常微与偏微课程作业

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题目1.

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 4y = e^{2t} \quad y(0) = 1 \quad y'(0) = -1$$

解答.

$$S^2Y(S) - S + 1 - 5(SY(S) - 1) + 4Y(S) = \frac{1}{S-2}$$

$$Y(S)(S^2 - 5S + 4) - S + 6 = \frac{1}{S-2}$$

$$Y(S)(S^2 - 5S + 4) = \frac{1}{S-2} + S - 6$$

$$Y(S) = \frac{1}{(S-2)(S^2-5S+4)} + \frac{S-6}{S^2-5S+4}$$

$$Y(S) = \frac{1}{(S-1)(S-2)(S-4)} + \frac{S-6}{(S-1)(S-4)}$$

$$\frac{1}{(S-1)(S-2)(S-4)} = \frac{A}{(S-1)} + \frac{B}{(S-2)} + \frac{C}{(S-4)}$$

$$A(S-2)(S-4) + B(S-1)(S-4) + C(S-1)(S-2) = 1$$

$$S = 1 \quad A = \frac{1}{3} \quad S = 2 \quad B = -\frac{1}{2} \quad S = 4 \quad C = \frac{1}{6}$$

$$\frac{1}{(S-1)(S-2)(S-4)} = \frac{1}{3}\frac{1}{(S-1)} - \frac{1}{2}\frac{1}{(S-2)} + \frac{1}{6}\frac{1}{(S-4)}$$

$$\frac{S-6}{(S-1)(S-4)} = \frac{D}{(S-1)} + \frac{E}{(S-4)}$$

$$D(S-4) + E(S-1) = S - 6$$

$$S = 1 \quad D = \frac{5}{3} \quad S = 4 \quad E = -\frac{2}{3}$$

$$\frac{S-6}{(S-1)(S-4)} = \frac{5}{3}\frac{1}{(S-1)} - \frac{2}{3}\frac{1}{(S-4)}$$
$$Y(S) = 2\frac{1}{(S-1)} - \frac{1}{2}\frac{1}{(S-2)} - \frac{1}{2}\frac{1}{(S-4)}$$
$$y(t) = 2e^{t} - \frac{1}{2}e^{2t} - \frac{1}{2}e^{4t}$$

题目2.

$$2\frac{d^2y}{dt^2} + \frac{dy}{dt} - y = e^{3t} \quad y(0) = 2 \quad y'(0) = 0$$

解答.

$$2[S^2Y(S) - 2S] + [SY(S) - 2] - Y(S) = \frac{1}{S-3}$$
$$Y(S)[2S^2 + S - 1] - 4S - 2 = \frac{1}{S-3}$$
$$Y(S)[2S^2 + S - 1] = \frac{1}{S-3} + 4S + 2$$
$$Y(S) = \frac{1}{(S-3)(2S^2+S-1)} + \frac{4S+2}{2S^2+S-1}$$
$$Y(S) = \frac{1}{(2S-1)(S+1)(S-3)} + \frac{2(2S+1)}{(2S-1)(S+1)}$$
$$\frac{1}{(2S-1)(S+1)(S-3)} = \frac{A}{(2S-1)} + \frac{B}{(S+1)} + \frac{C}{(S-3)}$$
$$A(S+1)(S-3) + B(2S-1)(S-3) + C(2S-1)(S+1) = 1$$
$$S = \frac{1}{2} \quad A = -\frac{4}{15} \quad S = -1 \quad B = \frac{1}{12} \quad S = 3 \quad C = \frac{1}{20}$$
$$\frac{1}{(2S-1)(S+1)(S-3)} = -\frac{4}{15}\frac{1}{(2S-1)} + \frac{1}{12}\frac{1}{(S+1)} + \frac{1}{20}\frac{1}{(S-3)}$$
$$\frac{2(2S+1)}{(2S-1)(S+1)} = \frac{D}{(2S-1)} + \frac{E}{(S+1)}$$
$$D(S+1) + E(2S-1) = 2(2S+1)$$
$$S = \frac{1}{2} \quad D = \frac{8}{3} \quad S = -1 \quad E = \frac{2}{3}$$
$$\frac{2(2S+1)}{(2S-1)(S+1)} = \frac{8}{3}\frac{1}{(2S-1)} + \frac{2}{3}\frac{1}{(S+1)}$$
$$Y(S) = \frac{12}{5}\frac{1}{(2S-1)} + \frac{3}{4}\frac{1}{(S+1)} + \frac{1}{20}\frac{1}{(S-3)}$$
$$y(t) = \frac{12}{5}e^{\frac{1}{2}t} + \frac{3}{4}e^{-t} + \frac{1}{20}e^{3t}$$

题目3.

