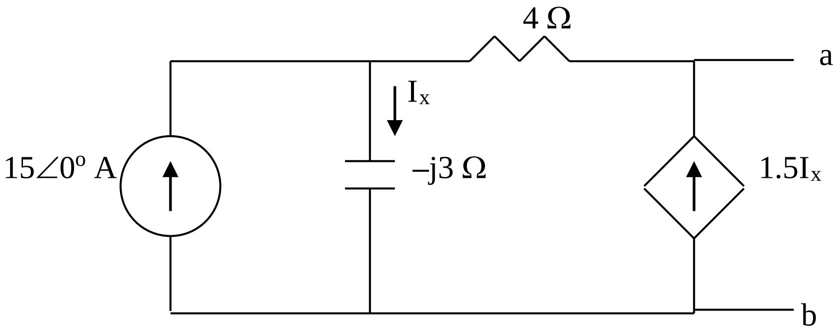
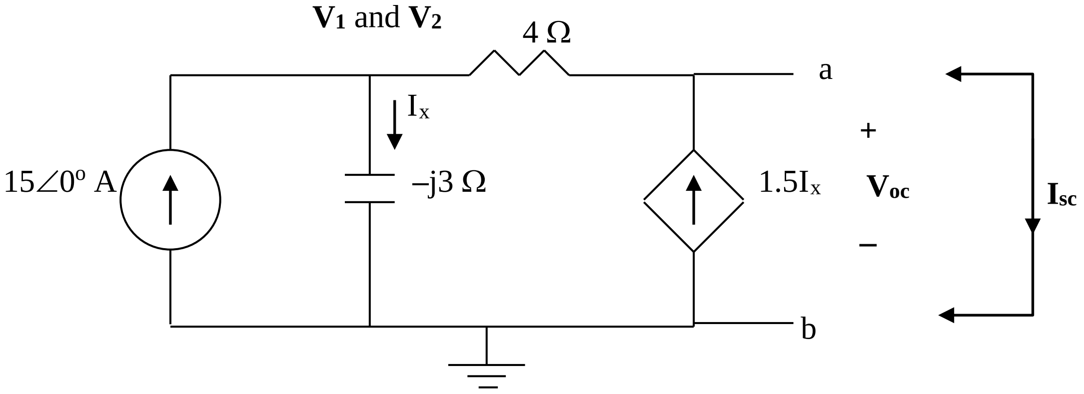


Figure 10.104
For Prob. 10.61.

Solution

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.



Step 1. First we solve for the open circuit voltage using the above circuit and writing two node equations. Then we solve for the short circuit current which only needs one node equation. For being able to solve for $\mathbf{V}_{oc}$, we need to solve these three equations,

$$-15 + [(\mathbf{V}_1-0)/(-j3)] + [(\mathbf{V}_1-\mathbf{V}_{oc})/4] = 0 \text{ and}$$

$$[(\mathbf{V}_{oc}-\mathbf{V}_1)/4] - 1.5\mathbf{I}_x = 0 \text{ where } \mathbf{I}_x = [(\mathbf{V}_1-0)/(-j3)].$$

To solve for $\mathbf{I}_{sc}$, all we need to do is to solve these three equations,

$$-15 + [(\mathbf{V}_2-0)/(-j3)] + [(\mathbf{V}_2-0)/4] = 0, \mathbf{I}_{sc} = [\mathbf{V}_2/4] + 1.5\mathbf{I}_x, \text{ and}$$

$\mathbf{I_x} = [\mathbf{V_2}/–j3]$.

Finally, $\mathbf{V_{Thev}} = \mathbf{V_{oc}}$ and $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$.

Step 2. Now all we need to do is to solve for the unknowns. For $\mathbf{V_{oc}}$,

$\mathbf{I_x} = j0.33333\mathbf{V_1}$ and $(0.25+(1.5)(j0.33333))\mathbf{V_1} = 0.25\mathbf{V_{oc}}$ or
$(0.25+j0.5)\mathbf{V_1} = (0.55902\angle 63.43°)\mathbf{V_1} = 0.25\mathbf{V_{oc}}$ or
$\mathbf{V_1} = (0.44721\angle –63.43°)\mathbf{V_{oc}}$ which leads to,

$(0.25+j0.33333)\mathbf{V_1} – 0.25\mathbf{V_{oc}} = 15$
$= (0.41666\angle +53.13°)(0.44721\angle –63.43°)V_{oc} – 0.25\mathbf{V_{oc}}$
$= (0.186335\angle –10.3°)\mathbf{V_{oc}} – 0.25\mathbf{V_{oc}} = (0.183333–0.25–j0.033333)\mathbf{V_{oc}}$
$= (–0.066667–j0.033333)\mathbf{V_{oc}} = (0.074536\angle –153.435°)\mathbf{V_{oc}} = 15$ or

$$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = \mathbf{201.2\angle 153.44° \ V = (–180+j90) \ V}.$$

Now for $\mathbf{I_{sc}}$,

$\mathbf{I_{sc}} = [\mathbf{V_2}/4] + 1.5\mathbf{I_x} = (0.25+(1.5)(j0.33333))\mathbf{V_2} = (0.25+j0.5)\mathbf{V_2}$.

$[(\mathbf{V_2}–0)/(–j3)] + [(\mathbf{V_2}–0)/4] = 15 = (0.25+j0.3333)\mathbf{V_2}$
$= (0.41667\angle 53.13°)\mathbf{V_2} = 15$ or $\mathbf{V_2} = 4.8\angle –53.13°$

$\mathbf{I_{sc}} = (0.25+j0.5)\mathbf{V_2} = (0.55901\angle 63.435°)(36\angle –53.13°)$
$= 20.124\angle 10.3°$ A

Finally,

$\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}} = 201.2\angle 153.435°/20.12\angle 10.305°$

$$= \mathbf{10\angle 143.13° \ \Omega} \text{ or } = \mathbf{(–8+j6) \ \Omega}.$$

Solution 10.62

Using Thevenin's theorem, find v_o in the circuit in Fig. 10.105.

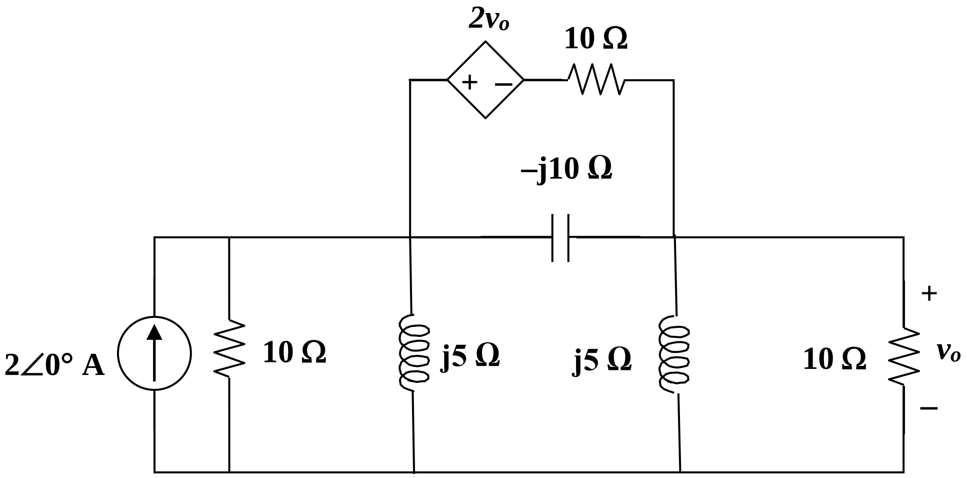
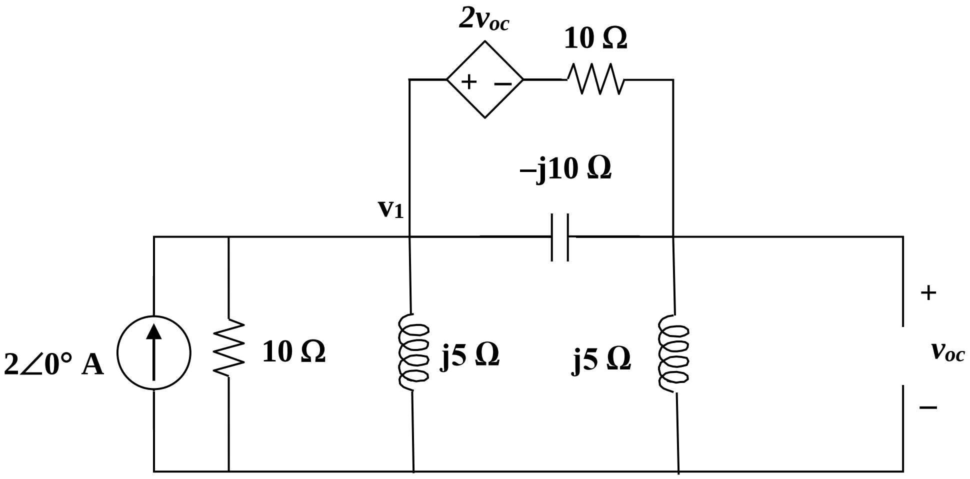


Figure 10.105
For Prob. 10.62.

Solution

We will take out the 10 Ω resistor and determine the Thevenin equivalent looking in from the right. We will calculate V_{oc} and I_{sc}.



We now have two node equations, the first on at v_1 is,

$$-2 + [(v_1-0)/10] + [(v_1-0)/j5] + [(v_1-v_{oc})/(-j10)] + [(v_1-2v_{oc}-v_{oc})/10] = 0 \text{ or}$$

$$[0.1-j0.2+j0.1+0.1]v_1 - [j0.1+0.3]v_{oc} = 2 = (0.22361\angle-26.565°)v_1 - (0.3+j0.1)v_{oc}.$$

The second equation, at v_{oc}, is, $[(v_{oc}–0)/j5] + [(v_{oc}–v_1)/(–j10)] + [(v_{oc}–v_1+2v_{oc})/10] = 0$ or

$$–(0.1+j0.1)v_1 + (–j0.2+j0.1+ 0.3)v_{oc} \text{ or } v_1 = [(0.3–j0.1)/(0.14142\angle 45°)]v_{oc}$$

$$= [(0.316228\angle –18.435°)/(0.14142\angle 45°)]v_{oc} = (2.2361\angle –63.435°)v_{oc}.$$

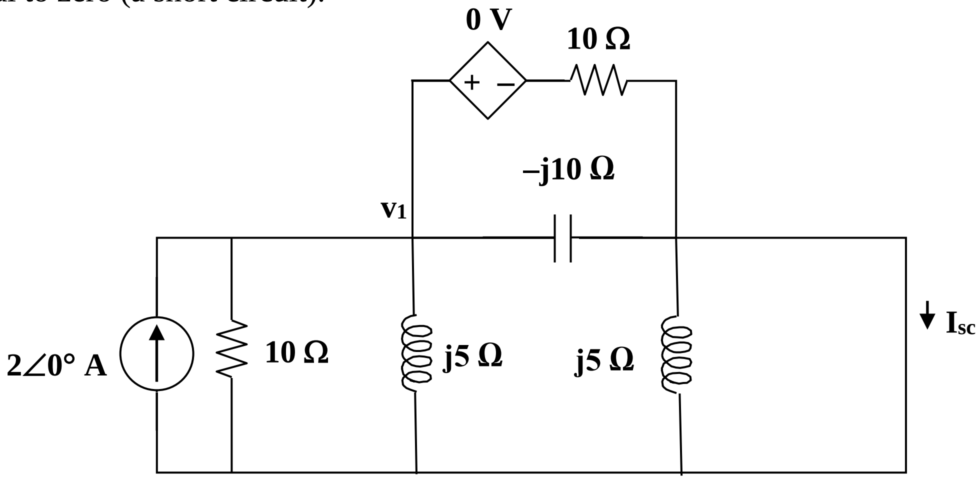
The first equation now becomes, $(0.22361\angle –26.565°)v_1 – (0.3+j0.1)v_{oc} = 2$ or

$$(0.22361\angle –26.565°)(2.2361\angle –63.435°)v_{oc} – (0.3+j0.1)v_{oc} = 2$$

$$= (0.5\angle –90° – 0.3 – j0.1)v_{oc} = (–0.3–j0.6)v_{oc} = 2 \text{ or } v_{oc} =2/(0.67082\angle –116.565°)$$

$$= 2.9814\angle 116.565° \text{ V}.$$

Now for I_{sc}, we have essentially the same equations with voltage across the second inductor equal to zero (a short circuit).



