Solution 13.1

For coil 1, $L_1 - M_{12} + M_{13} = 12 - 8 + 4 = 8$

For coil 2, $L_2 - M_{21} - M_{23} = 16 - 8 - 10 = -2$

For coil 3, $L_3 + M_{31} - M_{32} = 20 + 4 - 10 = 14$

$$L_T = 8 - 2 + 14 = 20H$$

or
$$L_T = L_1 + L_2 + L_3 - 2M_{12} - 2M_{23} + 2M_{13}$$

$$L_T = 12 + 16 + 20 - 2x8 - 2x10 + 2x4 = 48 - 16 - 20 + 8$$

= **20H**

Solution 13.2

Using Fig. 13.73, design a problem to help other students to better understand mutual inductance.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the inductance of the three series-connected inductors of Fig. 13.73.

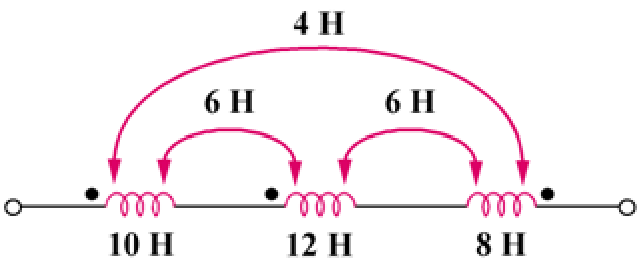


Figure 13.73

Solution

$L = L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31}$
$= 10 + 12 + 8 + 2x6 - 2x6 - 2x4$

= **22H**

Solution 13.3

$L_1 + L_2 + 2M = 500$ mH (1)

$L_1 + L_2 - 2M = 300$ mH (2)

Adding (1) and (2),

$2L_1 + 2L_2 = 800$ mH

But, $L_1 = 3L_2$,, or $8L_2 + 400$, and L_2 = **100 mH**

$L_1 = 3L_2$ = **300 mH**

From (2), $150 + 50 - 2M = 150$ leads to M = **50 mH**

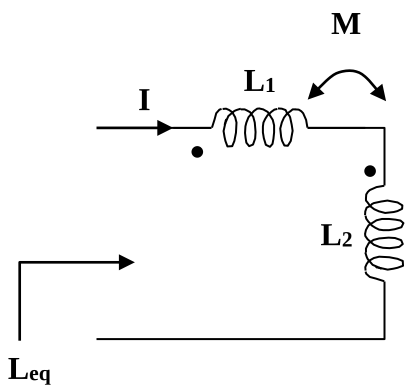
$k = M/\sqrt{L_1L_2} = 50/\sqrt{100x300}$ = **0.2887**

300 mH, 100 mH, 50 mH, 0.2887

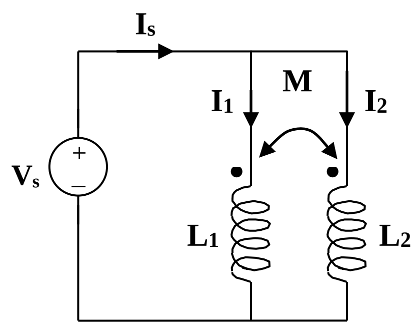
Solution 13.4

(a) For the series connection shown in Figure (a), the current I enters each coil from its dotted terminal. Therefore, the mutually induced voltages have the same sign as the self-induced voltages. Thus,

$$L_{eq} = \mathbf{L_1 + L_2 + 2M}$$



(a)



(b)

(b) For the parallel coil, consider Figure (b).

$$I_s = I_1 + I_2 \quad \text{and} \quad Z_{eq} = V_s/I_s$$

Applying KVL to each branch gives,

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_s = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

or

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2, \quad \Delta_1 = j\omega V_s(L_2 - M), \quad \Delta_2 = j\omega V_s(L_1 - M)$$

$$I_1 = \Delta_1/\Delta, \quad \text{and} \quad I_2 = \Delta_2/\Delta$$

$$I_s = I_1 + I_2 = (\Delta_1 + \Delta_2)/\Delta = j\omega(L_1 + L_2 - 2M)V_s/(-\omega^2(L_1L_2 - M^2))$$
$$= (L_1 + L_2 - 2M)V_s/(j\omega(L_1L_2 - M^2))$$

$$Z_{eq} = V_s/I_s = j\omega(L_1L_2 - M^2)/(L_1 + L_2 - 2M) = j\omega L_{eq}$$

i.e., $$L_{eq} = \mathbf{(L_1L_2 - M^2)/(L_1 + L_2 - 2M)}$$

Solution 13.5

(a) If the coils are connected in series,

$$L = L_1 + L_2 + 2M = 50 + 120 + 2(0.5)\sqrt{50x120} = \mathbf{247.4\ mH}$$

(b) If they are connected in parallel,

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = \frac{50x120 - 38.72^2}{50 + 120 - 2x38.72}\,\mathrm{mH} = \mathbf{48.62\ mH}$$

(a) 247.4 mH, (b) 48.62 mH

Problem 13.6

Given the circuit shown in Fig. 13.75. Determine the value of $\mathbf{V_1}$ and $\mathbf{I_2}$.

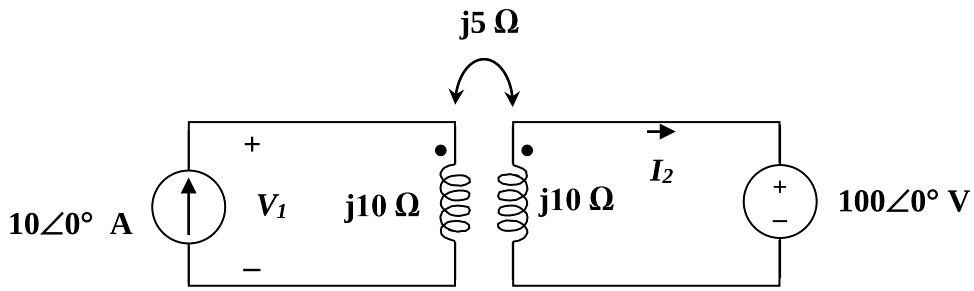
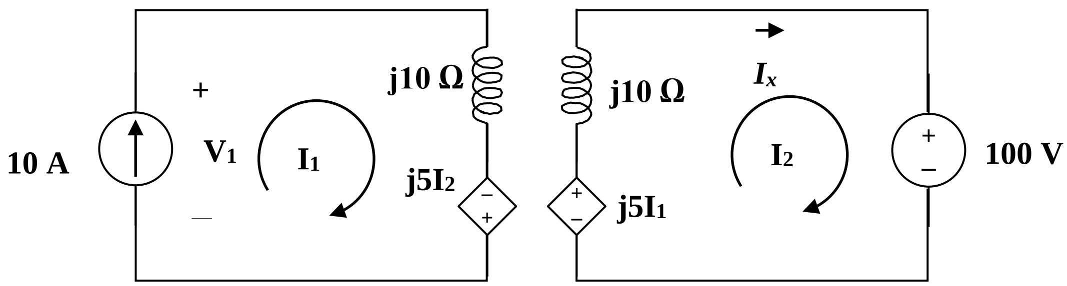


Figure 13.75
For Prob. 13.6.

Solution

Step 1. First we need to replace the coupled inductors with the dependent source model. Next we need to determine the signs on the dependent sources using the dot convention. Now we can write mesh equations around each loop and solve for $\mathbf{I_2}$ and $\mathbf{I_1}$. Once we have $\mathbf{I_1}$ and $\mathbf{I_2}$ we can find $\mathbf{V_1}$.

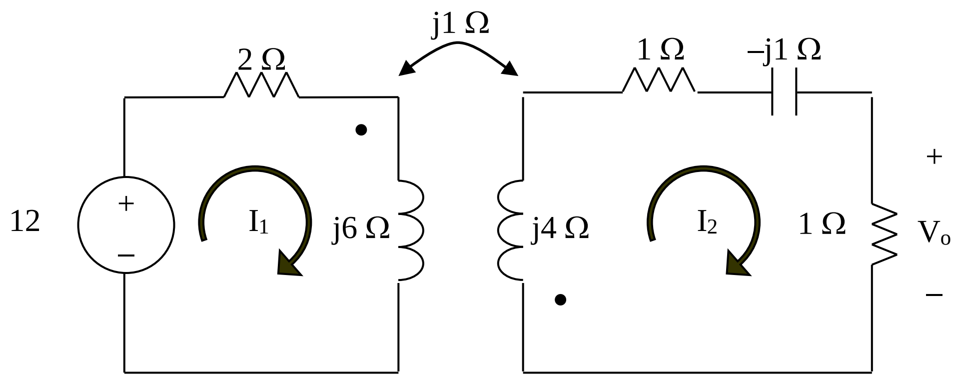


The mesh equations are $-\mathbf{V_1} + j10\mathbf{I_1} - j5\mathbf{I_2}$ where $\mathbf{I_1} = 10$ A and $-j5\mathbf{I_1} + j10\mathbf{I_2} + 100 = 0$. This then gives us two equations with two unknowns, $\mathbf{V_1}$ and $\mathbf{I_2}$.

Step 2. $-\mathbf{V_1} - j5\mathbf{I_2} = -j100$ and $j10\mathbf{I_2} = -100 + j50$ or $\mathbf{I_2} = 5 + j10$ $= \mathbf{11.1803\angle 63.435°}$. Next, $\mathbf{V_1} = -j5\mathbf{I_2} + j100 = 50 - j25 + j100 = 50 + j75$ $= \mathbf{90.139\angle 56.31°\ V}$.

Solution 13.7

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$(2+j6)I_1 + jI_2 = 24$$

For mesh 2,

$$jI_1 + (2-j+j4)I_2 = jI_1 + (2+j3)I_2 = 0 \text{ or } I_1 = (-3+j2)I_2$$

Substituting into the first equation results in $I_2 = (-0.8762+j0.6328)$ A.

$$V_o = I_2 x1 = \mathbf{1.081\angle 144.16^\circ\ V}.$$

Solution 13.8

Find v(t) for the circuit in Fig. 13.77.

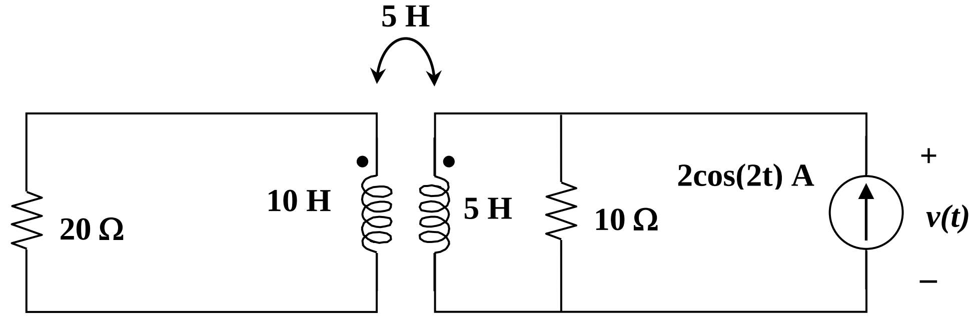
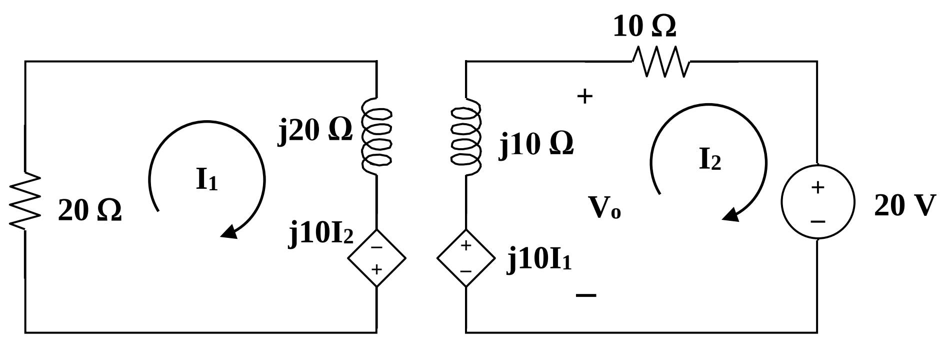


Figure 13.77
For Prob. 13.8.

Solution

Step 1. We need to transform the circuit into the frequency domain and replace the coupled inductors with the dependent source model. In addition, we need to replace the current source in parallel with the resistor with the equivalent voltage source in series with the resistor (source transformation).



The 10 H inductor becomes j2x10 = j20 Ω and the 5 H mutual coupling and 5 H inductor becomes j2x5 = j10 Ω. The source transformation converts the current source resistor combination of 10 Ω in parallel with the 2 A source with a 10 Ω resistor in series with a 20 V source.

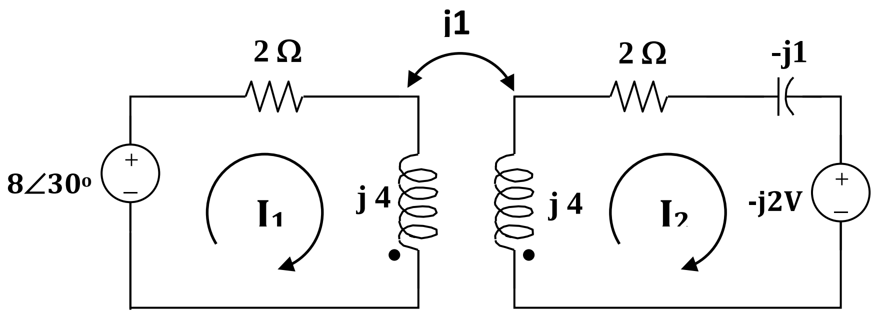
Loop 1, $20\mathbf{I}_1 + j20\mathbf{I}_1 - j10\mathbf{I}_2 = 0$ and loop 2, $-j10\mathbf{I}_1 + j10\mathbf{I}_2 + 10\mathbf{I}_2 + 20 = 0$. Finally $\mathbf{V}_o = -j10\mathbf{I}_2 + j10\mathbf{I}_1$. Note we are representing the frequency domain value of v(t) by $\mathbf{V}_o$.

Step 2. From the first loop equation we get $(20+j20)\mathbf{I}_1 = j10\mathbf{I}_2$ or $\mathbf{I}_1 = (0.25+j0.25)\mathbf{I}_2$.
This leads to $-j10(0.25+j0.25)\mathbf{I}_2 + (10+ j10)\mathbf{I}_2 = -20 = (12.5+j7.5)\mathbf{I}_2$ or
$\mathbf{I}_2 = 20\angle180°/(14.5774\angle30.964°) = 1.37199\angle149.036°$ and
$\mathbf{I}_1 = (0.35355\angle45°)(1.37199\angle149.039°) = 0.48507\angle{-165.961}°$. Finally,
$\mathbf{V}_o = -j10(-1.17647 + j0.70589) + j10(-0.47058-j0.117669)$
$= 8.2356+j7.0589 = 10.847\angle40.6°$ V or

v(t) = **10.847 cos(10t + 40.6°) A**.

Solution 13.9

Consider the circuit below.



For loop 1,

$$8\angle 30^\circ = (2 + j4)I_1 - jI_2 \qquad (1)$$

For loop 2, $(j4 + 2 - j)I_2 - jI_1 + (-j2) = 0$

or $$I_1 = (3 - j2)i_2 - 2 \qquad (2)$$

Substituting (2) into (1), $8\angle 30^\circ + (2 + j4)2 = (14 + j7)I_2$

$$I_2 = (10.928 + j12)/(14 + j7) = 1.037\angle 21.12^\circ$$

$$V_x = 2I_2 = \mathbf{2.074\angle 21.12^\circ\ V}$$

Solution 13.10

Find $v_o(t)$ in the circuit in Fig. 13.79.

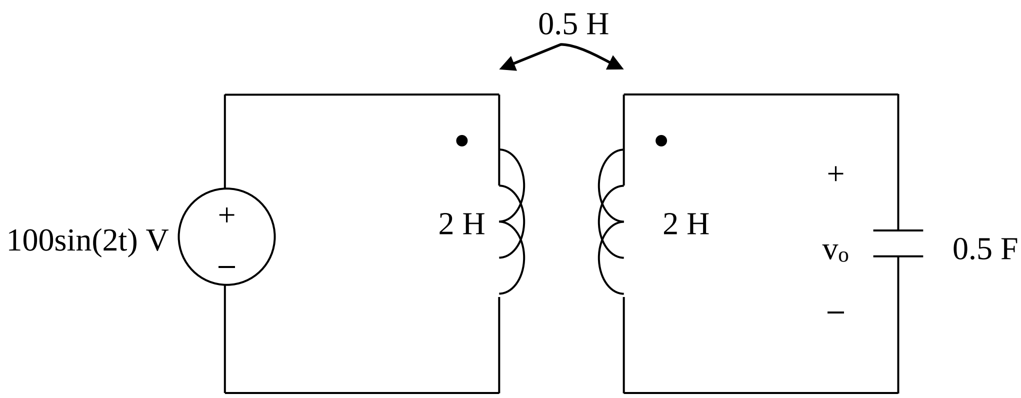
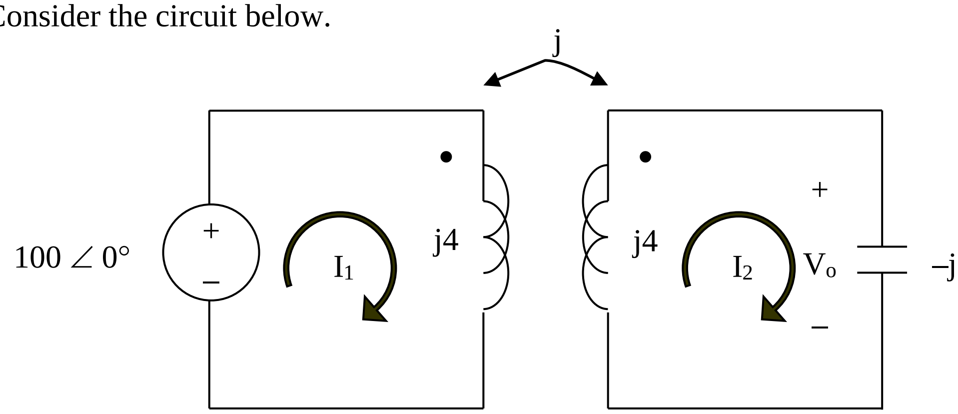


Figure 13.79
For Prob. 13.10.

Solution

$$2H \longrightarrow j\omega L = j2x2 = j4$$
$$0.5H \longrightarrow j\omega L = j2x0.5 = j$$
$$\frac{1}{2}F \longrightarrow 1/j\omega C = \frac{1}{j2x1/2} = -j$$

Consider the circuit below.



$j4I_1 – jI_2 = 100$ (1)
$–jI_1 + (j4–j)I_2 = 0 = –jI_1 + j3I_2$ (2)

In matrix form,

$$\begin{bmatrix} j4 & -j \\ -j & 3 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

Solving this, $\mathbf{I_2} = –j9.091$ A and $\mathbf{V_o} = –j\mathbf{I_2} = 9.091\angle 180°$ V.

$v_o(t) =$ **9.091sin(2t+180°) V**.

Solution 13.11

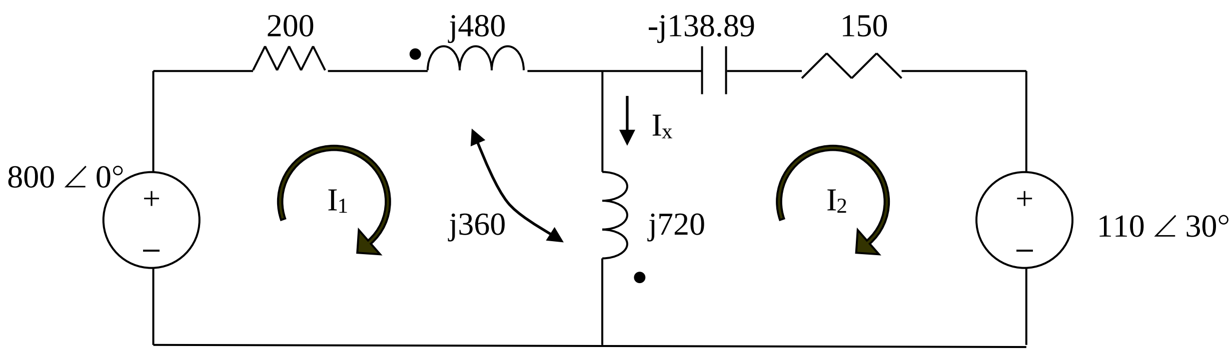
$$800mH \longrightarrow j\omega L = j600x800x10^{-3} = j480$$

$$600mH \longrightarrow j\omega L = j600x600x10^{-3} = j360$$

$$1200mH \longrightarrow j\omega L = j600x1200x10^{-3} = j720$$

$$12\mu F \rightarrow \frac{1}{j\omega C} = \frac{-j}{600x12x10^{-6}} = -j138.89$$

After transforming the current source to a voltage source, we get the circuit shown below.



For mesh 1,

$$800 = (200 + j480 + j720)I_1 + j360I_2 - j720I_2 \text{ or}$$

$$800 = (200 + j1200)I_1 - j360I_2 \qquad (1)$$

For mesh 2,

$$110\angle 30^\circ + 150–j138.89+j720)I_2 + j360I_1 = 0 \text{ or}$$

$$-95.2628 - j55 = -j360I_1 + (150 + j581.1)I_2 \qquad (2)$$

In matrix form,

$$\begin{bmatrix} 800 \\ -95.2628 - j55 \end{bmatrix} = \begin{bmatrix} 200 + j1200 & -j360 \\ -j360 & 150 + j581.1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Solving this using MATLAB leads to:

```
>> Z = [(200+1200i),-360i;-360i,(150+581.1i)]
Z =
  1.0e+003 *
   0.2000 + 1.2000i        0 - 0.3600i
        0 - 0.3600i   0.1500 + 0.5811i
>> V = [800;(-95.26-55i)]
V =
  1.0e+002 *
   8.0000
  -0.9526 - 0.5500i
>> I = inv(Z)*V
I =
   0.1390 - 0.7242i
   0.0609 - 0.2690i
```

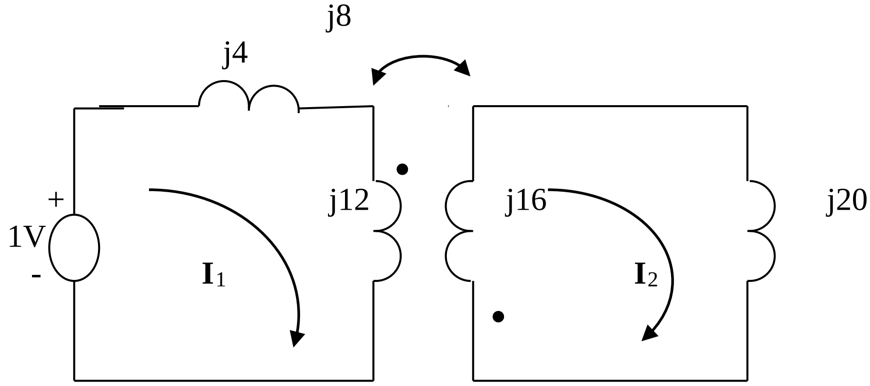
$I_x = I_1 - I_2 = 0.0781 - j0.4552 = 0.4619\angle{-80.26^\circ}$.

Hence,

$$i_x(t) = \mathbf{461.9\cos(600t-80.26^\circ)\ mA}.$$

Solution 13.12

Let $\omega = 1$.



Applying KVL to the loops,

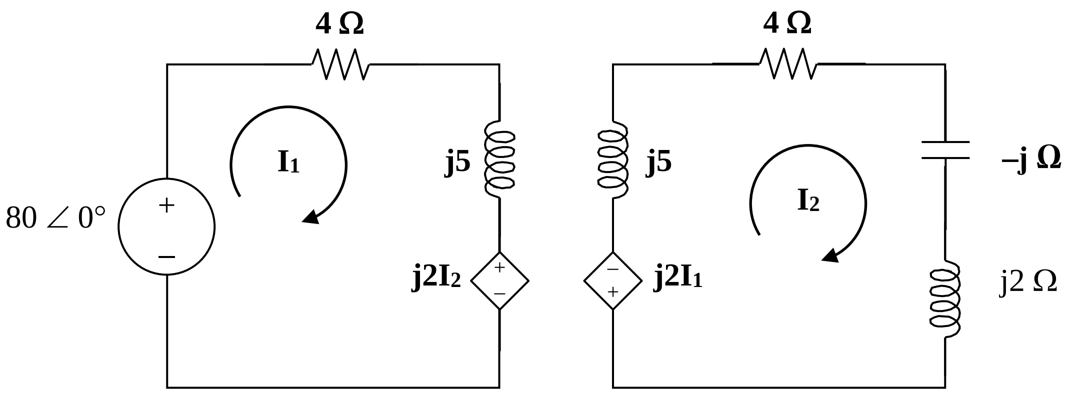
$$1 = j16I_1 + j8I_2 \qquad (1)$$
$$0 = j8I_1 + j36I_2 \qquad (2)$$

Solving (1) and (2) gives $\mathbf{I}_1 = -j0.0703$. Thus

$$Z = \frac{1}{I_1} = jL_{eq} \quad \longrightarrow \quad L_{eq} = \frac{1}{jI_1} = \mathbf{14.225\ H}.$$

We can also use the equivalent T-section for the transform to find the equivalent inductance.

Solution 13.13



$-80 + (4+j5)\mathbf{I_1} + j2\mathbf{I_2} = 0$ or $(4+j5)\mathbf{I_1} + j2\mathbf{I_2} = 80$

$j2\mathbf{I_1} + (4+j6)\mathbf{I_2} = 0$ or $\mathbf{I_2} = [-j2/(7.2111\angle 56.31°)]\mathbf{I_1} = (0.27735\angle -146.31°)\mathbf{J_1}$

$[4+j5 + j2(-0.230769-j0.153846)]I_1 = [4+j5+0.307692-j0.461538]I_1 = 80$

$[4.307692+j4.538462]I_1 = 80$ or $I_1 = 80/(6.2573\angle 46.494°)$
$= 12.78507\angle -46.494°$ A.

$Z_{in} = 80/I_1 = 6.2573\angle 46.494°\ \Omega =$ $\mathbf{(4.308 + j4.538)\ \Omega}$

An alternate approach would be to use the equation,

$$\mathbf{Z_{in}} = 4 + j(5) + \frac{4}{j5 + 4 - j + j2} = 4 + j5 + \frac{4}{7.2111\angle 56.31°}$$
$= 4+j5+0.5547\angle -56.31° = 4+0.30769+j(5-0.46154)$

$= \mathbf{[4.308+j4.538]\ \Omega}$.

Solution 13.14

Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals *a*-*b*.

