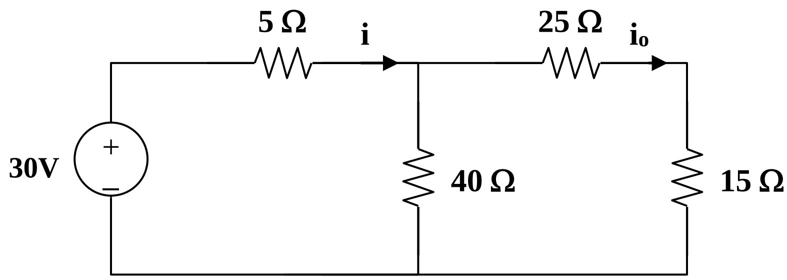
Solution 4.1



$40 \| (25+15) = 20\Omega$, i = [30/(5+20)] = 1.2 and i_o = i20/40 = **600 mA**.

Since the resistance remains the same we get can use linearity to find the new value of the voltage source = (30/0.6)5 = **250 V**.

Solution 4.2

Using Fig. 4.70, design a problem to help other students better understand linearity.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find v_o in the circuit of Fig. 4.70. If the source current is reduced to 1 μA, what is v_o?

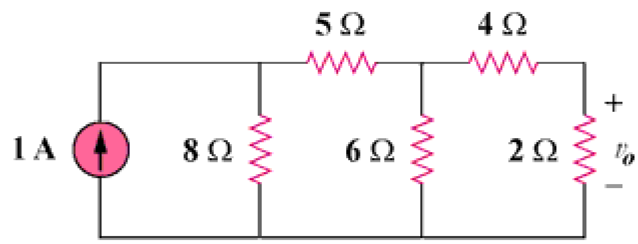
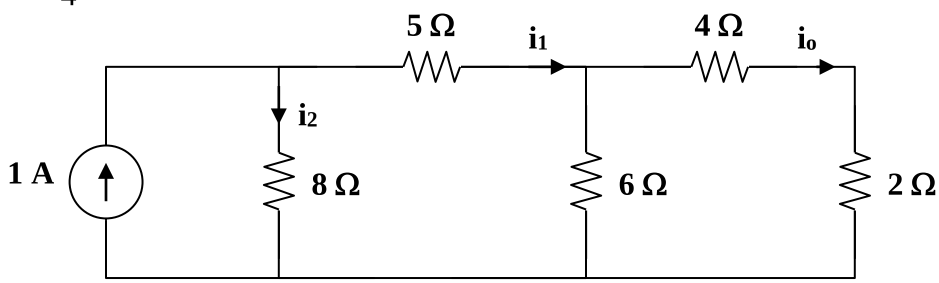


Figure 4.70

Solution

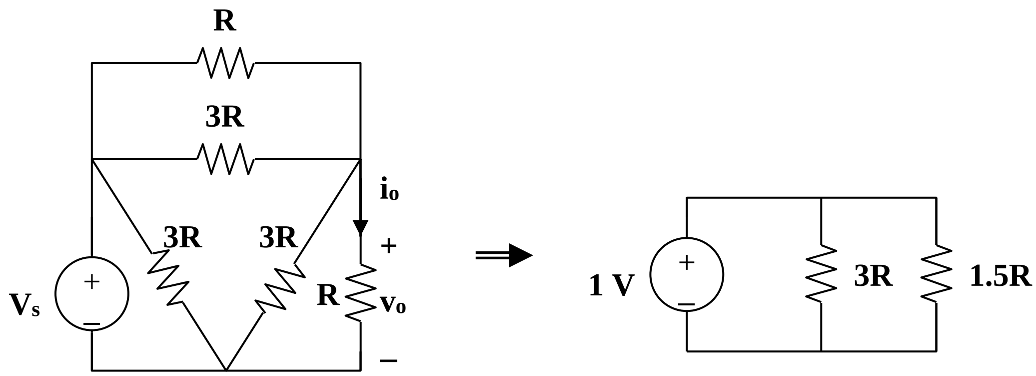
$6\|(4+2) = 3\Omega, \quad i_1 = i_2 = \frac{1}{2}\text{A}$

$i_o = \frac{1}{2} i_1 = \frac{1}{4}, \quad v_o = 2i_o = \underline{\mathbf{0.5V}}$



If i_s = 1μA, then v_o = **0.5μV**

Solution 4.3



(a) (b)

(a) We transform the Y sub-circuit to the equivalent Δ.

$$R \| 3R = \frac{3R^2}{4R} = \frac{3}{4}R, \quad \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$v_o = \dfrac{v_s}{2}$ independent of R

$i_o = v_o/(R)$

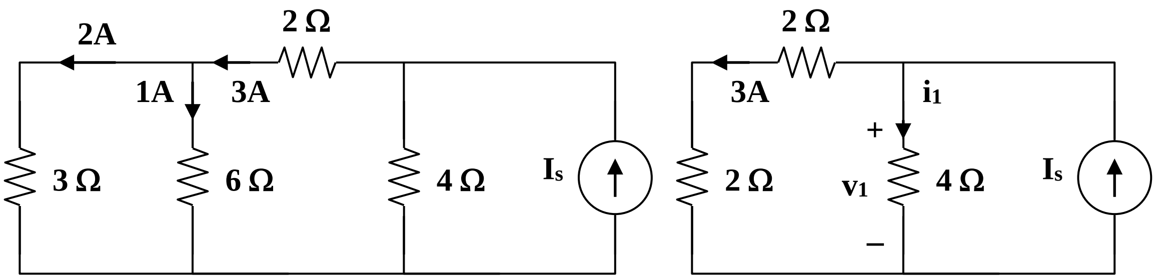
When v_s = 1V, v_o = **0.5V**, i_o = **0.5A**

(b) When v_s = 10V, v_o = **5V**, i_o = **5A**

(c) When v_s = 10V and R = 10Ω,
v_o = **5V**, i_o = 10/(10) = **500mA**

Solution 4.4

If $I_o = 1$, the voltage across the 6Ω resistor is 6V so that the current through the 3Ω resistor is 2A.



(a) (b)

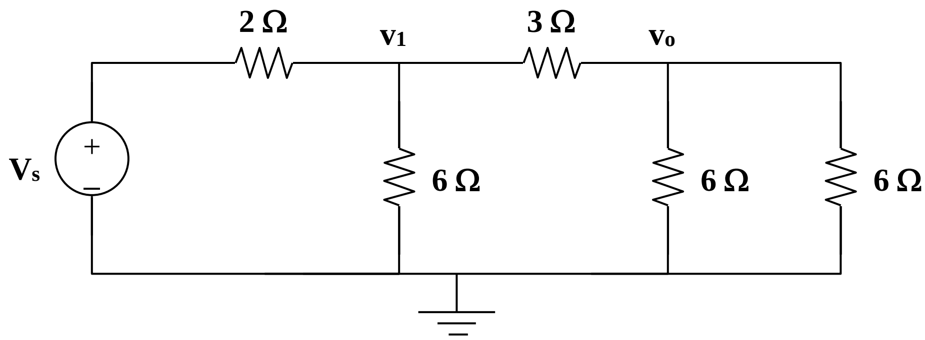
$3\|6 = 2\Omega$, $v_o = 3(4) = 12V$, $i_1 = \frac{v_o}{4} = 3A$.

Hence $I_s = 3 + 3 = 6A$

If $I_s = 6A \longrightarrow I_o = 1$

$I_s = 9A \longrightarrow I_o = 9/6 =$ **1.5A**

Solution 4.5



If v_o = 1V, $\quad V_1 = \left(\frac{1}{3}\right) + 1 = 2V$

$$V_s = 2\left(\frac{2}{3}\right) + v_1 = \frac{10}{3}$$

If $v_s = \frac{10}{3} \longrightarrow v_o = 1$

Then $v_s = 15 \longrightarrow v_o = \frac{3}{10} x15 =$ **4.5V**

Solution 4.6

Due to linearity, from the first experiment,

$$V_o = \frac{1}{3} V_s$$

Applying this to other experiments, we obtain:

Experiment	V_s	V_o
2	48	16 V
3	1 V	0.333 V
4	-6 V	-2V

Solution 4.7

Use linearity and the assumption that V_x = 1V to find the actual value of V_o in Fig. 4.75.

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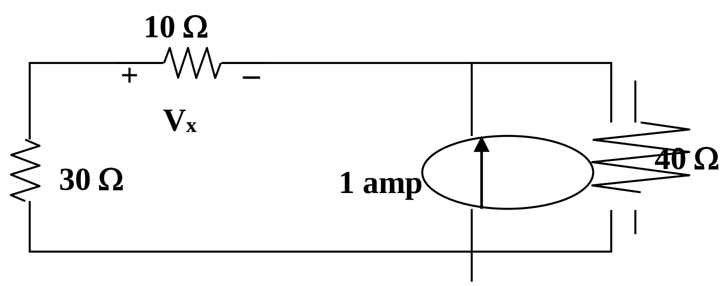


Figure 4.75
For Prob. 4.7.

Solution

Step 1. If we let V_x = 1 volt then I_{10} = 0.1 amp which leads to $V_{30\text{-}10}$ = 0.1x40 = 4 volts. Then I_{40} = 4/40 = 0.1 amp which would have required a current source equal to –0.1 – 0.1 = –0.2 amps.

Step 2. Since the current source is 1 amp which is –5(–0.2) then the voltage V_x = –5x1 or,

V_x = **–5volts**.

If V_o = 1V, then the current through the 2-Ω and 4-Ω resistors is ½ = 0.5. The voltage across the 3-Ω resistor is ½ (4 + 2) = 3 V. The total current through the 1-Ω resistor is 0.5 +3/3 = 1.5 A. Hence the source voltage

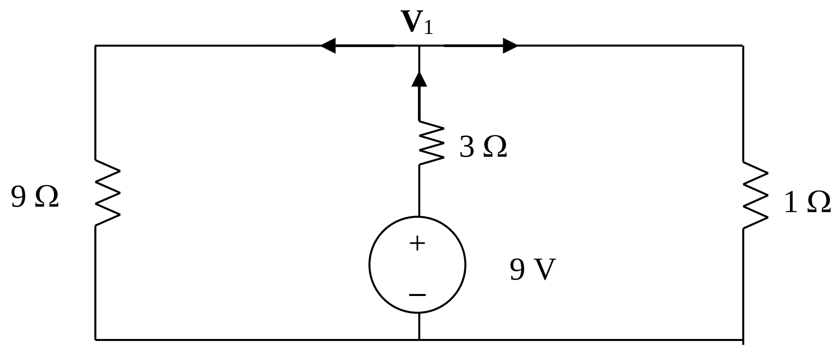
$$v_s = 1x1.5 + 3 = 4.5 \text{ V}$$

If $\quad v_s = 4.5 \quad \longrightarrow \quad 1V$

Then $\quad v_s = 4 \quad \longrightarrow \quad \frac{1}{4.5}x4 = \underline{0.8889 \text{ V}}$ = **888.9 mV**.

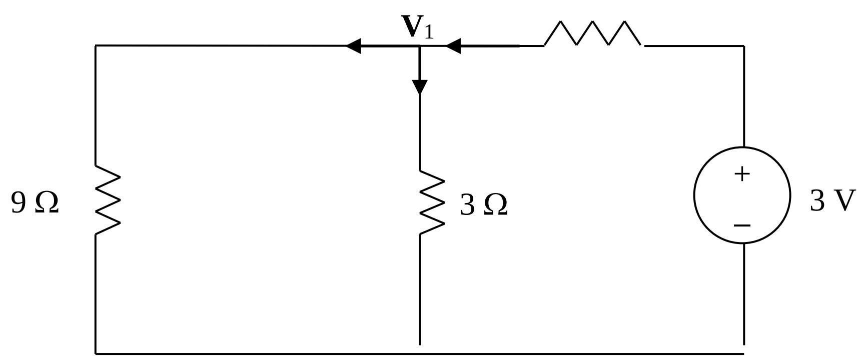
Solution 4.8

Let $V_o = V_1 + V_2$, where V_1 and V_2 are due to 9-V and 3-V sources respectively. To find V_1, consider the circuit below.



$$\frac{9-V_1}{3} = \frac{V_1}{9} + \frac{V_1}{1} \longrightarrow V_1 = 27/13 = 2.0769$$

To find V_2, consider the circuit below.



$$\frac{V_2}{9} + \frac{V_2}{3} = \frac{3-V_2}{1} \longrightarrow V_2 = 27/13 = 2.0769$$

$$V_o = V_1 + V_2 = \mathbf{4.1538\ \ V}$$

Solution 4.9

Given that I = 6 amps when V_s = 160 volts and I_s = –10 amps and I = 5 amp when V_s = 200 volts and I_s = 0, use superposition and linearity to determine the value of I when V_s = 120 volts and I_s = 5 amps.

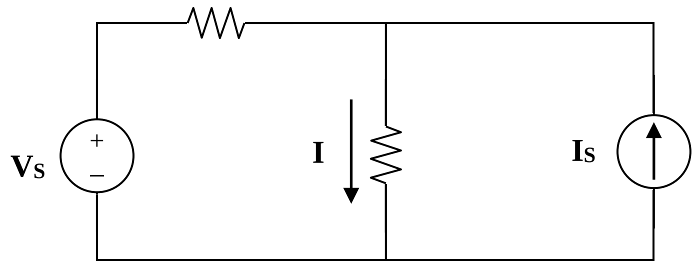


Figure 4.77
For Prob. 4.9.

Solution

At first this appears to be a difficult problem. However, if you take it one step at a time then it is not as hard as it seems. The important thing to keep in mind is that it is linear!

First superposition tells us that I = I' + I" where I' is the current contributed by the voltage source and I" is current contributed by the current source. Linearity tells us that I' = $(V_s)5/200 = V_s/40$. To find the relationship for I" we use superposition and linearity to find the value for I" = I_s(K) where I = 6 = (160/40) + (–10)(K) or –10K = 6 – 4 = 2 or K = –0.2.

This then leads to I = (120/40) – 5(0.2) = 3 – 1 = **2 A**.

Solution 4.10

Using Fig. 4.78, design a problem to help other students better understand superposition. Note, the letter *k* is a gain you can specify to make the problem easier to solve but must not be zero.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 4.78, find the terminal voltage V_{ab} using superposition.

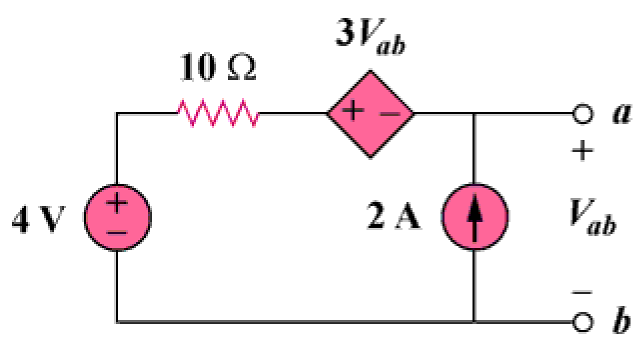
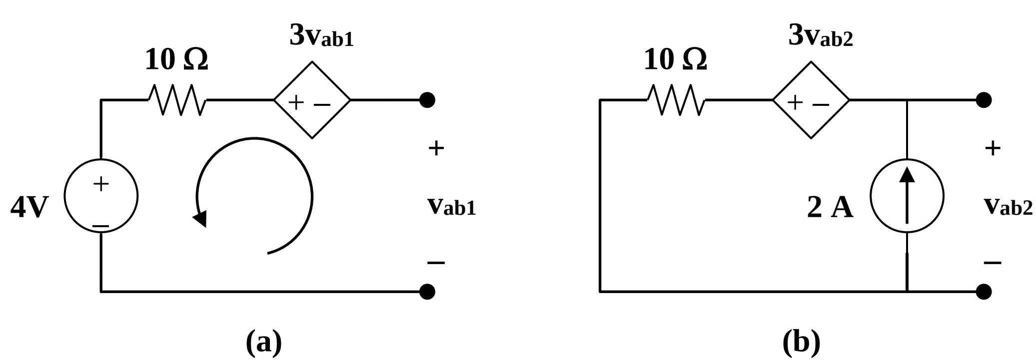


Figure 4.78
For Prob. 4.10.

Solution

Let $v_{ab} = v_{ab1} + v_{ab2}$ where v_{ab1} and v_{ab2} are due to the 4-V and the 2-A sources respectively.



For v_{ab1}, consider Fig. (a). Applying KVL gives,

$$- v_{ab1} - 3\,v_{ab1} + 10\text{x}0 + 4 = 0, \text{ which leads to } v_{ab1} = 1\text{ V}$$

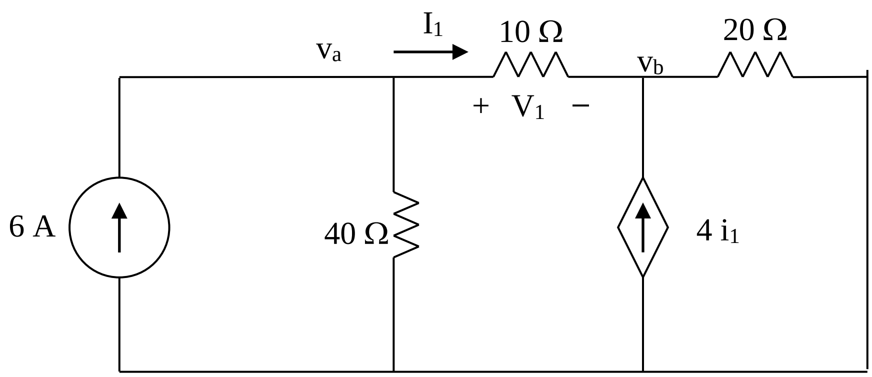
For v_{ab2}, consider Fig. (b). Applying KVL gives,

$$-v_{ab2} - 3v_{ab2} + 10x2 = 0, \text{ which leads to } v_{ab2} = 5$$

$$v_{ab} = 1 + 5 = \mathbf{6\ V}$$

Solution 4.11

Let $v_o = v_1 + v_2$, where v_1 and v_2 are due to the 6-A and 80-V sources respectively. To find v_1, consider the circuit below.



At node a,

$$6 = \frac{v_a}{40} + \frac{v_a - v_b}{10} \longrightarrow 240 = 5v_a - 4v_b \qquad (1)$$

At node b,

$$-I_1 - 4I_1 + (v_b - 0)/20 = 0 \text{ or } v_b = 100I_1$$

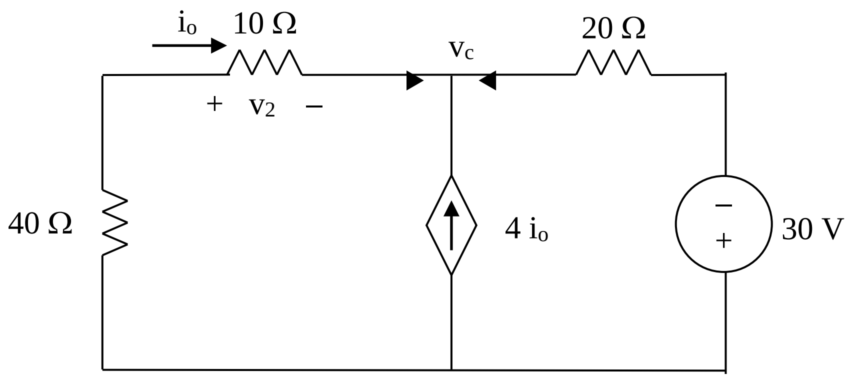
But $i_1 = \dfrac{v_a - v_b}{10}$ which leads to $100(v_a - v_b)10 = v_b$ or $v_b = 0.9091v_a$ (2)

Substituting (2) into (1),

$$5v_a - 3.636v_a = 240 \text{ or } v_a = 175.95 \text{ and } v_b = 159.96$$

However, $v_1 = v_a - v_b = 15.99$ V.

To find v_2, consider the circuit below.



$$\frac{0-v_c}{50}+4i_o+\frac{(-30-v_c)}{20}=0$$

But $\quad i_o=\dfrac{(0-v_c)}{50}$

$$-\frac{5v_c}{50}-\frac{(30+v_c)}{20}=0 \quad\longrightarrow\quad v_c=-10\text{ V}$$

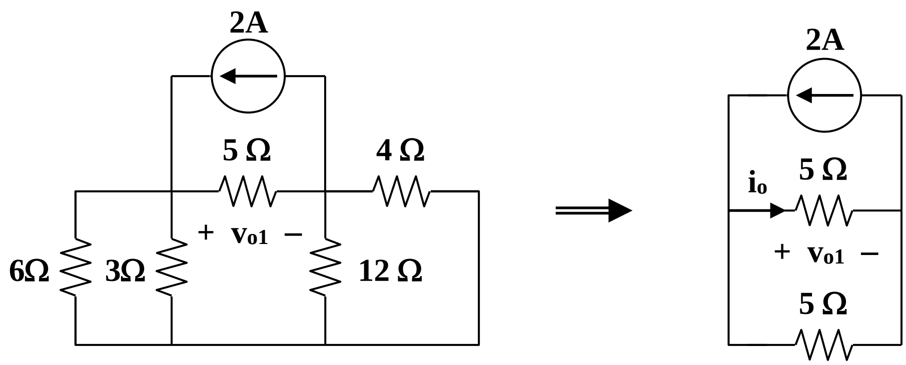
$$i_2=\frac{0-v_c}{50}=\frac{0+10}{50}=\frac{1}{5}$$

$$v_2=10i_2=2\text{ V}$$

$v_o = v_1 + v_2$ =15.99 + 2 = **17.99 V** and $i_o = v_o/10$= **1.799 A**.

Solution 4.12

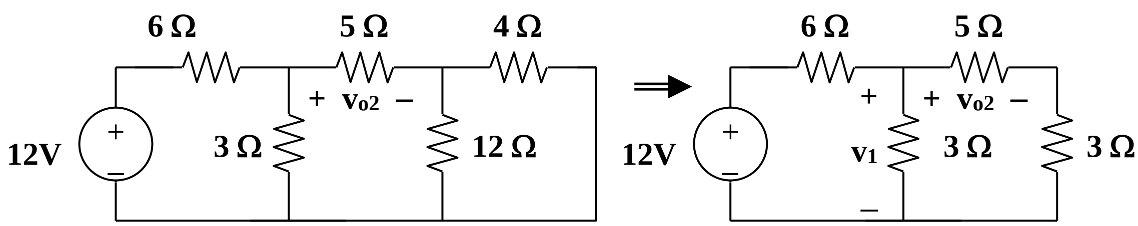
Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2}, and v_{o3} are due to the 2-A, 12-V, and 19-V sources respectively. For v_{o1}, consider the circuit below.



$$6||3 = 2 \text{ ohms}, \ 4||12 = 3 \text{ ohms. Hence,}$$

$$i_o = 2/2 = 1, v_{o1} = 5io = 5 \text{ V}$$

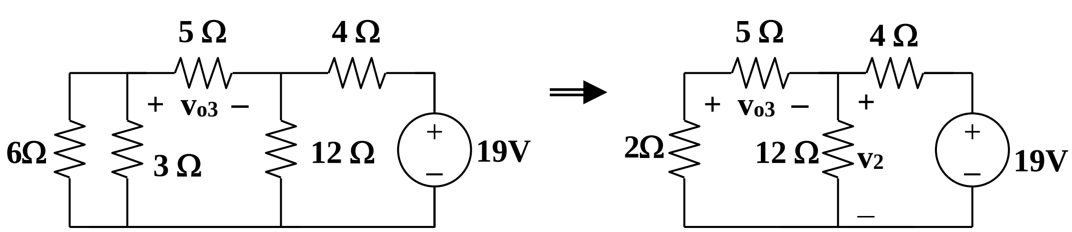
For v_{o2}, consider the circuit below.



$$3||8 = 24/11, \ v_1 = [(24/11)/(6 + 24/11)]12 = 16/5$$

$$v_{o2} = (5/8)v_1 = (5/8)(16/5) = 2 \text{ V}$$

For v_{o3}, consider the circuit shown below.

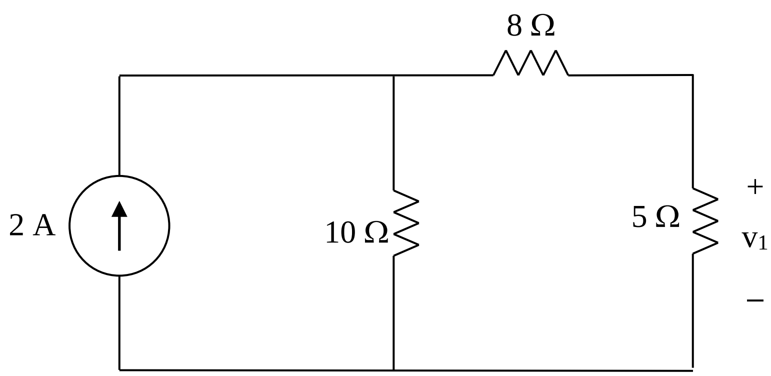


$$7||12 = (84/19) \text{ ohms}, \ v_2 = [(84/19)/(4 + 84/19)]19 = 9.975$$

$$v = (-5/7)v2 = -7.125$$
$$v_o = 5 + 2 – 7.125 = \mathbf{-125\ mV}$$

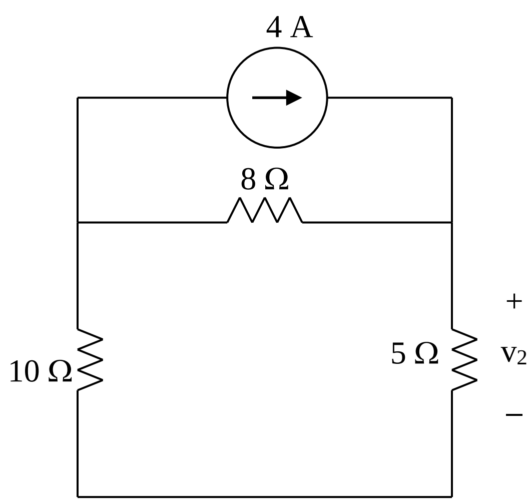
Solution 4.13

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are due to the independent sources. To find v_1, consider the circuit below.



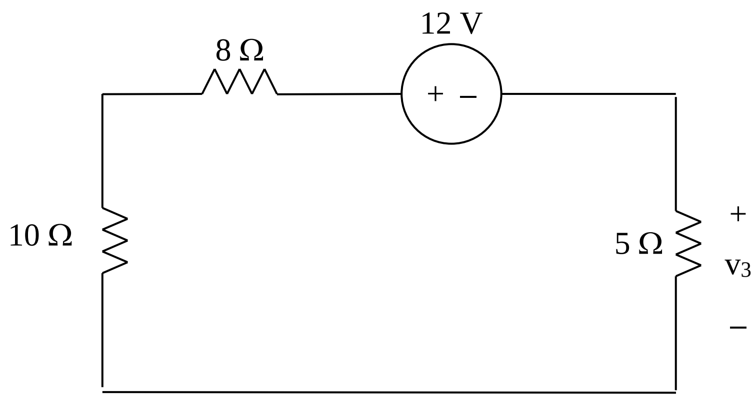
$$v_1 = 5x\frac{10}{10+8+5}x2 = 4.3478$$

To find v_2, consider the circuit below.



$$v_2 = 5x\frac{8}{8+10+5}x4 = 6.9565$$

To find v_3, consider the circuit below.

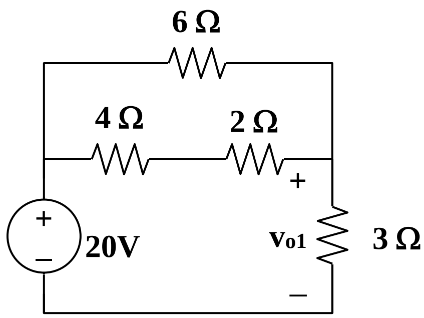


$$v_3 = -12\left(\frac{5}{5+10+8}\right) = -2.6087$$

$$v_o = v_1 + v_2 + v_3 = \underline{8.6956\ \text{V}} = \mathbf{8.696V}.$$

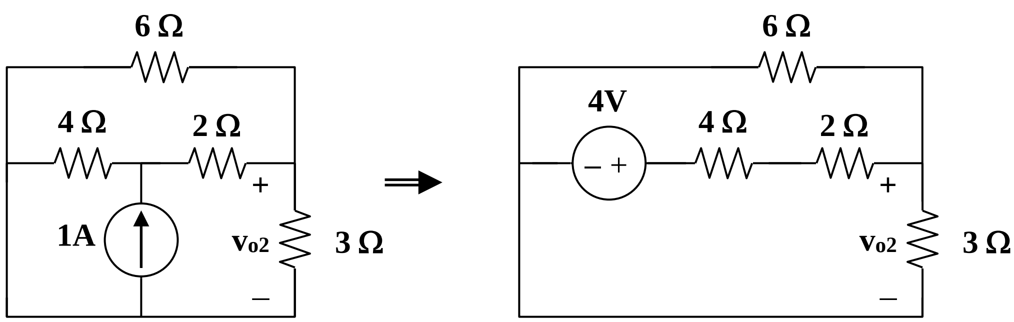
Solution 4.14

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2} , and v_{o3}, are due to the 20-V, 1-A, and 2-A sources respectively. For v_{o1}, consider the circuit below.



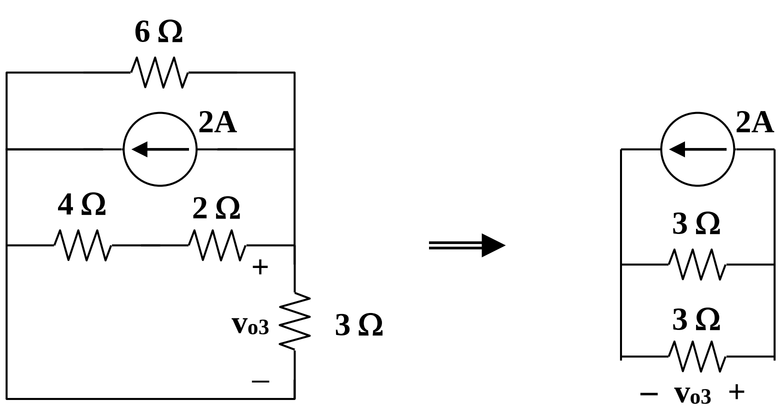
$$6||(4 + 2) = 3 \text{ ohms}, v_{o1} = (½)20 = 10 \text{ V}$$

For v_{o2}, consider the circuit below.



$$3||6 = 2 \text{ ohms}, v_{o2} = [2/(4 + 2 + 2)]4 = 1 \text{ V}$$

For v_{o3}, consider the circuit below.

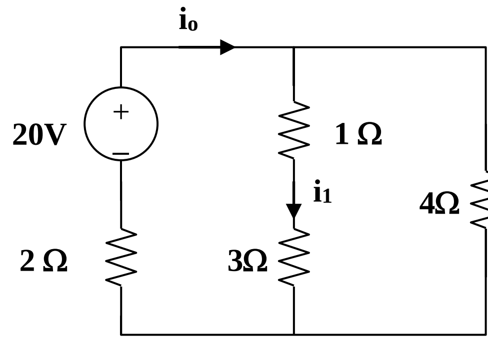


$$6||(4 + 2) = 3, v_{o3} = (-1)3 = –3$$

$$v_o = 10 + 1 – 3 = \mathbf{8\ V}$$

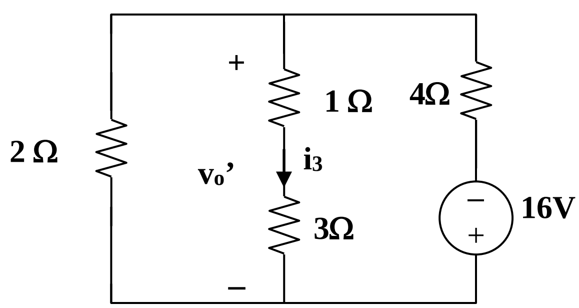
Solution 4.15

Let $i = i_1 + i_2 + i_3$, where i_1, i_2, and i_3 are due to the 20-V, 2-A, and 16-V sources. For i_1, consider the circuit below.



$$4\|(3 + 1) = 2 \text{ ohms, Then } i_o = [20/(2 + 2)] = 5 \text{ A}, \quad i_1 = i_o/2 = 2.5 \text{ A}$$

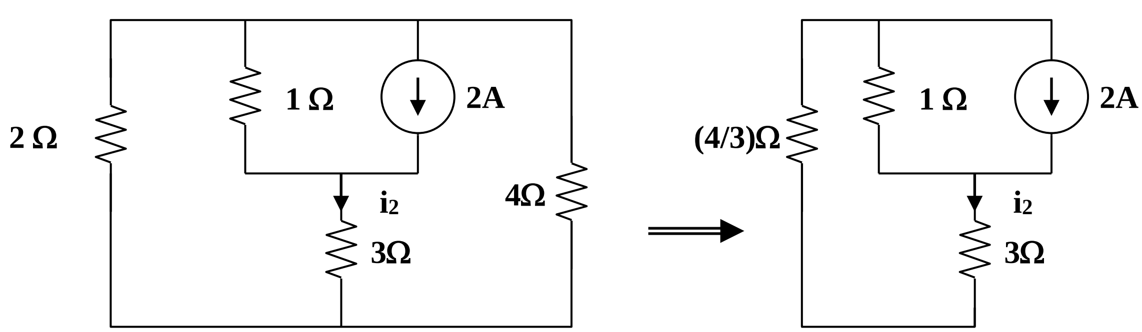
For i_3, consider the circuit below.



$$2\|(1 + 3) = 4/3, \quad v_o' = [(4/3)/((4/3) + 4)](-16) = -4$$

$$i_3 = v_o'/4 = -1$$

For i_2, consider the circuit below.



$$2\|4 = 4/3, \quad 3 + 4/3 = 13/3$$

Using the current division principle.

$$i_2 = [1/(1 + 13/2)]2 = 3/8 = 0.375$$

$$i = 2.5 + 0.375 - 1 = \mathbf{1.875\ A}$$

$$p = i^2R = (1.875)^2 3 = \mathbf{10.55\ watts}$$

Solution 4.16

Given the circuit in Fig. 4.84, use superposition to obtain $\boldsymbol{i_o}$.