Thus we get, $(0.22361\angle –26.565°)v_1 = 2$ or $v_1 = 8.9441\angle 26.565°$. From the above we get, $I_{sc} = [(v_1–0)/(–j10)] + [(v_1–0)/10] = 0.89441\angle 116.565° + 0.89441\angle 26.565°= –0.4+j0.8 + 0.8+j0.4 = 0.4+j1.2 = 1.2649\angle 71.565°$.
This now leads to, $Z_{eq} = 2.9814\angle 116.565°/1.2649\angle 71.565°$
$= 2.357\angle 45° = (1.66665+j1.66665)$ Ω.

V_{Thev} = 2.981∠116.56° V, Z_{eq} = (1.6666 + j1.6666) Ω.

Solution 10.63

Obtain the Norton equivalent of the circuit depicted in Fig. 10.106 at terminals *a-b*.

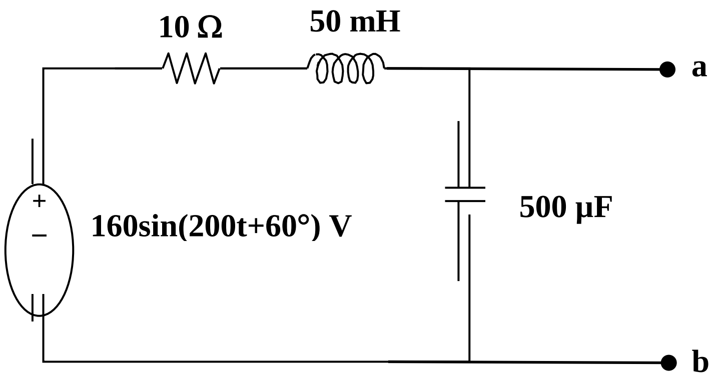
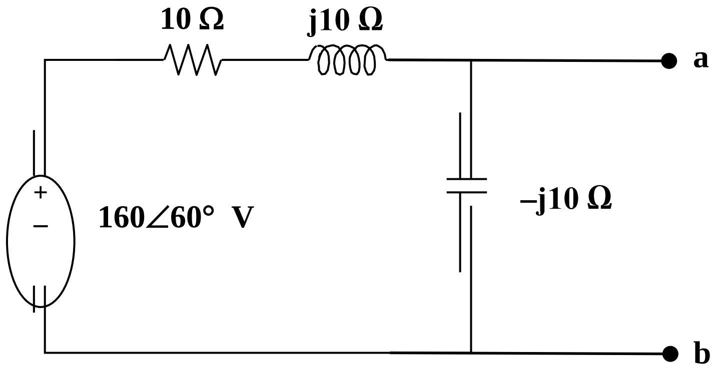


Figure 10.106
For Prob. 10.63.

Solution

First we need to transform this circuit into the frequency domain (where the Norton equivalent circuit exists) and then solve for $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$.

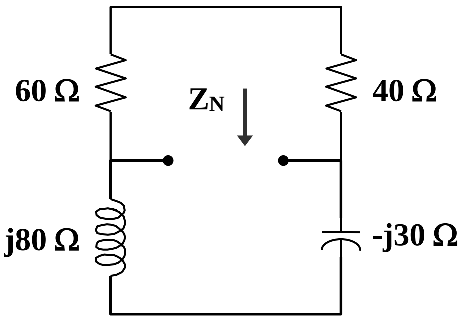


$\mathbf{V_{oc}}$ = [160∠60°/(10+j10–j10)](–j10) = 160∠–30° V.

I_{sc} = 160∠60°/(10+j10) = **11.314∠15° A** = $\mathbf{I_N}$ and Z_{eq} = V_{oc}/I_{sc}
= 160∠–30°/(11.314∠15°) = **14.142∠–45° Ω** = **(10–j10) Ω**.

Solution 10.64

$\mathbf{Z}_N$ is obtained from the circuit in Fig. (a).

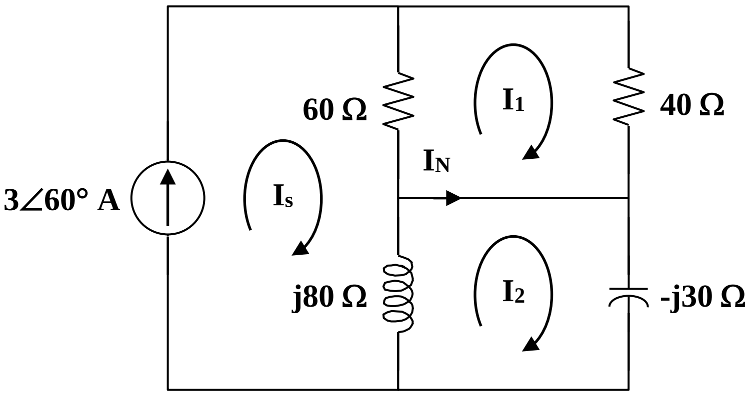


(a)

$$\mathbf{Z}_N = (60+40) \,||\, (j80-j30) = 100 \,||\, j50 = \frac{(100)(j50)}{100+j50}$$

$$\mathbf{Z}_N = 20 + j40 = \mathbf{44.72\angle 63.43^\circ\ \Omega}$$

To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_s = 3\angle 60^\circ$$

For mesh 1,

$$100\mathbf{I}_1 - 60\mathbf{I}_s = 0$$

$$\mathbf{I}_1 = 1.8\angle 60^\circ$$

For mesh 2,

$$(j80-j30)\mathbf{I}_2 - j80\mathbf{I}_s = 0$$

$$\mathbf{I}_2 = 4.8\angle 60^\circ$$

$I_N = I_2 - I_1 = \mathbf{3\angle 60^\circ\ A}$

Solution 10.65

Using Fig. 10.108, design a problem to help other students to better understand Norton's theorem.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Compute i_o in Fig. 10.108 using Norton's theorem.

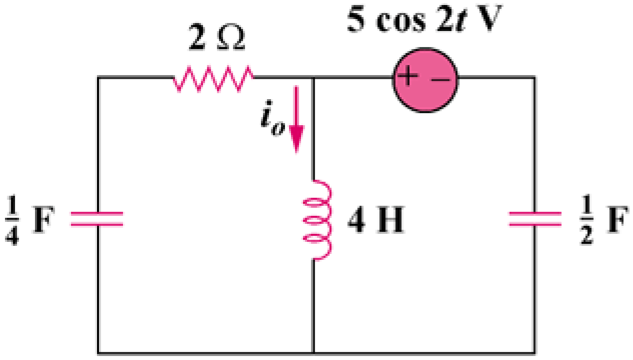


Figure 10.108

Solution

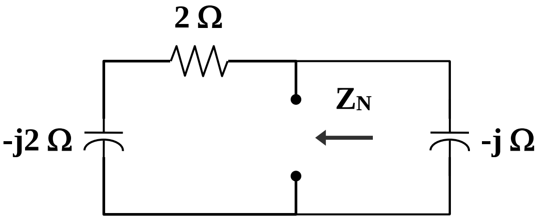
$$5\cos(2t) \longrightarrow 5\angle 0°, \quad \omega = 2$$

$$4\text{ H} \longrightarrow j\omega L = j(2)(4) = j8$$

$$\frac{1}{4}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/4)} = -j2$$

$$\frac{1}{2}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/2)} = -j$$

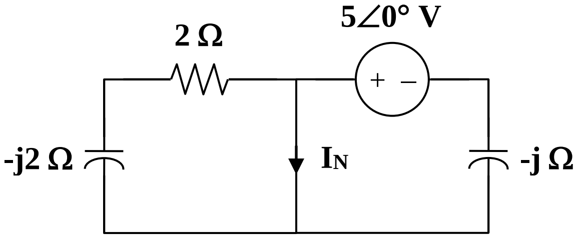
To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)

$$\mathbf{Z}_N = \text{-j} \,||\, (2 - j2) = \frac{\text{- j}(2 - j2)}{2 - j3} = \frac{1}{13}(2 - j10)$$

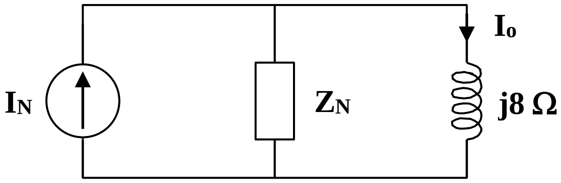
To find $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$\mathbf{I}_N = \frac{5\angle 0°}{\text{- j}} = j5$$

The Norton equivalent of the circuit is shown in Fig. (c).



(c)

Using current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + j8}\mathbf{I}_N = \frac{(1/13)(2 - j10)(j5)}{(1/13)(2 - j10) + j8} = \frac{50 + j10}{2 + j94}$$

$$\mathbf{I}_o = 0.1176 - j0.5294 = 0542\angle \text{-} 77.47°$$

Therefore, $i_o =$ **542 cos(2t – 77.47°) mA**

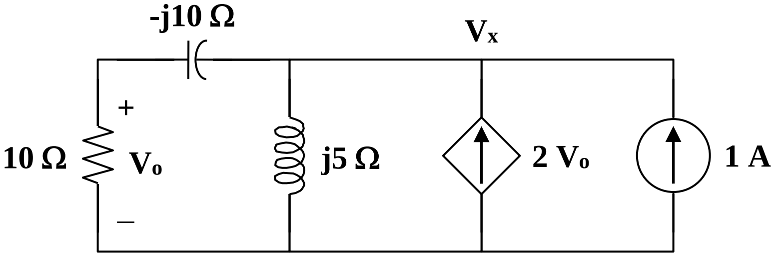
Solution 10.66

$$\omega = 10$$

$$0.5\text{ H} \longrightarrow j\omega L = j(10)(0.5) = j5$$

$$10\text{ mF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(10\times10^{-3})} = -j10$$

To find $\mathbf{Z}_{th}$, consider the circuit in Fig. (a).



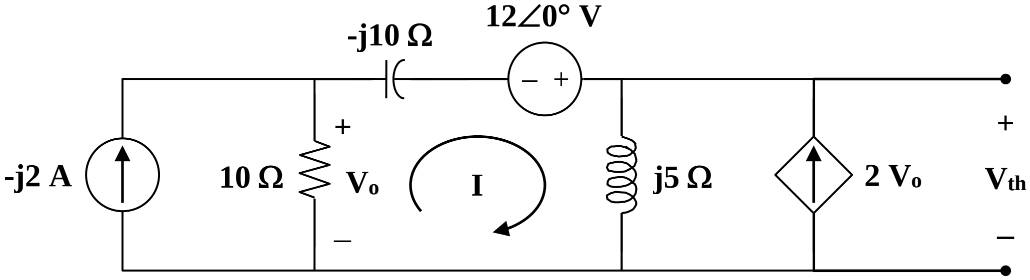
(a)

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_x}{j5} + \frac{\mathbf{V}_x}{10 - j10}, \qquad \text{where} \quad \mathbf{V}_o = \frac{10\mathbf{V}_x}{10 - j10}$$

$$1 + \frac{19\mathbf{V}_x}{10 - j10} = \frac{\mathbf{V}_x}{j5} \longrightarrow \mathbf{V}_x = \frac{-10 + j10}{21 + j2}$$

$$\mathbf{Z}_N = \mathbf{Z}_{th} = \frac{\mathbf{V}_x}{1} = \frac{14.142\angle135°}{21.095\angle5.44°} = \mathbf{670\angle129.56°\ m\Omega}$$

To find $\mathbf{V}_{th}$ and $\mathbf{I}_N$, consider the circuit in Fig. (b).



(b)

$$(10 - j10 + j5)\mathbf{I} - (10)(-j2) + j5(2\mathbf{V}_o) - 12 = 0$$

where $\mathbf{V}_o = (10)(-j2 - \mathbf{I})$

Thus,

$$(10 - j105)\mathbf{I} = -188 - j20$$

$$\mathbf{I} = \frac{188 + j20}{-10 + j105}$$

$$\mathbf{V}_{th} = j5(\mathbf{I} + 2\mathbf{V}_o) = j5(-19\mathbf{I} - j40) = -j95\mathbf{I} + 200$$

$$\mathbf{V}_{th} = \frac{-j95(188 + j20)}{-10 + j105} + 200 = \frac{(95\angle-90^\circ)(189.06\angle6.07^\circ)}{105.48\angle95.44} + 200$$

$$= 170.28\angle-179.37^\circ + 200 = -170.27 - j1.8723 + 200 = 29.73 - j1.8723$$

$$\mathbf{V}_{th} = \mathbf{29.79\angle{-3.6^\circ}\ V}$$

$$\mathbf{I}_N = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{29.79\angle-3.6^\circ}{0.67\angle129.56^\circ} = \mathbf{44.46\angle{-133.16^\circ}\ A}$$

Solution 10.67

Find the Thevenin and Norton equivalent circuits at terminals *a*-*b* of the circuit in Fig. 10.110.