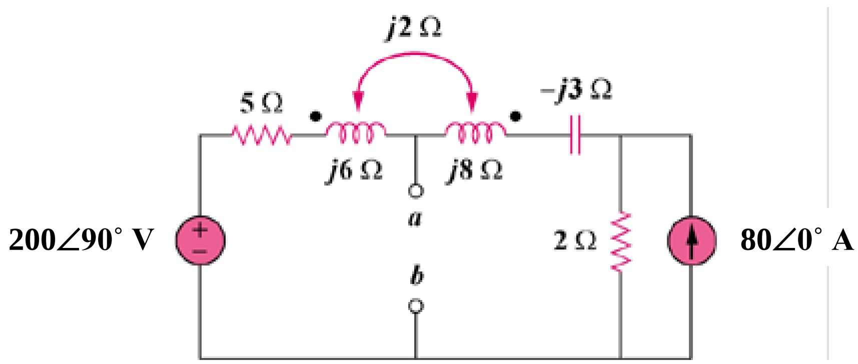
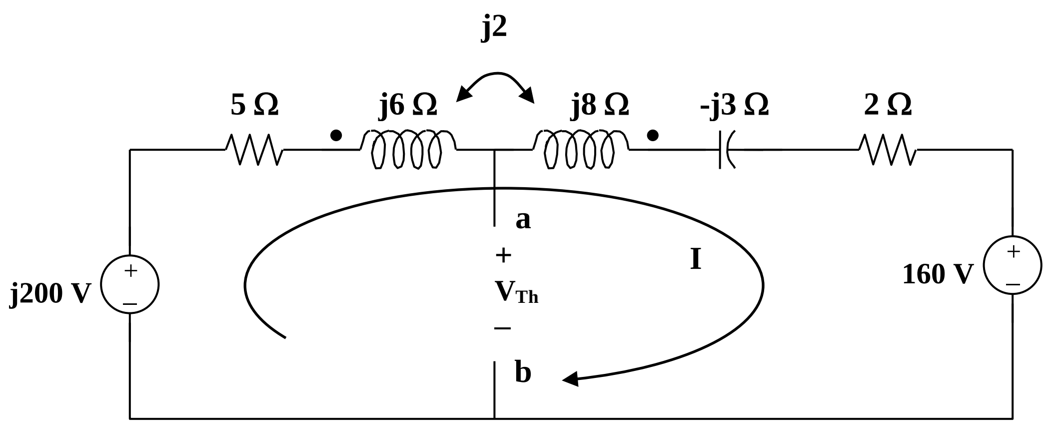


Figure 13.83
For Prob. 13.14.

Solution

To obtain V_{Th}, convert the current source to a voltage source as shown below.



Note that the two coils are connected series aiding.

$$\omega L = \omega L_1 + \omega L_2 - 2\omega M$$

$$j\omega L = j6 + j8 - j4 = j10$$

Thus,

$$-j200 + (5 + j10 - j3 + 2)\mathbf{I} + 160 = 0$$

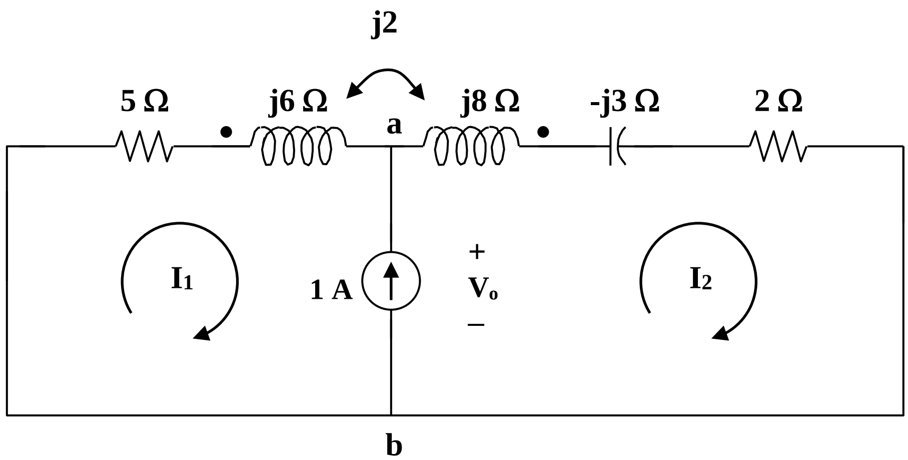
$$\mathbf{I} = (-160 + j200)/(7 + j7)$$

But, $-j200 + (5 + j6)\mathbf{I} - j2\mathbf{I} + \mathbf{V_{Th}} = 0$

$V_{Th} = j200 - (5 + j4)\mathbf{I} = j200 - (5 + j4)(-160 + j200)/(7 + j7)$
$= j200 - (6.4031\angle 38.66°)(256.12\angle 128.66°)/(9.8995\angle 45°) = j200 - 165.661\angle 122.32°$
$= j200 + 88.57 - j140 = 88.57 + j60 = 106.98\angle 34.115°$

$$V_{Th} = \mathbf{106.98\angle 34.12°\ V}$$

To obtain Z_{Th}, we set all the sources to zero and insert a 1-A current source at the terminals a–b as shown below.



Clearly, we now have only a super mesh to analyze.

$$(5 + j6)\mathbf{I_1} - j2\mathbf{I_2} + (2 + j8 - j3)\mathbf{I_2} - j2\mathbf{I_1} = 0$$

$$(5 + j4)\mathbf{I_1} + (2 + j3)\mathbf{I_2} = 0 \qquad (1)$$

But, $\mathbf{I_2} - \mathbf{I_1} = 1$ or $\mathbf{I_2} = \mathbf{I_1} - 1$ (2)

Substituting (2) into (1), $(5 + j4)\mathbf{I_1} + (2 + j3)(1 + \mathbf{I_1}) = 0$

$$\mathbf{I_1} = -(2 + j3)/(7 + j7)$$

Now, $(5 + j6)\mathbf{I_1} - j2\mathbf{I_1} + \mathbf{V_o} = 0$

$$\mathbf{V_o} = -(5 + j4)\mathbf{I_1} = (5 + j4)(2 + j3)/(7 + j7) = (-2 + j23)/(7 + j7) = 2.332\angle 50°$$

$$\mathbf{Z_{eq}} = \mathbf{V_o}/1 = \mathbf{2.332\angle 50°\ \Omega}.$$

Solution 13.15

Find the Norton equivalent for the circuit in Fig. 13.84 at terminals *a*-*b*.

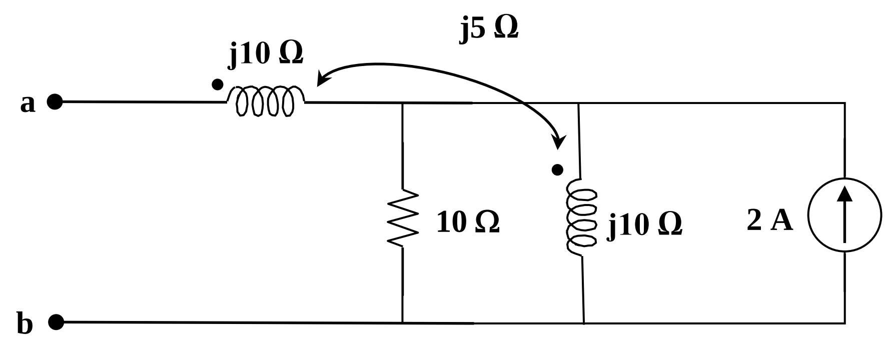
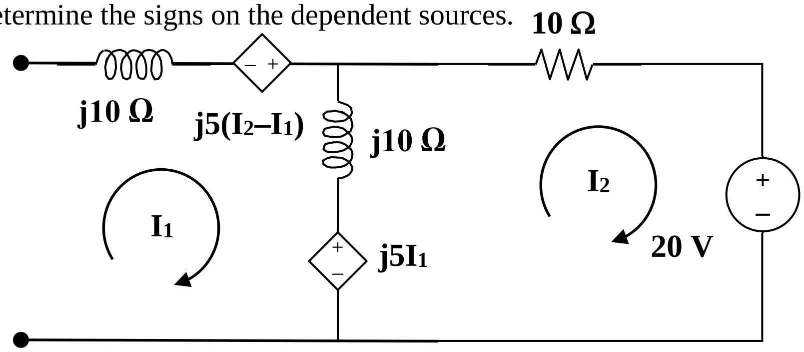


Figure 13.84
For Prob. 13.15.

Solution

Step 1. Since the current source is actually in parallel with the 10 Ω resistor, we can use source transformation to convert them into a resistance of 10Ω in series with a 20 V source. We next replace the mutually coupled inductors with their dependent source equivalent and establish the unknown loop currents. We use the dot convention to determine the signs on the dependent sources.



Clearly we need to find $\mathbf{V}_{oc}$ and $\mathbf{I}_{sc}$ since we have dependent sources. Thus, we have two circuits one with an open circuit at ab and the next is the short circuit at terminals ab.

For $\mathbf{V}_{oc}$ we solve the circuit with $\mathbf{I}_1 = 0$ we get loop 1 equal to $-\mathbf{V}_{oc} - j5\mathbf{I}_2 - j10\mathbf{I}_2 = 0$ or $\mathbf{V}_{oc} = -j15\mathbf{I}_2$ and loop 2 equal to $j10\mathbf{I}_2 + 10\mathbf{I}_2 + 20 = 0$ or $\mathbf{I}_2 = -20/(10+j10) = 1.4142\angle 135°$.

For $\mathbf{I}_{sc}$ we solve
$j10\mathbf{I}_1 - j5(\mathbf{I}_2-\mathbf{I}_1) + j10(\mathbf{I}_1-\mathbf{I}_2) + j5\mathbf{I}_1 = 0$ and $-j5\mathbf{I}_1 + j10(\mathbf{I}_2-\mathbf{I}_1) + 10\mathbf{I}_2 + 20 = 0$.

Now $\mathbf{I}_{sc} = -\mathbf{I}_1$.

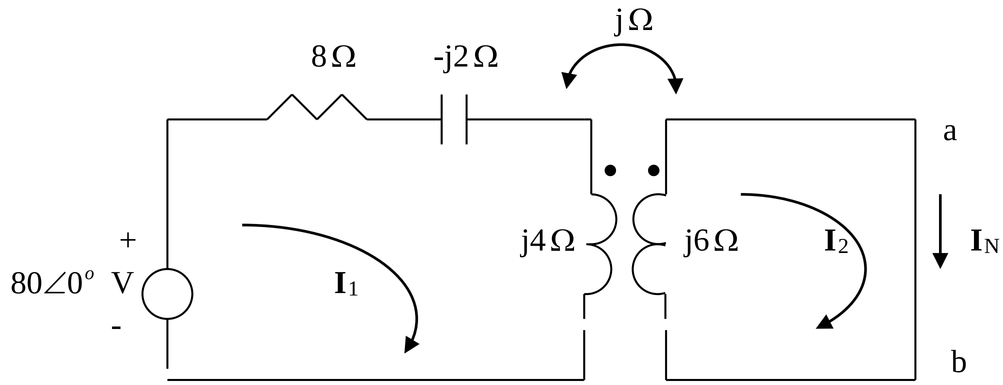
Step 2. $\mathbf{V_{oc}} = -j15(1.4142\angle 135°) = 21.213\angle 45°$ V.

Now for $\mathbf{I_{sc}}$ we solve $(j10+j5+j10+j5)\mathbf{I_1} + (-j5-j10)\mathbf{I_2} = j30\mathbf{I_1} - j15\mathbf{I_2} = 0$ or
$\mathbf{I_2} = 2\mathbf{I_1}$ and then $(-j5-j10)I_1 + (10+j10)I_2 = -20 = (-j15+20+j20)I_1 = (20+j5)I_1$ or
$\mathbf{I_1} = -20/(20+j5) = -20/(20.616\angle 14.036°) = 0.97012\angle 165.964°$ or
$\mathbf{I_{sc}} = \mathbf{I_N} =$ **$970.1\angle -14.04°$ A**.

Finally, $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}} = (21.213\angle 45°)/(0.97012\angle -14.04°) =$ **$21.87\angle 59.04°\ \Omega$**.

Solution 13.16

To find $\mathbf{I}_N$, we short-circuit a-b.



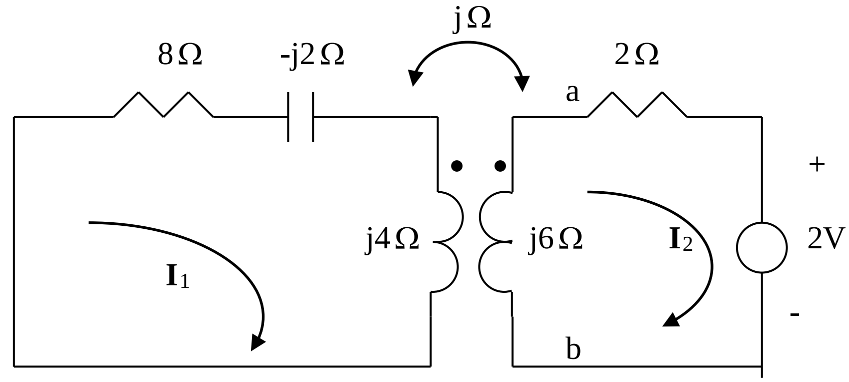
$$-80+(8-j2+j4)I_1-jI_2=0 \quad\longrightarrow\quad (8+j2)I_1-jI_2=80 \qquad (1)$$

$$j6I_2-jI_1=0 \quad\longrightarrow\quad I_1=6I_2 \qquad (2)$$

Solving (1) and (2) leads to

$$I_N=I_2=\frac{80}{48+j11}=1.584-j0.362=\mathbf{1.6246\angle{-12.91}^\circ\ A}$$

To find Z_N, insert a 1-A current source at terminals a-b. Transforming the current source to voltage source gives the circuit below.



$$0=(8+j2)I_1-jI_2 \quad\longrightarrow\quad I_1=\frac{jI_2}{8+j2}$$

$= [j/(8.24621\angle 14.036°)]\mathbf{I_2} = 0.121268\angle 75.964°\mathbf{I_2}$

$= (0.0294113+j0.117647)\mathbf{I_2}$ (3)

$$2+(2+j6)I_2-jI_1=0 \qquad (4)$$

Solving (3) and (4) leads to $(2+j6)\mathbf{I_2} - j(0.0294113+j0.117647)\mathbf{I_2} = -2$ or $(2.117647+j5.882353)\mathbf{I_2} = -2$ or $\mathbf{I_2} = -2/(6.25192\angle 70.201°) = 0.319902\angle 109.8°$.

$V_{ab} = 2(1+\mathbf{I_2}) = 2(1-0.1083629+j0.30099) = (1.78327+j0.601979)$ V $= 1\mathbf{Z_{eq}}$ or

$$\mathbf{Z_{eq}} = (1.78327+j0.601979) = \mathbf{1.8821\angle 18.65°\ \Omega}$$

An alternate approach would be to calculate the open circuit voltage.

$$-80 + (8+j2)\mathbf{I_1} - j\mathbf{I_2} = 0 \text{ or } (8+j2)\mathbf{I_1} - j\mathbf{I_2} = 80 \qquad (5)$$

$$(2+j6)\mathbf{I_2} - j\mathbf{I_1} = 0 \text{ or } \mathbf{I_1} = (2+j6)\mathbf{I_2}/j = (6-j2)\mathbf{I_2} \qquad (6)$$

Substituting (6) into (5) we get,

$(8.24621\angle 14.036°)(6.32456\angle -18.435°)\mathbf{I_2} - jI_2 = 80$ or

$[(52.1536\angle -4.399°)-j]I_2 = [52-j5]I_2 = (52.2398\angle -5.492°)I_2 = 80$ or

$I_2 = 1.5314\angle 5.492°$ A and $V_{oc} = 2I_2 = 3.0628\angle 5.492°$ V which leads to,

$Z_{eq} = V_{oc}/I_{sc} = (3.0628\angle 5.492°)/(1.6246\angle -12.91°) = \mathbf{1.8853\angle 18.4°\ \Omega}$

This is in good agreement with what we determined before.

Solution 3.17

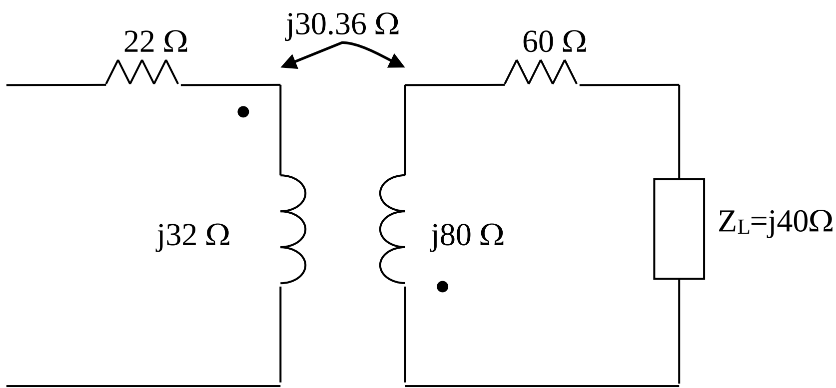
$$j\omega L = j40 \longrightarrow \omega = \frac{40}{L} = \frac{40}{15x10^{-3}} = \ 2667 \text{ rad/s}$$

$$M = k\sqrt{L_1L_2} = 0.6\sqrt{12x10^{-3}x30x10^{-3}} = \ 11.384 \text{ mH}$$

If 15 mH $\longrightarrow$ 40 Ω

Then 12 mH $\longrightarrow$ 32 Ω
30 mH $\longrightarrow$ 80 Ω
11.384 mH $\longrightarrow$ 30.36 Ω

The circuit becomes that shown below.



$$Z_{in} = 22 + j32 + \frac{\omega^2 M^2}{j80 + 60 + j40} = 22 + j32 + \frac{(30.36)^2}{60 + j120}$$

$$= 22 + j32 + \frac{921.7}{134.16\angle 63.43°} = 22 + j32 + 6.87\angle -63.43° = 22 + j32 + 3.073 - j6.144$$

= **[25.07 + j25.86] Ω**.

Solution 13.18

Find the Thevenin equivalent to the left of the load **Z** in the circuit in Fig. 13.87.

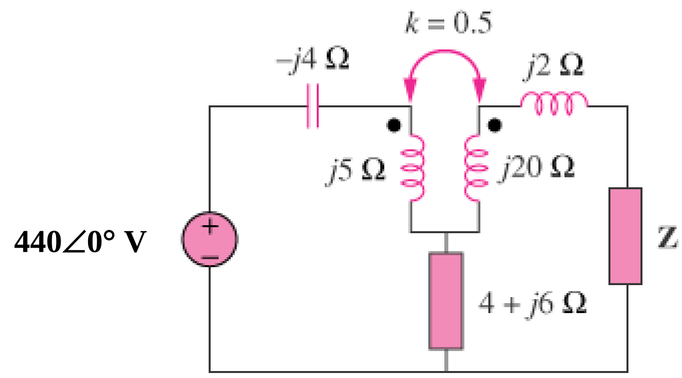
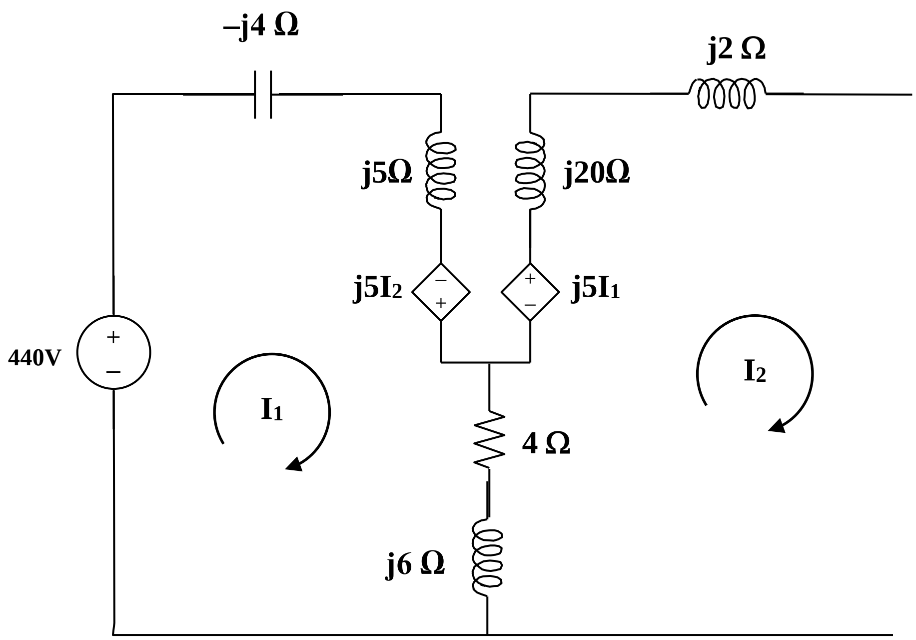


Figure 13.87
For Prob. 1318.

Solution

Replacing the mutually coupled circuit with the dependent source equivalent we get,



Now all we need to do is to find $\mathbf{V_{oc}}$ and $\mathbf{I_{sc}}$. To calculate the open circuit voltage, we note that I_2 is equal to zero. Thus,

$$-440 + (4 + j(-4+5+6))\mathbf{I_1} = 0 \text{ or } \mathbf{I_1} = 440/(4+j7) = 440/(8.06226\angle 60.255°)$$
$$= 54.575\angle -60.255°.$$

$$\mathbf{V_{oc}} = \mathbf{V_{Thev}} = j5\mathbf{I_1} + (4+j6)\mathbf{I_1} = (4+j11)\mathbf{I_1}$$
$$= (11.7047\angle 70.017°)(54.575\angle -60.255°) = \mathbf{638.79\angle 9.76°\ V}$$

To find the short circuit current ($\mathbf{I_{sc} = I_2}$), we need to solve the following mesh equations,

Mesh 1

$$-440 + (-j4+j5)\mathbf{I_1} - j5\mathbf{I_2} + (4+j6)(\mathbf{I_1}-\mathbf{I_2}) = 0 \text{ or}$$
$$(4+j7)\mathbf{I_1} - (4+j11)\mathbf{I_2} = 440 \qquad (1)$$

Mesh 2

$$(4+j6)(\mathbf{I_2}-\mathbf{I_1}) - j5\mathbf{I_1} + j22\mathbf{I_2} = 0 \text{ or } -(4+j11)\mathbf{I_1} + (4+j28)\mathbf{I_2} = 0 \text{ or}$$
$$\mathbf{I_1} = (28.2843\angle 81.87°)\mathbf{I_2}/(11.7047\angle 70.0169°) = (2.4165\angle 11.853°)\mathbf{I_2}$$

Substituting this into equation (1) we get,

$$(8.06226\angle 60.255°)(2.4165\angle 11.853°)\mathbf{I_2} - (4+j11)\mathbf{I_2} = 440 \text{ or}$$
$$[(19.4825\angle 72.108°) - 4 - j11]\mathbf{I_2} = 440 \text{ and}$$
$$[5.9855+j18.5403-4-j11]\mathbf{I_2} = (1.9855+j7.5403)\mathbf{I_2} = 440 \text{ or}$$

$$\mathbf{I_2 = I_{sc}} = 440/(7.79733\angle 75.248°) = 56.43\angle -75.248°\ \text{A}$$

Checking using MATLAB we get,

```
>> Z = [(4+7j) (-4-11j);(-4-11j) (4+28j)]

Z =

   4.0000 + 7.0000i  -4.0000 -11.0000i
  -4.0000 -11.0000i   4.0000 +28.0000i

>> V = [440;0]

V =

   440
     0

>> I = inv(Z)*V

I =
```

61.0687 -121.92583i ($\mathbf{I}_1$)
14.36893 -54.5706i ($\mathbf{I}_2 = \mathbf{I}_{sc}$) = 56.431∠−75.248° (answer checks)

Finally,

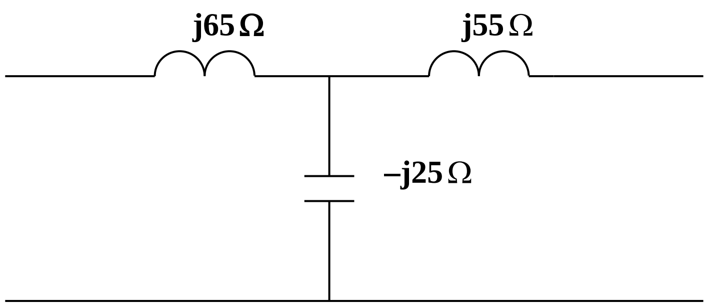
$$\mathbf{Z}_{eq} = \mathbf{V}_{Thev}/\mathbf{I}_{sc} = (638.79\angle 9.76°)/(56.43\angle -75.248°) = \mathbf{(11.32\angle 85.01°)\ \Omega}$$

$$\mathbf{V}_{Thev} = \mathbf{638.79\angle 9.76°\ V} \text{ and } \mathbf{Z}_{eq} = \mathbf{11.32\angle 85.01°\ \Omega}$$

Solution 13.19

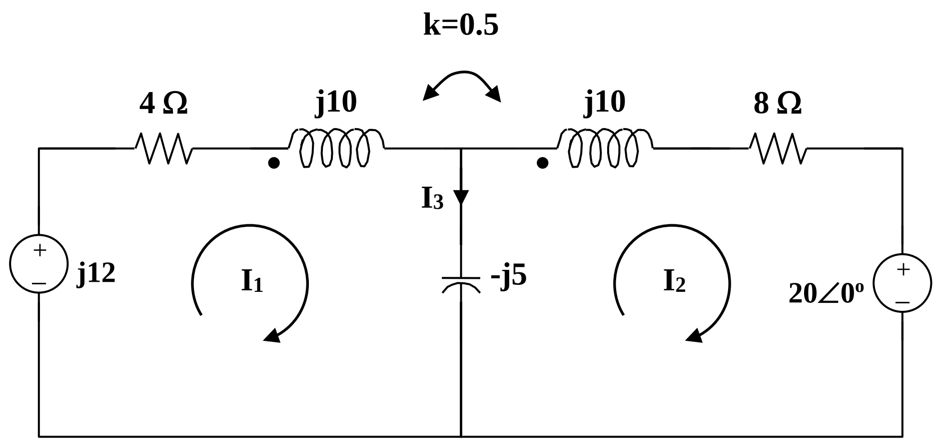
$X_{La} = X_{L1} - (-X_M) = 40 + 25 = 65\ \Omega$ and $X_{Lb} = X_{L2} - (-X_M) = 40 + 25 = 55\ \Omega$.

Finally, $X_C = X_M$ thus, the T-section is as shown below.



Solution 13.20

Transform the current source to a voltage source as shown below.



$$k = M/\sqrt{L_1L_2} \quad \text{or} \quad M = k\sqrt{L_1L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.5(10) = 5$$

For mesh 1, $j12 = (4 + j10 - j5)I_1 + j5I_2 + j5I_2 = (4 + j5)I_1 + j10I_2$ (1)

For mesh 2, $0 = 20 + (8 + j10 - j5)I_2 + j5I_1 + j5I_1$

$$-20 = +j10I_1 + (8 + j5)I_2 \quad (2)$$

From (1) and (2), $\begin{bmatrix} j12 \\ 20 \end{bmatrix} = \begin{bmatrix} 4 + j5 & +j10 \\ +j10 & 8 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 107 + j60, \quad \Delta_1 = -60 - j296, \quad \Delta_2 = 40 - j100$$

$$I_1 = \Delta_1/\Delta = \mathbf{2.462\angle 72.18° \ A}$$

$$I_2 = \Delta_2/\Delta = \mathbf{878\angle -97.48° \ mA}$$

$$I_3 = I_1 - I_2 = \mathbf{3.329\angle 74.89° \ A}$$

$$i_1 = 2.462 \cos(1000t + 72.18°) \text{ A}$$

$$i_2 = 0.878 \cos(1000t - 97.48°) \text{ A}$$

At $t = 2$ ms, $1000t = 2$ rad $= 114.6°$

$$i_1(0.002) = 2.462\cos(114.6° + 72.18°) = -2.445\text{A}$$

$$-2.445$$

$$i_2 = 0.878\cos(114.6° - 97.48°) = -0.8391$$

The total energy stored in the coupled coils is

$$w = 0.5L_1 i_1^2 + 0.5L_2 i_2^2 + Mi_1 i_2$$

Since $\omega L_1 = 10$ and $\omega = 1000$, $L_1 = L_2 = 10$ mH, $M = 0.5L_1 = 5$mH

$$w = 0.5(0.01)(-2.445)^2 + 0.5(0.01)(-0.8391)^2 + 0.05(-2.445)(-0.8391)$$

$$\mathbf{w = 43.67\ mJ}$$

Solution 13.21

Using Fig. 13.90, design a problem to help other students to better understand energy in a coupled circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit of Fig. 13.90. Calculate the power absorbed by the 4-Ω resistor.

