Solution 14.1

Find the transfer function $\mathbf{I}_o/\mathbf{I}_i$ of the *RL* circuit in Fig. 14.68. Express the transfer function using $\omega_o = R/L$.

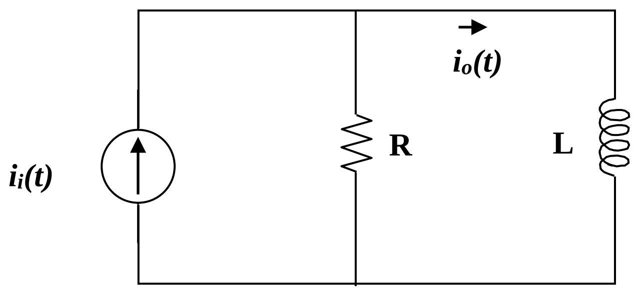


Figure 14.68
For Prob. 14.1.

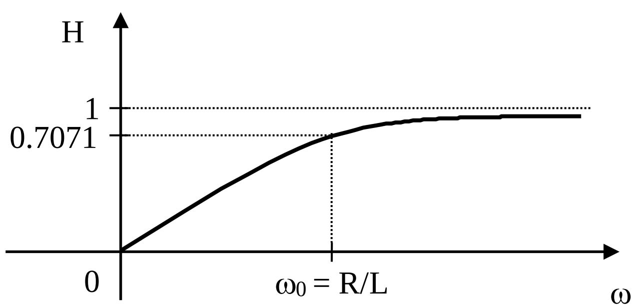
Solution

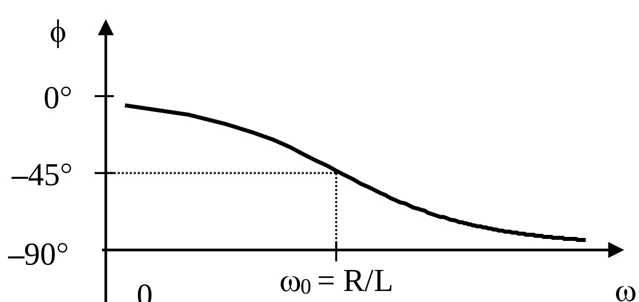
$\mathbf{H}(\omega) = \mathbf{I}_o/\mathbf{I}_i = [Rj\omega L/(R+j\omega L)]/(j\omega L) = 1/(1+j\omega L/R)$

If we let $\omega_o = R/L$ we get $\mathbf{H}(\omega) = 1/(1+j\omega/\omega_o)$.

$H = |\mathbf{H}(\omega)| = \dfrac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_o}\right)^2}}$ and $\theta = \angle\mathbf{H}(\omega) = -\tan^{-1}(\omega/\omega_o)$

This is a highpass filter. The frequency response is the same as that for P.P.14.1 except that $\omega_o = R/L$. Thus, the sketches of H and ϕ are shown below.





Solution 14.2

Using Fig. 14.69, design a problem to help other students to better understand how to determine transfer functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Obtain the transfer function $\mathbf{V}_o/\mathbf{V}_i$ of the circuit in Fig. 14.66.

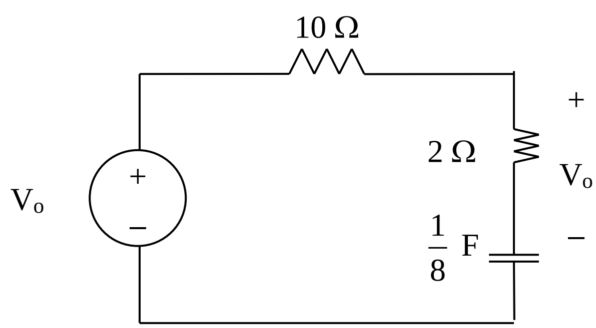


Figure 14.66 For Prob. 14.2.

Solution

$$H(s) = \frac{V_o}{V_i} = \frac{2 + \dfrac{1}{s/8}}{10 + 20 + \dfrac{1}{s/8}} = \frac{2 + 8/s}{12 + 8/s} = \frac{1}{6}\frac{s+4}{s+0.6667}$$

Solution 14.3

For the circuit shown in Fig. 14.67, find $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{I}_i(s)$.

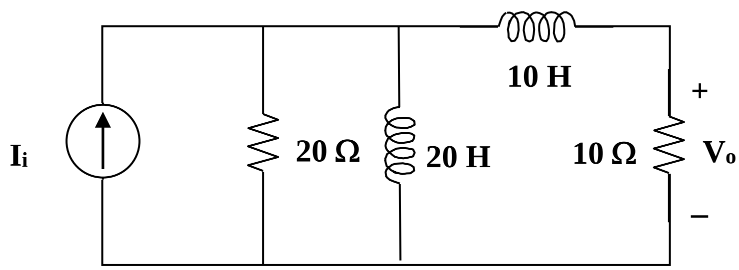
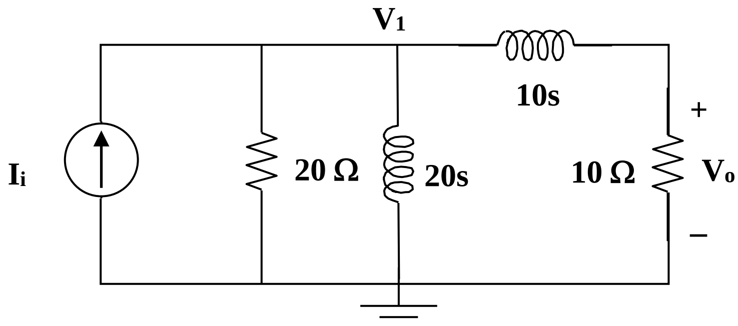


Figure 14.67
For Prob. 14.3.

Solution

Step 1. We can transform this circuit into the s-domain where 10 H becomes 10s and 20 H becomes 20s. This leads to the following circuit,



We can use nodal analysis to solve this problem. Clearly we can write the following nodal equation.

$-\mathbf{I}_i + [(\mathbf{V}_1-0)/20] + [(\mathbf{V}_1-0)/(20s)] + [(\mathbf{V}_1-0)/(10s+10)] = 0$. Finally,
$\mathbf{V}_o = [(\mathbf{V}_1-0)/(10s+10)]10$.

Step 2. $[0.05+(0.05/s)+0.1/(s+1)]\mathbf{V}_1 = 0.05[(s^2+s+s+1+2s)/(s(s+1))]\mathbf{V}_1 = \mathbf{I}_i$ and $\mathbf{V}_1 = 20[s(s+1)/(s^2+4s+1)]\mathbf{I}_i$. Finally,

$\mathbf{V}_o = \mathbf{V}_1/(s+1) = [20s/(s^2+4s+1)]\mathbf{I}_i$ or

$$\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{I}_i(s) = \mathbf{20s/(s^2+4s+1)}.$$

Solution 14.4

Find the transfer function **H(s)** = $\mathbf{V}_o/\mathbf{V}_i$ of the circuit shown in Fig. 14.71.

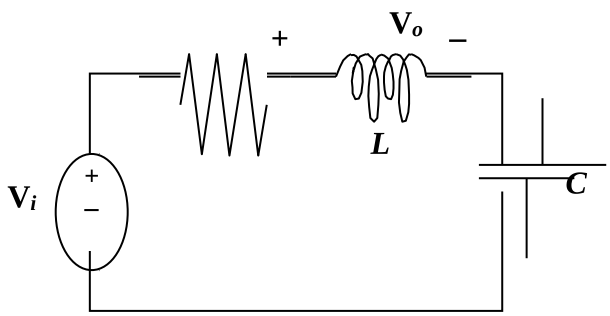
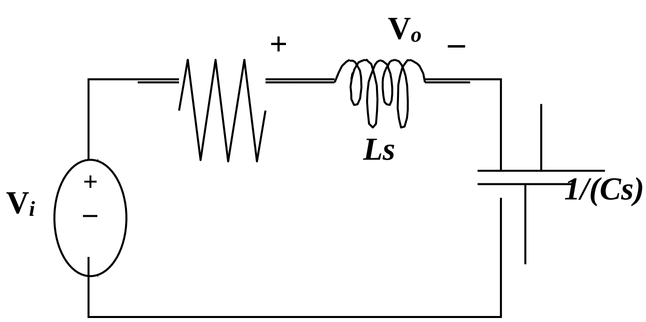


Figure 14.71
For Prob. 14.4.

Solution

Step 1. First we convert the circuit into the s-domain where the capacitor becomes 1/(Cs) and the inductor becomes Ls. Now we redraw the circuit as follows,



We can write a mesh equation, $-\mathbf{V}_i + R\mathbf{I} + Ls\mathbf{I} + [1/(Cs)]\mathbf{I} = 0$ and note that $\mathbf{V}_o = Ls\mathbf{I}$. This now leads to $\mathbf{V}_o/\mathbf{V}_i$.

Step 2. $[R+Ls+1/(Cs)]\mathbf{I} = \mathbf{V}_i$ or $\mathbf{I} = [Cs/(CLs^2+CRs+1]\mathbf{V}_i$. Thus, $\mathbf{V}_o = Ls\mathbf{I}$ or

$$\mathbf{H(s)} = \mathbf{V}_o/\mathbf{V}_i = \mathbf{LCs^2/(CLs^2+CRs+1)}.$$

Solution 14.5

For the circuit shown in Fig. 14.72, find $\mathbf{H}(s) = \mathbf{V}_o/\mathbf{I}_s$.

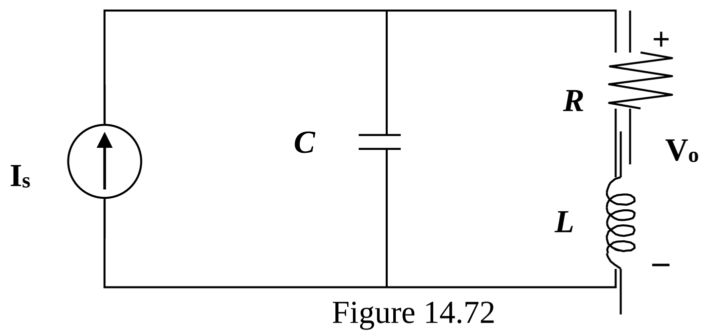
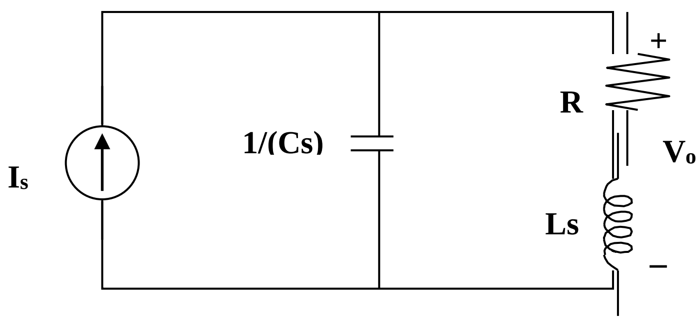


Figure 14.72
For Prob. 14.5.

Solution

Step 1. Let the capacitor be represented by 1/(Cs) and the inductor by Ls. Then convert the circuit into the s-domain.



Very simply by current division we can represent $\mathbf{V}_o$ as,
$\mathbf{I}_o = [1/(Cs)][\mathbf{I}_s/((1/(Cs))+R+Ls)]$ which leads to $\mathbf{V}_o = (Ls+R)\mathbf{I}_o$.

Step 2. $\mathbf{I}_o = \mathbf{I}_s/[1+RCs+LCs^2]$ or $\mathbf{V}_o = (Ls+R)\mathbf{I}_s/(LCs^2+RCs+1)$ or

$$\mathbf{H}(s) = \mathbf{V}_o/\mathbf{I}_s = (Ls+R)/(LCs^2+RCs+1).$$

Solution 14.6

For the circuit in Fig. 14.73, find $\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_s(s)$.

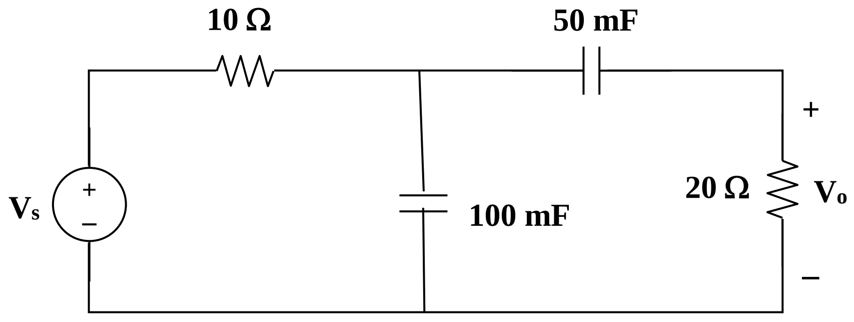
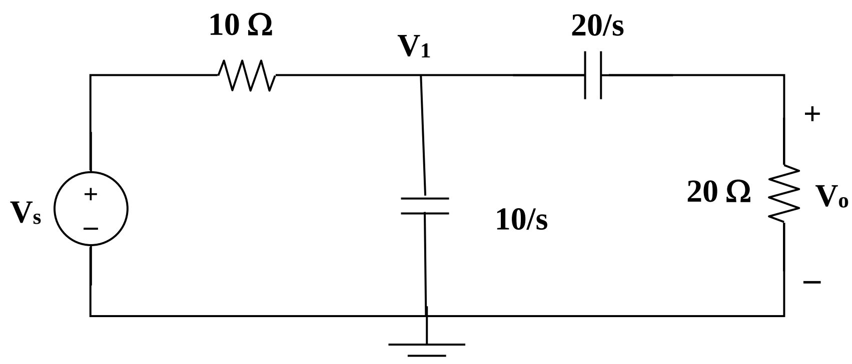


Figure 14.73
For Prob. 14.6.

Solution

Step 1. The 50 mF capacitor becomes 20/s and the 100 mF capacitor becomes 10/s. Now we convert the circuit into the s-domain and using nodal analysis we can solve for V_o.



$[(\mathbf{V_1}–\mathbf{V_s})/10] + [(\mathbf{V_1}–0)/(10/s)] + [(\mathbf{V_1}–0)/(20+20/s)] = 0$ and
$\mathbf{V_o} = \mathbf{V_1}/(20+20/s)$.

Step 2. $[(0.1+0.1s+0.05s/((s+1)]\mathbf{V_1} = 0.1\mathbf{V_s}$ or$[(s+1+s^2+s+0.5s)/(s+1)]\mathbf{V_1} = \mathbf{V_s}$ or $\mathbf{V_1} =[(s+1)/(s^2+2.5s+1)]V_s$ which then gives us
$\mathbf{V_o} = [(s+1)/(s^2+2.5s+1)][0.05s/(s+1)]\mathbf{V_s}$.

$$\mathbf{H}(s) = \mathbf{V}_o(s)/\mathbf{V}_s(s) = \mathbf{0.05s/(s^2+2.5s+1)}.$$

Solution 14.7

Calculate |**H**| if H_{dB} equals

(a) 0.1 dB
(b) –5 dB
(c) 215 dB

Solution

(a) 0.1 dB = $20\log_{10}|\mathbf{H}|$, thus, $|\mathbf{H}|$ = **1.0116**

(b) –5 dB = $20\log_{10}|\mathbf{H}|$, thus, $|\mathbf{H}|$ = **0.5623**

(c) 215 dB = $20\log_{10}|\mathbf{H}|$, thus, $|\mathbf{H}|$ = $\mathbf{5.623 x 10^{10}}$

Solution 14.8

Design a problem to help other students to better calculate the magnitude in dB and phase in degrees of a variety of transfer functions at a single value of ω.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the magnitude (in dB) and the phase (in degrees) of $\mathbf{H}(\omega)$ at $\omega = 1$ if $\mathbf{H}(\omega)$ equals

(a) 0.05
(b) 125
(c) $\dfrac{10\,j\omega}{2 + j\omega}$
(d) $\dfrac{3}{1 + j\omega} + \dfrac{6}{2 + j\omega}$

Solution

(a) $H = 0.05$

$H_{dB} = 20\log_{10} 0.05 = \mathbf{-26.02}$, $\quad \boldsymbol{\varphi = 0°}$

(b) $H = 125$

$H_{dB} = 20\log_{10} 125 = \mathbf{41.94}$, $\quad \boldsymbol{\varphi = 0°}$

(c) $H(1) = \dfrac{j10}{2 + j} = 4.472\angle 63.43°$

$H_{dB} = 20\log_{10} 4.472 = \mathbf{13.01}$, $\quad \boldsymbol{\varphi = 63.43°}$

(d) $H(1) = \dfrac{3}{1 + j} + \dfrac{6}{2 + j} = 3.9 - j2.7 = 4.743\angle -34.7°$

$H_{dB} = 20\log_{10} 4.743 = \mathbf{13.521}$, $\quad \boldsymbol{\varphi = -34.7°}$

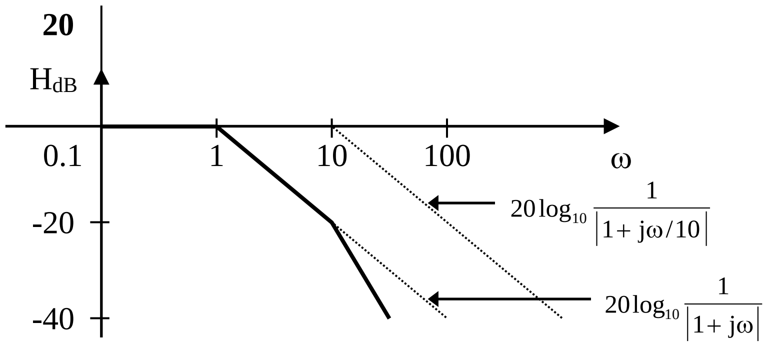
Solution 14.9

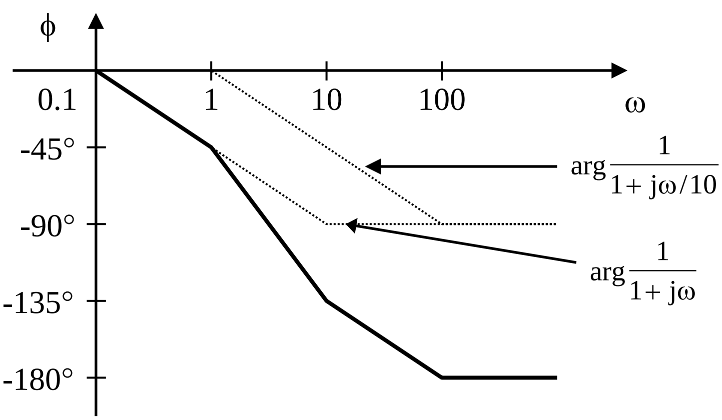
$$\mathbf{H}(\omega) = \frac{10}{10(1 + j\omega)(1 + j\omega/10)}$$

$$H_{dB} = 20\log_{10}|1| - 20\log_{10}|1 + j\omega| - 20\log_{10}|1 + j\omega/10|$$

$$\phi = -\tan^{-1}(\omega) - \tan^{-1}(\omega/10)$$

<u>The magnitude and phase plots are shown below</u>.





Solution 14.10

Design a problem to help other students to better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of jω.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

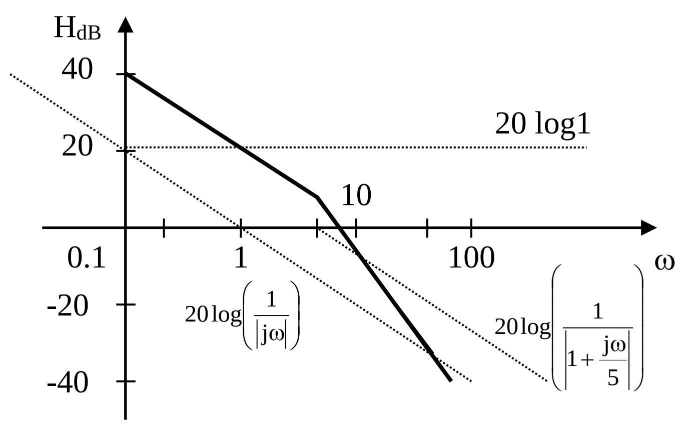
Problem

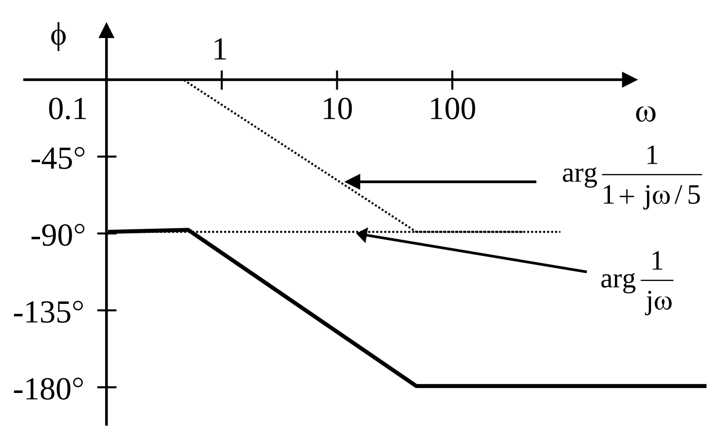
Sketch the Bode magnitude and phase plots of:

$$H(j\omega) = \frac{50}{j\omega(5 + j\omega)}$$

Solution

$$H(j\omega) = \frac{50}{j\omega(5 + j\omega)} = \frac{10}{1j\omega\left(1 + \dfrac{j\omega}{5}\right)}$$





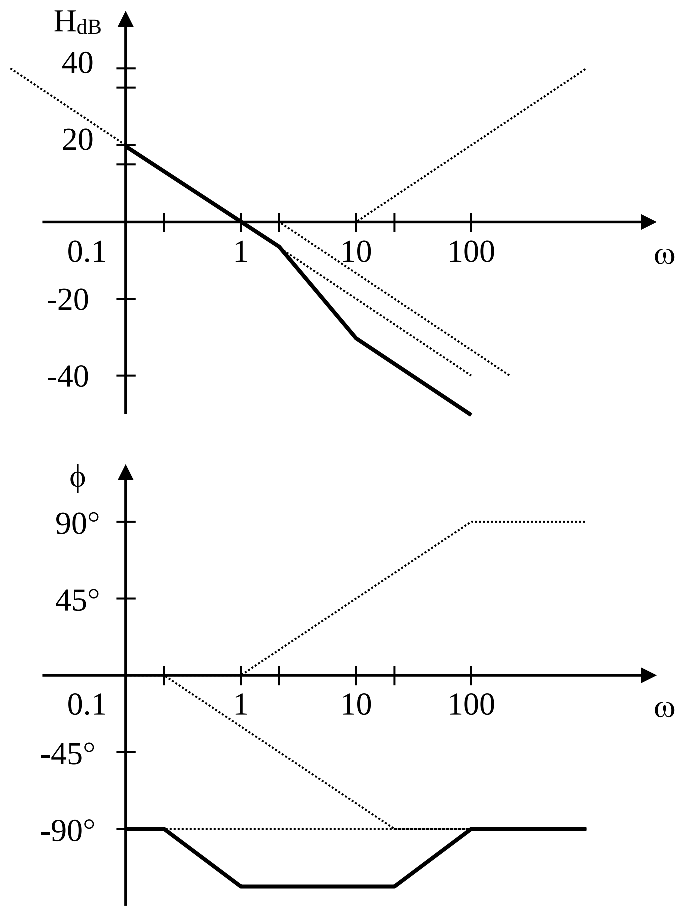
Solution 14.11

$$\mathbf{H}(\omega) = \frac{0.2x10(1 + j\omega/10)}{2[j\omega(1 + j\omega/2)]}$$

$$H_{dB} = 20\log_{10} 1 + 20\log_{10}\left|1 + j\omega/10\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1 + j\omega/2\right|$$

$$\phi = -90° + \tan^{-1}\omega/10 - \tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.

