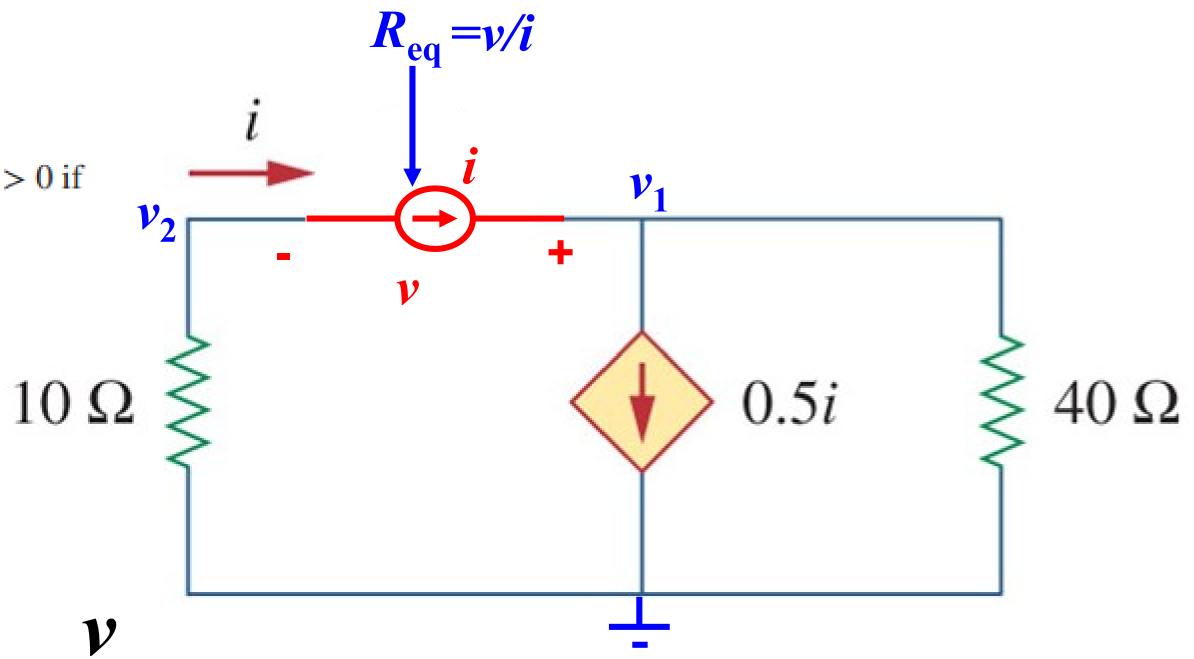
Problem 7.19 P301

7.19 In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 5$ A.



Solution:

Node v_1 : $\quad i = 0.5i + \dfrac{v_1}{40}$

$$\Rightarrow v_1 = 20i$$

Node v_2 : $\quad v_2 = -10i$

So : $v = v_1 - v_2 = 30i$

Hence $R_{eq} = \dfrac{v}{i} = 30$

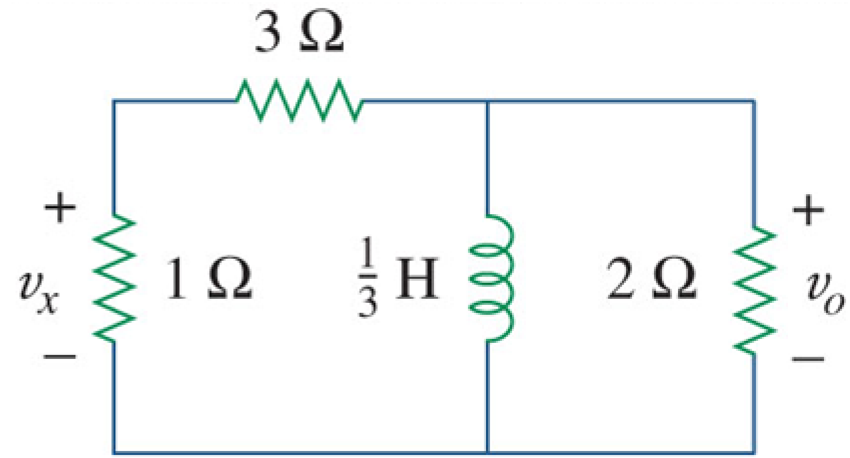
$$\tau = \frac{L}{R_{eq}} = \frac{6}{30} = \frac{1}{5}$$

$$i(t) = i(0)e^{-\frac{t}{\tau}}$$

$$= 5e^{-5t} \quad (t > 0)$$

Problem 7.23 P302

Consider the circuit in Fig. 7.103. Given that $v_o(0) = 10$ V, find v_o and v_x for $t > 0$.