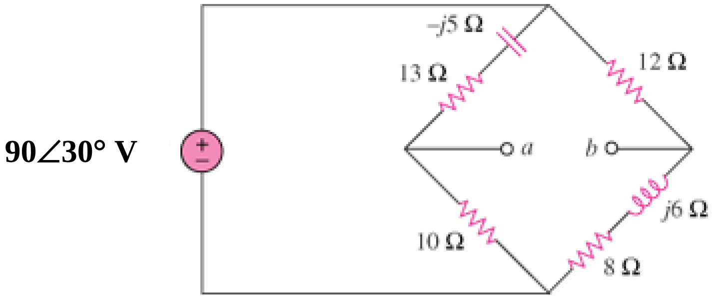


Figure 10.110
For Prob. 10.67.

Solution

$$\mathbf{Z_{eq}} = 10//(13-j5)+12//(8+j6) = \frac{10(13-j5)}{23-j5}+\frac{12(8+j6)}{20+j6} = \mathbf{(11.243 + j1.079)\ \Omega}.$$

$\mathbf{V_a}$ = [10/(23–j5)](90∠30°) = 900/(23.5372∠–12.265°) = 38.237∠42.265°
= 28.297 + j25.717 and $\mathbf{V_b}$ = [(8+j6)/(20+j6)](90∠30°)
= [(10∠36.87°)/(20.881∠16.699°)](90∠30°) = 43.1∠50.17° = (27.61 + j33.1) V.

Thus,
$\mathbf{V_{Thev}} = \mathbf{V_a} - \mathbf{V_b}$ = 0.687 – j7.383 = **7.415∠–84.68° V**.
$\mathbf{I_N} = \mathbf{V_{Thev}}/\mathbf{Z_{eq}}$ = (7.415∠–84.68°)/(11.2947∠5.482°) = **656.5∠–90.16° mA**.

Solution 10.68

For the circuit in Fig. 10.111, obtain the Thèvenin equivalent at terminals *a*-*b*.

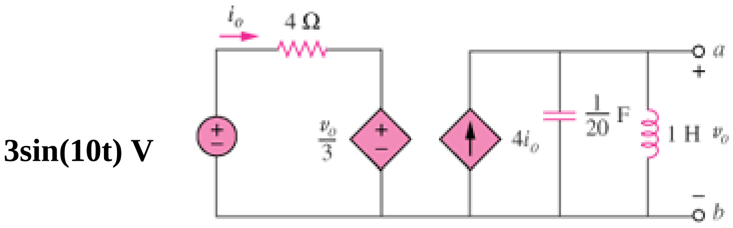


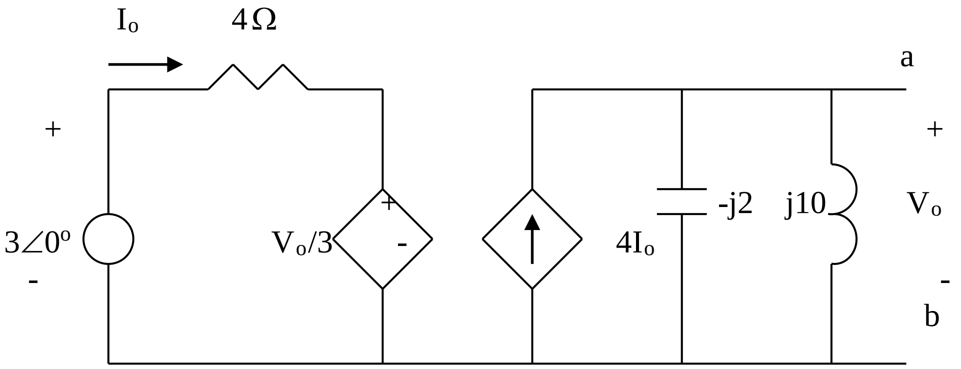
Figure 10.111
For Prob. 10.68.

Solution

$$1\text{H} \longrightarrow j\omega L = j10x1 = j10$$

$$\frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x\frac{1}{20}} = -j2$$

We obtain V_{Th} using the circuit below.



$$j10//(-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o x(-j2.5) = -j10I_o \qquad (1)$$

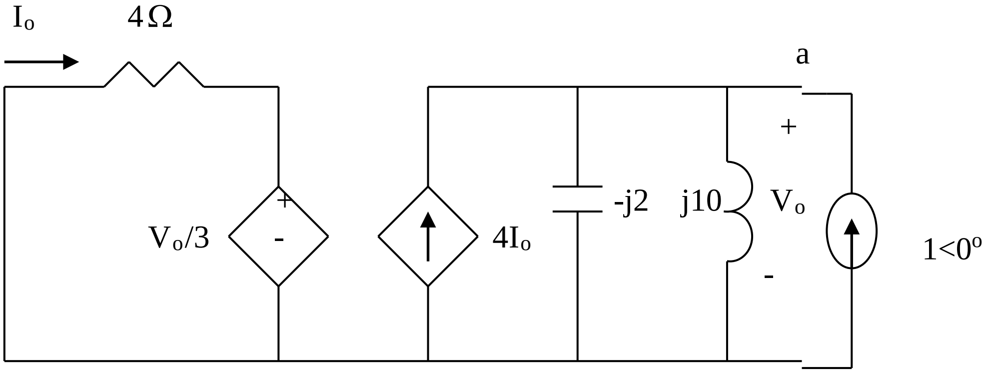
$$-3 + 4I_o + \frac{1}{3}V_o = 0 \qquad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{3}{4 - j10/3}, \qquad V_{Th} = V_o = -j10I_o = \frac{-j30}{4 - j10/3} = 5.7617\angle -50.1945^o$$

V_{Thev} = 5.762∠–50.19° V.

To find $R_{Th,}$ we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \quad \longrightarrow \quad I_o = -\frac{V_o}{12}$$

$1 + 4I_o = \frac{V_o}{-j2} + \frac{V_o}{j10}$ or $[(1/3)+(j0.5)–(j0.1)]\mathbf{V_o} = 1$ or $\mathbf{V_o} = 1/(0.33333+j0.4)$ or

$\mathbf{V_o}/1 = \mathbf{Z_{eq}} = 1/(0.5206812\angle 50.1947°) = 1.92056\angle –50.1947°$

$\mathbf{Z_{eq} = (1.2295–j1.4754)\ \Omega}$.

Solution 10.69

For the integrator shown in Fig. 10.112, obtain $\mathbf{V}_o/\mathbf{V}_s$. Find $v_o(t)$ when $v_s(t) = \mathbf{V}_m \sin\omega t$ and $\omega = 1/RC$.

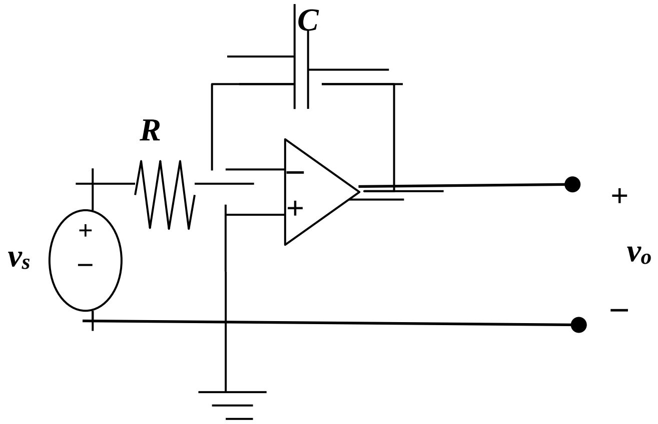


Figure 10.112
For Prob. 10.69.

Solution

This is an inverting op amp so that $\mathbf{V}_o/\mathbf{V}_s = -[1/(j\omega C)]/R = \mathbf{j[1/(\omega RC)]}$.

For $\mathbf{V}_s = V_m\angle 0°$ V and $\omega = 1/(RC)$ we get $\mathbf{V}_o = jV_m$ or

$v_o(t) = \mathbf{V_m\sin(\omega t+90°)\ V}$.

Solution 10.70

Using Fig. 10.113, design a problem to help other students to better understand op amps in AC circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The circuit in Fig. 10.113 is an integrator with a feedback resistor. Calculate $v_o(t)$ if $v_s = 2\cos 4 \times 10^4 t$ V.

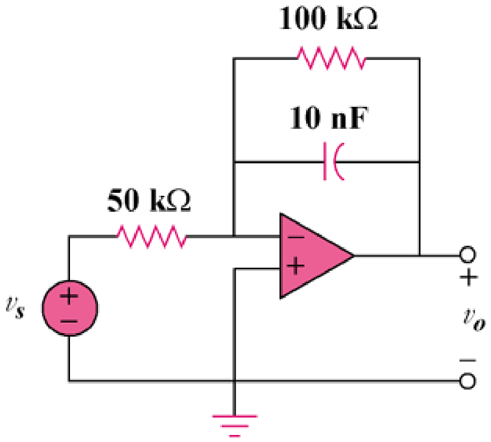


Figure 10.113

Solution

This may also be regarded as an inverting amplifier.

$$2\cos(4\times10^4 t) \longrightarrow 2\angle0°, \quad \omega = 4\times10^4$$

$$10\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4\times10^4)(10\times10^{-9})} = -j2.5\text{ k}\Omega$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i}$$

where $\mathbf{Z}_i = 50\text{ k}\Omega$ and $\mathbf{Z}_f = 100\text{k} \,||\, (-j2.5\text{k}) = \frac{-j100}{40-j}\text{ k}\Omega$.

Thus, $$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-(-j2)}{40-j}$$

If $\mathbf{V}_s = 2\angle0°$,

$$\mathbf{V}_o = \frac{j4}{40 - j} = \frac{4\angle 90°}{40.01\angle -1.43°} = 0.1\angle 91.43°$$

Therefore,

$$v_o(t) = \mathbf{100\ cos(4x10^4\ t + 91.43°)\ mV}$$

Solution 10.71

Find v_o in the op amp circuit shown in Fig. 114.

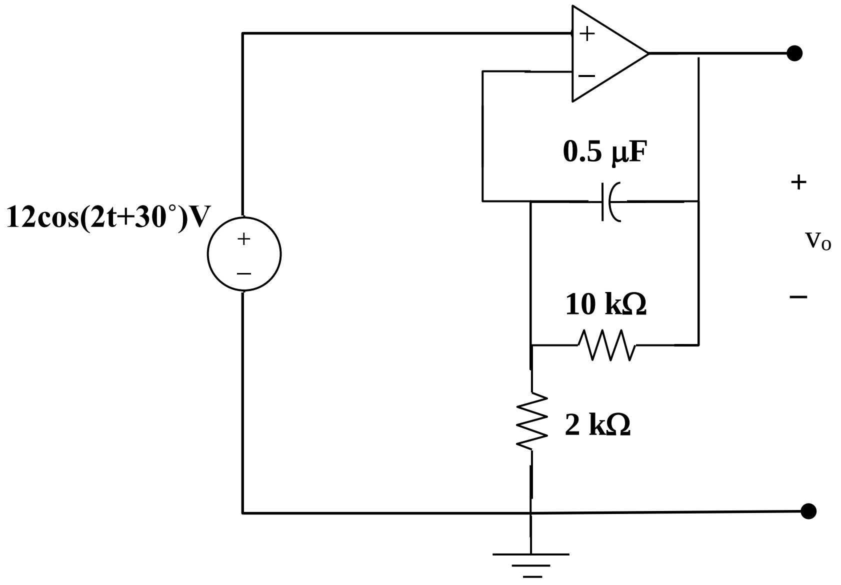


Figure 10.114
For Prob. 10.71.