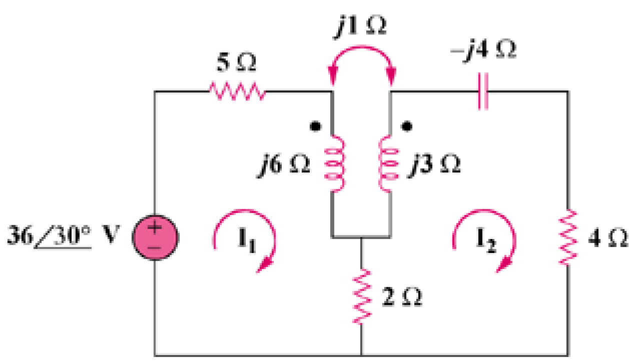


Figure 13.90

Solution

For mesh 1, $36\angle 30° = (7 + j6)I_1 - (2 + j)I_2$ (1)

For mesh 2, $0 = (6 + j3 - j4)I_2 - 2I_1 - jI_1 = -(2 + j)I_1 + (6 - j)I_2$ (2)

Placing (1) and (2) into matrix form, $\begin{bmatrix} 36\angle 30° \\ 0 \end{bmatrix} = \begin{bmatrix} 7 + j6 & -2 - j \\ -2 - j & 6 - j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = 45 + j25 = 51.48\angle 29.05°, \quad \Delta_1 = (6 - j)36\angle 30° = 219\angle 20.54°$$

$\Delta_2 = (2 + j)36\angle 30° = 80.5\angle 56.57°$, $I_1 = \Delta_1/\Delta =$ $\mathbf{4.254\angle{-8.51}°\ A}$, $I_2 = \Delta_2/\Delta =$ $\mathbf{1.5637\angle 27.52°\ A}$

Power absorbed by the 4-ohm resistor,

$$= 0.5(I_2)^2 4 = 2(1.5637)^2 = \mathbf{4.89\ watts}$$

Solution 13.22

Find current $\mathbf{I}_o$ in the circuit of Fig. 13.91.

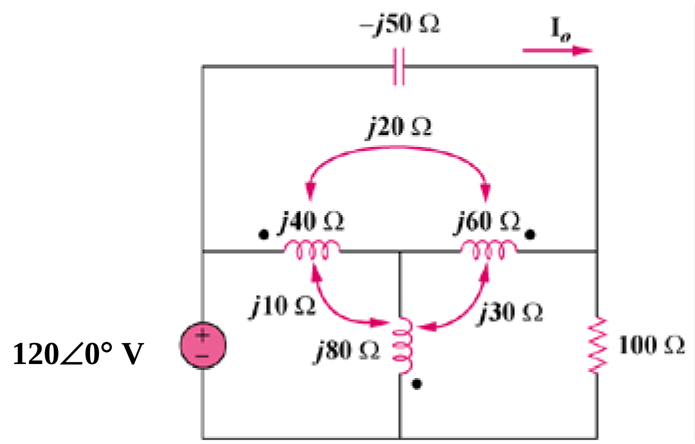
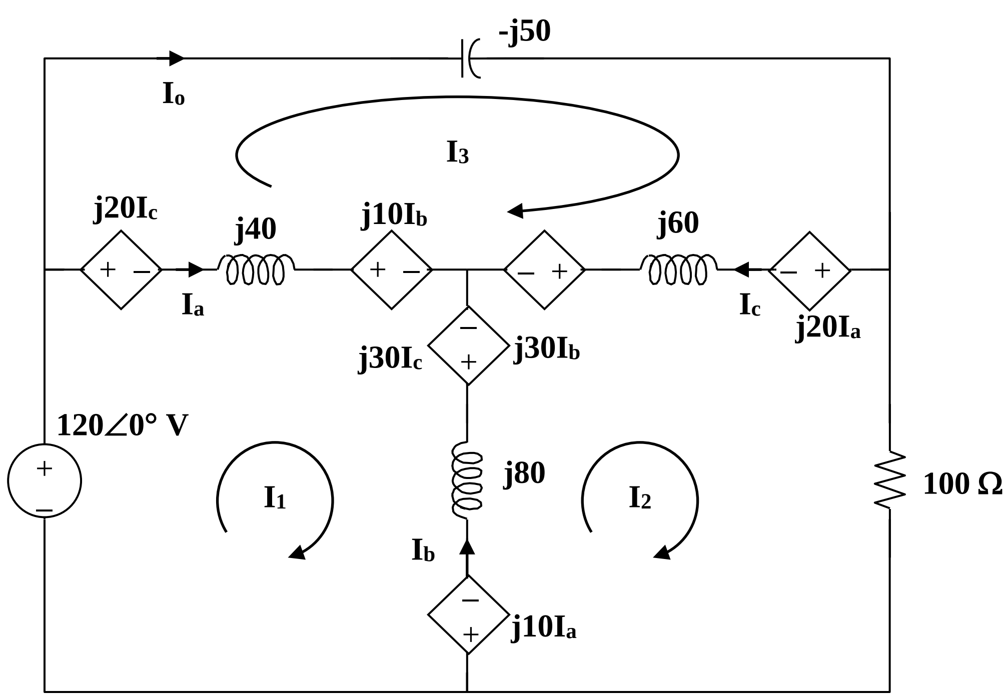


Figure 13.91
For Prob. 13.22.

Solution

With more complex mutually coupled circuits, it may be easier to show the effects of the coupling as sources in terms of currents that enter or leave the dot side of the coil. Figure 13.85 then becomes,



Note the following,

$$I_a = I_1 - I_3$$
$$I_b = I_2 - I_1$$
$$I_c = I_3 - I_2$$

$$\text{and} \quad I_o = I_3$$

Now all we need to do is to write the mesh equations and to solve for I_o.

Loop # 1,

$$-120 + j20(I_3 - I_2) + j40(I_1 - I_3) + j10(I_2 - I_1) - j30(I_3 - I_2)$$
$$+ j80(I_1 - I_2) - j10(I_1 - I_3) = 0$$

$$j100I_1 - j60I_2 - j40I_3 = 120$$

Multiplying everything by (1/j10) yields $\quad 10I_1 - 6I_2 - 4I_3 = -j12 \qquad (1)$

Loop # 2,

$$j10(I_1 - I_3) + j80(I_2 - I_1) + j30(I_3 - I_2) - j30(I_2 - I_1)$$
$$+ j60(I_2 - I_3) - j20(I_1 - I_3) + 100I_2 = 0$$

$$-j60I_1 + (100 + j80)I_2 - j20I_3 = 0 \qquad (2)$$

Loop # 3,

$$-j50I_3 + j20(I_1 - I_3) + j60(I_3 - I_2) + j30(I_2 - I_1)$$
$$-j10(I_2 - I_1) + j40(I_3 - I_1) - j20(I_3 - I_2) = 0$$

$$-j40I_1 - j20I_2 + j10I_3 = 0 \qquad (3)$$

Thus, $\begin{bmatrix} j100 & -j60 & -j40 \\ -j60 & 100+j80 & -j20 \\ -j40 & -j20 & j10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$

Using MATLAB we get,

```
>> Z=[100j,-60j,-40j;-60j,100+80j,-20j;-40j,-20j,10j]

Z =

   1.0e+02 *

   0.0000 + 1.0000i   0.0000 - 0.6000i   0.0000 - 0.4000i
   0.0000 - 0.6000i   1.0000 + 0.8000i   0.0000 - 0.2000i
```

```
   0.0000 - 0.4000i   0.0000 - 0.2000i   0.0000 + 0.1000i

>> V=[120;0;0]

V =

   120
     0
     0

>> I=inv(Z)*V

I =

   0.4523 + 0.3415i
  -0.1938 + 0.7108i
   1.4215 + 2.7877i
```

$$\mathbf{I_3 = I_o = 3.129\angle 62.98^o \text{ amp}}.$$

Chapter 13, Solution 23.

Let i_s = 5 cos(100t) A. Calculate the voltage across the capacitor, v_C. Also calculate the value of the energy stored in the coupled coils at t = 2.5π ms.

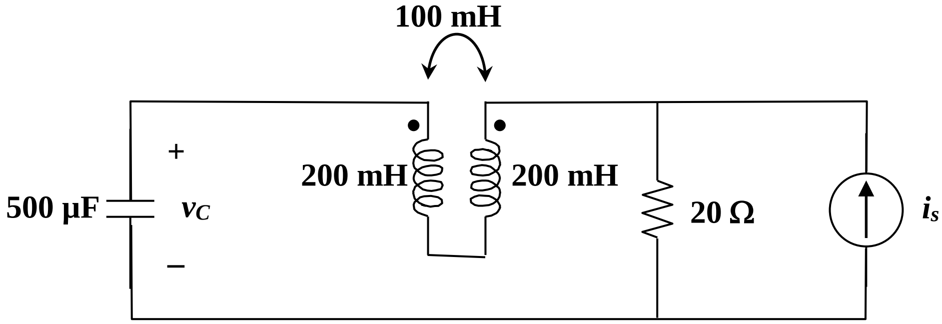
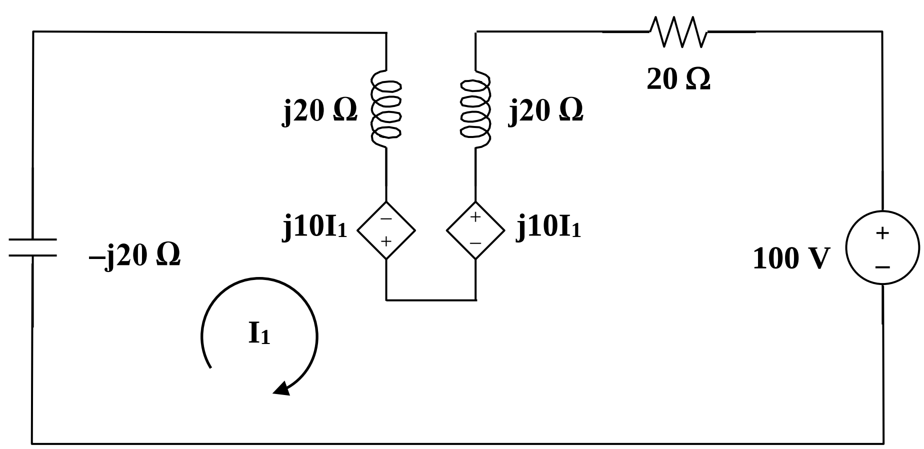


Figure 13.92
For Prob. 13.23.

Solution

Step 1. First we need to convert the circuit into the frequency domain and convert the coupled inductors into their dependent source equivalent. Then we can do source transformation to convert the 20 Ω resistor in parallel with the current source into a 20 Ω resistor in series with a 100 V voltage source. We note that ω = 100 rad/s which leads to the capacitor being equal to –j1/(0.0005x100) = –j20 Ω, the value of the individual inductors will be equal to j(0.2x100) = j20 Ω, and the mutual coupling equal to j(0.1x100) = j10 Ω. Now we have the following circuit,



$–j20I_1 + j20I_1 –j10I_1 – j10I_1 + j20I_1 + 20I_1 + 100 = 0$ and $V_C = j20I_1$. We then convert the value of V_C into the time domain and I_1 into the time domain. Thus, $w(0.25\pi) = 0.5(20–10–10+20)i(0.25\pi)^2$.

Step 2. $(-j20+j20-j10-j10+j20+20)\mathbf{I}_1 = -100$ or $\mathbf{I}_1 = -100/(20) = -5$ A. Thus, $\mathbf{V}_C = -j100 = 100\angle -90°$ or $v_C =$ **$100\cos(100t-90°)$ V** and $i_1 = 5\cos(100t-180°)$ A.

$$w(0.25\pi) = 0.5(20)[\cos(45°-180°)]^2 = 10(0.5) = \mathbf{5\ J}.$$

Solution 13.24

(a) $k = M/\sqrt{L_1L_2} = 1/\sqrt{4x2} = \mathbf{0.3535}$

(b) $\omega = 4$

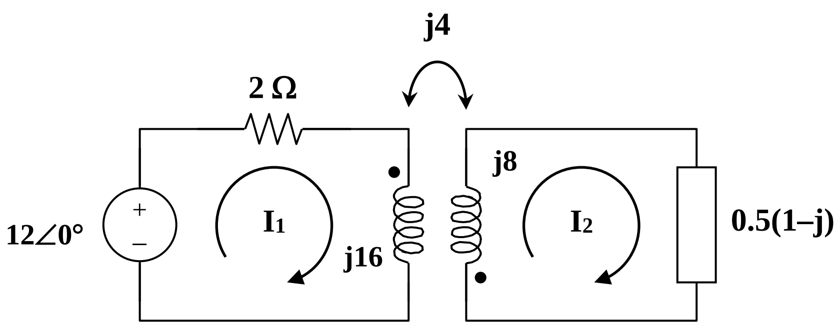
1/4 F leads to $1/(j\omega C) = -j/(4x0.25) = -j$

$1||(-j) = -j/(1 - j) = 0.5(1 - j)$

1 H produces $j\omega M = j4$

4 H produces j16

2 H becomes j8



$$12 = (2 + j16)I_1 + j4I_2$$

or $\quad 6 = (1 + j8)I_1 + j2I_2 \qquad (1)$

$0 = (j8 + 0.5 - j0.5)I_2 + j4I_1$ or $I_1 = (0.5 + j7.5)I_2/(-j4) \qquad (2)$

Substituting (2) into (1),

$$24 = (-11.5 - j51.5)I_2 \text{ or } I_2 = -24/(11.5 + j51.5) = -0.455\angle -77.41°$$

$$V_o = I_2(0.5)(1 - j) = 0.3217\angle 57.59°$$

$$v_o = \mathbf{321.7\cos(4t + 57.6°)\ mV}$$

(c) From (2), $I_1 = (0.5 + j7.5)I_2/(-j4) = 0.855\angle -81.21°$

$$i_1 = 0.885\cos(4t - 81.21°) \text{ A}, \quad i_2 = -0.455\cos(4t - 77.41°) \text{ A}$$

At t = 2s,

$$4t = 8 \text{ rad} = 98.37°$$

$$i_1 = 0.885\cos(98.37° - 81.21°) = 0.8169$$

$$i_2 = -0.455\cos(98.37° - 77.41°) = -0.4249$$

$$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2$$

$$= 0.5(4)(0.8169)^2 + 0.5(2)(-.4249)^2 + (1)(0.1869)(-0.4249) = \mathbf{1.168\ J}$$

Solution 3.25

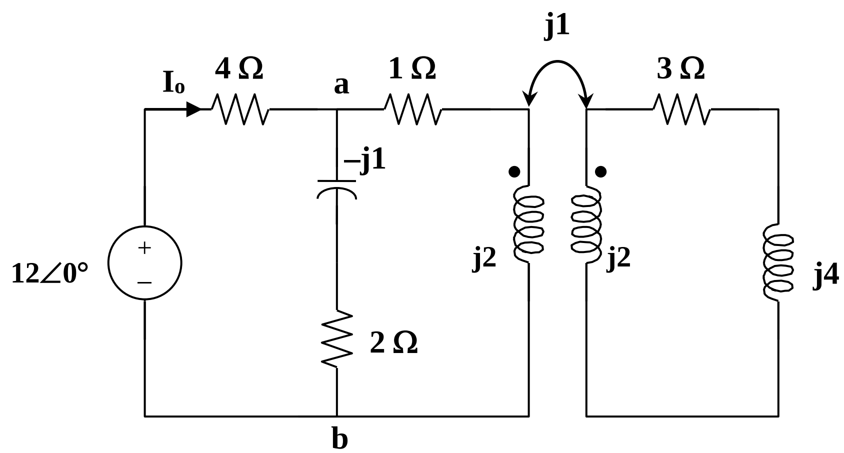
$m = k\sqrt{L_1 L_2} = 0.5$ H

We transform the circuit to frequency domain as shown below.

$$12\sin 2t \text{ converts to } 12\angle 0°, \ \omega = 2$$

$$0.5 \text{ F converts to } 1/(j\omega C) = -j$$

$$2 \text{ H becomes } j\omega L = j4$$



Applying the concept of reflected impedance,

$$Z_{ab} = (2 - j)||(1 + j2 + (1)^2/(j2 + 3 + j4))$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1 + j2 + (3/45) - j6/45)$$

$$= (2 - j)||(1.0667 + j1.8667)$$

$$=(2 - j)(1.0667 + j1.8667)/(3.0667 + j0.8667) = \mathbf{1.5085\angle 17.9° \ \Omega}$$

$$I_o = 12\angle 0°/(Z_{ab} + 4) = 12/(5.4355 + j0.4636) = 2.2\angle{-4.88°}$$

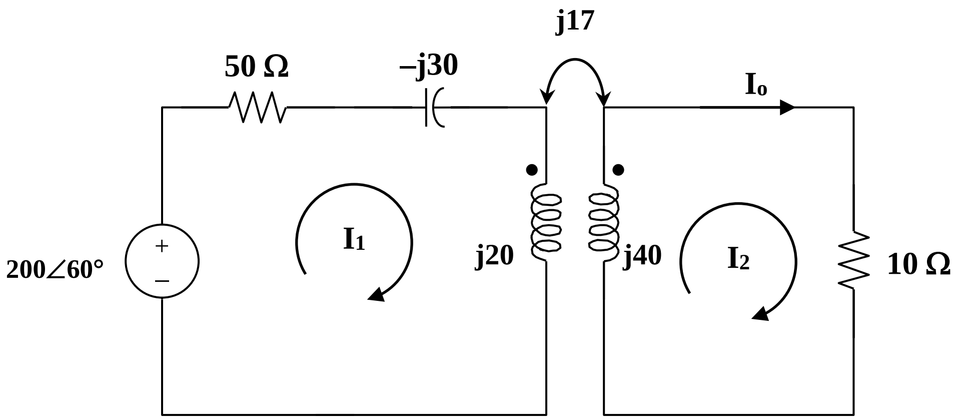
$$i_o = \mathbf{2.2\sin(2t - 4.88°) \ A}$$

Solution 13.26

$$M = k\sqrt{L_1 L_2}$$

$$\omega M = k\sqrt{\omega L_1 \omega L_2} = 0.601\sqrt{20x40} = 17$$

The frequency-domain equivalent circuit is shown below.



For mesh 1, $\quad -200\angle 60° + (50 - j30 + j20)I_1 - j17I_2 = 0$ or

$(50 - j10)I_1 - j17I_2 = 200\angle 60° \qquad (1)$

For mesh 2, $\quad (10 + j40)I_2 - j17I_1 = 0$ or $-j17I_1 + (10 + j40)I_2 = 0 \qquad (2)$

In matrix form,

$$\begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix} = \begin{bmatrix} 50 - j10 & -j17 \\ -j17 & 10 + j40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{or}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{\begin{bmatrix} 10 + j40 & j17 \\ j17 & 50 - j10 \end{bmatrix}}{500 + 400 + 289 - j100 + j2{,}000} \begin{bmatrix} 200\angle 60° \\ 0 \end{bmatrix}$$

$I_1 = (10+j40)(200\angle 60°)/(1{,}189+j1{,}900)$
$= (41.231\angle 75.964°)(200\angle 60°)/(2{,}241.4\angle 57.962°) = 3.679\angle 78°$ A and
$I_2 = j17(200\angle 60°)/(2{,}241.4\angle 57.962°) = 1.5169\angle 92.04°$ A

$$\mathbf{I_o = I_2 = 1.5169\angle 92.04°\ A}$$

It should be noted that switching the dot on the winding on the right only reverses the direction of $\mathbf{I_o}$.

Solution 13.27

Find the average power delivered to the 50-Ω resistor in the circuit of Fig. 13.96.

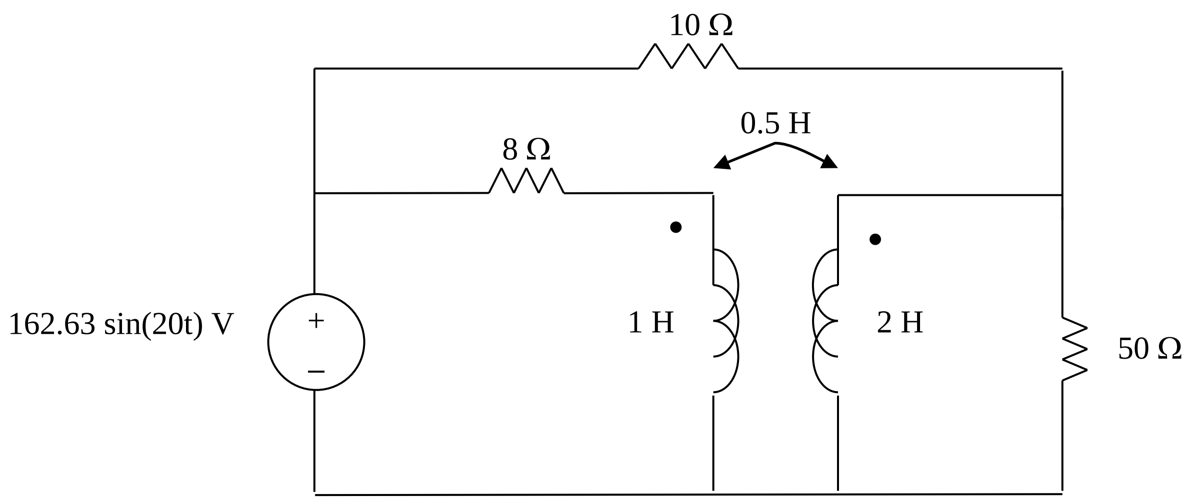


Figure 13.96
For Prob. 13.27.

Solution

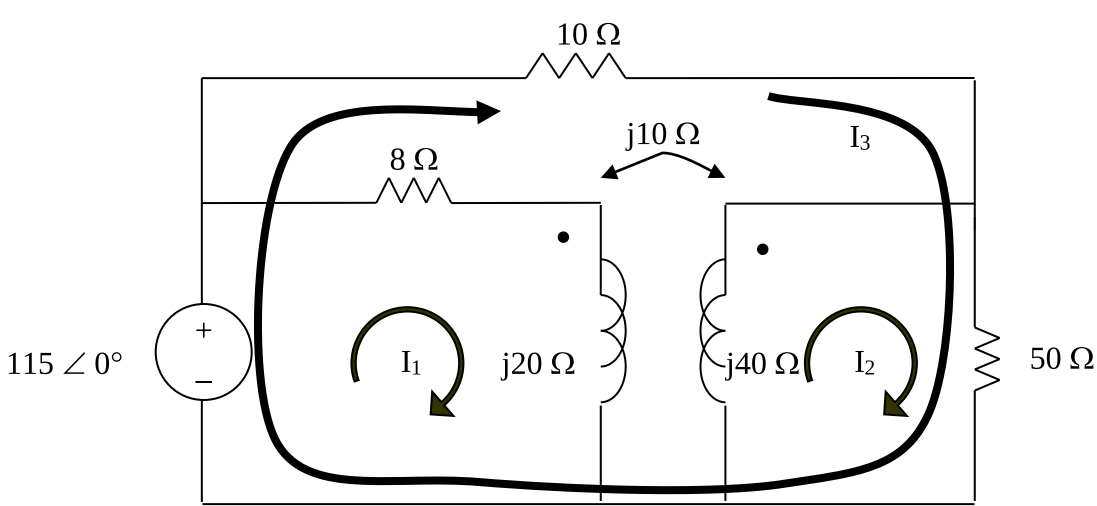
$V_s(\text{rms}) = 162.63/1.4142 = 115$ V

$1H \longrightarrow j\omega L = j20$

$2H \longrightarrow j\omega L = j40$

$0.5H \longrightarrow j\omega L = j10$

We apply mesh analysis to the circuit as shown below.



To make the problem easier to solve, let us have I_3 flow around the outside loop as shown.

For mesh 1,

$$-115 + 8\mathbf{I_1} + j20\mathbf{I_1} - j10\mathbf{I_2} = 0 \text{ or } (8+j20)\mathbf{I_1} - j10\mathbf{I_2} = 40 \qquad (1)$$

For mesh 2,

$$j40\mathbf{I_2} - j10\mathbf{I_1} + 50(\mathbf{I_2} + \mathbf{I_3}) = 0 \text{ or } -j10\mathbf{I_1} + (50+j40)\mathbf{I_2} + 50\mathbf{I_3} = 0 \qquad (2)$$

For mesh 3,

$$-115 + 10\mathbf{I_3} + 50(\mathbf{I_3} + \mathbf{I_2}) = 0 \text{ or } 50\mathbf{I_2} + 60\mathbf{I_3} = 40 \qquad (3)$$

In matrix form, (1) to (3) become

$$\begin{bmatrix} 8+j20 & -j10 & 0 \\ -j10 & 50+j40 & 50 \\ 0 & 50 & 60 \end{bmatrix} I = \begin{bmatrix} 115 \\ 0 \\ 115 \end{bmatrix}$$

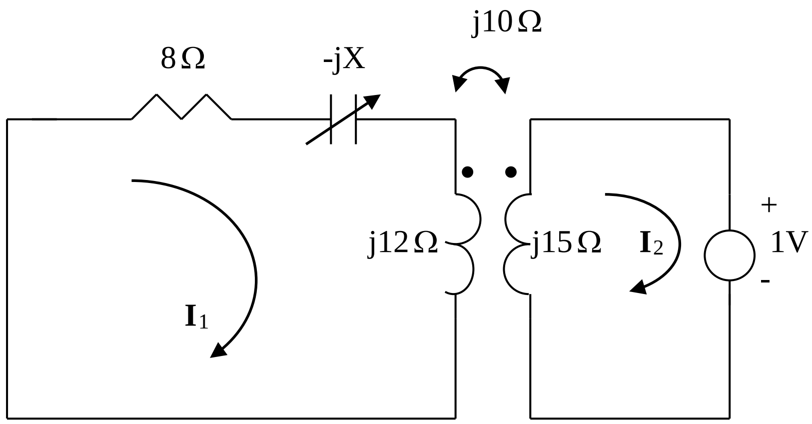
```
>> Z=[(8+20i),-10i,0;-10i,(50+40i),50;0,50,60]
Z =
   8.0000 +20.0000i        0 -10.0000i        0
        0 -10.0000i  50.0000 +40.0000i  50.0000
        0            50.0000            60.0000

>> V=[40;0;40]
V =
    115
     0
     115

>> I=inv(Z)*V
I =
   1.8268 – 4.3464i
   0.1762 + 1.3461i
   1.7698 – 1.1215i
```

Solving this leads to $\mathbf{I_{50}} = \mathbf{I_2} + \mathbf{I_3} = 0.1762 + j1.3461 + 1.7698 - j1.1215 = 1.946+j0.2246 = 1.9589\angle 6.58°$ A. This is already an rms value so,

$$P_{50} = (1.9589)^2(50) = \mathbf{191.86\ W}.$$

Solution 13.28

We find Z_{Th} by replacing the 20-ohm load with a unit source as shown below.



