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UWA – ENSC3015 Signals and Systems

Please complete your details below:

Surname: ______________________________ Number: ____________________

Signature: _____________________________ Date: __________________

10:58am, Monday, September 4, 2017 in **Tattersall LT**

Class Test 1:
Introduction and Time-Domain Analysis

Time allowed: 45 minutes
Max mark: **48**, Assessment: **5%**[1]

This paper contains:
X pages, 6 questions

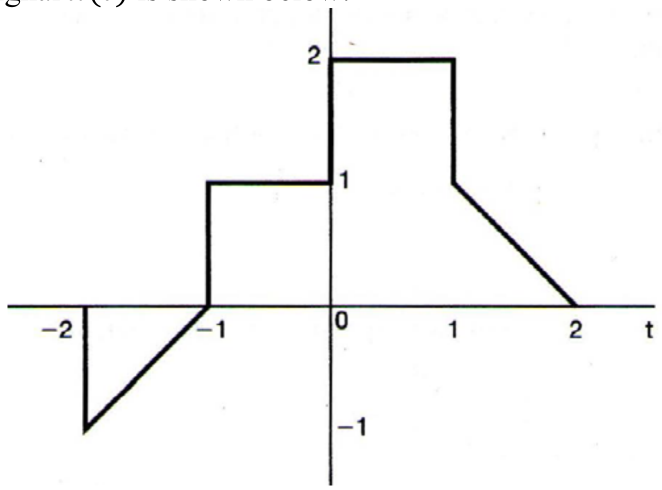
Candidates should attempt **all** questions and show **all** working with numerical answers to **3** decimal places in the spaces provided after each question, show as much working as possible to gain maximum marks. You can use the blank page on the reverse side for rough working, but these pages will not be marked

FOR THE ATTACHMENTS PLEASE REFER TO THE PAGES AT THE END

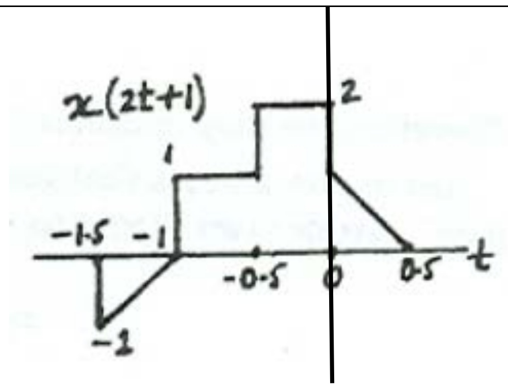
[1] If you do better in the exam this test will not contribute to your unit marks and the 5% will come from the final exam performance. However if you do better in this test compared to the final exam then this test will be included in the unit marks.

Question 1 (8 marks)

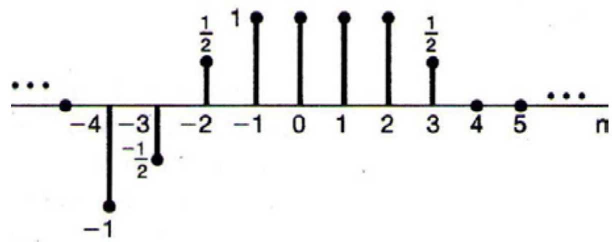
(a) A continuous-time signal $x(t)$ is shown below:



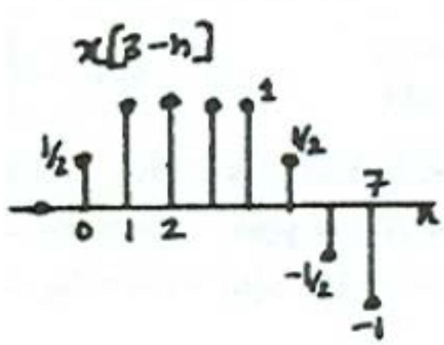
Carefully sketch the signal $x(2t + 1)$:



(b) A discrete-time signal $x[n]$ is shown below:



Carefully sketch the signal $x[3 - n]$:



Question 2 (8 marks)

(a) Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine the fundamental period

(i) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$

(ii) $x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$, where $x(t) = \cos(4\pi t)\,u(t)$

(i) is periodic with period: $4T = 2\pi \rightarrow T = \frac{\pi}{2}$

(ii) evaluates to $x_e(t) = \frac{\cos(4\pi t)}{2}$ which is periodic with period: $4\pi T = 2\pi \rightarrow T = \frac{1}{2}$

(b) Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine the fundamental period (**HINT**: For discrete-time signals the period must be an integer number of samples).

(i) $x[n] = \sin\left(\frac{6\pi}{7}n + 1\right)$

(ii) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$

(i) is periodic since $F = \frac{\Omega}{2\pi} = \frac{3}{7} = \frac{m}{N_0}$ with period $N_0 = 7$

(ii) is NOT periodic since $F = \frac{\Omega}{2\pi} = \frac{1}{16\pi}$ is not a rational number.

Question 3 (8 marks)

Determine for each of the systems below (where $x(t)$ is the input and $y(t)$ is the output) which and all of the properties that apply: Memoryless, Time invariant, Linear, Causal, Stable.

