Solution 3.1

Using Fig. 3.50, design a problem to help other students to better understand nodal analysis.

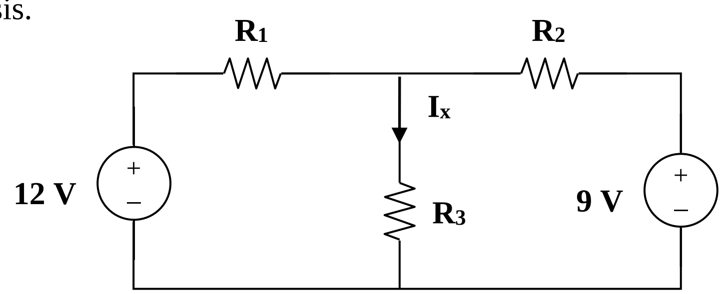


Figure 3.50
For Prob. 3.1 and Prob. 3.39.

Solution

Given R_1 = 4 kΩ, R_2 = 2 kΩ, and R_3 = 2 kΩ, determine the value of I_x using nodal analysis.

Let the node voltage in the top middle of the circuit be designated as V_x.

$[(V_x–12)/4k] + [(V_x–0)/2k] + [(V_x–9)/2k] = 0$ or (multiply this by 4 k)

$(1+2+2)V_x = 12+18 = 30$ or $V_x = 30/5 = 6$ volts and

$I_x = 6/(2k) =$ **3 mA**.

Solution 3.2

At node 1,

$$\frac{-v_1}{10} - \frac{v_1}{5} = 6 + \frac{v_1 - v_2}{2} \longrightarrow 60 = -8v_1 + 5v_2 \qquad (1)$$

At node 2,

$$\frac{v_2}{4} = 3 + 6 + \frac{v_1 - v_2}{2} \longrightarrow 36 = -2v_1 + 3v_2 \qquad (2)$$

Solving (1) and (2),

v_1 = **0 V**, v_2 = **12 V**

Solution 3.3

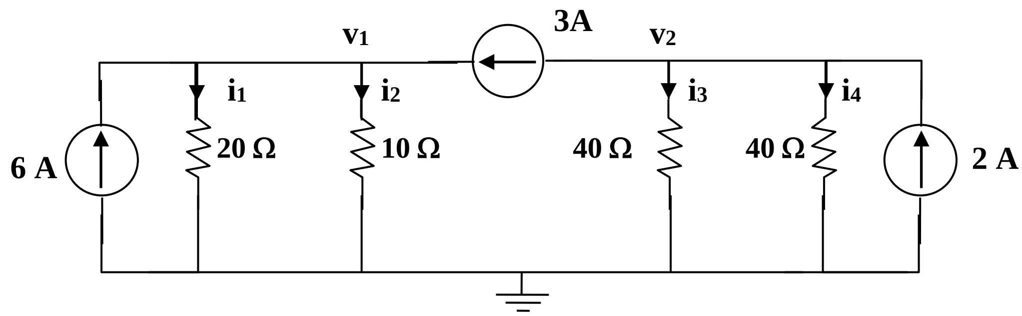
Applying KCL to the upper node,

$$-8+\frac{v_0}{10}+\frac{v_o}{20}+\frac{v_o}{30}+20+\frac{v_0}{60}=0 \text{ or } v_0 = \mathbf{-60\ V}$$

$i_1 = \dfrac{v_0}{10} = \mathbf{-6\ A}$, $i_2 = \dfrac{v_0}{20} = \mathbf{-3\ A}$,

$i_3 = \dfrac{v_0}{30} = \mathbf{-2\ A}$, $i_4 = \dfrac{v_0}{60} = \mathbf{1\ A}$.

Solution 3.4



At node 1,

$$-6 - 3 + v_1/(20) + v_1/(10) = 0 \text{ or } v_1 = 9(200/30) = 60 \text{ V}$$

At node 2,

$$3 - 2 + v_2/(10) + v_2/(5) = 0 \text{ or } v_2 = -1(1600/80) = -20 \text{ V}$$

$i_1 = v_1/(20) =$ **3 A**, $i_2 = v_1/(10) =$ **6 A**,
$i_3 = v_2/(40) =$ **–500 mA**, $i_4 = v_2/(40) =$ **–500 mA**.

Solution 3.5

Obtain v_o in the circuit of Fig. 3.54.

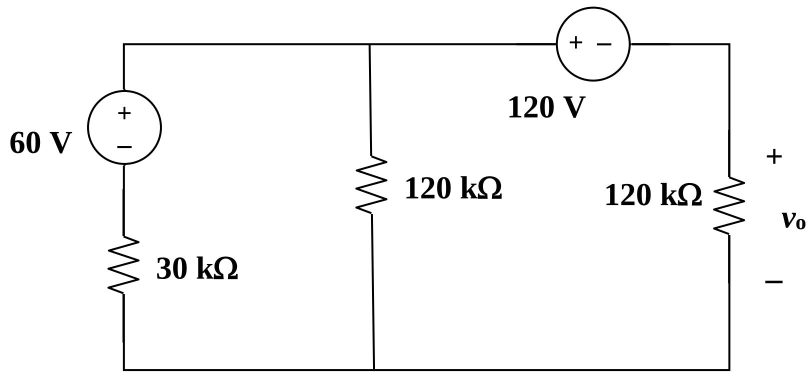
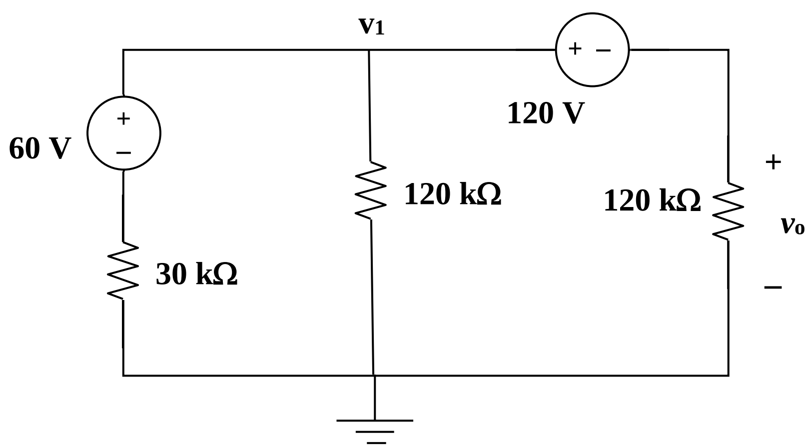


Figure 3.54
For Prob. 3.5.

Step 1. First you need to pick a reference, so we place a ground at the bottom of the circuit. Then we identify the unknown node and then write our nodal equations. Next we apply a constraint equation to solve for v_o.



At node 1, $[((v_1–60)–0)/30k] + [(v_1–0)/120k] + [((v_1–120)/120k] = 0$ and $v_o = (v_1–120)–0$.

Step 2. $[(1/30k)+(1/120k)+(1/120k)]v_1 = (60/30k)+(120/120k) = 0.002+0.001 = 0.003 = (6/120k)v_1$ or $v_1 = 0.003x20k = 60$ volts.

Therefore,

$v_o = 60–120 =$ **–60 V**.

Solution 3.6

Solve for V_1 using nodal analysis.

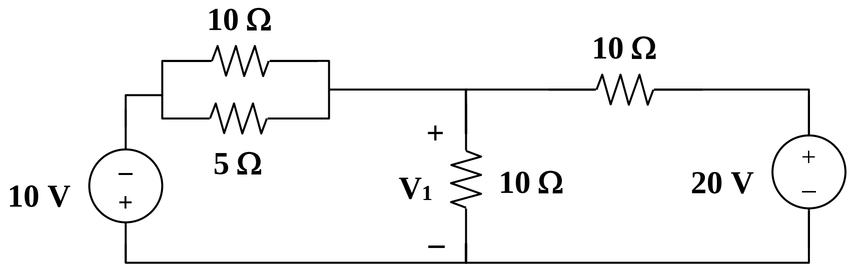


Figure 3.55
For Prob. 3.6.

Step 1. The first thing to do is to select a reference node and to identify all the unknown nodes. We select the bottom of the circuit as the reference node. The only unknown node is the one connecting all the resistors together and we will call that node V_1. The other two nodes are at the top of each source. Relative to the reference, the one at the top of the 10-volt source is –10 V. At the top of the 20-volt source is +20 V.

Step 2. Setup the nodal equation (there is only one since there is only one unknown).

$$\frac{(V_1-(-10))}{5}+\frac{(V_1-(-10))}{10}+\frac{(V_1-0)}{10}+\frac{(V_1-20)}{10}=0$$

Step 3. Simplify and solve.

$$\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{10}+\frac{1}{10}\right)V_1=-\frac{10}{5}-\frac{10}{10}+\frac{20}{10}$$
$$(0.2+0.1+0.1+0.1)V_1=0.5V_1=-2-1+2=-1$$

or

$$V_1 = \mathbf{-2\ V}.$$

The answer can be checked by calculating all the currents and see if they add up to zero. The top two currents on the left flow right to left and are 0.8 A and 1.6 A respectively. The current flowing up through the 10-ohm resistor is 0.2 A. The current flowing right to left through the 10-ohm resistor is 2.2 A. Summing all the currents flowing out of the node, V_1, we get, +0.8+1.6 –0.2–2.2 = 0. The answer checks.

Solution 3.7

Apply nodal analysis to solve for V_x in the circuit in Fig. 3.56.

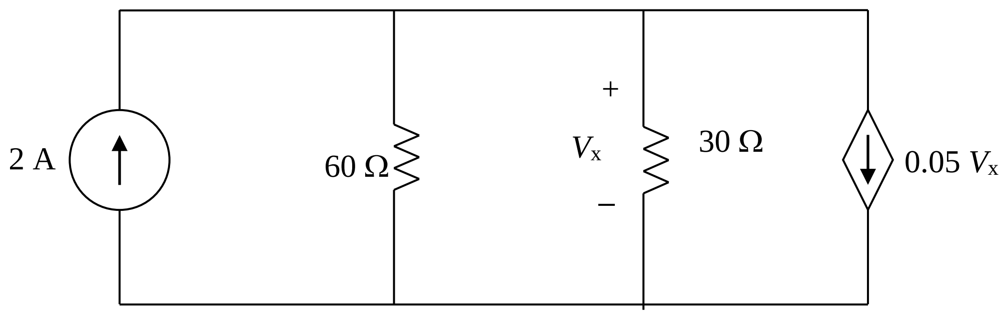


Figure 3.56
For Prob. 3.7.

