



12:00, Monday, October 18, 2021

Class Test 2:
Introduction and Systems AnalysisTime allowed: 45 minutes
Max mark: 30, Assessment: 10%This paper contains:
2 pages, 4 questions**IMPORTANT INSTRUCTIONS:**

Candidates should attempt **all** questions with numerical answers to **3** decimal places for each question, you need to show **your working to the final answer to gain maximum marks**.

Properly space solutions to ensure high quality image scans, **use black/blue pen or 2B pencil on white ruled/plain paper** to ensure sufficient contrast, and ensure **you are in a well-lit area**. You will also need a **scientific calculator** and scratch pad to for draft working.

Solutions will be marked page by page, so **start questions on new page**.

Question 1 (10 marks)

Determine the Fourier series representation, $X[k]$, of:

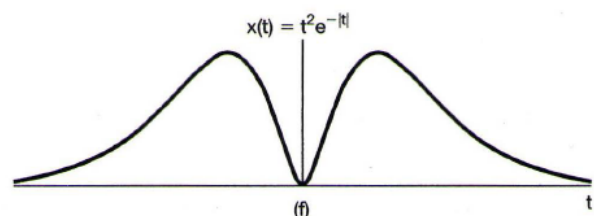
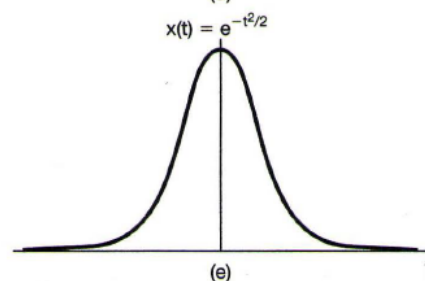
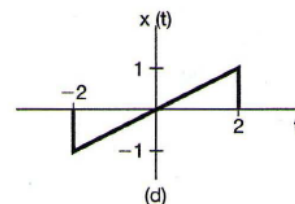
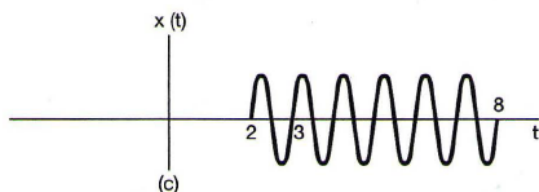
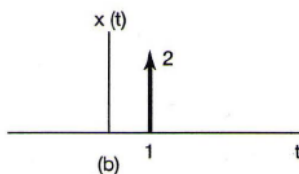
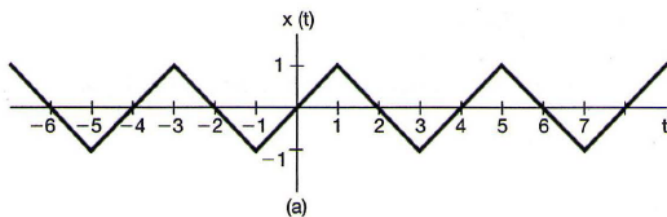
$$x(t) = 2 \sin(2\pi t - 3) + \sin(6\pi t)$$

HINT: Expand using *Euler's relation* and match with the *inverse Fourier series* expression.

Question 2 (6 marks)

For each condition listed below determine which, if any, of the real signals depicted below have Fourier transforms that satisfy that condition:

- (A) $\text{Re}\{X(j\omega)\} = 0$
- (B) $\text{Im}\{X(j\omega)\} = 0$
- (C) $|X(j\omega)| = 2$, for all ω
- (D) $X(j\omega)$ is non-zero only for certain values of ω

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Question 3 (6 marks)

- (i) Consider the continuous-time signal:

$$x(t) = 3 \cos 6\pi t + \sin 18\pi t + 2 \cos 28\pi t$$

What is the expression for the magnitude spectrum, $|X(\omega)|$, and sketch it as a function of ω .

HINT: You have three sinusoids so you will have just three spectral harmonics.

- (ii) What is the Nyquist rate, that is, determine the range of possible sampling frequencies (in Hz), f_s , required to be able to reconstruct $x(t)$ from these samples without error?
- (iii) What is the sampling frequency if you sample $x(t)$ at 25% above the Nyquist rate (i.e. your answer in (ii) x 1.25)? For this sampling frequency carefully sketch the magnitude spectrum of the sampled signal (in Hz) over the range ± 50 Hz.

Question 4 (8 marks)

Consider an LTI system frequency response:

$$H(j\omega) = \frac{3(3 + j\omega)}{8 - \omega^2 + 6j\omega}$$

- (a) What is the magnitude response, $|H(j\omega)|$?
- (b) What is the phase response, $\angle H(j\omega)$? HINT: Use the $\text{atan2}(y,x)$ function for the phase of $x + iy$
- (c) If the input is $x(t) = 3 \cos(2t)$, what is the output $y(t)$?
- (d) By evaluating $|H(j\omega)|$ at $\omega = 0$, $\omega^2 = 8$ and $\omega \rightarrow \infty$ indicate whether this LTI system represents a low-pass, high-pass or band-pass response?