For mesh 1, $$0 = (8 - jX + j12)I_1 - j10I_2 \qquad (1)$$

For mesh 2,
$$1 + j15I_2 - j10I_1 = 0 \quad \longrightarrow \quad I_1 = 1.5I_2 - 0.1j \qquad (2)$$

Substituting (2) into (1) leads to
$$I_2 = \frac{-1.2 + j0.8 + 0.1X}{12 + j8 - j1.5X}$$

$$Z_{Th} = \frac{1}{-I_2} = \frac{12 + j8 - j1.5X}{1.2 - j0.8 - 0.1X}$$

$$|Z_{Th}| = 20 = \frac{\sqrt{12^2 + (8 - 1.5X)^2}}{\sqrt{(1.2 - 0.1X)^2 + 0.8^2}} \quad \longrightarrow \quad 0 = 1.75X^2 + 72X - 624$$

Solving the quadratic equation yields X = **6.425 Ω**

Solution 13.29

In the circuit of Fig. 13.97, find the value of the coupling coefficient *k* that will make the 10-Ω resistor dissipate 1.28 kW. For this value of *k*, find the energy stored in the coupled coils at $t = 1.5$ s.

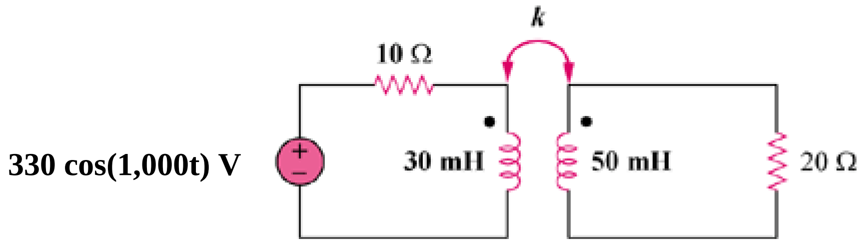


Figure 13.97
For Prob. 13.29.

Solution

30 mH becomes $j\omega L = j30x10^{-3}x10^{3} = j30\ \Omega$

50 mH becomes $j50\ \Omega$

Let $X = \omega M = 1{,}000M$

Using the concept of reflected impedance,

$\mathbf{Z_{in}} = 10 + j30 + X^2/(20 + j50)$

$\mathbf{I_1} = \mathbf{V}/\mathbf{Z_{in}} = 330/(10 + j30 + X^2/(20 + j50))$

$p = 0.5|\mathbf{I_1}|^2(10) = 1{,}280$ leads to $|\mathbf{I_1}|^2 = 256$ or $|\mathbf{I_1}| = 16$

$16 = |330(20 + j50)/(X^2 + (10 + j30)(20 + j50))|$

$= |330(20 + j50)/(X^2 - 1300 + j1100)|$ or

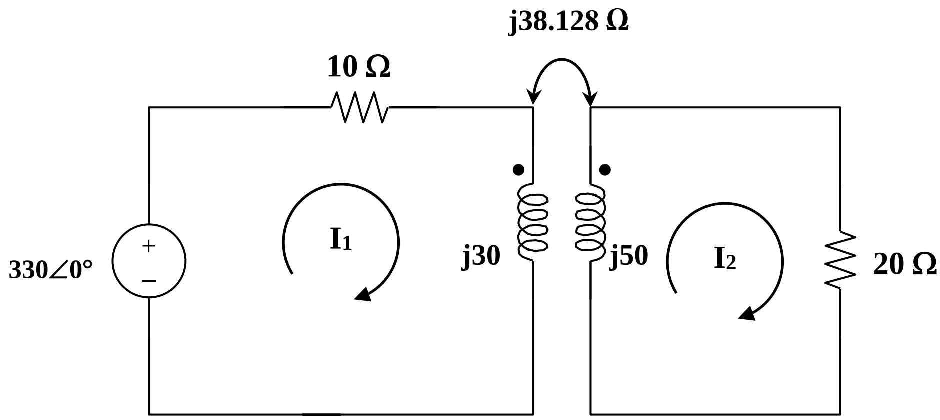
$256 = 108{,}900(400 + 2500)/((X^2 - 1300)^2 + 1{,}210{,}000)$

$(X^2 - 1300)^2 + 1{,}210{,}000 = 1{,}233{,}632.8$ or $(X^2 - 1{,}300)^2 = 23{,}632.8$ or

$X^2 - 1300 = \pm 153.73$ which means that there are two positive values of X which solve this equation. X = 33.857 and 38.128. Let us use the value of X = 38.128.

For X = 38.128 = ωM or M = **38.128 mH**.

$$k = M/\sqrt{L_1L_2} = 38.128/\sqrt{30x50} = \mathbf{0.9845}$$



$$330 = (10 + j30)\mathbf{I_1} - j38.128\mathbf{I_2} \qquad (1)$$

$$0 = (20 + j50)\mathbf{I_2} - j38.128\mathbf{I_1} \qquad (2)$$

In matrix form,
$$\begin{bmatrix} 330 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 + j30 & -j38.128 \\ -j38.128 & 20 + j50 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$\Delta = 200 - 1500 + j(600+500) + 1453.744 = 153.74 + j1100 = 1{,}110.69\angle 82.04°$,
$\Delta_1 = 6{,}600 + j16{,}500 = 17{,}771\angle 68.2°$, $\Delta_2 = j12{,}582$

$I_1 = \Delta_1/\Delta = 16\angle -13.81°$, $I_2 = \Delta_2/\Delta = 11.328\angle 7.97°$

$i_1 = 16\cos(1000t - 13.83°)$, $i_2 = 11.328\cos(1000t + 7.97°)$

At t = 1.5 ms, 1000t = 1.5 rad = 85.94°

$i_1 = 16\cos(85.94° - 13.83°) = 4.915$ A

$i_2 = 11.328\cos(85.94° + 7.97°) = -0.77245$ A

$w = 0.5L_1i_1^2 + 0.5L_2i_2^2 + Mi_1i_2 = 0.5(0.03)(4.915)^2 + 0.5(0.05)(-0.77245)^2$
$- 0.038128(4.915)(-0.77245) = 0.36236 + 0.014917 + 0.144756 = 0.5216$

$= \mathbf{521.6\ mJ}$

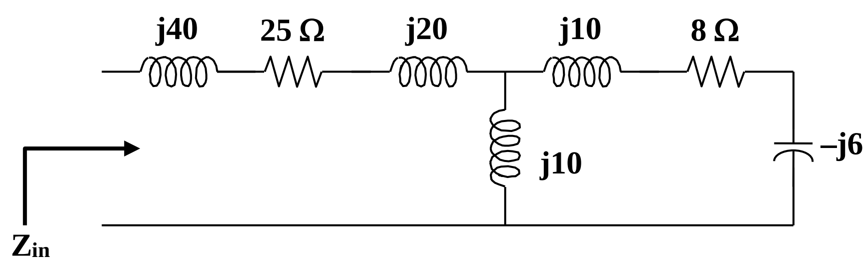
Solution 13.30

(a) $Z_{in} = j40 + 25 + j30 + (10)^2/(8 + j20 - j6)$

$= 25 + j70 + 100/(8 + j14) =$ **(28.08 + j64.62) ohms**

(b) $j\omega L_a = j30 - j10 = j20,\ j\omega L_b = j20 - j10 = j10,\ j\omega L_c = j10$

Thus the Thevenin Equivalent of the linear transformer is shown below.



$Z_{in} = j40 + 25 + j20 + j10\|(8 + j4) = 25 + j60 + j10(8 + j4)/(8 + j14)$

$=$ **(28.08 + j64.62) ohms**

Solution 13.31

Using Fig. 13.100, design a problem to help other students to better understand linear transformers and how to find T-equivalent and Π–equivalent circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 13.99, find:

(a) the *T*-equivalent circuit,

(b) the Π-equivalent circuit.

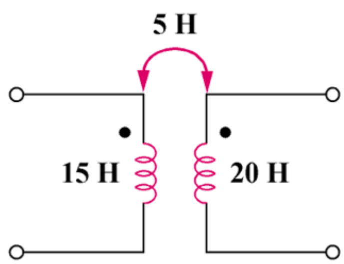


Figure 13.99

Solution

(a) $L_a = L_1 - M = \mathbf{10\ H}$

$L_b = L_2 - M = \mathbf{15\ H}$

$L_c = M = \mathbf{5\ H}$

(b) $L_1L_2 - M^2 = 300 - 25 = 275$

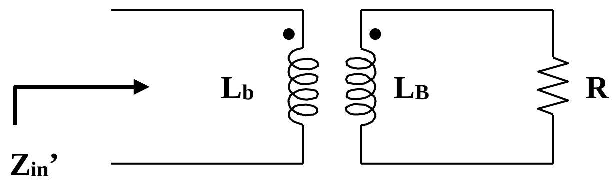
$L_A = (L_1L_2 - M^2)/(L_1 - M) = 275/15 = \mathbf{18.33\ H}$

$L_B = (L_1L_2 - M^2)/(L_1 - M) = 275/10 = \mathbf{27.5\ H}$

$L_C = (L_1L_2 - M^2)/M = 275/5 = \mathbf{55\ H}$

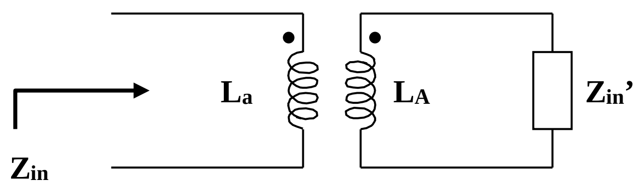
Solution 13.32

We first find Z_{in} for the second stage using the concept of reflected impedance.



$Z_{in}' = j\omega L_b + \omega^2 M_b^2/(R + j\omega L_b) = (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)/(R + j\omega L_b)$
(1)

For the first stage, we have the circuit below.



$$Z_{in} = j\omega L_a + \omega^2 M_a^2/(j\omega L_a + Z_{in})$$

$$= (-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a Z_{in})/(j\omega L_a + Z_{in}) \qquad (2)$$

Substituting (1) into (2) gives,

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \dfrac{(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_b L_a + j\omega^3 L_b M_a^2 + j\omega L_a(j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega R La - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \frac{\boldsymbol{\omega^2 R(L_a^2 + L_a L_b - M_a^2) + j\omega^3(L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}}{\boldsymbol{\omega^2(L_a L_b + L_b^2 - M_b^2) - j\omega R(L_a + L_b)}}$$

Solution 13.33

$$
\begin{aligned}
Z_{in} &= 10 + j12 + (15)^2/(20 + j\,40 - j5) = 10 + j12 + 225/(20 + j35) \\
&= 10 + j12 + 225(20 - j35)/(400 + 1225) \\
&= (12.769 + j7.154)\ \Omega
\end{aligned}
$$

Solution 13.34

Using Fig. 13.103, design a problem to help other students to better understand how to find the input impedance of circuits with transformers.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the input impedance of the circuit in Fig. 13.102.

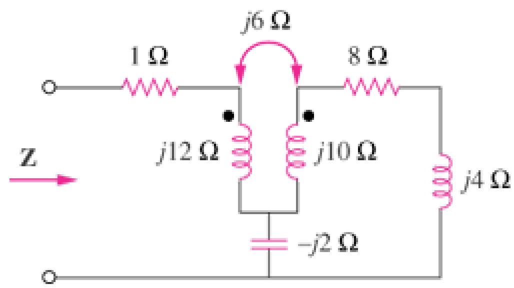
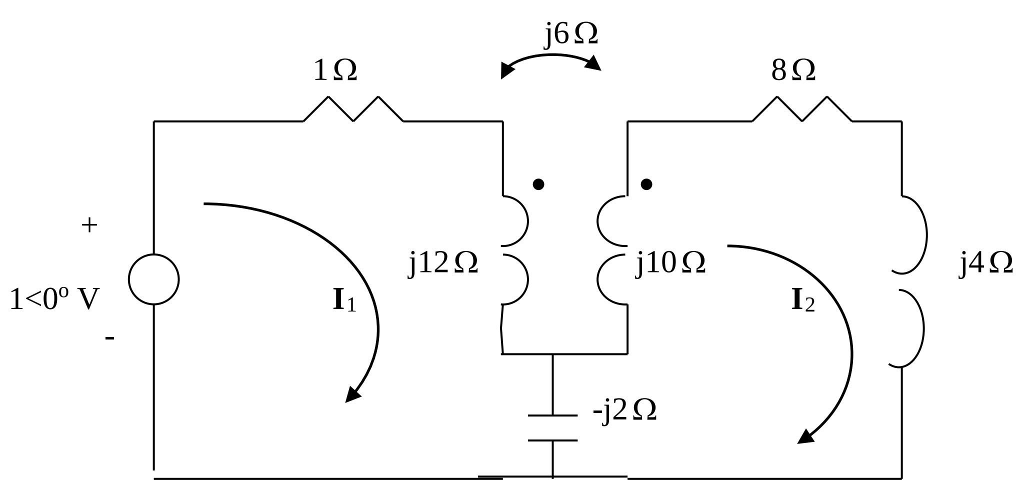


Figure 13.102

Solution

Insert a 1-V voltage source at the input as shown below.



For loop 1,

$$1 = (1 + j10)I_1 - j4I_2 \qquad (1)$$

For loop 2,

$$0=(8+j4+j10-j2)I_2+j2I_1-j6I_1 \quad\longrightarrow\quad 0=-jI_1+(2+j3)I_2 \qquad (2)$$

Solving (1) and (2) leads to $\mathbf{I}_1$=0.019 –j0.1068

$$\boldsymbol{Z=\frac{1}{I_1}=1.6154+j9.077=\underline{9.219\angle 79.91^o\,\Omega}}$$

Alternatively, an easier way to obtain **Z** is to replace the transformer with its equivalent T circuit and use series/parallel impedance combinations. This leads to exactly the same result.

Solution 13.35

For mesh 1,

$$16 = (10 + j4)I_1 + j2I_2 \qquad (1)$$

For mesh 2,
$$0 = j2I_1 + (30 + j26)I_2 - j12I_3 \qquad (2)$$

For mesh 3,
$$0 = -j12I_2 + (5 + j11)I_3 \qquad (3)$$

We may use MATLAB to solve (1) to (3) and obtain

$$\mathbf{I_1} = 1.3736 - \mathrm{j}0.5385 = \mathbf{1.4754\angle{-21.41}^\circ\ A}$$
$$\mathbf{I_2} = -0.0547 - \mathrm{j}0.0549 = \mathbf{77.5\angle{-134.85}^\circ\ mA}$$
$$\mathbf{I_3} = -0.0268 - \mathrm{j}0.0721 = \mathbf{77\angle{-110.41}^\circ\ mA}$$

1.4754∠–21.41° A, 77.5∠–134.85° mA, 77∠–110.41° mA

Solution 13.36

Following the two rules in section 13.5, we obtain the following:

(a) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$ $(n = V_2/V_1)$

(b) $V_2/V_1 = \mathbf{-n}$, $I_2/I_1 = \mathbf{-1/n}$

(c) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{1/n}$

(d) $V_2/V_1 = \mathbf{n}$, $I_2/I_1 = \mathbf{-1/n}$

Solution 13.37

A 240/2400 V (rms) step-up ideal transformer delivers 50 kW to a resistive load. Calculate: (a) the turns ratio, (b) the primary current, (c) the secondary current.

Solution

(a) $n = \dfrac{V_2}{V_1} = \dfrac{2400}{240} = \mathbf{10}$

(b) $\mathbf{S_1} = \mathbf{V_1}(\mathbf{I_1})^* = \mathbf{S_2} = \mathbf{V_2}(\mathbf{I_2})^* = 50{,}000$ which leads to

$\mathbf{I_1} = 50{,}000/240 = \mathbf{208.3\ A}$.

(c) $\mathbf{I_2} = 50{,}000/2{,}400 = \mathbf{20.83\ A}$.

Solution 13.38

Design a problem to help other students to better understand ideal transformers.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A 4-kVA, 2300/230-V rms transformer has an equivalent impedance of $2\angle 10°\Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

Solution

$Z_{in} = Z_p + Z_L/n^2$, $n = v_2/v_1 = 230/2300 = 0.1$

$v_2 = 230$ V, $s_2 = v_2 I_2^*$

$I_2^* = s_2/v_2 = 17.391\angle -53.13°$ or $I_2 = 17.391\angle 53.13°$ A

$Z_L = v_2/I_2 = 230\angle 0°/17.391\angle 53.13° = 13.235\angle -53.13°$

$Z_{in} = 2\angle 10° + 1323.5\angle -53.13°$

$= 1.97 + j0.3473 + 794.1 - j1058.8$

$Z_{in} = \mathbf{1.324\angle -53.05°\ k\Omega}$

Solution 13.39

Referred to the high-voltage side,

$Z_L = (1200/240)^2(0.8\angle 10°) = 20\angle 10°$

$Z_{in} = 60\angle -30° + 20\angle 10° = 76.4122\angle -20.31°$

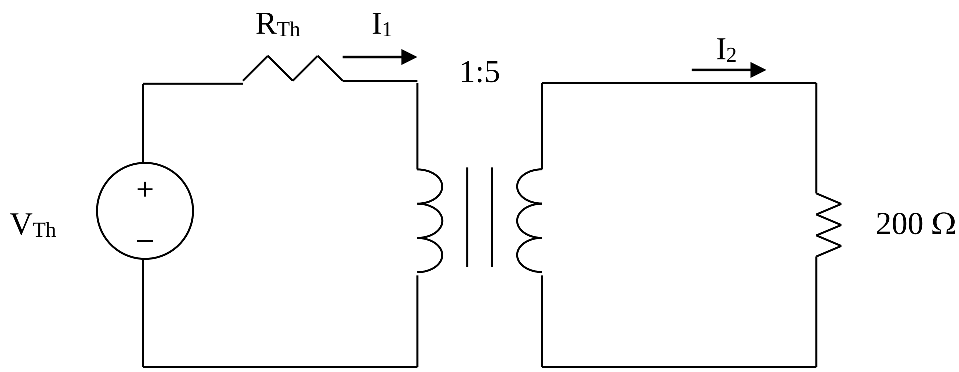
$I_1 = 1200/Z_{in} = 1200/76.4122\angle -20.31° = \mathbf{15.7\angle 20.31°\ A}$

Since $S = I_1 v_1 = I_2 v_2$, $I_2 = I_1 v_1/v_2$

$= (1200/240)(15.7\angle 20.31°) = \mathbf{78.5\angle 20.31°\ A}$

Solution 13.40

Consider the circuit as shown below.



We reflect the 200-Ω load to the primary side.

$$Z_p = 100 + \frac{200}{5^2} = 108$$

$$I_1 = \frac{10}{108}, \qquad I_2 = \frac{I_1}{n} = 2/108$$

$$P = \frac{1}{2}|I_2|^2 R_L = \frac{1}{2}(\frac{2}{108})^2(200) = \underline{34.3 \text{ mW}}$$

Solution 13.41

Given $\mathbf{I}_2$ = 2 A, determine the value of $\mathbf{I}_s$.

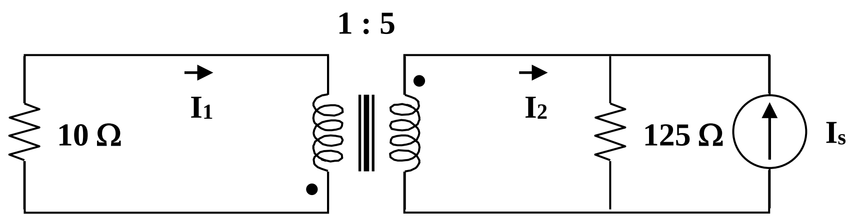


Figure 13.105
For Prob. 13.41.

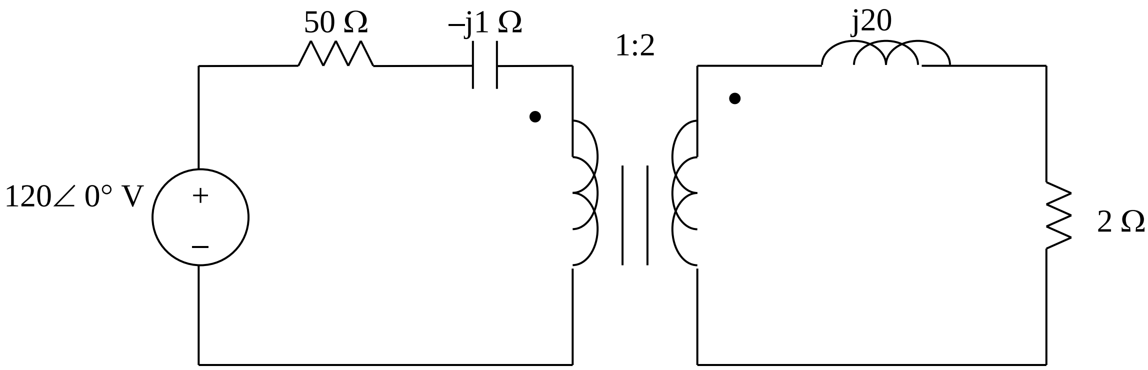
Solution

Step 1. First we note that the dots are not relevant for this problem (the value of I_2 is independent of the location of the dots). Thus, all we need to do is to reflect the 10 Ω to the right hand side of the circuit. The value of the reflected resistance is equal to 25x10 = 250 Ω. Current division gives us $\mathbf{I}_2 = 125(-\mathbf{I}_s)/(250+125) = 2$. Now all we need to do is to solve for $\mathbf{I}_s$.

Step 2. $2\text{x}375 = 125(-\mathbf{I}_s)$ or $\mathbf{I}_s = \mathbf{-6\ A}$.

Solution 13.42

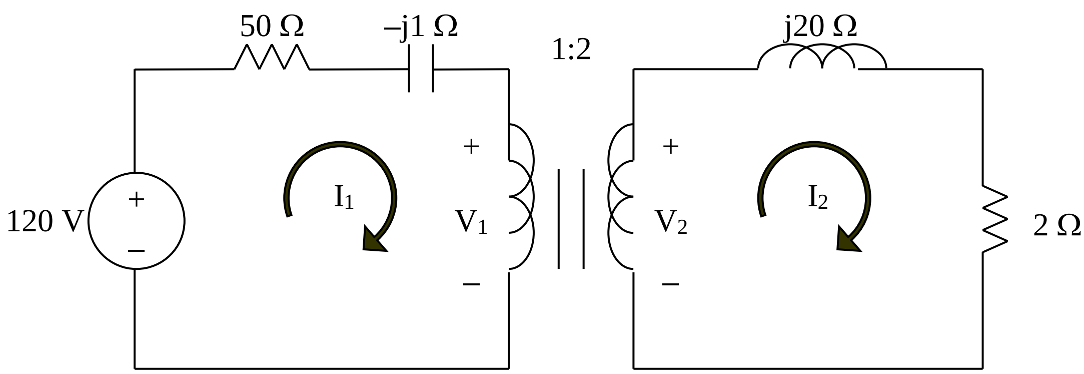
For the circuit in Fig. 13.106, determine the power absorbed by the 2 Ω resistor. Assume the 120 V source is an rms value.



Ideal
Figure 13.106
For Prob. 13.42.

Solution

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-120 + (50\text{–}j)I_1 + V_1 = 0 \qquad (1)$$

For mesh 2,

$$-V_2 + (2+j20)I_2 = 0 \qquad (2)$$

At the transformer terminals,

$$V_2 = 2V_1 \text{ or } 2V_1 - V_2 = 0 \qquad (3)$$

$$I_1 = 2I_2 \text{ or } I_1 - 2I_2 = 0 \qquad (4)$$

From (1) to (4),

$$\begin{bmatrix} 50-j & 0 & 1 & 0 \\ 0 & 2+j20 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 1 & -2 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this with MATLAB,

```
>> A = [(50-j) 0 1 0;0 (2+20j) 0 -1;0 0 2 -1;1 -2 0 0]

A =

  Columns 1 through 3

  50.0000 - 1.0000i        0              1.0000
       0              2.0000 +20.0000i        0
       0                   0            2.0000
   1.0000            -2.0000                   0

  Column 4

       0
  -1.0000
  -1.0000
       0

>> B = [120;0;0;0]

B =

   120
    0
    0
    0

>> C = inv(A)*B

C =

   2.3614 – 0.8170i                (I1)
   1.1806 – 0.0934i                (I2)
   2.1159 + 11.7136i               (V1)
   4.2318 + 23.4272i               (V2)
```

$$I_2 = (1.1806 - j0.0934) \text{ A or } 1.1843\angle -4.52° \text{ A}$$

The power absorbed by the 2-Ω resistor is

$$P = |I_2|^2R = (1.1843)^2 2 = \mathbf{2.805\ W}.$$

Solution 13.43

Obtain $\mathbf{V}_1$ and $\mathbf{V}_2$ in the ideal transformer circuit of Fig. 13.107.

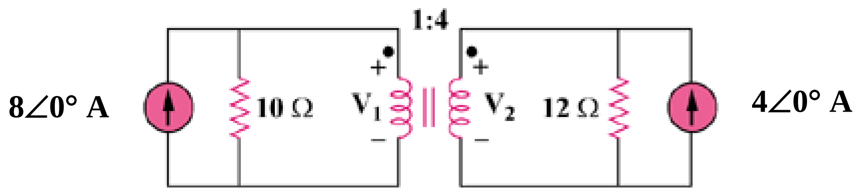
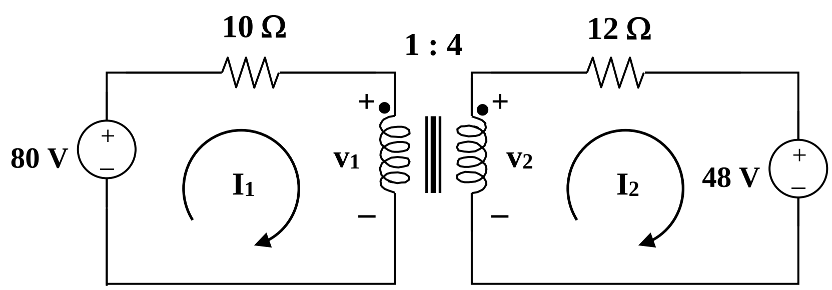


Figure 13.107
For Prob. 13.43.

Solution

Transform the two current sources to voltage sources, as shown below.



