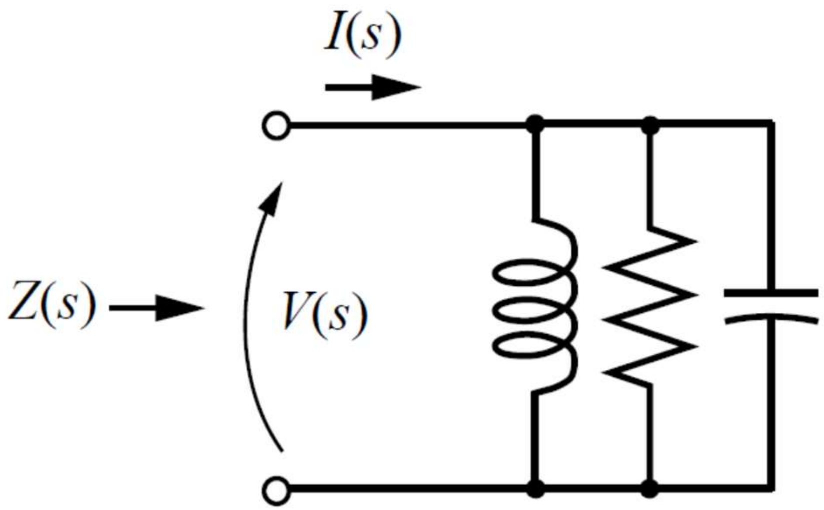
Lecture 5

Two Port Networks

Why two port networks?

So far we have looked at network elements with just one port. Furthermore we have also looked at small networks which have one port, for example for driving point impedance.

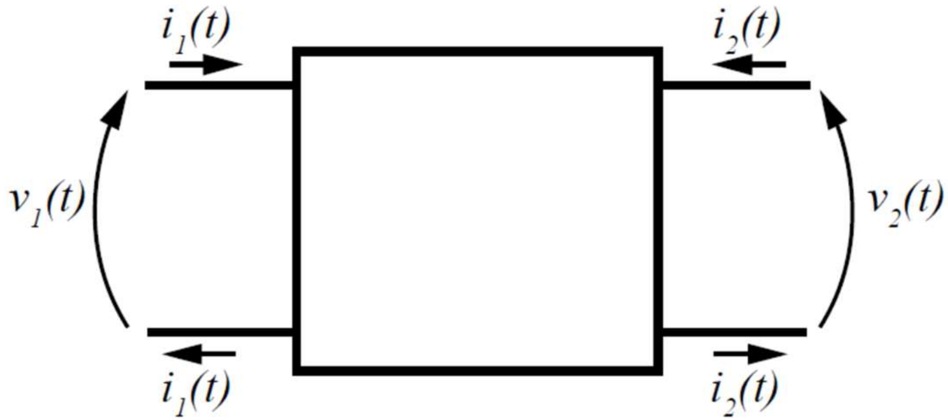


Two ports are network elements which have 2 ports. The two port element may be composed of a single component or some subnetwork consisting of many components. The concept can be extended to multiports elements, but we just consider two ports here.

Why two port networks? (cont'd)

Each port is associated with just 2 directed variables – a flow variable – current, and a rise variable – potential difference. Like in a one port, at each port of the two port, the current going in is identical to the current coming out.

So a two port has four directed variables associated with it: $v_1(t)$, $i_1(t)$, $v_2(t)$, $i_2(t)$.

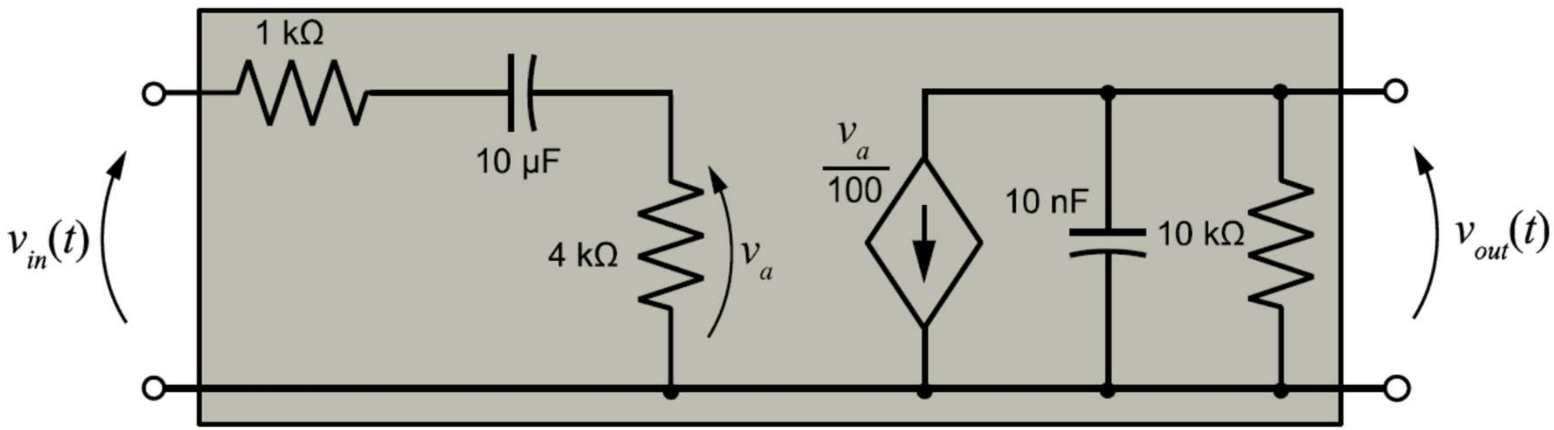


The two terminals of each port are located close together so that the axioms of circuit theory apply in the localized region of each port. **However,** the ports may be separated by a large distance depending on the two port element and its use in the larger network.

Why two port networks? (cont'd)

Two ports can be employed ***to represent a subnetwork or functional block***. (e.g. an amplifier or a filter). In many systems we have networks which say take an input voltage and produce an output voltage.

Representing a whole subnetwork by a two port can simply analysis and design.



