Solution 6.1

$$i = C\frac{dv}{dt} = 7.5\left(2e^{-3t} - 6te^{-3t}\right) = \mathbf{15(1 - 3t)e^{-3t}\ A}$$

p = vi = $15(1–3t)e^{–3t} \cdot 2t\ e^{–3t}$ = $\mathbf{30t(1 - 3t)e^{-6t}}$ **W**.

$15(1 - 3t)e^{-3t}$ A, $30t(1 - 3t)e^{-6t}$ W

Solution 6.2

$w(t) = (1/2)C(v(t))^2$ or $(v(t))^2 = 2w(t)/C = (20\cos^2(377t))/(50\text{x}10^{-6}) = 0.4\text{x}10^6\cos^2(377t)$

or

$v(t) = \pm 632.5\cos(377t)$ V.

Let us assume that $v(t) = 632.5\cos(377t)$ V, which leads to
$i(t) = C(dv/dt) = 50\text{x}10^{-6}(632.5)(-377\sin(377t))$

$= \mathbf{-11.923\sin(377t)\ A}$.

Please note that if we had chosen the negative value for v, then i(t) would have been positive.

Solution 6.3

Design a problem to help other students to better understand how capacitors work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

In 5 s, the voltage across a 40-mF capacitor changes from 160 V to 220 V. Calculate the average current through the capacitor.

Solution

$$i = C\frac{dv}{dt} = 40x10^{-3}\frac{220-160}{5} = \mathbf{480\ mA}$$

Solution 6.4

A voltage across a capacitor is equal to [2–2 cos(4t)] V and the current flowing through it is equal to 2sin(4t) μA, determine the value of the capacitance. Calculate the power being stored by the capacitor.

Solution

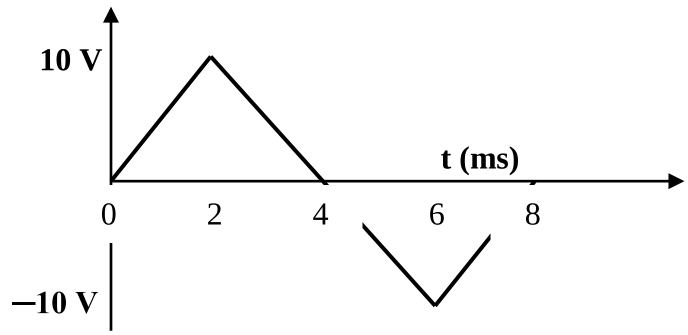
Starting with $i_C = Cdv_C/dt$ and $v_C = [2–2\cos(4t)]$ V and that $i_C = 2\sin(4t)$ μA,

we get $c = 2\sin(4t)x10^{–6}/8\sin(4t)$ = **0.25 μF**.

$P_C = v_Ci_C = [2–2\cos(4t)][2\sin(4t)x10^{–6}]$ = **[4sin(4t)–4cos(4t)sin(4t)] μW**.

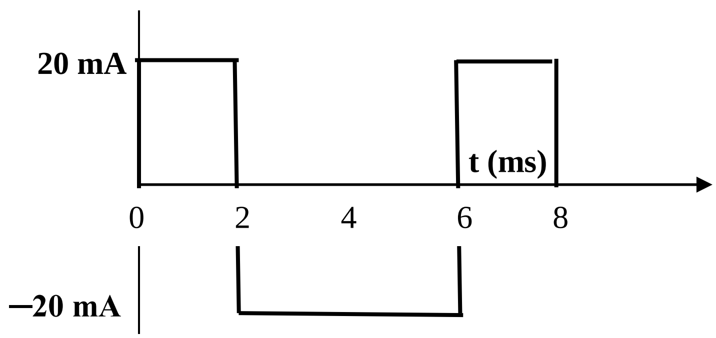
Solution 6.5

The voltage across a 4-μF capacitor is shown below. Find the current waveform.



Step 1. $$v = \begin{cases} 5000t, & 0 < t < 2ms \\ 20 - 5000t, & 2 < t < 6ms \\ -40 + 5000t, & 6 < t < 8ms \end{cases}$$ and $i_C(t) = Cdv_C(t)/dt$.

Step 2. For 0<t<2ms, $i_C(t) = 4x10^{-6}d(5000t)/dt = 20$ mA;
for 2ms<t<6ms, $i_C(t) = 4x10^{-6}d(20-5000t)/dt = -20$ mA;
and for 6ms<t<8ms, $i_C(t) = 4x10^{-6}d(-40+5000t)/dt = 20$ mA.



Solution 6.6

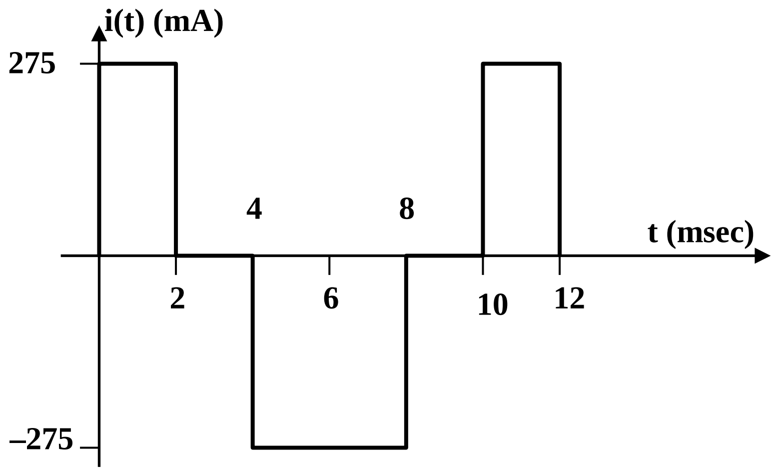
$i = C\dfrac{dv}{dt} = 55x10^{-6}$ times the slope of the waveform.

For example, for 0 < t < 2,

$$\frac{\mathrm{dv}}{\mathrm{dt}} = \frac{10}{2\mathrm{x}10^{-3}}$$

i = $C\dfrac{dv}{dt} = (55x10^{-6})\dfrac{10}{2x10^{-3}} = 275mA$

Thus the current i(t) is sketched below.



Solution 6.7

$$v = \frac{1}{C}\int idt + v(t_o) = \frac{1}{25x10^{-3}}\int_o^t 5tx10^{-3}\,dt + 10$$

$$= \frac{2.5t^2}{25} + 10 = \mathbf{[0.1t^2 + 10]\ V}.$$

Solution 6.8

(a) $i = C\dfrac{dv}{dt} = -100ACe^{-100t} - 600BCe^{-600t}$ (1)

$i(0) = 2 = -100AC - 600BC \longrightarrow 5 = -A - 6B$ (2)

$v(0^+) = v(0^-) \longrightarrow 50 = A + B$ (3)

Solving (2) and (3) leads to

$\underline{A=61,\ B=-11}$

(b) $\text{Energy} = \dfrac{1}{2}Cv^2(0) = \dfrac{1}{2}x4x10^{-3}x2500 = \underline{5\text{ J}}$

(c) From (1),

$$\boldsymbol{i = -100x61x4x10^{-3}e^{-100t} - 600x11x4x10^{-3}e^{-600t} = \underline{-24.4e^{-100t} - 26.4e^{-600t}\ \ \text{A}}}$$

Solution 6.9

$$v(t) = \frac{1}{1/2}\int_o^t 6\left(1 - e^{-t}\right)dt + 0 = 12\left(t + e^{-t}\right)\Big|_0^t \text{ V} = 12(t + e^{-t}) - 12$$

$v(2) = 12(2 + e^{-2}) - 12 =$ **13.624 V**

$p = iv = [12\ (t + e^{-t}) - 12]6(1-e^{-t})$

$p(2) = [12\ (2 + e^{-2}) - 12]6(1-e^{-2})$ = **70.66 W**

Solution 6.10

$$i = C\frac{dv}{dt} = 5x10^{-3}\frac{dv}{dt}$$

$$v = \begin{cases} 16t, & 0 < t < 1\mu\text{s} \\ 16, & 1 < \text{t} < 3\ \mu\text{s} \\ 64 - 16\text{t}, & 3 < \text{t} < 4\mu\text{s} \end{cases}$$

$$\frac{dv}{dt} = \begin{cases} 16x10^{6}, & 0 < t < 1\mu\text{s} \\ 0, & 1 < \text{t} < 3\ \mu\text{s} \\ -16\text{x}10^{6}, & 3 < \text{t} < 4\mu\text{s} \end{cases}$$

$$\boldsymbol{i(t)} = \begin{cases} \mathbf{80\,kA,} & \mathbf{0 < \boldsymbol{t} < 1\boldsymbol{\mu}s} \\ \mathbf{0,} & \mathbf{1 < t < 3\ \boldsymbol{\mu}s} \\ \mathbf{-\,80\,kA,} & \mathbf{3 < t < 4\boldsymbol{\mu}s} \end{cases}$$

Solution 6.11

$$v = \frac{1}{C}\int_0^t i dt + v(0) = 10 + \frac{1}{4x10^{-3}}\int_0^t i(t) dt$$

For 0<t <2, i(t)=15mA, V(t)= 10+ $v = 10 + \frac{10^3}{4x10^{-3}}\int_0^t 15 dt = 10 + 3.76t$

v(2) = 10+7.5 =17.5

For 2 < t <4, i(t) = –10 mA

$$v(t) = \frac{1}{4x10^{-3}}\int_2^t i(t) dt + v(2) = -\frac{10x10^{-3}}{4x10^{-3}}\int_2^t dt + 17.5 = 22.5 + 2.5t$$

v(4)=22.5-2.5x4 =12.5

For 4<t<6, i(t) = 0, $v(t) = \frac{1}{4x10^{-3}}\int_2^t 0 dt + v(4) = 12.5$

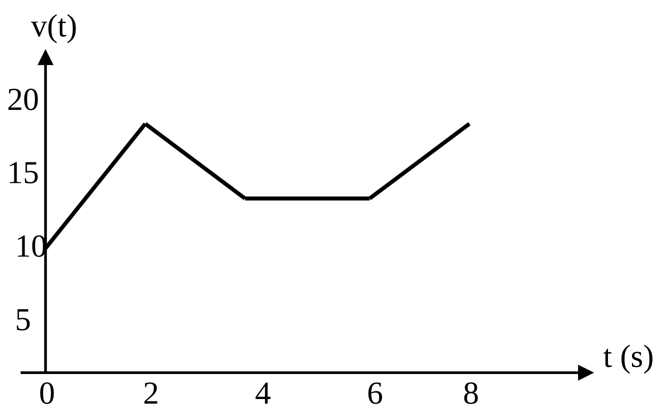
For 6<t<8, i(t) = 10 mA

$$v(t) = \frac{10x10^{3}}{4x10^{-3}}\int_4^t dt + v(6) = 2.5(t-6) + 12.5 = 2.5t - 2.5$$

Hence,

$$v(t) = \begin{cases} 10 + 3.75tV, & 0 < t < 2s \\ 22.5 - 2.5tV, & 2 < t < 4s \\ 12.5V, & 4 < t < 6s \\ 2.5t - 2.5V, & 6 < t < 8s \end{cases}$$

which is sketched below.



Solution 6.12

A voltage of $45e^{-2000t}$ V appears across a parallel combination of a 100-mF capacitor and a 12-Ω resistor. Calculate the power absorbed by the parallel combination.

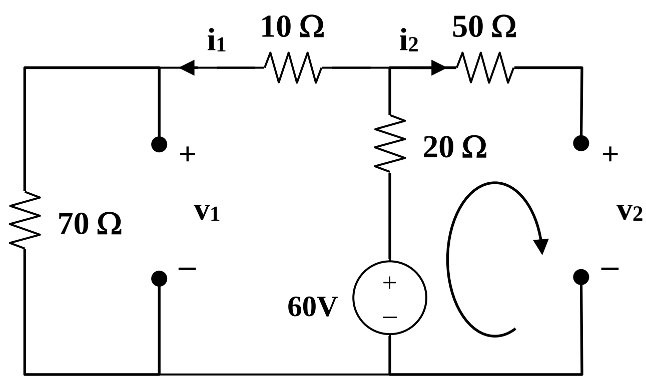
Solution

$i_R = V/R = (45/12)e^{-2000t} = 3.75\ e^{-2000t}$ and
$i_C = C(dv/dt) = 0.1x45(-2000)\ e^{-2000t} = -9000\ e^{-2000t}$ A.

Thus, $i = i_R + i_C = -8{,}996.25e^{-2000t}$. The power is equal to:
$vi =$ **$-40.48\ 179.925\ e^{-4000t}$ kW**.