Using mesh analysis, $\quad -80 + 10I_1 + v_1 = 0$

$$80 = v_1 + 10I_1 \qquad (1)$$

$$48 + 12I_2 - v_2 = 0 \text{ or } 48 = v_2 - 12I_2 \qquad (2)$$

At the transformer terminal, $\quad v_2 = nv_1 = 4v_1 \qquad (3)$

$$I_1 = nI_2 = 4I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we get,

$$80 = v_1 + 40I_2 \qquad (5)$$

$$48 = 4v_1 - 12I_2 \qquad (6)$$

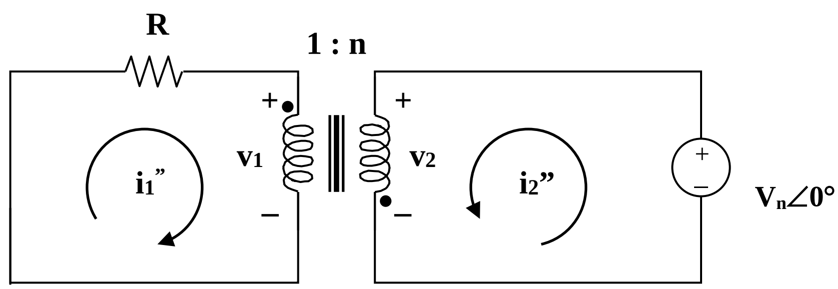
Solving (5) and (6) gives $\quad v_1 = \mathbf{16.744\ V}$ and $v_2 = 4v = \mathbf{66.98\ V}$

Solution 13.44

We can apply the superposition theorem. Let $i_1 = i_1' + i_1''$ and $i_2 = i_2' + i_2''$ where the single prime is due to the DC source and the double prime is due to the AC source. Since we are looking for the steady-state values of i_1 and i_2,

$$i_1' = i_2' = 0.$$

For the AC source, consider the circuit below.



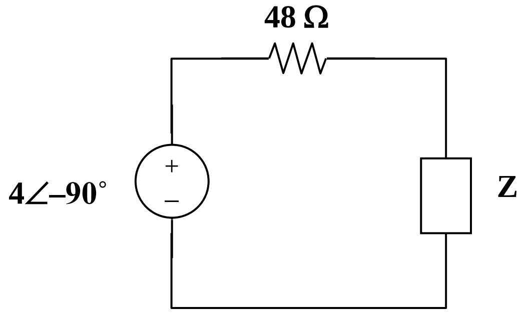
$$v_2/v_1 = -n, \quad I_2''/I_1'' = -1/n$$

But $v_2 = v_m$, $v_1 = -v_m/n$ or $I_1'' = v_m/(Rn)$

$$I_2'' = -I_1''/n = -v_m/(Rn^2)$$

Hence, $i_1(t) = \mathbf{(v_m/Rn)\cos\omega t\ A}$, and $i_2(t) = \mathbf{(-v_m/(n^2R))\cos\omega t\ A}$

Solution 13.45



$$Z_L = 8 - \frac{j}{\omega C} = 8 - j4, \;\; n = 1/3$$

$$Z = \frac{Z_L}{n^2} = 9Z_L = 72 - j36$$

$$I = \frac{4\angle -90^\circ}{48 + 72 - j36} = \frac{4\angle -90^\circ}{125.28\angle -16.7^\circ} = 0.03193\angle -73.3^\circ$$

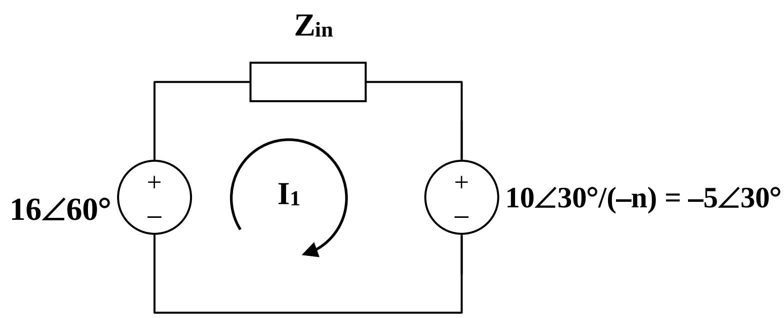
We now have some choices, we can go ahead and calculate the current in the second loop and calculate the power delivered to the 8-ohm resistor directly or we can merely say that the power delivered to the equivalent resistor in the primary side must be the same as the power delivered to the 8-ohm resistor. Therefore,

$$P_{8\Omega} = \left|\frac{I^2}{2}\right| 72 = 0.5098\text{x}10^{-3} 72 = \mathbf{36.71\ mW}$$

The student is encouraged to calculate the current in the secondary and calculate the power delivered to the 8-ohm resistor to verify that the above is correct.

Solution 13.46

(a) Reflecting the secondary circuit to the primary, we have the circuit shown below.



$\mathbf{Z_{in}} = 10 + j16 + (1/4)(12 - j8) = 13 + j14$

$-16\angle 60° + \mathbf{Z_{in}I_1} - 5\angle 30° = 0$ or $\mathbf{I_1} = (16\angle 60° + 5\angle 30°)/(13 + j14)$

Hence, $\mathbf{I_1} = \mathbf{1.072\angle 5.88°\ A}$, and $\mathbf{I_2} = -0.5\mathbf{I_1} = \mathbf{0.536\angle 185.88°\ A}$

(b) Switching a dot will not affect Z_{in} but will affect $\mathbf{I_1}$ and $\mathbf{I_2}$.

$\mathbf{I_1} = (16\angle 60° - 5\angle 30°)/(13 + j14) = \mathbf{0.625\ \angle 25\ A}$

and $\mathbf{I_2} = 0.5\mathbf{I_1} = \mathbf{0.3125\angle 25°\ A}$

Solution 13.47

Find v(t) for the circuit in Fig. 13.111.

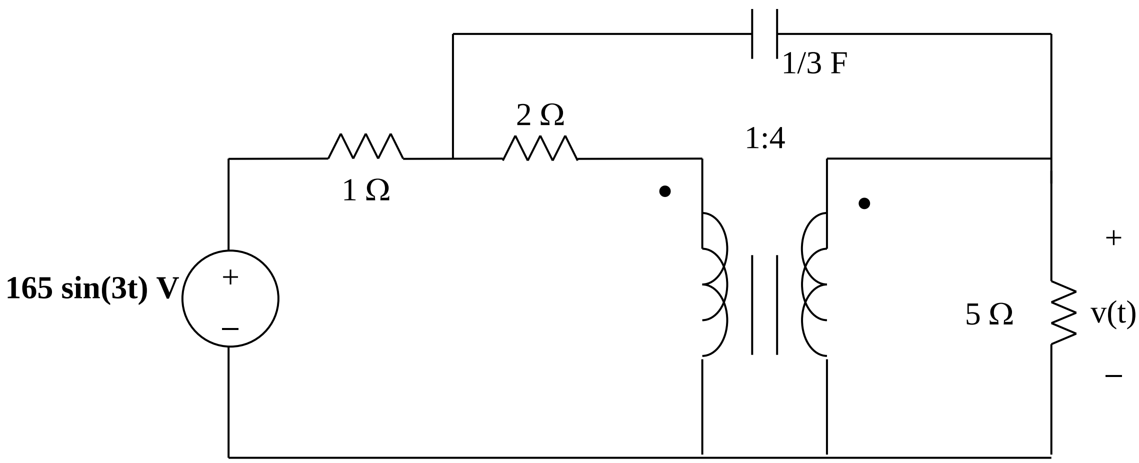
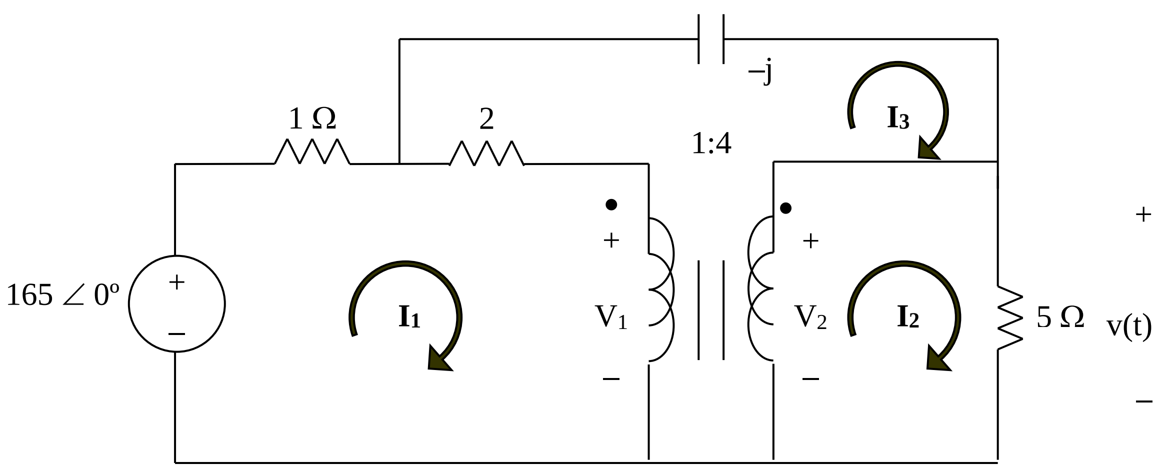


Figure 13.111
For Prob. 13.47.

Solution

$$\text{(1/3) F } 1F \longrightarrow \frac{1}{j\omega C} = \frac{1}{j3x1/3} = -j1$$

Consider the circuit shown below.



For mesh 1,

$$3I_1 - 2I_3 + V_1 = 165 \qquad (1)$$

For mesh 2,

$$5I_2 - V_2 = 0 \qquad (2)$$

For mesh 3,

$$-2I_1 + (2-j)I_3 - V_1 + V_2 = 0 \qquad (3)$$

At the terminals of the transformer,

$$V_2 = nV_1 = 4V_1 \qquad (4)$$

$$I_1 - I_3 = 4(I_2 - I_3) \qquad (5)$$

In matrix form,

$$\begin{bmatrix} 3 & 0 & -2 & 1 & 0 \\ 0 & 5 & 0 & 0 & -1 \\ -2 & 0 & 2-j & -1 & 1 \\ 0 & 0 & 0 & -4 & 1 \\ 1 & -4 & 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 165 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving this using MATLAB yields

```
>>A = [3,0,-2,1,0; 0,5,0,0,-1; -2,0,(2-j),-1,1 ;0,0,0,-4,1; 1,-4,3,0,0]


A =

   3.0000         0         -2.0000          1.0000          0
        0    5.0000               0               0    -1.0000
  -2.0000         0   2.0000 - 1.0000i  -1.0000         1.0000
        0         0               0         -4.0000     1.0000
   1.0000   -4.0000          3.0000               0          0

>>U = [165;0;0;0;0]

>>X = inv(A)*U

X =

   53.427 + 0.8085i
   21.809 + 2.0914i
   11.274 + 2.5204i
   27.262 + 2.6152i
   109.053 + 10.465i
```

$\mathbf{V} = 5\mathbf{I_2} = \mathbf{V_2} = 109.053 + j10.465 = 109.55\angle 5.48°$ V, therefore,

v(t) = **109.55 sin(3t+5.48°) V**

Solution 13.48

Using Fig. 13.113, design a problem to help other students to better understand how ideal transformers work.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find $\mathbf{I}_x$ in the ideal transformer circuit of Fig. 13.112.

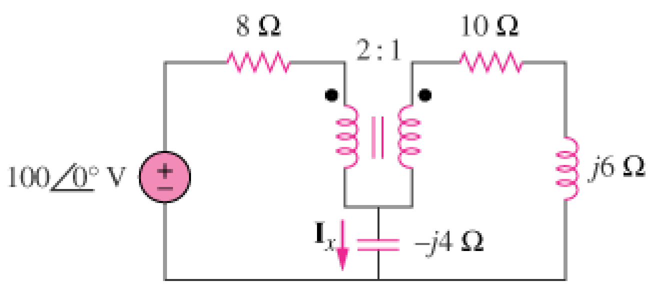
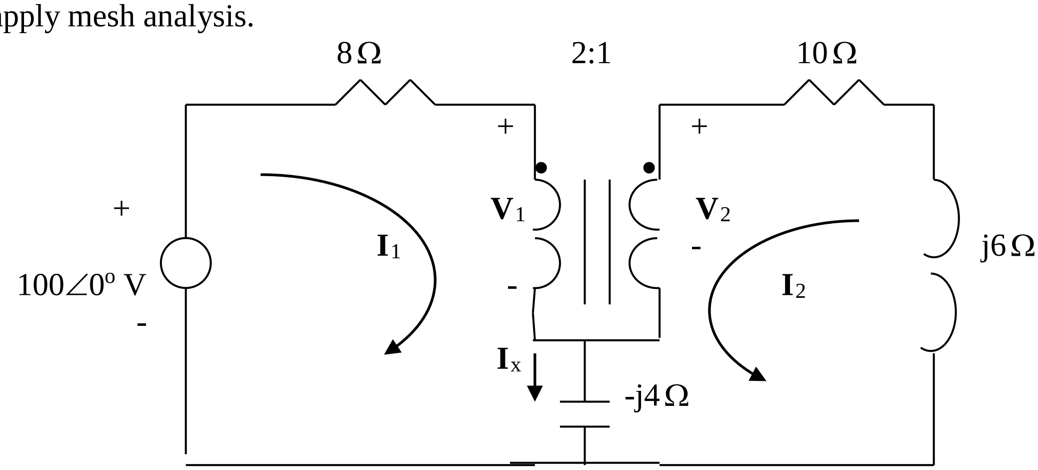


Figure 13.112

Solution

We apply mesh analysis.



$$100 = (8 - j4)I_1 - j4I_2 + V_1 \qquad (1)$$

$$0 = (10 + j2)I_2 - j4I_1 + V_2 \qquad (2)$$

But

$$\frac{V_2}{V_1} = n = \frac{1}{2} \longrightarrow V_1 = 2V_2 \qquad (3)$$

$$\frac{I_2}{I_1} = -\frac{1}{n} = -2 \quad \longrightarrow \quad I_1 = -0.5 I_2 \qquad (4)$$

Substituting (3) and (4) into (1) and (2), we obtain

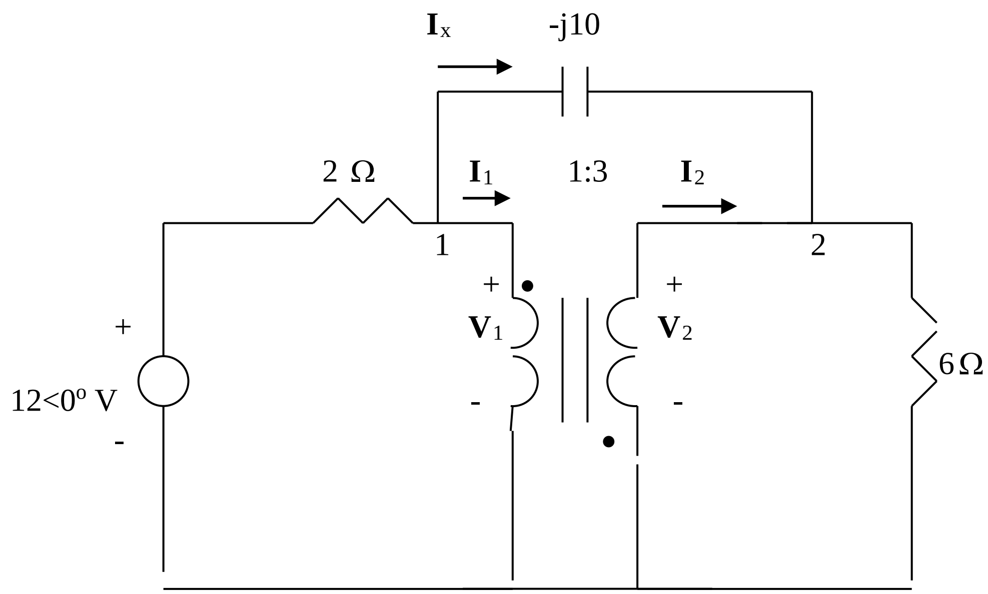
$$100 = (-4 - j2)I_2 + 2V_2 \qquad (1)a$$
$$0 = (10 + j4)I_2 + V_2 \qquad (2)a$$

Solving (1)a and (2)a leads to **I**$_2$ = -3.5503 +j1.4793

$$\boldsymbol{I_x = I_1 + I_2 = 0.5I_2 = \underline{1.923\angle 157.4^o \text{ A}}}$$

Solution 13.49

$$\omega = 2, \quad \frac{1}{20}\text{F} \quad \longrightarrow \quad \frac{1}{j\omega C} = -j10$$



At node 1,

$$\frac{12 - V_1}{2} = \frac{V_1 - V_2}{-j10} + I_1 \quad \longrightarrow \quad 12 = 2I_1 + V_1(1 + j0.2) - j0.2V_2 \qquad (1)$$

At node 2,

$$I_2 + \frac{V_1 - V_2}{-j10} = \frac{V_2}{6} \quad \longrightarrow \quad 0 = 6I_2 + j0.6V_1 - (1 + j0.6)V_2 \qquad (2)$$

At the terminals of the transformer, $V_2 = -3V_1, \quad I_2 = -\frac{1}{3}I_1$

Substituting these in (1) and (2),

$$12 = -6I_2 + V_1(1 + j0.8), \quad 0 = 6I_2 + V_1(3 + j2.4)$$

Adding these gives **V**$_1$=1.829 –j1.463 and

$$I_x = \frac{V_1 - V_2}{-j10} = \frac{4V_1}{-j10} = 0.937\angle 51.34^o$$

$i_x(t)$ = **937cos(2t+51.34˚) mA**.

Solution 13.50

The value of Z_{in} is not effected by the location of the dots since n^2 is involved.

$$Z_{in}' = (6 - j10)/(n')^2, \quad n' = 1/4$$

$$Z_{in}' = 16(6 - j10) = 96 - j160$$

$$Z_{in} = 8 + j12 + (Z_{in}' + 24)/n^2, \quad n = 5$$

$$Z_{in} = 8 + j12 + (120 - j160)/25 = 8 + j12 + 4.8 - j6.4$$

$$Z_{in} = \mathbf{(12.8 + j5.6)\ \Omega}$$

Solution 13.51

Use the concept of reflected impedance to find the input impedance and current $\mathbf{I}_1$ in Fig. 13.115 below.

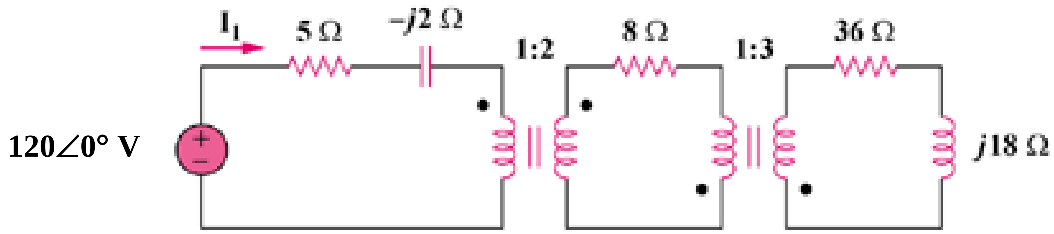


Figure 13.115
For Prob. 13.51.

Solution

Let $\mathbf{Z}_3 = 36 + j18$, where $\mathbf{Z}_3$ is reflected to the middle circuit.

$\mathbf{Z}_R' = \mathbf{Z}_L/n^2 = (12 + j2)/4 = 3 + j0.5$

$$\mathbf{Z}_{in} = 5 - j2 + \mathbf{Z}_R' = \mathbf{[8 - j1.5]\ \Omega}$$

$$\mathbf{I}_1 = 120\angle 0°/\mathbf{Z}_{eq} = 120\angle 0°/(8 - j1.5) = 120\angle 0°/8.1394\angle -10.62°$$
$$= \mathbf{14.743\angle 10.62°\ A}$$

$$\mathbf{[8 - j1.5]\ \Omega,\ 14.743\angle 10.62°\ A}$$

Solution 13.52

For maximum power transfer,

$$40 = Z_L/n^2 = 10/n^2 \quad \text{or} \quad n^2 = 10/40 \quad \text{which yields } n = 1/2 = 0.5$$

$$I = 120/(40 + 40) = 3/2$$

$p = I^2R = (9/4)x40 =$ **90 watts**.

Solution 13.53

Refer to the network in Fig. 13.117.

(a) Find *n* for maximum power supplied to the 200-Ω load.

(b) Determine the power in the 200-Ω load if *n* = 10.

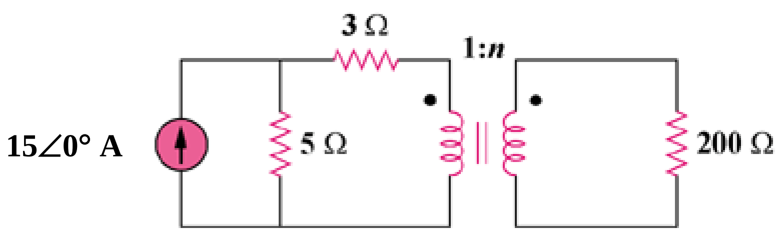
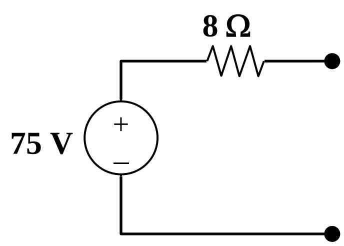


Figure 13.117
For Prob. 13.53.

Solution

(a) The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is $\mathbf{Z_L}' = \mathbf{Z_L}/n^2 = 200/n^2$.

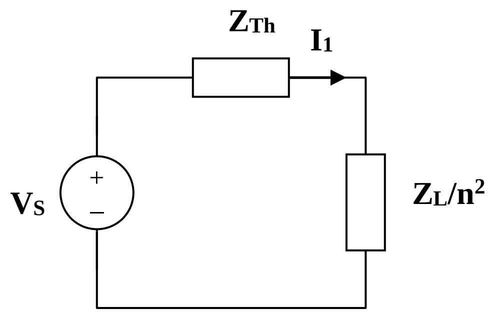
For maximum power transfer, $8 = 200/n^2$ produces $n = \mathbf{5}$.

(b) If $n = 10$, $\mathbf{Z_L}' = 200/100 = 2\ \Omega$ and $I = 75/(8 + 2) = 7.5$ A

$$p = I^2\mathbf{Z_L}' = (7.5)^2(2) = \mathbf{112.5\ W}.$$

Solution 13.54

(a)



For maximum power transfer,

$$Z_{Th} = Z_L/n^2, \text{ or } n^2 = Z_L/Z_{Th} = 8/128$$

$$n = \mathbf{0.25}$$

(b) $I_1 = V_{Th}/(Z_{Th} + Z_L/n^2) = 10/(128 + 128) = \mathbf{39.06\ mA}$

(c) $v_2 = I_2Z_L = 156.24\text{x}8 \text{ mV} = 1.25 \text{ V}$

But $v_2 = nv_1$ therefore $v_1 = v_2/n = 4(1.25) = \mathbf{5\ V}$

Solution 13.55

For the circuit in Fig. 13.119, calculate the equivalent resistance.

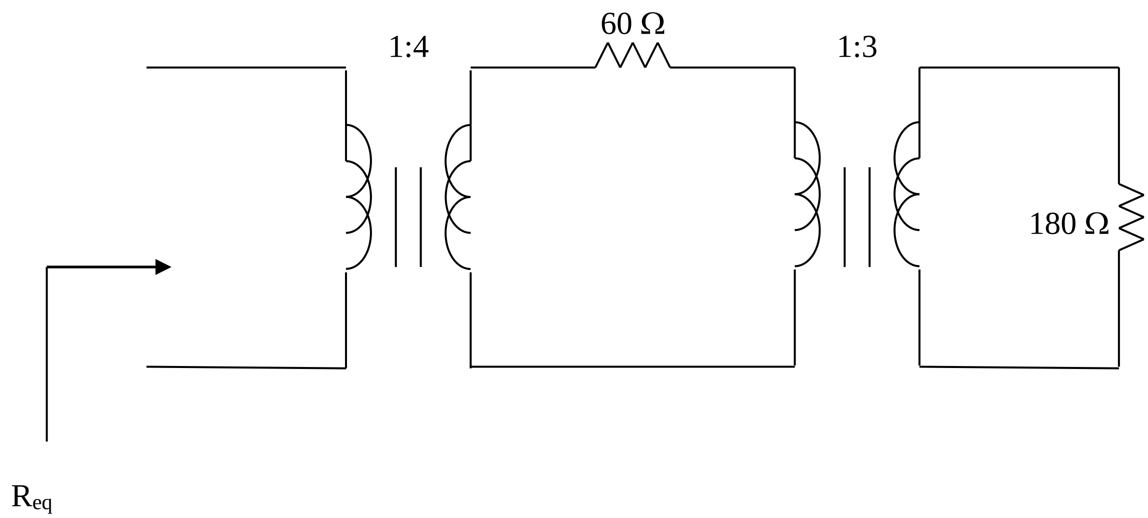


Figure 13.119
For Prob. 13.55.

Solution

We first reflect the 80-Ω resistance to the middle circuit.

$$\mathbf{Z'} = 60 + [180/(3)^2] = 60 + 20 = 80\ \mathbf{\Omega}$$

We now reflect this to the primary side.

$$\mathbf{R_{eq}} = \mathbf{Z'}/(4)^2 = \mathbf{5\ \Omega}.$$

Solution 13.56

Find the power absorbed by the 100-Ω resistor in the ideal transformer circuit of Fig. 13.120.

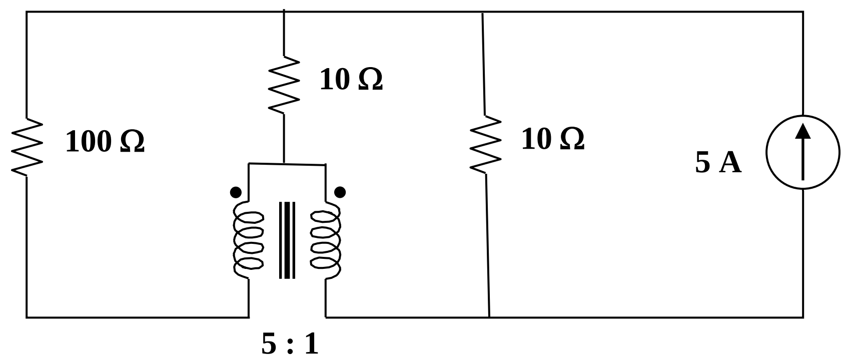
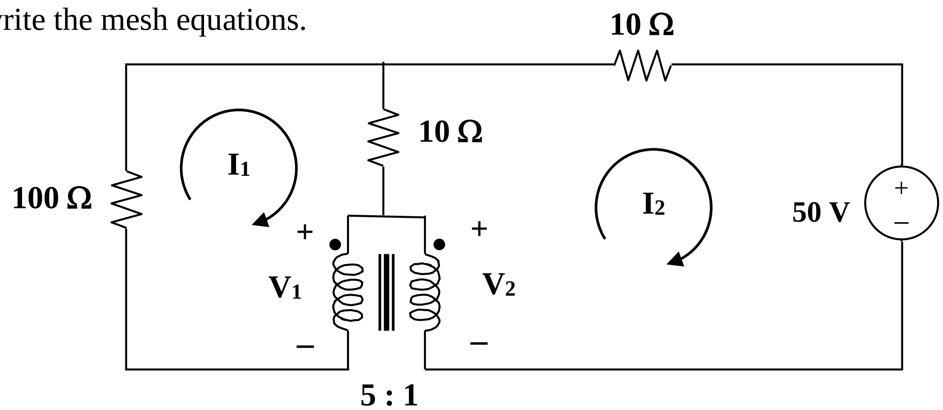


Figure 13.120
For Prob. 13.56.