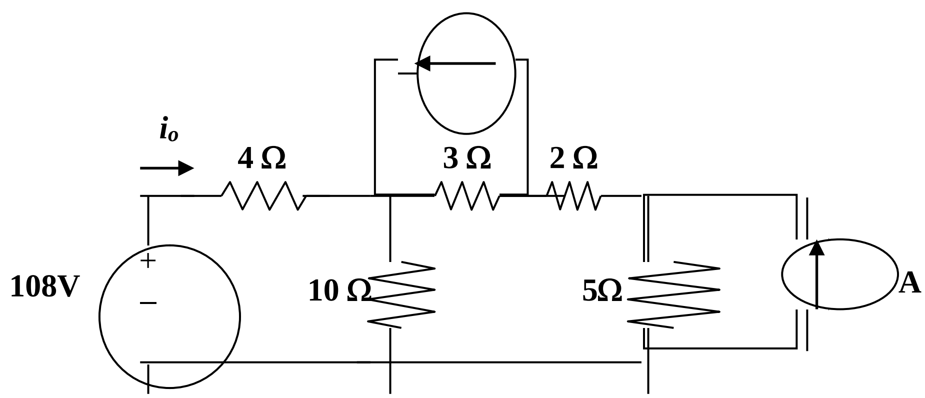
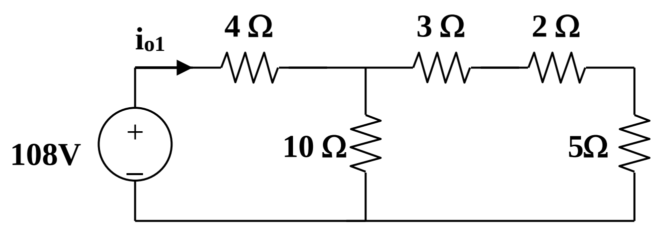


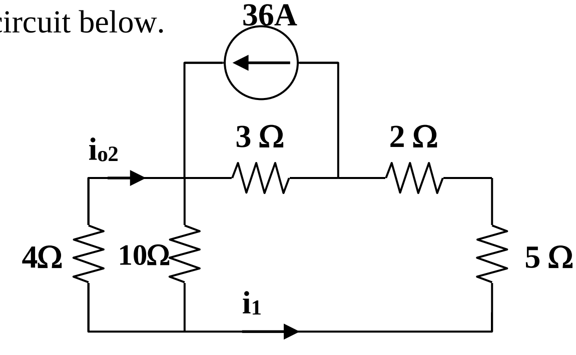
Figure 4.84
For Prob. 4.16.

Let $i_o = i_{o1} + i_{o2} + i_{o3}$, where i_{o1}, i_{o2}, and i_{o3} are due to the 108 V, 36 A, and 18 A sources. For i_{o1}, consider the circuit below.



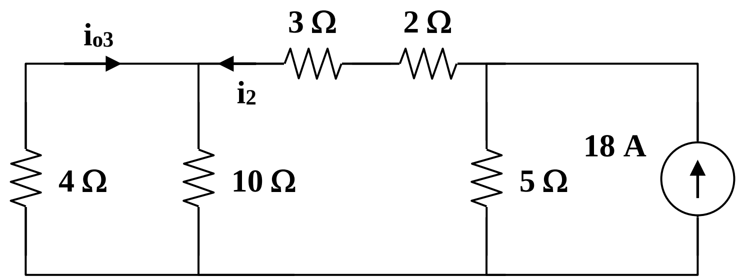
$$10||(3 + 2 + 5) = 5 \text{ ohms}, \quad i_{o1} = 108/(5 + 4) = 12 \text{ A}$$

For i_{o2}, consider the circuit below.



$$2 + 5 + 4||10 = 7 + 40/14 = 69/7$$
$$i_1 = [3/(3 + 69/7)]36 = 756/90 = 8.4, \quad i_{o2} = [-10/(4 + 10)]i_1 = -6 \text{ A}$$

For i_{o3}, consider the circuit below.



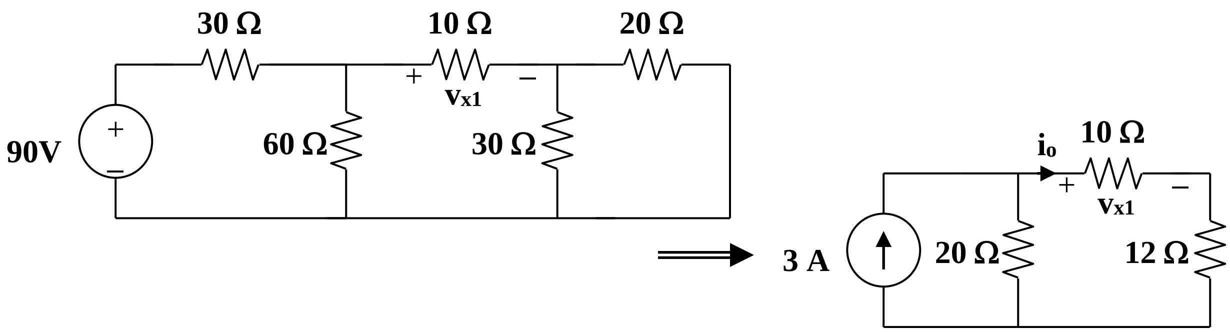
$$3 + 2 + 4||10 = 5 + 20/7 = 55/7$$

$$i_2 = [5/(5 + 55/7)]18 = 7, \quad i_{o3} = [-10/(10 + 4)]i_2 = -5$$

$$i_o = 12 - 6 - 5 = 1 = \mathbf{1\ A}.$$

Solution 4.17

Let $v_x = v_{x1} + v_{x2} + v_{x3}$, where v_{x1}, v_{x2}, and v_{x3} are due to the 90-V, 6-A, and 40-V sources. For v_{x1}, consider the circuit below.

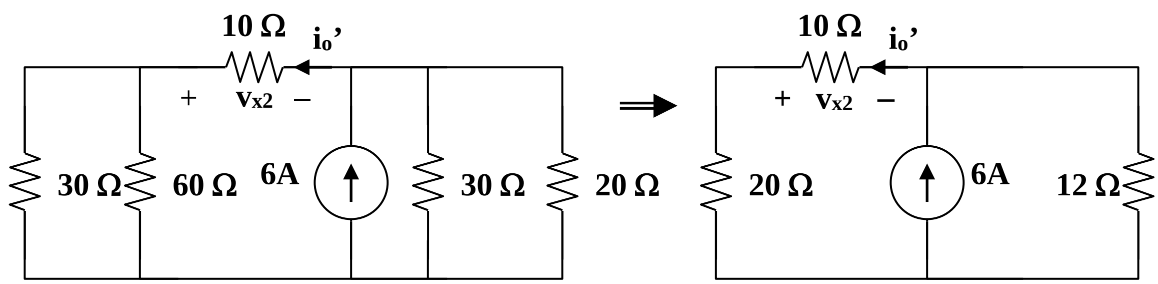


$$20||30 = 12 \text{ ohms}, \quad 60||30 = 20 \text{ ohms}$$

By using current division,

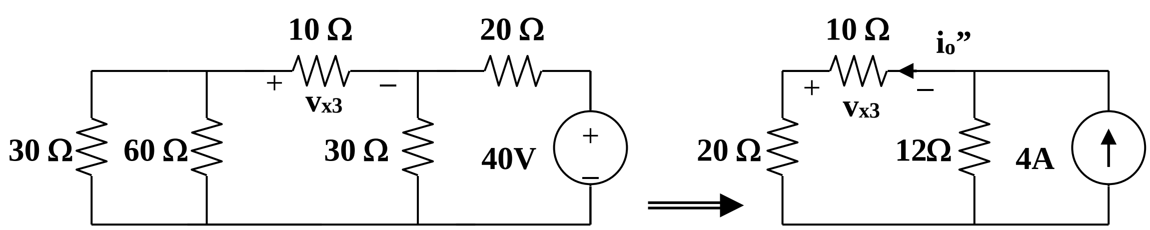
$$i_o = [20/(22 + 20)]3 = 60/42, \quad v_{x1} = 10i_o = 600/42 = 14.286 \text{ V}$$

For v_{x2}, consider the circuit below.



$$i_o' = [12/(12 + 30)]6 = 72/42, \quad v_{x2} = -10i_o' = -17.143 \text{ V}$$

For v_{x3}, consider the circuit below.



$$i_o" = [12/(12 + 30)]2 = 24/42, \quad v_{x3} = -10i_o" = -5.714 = [12/(12 + 30)]2 = 24/42,$$
$$v_{x3} = -10i_o" = -5.714$$
$$= [12/(12 + 30)]2 = 24/42, \quad v_{x3} = -10i_o" = -5.714$$
$$v_x = 14.286 - 17.143 - 5.714 = \mathbf{-8.571\ V}$$

Solution 4.18

Use superposition to find V_o in the circuit of Fig. 4.86.

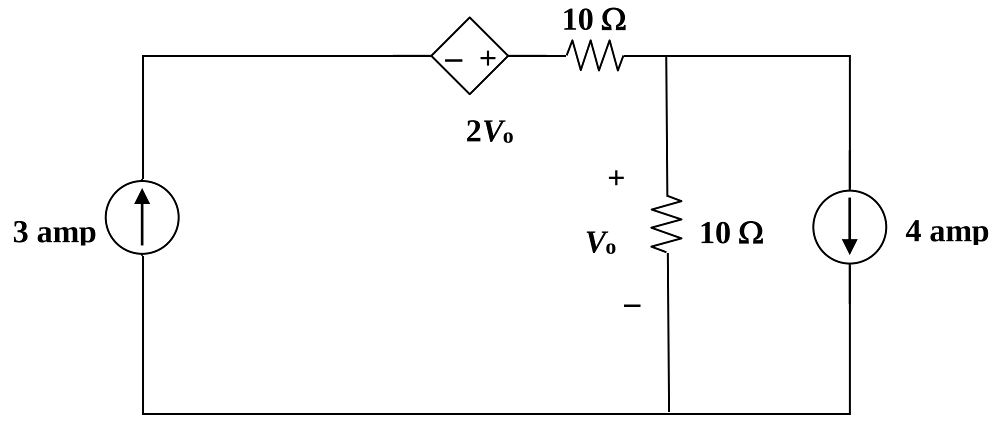
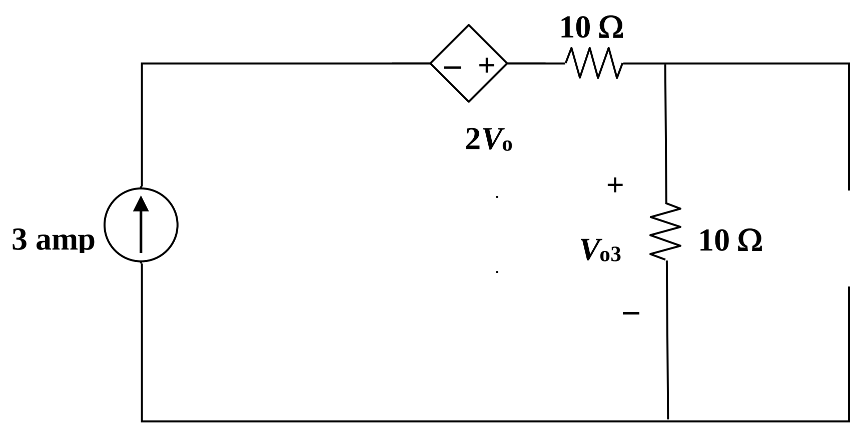
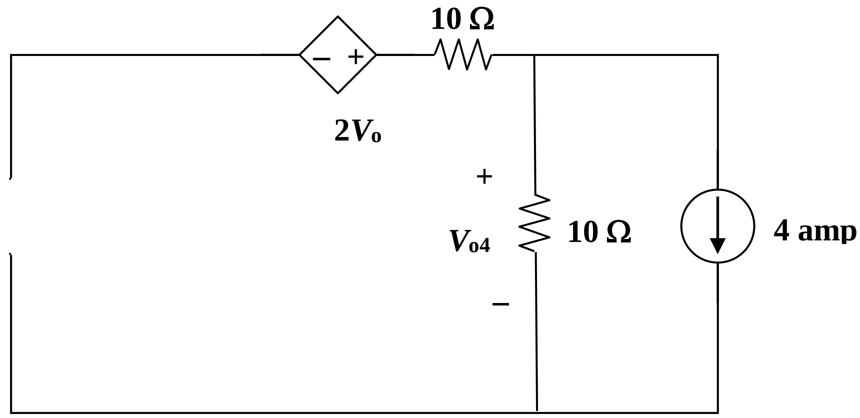


Figure 4.86
For Prob. 4.18.

Solution

Step 1. Since we only have two independent sources, we need to look at the contributions from each of these sources. Next we create two circuits.





Step 2. V_{o3} = 10x3 = 30 volts and V_{o4} = 10(–4) = –40 volts which leads to,

$V_o = V_{o3} + V_{o4}$ = 30 – 40 = **–10 volts**.

Solution 4.19

Use superposition to solve for v_x in the circuit of Fig. 4.87.

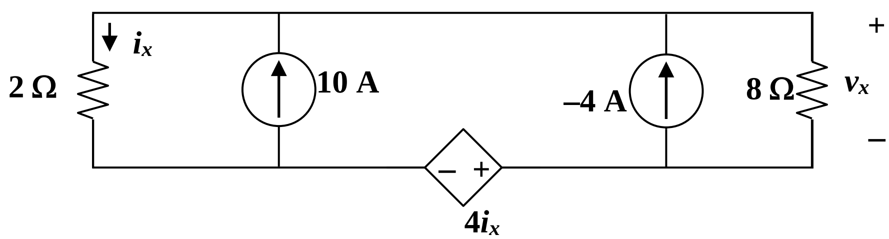
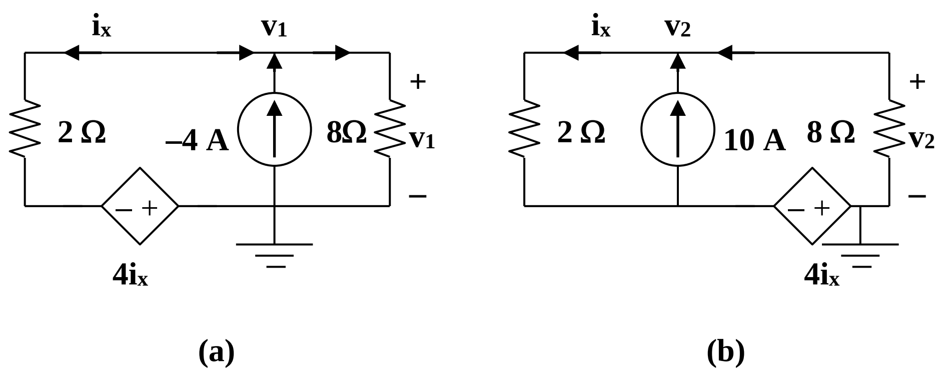


Figure 4.87
For Prob. 4.19.

Solution

Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the 4-A and 6-A sources respectively.



To find v_1, consider the circuit in Fig. (a).

$$[(v_1–0)/8] – (–4) + [(v_1 – (–4i_x))/2] = 0 \text{ or } (0.125+0.5)v_1 = –4 – 2i_x \text{ or } v_1 = –6.4 – 3.2i_x$$

But, $i_x = (v_1 – (–4i_x))/2$ or $i_x = –0.5v_1$. Thus,

$$v_1 = –6.4 + 3.2(0.5v_1), \text{ which leads to } v_1 = 6.4/0.6 = 10.667$$

To find v_2, consider the circuit shown in Fig. (b).

$$v_2/8 – 10 + (v_2 – (–4i_x))/2 = 0 \text{ or } v_2 + 3.2i_x = 16$$

But $i_x = –0.5v_2$. Therefore,

$$v_2 + 3.2(–0.5v_2) = 16 \text{ which leads to } v_2 = –26.67$$

Hence, $v_x = 10.667 - 26.667 = \mathbf{-16\ V}$.

Checking,

$$i_x = -0.5v_x = 8 \text{ A}$$

Now all we need to do now is sum the currents flowing out of the top node.

$$8 - 10 + 4 + (-16/8) = 0.$$

Solution 4.20

Use source transformations to reduce the circuit between terminals a and b shown in Fig. 4.88 to a single voltage source in series with a single resistor.

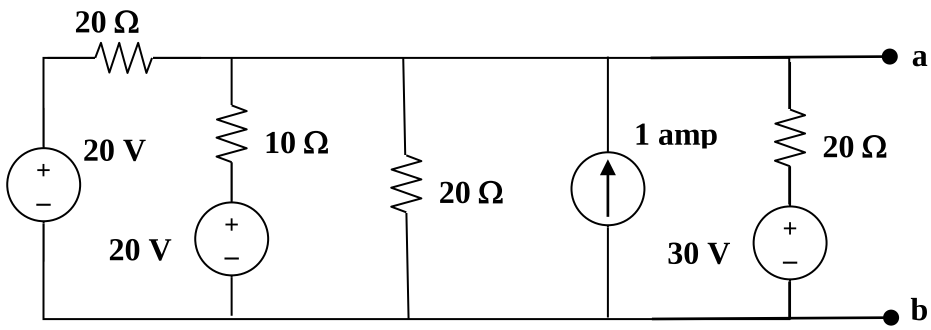
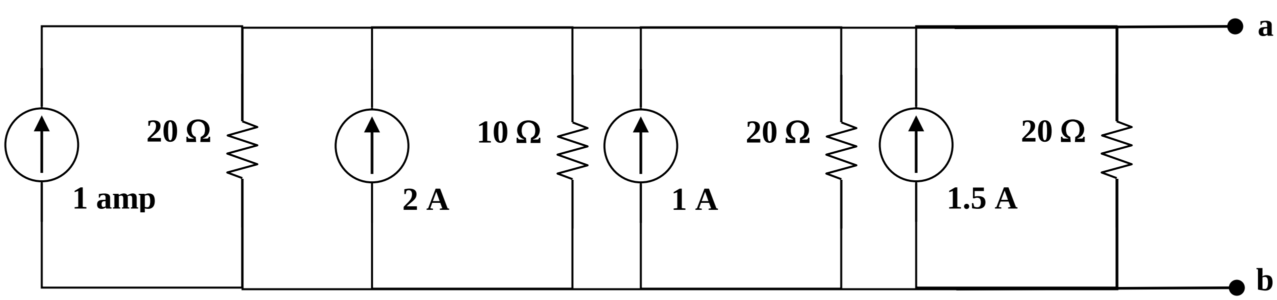


Figure 4.88
For Prob. 4.20.

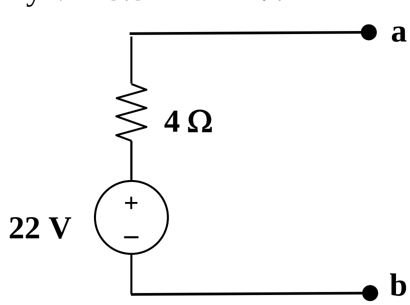
Solution

Step 1. This problem is most easily solved by converting all the voltage sources in series with resistors to current sources in parallel with resistors.



Now all we need is to add the current sources together algebraically and place all the resistors in parallel and combine them. Finally all we need to do is to change the current source and resistance back into a single voltage source in series with a resistor.

Step 2. $I = 1+2+1+1.5 = 5.5$ A and $(1/R_{eq}) = 0.05+0.1+0.05+0.05 = 0.25$ or R_{eq} = **4 Ω**. Finally V = 5.5x4 = **22 V**.



Solution 4.21

Using Fig. 4.89, design a problem to help other students to better understand source transformation.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Apply source transformation to determine $\boldsymbol{v_o}$ and $\boldsymbol{i_o}$ in the circuit in Fig. 4.89.

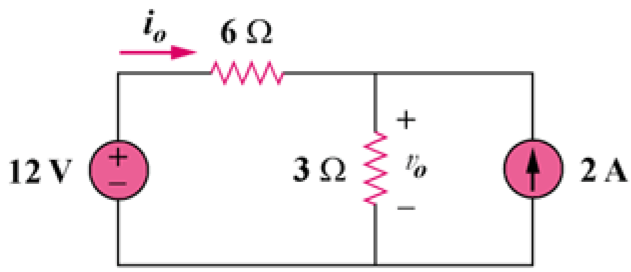
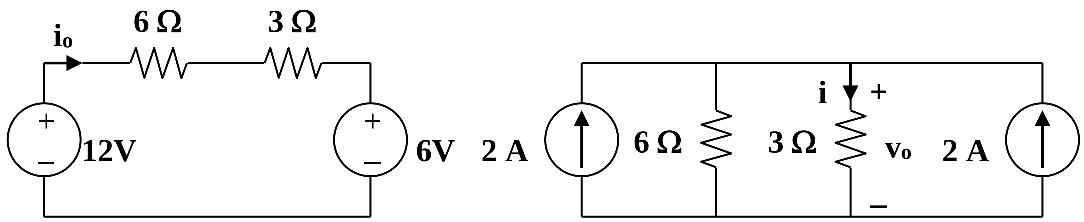


Figure 4.89

Solution

To get i_o, transform the current sources as shown in Fig. (a).



(a) (b)

From Fig. (a), $-12 + 9i_o + 6 = 0$, therefore $i_o = \mathbf{666.7\ mA}$

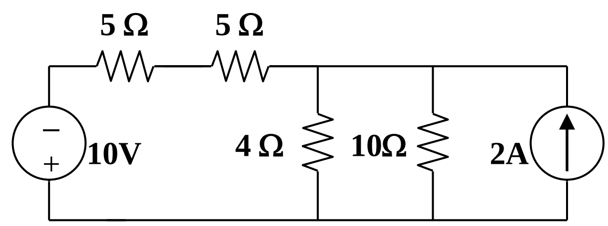
To get v_o, transform the voltage sources as shown in Fig. (b).

$$i = [6/(3 + 6)](2 + 2) = 8/3$$

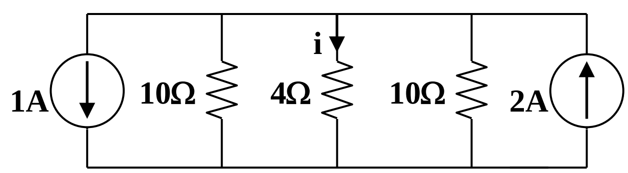
$$v_o = 3i = \mathbf{8\ V}$$

Solution 4.22

We transform the two sources to get the circuit shown in Fig. (a).



(a)



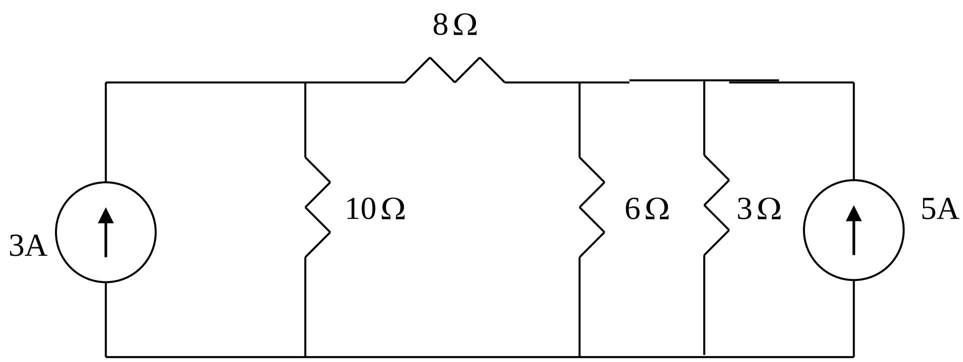
(b)

We now transform only the voltage source to obtain the circuit in Fig. (b).

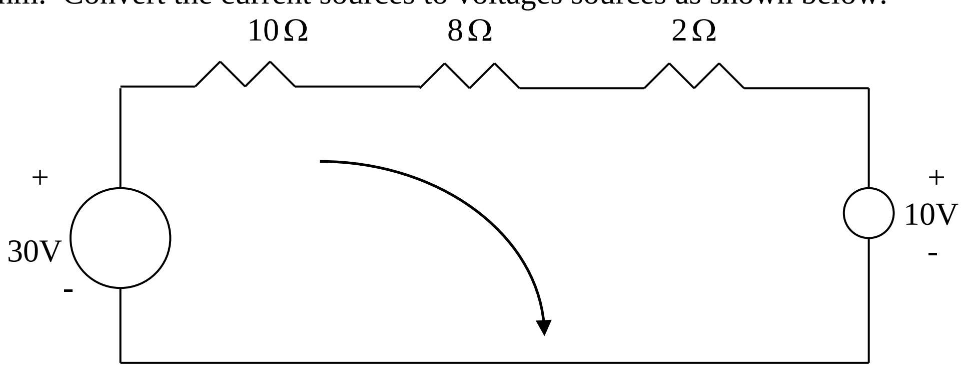
$$10||10 = 5 \text{ ohms}, \ i = [5/(5+4)](2-1) = 5/9 = \mathbf{555.5\ mA}$$

Solution 4.23

If we transform the voltage source, we obtain the circuit below.



3//6 = 2-ohm. Convert the current sources to voltages sources as shown below.



Applying KVL to the loop gives

$-30+10+I(10+8+2)=0 \quad \longrightarrow \quad$ I = **1 A**

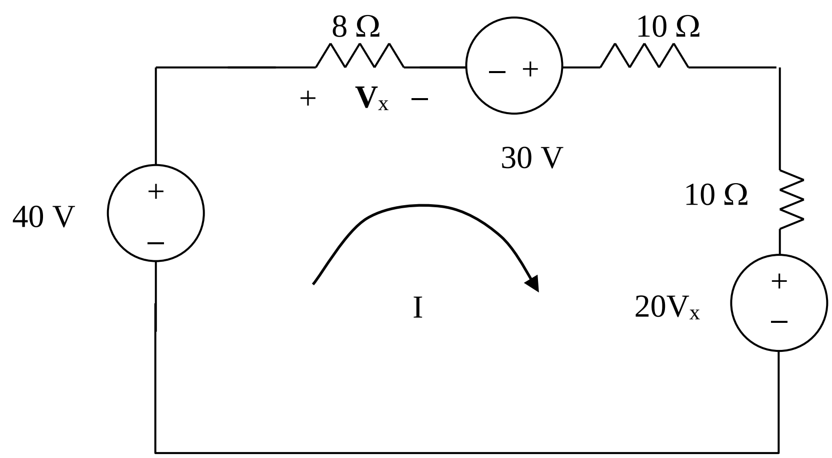
$$p = VI = I^2R = \mathbf{8\ W}$$

Solution 4.24

Transform the two current sources in parallel with the resistors into their voltage source equivalents yield,

> a 30-V source in series with a 10-Ω resistor and a $20V_x$-V sources in series with a 10-Ω resistor.

We now have the following circuit,

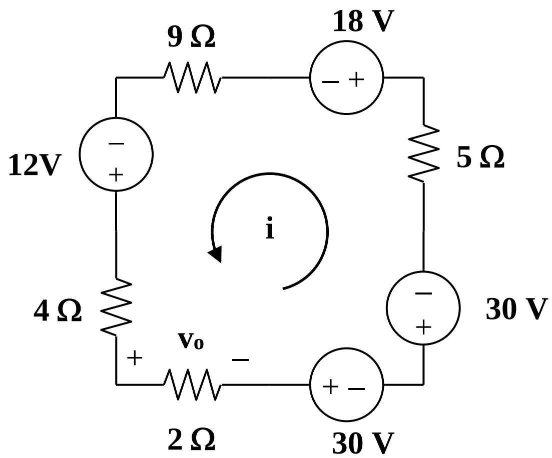


We now write the following mesh equation and constraint equation which will lead to a solution for V_x,

> $28I – 70 + 20V_x = 0$ or $28I + 20V_x = 70$, but $V_x = 8I$ which leads to
>
> $28I + 160I = 70$ or $I = 0.3723$ A or V_x = **2.978 V**.

Solution 4.25

Transforming only the current source gives the circuit below.



Applying KVL to the loop gives,

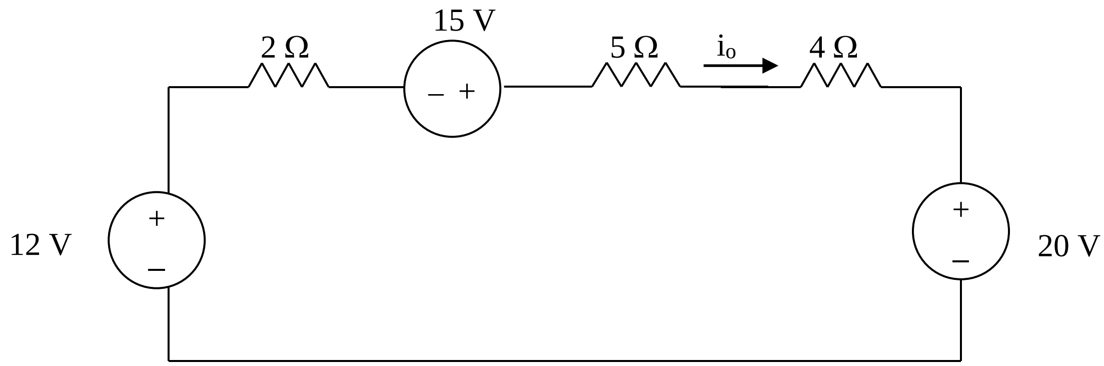
$$-(4 + 9 + 5 + 2)i + 12 - 18 - 30 - 30 = 0$$

$$20i = -66 \text{ which leads to } i = -3.3$$

$v_o = 2i =$ **-6.6 V**

Solution 4.26

Transforming the current sources gives the circuit below.



$-12 + 11i_o - 15 + 20 = 0$ or $11i_o = 7$ or $i_o =$ **636.4 mA**.

Solution 4.27

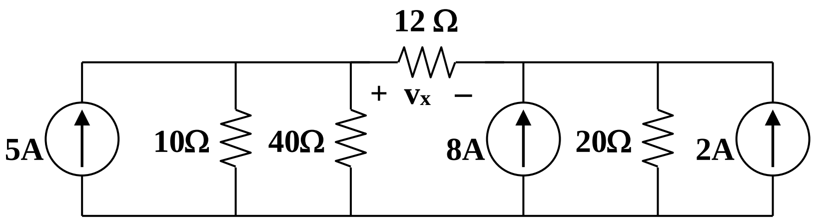
Transforming the voltage sources to current sources gives the circuit in Fig. (a).

$$10||40 = 8 \text{ ohms}$$

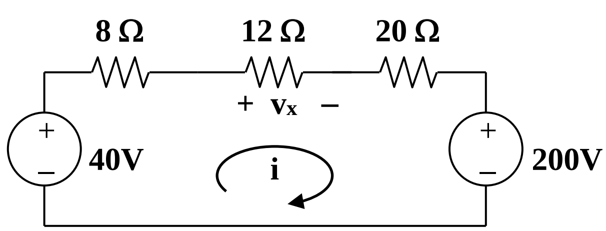
Transforming the current sources to voltage sources yields the circuit in Fig. (b). Applying KVL to the loop,

$$-40 + (8 + 12 + 20)i + 200 = 0 \text{ leads to } i = -4$$

$$v_x \;\; 12i = \mathbf{-48\ V}$$



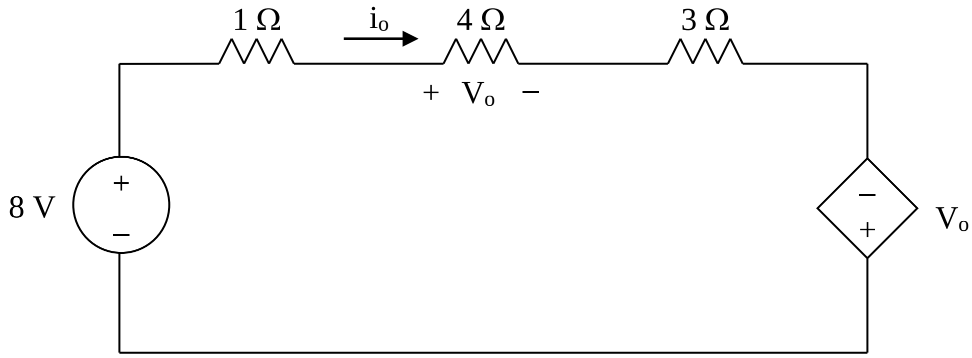
(a)



(b)

Solution 4.28

Convert the dependent current source to a dependent voltage source as shown below.