Solution 6.13

Under dc conditions, the circuit becomes that shown below:



$i_2 = 0$, $i_1 = 60/(70+10+20) = 0.6$ A

$v_1 = 70i_1 = 42$ V, $v_2 = 60–20i_1 = 48$ V

Thus, $v_1 =$ **42 V**, $v_2 =$ **48 V**.

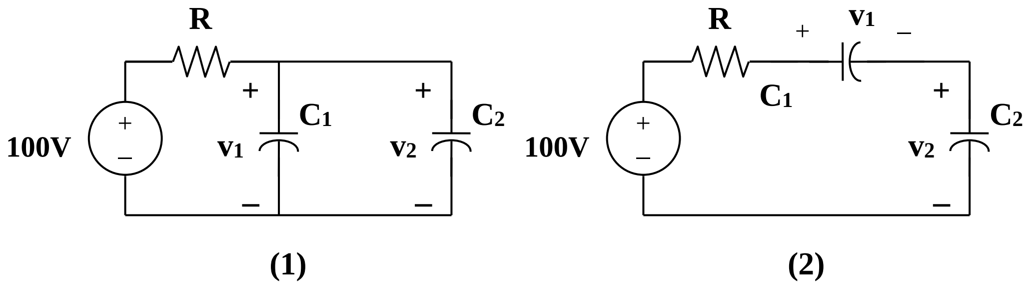
Solution 6.14

20 pF is in series with 60pF = 20*60/80=15 pF
30-pF is in series with 70pF = 30x70/100=21pF
15pF is in parallel with 21pF = 15+21 = **36 pF**

Solution 6.15

Arranging the capacitors in parallel results in circuit shown in Fig. (1) (It should be noted that the resistors are in the circuits only to limit the current surge as the capacitors charge. Once the capacitors are charged the current through the resistors are obviously equal to zero.):

$$v_1 = v_2 = 100$$



$$w_{20} = \frac{1}{2}Cv^2 = \frac{1}{2}x25x10^{-6}x100^2 = \textbf{125 mJ}$$

$$w_{30} = \frac{1}{2}x75x10^{-6}x100^2 = \textbf{375 mJ}$$

(b) Arranging the capacitors in series results in the circuit shown in Fig. (2):

$$v_1 = \frac{C_2}{C_1 + C_2}V = \frac{75}{100}x100 = 75 \text{ V}, \; v_2 = 25 \text{ V}$$

$$w_{25} = \frac{1}{2}x25x10^{-6}x75^2 = \textbf{70.31 mJ}$$

$$w_{75} = \frac{1}{2}x75x10^{-6}x25^2 = \textbf{23.44 mJ}.$$

(a) **125 mJ, 375 mJ (b) 70.31 mJ, 23.44 mJ**

Solution 6.16

The equivalent capacitance at terminals *a*-*b* in the circuit in Fig. 6.50 is 20 μF. Calculate the value of *C*.

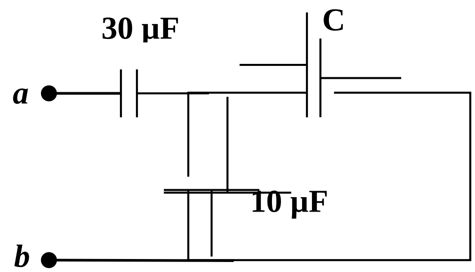


Figure 6.50
For Prob. 6.16.

Solution

The capacitance looking into terminals a and b is equal to,

$C_{eq} = 20\text{x}10^{-6} = 30\text{x}10^{-6}(10\text{x}10^{-6}+C)/(30\text{x}10^{-6}+10\text{x}10^{-6}+C)$.

$30\text{x}10^{-6}+10\text{x}10^{-6}+C = (30/20)(10\text{x}10^{-6}+C)$ or
$(1.5-1)C = 30\text{x}10^{-6}+10\text{x}10^{-6}-15\text{x}10^{-6} = 25\text{x}10^{-6}$ or

$C = 25\text{x}10^{-6}/0.5 =$ **50 μF**.

Solution 6.17

(a) 4F in series with 12F = 4 x 12/(16) = 3F

3F in parallel with 6F and 3F = 3+6+3 = 12F

4F in series with 12F = 3F

i.e. C_{eq} = **3F**

(b) C_{eq} = 5 + [6x(4 + 2)/(6+4+2)] = 5 + (36/12) = 5 + 3 = **8F**

(c)

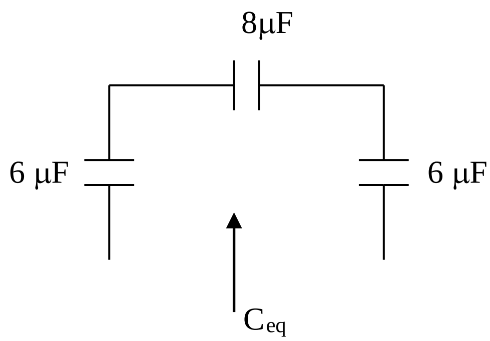
(d) 3F in series with 6F = (3 x 6)/9 = 2F

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 1$$

C_{eq} = **1F**

Solution 6.18

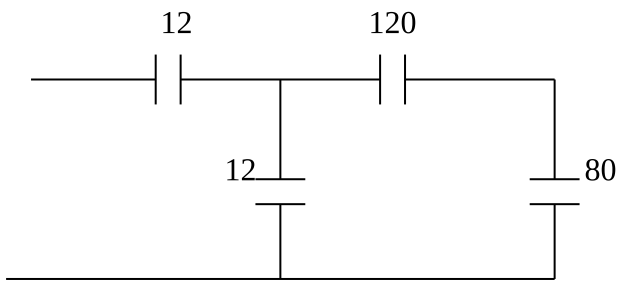
4 μF in parallel with 4 μF = 8μF
4 μF in series with 4 μF = 2 μF
2 μF in parallel with 4 μF = 6 μF
Hence, the circuit is reduced to that shown below.



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = 0.4583 \quad \longrightarrow \quad C_{eq} = \underline{2.1818\ \mu F}$$

Solution 6.19

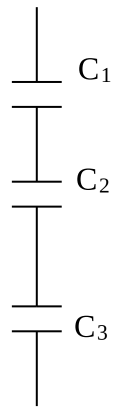
We combine 10-, 20-, and 30- μF capacitors in parallel to get 60 μF. The 60 - μF capacitor in series with another 60- μF capacitor gives 30 μF.
30 + 50 = 80 μF, 80 + 40 = 120 μF
The circuit is reduced to that shown below.



120- μF capacitor in series with 80 μF gives (80x120)/200 = 48
48 + 12 = 60
60- μF capacitor in series with 12 μF gives (60x12)/72 = **10 μF**

Solution 6.20

Consider the circuit shown below.



$C_1 = 1 + 1 = 2\mu F$
$C_2 = 2 + 2 + 2 = 6\mu F$

$C_3 = 4x3 = 12\mu F$

$1/C_{eq} = (1/C_1) + (1/C_2) + (1/C_3) = 0.5 + 0.16667 + 0.08333 = 0.75x10^6$

C_{eq} = **1.3333 μF**.

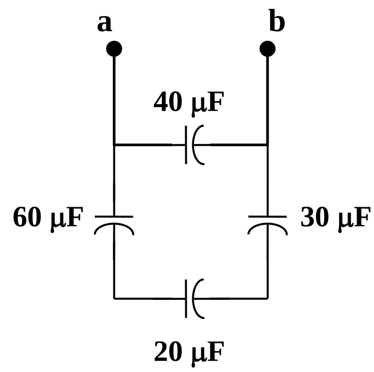
Solution 6.21

$4\mu F$ in series with $12\mu F = (4x12)/16 = 3\mu F$
$3\mu F$ in parallel with $3\mu F = 6\mu F$
$6\mu F$ in series with $6\mu F = 3\mu F$
$3\mu F$ in parallel with $2\mu F = 5\mu F$
$5\mu F$ in series with $5\mu F = 2.5\mu F$

Hence $C_{eq} = \mathbf{2.5\mu F}$

Solution 6.22

Combining the capacitors in parallel, we obtain the equivalent circuit shown below:



Combining the capacitors in series gives C_{eq}^{1}, where

$$\frac{1}{C_{eq}^{1}} = \frac{1}{60} + \frac{1}{20} + \frac{1}{30} = \frac{1}{10} \longrightarrow C_{eq}^{1} = 10\mu F$$

Thus

$$C_{eq} = 10 + 40 = \mathbf{50\ \mu F}$$

Solution 6.23

Using Fig. 6.57, design a problem to help other students better understand how capacitors work together when connected in series and parallel.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

For the circuit in Fig. 6.57, determine:

(a) the voltage across each capacitor,
(b) the energy stored in each capacitor.

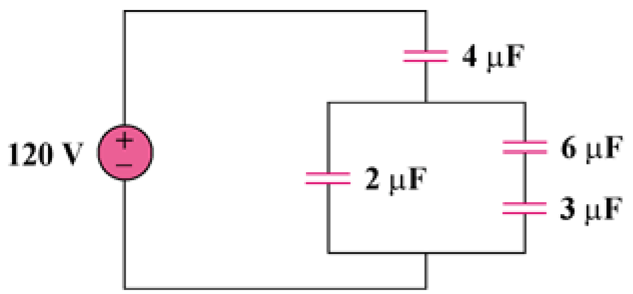


Figure 6.57

Solution

(a) 3μF is in series with 6μF $3x6/(9) = 2\mu F$

$v_{4\mu F} = 1/2 \text{ x } 120 = \mathbf{60V}$

$v_{2\mu F} = \mathbf{60V}$

$$v_{6\mu F} = \frac{3}{6+3}(60) = \mathbf{20V}$$

$v_{3\mu F} = 60 - 20 = \mathbf{40V}$

(b) Hence $w = 1/2\ Cv^2$

$w_{4\mu F} = 1/2 \text{ x } 4 \text{ x } 10^{-6} \text{ x } 3600 = \mathbf{7.2mJ}$

$w_{2\mu F} = 1/2 \text{ x } 2 \text{ x } 10^{-6} \text{ x } 3600 = \mathbf{3.6mJ}$

$w_{6\mu F} = 1/2 \text{ x } 6 \text{ x } 10^{-6} \text{ x } 400 = \mathbf{1.2mJ}$

$w_{3\mu F} = 1/2 \text{ x } 3 \text{ x } 10^{-6} \text{ x } 1600 = \mathbf{2.4mJ}$

Solution 6.24

In the circuit shown in Fig. 6.58 assume that the capacitors were initially uncharged and that the current source has been connected to the circuit long enough for all the capacitors to reach steady-state (no current flowing through the capacitors), determine the voltage across each capacitor and the energy stored in each.

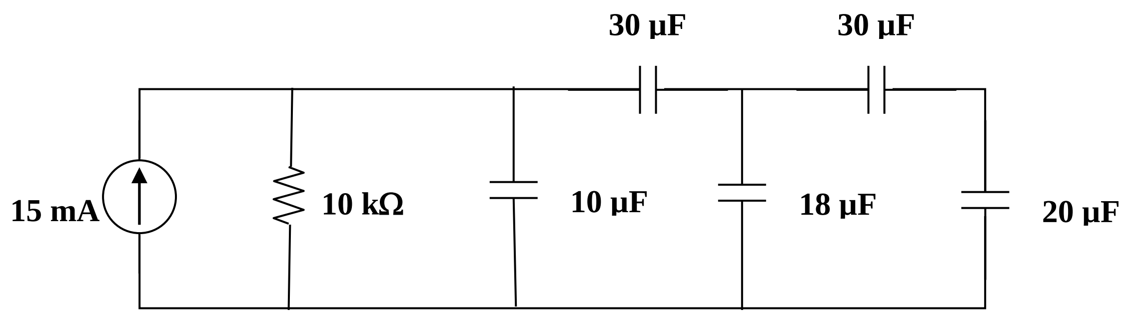


Figure 6.58
For Prob. 6.24.