Solution:

$$v_x(0) = \frac{1}{3+1} v_o(0) = \frac{5}{2}$$

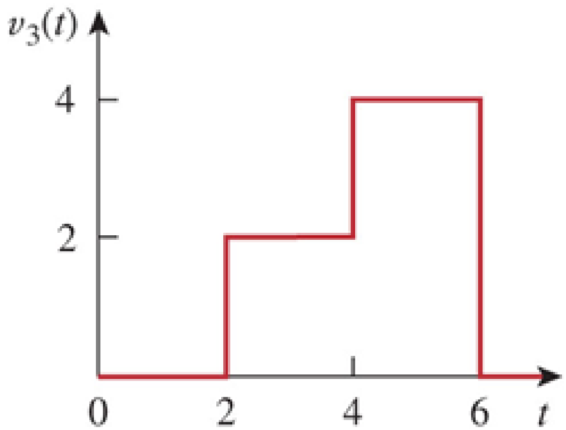
$$R_{eq} = (1+3) \| 2 = \frac{4}{3}$$

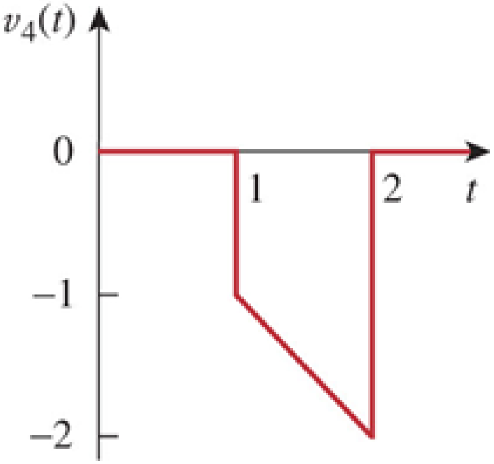
$$\tau = \frac{L}{R_{eq}} = \frac{1}{4}$$

$$v_0 = v_0(0) e^{-\frac{t}{\tau}} = 10\ e^{-4t} \quad (t > 0)$$

$$v_x = v_x(0) e^{-\frac{t}{\tau}} = \frac{5}{2} e^{-4t} \quad (t > 0)$$

Problem 7.26 P302 Express the signals in Fig. 7.104 in terms of singularity functions.





Solution:

$$v_3(t) = 2u(t-2) + 2u(t-4) - 4u(t-6)$$

Or $$v_3(t) = 2[u(t-2) - u(t-4)] + 4[u(t-4) - u(t-6)]$$

$$v_4(t) = -t[u(t-1) - u(t-2)]$$

$$= -(t-1+1)u(t-1) + (t-2+2)u(t-2)$$

$$= -r(t-1) + r(t-2) - u(t-1) + 2u(t-2)$$

Problem 7.31 P303 Evaluate the following integrals:

$$(a)\quad \int_{-\infty}^{+\infty} e^{-4t^2}\delta(t-2)dt = e^{-4t^2}\Big|_{t=2} = e^{-16}$$