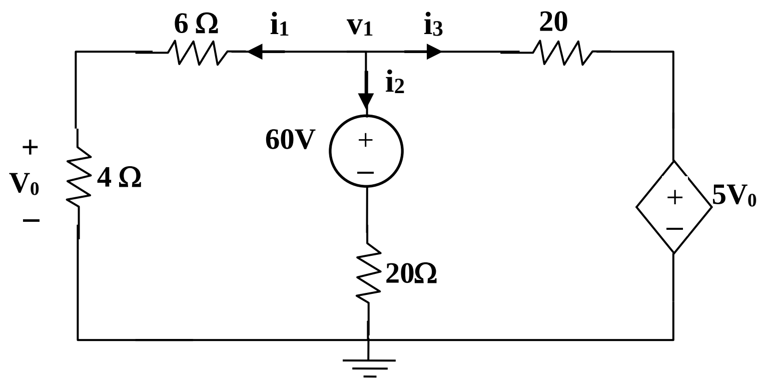
Step 1. First we identify all of the unknown nodes and in this case, we only have one and that is V_x. Next we write one nodal equation.

$$-2+\frac{V_x-0}{60}+\frac{V_x-0}{30}+0.05V_x=0\,.$$

Step 2. $[0.05+(1/60)+(1/30)]V_x = [0.05+0.05]V_x = 0.1V_x = 2$ or $V_x =$ **20 V**.

Substituting into the original equation for a check we get, –2 + (20/60) + (20/30) + (0.05)(20) = 0 = –2 + 0.33333 + 0.66667 + 1 = 0. The answer checks!

Solution 3.8



$i_1 + i_2 + i_3 = 0 \longrightarrow \quad \dfrac{v_1}{10} + \dfrac{(v_1 - 60) - 0}{20} + \dfrac{v_1 - 5v_0}{20} = 0$

But $\quad v_0 = \dfrac{2}{5}v_1$ so that $2v_1 + v_1 - 60 + v_1 - 2v_1 = 0$

or $v_1 = 60/2 = 30$ V, therefore $v_o = 2v_1/5 =$ **12 V**.

Solution 3.9

Let V_1 be the unknown node voltage to the right of the 250-Ω resistor. Let the ground reference be placed at the bottom of the 50-Ω resistor. This leads to the following nodal equation:

$$\frac{V_1 - 24}{250} + \frac{V_1 - 0}{50} + \frac{V_1 - 60I_b - 0}{150} = 0$$

simplifying we get

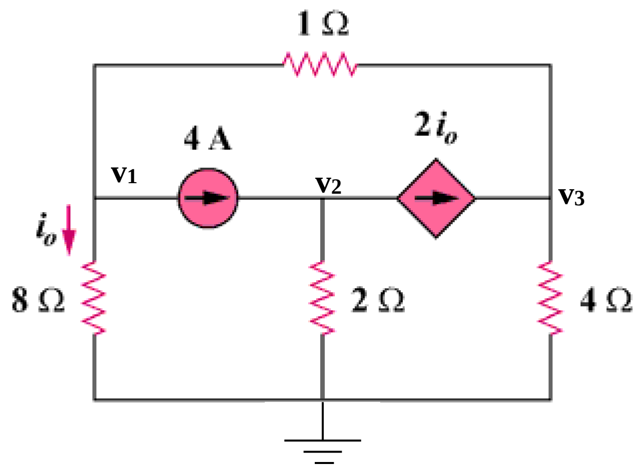
$$3V_1 - 72 + 15V_1 + 5V_1 - 300I_b = 0$$

But $I_b = \dfrac{24 - V_1}{250}$. Substituting this into the nodal equation leads to

$$24.2V_1 - 100.8 = 0 \quad \text{or } V_1 = 4.165 \text{ V}.$$

Thus, $I_b = (24 – 4.165)/250 =$ **79.34 mA**.

Solution 3.10



At node 1. $[(v_1–0)/8] + [(v_1–v_3)/1] + 4 = 0$

At node 2. $–4 + [(v_2–0)/2] + 2i_o = 0$

At node 3. $–2i_o + [(v_3–0)/4] + [(v_3–v_1)/1] = 0$

Finally, we need a constraint equation, $i_o = v_1/8$

This produces,

$$1.125v_1 – v_3 = –4 \qquad (1)$$

$$0.25v_1 + 0.5v_2 = 4 \qquad (2)$$

$$–1.25v_1 + 1.25v_3 = 0 \text{ or } v_1 = v_3 \qquad (3)$$

Substituting (3) into (1) we get $(1.125–1)v_1 = –4$ or $v_1 = –4/0.125 = –32$ volts. This leads to,

$i_o = 32/8 =$ **–4 amps**.

Solution 3.11

Find V_o and the power absorbed by all the resistors in the circuit of Fig. 3.60.

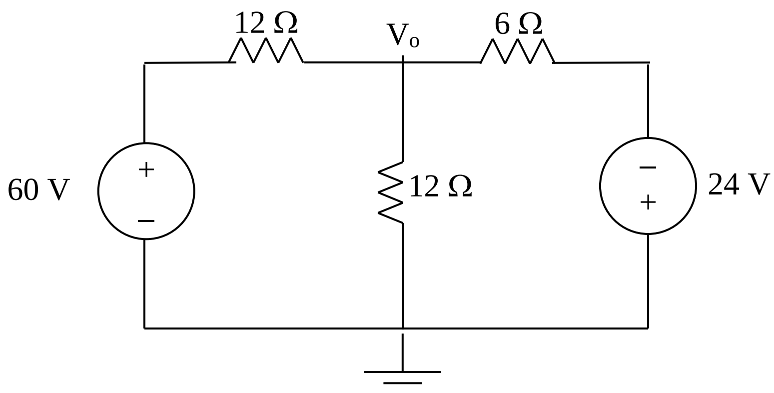


Figure 3.60
For Prob. 3.11.

Solution

At the top node, KCL produces $\dfrac{V_o - 60}{12} + \dfrac{V_o - 0}{12} + \dfrac{V_o - (-24)}{6} = 0$

$(1/3)V_o = 1$ or $V_o =$ **3 V**.

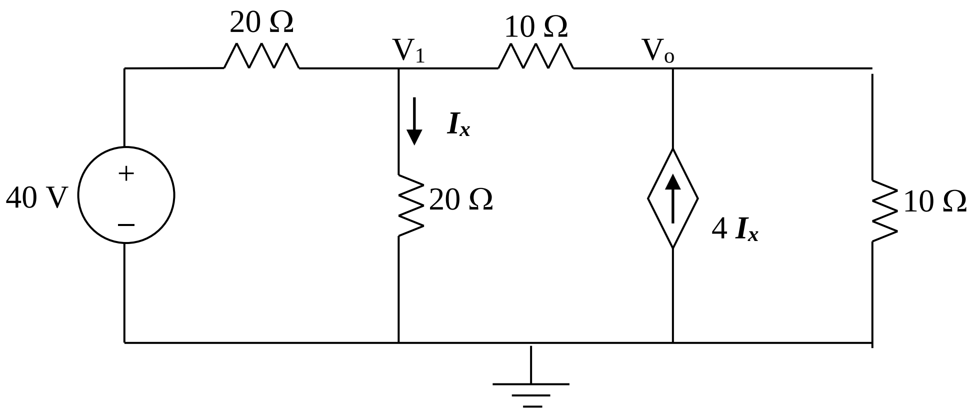
$P_{12\Omega} = (3–60)^2/1 =$ **293.9 W** (this is for the 12 Ω resistor in series with the 60 V source)

$P_{12\Omega} = (V_o)^2/12 = 9/12 =$ **750 mW** (this is for the 12 Ω resistor connecting V_o to ground)

$P_{4\Omega} = (3–(–24))^2/6 =$ **121.5 W**.

Solution 3.12

There are two unknown nodes, as shown in the circuit below.



At node 1,

$$\frac{V_1 - 40}{20} + \frac{V_1 - 0}{20} + \frac{V_1 - V_o}{10} = 0 \text{ or}$$

$$(0.05+0.05+.1)V_1 - 0.1V_o = 0.2V_1 - 0.1V_o = 2 \qquad (1)$$

At node o,

$$\frac{V_o - V_1}{10} - 4I_x + \frac{V_o - 0}{10} = 0 \text{ and } I_x = V_1/20$$

$$-0.1V_1 - 0.2V_1 + 0.2V_o = -0.3V_1 + 0.2V_o = 0 \text{ or} \qquad (2)$$

$$V_1 = (2/3)V_o \qquad (3)$$

Substituting (3) into (1),

$$0.2(2/3)V_o - 0.1V_o = 0.03333V_o = 2 \text{ or}$$

$$V_o = \mathbf{60\ V}.$$

Solution 3.13

Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

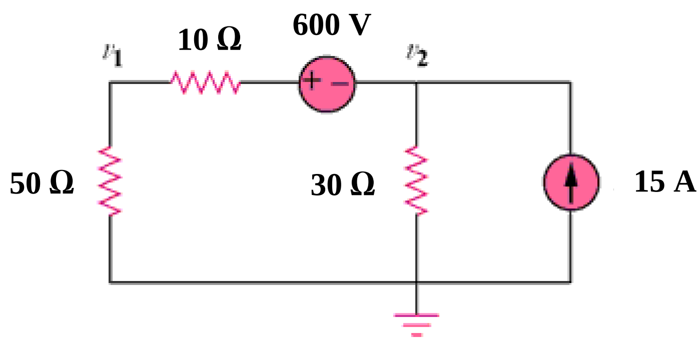


Figure 3.62
For Prob. 3.13.

Solution

Step 1. We note that the 10 ohm resistor is in series with the 50 ohm resistor which can be replaced by a 60 ohm resistor. The then gives us a circuit with one unknown node and we can write one nodal equation to let us solve for v_2.

Once we have v_2 we can use voltage division to solve for v_1.

$$\frac{(v_2 + 600) - 0}{60} + \frac{v_2 - 0}{30} - 15 = 0$$ and $v_2 = [(v_1+600)–0](50/60)$.

Step 2. $[(1/60)+(1/30)]v_2 = –(600/60)+15 = 5 = 0.05v_2$ or $v_2 =$ **100 V**.

Now $v_1 = 700(50/60) =$ **583.3 V**.

Solution 3.14

Using nodal analysis, find v_o in the circuit of Fig. 3.63.