Solution
Reducing the capacitance starting from right to left. 30µF in series with 20µF we get, 30x20µF/(30+20) = 12µF in parallel with 18µF we get (12+18)µF = 30µF.
Now

v_{10} = 15x10 = **150 V** and w_{10} = $0.5x10x150^2x10^{-6}$ = **112.5 mJ**
v_{30} = **75 V** and w_{30} = $0.5x30x75^2x10^{-6}$ = **84.38 mJ**
v_{18} = **75 V** and w_{18} = $0.5x18x75^2x10^{-6}$ = **50.62 mJ**
v_{30} = [20/(30+20)]75 = **30 V** and w_{30} = $0.5x30x30^2x10^{-6}$ = **13.5 mJ**
v_{20} = [30/(30+20)]75 = **45 V** and w_{20} = $0.5x20x45^2x10^{-6}$ = **20.25 mJ**

Solution 6.25

(a) For the capacitors in series,

$$Q_1 = Q_2 \longrightarrow C_1 v_1 = C_2 v_2 \longrightarrow \frac{v_1}{v_2} = \frac{C_2}{C_1}$$

$$v_s = v_1 + v_2 = \frac{C_2}{C_1} v_2 + v_2 = \frac{C_1 + C_2}{C_1} v_2 \longrightarrow \boldsymbol{v_2 = \frac{C_1}{C_1 + C_2} v_s}$$

Similarly, $\boldsymbol{v_1 = \frac{C_2}{C_1 + C_2} v_s}$

(b) For capacitors in parallel

$$v_1 = v_2 = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_s = Q_1 + Q_2 = \frac{C_1}{C_2} Q_2 + Q_2 = \frac{C_1 + C_2}{C_2} Q_2$$

or

$$Q_2 = \frac{C_2}{C_1 + C_2}$$

$$Q_1 = \frac{C_1}{C_1 + C_2} Q_s$$

$$i = \frac{dQ}{dt} \longrightarrow \boldsymbol{i_1 = \frac{C_1}{C_1 + C_2} i_s}, \quad \boldsymbol{i_2 = \frac{C_2}{C_1 + C_2} i_s}$$

Solution 6.26

Three capacitors, $\boldsymbol{C}_1 = 5\ \mu$F, $\boldsymbol{C}_2 = 10\ \mu$F, and $\boldsymbol{C}_3 = 20\ \mu$F, are connected in parallel across a 200-V source. Determine:
(a) the total capacitance,
(b) the charge on each capacitor,
(c) the total energy stored in the parallel combination.

Solution

(a) $C_{eq} = C_1 + C_2 + C_3 = \mathbf{35\mu F}$

(b) $Q_1 = C_1 v = 5x200\ \mu C = \mathbf{1\ mC}$
$Q_2 = C_2 v = 10x200\ \mu C = \mathbf{2\ mC}$
$Q_3 = C_3 v = 20x200\ \mu C = \mathbf{4\ mC}$

(c) $w = \frac{1}{2} C_{eq} v^2 = \frac{1}{2} x 35 x 200^2\ \mu J = \mathbf{700\ mJ}$

Solution 6.27

Given that four 10-μF capacitors can be connected in series and in parallel, find the minimum and maximum values that can be obtained by such series/parallel combinations.

Solution

If they are all connected in parallel, we get C_{total} = 4x10 μF = 40 μF.

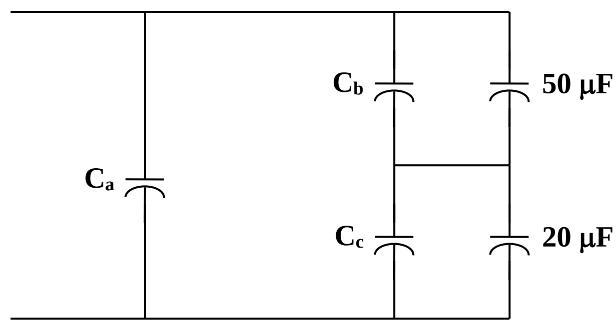
If they are all connected in series, we get C_{total} = 1/(4/10 μF) = 2.5 μF.

Since all other combinations fall within these two extreme cases. Thus,

C_{min} = **2.5 μF**, C_{max} = **40 μF**.

Solution 6.28

We may treat this like a resistive circuit and apply delta-wye transformation, except that R is replaced by 1/C.



$$\frac{1}{C_a} = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{40}\right)+\left(\frac{1}{10}\right)\left(\frac{1}{30}\right)+\left(\frac{1}{30}\right)\left(\frac{1}{40}\right)}{\frac{1}{30}}$$

$$= \frac{3}{40}+\frac{1}{10}+\frac{1}{40}=\frac{2}{10}$$

$C_a = 5\mu F$

$$\frac{1}{C_b} = \frac{\frac{1}{400}+\frac{1}{300}+\frac{1}{1200}}{\frac{1}{10}} = \frac{2}{30}$$

$C_b = 15\mu F$

$$\frac{1}{C_c} = \frac{\frac{1}{400}+\frac{1}{300}+\frac{1}{1200}}{\frac{1}{40}} = \frac{4}{15}$$

$C_c = 3.75\mu F$

C_b in parallel with $50\mu F = 50 + 15 = 65\mu F$

C_c in series with $20\mu F = 23.75\mu F$

$65\mu F$ in series with $23.75\mu F = \dfrac{65x23.75}{88.75} = 17.39\mu F$

$17.39\mu F$ in parallel with $C_a = 17.39 + 5 = 22.39\mu F$

Hence C_{eq} = $\mathbf{22.39\mu F}$

Solution 6.29

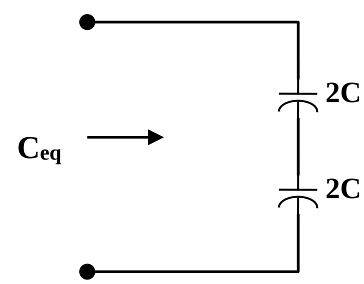
(a) C in series with C = C/(2)

C/2 in parallel with C = 3C/2

$$\frac{3C}{2} \text{ in series with } C = \frac{C\text{x}\,\frac{3C}{2}}{5\frac{C}{2}} = \frac{3C}{5}$$

$$3\frac{C}{5} \text{ in parallel with } C = C + 3\frac{C}{5} = \mathbf{1.6\ C}$$

(b)



$$\frac{1}{C_{eq}} = \frac{1}{2C} + \frac{1}{2C} = \frac{1}{C}$$

C_{eq} = **1 C**

Solution 6.30

$v_o = \dfrac{1}{C}\int_o^t i\,dt + i(0)$

For $0 < t < 1$, $i = 90t$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_o^t 90t\,dt + 0 = 15t^2 kV$$

$v_o(1) = 15$ kV

For $1 < t < 2$, $i = (180 – 90t)$ mA,

$$v_o = \frac{10^{-3}}{3x10^{-6}}\int_1^t (180 - 90t)dt + v_o(1)$$

$$= [60t – 15t^2]\Big|_1^t + 15kV$$

$$= [60t – 15t^2 – (60–15) + 15] \text{ kV} = [60t – 15t^2 – 30] \text{ kV}$$

$$\boldsymbol{v_o(t) = \begin{cases} 15t^2 kV, & 0 < t < 1 \\ [60t - 15t^2 - 30]kV, & 1 < t < 2 \end{cases}}$$

Solution 6.31

$$i_s(t) = \begin{bmatrix} 30tmA, & 0 < t < 1 \\ 30mA, & 1 < t < 3 \\ -75 + 15t, & 3 < t < 5 \end{bmatrix}$$

C_{eq} = 4 + 6 = 10μF

$$\text{v} = \frac{1}{\text{C}_{\text{eq}}}\int_{\text{o}}^{\text{t}} \text{idt} + \text{v(0)}$$

For 0 < t < 1,

$$v = \frac{10^{-3}}{10x10^{-6}}\int_{o}^{t} 30t \text{ dt} + 0 = 1.5\text{t}^2 \text{ kV}$$

For 1 < t < 3,

$$v = \frac{10^3}{10}\int_{1}^{t} 20dt + v(1) = [3(t-1) + 1.5]kV$$
$$= [3t - 1.5]kV$$

For 3 < t < 5,

$$v = \frac{10^3}{10}\int_{3}^{t} 15(t-5)dt + v(3)$$
$$= \left[1.5\frac{t^2}{2} - 7.5t\right]\Bigg|_3^t + 7.5kV = [0.75t^2 - 7.5t + 23.25]kV$$

$$\boldsymbol{v(t) = \begin{bmatrix} 1.5t^2kV, & 0 < t < 1s \\ [3t - 1.5]kV, & 1 < t < 3s \\ [0.75t^2 - 7.5t + 23.25]kV, & 3 < t < 5s \end{bmatrix}}$$

$$\text{i}_1 = \text{C}_1\frac{\text{dv}}{\text{dt}} = 6\text{x}10^{-6}\frac{\text{dv}}{\text{dt}}$$

$$\boldsymbol{i_1 = \begin{bmatrix} 18tmA, & 0 < t < 1s \\ 18mA, & 1 < t < 3s \\ [9t - 45]mA, & 3 < t < 5s \end{bmatrix}}$$

$$\text{i}_2 = \text{C}_2\frac{\text{dv}}{\text{dt}} = 4\text{x}10^{-6}\frac{\text{dv}}{\text{dt}}$$

$$i_2 = \begin{cases} 12tmA, & 0 < t < 1s \\ 12mA, & 1 < t < 3s \\ [6t - 30]mA, & 3 < t < 5s \end{cases}$$

Solution 6.32

In the circuit in Fig. 6.64, let $i_s = 4.5e^{-2t}$ mA and the voltage across each capacitor is equal to zero at t = 0. Determine v_1 and v_2 and the energy stored in each capacitor for all t > 0.

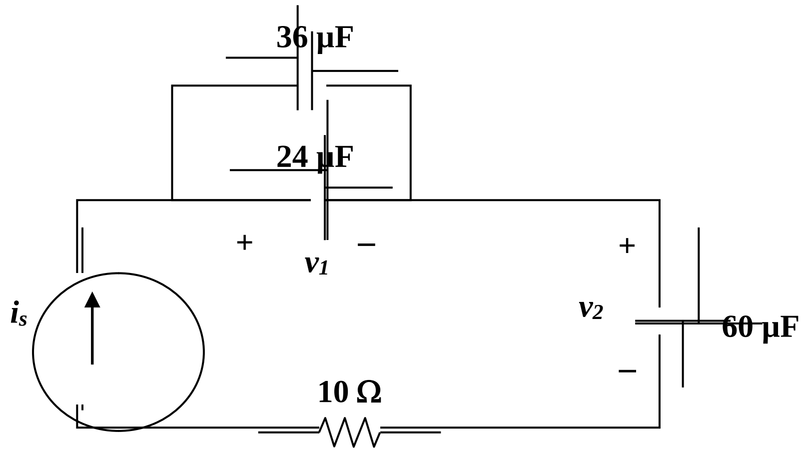


Figure 6.64
For Prob. 6.32.

Solution

Combining the 36 μF with the 24 μF we get 60 μF which leads to $v_1 = \dfrac{1}{60\mu}\int_0^t 4.5e^{-2\tau}m\,d\tau$

= $\mathbf{[37.5–37.5e^{-2t}]\ V}$ = v_2.

$(v_1)^2 = [(37.5)^2 – 2(37.5)^2e^{-2t} + (37.5)^2e^{-4t}] = 1406.25[1 – 2e^{-2t} + e^{-4t}] = (v_2)^2$

$w_{24} = 0.5\text{x}24\text{x}10^{-6}(v_1)^2 = \mathbf{16.875[1 – 2e^{-2t} + e^{-4t}]\ mJ}$
$w_{36} = 0.5\text{x}36\text{x}10^{-6}(v_1)^2 = \mathbf{25.31[1 – 2e^{-2t} + e^{-4t}]\ mJ}$
$w_{60} = 0.5\text{x}60\text{x}10^{-6}(v_2)^2 = \mathbf{42.19[1 – 2e^{-2t} + e^{-4t}]\ mJ}$

Solution 6.33

Because this is a totally capacitive circuit, we can combine all the capacitors using the property that capacitors in parallel can be combined by just adding their values and we combine capacitors in series by adding their reciprocals. However, for this circuit we only have the three capacitors in parallel.

3 F + 2 F = 5 F (we need this to be able to calculate the voltage)

$C_{Th} = C_{eq} = 5+3+2 = 10$ F

The voltage will divide equally across the two 5 F capacitors. Therefore, we get:

$$V_{Th} = \mathbf{15\ V}, \quad C_{Th} = \mathbf{10\ F}.$$

15 V, **10 F**

Solution 6.34

The current through a 25-mH inductor is $10e^{-t/2}$ A. Find the voltage and the power at $t = 3$ s.

Solution

$i = 10e^{-t/2}$

$$v = L\frac{di}{dt} = 25x10^{-3}(10)\left(\frac{-1}{2}\right)e^{-t/2}$$

$= -125e^{-t/2}$ mV

$v(3) = -125e^{-3/2}$ mV = **-27.89 mV**

$p = vi = -1.25e^{-t}$ W

$p(3) = -1.25e^{-3}$ W = **-62.23 mW**.

Solution 6.35

An inductor has a linear change in current from 100 mA to 200 mA in 2 ms and induces a voltage of 160 mV. Calculate the value of the inductor.

Solution

$v = L(di/dt)$ or $L = v/(di/dt)$

Clearly $di/dt = (0.2–0.1)/0.002 = 50$ amp/sec. Thus,

$L = 0.16/50 =$ **3.2 mH**.