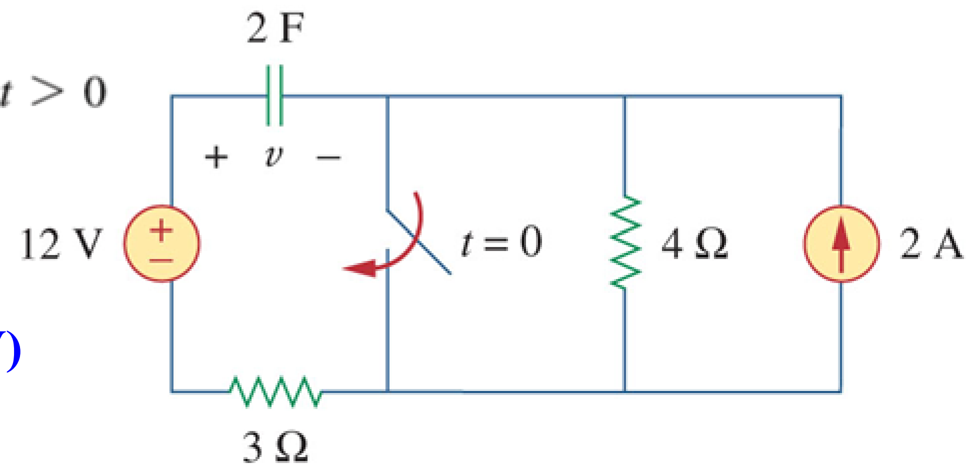
$$(b)\quad \int_{-\infty}^{+\infty}\left[5\delta(t)+e^{-t}\delta(t)+\cos 2\pi t\delta(t)\right]dt$$

$$= 5 + e^{-t}\Big|_{t=0} + \cos 2\pi t\Big|_{t=0}$$

$$= 5+1+1=7$$

Problem 7.39（b） P303

Calculate the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.106.



Solution:

$t < 0 \quad v = 12 - 4 \times 2 = 4\cancel{V}$ (V)

$t > 0 \quad v(0^+) = 4\text{V}$

$v(\infty) = 12\text{V}$

$R_{eq} = 3\Omega$

$\tau = R_{eq}C = 3 \times 2 = 6$ (s)

So $\quad v = v(\infty) + \left[v(0^+) - v(\infty)\right]e^{-\frac{t}{\tau}}$

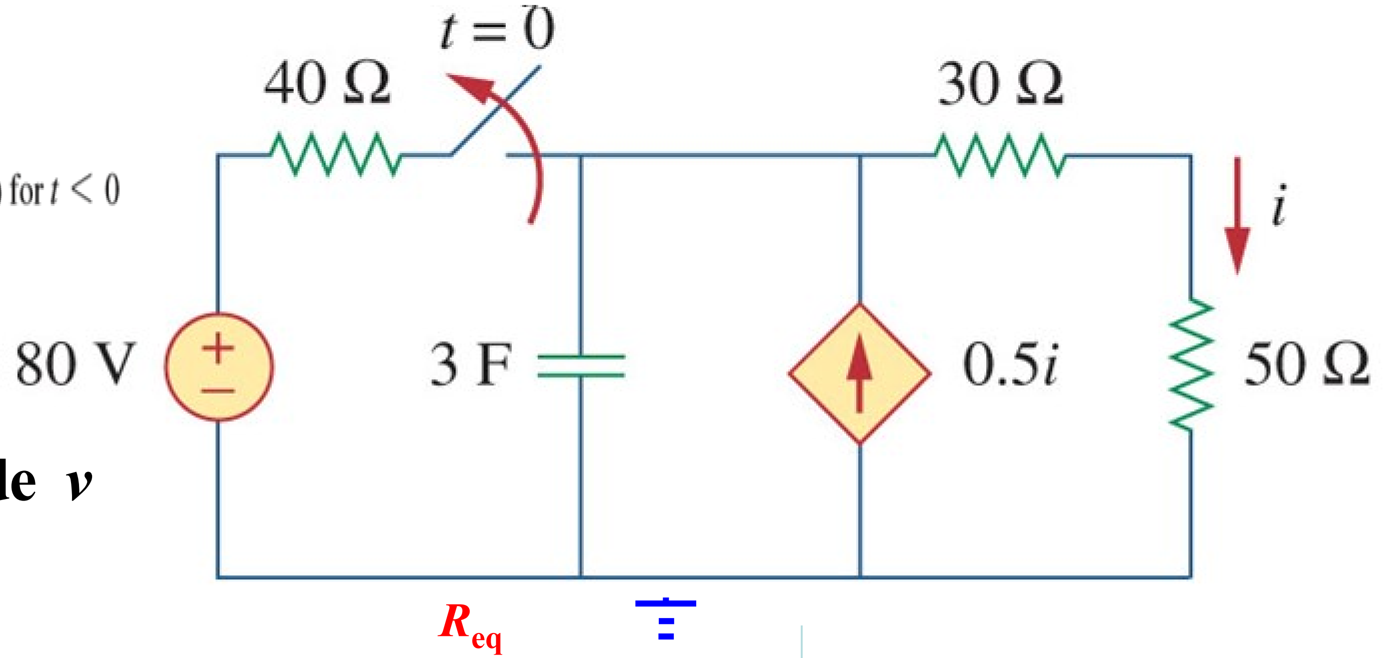
$= 12 - 8e^{-\frac{t}{6}}$ (V)