Solution

$$12\cos(2t+30^o) \quad \longrightarrow \quad 12\angle 30^o$$

$$0.5\mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j2x0.5x10^{-6}} = -j1M\Omega$$

At the inverting terminal,

$$\frac{V_o - 12\angle 30^o}{-j1000k} + \frac{V_o - 12\angle 30^o}{10k} = \frac{12\angle 30^o}{2k} \qquad \longrightarrow$$

$$V_o(1 - j100) = 12\angle 30 + 1200\angle -60^\circ + 6000\angle -60^\circ$$

$$V_o = \frac{10.3923 + j6 + 3600 - j6235.38}{1 - j100} = \frac{7200\angle -59.9045^\circ}{100\angle -89.427^\circ} = 72\angle 29.52^o$$

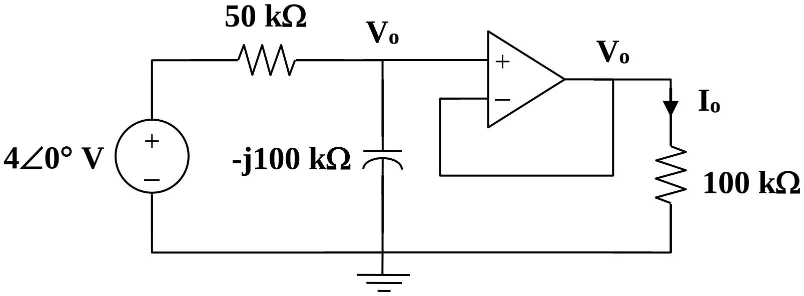
$v_o(t)$ = **72cos(2t+29.52°) V**

Solution 10.72

$$4\cos(10^4 t) \longrightarrow 4\angle 0°, \quad \omega = 10^4$$

$$1\text{ nF} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10^4)(10^{-9})} = -j100\text{ k}\Omega$$

Consider the circuit as shown below.



At the noninverting node,

$$\frac{4-\mathbf{V}_o}{50} = \frac{\mathbf{V}_o}{-j100} \longrightarrow \mathbf{V}_o = \frac{4}{1+j0.5}$$

$$\mathbf{I}_o = \frac{\mathbf{V}_o}{100k} = \frac{4}{(100)(1+j0.5)}\text{mA} = 35.78\angle -26.56°\ \mu\text{A}$$

Therefore,

$i_o(t) =$ **35.78cos(10^{4}t–26.56°) μA**

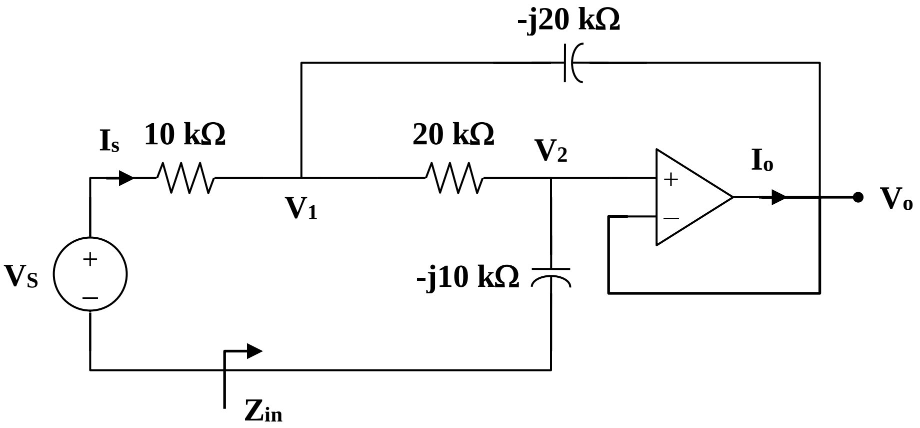
Solution 10.73

As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

$$C_1 = 10 \text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times10^3)(10\times10^{-9})} = \text{-j20 k}\Omega$$

$$C_2 = 20 \text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times10^3)(20\times10^{-9})} = \text{-j10 k}\Omega$$

Consider the circuit in the frequency domain as shown below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-\,j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$2\mathbf{V}_s = (3+j)\mathbf{V}_1 - (1+j)\mathbf{V}_o$$

(1)

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-\,j10}$$

$$\mathbf{V}_1 = (1+j2)\mathbf{V}_o$$

(2)

Substituting (2) into (1) gives

$$2\mathbf{V}_s = j6\mathbf{V}_o \qquad \text{or} \qquad \mathbf{V}_o = \text{-}j\frac{1}{3}\mathbf{V}_s$$

$$\mathbf{V}_1 = (1+j2)\mathbf{V}_o = \left(\frac{2}{3} - j\frac{1}{3}\right)\mathbf{V}_s$$

$$\mathbf{I}_s = \frac{\mathbf{V}_s - \mathbf{V}_1}{10k} = \frac{(1/3)(1+j)}{10k}\mathbf{V}_s$$

$$\frac{\mathbf{I}_s}{\mathbf{V}_s} = \frac{1+j}{30k}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{30k}{1+j} = 15(1-j)\,k$$

$$\mathbf{Z}_{in} = \mathbf{21.21\angle{-45^\circ}\ k\Omega}$$

Solution 10.74

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1}, \qquad \mathbf{Z}_f = R_2 + \frac{1}{j\omega C_2}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = -\frac{R_2 + \frac{1}{j\omega C_2}}{R_1 + \frac{1}{j\omega C_1}} = -\left(\frac{C_1}{C_2}\right)\left(\frac{1 + j\omega R_2 C_2}{1 + j\omega R_1 C_1}\right)$$

At $\omega = 0$, $\quad \mathbf{A}_v = -\frac{\mathbf{C_1}}{\mathbf{C_2}}$

As $\omega \rightarrow \infty$, $\quad \mathbf{A}_v = -\frac{\mathbf{R_2}}{\mathbf{R_1}}$

At $\omega = \frac{1}{R_1 C_1}$, $\quad \mathbf{A}_v = -\left(\frac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\frac{\mathbf{1 + jR_2C_2/R_1C_1}}{\mathbf{1 + j}}\right)$

At $\omega = \frac{1}{R_2 C_2}$, $\quad \mathbf{A}_v = -\left(\frac{\mathbf{C_1}}{\mathbf{C_2}}\right)\left(\frac{\mathbf{1 + j}}{\mathbf{1 + jR_1C_1/R_2C_2}}\right)$

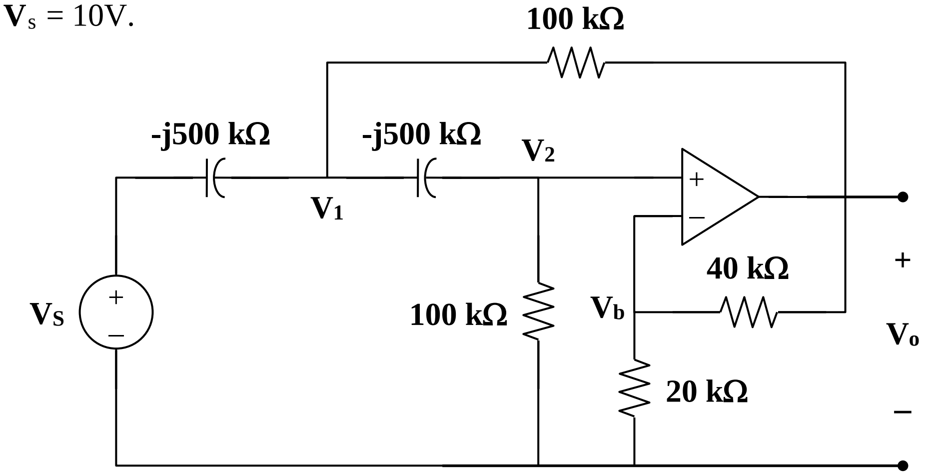
Solution 10.75

$$\omega = 2 \times 10^3$$

$$C_1 = C_2 = 1\,\text{nF} \quad \longrightarrow \quad \frac{1}{j\omega C_1} = \frac{1}{j(2\times10^3)(1\times10^{-9})} = \text{-j}500\ \text{k}\Omega$$

Consider the circuit shown below.

Let $\mathbf{V}_s$ = 10V.



At node 1,

$[(\mathbf{V}_1–10)/(–j500k)] + [(\mathbf{V}_1–\mathbf{V}_o)/10^5] + [(\mathbf{V}_1–\mathbf{V}_2)/(–j500k)] = 0$

or $(1+j0.4)\mathbf{V}_1 – j0.2\mathbf{V}_2 – \mathbf{V}_o = j2$ (1)

At node 2,

$[(\mathbf{V}_2–\mathbf{V}_1)/(–j500k)] + [(\mathbf{V}_2–0)/100k] + 0 = 0$ or

$–j0.2\mathbf{V}_1 + (1+j0.2)\mathbf{V}_2 = 0$ or $\mathbf{V}_1 = [–(1+j0.2)/(–j0.2)]V_2$

$= (1–j5)\mathbf{V}_2$ (2)

At node b,

$$\mathbf{V_b} = \frac{R_3}{R_3+R_4}\mathbf{V}_o = \frac{\mathbf{V}_o}{3} = \mathbf{V_2} \qquad (3)$$

From (2) and (3),

$\mathbf{V}_1 = (0.3333–j1.6667)\mathbf{V}_o$ (4)

Substituting (3) and (4) into (1),

$(1+j0.4)(0.3333–j1.6667)\mathbf{V}_o – j0.06667\mathbf{V}_o – \mathbf{V}_o = j2$

$(1+j0.4)(0.3333–j1.6667) = (1.077\angle 21.8^\circ)(1.6997\angle –78.69^\circ)$

$= 1.8306\angle –56.89^\circ = 1–j1.5334$

$$(1-1+j(-1.5334-0.06667))\mathbf{V_o} = (-j1.6001)\mathbf{V_o} = 1.6001\angle{-90^\circ}$$

Therefore,

$$\mathbf{V}_o = 2\angle 90^\circ/(1.6001\angle{-90^\circ}) = 1.2499\angle 180^\circ$$

Since $\mathbf{V}_s = 10$,

$$\mathbf{V}_o/\mathbf{V}_s = \mathbf{0.12499\angle 180^\circ}.$$

Solution 10.76

Determine $\mathbf{V}_o$ and $\mathbf{I}_o$ in the op amp circuit of Fig. 10.119.

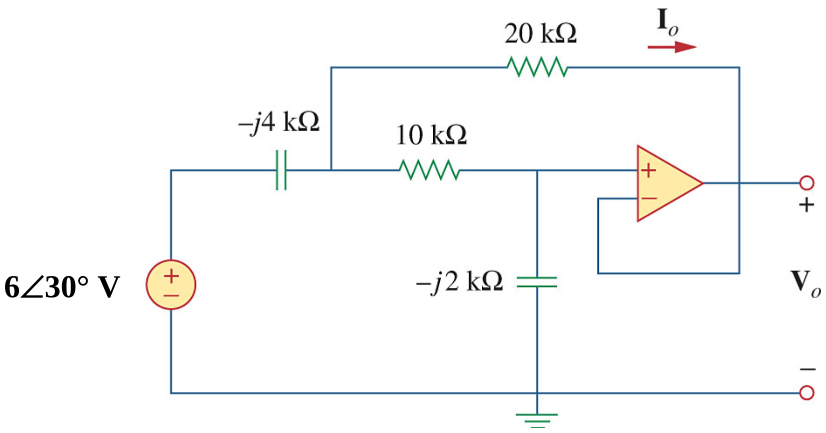


Figure 10.119
For Prob. 10.76.

Solution

Let the voltage between the –j4 kΩ capacitor and the 10 kΩ resistor be $\mathbf{V}_1$.

$$\frac{6\angle 30^o - V_1}{-j4k} = \frac{V_1 - V_o}{10k} + \frac{V_1 - V_o}{20k} \quad \longrightarrow \qquad (1)$$

$$6\angle 30^o = (1 - j0.6)V_1 + j0.6V_o$$
$$= 5.196+j3$$

Also,

$$\frac{V_1 - V_o}{10k} = \frac{V_o}{-j2k} \quad \longrightarrow \quad V_1 = (1 + j5)V_o \qquad (2)$$

Solving (2) into (1) yields

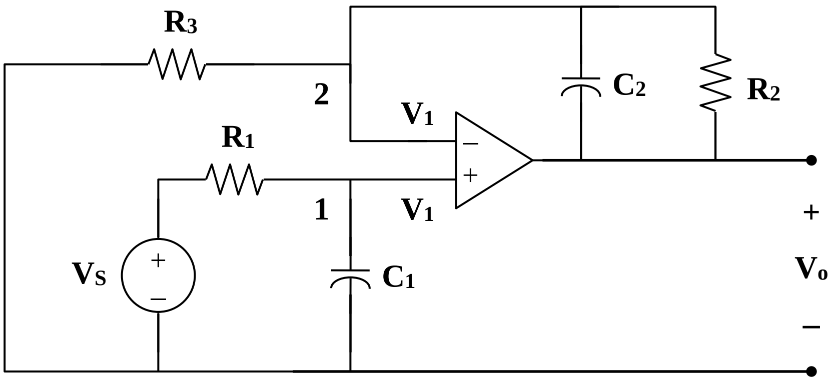
$$6\angle 30° = (1 - j0.6)(1 + j5)V_o + j0.6V_o = (1 + 3 - j0.6 + j5 + j6)V_o$$
$$= (4+j5)\mathbf{V_o}$$
$$V_o = \frac{6\angle 30°}{6.403\angle 51.34°} = \underline{0.9371\angle -21.34^o \text{ V}}$$

$$= \mathbf{937.1\angle{–}21.34^\circ\ mV}$$

$$\mathbf{I}_o = (\mathbf{V}_1–\mathbf{V}_o)/20k = \mathbf{V}_o/(–j4k) = (0.9371/4k)\angle(–21.43+90)^\circ$$
$$= \mathbf{234.3\angle 68.57^\circ\ \mu A}$$

Solution 10.77

Consider the circuit below.