Solution 6.36

Design a problem to help other students to better understand how inductors work.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

The current through a 12-mH inductor is $i(t) = 30te^{-2t}$ A, $t \geq 0$. Determine: (a) the voltage across the inductor, (b) the power being delivered to the inductor at t = 1 s, (c) the energy stored in the inductor at t = 1 s.

Solution

(a) $v = L\frac{di}{dt} = 12x10^{-3}(30e^{-2t} - 60te^{-2t}) = \underline{(0.36 - 0.72t)e^{-2t}\text{ V}}$

(b) $p = vi = (0.36 - 0.72x1)e^{-2}x30x1e^{-2} = 0.36x30e^{-4} = \underline{-0.1978\text{ W}}$

(c) $w = \frac{1}{2}Li^2 = 0.5x12x10^{-3}(30x1xe^{-2})^2 =$ **98.9 mJ**.

Solution 6.37

$$v = L\frac{di}{dt} = 12x10^{-3}\,x4(100)\cos 100t$$

$$= \mathbf{4.8\ cos\ (100t)\ V}$$

p = vi = 4.8 x 4 sin 100t cos 100t = 9.6 sin 200t

$$w = \int_o^t pdt = \int_o^{11/200} 9.6\sin 200t$$

$$= -\frac{9.6}{200}\cos 200t\Big|_o^{11/200}\,J$$

$$= -48(\cos\pi - 1)mJ = \mathbf{96\ mJ}$$

Please note that this problem could have also been done by using (½)Li^2.

Chapter 6.38

$$v = L\frac{di}{dt} = 40x10^{-3}\left(e^{-2t} - 2te^{-2t}\right)dt$$

$$= \boldsymbol{40(1-2t)e^{-2t}mV\,,t > 0}$$

Solution 6.39

The voltage across a 50-mH inductor is given by

$$v(t) = [5e^{-2t} + 2t + 4] \text{ V for } t > 0.$$

Determine the current $i(t)$ through the inductor. Assume that $i(0) = 0$ A.

Solution

$$v = L\frac{di}{dt} \longrightarrow i = \frac{1}{L}\int_0^t i\,d\tau + i(0)$$

$$i = \frac{1}{50x10^{-3}}\int_0^t (5e^{-2\tau} + 2\tau + 4)d\tau + 0$$

$$= 20(-2.5e^{-2\tau} + \tau^2 + 4\tau)\Big|_0^t$$

i(t) = $\mathbf{[-50e^{-2t} + 50 + 20t^2 + 80t]}$ **A**.

Solution 6.40

$$i = \begin{cases} 5t, & 0 < t < 2ms \\ 10, & 2 < t < 4ms \\ 30 - 5t, & 4 < t < 6ms \end{cases}$$

$$v = L\frac{di}{dt} = \frac{5x10^{-3}}{10^{-3}}\begin{cases} 5, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -5, & 4 < t < 6ms \end{cases} = \begin{cases} 25, & 0 < t < 2ms \\ 0, & 2 < t < 4ms \\ -25, & 4 < t < 6ms \end{cases}$$

At t = 1ms, v = **25 V**
At t = 3ms, v = **0 V**
At t = 5ms, v= **–25 V**

Solution 6.41

$$i = \frac{1}{L}\int_0^t v dt + C = \left(\frac{1}{2}\right)\int_o^t 20\left(1 - e^{-2t}\right)dt + C$$

$$= 10\left(t + \frac{1}{2}e^{-2t}\right)\Big|_o^t + C = 10t + 5e^{-2t} - 4.7A$$

Note, we get C = –4.7 from the initial condition for i needing to be 0.3 A.

We can check our results be solving for v = Ldi/dt.

$v = 2(10 – 10e^{-2t})V$ which is what we started with.

At t = l s, $i = 10 + 5e^{-2} – 4.7 = 10 + 0.6767 – 4.7 =$ **5.977 A**

$w = \frac{1}{2}L\,i^2 =$ **35.72J**

Solution 6.42

$$i = \frac{1}{L}\int_o^t vdt + i(0) = \frac{1}{5}\int_o^t v(t)dt - 1$$

For $0 < t < 1$, $i = \frac{10}{5}\int_0^t dt - 1 = 2t - 1$ A

For $1 < t < 2$, $i = 0 + i(1) = 1A$

For $2 < t < 3$, $i = \frac{1}{5}\int 10dt + i(2) = 2t\Big|_t^2 + 1$
$= 2t - 3$ A

For $3 < t < 4$, $i = 0 + i(3) = 3$ A

For $4 < t < 5$, $i = \frac{1}{5}\int_4^t 10dt + i(4) = 2t\Big|_4^t + 3$
$= 2t - 5$ A

Thus,

$$\boldsymbol{i(t) = \begin{cases} 2t - 1A, & 0 < t < 1 \\ 1A, & 1 < t < 2 \\ 2t - 3A, & 2 < t < 3 \\ 3A, & 3 < t < 4 \\ 2t - 5, & 4 < t < 5 \end{cases}}$$

Solution 6.43

The current in a 150-mH inductor increases from 0 to 60 mA (steady-state). How much energy is stored in the inductor?

Solution

$w = (1/2)L(i(t))^2 = 0.5x0.15x0.06^2 =$ **270 μJ**.

Solution 6.44

A 100-mH inductor is connected in parallel with a 2-kΩ resistor. The current through the inductor is $i(t) = 35e^{-400t}$ mA.

(a) Find the voltage $v_L(t)$ across the inductor. (b) Find the voltage $v_R(t)$ across the resistor. (c) Is $v_R(t) + v_L(t) = 0$? (d) Calculate the energy stored in the inductor at t=0.

Solution

(a) $v_L(t) = L di/dt = 0.1(-400)0.035e^{-400t}$ = $\mathbf{1.4e^{-400t}}$ **V**.

(b) Since R and L are in parallel, $v_R(t) = v_L(t)$ = $\mathbf{1.4e^{-400t}}$ **V**.

(c) Again, since the two elements are in parallel, $v_R(t) = v_L(t) = 1.4e^{-400t}$ V thus, $v_R(t) + v_L(t) = 2.8e^{-400t}$ and **not equal to zero**!

(d) $w = 0.5Li^2 = 0.5(0.1)(0.035)^2$ = **61.25 μJ**.

Solution 6.45

If the voltage waveform in Fig. 6.68 is applied to a 25-mH inductor, find the inductor current ***i*(*t*)** for 0 < t < 2 seconds. Assume ***i*(0)** = 0.

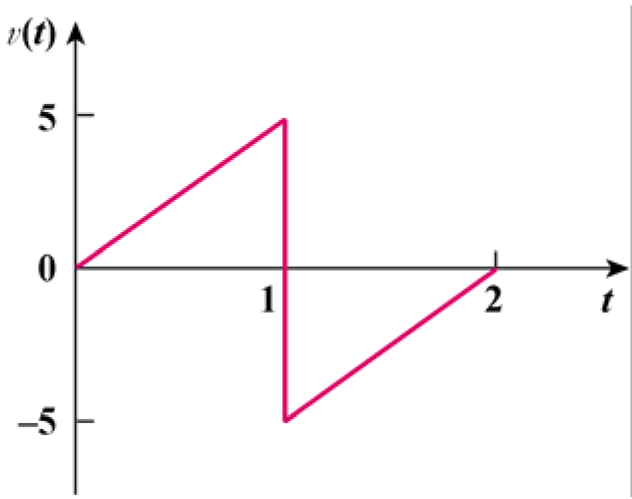


Figure 6.68
For Prob. 6.45.

Solution

i(t) = $\frac{1}{L}\int_o^t v(t) + i(0)$

For 0 < t < 1, v = 5t

$$i = \frac{1}{25x10^{-3}}\int_o^t 5t \text{ dt} + 0$$

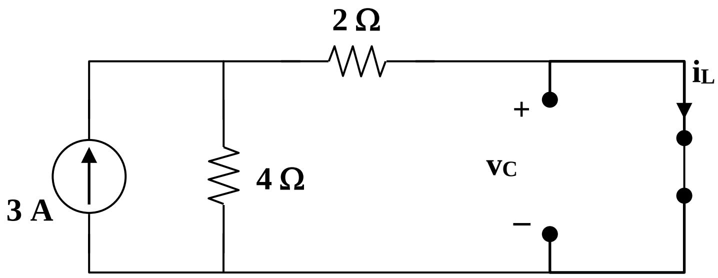
$$= \mathbf{100t^2 \ A}.$$

For 1 < t < 2, v = –10 + 5t

$$i = \frac{1}{25x10^{-3}}\int_1^t (-10+5t)dt + i(1)$$
$$= \left(\int_1^t (0.2\tau - 0.4)d\tau + 0.1\right)kA = \{\left[0.1\tau^2 - 0.4\tau\right]_1^t + 0.1\} \text{ kA}$$
$$= \mathbf{[400 - 400t + 100t^2]\ A}.$$

Solution 6.46

Under dc conditions, the circuit is as shown below:



By current division,

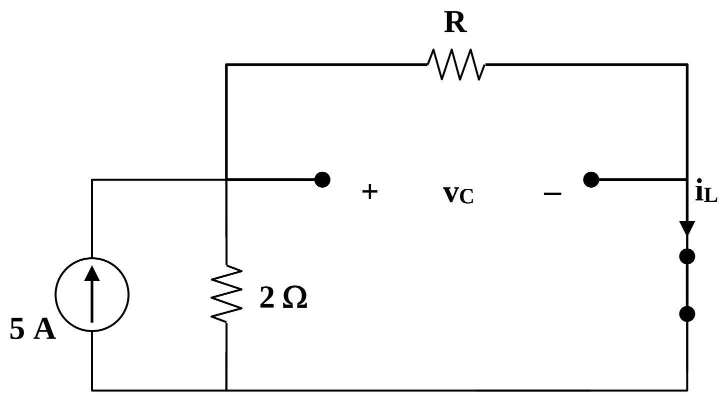
$$i_L = \frac{4}{4+2}(3) = \mathbf{2A}, \quad v_c = \mathbf{0V}$$

$$w_L = \frac{1}{2} L\, i_L^2 = \frac{1}{2}\left(\frac{1}{2}\right)(2)^2 = \mathbf{1J}$$

$$w_c = \frac{1}{2} C\, v_c^2 = \frac{1}{2}(2)(v) = \mathbf{0J}$$

Solution 6.47

Under dc conditions, the circuit is equivalent to that shown below:



$$i_L = \frac{2}{R+2}(5) = \frac{10}{R+2}, \quad v_c = Ri_L = \frac{10R}{R+2}$$

$$w_c = \frac{1}{2}Cv_c^2 = 80x10^{-6}x\frac{100R^2}{(R+2)^2}$$

$$w_L = \frac{1}{2}Li_1^2 = 2x10^{-3}x\frac{100}{(R+2)^2}$$

If $w_c = w_L$,

$$80x10^{-6}x\frac{100R^2}{(R+2)^2} = \frac{2x10^{-3}x100}{(R+2)^2} \longrightarrow 80 \text{ x } 10^{-3}R^2 = 2$$

R = **5Ω**

Solution 6.48

Under steady-state dc conditions, find *i* and *v* in the circuit in Fig. 6.71.

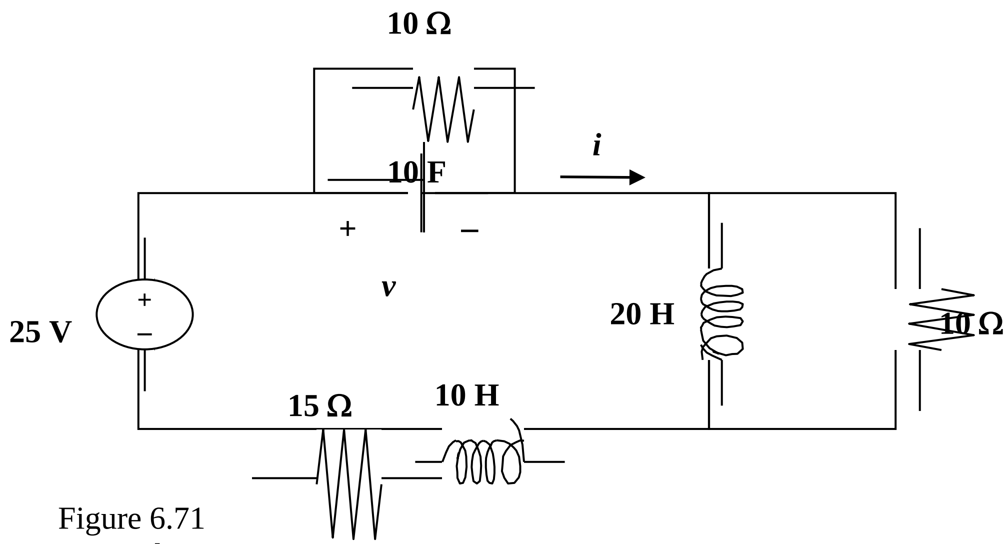


Figure 6.71
For Prob. 6.48.