Applying KVL,

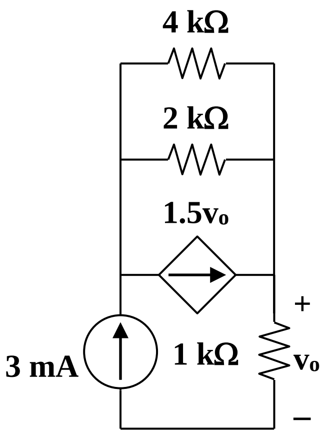
$$-8 + i_o(1 + 4 + 3) - V_o = 0$$

But $V_o = 4i_o$

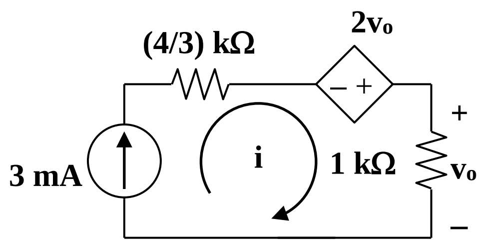
$$-8 + 8i_o - 4i_o = 0 \quad \longrightarrow \quad i_o = \underline{2\ \text{A}}$$

Solution 4.29

Transform the dependent voltage source to a current source as shown in Fig. (a). 2||4 = (4/3) k ohms



(a)



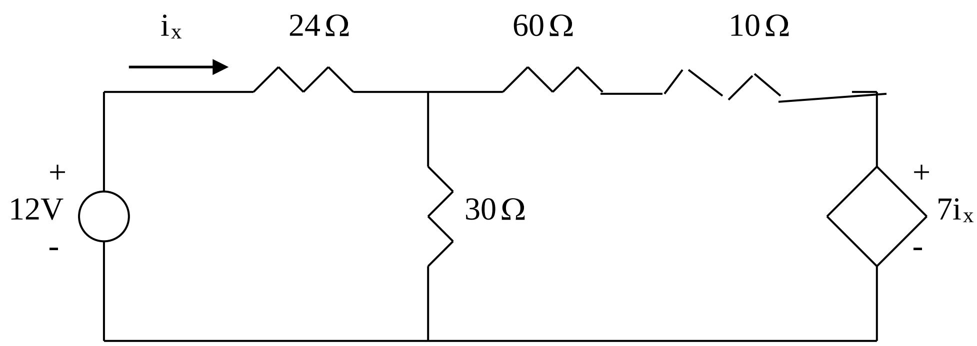
(b)

It is clear that i = 3 mA which leads to v_o = 1000i = **3 V**

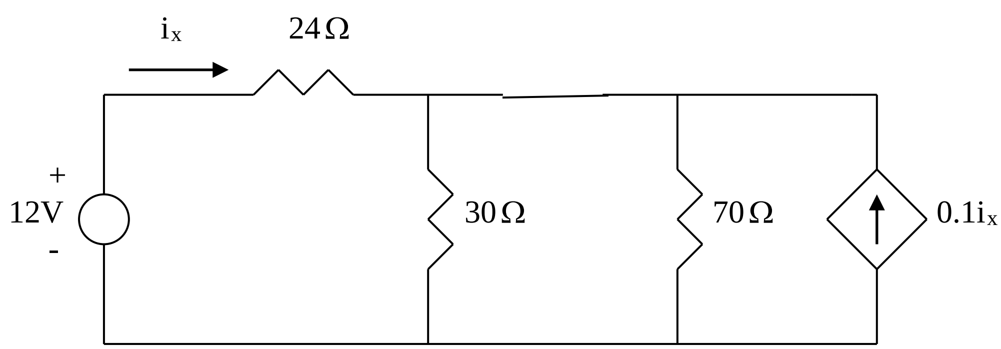
If the use of source transformations was not required for this problem, the actual answer could have been determined by inspection right away since the only current that could have flowed through the 1 k ohm resistor is 3 mA.

Solution 4.30

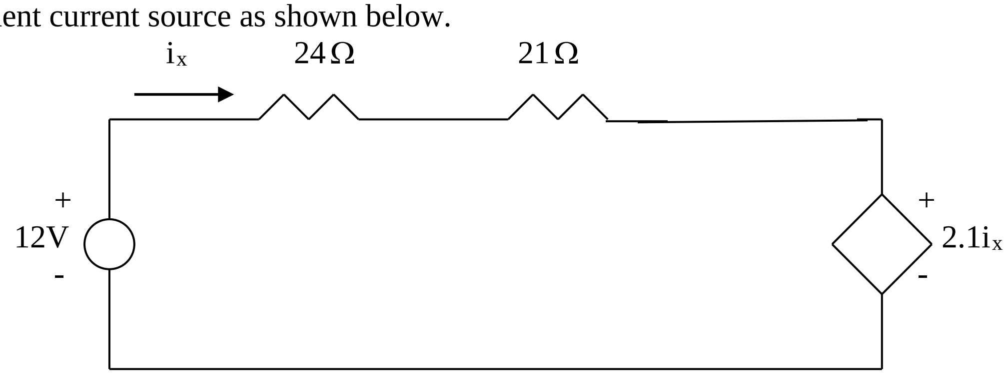
Transform the dependent current source as shown below.



Combine the 60-ohm with the 10-ohm and transform the dependent source as shown below.



Combining 30-ohm and 70-ohm gives 30//70 = 70x30/100 = 21-ohm. Transform the dependent current source as shown below.



Applying KVL to the loop gives

$$\mathbf{45}\boldsymbol{i_x} - \mathbf{12} + \mathbf{2.1}\boldsymbol{i_x} = \mathbf{0} \quad \longrightarrow \quad \boldsymbol{i_x} = \frac{\mathbf{12}}{\mathbf{47.1}} = \mathbf{254.8\ mA}.$$

Solution 4.31

Determine v_x in the circuit of Fig. 4.99 using source transformation.

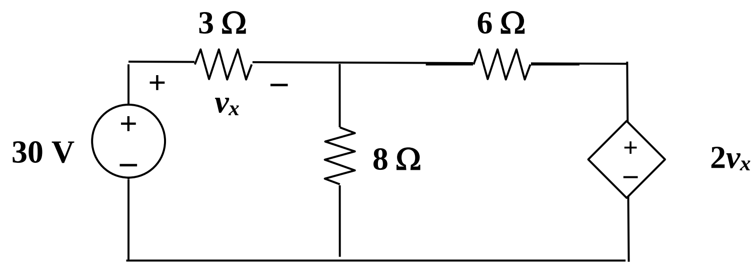
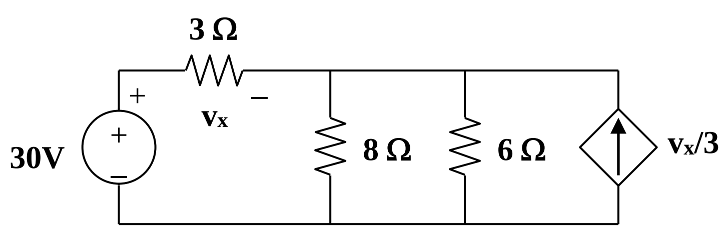
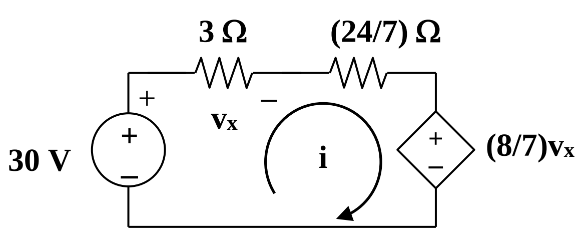


Figure 4.99
For Prob. 4.31.

Solution Transform the dependent source so that we have the circuit in Fig. (a). 6||8 = (24/7) ohms. Transform the dependent source again to get the circuit in Fig. (b).



(a)



(b)

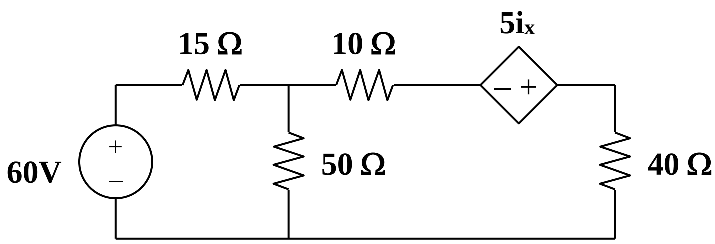
From Fig. (b), $v_x = 3i$, or $i = v_x/3$.

Applying KVL, $-30 + (3 + 24/7)i + (8/7)v_x = 0$

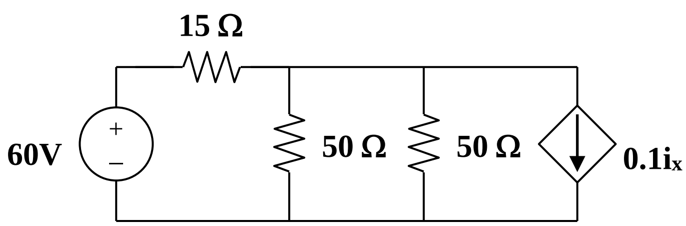
$$[(21 + 24)/7]v_x/3 + (8/7)v_x = 30 \text{ leads to } v_x = 30/3.2857 = \mathbf{9.13\ V}.$$

Solution 4.32

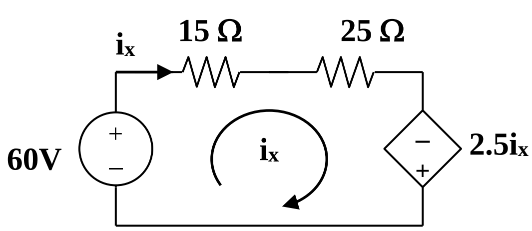
As shown in Fig. (a), we transform the dependent current source to a voltage source,



(a)



(b)



(c)

In Fig. (b), 50||50 = 25 ohms. Applying KVL in Fig. (c),

$-60 + 40i_x - 2.5i_x = 0$, or $i_x =$ **1.6 A**

Solution 4.33

Determine the Thevenin equivalent circuit, shown in Fig. 4.101, as seen by the 7-ohm resistor. Then calculate the current flowing through the 7-ohm resistor.

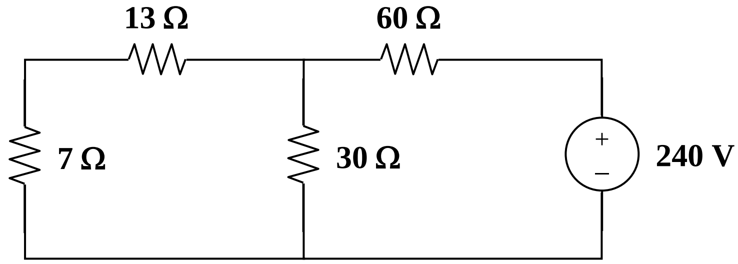
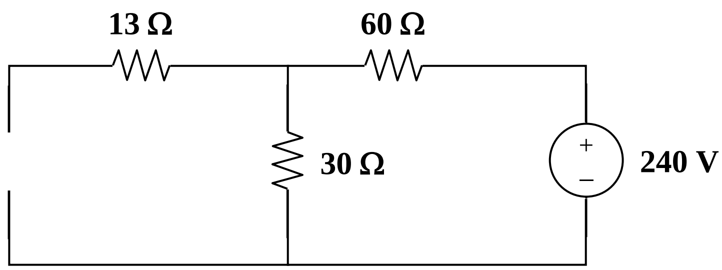
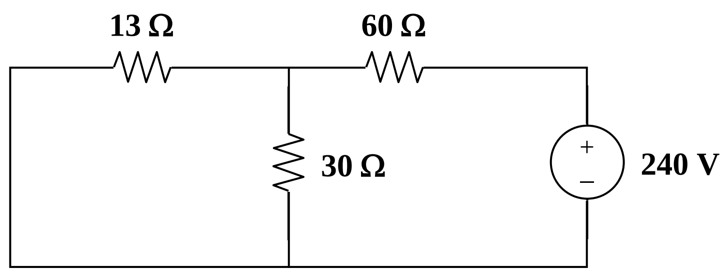


Figure 4.101
For Prob. 4.33.

Solution

Step 1. We need to find V_{oc} and I_{sc}. To do this, we will need two circuits, label the appropriate unknowns and solve for V_{oc}, I_{sc}, and then R_{eq} which is equal to V_{oc}/I_{sc}.





For the open circuit voltage all we need to do is to recognize that there is no voltage drop across the 13 Ω resistor so that $V_{oc} = 240x30/(30+60)$.
Clearly I_{sc} is equal to the current through the 13 Ω resistor or
$I_{sc} = [240/(60+(13x30/(13+30))][13x30/(13+30)]/13$.

Step 2. $V_{oc} = V_{Thev} =$ **80 V**. $I_{sc} = [240/69.07]9.07/13 = 2.4242$ which leads to $R_{eq} =$ **33 Ω**. Clearly,

$I_7 = 80/(7+33) =$ **2 A**.

Solution 4.34

Using Fig. 4.102, design a problem that will help other students better understand Thevenin equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Thevenin equivalent at terminals ***a-b*** of the circuit in Fig. 4.102.

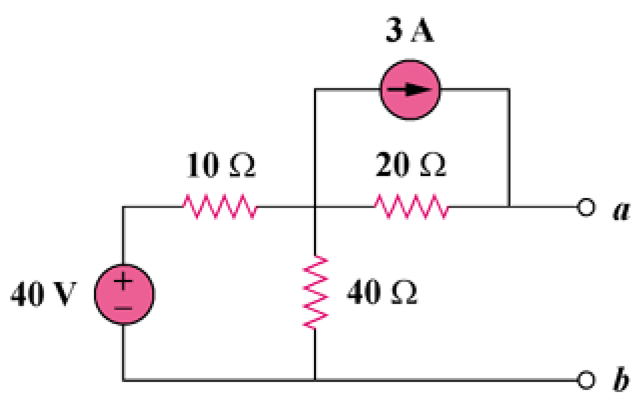
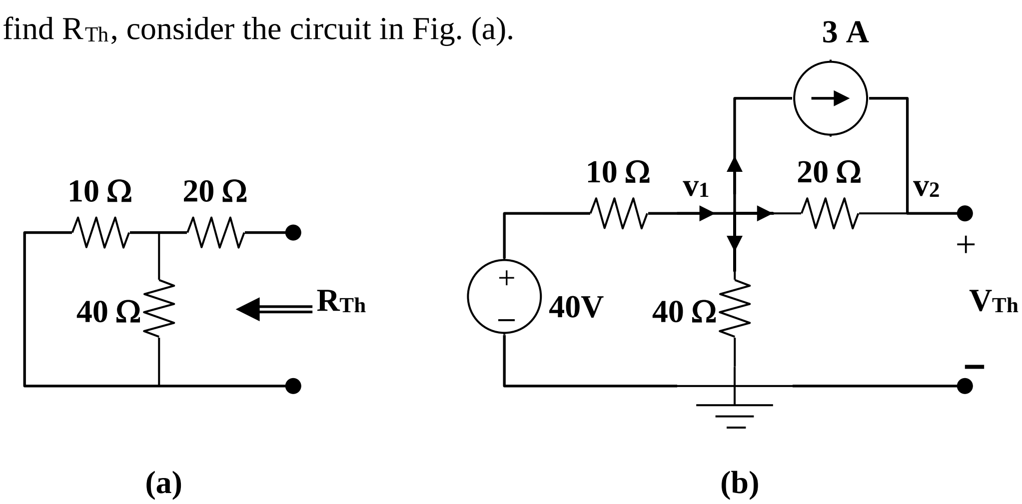


Figure 4.102

Solution

To find R_{Th}, consider the circuit in Fig. (a).



$$R_{Th} = 20 + 10||40 = 20 + 400/50 = \mathbf{28\ ohms}$$

To find V_{Th}, consider the circuit in Fig. (b).

At node 1, $(40 - v_1)/10 = 3 + [(v_1 - v_2)/20] + v_1/40$, $40 = 7v_1 - 2v_2$ (1)

At node 2, $3 + (v1 - v2)/20 = 0$, or $v1 = v2 - 60$ (2)

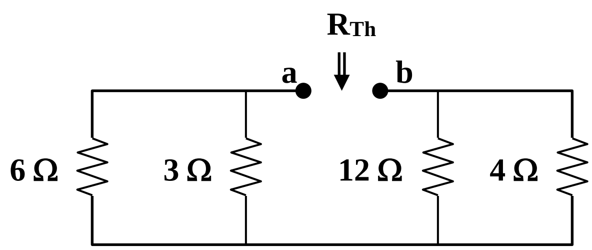
Solving (1) and (2), $v_1 = 32$ V, $v_2 = 92$ V, and $V_{Th} = v_2 =$ **92 V**

Solution 4.35

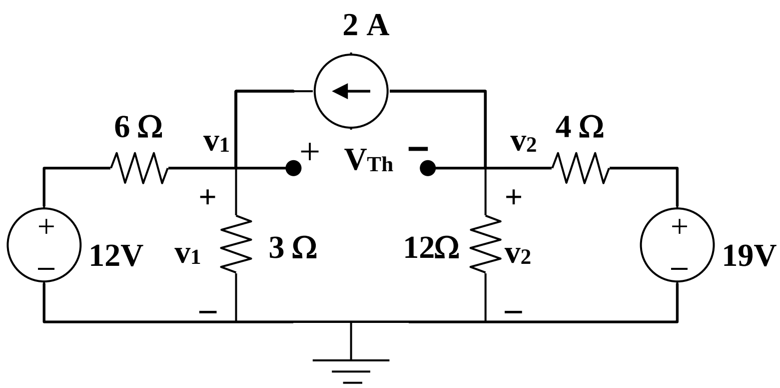
To find R_{Th}, consider the circuit in Fig. (a).

$$R_{Th} = R_{ab} = 6||3 + 12||4 = 2 + 3 = 5 \text{ ohms}$$

To find V_{Th}, consider the circuit shown in Fig. (b).



(a)

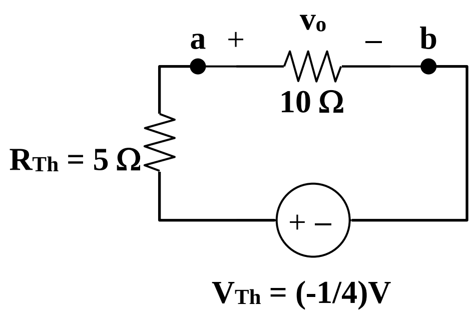


(b)

At node 1, $\quad 2 + (12 - v_1)/6 = v_1/3$, or $v_1 = 8$

At node 2, $\quad (19 - v_2)/4 = 2 + v_2/12$, or $v_2 = 33/4$

But, $\quad -v_1 + V_{Th} + v_2 = 0$, or $V_{Th} = v_1 - v_2 = 8 - 33/4 = -0.25$



$v_o = V_{Th}/2 = -0.25/2 =$ **–125 mV**

Solution 4.36

Solve for the current ***i*** in the circuit of Fig. 4.103 using Thevenin's theorem. (*Hint:* Find the Thevenin equivalent as seen by the 12-Ω resistor.)

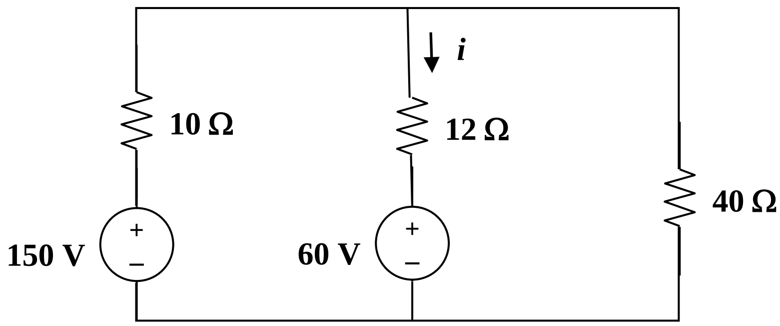
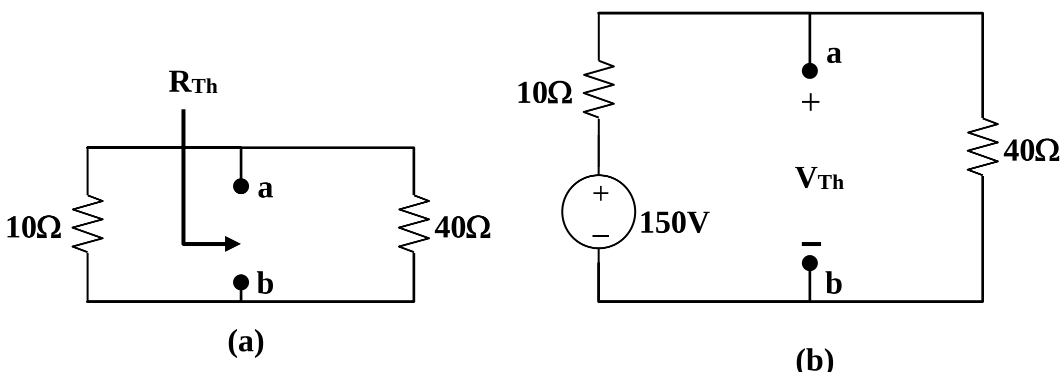


Figure 4.103
For Prob. 4.36.

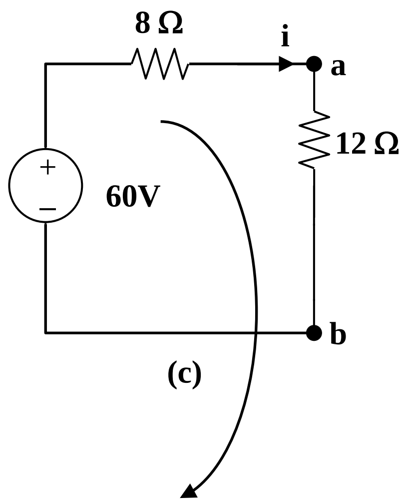
Solution

Although we could just remove the 12 Ω resistor and find the Thevenin equivalent, let us remove the 60 V voltage source and the 12 Ω resistor (b) and then know that the Thevenin voltage is equal to the Thevenin voltage we find below –60 volts. To find the Thevenin resistance set the 150 V source to 0.



From Fig. (a), $R_{Th} = 10||40 = 8$ ohms

From Fig. (b), $V_{Th} = (40/(10 + 40))150 = 120$V or the actual Thevenin voltage is equal to 120–60 = 60 V.



The equivalent circuit of the original circuit is shown in Fig. (c). Applying KVL,

$$-60 + (8 + 12)i = 0, \text{ which leads to } i = \mathbf{3\ A}.$$

Solution 4.37

Find the Norton equivalent with respect to terminals *a*-*b* in the circuit shown in Fig. 4.104.

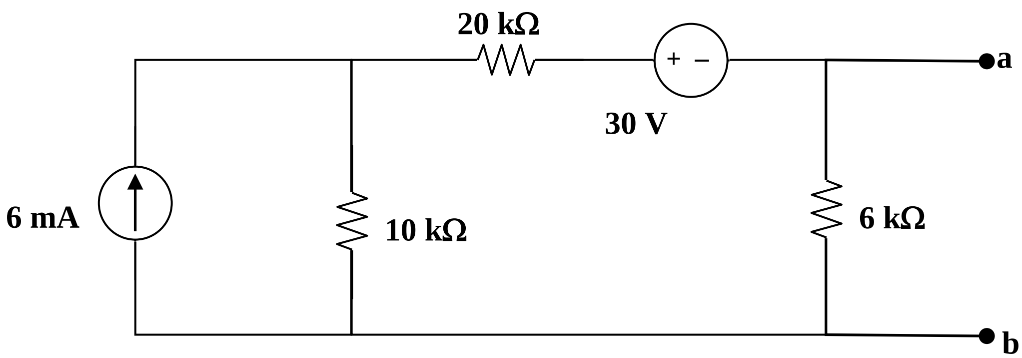


Figure 4.104
For Prob. 4.37.

Solution

Step 1. Since we do not have a dependent source we can find the equivalent resistance by setting the independent sources equal to zero. Therefore, $R_{eq} = 6k(30k)/(6k+30k)$.

Now all we need to do is to find the value of $I_{sc} = I_N$. $10k(I_{sc}–0.006) + 20kI_{sc} + 30 = 0$.

Step 2. $R_{eq} = 180k/36 =$ **5 kΩ**. $30kI_{sc} = 30$ or $I_{sc} = I_N =$ **1 mA**.

Solution 4.38

Apply Thevenin's theorem to find V_o in the circuit of Fig. 4.105.

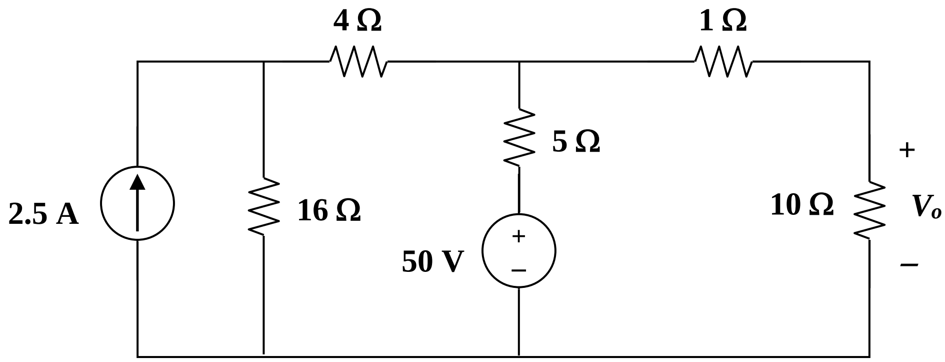
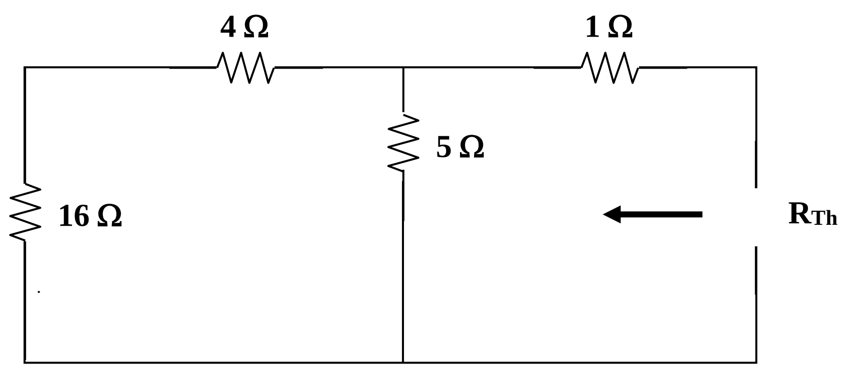


Figure 4.105
For Prob. 4.38.

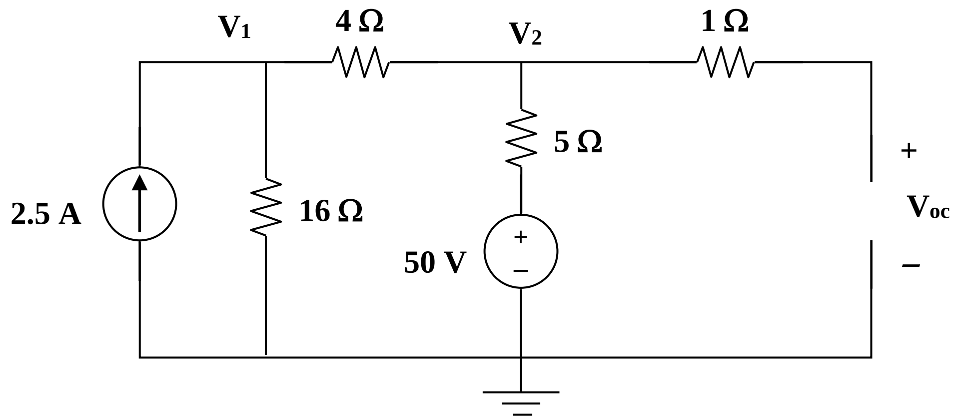
Solution

We find Thevenin equivalent at the terminals of the 10-ohm resistor. For R_{Th}, consider the circuit below.



$R_{Th} = 1 + 5//(4+16) = 1 + 4 =$ **5 Ω.**

For V_{Th}, consider the circuit below.



At node 1,

$$-2.5+\frac{V_1-0}{16}+\frac{V_1-V_2}{4}=0 \quad \longrightarrow \quad 0.3125V_1-0.25V_2=2.5 \qquad (1)$$

At node 2,

$$\frac{V_2-V_1}{4}+\frac{V_2-50}{5}=0 \quad \longrightarrow \quad -0.25V_1+0.45V_2=10 \qquad (2)$$

Solving (1) and (2) leads to $V_{Th}=V_2=48$ V. Thus we get,

V_o = 48[10/(5+10)] = **32 V**.

Solution 4.39

Obtain the Thevenin equivalent at terminals a-b of the circuit shown in Fig. 4.106.

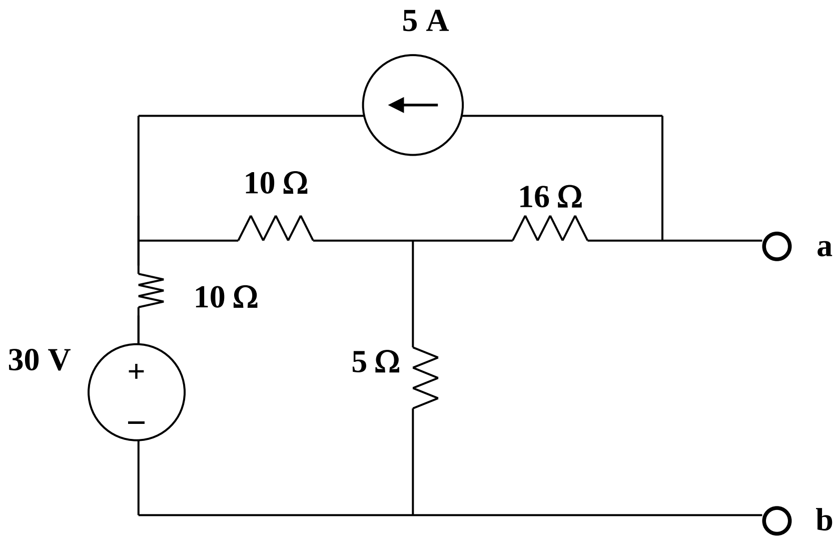
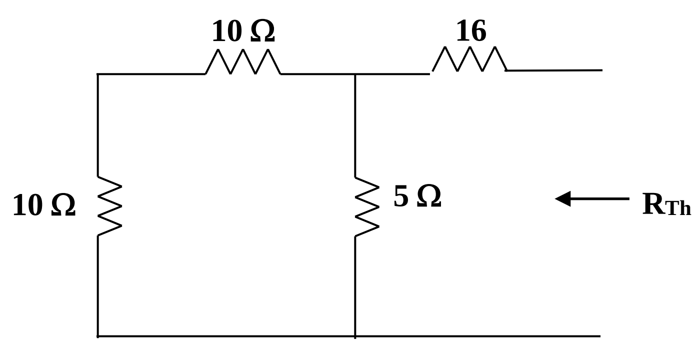


Figure 4.106
For Prob. 4.39.

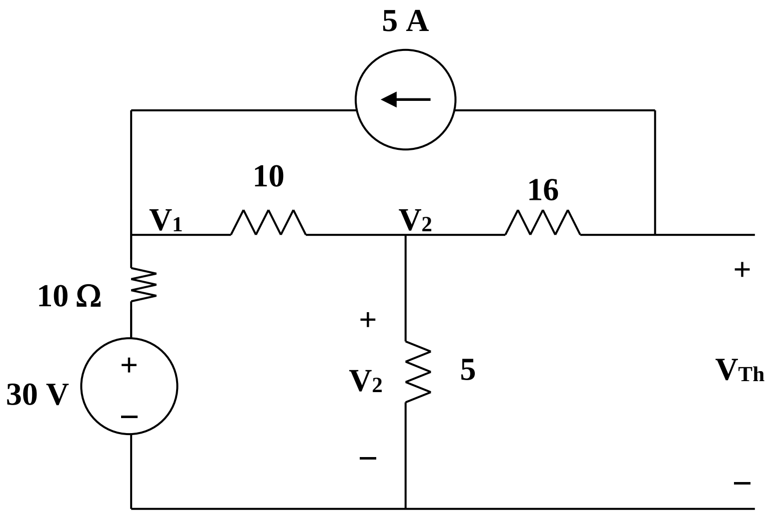
Solution

We obtain R_{Th} using the circuit below.



$$R_{Thev} = 16 + (20||5) = 16 + (20\text{x}5)/(20+5) = \mathbf{20\ \Omega}$$

To find V_{Th}, we use the circuit below.

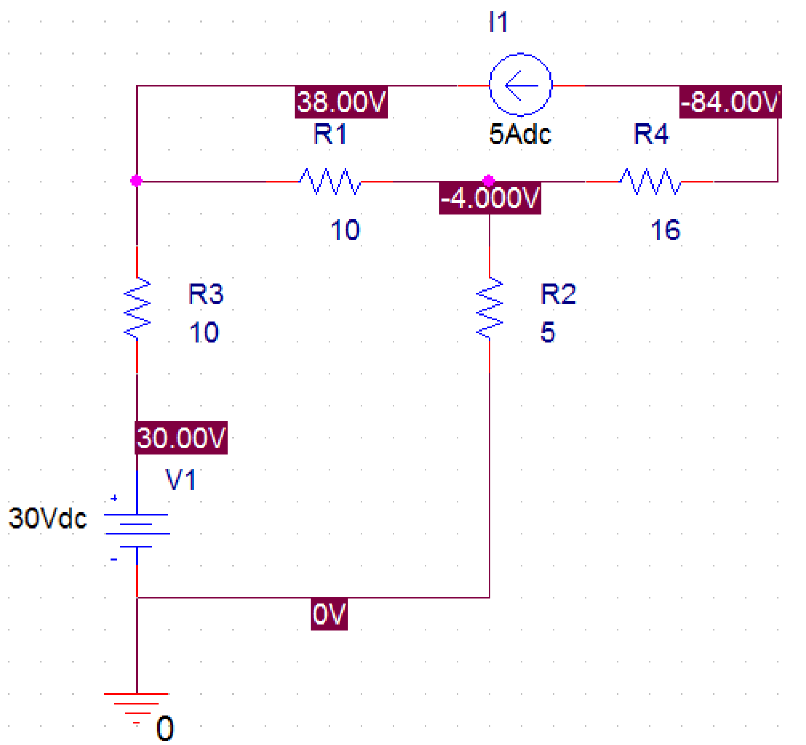


At node 1, $[(V_1–30)/10] +[(V_1–V_2)/10] – 5 = 0$ or
$[0.1+0.1]v_1 – 0.1V_2 = 8$ (1)

At node 2, $[(V_2–V_1)/10] + [(V_2–0)/5] + 5 = 0$ or
$–0.1V_1 + 0.3V_2 = –5$ (2)

Adding 3x(1) to (2) gives $(0.6–0.1)V_1 = 19$ or $V_1 = 19/0.5 = 38$ and $V_2 = (–5+0.1x38)/0.3 = –4$ V.