At node 1,

$$\frac{\mathbf{V}_s - \mathbf{V}_1}{R_1} = j\omega C\,\mathbf{V}_1$$

$$\mathbf{V}_s = (1 + j\omega R_1 C_1)\,\mathbf{V}_1 \qquad (1)$$

At node 2,

$$\frac{0 - \mathbf{V}_1}{R_3} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{R_2} + j\omega C_2\,(\mathbf{V}_1 - \mathbf{V}_o)$$

$$\mathbf{V}_1 = (\mathbf{V}_o - \mathbf{V}_1)\left(\frac{R_3}{R_2} + j\omega C_2 R_3\right)$$

$$\mathbf{V}_o = \left(1 + \frac{1}{(R_3/R_2) + j\omega C_2 R_3}\right)\mathbf{V}_1 \qquad (2)$$

From (1) and (2),

$$\mathbf{V}_o = \frac{\mathbf{V}_s}{1 + j\omega R_1 C_1}\left(1 + \frac{R_2}{R_3 + j\omega C_2 R_2 R_3}\right)$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{\mathbf{R_2 + R_3 + j\omega C_2 R_2 R_3}}{\mathbf{(1 + j\omega R_1 C_1)(R_3 + j\omega C_2 R_2 R_3)}}$$

Solution 10.78

Determine $v_o(t)$ in the op amp circuit in Fig. 10.121 below.

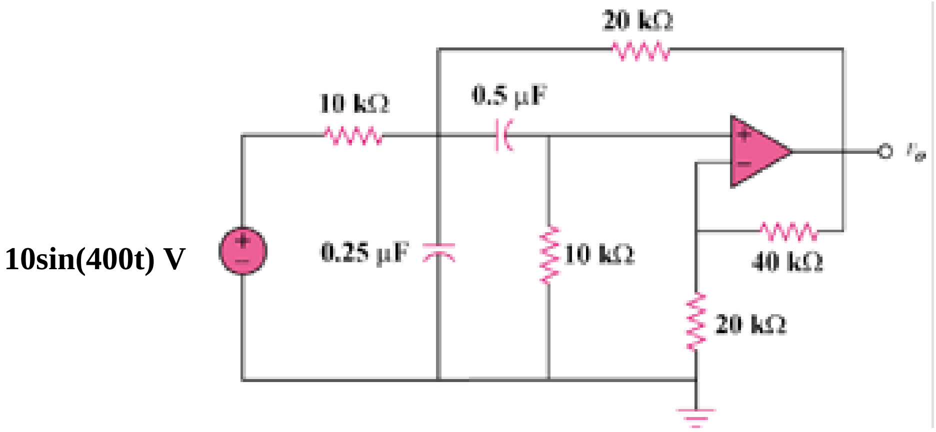


Figure 10.121
For Prob. 10.78.

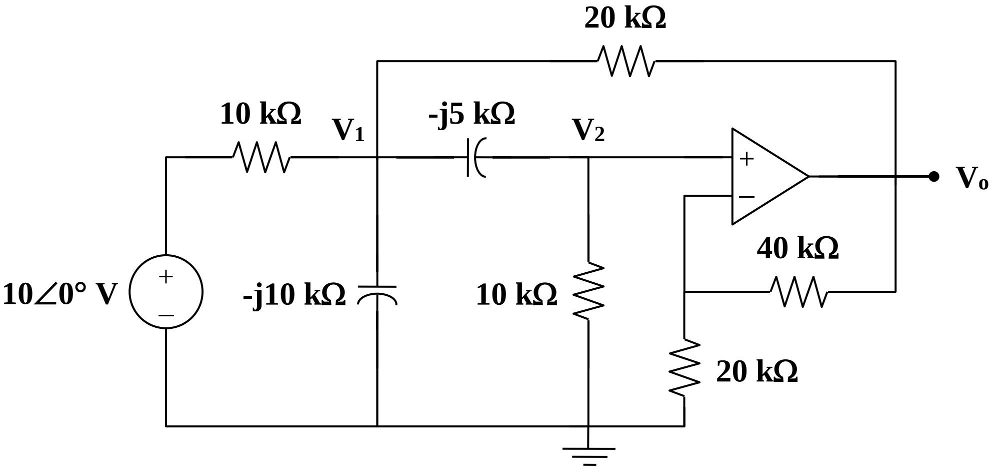
Solution

$$10\sin(400t) \longrightarrow 10\angle 0°, \quad \omega = 400$$

$$0.5\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.5\times 10^{-6})} = \text{-j}5\ \text{k}\Omega$$

$$0.25\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(400)(0.25\times 10^{-6})} = \text{-j}10\ \text{k}\Omega$$

Consider the circuit as shown below.



At node 1,

$$\frac{10-\mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1-\mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1-\mathbf{V}_o}{20}$$

$$20 = (3+j6)\mathbf{V}_1 - j4\mathbf{V}_2 - \mathbf{V}_o \quad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{-\mathrm{j}5} = \frac{\mathbf{V}_2}{10}$$

$$\mathbf{V}_1 = (1-\mathrm{j}0.5)\mathbf{V}_2 \quad (2)$$

But

$$\mathbf{V}_2 = \frac{20}{20+40}\mathbf{V}_\mathrm{o} = \frac{1}{3}\mathbf{V}_\mathrm{o} \quad (3)$$

From (2) and (3),

$$\mathbf{V}_1 = \frac{1}{3}\cdot(1-\mathrm{j}0.5)\mathbf{V}_\mathrm{o} \quad (4)$$

Substituting (3) and (4) into (1) gives

$$20 = (3+j6)\cdot\frac{1}{3}\cdot(1-j0.5)\mathbf{V}_o - j\frac{4}{3}\mathbf{V}_o - \mathbf{V}_o = \left(1+j\frac{1}{6}\right)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{120}{6+j} = \frac{120}{6.08276\angle 9.4623^\circ} = 19.728\angle -9.46^\circ$$

Therefore,

$$\mathrm{v}_\mathrm{o}(\mathrm{t}) = \mathbf{19.728sin(400t–9.46°)\ V}$$

Solution 10.79

For the op amp circuit in Fig. 10.122, obtain $\mathbf{V_o}$.

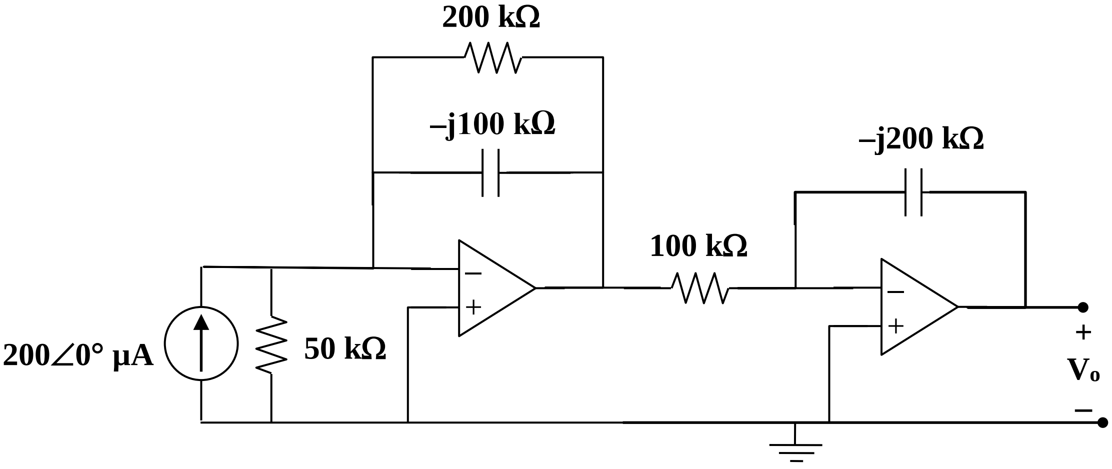
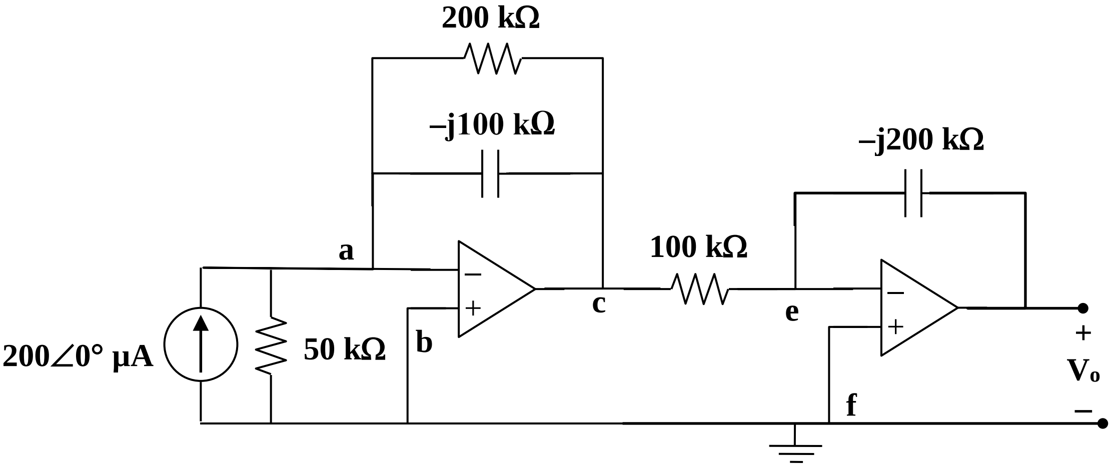


Figure 10.122
For Prob. 10.79.

Solution

First we label all the unknown nodes in the circuit.



At node a we have, –200μ + [($\mathbf{V_a}$–0)/50k] + [($\mathbf{V_a}$–$\mathbf{V_c}$)/200k] + [($\mathbf{V_a}$–$\mathbf{V_c}$)/(–j100k)] = 0. At b we have $\mathbf{V_b}$ = 0. At e we have [($\mathbf{V_e}$–$\mathbf{V_c}$)/100k] + [($\mathbf{V_e}$–$\mathbf{V_o}$)/(–j200k)] = 0. At f we have $\mathbf{V_f}$ = 0. Now we need to use the constraint equations, $\mathbf{V_a}$ = $\mathbf{V_b}$ = 0 and $\mathbf{V_e}$ = $\mathbf{V_f}$ = 0.

This leads to the following,

$\{[1/200\text{k}] + [1/(–j100\text{k})]\}\mathbf{V_c} = –200\mu$ or $(0.5+j)\mathbf{V_c} = –20$ or
$\mathbf{V_c} = –20/(1.118034\angle 63.435°) = –17.88854\angle –63.435°$.

Now for the second op amp, $[(–\mathbf{V_c})/100\text{k}] + [(–\mathbf{V_o})/(–j200\text{k})] = 0$ or
$\mathbf{V_o}/(–j2) = –\mathbf{V_c}$ or $\mathbf{V_o} = –(–j2)(–17.88854\angle –63.435°)$
$= 35.777\angle(90°–180°–63.44°) = 35.78\angle –153.44°$ or

$$\mathbf{V_o} = \mathbf{35.78\angle –153.44°\ V}.$$

Solution 10.80

Obtain $v_o(t)$ for the op amp circuit in Fig. 10.123 if v_s = 12cos (1000t – 60°) V.