Solution

Step 1. First we transform the current source in parallel with the 10 Ω into a voltage source, equal to 5x10 = 50 V, in series with a 10 Ω resistor. Then we can write the mesh equations.



$100\mathbf{I_1} + 10(\mathbf{I_1}–\mathbf{I_2}) + \mathbf{V_1} = 0$ and $–\mathbf{V_2} + 10(\mathbf{I_2}–\mathbf{I_1}) + 10\mathbf{I_2} + 50 = 0$. Now for the constraint equations, $\mathbf{V_1} = 5\mathbf{V_2}$ and $\mathbf{I_2} = 5\mathbf{I_1}$. Now all we need to do is to solve for $\mathbf{I_1}$ and then calculate the power. $P_{100} = |\mathbf{I_1}|^2(100)$.

Step 2. Replacing $\mathbf{V_2}$ and $\mathbf{I_2}$ in the above equations gives us, $(100+10–50)\mathbf{I_1} + \mathbf{V_1} = 0$ and $–0.2\mathbf{V_1} + 10(5\mathbf{I_1}–\mathbf{I_1}) + 50\mathbf{I_1} + 50 = 0$ or $0.2\mathbf{V_1} + (40+50)\mathbf{I_1} = –50$. From the first equation we get $\mathbf{V_1} = –60\mathbf{I_1}$ which now can be put into the second equation. $–0.2(–60\mathbf{I_1}) +90\mathbf{I_1} = –50$ or $(12+90)\mathbf{I_1} = –50$ or $\mathbf{I_1} = –0.490196$ A. This then gives us,

$$P_{100} = 100x(–0.4902)^2 = \mathbf{24.03\ W}.$$

Solution 13.57

(a) $Z_L = j3||(12 – j6) = j3(12 – j6)/(12 – j3) = (12 + j54)/17$

Reflecting this to the primary side gives

$Z_{in} = 2 + Z_L/n^2 = 2 + (3 + j13.5)/17 = 2.3168\angle 20.04°$

$I_1 = v_s/Z_{in} = 60\angle 90°/2.3168\angle 20.04° =$ **$25.9\angle 69.96°$ A(rms)**

$I_2 = I_1/n =$ **$12.95\angle 69.96°$ A(rms)**

(b) $60\angle 90° = 2I_1 + v_1$ or $v_1 = j60 – 2I_1 = j60 – 51.8\angle 69.96°$

$v_1 =$ **$21.06\angle 147.44°$ V(rms)**

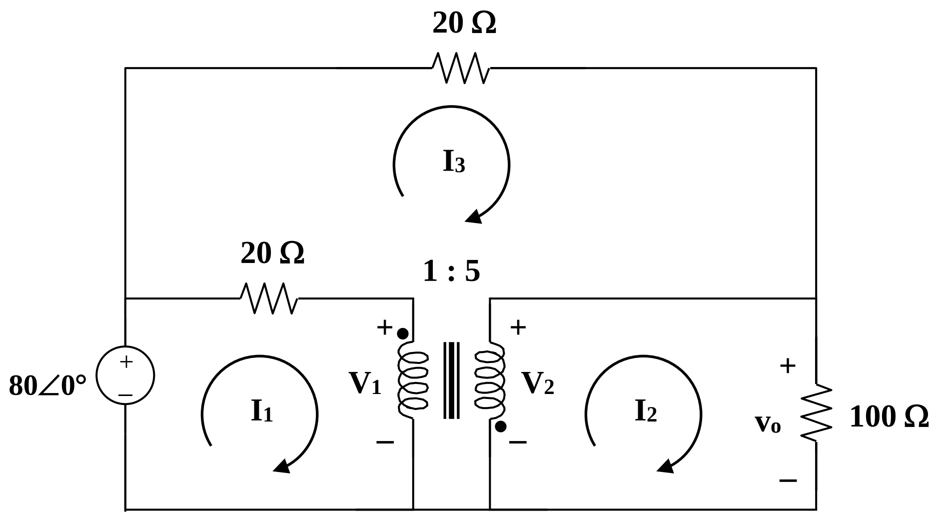
$v_2 = nv_1 =$ **$42.12\angle 147.44°$ V(rms)**

$v_o = v_2 =$ **$42.12\angle 147.44°$ V(rms)**

(c) $S = v_sI_1^* = (60\angle 90°)(25.9\angle –69.96°) =$ **$1.554\angle 20.04°$ kVA**

Solution 13.58

Consider the circuit below.



For mesh1, $-80 + 20I_1 - 20I_3 + V_1 = 0$ or

$$20I_1 - 20I_3 + V_1 = 80 \quad (1)$$

For mesh 2, $V_2 = 100I_2$ or $100I_2 - V_2 = 0 \quad (2)$

For mesh 3, $40I_3 - 20I_1 + V_2 - V_1 = 0$ which leads to

$$-20\,I_1 + 40I_3 - V_1 + V_2 = 0 \quad (3)$$

At the transformer terminals, $V_2 = -nV_1 = -5V_1$ or $5V_1 + V_2 \quad (4)$

$$I_1 - I_3 = -n(I_2 - I_3) = -5(I_2 - I_3) \text{ or}$$
$$I_1 + 5I_2 - 6I_3 = 0 \quad (5)$$

Solving using MATLAB,

```
>>A = [ 20  0  -20  1  0 ; 0  100  0  0  -1; -20  0  40  -1  1; 0  0  0  5  1;  1  5  -6  0
0 ]
```

```
A =

    20     0   -20     1     0
     0   100     0     0    -1
   -20     0    40    -1     1
     0     0     0     5     1
     1     5    -6     0     0

>> B = [ 80  0  0  0  0 ]'

B =

    80
     0
     0
     0
     0

>> Y = inv(A)*B

Y =

    5.9355
    0.5161
    1.4194
  -10.3226
   51.6129
```

P20,1 = 0.5*(I1 – I3)^2*20 = 0.5*(5.9355– 1.4194)^2*20 = 203.95

p_{20}(the one between 1 and 3) $= 0.5(20)(I_1 - I_3)^2 = 10(5.9355-1.4194)^2$

$=$ **203.95 watts**

p_{20}(at the top of the circuit) $= 0.5(20)I_3^2 =$ **20.15 watts**

$p_{100} = 0.5(100)I_2^2 =$ **13.318 watts**

Solution 13.59

In the circuit in Fig. 13.123, let $v_s = 165\sin(1{,}000t)$ V. Find the average power delivered to each resistor.

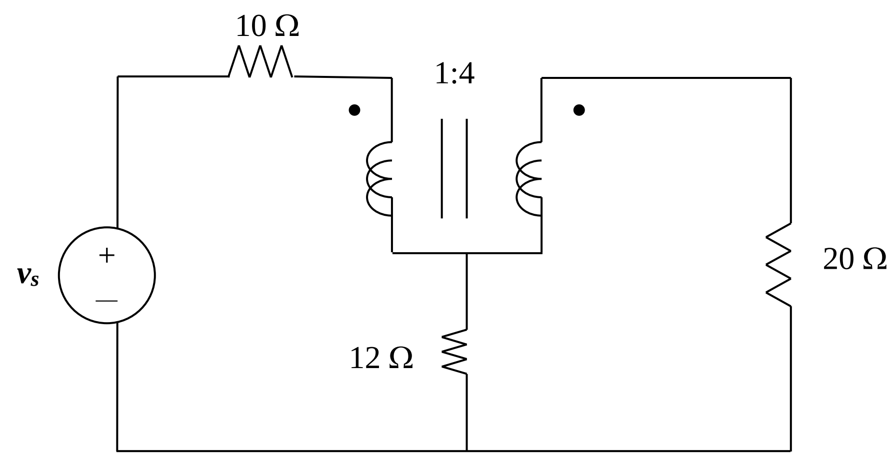
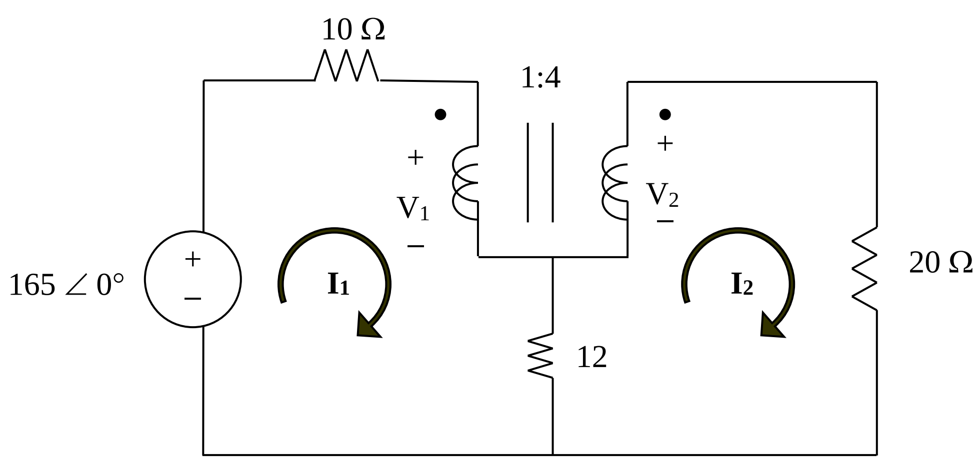


Figure 13.123
For Prob. 13.59.

Solution

We apply mesh analysis to the circuit as shown below.



For mesh 1,

$$-165 + 22\mathbf{I_1} - 12\mathbf{I_2} + \mathbf{V_1} = 0 \qquad (1)$$

For mesh 2,

$$-12\mathbf{I_1} + 32\mathbf{I_2} - \mathbf{V_2} = 0 \qquad (2)$$

At the transformer terminals,

$$-4\mathbf{V_1} + \mathbf{V_2} = 0 \qquad (3)$$

$$\mathbf{I_1} - 4\mathbf{I_2} = 0 \qquad (4)$$

Putting (1), (2), (3), and (4) in matrix form, we obtain

$$\begin{bmatrix} 22 & -12 & 1 & 0 \\ -12 & 32 & 0 & -1 \\ 0 & 0 & -4 & 1 \\ 1 & -4 & 0 & 0 \end{bmatrix} I = \begin{bmatrix} 165 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

```
>> A=[22,-12,1,0;-12,32,0,-1;0,0,-4,1;1,-4,0,0]
A =
   22   -12    1    0
  -12    32    0   -1
    0     0   -4    1
    1    -4    0    0
>> U=[165;0;0;0]
U =
   165
    0
    0
    0
>> X=inv(A)*U
X =
   9.1666
   2.2918
  –9.1666
  –36.6667
```

For 10-Ω resistor,

$$P_{10} = [(9.1666)^2/2]10 = \mathbf{420.1\ W}$$

For 12-Ω resistor,

$$P_{12} = [(9.1666–2.2918)^2/2]12 = \mathbf{283.6\ W}$$

For 20-Ω resistor,

$$P_{20} = [(2.2918)^2/2]20 = \mathbf{52.52\ W}.$$

Solution 13.60

(a) Transferring the 40-ohm load to the middle circuit,

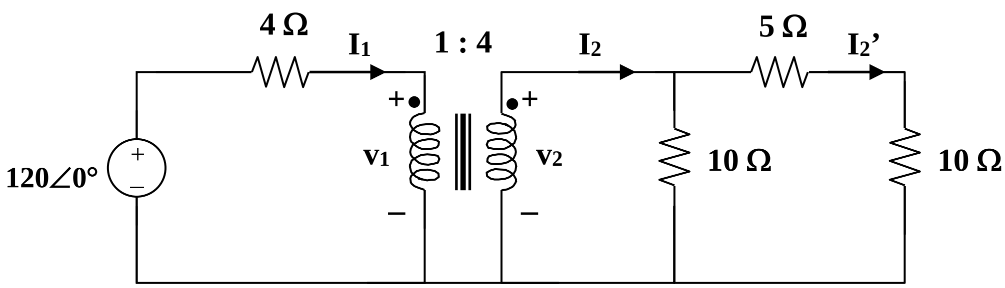
$Z_L' = 40/(n')^2 = 10$ ohms where $n' = 2$

$10||(5 + 10) = 6$ ohms

We transfer this to the primary side.

$Z_{in} = 4 + 6/n^2 = 4 + 0.375 = 4.375$ ohms, where $n = 4$

$I_1 = 120/4.375 =$ **27.43 A** and $I_2 = I_1/n =$ **6.857 A**



Using current division, $I_2' = (10/25)I_2 = 2.7429$ and

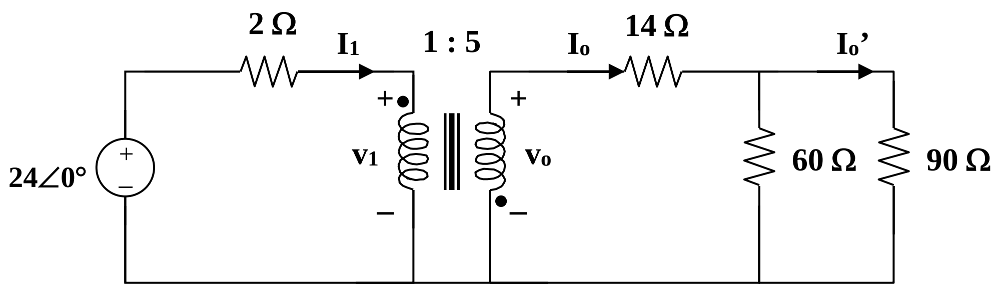
$I_3 = I_2'/n' =$ **1.3714 A**

(b) $p = 0.5(I_3)^2(40) =$ **37.62 watts**

Solution 13.61

We reflect the 160-ohm load to the middle circuit.

$$Z_R = Z_L/n^2 = 160/(4/3)^2 = 90 \text{ ohms, where } n = 4/3$$



$$14 + 60||90 = 14 + 36 = 50 \text{ ohms}$$

We reflect this to the primary side.

$$Z_R' = Z_L'/(n')^2 = 50/5^2 = 2 \text{ ohms when } n' = 5$$

$I_1 = 24/(2 + 2) = \mathbf{6A}$

$24 = 2I_1 + v_1$ or $v_1 = 24 - 2I_1 = 12$ V

$v_o = -nv_1 = \mathbf{-60\ V}$, $I_o = -I_1/n_1 = -6/5 = -1.2$

$I_o' = [60/(60 + 90)]I_o = -0.48A$

$I_2 = -I_o'/n = 0.48/(4/3) = \mathbf{360\ mA}$

Solution 13.62

(a) Reflect the load to the middle circuit.

$$\mathbf{Z_L}' = 8 - j20 + (18 + j45)/3^2 = 10 - j15$$

We now reflect this to the primary circuit so that

$\mathbf{Z_{in}} = 6 + j4 + (10 - j15)/n^2 = 7.6 + j1.6 = 7.767\angle 11.89°$, where
$n = 5/2 = 2.5$

$$\mathbf{I_1} = 40/\mathbf{Z_{in}} = 40/7.767\angle 11.89° = 5.15\angle -11.89°$$

$$\mathbf{S} = \mathbf{v_s I_1}^* = (40\angle 0°)(5.15\angle 11.89°) = \mathbf{206\angle 11.89°\ VA}$$

(b)
$$\mathbf{I_2} = -\mathbf{I_1}/n, \quad n = 2.5$$

$$\mathbf{I_3} = -\mathbf{I_2}/n', \quad n = 3$$

$$\mathbf{I_3} = \mathbf{I_1}/(nn') = 5.15\angle -11.89°/(2.5x3) = 0.6867\angle -11.89°$$

$$p = |\mathbf{I_2}|^2(18) = 18(0.6867)^2 = \mathbf{8.488\ watts}$$

Solution 13.63

Find the mesh currents in the circuit of Fig. 13.128 below.

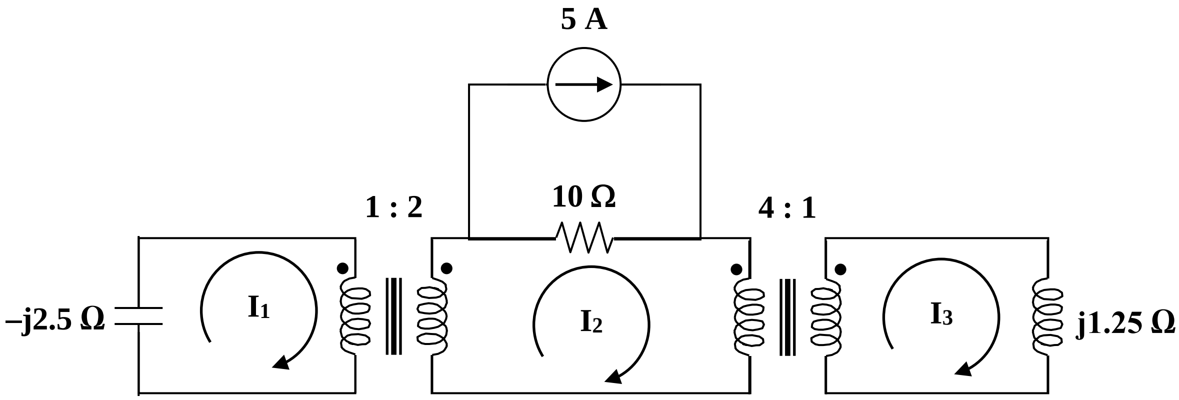


Figure 13.127
For Prob. 13.63.

Solution

Step 1. We can start by reflecting the –j2.5 Ω and the j1.25 Ω impedances into the center circuit and then solve for $\mathbf{I_2}$. The capacitor becomes –j2.5x4 = –j10 Ω and the inductor becomes j1.25x16 = j20 Ω. The mesh equation now becomes $-j10\mathbf{I_2} + 10(\mathbf{I_2}-5) + j20\mathbf{I_2} = 0$. Once we solve for $\mathbf{I_2}$ we can find $\mathbf{I_1} = 2\mathbf{I_2}$ and $\mathbf{I_3} = 4\mathbf{I_2}$.

Step 2. $(10+j10)\mathbf{I_2} = 50$ or $\mathbf{I_2} = 5/(1.4142\angle 45°) =$ **3.536∠–45° A**, $\mathbf{I_1} =$ **7.071∠–45° A**, and $\mathbf{I_3} =$ **14.1402∠–45° A**.

Solution 13.64

For the circuit in Fig. 13.128, find the turn ratio so that the maximum power is delivered to the 30-kΩ resistor.

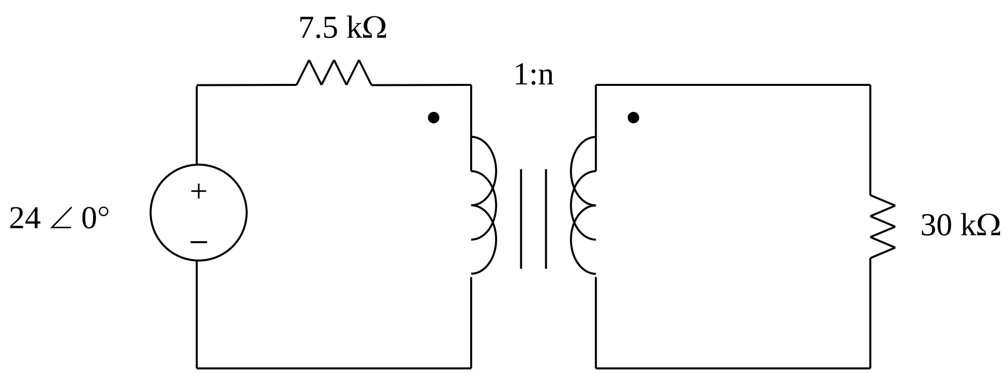
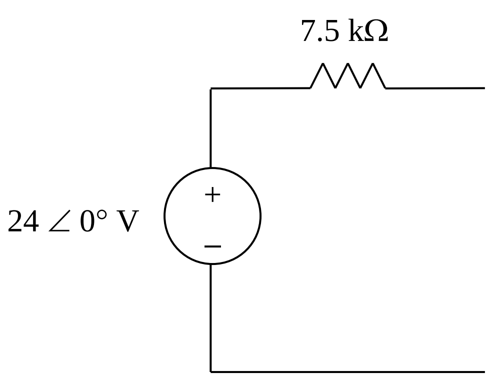


Figure 13.128
For Prob. 13.64.

Solution

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

$$\mathbf{Z'} = \mathbf{Z_L}/n^2 = 30k/n^2$$

For maximum power transfer,

$$7.5k = 30k/n^2 \text{ or } n^2 = 30/7.5 = 4 \text{ or } n = \mathbf{2}.$$

Solution 13.65

For the circuit in Fig. 13.128, find the turn ratio so that the maximum power is delivered to the 30-kΩ resistor.

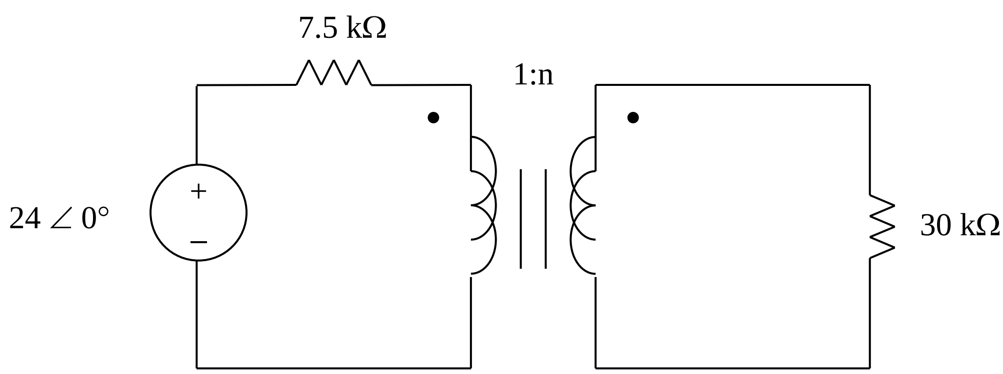
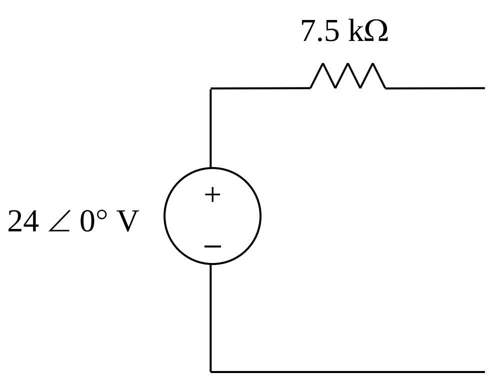


Figure 13.128
For Prob. 13.64.

Solution

The Thevenin equivalent to the left of the transformer is shown below.



The reflected load impedance is

$$\mathbf{Z'} = \mathbf{Z_L}/n^2 = 30k/n^2$$

For maximum power transfer,

$$7.5k = 30k/n^2 \text{ or } n^2 = 30/7.5 = 4 \text{ or } n = \mathbf{2}.$$

Solution 13.66

Design a problem to help other students to better understand how the ideal autotransformer works.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

An ideal autotransformer with a 1:4 step-up turns ratio has its secondary connected to a 120-Ω load and the primary to a 420-V source. Determine the primary current.

Solution

$$v_1 = 420 \text{ V} \tag{1}$$

$$v_2 = 120I_2 \tag{2}$$

$$v_1/v_2 = 1/4 \text{ or } v_2 = 4v_1 \tag{3}$$

$$I_1/I_2 = 4 \text{ or } I_1 = 4 I_2 \tag{4}$$

Combining (2) and (4),

$$v_2 = 120[(1/4)I_1] = 30 I_1$$

$$4v_1 = 30I_1$$

$$4(420) = 1680 = 30I_1 \text{ or } I_1 = \mathbf{56\ A}$$

Solution 13.67

An autotransformer with a 40% tap is supplied by an 880-V, 60-Hz source and is used for step-down operation. A 5-kVA load operating at unity power factor is connected to the secondary terminals. Find: (a) the secondary voltage, (b) the secondary current, (c) the primary current.

Solution

(a) $\dfrac{V_1}{V_2} = \dfrac{N_1 + N_2}{N_2} = \dfrac{1}{0.4} \longrightarrow V_2 = 0.4V_1 = 0.4x880 =$ **352 V**.

(b) $\mathbf{S_2} = \mathbf{V_2}(\mathbf{I_2})^* = 5$ kVA or $\mathbf{I_2} = 5{,}000/352 =$ **14.205 A**.

(c) $\mathbf{S_1} = \mathbf{V_1}(\mathbf{I_1})^* = \mathbf{S_2} = 5$ kVA or $\mathbf{I_1} = 5{,}000/880 =$ **5.682 A**.

Solution 13.68

In the ideal autotransformer of Fig. 13.130, calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_o$. Find the average power delivered to the load.

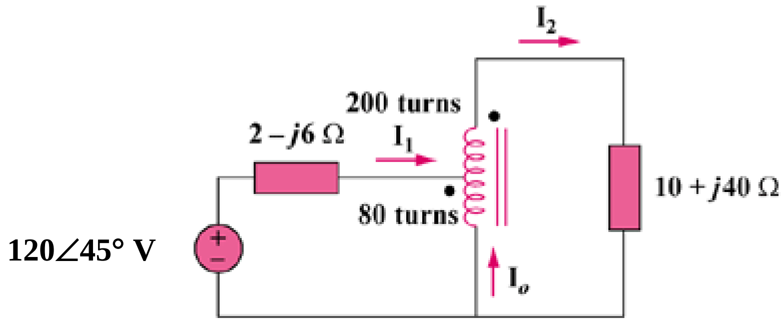
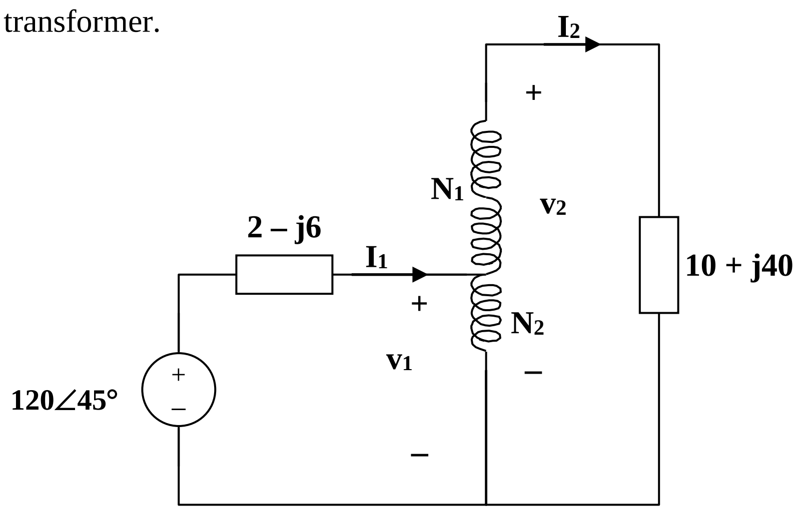


Figure 13.130
For Prob. 13.68.

Solution

This is a step-up transformer.