Solution

Under steady-state, the inductor acts like a short-circuit, while the capacitor acts like an open circuit. Thus the resistor on the right is shorted out and the voltage source only sees the top 10 Ω resistor in series with the 15 Ω resistor for a total of 25 Ω.

Thus i = 25/25 = **1 A** and v = 10x1 = **10 V**.

Solution 6.49

Find the equivalent inductance of the circuit in Fig. 6.72. Assume all inductors are 40 mH.

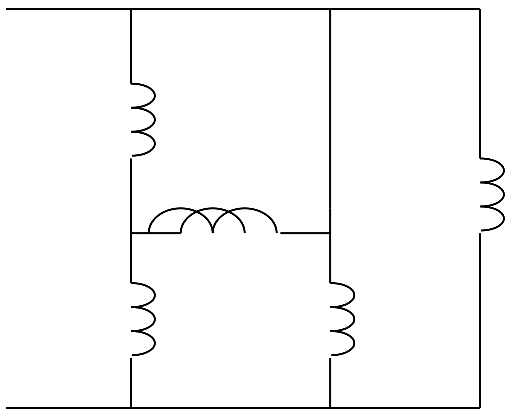
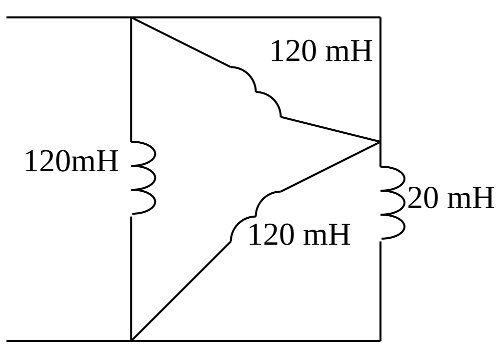


Figure 6.72
For Prob. 6.49.

Solution

Converting the wye-subnetwork to its equivalent delta gives the circuit below.



$120||0 = 0$, $120||20 = 120x20/140= 17.14286$ mH.

Finally,

$L_{eq} = 0.12x0.01714286/(0.12+0.01714286) =$ **15 mH**.

Solution 6.50

16mH in series with 14 mH = 16+14=30 mH
24 mH in series with 36 mH = 24+36=60 mH
30mH in parallel with 60 mH = 30x60/90 = **20 mH**

Solution 6.51

$$\frac{1}{L}=\frac{1}{60}+\frac{1}{20}+\frac{1}{30}=\frac{1}{10}$$ L = 10 mH

$$L_{eq}=10\|(25+10)=\frac{10x35}{45}$$

= **7.778 mH**

Solution 6.52

Using Fig. 6.74, design a problem to help other students better understand how inductors behave when connected in series and when connected in parallel.

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find L_{eq} in the circuit of Fig. 6.74.

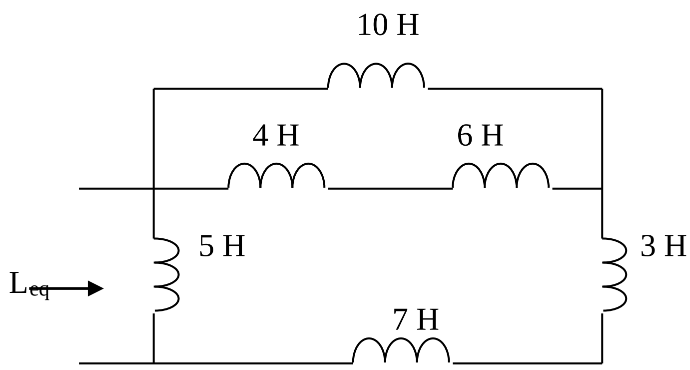


Figure 6.74 For Prob. 6.52.

Solution

$$L_{eq} = 5//(7+3+10//(4+6)) == 5//(7+3+5)) = \frac{5x15}{20} = \underline{3.75\ \ H}$$

Solution 6.53

$$L_{eq} = 6 + 10 + 8 \| \left[5 \| (8 + 12) + 6 \| (8 + 4) \right]$$

$$= 16 + 8 \| (4 + 4) = 16 + 4$$

L_{eq} = **20 mH**

Solution 6.54

Find the equivalent inductance looking into the terminals of the circuit in Fig. 6.76.

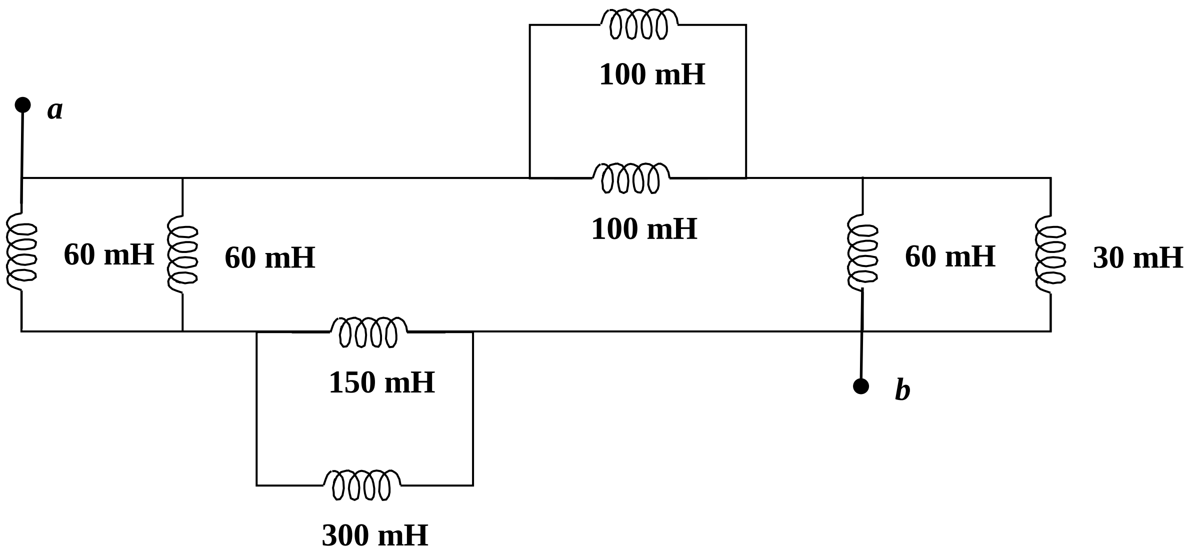


Figure 6.76
For Prob. 6.54.

Solution

The parallel combinations gives us L_{6060} = [60x60/(60+60)]mH = 30 mH; L_{150300} = [150x300/(150+300)]mH = 100 mH; L_{6030} = [60x30/(60+30)]mH = 20 mH; and L_{100100} = [100x100/(100+100)]mH = 50 mH.

We now have inductors in series and then in parallel or L_{30100} = 130 mH and L_{2050} = 70 mH and finally we get L_{ab} = [130x70/(130+70)]mH = **45.5 mH**.

Solution 6.55

(a) L//L = 0.5L, L + L = 2L

$$L_{eq} = L + 2L\,//\,0.5L = L + \frac{2Lx0.5L}{2L + 0.5L} = \underline{1.4L} = \mathbf{1.4\ L}.$$

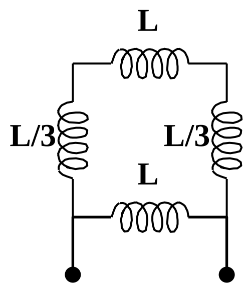
(b) L//L = 0.5L, L//L + L//L = L

L_{eq} = L//L = **500 mL**

Solution 6.56

$$L\|L\|L = \frac{1}{\frac{3}{L}} = \frac{L}{3}$$

Hence the given circuit is equivalent to that shown below:



$$L_{eq} = L \left\| \left(L + \frac{2}{3}L \right) \right. = \frac{L x \frac{5}{3} L}{L + \frac{5}{3} L} = \frac{\mathbf{5}}{\mathbf{8}}\boldsymbol{L}$$

Solution 6.57

$$\text{Let } v = L_{eq}\frac{di}{dt} \qquad (1)$$

$$v = v_1 + v_2 = 4\frac{di}{dt} + v_2 \qquad (2)$$

$$i = i_1 + i_2 \longrightarrow i_2 = i - i_1 \qquad (3)$$

$$v_2 = 3\frac{di_1}{dt} \text{ or } \frac{di_1}{dt} = \frac{v_2}{3} \qquad (4)$$

and

$$-v_2 + 2\frac{di}{dt} + 5\frac{di_2}{dt} = 0$$

$$v_2 = 2\frac{di}{dt} + 5\frac{di_2}{dt} \qquad (5)$$

Incorporating (3) and (4) into (5),

$$v_2 = 2\frac{di}{dt} + 5\frac{di}{dt} - 5\frac{di_1}{dt} = 7\frac{di}{dt} - 5\frac{v_2}{3}$$

$$v_2\left(1 + \frac{5}{3}\right) = 7\frac{di}{dt}$$

$$v_2 = \frac{21}{8}\frac{di}{dt}$$

Substituting this into (2) gives

$$v = 4\frac{di}{dt} + \frac{21}{8}\frac{di}{dt}$$

$$= \frac{53}{8}\frac{di}{dt}$$

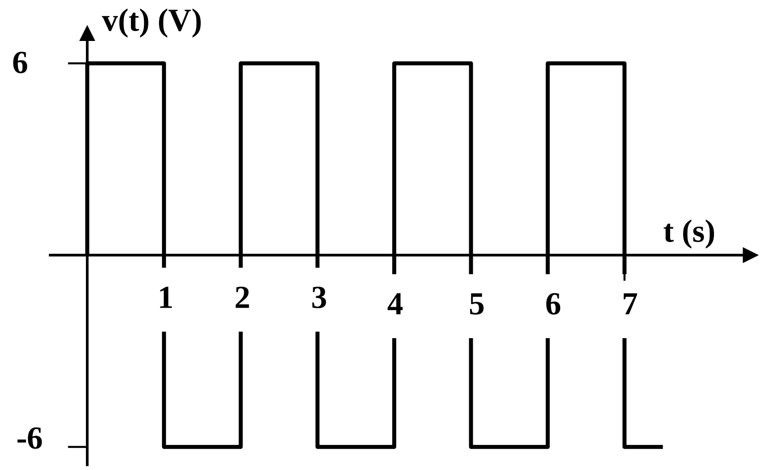
Comparing this with (1),

$$L_{eq} = \frac{53}{8} = \mathbf{6.625\ H}$$

Solution 6.58

$$v = L\frac{di}{dt} = 3\frac{di}{dt} = 3 \text{ x slope of i(t).}$$

Thus v is sketched below:



Solution 6.59

(a) $v_s = (L_1 + L_2)\frac{di}{dt}$

$$\frac{di}{dt} = \frac{v_s}{L_1 + L_2}$$

$$v_1 = L_1 \frac{di}{dt},\ \ v_2 = L_2 \frac{di}{dt}$$

$$\boldsymbol{v_1 = \frac{L_1}{L_1 + L_2} v_s\,,\ v_L = \frac{L_2}{L_1 + L_2} v_s}$$

(b) $v_i = v_2 = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$

$$i_s = i_1 + i_2$$

$$\frac{di_s}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{v}{L_1} + \frac{v}{L_2} = v\frac{(L_1 + L_2)}{L_1 L_2}$$

$$i_1 = \frac{1}{L_1}\int v dt = \frac{1}{L_1}\int \frac{L_1 L_2}{L_1 + L_2}\frac{di_s}{dt} dt = \boldsymbol{\frac{L_2}{L_1 + L_2} i_s}$$

$$i_2 = \frac{1}{L_2}\int v dt = \frac{1}{L_2}\int \frac{L_1 L_2}{L_1 + L_2}\frac{di_s}{dt} dt = \boldsymbol{\frac{L_1}{L_1 + L_2} i_s}$$

Solution 6.60

$$L_{eq} = 3 // 5 = \frac{15}{8}$$

$$v_o = L_{eq}\frac{di}{dt} = \frac{15}{8}\frac{d}{dt}\left(4e^{-2t}\right) = \underline{-15e^{-2t}}$$

$$i_o = \frac{I}{L}\int_0^t v_o(t)dt + i_o(0) = 2 + \frac{1}{5}\int_0^t (-15)e^{-2t}dt = 2 + 1.5e^{-2t}\Big|_0^t$$

i_o = $\mathbf{(0.5 + 1.5e^{-2t})}$ **A**

Solution 6.61

(a) $L_{eq} = 20//(4+6) = 20x10/30 = \underline{6.667\ \text{mH}}$

Using current division,

$$i_1(t) = \frac{10}{10+20} i_s = \underline{e^{-t}\ \text{mA}}$$

$$i_2(t) = \underline{2e^{-t}\ \text{mA}}$$

(b) $v_o = L_{eq} \dfrac{di_s}{dt} = \dfrac{20}{3} x10^{-3}(-3e^{-t}x10^{-3}) = \underline{-20e^{-t}\ \mu\text{V}}$

(c) $w = \dfrac{1}{2} L i_1^2 = \dfrac{1}{2} x20x10^{-3} xe^{-2}x10^{-6} = \underline{1.3534\ \text{nJ}}$

Solution 6.62

Consider the circuit in Fig. 6.84. Given that $v(t) = 12e^{-3t}$ mV for $t > 0$ and $i_1(0) = -30$ mA, find: (a) $i_2(0)$, (b) $i_1(t)$ and $i_2(t)$.