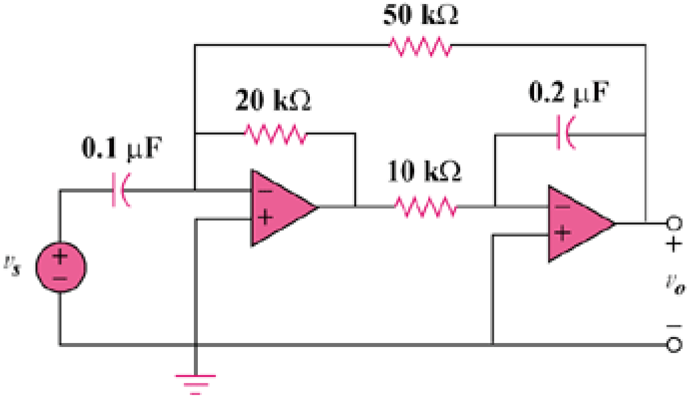


Figure 10.123
For Prob. 10.80.

Solution

$$12\cos(1000t - 60°) \longrightarrow 12\angle -60°, \quad \omega = 1000$$

$$0.1\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.1\times 10^{-6})} = -j10\ \text{k}\Omega$$

$$0.2\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(1000)(0.2\times 10^{-6})} = -j5\ \text{k}\Omega$$

Let the input to the inverting terminal of the first op amp be $\mathbf{V_a}$, the output of the first op amp be $\mathbf{V_1}$, and the input to the inverting terminal of the second op amp be $\mathbf{V_b}$. This then gives us the following node equations,

[($\mathbf{V_a}$–$\mathbf{V_s}$)/(–j10k)] + [($\mathbf{V_a}$–$\mathbf{V_o}$)/50k] + [($\mathbf{V_a}$–$\mathbf{V_1}$)/20k] + 0 = 0 where $\mathbf{V_a}$ = 0 or
$\mathbf{V_1}$ = 20k{[–$\mathbf{V_s}$/(–j10k)] + [–$\mathbf{V_o}$/50k]} = –2j$\mathbf{V_s}$ – 0.4$\mathbf{V_o}$.

[($\mathbf{V_b}$–$\mathbf{V_1}$)/10k] + [($\mathbf{V_b}$–$\mathbf{V_o}$)/(–j5k)] + 0 = 0 where $\mathbf{V_b}$ = 0 or
$\mathbf{V_o}$ = –j5k[–$\mathbf{V_1}$/10k] = j0.5$\mathbf{V_1}$ = j0.5[–j2$\mathbf{V_s}$ – 0.4$\mathbf{V_o}$] = $\mathbf{V_s}$ – j0.2$\mathbf{V_o}$ or
(1+j0.2)$\mathbf{V_o}$ = $\mathbf{V_s}$ or $\mathbf{V_o}$ = $\mathbf{V_s}$/(1.0198∠11.31°) = (0.9806∠–11.31°)$\mathbf{V_s}$.

Since $\mathbf{V_s}$ = 12∠–60° V, which leads to, $\mathbf{V_o}$ = 11.767∠–71.31° V or

$v_o(t)$ = **11.767cos(1,000t–71.31°)** V.

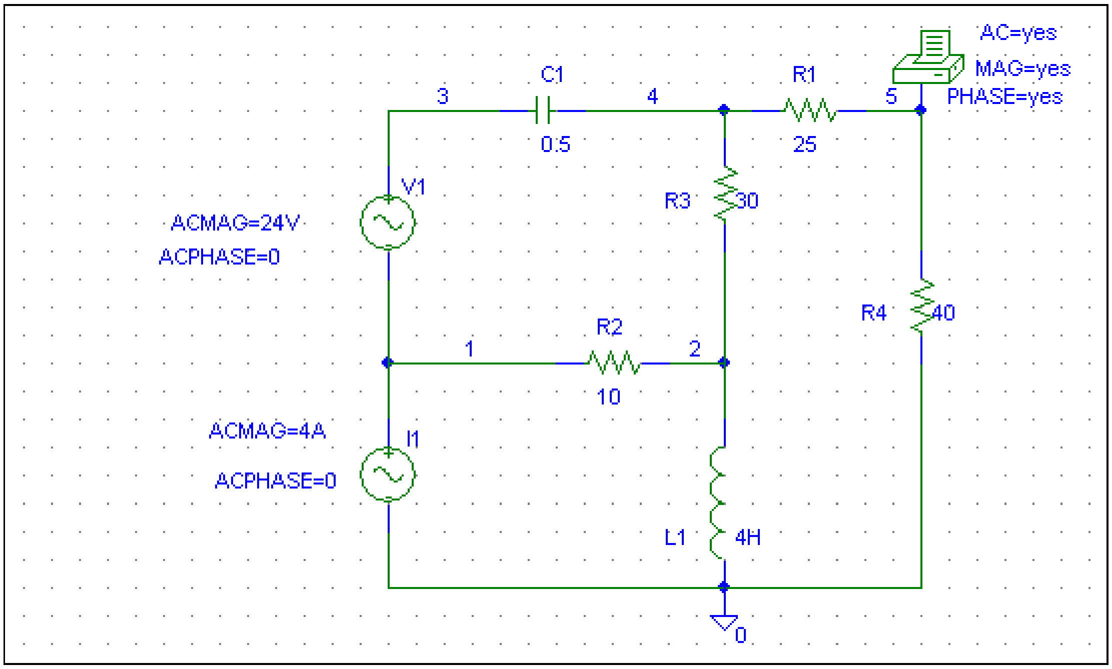
Solution 10.81

We need to get the capacitance and inductance corresponding to –j2 Ω and j4 Ω.

$$-j2 \quad \longrightarrow \quad C = \frac{1}{\omega X_c} = \frac{1}{1x2} = 0.5F$$

$$j4 \quad \longrightarrow \quad L = \frac{X_L}{\omega} = 4H$$

The schematic is shown below.



When the circuit is simulated, we obtain the following from the output file.

```
FREQ        VM(5)       VP(5)
1.592E-01  1.127E+01  -1.281E+02
```

From this, we obtain

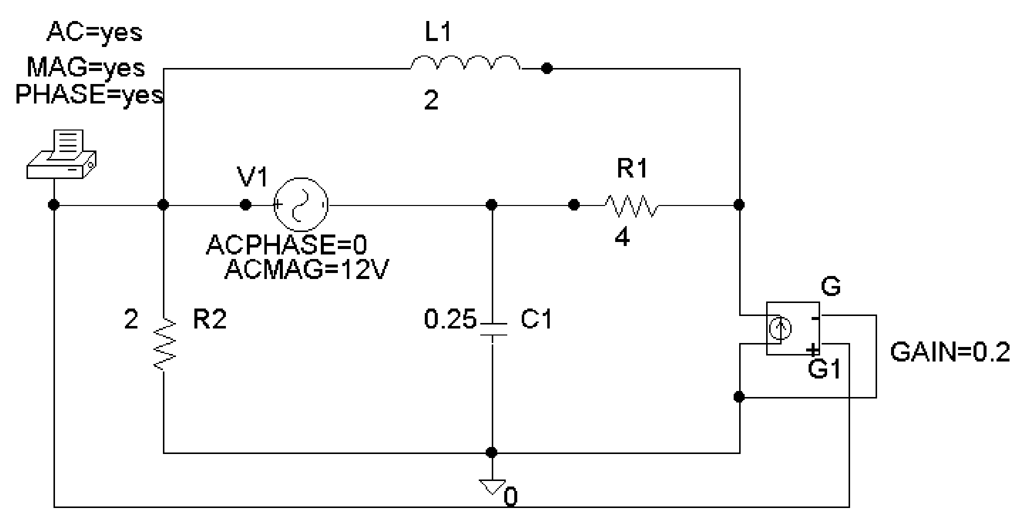
V_o = **$11.27\angle 128.1^o$ V**.

Solution 10.82

The schematic is shown below. We insert PRINT to print V_o in the output file. For AC Sweep, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we print out the output file which includes:

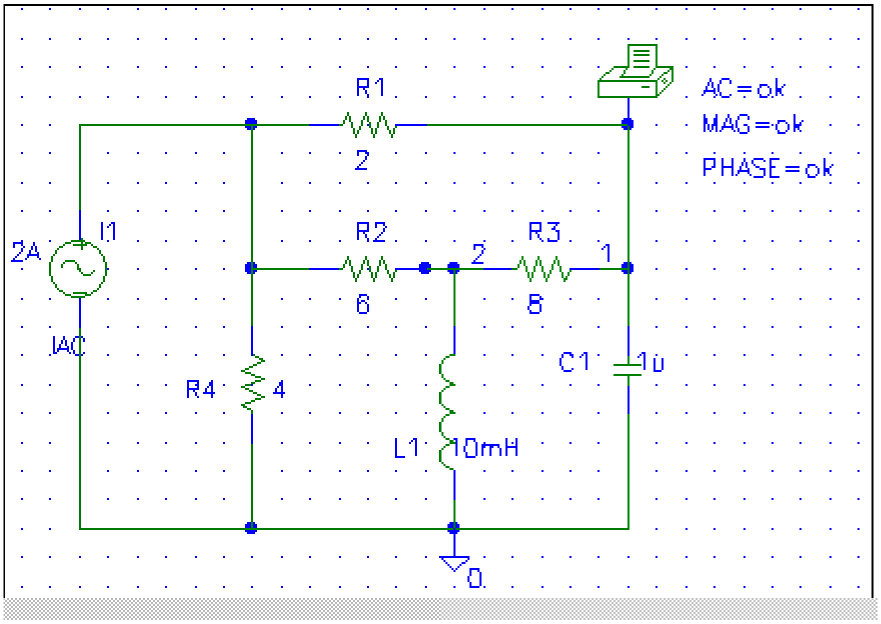
```
FREQ              VM($N_0001)            VP($N_0001)
1.592 E-01        7.684 E+00             5.019 E+01
```

which means that $V_o = \mathbf{7.684\angle 50.19^o\ V}$



Solution 10.83

The schematic is shown below. The frequency is $f = \omega / 2\pi = \dfrac{1000}{2\pi} = 159.15$



When the circuit is saved and simulated, we obtain from the output file

```
FREQ          VM(1)              VP(1)
1.592E+02     6.611E+00          -1.592E+02
```

Thus,

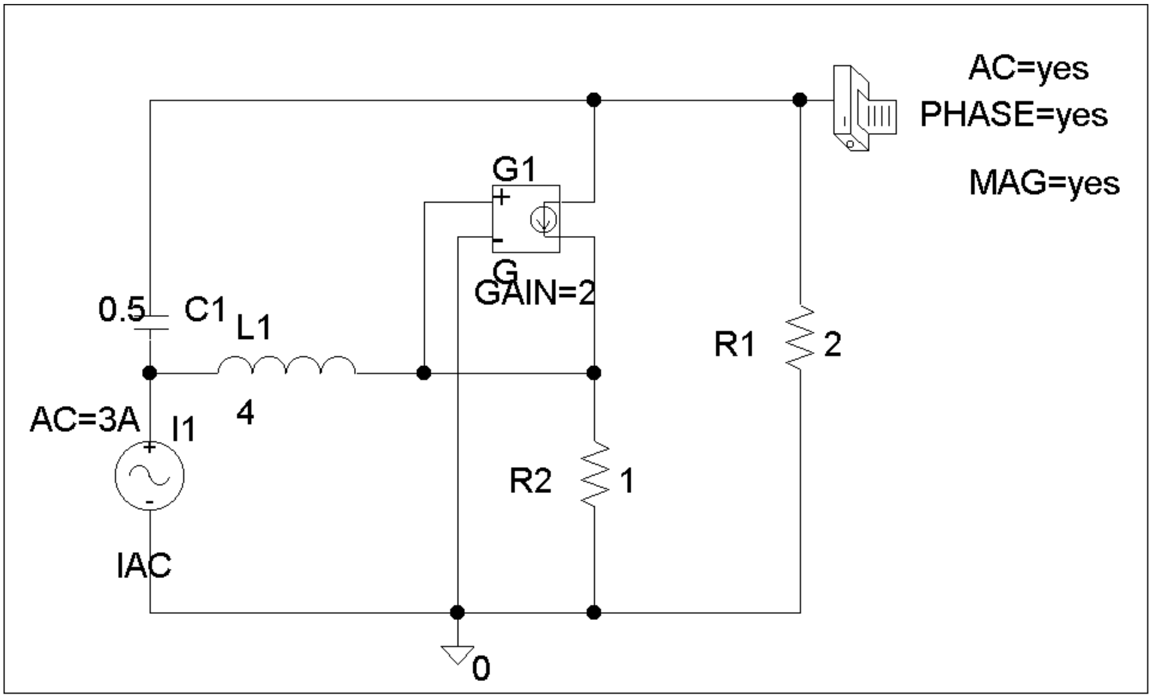
$$v_o = \mathbf{6.611cos(1000t - 159.2^o)\ \ V}$$

Solution 10.84

The schematic is shown below. We set PRINT to print V_o in the output file. In AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes:

```
                         FREQ               VM($N_0003)
      VP($N_0003)
                         1.592 E-01         1.664 E+00          –1.646 E+02
```

Namely, V_o = **1.664∠–146.4º V**



Solution 10.85

Using Fig. 10.127, design a problem to help other students to better understand performing AC analysis with *PSpice*.