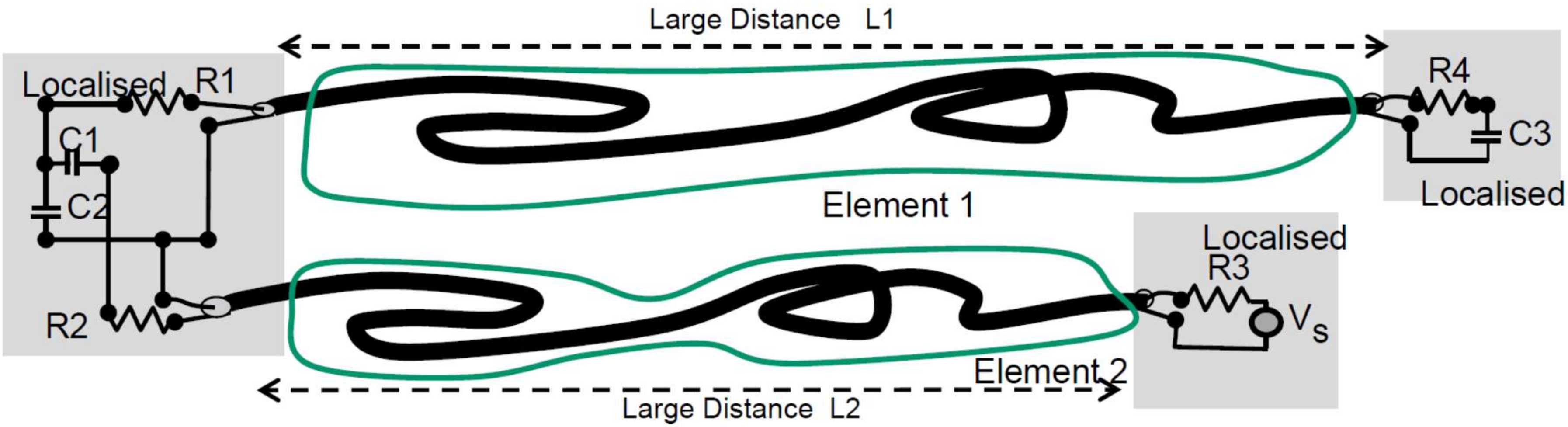
Why two port networks? (cont'd)

Two ports can also represent a ***single component or network element*** which is naturally represented as a two port.

For example: a transformer, or a transmission line.

The ***transmission line*** is an important example of a two port element. It is a distributed element, so the lumped circuit approximation is not valid for the element itself. However, we can obtain a two port description or model of the transmission line, and this two port can be used in networks, and all currents and voltages in those networks can be found using circuit theory.

Why two port networks? (cont'd)



In the above network we have a number of localized regions which are connected via transmission lines which are modelled by two ports.

In each localized region all nodes, circuit loops and the ports of all elements are close together compared to the shortest wavelength of any sources in the network. Hence network/circuit theory can be employed to solve for all voltages and currents in the network.

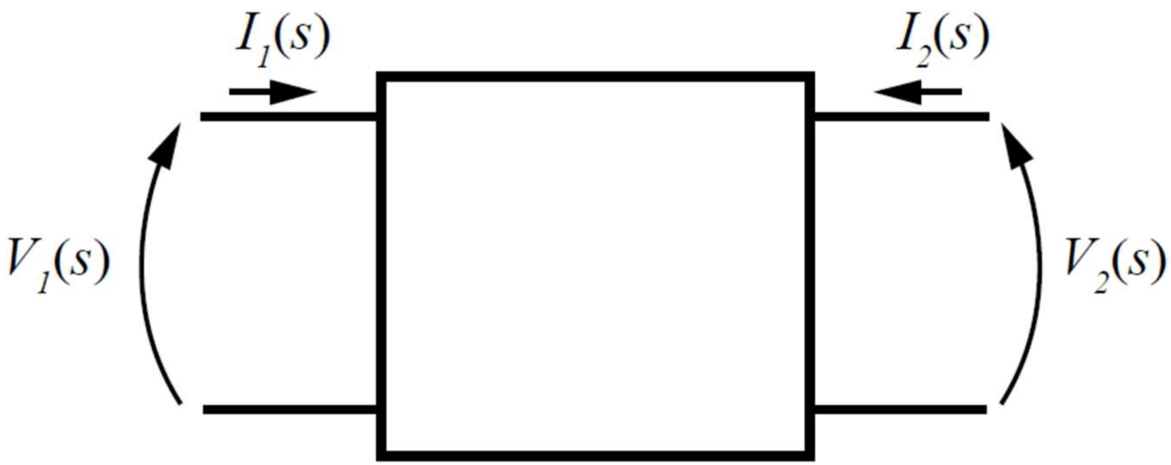
We will not study transmissions lines in this course. However, they demonstrate the power of two port models.

Two port networks

In this course we will consider just two ports which are composed of linear elements and contain no independent sources. Dependent sources are permitted. Being linear networks we can represent the potential differences and currents at the ports in the Laplace domain: $V_1(s)$, $I_1(s)$, $V_2(s)$, $I_2(s)$.

We may also use a representation in the Frequency domain.

Note also the direction of the currents, with the inward directed current at the higher potential side of each port.



Two port networks (cont'd)

Considering $I_1(s)$. Since the network is linear and there are no independent sources inside, then $I_1(s)$ can be considered the superposition of two components, one caused by $V_1(s)$ and another caused by $V_2(s)$. Similarly for $I_2(s)$.

So only two of the four variables $V_1(s)$, $I_1(s)$, $V_2(s)$, $I_2(s)$ are independent. We can express any two of the four in terms of the other two.

In fact there turns out to be six ways we can express two of the variables in terms of the other two. Which one we choose will depend on the application of the two port and also may depend on how the physical component modelled by the two port can be measured.