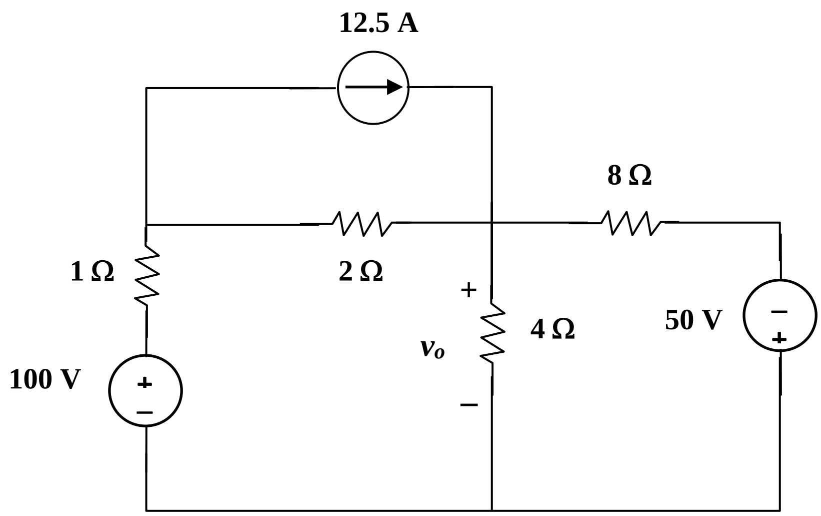
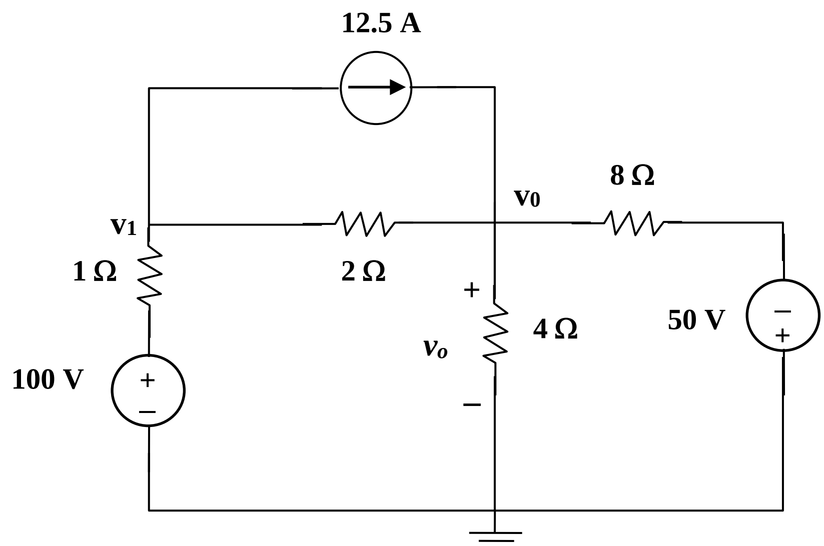


Figure 3.63
For Prob. 3.14.

Solution



At node 1,

$$[(v_1–100)/1] + [(v_1–v_o)/2] + 12.5 = 0 \text{ or } 3v_1 – v_o = 200–25 = 175 \qquad (1)$$

At node o,

$$[(v_o–v_1)/2] – 12.5 + [(v_o–0)/4] + [(v_o+50)/8] = 0 \text{ or } –4v_1 + 7v_o = 50 \qquad (2)$$

Adding 4x(1) to 3x(2) yields,

$$4(1) + 3(2) = -4v_o + 21v_o = 700 + 150 \text{ or } 17v_o = 850 \text{ or}$$

$$v_o = \mathbf{50\ V}.$$

Checking, we get $v_1 = (175+v_o)/3 = 75$ V.

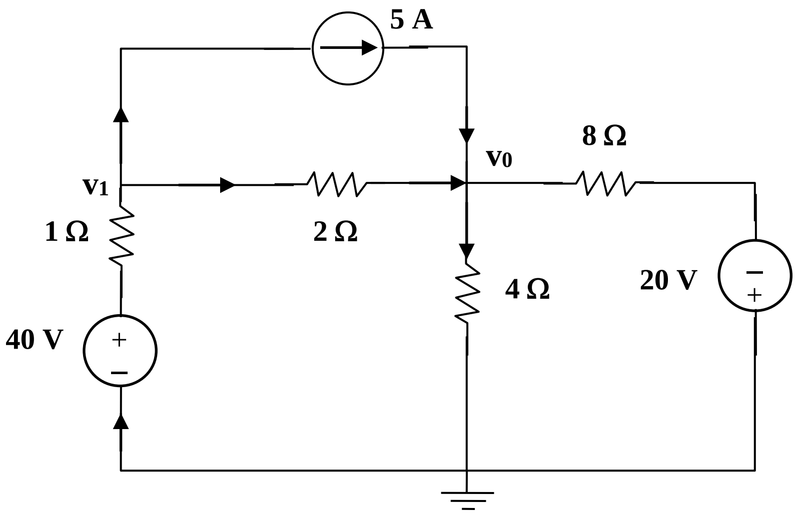
At node 1,

$$[(75-100)/1] + [(75-50)/2] + 12.5 = -25 + 12.5 + 12.5 = 0!$$

At node o,

$$[(50-75)/2] + [(50-0)/4] + [(50+50)/8] - 12.5 = -12.5 + 12.5 + 12.5 - 12.5 = 0!$$

Solution 3.15



Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3(v_3 - v_2) \longrightarrow \quad 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3(v_3 - v_2) \longrightarrow \quad v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \longrightarrow v_2 = \frac{-56}{11}$$

$$v_1 = v_2 + 10 = \frac{54}{11}$$

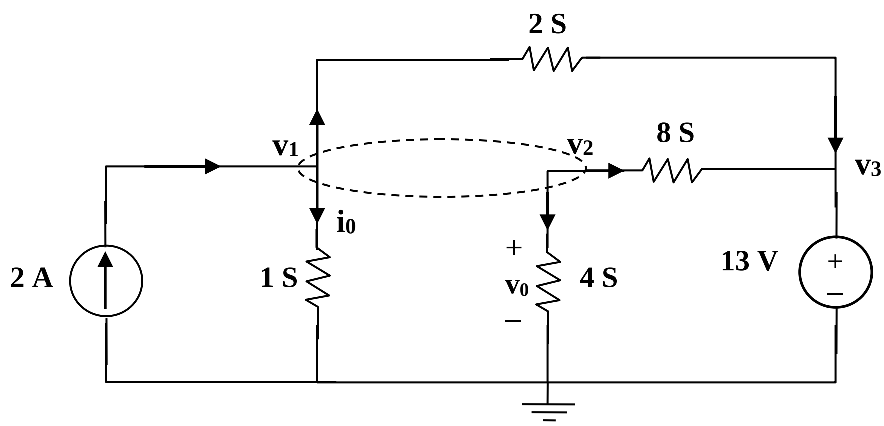
$$i_0 = 6v_i = \mathbf{29.45\ A}$$

$$P_{65} = \frac{v_1^2}{R} = v_1^2 G = \left(\frac{54}{11}\right)^2 6 = \mathbf{144.6\ W}$$

$$P_{55} = v_2^2 G = \left(\frac{-56}{11}\right)^2 5 = \mathbf{129.6\ W}$$

$$P_{35} = (v_L - v_3)^2 G = (2)^2 3 = \mathbf{12\ W}$$

Solution 3.16



At the supernode,

$$2 = v_1 + 2\,(v_1 - v_3) + 8(v_2 - v_3) + 4v_2, \text{ which leads to } 2 = 3v_1 + 12v_2 - 10v_3 \quad (1)$$

But

$$v_1 = v_2 + 2v_0 \text{ and } v_0 = v_2.$$

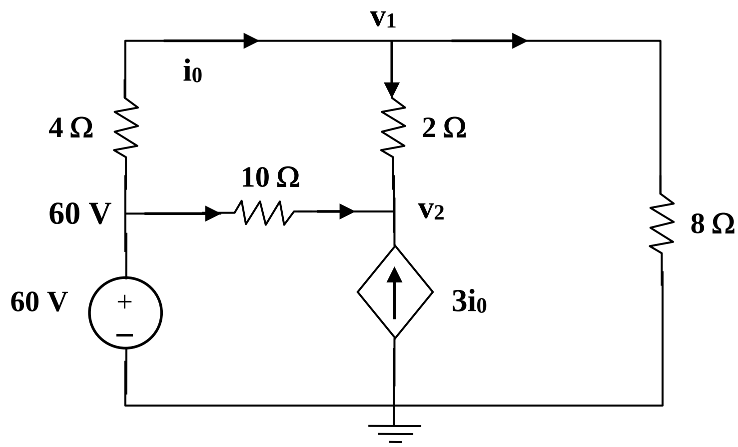
Hence

$$v_1 = 3v_2 \qquad (2)$$
$$v_3 = 13V \qquad (3)$$

Substituting (2) and (3) with (1) gives,

$$v_1 = \mathbf{18.858\ V},\ v_2 = \mathbf{6.286\ V},\ v_3 = \mathbf{13\ V}$$

Solution 3.17



At node 1, $\dfrac{60 - v_1}{4} = \dfrac{v_1}{8} + \dfrac{v_1 - v_2}{2}$ $\qquad 120 = 7v_1 - 4v_2 \qquad (1)$

At node 2, $3i_0 + \dfrac{60 - v_2}{10} + \dfrac{v_1 - v_2}{2} = 0$

But $i_0 = \dfrac{60 - v_1}{4}$.

Hence

$$\frac{3(60 - v_1)}{4} + \frac{60 - v_2}{10} + \frac{v_1 - v_2}{2} = 0 \longrightarrow 1020 = 5v_1 + 12v_2 \qquad (2)$$

Solving (1) and (2) gives $v_1 = 53.08$ V. Hence $i_0 = \dfrac{60 - v_1}{4} =$ **1.73 A**

Solution 3.18

Determine the node voltages in the circuit in Fig. 3.67 using nodal analysis.

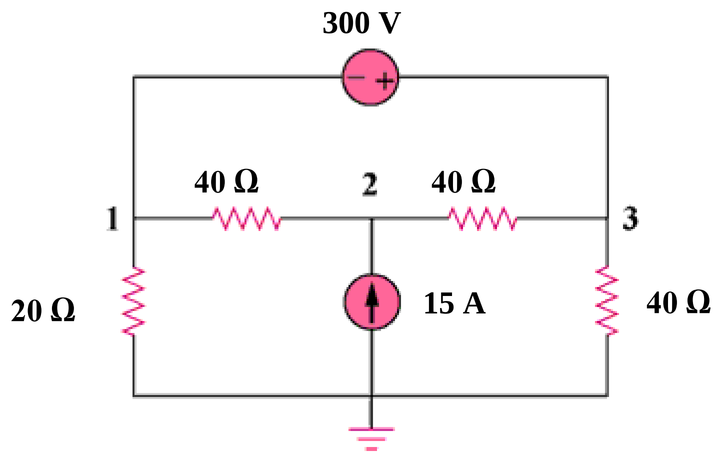


Figure 3.67
For Prob. 3.18.