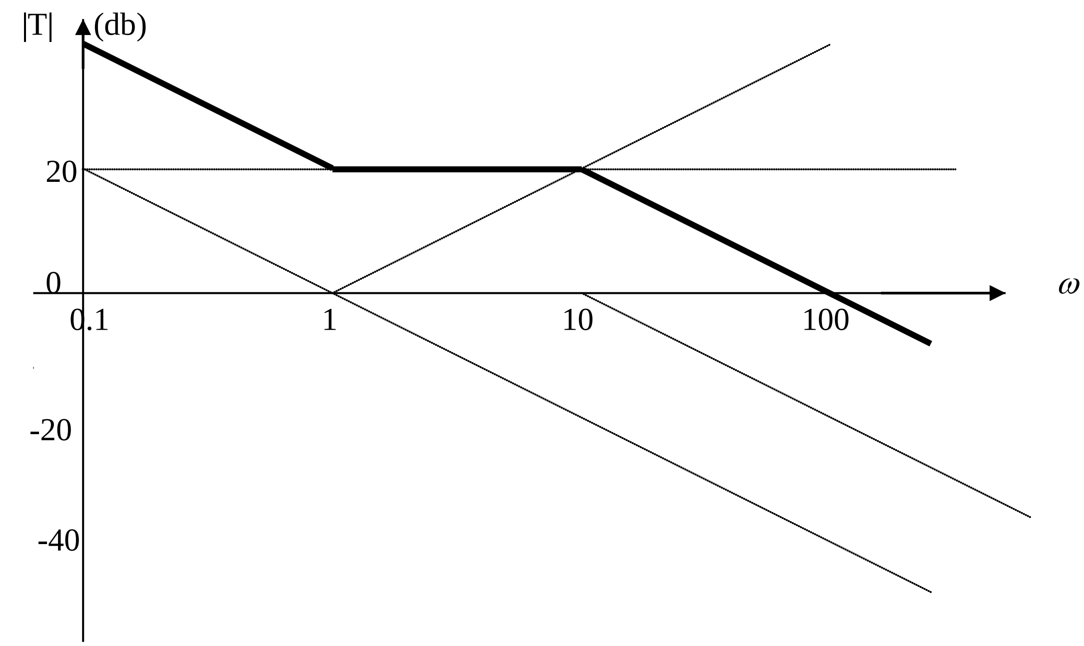


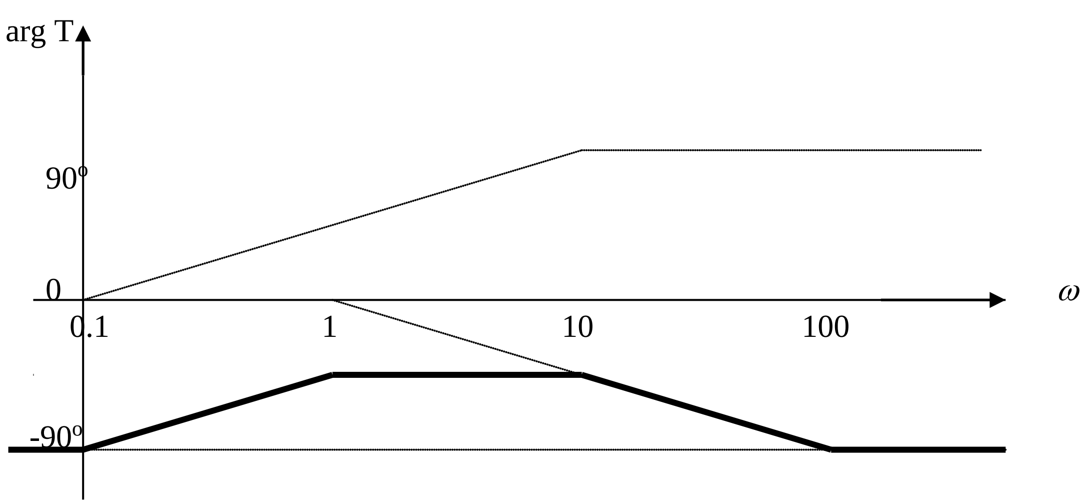
Solution 14.12

$$T(\omega) = \frac{10(1 + j\omega)}{j\omega(1 + j\omega/10)}$$

To sketch this we need $20\log_{10}|T(\omega)| = 20\log_{10}|10| + 20\log_{10}|1+j\omega| - 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10|$ and the phase is equal to $\tan^{-1}(\omega) - 90° - \tan^{-1}(\omega/10)$.

The plots are shown below.



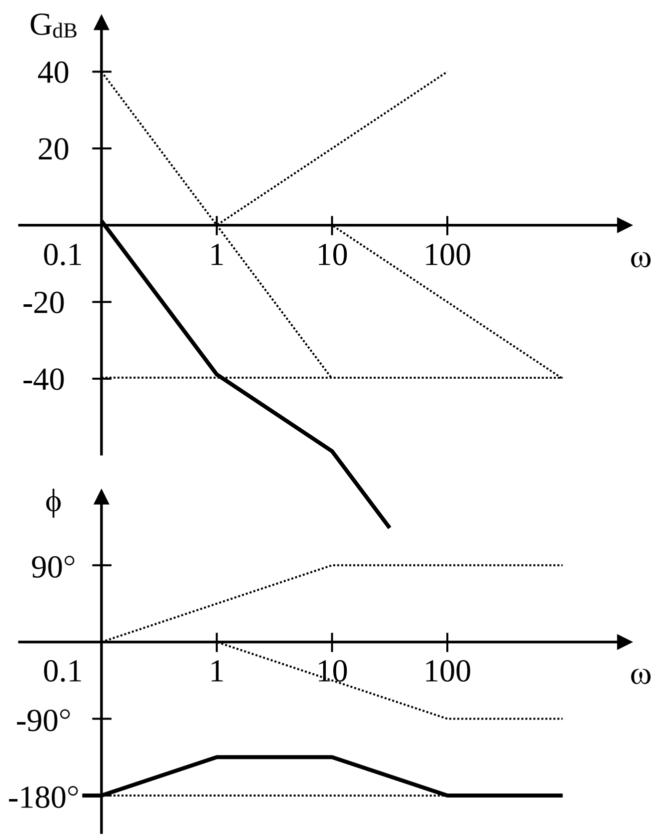


Solution 14.13

$$\mathbf{G}(\omega)=\frac{0.1(1+j\omega)}{(j\omega)^2(10+j\omega)}=\frac{(1/100)(1+j\omega)}{(j\omega)^2(1+j\omega/10)}$$

$$G_{dB}=-40+20\log_{10}\left|1+j\omega\right|-40\log_{10}\left|j\omega\right|-20\log_{10}\left|1+j\omega/10\right|$$
$$\phi=-180^\circ+\tan^{-1}\omega-\tan^{-1}\omega/10$$

The magnitude and phase plots are shown below.



Solution 14.14

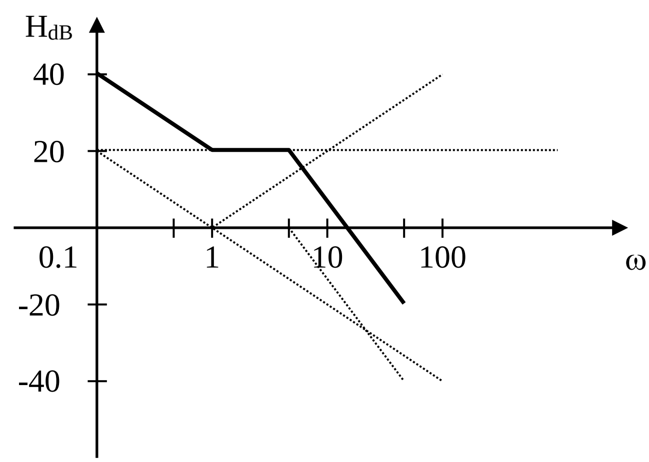
$$\mathbf{H}(\omega) = \frac{250}{25} \frac{1 + j\omega}{j\omega\left(1 + \frac{j\omega 10}{25} + \left(\frac{j\omega}{5}\right)^2\right)}$$

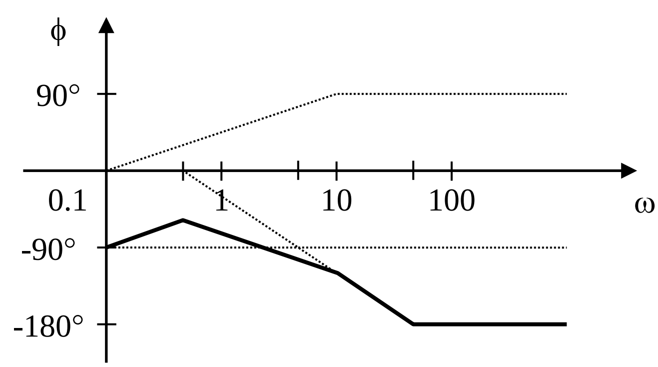
$$H_{dB} = 20\log_{10} 10 + 20\log_{10}\left|1 + j\omega\right| - 20\log_{10}\left|j\omega\right|$$

$$-20\log_{10}\left|1 + j\omega 2/5 + (j\omega/5)^2\right|$$

$$\phi = -90° + \tan^{-1}\omega - \tan^{-1}\left(\frac{\omega 10/25}{1 - \omega^2/5}\right)$$

The magnitude and phase plots are shown below.



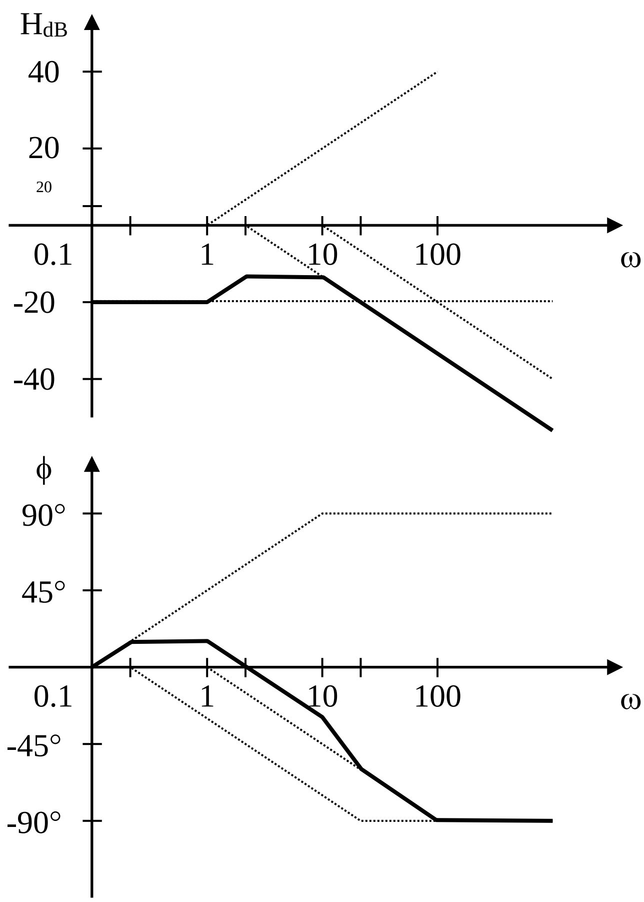


Solution 14.15

$$\mathbf{H}(\omega) = \frac{2(1+j\omega)}{(2+j\omega)(10+j\omega)} = \frac{0.1(1+j\omega)}{(1+j\omega/2)(1+j\omega/10)}$$

$$H_{dB} = 20\log_{10} 0.1 + 20\log_{10}\left|1+j\omega\right| - 20\log_{10}\left|1+j\omega/2\right| - 20\log_{10}\left|1+j\omega/10\right|$$
$$\phi = \tan^{-1}\omega - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$$

The magnitude and phase plots are shown below.

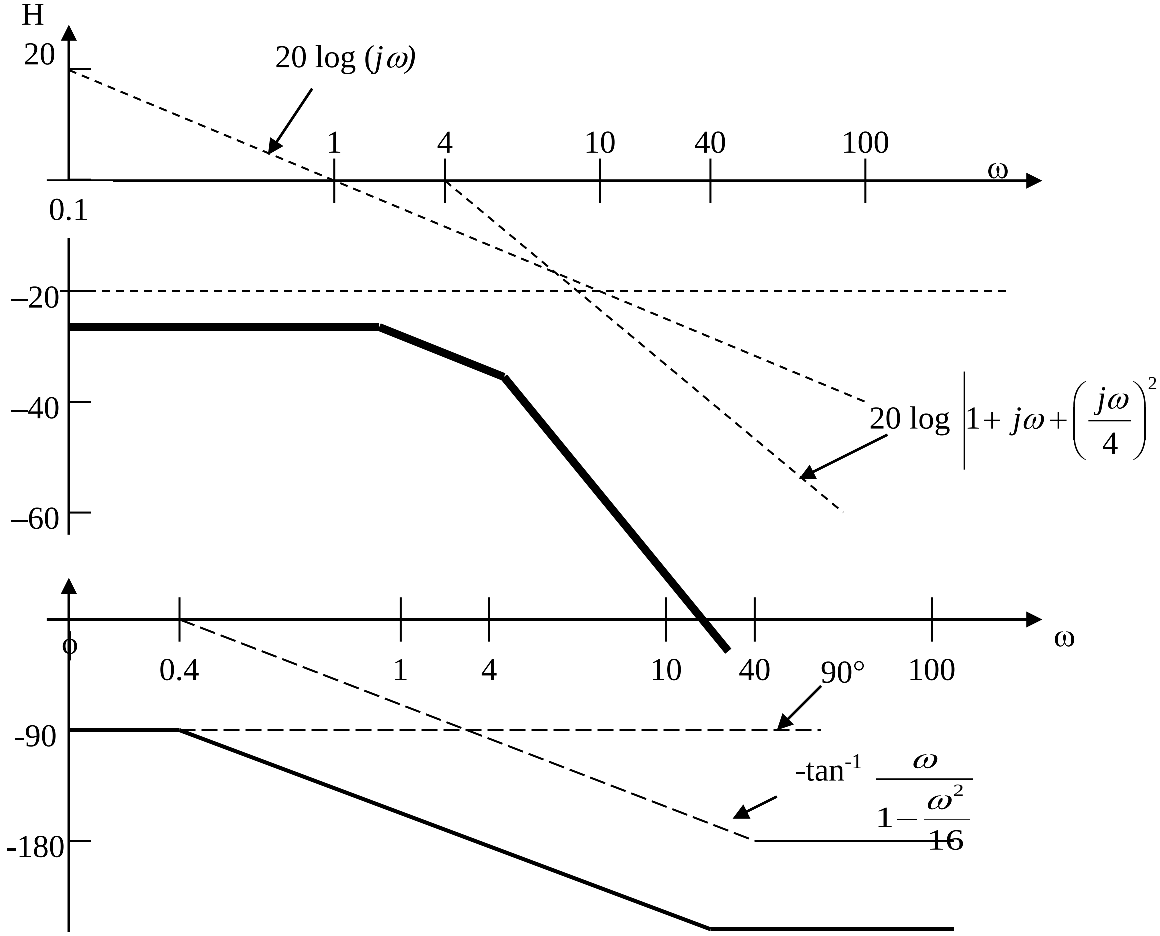


Solution 14.16

$$H(\omega) = \frac{\frac{1.6}{16}}{j\omega\left[1+j\omega+\left(\frac{j\omega}{4}\right)^2\right]} = \frac{0.1}{j\omega\left[1+j\omega+\left(\frac{j\omega}{4}\right)^2\right]}$$

$H_{db} = 20\log_{10}|0.1| - 20\text{olg}_{10}|j\omega| - 20\log_{10}|1+j\omega+(j\omega/4)^2|$

The magnitude and phase plots are shown below.



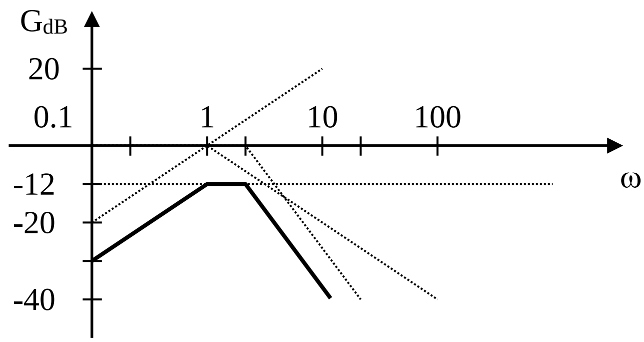
Solution 14.17

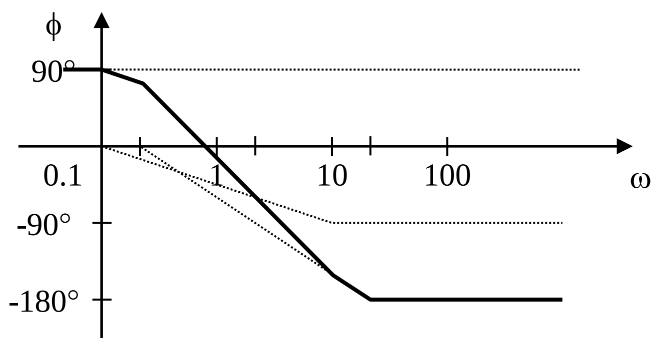
$$\mathbf{G}(\omega) = \frac{(1/4)\,j\omega}{(1+j\omega)(1+j\omega/2)^2}$$

$$G_{dB} = -20\log_{10}4 + 20\log_{10}\left|\,j\omega\right| - 20\log_{10}\left|1+j\omega\right| - 40\log_{10}\left|1+j\omega/2\,\right|$$

$$\phi = -90° - \tan^{-1}\omega - 2\tan^{-1}\omega/2$$

The magnitude and phase plots are shown below.

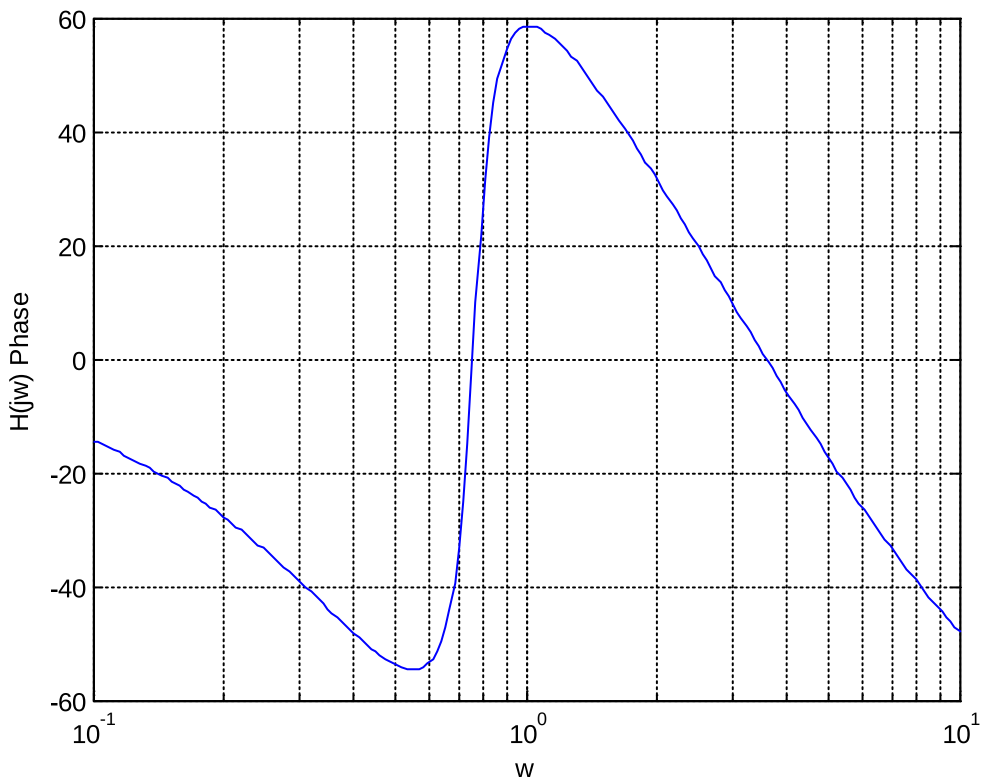




Solution 14.18

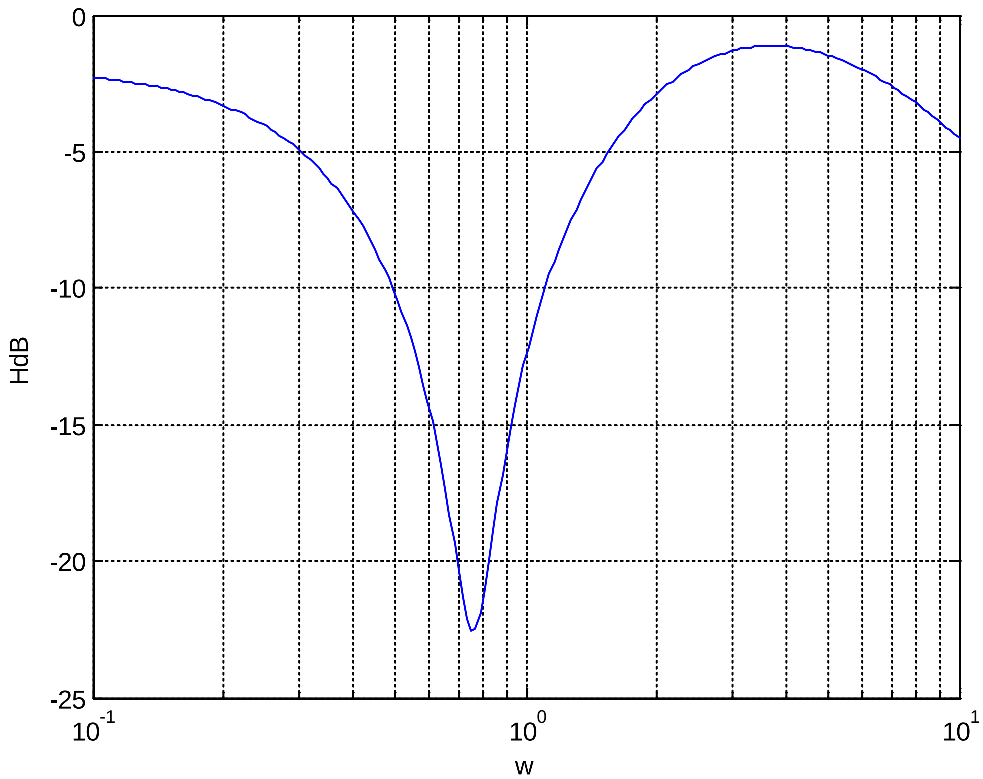
The MATLAB code is shown below.

```
>> w=logspace(-1,1,200);
>> s=i*w;
>> h=(7*s.^2+s+4)./(s.^3+8*s.^2+14*s+5);
>> Phase=unwrap(angle(h))*57.23;
>> semilogx(w,Phase)
>> grid on
```



Now for the magnitude, we need to add the following to the above,

```
>> H=abs(h);
>> HdB=20*log10(H);
>> semilogx(w,HdB);
>> grid on
```


0
-5
-10
-15
-20
-25
HdB
10^{-1}
10^{0}
10^{1}
w

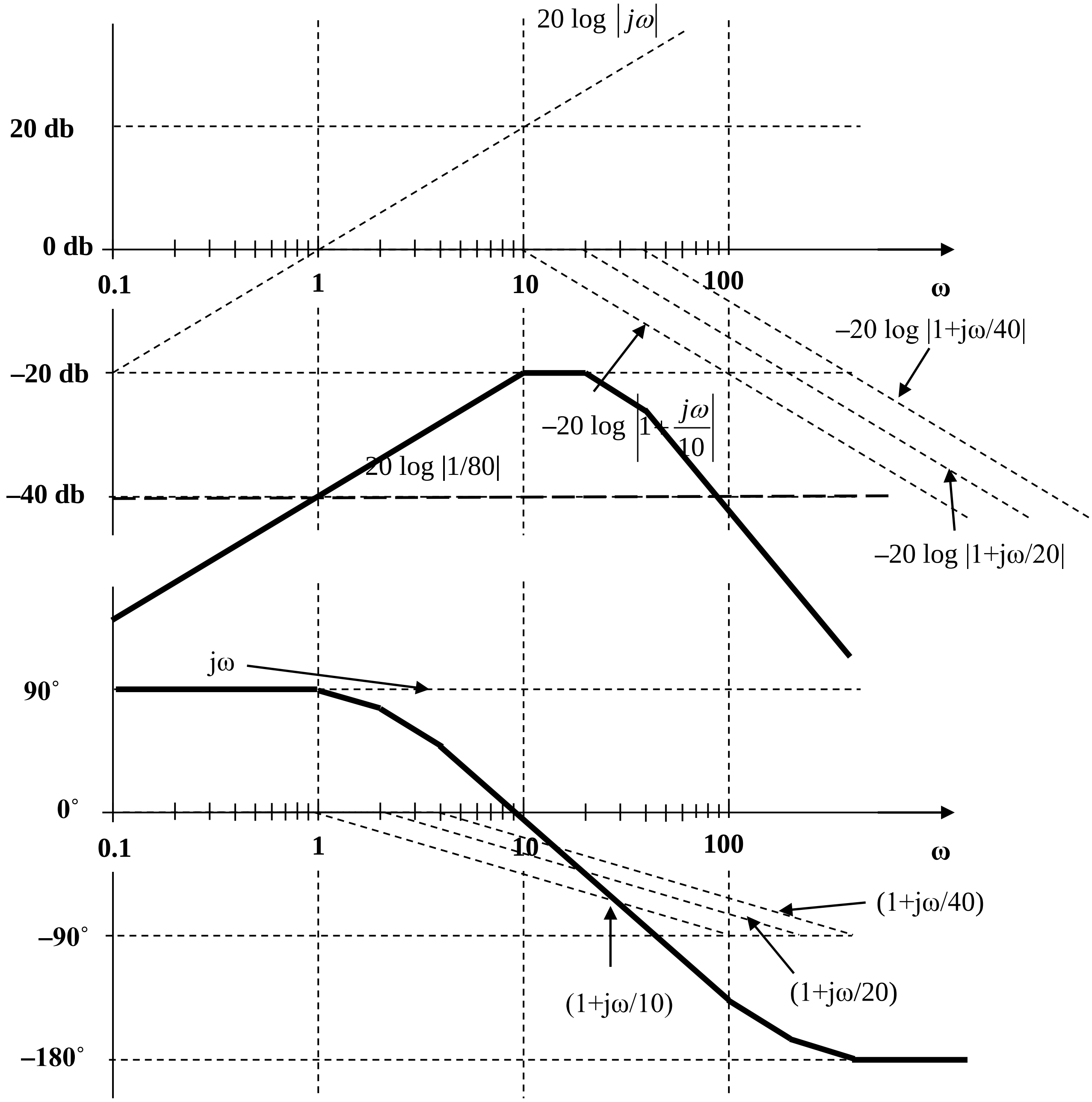
Solution 14.19

$H(\omega) = 80j\omega/[(10+j\omega)(20+j\omega)(40+j\omega)]$

$= [80/(10\text{x}20\text{x}40)](j\omega)/[(1+j\omega/10)(1+j\omega/20)(1+j\omega/40)]$

$H_{db} = 20\log_{10}|0.01| + 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/10| - 20\log_{10}|1+j\omega/20| - 20\log_{10}|1+j\omega/40|$

The magnitude and phase plots are shown below.



