Tutorial 7 solutions

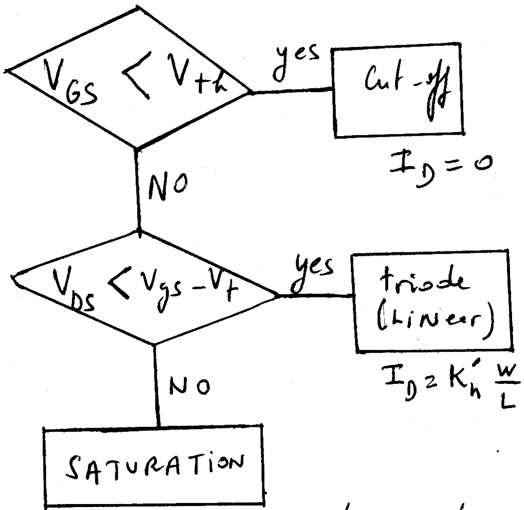
Problem 1:

The condition required for the transistor to be ON is $|V_{GS}| \geq |V_T|$. It follows that:

- For an NMOS to be ON requires $V_{GS} > V_T$, since V_{GS} and V_T are positive for NMOS.
- For a PMOS to be ON requires $V_{GS} < V_T$, since V_{GS} and V_T are negative for PMOS.

In order to find the operating mode of a transistor and to determine the current I_D, we must proceed as follow:

For NMOS transistor:



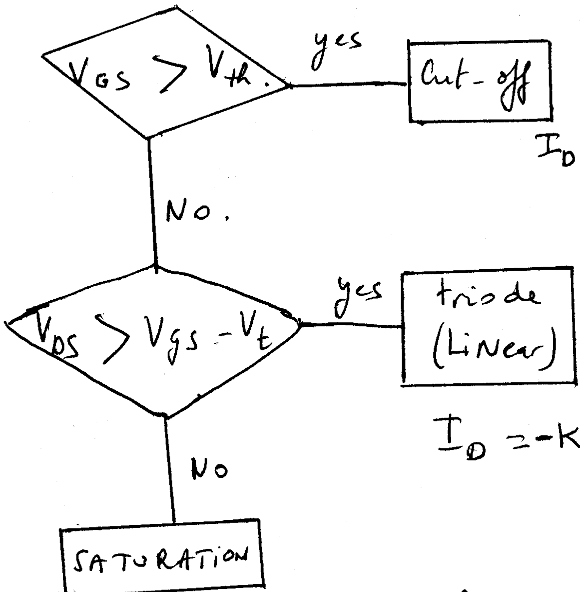
$$K_n' = \mu_n C_{ox} = \mu_n \cdot \frac{\varepsilon_{ox}}{t_{ox}}$$

μ_n : mobility of e^-.
ε_{ox} : oxide permittivity
t_{ox} : oxide thikness

$$I_D = K_n' \frac{W}{L}\left[(V_{gs} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = K_n' \cdot \frac{W}{2L}(V_{gs} - V_t)^2 (1 + \lambda V_{DS}).$$

For pmos



$$K_p' = \mu_p \cdot C_{ox} = \mu_p \frac{\varepsilon_{ox}}{t_{ox}}.$$

μ_p : mobility of holes.

$$I_D = -K_p' \frac{W}{L}\left[(V_{gs} - V_t)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$I_D = -K_p' \cdot \frac{W}{2L}(V_{gs} - V_t)^2 (1 + \lambda \cdot V_{DS})$$

Case (a). $V_{gs} = 3.3\text{ V} > V_{th} \Rightarrow$ Transistor is ON

NMOS:

$V_{DS} = 3.3\text{ V} > V_{gs} - V_{t0} = 3.3 - 0.7$

$= 2.6$

$\Rightarrow$ Transistor in the SATURATION Mode.

$I_{DS_N} = \frac{k_n'}{2} \cdot \frac{W}{L} (V_{gs} - V_t)^2 (1 + \lambda V_{DS})$.

$= \frac{60\,\mu A}{V^2} \cdot \frac{1}{2} \times 1 (3.3 - 0.7)^2 (1 + 0.1 \times 3.3)$

$\boxed{I_{DS_N} = 269.8\ \mu A}$.

PMOS:

$V_{gs} = -0.5\text{ V} > V_{T0} = -0.8\text{ V}$

$\Rightarrow$ PMOS is Cut-off $\boxed{I_{DS_p} = 0}$.

Case (b)

NMOS: $V_{gs} = 3.3\text{ V} > V_{th} = 0.7 \Rightarrow$ ON.