Step 1. First we identify the unknown nodes and find that there really are only two unknown nodes, v_1 and v_2 since $v_3 = v_1 + 300$ V (essentially a supernode).

$$\frac{v_1 - 0}{20} + \frac{v_1 - v_2}{40} + \frac{(v_1 + 300) - v_2}{40} + \frac{(v_1 + 300) - 0}{40} = 0 \text{ and}$$

$$\frac{v_2 - v_1}{40} - 15 + \frac{v_2 - (v_1 + 300)}{40} = 0.$$ Finally we need, $v_3 = v_1 + 300$.

Step 2. $(0.05+0.025+0.025+0.025)v_1 - (0.025+0.025)v_2 = -15$ or $0.125v_1 - 0.05v_2 = -15$ and $-(0.025+0.025)v_1 + (0.025+0.025)v_2 = 22.5$ or $-0.05v_1 + 0.05v_2 = 22.5$. Adding the two equations together we get, $0.075v_1 = 7.5$ or $v_1 =$ **100 V**. Since $0.05v_2 = 0.05v_1 + 22.5 = 27.5$ or $v_2 =$ **550 V**.

Finally $v_3 = v_1 + 300 =$ **400 V**.

Solution 3.19

At node 1,

$$5 = 3 + \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{8} + \frac{V_1}{4} \quad \longrightarrow \quad 16 = 7V_1 - V_2 - 4V_3 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{8} = \frac{V_2}{2} + \frac{V_2 - V_3}{4} \quad \longrightarrow \quad 0 = -V_1 + 7V_2 - 2V_3 \qquad (2)$$

At node 3,

$$3 + \frac{12 \; - V_3}{8} + \frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{4} = 0 \quad \longrightarrow \quad -36 = 4V_1 + 2V_2 - 7V_3 \quad (3)$$

From (1) to (3),

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ 4 & 2 & -7 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ -36 \end{pmatrix} \quad \longrightarrow \quad AV = B$$

Using MATLAB,

$$\boldsymbol{V = A^{-1}B} = \begin{bmatrix} \mathbf{10} \\ \mathbf{4.933} \\ \mathbf{12.267} \end{bmatrix} \quad \longrightarrow \quad \underline{\boldsymbol{V_1 = 10\text{ V},\ V_2 = 4.933\text{ V},\ V_3 = 12.267\text{ V}}}$$

Solution 3.20

For the circuit in Fig. 3.69, find v_1, v_2, and v_3 using nodal analysis.

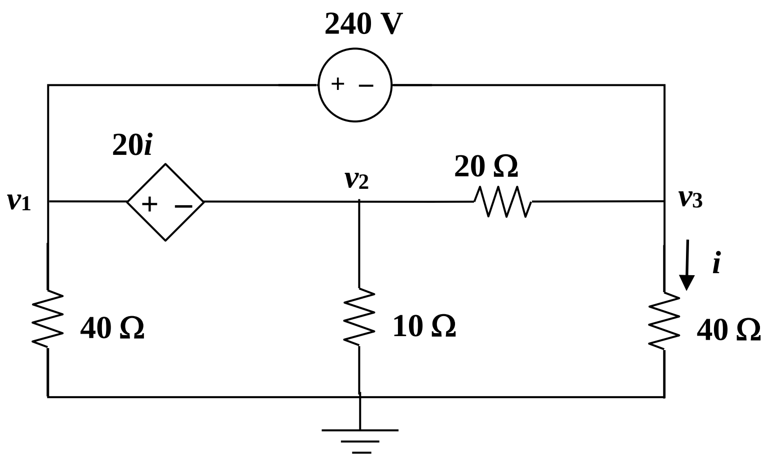


Figure 3.69
For Prob. 3.20.

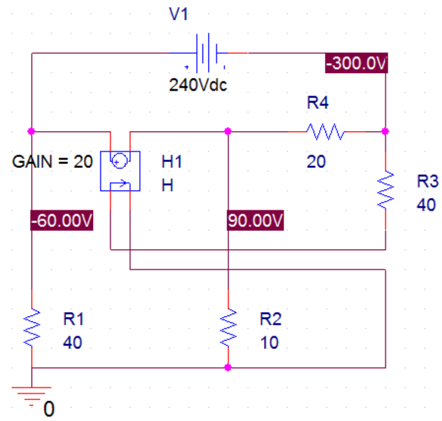
Step 1. This is an interesting problem, once we choose, say v_1, the other two nodes are really known i.e. $v_2 = v_1 – 20i$ and $v_3 = v_1 – 240$ volts.

Obviously we have one supernodes. Additionally we need the constraint equation, $i = (v_3–0)/40 = (v_1–240)/40$.

$[(v_1–0)/40] + [((v_1–20i)–0)/10] + [((v_1–240)–0)/40] = 0$

Step 2. $(0.025+0.1+0.025)v_1 – 0.05v_1 + 12 – 6 = 0$ or $0.1v_1 = –6$ or $v_1 =$ **–60 V**. Now $v_3 = –60–240 =$ **–300 V**. This leads to $i = –300/40 = –7.5$ and $v_2 = –60+150 =$ **90 V**.

Checking with PSpice we get,



Solution 3.21

For the circuit in Fig. 3.70, find v_1 and v_2 using nodal analysis.

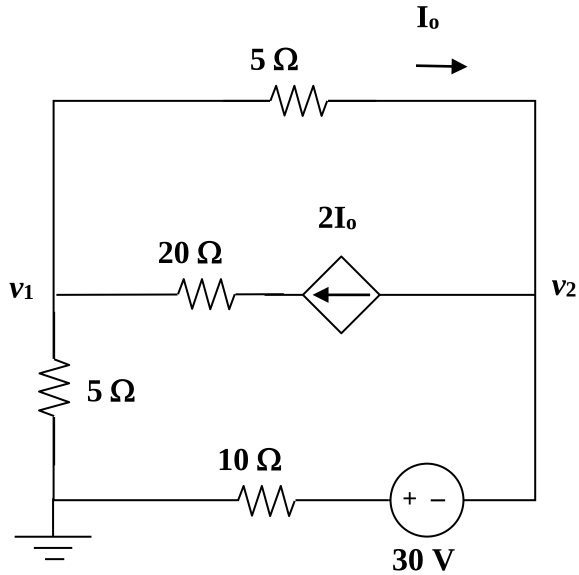
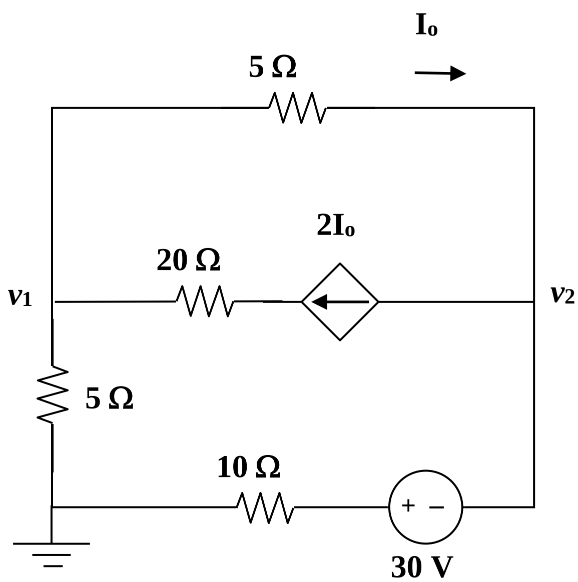


Figure 3.70
For Prob. 3.21.

Step 1. We start by writing the nodal equations. Then we need a constraint equation. This then will allow us to solve for v_1 and v_2.



Node 1. $[(v_1–0)/5] – 2I_o + [(v_1–v_2)/5] = 0$ or $(0.2+0.2)v_1 – 0.2v_2 – 2I_o = 0$

Node 2. $[(v_2–v_1)/5] + 2I_o + [(v_2+30–0)/10] = 0$ or $–0.2v_1 + 0.3v_2 + 2I_o = –3$

Constraint equation, $I_o = [(v_1–v_2)/5]$.

Step 2. $2I_o = 0.4(v_1–v_2)$ or $(0.2+0.2)v_1 – 0.2v_2 – 0.4(v_1–v_2) = 0$ or

$0v_1 + 0.2v_2 = 0$ or $v_2 = 0$. Now, $–0.2v_1 + 0.3v_2 + 0.4v_1 – 0.4v_2 = 0.2v_1 – 0.1v_2 = 0.2v_1 = –3$ or $v_1 = –15$ volts. Therefore,

$v_1 =$ **–15 V** and $v_2 =$ **0 V**.

Solution 3.22

Determine v_1 and v_2 in the circuit in Fig. 3.71.

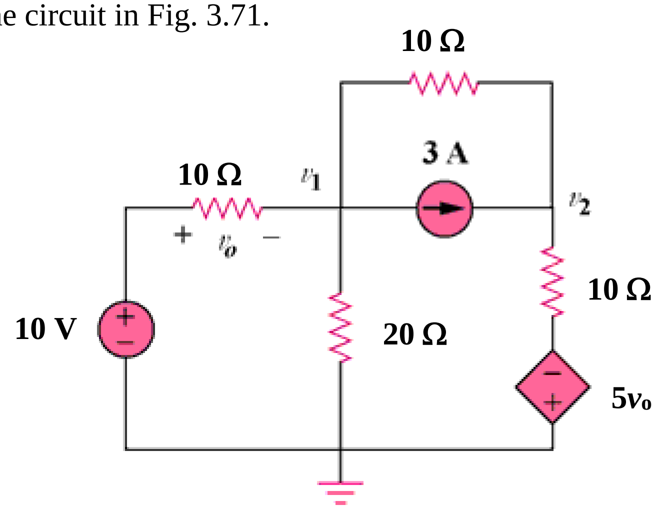


Figure 3.71
For Prob. 3.22.