Finally, $V_{Th} = V_2 + (–5)16 –4–80 =$ **–84 V**. Checking with PSpice we get,



Solution 4.40

To obtain V_{Th}, we apply KVL to the loop.

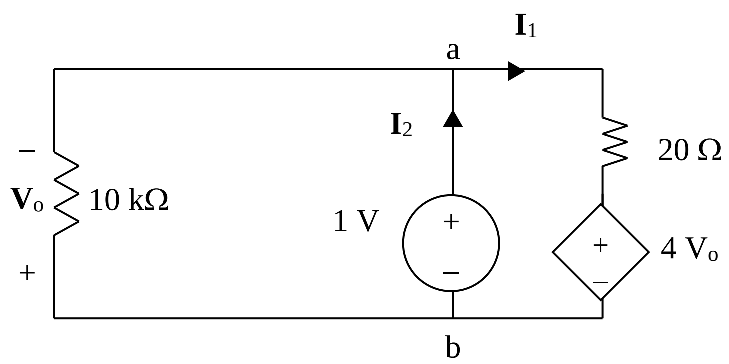
$$-70+(10+20)kI+4V_o=0$$

But $V_o=10kI$

$$70=70kI \longrightarrow I=1mA$$

$$-70+10kI+V_{Th}=0 \longrightarrow V_{Th}=\underline{60\text{ V}}$$

To find R_{Th}, we remove the 70-V source and apply a 1-V source at terminals a-b, as shown in the circuit below.



We notice that $V_o = -1$ V.

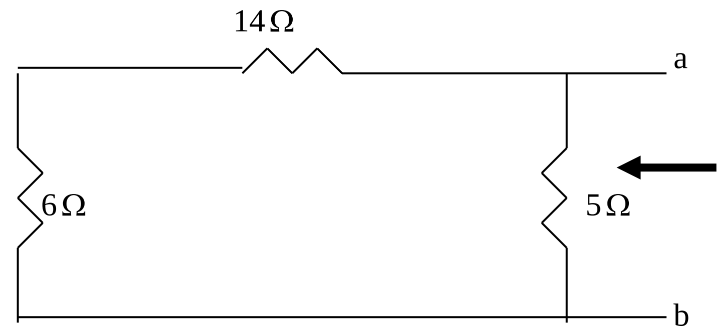
$$-1+20kI_1+4V_o=0 \longrightarrow I_1=0.25\text{ mA}$$

$$I_2=I_1+\frac{1V}{10k}=0.35\text{ mA}$$

$$R_{Th}=\frac{1V}{I_2}=\frac{1}{0.35}k\Omega=\underline{2.857\text{ k}\Omega}$$

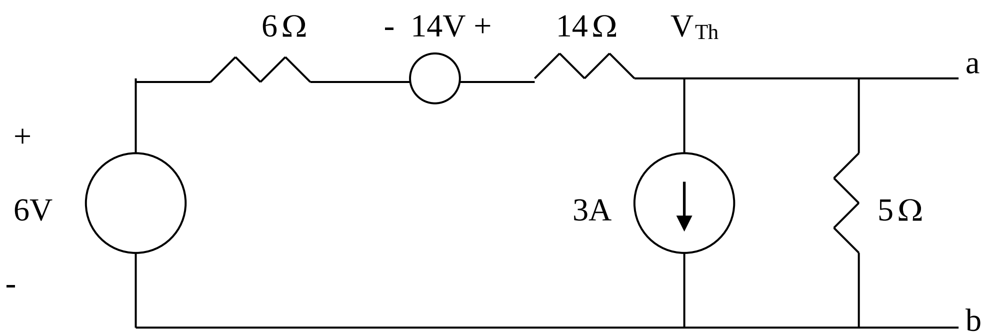
Solution 4.41

To find R_{Th}, consider the circuit below



$$R_{Th} = 5//(14+6) = 4\Omega = R_N$$

Applying source transformation to the 1-A current source, we obtain the circuit below.



At node a,

$$\frac{14+6-V_{Th}}{6+14} = 3 + \frac{V_{Th}}{5} \longrightarrow V_{Th} = -8\text{ V}$$

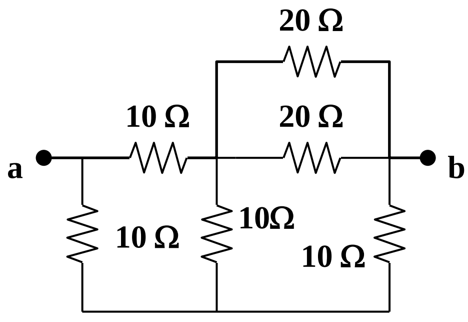
$$I_N = \frac{V_{Th}}{R_{Th}} = (-8)/4 = -2\text{ A}$$

Thus,

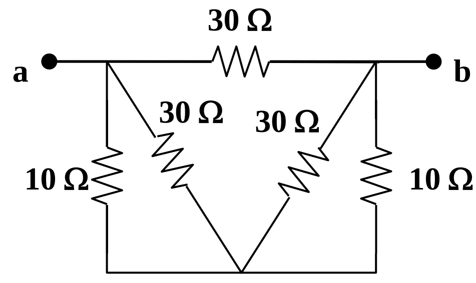
$$\underline{\boldsymbol{R_{Th} = R_N = 4\Omega, \quad V_{Th} = -8\text{V}, \quad I_N = -2\text{ A}}}$$

Solution 4.42

To find R_{Th}, consider the circuit in Fig. (a).



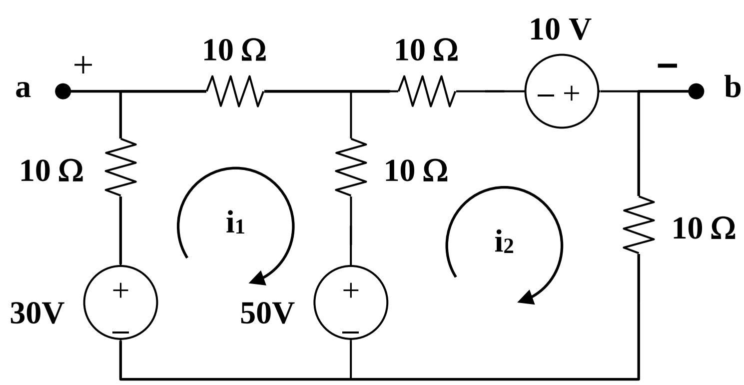
(a)



(b)

20||20 = 10 ohms. Transform the wye sub-network to a delta as shown in Fig. (b).
10||30 = 7.5 ohms. R_{Th} = R_{ab} = 30||(7.5 + 7.5) = **10 ohms**.

To find V_{Th}, we transform the 20-V (to a current source in parallel with the 20 Ω resistor and then back into a voltage source in series with the parallel combination of the two 20 Ω resistors) and the 5-A sources. We obtain the circuit shown in Fig. (c).



(c)

For loop 1, $-30 + 50 + 30i_1 – 10i_2 = 0$, or $-2 = 3i_1 – i_2$ (1)

For loop 2, $-50 – 10 + 30i_2 – 10i_1 = 0$, or $6 = -i_1 + 3i_2$ (2)

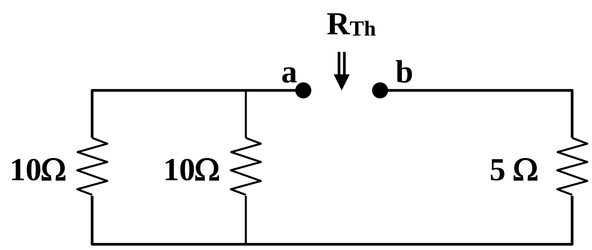
Solving (1) and (2), $i_1 = 0$, $i_2 = 2$ A

Applying KVL to the output loop, $-v_{ab} – 10i_1 + 30 – 10i_2 = 0$, $v_{ab} = 10$ V

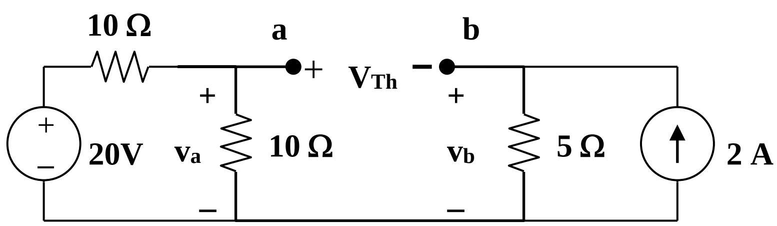
$$V_{Th} = v_{ab} = \textbf{10 volts}$$

Solution 4.43

To find R_{Th}, consider the circuit in Fig. (a).



(a)



(b)

$$R_{Th} = 10\|10 + 5 = \mathbf{10\ ohms}$$

To find V_{Th}, consider the circuit in Fig. (b).

$$v_b = 2x5 = 10\text{ V},\ \ v_a = 20/2 = 10\text{ V}$$

But, $\quad -v_a + V_{Th} + v_b = 0$, or $V_{Th} = v_a - v_b =$ **0 volts**

Solution 4.44

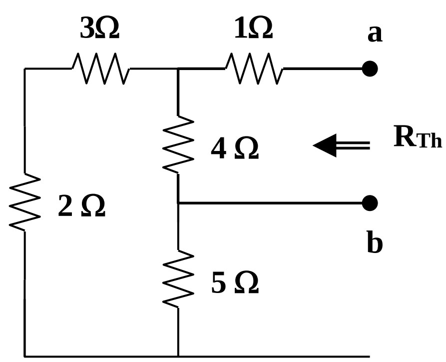
(a) For R_{Th}, consider the circuit in Fig. (a).

$$R_{Th} = 1 + 4||(3 + 2 + 5) = \mathbf{3.857\ ohms}$$

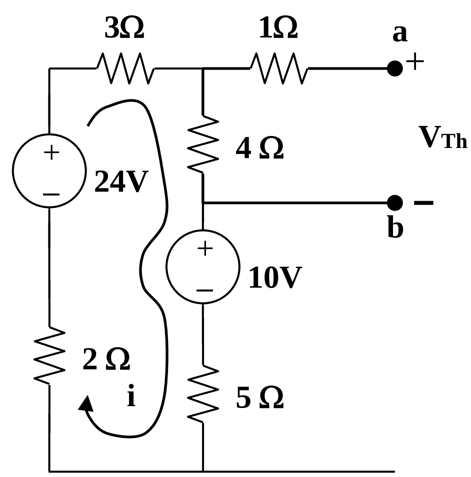
For V_{Th}, consider the circuit in Fig. (b). Applying KVL gives,

$$10 - 24 + i(3 + 4 + 5 + 2), \text{ or } i = 1$$

$$V_{Th} = 4i = \mathbf{4\ V}$$

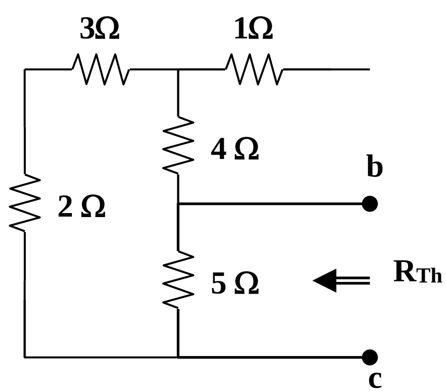


(a)

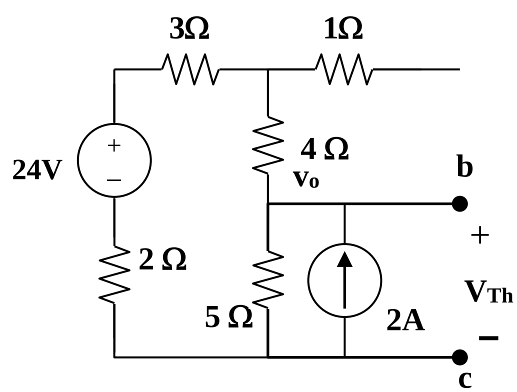


(b)

(b) For R_{Th}, consider the circuit in Fig. (c).



(c)



(d)

$$R_{Th} = 5||(2 + 3 + 4) = \mathbf{3.214\ ohms}$$

To get V_{Th}, consider the circuit in Fig. (d). At the node, KCL gives,

$$[(24 - vo)/9] + 2 = vo/5, \text{ or } vo = 15$$

$$V_{Th} = vo = \mathbf{15\ V}$$

Solution 4.45

Find the Thevenin equivalent of the circuit in Fig. 4.112 as seen by looking into terminals *a* and *b*.

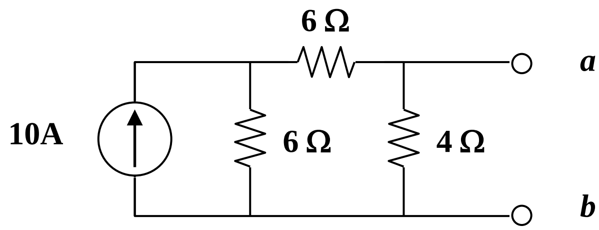
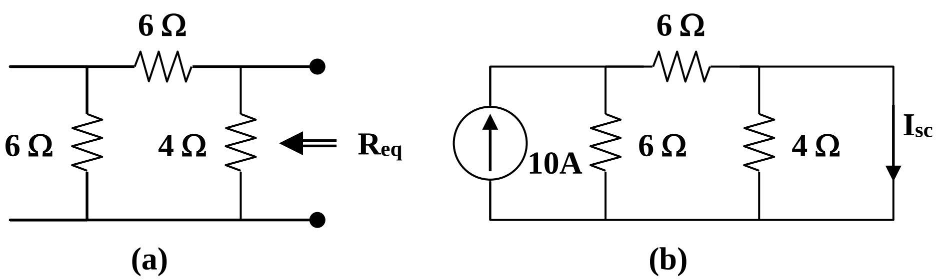


Figure 4.112
For Prob. 4.45.

Solution

For R_{eq}, consider the circuit in Fig. (a).



$$R_{eq} = (6 + 6)||4 = \mathbf{3\ \Omega}$$

For V_{Thev}, we first find I_{sc} and then $V_{Thev} = I_{sc}R_{eq}$. For I_{sc}, consider the circuit in Fig. (b). The 4-ohm resistor is shorted so that 10-A current is equally divided between the two 6-ohm resistors. Hence, $I_{sc} = 10/2 = 5$ A. Thus,

$$V_{Thev} = 5x3 = \mathbf{15\ V}.$$

Solution 4.46

Using Fig. 4.113, design a problem to help other students better understand Norton equivalent circuits.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the Norton equivalent at terminals a-b of the circuit in Fig. 4.113.

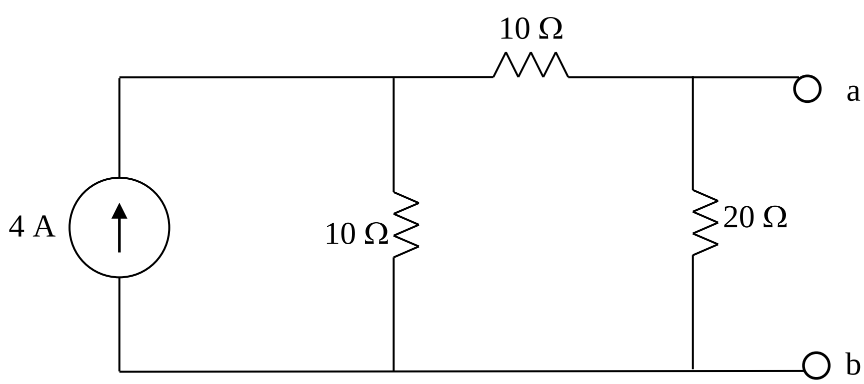
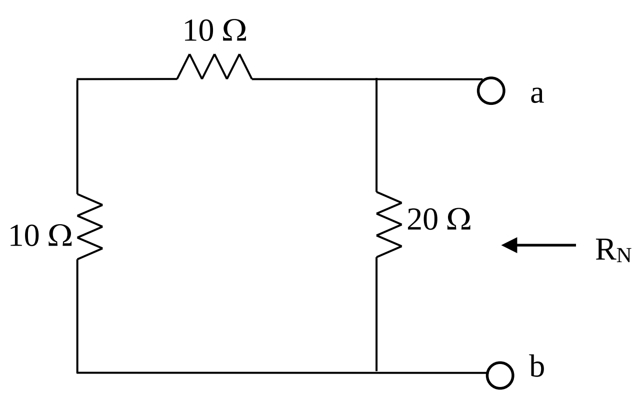


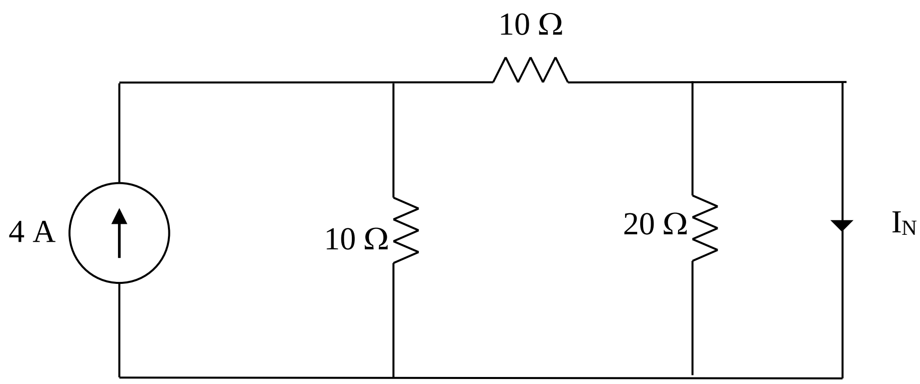
Figure 4.113 For Prob. 4.46.

Solution

R_N is found using the circuit below.



$R_N = 20//(10+10) = 10\ \Omega$
To find I_N, consider the circuit below.



The 20-Ω resistor is short-circuited and can be ignored.

$$I_N = \tfrac{1}{2} \times 4 = \mathbf{2\ A}$$

Solution 4.47

Obtain the Thevenin and Norton equivalent circuits of the circuit in Fig. 4.114 with respect to terminals *a* and *b*.

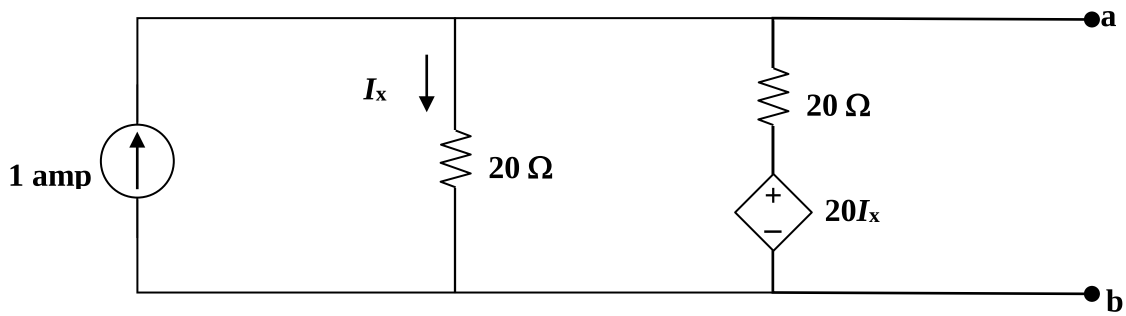


Figure 4.114
For Prob. 4.47.

Solution

Step 1. We note that there is a dependent source which means to best way to identify the equivalent circuits is to find $V_{oc} = V_{Thev}$ and $I_{sc} = I_N$ and $R_{eq} = V_{oc}/I_{sc}$.

$-1 + [(V_{oc}-0)/20] + [(V_{oc}-20I_x)/20] = 0$ and $I_x = V_{oc}/20$.

$I_{sc} = 1$ amp ($I_x = 0$ because of shorting a to b and the dependent voltage source is equal to zero because $I_x = 0$).

Step 2. $-1 + [V_{oc}/20] + [V_{oc}/20] - [V_{oc}/20] = 0$ or $V_{oc} = 20$ volts. Therefore,

$V_{Thev} =$ **20 V**, $I_N =$ **1 A**, and $R_{eq} =$ **20Ω**.

Solution 4.48

Determine the Norton equivalent at terminals ***a-b*** for the circuit in Fig. 4.115.

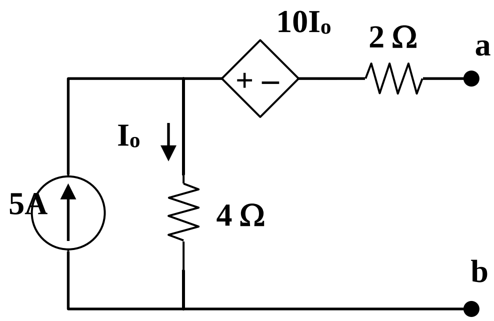
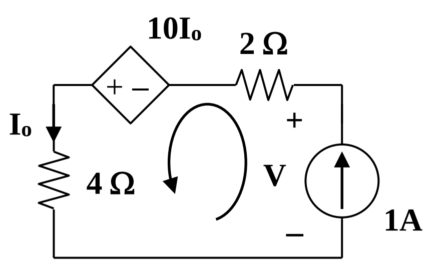


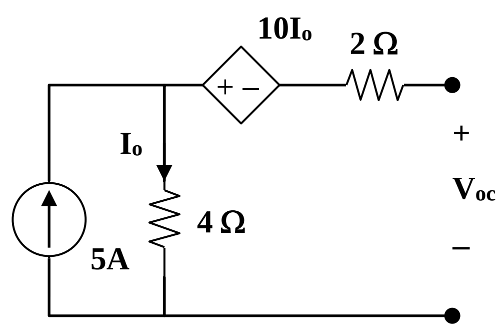
Figure 4.115
For Prob. 4.48.

Solution

To get R_{Th}, consider the circuit in Fig. (a).



(a)



(b)

From Fig. (a), $I_o = 1$, $6 - 10 - V = 0$, or $V = -4$

$$R_{eq} = V/1 = \mathbf{-4\ ohms}$$

Note that the negative value of R_{eq} indicates that we have an active device in the circuit since we cannot have a negative resistance in a purely passive circuit.

To solve for I_N we first solve for V_{oc}, consider the circuit in Fig. (b),

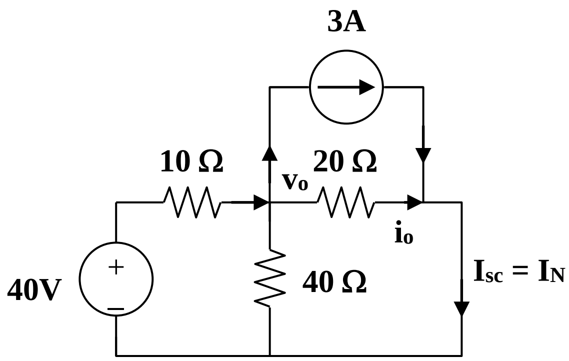
$$I_o = 5,\ V_{oc} = -10I_o + 4I_o = -30\text{ V}$$

$$I_N = V_{oc}/R_{eq} = \mathbf{7.5\ A}.$$

Solution 4.49

$R_N = R_{Th} =$ **28 ohms**

To find I_N, consider the circuit below,

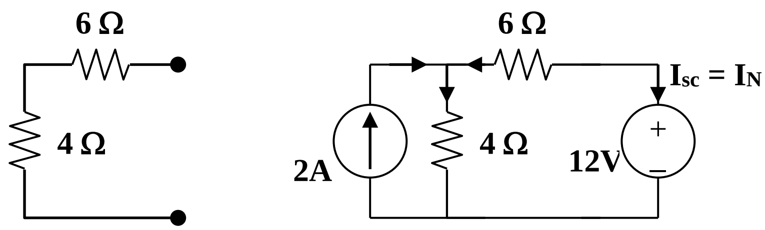


At the node, $(40 - v_o)/10 = 3 + (v_o/40) + (v_o/20)$, or $v_o = 40/7$

$i_o = v_o/20 = 2/7$, but $I_N = I_{sc} = i_o + 3 =$ **3.286 A**

Solution 4.50

From Fig. (a), $R_N = 6 + 4 =$ **10 ohms**

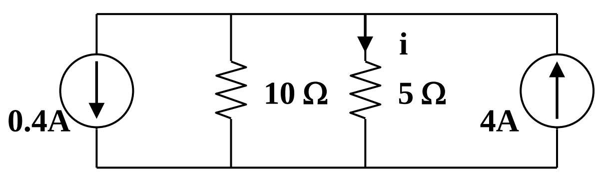


(a) (b)

From Fig. (b), $2 + (12 - v)/6 = v/4$, or $v = 9.6$ V

$-I_N = (12 - v)/6 = 0.4$, which leads to $I_N =$ **-0.4 A**

Combining the Norton equivalent with the right-hand side of the original circuit produces the circuit in Fig. (c).



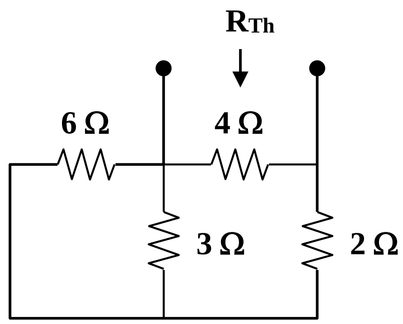
(c)

$i = [10/(10 + 5)] (4 - 0.4) =$ **2.4 A**

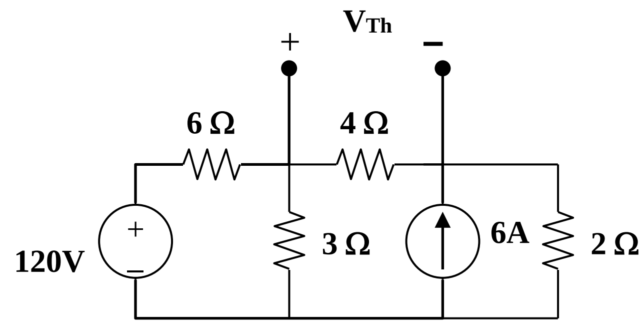
Solution 4.51

(a) From the circuit in Fig. (a),

$$R_N = 4\|(2 + 6\|3) = 4\|4 = \mathbf{2\ ohms}$$

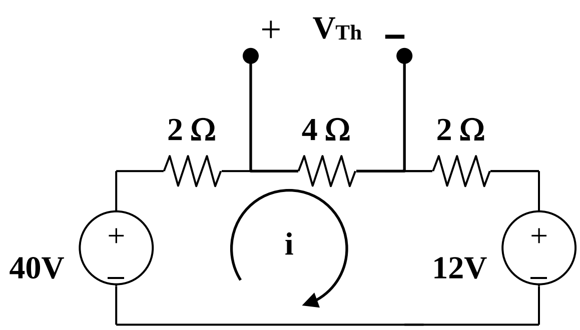


(a)



(b)

For I_N or V_{Th}, consider the circuit in Fig. (b). After some source transformations, the circuit becomes that shown in Fig. (c).



(c)

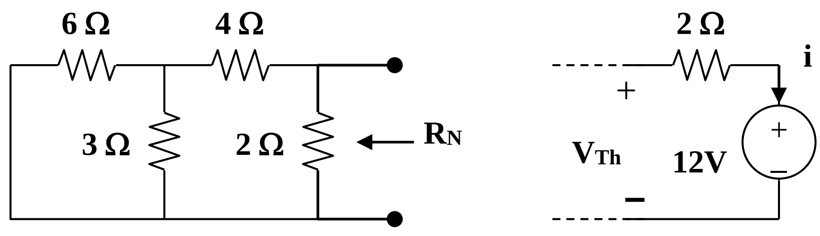
Applying KVL to the circuit in Fig. (c),

$$-40 + 8i + 12 = 0 \text{ which gives } i = 7/2$$

$$V_{Th} = 4i = 14 \text{ therefore } I_N = V_{Th}/R_N = 14/2 = \mathbf{7\ A}$$

(b) To get R_N, consider the circuit in Fig. (d).

$$R_N = 2\|(4 + 6\|3) = 2\|6 = \mathbf{1.5\ ohms}$$



(d) **(e)**

To get I_N, the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).

$$i = 7/2,\ V_{Th} = 12 + 2i = 19,\ I_N = V_{Th}/R_N = 19/1.5 = \mathbf{12.667\ A}$$

Solution 4.52

For the transistor model in Fig. 4.118, obtain the Thevenin equivalent at terminals ***a-b***.

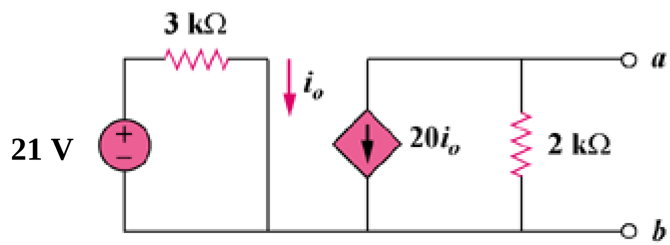
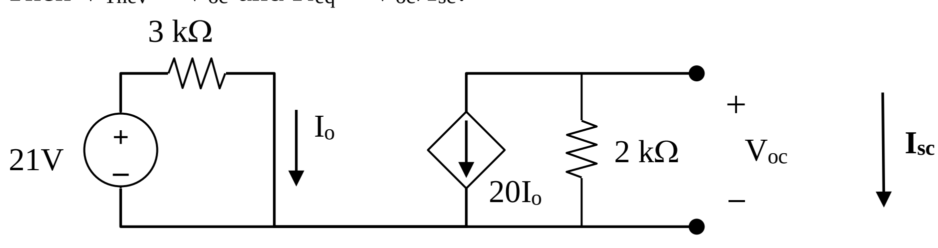


Figure 4.118
For Prob. 4.52.

Solution

Step 1. To find the Thevenin equivalent for this circuit we need to find V_{oc} and I_{sc}.

Then $V_{Thev} = V_{oc}$ and $R_{eq} = V_{oc}/I_{sc}$.



For V_{oc}, $I_o = (21–0)/3k = 7$ mA and $20I_o + (V_{oc}–0)/2k = 0$.

For I_{sc}, $I_{sc} = –20I_o$.

Step 2. $V_{oc} = –2k(20I_o) = –40x7 =$ **–280 volts** $= V_{Thev}$

$i_{sc} = –20x7x10^{–3} = –140$ mA or

$R_{eq} = –280/(–140x10^{–3}) =$ **2 kΩ**.

Solution 4.53

Find the Norton equivalent at terminals ***a-b*** of the circuit in Fig. 4.119.

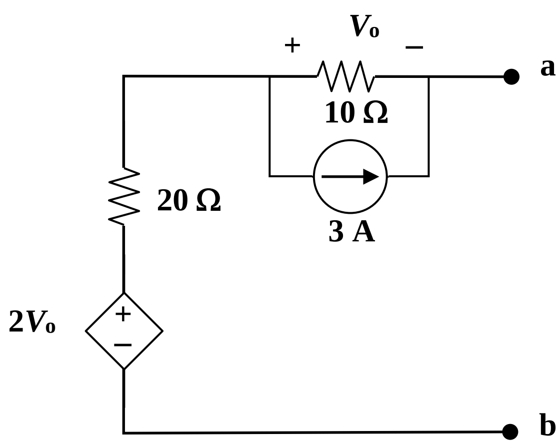


Figure 4.119
For Prob. 4.53.

Solution

Step 1. Since we have a dependent source, we need to determine $I_{sc} = I_N$ and $V_{oc} = V_{Thev}$ and $R_{eq} = V_{oc}/I_{sc}$. $V_{oc} = V_{ab} = 3(10) + 2V_o$ where $V_o = –3(10) = –30$ volts.

For I_{sc} we need to solve this mesh equation, $–2V_o + 20I_{sc} + 10(I_{sc}–3) = 0$ and $V_o = 10(I_{sc}–3)$.

Step 2. $V_{oc} = 30 – 60 = –30$ volts. $–20(I_{sc}–3) +20I_{sc} + 10I_{sc} – 30 = 0$ or

$I_{sc} =$ **$–3$ amps**. Therefore $R_{eq} = –30/–3 =$ **10 Ω**.