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}+y=H\pi(t)-H2\pi(t)\quad y(0)=1\quad y'(0)=0$$

解答.

$$S^2Y(S)-S+SY(S)-1+Y(S)=\frac{e^{-\pi s}-e^{-2\pi s}}{S}$$

$$Y(S)(S^2+S+1)-S-1=\frac{e^{-\pi s}-e^{-2\pi s}}{S}$$

$$Y(S)=\frac{e^{-\pi s}-e^{-2\pi s}}{S(S^2+S+1)}+\frac{S+1}{S^2+S+1}$$

$$\frac{S+1}{S^2+S+1}=\frac{S+1}{(S+\frac{1}{2})^2+\frac{3}{4}}=\frac{S+\frac{1}{2}}{(S+\frac{1}{2})^2+\frac{3}{4}}+\frac{\frac{1}{2}}{(S+\frac{1}{2})^2+\frac{3}{4}}$$

$$\frac{S+1}{S^2+S+1}=e^{-\frac{S}{2}}[\cos(\frac{\sqrt{3}}{2}S)+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}S)]$$

$$\frac{1}{S(S^2+S+1)}=\frac{AS+B}{S(S^2+S+1)}+\frac{C}{S}$$

$$(AS+B)S+C(S^2+S+1)=1\quad C=1$$

$$B+C=0\quad B=-1\quad A+C=0\quad A=-1$$

$$\frac{1}{S(S^2+S+1)}=\frac{-(S+1)}{S(S^2+S+1)}+\frac{1}{S}$$

$$\frac{1}{S(S^2+S+1)}=\frac{-(S+1)}{(S+\frac{1}{2})^2+\frac{3}{4}}+\frac{1}{S}$$

$$=e^{-\frac{S}{2}}[\cos(\frac{\sqrt{3}}{2}S)+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}S)]+1$$

$$\frac{e^{-\pi s}-e^{-2\pi s}}{S(S^2+S+1)}=\{-e^{-\frac{-(S-\pi)}{2}}[\cos(\frac{\sqrt{3}}{2}(S-\pi))+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}(S-\pi))]+1\}H\pi(S)$$

$$+\{-e^{-\frac{-(S-2\pi)}{2}}[\cos(\frac{\sqrt{3}}{2}(S-2\pi))+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}(S-2\pi))]+1\}H2\pi(S)$$

$$y(t)=e^{-\frac{t}{2}}[\cos(\frac{\sqrt{3}}{2}t)+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}t)]$$

$$+\{-e^{-\frac{-(t-\pi)}{2}}[\cos(\frac{\sqrt{3}}{2}(t-\pi))+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}(t-\pi))]+1\}H\pi(t)$$

$$+\{-e^{-\frac{-(t-2\pi)}{2}}[\cos(\frac{\sqrt{3}}{2}(t-2\pi))+\frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}(t-2\pi))]+1\}H2\pi(t)$$

题目4.

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}+7y=t,0\leqq t<2\quad 0,2\leqq t<\infty$$

解答.

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}+7y=t[H0(t)-H2(t)]$$

$$Y(S)(S^2+S+7)=\frac{1}{S^2}-\frac{e^{-2S}}{S^2}-\frac{2e^{-2S}}{S}$$

$$Y(S)=\frac{1}{S^2(S^2+S+7)}-\frac{e^{-2S}}{S^2(S^2+S+7)}-\frac{2e^{-2S}}{S(S^2+S+7)}$$

$$\frac{1}{S^2(S^2+S+7)}=\frac{A}{S}+\frac{B}{S^2}+\frac{CS+D}{S^2+S+7}$$

$$AS(S^2+S+7)+B(S^2+S+7)+(CS+D)(S^2)=1$$

$$7B=1\quad B=\frac{1}{7}\quad B+7A=0\quad A=-\frac{1}{49}$$

$$D+B+A=0\quad D=-\frac{6}{49}\quad C+A=0\quad C=\frac{1}{49}$$

$$\frac{1}{S^2(S^2+S+7)}=-\frac{1}{49}\frac{1}{S}+\frac{1}{7}\frac{1}{S^2}+\frac{\frac{1}{49}S-\frac{6}{49}}{S^2+S+7}$$