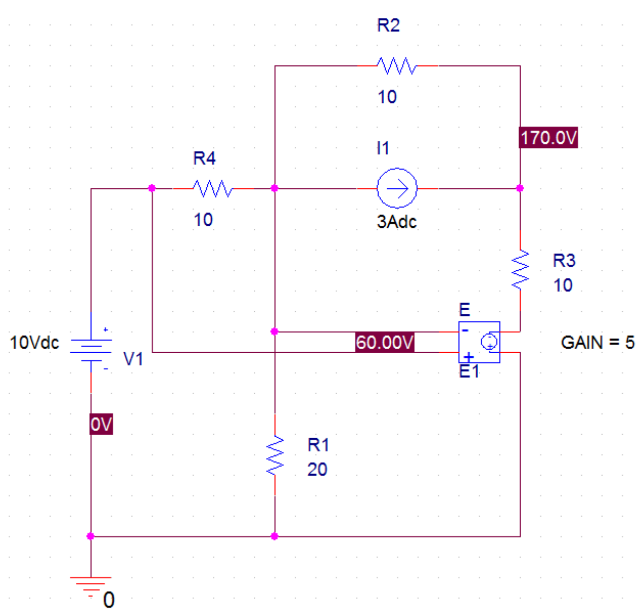
Solution

Step 1. We have two unknown so we end up with two nodal equations,
$[(v_1–10)/10] + [(v_1–0)/20] + 3 + [(v_1–v_2)/10] = 0$ and
$[(v_2–v_1)/10] – 3 + [(v_2–(–5v_o))/10] = 0$. We now have two equations with three unknowns so we need a constraint equation, $v_o = 10–v_1$ or $5v_o = 50–5v_1$.

Step 2. $(0.1+0.05+0.1)v_1 – 0.1v_2 = –2 = 0.25v_1 – 0.1v_2$ and
$–0.1v_1 + (0.1+0.1)v_2 + 5 – 0.5v_1 = 3$ or $–0.6v_1 + 0.2v_2 = –2$. Now we can combine the two equations after having multiplied the first one by 2.

$0.5v_1 –0.2v_2 = –4$ and
$–0.6v_1 + 0.2v_2 = –2$ or $–0.1v_1 = –6$ or $v_1 =$ **60 V** and $0.2v_2 = 0.6v_1 – 2 = 34$ or
$v_2 =$ **170 V**.

Checking with PSpice we get,



Solution 3.23

Use nodal analysis to find V_o in the circuit of Fig. 3.72.

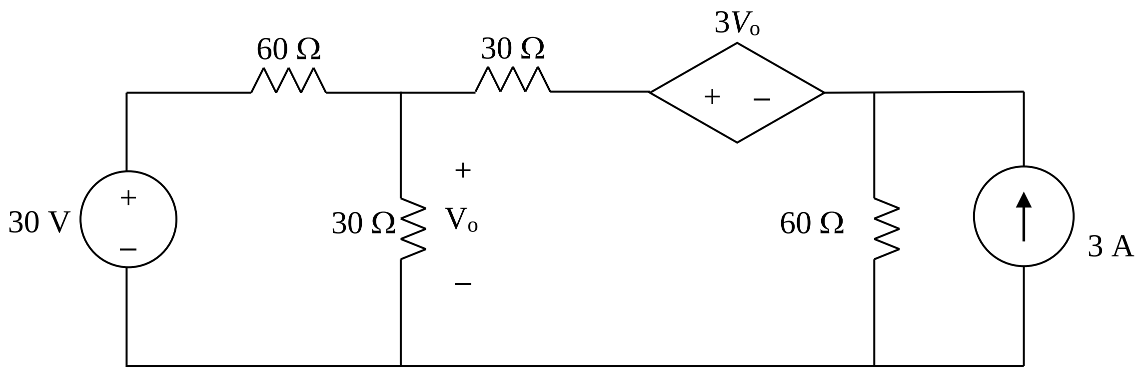
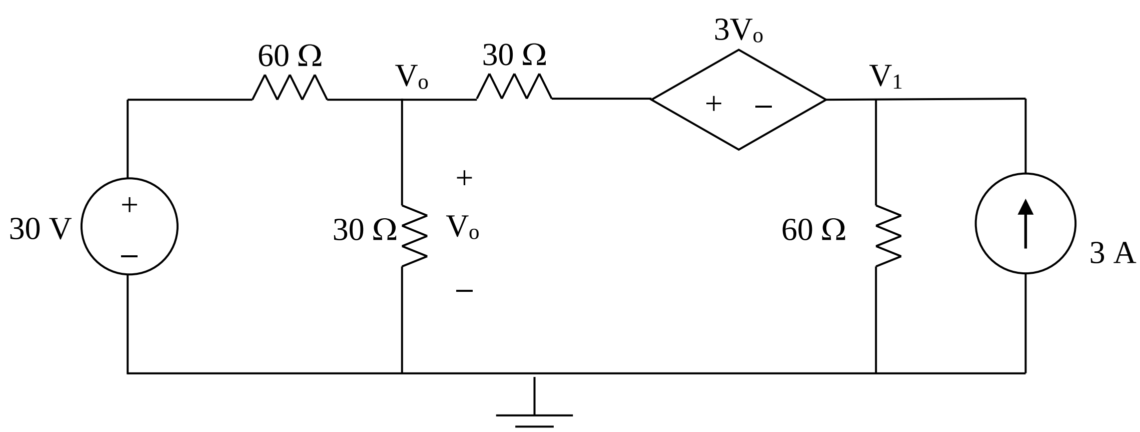


Figure 3.72
For Prob. 3.23.

Solution

Step 1. We apply nodal analysis to the circuit shown below.



At node o, $\frac{V_o - 30}{60} + \frac{V_o - 0}{30} + \frac{V_o - (3V_o + V_1)}{30} = 0$ and at node 1, we get,

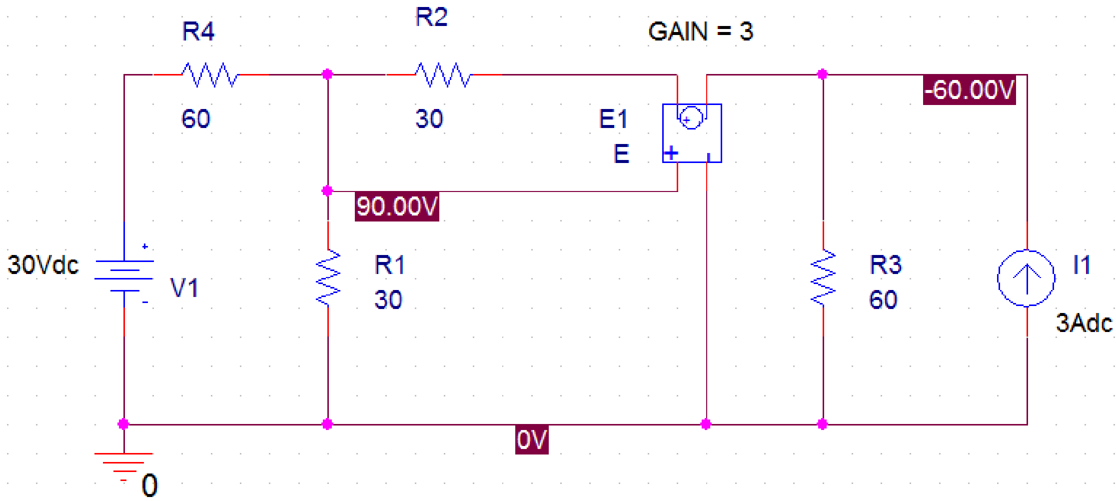
$$\frac{(3V_o + V_1) - V_o}{30} + \frac{V_1 - 0}{60} - 3 = 0$$

Step 2. $[(1/60)+(1/30)+(1/30)–(3/30)]V_o – (1/30)v_1 = 0.5$ or
$(0.08333–0.1)V_o – 0.033333V_1 = –0.0166667V_o – 0.033333V_1 = 0.5$ and

$(0.1–0.033333)V_o + 0.05V_1 = 0.06667V_o + 0.05V_1 = 3$. To find V_o all we need to do is to multiply the first equation by 3 and multiply the second equation by 2 and then combine them.

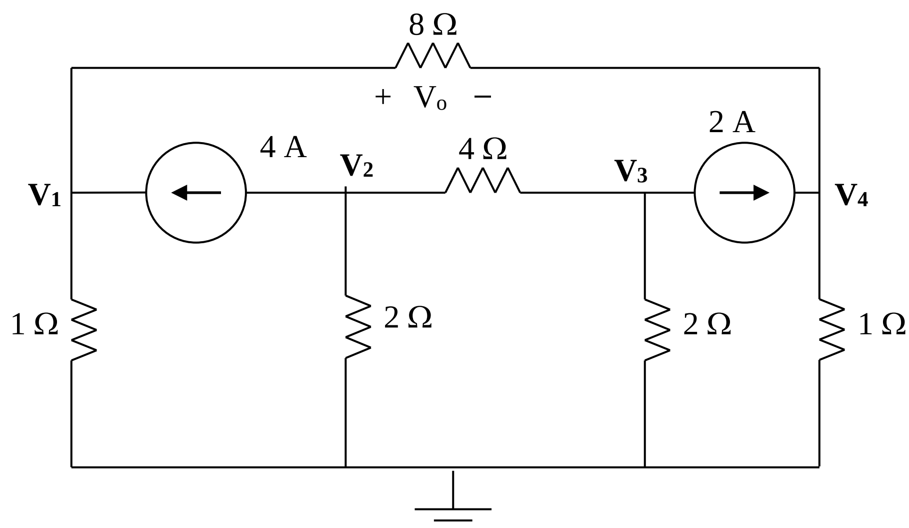
$–0.05V_o – 0.1V_1 = 1.5$ and $0.13333V_o + 0.1V_1 = 6$ which leads to $0.083333V_o = 7.5$ or $V_o =$ **90 V**.