Admittance or y-parameters

Here $I_1(s)$, $I_2(s)$ are a function of $V_1(s)$ and $V_2(s)$.

$$I_1(s) = y_{11}V_1(s) + y_{12}V_2(s)$$
$$I_2(s) = y_{21}V_1(s) + y_{22}V_2(s)$$

In Matrix form: $$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = [y] \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$$

$$y_{11} = \left. \frac{I_1(s)}{V_1(s)} \right|_{V_2(s)=0}$$ $\equiv$ short circuit input admittance

$$y_{22} = \left. \frac{I_2(s)}{V_2(s)} \right|_{V_1(s)=0}$$ $\equiv$ short circuit output admittance

$$y_{12} = \left. \frac{I_1(s)}{V_2(s)} \right|_{V_1(s)=0}$$ $\equiv$ short circuit reverse transfer admittance

$$y_{21} = \left. \frac{I_2(s)}{V_1(s)} \right|_{V_2(s)=0}$$ $\equiv$ open circuit forward transfer admittance

Impedance or z-parameters

Here $V_1(s)$, $V_2(s)$ are a function of $I_1(s)$ and $I_2(s)$.

$$V_1(s) = z_{11}I_1(s) + z_{12}I_2(s)$$
$$V_2(s) = z_{21}I_1(s) + z_{22}I_2(s)$$

In Matrix form: $$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = [z] \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$z_{11} = \left.\frac{V_1(s)}{I_1(s)}\right|_{I_2(s)=0} \equiv \text{ open circuit input impedance}$$

$$z_{22} = \left.\frac{V_2(s)}{I_2(s)}\right|_{I_1(s)=0} \equiv \text{ open circuit output impedance}$$

$$z_{12} = \left.\frac{V_1(s)}{I_2(s)}\right|_{I_1(s)=0} \equiv \text{ open circuit reverse transfer impedance}$$

$$z_{21} = \left.\frac{V_2(s)}{I_1(s)}\right|_{I_2(s)=0} \equiv \text{ open circuit forward transfer impedance}$$

Two port matrix description

Listing the six possible ways to express two of the variables in terms of the other two:

Name	Function		Equation
	Express	In terms of	
Impedance (z)	V_1, V_2	I_1, I_2	$V_1 = z_{11}I_1 + z_{12}I_2$ $V_2 = z_{21}I_1 + z_{22}I_2$
Admittance (y)	I_1, I_2	V_1, V_2	$I_1 = y_{11}V_1 + y_{12}V_2$ $I_2 = y_{21}V_1 + y_{22}V_2$
Transmission (T)	V_1, I_1	$V_2, -I_2$	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse transmission (t)	V_2, I_2	$V_1, -I_1$	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$
Hybrid (h)	V_1, I_2	V_2, I_1	$V_1 = h_{11}I_1 + h_{12}V_2$ $I_2 = h_{21}I_1 + h_{22}V_2$
Inverse hybrid (g)	V_2, I_1	V_1, I_2	$I_1 = g_{11}V_1 + g_{12}I_2$ $V_2 = g_{21}V_1 + g_{22}I_2$

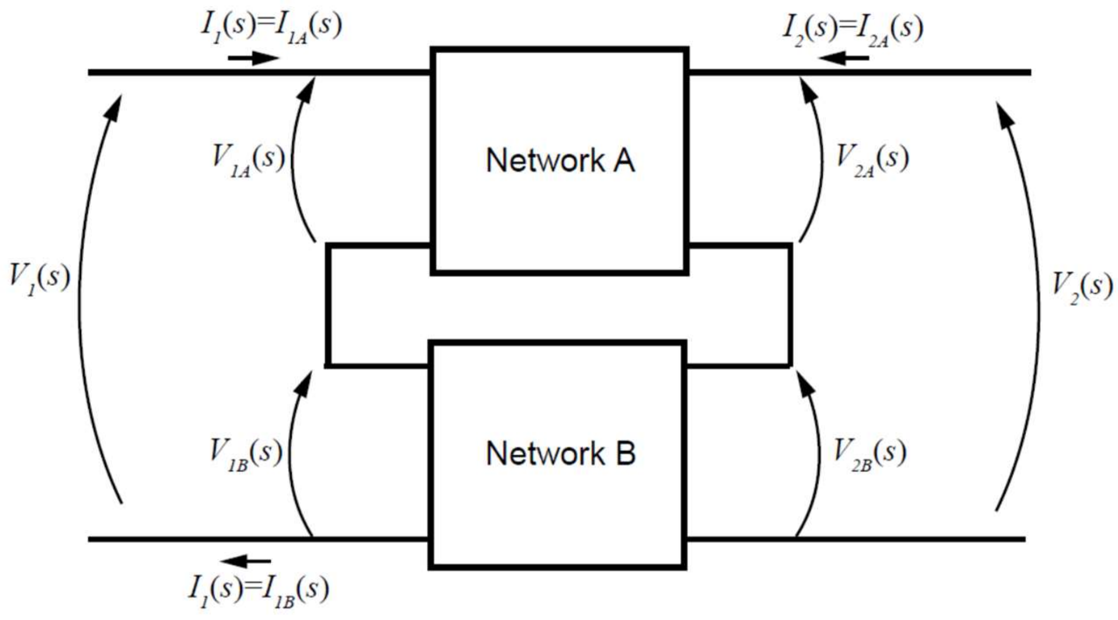
Connecting two ports together

Two ports connected together in various configurations can be represented by a single set of parameters:

For example the series connection of two z-parameter networks.

$$[V]=[V_A]+[V_B]=[z_A][I_A]+[z_B][I_B]$$
$$=([z_A]+[z_B])[I]=[z][I]$$

so: $[z]=[z_A]+[z_B]$



Connecting two ports together

For a parallel connection of two y-parameter networks.