Solution 14.20

Design a more complex problem than given in Prob. 14.10, to help other students to better understand how to determine the Bode magnitude and phase plots of a given transfer function in terms of jω. Include at least a second order repeated root.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Sketch the magnitude phase Bode plot for the transfer function

$$H(\omega) = \frac{25j\omega}{(j\omega + 1)(j\omega + 5)^2(j\omega + 10)}$$

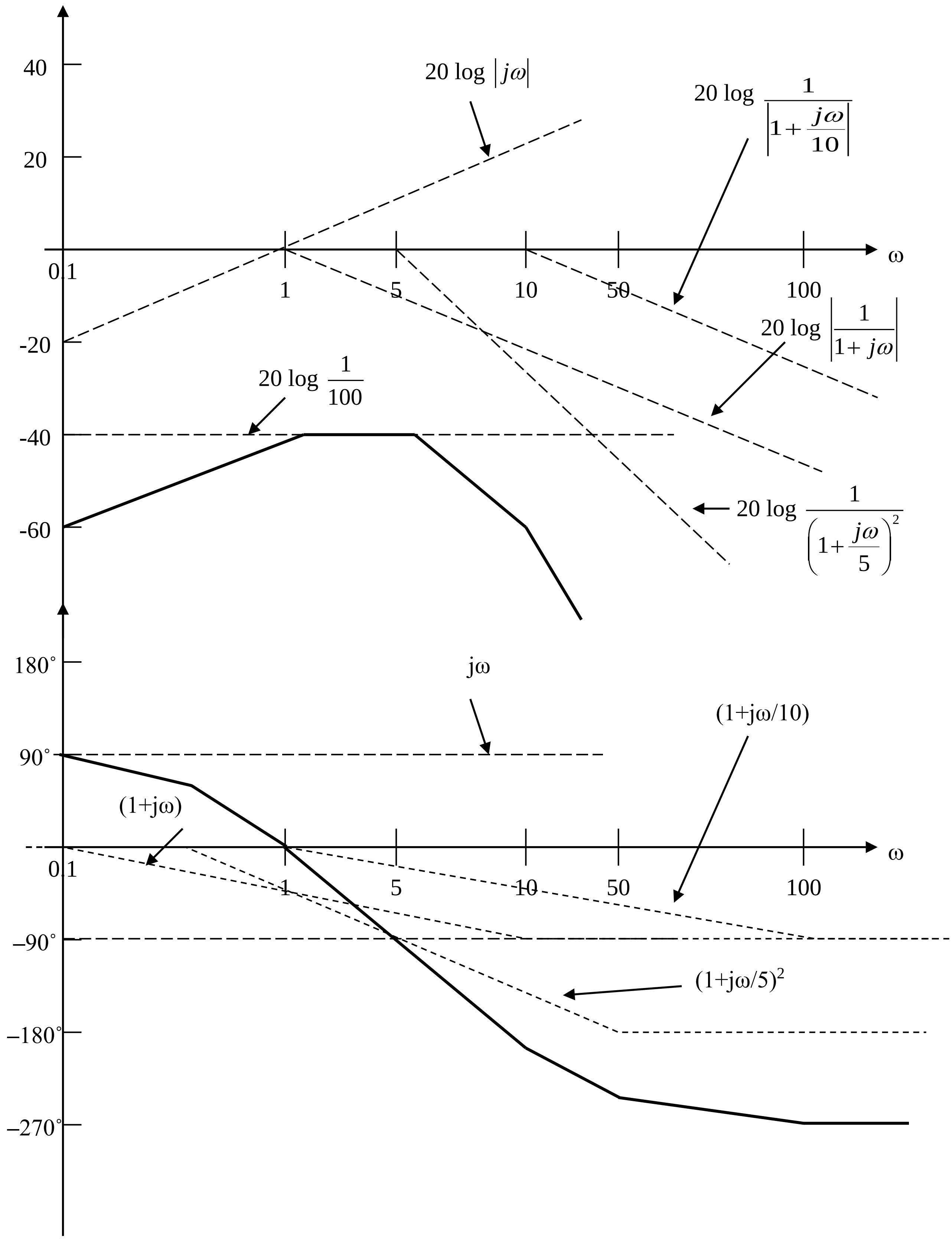
Solution

$$H(\omega) = \frac{\left(\frac{1}{100}\right)j\omega}{(1 + j\omega)\left(1 + \frac{j\omega}{5}\right)^2\left(1 + \frac{j\omega}{10}\right)}$$

20log(1/100) = -40

For the plots, see the next page.

The magnitude and phase plots are shown below.



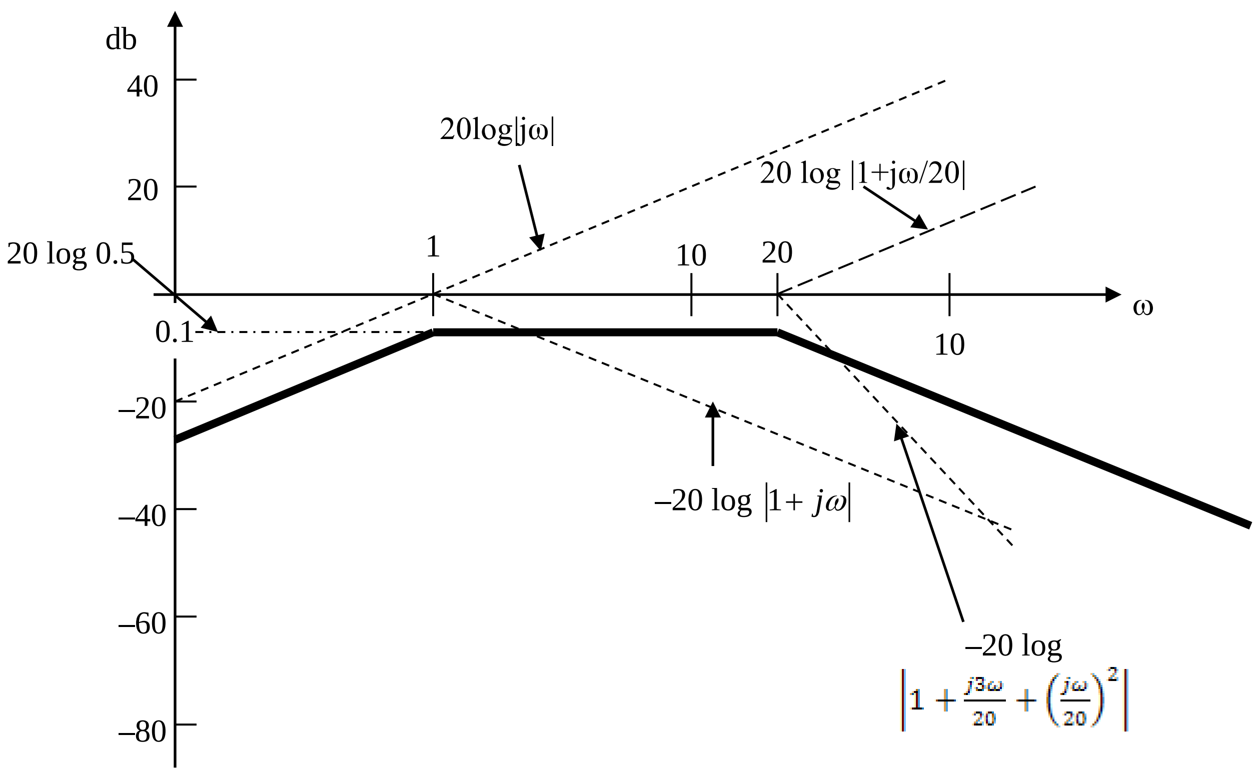
Solution 14.21

$H(\omega) = 10(j\omega)(20+j\omega)/[(1+j\omega)(400+60j\omega-\omega^2)]$

$= [10x20/400](j\omega)(1+j\omega/20)/[(1+j\omega)(1+(3j\omega/20)+(j\omega/20)^2)]$

$$H_{dB} = 20\log(0.5) + 20\log|j\omega| + 20\log\left|1+\frac{j\omega}{20}\right| - 20\log|1+j\omega| - 20\log\left|1+\frac{j3\omega}{20}+\left(\frac{j\omega}{20}\right)^2\right|$$

The magnitude plot is as sketched below. $20\log_{10}|0.5| = -6$ db



Solution 14.22

Find the transfer function $\mathbf{H}(\omega)$ with the Bode magnitude plot shown in Fig. 14.74.

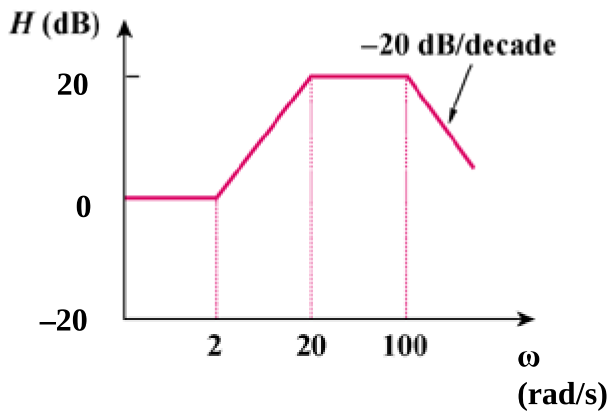


Figure 14.74
For Prob. 14.22.

Solution

$$0 = 20\log_{10} k \quad \longrightarrow \quad k = 1$$

A zero of slope $+20$ dB/dec at $\omega = 2 \quad \longrightarrow \quad 1 + j\omega/2$

A pole of slope -20 dB/dec at $\omega = 20 \quad \longrightarrow \quad \dfrac{1}{1 + j\omega/20}$

A pole of slope -20 dB/dec at $\omega = 100 \quad \longrightarrow \quad \dfrac{1}{1 + j\omega/100}$

Hence,

$$\mathbf{H}(\omega) = \frac{1(1 + j\omega/2)}{(1 + j\omega/20)(1 + j\omega/100)} = \frac{1{,}000(2 + j\omega)}{(20 + j\omega)(100 + j\omega)}$$

Solution 14.23

The Bode magnitude plot of **H**(ω) is shown in Fig. 14.75. Find **H**(ω).

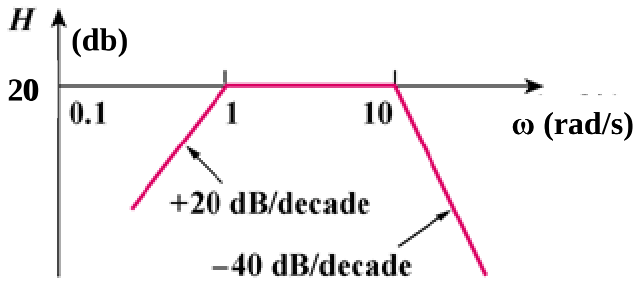


Figure 14.75
For Prob. 14.23.

Solution

The initial slope indicates we have jω in the numerator. Our approach to plotting requires the plot of jω to cross 0db at ω = 1 rad/s. Since it crosses at 20db, that indicates that the overall gain is 20db or,

$20 = 20\log_{10}|\text{gain}|$ the gain has to be 10.

A zero of slope $+20\ \text{dB/dec}$ at the origin $\longrightarrow$ $j\omega$

A pole of slope $-20\ \text{dB/dec}$ at $\omega = 1$ $\longrightarrow$ $\dfrac{1}{1 + j\omega/1}$

A pole of slope $-40\ \text{dB/dec}$ at $\omega = 10$ $\longrightarrow$ $\dfrac{1}{(1 + j\omega/10)^2}$

Hence,

$$\mathbf{H}(\omega) = \frac{10\,j\omega}{(1 + j\omega)(1 + j\omega/10)^2}$$

$$\mathbf{H}(\omega) = \frac{1{,}000\,j\omega}{(1 + j\omega)(10 + j\omega)^2}$$

(It should be noted that this function could also have a minus sign out in front and still be correct. The magnitude plot does not contain this information. It can only be obtained from the phase plot.)

Solution 14.24

The magnitude plot in Fig. 14.76 represents the transfer function of a preamplifier. Find H(s).

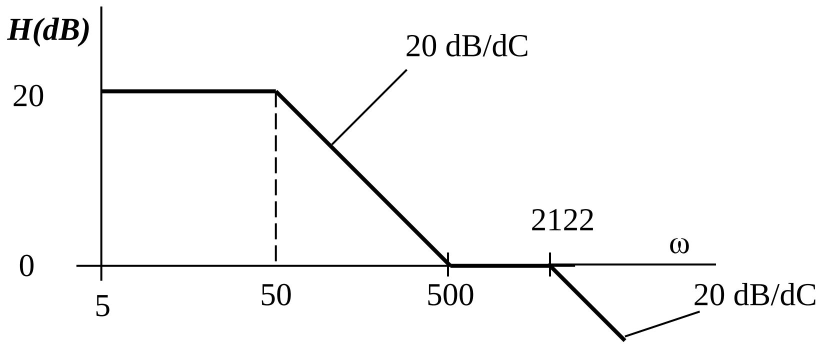


Figure 14.76
For Prob. 14.24.

Solution

$20 = 20\log_{10}|\text{gain}|$ or gain = 10.

There is a pole at $\omega=50$ giving $1/(1+j\omega/50)$
There is a zero at $\omega=500$ giving $(1 + j\omega/500)$.
There is another pole at $\omega=2122$ giving $1/(1 + j\omega/2122)$.
Thus,

$$H(j\omega) = 10(1+j\omega/500)/[(1+j\omega/50)(1+j\omega/2122)]$$

$$= [10(50\text{x}2122)/500](j\omega+500)/[(j\omega+50)(j\omega+2122)]$$

or

$$H(s) = \mathbf{2{,}122(s+500)/[(s+50)(s+2122)]}.$$

Solution 14.25

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40\times10^{-3})(1\times10^{-6})}} = 5\ \text{krad/s}$$

$$\mathbf{Z}(\omega_0) = R = \mathbf{2\ k\Omega}$$

$$\mathbf{Z}(\omega_0/4) = R + j\left(\frac{\omega_0}{4}L - \frac{4}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j\left(\frac{5\times10^3}{4}\cdot 40\times10^{-3} - \frac{4}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/4) = 2000 + j(50 - 4000/5)$$

$$\mathbf{Z}(\omega_0/4) = \mathbf{2 - j0.75\ k\Omega}$$

$$\mathbf{Z}(\omega_0/2) = R + j\left(\frac{\omega_0}{2}L - \frac{2}{\omega_0 C}\right)$$

$$\mathbf{Z}(\omega_0/2) = 2000 + j\left(\frac{(5\times10^3)}{2}(40\times10^{-3}) - \frac{2}{(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(\omega_0/2) = 200+j(100-2000/5)$$

$$\mathbf{Z}(\omega_0/2) = \mathbf{2 - j0.3\ k\Omega}$$

$$\mathbf{Z}(2\omega_0) = R + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right)$$

$$\mathbf{Z}(2\omega_0) = 2000 + j\left((2)(5\times10^3)(40\times10^{-3}) - \frac{1}{(2)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(2\omega_0) = \mathbf{2 + j0.3\ k\Omega}$$

$$\mathbf{Z}(4\omega_0) = R + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right)$$

$$\mathbf{Z}(4\omega_0) = 2000 + j\left((4)(5\times10^3)(40\times10^{-3}) - \frac{1}{(4)(5\times10^3)(1\times10^{-6})}\right)$$

$$\mathbf{Z}(4\omega_0) = \mathbf{2 + j0.75\ k\Omega}$$

Solution 14.26

Design a problem to help other students to better understand ω_o, Q, and B at resonance in series *RLC* circuits.

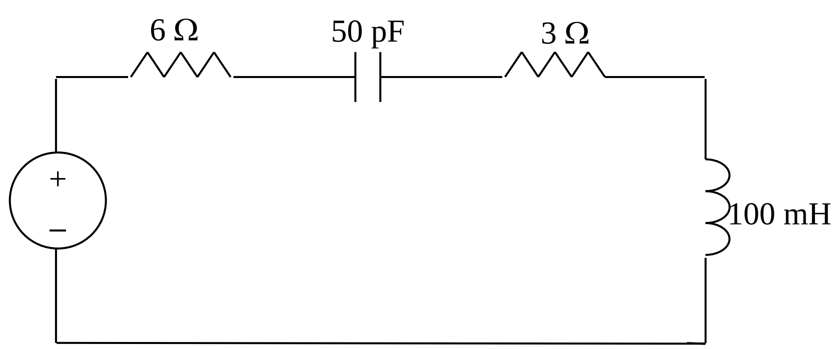
Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A coil with resistance 3 Ω and inductance 100 mH is connected in series with a capacitor of 50 pF, a resistor of 6 Ω, and a signal generator that gives 110V-rms at all frequencies. Calculate ω_o, Q, and B at resonance of the resultant series RLC circuit.

Solution

Consider the circuit as shown below. This is a series RLC resonant circuit.



R = 6 + 3 = 9 Ω

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100x10^{-3}x50x10^{-12}}} = \underline{447.21 \text{ krad/s}}$$

$$Q = \frac{\omega_o L}{R} = \frac{447.21x10^3 x100x10^3}{9} = \underline{4969}$$

$$B = \frac{\omega_o}{Q} = \frac{447.21x10^3}{4969} = \underline{90 \text{ rad/s}}$$

Solution 14.27

$$\omega_o = \frac{1}{\sqrt{LC}} = 40 \quad \longrightarrow \quad LC = \frac{1}{40^2}$$

$$B = \frac{R}{L} = 10 \quad \longrightarrow \quad R = 10L$$

If we select R =**1 Ω**, then L = R/10 = **100 mH** and

$$C = \frac{1}{40^2 L} = \frac{1}{40^2 x 0.1} = \underline{6.25 \text{ mF}}$$

Solution 14.28

Design a series *RLC* circuit with $\boldsymbol{B}$ = 20 rad/s and ω_0 = 1,000 rad/s. Find the circuit's Q. Let R = 10 Ω.

Solution

$$R = 10\,\Omega.$$

$$L = \frac{R}{B} = \frac{10}{20} = 0.5\text{ H}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(1000)^2(0.5)} = 2\ \mu\text{F}$$

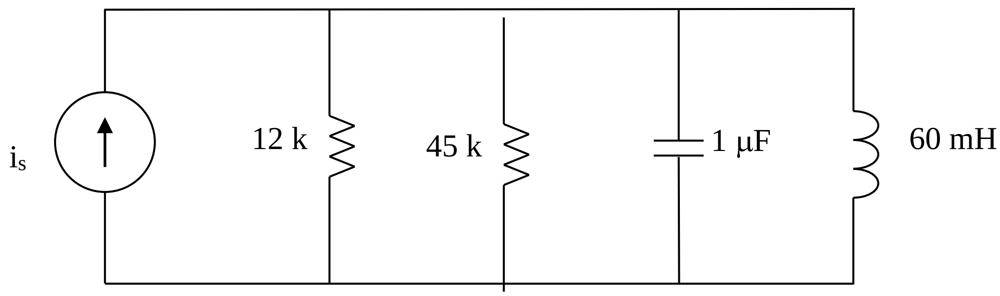
$$Q = \frac{\omega_0}{B} = \frac{1000}{20} = 50$$

Therefore, if $R = 10\,\Omega$ then

$L = \mathbf{500\ mH}$, $C = \mathbf{2\ \mu F}$, $Q = \mathbf{50}$

Solution 14.29

We convert the voltage source to a current source as shown below.



$$i_s = \frac{20}{12}\cos\omega t\,,\quad \text{R = 12//45= 12x45/57 = 9.4737 k}\Omega$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{60x10^{-3}x1x10^{-6}}} = \underline{4.082\text{ krad/s}} = \mathbf{4.082\text{ krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{9.4737x10^{3}x10^{-6}} = \underline{105.55\ \text{ rad/s}} = \mathbf{105.55\text{ rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4082}{105.55} = \underline{38.674} = \mathbf{38.67}$$

4.082 krads/s, 105.55 rad/s, 38.67

Solution 14.30

(a) $f_o = 15{,}000$ Hz leads to $\omega_o = 2\pi f_o = 94.25$ krad/s $= 1/(LC)^{0.5}$ or

$LC = 1/8.883\text{x}10^9$ or $C = 1/(8.883\text{x}10^9\text{x}10^{-2}) = 11.257\text{x}10^{-9}$ F = **11.257 pF**.

(b) since the capacitive reactance cancels out the inductive reactance at resonance, the current through the series circuit is given by

$I = 120/20 =$ **6 A**.

(c) $Q = \omega_o L/R = 94.25\text{x}10^3(0.01)/20 =$ **47.12**.