Checking with PSpice we get,



Solution 3.24

Consider the circuit below.



$$\frac{V_1-0}{1}-4+\frac{V_1-V_4}{8}=0\rightarrow 1.125V_1-0.125V_4=4 \qquad (1)$$

$$+4+\frac{V_2-0}{2}+\frac{V_2-V_3}{4}=0\rightarrow 0.75V_2-0.25V_3=-4 \qquad (2)$$

$$\frac{V_3-V_2}{4}+\frac{V_3-0}{2}+2=0\rightarrow -0.25V_2+0.75V_3=-2 \qquad (3)$$

$$-2+\frac{V_4-V_1}{8}+\frac{V_4-0}{1}=0\rightarrow -0.125V_1+1.125V_4=2 \qquad (4)$$

$$\begin{bmatrix} 1.125 & 0 & 0 & -0.125 \\ 0 & 0.75 & -0.25 & 0 \\ 0 & -0.25 & 0.75 & 0 \\ -0.125 & 0 & 0 & 1.125 \end{bmatrix} V = \begin{bmatrix} 4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

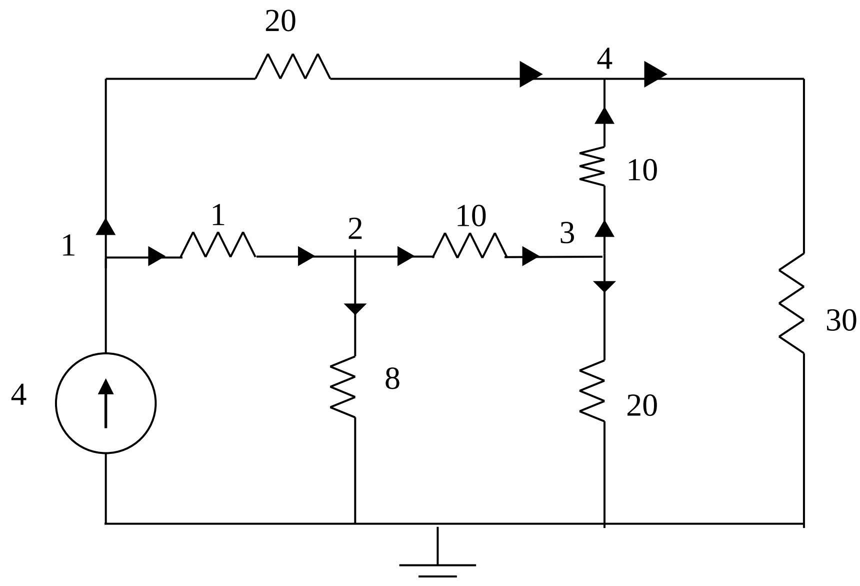
Now we can use MATLAB to solve for the unknown node voltages.

```
>> Y=[1.125,0,0,-0.125;0,0.75,-0.25,0;0,-0.25,0.75,0;-0.125,0,0,1.125]

Y =

    1.1250         0         0   -0.1250
         0    0.7500   -0.2500         0
         0   -0.2500    0.7500         0
   -0.1250         0         0    1.1250

>> I=[4,-4,-2,2]'

I =

     4
    -4
    -2
     2

>> V=inv(Y)*I

V =

    3.8000
   -7.0000
   -5.0000
    2.2000
```

$V_o = V_1 - V_4 = 3.8 - 2.2 =$ **1.6 V**.

Solution 3.25

Consider the circuit shown below.



At node 1.

$$4 = \frac{V_1 - V_2}{1} + \frac{V_1 - V_4}{20} \longrightarrow 80 = 21V_1 - 20V_2 - V_4 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{1} = \frac{V_2}{8} + \frac{V_2 - V_3}{10} \longrightarrow 0 = -80V_1 + 98V_2 - 8V_3 \qquad (2)$$

At node 3,

$$\frac{V_2 - V_3}{10} = \frac{V_3}{20} + \frac{V_3 - V_4}{10} \longrightarrow 0 = -2V_2 + 5V_3 - 2V_4 \qquad (3)$$

At node 4,

$$\frac{V_1 - V_4}{20} + \frac{V_3 - V_4}{10} = \frac{V_4}{30} \longrightarrow 0 = 3V_1 + 6V_3 - 11V_4 \qquad (4)$$

Putting (1) to (4) in matrix form gives:

$$\begin{bmatrix} 80 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 21 & -20 & 0 & -1 \\ -80 & 98 & -8 & 0 \\ 0 & -2 & 5 & -2 \\ 3 & 0 & 6 & -11 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

$$B = A\ V \longrightarrow V = A^{-1} B$$

Using MATLAB leads to

$$V_1 = \mathbf{25.52\ V}, \quad V_2 = \mathbf{22.05\ V}, \quad V_3 = \mathbf{14.842\ V}, \quad V_4 = \mathbf{15.055\ V}$$

Solution 3.26

At node 1,

$$\frac{15-V_1}{20}=3+\frac{V_1-V_3}{10}+\frac{V_1-V_2}{5} \longrightarrow -45=7V_1-4V_2-2V_3 \qquad (1)$$

At node 2,

$$\frac{V_1-V_2}{5}+\frac{4I_o-V_2}{5}=\frac{V_2-V_3}{5} \qquad (2)$$

But $I_o=\dfrac{V_1-V_3}{10}$. Hence, (2) becomes

$$0=7V_1-15V_2+3V_3 \qquad (3)$$

At node 3,

$$3+\frac{V_1-V_3}{10}+\frac{-10-V_3}{15}+\frac{V_2-V_3}{5}=0 \longrightarrow 70=-3V_1-6V_2+11V_3 \qquad (4)$$

Putting (1), (3), and (4) in matrix form produces

$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}=\begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \longrightarrow AV=B$$

Using MATLAB leads to

$$V=A^{-1}B=\begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus,

V_1 = **–7.19V**; V_2 = **–2.78V**; V_3 = **2.89V**.

Solution 3.27

At node 1,

$$2 = 2v_1 + v_1 - v_2 + (v_1 - v_3)4 + 3i_0, \quad i_0 = \quad 4v_2. \text{ Hence,}$$

$$2 = 7v_1 + 11v_2 - 4v_3 \qquad (1)$$

At node 2,

$$v_1 - v_2 = 4v_2 + v_2 - v_3 \longrightarrow \qquad 0 = - \ v_1 + 6v_2 - v_3 \qquad (2)$$

At node 3,

$$2v_3 = 4 + v_2 - v_3 + 12v_2 + 4(v_1 - v_3)$$

or

$$-4 = 4v_1 + 13v_2 - 7v_3 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{vmatrix} = 176, \quad \Delta_1 = \begin{vmatrix} 2 & 11 & -4 \\ 0 & -6 & 1 \\ -4 & 13 & -7 \end{vmatrix} = 110$$

$$\Delta_2 = \begin{vmatrix} 7 & 2 & -4 \\ 1 & 0 & 1 \\ 4 & -4 & -7 \end{vmatrix} = 66, \quad \Delta_3 = \begin{vmatrix} 7 & 11 & 2 \\ 1 & -6 & 0 \\ 4 & 13 & -4 \end{vmatrix} = 286$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{110}{176} = 0.625\text{V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{66}{176} = 0.375\text{V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{286}{176} = 1.625\text{V}.$$

v_1 = **625 mV**, v_2 = **375 mV**, v_3 = **1.625 V**.

Solution 3.28

At node c,

$$\frac{V_d - V_c}{10} = \frac{V_c - V_b}{4} + \frac{V_c}{5} \longrightarrow 0 = -5V_b + 11V_c - 2V_d \qquad (1)$$

At node b,

$$\frac{V_a + 90 - V_b}{8} + \frac{V_c - V_b}{4} = \frac{V_b}{8} \longrightarrow -90 = V_a - 4V_b + 2V_c \quad (2)$$

At node a,

$$\frac{V_a - 60 - V_d}{4} + \frac{V_a}{16} + \frac{V_a + 90 - V_b}{8} = 0 \longrightarrow 60 = 7V_a - 2V_b - 4V_d \quad (3)$$

At node d,

$$\frac{V_a - 60 - V_d}{4} = \frac{V_d}{20} + \frac{V_d - V_c}{10} \longrightarrow 300 = 5V_a + 2V_c - 8V_d \quad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 0 & -5 & 11 & -2 \\ 1 & -4 & 2 & 0 \\ 7 & -2 & 0 & -4 \\ 5 & 0 & 2 & -8 \end{pmatrix} \begin{pmatrix} V_a \\ V_b \\ V_c \\ V_d \end{pmatrix} = \begin{pmatrix} 0 \\ -90 \\ 60 \\ 300 \end{pmatrix} \longrightarrow AV = B$$

We use MATLAB to invert A and obtain

$$V = A^{-1}B = \begin{pmatrix} -10.56 \\ 20.56 \\ 1.389 \\ -43.75 \end{pmatrix}$$

Thus,

V_a = **–10.56 V**; V_b = **20.56 V**; V_c = **1.389 V**; VC_d = **–43.75 V**.

Solution 3.29

At node 1,

$$5+V_1-V_4+2V_1+V_1-V_2=0 \quad \longrightarrow \quad -5=4V_1-V_2-V_4 \qquad (1)$$

At node 2,

$$V_1-V_2=2V_2+4(V_2-V_3)=0 \quad \longrightarrow \quad 0=-V_1+7V_2-4V_3 \qquad (2)$$

At node 3,

$$6+4(V_2-V_3)=V_3-V_4 \quad \longrightarrow \quad 6=-4V_2+5V_3-V_4 \qquad (3)$$

At node 4,

$$2+V_3-V_4+V_1-V_4=3V_4 \quad \longrightarrow \quad 2=-V_1-V_3+5V_4 \qquad (4)$$

In matrix form, (1) to (4) become

$$\begin{pmatrix} 4 & -1 & 0 & -1 \\ -1 & 7 & -4 & 0 \\ 0 & -4 & 5 & -1 \\ -1 & 0 & -1 & 5 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix}=\begin{pmatrix} -5 \\ 0 \\ 6 \\ 2 \end{pmatrix} \quad \longrightarrow \quad AV=B$$

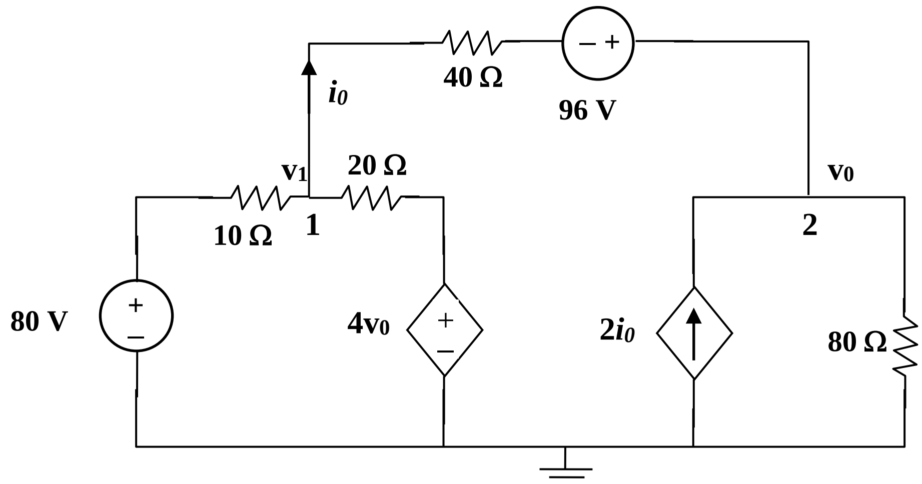
Using MATLAB,

$$V=A^{-1}B=\begin{pmatrix} -0.7708 \\ 1.209 \\ 2.309 \\ 0.7076 \end{pmatrix}$$

i.e.