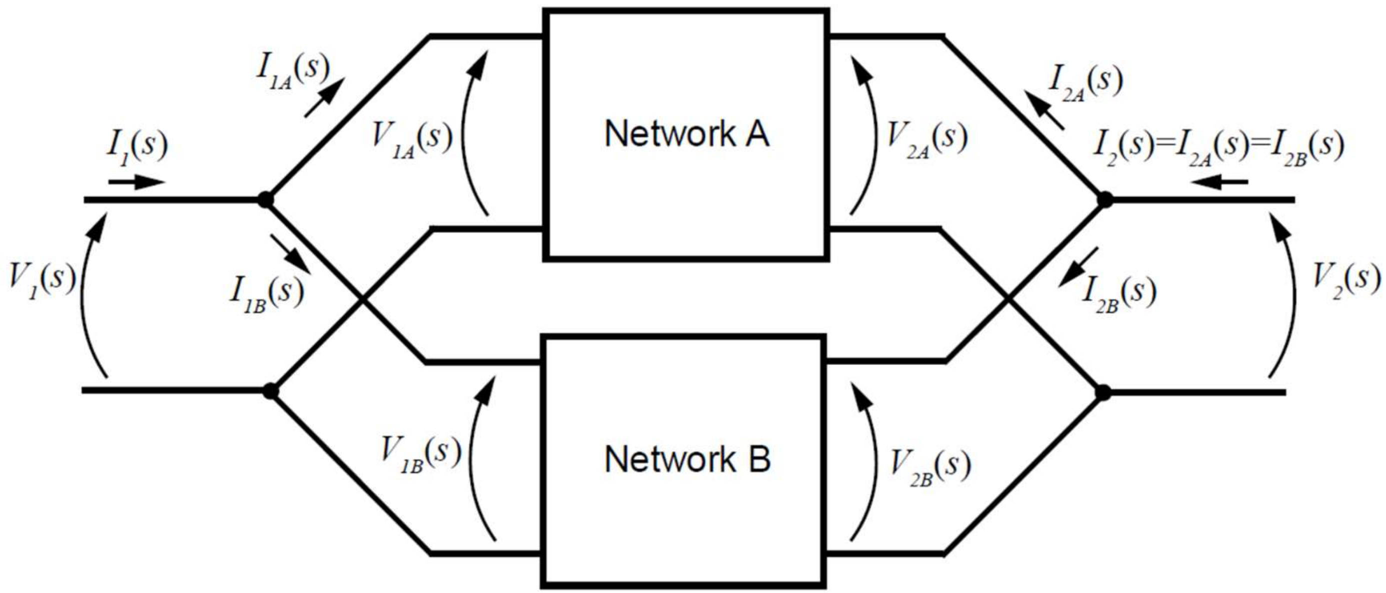
$$[V]=[V_A]=[V_B] \text{ and } [I]=[I_A]+[I_B]$$

$$[I_A]=[y_A][V_A] \text{ and } [I_B]=[y_B][V_B]$$

so:

$$[I]=([y_A]+[y_B])[V]$$

$$[y]=[y_A]+[y_B]$$



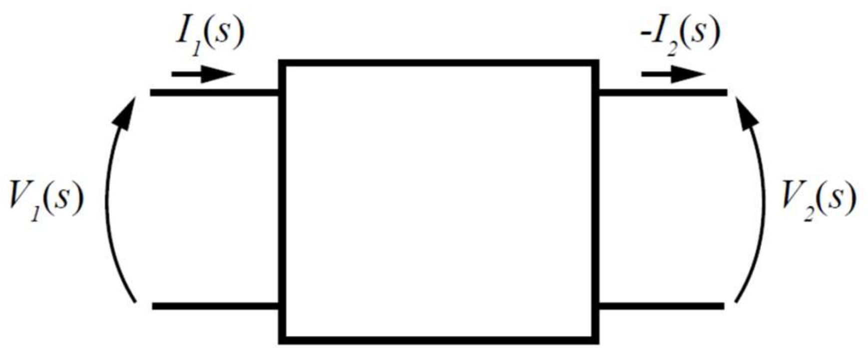
ABCD parameters

We also consider here one other two port matrix description, ABCD parameters or also called t or transmission parameters.

$$V_1(s) = AV_2(s) - BI_2(s)$$
$$I_1(s) = CV_2(s) - DI_2(s)$$

$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} A\,B \\ C\,D \end{bmatrix} \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = \begin{bmatrix} t_{11}\,t_{12} \\ t_{21}\,t_{22} \end{bmatrix} \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = [t] \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix}$$

Note that the current direction on the right is reversed, so a negative sign in front of I2(s). Quantities on the left are often thought of as given or independent variables, i.e. input voltage and current. Quantities on the right are the output variables. Major use of ABCD parameters in cascaded networks and transmission line analysis



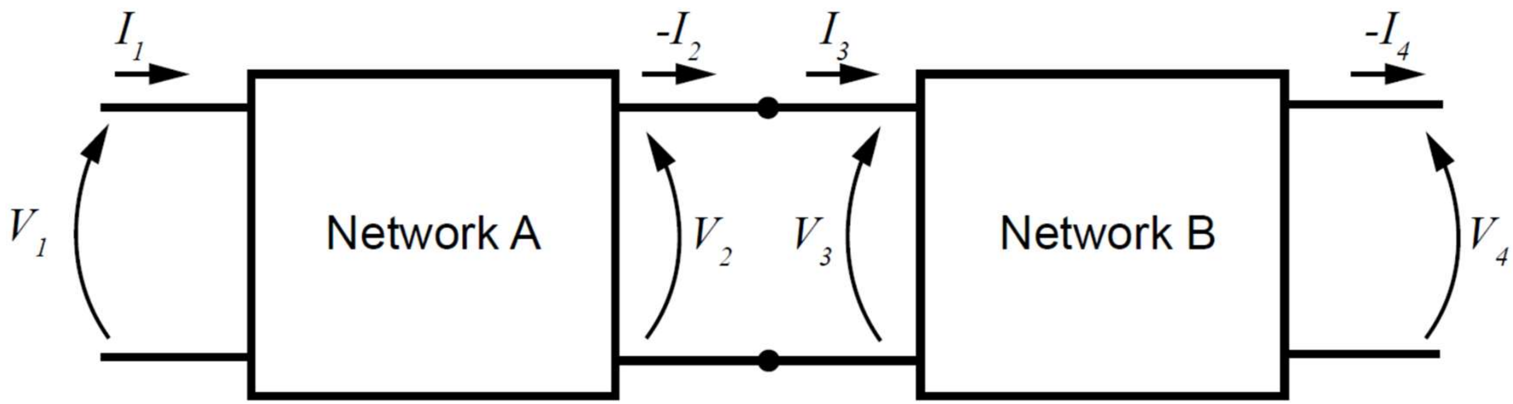
Cascading ABCD parameters two ports

Cascading two ABCD or t networks:

For network A:
$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = [t_A]\begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix} = [t_A]\begin{bmatrix} V_3(s) \\ I_3(s) \end{bmatrix}$$

For network B:
$$\begin{bmatrix} V_3(s) \\ I_3(s) \end{bmatrix} = [t_B]\begin{bmatrix} V_4(s) \\ -I_4(s) \end{bmatrix}$$

Combining the two:
$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = [t_A][t_B]\begin{bmatrix} V_4(s) \\ -I_4(s) \end{bmatrix}, \qquad [t] = [t_A][t_B]$$



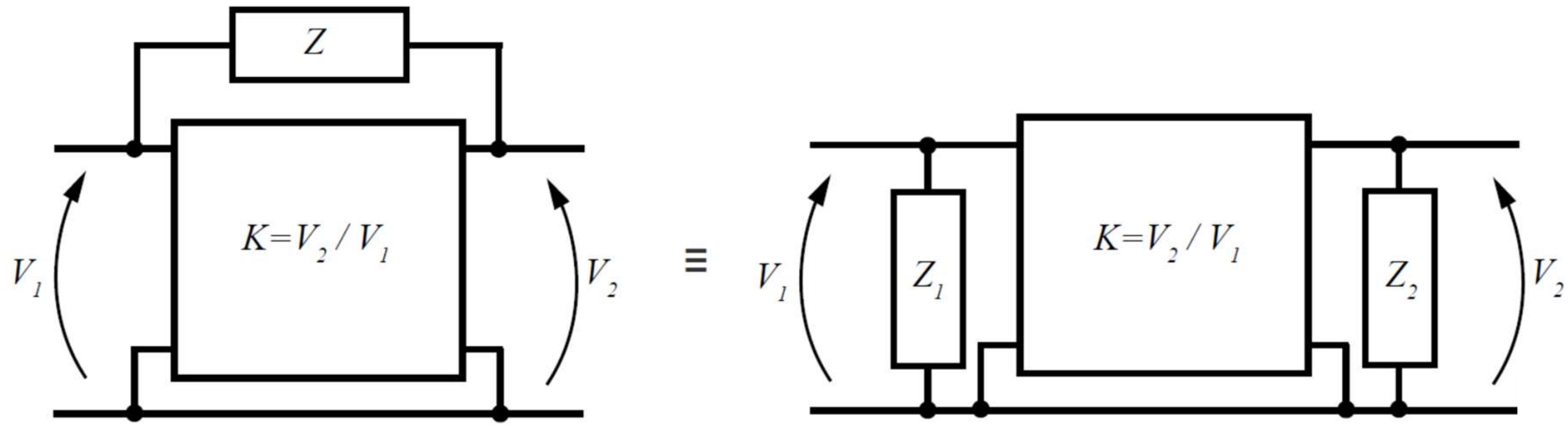
Miller effect

Consider a network which provides voltage amplification *K* (which may be complex or a function of *s*). Represented by the two port below with the lower port terminals connected together.

An impedance *Z* which is connected between the input and output ports as shown below can be replaced by equivalent impedances Z_1 at the input port and Z_2 at the output port.

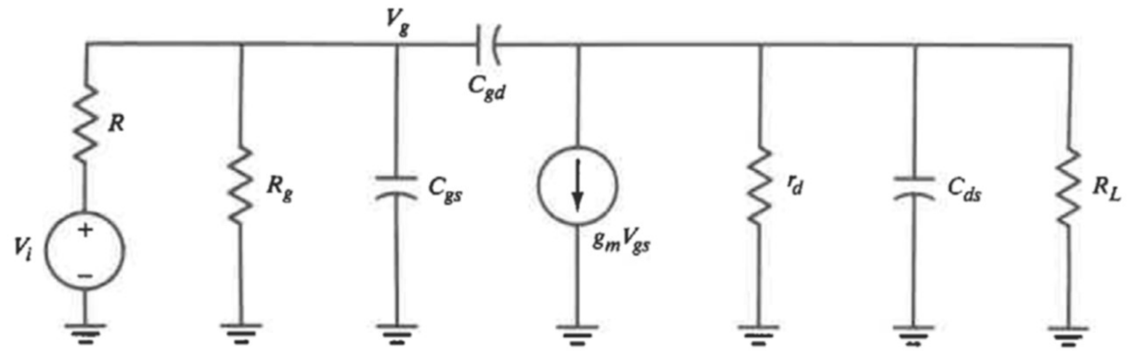
, where: $$Z_1 = \frac{Z}{1-K} \qquad Z_2 = \frac{KZ}{K-1}$$

This result is known as Miller's Theorem. For proof see tutorial questions

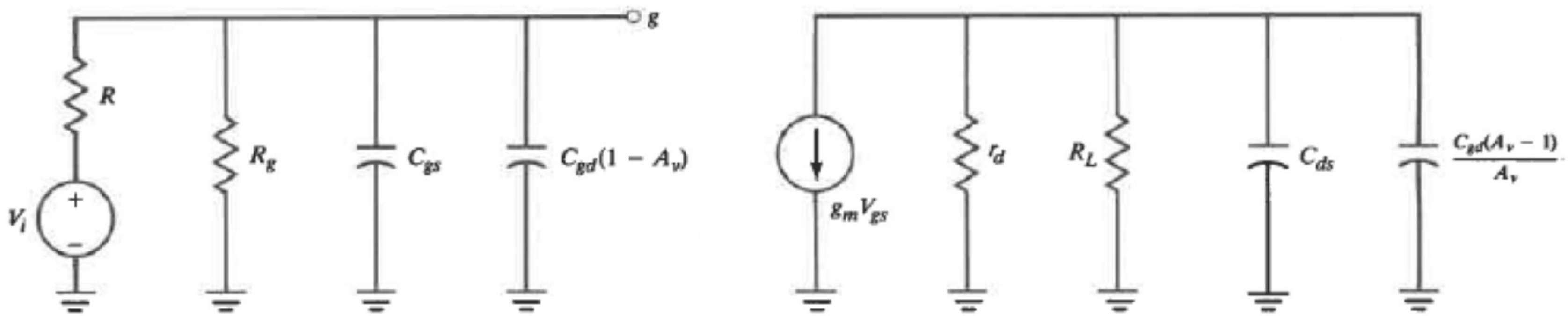


Miller effect often encountered in transistor amplifiers.