Solution 14.31

Design a parallel resonant *RLC* circuit with ω_0 = 100 krad/s and a bandwidth of 10 krad/s.
Additionally what is the value of Q?

Solution

Step 1. We note that,

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$Q = R/(\omega_o L) = \omega_o RC$$

$$B = \omega_o/Q.$$

Since this is a design problem, we need to find out where to start. Let us pick a value of L = **10 mH**. Now all we need to do is to solve for Q, R, and C and make sure we meet the design criterion.

Step 2. Q = 100/10 = **10**. Next

$$R/L = \omega_0 Q = 10^5 x10 = 10^6 \text{ and } RC = Q/\omega_0 = 10/10^5 = 10^{-4}$$

Since L = 10 mH we get $R = \omega_0 QL = 10^6 x0.01 =$ **10 kΩ**. Next we get,

$$C = 10^{-4}/R = 10^{-4}/10^4 = 10^{-8} = \mathbf{10\ nF}.$$

Solution 14.32

Design a problem to help other students to better understand the quality factor, the resonant frequency, and bandwidth of a parallel *RLC* circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

A parallel RLC circuit has the following values:

R = 60 Ω,   L = 1 mH, and C = 50 μF

Find the quality factor, the resonant frequency, and the bandwidth of the RLC circuit.

Solution

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}x50x10^{-6}}} = \underline{4.472\ \text{ krad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{60x50x10^{-6}} = \underline{333.33\ \text{ rad/s}}$$

$$Q = \frac{\omega_o}{B} = \frac{4472}{333.33} = \underline{13.42}$$

Solution 14.33

A parallel resonant circuit has a bandwidth of 40 krad/s and the half power frequencies are ω_1 = 4.98 Mrad/s and ω_2 = 5.02 Mrad/s, calculate the quality factor and resonant frequency.

Solution

Since $\omega_1 = \omega_o - B/2$ then ω_o = (4.98+0.04/2) M = **5 Mrad/s**. Since $B = \omega_o/Q$ then Q = 5 M/0.04 M = **125**.

Solution 14.34

A parallel RLC circuit has an R = 100 kΩ, L = 100 mH, and a C = 10 µF, determine the value of Q, the resonant frequency, and the bandwidth. If R = 200 kΩ, how does that effect the values of Q, resonant frequency, and the bandwidth?

Solution

Since,

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$Q = R/(\omega_o L) = \omega_o RC$
$B = \omega_o/Q$.

$\omega_o = 1/\sqrt{0.1x10^{-5}}$ = **1,000 rad/s** and $Q = \omega_o RC = 10^3x10^5x10^{-5}$ = **1,000**. Finally,

$B = \omega_o/Q = 1{,}000/1{,}000$ = **1 rad/s**.

When R = 200 kΩ the value of **ω_o does not change** since it is only dependent on L and C.

$Q = \omega_o RC = 1{,}000x2x10^5x10^{-5} = 2{,}000$ and $B = 1{,}000/2{,}000$ = **0.5**.

Solution 15.35

A parallel RLC circuit has an R = 10 kΩ, an L = 100 mH, and a resonant frequency of 200 krad/s, calculate the value of C, the value of the quality factor, and the bandwidth.

Solution

Since,

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ or } LC = 1/(\omega_o)^2$$
$$Q = R/(\omega_o L) = \omega_o RC$$
$$B = \omega_o/Q$$

we get $C = 1/(4x10^{10}x0.1) = 0.25x10^{-9}$ = **0.25 nF** and $Q = 2x10^5x10^4x0.25x10^{-9}$ = **0.5**.

Finally, B = 200k/0.5 = **400 krad/s**. Note, since the bandwidth is equal to 400 krad/s, the lower frequency must be equal to 0 Hz! Clearly the bandwidth goes from DC to 400 krad/s.

Solution 14.36

It is expected that a parallel *RLC* resonant circuit has a midband admittance of 25×10^{-3} S, quality factor of 120, and a resonant frequency of 200 krad/s. Calculate the values of *R*, *L*, and *C*. Find the bandwidth and the half-power frequencies.

Solution

At resonance,

$$\mathbf{Y} = \frac{1}{\mathrm{R}} \quad \longrightarrow \quad \mathrm{R} = \frac{1}{\mathrm{Y}} = \frac{1}{25 \times 10^{-3}} = \mathbf{40\ \Omega}$$

$$Q = \omega_0 RC \quad \longrightarrow \quad C = \frac{Q}{\omega_0 R} = \frac{120}{(200 \times 10^3)(40)} = \mathbf{15\ \mu F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \longrightarrow \quad L = \frac{1}{\omega_0^2 C} = \frac{1}{(4 \times 10^{10})(15 \times 10^{-6})} = \mathbf{1.6667\ \mu H}$$

$$B = \frac{\omega_0}{Q} = \frac{200 \times 10^3}{120} = \mathbf{1.6667\ krad/s}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 200 - 0.8333 = \mathbf{199.167\ krad/s}$$

$$\omega_2 = \omega_0 + \frac{\mathrm{B}}{2} = 200 + 0.8333 = \mathbf{200.833\ krad/s}$$

Solution 4.37

$$\omega_0 = \frac{1}{\sqrt{LC}} = 5000 \text{ rad/s}$$

$$\mathbf{Y}(\omega_0) = \frac{1}{R} \longrightarrow \mathbf{Z}(\omega_0) = R = \mathbf{2\ k\Omega}$$

$$\mathbf{Y}(\omega_0/4) = \frac{1}{R} + j\left(\frac{\omega_0}{4}C - \frac{4}{\omega_0 L}\right) = 0.5 - j18.75 \text{ mS}$$

$$\mathbf{Z}(\omega_0/4) = \frac{1}{0.0005 - j0.01875} = \mathbf{(1.4212 + j53.3)\ \Omega}$$

$$\mathbf{Y}(\omega_0/2) = \frac{1}{R} + j\left(\frac{\omega_0}{2}C - \frac{2}{\omega_0 L}\right) = 0.5 - j7.5 \text{ mS}$$

$$\mathbf{Z}(\omega_0/2) = \frac{1}{0.0005 - j0.0075} = \mathbf{(8.85 + j132.74)\ \Omega}$$

$$\mathbf{Y}(2\omega_0) = \frac{1}{R} + j\left(2\omega_0 L - \frac{1}{2\omega_0 C}\right) = 0.5 + j7.5 \text{ mS}$$

$$\mathbf{Z}(2\omega_0) = \mathbf{(8.85 - j132.74)\ \Omega}$$

$$\mathbf{Y}(4\omega_0) = \frac{1}{R} + j\left(4\omega_0 L - \frac{1}{4\omega_0 C}\right) = 0.5 + j18.75 \text{ mS}$$

$$\mathbf{Z}(4\omega_0) = \mathbf{(1.4212 - j53.3)\ \Omega}$$

Solution 14.38

Find the resonant frequency of the circuit in Fig. 14.78.

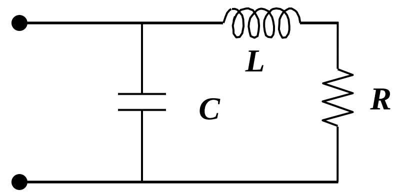


Figure 14.78
For Prob. 14.38.

Solution

$$Z = (1/(j\omega C))(R+j\omega L)/[(1/(j\omega C))+R+j\omega L] = (R+j\omega L)/(1+j\omega RC-\omega^2LC)$$

$$= (R+j\omega L)(1-\omega^2LC-j\omega RC)/[(1-\omega^2LC)^2+(\omega RC)^2]$$

$$=[R-\omega^2RLC+\omega^2RLC+j(\omega L-\omega^3L^2C-\omega R^2C)]/[(1-\omega^2LC)^2+(\omega RC)^2]$$

To find the resonant frequency all we need to do is to set the imaginary part to zero.

Thus, $(\omega L-\omega^3L^2C-\omega R^2C) = 0 = (L-\omega^2L^2C-R^2C)$ gives us $(\omega_o)^2 = (L-R^2C)/(L^2C)$ or

$$\omega_o = \sqrt{\frac{L-R^2C}{L^2C}} \text{ rad/s}$$

Solution 4.39

$$Y = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\mathrm{Im}(\mathbf{Y}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40\times 10^{-3})(1\times 10^{-6})} - \left(\frac{50}{40\times 10^{-3}}\right)^2}$$

$$\omega_0 = \mathbf{4.841\ krad/s}$$

Solution 14.40

(a) $B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(90-86)x10^3 = 8\pi krad/s$

$$\omega_o = \frac{1}{2}(\omega_1 + \omega_2) = 2\pi(88)x10^3 = 176\pi X10^3$$

$$B = \frac{1}{RC} \longrightarrow C = \frac{1}{BR} = \frac{1}{8\pi x10^3 x2x10^3} = \underline{19.89nF}$$

(b) $\omega_o = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega^2{}_o C} = \frac{1}{(176\pi X10^3)^2 x19.89x10^{-9}}$ = **164.45 μH**

(c) $\omega_o = 176\pi = \underline{552.9krad/s}$

(d) $B = 8\pi = \underline{25.13krad/s}$

(e) $Q = \frac{\omega_o}{B} = \frac{176\pi}{8\pi} = \underline{22}$

Solution 14.41

Using Fig. 14.80, design a problem to help other students to better understand the quality factor, the resonant frequency, and bandwidth of an *RLC* circuit.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in Example 14.9.

Problem

For the circuits in Fig. 14.80, find the resonant frequency ω_0, the quality factor Q, and the bandwidth B. Let C = 0.1 F, R_1 = 10 Ω, R_2 = 2 Ω, and L = 2 H.

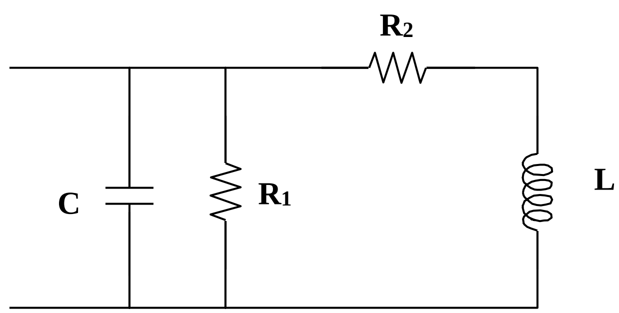


Figure 14.80
For Prob. 14.41.

Solution

To find ω_o, we need to find the input impedance or input admittance and set imaginary component equal to zero. Finding the input admittance seems to be the easiest approach.

$$\mathbf{Y} = j\omega 0.1 + 0.1 + 1/(2+j\omega 2) = j\omega 0.1 + 0.1 + [2/(4+4\omega^2)] - [j\omega 2/(4+4\omega^2)]$$

At resonance,

$$0.1\omega = 2\omega/(4+4\omega^2) \text{ or } 4\omega^2 + 4 = 20 \text{ or } \omega^2 = 4 \text{ or } \omega_o = \mathbf{2\ rad/s}$$

and,

$$\mathbf{Y} = 0.1 + 2/(4+16) = 0.1 + 0.1 = 0.2 \text{ S}$$

The bandwidth is define as the two values of ω such that $|\mathbf{Y}| = 1.4142(0.2) = 0.28284$ S.

I do not know about you, but I sure would not want to solve this analytically. So how about using MATLAB or excel to solve for the two values of ω?

Using Excel, we get $\omega_1 = 1.414$ rad/s and $\omega_2 = 3.741$ rad/s or B = **2.327 rad/s**

We can now use the relationship between ω_o and the bandwidth.

$$Q = \omega_o/B = 2/2.327 = \mathbf{0.8595}$$

Solution 14.42

(a) This is a series RLC circuit.

$$R = 2 + 6 = 8\,\Omega, \qquad L = 1\,\text{H}, \qquad C = 0.4\,\text{F}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.4}} = \mathbf{1.5811\ rad/s}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1.5811}{8} = \mathbf{0.1976}$$

$$B = \frac{R}{L} = \mathbf{8\ rad/s}$$

(b) This is a parallel RLC circuit.

$$3\ \mu\text{F and } 6\ \mu\text{F} \longrightarrow \frac{(3)(6)}{3+6} = 2\ \mu\text{F}$$

$$C = 2\ \mu\text{F}, \qquad R = 2\ \text{k}\Omega, \qquad L = 20\ \text{mH}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2\times 10^{-6})(20\times 10^{-3})}} = \mathbf{5\ krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{2\times 10^{3}}{(5\times 10^{3})(20\times 10^{-3})} = \mathbf{20}$$

$$B = \frac{1}{RC} = \frac{1}{(2\times 10^{3})(2\times 10^{-6})} = \mathbf{250\ rad/s}$$

Solution 14.43

(a) $\mathbf{Z}_{in} = (1/j\omega C) \,||\, (R + j\omega L)$

$$\mathbf{Z}_{in} = \frac{\dfrac{R + j\omega L}{j\omega C}}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{R + j\omega L}{1 - \omega^2 LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R + j\omega L)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega_0 L(1 - \omega_0^2 LC) - \omega_0 R^2 C$$

$$\omega_0^2 L^2 C = L - R^2 C$$

$$\omega_0 = \sqrt{\frac{L - R^2 C}{L^2 C}} = \sqrt{\frac{\mathbf{1}}{\mathbf{LC}} - \frac{\mathbf{R^2}}{\mathbf{L^2}}}$$

(b) $\mathbf{Z}_{in} = R \,||\, (j\omega L + 1/j\omega C)$

$$\mathbf{Z}_{in} = \frac{R\,(j\omega L + 1/j\omega C)}{R + j\omega L + 1/j\omega C} = \frac{R\,(1 - \omega^2 LC)}{(1 - \omega^2 LC) + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{R\,(1 - \omega^2 LC)[(1 - \omega^2 LC) - j\omega RC]}{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

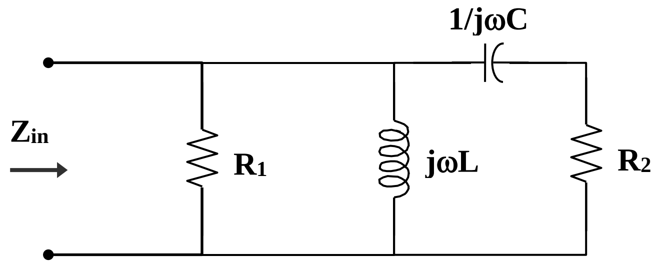
$$0 = R\,(1 - \omega^2 LC)\,\omega RC$$

$$1 - \omega^2 LC = 0$$

$$\omega_0 = \frac{\mathbf{1}}{\sqrt{\mathbf{LC}}}$$

Solution 14.44

Consider the circuit below.



(a) $\mathbf{Z}_{in} = (R_1 \parallel j\omega L) \parallel (R_2 + 1/j\omega C)$

$$\mathbf{Z}_{in} = \left(\frac{R_1 j\omega L}{R_1 + j\omega L}\right) \parallel \left(R_2 + \frac{1}{j\omega C}\right)$$

$$\mathbf{Z}_{in} = \frac{\dfrac{j\omega R_1 L}{R_1 + j\omega L} \cdot \left(R_2 + \dfrac{1}{j\omega C}\right)}{R_2 + \dfrac{1}{j\omega C} + \dfrac{jR_1\omega L}{R_1 + j\omega L}}$$

$$\mathbf{Z}_{in} = \frac{j\omega R_1 L\,(1 + j\omega R_2 C)}{(R_1 + j\omega L)(1 + j\omega R_2 C) - \omega^2 LCR_1}$$

$$\mathbf{Z}_{in} = \frac{-\omega^2 R_1 R_2 LC + j\omega R_1 L}{R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 + j\omega(L + R_1 R_2 C)}$$

$$\mathbf{Z}_{in} = \frac{(-\omega^2 R_1 R_2 LC + j\omega R_1 L)[R_1 - \omega^2 LCR_1 - \omega^2 LCR_2 - j\omega(L + R_1 R_2 C)]}{(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)^2 + \omega^2 (L + R_1 R_2 C)^2}$$

At resonance, $\mathrm{Im}(\mathbf{Z}_{in}) = 0$, i.e.

$$0 = \omega^3 R_1 R_2 LC\,(L + R_1 R_2 C) + \omega R_1 L\,(R_1 - \omega^2 LCR_1 - \omega^2 LCR_2)$$

$$0 = \omega^3 R_1^2 R_2^2 LC^2 + R_1^2 \omega L - \omega^3 R_1^2 L^2 C$$

$$0 = \omega^2 R_2^2 C^2 + 1 - \omega^2 LC$$

$$\omega^2 (LC - R_2^2 C^2) = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC - R_2^2 C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{(0.02)(9 \times 10^{-6}) - (0.1)^2 (9 \times 10^{-6})^2}}$$

$$\omega_0 = \mathbf{2.357\ krad/s}$$

(b) At $\omega = \omega_0 = 2.357\ \text{krad/s}$,

$$j\omega L = j(2.357\times 10^{3})(20\times 10^{-3}) = j47.14$$

$$R_1 \parallel j\omega L = \frac{j47.14}{1+j47.14} = 0.9996 + j0.0212$$

$$R_2 + \frac{1}{j\omega C} = 0.1 + \frac{1}{j(2.357\times 10^{3})(9\times 10^{-6})} = 0.1 - j47.14$$

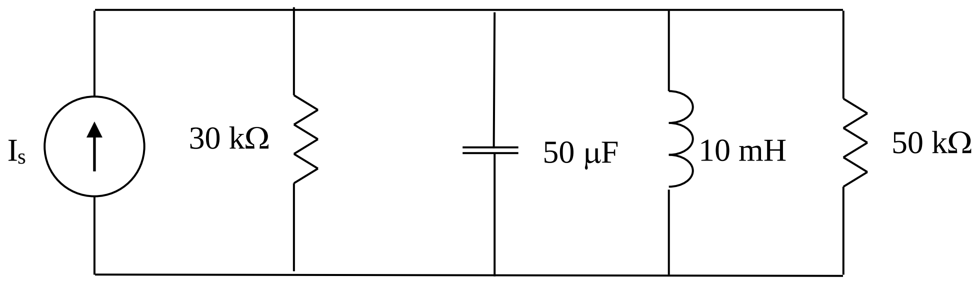
$$\mathbf{Z}_{in}(\omega_0) = (R_1 \parallel j\omega L) \parallel (R_2 + 1/j\omega C)$$

$$\mathbf{Z}_{in}(\omega_0) = \frac{(0.9996 + j0.0212)(0.1 - j47.14)}{(0.9996 + j0.0212) + (0.1 - j47.14)}$$

$$\mathbf{Z}_{in}(\omega_0) = \mathbf{1\,\Omega}$$

Solution 14.45

Convert the voltage source to a current source as shown below.



R = 30//50 = 30x50/80 = 18.75 kΩ

This is a parallel resonant circuit.

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10x10^{-3}x50x10^{-6}}} = \underline{447.21 \text{ rad/s}}$$

$$B = \frac{1}{RC} = \frac{1}{18.75x10^{3}x50x10^{-6}} = \underline{1.067 \text{ rad/s}}$$

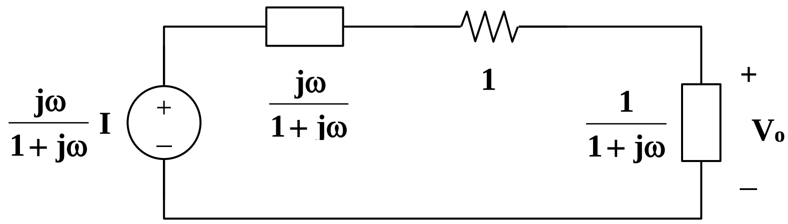
$$Q = \frac{\omega_o}{B} = \frac{447.21}{1.067} = \underline{419.13}$$

447.2 rad/s, 1.067 rad/s, 419.1

Solution 14.46

(a) $$1 \,\|\, j\omega = \frac{j\omega}{1+j\omega}, \qquad 1 \,\|\, \frac{1}{j\omega} = \frac{1/j\omega}{1+1/j\omega} = \frac{1}{1+j\omega}$$

Transform the current source gives the circuit below.



$$\mathbf{V}_o = \frac{\dfrac{1}{1+j\omega}}{1+\dfrac{1}{1+j\omega}+\dfrac{j\omega}{1+j\omega}} \cdot \frac{j\omega}{1+j\omega}\,\mathbf{I}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{I}} = \boldsymbol{\frac{j\omega}{2(1+j\omega)^2}}$$

(b) $$\mathbf{H}(1) = \frac{1}{2(1+j)^2}$$

$$|\mathbf{H}(1)| = \frac{1}{2(\sqrt{2})^2} = \mathbf{0.25}$$

Solution 14.47

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$

$H(0) = 1$ and $H(\infty) = 0$ showing that this circuit is a lowpass filter.

At the corner frequency, $\left|H(\omega_c)\right| = \frac{1}{\sqrt{2}}$, i.e.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega_c L}{R}\right)^2}} \longrightarrow 1 = \frac{\omega_c L}{R} \qquad \text{or} \qquad \omega_c = \frac{R}{L}$$

Hence,

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$f_c = \frac{1}{2\pi} \cdot \frac{R}{L} = \frac{1}{2\pi} \cdot \frac{10 \times 10^3}{2 \times 10^{-3}} = \mathbf{796\ kHz}$$

Solution 14.48

Find the transfer function $\mathbf{V}_o/\mathbf{V}_s$ of the circuit in Fig. 14.86. Show that the circuit is a lowpass filter.

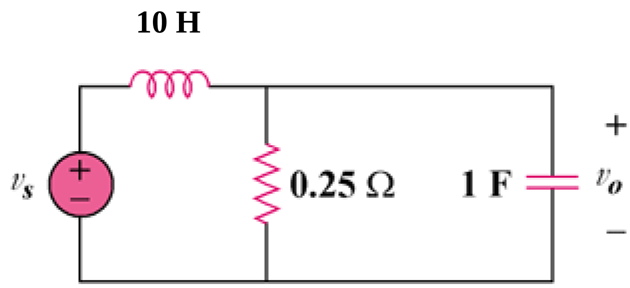


Figure 14.86
For Prob. 14.48.

Solution

$$\mathbf{H}(\omega) = \frac{R \,||\, \dfrac{1}{j\omega C}}{j\omega L + R \,||\, \dfrac{1}{j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{\dfrac{R/j\omega C}{R+1/j\omega C}}{j\omega L + \dfrac{R/j\omega C}{R+1/j\omega C}} = \frac{\dfrac{R}{1+j\omega RC}}{j\omega L + \dfrac{R}{1+j\omega RC}} = \frac{R}{R + j\omega L + \dfrac{R/j\omega C}{R+1/j\omega C}}$$

$$\mathbf{H}(\omega) = \frac{0.25}{(0.25-\omega^2 2.5) + j\omega 10}$$

$\mathrm{H}(0) = 1$ and $\mathrm{H}(\infty) = 0$ showing that **this circuit is a lowpass filter**.

Solution 14.49

Design a problem to help other students to better understand lowpass filters described by transfer functions.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the cutoff frequency of the lowpass filter described by

$$\mathrm{H}(\omega) = \frac{4}{2 + j\omega 10}$$

Find the gain in dB and phase of $\mathbf{H}(\omega)$ at $\omega = 2$ rad/s.

Solution

At dc, $\mathrm{H}(0) = \frac{4}{2} = 2$.

Hence, $$\left|\mathrm{H}(\omega)\right| = \frac{1}{\sqrt{2}}\mathrm{H}(0) = \frac{2}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} = \frac{4}{\sqrt{4 + 100\omega_c^2}}$$

$$4 + 100\omega_c^2 = 8 \longrightarrow \omega_c = 0.2$$

$$\mathrm{H}(2) = \frac{4}{2 + j20} = \frac{2}{1 + j10}$$

$$\left|\mathrm{H}(2)\right| = \frac{2}{\sqrt{101}} = 0.199$$

In dB, $20\log_{10}\left|\mathrm{H}(2)\right| = \mathbf{-14.023}$

$\arg \mathrm{H}(2) = -\tan^{-1}10 = -84.3°$ or ω_c = **1.4713 rad/sec**.