$V_{DS} = 2.2\text{ V} < V_{gs} - V_t = 3.3 - 0.7 = 2.6$

$\Rightarrow$ Transistor in the Triode Mode.

$I_{D_S} = k_n' \cdot \frac{W}{L} \left[(V_{gs} - V_t) \cdot V_{DS} - \frac{V_{DS}^2}{2} \right]$

$\boxed{I_{D_{SN}} = 198\ \mu A}$

PMOS: $V_{gs} = -3.3\text{ V} < -0.8\text{ V} \Rightarrow$ ON

$V_{DS} = -2.6 < V_{gs} - V_t = -3.3\text{ V} - (-0.8)$ =-2.5V

$\Rightarrow$ Transistor is in the SATURATION Mode

$$I_{DS_p} = -K_p' \cdot \frac{W}{2L}\left[(V_{gs}-V_t)^2\,(1+\lambda V_{DS})\right].$$

$$= -\frac{20}{2}\left[(-2.5)^2\,(1 \quad 0.1\,(-2.6))\right]$$

$$\boxed{I_{DSp} = -78.75\ \mu A}$$

Case (c) : NMOS :

$V_{GS} = 0.6\ V \qquad V_{DS} = 0.1\ V$

$V_{GS} = 0.6 < V_{th} = 0.7V \Rightarrow$ Transistor is off $\boxed{I_{DS} = 0}$.

pmos : $V_{GS} = -3.3\ V \qquad V_{DS} = -0.5\ V$

$V_{GS} < V_{th} \Rightarrow$ ON.

$V_{DS} = -0.5V > V_{gs} - V_{th} = -3.3\ V + 0.8$
$= -2.5\ V$.

$\Rightarrow$ transistor is in the triode mode.

$$I_{DS_p} = -K_p' \cdot \frac{W}{L}\left[(V_{gs}-V_t)\,V_{DS} - \frac{V_{DS}^2}{2}\right]$$

$$= -20\left[(-2.5).(-0.5) - \frac{(-0.5)^2}{2}\right]$$

$$\boxed{I_{DS_p} = -22.5\ \mu A}$$

Problem 2:

Transistor	V_S	V_G	V_D	I_D	type	mode	$\mu C_{ox}\frac{W}{L}$	V_t
1	0	2	5	100	N	Sat.	200	1
1	0	3	5	400	N	Sat.	200	1
2	5	3	-45	50	P	Sat.	400	-1.5
2	5	2	-05	450	P	Sat.	400	-1.5
3	5	3	4	200	P	Sat.	400	-1
3	5	2	0	800	P	Sat.	400	-1
4	-2	0	0	72	N	Sat.	100	0.8
4	-4	0	-3	270	N	Triode	100	0.8

Case a) transistor $\underline{1}$: $V_{GS} = 2^V$ $V_{DS} = 5^V$ $I_D = 100\mu A$

This must be an NMOS operating in Saturation

$$I_D = 100 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2-V_t)^2$$

when $V_{GS}=3V$: $400 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-V_t)^2 \Rightarrow 4 = \frac{(3-V_t)^2}{(2-V_t)^2}$

$$\Rightarrow 2(2-V_t) = 3-V_t \Rightarrow \underline{V_t = 1^V}, \ \underline{\mu_n C_{ox}\frac{W}{L} = 200\frac{\mu A}{V^2}}$$

Case b) transistor 2: $V_{GS} = 3-5 = -2^V$ $V_{DS} = -9.5^V$

Therefore $V_{DS} < V_{GS} - V_t$ regardless of value of V_t, and the device operates in saturation. (PMOS)

$$I_D = 50 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-2-V_t)^2$$
$$450 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-3-V_t)^2 \Rightarrow 9 = \frac{(3+V_t)^2}{(2+V_t)^2}$$

$$\Rightarrow 3(2+V_t) = 3+V_t \Rightarrow V_t = -1.5V, \ \mu_p C_{ox}\frac{W}{L} = 400\frac{\mu A}{V^2}$$

case c) transistor 3: $V_{GS} = -2^V$ $V_{DS} = -1^V \Rightarrow$ PMOS

This device can be either in saturation or triode region. First, we assume saturation region:

$$I_D = 200 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-2-V_t)^2$$
$$800 = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(-3-V_t)^2 \Rightarrow 4 = \frac{(3+V_t)^2}{(2+V_t)^2}$$

$$\Rightarrow 2(2+V_t) = 3+V_t \Rightarrow V_t = -1V, \ \mu_p C_{ox}\frac{W}{L} = 400\frac{\mu A}{V^2}$$