$$\underline{\boldsymbol{V_1=-0.7708\text{ V},\ V_2=1.209\text{ V},\ V_3=2.309\text{ V},\ V_4=0.7076\text{ V}}}$$

Solution 3.30



At node 1,

$[(v_1–80)/10]+[(v_1–4v_o)/20]+[(v_1–(v_o–96))/40] = 0$ or
$(0.1+0.05+0.025)v_1 – (0.2+0.025)v_o =$
$0.175v_1 – 0.225v_o = 8–2.4 = 5.6$ (1)

At node 2,

$–2\boldsymbol{i_o} + [((v_o–96)–v_1)/40] + [(v_o–0)/80] = 0$ and $\boldsymbol{i_o} = [(v_1–(v_o–96))/40]$
$–2[(v_1–(v_o–96))/40] + [((v_o–96)–v_1)/40] + [(v_o–0)/80] = 0$
$–3[(v_1–(v_o–96))/40] + [(v_o–0)/80] = 0$ or
$–0.0.075v_1 + (0.075+0.0125)v_o = 7.2 =$
$–0.075v_1 + 0.0875v_o = 7.2$ (2)

Using (1) and (2) we get,

$$\begin{bmatrix} 0.175 & -0.225 \\ -0.075 & 0.0875 \end{bmatrix}\begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} \; or$$

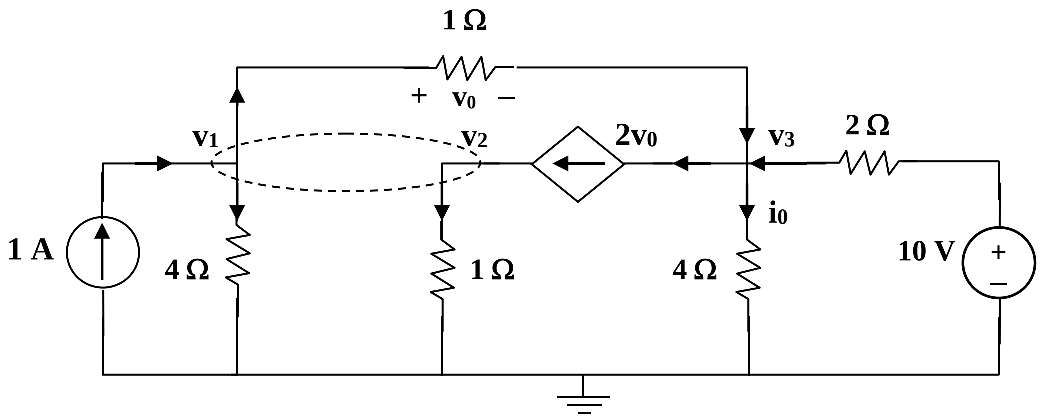
$$\begin{bmatrix} v_1 \\ v_o \end{bmatrix} = \frac{\begin{bmatrix} 0.0875 & 0.225 \\ 0.075 & 0.175 \end{bmatrix}}{0.0153125 - 0.016875}\begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix} = \frac{\begin{bmatrix} 0.0875 & 0.225 \\ 0.075 & 0.175 \end{bmatrix}}{-0.0015625}\begin{bmatrix} 5.6 \\ 7.2 \end{bmatrix}$$

$v_1 = –313.6–1036.8 = –1350.4$

$v_o = –268.8–806.4 =$ **$–1.0752$ kV**

and $\boldsymbol{i_o} = [(v_1–(v_o–96))/40] = [(–1350.4–(–1075.2–96))/40] =$ **–4.48 amps**.

Solution 3.31



At the supernode,

$$1 + 2v_0 = \frac{v_1}{4} + \frac{v_2}{1} + \frac{v_1 - v_3}{1} \qquad (1)$$

But $v_o = v_1 - v_3$. Hence (1) becomes,

$$4 = -3v_1 + 4v_2 + 4v_3 \qquad (2)$$

At node 3,

$$2v_o + \frac{v_3}{4} = v_1 - v_3 + \frac{10 - v_3}{2}$$

or

$$20 = 4v_1 + 0v_2 - v_3 \qquad (3)$$

At the supernode, $v_2 = v_1 + 4i_o$. But $i_o = \frac{v_3}{4}$. Hence,

$$v_2 = v_1 + v_3 \qquad (4)$$

Solving (2) to (4) leads to,

$v_1 =$ **4.97V**, $v_2 =$ **4.85V**, $v_3 =$ **–0.12V**.

Solution 3.32

Obtain the node voltages $\boldsymbol{v_1}$, $\boldsymbol{v_2}$, and $\boldsymbol{v_3}$ in the circuit of Fig. 3.81.

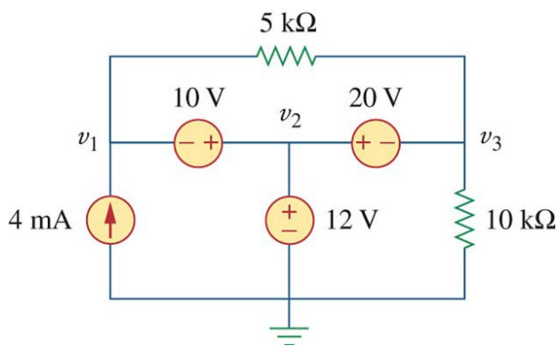
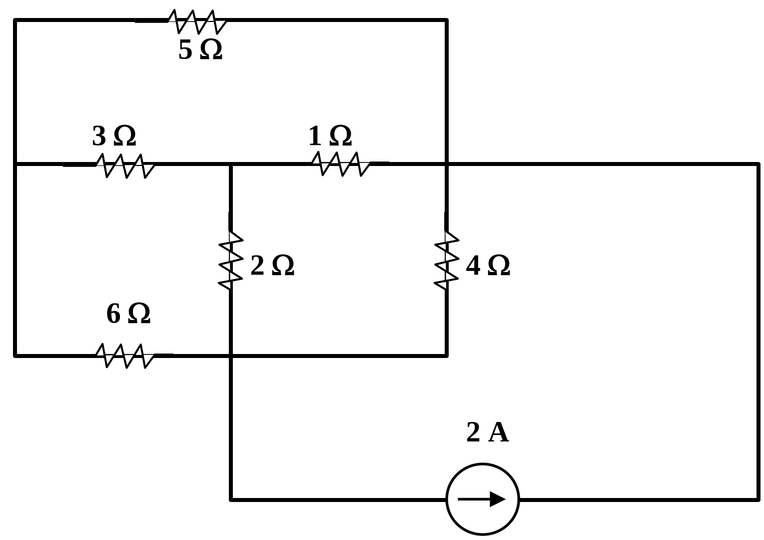


Figure 3.81
For Prob. 3.32.

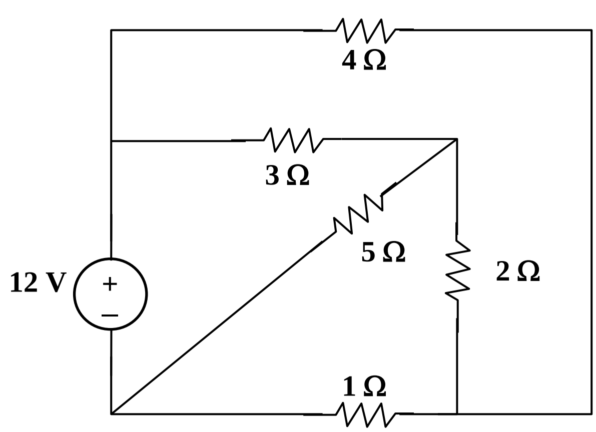
Step 1. and 2. This is an interesting problem. Clearly we have one supernode and that all the node voltages are known! From the circuit, v_2 = **120 V**; $v_1 = v_2 - 50$ = **70 V**; and $v_3 = v_2 - 75$ = **45 V**.

Solution 3.33

(a) This is a **planar** circuit. It can be redrawn as shown below.

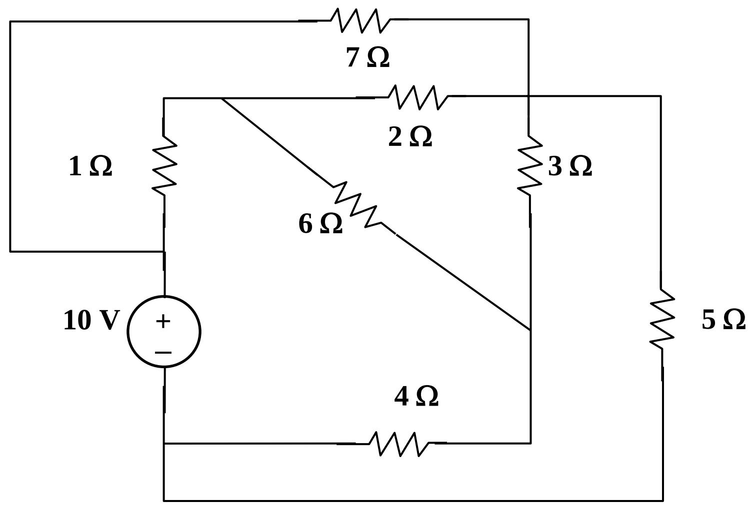


(b) This is a **planar** circuit. It can be redrawn as shown below.



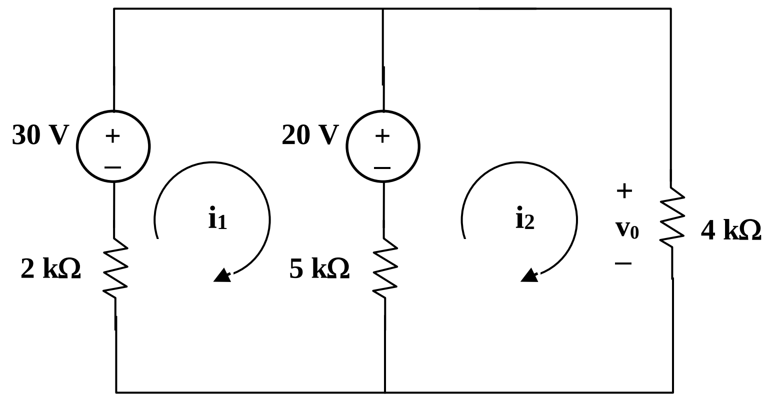
Solution 3.34

(a) This is a **planar** circuit because it can be redrawn as shown below,



(b) This is a **non-planar** circuit.