Solution 14.50

Determine what type of filter is in Fig. 14.87. Calculate the corner frequency f_c.

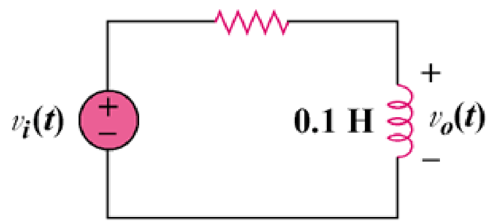


Figure 14.87
For Prob. 14.50.

Solution

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j\omega L}{R + j\omega L}$$

$H(0) = 0$ and $H(\infty) = 1$ showing that **this circuit is a highpass filter**.

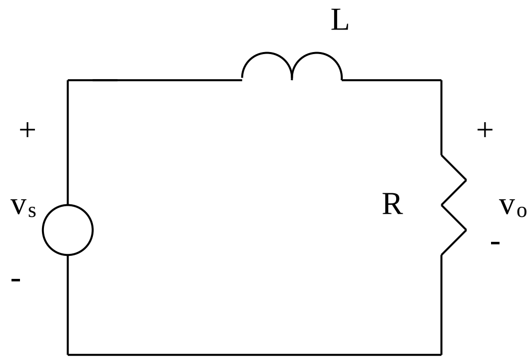
$$\left|\mathbf{H}(\omega_c)\right| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\left(\dfrac{R}{\omega_c L}\right)^2}} \longrightarrow 1 = \frac{R}{\omega_c L}$$

or $\qquad \omega_c = \dfrac{R}{L} = 2\pi f_c$

$$f_c = \frac{1}{2\pi}\cdot\frac{R}{L} = \frac{1}{2\pi}\cdot\frac{1{,}000}{0.1} = \mathbf{1.5915\ kHz}.$$

Solution 14.51

The lowpass RL filter is shown below.



$$H = \frac{V_o}{V_s} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L / R}$$

$$\boldsymbol{\omega_c = \frac{R}{L} = 2\pi f_c \quad \longrightarrow \quad R = 2\pi f_c L = 2\pi x 5x10^3 x 40x10^{-3} = \underline{1.256k\Omega}}$$

Solution 14.52

Design a problem to help other students to better understand passive highpass filters.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In a highpass *RL* filter with a cutoff frequency of 100 kHz, $L = 40$ mH. Find *R*.

Solution

$$\omega_c = \frac{R}{L} = 2\pi f_c$$

$$R = 2\pi f_c L = (2\pi)(10^5)(40\times10^{-3}) = \mathbf{25.13\ k\Omega}$$

Solution 14.53

$$\omega_1 = 2\pi f_1 = 20\pi \times 10^3$$
$$\omega_2 = 2\pi f_2 = 22\pi \times 10^3$$

$$B = \omega_2 - \omega_1 = 2\pi \times 10^3$$
$$\omega_0 = \frac{\omega_2 + \omega_1}{2} = 21\pi \times 10^3$$
$$Q = \frac{\omega_0}{B} = \frac{21\pi}{2\pi} = \mathbf{10.5}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow L = \frac{1}{\omega_0^2 C}$$
$$L = \frac{1}{(21\pi \times 10^3)^2 (80 \times 10^{-12})} = \mathbf{2.872\ H}$$

$$B = \frac{R}{L} \longrightarrow R = BL$$
$$R = (2\pi \times 10^3)(2.872) = \mathbf{18.045\ k\Omega}$$

Solution 14.54

We start with a series RLC circuit and the use the equations related to the circuit and the values for a bandstop filter.

$Q = \omega_o L/R = 1/(\omega_o CR) = 20$; $B = R/L = \omega_o/Q = 10/20 = 0.5$; $\omega_o = 1/(LC)^{0.5} = 10$

$(LC)^{0.5} = 0.1$ or $LC = 0.01$. Pick L = **10 H** then C = **1 mF**.

$Q = 20 = \omega_o L/R = 10x10/R$ or R = 100/20 = **5 Ω**.

Solution 14.55

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times10^{-3})(0.4\times10^{-6})}} = 10 \text{ krad/s}$$

$$B = \frac{R}{L} = \frac{10}{25\times10^{-3}} = 0.4 \text{ krad/s}$$

$$Q = \frac{10}{0.4} = \mathbf{25}$$

$$\omega_1 = \omega_o - B/2 = 10 - 0.2 = 9.8 \text{ krad/s} \qquad \text{or} \qquad f_1 = \frac{9.8}{2\pi} = 1.56 \text{ kHz}$$

$$\omega_2 = \omega_o + B/2 = 10 + 0.2 = 10.2 \text{ krad/s} \qquad \text{or} \qquad f_2 = \frac{10.2}{2\pi} = 1.62 \text{ kHz}$$

Therefore,

$$\mathbf{1.56 \text{ kHz} < f < 1.62 \text{ kHz}}$$

Solution 14.56

(a) From Eq. 14.54,

$$\mathbf{H}(s) = \frac{R}{R + sL + \frac{1}{sC}} = \frac{sRC}{1 + sRC + s^2LC} = \frac{s\frac{R}{L}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

Since $B = \frac{R}{L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$,

$$\mathbf{H}(s) = \frac{\mathbf{sB}}{\mathbf{s^2 + sB + \omega_0^2}}$$

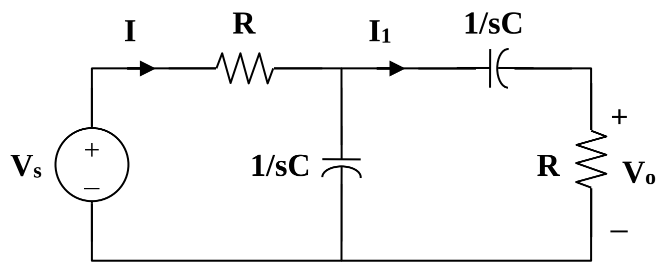
(b) From Eq. 14.56,

$$\mathbf{H}(s) = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{s^2 + \frac{1}{LC}}{s^2 + s\frac{R}{L} + \frac{1}{LC}}$$

$$\mathbf{H}(s) = \frac{\mathbf{s^2 + \omega_0^2}}{\mathbf{s^2 + sB + \omega_0^2}}$$

Solution 14.57

(a) Consider the circuit below.



$$\mathbf{Z}(s) = R + \frac{1}{sC} \,\|\, \left(R + \frac{1}{sC}\right) = R + \frac{\frac{1}{sC}\left(R + \frac{1}{sC}\right)}{R + \frac{2}{sC}}$$

$$\mathbf{Z}(s) = R + \frac{1 + sRC}{sC\,(2 + sRC)}$$

$$\mathbf{Z}(s) = \frac{1 + 3sRC + s^2R^2C^2}{sC\,(2 + sRC)}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}$$

$$\mathbf{I}_1 = \frac{1/sC}{2/sC + R}\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}\,(2 + sRC)}$$

$$\mathbf{V}_o = \mathbf{I}_1 R = \frac{R\,\mathbf{V}_s}{2 + sRC} \cdot \frac{sC\,(2 + sRC)}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRC}{1 + 3sRC + s^2R^2C^2}$$

$$\mathbf{H}(s) = \frac{1}{3}\left[\frac{\frac{3}{RC}s}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}\right]$$

Thus, $\omega_0^2 = \dfrac{1}{R^2C^2}$ or $\omega_0 = \dfrac{1}{RC} = \mathbf{1\,rad/s}$

$$B = \frac{3}{RC} = \mathbf{3\,rad/s}$$

(b) Similarly,

$$\mathbf{Z}(s) = sL + R \,||\, (R + sL) = sL + \frac{R(R + sL)}{2R + sL}$$

$$\mathbf{Z}(s) = \frac{R^2 + 3sRL + s^2L^2}{2R + sL}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}}, \qquad \mathbf{I}_1 = \frac{R}{2R + sL}\mathbf{I} = \frac{R\,\mathbf{V}_s}{\mathbf{Z}(2R + sL)}$$

$$\mathbf{V}_o = \mathbf{I}_1 \cdot sL = \frac{sLR\,\mathbf{V}_s}{2R + sL} \cdot \frac{2R + sL}{R^2 + 3sRL + s^2L^2}$$

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{sRL}{R^2 + 3sRL + s^2L^2} = \frac{\frac{1}{3}\left(\frac{3R}{L}s\right)}{s^2 + \frac{3R}{L}s + \frac{R^2}{L^2}}$$

Thus, $\omega_0 = \frac{R}{L} = \mathbf{1\,rad/s}$

$B = \frac{3R}{L} = \mathbf{3\,rad/s}$

Solution 14.58

The circuit parameters for a series *RLC* bandstop filter are R = 250 Ω, L = 1 mH, C = 40 pF. Calculate:

(a) the center frequency
(b) the half-power frequencies
(c) the quality factor.

Solution

(a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.001)(40\times10^{-12})}} = \mathbf{5\ Mrad/s}$

(b) $B = \frac{R}{L} = \frac{250}{0.001} = 0.25\times10^6$ rad/s

$$Q = \frac{\omega_0}{B} = \frac{5\times10^6}{0.25\times10^6} = 20$$

As a high Q (Q greater than 10) circuit,

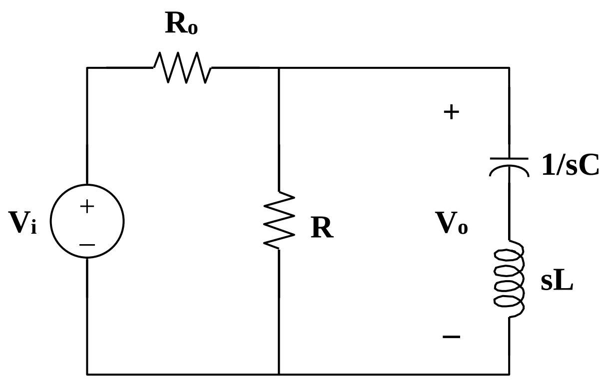
$$\omega_1 = \omega_0 - \frac{B}{2} = 10^6(5-0.125) = \mathbf{4.875\ Mrad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 10^6(5+0.125) = \mathbf{5.125\ Mrad/s}$$

(c) As seen in part (b), $\mathrm{Q} = \mathbf{20}$

Solution 14.59

Consider the circuit below.



where L = 1 mH, C = 4 μF, R_o = 6 Ω, and R = 4 Ω.

$$\mathbf{Z}(s) = R \,\|\, \left(sL + \frac{1}{sC}\right) = \frac{R\,(sL + 1/sC)}{R + sL + 1/sC}$$

$$\mathbf{Z}(s) = \frac{R\,(1 + s^2 LC)}{1 + sRC + s^2 LC}$$

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}}{\mathbf{Z} + R_o} = \frac{R\,(1 + s^2 LC)}{R_o + sRR_oC + s^2 LCR_o + R + s^2 LCR}$$

$$\mathbf{Z}_{in} = R_o + \mathbf{Z} = R_o + \frac{R\,(1 + s^2 LC)}{1 + sRC + s^2 LC}$$

$$\mathbf{Z}_{in} = \frac{R_o + sRR_oC + s^2 LCR_o + R + s^2 LCR}{1 + sRC + s^2 LC}$$

$s = j\omega$

$$\mathbf{Z}_{in} = \frac{R_o + j\omega RR_oC - \omega^2 LCR_o + R - \omega^2 LCR}{1 - \omega^2 LC + j\omega RC}$$

$$\mathbf{Z}_{in} = \frac{(R_o + R - \omega^2 LCR_o - \omega^2 LCR + j\omega RR_oC)(1 - \omega^2 LC - j\omega RC)}{(1 - \omega^2 LC)^2 + (\omega RC)^2}$$

$\text{Im}(\mathbf{Z}_{in}) = 0$ implies that

$$-\omega RC[R_o + R - \omega^2 LCR_o - \omega^2 LCR] + \omega RR_oC\,(1 - \omega^2 LC) = 0$$

$$R_o + R - \omega^2 LCR_o - \omega^2 LCR - R_o + \omega^2 LCR_o = 0$$
$$\omega^2 LCR = R$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1\times10^{-3})(4\times10^{-6})}} = \mathbf{15.811\ krad/s}$$

$$\mathbf{H} = \frac{R\,(1-\omega^2 LC)}{R_o + j\omega RR_o C + R - \omega^2 LCR_o - \omega^2 LCR}$$

$$H_{max} = H(0) = \frac{R}{R_o + R}$$

or $$H_{max} = H(\infty) = \lim_{\omega\to\infty} \frac{R\left(\frac{1}{\omega^2} - LC\right)}{\frac{R_o + R}{\omega^2} + j\frac{RR_oC}{\omega} - LC(R+R_o)} = \frac{R}{R+R_o}$$

At ω_1 and ω_2, $\left|\mathbf{H}\right| = \frac{1}{\sqrt{2}} H_{mzx}$

$$\frac{R}{\sqrt{2}(R_o + R)} = \left|\frac{R\,(1-\omega^2 LC)}{R_o + R - \omega^2 LC(R_o + R) + j\omega RR_o C}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{(R_o + R)(1-\omega^2 LC)}{\sqrt{(\omega RR_o C)^2 + (R_o + R - \omega^2 LC(R_o + R))^2}}$$

$$\frac{1}{\sqrt{2}} = \frac{10\,(1-\omega^2 \cdot 4\times10^{-9})}{\sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2}}$$

$$0 = \frac{10\,(1-\omega^2 \cdot 4\times10^{-9})}{\sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2}} - \frac{1}{\sqrt{2}}$$

$$(10-\omega^2 \cdot 4\times10^{-8})(\sqrt{2}) - \sqrt{(96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2} = 0$$

$$(2)(10-\omega^2 \cdot 4\times10^{-8})^2 = (96\times10^{-6}\omega)^2 + (10-\omega^2 \cdot 4\times10^{-8})^2$$

$$(96\times10^{-6}\omega)^2 - (10-\omega^2 \cdot 4\times10^{-8})^2 = 0$$

$$1.6\times10^{-15}\omega^4 - 8.092\times10^{-7}\omega^2 + 100 = 0$$

$$\omega^4 - 5.058\times10^8 + 6.25\times10^{16} = 0$$

$$\omega^2 = \begin{cases} 2.9109\times10^8 \\ 2.1471\times10^8 \end{cases}$$

Hence,

$$\omega_1 = 14.653 \text{ krad/s}$$

$$\omega_2 = 17.061 \text{ krad/s}$$

$$B = \omega_2 - \omega_1 = 17.061 - 14.653 = \mathbf{2.408\ krad/s}$$

Solution 14.60

Obtain the transfer function of a highpass filter with a passband gain of 100 and a cutoff frequency of 40 rad/s.

Solution

$$\mathbf{H}'(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega}{j\omega + 1/RC} \qquad \text{(from Eq. 14.52)}$$

This has a unity passband gain, i.e. $H(\infty) = 1$.

$$\frac{1}{RC} = \omega_c = 40$$

$$\mathbf{H}^{\wedge}(\omega) = 100\,\mathbf{H}'(\omega) = \frac{j100\omega}{40 + j\omega}$$

$$\mathbf{H}(\omega) = \mathbf{j100\omega/(40+j\omega)}$$

Solution 14.61

(a) $\quad \mathbf{V}_+ = \dfrac{1/j\omega C}{R + 1/j\omega C}\mathbf{V}_i, \qquad \mathbf{V}_- = \mathbf{V}_o$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{\frac{1}{1 + j\omega RC}}$$

(b) $\quad \mathbf{V}_+ = \dfrac{R}{R + 1/j\omega C}\mathbf{V}_i, \qquad \mathbf{V}_- = \mathbf{V}_o$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i = \mathbf{V}_o$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{\frac{j\omega RC}{1 + j\omega RC}}$$

Solution 14.62

This is a highpass filter.

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} = \frac{1}{1 - j/\omega RC}$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j\omega_c/\omega}, \qquad \omega_c = \frac{1}{RC} = 2\pi(1000)$$

$$\mathbf{H}(\omega) = \frac{1}{1 - j f_c/f} = \frac{1}{1 - j1000/f}$$

(a) $$\mathbf{H}(f = 200 \text{ Hz}) = \frac{1}{1 - j5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120 \text{ mV}}{\left|1 - j5\right|} = \mathbf{23.53\ mV}$$

(b) $$\mathbf{H}(f = 2 \text{ kHz}) = \frac{1}{1 - j0.5} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120 \text{ mV}}{\left|1 - j0.5\right|} = \mathbf{107.3\ mV}$$

(c) $$\mathbf{H}(f = 10 \text{ kHz}) = \frac{1}{1 - j0.1} = \frac{\mathbf{V}_o}{\mathbf{V}_i}$$

$$\left|\mathbf{V}_o\right| = \frac{120 \text{ mV}}{\left|1 - j0.1\right|} = \mathbf{119.4\ mV}$$

Solution 14.63

For an active highpass filter,

$$H(s) = -\frac{sC_iR_f}{1+sC_iR_i} \qquad (1)$$

But

$$H(s) = -\frac{10s}{1+s/10} \qquad (2)$$

Comparing (1) and (2) leads to:

$$\mathbf{C_iR_f = 10 \quad \longrightarrow \quad R_f = \frac{10}{C_i} = \underline{10M\Omega}}$$

$$\mathbf{C_iR_i = 0.1 \quad \longrightarrow \quad R_i = \frac{0.1}{C_i} = \underline{100k\Omega}}$$

Solution 14.64

$$Z_f = R_f \parallel \frac{1}{j\omega C_f} = \frac{R_f}{1 + j\omega R_f C_f}$$

$$Z_i = R_i + \frac{1}{j\omega C_i} = \frac{1 + j\omega R_i C_i}{j\omega C_i}$$

Hence,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{-\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{\mathbf{-\,j\omega R_f C_i}}{\mathbf{(1 + j\omega R_f C_f)(1 + j\omega R_i C_i)}}$$

This is a bandpass filter. $\mathbf{H}(\omega)$ is similar to the product of the transfer function of a lowpass filter and a highpass filter.

Solution 14.65

$$\mathbf{V}_+ = \frac{R}{R + 1/j\omega C}\mathbf{V}_i = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{V}_- = \frac{R_i}{R_i + R_f}\mathbf{V}_o$$

Since $\mathbf{V}_+ = \mathbf{V}_-$,

$$\frac{R_i}{R_i + R_f}\mathbf{V}_o = \frac{j\omega RC}{1 + j\omega RC}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}\right)\left(\frac{\mathbf{j\omega RC}}{\mathbf{1 + j\omega RC}}\right)$$

It is evident that as $\omega \to \infty$, the gain is $\mathbf{1} + \frac{\mathbf{R_f}}{\mathbf{R_i}}$ and that the corner frequency is $\frac{\mathbf{1}}{\mathbf{RC}}$.

Solution 14.66

(a) **Proof**

(b) When $\mathbf{R_1R_4 = R_2R_3}$,

$$\mathbf{H}(s) = \frac{R_4}{R_3 + R_4} \cdot \frac{s}{s + 1/R_2C}$$

(c) When $\mathbf{R_3} \to \infty$,

$$\mathbf{H}(s) = \frac{-1/R_1C}{s + 1/R_2C}$$

Solution 14.67

$$\text{DC gain} = \frac{R_f}{R_i} = \frac{1}{4} \longrightarrow R_i = 4R_f$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_f C_f} = 2\pi(500)\ \text{rad/s}$$

If we select $R_f = 20\ \text{k}\Omega$, then $R_i = 80\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(500)(20\times10^3)} = 15.915\ \text{nF}$$

Therefore, if $R_f = \mathbf{20\ k\Omega}$, then $R_i = \mathbf{80\ k\Omega}$ and $C = \mathbf{15.915\ nF}$

Solution 14.68

Design a problem to help other students to better understand the design of active highpass filters when specifying a high-frequency gain and a corner frequency.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Design an active highpass filter with a high-frequency gain of 5 and a corner frequency of 200 Hz.

Solution

$$\text{High frequency gain} = 5 = \frac{R_f}{R_i} \quad \longrightarrow \quad R_f = 5R_i$$

$$\text{Corner frequency} = \omega_c = \frac{1}{R_i C_i} = 2\pi(200) \text{ rad/s}$$

If we select $R_i = 20\ \text{k}\Omega$, then $R_f = 100\ \text{k}\Omega$ and

$$C = \frac{1}{(2\pi)(200)(20 \times 10^3)} = 39.8 \text{ nF}$$

Therefore, if $R_i = \mathbf{20\ k\Omega}$, then $R_f = \mathbf{100\ k\Omega}$ and $C = \mathbf{39.8\ nF}$

Solution 14.69

This is a highpass filter with $f_c = 2$ kHz.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$RC = \frac{1}{2\pi f_c} = \frac{1}{4\pi \times 10^3}$$

10^8 Hz may be regarded as high frequency. Hence the high-frequency gain is

$$\frac{-R_f}{R} = \frac{-10}{4} \qquad \text{or} \qquad R_f = 2.5R$$

If we let $R = \mathbf{10\ k\Omega}$, then $R_f = \mathbf{25\ k\Omega}$, and $C = \dfrac{1}{4000\pi \times 10^4} = \mathbf{7.96\ nF}$.

Solution 14.70

(a) $\mathbf{H}(s) = \dfrac{\mathbf{V}_o(s)}{\mathbf{V}_i(s)} = \dfrac{Y_1 Y_2}{Y_1 Y_2 + Y_4 (Y_1 + Y_2 + Y_3)}$

where $Y_1 = \dfrac{1}{R_1} = G_1,\ Y_2 = \dfrac{1}{R_2} = G_2,\ Y_3 = sC_1,\ Y_4 = sC_2$.

$$\mathbf{H}(s) = \frac{\mathbf{G_1 G_2}}{\mathbf{G_1 G_2 + sC_2 (G_1 + G_2 + sC_1)}}$$

(b) $H(0) = \dfrac{G_1 G_2}{G_1 G_2} = 1, \qquad H(\infty) = 0$

showing that **this circuit is a lowpass filter**.

Solution 14.71

$R = 50\,\Omega,\; L = 40\text{ mH},\; C = 1\,\mu\text{F}$

$$L' = \frac{K_m}{K_f} L \quad \longrightarrow \quad 1 = \frac{K_m}{K_f} \cdot (40 \times 10^{-3})$$

$$25K_f = K_m \qquad (1)$$

$$C' = \frac{C}{K_m K_f} \quad \longrightarrow \quad 1 = \frac{10^{-6}}{K_m K_f}$$

$$10^6 K_f = \frac{1}{K_m} \qquad (2)$$

Substituting (1) into (2),

$$10^6 K_f = \frac{1}{25K_f}$$

$$K_f = \mathbf{2x10^{-4}}$$

$$K_m = 25K_f = \mathbf{5 \times 10^{-3}}$$

Solution 14.72

Design a problem to help other students to better understand magnitude and frequency scaling.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

What values of K_m and K_f will scale a 4-mH inductor and a 20-μF capacitor to 1 H and 2 F respectively?