So our assumption was right:

$$(V_{DS} = -1^V) \leq (V_{GS} - V_t) = -2+1 = -1^V$$

edge of saturation

Case d) transistor 4: $V_{GS} = 2^V$ $V_{DS} = 2^V$ $\Rightarrow$ NMOS

So $V_{DS} > V_{GS} - V_t$ $\Rightarrow$ Saturation region

$$① \; I_D = 72 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_t)^2$$

But for $V_{GS} = 4^V, V_{DS} = 1^V$ and considering that $V_t < 2V$, then the device is in triode region:

$$270 = \mu_n C_{ox} \frac{W}{L} \left[(4 - V_t) \times 1 - \frac{1}{2} \times 1 \right]$$

$$② \; 270 = \mu_n C_{ox} \frac{W}{L} (3.5 - V_t)$$

①,② $\Rightarrow$ $V_t = 0.8V$ $\mu_n C_{ox} \frac{W}{L} = 100 \, \mu A/V^2$

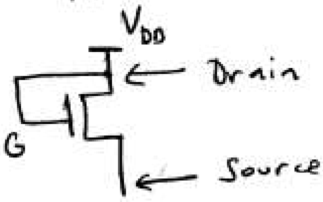
Problem 3

a) before finding out the operating mode of the transistors, we need to figure out the drain and the source terminals of the transistor.

Remember ! - the source of an NMOS is at LOWER voltage compared to the drain $V_S < V_D$ for NMOS

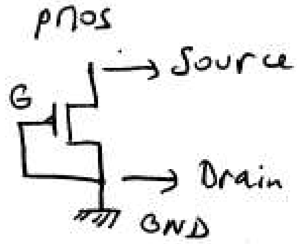
and - the source of a PMOS is at Higher voltage compared to the drain $V_S > V_D$ for PMOS.

NMOS



$V_D > V_S$

PMOS



$V_S > V_D$.

for both transistors the gate is connected to the drain

$\Rightarrow \quad V_{gs} = V_{ds}$.

For the NMOS transistor $V_g = V_{DD} \Rightarrow$ Transistor is ON when $V_{gs} > V_{th_N} \Rightarrow V_g - V_s > V_{th_N}$

$\Rightarrow V_{DD} - V_s > V_{th_N} \Rightarrow \boxed{V_S < V_{DD} - V_{th_N}} \rightarrow$ ON.

when the NMOS is ON $\quad V_{DS} > V_{gs} - V_{th_N}$ is always true. because $V_{DS} = V_{GS}$

$\Rightarrow$ SATURATION.

So

For $V_S < V_{DD} - V_{th_N}$ the transistor is SATURATED

For $V_S > V_{DD} - V_{th_N}$ the transistor is Cut-off

For the pMOS transistor $V_g = 0 \Rightarrow$ transistor is ON when $V_{gs} < V_{th} \Rightarrow \cancel{V_g} - V_s < V_{th_p}$

$\Rightarrow V_s > -V_{th_p}$ (with $V_{th_p} < 0$)

when the NMOS is ON $V_{DS} < V_{gs} - V_{th_p}$ is always true because $V_{DS} = V_{gs}$

$\Rightarrow$ SATURATION.

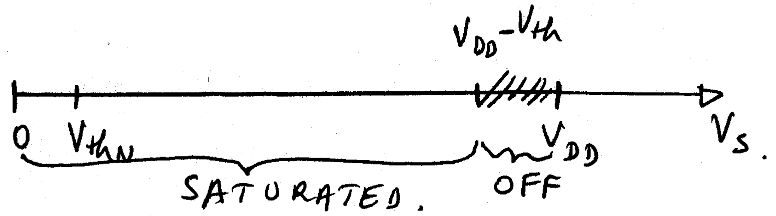
So For the pMOS

For $V_S > -V_{th_p}$ SATURATION

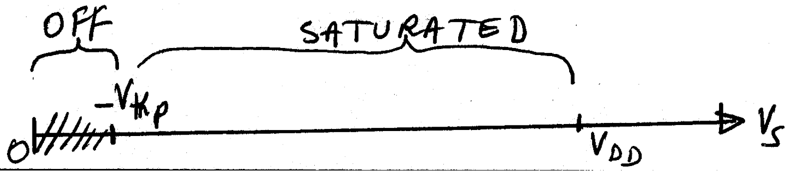
$V_S < -V_{th_p}$ the transistor is Cut-off.