So

$$v = \begin{cases} 4 \text{ V} & (t < 0) \\ \cancel{12 - 8e^{-\frac{t}{6}}} \quad (12 - 8e^{-\frac{t}{6}})\text{V} & (t > 0) \end{cases}$$

Problem 7.43

Consider the circuit in Fig. 7.110. Find $i(t)$ for $t < 0$ and $t > 0$.



Solution:

When $t < 0$, at node v

$$\frac{80 - v}{40} + 0.5i = i$$

$$i = \frac{v}{30+50}$$

Find $v = 64\text{V}, i = \frac{4}{5}\text{A}$

At $t = 0^+, v(0^+) = 64$

$$i(0^+) = \frac{v(0^+)}{30+50} = \frac{4}{5}\text{A}$$

$$i\ (\infty) = 0$$

For R_{eq} $\quad i_s + 0.5i = i$

$$v = (30+50)i$$

Find $v = 160 i_s$

So $R_{eq} = \dfrac{v}{i_s} = 160\Omega$

So $\tau = R_{eq}C = 480\text{s}$

So $t > 0$

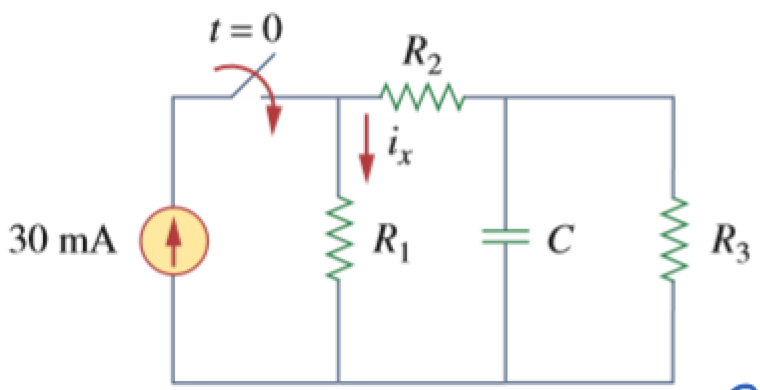
$$i = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{-\frac{t}{\tau}} = \frac{4}{5}e^{-\frac{t}{480}}$$

So

$$i = \begin{cases} \frac{4}{5} & (t < 0) \\ \frac{4}{5}e^{-\frac{t}{480}} & (t > 0) \end{cases}$$

Problem 7.50 P305

In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1\text{ k}\Omega$, $R_3 = 2\text{ k}\Omega$, and $C = 0.25$ mF.



$$v(\infty) = \frac{R_1}{R_1+R_2+R_3} \times 30 \times R_3 = 15V$$

$$R_{eq} = (R_1 + R_2) \parallel R_3 = 1\text{ k}\Omega$$

$$\tau = R_{eq} \cdot C = \frac{1}{4} s$$

So, $v(t) = v(\infty) + [v(0^+) - v(\infty)] e^{-4t}$

$$= 15 - 15e^{-4t} \; V$$

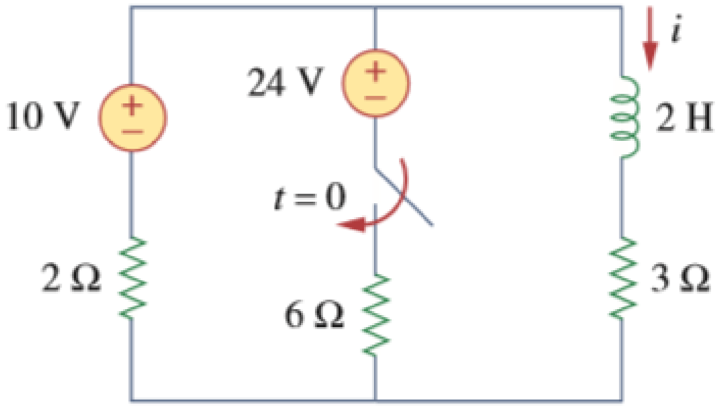
Then, $i(t) = 30 - \frac{v(t)}{R_3} - C\frac{dv(t)}{dt}$

$$= 30 - \frac{15 - 15e^{-4t}}{2} - \frac{1}{4} \times (-4) \times (-15) e^{-4t}$$

$$= \frac{45}{2} - \frac{15}{2} e^{-4t} \; mA$$

Problem 7.54 P305

Obtain the inductor current for both $t < 0$ and $t > 0$ in each of the circuits in Fig. 7.120.