Solution

$$L'C' = \frac{LC}{K_f^2} \longrightarrow K_f^2 = \frac{LC}{L'C'}$$

$$K_f^2 = \frac{(4\times10^{-3})(20\times10^{-6})}{(1)(2)} = 4\times10^{-8}$$

$$K_f = \mathbf{2\times10^{-4}}$$

$$\frac{L'}{C'} = \frac{L}{C}K_m^2 \longrightarrow K_m^2 = \frac{L'}{C'}\cdot\frac{C}{L}$$

$$K_m^2 = \frac{(1)(20\times10^{-6})}{(2)(4\times10^{-3})} = 2.5\times10^{-3}$$

$$K_m = \mathbf{5\times10^{-2}}$$

Solution 14.73

$$R' = K_m R = (12)(800 \times 10^3) = \mathbf{9.6\ M\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{800}{1000}(40 \times 10^{-6}) = \mathbf{32\ \mu F}$$

$$C' = \frac{C}{K_m K_f} = \frac{300 \times 10^{-9}}{(800)(1000)} = \mathbf{0.375\ pF}$$

Solution 14.74

$$\mathbf{R'_1 = K_m R_1 = 3x100 = \underline{300\Omega}}$$

$$\mathbf{R'_2 = K_m R_2 = 10x100 = \underline{1\,k\Omega}}$$

$$\mathbf{L' = \frac{K_m}{K_f} L = \frac{10^2}{10^6}(2) = \underline{200\,\mu H}}$$

$$\mathbf{C' = \frac{C}{K_m K_f} = \frac{\frac{1}{10}}{10^8} = \underline{1\,nF}}$$

Solution 14.75

$$R' = K_m R = 20x10 = \underline{200\,\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{10^5}(4) = \underline{400\,\mu H}$$

$$C' = \frac{C}{K_m K_f} = \frac{1}{10x10^5} = \underline{1\,\mu F}$$

Solution 14.76

$$R' = K_m R = 500x5x10^3 = \underline{25\ \text{M}\Omega}$$

$$L' = \frac{K_m}{K_f} L = \frac{500}{10^5}(10mH) = \underline{50\ \mu\text{H}}$$

$$C' = \frac{C}{K_m K_f} = \frac{20x10^{-6}}{500x10^5} = \underline{0.4\ \text{pF}}$$

Solution 14.77

L and C are needed before scaling.

$$B = \frac{R}{L} \longrightarrow L = \frac{R}{B} = \frac{10}{5} = 2\text{ H}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(1600)(2)} = 312.5\ \mu\text{F}$$

(a) $L' = K_m L = (600)(2) = \mathbf{1.200\ kH}$

$$C' = \frac{C}{K_m} = \frac{3.125\times10^{-4}}{600} = \mathbf{0.5208\ \mu F}$$

(b) $L' = \dfrac{L}{K_f} = \dfrac{2}{10^3} = \mathbf{2\ mH}$

$$C' = \frac{C}{K_f} = \frac{3.125\times10^{-4}}{10^3} = \mathbf{312.5\ nF}$$

(c) $L' = \dfrac{K_m}{K_f} L = \dfrac{(400)(2)}{10^5} = \mathbf{8\ mH}$

$$C' = \frac{C}{K_m K_f} = \frac{3.125\times10^{-4}}{(400)(10^5)} = \mathbf{7.81\ pF}$$

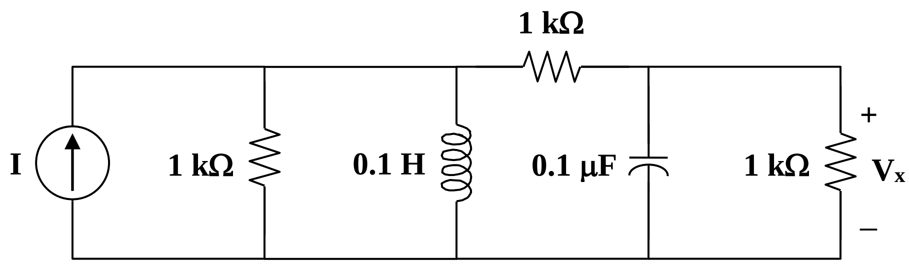
Solution 14.78

$$R' = K_m R = (1000)(1) = 1\,\text{k}\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10^3}{10^4}(1) = 0.1\,\text{H}$$

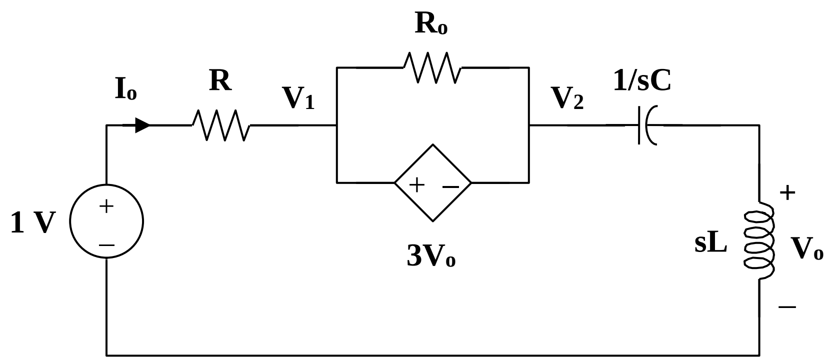
$$C' = \frac{C}{K_m K_f} = \frac{1}{(10^3)(10^4)} = 0.1\,\mu\text{F}$$

The new circuit is shown below.



Solution 14.79

(a) Insert a 1-V source at the input terminals.



There is a supernode.

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_2}{sL + 1/sC} \qquad (1)$$

But $\mathbf{V}_1 = \mathbf{V}_2 + 3\mathbf{V}_o \longrightarrow \mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o$ (2)

Also, $\mathbf{V}_o = \dfrac{sL}{sL + 1/sC}\mathbf{V}_2 \longrightarrow \dfrac{\mathbf{V}_o}{sL} = \dfrac{\mathbf{V}_2}{sL + 1/sC}$ (3)

Combining (2) and (3)

$$\mathbf{V}_2 = \mathbf{V}_1 - 3\mathbf{V}_o = \frac{sL + 1/sC}{sL}\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{s^2LC}{1 + 4s^2LC}\mathbf{V}_1 \qquad (4)$$

Substituting (3) and (4) into (1) gives

$$\frac{1-\mathbf{V}_1}{R} = \frac{\mathbf{V}_o}{sL} = \frac{sC}{1+4s^2LC}\mathbf{V}_1$$

$$1 = \mathbf{V}_1 + \frac{sRC}{1+4s^2LC}\mathbf{V}_1 = \frac{1+4s^2LC+sRC}{1+4s^2LC}\mathbf{V}_1$$

$$\mathbf{V}_1 = \frac{1+4s^2LC}{1+4s^2LC+sRC}$$

$$\mathbf{I}_o = \frac{1-\mathbf{V}_1}{R} = \frac{sRC}{R\,(1+4s^2LC+sRC)}$$

$$\mathbf{Z}_{in} = \frac{1}{\mathbf{I}_o} = \frac{1+sRC+4s^2LC}{sC}$$

$$\mathbf{Z}_{in} = 4sL + R + \frac{1}{sC} \tag{5}$$

When $R = 5$, $L = 2$, $C = 0.1$,

$$\mathbf{Z}_{in}(s) = \mathbf{8s + 5 + \frac{10}{s}}$$

At resonance,

$$\mathrm{Im}(\mathbf{Z}_{in}) = 0 = 4\omega L - \frac{1}{\omega C}$$

or $$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.1)(2)}} = \mathbf{1.118\ rad/s}$$

(b) After scaling,

$$R' \longrightarrow K_m R$$
$$4\,\Omega \longrightarrow 40\,\Omega$$
$$5\,\Omega \longrightarrow 50\,\Omega$$

$$L' = \frac{K_m}{K_f} L = \frac{10}{100}(2) = 0.2\ \mathrm{H}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.1}{(10)(100)} = 10^{-4}$$

From (5),

$$\mathbf{Z}_{in}(s) = \mathbf{0.8s + 50 + \frac{10^4}{s}}$$

$$\omega_0 = \frac{1}{2\sqrt{LC}} = \frac{1}{2\sqrt{(0.2)(10^{-4})}} = \mathbf{111.8\ rad/s}$$

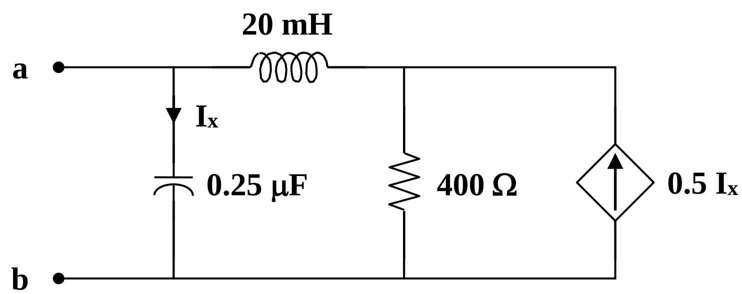
Solution 14.80

(a) $R' = K_m R = (200)(2) = 400\ \Omega$

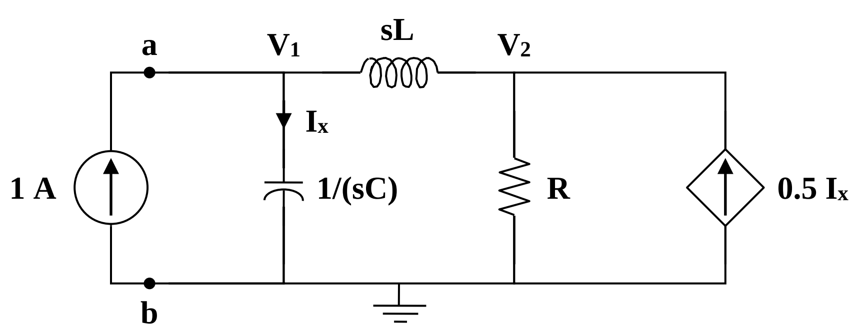
$$L' = \frac{K_m L}{K_f} = \frac{(200)(1)}{10^4} = 20\text{ mH}$$

$$C' = \frac{C}{K_m K_f} = \frac{0.5}{(200)(10^4)} = 0.25\ \mu\text{F}$$

The new circuit is shown below.



(b) Insert a 1-A source at the terminals a-b.



At node 1,

$$1 = sC\mathbf{V}_1 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5\mathbf{I}_x = \frac{\mathbf{V}_2}{R}$$

But, $\mathbf{I}_x = sC\mathbf{V}_1$.

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{sL} + 0.5sC\mathbf{V}_1 = \frac{\mathbf{V}_2}{R} \qquad (2)$$

Solving (1) and (2),

$$\mathbf{V}_1 = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_1}{1} = \frac{sL + R}{s^2LC + 0.5sCR + 1}$$

At $\omega = 10^4$,

$$\mathbf{Z}_{Th} = \frac{(j10^4)(20\times10^{-3}) + 400}{(j10^4)^2(20\times10^{-3})(0.25\times10^{-6}) + 0.5(j10^4)(0.25\times10^{-6})(400) + 1}$$

$$\mathbf{Z}_{Th} = \frac{400 + j200}{0.5 + j0.5} = 600 - j200$$

$$\mathbf{Z}_{Th} = \mathbf{632.5\angle -18.435^\circ \ ohms}$$

Solution 14.81

(a)

$$\frac{1}{Z} = G + j\omega C + \frac{1}{R + j\omega L} = \frac{(G + j\omega C)(R + j\omega L) + 1}{R + j\omega L}$$

which leads to $$Z = \frac{j\omega L + R}{-\omega^2 LC + j\omega(RC + LG) + GR + 1}$$

$$Z(\omega) = \frac{j\frac{\omega}{C} + \frac{R}{LC}}{-\omega^2 + j\omega\left(\frac{R}{L} + \frac{G}{C}\right) + \frac{GR + 1}{LC}} \qquad (1)$$

We compare this with the given impedance:

$$Z(\omega) = \frac{1000(j\omega + 1)}{-\omega^2 + 2j\omega + 1 + 2500} \qquad (2)$$

Comparing (1) and (2) shows that

$$\frac{1}{C} = 1000 \longrightarrow C = 1\,\text{mF}, \quad R/L = 1 \longrightarrow R = L$$

$$\frac{R}{L} + \frac{G}{C} = 2 \longrightarrow G = C = 1\ \text{mS}$$

$$2501 = \frac{GR + 1}{LC} = \frac{10^{-3}R + 1}{10^{-3}R} \longrightarrow R = 0.4 = L$$

Thus,

R = **0.4Ω**, L = **0.4 H**, C = **1 mF**, G = **1 mS**

(b) By frequency-scaling, K_f =1000.

R’ = **0.4 Ω**, G’ = **1 mS**

$$\mathbf{L' = \frac{L}{K_f} = \frac{0.4}{10^3} = \underline{0.4mH}}, \quad \mathbf{C' = \frac{C}{K_f} = \frac{10^{-3}}{10^{-3}} = \underline{1\mu F}}$$

Solution 14.82

$$C' = \frac{C}{K_m K_f}$$

$$K_f = \frac{\omega'_c}{\omega} = \frac{200}{1} = 200$$

$$K_m = \frac{C}{C'} \cdot \frac{1}{K_f} = \frac{1}{10^{-6}} \cdot \frac{1}{200} = 5000$$

$R' = K_m R =$ **5 kΩ**, thus, $R'_f = 2R_i =$ **10 kΩ**

Solution 14.83

$$1\mu F \longrightarrow \quad C' = \frac{1}{K_m K_f} C = \frac{10^{-6}}{100x10^5} = \underline{0.1\,pF}$$

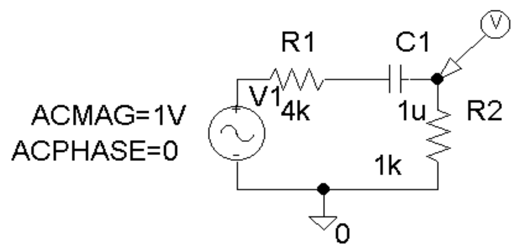
$$5\mu F \longrightarrow \quad C' = \underline{0.5\,pF}$$

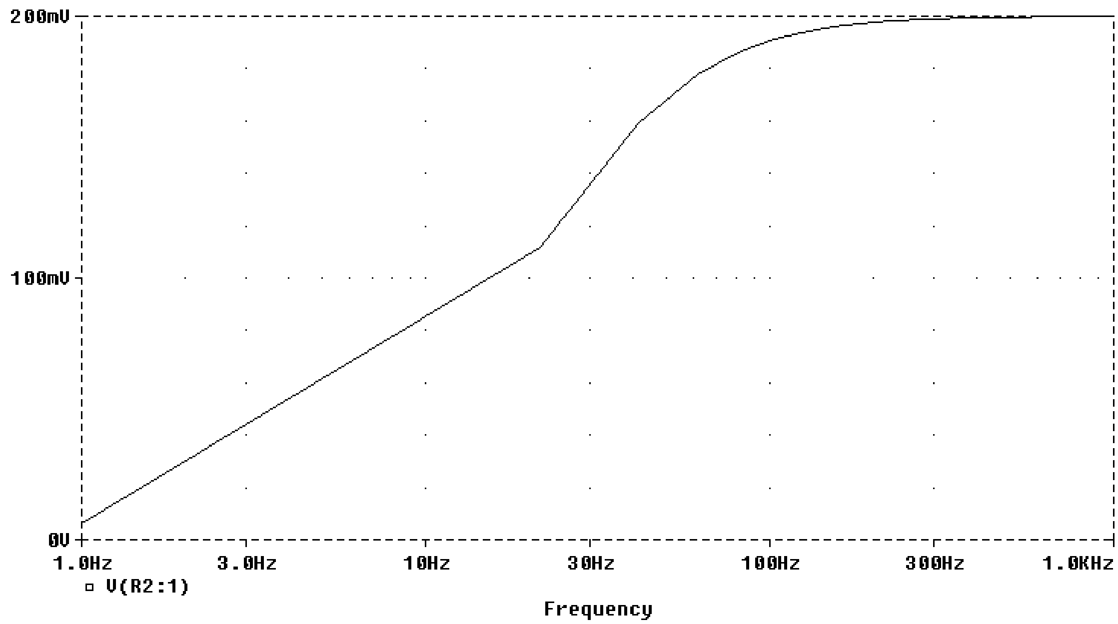
$$10\,k\Omega \longrightarrow \quad R' = K_m R = 100x10\,k\Omega = \underline{1\,M\Omega}$$

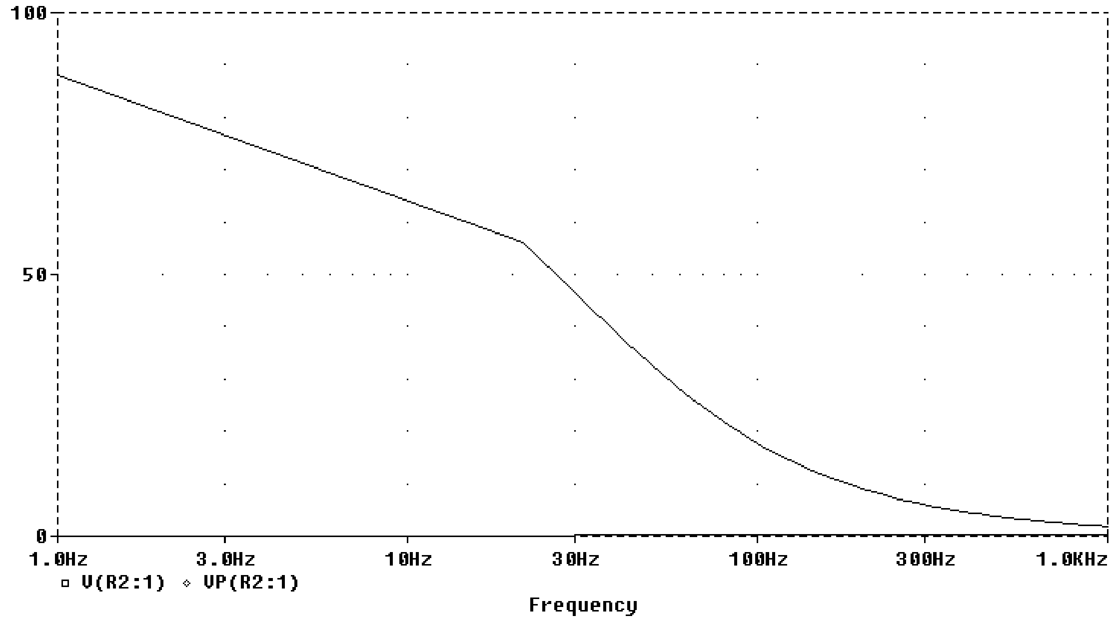
$$20\,k\Omega \longrightarrow \quad R' = \underline{2\,M\Omega}$$

Solution 14.84

The schematic is shown below. A voltage marker is inserted to measure v_o. In the AC sweep box, we select Total Points = 50, Start Frequency = 1, and End Frequency = 1000. After saving and simulation, we obtain the magnitude and phase plots in the probe menu as shown below.

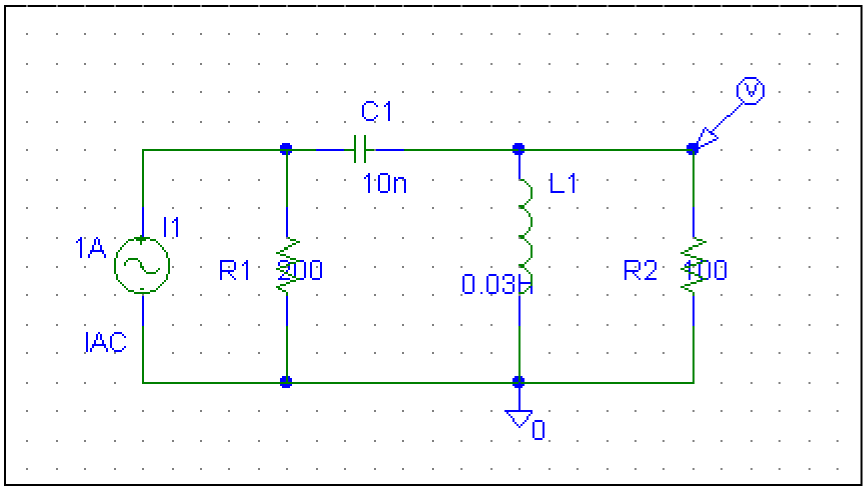


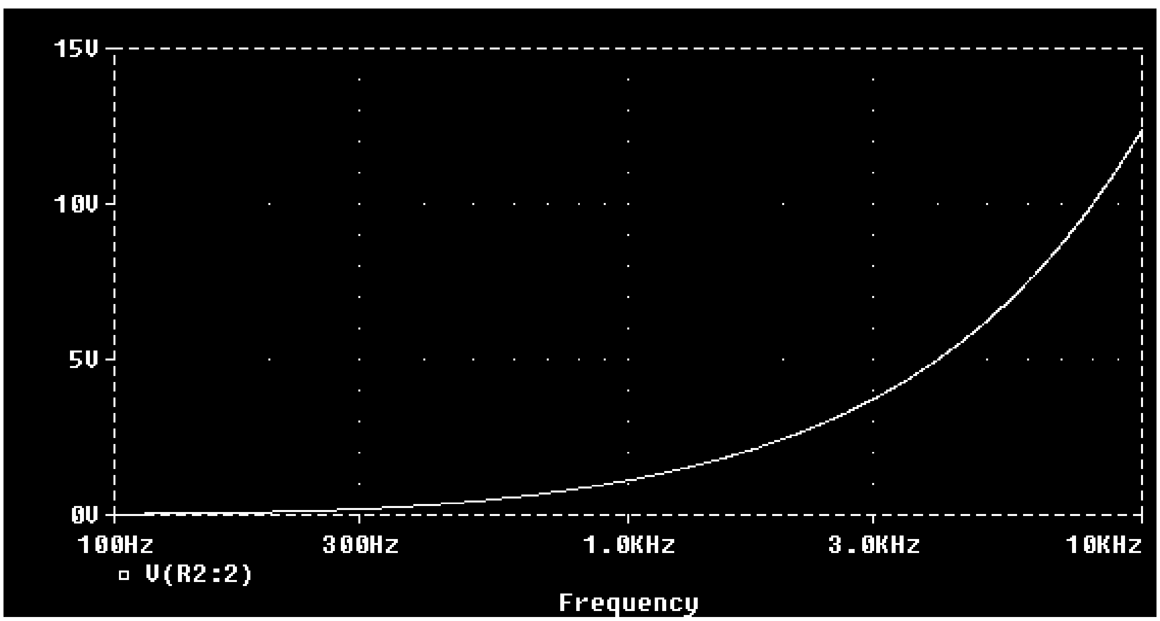


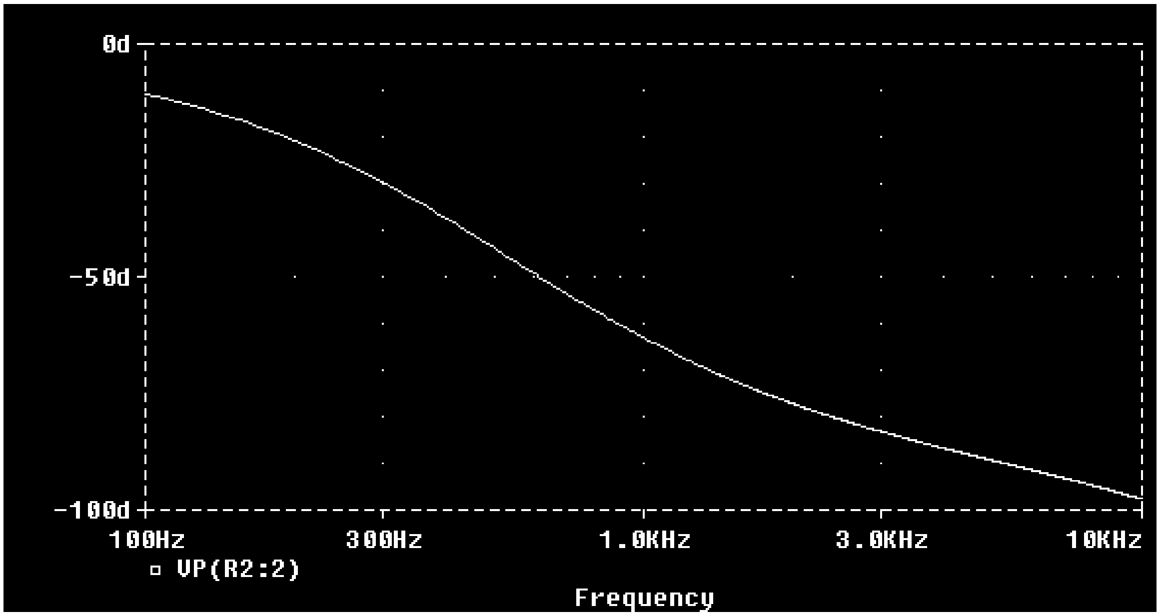

100
50
0
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1)
VP(R2:1)
Frequency

Solution 14.85

We let $I_s = 1\angle 0^o$ A so that $V_o / I_s = V_o$. The schematic is shown below. The circuit is simulated for $100 < f < 10$ kHz.