Solution 4.54

To find $V_{Th} = V_x$, consider the left loop.

$$-3 + 1000i_o + 2V_x = 0 \longrightarrow 3 = 1000i_o + 2V_x \qquad (1)$$

For the right loop,

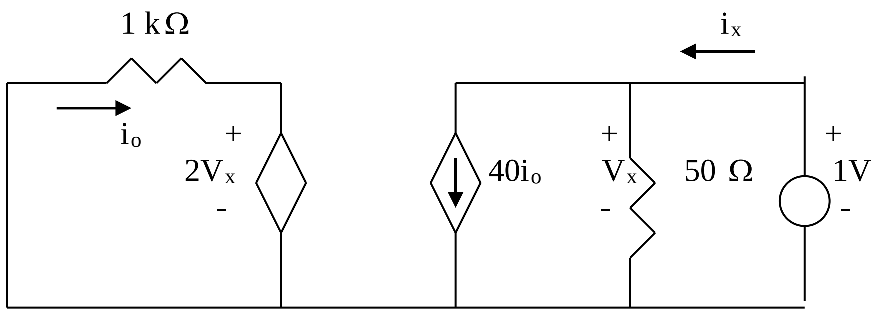
$$V_x = -50x40i_o = -2000i_o \qquad (2)$$

Combining (1) and (2),

$$3 = 1000i_o - 4000i_o = -3000i_o \longrightarrow i_o = -1\text{mA}$$

$$V_x = -2000i_o = 2 \longrightarrow \underline{V_{Th} = 2}$$

To find R_{Th}, insert a 1-V source at terminals a-b and remove the 3-V independent source, as shown below.



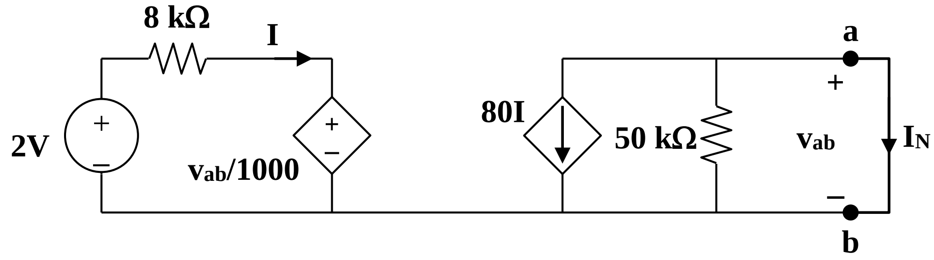
$$V_x = 1, \quad i_o = -\frac{2V_x}{1000} = -2\text{mA}$$

$$i_x = 40i_o + \frac{V_x}{50} = -80\text{mA} + \frac{1}{50}\text{A} = \text{-60mA}$$

$$R_{Th} = \frac{1}{i_x} = -1/0.060 = \underline{-16.67\Omega}$$

Solution 4.55

To get R_N, apply a 1 mA source at the terminals a and b as shown in Fig. (a).



(b)

We assume all resistances are in k ohms, all currents in mA, and all voltages in volts. At node a,

$$(v_{ab}/50) + 80I = 1 \qquad (1)$$

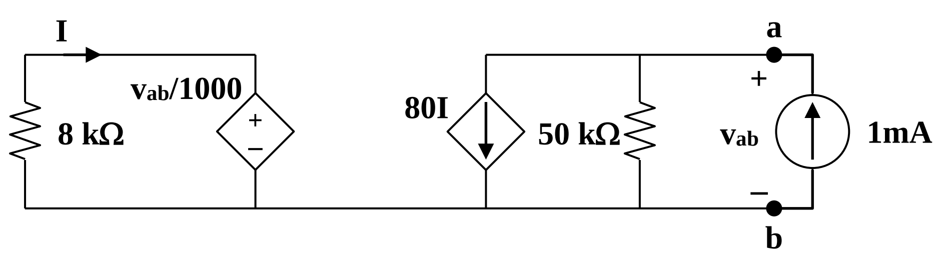
Also,

$$-8I = (v_{ab}/1000), \text{ or } I = -v_{ab}/8000 \qquad (2)$$

From (1) and (2), $(v_{ab}/50) - (80v_{ab}/8000) = 1$, or $v_{ab} = 100$

$$R_N = v_{ab}/1 = \mathbf{100\ k\ ohms}$$

To get I_N, consider the circuit in Fig. (b).



(a)

Since the 50-k ohm resistor is shorted,

$$I_N = -80I, \ v_{ab} = 0$$

Hence, $8i = 2$ which leads to $I = (1/4)$ mA

$$I_N = \mathbf{-20\ mA}$$

Solution 4.56

Use Norton's theorem to find V_o in the circuit of Fig. 4.122.

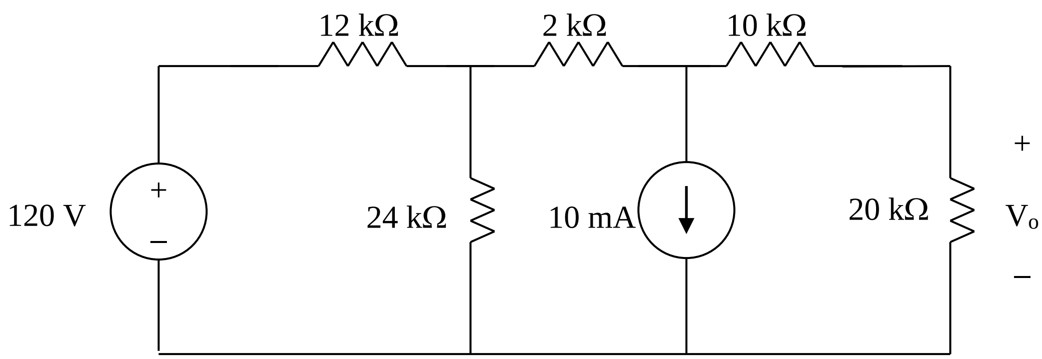
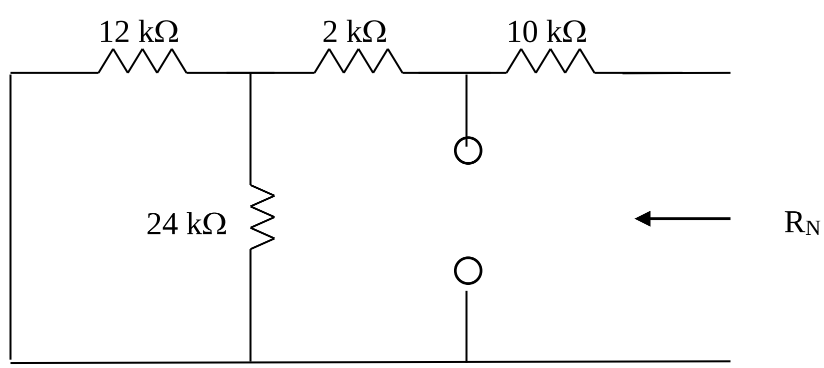


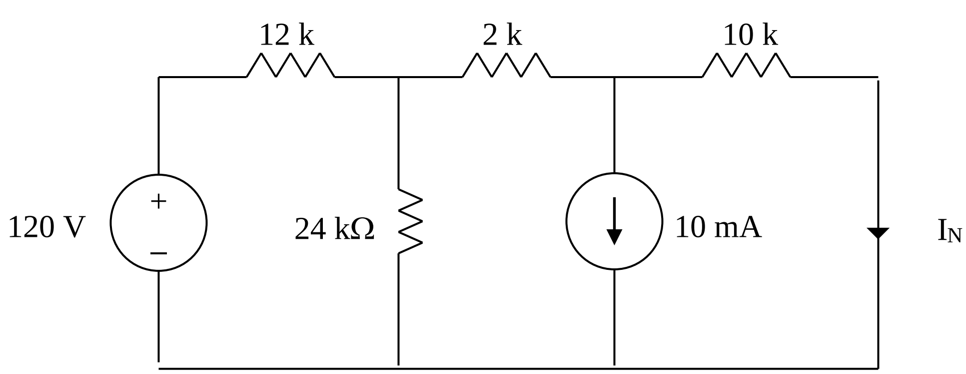
Figure 4.122
For Prob. 4.56.

Solution

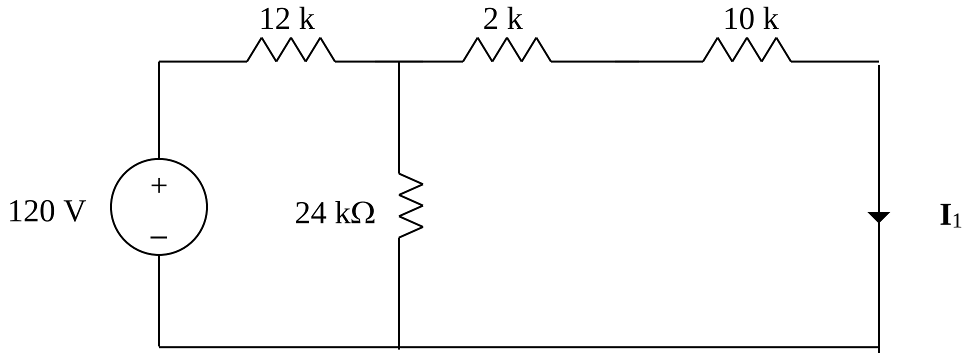
We remove the 20 kΩ resistor temporarily and find the Norton equivalent across its terminals. R_{eq} is obtained from the circuit below.



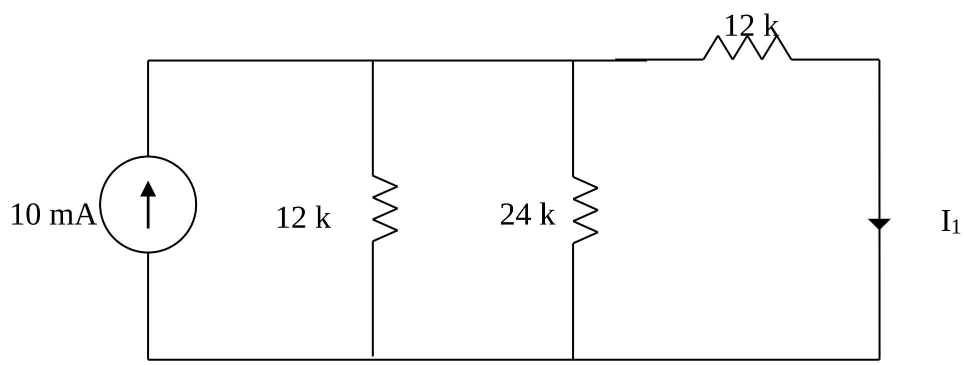
$R_{eq} = 10 + 2 + (12//24) = 12+8 = 20$ kΩ
I_N is obtained from the circuit below.



We can use superposition theorem to find I_N. Let $I_N = I_1 + I_2$, where I_1 and I_2 are due to 16-V and 3-mA sources respectively. We find I_1 using the circuit below.

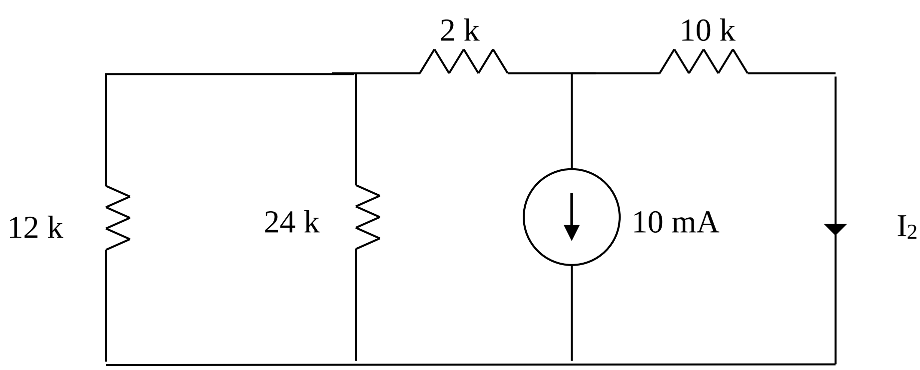


Using source transformation, we obtain the circuit below.



$12//24 = 8$ kΩ and $I_1 = [8k/(8k+12k)]0.01 = 4$ mA.

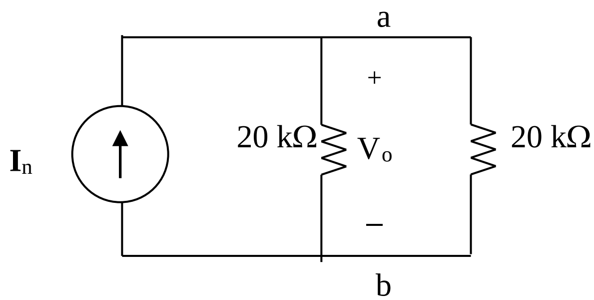
To find I_2, consider the circuit below.



$2k + 12k//24\text{ k} = 10$ kΩ
$I_2=0.5(-10\text{mA}) = -5$ mA

$I_N = 4–5 = –1$ mA

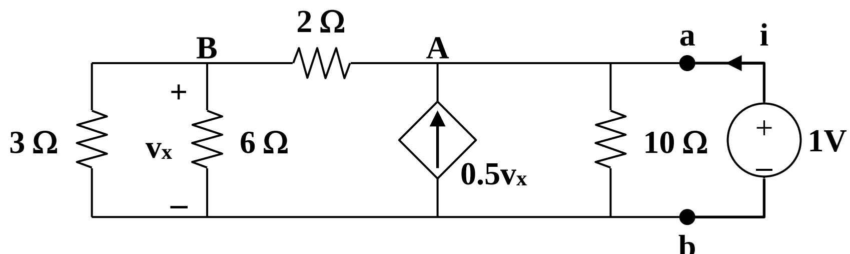
The Norton equivalent with the 20 kΩ resistor is shown below



$V_o = 20k(20k/(20k+20k))(–1 \text{ mA}) = \mathbf{–10\ V}$.

Solution 4.57

To find R_{Th}, remove the 50V source and insert a 1-V source at a – b, as shown in Fig. (a).



(a)

We apply nodal analysis. At node A,

$$i + 0.5v_x = (1/10) + (1 - v_x)/2, \text{ or } i + v_x = 0.6 \qquad (1)$$

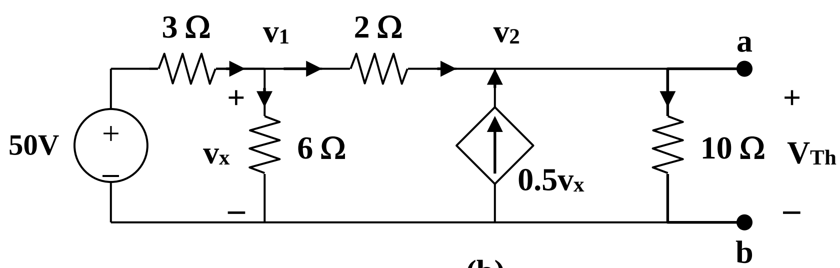
At node B,

$$(1 - v_o)/2 = (v_x/3) + (v_x/6), \text{ and } v_x = 0.5 \qquad (2)$$

From (1) and (2), $i = 0.1$ and

$$R_{Th} = 1/i = \mathbf{10\ ohms}$$

To get V_{Th}, consider the circuit in Fig. (b).



(b)

At node 1, $(50 - v_1)/3 = (v_1/6) + (v_1 - v_2)/2$, or $100 = 6v_1 - 3v_2 \qquad (3)$

At node 2, $0.5v_x + (v_1 - v_2)/2 = v_2/10$, $v_x = v_1$, and $v_1 = 0.6v_2 \qquad (4)$

From (3) and (4),

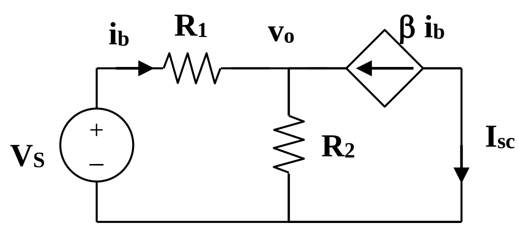
$$v_2 = V_{Th} = \mathbf{166.67\ V}$$

$$I_N = V_{Th}/R_{Th} = \mathbf{16.667\ A}$$

$$R_N = R_{Th} = \mathbf{10\ ohms}$$

Solution 4.58

This problem does not have a solution as it was originally stated. The reason for this is that the load resistor is in series with a current source which means that the only equivalent circuit that will work will be a Norton circuit where the value of R_N = **infinity**. I_N can be found by solving for I_{sc}.



Writing the node equation at node vo,

$$i_b + \beta i_b = v_o/R_2 = (1 + \beta)i_b$$

But $$i_b = (V_s - v_o)/R_1$$

$$v_o = V_s - i_b R_1$$

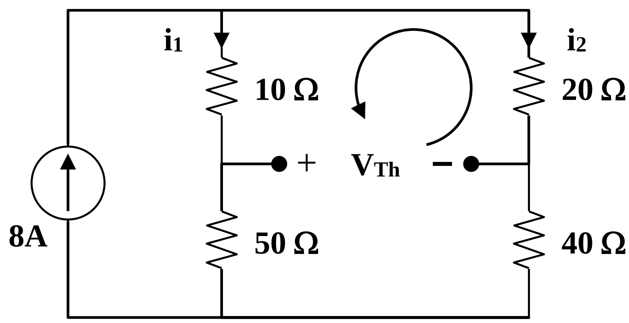
$$V_s - i_b R_1 = (1 + \beta)R_2 i_b, \text{ or } i_b = V_s/(R_1 + (1 + \beta)R_2)$$

$$I_{sc} = I_N = -\beta i_b = \mathbf{-\beta V_s/(R_1 + (1 + \beta)R_2)}$$

Solution 4.59

R_{Th} = (10 + 20)||(50 + 40) 30||90 = **22.5 ohms**

To find V_{Th}, consider the circuit below.

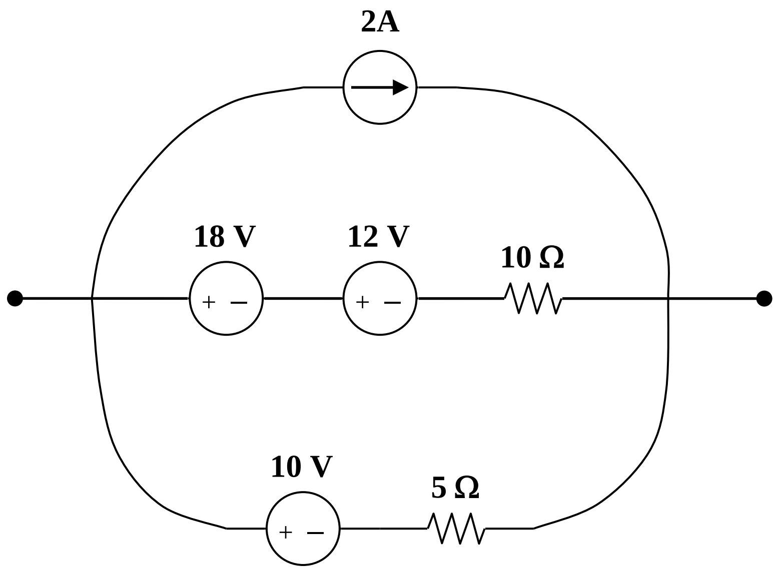


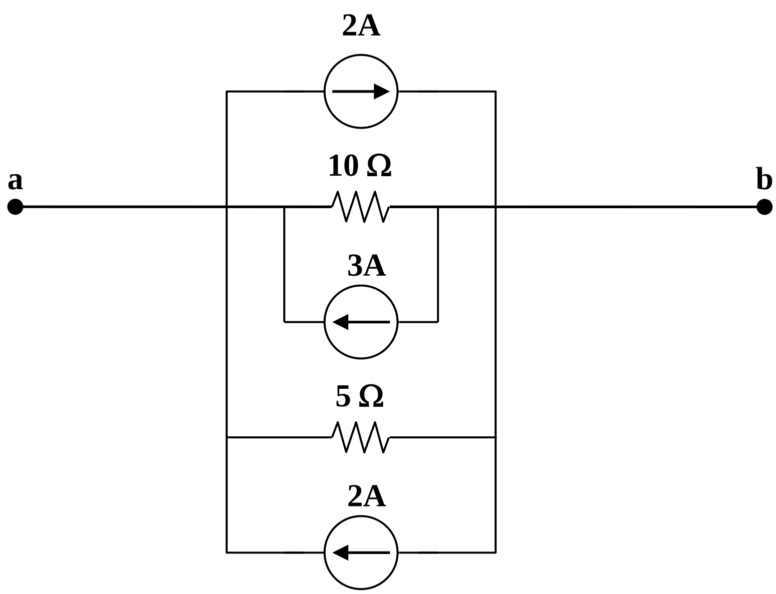
$i_1 = i_2 = 8/2 = 4$, $10i_1 + V_{Th} - 20i_2 = 0$, or $V_{Th} = 20i_2 - 10i_1 = 10i_1 = 10 \times 4$

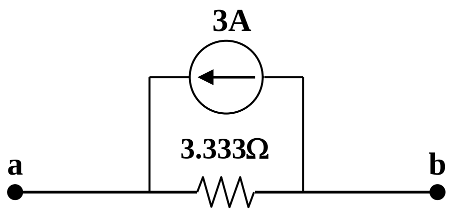
V_{Th} = **40V**, and $I_N = V_{Th}/R_{Th} = 40/22.5 =$ **1.7778 A**

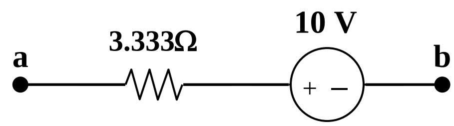
Solution 4.60

The circuit can be reduced by source transformations.









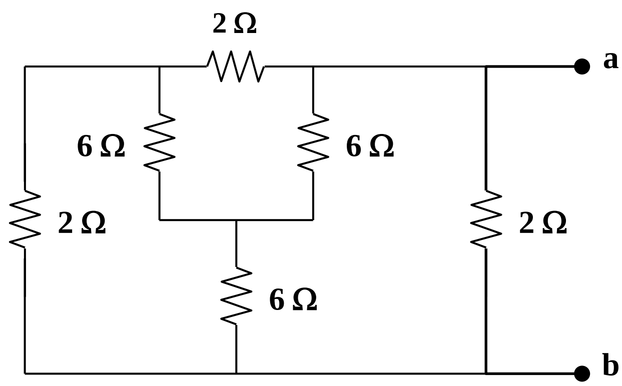
Norton Equivalent Circuit **Thevenin Equivalent Circuit**

Solution 4.61

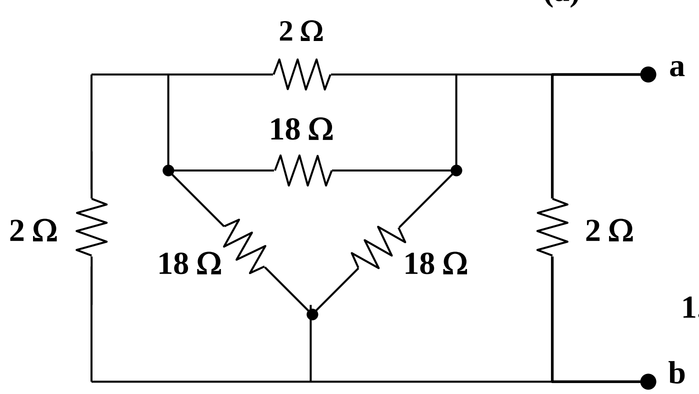
To find R_{Th}, consider the circuit in Fig. (a).

Let $R = 2||18 = 1.8$ ohms, $R_{Th} = 2R||R = (2/3)R =$ **1.2 ohms**.

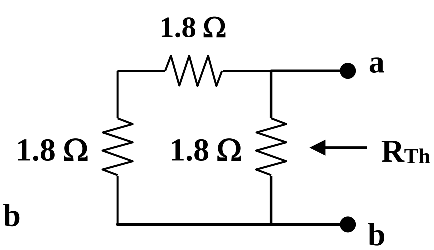
To get V_{Th}, we apply mesh analysis to the circuit in Fig. (d).



(a)

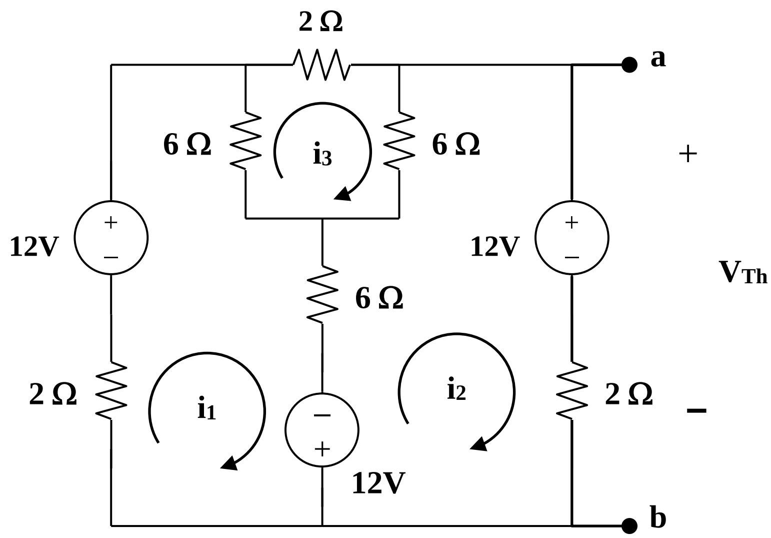


(b)



(c)

Solution continued on the next page...



(d)

$-12 - 12 + 14i_1 - 6i_2 - 6i_3 = 0$, and $7i_1 - 3i_2 - 3i_3 = 12$ (1)

$12 + 12 + 14i_2 - 6i_1 - 6i_3 = 0$, and $-3i_1 + 7i_2 - 3i_3 = -12$ (2)

$14i_3 - 6i_1 - 6i_2 = 0$, and $-3i_1 - 3i_2 + 7i_3 = 0$ (3)

This leads to the following matrix form for (1), (2) and (3),

$$\begin{bmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -3 & -3 \\ -3 & 7 & -3 \\ -3 & -3 & 7 \end{vmatrix} = 100, \qquad \Delta_2 = \begin{vmatrix} 7 & 12 & -3 \\ -3 & -12 & -3 \\ -3 & 0 & 7 \end{vmatrix} = -120$$

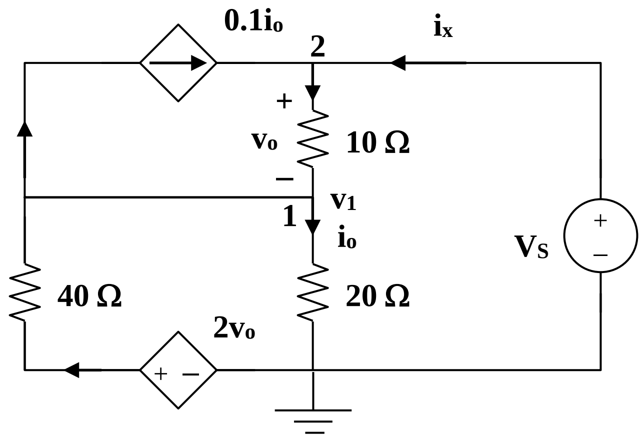
$i_2 = \Delta/\Delta_2 = -120/100 = -1.2$ A

$V_{Th} = 12 + 2i_2 =$ **9.6 V**, and $I_N = V_{Th}/R_{Th} =$ **8 A**

Solution 4.62

Since there are no independent sources, $V_{Th} = 0$ V

To obtain R_{Th}, consider the circuit below.



At node 2,

$$i_x + 0.1i_o = (1 - v_1)/10, \text{ or } 10i_x + i_o = 1 - v_1 \qquad (1)$$

At node 1,

$$(v_1/20) + 0.1i_o = [(2v_o - v_1)/40] + [(1 - v_1)/10] \qquad (2)$$

But $i_o = (v_1/20)$ and $v_o = 1 - v_1$, then (2) becomes,

$$1.1v_1/20 = [(2 - 3v_1)/40] + [(1 - v_1)/10]$$

$$2.2v_1 = 2 - 3v_1 + 4 - 4v_1 = 6 - 7v_1$$

or

$$v_1 = 6/9.2 \qquad (3)$$

From (1) and (3),

$$10i_x + v_1/20 = 1 - v_1$$

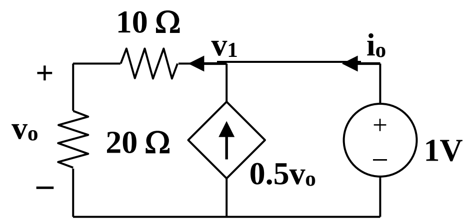
$$10i_x = 1 - v_1 - v_1/20 = 1 - (21/20)v_1 = 1 - (21/20)(6/9.2)$$

$$i_x = 31.52 \text{ mA}, \ R_{Th} = 1/i_x = \mathbf{31.73 \text{ ohms.}}$$

Solution 4.63

Because there are no independent sources, $I_N = I_{sc} =$ **0 A**

R_N can be found using the circuit below.



Applying KCL at node 1, $v_1 = 1$, and $v_o = (20/30)v_1 = 2/3$

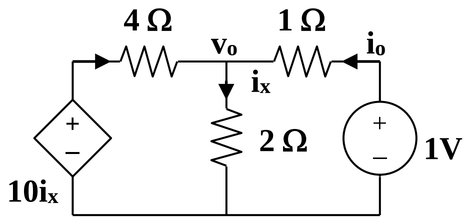
$i_o = (v_1/30) - 0.5v_o = (1/30) - 0.5x2/3 = 0.03333 - 0.33333 = -0.3$ A.

Hence,

$$R_N = 1/(-0.3) = \mathbf{-3.333\ ohms}$$

Solution 4.64

With no independent sources, $V_{Th} = \mathbf{0\ V}$. To obtain R_{Th}, consider the circuit shown below.



$$i_x = [(1 - v_o)/1] + [(10i_x - v_o)/4], \text{ or } 5v_o = 4 + 6i_x \qquad (1)$$

But $i_x = v_o/2$. Hence,

$$5v_o = 4 + 3v_o, \text{ or } v_o = 2, \; i_o = (1 - v_o)/1 = -1$$

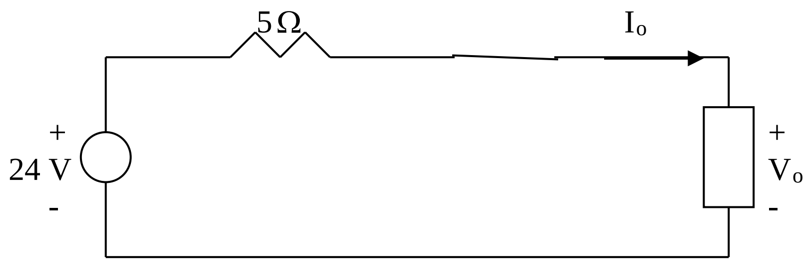
$$\text{Thus, } R_{Th} = 1/i_o = \mathbf{-1\ ohm}$$

Solution 4.65

At the terminals of the unknown resistance, we replace the circuit by its Thevenin equivalent.

$R_{eq} = 2 + (4 \| 12) = 2 + 3 = 5\Omega, \qquad V_{Th} = \frac{12}{12+4}(32) = 24 \text{ V}$

Thus, the circuit can be replaced by that shown below.



Applying KVL to the loop,

$$-24 + 5I_o + V_o = 0 \qquad \longrightarrow \qquad \mathbf{V_o = 24 - 5I_o.}$$

Solution 4.66

Find the maximum power that can be delivered to the resistor ***R*** in the circuit in Fig. 4.132.

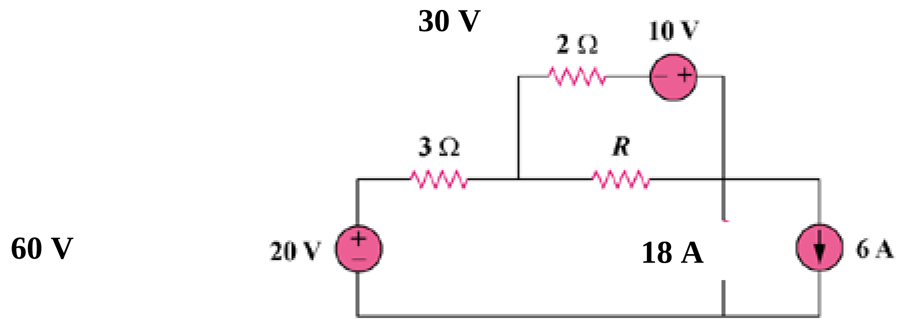
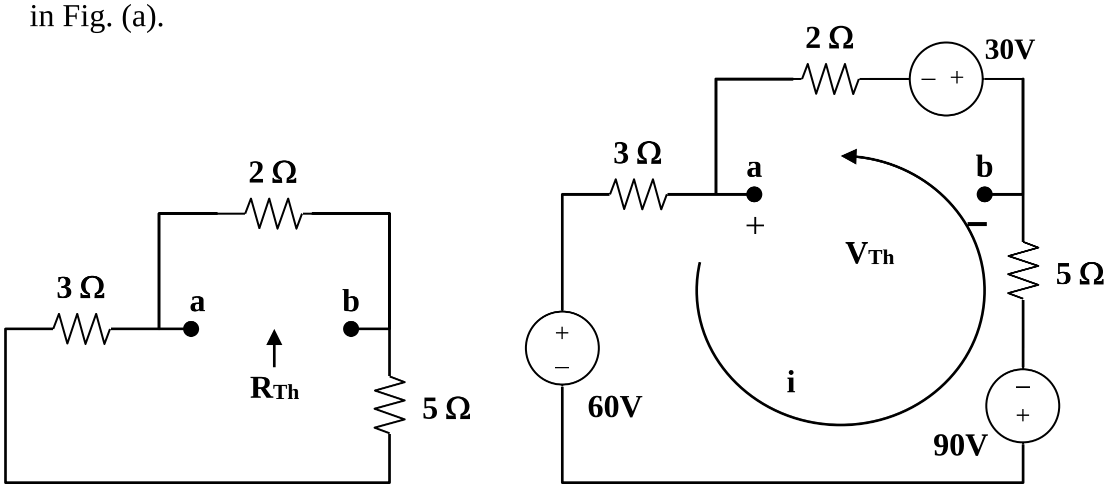


Figure 4.132
For Prob. 4.66.

