

# 常微与偏微课程作业

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题目1.

$$\frac{d^2y}{dt^2} - y = 0$$

解答.

$$t^2 - 1 = 0$$

$$t_1 = 1 \quad t_2 = -1$$

$$y_1(t) = e^t \quad y_2(t) = e^{-t}$$

$$y(t) = C_1 e^t + C_2 e^{-t}$$

题目2.

$$6\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + y = 0$$

解答.

$$6t^2 - 7t + 1 = 0$$

$$t_1 = 1 \quad t_2 = \frac{1}{6}$$

$$y_1(t) = e^t \quad y_2(t) = e^{\frac{1}{6}t}$$

$$y(t) = C_1 e^t + C_2 e^{\frac{1}{6}t}$$

题目3.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 0$$

解答.

$$\begin{aligned} t^2 + t + 1 &= 0 \\ t_1 &= \frac{-1 + \sqrt{3}}{2} \quad t_2 = \frac{-1 - \sqrt{3}}{2} \\ y(t) &= e^{r_1 t} = e^{\frac{-1 + \sqrt{3}}{2} t} \\ &= e^{\frac{-t}{2}} C_1 \cos \frac{\sqrt{3}t}{2} + i e^{\frac{-t}{2}} C_2 \sin \frac{\sqrt{3}t}{2} \end{aligned}$$

题目4.

$$2 \frac{d^2y}{dt^2} + 3 \frac{dy}{dt} + 4y = 0$$

解答.

$$\begin{aligned} 2t^2 + 3t + 4 &= 0 \\ t_1 &= \frac{-3 + \sqrt{23}}{4} \quad t_2 = \frac{-3 - \sqrt{23}}{4} \\ y(t) &= e^{r_1 t} = e^{\frac{-3 + \sqrt{23}}{4} t} \\ &= e^{\frac{-3t}{4}} C_1 \cos \frac{\sqrt{23}t}{4} + i e^{\frac{-3t}{4}} C_2 \sin \frac{\sqrt{23}t}{4} \end{aligned}$$

题目5.

$$\frac{d^2y}{dt^2} - 6 \frac{dy}{dt} + 9y = 0$$

解答.

$$\begin{aligned} t^2 - 6t + 9 &= 0 \\ (t - 3)^2 &= 0 \\ r_1 &= r_2 = 3 \\ y(t) &= C_1 e^{3t} + C_2 t e^{3t} \end{aligned}$$

题目6.

$$4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = 0$$

解答.

$$4t^2 - 12t + 9 = 0$$

$$(2t - 3)^2 = 0$$

$$r_1 = r_2 = \frac{3}{2}$$

$$y(t) = C_1 e^{\frac{3}{2}t} + C_2 t e^{\frac{3}{2}t}$$