<u>To Summarize:</u>

For NMOS



For pMOS.



b) For $V_s = \frac{V_{dd}}{2}$ both transistors are saturated. (3)

NMOS: $\Rightarrow I_{DS_N} = \frac{k_n'}{2} \frac{W}{L} \left(V_{gs} - V_t\right)^2 \left(1 + \lambda V_{DS}\right)$.

$$V_{gs} = V_g - V_s = V_{dd} - \frac{V_{dd}}{2} = \frac{V_{dd}}{2} = 2.5\,V$$

$$V_{ds} = V_d - V_s = V_{dd} - \frac{V_{dd}}{2} = \frac{V_{dd}}{2} = 2.5\,V$$

$$I_{DS_N} = \frac{60\,\mu A/V^2}{2} \cdot 1 \left(2.5 - 0.7\right)^2 \left(1 + 0.1 \times 2.5\right)$$

$$\boxed{I_{DS} = 121.5\,\mu A.}$$

PMOS: $I_{DS_P} = -\frac{k_p'}{2} \frac{W}{L} \left(V_{gs} - V_t\right)^2 \left(1 + \lambda V_{ds}\right)$.

$$V_{gs} = V_g - V_s = 0 - \frac{V_{dd}}{2} = -2.5\,V.$$

$$V_{ds} = V_d - V_s = 0 - \frac{V_{dd}}{2} = -2.5\,V$$

$$I_{DS_P} = -\frac{20\,\mu A/V^2}{2} \cdot 1 \left(-2.5 - (-0.8)\right)^2 \left(1 - 0.1 \cdot (-2.5)\right)$$

$$\boxed{I_{DS_P} = -\ 36.125\ \mu A}$$

Problem 4:

a)

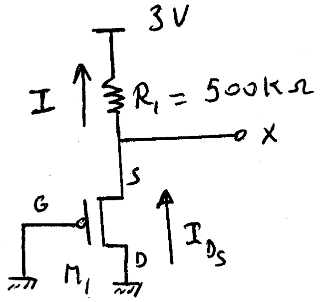


Fig2.

$\rightarrow M_1$ is in the same config as the one of figure 1.B

$\Rightarrow$ for $V_x > -V_{thp}$: M_1 SATURATED.

$$I_{DS} = -\frac{k_p'}{2}\cdot\frac{W}{L}\cdot\left(V_{gs} - V_{thp}\right)^2\left(1 + \lambda V_{DS}\right).$$

$$V_{gs} = V_g - V_s = 0 - V_x.$$

$$V_{DS} = V_D - V_S = 0 - V_x.$$

$$I_{DS} = -\frac{k_p'}{2}\cdot\frac{W}{L}\left(-V_x - V_{thp}\right)^2\left(1 - \lambda V_x\right).$$

V_{thp} is negative $\Rightarrow$ $V_{thp} = -|V_{thp}|$.

$$\Rightarrow I_{DS} = -\frac{k_p'}{2}\cdot\frac{W}{L}\left(-\left(V_x - |V_{thp}|\right)\right)^2\cdot\left(1 - \lambda V_x\right).$$

$$\boxed{I_{DS} = -\frac{k_p'}{2}\cdot\frac{W}{L}\cdot\left(V_x - |V_{thp}|\right)^2\left(1 - \lambda V_x\right).}$$

b) Voltage V_x = ?

From figure 2, we can write that $I_{DS} = I$

$$I = -\frac{3V - V_x}{R} = I_{DS} = -\frac{k_p'}{2}\cdot\frac{W}{L}\left(V_x - |V_{thp}|\right)^2\left(1 - \lambda V_x\right)$$

For $\lambda = 0$ $\Rightarrow$ $$\boxed{\left(3V - V_x\right) = \frac{R}{2}k_p'\cdot\frac{W}{L}\left(V_x - |V_{thp}|\right)^2.}$$

c) The required width of the transistor : W.

$$\frac{3 - V_x}{R} = \frac{K'_p}{2} \cdot \frac{W}{L} \left(V_x - |V_{thp}| \right)^2$$

$$\Rightarrow \quad \frac{1.5\,V}{500\,k\Omega} = \frac{5.4 \cdot 10^{-6}\,mA/V^2}{2} \cdot \frac{W}{L} \left(1.5 - |V_{thp}| \right)^2 .$$

for $L = 1.2\,\mu m$ and $|V_{thp}| = 0.739$

$$\Rightarrow \quad \boxed{W = 2.3\,\mu m.}$$