Solution

We first find the Thevenin equivalent at terminals a and b. We find R_{Th} using the circuit in Fig. (a).



(a) **(b)**

$$R_{Th} = 2\|(3 + 5) = 2\|8 = \mathbf{1.6\ \Omega}$$

By performing a source transformation on the given circuit, we obtain the circuit in (b). We now use this to find V_{Th}.

$$10i + 90 + 60 + 30 = 0, \text{ or } i = -18$$

$$V_{Th} + 30 + 2i = 0, \text{ or } V_{Th} = 6 \text{ V}$$

$$p = V_{Th}^2/(4R_{Th}) = (6)^2/[4(1.6)] = \mathbf{5.625\ watts}.$$

Solution 4.67

The variable resistor R in Fig. 4.133 is adjusted until it absorbs the maximum power from the circuit. (a) Calculate the value of R for maximum power. (b) Determine the maximum power absorbed by R.

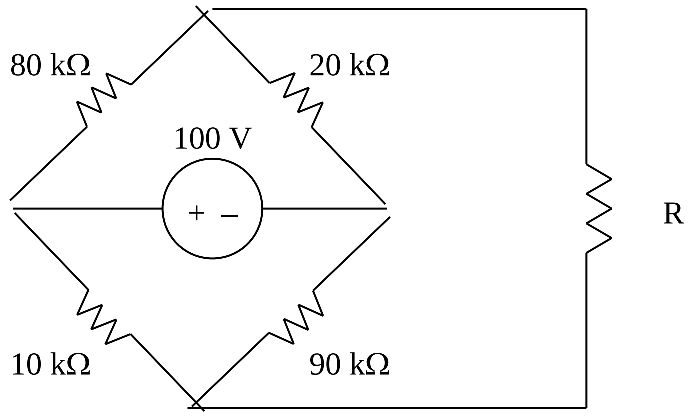
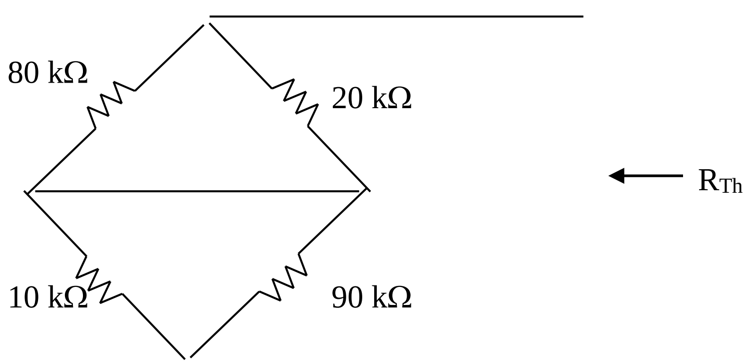


Figure 4.133
For Prob. 4.67.

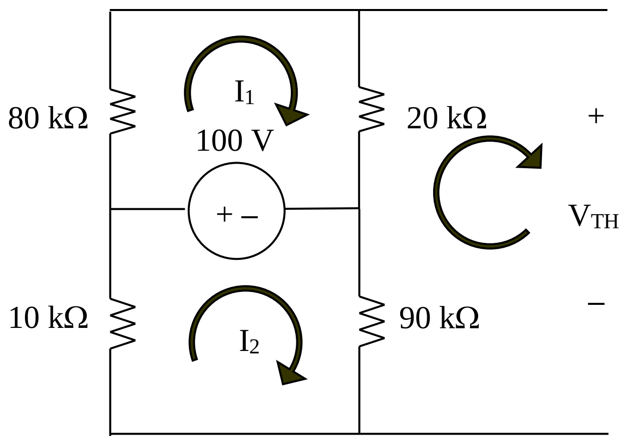
Solution

We first find the Thevenin equivalent. We find R_{Th} using the circuit below.



R_{Thev} = [80k20k/(80k+20k)] + [10k90k/(10k+90k)] = [(1600k/100)+(900k/100)]
= 16k+9k = 25 kΩ.

We find V_{Th} using the circuit below. We apply mesh analysis.



Loop 1, $-100 + (80k+20k)I_1 = 0$ or $I_1 = 100/100k = 1$ mA.

Loop 2, $(10k+90k)I_2 + 100 = 0$ or $I_2 = -100/100k = -1$ mA.

Finally, $V_{Thev} = 20k(0.001) + 90k(-0.001) = 20-90 = -70$ V.

(a) $R = R_{Th} =$ **25 kΩ**

(b) $P_{max} = (V_{Thev})^2/(4R_{Thev}) = (-70)^2/(4x25k) =$ **49 mW**.

Solution 4.68

Consider the 30 Ω resistor. First compute the Thevenin equivalent circuit as seen by the 30 Ω resistor. Compute the value of ***R*** that results in Thevenin equivalent resistance equal to the 30 Ω resistance and then calculate power delivered to the 30 Ω resistor. Now let R = 0 Ω, 110 Ω, and ∞, calculate the power delivered to the 30 Ω resistor in each case. What can you say about the value of R that will result in the maximum power that can be delivered to the 30 Ω resistor?

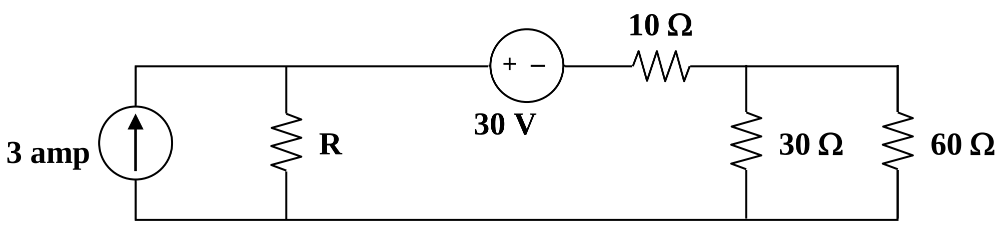
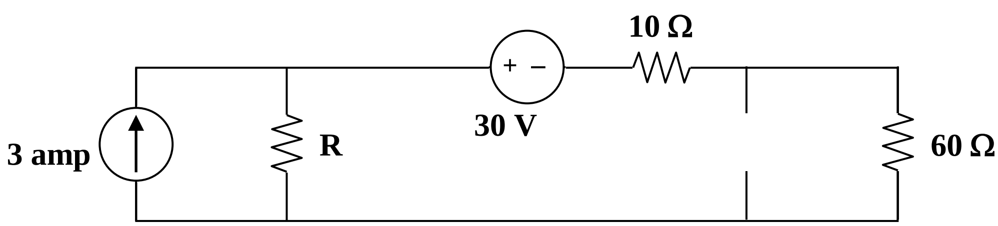


Figure 4.134
For Prob. 4.68.

Step 1. The first thing we need to do is to solve for V_{Thev} and R_{eq} as seen by the 30 ohm resistor.



We can replace the current source and parallel R into a voltage source of 3R in series with R. Now we can find the open circuit voltage equal to $60[(3R–30)/(R+10+60)]$. Since we do not have any dependent sources we can calculate $R_{eq} = 60(R+10)/(R+10+60)$. Finally the power delivered to the 30 ohm resistor is equal to $[V_{oc}/(R_{eq}+30)]^2(30)$. We also need to know the value of R that makes $R_{eq} = 30$ or $30 = [(60R+600)/(R+70)]$ or $(R+70) = 2R+20$ or R = **50 Ω**.

Step 2. Let us look at the Thevenin equivalent resistance first. When R = 0, $R_{eq} = 600/70 = 8.571$ ohms, when R = 110, $R_{eq} = 40$ Ω, and when R = ∞, $R_{eq} = 60$ Ω. Now look at the Thevenin voltage. When R = 0 we get $V_{Thev} = –60\text{x}30/70 = –25.71$ volts, when R = 50 Ω we get $V_{Thev} = 60$ volts, when R = 110 Ω, $V_{Thev} = 100$ volts, and when R = ∞ $V_{Thev} = 180$ volts.

Now we can calculate the values of power, R = 0 we get $P_{30} = [–25.71/38.571]^2 30 =$ **13.329 watts**, when R = 50 we get $P_{30} = [60/60]^2 30 =$ **30 watts**, $P_{30} = [100/70]^2 30 =$ **61.22 watts**, and when R = ∞ $P_{30} = [180/90]^2 30 =$ **120 watts**. It would appear that the value of R that causes the maximum power to be delivered to the 30 Ω resistor in R = ∞.

Solution 4.69

Find the maximum power transferred to resistor ***R*** in the circuit of Fig. 4.135.

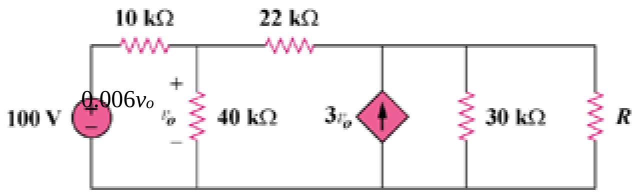
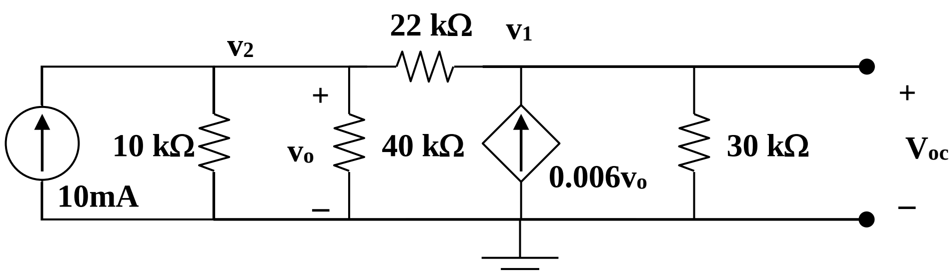


Figure 4.135
For Prob. 4.69.

Solution

First we need the Thevenin equivalent seen by the resistor R. To find the Thevenin equivalent circuit we only need to find V_{oc} and I_{sc}.



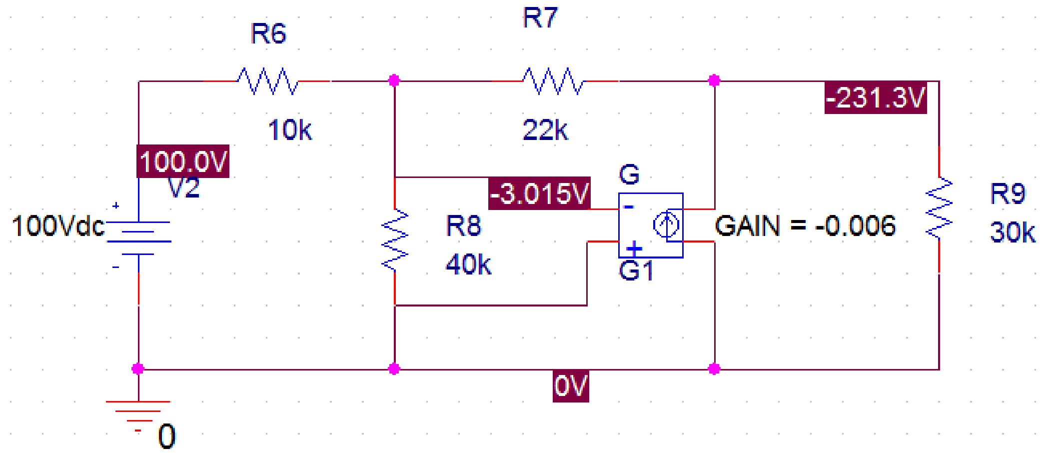
Now we have $[(v_1–v_2)/22k] – 0.006v_2 + [(v_1–0)/30k] = 0$ and
$–0.01 + [(v_2–0)/10k] + [(v_2–0)/40k] + [(v_2–v_1)/22k] = 0$ which leads to
$[0.0000454545+0.006]v_2 = 0.00604545v_2 = (0.0454545+0.03333)v_1/1000$ or
$v_2 = [0.0787878/0.00604545]v_1/1000 = 0.0130326v_1$. Finally,
$\{[(0.1+0.025+0.0454545)(0.0130326)/1000]–[0.0454545/1000]\}v_1 =$
$\{[(0.1704545)(0.0130326)–0.0454545]/1000\}v_1 = 0.01$ or v_1
$= 10/(–0.043233) = –231.3$ V.

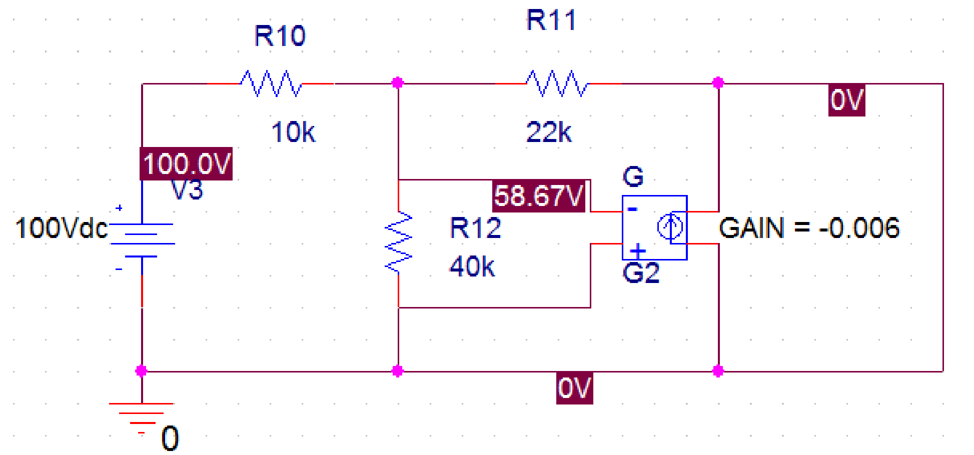
To determine I_{sc} we set $v_1 = 0$ and find the current through the short. We have
$–0.01 + [(v_2–0)/10k] + [(v_2–0/40k] + [(v_2–0)/22k] = 0$ or
$[(0.1+0.025+0.0454545)/1000]v_2 = 0.01$ or $v_2 = 10/0.1704545 = 58.667$ V.

Now we can find $I_{sc} = [(v_2–0)/22k] + 0.006(58.667) = [(58.667)/22k] + 0.006(58.667) =$
$0.0026667+0.352 = 0.35467$ A or
$R_{Thev} = –231.3/0.35467 = 16.9443/0.751414 =$ **–652.2 Ω**.

This is now an interesting problem since the equivalent resistance is negative. Obviously the correct answer is let R = 652.2 Ω which then means the current through R is infinite and the power delivered to R is infinity. The negative resistance for the equivalent circuit means that both the source and resistance effectively deliver power to the load. Please note that a negative equivalent resistance indicates that we have a dependent source.

We can check the voltages by using PSpice and we get,





Solution 4.70

Determine the maximum power delivered to the variable resistor *R* shown in the circuit of Fig. 4.136.

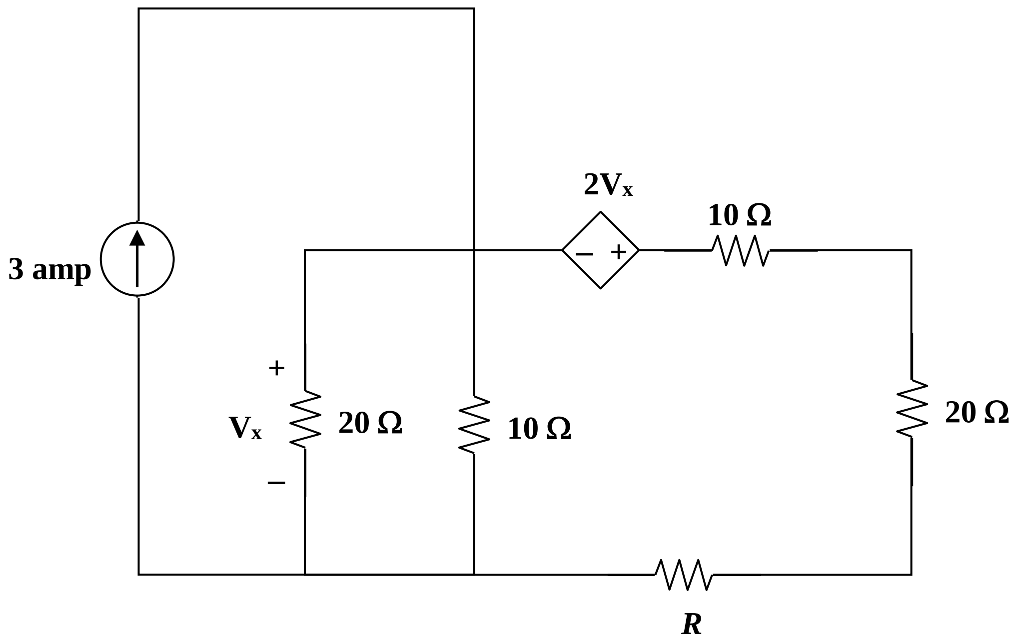
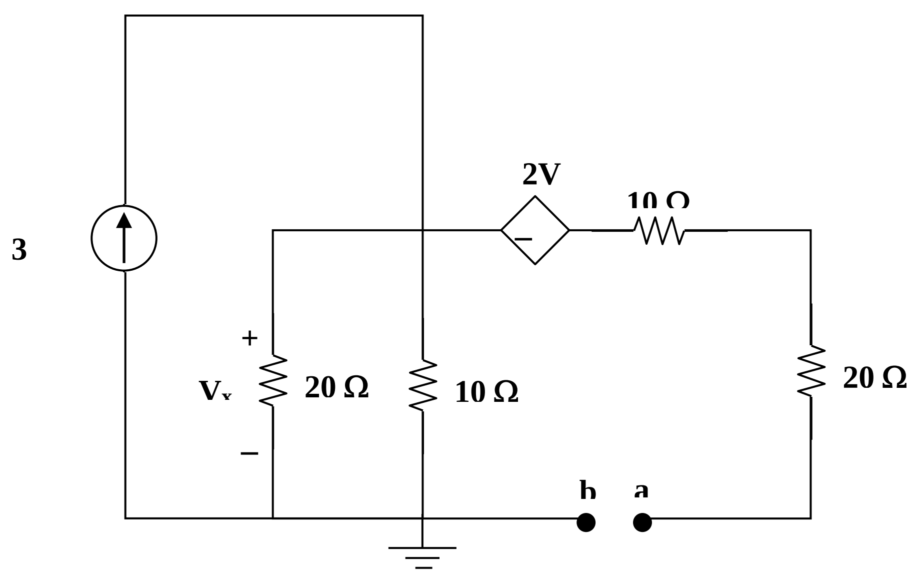


Figure 4.136
For Prob. 4.70.

Step 1. We need to start with finding the Thevenin equivalent circuit looking into the terminals connected to the resistor R. Once we find the equivalent circuit we know that the maximum power will be delivered to R when $R = R_{eq}$. To find the Thevenin equivalent we need to find V_{oc} and I_{sc} (see the circuit below).



Let $V_{oc} = V_{ab}$ and let $I_{sc} = I_{ab}$. We then need to analyze the separate circuits.

To solve for V_{oc}, $V_{ab} = 2V_x + V_x = 3V_x$. To solve for V_x we solve this nodal equation $-3 + [(V_x\text{-}0)/20] + [(V_x-0)/10] = 0$.

To solve for I_{sc} we know that $I_{ab} = [(V_x+2V_x-0)/(10+20)] = [(3V_x)/30] = V_x/10$ again we solve this nodal equation to find V_x, $-3 + [(V_x\text{-}0)/20] + [(V_x-0)/10] + [(3V_x-0)/30] = 0$.

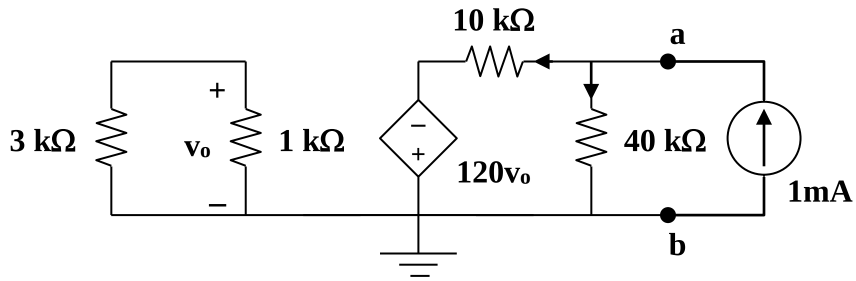
Step 2. For V_{oc}, $[(1/20)+(1/10)]V_x = 3$ or $V_x = 3(20/3) = 20$ volts which leads to $V_{oc} = 3V_x = 60$ volts $= V_{Thev}$.

For I_{sc} $[(1/20)+(1/10)+(1/10)]V_x = (5/20)V_x = 3$ or $V_x = 12$ volts. Therefore, $I_{sc} = V_x/10 = 1.2$ amps. Finally, $R_{eq} = R = 60/1.2 = 50\ \Omega$.

Now, $P_R = (60/100)^2 50 =$ **18 watts**.

Solution 4.71

We need R_{Th} and V_{Th} at terminals a and b. To find R_{Th}, we insert a 1-mA source at the terminals a and b as shown below.



Assume that all resistances are in k ohms, all currents are in mA, and all voltages are in volts. At node a,

$$1 = (v_a/40) + [(v_a + 120v_o)/10], \text{ or } 40 = 5v_a + 480v_o \qquad (1)$$

The loop on the left side has no voltage source. Hence, $v_o = 0$. From (1), $v_a = 8$ V.

$$R_{Th} = v_a/1 \text{ mA} = 8 \text{ kohms}$$

To get V_{Th}, consider the original circuit. For the left loop,

$$v_o = (1/4)8 = 2 \text{ V}$$

For the right loop, $v_R = V_{Th} = (40/50)(-120v_o) = -192$

The resistance at the required resistor is

$$R = R_{Th} = \mathbf{8\ k\Omega}$$

$$p = V_{Th}^2/(4R_{Th}) = (-192)^2/(4\text{x}8\text{x}10^3) = \mathbf{1.152\ watts}$$

Solution 4.72

(a) For the circuit in Fig. 4.138, obtain the Thevenin equivalent at terminals ***a-b***.
(b) Calculate the current in $R_L = 13\ \Omega$.
(c) Find R_L for maximum power deliverable to R_L.
(d) Determine that maximum power.

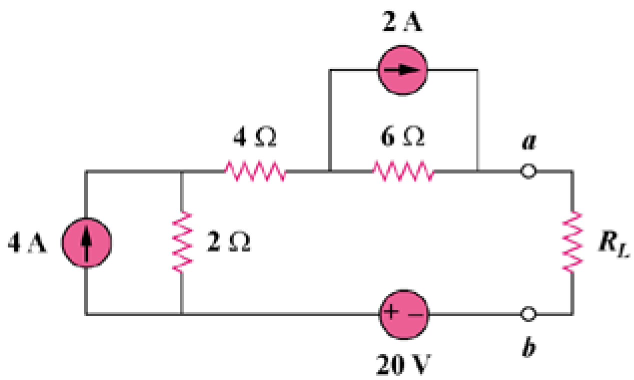


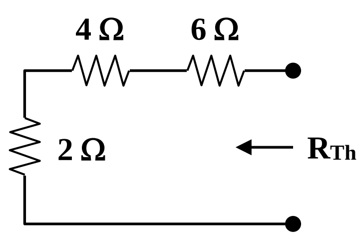
Figure 4.138
For Prob. 4.72.

Solution

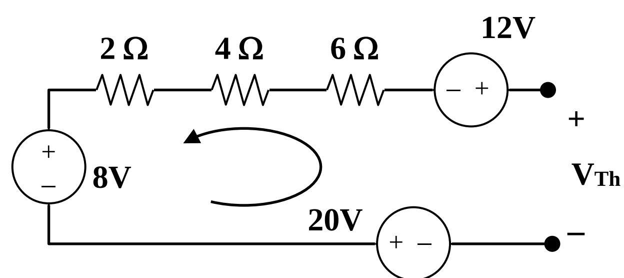
(a) R_{Th} and V_{Th} are calculated using the circuits shown in Fig. (a) and (b) respectively.

From Fig. (a), $R_{Th} = 2 + 4 + 6 = \mathbf{12\ ohms}$

From Fig. (b), $-V_{Th} + 12 + 8 + 20 = 0$, or $V_{Th} = \mathbf{40\ V}$



(a)



(b)

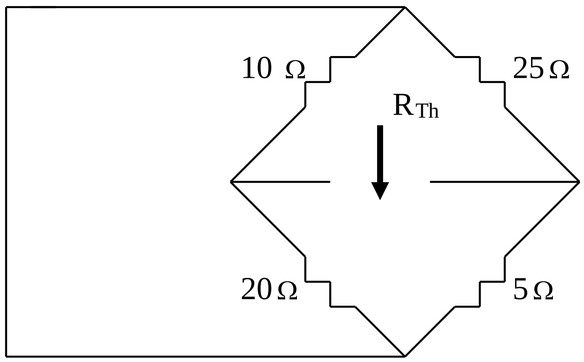
(b) $i = V_{Th}/(R_{Th} + R) = 40/(12 + 13) = \mathbf{1.6\ A}$

(c) For maximum power transfer, $R_L = R_{Th} = \mathbf{12\ ohms}$

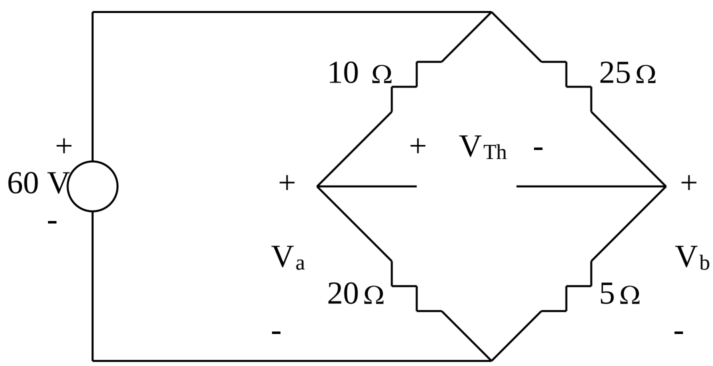
(d) $p = V_{Th}^2/(4R_{Th}) = (40)^2/(4\text{x}12) = \mathbf{33.33\ watts}$.

Solution 4.73

Find the Thevenin's equivalent circuit across the terminals of R.



$$R_{Th} = 10 // 20 + 25 // 5 = 325 / 30 = 10.833 \Omega$$



$$V_a = \frac{20}{30}(60) = 40, \qquad V_b = \frac{5}{30}(60) = 10$$

$$-V_a + V_{Th} + V_b = 0 \quad \longrightarrow \quad V_{Th} = V_a - V_b = 40 - 10 = 30 \text{ V}$$

$$\boldsymbol{p_{\max} = \frac{V_{Th}{}^2}{4R_{Th}} = \frac{30^2}{4x10.833}} = \textbf{20.77 W}.$$

Solution 4.74

When R_L is removed and V_s is short-circuited,

$$R_{Th} = R_1 \| R_2 + R_3 \| R_4 = [R_1 R_2/(R_1 + R_2)] + [R_3 R_4/(R_3 + R_4)]$$

$$R_L = R_{Th} = \mathbf{(R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)/[(R_1 + R_2)(R_3 + R_4)]}$$

When R_L is removed and we apply the voltage division principle,

$$V_{oc} = V_{Th} = v_{R2} - v_{R4}$$

$$= ([R_2/(R_1 + R_2)] - [R_4/(R_3 + R_4)])V_s = \{[(R_2R_3) - (R_1R_4)]/[(R_1 + R_2)(R_3 + R_4)]\}V_s$$

$$p_{max} = V_{Th}^2/(4R_{Th})$$

$$= \{[(R_2R_3) - (R_1R_4)]^2/[(R_1 + R_2)(R_3 + R_4)]^2\}V_s^2[(R_1 + R_2)(R_3 + R_4)]/[4(a)]$$

$$\text{where } a = (R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)$$

$$p_{max} =$$

$$\mathbf{[(R_2R_3) - (R_1R_4)]^2V_s^2/[4(R_1 + R_2)(R_3 + R_4)(R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4)]}$$

Solution 4.75

For the circuit in Fig. 4.141, determine the value of ***R*** such that the maximum power delivered to the load is 12 mW.

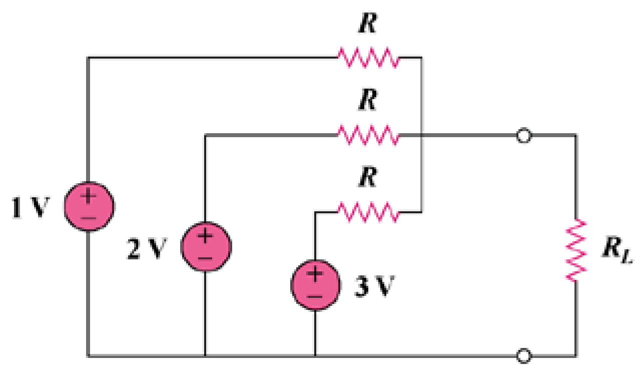
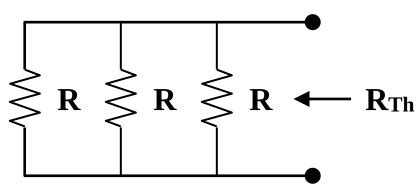


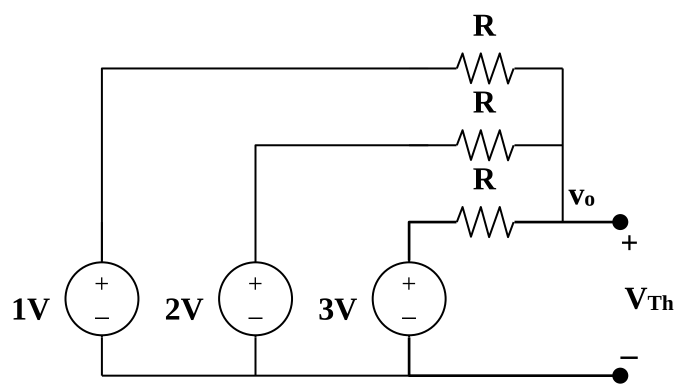
Figure 4.141
For Prob. 4.75.

Solution

We need to first find R_{Th} and V_{Th}.