For the primary circuit, $\quad 120\angle 45° = (2 - j6)\mathbf{I}_1 + \mathbf{v}_1 \qquad (1)$

For the secondary circuit, $\quad \mathbf{v}_2 = (10 + j40)\mathbf{I}_2 \qquad (2)$

At the autotransformer terminals,

$$\mathbf{v}_1/\mathbf{v}_2 = N_1/(N_1 + N_2) = 200/280 = 5/7,$$

$$\text{thus } \mathbf{v}_2 = 7\mathbf{v}_1/5 \qquad (3)$$

Also, $\quad \mathbf{I}_1/\mathbf{I}_2 = 7/5 \text{ or } \mathbf{I}_2 = 5\mathbf{I}_1/7 \qquad (4)$

Substituting (3) and (4) into (2), $\mathbf{v_1} = (10 + j40)25\mathbf{I_1}/49$

Substituting that into (1) gives $120\angle 45° = (7.102 + j14.408)\mathbf{I_1}$

$$\mathbf{I_1} = 120\angle 45°/16.063\angle 63.76° = \mathbf{7.4706\angle{-18.76°}\ A}$$

$$\mathbf{I_2} = 5\mathbf{I_1}/7 = \mathbf{5.3361\angle{-18.76°}\ A}$$

$$\mathbf{I_o} = \mathbf{-I_1 + I_2} = [-(5/7) + 1]\mathbf{I_1} = 2\mathbf{I_1}/7 = \mathbf{2.1345\angle{-18.76°}\ A}$$

$$p = |\mathbf{I_2}|^2 R = (5.3361)^2(10) = \mathbf{284.7\ W}.$$

Solution 13.69

In the circuit of Fig. 13.131, N_1 = 190 turns and N_2 = 10 turns, determine the Thevenin equivalent circuit looking into terminals a and b. What would be the value of $\mathbf{Z_L}$ that would absorb maximum power from the circuit?

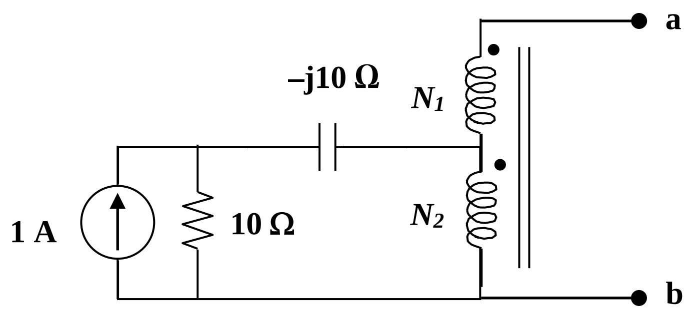
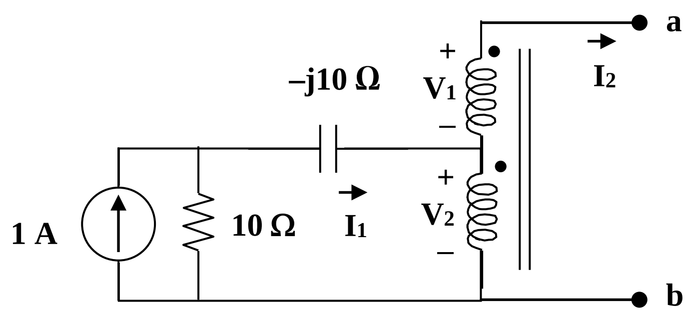


Figure 13.131
For Prob. 13.69.

Solution

Step 1. For the open circuit, $\mathbf{I_1} = 0$ and $\mathbf{I_2} = 0$. Thus, $\mathbf{V_{oc}} = [(190+10)/10](1\text{x}10) = \mathbf{V_{Thev}}$. For the short circuit current, V_1 and V_2 are equal to zero.



$\mathbf{I_1} = 1\text{x}10/(10–j10) = 0.7071\angle 45°$ A. $\mathbf{I_2} = \mathbf{I_{sc}} = [10/200]\mathbf{I_1}$. $\mathbf{Z_{eq}} = \mathbf{V_{oc}}/\mathbf{I_{sc}}$.

Step 2. $\mathbf{V_{oc}} = \mathbf{V_{Thev}} =$ **200 V** and $\mathbf{I_{sc}} = 0.035355\angle 45°$ or
$\mathbf{Z_{eq}} = 200/(0.035355\angle 45°) = 5{,}657\angle –45°\ \Omega =$ **(4 – j4) kΩ**.

For maximum power transfer we need $\mathbf{Z_L} =$ **(4 + j4) kΩ**.

Solution 13.70

In the ideal transformer circuit shown in Fig. 13.132, determine the average power delivered to the load.

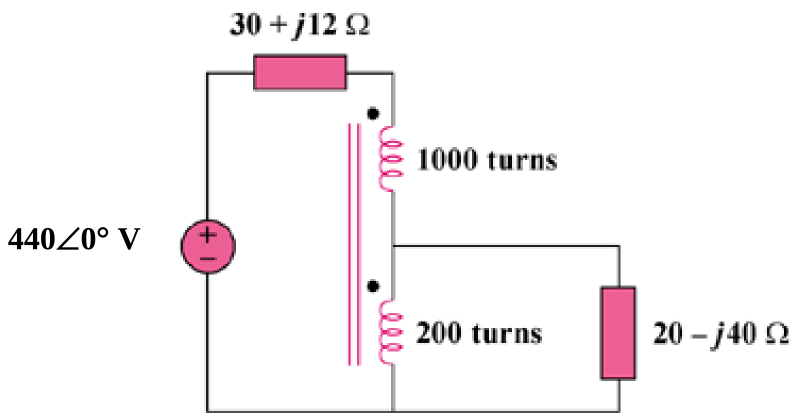
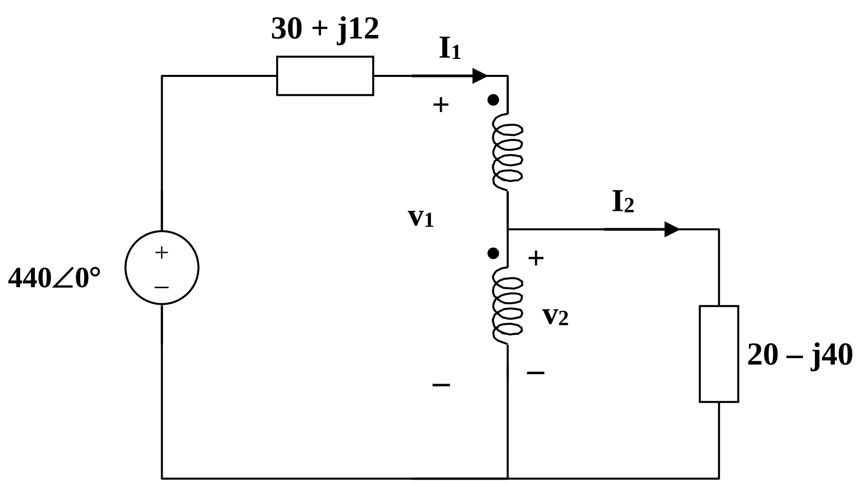


Figure 13.132
For Prob. 13.70.

Solution

This is a step-down transformer.



$$\mathbf{I_1}/\mathbf{I_2} = N_2/(N_1 + N_2) = 200/1200 = 1/6, \text{ or } \mathbf{I_1} = \mathbf{I_2}/6 \qquad (1)$$

$$\mathbf{v_1}/\mathbf{v_2} = (N_2 + N_2)/N_2 = 6, \text{ or } \mathbf{v_1} = 6\mathbf{v_2} \qquad (2)$$

For the primary loop, $440 = (30 + j12)\mathbf{I_1} + \mathbf{v_1}$ (3)

For the secondary loop, $\mathbf{v_2} = (20 - j40)\mathbf{I_2}$ (4)

Substituting (1) and (2) into (3),

$$440 = (30 + j12)(\mathbf{I_2}/6) + 6\mathbf{v_2}$$

and substituting (4) into this yields

$$440 = (5+j2)I_2 + 6(20–j40)I_2 = (125 – j238)\mathbf{I_2} \quad \text{or}$$
$$\mathbf{I_2} = 440/(268.83\angle–62.291° = 1.63672\angle 62.291° \text{ A.}$$

$$P_{20} = |\mathbf{I_2}|^2(20) = \mathbf{53.58\ W}.$$

Solution 13.71

When individuals travel, their electrical appliances need to have converters to match the voltages required by their appliances to the local voltage available to power their appliances. Today these converters use power electronics to convert voltages. In the past these converters were auto transformers. The auto transformer shown in Fig. 134 is used to convert 115 volts to 220 volts. What is the value of the turns? If the maximum current available from the 115 V source is 15 amps, what will be the maximum current available for the 220 V appliance?

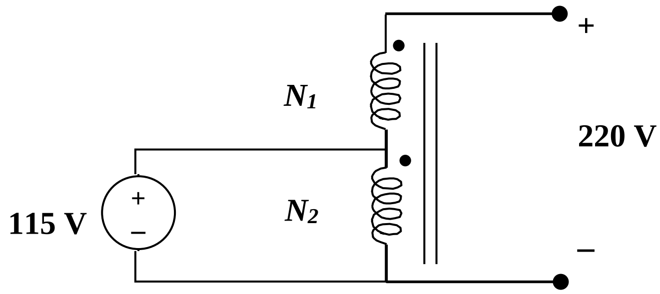


Figure 13.133
For Prob. 13.71.

Solution

Step 1. The relationship between the 115 V and the 220 V is equal to
$115/220 = N_2/(N_1+N_2)$ and $\mathbf{I_{115}} = 15$ A $= [(N_1+N_2)/N_2]\mathbf{I_{220}}$.

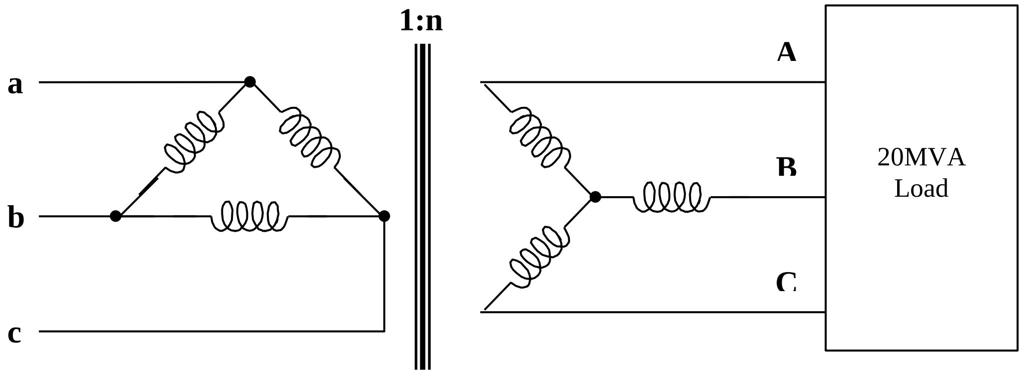
Step 2. $0.52273(N_1+N_2) = N_2$ or $0.52273N_1 = 0.47727N_2$ or

$$N_1/N_2 = \mathbf{0.913}$$

$\mathbf{I_{220}} = 0.52273 \text{x} 15 =$ **7.841 A**.

Solution 13.72

(a) Consider just one phase at a time.



$$n = V_L/\sqrt{3}V_{Lp} = 7200/(12470\sqrt{3}) = \mathbf{1/3}$$

(b) The load carried by each transformer is 60/3 = 20 MVA.

Hence I_{Lp} = 20 MVA/12.47 k = **1604 A**

I_{Ls} = 20 MVA/7.2 k = **2778 A**

(c) The current in incoming line a, b, c is

$$\sqrt{3}I_{Lp} = \sqrt{3}\text{x}1603.85 = \mathbf{2778\ A}$$

Current in each outgoing line A, B, C is

$$2778/(n\sqrt{3}) = \mathbf{4812\ A}$$

Solution 13.73

(a) This is a **three-phase Δ-Y transformer**.

(b) $V_{Ls} = nv_{Lp}/\sqrt{3} = 450/(3\sqrt{3}) = 86.6$ V, where $n = 1/3$

As a Y-Y system, we can use per phase equivalent circuit.

$I_a = V_{an}/Z_Y = 86.6\angle 0°/(8 - j6) = 8.66\angle 36.87°$

$I_c = I_a\angle 120° = \mathbf{8.66\angle 156.87°\ A}$

$I_{Lp} = n\sqrt{3}\,I_{Ls}$

$I_1 = (1/3)\sqrt{3}\,(8.66\angle 36.87°) = 5\angle 36.87°$

$I_2 = I_1\angle -120° = \mathbf{5\angle -83.13°\ A}$

(c) $p = 3|I_a|^2(8) = 3(8.66)^2(8) =$ **1.8 kw**.

Solution 13.74

(a) This is a **Δ-Δ connection**.

(b) The easy way is to consider just one phase.

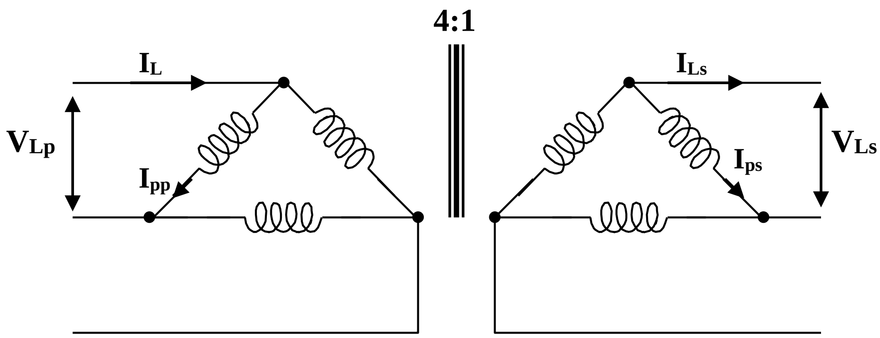
$$1:n = 4:1 \text{ or } n = 1/4$$

$$n = V_2/V_1 \text{ which leads to } V_2 = nV_1 = 0.25(2400) = 600$$

$$\text{i.e. } V_{Lp} = 2400 \text{ V and } V_{Ls} = 600 \text{ V}$$

$$S = p/\cos\theta = 120/0.8 \text{ kVA} = 150 \text{ kVA}$$

$$p_L = p/3 = 120/3 = 40 \text{ kw}$$



But $p_{Ls} = V_{ps}I_{ps}$

For the Δ-load, $I_L = \sqrt{3}\, I_p$ and $V_L = V_p$

Hence, $I_{ps} = 40{,}000/600 = 66.67$ A

$$I_{Ls} = \sqrt{3}\, I_{ps} = \sqrt{3}\text{x}66.67 = \mathbf{115.48\ A}$$

(c) Similarly, for the primary side

$$p_{pp} = V_{pp}I_{pp} = p_{ps} \text{ or } I_{pp} = 40{,}000/2400 = \mathbf{16.667\ A}$$

$$\text{and} \quad I_{Lp} = \sqrt{3}\, I_p = \mathbf{28.87\ A}$$

(d) Since $S = 150$ kVA therefore $S_p = S/3 =$ **50 kVA**

Solution 13.75

(a) $n = V_{Ls}/(\sqrt{3}\,V_{Lp}) = 900/(4500\sqrt{3}) = \mathbf{0.11547}$

(b) $S = \sqrt{3}\,V_{Ls}I_{Ls}$ or $I_{Ls} = 120{,}000/(900\sqrt{3}) = \mathbf{76.98\ A}$

$I_{Ls} = I_{Lp}/(n\sqrt{3}) = 76.98/(2.887\sqrt{3}) = \mathbf{15.395\ A}$

Solution 13.76

Using Fig. 13.138, design a problem to help other students to better understand a wye-delta, three-phase transformer and how they work.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A Y-Δ three-phase transformer is connected to a 60-kVA load with 0.85 power factor (leading) through a feeder whose impedance is 0.05 + j0.1Ω per phase, as shown in Fig. 13.137 below. Find the magnitude of:

(a) the line current at the load,

(b) the line voltage at the secondary side of the transformer,

(c) the line current at the primary side of the transformer.

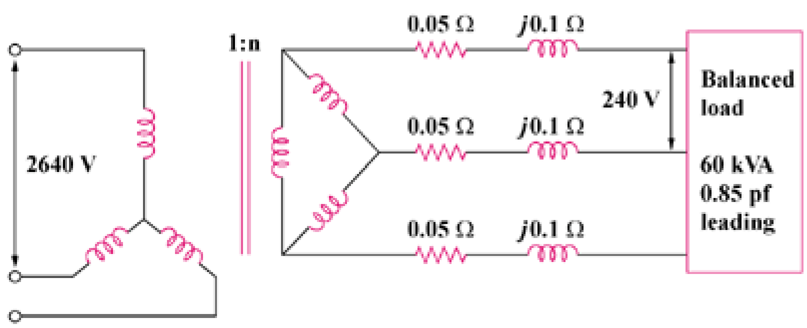


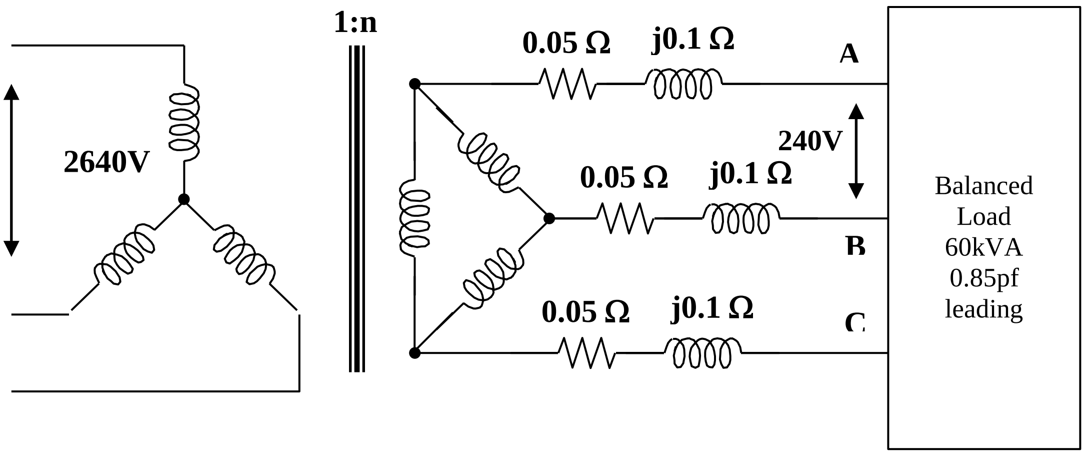
Figure 13.137

Solution

(a) At the load, $V_L = 240\text{ V} = V_{AB}$

$$V_{AN} = V_L/\sqrt{3} = 138.56\text{ V}$$

Since $S = \sqrt{3}V_LI_L$ then $I_L = 60{,}000/(240\sqrt{3}) = \mathbf{144.34\ A}$



(b) Let $V_{AN} = |V_{AN}|\angle 0° = 138.56\angle 0°$

$\cos\theta = pf = 0.85$ or $\theta = 31.79°$

$I_{AA'} = I_L\angle\theta = 144.34\angle 31.79°$

$V_{A'N'} = ZI_{AA'} + V_{AN}$

$= 138.56\angle 0° + (0.05 + j0.1)(144.34\angle 31.79°)$

$= 138.03\angle 6.69°$

$V_{Ls} = V_{A'N'}\sqrt{3} = 138.03\sqrt{3} = \mathbf{239.1\ V}$

(c) For Y-Δ connections,

$n = \sqrt{3}\,V_{Ls}/V_{ps} = \sqrt{3}\,x238.7/2640 = 0.1569$

$f_{Lp} = nI_{Ls}/\sqrt{3} = 0.1569x144.34/\sqrt{3} = \mathbf{13.05\ A}$

Solution 13.77

(a) This is a single phase transformer. $V_1 = 13.2$ kV, $V_2 = 120$ V

$n = V_2/V_1 = 120/13{,}200 = 1/110$, therefore $n = \mathbf{1/110}$

or 110 turns on the primary to every turn on the secondary.

(b) $P = VI$ or $I = P/V = 100/120 = 0.8333$ A

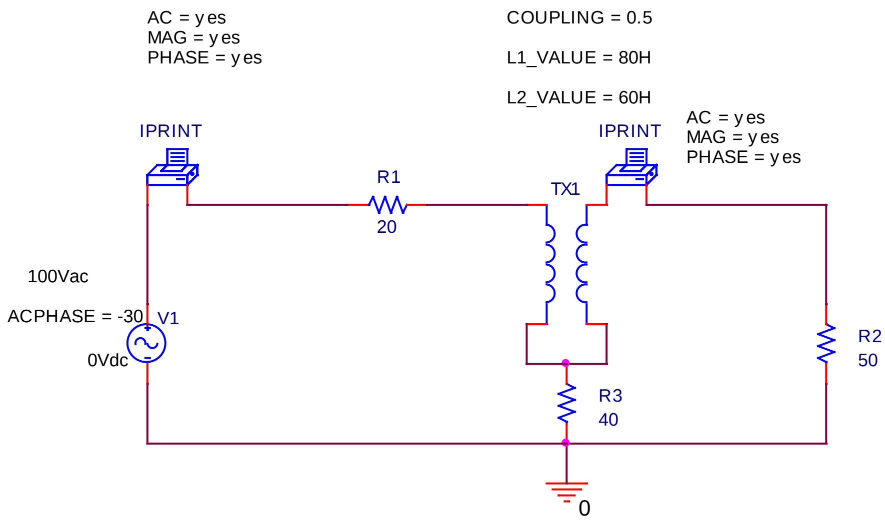
$I_1 = nI_2 = 0.8333/110 = \mathbf{7.576\ mA}$

Solution 13.78

We convert the reactances to their inductive values.

$$X = \omega L \quad \longrightarrow \quad L = \frac{X}{\omega}$$

The schematic is as shown below.



FREQ IM(V_PRINT1)IP(V_PRINT1)

1.592E-01 1.347E+00 -8.489E+01

FREQ IM(V_PRINT2)IP(V_PRINT2)

1.592E-01 6.588E-01 -7.769E+01

Thus,

$$\mathbf{I_1 = 1.347\angle{-84.89^\circ}\ amps} \text{ and } \mathbf{I_2 = 658.8\angle{-77.69^\circ}\ mA}$$

Solution 13.79

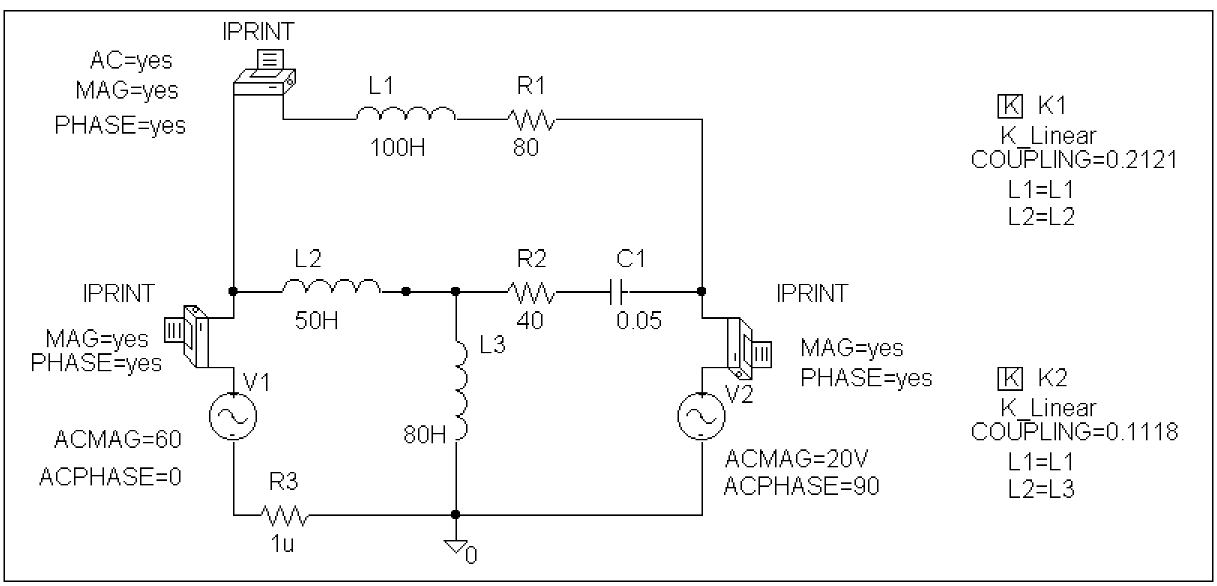
The schematic is shown below.

$$k_1 = 15/\sqrt{5000} = 0.2121, \; k_2 = 10/\sqrt{8000} = 0.1118$$

In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the circuit is saved and simulated, the output includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.068 E–01	–7.786 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	1.306 E+00	–6.801 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.336 E+00	–5.492 E+01

Thus, $\mathbf{I_1 = 1.306\angle{-68.01^\circ}\ A}$, $\mathbf{I_2 = 406.8\angle{-77.86^\circ}\ mA}$, $\mathbf{I_3 = 1.336\angle{-54.92^\circ}\ A}$



Solution 13.80

The schematic is shown below.