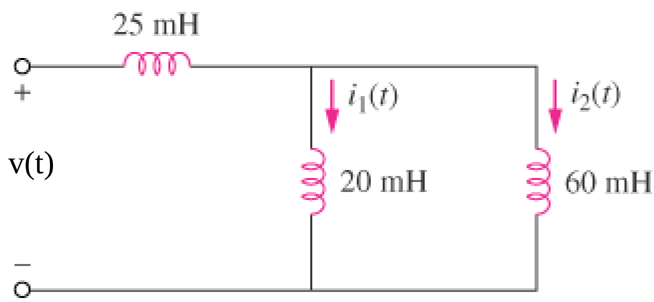


Figure 6.84
For Prob. 6.62.

Solution

(a) $L_{eq} = 25 + 20 \,||\, 60 = 25 + \dfrac{20x60}{80} = 40 \text{ mH}$

$$v = L_{eq}\frac{di}{dt} \quad \longrightarrow \quad i = \frac{1}{L_{eq}}\int v(t)dt + i(0) = \frac{10^{-3}}{40x10^{-3}}\int_0^t 12e^{-3t}dt + i(0) = -0.1(e^{-3t} - 1) + i(0)$$

Using current division and the fact that all the currents were zero when the circuit was put together, we get,

$$i_1 = \frac{60}{80}i = \frac{3}{4}i, \qquad i_2 = \frac{1}{4}i$$

$$i_1(0) = \frac{3}{4}i(0) \quad \longrightarrow \quad 0.75i(0) = -0.03 \quad \longrightarrow \quad i(0) = -0.04$$

$$i_2 = \frac{1}{4}(-0.1e^{-3t} + 0.06) \text{ A} = (-25e^{-3t} + 15) \text{ mA}$$

$i_2(0) = -25+15 =$ **-10 mA**.

(b) $i_1(t) = 0.75(-0.1e^{-3t} + 0.06) =$ **$(-75e^{-3t} + 45)$ mA** and $i_2(t) =$ **$(-25e^{-3t} + 15)$ mA**.

Solution 6.63

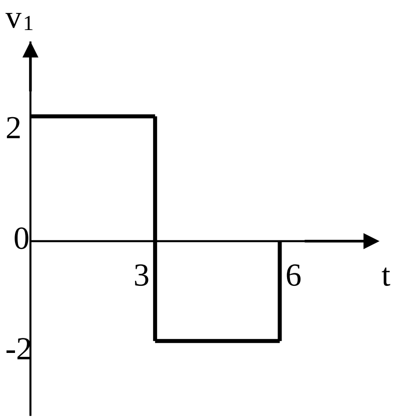
We apply superposition principle and let

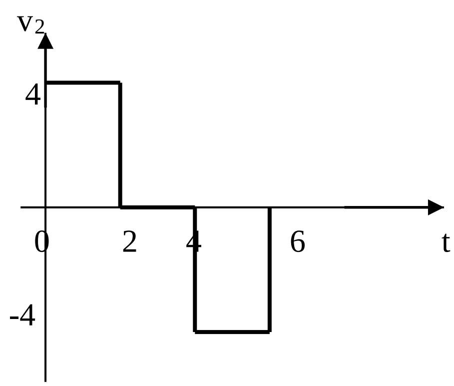
$$v_o = v_1 + v_2$$

where v_1 and v_2 are due to i_1 and i_2 respectively.

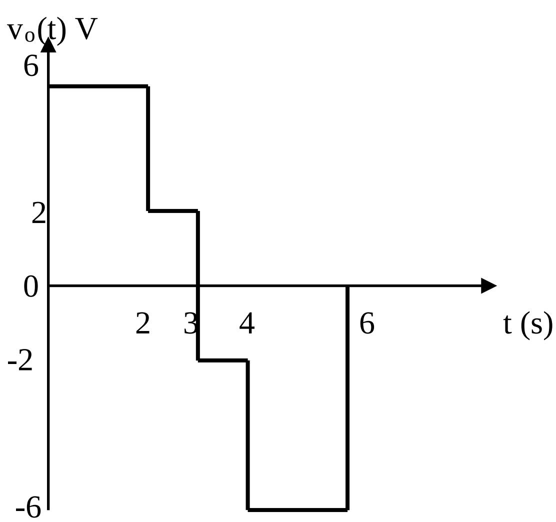
$$v_1 = L\frac{di_1}{dt} = 2\frac{di_1}{dt} = \begin{cases} 2, & 0 < t < 3 \\ -2, & 3 < t < 6 \end{cases}$$

$$v_2 = L\frac{di_2}{dt} = 2\frac{di_2}{dt} = \begin{cases} 4, & 0 < t < 2 \\ 0, & 2 < t < 4 \\ -4, & 4 < t < 6 \end{cases}$$





Adding v_1 and v_2 gives v_o, which is shown below.



Solution 6.64

(a) When the switch is in position A, i= –6 = i(0)

When the switch is in position B, $i(\infty) = 12/4 = 3$, $\tau = L/R = 1/8$

$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$

i(t) = $\mathbf{(3 - 9e^{-8t})}$ **A**

(b) -12 + 4i(0) + v=0, i.e. v = 12 – 4i(0) = **36 V**

(c) At steady state, the inductor becomes a short circuit so that v = **0 V.**

Solution 6.65

(a) $w_5 = \frac{1}{2}L_1 i_1^2 = \frac{1}{2}x5x(4)^2 = \mathbf{40\ J}$

$w_{20} = \frac{1}{2}(20)(-2)^2 = \mathbf{40\ J}$

(b) $w = w_5 + w_{20} =$ **80 J**

(c) $i_1 = \frac{1}{L_1}\int_0^t -50e^{-200t}dt + i_1(0) = \frac{1}{5}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t + 4$

= **[5x10^{-5}(e^{-200t} – 1) + 4] A**

$i_2 = \frac{1}{L_2}\int_0^t -50e^{-200t}dt + i_2(0) = \frac{1}{20}\left(\frac{1}{200}\right)\left(50e^{-200t}x10^{-3}\right)\Big|_0^t - 2$

= **[1.25x10^{-5} (e^{-200t} – 1) – 2] A**

(d) $i = i_1 + i_2 =$ **[6.25x10^{-5} (e^{-200t}– 1) + 2] A**

Solution 6.66

If v=i, then

$$i = L\frac{di}{dt} \longrightarrow \frac{dt}{L} = \frac{di}{i}$$

Integrating this gives

$$\frac{t}{L} = \ln(i) - \ln(C_o) = \ln\left(\frac{i}{C_o}\right) \rightarrow i = C_o e^{t/L}$$

$i(0) = 2 = C_o$

$i(t) = 2e^{t/0.02} =$ **$2e^{50t}$ A**.

Solution 6.67

$v_o = -\frac{1}{RC}\int vi\,dt$, RC = 50 x 10^3 x 0.04 x 10^{-6} = 2 x 10^{-3}

$v_o = \frac{-10^3}{2}\int 10\sin 50t\ dt$

v_o = **100cos(50t) mV**

Solution 6.68

A 6-V dc voltage is applied to an integrator with $\boldsymbol{R}$ = 50 kΩ, $\boldsymbol{C}$ = 100 μF at $\boldsymbol{t}$ = 0. How long will it take for the op amp to saturate if the saturation voltages are +12 V and –12 V? Assume that the initial capacitor voltage was zero.

Solution

$$v_o = -\frac{1}{RC}\int vi\, dt + v_o(0),\ RC = 50 \times 10^3 \times 100 \times 10^{-6} = 5$$

$$v_o = -\frac{1}{5}\int_0^t 6d\tau + 0 = -\frac{6t}{5} = -1.2t\ .$$

The op amp will saturate at $v_o = \pm 12$

$$-12 = -1.2t \longrightarrow t = \mathbf{10\ s}.$$

Solution 6.69

RC = 4 x 10^6 x 1 x 10^{-6} = 4

$$v_o = -\frac{1}{RC}\int v_i dt = -\frac{1}{4}\int v_i dt$$

For 0 < t < 1, v_i = 20, $v_o = -\frac{1}{4}\int_o^t 20dt =$ -5t mV

For 1 < t < 2, v_i = 10, $v_o = -\frac{1}{4}\int_1^t 10dt + v(1) = -2.5(t-1) - 5$

$= -2.5t - 2.5mV$

For 2 < t < 4, v_i = - 20, $v_o = +\frac{1}{4}\int_2^t 20dt + v(2) = 5(t-2) - 7.5$

$= 5t - 17.5$ mV

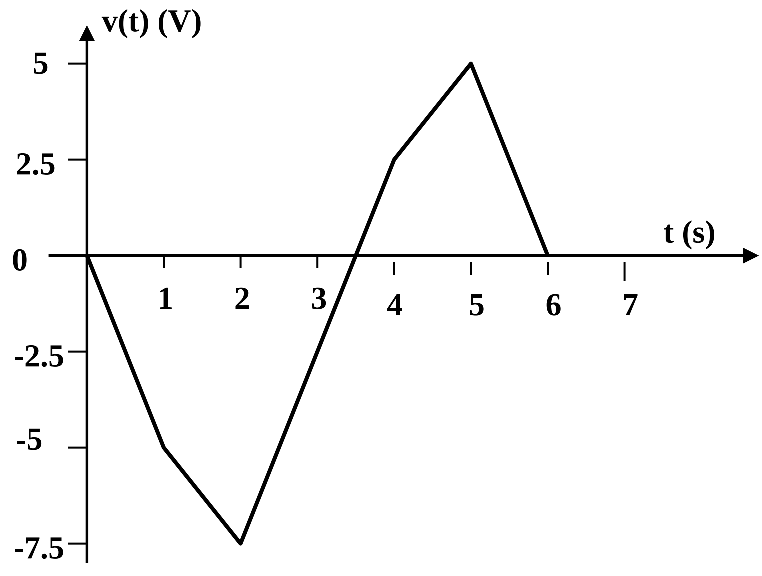
For 4 < t < 5m, v_i = -10, $v_o = \frac{1}{4}\int_4^t 10dt + v(4) = 2.5(t-4) + 2.5$

$= 2.5t - 7.5$ mV

For 5 < t < 6, v_i = 20, $v_o = -\frac{1}{4}\int_5^t 20dt + v(5) = -5(t-5) + 5$

$= - 5t + 30$ mV

Thus $v_o(t)$ is as shown below:



Solution 6.70

Using a single op amp, a capacitor, and resistors of 100 kΩ or less, design a circuit to implement

$$v_o = -2\int_0^t \mathrm{vi}(\tau)\,d\tau$$

Assume $\boldsymbol{v_o}$ = 0 at $\boldsymbol{t}$ = 0.

Solution

One possibility is as follows:

Let RC = 0.5 (which produces (1/RC) = 2.

Next we need to either pick R or C. Let us choose R = **100 kΩ** a practical value. This means that C = $0.5/10^5$ = **5 μF**.

Solution 6.71

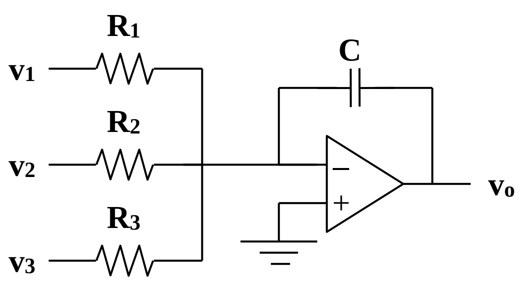
Show how you would use a single op amp to generate

$$v_0 = -\int (v_1 + 4v_2 + 10v_3)\,dt$$

If the integrating capacitor is ***C*** = 5 μF, obtain other component values.