See below example FET amplifier small signal model from J.Smith, Modern Communications Circuits, McGraw-Hill.

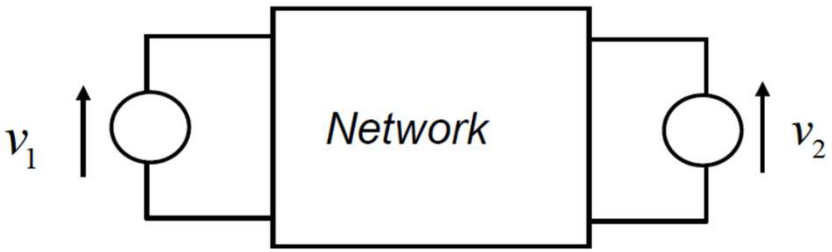


The capacitor connecting the input and output sides can be replaced by two capacitors, allowing simpler analysis and better insight into performance.



Bisection Theorem

The Bisection theorem is often used to simplify the analysis of electronic circuits that have two input (driven) ports, contain **no internal independent** sources and exhibit mirror symmetry. You should make use of this theorem in the next part of the course.



For simplicity we have not exposed any output ports.

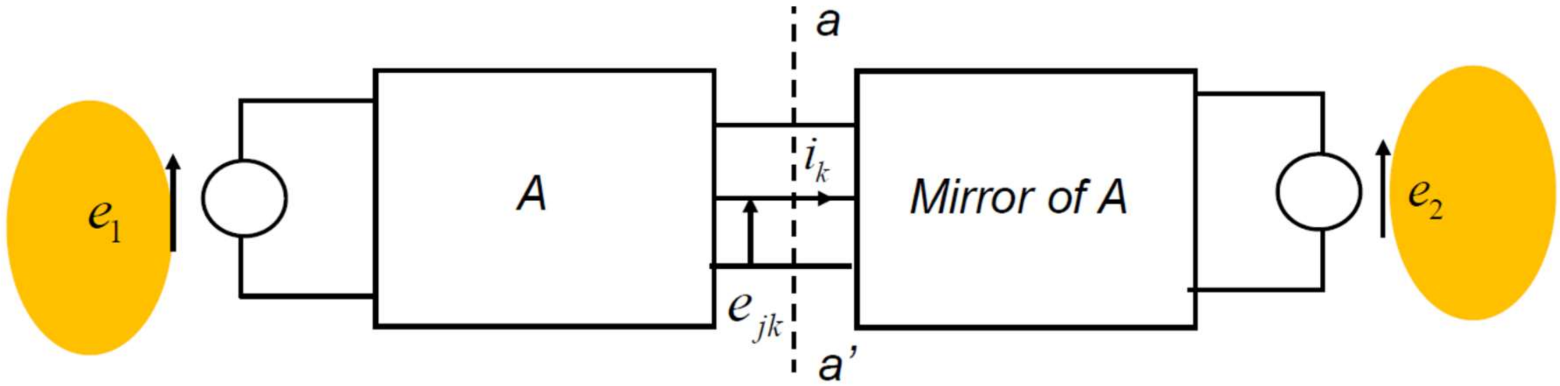
Alternate unique representation of the two input voltages:

The *differential* component $$e_d = \frac{v_1 - v_2}{2}$$

The *common mode* component $$e_c = \frac{v_1 + v_2}{2}$$

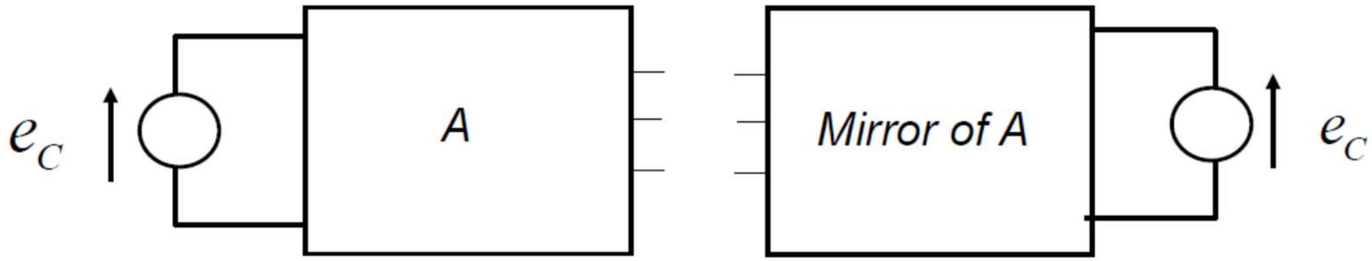
Note: $v_1 = e_c + e_d$ & $v_2 = e_c - e_d$

Bisection Theorem (cont'd)



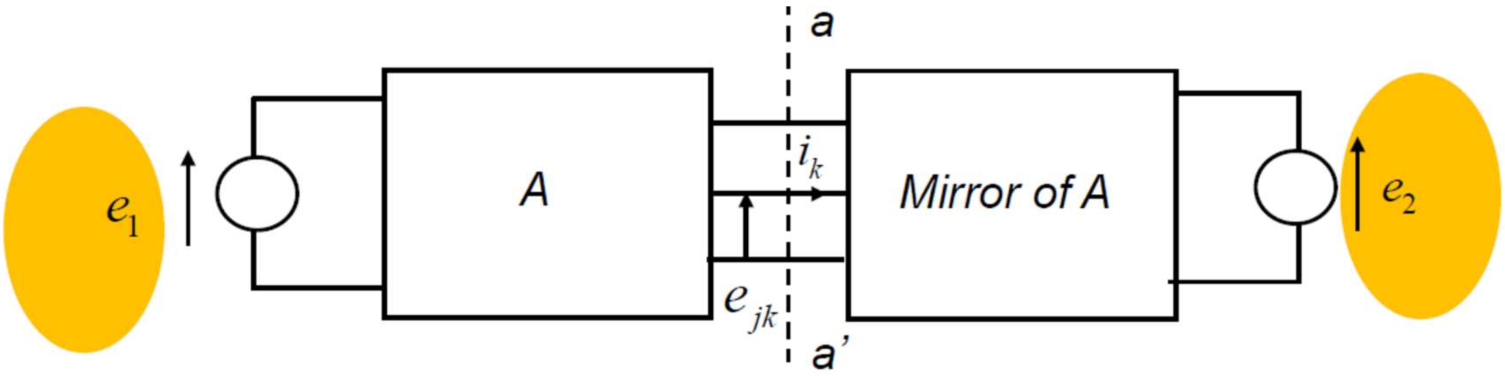
If the network is excited in the *common mode* so that $e_1 = e_2 = e_c \quad e_d = 0$

The currents and voltages throughout the network are not changed if the links joining the two parts of the network are cut.