(b)

Solution:

$t<0$: $i(t) = \frac{10}{2+3} = 2\,A$

So: $i(0^+) = 2A$

$t \to \infty$: $\frac{v-10}{2} + \frac{v-24}{6} + \frac{v}{3} = 0$

$\Rightarrow v = 9v$

$i(\infty) = \frac{v}{3} = 3\,A$

$t>0$: $R_{eq} = 3 + 2 // 6 = \frac{9}{2}$

So: $\tau = \frac{L}{R_{eq}} = \frac{4}{9}$

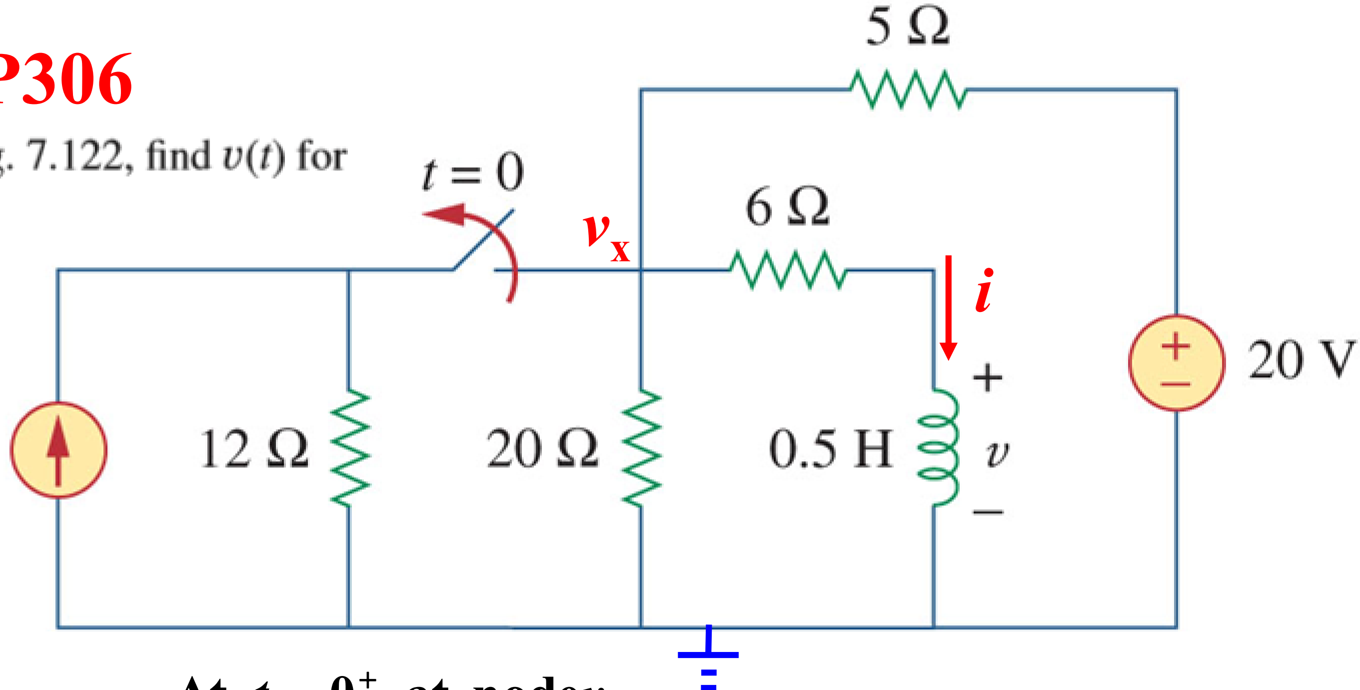
So: $i(t) = i(\infty) + [i(0^+) - i(\infty)]\, e^{-t/\tau}$

$= 3 - e^{-\frac{9}{4}t}\ A$

So: $i(t) = \begin{cases} 2\,A & t<0 \\ 3 - e^{-\frac{9}{4}t}\ A & t>0 \end{cases}$

Problem 7.56 P306

For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.