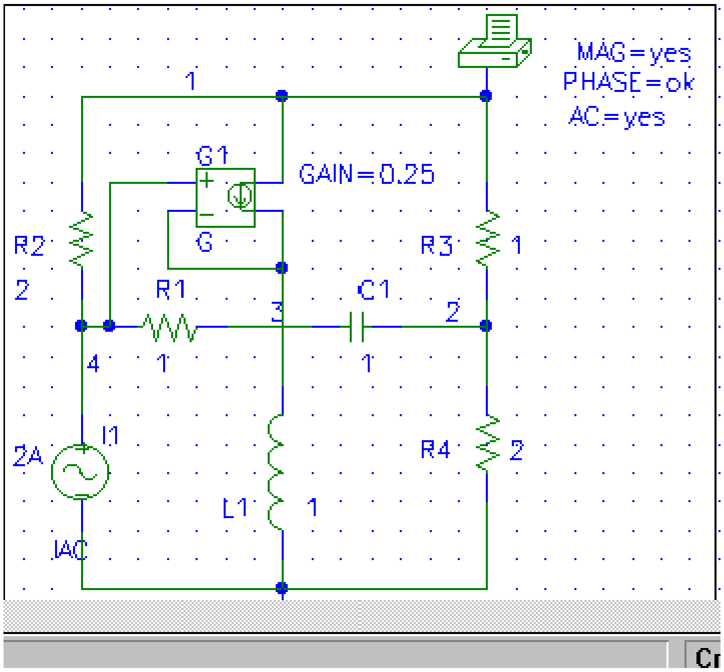
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use *PSpice* to find V_o in the circuit of Fig. 10.127. Let $R_1 = 2\ \Omega$, $R_2 = 1\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 2\ \Omega$, $I_s = 2\angle 0^\circ$ A, $X_L = 1\ \Omega$, and $X_C = 1\ \Omega$.

Solution

The schematic is shown below. We let $\omega = 1$ rad/s so that L=1H and C=1F.



When the circuit is saved and simulated, we obtain from the output file

```
FREQ            VM(1)       VP(1)
1.591E-01       2.228E+00   -1.675E+02
```

From this, we conclude that

$$V_o = \mathbf{2.228\angle{-167.5}^\circ\ V}.$$

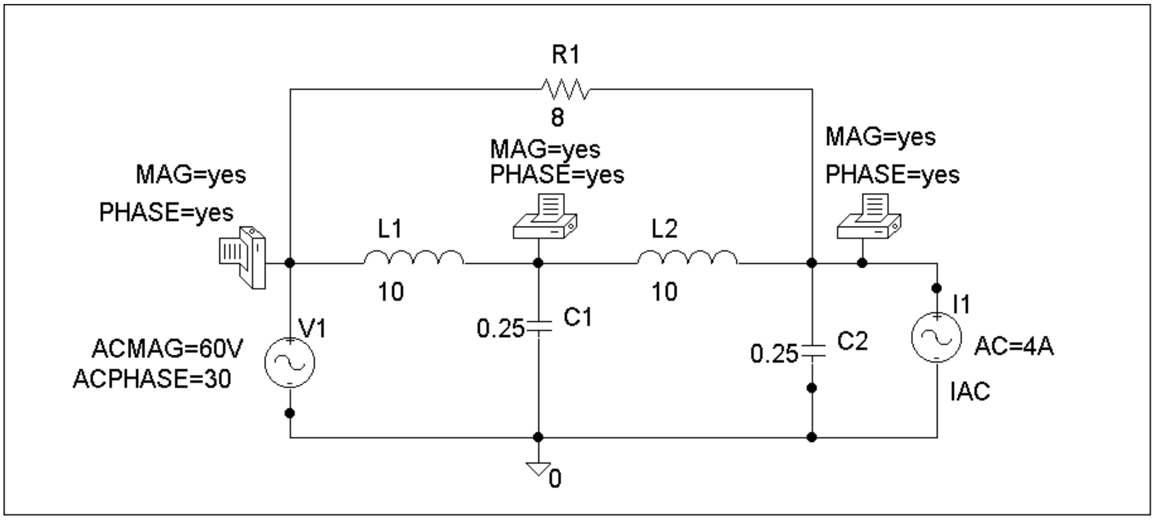
Solution 10.86

The schematic is shown below. We insert three pseudo-component PRINTs at nodes 1, 2, and 3 to print V_1, V_2, and V_3, into the output file. Assume that w = 1, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After saving and simulating the circuit, we obtain the output file which includes:

```
                         FREQ              VM($N_0002)
      VP($N_0002)
                         1.592 E-01        6.000 E+01           3.000
E+01

                         FREQ              VM($N_0003)
      VP($N_0003)
                         1.592 E-01        2.367 E+02           -8.483
E+01


                         FREQ              VM($N_0001)
      VP($N_0001)
                         1.592 E-01        1.082 E+02           1.254
E+02
```

Therefore,

$$V_1 = \mathbf{60\angle 30^{o}\ V} \quad V_2 = \mathbf{236.7\angle{-84.83^{o}}\ V} \quad V_3 = \mathbf{108.2\angle 125.4^{o}\ V}$$



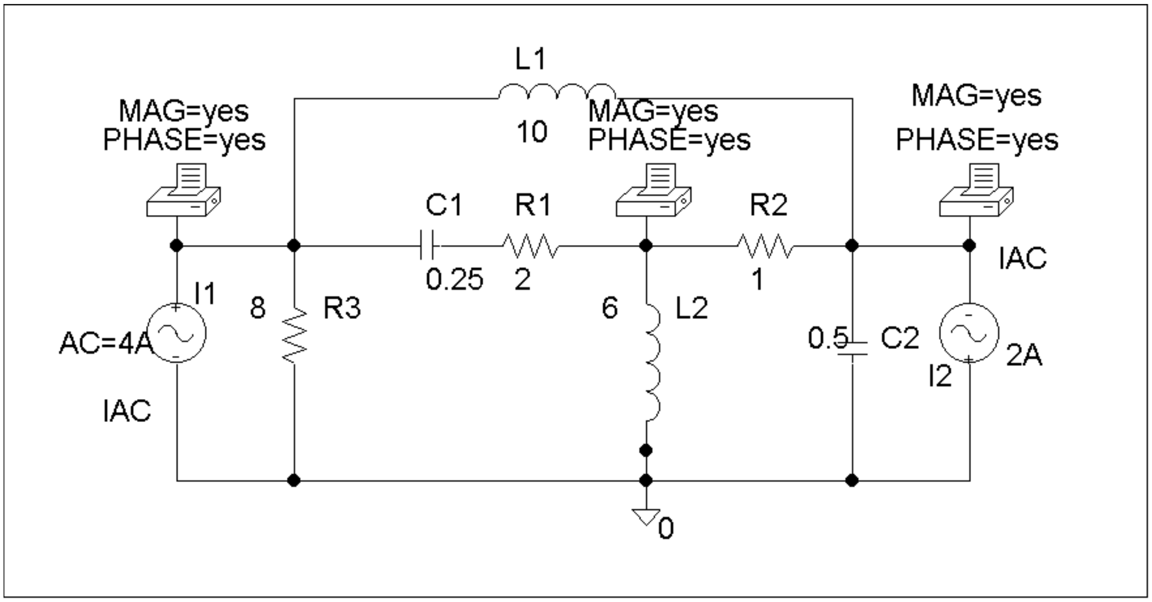
Solution 10.87

The schematic is shown below. We insert three PRINTs at nodes 1, 2, and 3. We set Total Pts = 1, Start Freq = 0.1592, End Freq = 0.1592 in the AC Sweep box. After simulation, the output file includes:

```
                    FREQ            VM($N_0004)
     VP($N_0004)
                    1.592 E-01      1.591 E+01            1.696
E+02

                    FREQ            VM($N_0001)
     VP($N_0001)
                    1.592 E-01      5.172 E+00            -1.386
E+02


                    FREQ            VM($N_0003)
     VP($N_0003)
                    1.592 E-01      2.270 E+00            -1.524
E+02
```

Therefore,

$$V_1 = \mathbf{15.91\angle 169.6^o\ V} \quad V_2 = \mathbf{5.172\angle -138.6^o\ V} \quad V_3 = \mathbf{2.27\angle -152.4^o\ V}$$



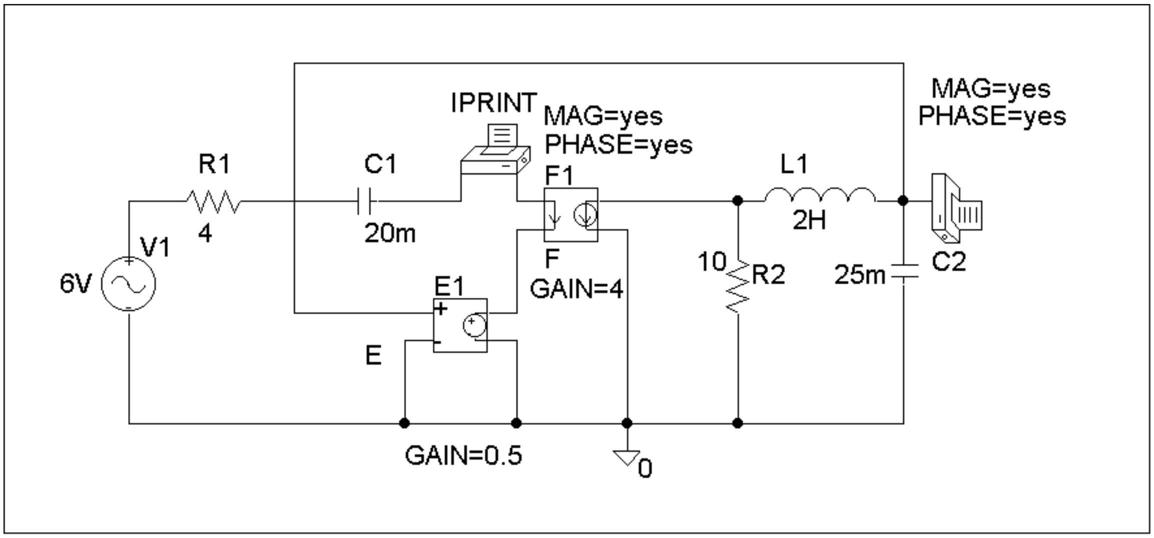
Solution 10.88

The schematic is shown below. We insert IPRINT and PRINT to print I_o and V_o in the output file. Since w = 4, f = w/2π = 0.6366, we set Total Pts = 1, Start Freq = 0.6366, and End Freq = 0.6366 in the AC Sweep box. After simulation, the output file includes:

```
                                FREQ                VM($N_0002)
        VP($N_0002)
                                6.366 E-01          3.496 E+01                1.261
E+01


                                FREQ                IM(V_PRINT2)              IP
(V_PRINT2)
                                6.366 E-01          8.912 E-01
        -8.870 E+01
```

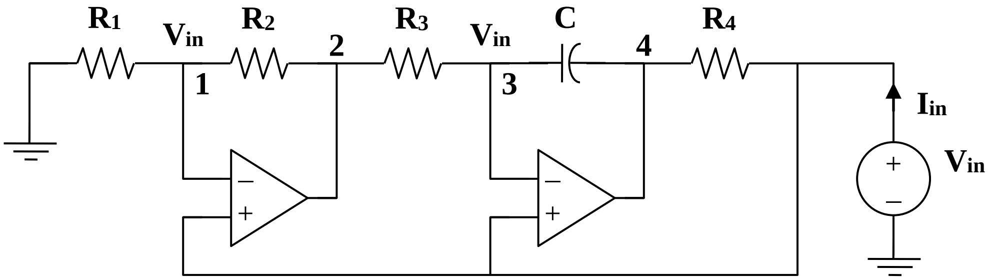
Therefore, $V_o = 34.96\angle 12.6^o$ V, $I_o = 0.8912\angle -88.7^o$ A

v_o = **$34.96 \cos(4t + 12.6^o)$V**, i_o = **$0.8912\cos(4t – 88.7^o)$A**



Solution 10.89

Consider the circuit below.