Solution

By combining a summer with an integrator, we have the circuit below:



$$v_o = -\frac{1}{R_1C}\int v_1 dt - \frac{1}{R_2C}\int v_2 dt - \frac{1}{R_3C}\int v_2 dt$$

For the given problem, C = 5 μF,

$R_1C = 1$ gives us $R_1 = 1/C = 10^6/5 =$ **200 kΩ**
$R_2C = 1/4$ gives us $R_2 = 0.25/(C) = 0.25\text{x}200\text{k}\Omega =$ **50 kΩ**
$R_3C = 1/10$ gives us $R_3 = 0.1\text{x}200\text{k}\Omega =$ 2**0 kΩ**

Solution 6.72

The output of the first op amp is

$$v_1 = -\frac{1}{RC}\int v_i\, dt = -\frac{1}{10x10^3 x2x10^{-6}}\int_o^t v_i dt = -\frac{100t}{2}$$

$$= -50t$$

$$v_o = -\frac{1}{RC}\int v_i\, dt = -\frac{1}{20x10^3 x0.5x10^{-6}}\int_o^t (-50t)dt$$

$$= 2500t^2$$

At t = 1.5ms,

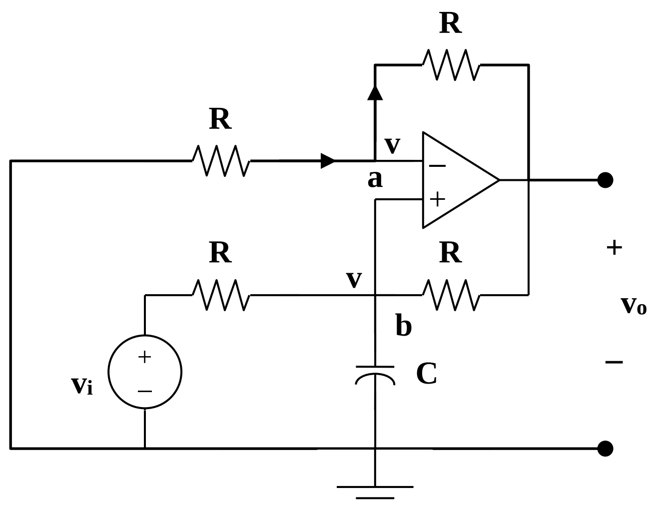
$$v_o = 2500(1.5)^2 x10^{-6} = \mathbf{5.625\ mV}$$

Solution 6.73

Consider the op amp as shown below:

Let $v_a = v_b = v$

At node a, $\dfrac{0-v}{R} = \dfrac{v-v_o}{R} \longrightarrow \mathbf{2v - v_o = 0}$ (1)



At node b, $\dfrac{v_i - v}{R} = \dfrac{v-v_o}{R} + C\dfrac{dv}{dt}$

$$\boldsymbol{v_i = 2v - v_o + RC\frac{dv}{dt}} \qquad (2)$$

Combining (1) and (2),

$$\boldsymbol{v_i = v_o - v_o + \frac{RC}{2}\frac{dv_o}{dt}}$$

or

$$\boldsymbol{v_o = \frac{2}{RC}\int v_i\, dt}$$

showing that the circuit is a noninverting integrator.

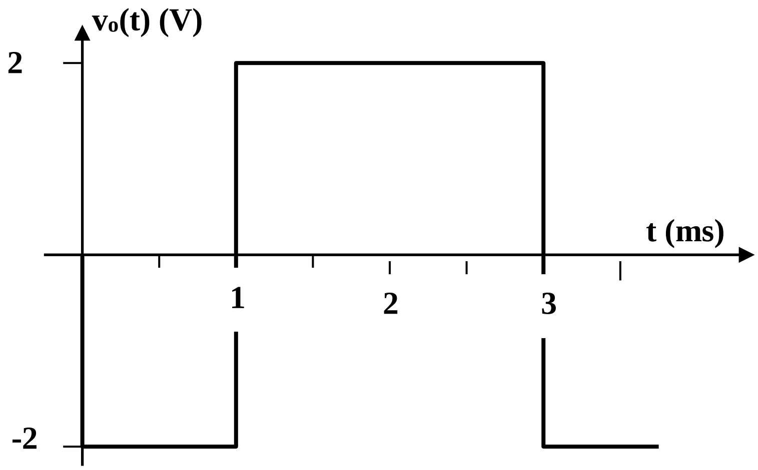
Solution 6.74

RC = 0.01 x 20 x 10^{-3} sec

$$v_o = -RC\frac{dv_i}{dt} = -0.2\frac{dv}{dt}\,m\sec$$

$$v_o = \begin{bmatrix} -2V, & 0 < t < 1 \\ 2V, & 1 < t < 3 \\ -2V, & 3 < t < 4 \end{bmatrix}$$

Thus $v_o(t)$ is as sketched below:



Solution 6.75

An op amp differentiator has $\boldsymbol{R}$= 250 kΩ and $\boldsymbol{C}$ = 10 μF. The input voltage is a ramp $\boldsymbol{r(t)}$ = 7 $\boldsymbol{t}$ mV. Find the output voltage.

Solution

$$v_0 = -RC\frac{dv_i}{dt}, \quad RC = 250x10^3 x10x10^{-6} = 2.5$$

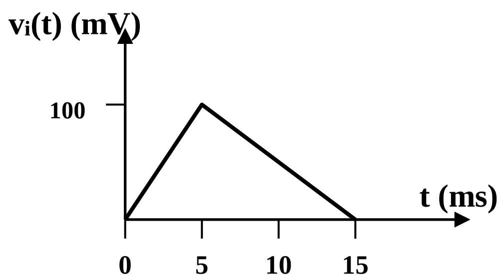
$$v_o = -2.5\frac{d}{dt}(7t) = \mathbf{-17.5\ mV}.$$

Solution 6.76

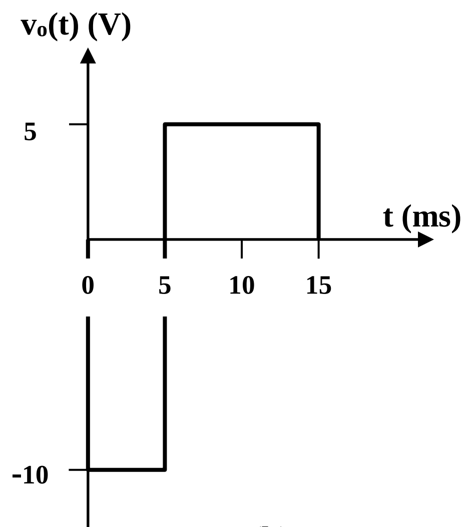
$$v_o = -RC\frac{dv_i}{dt},\ RC = 50 \text{ x } 10^3 \text{ x } 10 \text{ x } 10^{-6} = 0.5$$

$$v_o = -0.5\frac{dv_i}{dt} = \begin{cases} -10, & 0 < t < 5 \\ 5, & 5 < t < 15 \end{cases}$$

The input is sketched in Fig. (a), while the output is sketched in Fig. (b).



(a)



(b)

Solution 6.77

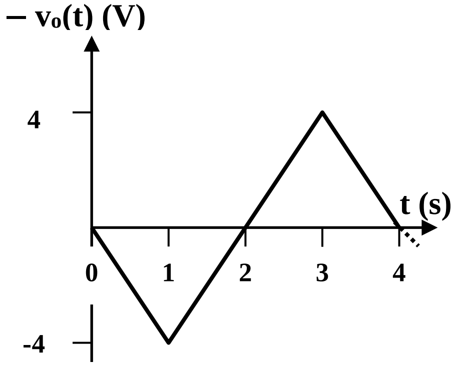
$i = i_R + i_C$

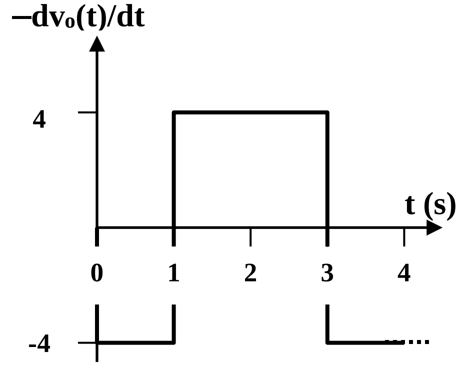
$$\frac{v_i - 0}{R} = \frac{0 - v_0}{R_F} + C\frac{d}{dt}(0 - v_o)$$

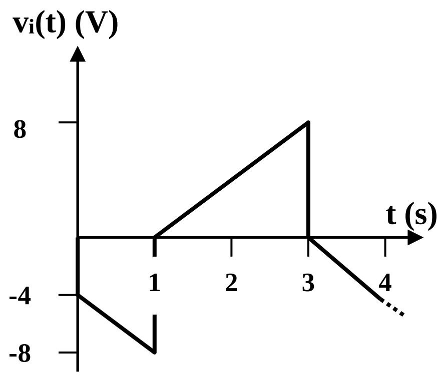
$$R_F C = 10^6 x 10^{-6} = 1$$

Hence $v_i = -\left(v_o + \frac{dv_o}{dt}\right)$

Thus v_i is obtained from v_o as shown below:







Solution 6.78

Design an analog computer to simulate

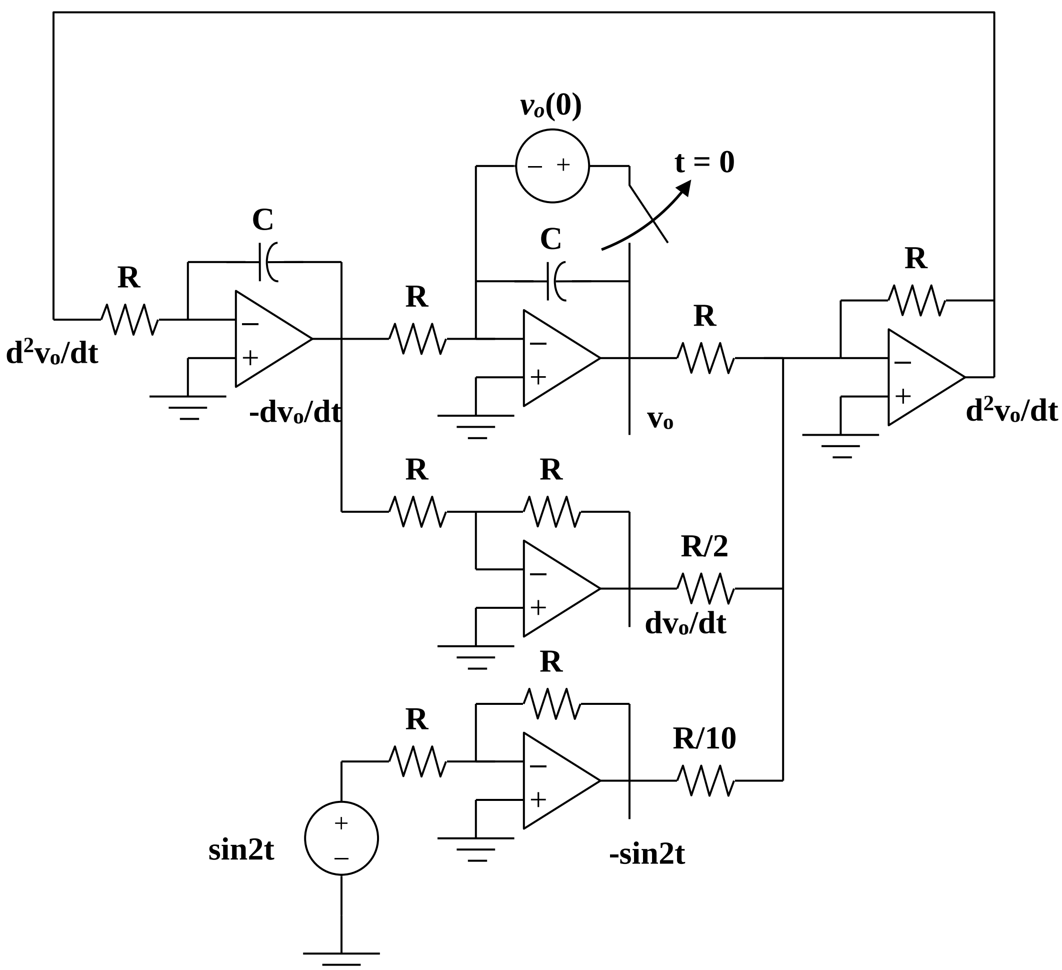
$$\frac{d^2 v_0}{dt^2} + 2\frac{dv_0}{dt} + v_0 = 10\sin 2t$$

where $\boldsymbol{v_0(0)} = -6$ V and $v'_0\mathbf{(0)} = 0$.