Solution:

When $t < 0$, at node v_x

$$\frac{v_x}{12} + \frac{v_x}{20} + \frac{v_x}{6} + \frac{v_x - 20}{5} = 2$$

Find $v_x = 12\text{V}$

So $i = \dfrac{v_x}{6} = 2\text{A}$

So $i\ (0^+) = 2\text{A}$

At $t = 0^+$, at node v_x

$$\frac{v_x(0^+)}{20} + 2 + \frac{v_x(0^+) - 20}{5} = 0 \quad \text{Find } v_x(0^+) = 8\text{V}$$

So $v(0^+) = v_x(0^+) - 6i(0^+) = \text{-4V}$

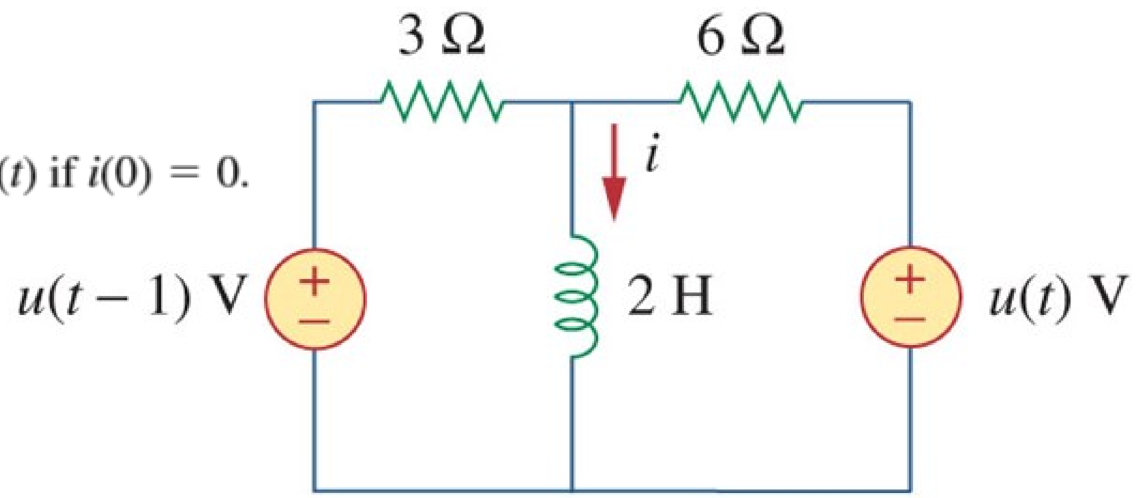
Since $v(\infty) = 0\text{V}$ $\quad R_{eq} = 6 + 20 \| 5 = 10\Omega$

So $\tau = L/R_{eq} = 1/20$

So $v = v(\infty) + [v(0^+) - v(\infty)]e^{-\frac{t}{\tau}} = -4e^{-20t}$

Problem 7.62 P306

For the circuit in Fig. 7.127, calculate $i(t)$ if $i(0) = 0$.



Solution:

For $0 < t < 1$

$$i(0^+) = 0$$

$$i(\infty) = \frac{1}{6}$$

$$R_{eq} = 3 \| 6 = 2\Omega$$

So $\tau = L/R_{eq} = 1$

So $i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{-\frac{t}{\tau}}$

$$= \frac{1}{6}\left(1 - e^{-t}\right) \qquad (0 < t < 1)$$

For $t = 1$

$$i(1) = \frac{1}{6}(1 - e^{-1}) = 0.1054$$

For $t > 1$ $\quad i(\infty) = \frac{1}{6} + \frac{1}{3} = 0.5$

$$\tau' = L/R_{eq} = 1$$

$$i(t) = i(\infty) + \left[i(1) - i(\infty)\right]e^{-\frac{t-1}{\tau'}}$$

$$= 0.5 - 0.3946e^{-t+1} \qquad (t > 1)$$