0d
-50d
-100d
100Hz
300Hz
1.0KHz
3.0KHz
10KHz
VP(R2:2)
Frequency

Solution 14.86

Using Fig. 14.103, design a problem to help other students to better understand how to use PSpice to obtain the frequency response (magnitude and phase of I) in electrical circuits.

Although there are many ways to solve this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Use *PSpice* to provide the frequency response (magnitude and phase of *i*) of the circuit in Fig. 14.103. Use linear frequency sweep from 1 to 10,000 Hz.

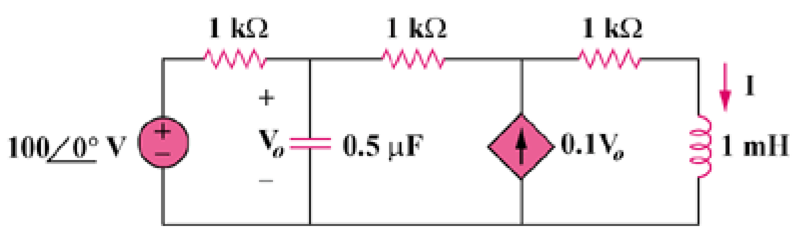
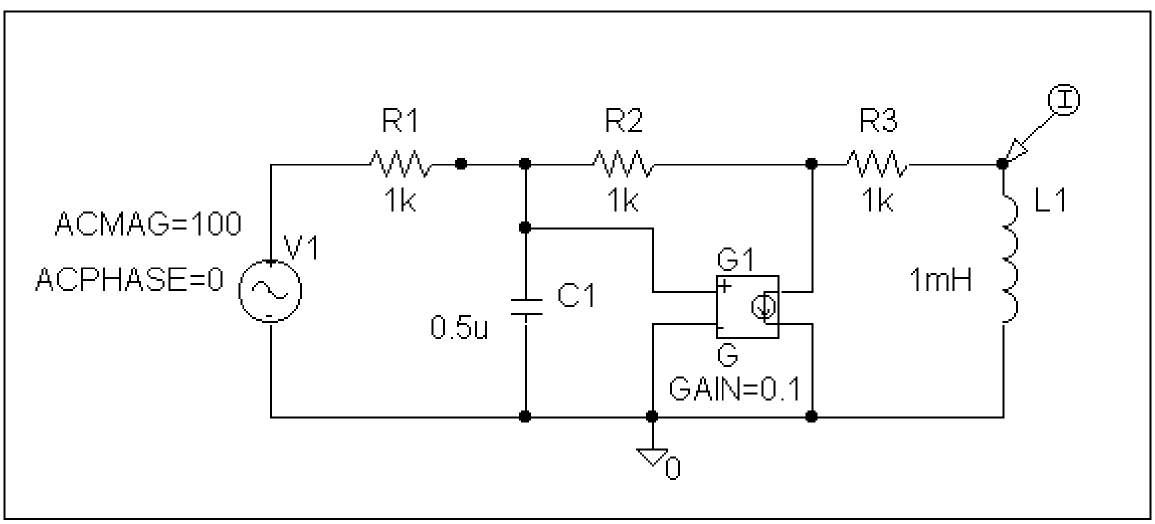
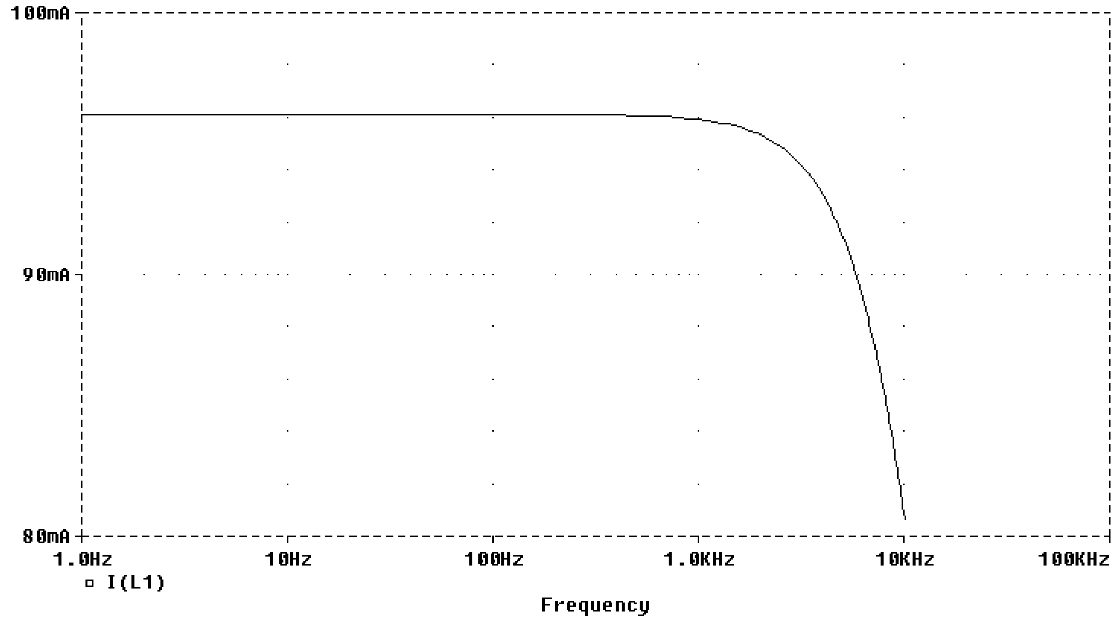


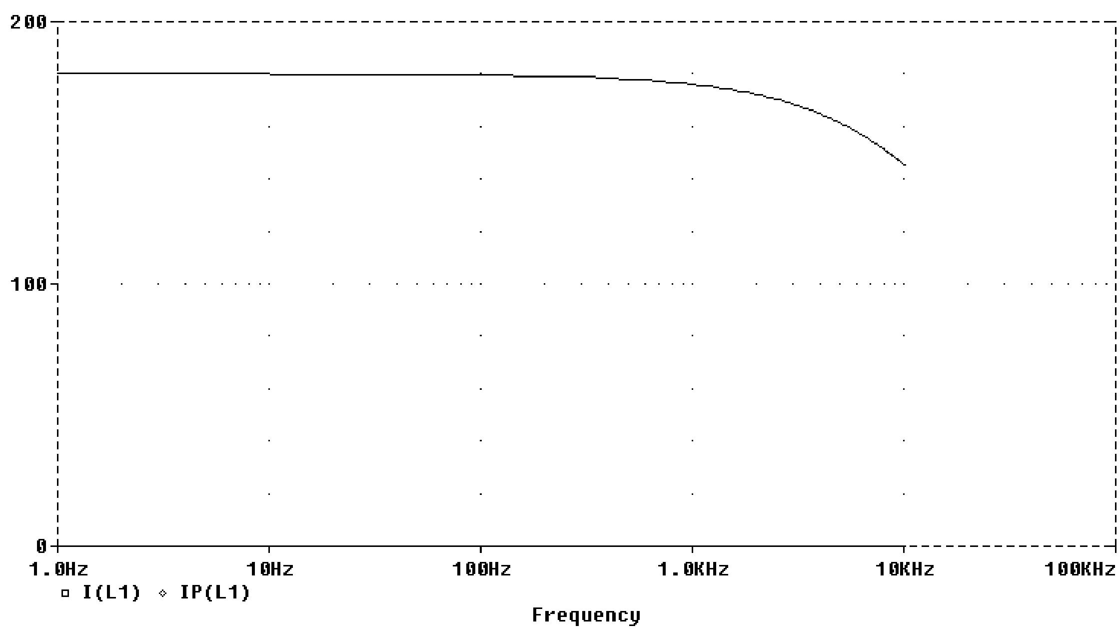
Figure 14.103

Solution

The schematic is shown below. A current marker is inserted to measure **I**. We set Total Points = 101, start Frequency = 1, and End Frequency = 10 kHz in the AC sweep box. After simulation, the magnitude and phase plots are obtained in the Probe menu as shown below.

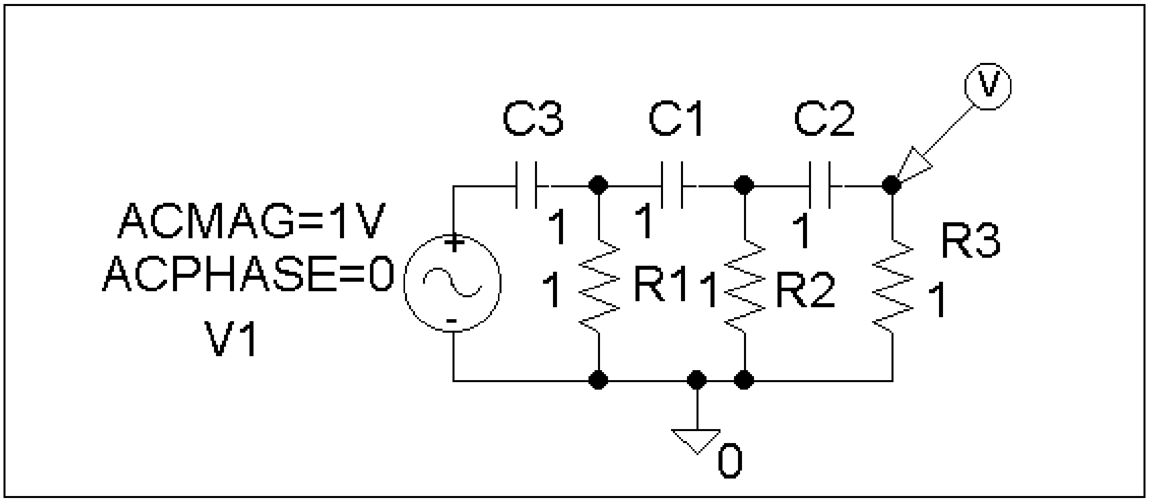


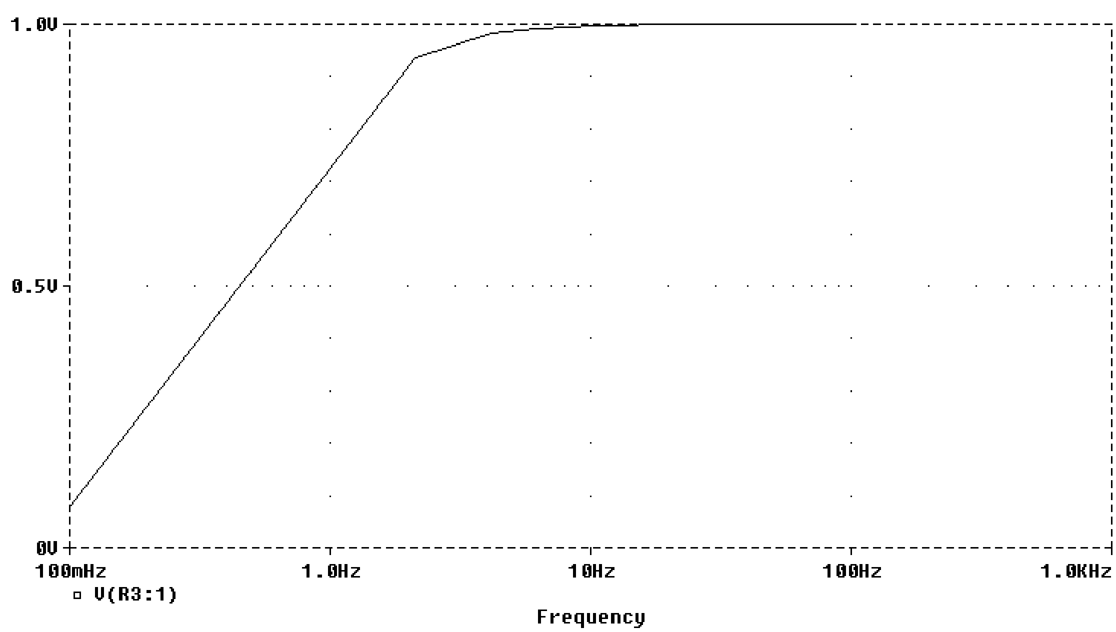

100mA
90mA
80mA
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
I(L1)
Frequency


200
100
0
1.0Hz
10Hz
100Hz
1.0KHz
10KHz
100KHz
I(L1) IP(L1)
Frequency

Solution 14.87

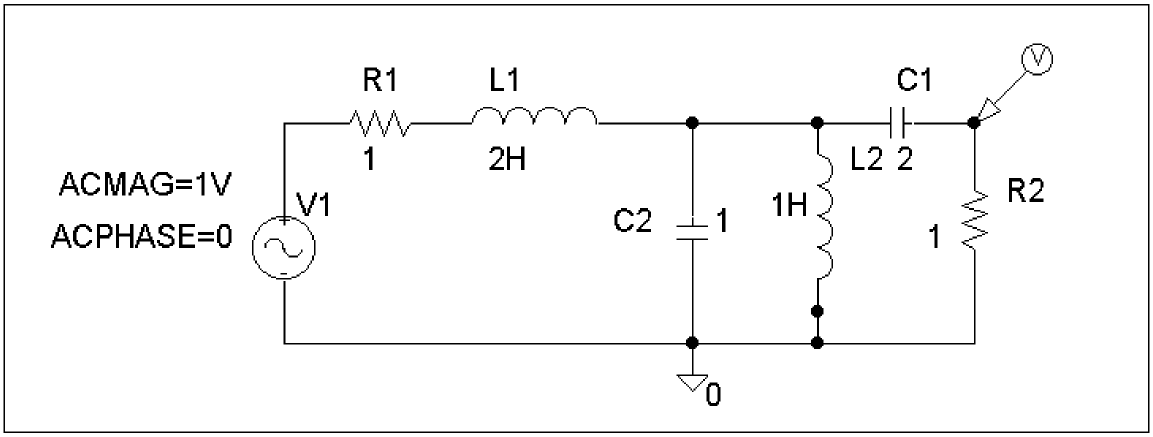
The schematic is shown below. I_n the AC Sweep box, we set Total Points = 50, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude response as shown below. It is evident from the response that the circuit represents a high-pass filter.

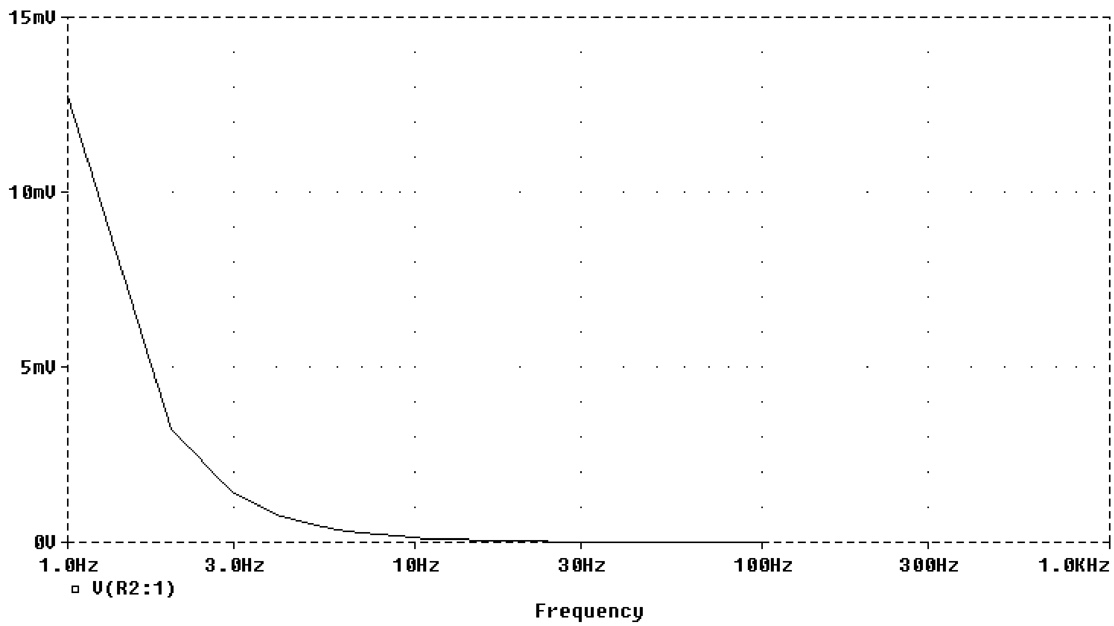


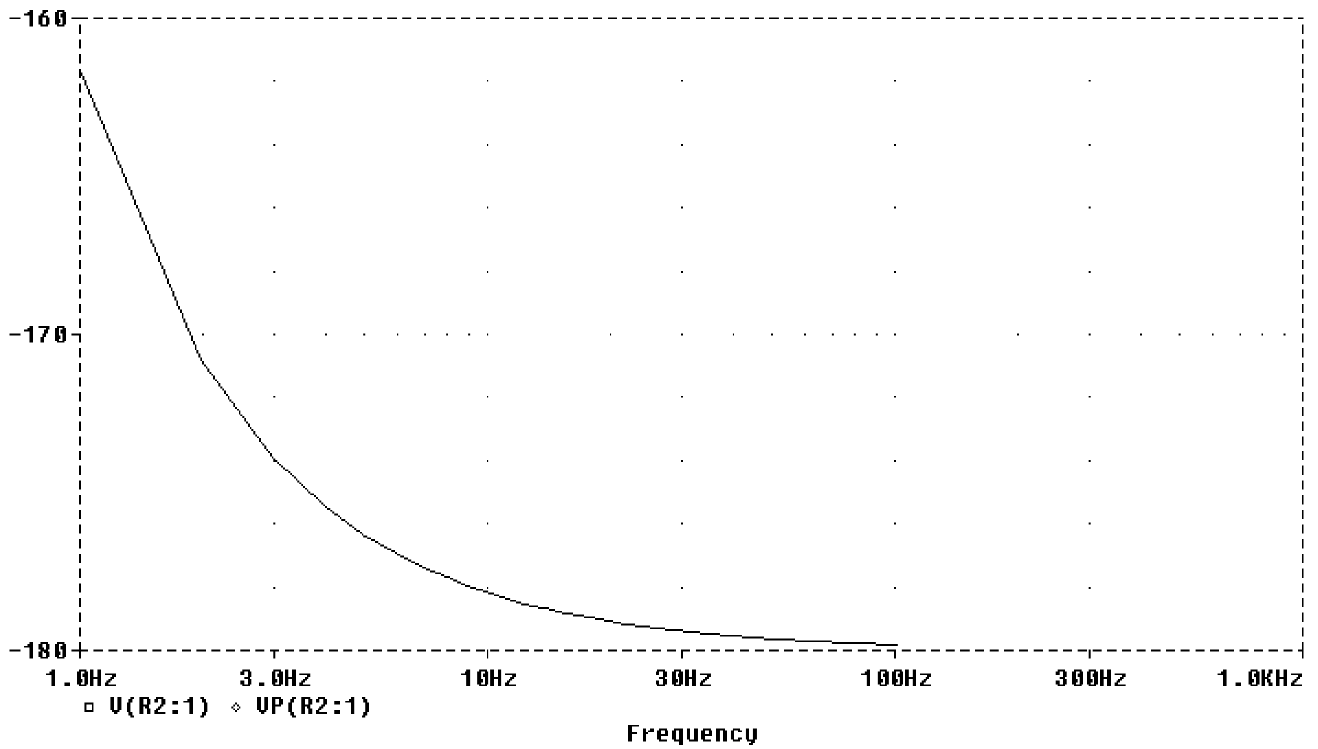


Solution 14.88

The schematic is shown below. We insert a voltage marker to measure V_o. In the AC Sweep box, we set Total Points = 101, Start Frequency = 1, and End Frequency = 100. After simulation, we obtain the magnitude and phase plots of V_o as shown below.

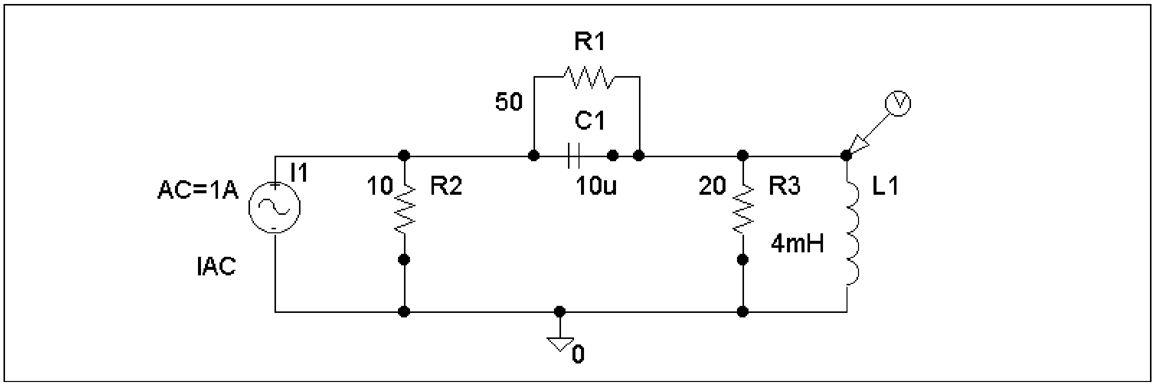


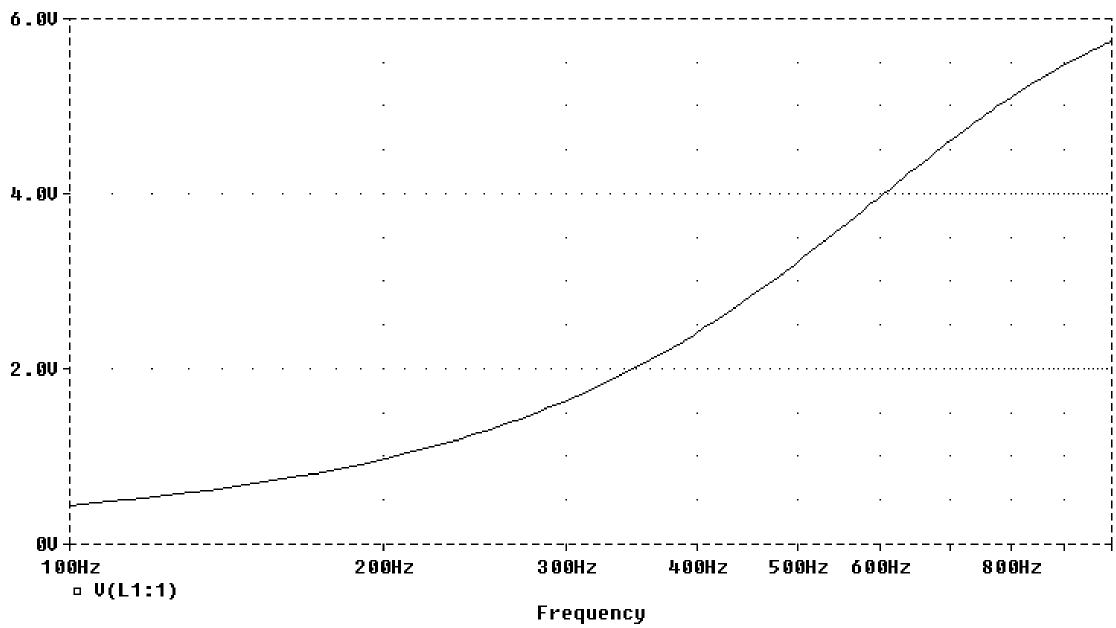



-160
-170
-180
1.0Hz
3.0Hz
10Hz
30Hz
100Hz
300Hz
1.0KHz
V(R2:1) VP(R2:1)
Frequency

Solution 14.89

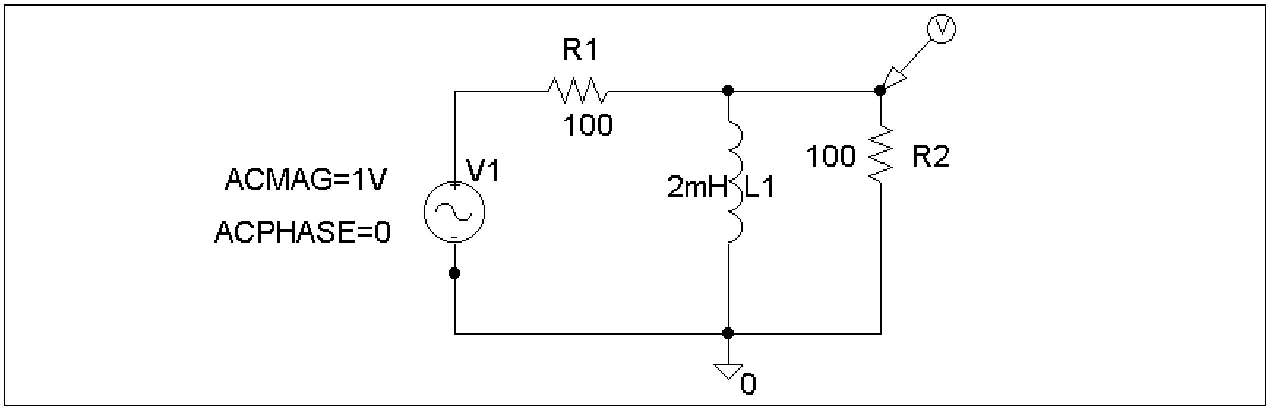
The schematic is shown below. In the AC Sweep box, we type Total Points = 101, Start Frequency = 100, and End Frequency = 1 k. After simulation, the magnitude plot of the response V_o is obtained as shown below.