Solution

$$\frac{d^2 v_o}{dt} = 10\sin 2t - \frac{2dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below, where $v_o(0) = -6$ V:



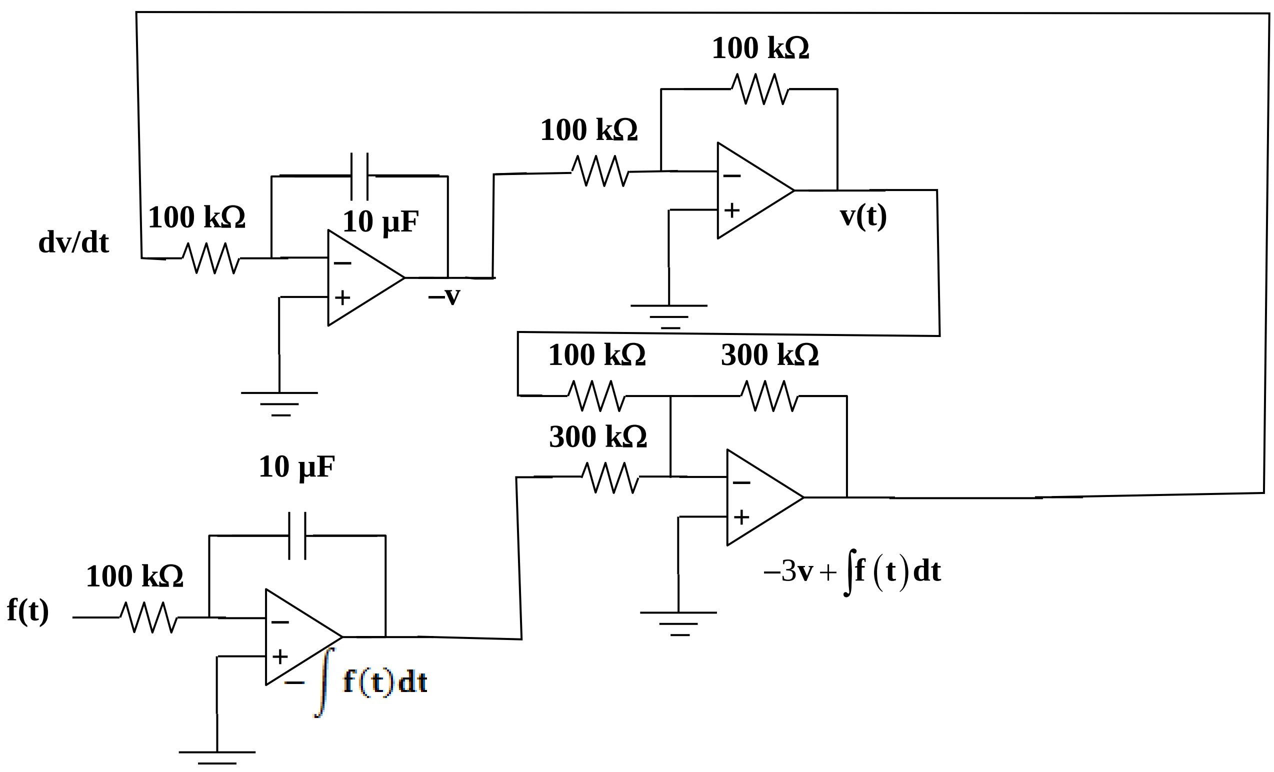
Solution 6.79

Design an analog computer circuit to solve for v(t) given the following equation and a value for f(t) and that v(0) = 0 V.

$$\frac{dv(t)}{dt} + 3v(t) = \int f(t)\,dt$$

Solution

We can write the equation as $\frac{dv(t)}{dt} = \int f(t)\,dt - 3v(t)$. As with any design problem, there are many acceptable solutions, this is just one of them.



Solution 6.80

From the given circuit,

$$\frac{d^2 v_o}{dt^2} = f(t) - \frac{1000k\Omega}{5000k\Omega} v_o - \frac{1000k\Omega}{200k\Omega} \frac{dv_o}{dt}$$

or

$$\boldsymbol{\frac{d^2 v_o}{dt^2} + 5\frac{dv_o}{dt} + 2v_o = f(t)}$$

Solution 6.81

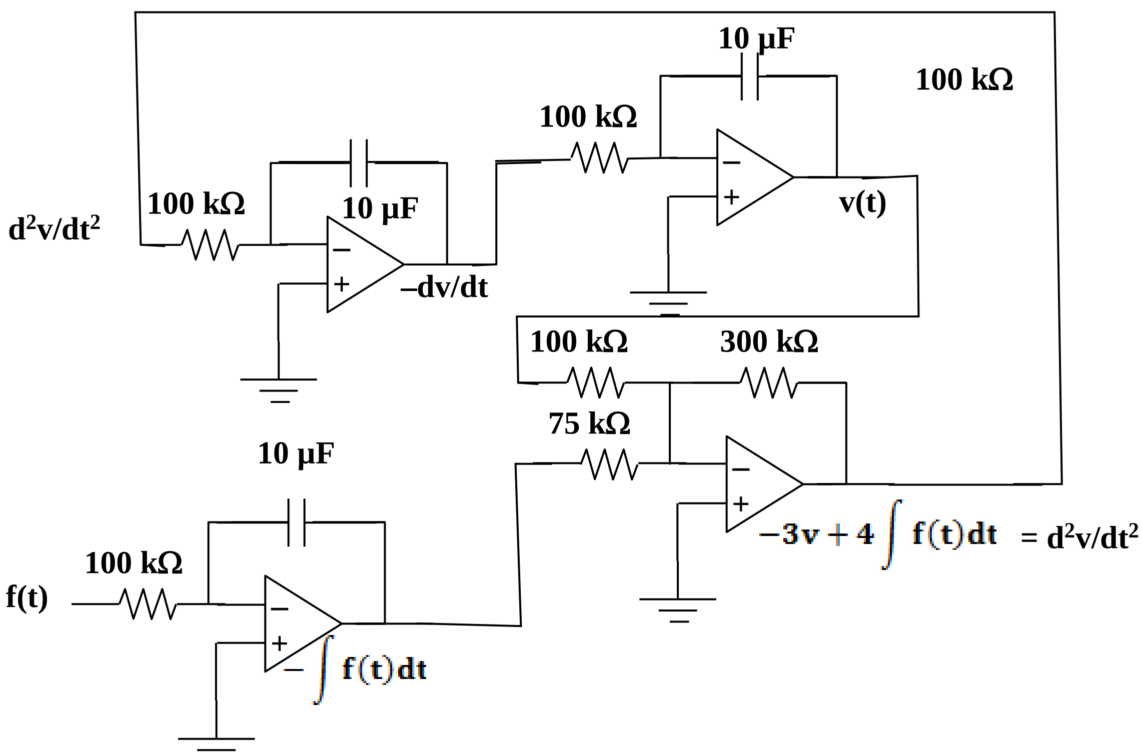
Design an analog computer to simulate the following equation to solve for v(t) (assume the initial conditions are zero):

$$\frac{d^3v(t)}{dt^3} + 3\frac{dv(t)}{dt} = 4f(t).$$

Solution

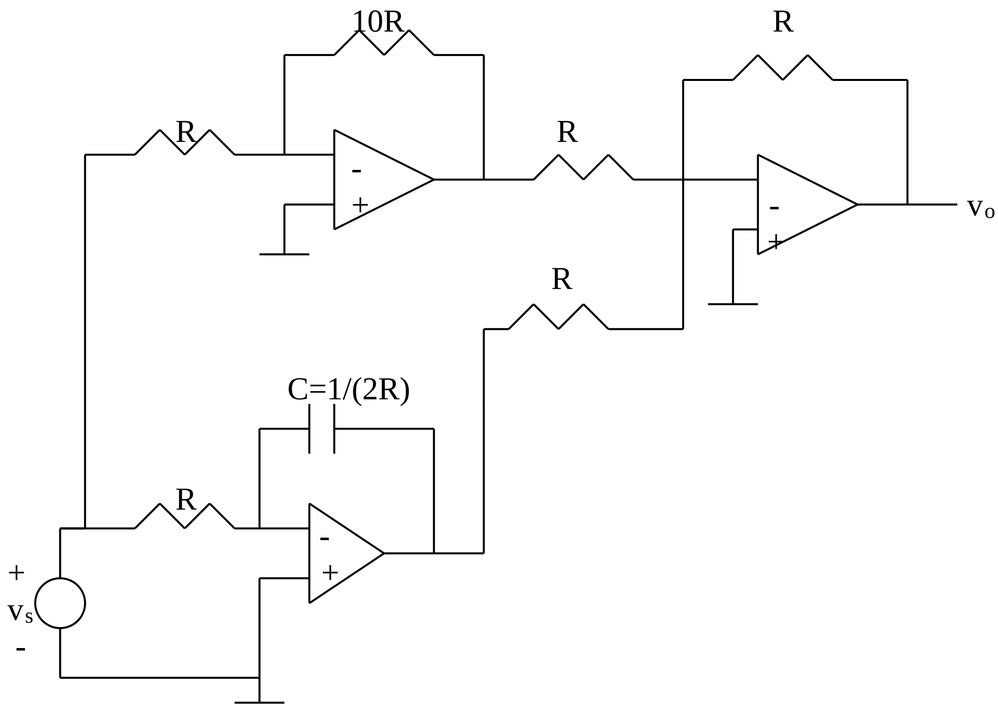
If we integrate both sides we can end up with $\frac{d^2v(t)}{dt^2} = -3v(t) + 4\int f(t)dt$.

As with any design problem there are many acceptable solutions, this is one of them. Note, the above can also be solved without the integration step and would also be an accurate solution.



Solution 6.82

The circuit consists of a summer, an inverter, and an integrator. Such circuit is shown below.

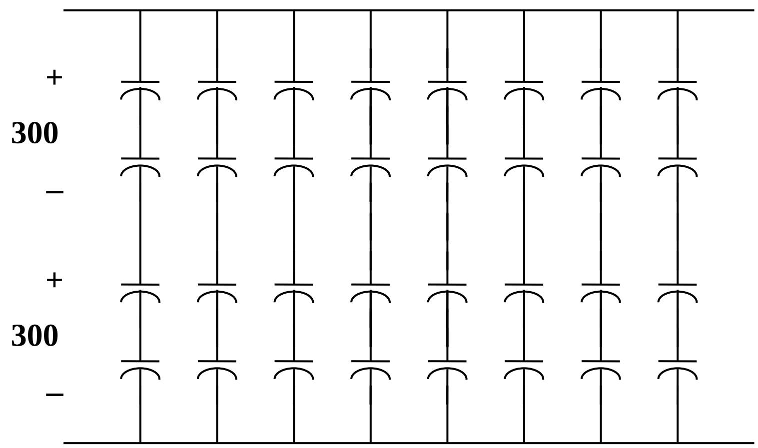


Solution 6.83

Your laboratory has available a large number of 5-μF capacitors rated at 150 V. To design a capacitor bank of 10-μF rated at 600 V, how many 5-μF capacitors are needed and how would you connect them?

Solution

Since four 5 μF capacitors in series gives 1.25 μF, rated at 600V, it requires 8 groups in parallel with each group consisting of four capacitors in series, as shown below:



Answer: **Eight groups in parallel with each group made up of four capacitors in series.**

Solution 6.84

An 8-mH inductor is used in a fusion power experiment. If the current through the inductor is $i(t) = 10\cos^2(\pi t)$ mA, for all $t > 0$ sec, find the power being delivered to the inductor and the energy stored in it at t=0.5s.

Solution

$v = L(di/dt) = 8x10^{-3}x10x(-2\pi)\cos(\pi t)\sin(\pi t)10^{-3} = -80\pi\sin(2\pi t)\ \mu V$.

$p = vi =$ $\mathbf{-80\pi\sin(2\pi t)10\cos^2(\pi t)10^{-9}\ W}$. At t = 0.5 s, p(0.5) = **0 W**.

At t = 0.5 s, $w(0.5) = (1/2)Li(0.5)^2 = 0.5x8x10^{-3}(10x10^{-3}x0)^2 =$ **0 J**.

Solution 6.85

It is evident that differentiating i will give a waveform similar to v. Hence,

$$v = L\frac{di}{dt}$$

$$i = \begin{bmatrix} 4t, 0 < t < 1ms \\ 8 - 4t, 1 < t < 2ms \end{bmatrix}$$

$$v = L\left[\frac{di}{dt} = \begin{bmatrix} 4000L, 0 < t < 1ms \\ -4000L, 1 < t < 2ms \end{bmatrix}\right.$$

But, $$v = \begin{bmatrix} 5V, 0 < t < 1ms \\ -5V, 1 < t < 2ms \end{bmatrix}$$

Thus, 4000L = 5 $\longrightarrow$ L = 1.25 mH in a **1.25 mH inductor**

Solution 6.86

$$v = v_R + v_L = Ri + L\frac{di}{dt} = 12x2te^{-10t} + 200x10^{-3}x(-20te^{-10t} + 2e^{-10t}) = \underline{(0.4 - 20t)e^{-10t} \text{ V}}$$