常微与偏微课程作业

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题目1.

$$2t\sin y + y^3 e^t + (t^2\cos y + 3y^2 e^t)\frac{dy}{dt} = 0$$

解答.

$$M(t,y) = 2t\sin y + y^3 e^t$$

$$N(t,y) = t^2\cos y + 3y^2 e^t$$

$$\frac{\partial M}{\partial y} = 2t\cos y + 3y^2 e^t$$

$$\frac{\partial N}{\partial t} = 2t\cos y + 3y^2 e^t$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\phi(t,y) = \int M(t,y)dt + h(y)$$

$$\phi(t,y) = t^2\sin y + y^3 e^t + h(y)$$

$$h'(y) + 3y^2 e^t + t^2\cos y = t^2\cos y + 3y^2 e^t$$

$$h'(y) = 0$$

$$h(y) = C$$

$$\phi(t,y) = \int N(t,y)dy + k(t)$$

$$\phi(t,y) = t^2\sin y + y^3 e^t + k(t)$$

$$k'(t) + 2t\sin y + y^3 e^t = 2t\sin y + y^3 e^t$$

$$k'(t) = 0$$

$$k(t) = C$$

$$\phi(t,y) = t^2 \sin y + y^3 e^t + C$$

题目2.

$$1 + (1+ty)e^{ty} + (1+t^2e^{ty})\frac{dy}{dt} = 0$$

解答.

$$M(t,y) = 1 + (1+ty)e^{ty}$$

$$N(t,y) = 1 + t^2e^{ty}$$

$$\frac{\partial M}{\partial y} = e^{ty}(2t + t^2y)$$

$$\frac{\partial N}{\partial t} = e^{ty}(2t + t^2y)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\phi(t,y) = \int M(t,y)dt + h(y)$$

$$\phi(t,y) = t + te^t y + h(y)$$

$$h'(y) + t^2e^{ty} = 1 + t^2e^t y$$

$$h'(y) = 1$$

$$h(y) = y$$

$$\phi(t,y) = \int N(t,y)dy + k(t)$$

$$\phi(t,y) = y + te^{ty} + k(t)$$

$$k'(t) + e^{ty} + yte^{ty} = 1 + (1+ty)e^{ty}$$

$$k'(t) = 1$$

$$k(t) = t$$

$$\phi(t,y) = t + y + te^{ty}$$

题目3.

$$y\sec^2 t + \sec t \tan t + (2y + \tan t)\frac{dy}{dt} = 0$$

解答.

$$M(t,y) = y\sec^2 t + \sec t \tan t$$

$$N(t,y) = 2y + \tan t$$

$$\frac{\partial M}{\partial y} = \sec^2 t$$

$$\frac{\partial N}{\partial t} = \sec^2 t$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\phi(t,y) = \int M(t,y)dt + h(y)$$

$$\phi(t,y) = y\tan t + \sec t + h(y)$$

$$h'(y) + \tan t = 2y + \tan t$$

$$h'(y) = 2y$$

$$h(y) = y^2$$

$$\phi(t,y) = \int N(t,y)dy + k(t)$$

$$\phi(t,y) = y^2 + k(t)$$

$$k'(t) = y\sec^2 t + \sec t \tan t$$

$$k(t) = y\tan t + \sec(t)$$

$$\phi(t,y) = y^2 + y\tan t + \sec t$$

题目4.

$$\frac{y^2}{2} - 2ye^t + (y - e^t)\frac{dy}{dt} = 0$$

解答.

$$M(t,y)=\frac{y^2}{2}$$

$$N(t,y)=y-e^t$$

$$\frac{\partial M}{\partial y}=y-2e^t$$

$$\frac{\partial N}{\partial t}=-e^t$$

$$\frac{1}{N}(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial y})=\frac{y-e^t}{y-e^t}=1$$

$$\mu(t)=e^{\int dt}=e^t$$

$$\frac{y^2e^t}{2}-2ye^2t+(ye^t-e^2t)\frac{dy}{dt}=0$$

$$\frac{\partial\phi}{\partial y}=\frac{y^2e^t}{2}=m(t,y)$$

$$\frac{\partial\phi}{\partial t}=ye^t-e^2t=n(t,y)$$

$$\phi(t,y)=\int m(t,y)dt+h(y)$$

$$h'(y)+ye^t-e^2t=ye^t-e^2t$$

$$h'(y)=0$$

$$h(y)=C$$

$$\phi(t,y)=\int n(t,y)dy+k(t)$$

$$k'(t)-2ye^2t+\frac{y^2}{2}=\frac{y^2}{2}-2ye^2t$$

$$k'(t)=0$$

$$k(t)=C$$

$$\phi(t,y)=\frac{y^2e^t}{2}-ye^2t+C$$

题目5.

$$2ty^3+3t^2y^2\frac{dy}{dt}=0,\quad y(1)=1$$

解答.

$$M(t,y)=2ty^3$$

$$N(t,y)=3t^2y^2$$

$$\frac{\partial M}{\partial y}=6ty^2$$

$$\frac{\partial N}{\partial t}=6ty^2$$

$$\phi(t,y)=\int M(t,y)dt+h(y)$$

$$h'(y)+3t^2y^2=3t^2y^2$$

$$h'(y)=0$$

$$h(y)=C$$

$$\phi(t,y)=\int N(t,y)dy+k(t)$$

$$k'(t)+2ty^3=2ty^3$$

$$k'(t)=0$$

$$k(t)=C$$

$$y(1)=1$$

$$\phi(t,y)=t^2y^3$$

$$\phi(t,y)=1$$

题目6.