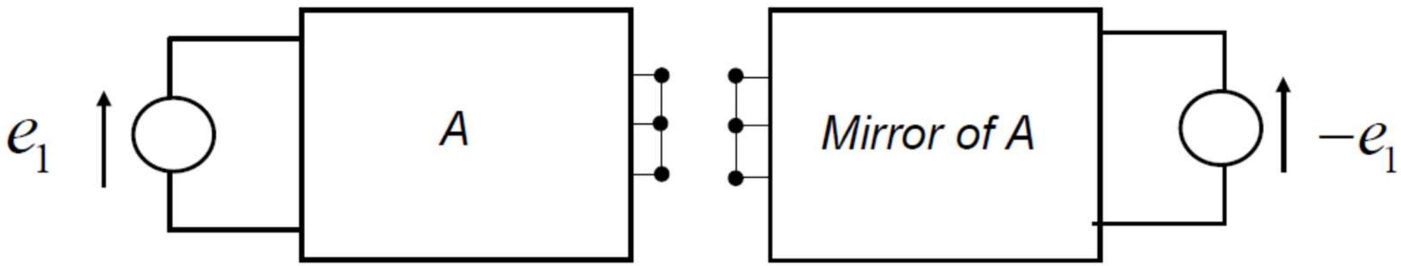
Analyse half the circuit !

Bisection Theorem (cont'd)



If the network is excited in the differential mode so that $e_1 = -e_2 = e_d \quad e_c = 0$

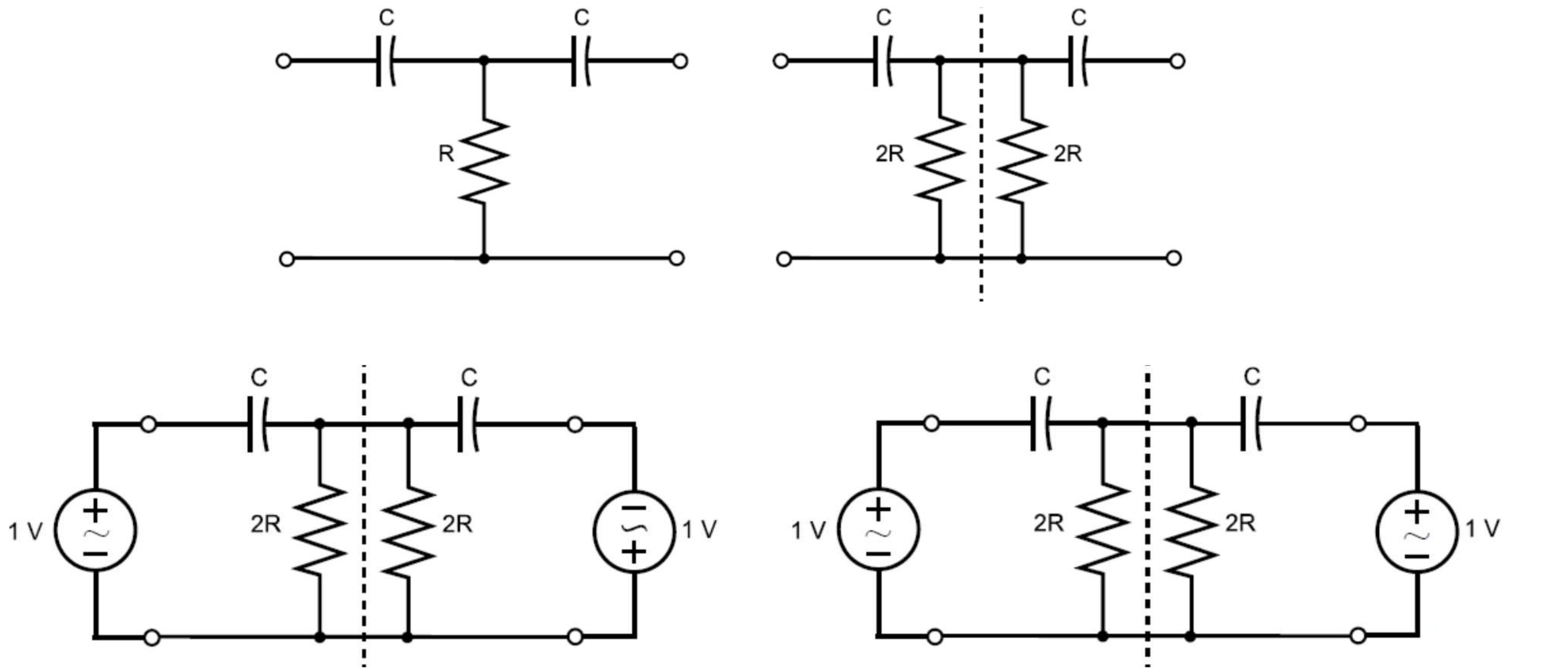
The currents and voltages are not changed if the links are cut and joined together separately in each of the two parts.



Analyse half the circuit !