At node 1,

$$\frac{0-\mathbf{V}_{in}}{R_1}=\frac{\mathbf{V}_{in}-\mathbf{V}_2}{R_2}$$

$$-\mathbf{V}_{in}+\mathbf{V}_2=\frac{R_2}{R_1}\mathbf{V}_{in} \qquad (1)$$

At node 3,

$$\frac{\mathbf{V}_2-\mathbf{V}_{in}}{R_3}=\frac{\mathbf{V}_{in}-\mathbf{V}_4}{1/j\omega C}$$

$$-\mathbf{V}_{in}+\mathbf{V}_4=\frac{\mathbf{V}_{in}-\mathbf{V}_2}{j\omega CR_3} \qquad (2)$$

From (1) and (2),

$$-\mathbf{V}_{in}+\mathbf{V}_4=\frac{-R_2}{j\omega CR_3R_1}\mathbf{V}_{in}$$

Thus,

$$\mathbf{I}_{in}=\frac{\mathbf{V}_{in}-\mathbf{V}_4}{R_4}=\frac{R_2}{j\omega CR_3R_1R_4}\mathbf{V}_{in}$$

$$\mathbf{Z}_{in}=\frac{\mathbf{V}_{in}}{\mathbf{I}_{in}}=\frac{j\omega CR_1R_3R_4}{R_2}=j\omega L_{eq}$$

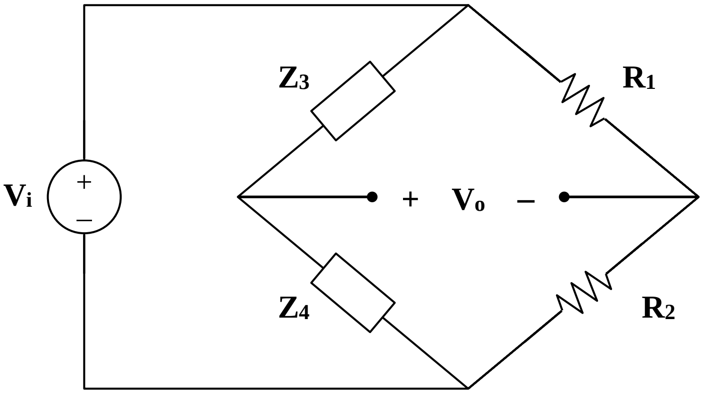
where $$\mathbf{L_{eq}=\frac{R_1R_3R_4C}{R_2}}$$

Solution 10.90

Let $$\mathbf{Z}_4 = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\mathbf{Z}_3 = R + \frac{1}{j\omega C} = \frac{1 + j\omega RC}{j\omega C}$$

Consider the circuit shown below.



$$\mathbf{V}_o = \frac{\mathbf{Z}_4}{\mathbf{Z}_3 + \mathbf{Z}_4}\mathbf{V}_i - \frac{R_2}{R_1 + R_2}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\dfrac{R}{1 + j\omega C}}{\dfrac{R}{1 + j\omega C} + \dfrac{1 + j\omega RC}{j\omega C}} - \frac{R_2}{R_1 + R_2}$$

$$= \frac{j\omega RC}{j\omega RC + (1 + j\omega RC)^2} - \frac{R_2}{R_1 + R_2}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega RC}{1 - \omega^2 R^2 C^2 + j3\omega RC} - \frac{R_2}{R_1 + R_2}$$

For $\mathbf{V}_o$ and $\mathbf{V}_i$ to be in phase, $\dfrac{\mathbf{V}_o}{\mathbf{V}_i}$ must be purely real. This happens when

$$1 - \omega^2 R^2 C^2 = 0$$

$$\omega = \frac{1}{RC} = 2\pi f$$

or $$f = \frac{1}{2\pi RC}$$

At this frequency,

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{3} - \frac{R_2}{R_1 + R_2}$$

Solution 10.91

(a) Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$\mathbf{Z}_p = 10\ \text{k}\Omega = \text{R}_o$

$$\mathbf{Z}_s = R + j\omega L + \frac{1}{j\omega C}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{R_o}{R + R_o + j\omega L - \dfrac{j}{\omega C}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega C R_o}{\omega C (R + R_o) + j(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC - 1 = 0 \quad \longrightarrow \quad \omega_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.4\times10^{-3})(2\times10^{-9})}}$$

$$f_o = \mathbf{180\ kHz}$$

(b) At oscillation,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o C R_o}{\omega_o C (R + R_o)} = \frac{R_o}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{80}{20} = 5$$

$$\frac{R_o}{R + R_o} = \frac{1}{5} \quad \longrightarrow \quad R = 4R_o = \mathbf{40\ k\Omega}$$

Solution 10.92

Let $\mathbf{V}_2 =$ voltage at the noninverting terminal of the op amp

$\mathbf{V}_o =$ output voltage of the op amp

$$\mathbf{Z}_s = R_o$$

$$\mathbf{Z}_p = j\omega L \parallel \frac{1}{j\omega C} \parallel R = \frac{1}{\dfrac{1}{R} + j\omega C + \dfrac{1}{j\omega L}} = \frac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}$$

As in Section 10.9,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{Z}_p}{\mathbf{Z}_s + \mathbf{Z}_p} = \frac{\dfrac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}{R_o + \dfrac{\omega RL}{\omega L + jR(\omega^2 LC - 1)}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega RL}{\omega RL + \omega R_o L + jR_o R\,(\omega^2 LC - 1)}$$

For this to be purely real,

$$\omega_o^2 LC = 1 \longrightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$$

(a) At $\omega = \omega_o$,

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\omega_o RL}{\omega_o RL + \omega_o R_o L} = \frac{R}{R + R_o}$$

This must be compensated for by

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_2} = 1 + \frac{R_f}{R_o} = 1 + \frac{1000k}{100k} = 11$$

Hence,

$$\frac{R}{R + R_o} = \frac{1}{11} \longrightarrow R_o = 10R = \mathbf{100\ k\Omega}$$

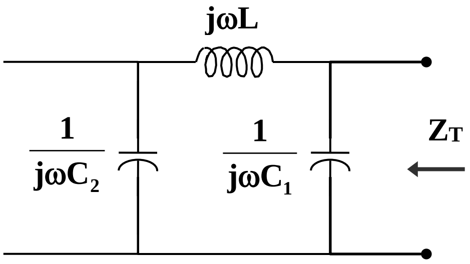
(b)

$$f_o = \frac{1}{2\pi\sqrt{(10 \times 10^{-6})(2 \times 10^{-9})}}$$

$$f_o = \mathbf{1.125\ MHz}$$

Solution 10.93

As shown below, the impedance of the feedback is



$$\mathbf{Z}_T = \frac{1}{j\omega C_1} \,||\, \left(j\omega L + \frac{1}{j\omega C_2} \right)$$

$$\mathbf{Z}_T = \frac{\frac{-j}{\omega C_1}\left(j\omega L + \frac{-j}{\omega C_2} \right)}{\frac{-j}{\omega C_1} + j\omega L + \frac{-j}{\omega C_2}} = \frac{\frac{1}{\omega} - \omega L C_2}{j(C_1 + C_2 - \omega^2 L C_1 C_2)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

$$C_1 + C_2 - \omega_o^2 L C_1 C_2 = 0$$

$$\omega_o^2 = \frac{C_1 + C_2}{L C_1 C_2} = \frac{1}{L C_T}$$

$$\mathbf{f_o = \frac{1}{2\pi\sqrt{LC_T}}}$$

Solution 10.94

If we select $C_1 = C_2 = 20 \text{ nF}$

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{C_1}{2} = 10 \text{ nF}$$

Since $f_o = \dfrac{1}{2\pi\sqrt{LC_T}}$,

$$L = \frac{1}{(2\pi f)^2 C_T} = \frac{1}{(4\pi^2)(2500 \times 10^6)(10 \times 10^{-9})} = 10.13 \text{ mH}$$

$$X_c = \frac{1}{\omega C_2} = \frac{1}{(2\pi)(50 \times 10^3)(20 \times 10^{-9})} = 159\ \Omega$$

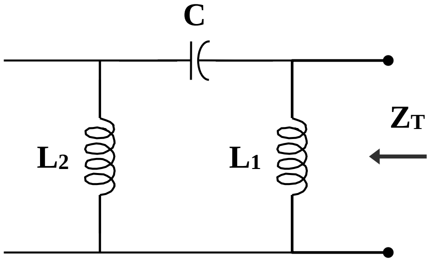
We may select $R_i = 20 \text{ k}\Omega$ and $R_f \geq R_i$, say $R_f = 20 \text{ k}\Omega$.

Thus,

$C_1 = C_2 =$ **20 nF**, $L =$ **10.13 mH** $R_f = R_i =$ **20 kΩ**

Solution 10.95

First, we find the feedback impedance.



$$\mathbf{Z}_T = j\omega L_1 \,||\left(j\omega L_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_T = \frac{j\omega L_1\left(j\omega L_2 - \dfrac{j}{\omega C}\right)}{j\omega L_1 + j\omega L_2 - \dfrac{j}{\omega C}} = \frac{\omega^2 L_1 C(1-\omega L_2)}{j(\omega^2 C(L_1+L_2)-1)}$$

In order for $\mathbf{Z}_T$ to be real, the imaginary term must be zero; i.e.

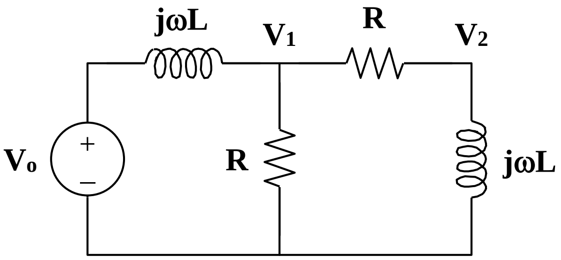
$$\omega_o^2 C(L_1+L_2)-1=0$$

$$\omega_o = 2\pi f_o = \frac{1}{C(L_1+L_2)}$$

$$\boldsymbol{f_o = \frac{1}{2\pi\sqrt{C(L_1+L_2)}}}$$

Solution 10.96

(a) Consider the feedback portion of the circuit, as shown below.



$$\mathbf{V}_2 = \frac{j\omega L}{R + j\omega L}\mathbf{V}_1 \quad \longrightarrow \quad \mathbf{V}_1 = \frac{R + j\omega L}{j\omega L}\mathbf{V}_2 \qquad (1)$$

Applying KCL at node 1,

$$\frac{\mathbf{V}_o - \mathbf{V}_1}{j\omega L} = \frac{\mathbf{V}_1}{R} + \frac{\mathbf{V}_1}{R + j\omega L}$$

$$\mathbf{V}_o - \mathbf{V}_1 = j\omega L\, \mathbf{V}_1\left(\frac{1}{R} + \frac{1}{R + j\omega L}\right)$$

$$\mathbf{V}_o = \mathbf{V}_1\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)$$

(2)

From (1) and (2),

$$\mathbf{V}_o = \left(\frac{R + j\omega L}{j\omega L}\right)\left(1 + \frac{j2\omega RL - \omega^2 L^2}{R\,(R + j\omega L)}\right)\mathbf{V}_2$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{R^2 + j\omega RL + j2\omega RL - \omega^2 L^2}{j\omega RL}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3 + \dfrac{R^2 - \omega^2 L^2}{j\omega RL}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{\mathbf{1}}{\mathbf{3 + j(\omega L/R - R/\omega L)}}$$

(b) Since the ratio $\frac{\mathbf{V}_2}{\mathbf{V}_o}$ must be real,

$$\frac{\omega_o L}{R} - \frac{R}{\omega_o L} = 0$$

$$\omega_o L = \frac{R^2}{\omega_o L}$$

$$\omega_o = 2\pi f_o = \frac{R}{L}$$

$$\mathbf{f_o = \frac{R}{2\pi L}}$$

(c) When $\omega = \omega_o$

$$\frac{\mathbf{V}_2}{\mathbf{V}_o} = \frac{1}{3}$$

This must be compensated for by $\mathbf{A}_v = 3$. But

$$\mathbf{A}_v = 1 + \frac{R_2}{R_1} = 3$$

$$\mathbf{R_2 = 2R_1}$$