$$\frac{1}{S^2(S^2+S+7)}=\frac{1}{49}\{-1+7S+[\cos(\frac{\sqrt{27}S}{2})-\frac{13}{\sqrt{27}}\sin(\frac{\sqrt{27}S}{2})]e^{-\frac{S}{2}}\}$$

$$\frac{e^{-2S}}{S^2(S^2+S+7)}=\frac{1}{49}\{-1+7(S-2)+[\cos(\frac{\sqrt{27}(S-2)}{2})-\frac{13}{\sqrt{27}}\sin(\frac{\sqrt{27}(S-2)}{2})]e^{-\frac{S-2}{2}}\}$$

$$\frac{2}{S(S^2+S+7)}=\frac{A}{S}+\frac{BS+C}{S^2+S+7}$$

$$A(S^2+S+7)+(BS+C)(S)=2$$

$$7A=2\quad A=\frac{2}{7}\quad A+B=0\quad B=-\frac{2}{7}\quad A+C=0\quad C=-\frac{2}{7}$$

$$\frac{2}{S(S^2+S+7)}=\frac{2}{7}\frac{1}{S}+\frac{-\frac{2}{7}S-\frac{2}{7}}{S^2+S+7}$$

$$\frac{2e^{-2S}}{S(S^2+S+7)}=(-\frac{2}{7})\{-1+[\cos(\frac{\sqrt{27}(S-2)}{2})-\frac{1}{\sqrt{27}}\sin(\frac{\sqrt{27}(S-2)}{2})]e^{-\frac{S-2}{2}}\}$$

$$y(t)=\frac{1}{49}\{H0(t)[7(t-1)+[\cos(\frac{\sqrt{27}t}{2})-\frac{13}{\sqrt{27}}\sin(\frac{\sqrt{27}t}{2})]e^{-\frac{t}{2}}]$$

$$-H2(t)\{7(t-1)+[\cos(\frac{\sqrt{27}(t-2)}{2})-\frac{13}{\sqrt{27}}\sin(\frac{\sqrt{27}(t-2)}{2})]e^{-\frac{t-2}{2}}\}$$

$$y(t) = \frac{1}{49}\{[7(t-1) + [\cos(\frac{\sqrt{27}t}{2}) - \frac{13}{\sqrt{27}}\sin(\frac{\sqrt{27}t}{2})]e^{-\frac{t}{2}}]$$

$$-H2(t)\{7(t-1) + [\cos(\frac{\sqrt{27}(t-2)}{2}) - \frac{13}{\sqrt{27}}\sin(\frac{\sqrt{27}(t-2)}{2})]e^{-\frac{t-2}{2}}\}$$

题目5.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 2\delta(t-2) - \delta(t-2) \quad y(0) = 1 \quad y'(0) = 0$$

解答.

$$S^2Y(S) - S + SY(S) - 1 + Y(S) = 2e^{-s} - e^{-2s}$$

$$Y(S)(S^2 + S + 1) = S + 1 + 2e^{-s} - e^{-2s}$$

$$Y(S) = \frac{2e^{-s}}{S^2+S+1} - \frac{e^{-2s}}{S^2+S+1} + \frac{S+1}{S^2+S+1}$$

$$\frac{2e^{-s}}{S^2+S+1} = \frac{2e^{-s}}{(S+\frac{1}{2})^2+\frac{3}{4}} = \frac{4}{\sqrt{3}}e^{-\frac{s-1}{2}}\sin[\frac{\sqrt{3}}{2}(s-1)]H1(t)$$

$$\frac{e^{-2s}}{S^2+S+1} = \frac{e^{-2s}}{(S+\frac{1}{2})^2+\frac{3}{4}} = \frac{2}{\sqrt{3}}e^{-\frac{s-2}{2}}\sin[\frac{\sqrt{3}}{2}(s-2)]H2(t)$$

$$\frac{S+1}{S^2+S+1} = \frac{(S+\frac{1}{2})+\frac{1}{2}}{S^2+S+1} = e^{-\frac{s}{2}}[\cos(\frac{\sqrt{3}}{2}S) + \frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}S)]$$

$$y(t) = e^{-\frac{t}{2}}[\cos(\frac{\sqrt{3}}{2}t) + \frac{1}{\sqrt{3}}\sin(\frac{\sqrt{3}}{2}t)] + \frac{4}{\sqrt{3}}e^{-\frac{t-1}{2}}\sin[\frac{\sqrt{3}}{2}(t-1)]H1(t)$$

$$-\frac{2}{\sqrt{3}}e^{-\frac{t-2}{2}}\sin[\frac{\sqrt{3}}{2}(t-2)]H2(t)$$