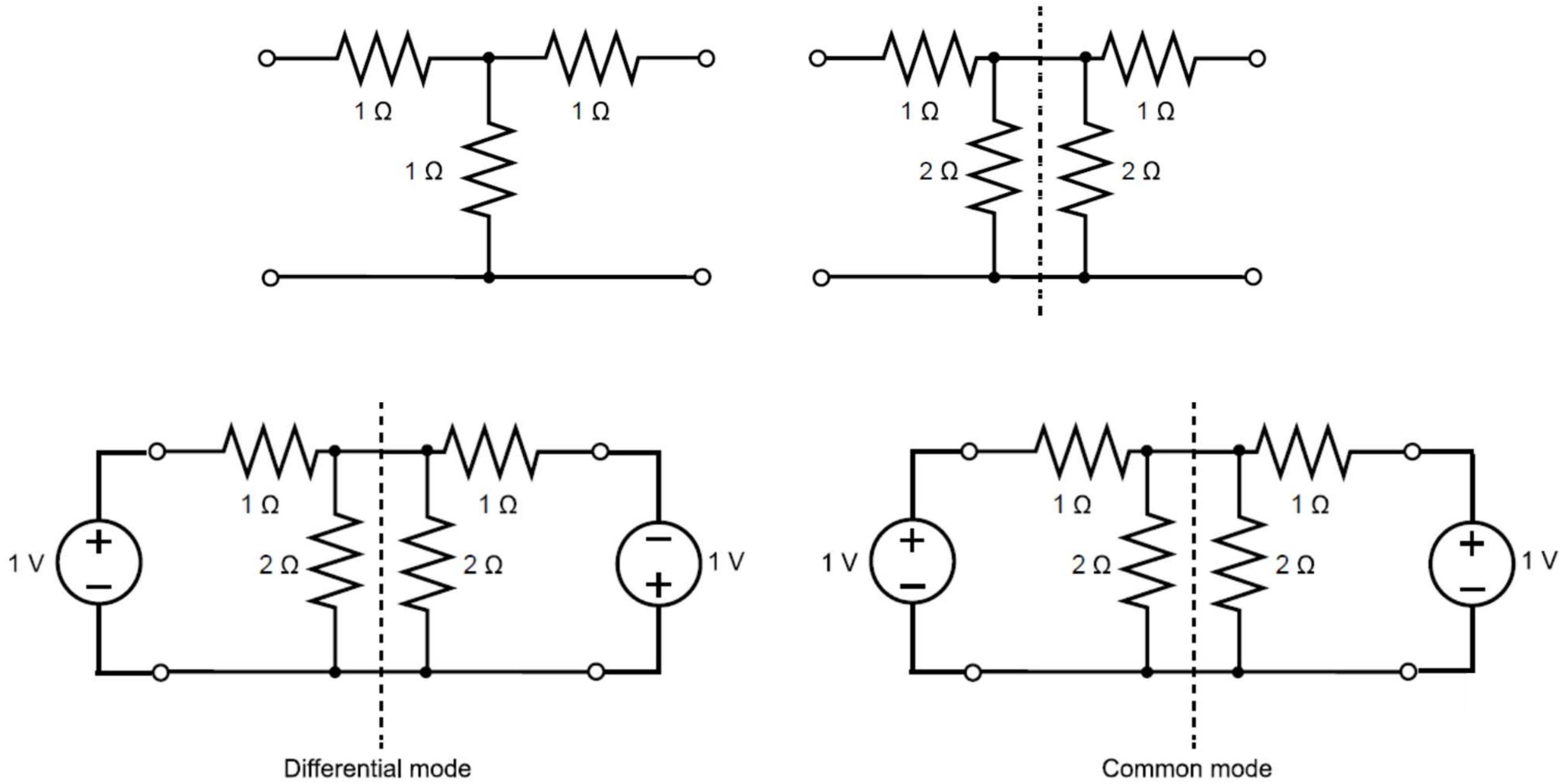
Bisection Theorem (cont'd)

To help understand the concept consider the symmetric network of a resistor and two capacitors. It can be split in the center. Now consider the potentials on the connections bisected. For differential mode all potentials forced equal. For common mode no current flows through the connections as potentials on each side the same, so they can be cut and left open circuit.



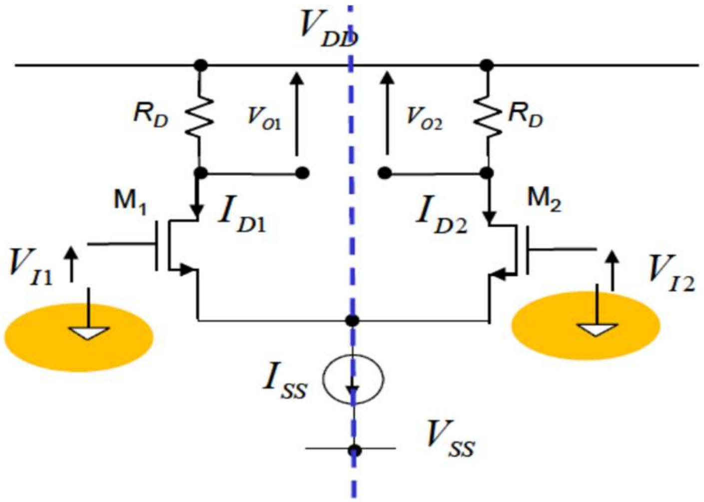
Example

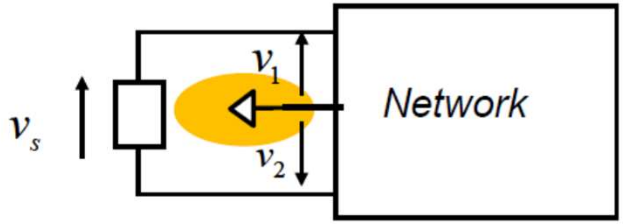
Even simpler example: do the DC circuit analysis for the symmetric network of the 3 resistors. Do for the differential and common mode and check that currents in and out of the network are the same when cut it in two and leave open circuit for common mode, or short together for differential mode.



Example

FET Differential Amplifier





Note the common input reference node

What about the power supply sources ?

Power supplies (all independent sources) must be removed !

=> Small signal AC analysis ! (or for linear circuit ac analysis)

Acknowledgments

- Lecture slides are based on lecture materials from various sources, including M. Hill, T. Cantoni, F. Boussaid, and R. Togneri.

- Credit is acknowledged where credit is due. Please refer to the full list of references.