(a)



(b)

Consider the circuit in Fig. (a).

$$(1/R_{eq}) = (1/R) + (1/R) + (1/R) = 3/R$$

$$R_{eq} = R/3$$

From the circuit in Fig. (b),

$$((1 - v_o)/R) + ((2 - v_o)/R) + ((3 - v_o)/R) = 0$$

$$v_o = 2 = V_{Th}$$

For maximum power transfer,

$$R_L = R_{Th} = R/3$$

$$P_{max} = [(V_{Th})^2/(4R_{Th})] = \mathbf{12\ mW}$$

$$R_{Th} = [(V_{Th})^2/(4P_{max})] = 4/(4xP_{max}) = 1/P_{max} = R/3$$

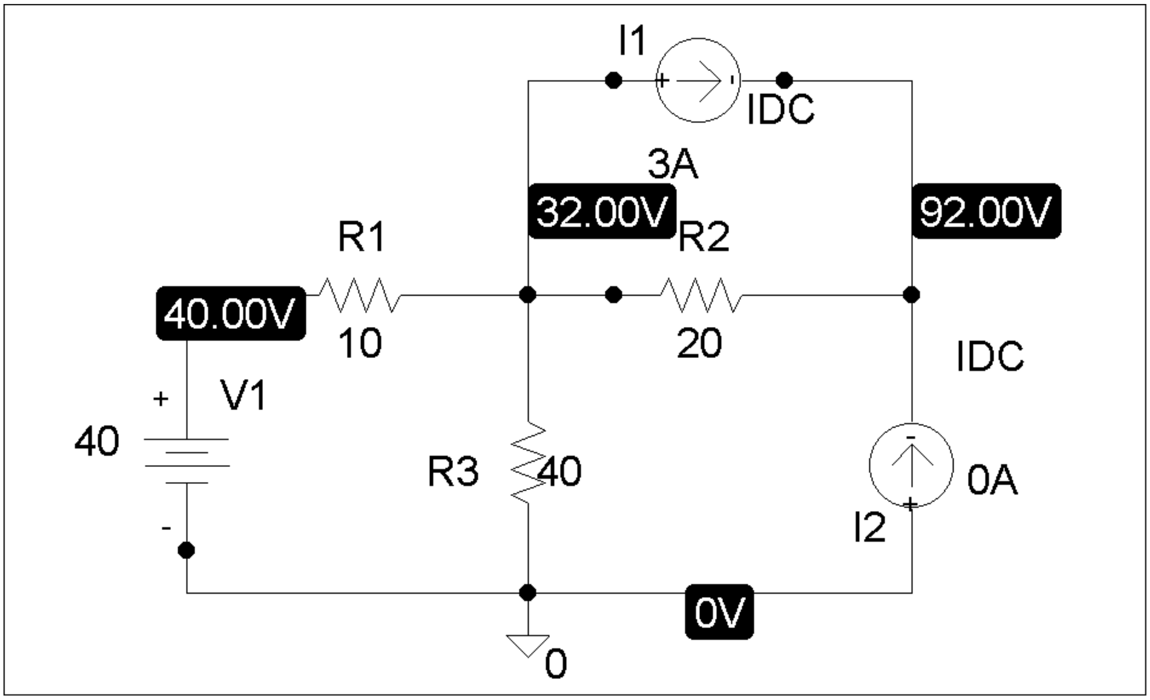
$$R = 3/(0.012) = \mathbf{250\ \Omega}.$$

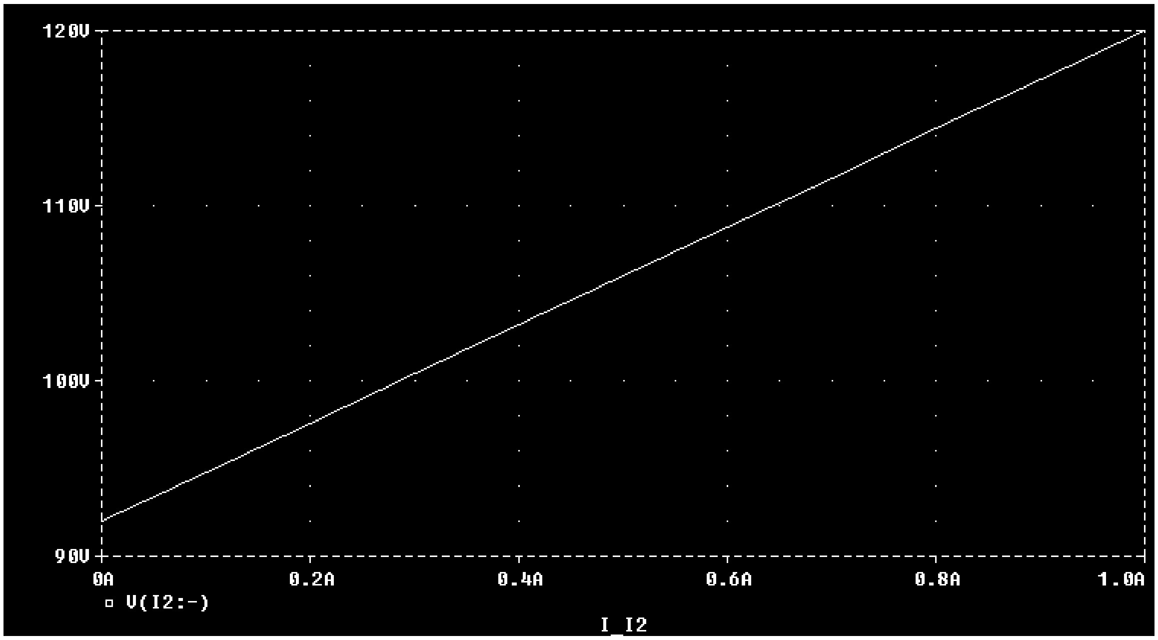
Solution 4.76

Follow the steps in Example 4.14. The schematic and the output plots are shown below. From the plot, we obtain,

$$V = \mathbf{92\ V}\ [i = 0,\ \text{voltage axis intercept}]$$

$$R = \text{Slope} = (120 - 92)/1 = \mathbf{28\ ohms}$$





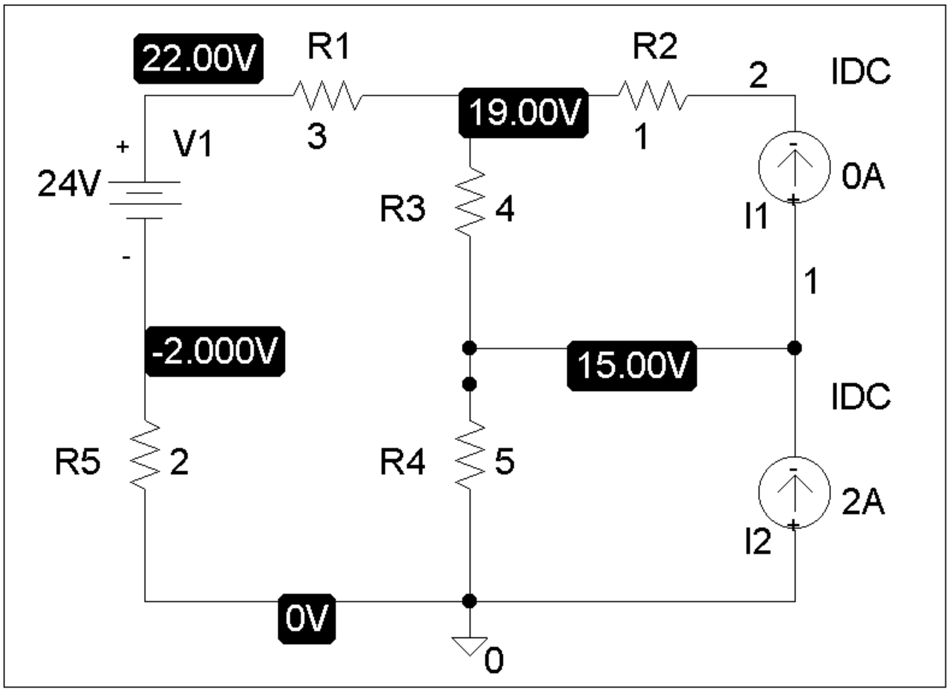
120V
110V
100V
90V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(I2:-)
I_I2

Solution 4.77

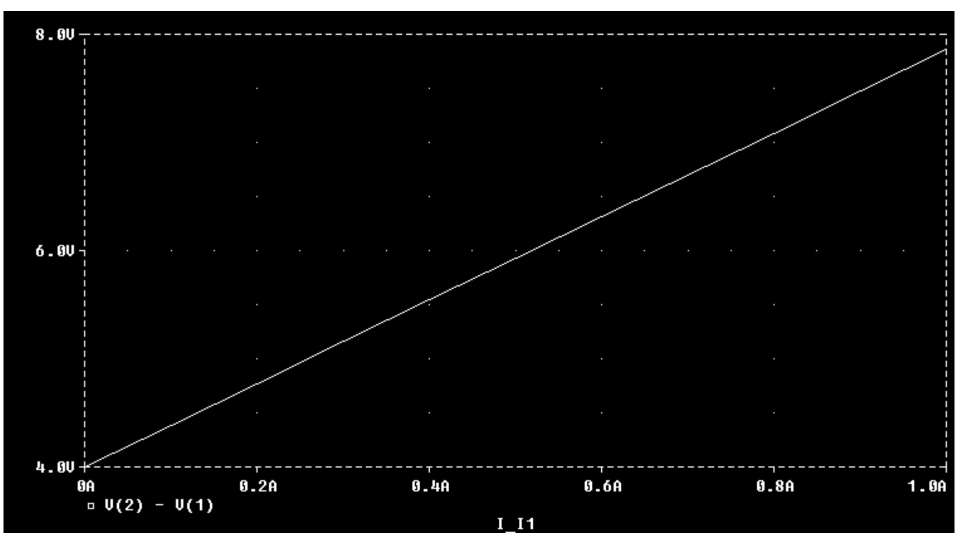
(a) The schematic is shown below. We perform a dc sweep on a current source, I1, connected between terminals a and b. We label the top and bottom of source I1 as 2 and 1 respectively. We plot V(2) – V(1) as shown.

$$V_{Th} = \mathbf{4\ V} \text{ [zero intercept]}$$

$$R_{Th} = (7.8 - 4)/1 = \mathbf{3.8\ ohms}$$

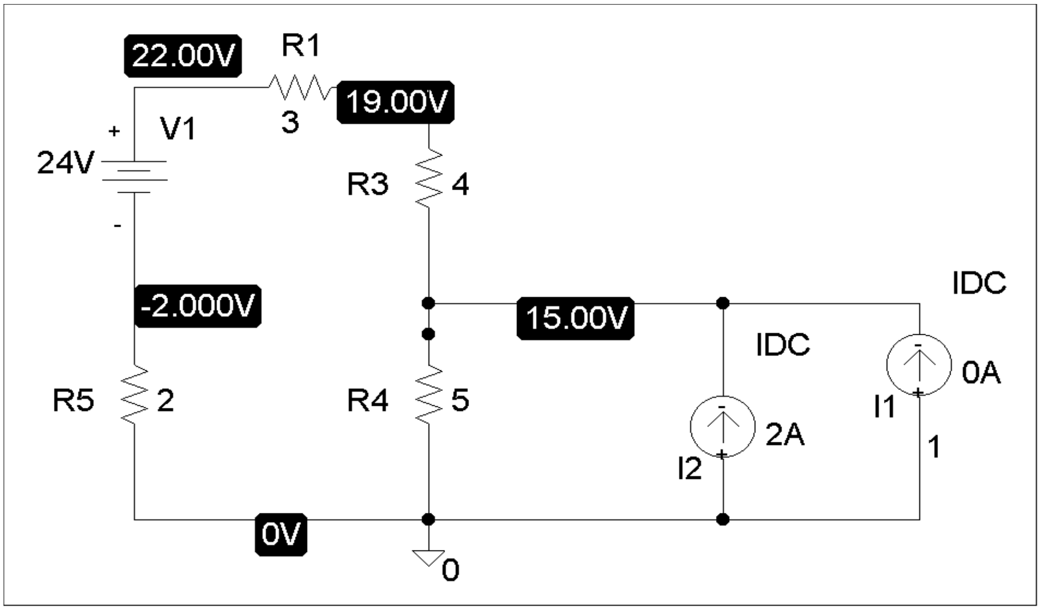


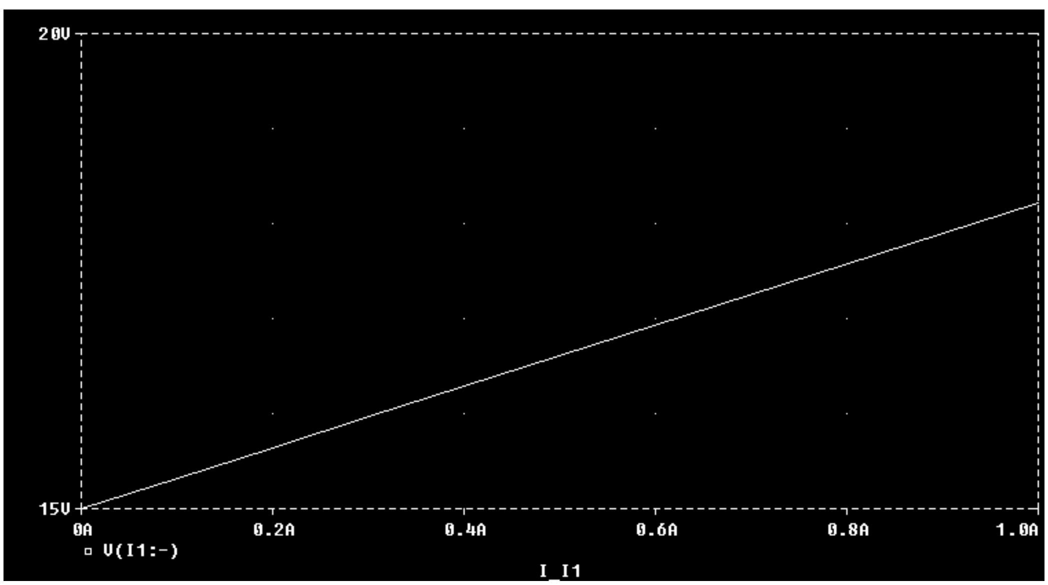
(b) Everything remains the same as in part (a) except that the current source, I1, is connected between terminals b and c as shown below. We perform a dc sweep on I1 and obtain the plot shown below. From the plot, we obtain,



V = **15 V** [zero intercept]

R = (18.2 – 15)/1 = **3.2 ohms**



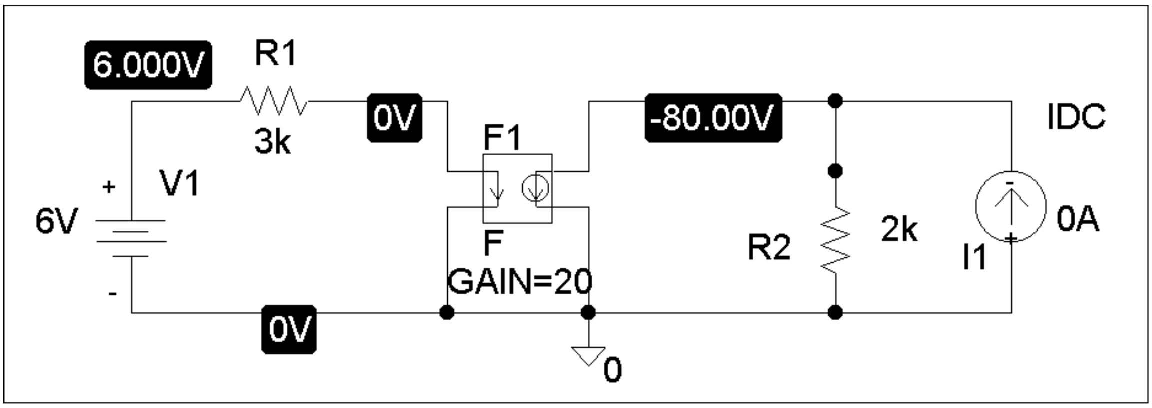


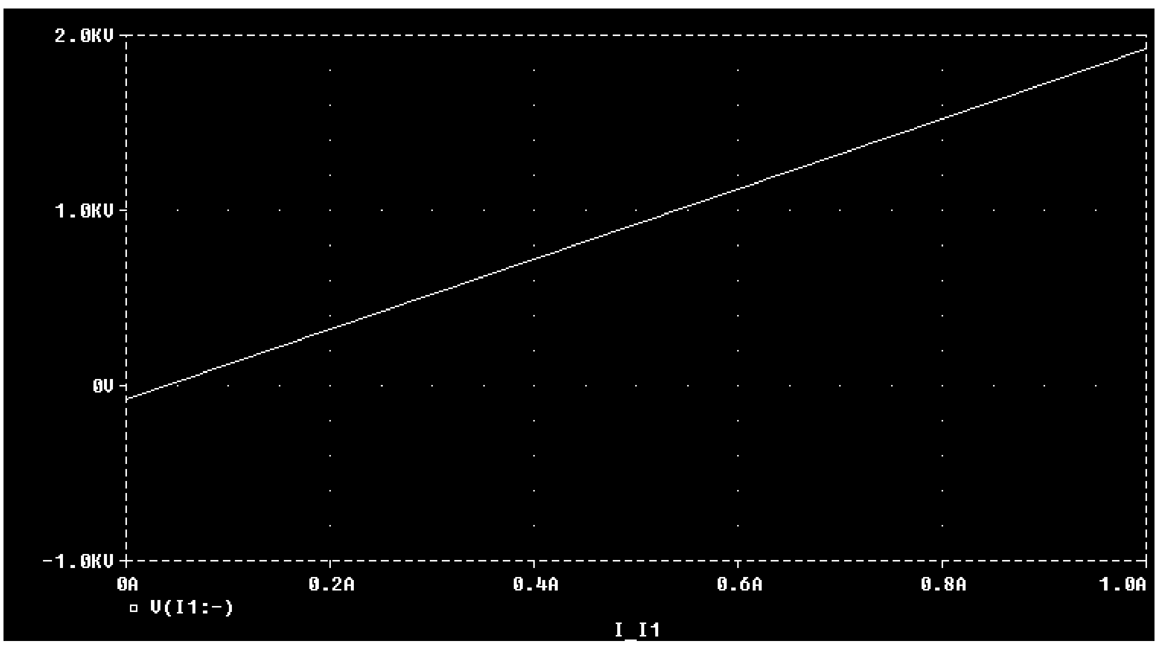
Solution 4.78

The schematic is shown below. We perform a dc sweep on the current source, I1, connected between terminals a and b. The plot is shown. From the plot we obtain,

$$V_{Th} = \mathbf{-80\ V} \text{ [zero intercept]}$$

$$R_{Th} = (1920 - (-80))/1 = \mathbf{2\ k\ ohms}$$



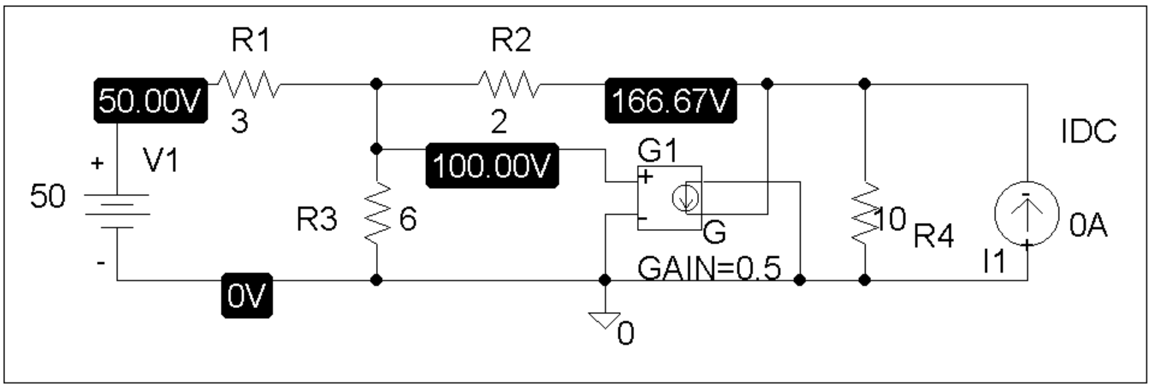


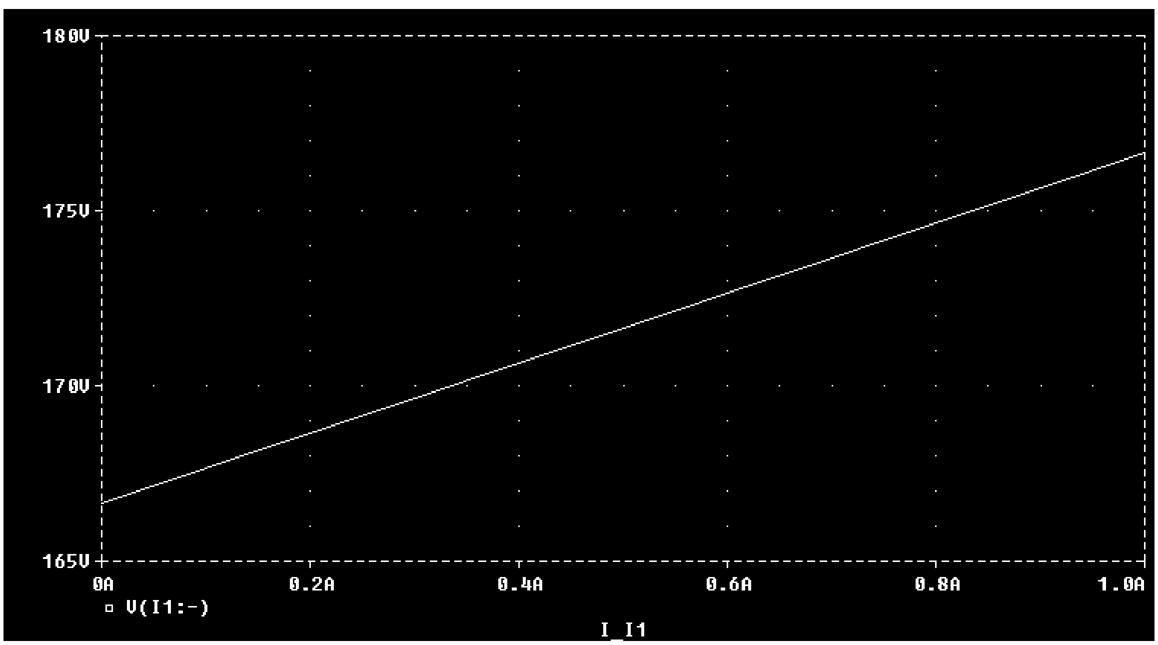
Solution 4.79

After drawing and saving the schematic as shown below, we perform a dc sweep on I1 connected across a and b. The plot is shown. From the plot, we get,

$$V = \mathbf{167\ V} \quad \text{[zero intercept]}$$

$$R = (177 - 167)/1 = \mathbf{10\ ohms}$$



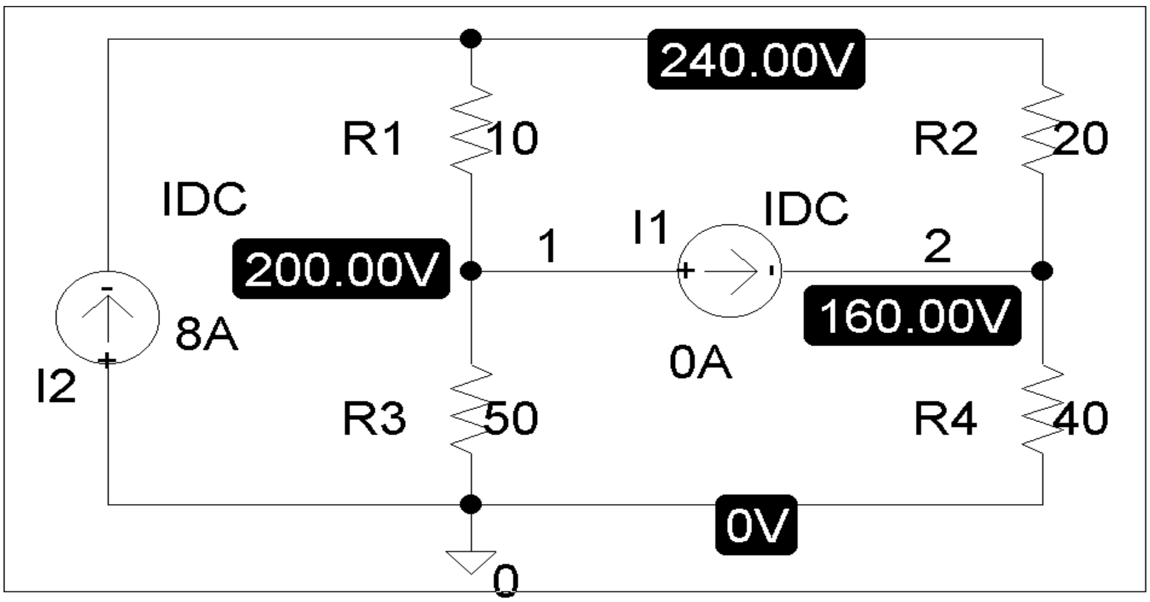


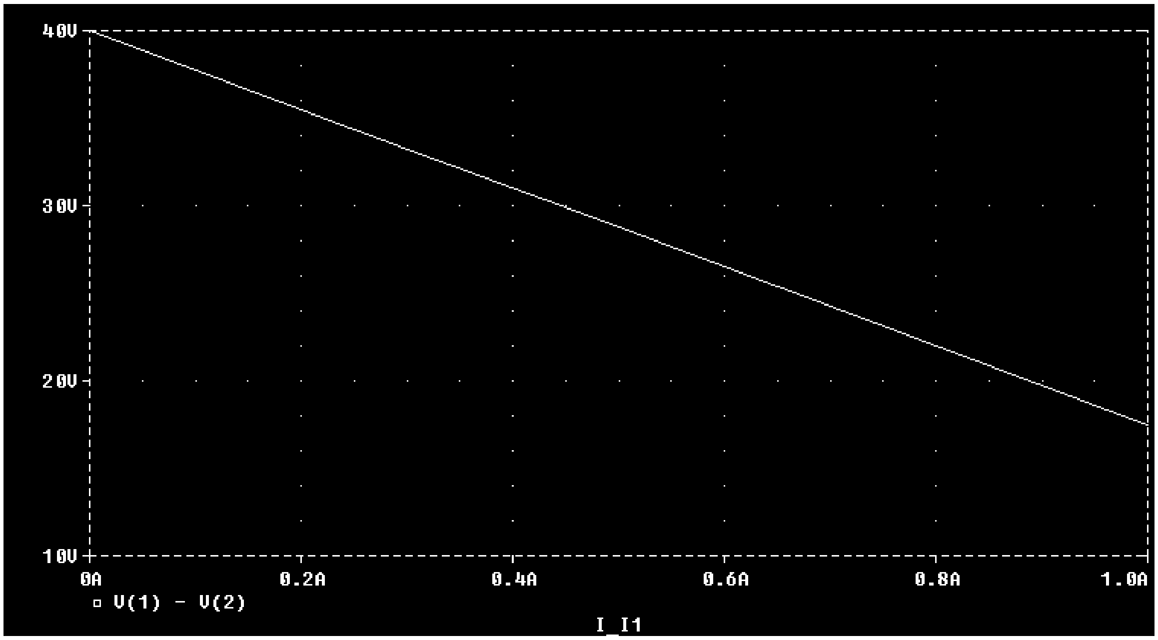
Solution 4.80

The schematic in shown below. We label nodes a and b as 1 and 2 respectively. We perform dc sweep on I1. In the Trace/Add menu, type v(1) – v(2) which will result in the plot below. From the plot,

$$V_{Th} = \mathbf{40\ V} \quad \text{[zero intercept]}$$

$$R_{Th} = (40 - 17.5)/1 = \mathbf{22.5\ ohms} \quad \text{[slope]}$$





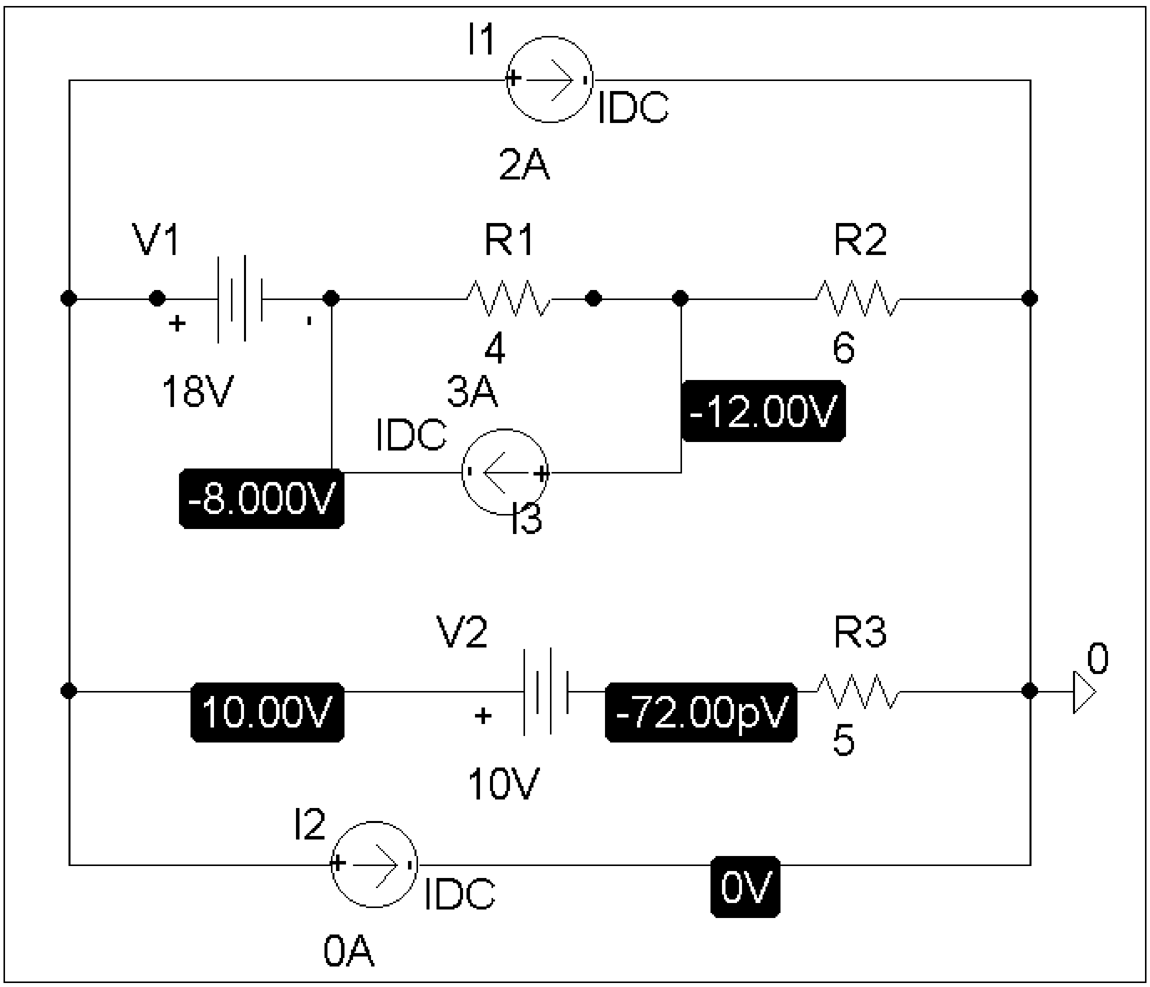
40V
30V
20V
10V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(1) - V(2)
I_I1

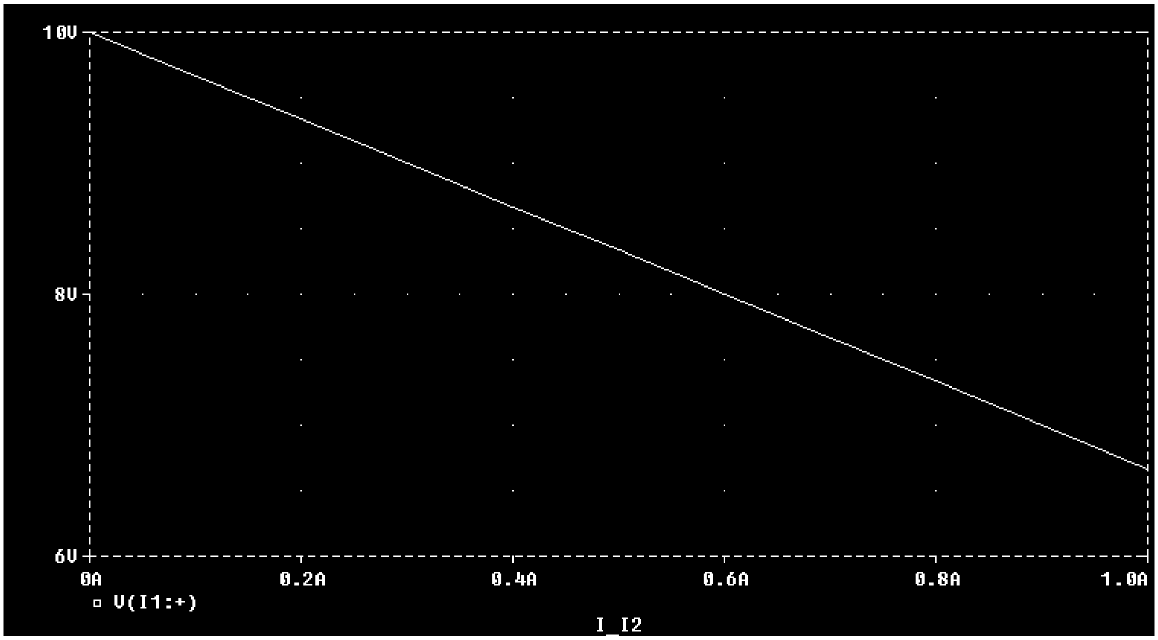
Solution 4.81

The schematic is shown below. We perform a dc sweep on the current source, I2, connected between terminals a and b. The plot of the voltage across I2 is shown below. From the plot,

$$V_{Th} = \mathbf{10\ V} \quad \text{[zero intercept]}$$

R_{Th} = (10 – 6.7)/1 = **3.3 ohms**. Note that this is in good agreement with the exact value of 3.333 ohms.




10V
8V
6V
0A
0.2A
0.4A
0.6A
0.8A
1.0A
V(I1:+)
I_I2

Solution 4.82

An automobile battery has an open circuit voltage of 14.7 volts which drops to 12 volts when connected to two 65 watt headlights. What is the resistance of each headlight and the value of the internal resistance of the battery?

Step 1. Basically we can treat this like a Thevenin equivalent circuit problem.

Clearly $V_{oc} = V_{Thev} = 14.7$ volts and R_{HP} is equal to the resistance of each bulb in parallel or $R_{HP} = R_B R_B/(R_B+R_B)$. In addition, 2x65 = 130 = 12i and
$R_{HB} = 12/i$ and $i = 14.7/(R_s + R_{HB})$.

Step 2. $i = 130/12 = 10.8333$ amps and $R_{HB} = 12/10.8333 = 1.107696$ ohms = $R_B/2$ or R_B = **2.215 Ω** per bulb. Finally $(R_s+1.107696) = 14.7/10.8333$ or

R_s = **249.2 mΩ**.

Solution 4.83

The following results were obtained from measurements taken between the two terminals of a resistive network.

Terminal Voltage	72 V	0 V
Terminal Current	0 A	9A

Find the Thevenin equivalent of the network.

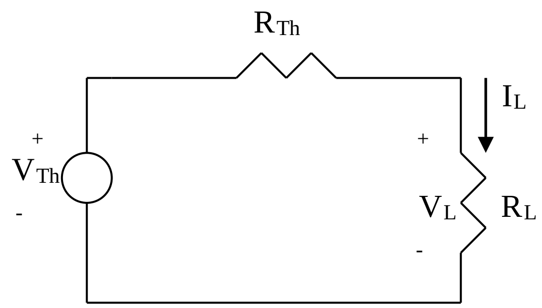
Solution

Step 1. We note that $V_{Thev} = V_{oc}$ and $I_N = I_{sc}$ and $R_{eq} = V_{oc}/I_{sc} = R_{Thev} = R_N$.

Step 2. V_{Thev} = **72 V** and $R_{Thev} = 72/9 =$ **8 Ω**.

Solution 4.84

Let the equivalent circuit of the battery terminated by a load be as shown below.