(a) $y(t) = \begin{cases} 0 & x(t) < 0 \\ x(t) + x(t-2) & x(t) \geq 0 \end{cases}$

(b) $y(t) = x(t/3)$

(c) $y[n] = nx[n]$

(d) $y[n] = x[4n+1]$

(a) Time-invariant, Causal, Stable

(b) Linear, Stable

(c) Memoryless, Linear, Causal

(d) Linear, Stable

Question 4 (8 marks)

Compute the convolution $y[n] = x[n] * h[n]$ where $x[n] = \alpha^n u[n]$ and $h[n] = \beta^n u[n]$ and $\alpha \neq \beta$.
Use $\sum_{m=0}^{M-1} \gamma^m = \frac{1-\gamma^M}{1-\gamma}$, for $\gamma \neq 1$ and simplify your expression.

$$
\begin{aligned}
y[n] = x[n] * h[n] &= \sum_{m=-\infty}^{\infty} x[m]h[n-m] \\
&= \sum_{m=-\infty}^{\infty} \alpha^m u[m]\beta^{n-m}u[n-m] \\
&= \sum_{m=0}^{n} \alpha^m \beta^{n-m} = \beta^n \sum_{m=0}^{n} (\alpha/\beta)^m \\
&= \beta^n \frac{1-(\alpha/\beta)^{n+1}}{1-(\alpha/\beta)} = \frac{\beta^n - \alpha^{n+1}/\beta}{1-(\alpha/\beta)} = \frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}, \qquad n \geq 0
\end{aligned}
$$

Question 5 (8 marks)

A Linear Time-Invariant (LTI) continuous-time system is specified by the differential equation

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) = \frac{d}{dt}x(t) + 2x(t)$$

with initial conditions $y_0(0) = \dot{y}_0(0) = 1$. Find $y_0(t)$, the zero-input response. Do NOT use the Laplace transform for this.

Characteristic equation is:

$$\lambda^2 + 3\lambda = 0 \rightarrow \lambda = 0, -3$$

Hence:

$$y_0(t) = c_1 + c_2 e^{-3t}$$
$$\dot{y}_0(t) = -3c_2 e^{-3t}$$

And:

$$y_0(0) = c_1 + c_2 = 1$$
$$\dot{y}_0(0) = -3c_2 = 1$$

Hence:

$$c_2 = -\frac{1}{3}, \quad c_1 = \frac{4}{3}$$

Finally:

$$y_0(t) = \left(\frac{4}{3} - \frac{1}{3}e^{-3t}\right)u(t)$$

Question 6 (8 marks)

For the following LTI system impulse responses determine whether the system is casual and also whether it is stable. Justify your answers.

(a) $h(t) = e^{-4t}u(t-2)$

(b) $h(t) = e^{-2t}u(t+50)$

(c) $h[n] = (0.5)^n u[-n]$

(d) $h[n] = (-0.5)^n u[n] + (1.01)^n u[n-1]$

(a) Causal because $h(t) = 0$ for $t < 0$; Stable because $\int_{-\infty}^{\infty} |h(t)|dt \rightarrow \int_{2}^{\infty} e^{-4t} < \infty$

(b) Not causal because $h(t) \neq 0$ for $-50 < t < 0$; Stable because $\int_{-\infty}^{\infty} |h(t)|dt \rightarrow \int_{-50}^{\infty} e^{-2t} < \infty$

(c) Not causal because $h[n] \neq 0$ for $n < 0$; Unstable $\sum_{k=-\infty}^{\infty} |h[k]| \rightarrow \sum_{k=-\infty}^{0} (0.5)^k \equiv \sum_{l=0}^{\infty} (2)^l \nless \infty$

(d) Causal because $h[n] = 0$ for $n < 0$; Unstable $\sum_{k=-\infty}^{\infty} |h[k]| \rightarrow \sum_{k=1}^{\infty} (1.01)^k + \cdots \nless \infty$

YOU CAN TEAR THIS LAST PAGE AND KEEP

Periodic Signals

For discrete signals the sinusoid is a periodic function only for certain values of Ω given by $\Omega = 2\pi\frac{m}{N_0} = 2\pi F$ where $F = \frac{m}{N_0}$

LTI System Properties

Linearity: $\mathbf{T}\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2$

Time-invariant: $\mathbf{T}\{x(t-\tau)\} = y(t-\tau)$

Impulse Response

Memoryless System:	$h(t) = K\delta(t)$	$h[n] = K\delta[n]$
Causal System:	$h(t) = 0 \quad t < 0$	$h[n] = 0 \quad n < 0$
BIBO Stable System:	$\int_{-\infty}^{\infty} \lvert h(\tau)\rvert d\tau < \infty$	$\sum_{k=-\infty}^{\infty} \lvert h[k]\rvert < \infty$

Convolution Integral / Sum

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau$$

$$x_1[n] * x_2[n] = \sum_{m=-\infty}^{\infty} x_1[m]x_2[n-m]$$

Zero-input response for real roots of a continuous-time LTI system

Assume *r* of the *N* roots are real but identical, $(=\lambda)$ with the remaining $(N-r)$ roots being distinct:

$$y_0(t) = (c_1 + c_2 t + \cdots + c_r t^{r-1})e^{\lambda t} + c_{r+1}e^{\lambda_{r+1}t} + c_{r+2}e^{\lambda_{r+2}t} + \cdots + c_N e^{\lambda_N t}$$

Zero-input response for real roots of a discrete-time LTI system

Assume *r* of the *N* roots are real but identical $(=\lambda)$, with the remaining $(N-r)$ roots being distinct:

$$y_0[n] = (c_1 + c_2 n + \cdots + c_r n^{r-1})\lambda^n + \sum_{k=r+1}^{N} c_k \lambda_k^n$$