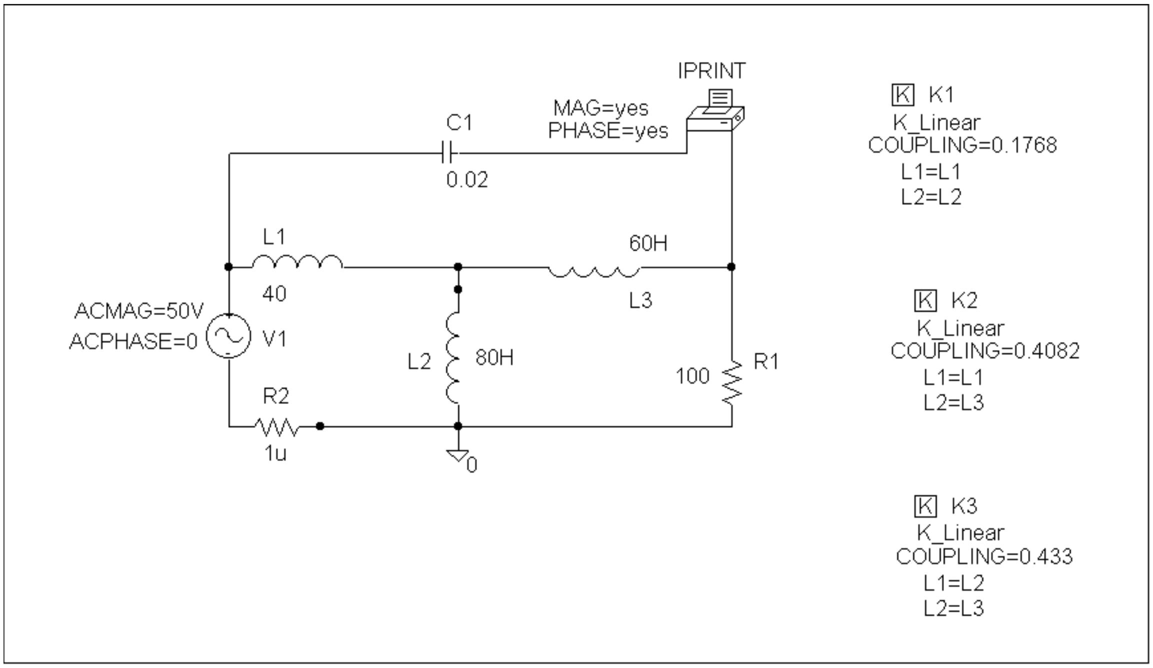
$$k_1 = 10/\sqrt{40x80} = 0.1768, \quad k_2 = 20/\sqrt{40x60} = 0.4082$$

$$k_3 = 30/\sqrt{80x60} = 0.433$$

In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After the simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.304 E+00	6.292 E+01

i.e. I_o = **1.304∠62.92° A**



Solution 13.81

The schematic is shown below.

$$k_1 = 2/\sqrt{4x8} = 0.3535, \; k_2 = 1/\sqrt{2x8} = 0.25$$

In the AC Sweep box, we let Total Pts = 1, Start Freq = 100, and End Freq = 100. After simulation, the output file includes

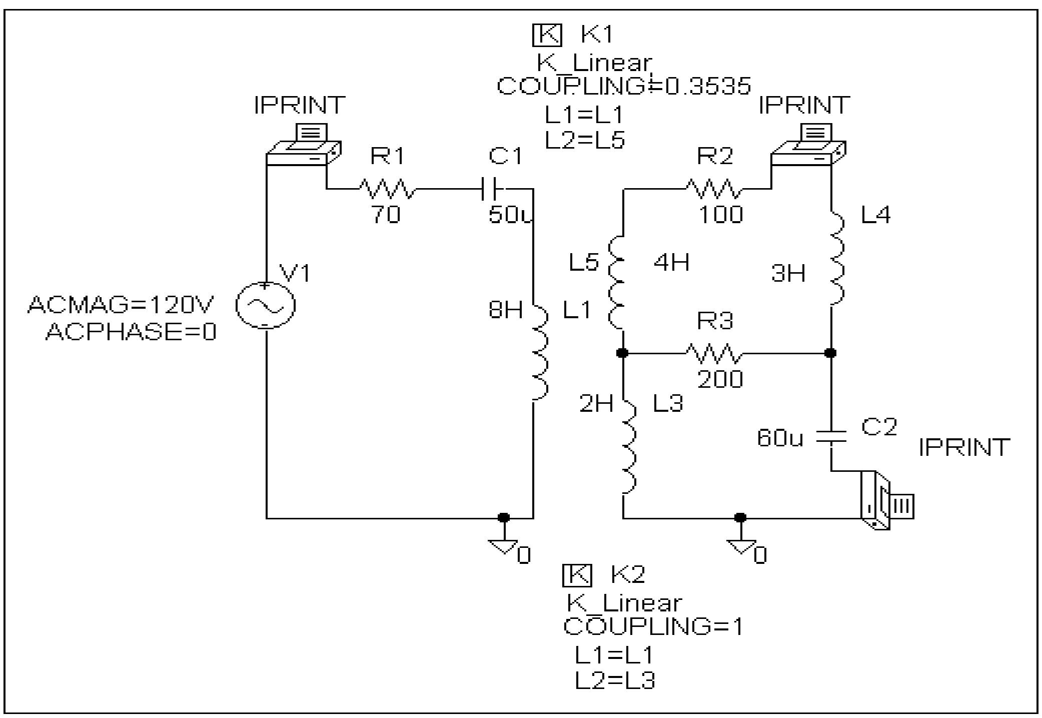
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000 E+02	1.0448 E–01	1.396 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.000 E+02	2.954 E–02	–1.438 E+02

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.000 E+02	2.088 E–01	2.440 E+01

i.e. I_1 = **104.5∠13.96° mA**, I_2 = **29.54∠–143.8° mA**,

I_3 = **208.8∠24.4° mA**.



Solution 13.82

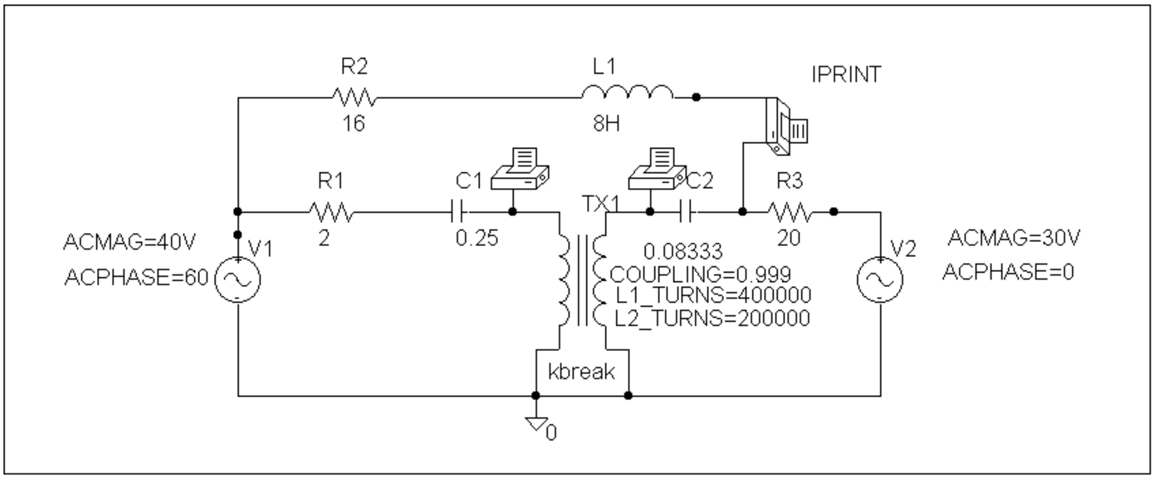
The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain the output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.955 E+01	8.332 E+01
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	6.847 E+01	4.640 E+01
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	4.434 E–01	–9.260 E+01

i.e. V_1 = **19.55∠83.32° V**, V_2 = **68.47∠46.4° V**,

I_o = **443.4∠–92.6° mA**.

These answers are incorrect, we need to adjust the magnitude of the inductances.



Solution 13.83

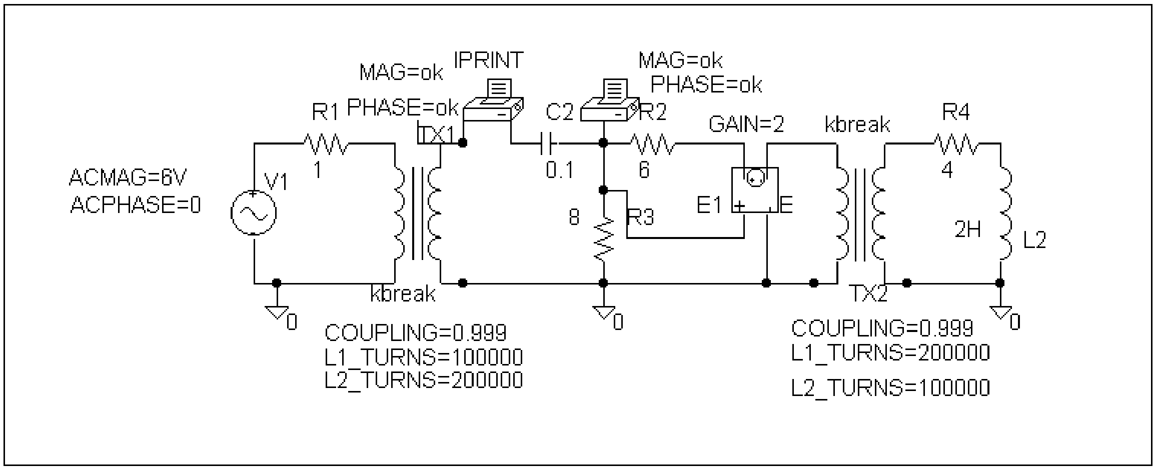
The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	1.080 E+00	3.391 E+01

FREQ	VM($N_0001)	VP($N_0001)
1.592 E–01	1.514 E+01	–3.421 E+01

i.e. **ix** = **1.08∠33.91° A**, **V$_x$** = **15.14∠–34.21° V**.

This is most likely incorrect and needs to have the values of turns changed. Clearly the turns ratio makes L2 = 400000 and L1 = 4000000.



Checking with hand calculations.

Loop 1. $-6 + 1I_1 + V_1 = 0$ or $I_1 + V_1 = 6$ (1)

Loop 2. $-V_2 - j10I_2 + 8(I_2-I_3) = 0$ or $(8-j10)I_2 - 8I_3 - V_2 = 0$ (2)

Loop 3. $8(I_3-I_2) + 6I_3 + 2V_x + V_3 = 0$ or $-8I_2 + 14I_3 + V_3 + 2V_x = 0$ but $V_x = 8(I_2-I_3)$, therefore we get $8I_2 - 2I_3 + V_3 = 0$ (3)

Loop 4. $-V_4 + (4+j2)I_4 = 0$ or $(4+j2)I_4 - V_4 = 0$ (4)

We also need the constraint equations, $V_2 = 2V_1$, $I_1 = 2I_2$, $V_3 = 2V_4$, and $I_4 = 2I_3$. Finally,

$$I_x = I_2 \text{ and } V_x = 8(I_2 - I_3).$$

We can eliminate the voltages from the equations (we only need I_2 and I_3 to obtain the required answers) by,

$$(1)+0.5(2) = I_1 + (4–j5)I_2 – 4I_3 = 6 \text{ and}$$

$$0.5(3) + (4) = 4I_2 – I_3 + (4+j2)I_4 = 0.$$

Next we use $I_1 = 2I_2$ and $I_4 = 2I_3$ to end up with the following equations,

$(6–j5)I_2 – 4I_3 = 6$ and $4I_2 + (7+j4)I_3 = 0$ or $I_2 = –[(7+j4)I_3]/4 = (–1.75–j)I_3$
$= (2.01556\angle–150.255°)I_3$

This leads to or $(6–j5)(–1.75–j)I_3 – 4I_3 = (–10.5–5–4+j(8.75–6))I_3 = (–19.5+j2.75)I_3 = 6$

$I_3 = 6/(19.69296\angle171.973°) = 0.304677\angle–171.973°$ amps
$= –0.301692–j0.042545.$
$I_2 = (–1.75–j)(0.304677\angle–171.973°)$
$= (2.01556\angle–150.255°)(0.304677\angle–171.973°)$
$= 614.096\angle37.772°$ mA $= 0.48541+j0.37615$
and $I_2 – I_3 = 0.7871+j0.4187 = 0.89154\angle28.01°.$

Therefore,

$$V_x = 8(0.854876\angle22.97°) = \mathbf{7.132\angle28.01°\ V}$$

$$I_x = I_2 = \mathbf{614.1\angle37.77°\ mA}.$$

Checking with MATLAB we get A and X from equations (1) – (4) and the four constraint equations.

```
>> A = [1 0 0 0 1 0 0 0;0 (8-10j) -8 0 0 -1 0 0;0 8 -2 0 0 0 1 0;0 0 0 (4+2j) 0 0 0 -1;0 0 0 0
-2 1 0 0;1 -2 0 0 0 0 0 0;0 0 0 0 0 0 1 -2;0 0 -2 1 0 0 0 0]

A =

   1.0000                  0                  0                  0             1.0000                  0                  0
0
        0             8.0000 -10.0000i   -8.0000                  0                  0            -1.0000
0                  0
```

```
       0        8.0000       -2.0000          0             0             0          1.0000
0
       0           0            0        4.0000 + 2.0000i   0             0             0
-1.0000
       0           0            0            0          -2.0000       1.0000            0
0
  1.0000      -2.0000            0            0             0             0             0
0
       0           0            0            0             0             0          1.0000
-2.0000
       0           0        -2.0000      1.0000              0             0             0
0

>> X = [6;0;0;0;0;0;0;0]

X =

     6
     0
     0
     0
     0
     0
     0
     0

>> Y = inv(A)*X

Y =
```

0.9708 + 0.7523i = $I_1 = 1.2817\angle 37.773^\circ$ amps
0.4854 + 0.3761i = $I_2 = 614.056\angle 37.769^\circ$ mA = I_x
-0.3017 - 0.0425i = $I_3 = 0.30468\angle -171.982^\circ$ amps
-0.6034 - 0.0851i = I_4
5.0292 - 0.7523i = V_1
10.0583 - 1.5046i = V_2
-4.4867 - 3.0943i = V_3
-2.2434 - 1.5471i = V_4

$$\mathbf{I_x = 614.1\angle 37.77^\circ \text{ mA}}$$

Finally, $V_x = 8(I_2 - I_3) = 8(0.7871+j0.4186) = 8(0.891489\angle 28.01^\circ)$

$$= \mathbf{7.132\angle 28.01^\circ \text{ volts}}$$

Solution 13.84

The schematic is shown below. we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E–01	4.028 E+00	–5.238 E+01

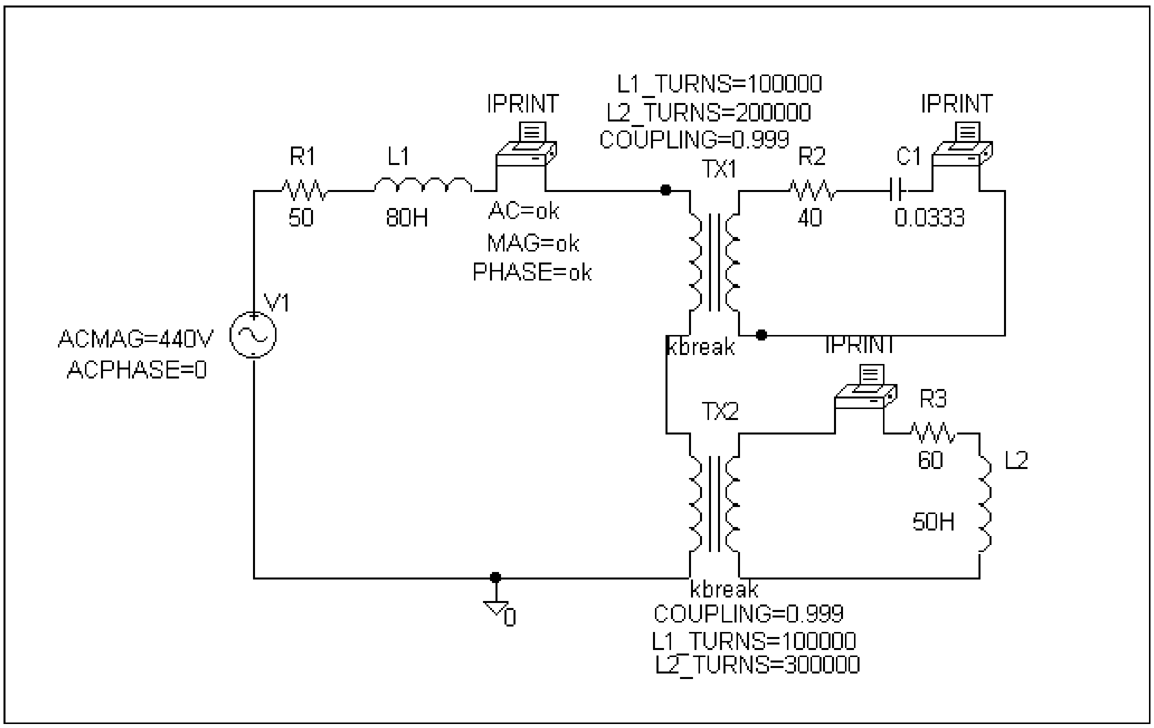
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E–01	2.019 E+00	–5.211 E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E–01	1.338 E+00	–5.220 E+01

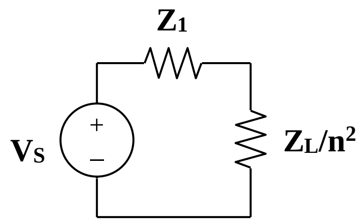
i.e. $I_1 = \mathbf{4.028\angle{-52.38^\circ}\ A}$, $I_2 = \mathbf{2.019\angle{-52.11^\circ}\ A}$,

$I_3 = \mathbf{1.338\angle{-52.2^\circ}\ A}$.

Dot convention is wrong.



Solution 13.85



For maximum power transfer,

$$Z_1 = Z_L/n^2 \text{ or } n^2 = Z_L/Z_1 = 8/7200 = 1/900$$

$n = 1/30 = N_2/N_1$. Thus $N_2 = N_1/30 = 3000/30 =$ **100 turns**.

Solution 13.86

$n = N_2/N_1 = 48/2400 = 1/50$

$Z_{Th} = Z_L/n^2 = 3/(1/50)^2 =$ **7.5 kΩ**

Solution 13.87

$Z_{Th} = Z_L/n^2$ or $n = \sqrt{Z_L / Z_{Th}} = \sqrt{75/300} = \mathbf{0.5}$

Solution 13.88

$n = V_2/V_1 = I_1/I_2$ or $I_2 = I_1/n = 2.5/0.1 = 25$ A

$p = IV = 25 \times 12.6 =$ **315 watts**

Solution 13.89

$n = V_2/V_1 = 120/240 =$ **0.5**

$S = I_1V_1$ or $I_1 = S/V_1 = 10x10^3/240 =$ **41.67 A**

$S = I_2V_2$ or $I_2 = S/V_2 = 10^4/120 =$ **83.33 A**

Solution 13.90

(a) $n = V_2/V_1 = 240/2400 = \mathbf{0.1}$

(b) $n = N_2/N_1$ or $N_2 = nN_1 = 0.1(250) =$ **25 turns**

(c) $S = I_1V_1$ or $I_1 = S/V_1 = 4\text{x}10^3/2400 =$ **1.6667 A**

$S = I_2V_2$ or $I_2 = S/V_2 = 4\text{x}10^4/240 =$ **16.667 A**

Solution 13.91

(a) The kVA rating is $S = VI = 25{,}000 \text{x} 75 = \mathbf{1.875\ MVA}$

(b) Since $S_1 = S_2 = V_2 I_2$ and $I_2 = 1875 \text{x} 10^3/240 = \mathbf{7.812\ kA}$

Solution 13.92

(a) $V_2/V_1 = N_2/N_1 = n$, $V_2 = (N_2/N_1)V_1 = (28/1200)4800 =$ **112 V**

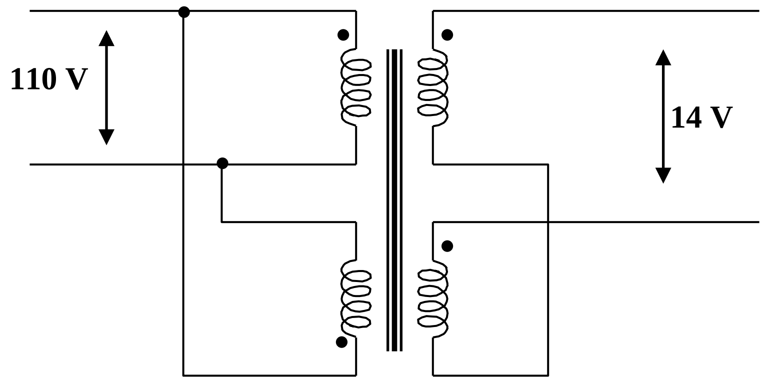
(b) $I_2 = V_2/R = 112/10 =$ **11.2 A** and $I_1 = nI_2$, $n = 28/1200$

$I_1 = (28/1200)11.2 =$ **261.3 mA**

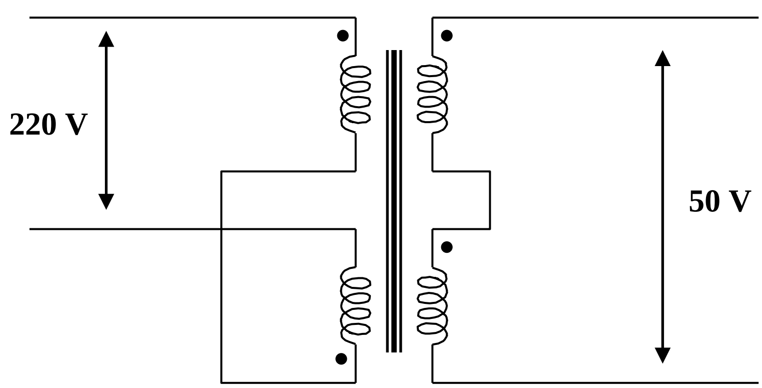
(c) $p = |I_2|^2R = (11.2)^2(10) =$ **1254 watts**.

Solution 13.93

(a) For an input of 110 V, the primary winding must be connected in parallel, with series aiding on the secondary. The coils must be series opposing to give 14 V. Thus, the connections are shown below.



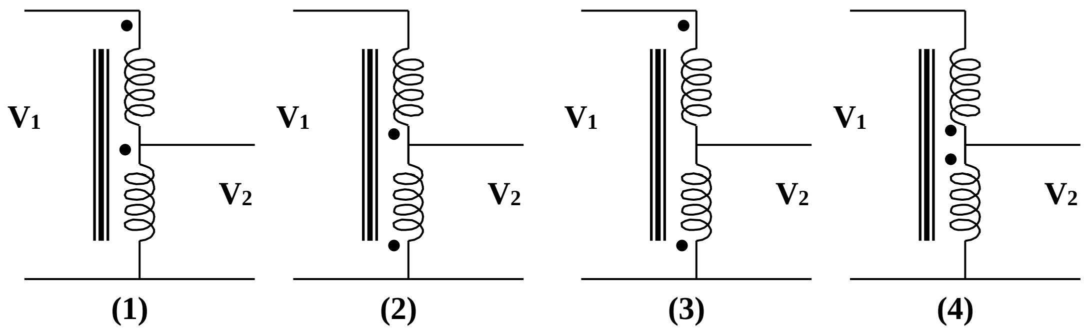
(b) To get 220 V on the primary side, the coils are connected in series, with series aiding on the secondary side. The coils must be connected series aiding to give 50 V. Thus, the connections are shown below.



Solution 13.94

$V_2/V_1 = 110/440 = 1/4 = I_1/I_2$

There are four ways of hooking up the transformer as an auto-transformer. However it is clear that there are only two outcomes.



(1) and (2) produce the same results and (3) and (4) also produce the same results. Therefore, we will only consider Figure (1) and (3).

(a) For Figure (3), $V_1/V_2 = 550/V_2 = (440 – 110)/440 = 330/440$

Thus, $V_2 = 550x440/330 =$ **733.4 V (not the desired result)**

(b) For Figure (1), $V_1/V_2 = 550/V_2 = (440 + 110)/440 = 550/440$

Thus, $V_2 = 550x440/550 =$ **440 V (the desired result)**

Solution 13.95

(a) $n = V_s/V_p = 120/7200 = \mathbf{1/60}$

(b) $I_s = 10x120/144 = 1200/144$

$$S = V_pI_p = V_sI_s$$

$$I_p = V_sI_s/V_p = (1/60)x1200/144 = \mathbf{139\ mA}$$

Solution 13.96*

Problem

Some modern power transmission systems now have major, high voltage DC transmission segments. There are a lot of good reasons for doing this but we will not go into them here. To go from the AC to DC, power electronics are used. We start with three-phase AC and then rectify it (using a full-wave rectifier). It was found that using a delta to wye and delta combination connected secondary would give us a much smaller ripple after the full-wave rectifier. How is this accomplished? Remember that these are real devices and are wound on common cores. Hint, using Figures 13.47 and 13.49, and the fact that each coil of the wye connected secondary and each coil of the delta connected secondary are wound around the same core of each coil of the delta connected primary so the voltage of each of the corresponding coils are in phase. When the output leads of both secondaries are connected through full-wave rectifiers with the same load, you will see that the ripple is now greatly reduced. Please consult the instructor for more help if necessary.

Solution

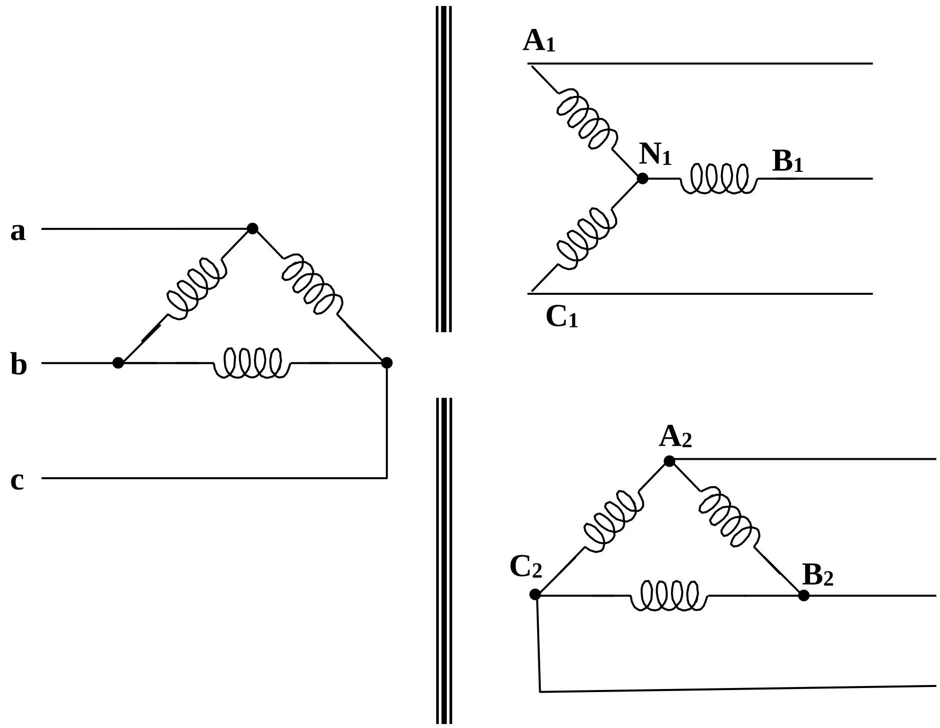
This is a most interesting and very practical problem. The solution is actually quite easy, you are creating a second set of sine waves to send through the full-wave rectifier, 30˚ out of phase with the first set. We will look at this graphically in a minute. We begin by showing the transformer components.

The key to making this work is to wind the secondary coils with each phase of the primary. Thus, a-b is wound around the same core as A_1-N_1 and A_2-B_2. The next thing we need to do is to make sure the voltages come out equal. We need to work the number of turns of each secondary so that the peak of $V_{A1}- V_{B1}$ is equal to $V_{A2}–V_{B2}$. Now, let us look at some of the equations involved.

If we let $v_{ab}(t) = 100\sin(t)$ V, assume that we have an ideal transformer, and the turns ratios are such that we get $v_{A1\text{-}N1}(t) = 57.74\sin(t)$ V and $V_{A2\text{-}B2}(t) = 100\sin(t)$ V. Next, let us look at $V_{bc}(t) = 100\sin(t+120^\circ)$ V. This leads to $V_{B1\text{-}N1}(t) = 57.74\sin(t+120^\circ)$ V. We now need to determine $V_{A1\text{-}B1}(t)$.

$$V_{A1\text{-}B1}(t) = 57.74\sin(t) – 57.74\sin(t+120^\circ) = 100\sin(t–30^\circ) \text{ V.}$$

This then leads to the output per phase voltage being equal to $v_{out}(t) = [100\sin(t) + 100\sin(t–30^\circ)]$ V. We can do this for each phase and end up with the output being sent to the full-wave rectifier. This looks like $v_{out}(t) = [|100\sin(t)| + |100\sin(t–30^\circ)| + |\sin(t+120^\circ)|\ |100\sin(t+90^\circ)| + |100\sin(t–120^\circ)| + |100\sin(t–150^\circ)|]$ V. The end result will be more obvious if we look at plots of the rectified output.



In the plot below we see the normalized (1 corresponds to 100 volts) ripple with only one of the secondary sets of windings and then the plot with both. Clearly the ripple is greatly reduced!