For open circuit,

$R_L = \infty, \quad \longrightarrow \quad V_{Th} = V_{oc} = V_L = \underline{10.8\text{ V}}$

When R_L = 4 ohm, V_L=10.5,

$$I_L = \frac{V_L}{R_L} = 10.8/4 = 2.7$$

But

$$V_{Th} = V_L + I_L R_{Th} \quad \longrightarrow \quad R_{Th} = \frac{V_{Th} - V_L}{I_L} = \frac{12 - 10.8}{2.7} = \underline{0.4444\Omega}$$

= **444.4 mΩ**.

Solution 4.85

The Thevenin equivalent at terminals *a-b* of the linear network shown in Fig. 4.142 is to be determined by measurement. When a 10-kΩ resistor is connected to terminals *a-b*, the voltage V_{ab} is measured as 20 V. When a 30-kΩ resistor is connected to the terminals, V_{ab} is measured as 40 V. Determine: (a) the Thevenin equivalent at terminals *a-b*, (b) V_{ab} when a 20-kΩ resistor is connected to terminals *a-b*.

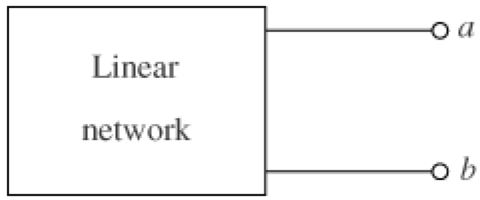
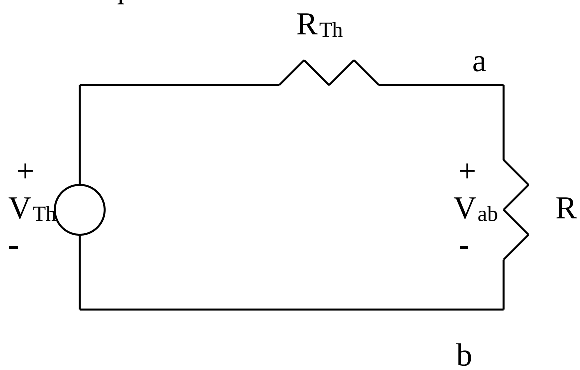


Figure 4.142
For Prob. 4.85.

Solution

(a) Consider the equivalent circuit terminated with R as shown below.



$V_{ab} = \frac{R}{R + R_{Th}} V_{Th} \quad \longrightarrow \quad 20 = \frac{10}{10 + R_{Th}} V_{Th}$

or 200 + 20R_{Thev} = 10V_{Thev} where R_{Th} is in k-ohm.

Similarly, $40 = \frac{30}{30 + R_{Th}} V_{Th}$ or 1200 + 40R_{Thev} = 30V_{Thev}. We now have two equations with two unknowns or V_{Thev} = 20+2R_{Thev} and 1200 + 40R_{Thev} = 30(20+2R_{Thev}) or 20R_{Thev} = 1200–600 or R_{Thev} = **30 kΩ**. Next we get V_{Thev} = 20 + 2(30) = **80 V**.

(b) Clearly V_{ab} = 80[20/(20+30)] = **32 V**.

Solution 4.86

A black box with a circuit in it is connected to a variable resistor. An ideal ammeter (with zero resistance) and an ideal voltmeter (with infinite resistance) are used to measure current and voltage as shown in Fig. 4.143. The results are shown in the table below.

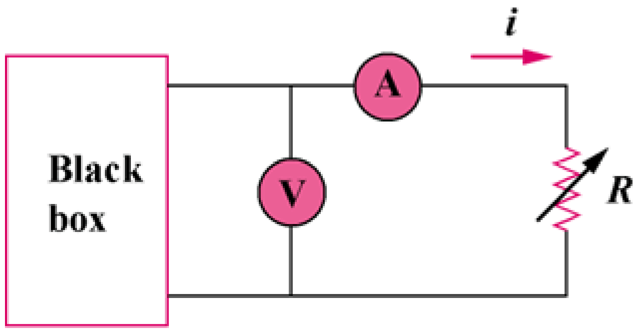


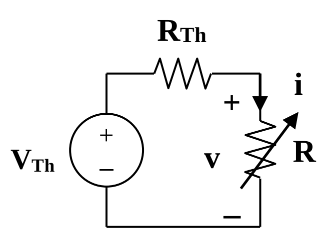
Figure 4.143
For Prob. 4.86.

(a) Find $\boldsymbol{i}$ when $\boldsymbol{R} = 12\ \Omega$.
(b) Determine the maximum power from the box.

$\boldsymbol{R}(\Omega)$	$\boldsymbol{V}$(V)	$\boldsymbol{i}$(A)
2	6	3
8	16	2
14	21	1.5

Solution

Step 1. We replace the box with the Thevenin equivalent.



$V_{Th} = v + iR_{Th}$

When $i = 3$, $v = 6$, which implies that $V_{Th} = 6 + 3R_{Th}$ (1)

When $i = 2$, $v = 16$, which implies that $V_{Th} = 16 + 2R_{Th}$ (2)

Step 2. From (1) – (2), we get $0 = -10 + R_{Thev}$ or $R_{Thev} = 10\ \Omega$ and $V_{Thev} = 6+30$ or 36 V. It is interesting to note that we really only need the two data points, the third is redundant.

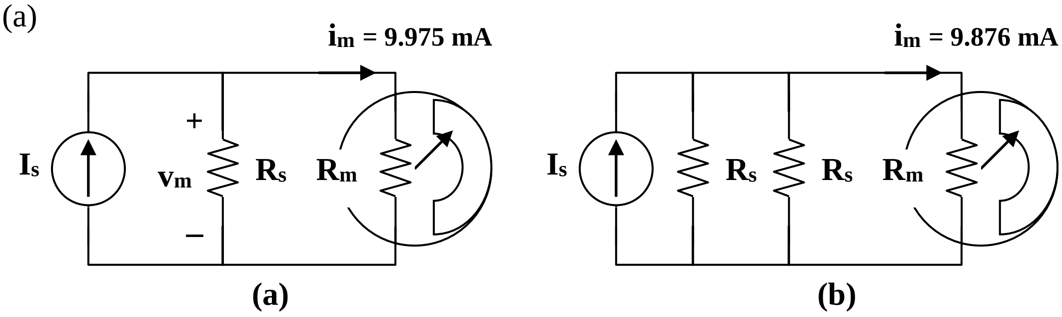
(a) When $R = 12\ \Omega$, $i = V_{Th}/(R + R_{Th}) = 36/(12 + 10) =$ **1.6364 A**

(b) For maximum power, $R = R_{TH}$

$$P_{max} = (V_{Th})^2/4R_{Th} = 36^2/(4\text{x}10) = \mathbf{32.4\ watts}.$$

Solution 4.87

(a)



From Fig. (a),

$$v_m = R_m i_m = 9.975 \text{ mA x } 20 = 0.1995 \text{ V}$$

$$I_s = 9.975 \text{ mA} + (0.1995/R_s) \qquad (1)$$

From Fig. (b),

$$v_m = R_m i_m = 20\text{x}9.876 = 0.19752 \text{ V}$$

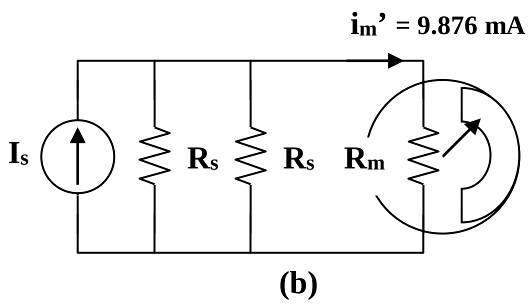
$$I_s = 9.876 \text{ mA} + (0.19752/2\text{k}) + (0.19752/R_s)$$

$$= 9.975 \text{ mA} + (0.19752/R_s) \qquad (2)$$

Solving (1) and (2) gives,

R_s = **8 k ohms**, I_s = **10 mA**

(b)

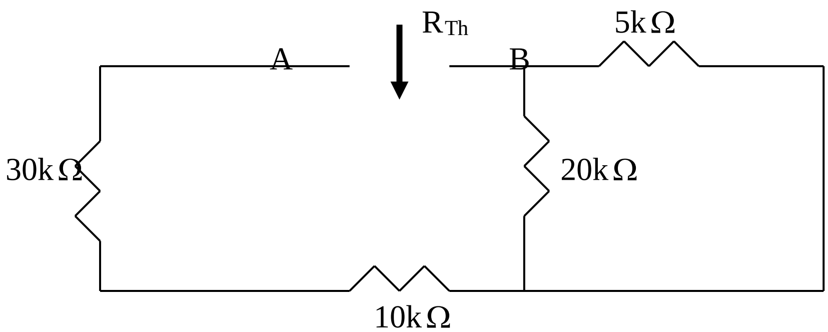


8k||4k = 2.667 k ohms

i_m' = [2667/(2667 + 20)](10 mA) = **9.926 mA**

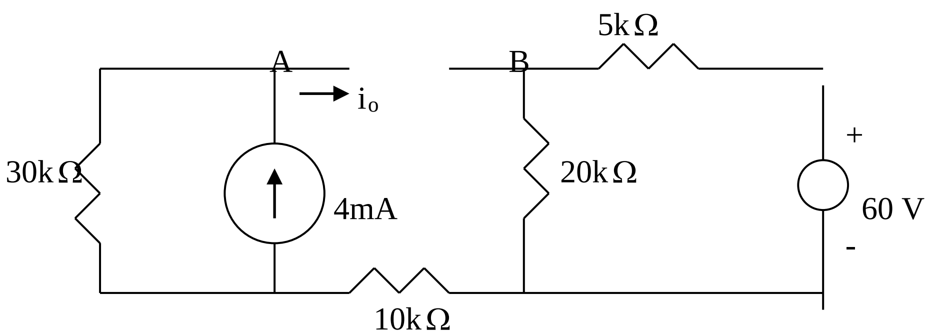
Solution 4.88

To find R_{Th}, consider the circuit below.



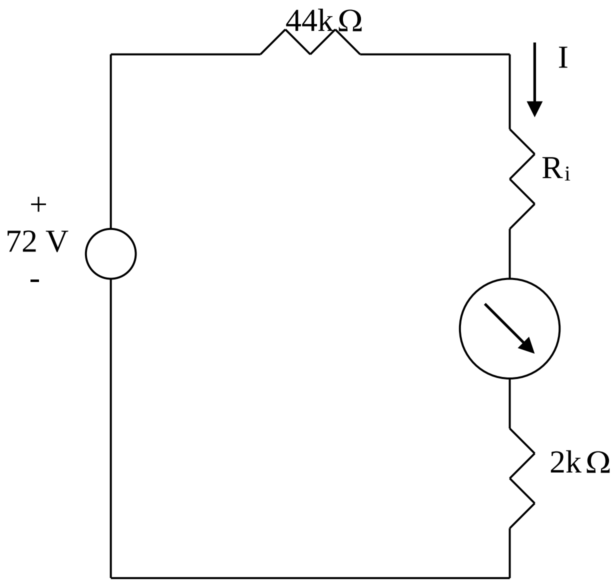
$R_{Th} = 30 + 10 + 20 // 5 = 44k\Omega$

To find V_{Th} , consider the circuit below.



$$V_A = 30x4 = 120, \qquad V_B = \frac{20}{25}(60) = 48, \qquad V_{Th} = V_A - V_B = 72 \text{ V}$$

The Thevenin equivalent circuit is shown below.



$$I = \frac{72}{44 + 2 + R_i} \text{ mA}$$

assuming R_i is in k-ohm.

(a) When $R_i = 500\,\Omega$,

$$\boldsymbol{I = \frac{72}{44 + 2 + 0.5} = \underline{1.548 \text{ mA}}}$$

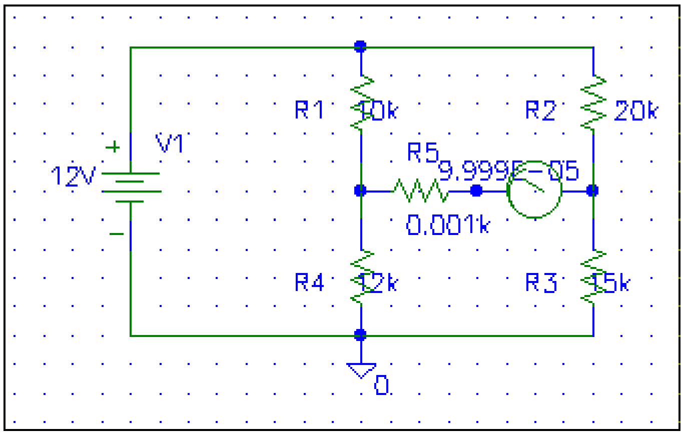
(b) When $R_i = 0\,\Omega$,

$$\boldsymbol{I = \frac{72}{44 + 2 + 0} = \underline{1.565 \text{ mA}}}$$

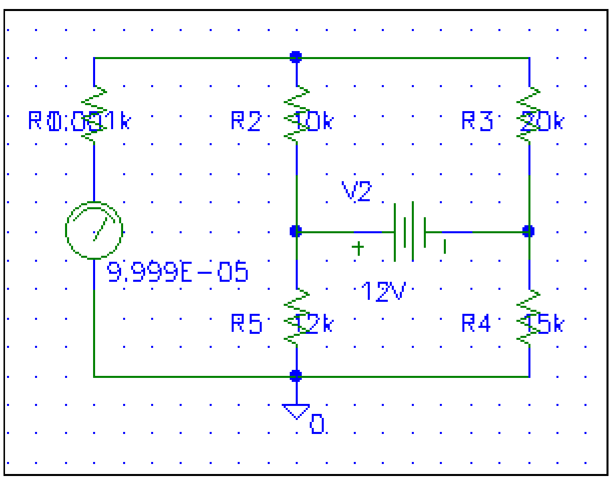
Solution 4.89

It is easy to solve this problem using Pspice.

(a) The schematic is shown below. We insert IPROBE to measure the desired ammeter reading. We insert a very small resistance in series IPROBE to avoid problem. After the circuit is saved and simulated, the current is displaced on IPROBE as **99.99μA** .



(b) By interchanging the ammeter and the 12-V voltage source, the schematic is shown below. We obtain exactly the same result as in part (a).



Solution 4.90

$R_x = (R_3/R_1)R_2 = (4/2)R_2 = 42.6, R_2 = 21.3$

which is $(21.3\text{ohms}/100\text{ohms})\% =$ **21.3%**

Solution 4.91

(a) In the Wheatstone bridge circuit of Fig. 4.147 select the values of $\boldsymbol{R_a}$ and $\boldsymbol{R_b}$ such that the bridge can measure $\boldsymbol{R_x}$ in the range of 0-25 Ω.
(b) Repeat for the range of 0-250 Ω.

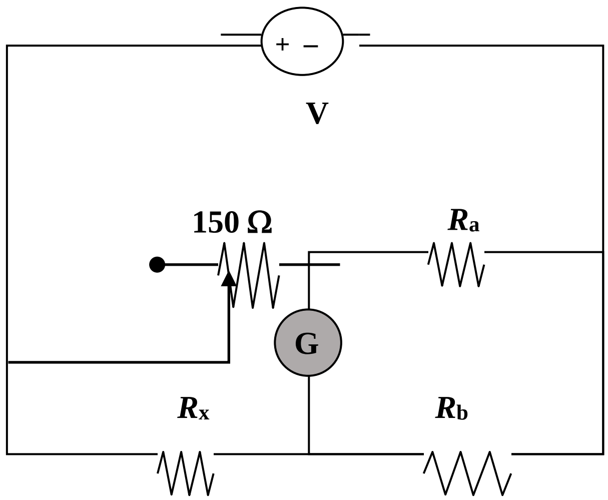


Figure 4.147
For Prob. 4.91.

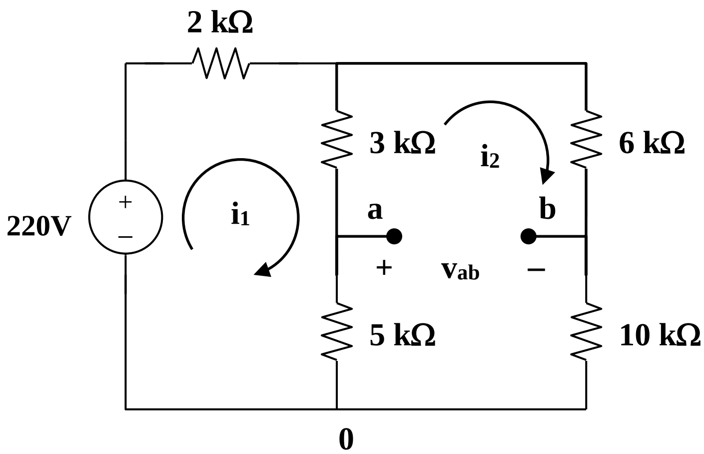
Step 1. We start from $[R_a/(k(150))] = [R_b/R_x]$ when the current through G is equal to zero where $0<k<1$. Since R_x can range from 0 to 25 Ω (a) and from 0 to 250 Ω (b), we can then calculate R_a and R_b. We will use the limits to do this, in other words when $R_x = 25\ \Omega$ then $k = 1$ and when $R_x = 250\ \Omega$ $k = 1$.

Step 2. (a) $R_a/150 = R_b/25$ gives one equation and two unknowns so we need to select one of them to solve for the other one. A good value to choose is $R_b =$ **25 Ω** which leads to $R_a =$ **150 Ω**.

(b) Like (a), we can choose any value we wish but a good value might be $R_b =$ **250 Ω** which leads to $R_a =$ **150 Ω**.

Solution 4.92

For a balanced bridge, $v_{ab} = 0$. We can use mesh analysis to find v_{ab}. Consider the circuit in Fig. (a), where i_1 and i_2 are assumed to be in mA.



(a)

$$220 = 2i_1 + 8(i_1 - i_2) \quad \text{or} \quad 220 = 10i_1 - 8i_2 \quad (1)$$

$$0 = 24i_2 - 8i_1 \quad \text{or} \quad i_2 = (1/3)i_1 \quad (2)$$

From (1) and (2),

$$i_1 = 30 \text{ mA and } i_2 = 10 \text{ mA}$$

Applying KVL to loop 0ab0 gives

$$5(i_2 - i_1) + v_{ab} + 10i_2 = 0 \text{ V}$$

Since $v_{ab} = 0$, the bridge is balanced.

When the 10 k ohm resistor is replaced by the 18 k ohm resistor, the bridge becomes unbalanced. (1) remains the same but (2) becomes

$$0 = 32i_2 - 8i_1, \quad \text{or} \quad i_2 = (1/4)i_1 \quad (3)$$

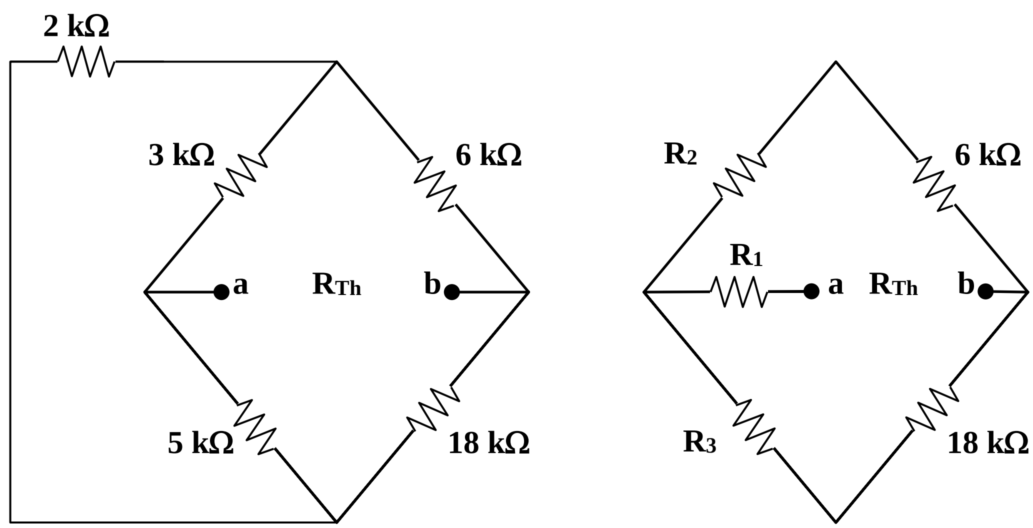
Solving (1) and (3),

$$i_1 = 27.5 \text{ mA}, \quad i_2 = 6.875 \text{ mA}$$

$$v_{ab} = 5(i_1 - i_2) - 18i_2 = -20.625 \text{ V}$$

$$V_{Th} = v_{ab} = -20.625 \text{ V}$$

To obtain R_{Th}, we convert the delta connection in Fig. (b) to a wye connection shown in Fig. (c).



(b) (c)

$$R_1 = 3x5/(2 + 3 + 5) = 1.5 \text{ k ohms}, \quad R_2 = 2x3/10 = 600 \text{ ohms},$$

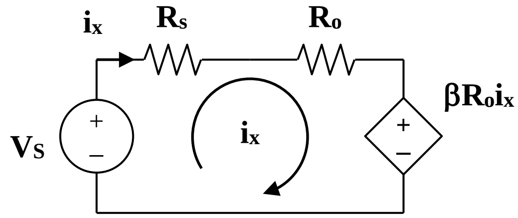
$$R_3 = 2x5/10 = 1 \text{ k ohm.}$$

$$R_{Th} = R_1 + (R_2 + 6)||(R_3 + 18) = 1.5 + 6.6||9 = 6.398 \text{ k ohms}$$

$$R_L = R_{Th} = \mathbf{6.398\ k\ ohms}$$

$$P_{max} = (V_{Th})^2/(4R_{Th}) = (20.625)^2/(4x6.398) = \mathbf{16.622\ mWatts}$$

Solution 4.93



$$-V_s + (R_s + R_o)i_x + \beta R_o i_x = 0$$

$$i_x = \mathbf{V_s/(R_s + (1 + \beta)R_o)}$$

Solution 4.94

(a) $$V_o/V_g = R_p/(R_g + R_s + R_p) \quad (1)$$

$$R_{eq} = R_p \| (R_g + R_s) = R_g$$

$$R_g = R_p(R_g + R_s)/(R_p + R_g + R_s)$$

$$R_gR_p + R_g^2 + R_gR_s = R_pR_g + R_pR_s$$

$$R_pR_s = R_g(R_g + R_s) \quad (2)$$

From (1), $$R_p/\alpha = R_g + R_s + R_p$$

$$R_g + R_s = R_p((1/\alpha) - 1) = R_p(1 - \alpha)/\alpha \quad (1a)$$

Combining (2) and (1a) gives,

$$R_s = [(1 - \alpha)/\alpha]R_{eq} \quad (3)$$

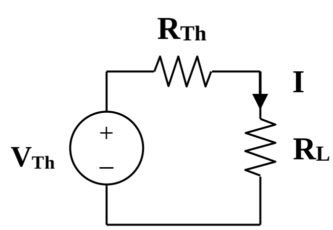
$$= (1 - 0.125)(100)/0.125 = \mathbf{700\ ohms}$$

From (3) and (1a),

$$R_p(1 - \alpha)/\alpha = R_g + [(1 - \alpha)/\alpha]R_g = R_g/\alpha$$

$$R_p = R_g/(1 - \alpha) = 100/(1 - 0.125) = \mathbf{114.29\ ohms}$$

(b)



$$V_{Th} = V_s = 0.125V_g = 1.5\ \text{V}$$

$$R_{Th} = R_g = 100\ \text{ohms}$$

$$I = V_{Th}/(R_{Th} + R_L) = 1.5/150 = \mathbf{10\ mA}$$

Solution 4.95

A dc voltmeter with a sensitivity of 10 kΩ/V is used to find the Thevenin equivalent of a linear network. Readings on two scales are as follows:
(a) 0-10 V scale: 8 V
(b) 0-50 V scale: 10 V

Obtain the Thevenin voltage and the Thevenin resistance of the network.

Solution

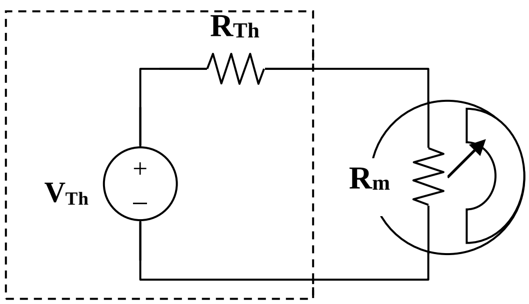
Let 1/sensitivity = 1/(10 k ohms/volt) = 100 μA

For the 0 – 10 V scale,

$$R_m = V_{fs}/I_{fs} = 10 \text{ V}/100 \text{ μA} = 100 \text{ k}\Omega$$

For the 0 – 50 V scale,

$$R_m = 50 \text{ V}/100 \text{ μA} = 500 \text{ k}\Omega$$



$$V_{Th} = I(R_{Th} + R_m)$$

(a) A 8 V reading corresponds to

$$I = (8/10)I_{fs} = 0.8\text{x}100 \text{ μA} = 80 \text{ μA}$$

$$V_{Th} = (80 \text{ μA})\, R_{Th} + (80 \text{ μA})\, 100 \text{ k}\Omega$$

$$= 8 + (80 \text{ μA})\, R_{Th} \qquad (1)$$

(b) A 10 V reading corresponds to

$$I = (10/50)I_{fs} = 0.2 \text{ x } 100 \text{ μA} = 20 \text{ μA}$$

$$V_{Th} = (20 \text{ μA})\, R_{Th} + (20 \text{ μA})\, 500 \text{ k}\Omega$$

$$V_{Th} = 10 + (20 \text{ μA})\, R_{Th} \qquad (2)$$

From (1) – (2)

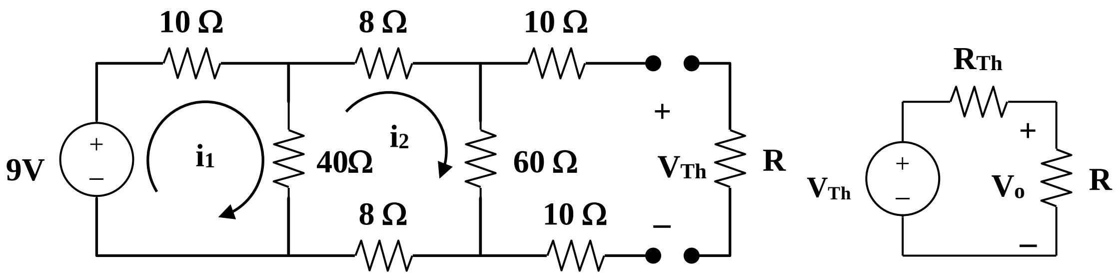
$$0 = -2 + (60\ \mu A)\, R_{Th} \quad \text{which leads to} \quad R_{Th} = \mathbf{33.33\ k\Omega}$$

From (1),

$$V_{Th} = 8 + (80x10^{-6}x33.333x10^{3}) = \mathbf{10.667\ V}$$

Solution 4.96

(a) The resistance network can be redrawn as shown in Fig. (a),



(a) (b)

$$R_{Th} = 10 + 10 + [60\|(8 + 8 + [10\|40])] = 20 + (60\|24) = 37.14 \text{ ohms}$$

Using mesh analysis,

$$-9 + 50i_1 - 40i_2 = 0 \qquad (1)$$
$$116i_2 - 40i_1 = 0 \text{ or } i_1 = 2.9i_2 \qquad (2)$$

From (1) and (2), $i_2 = 9/105 = 0.08571$

$$V_{Th} = 60i_2 = 5.143 \text{ V}$$

From Fig. (b),

$$V_o = [R/(R + R_{Th})]V_{Th} = 1.8 \text{ V}$$

$$R/(R + 37.14) = 1.8/5.143 = 0.35 \text{ or } R = 0.35R + 13 \text{ or } R = (13)/(1–0.35)$$

which leads to $R = \mathbf{20\ \Omega}$ (note, this is just for the $V_o = 1.8$ V)

(b) Asking for the value of R for maximum power would lead to $R = R_{Th} = \mathbf{37.14\ \Omega}$.

However, the problem asks for the value of R for maximum current. This happens when the value of resistance seen by the source is a minimum thus R = 0 is the correct value.

$$I_{max} = V_{Th}/(R_{Th}) = 5.143/(37.14) = \mathbf{138.48\ mA}.$$

Solution 4.97

A common-emitter amplifier circuit is shown in Fig. 4.152. Obtain the Thevenin equivalent to the left of points ***B*** and ***E***.

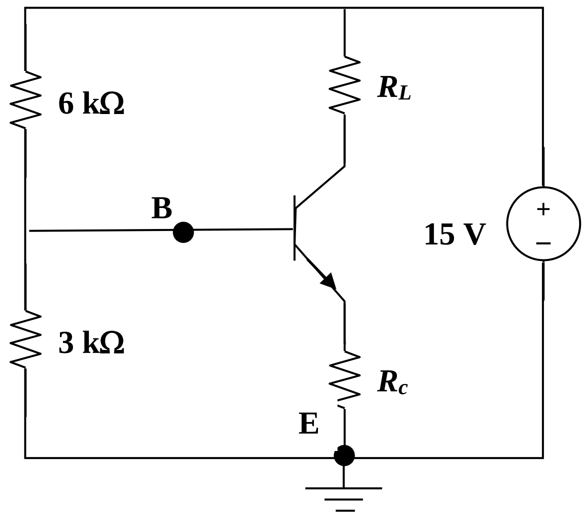
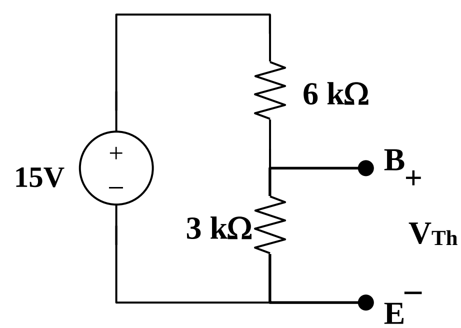


Figure 4.152
For Prob. 4.97.

Solution

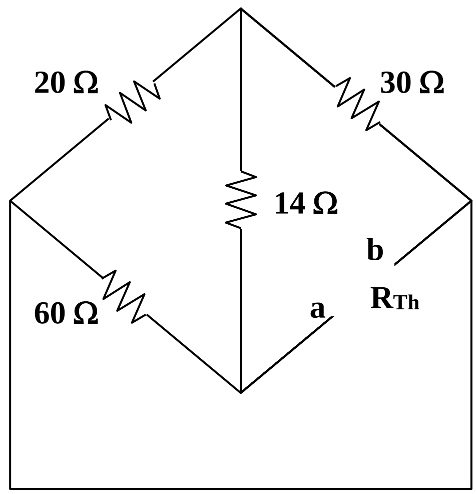


$$R_{Th} = R_1 \| R_2 = 6\|3 = \mathbf{2\ k\ ohms}$$

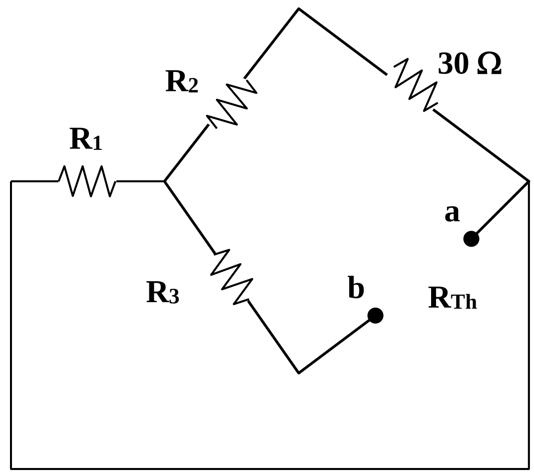
$$V_{Th} = [R_2/(R_1 + R_2)]v_s = [3/(6+3)](15) = \mathbf{5\ V}$$

Solution 4.98

The 20-ohm, 60-ohm, and 14-ohm resistors form a delta connection which needs to be connected to the wye connection as shown in Fig. (b),



(a)



(b)

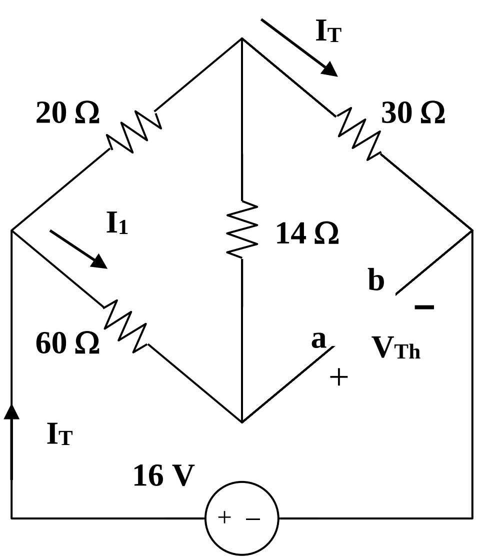
$$R_1 = 20x60/(20 + 60 + 14) = 1200/94 = 12.766 \text{ ohms}$$

$$R_2 = 20x14/94 = 2.979 \text{ ohms}$$

$$R_3 = 60x14/94 = 8.936 \text{ ohms}$$

$$R_{Th} = R_3 + R_1||(R_2 + 30) = 8.936 + 12.766||32.98 = 18.139 \text{ ohms}$$

To find V_{Th}, consider the circuit in Fig. (c).



(c)

$$I_T = 16/(30 + 15.745) = 349.8 \text{ mA}$$

$$I_1 = [20/(20 + 60 + 14)]I_T = 74.43 \text{ mA}$$

$$V_{Th} = 14I_1 + 30I_T = 11.536 \text{ V}$$

$$I_{40} = V_{Th}/(R_{Th} + 40) = 11.536/(18.139 + 40) = 198.42 \text{ mA}$$

$$P_{40} = I_{40}^2R = \mathbf{1.5748\ watts}$$