$$2t\cos y+3t^2y+(t^3-t^2\sin y-y)\frac{dy}{dt}=0,\quad y(0)=2$$

解答.

$$M(t,y)=2t\cos y+3t^2y$$

$$N(t,y)=t^3-t^2\sin y-y$$

$$\frac{\partial M}{\partial y}=-2t\sin y+3t^2$$

$$\frac{\partial N}{\partial t}=-2t\sin y+3t^2$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\phi(t,y) = \int M(t,y)dt + h(y)$$

$$h'(y) + t^3 - t^2 \sin y = t^3 - t^2 \sin y - y$$

$$h(y) = \frac{-y^2}{2}$$

$$\phi(t,y) = \int N(t,y)dy + k(t)$$

$$k'(t) + 3t^2 y + 2t\cos y = 2t\cos y + 3t^2 y$$

$$k'(t) = 0$$

$$k(t) = C$$

$$\phi(t,y) = t^2 \cos y + t^3 y - \frac{y^2}{2}$$

$$\phi(t,y) = -2$$

题目7.

$$3t^2 + 4ty + (2y + 2t^2)\frac{dy}{dt} = 0, \quad y(0) = 1$$

解答.

$$M(t,y) = 3t^2 + 4ty$$

$$N(t,y) = 2y + 2t^2$$

$$\frac{\partial M}{\partial y} = 4t$$

$$\frac{\partial N}{\partial t} = 4t$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\phi(t,y) = \int M(t,y)dt + h(y)$$

$$h'(y) + 2t^2 = 2y + 2t^2$$

$$h'(y) = 2y$$

$$h(y) = y^2$$

$$\phi(t,y) = \int N(t,y)dy + k(t)$$

$$k'(t) + 4ty = 4ty + 3t^2$$

$$k'(t) = 3t^2$$

$$k(t) = t^3$$

$$\phi(t,y) = y^2 + 2t^3y + t^3$$

$$\phi(t,y) = y^2 + 2t^3y + t^3 = 1 = 1$$

$$y(0) = 1$$

$$so \quad \phi(t,y) = 1$$

题目8.

$$y(\cos 2t)e^{ty} - 2(\sin 2t)e^{ty} + 2t + (t(\cos 2t)e^{ty} - 3)\frac{dy}{dt} = 0, \quad y(0) = 0$$

解答.

$$M(t,y) = y(\cos 2t)e^{ty} - 2(\sin 2t)e^{ty} + 2t$$

$$N(t,y) = t(\cos 2t)e^{ty} - 3$$

$$\frac{\partial M}{\partial y} = \cos 2te^{ty} + ty(\cos 2t)e^{ty} - 2t(\sin 2t)e^{ty}$$

$$\frac{\partial N}{\partial t} = (\cos 2t) - 2t\sin 2t)e^{ty} + ty(\cos 2t)e^{ty}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$\phi(t,y) = \int M(t,y)dt + h(y)$$

$$h'(y) + t(\cos 2t)e^{ty} = t(\cos 2t)e^{ty} - 3$$

$$h'(y) = -3$$

$$h(y) = -3y$$

$$\phi(t,y) = \int N(t,y)dy + k(t)$$

$$k'(t) + y(\cos 2t)e^{ty} - 2(\sin 2t)e^{ty} = y(\cos 2t)e^{ty} - 2(\sin 2t)e^{ty} + 2t$$

$$k'(t) = 2t$$

$$k(t) = t^2$$

$$\phi(t, y) = t^2 + (\cos 2t)e^{ty} - 3y$$

$$\phi(t, y) = 1$$

题目9.

$$Show \quad that \quad if \quad ((\frac{\partial N}{\partial t}) - (\frac{\partial M}{\partial y}))/M = Q(y), then \quad the \quad differential$$

$$equation \quad M(t, y) + N(t, y)\frac{dy}{dt} = 0 \quad has \quad an \quad integrating \quad factor$$

$$\mu(y) = exp(\int Q(y)dy)$$

解答.

$$M(t, y) + N(t, y)\frac{dy}{dt} = 0$$

$$\mu(t)M(t, y) + \mu(t)N(t, y)\frac{dy}{dt} = 0$$

$$\frac{\partial}{\partial y}(\mu(t)M(t, y)) = \frac{\partial}{\partial t}(\mu(t)N(t, y))$$

$$M\frac{\partial \mu}{\partial y} + \mu\frac{\partial M}{\partial y}$$

$$= N\frac{\partial \mu}{\partial t} + \mu\frac{\partial N}{\partial t}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial t}$$

$$M\frac{\partial \mu}{\partial y} = \mu(\frac{\partial N}{\partial t} - \frac{\partial M}{\partial y})$$

$$\frac{\partial \mu}{\partial y} = \frac{\mu(\frac{\partial N}{\partial t} - \frac{\partial M}{\partial y})}{M}$$

$$Q(y) = \frac{\frac{\partial N}{\partial t} - \frac{\partial M}{\partial y}}{M}$$

$$\mu(t) = e^{\int Q(y)dy}$$