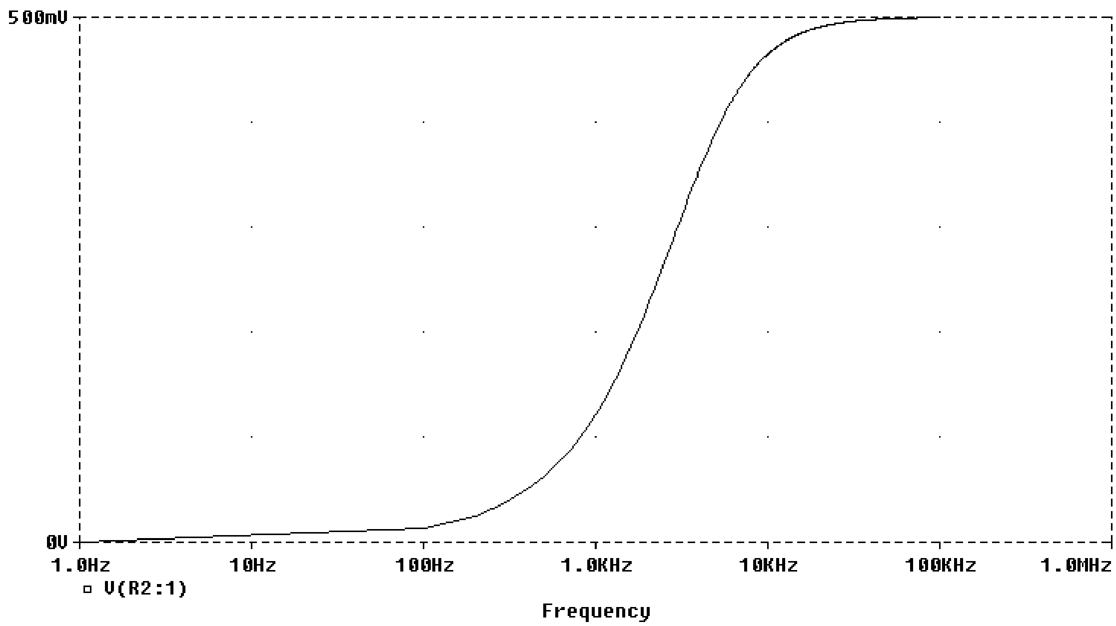




Solution 14.90

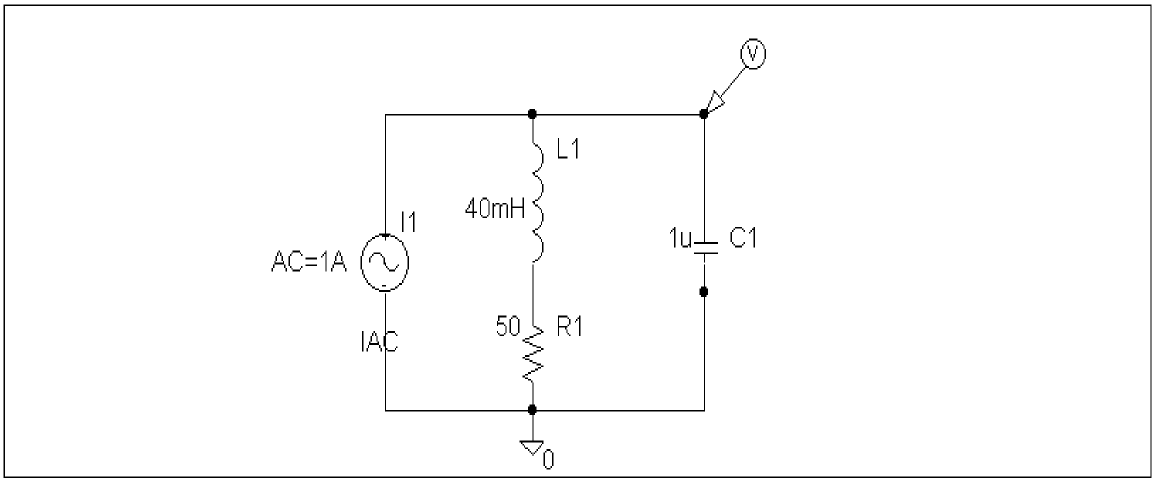
The schematic is shown below. In the AC Sweep box, we set
Total Points = 1001, Start Frequency = 1, and End Frequency = 100k. After simulation, we obtain the magnitude plot of the response as shown below. The response shows that the circuit is a high-pass filter.

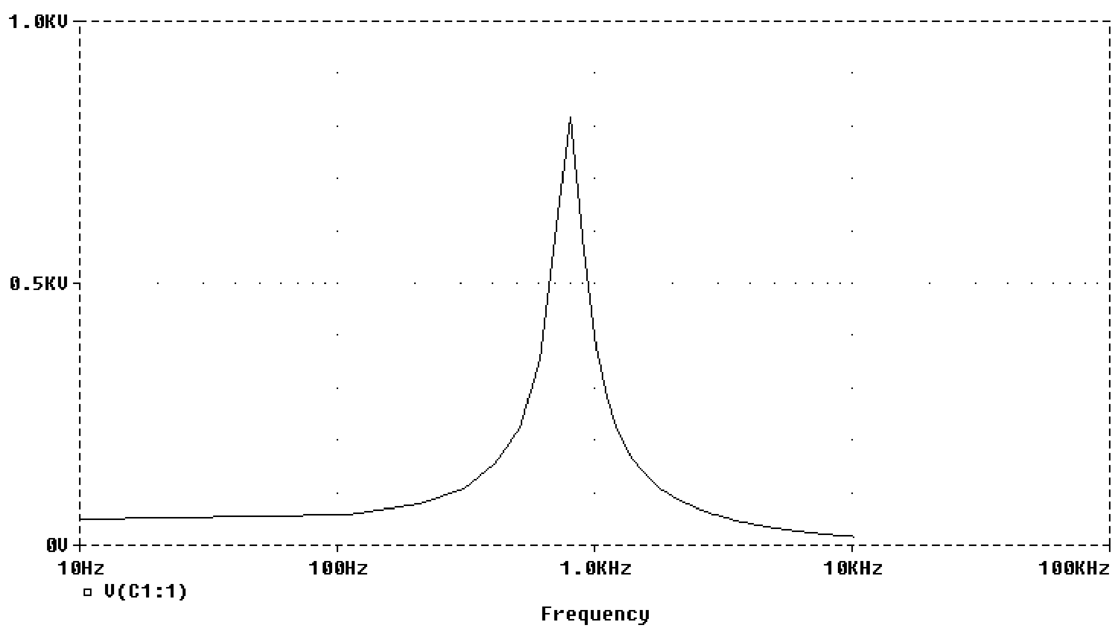




Solution 14.91

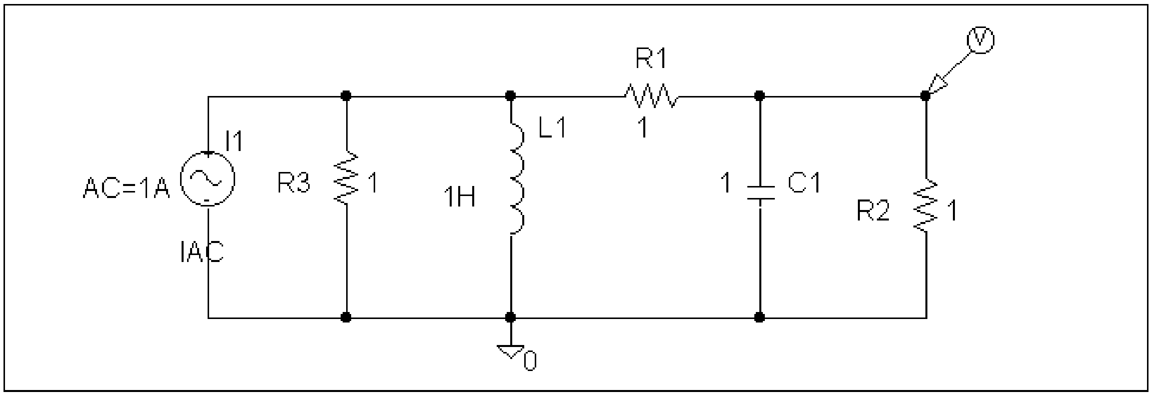
The schematic is shown below. In the AC Sweep box, we select Total Points = 101, Start Frequency = 10, and End Frequency = 10 k. After simulation, the magnitude plot of the frequency response is obtained. From the plot, we obtain the resonant frequency f_o is approximately equal to **800 Hz** so that $\omega_o = 2\pi f_o = 5026$ rad/s.

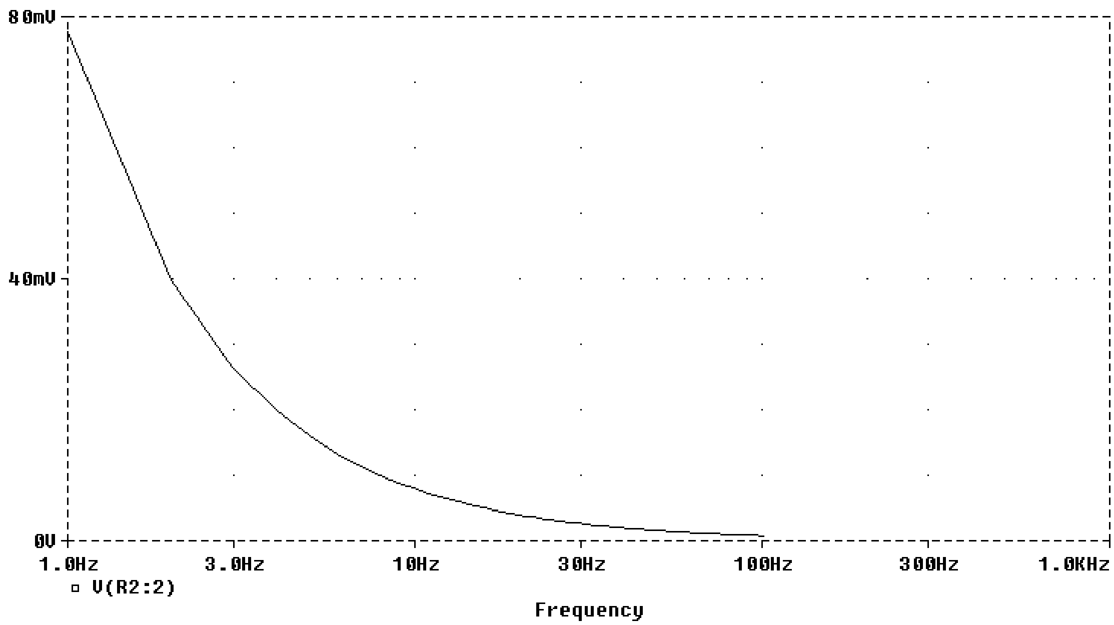




Solution 14.92

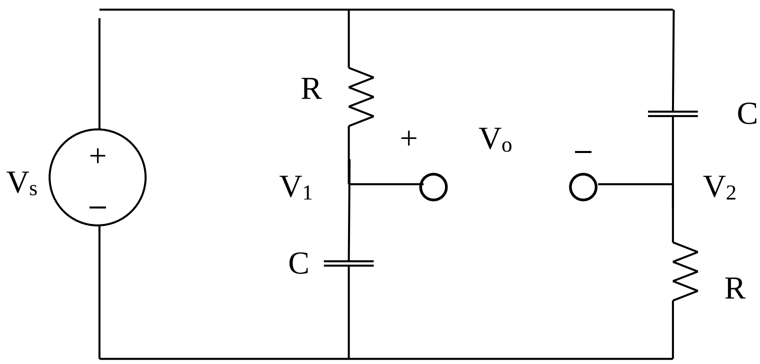
The schematic is shown below. We type Total Points = 101, Start Frequency = 1, and End Frequency = 100 in the AC Sweep box. After simulating the circuit, the magnitude plot of the frequency response is shown below.





Solution 14.93

Consider the circuit as shown below.



$$V_1 = \frac{\frac{1}{sC}}{R+\frac{1}{sC}}V_s = \frac{V}{1+sRC}$$

$$V_2 = \frac{R}{R+sC}V_s = \frac{sRC}{1+sRC}V_s$$

$$V_o = V_1 - V_2 = \frac{1-sRC}{1+sRC}V_s$$

Hence,

$$H(s) = \frac{V_o}{V_s} = \underline{\frac{1-sRC}{1+sRC}}$$

Solution 14.94

$$\omega_c = \frac{1}{RC}$$

We make R and C as small as possible. To achieve this, we connect 1.8 $k\Omega$ and 3.3 $k\Omega$ in parallel so that

$$R = \frac{1.8x3.3}{1.8+3.3} = 1.164\ k\Omega$$

We place the 10-pF and 30-pF capacitors in series so that

$$C = (10x30)/40 = 7.5\ pF$$

Hence,

$$\boldsymbol{\omega_c = \frac{1}{RC} = \frac{1}{1.164x10^3 x7.5x10^{-12}} = \underline{114.55x10^6\ rad/s}}$$

Solution 14.95

(a) $$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

When $C = 360\ \text{pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(360\times10^{-12})}} = 0.541\ \text{MHz}$$

When $C = 40\ \text{pF}$,

$$f_0 = \frac{1}{2\pi\sqrt{(240\times10^{-6})(40\times10^{-12})}} = 1.624\ \text{MHz}$$

Therefore, the frequency range is

$$\mathbf{0.541\ MHz < f_0 < 1.624\ MHz}$$

(b) $$Q = \frac{2\pi fL}{R}$$

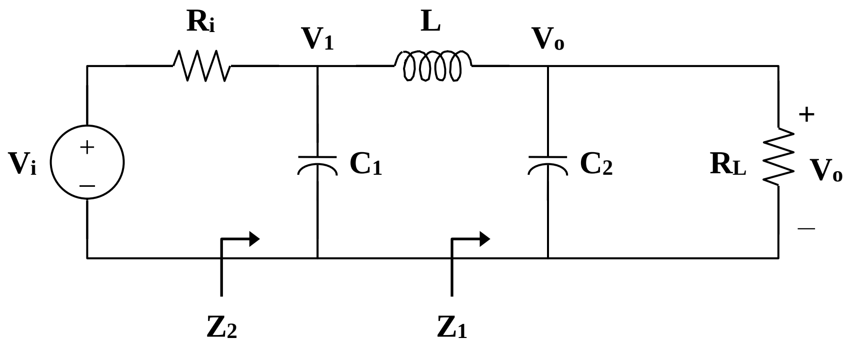
At $f_0 = 0.541\ \text{MHz}$,

$$Q = \frac{(2\pi)(0.541\times10^{6})(240\times10^{-6})}{12} = \mathbf{67.98}$$

At $f_0 = 1.624\ \text{MHz}$,

$$Q = \frac{(2\pi)(1.624\times10^{6})(240\times10^{-6})}{12} = \mathbf{204.1}$$

Solution 14.96



$$\mathbf{Z}_1 = R_L \parallel \frac{1}{sC_2} = \frac{R_L}{1+sR_2C_2}$$

$$\mathbf{Z}_2 = \frac{1}{sC_1} \parallel (sL+\mathbf{Z}_1) = \frac{1}{sC_1} \parallel \left(\frac{sL+R_L+s^2R_LC_2L}{1+sR_LC_2}\right)$$

$$\mathbf{Z}_2 = \frac{\dfrac{1}{sC_1}\cdot\dfrac{sL+R_L+s^2R_LC_2L}{1+sR_LC_2}}{\dfrac{1}{sC_1}+\dfrac{sL+R_L+s^2R_LC_2L}{1+sR_LC_2}}$$

$$\mathbf{Z}_2 = \frac{sL+R_L+s^2R_LLC_2}{1+sR_LC_2+s^2LC_1+sR_LC_1+s^3R_LLC_1C_2}$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2+R_i}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{\mathbf{Z}_1}{\mathbf{Z}_1+sL}\mathbf{V}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_2+R_2}\cdot\frac{\mathbf{Z}_1}{\mathbf{Z}_1+sL}\mathbf{V}_i$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{Z}_2}{\mathbf{Z}_2+R_2}\cdot\frac{\mathbf{Z}_1}{\mathbf{Z}_1+sL}$$

where

$$\frac{\mathbf{Z}_2}{\mathbf{Z}_2+R_2} =$$

$$\frac{sL+R_L+s^2R_LLC_2}{sL+R_L+s^2R_LLC_2+R_i+sR_iR_LC_2+s^2R_iLC_1+sR_iR_LC_1+s^3R_iR_LLC_1C_2}$$

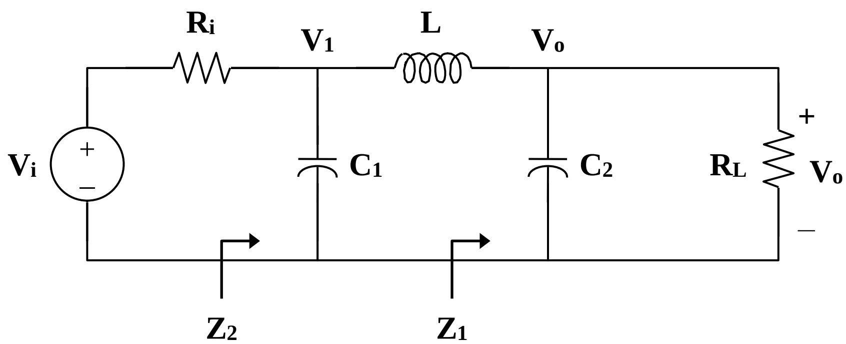
and $$\frac{\mathbf{Z}_1}{\mathbf{Z}_1+sL} = \frac{R_L}{R_L+sL+s^2R_LLC_2}$$

Therefore,

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\mathbf{R_L(sL + R_L + s^2 R_L LC_2)}}{\begin{gathered}\mathbf{(sL + R_L + s^2 R_L LC_2 + R_i + sR_i R_L C_2 + s^2 R_i LC_1 + sR_i R_L C_1} \\ \mathbf{+ s^3 R_i R_L LC_1 C_2)(R_L + sL + s^2 R_L LC_2)}\end{gathered}}$$

where $s = j\omega$.

Solution 14.97



$$\mathbf{Z} = sL \parallel \left(R_L + \frac{1}{sC_2} \right) = \frac{sL(R_L + 1/sC_2)}{R_L + sL + 1/sC_2}, \qquad s = j\omega$$

$$\mathbf{V}_1 = \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{V}_o = \frac{R_L}{R_L + 1/sC_2}\mathbf{V}_1 = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{\mathbf{Z}}{\mathbf{Z} + R_i + 1/sC_1}\mathbf{V}_i$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_L}{R_L + 1/sC_2} \cdot \frac{sL(R_L + 1/sC_2)}{sL(R_L + 1/sC_2) + (R_i + 1/sC_1)(R_L + sL + 1/sC_2)}$$

$$\mathbf{H}(\omega) = \mathbf{\frac{s^3 L R_L C_1 C_2}{(sR_i C_1 + 1)(s^2 L C_2 + sR_L C_2 + 1) + s^2 L C_1 (sR_L C_2 + 1)}}$$

where $s = j\omega$.

Solution 14.98

$$B = \omega_2 - \omega_1 = 2\pi(f_2 - f_1) = 2\pi(454 - 432) = 44\pi$$

$$\omega_0 = 2\pi f_0 = QB = (20)(44\pi)$$

$$f_0 = \frac{(20)(44\pi)}{2\pi} = (20)(22) = \mathbf{440\ Hz}$$

Solution 14.99

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f\, C}$$

$$C = \frac{1}{2\pi f\, X_c} = \frac{1}{(2\pi)(2\times 10^6)(5\times 10^3)} = \frac{10^{-9}}{20\pi}$$

$$X_L = \omega L = 2\pi f\, L$$

$$L = \frac{X_L}{2\pi f} = \frac{300}{(2\pi)(2\times 10^6)} = \frac{3\times 10^{-4}}{4\pi}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\dfrac{3\times 10^{-4}}{4\pi}\cdot\dfrac{10^{-9}}{20\pi}}} = \textbf{8.165 MHz}$$

$$B = \frac{R}{L} = (100)\left(\frac{4\pi}{3\times 10^{-4}}\right) = \mathbf{4.188\times 10^6\ rad/s}$$

Solution 14.100

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(20\times 10^3)(0.5\times 10^{-6})} = \mathbf{15.91\,\Omega}$$

Solution 14.101

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$R = \frac{1}{2\pi f_c C} = \frac{1}{(2\pi)(15)(10\times10^{-6})} = \mathbf{1.061\ k\Omega}$$

Solution 14.102

(a) When $R_s = 0$ and $R_L = \infty$, we have a low-pass filter.

$$\omega_c = 2\pi f_c = \frac{1}{RC}$$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(4\times 10^3)(40\times 10^{-9})} = \mathbf{994.7\ Hz}$$

(b) We obtain R_{Th} across the capacitor.

$$R_{Th} = R_L \parallel (R + R_s)$$

$$R_{Th} = 5 \parallel (4+1) = 2.5\ \text{k}\Omega$$

$$f_c = \frac{1}{2\pi R_{Th} C} = \frac{1}{(2\pi)(2.5\times 10^3)(40\times 10^{-9})}$$

$$f_c = \mathbf{1.59\ kHz}$$

Solution 14.103

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{R_2 + R_1 \parallel 1/j\omega C}, \qquad s = j\omega$$

$$\mathbf{H}(s) = \frac{R_2}{R_2 + \dfrac{R_1(1/sC)}{R_1 + 1/sC}} = \frac{R_2(R_1 + 1/sC)}{R_1 R_2 + (R_1 + R_2)(1/sC)}$$

$$\mathbf{H}(s) = \frac{\mathbf{R_2(1 + sCR_1)}}{\mathbf{R_1 + R_2 + sCR_1R_2}}$$

Solution 14.104

The schematic is shown below. We click Analysis/Setup/AC Sweep and enter Total Points = 1001, Start Frequency = 100, and End Frequency = 100 k. After simulation, we obtain the magnitude plot of the response as shown.