Solution 3.35



Assume that i_1 and i_2 are in mA. We apply mesh analysis. For mesh 1,

$$-30 + 20 + 7i_1 - 5i_2 = 0 \quad \text{or} \quad 7i_1 - 5i_2 = 10 \qquad (1)$$

For mesh 2,

$$-20 + 9i_2 - 5i_1 = 0 \quad \text{or} \quad -5i_1 + 9i_2 = 20 \qquad (2)$$

Solving (1) and (2), we obtain, $i_2 = 5$.

$$v_0 = 4i_2 = \mathbf{20\ volts}.$$

Solution 3.36

Use mesh analysis to obtain i_a, i_b, and i_c in the circuit shown in Fig. 3.84.

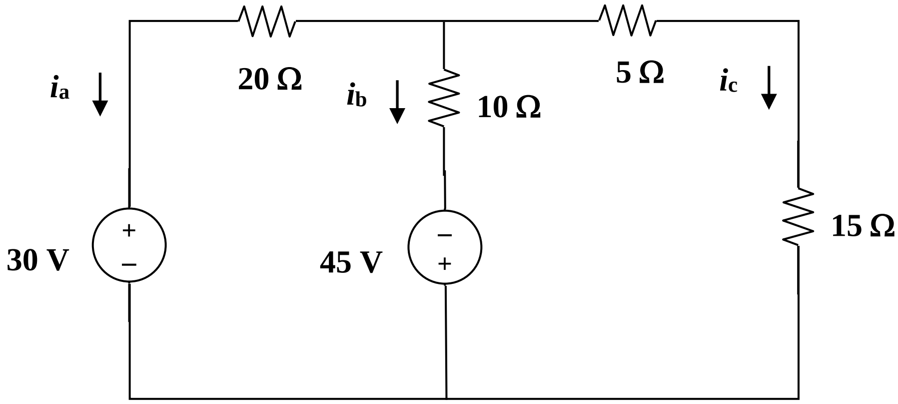
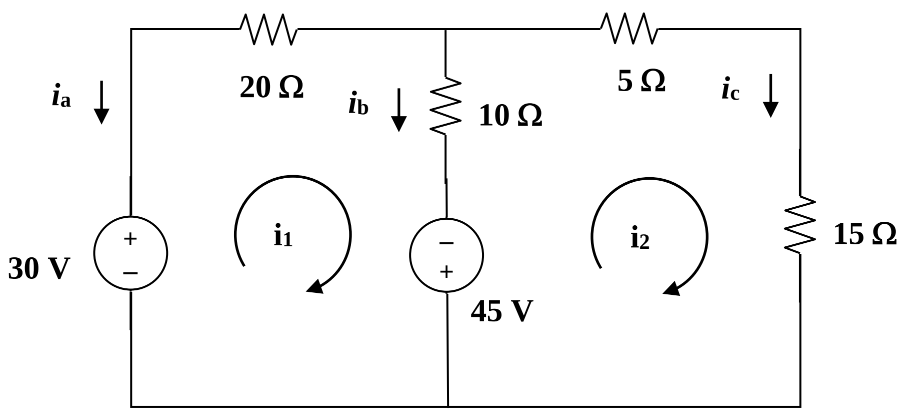


Figure 3.84
For Prob. 3.36.

Step 1. Establish two unknown loop currents and write the mesh equations. Then solve the mesh equations for the two unknown loop currents which will allow us to solve for the unknown branch currents.



Loop 1. $-30 + 20i_1 + 10(i_1 - i_2) - 45 = 0$ or $30i_1 - 10i_2 = 75$

Loop 2. $45 + 10(i_2 - i_1) + 5i_2 + 15i_2 = 0$ or $-10i_1 + 30i_2 = -45$

Finally $i_a = -i_1$; $i_b = i_1 - i_2$; and $i_c = i_2$.

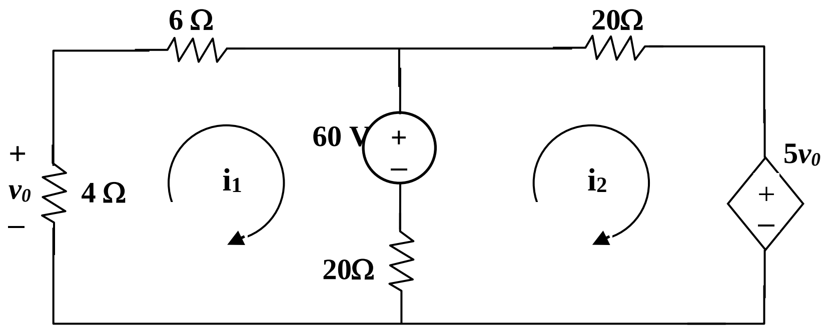
Step 2. The matrix equation is,

$$\begin{bmatrix} 30 & -10 \\ -10 & 30 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 75 \\ -45 \end{bmatrix} \text{or} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{\begin{bmatrix} 30 & 10 \\ 10 & 30 \end{bmatrix}}{900-100}\begin{bmatrix} 75 \\ -45 \end{bmatrix}$$

i_1 = (2250–450)/800 = 2.25 A and i_2 = (750–1350)/800 = –750 mA.

Finally, i_a = **–2.25 A**; i_b = **3 A**; and i_c = **–750 mA**.

Solution 3.37



Applying mesh analysis to loops 1 and 2, we get,

$30i_1 - 20i_2 + 60 = 0$ which leads to $i_2 = 1.5i_1 + 3$ (1)

$-20i_1 + 40i_2 - 60 + 5v_0 = 0$ (2)

But, $v_0 = -4i_1$ (3)

Using (1), (2), and (3) we get $-20i_1 + 60i_1 + 120 - 60 - 20i_1 = 0$ or

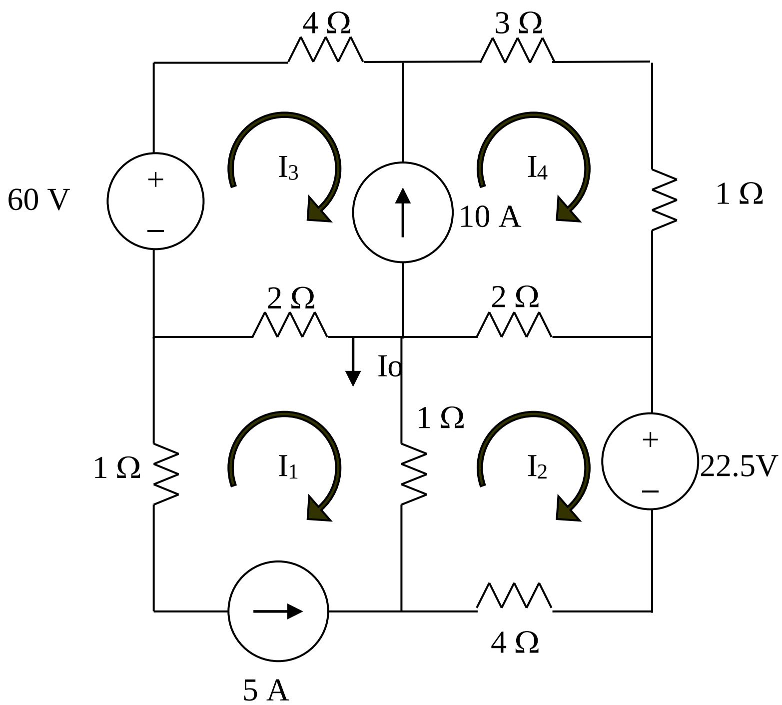
$20i_1 = -60$ or $i_1 = -3$ amps and $i_2 = 7.5$ amps.

Therefore, we get,

$\boldsymbol{v_0} = -4i_1 =$ **12 volts**.

Solution 3.38

Consider the circuit below with the mesh currents.



$I_1 = -5$ A (1)

$1(I_2-I_1) + 2(I_2-I_4) + 22.5 + 4I_2 = 0$

$7I_2 - 2I_4 = -27.5$ (2)

$-60 + 4I_3 + 3I_4 + 1I_4 + 2(I_4-I_2) + 2(I_3 - I_1) = 0$ (super mesh)

$-2I_2 + 6\,I_3 + 6I_4 = +60 - 10 = 50$ (3)

But, we need one more equation, so we use the constraint equation $-I_3 + I_4 = 10$. This now gives us three equations with three unknowns.

$$\begin{bmatrix} 7 & 0 & -2 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -27.5 \\ 50 \\ 10 \end{bmatrix}$$

We can now use MATLAB to solve the problem.

```
>> Z=[7,0,-1;-2,6,6;0,-1,0]
```

```
Z =

     7     0    -1
    -2     6     6
     0    -1     0
>> V=[–27.5,50,10]'

V =
  –27.5
   50
   10
>> I=inv(Z)*V

I =
  –1.3750
  –10.0000
   17.8750
```

$I_o = I_1 - I_2 = -5 - 1.375 =$ **–6.375 A**.

Check using the super mesh (equation (3)):

$-2I_2 + 6\,I_3 + 6I_4 = 2.75 - 60 + 107.25 = 50!$

Solution 3.39

Using Fig. 3.50 from Prob. 3.1, design a problem to help other students to better understand mesh analysis.

Solution

Given $R_1 = 4$ kΩ, $R_2 = 2$ kΩ, and $R_3 = 2$ kΩ, determine the value of I_x using mesh analysis.

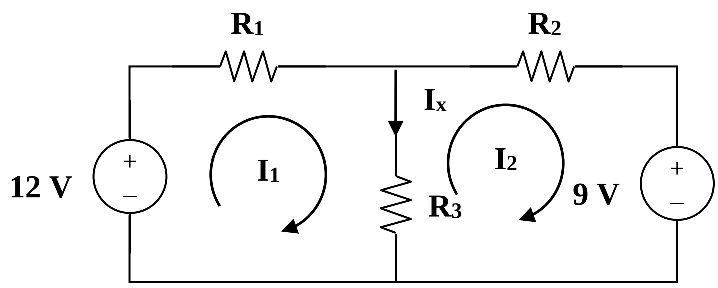


Figure 3.50
For Prob. 3.1 and 3.39.

For loop 1 we get $-12 + 4kI_1 + 2k(I_1 - I_2) = 0$ or $6I_1 - 2I_2 = 0.012$ and at

loop 2 we get $2k(I_2 - I_1) + 2kI_2 + 9 = 0$ or $-2I_1 + 4I_2 = -0.009$.

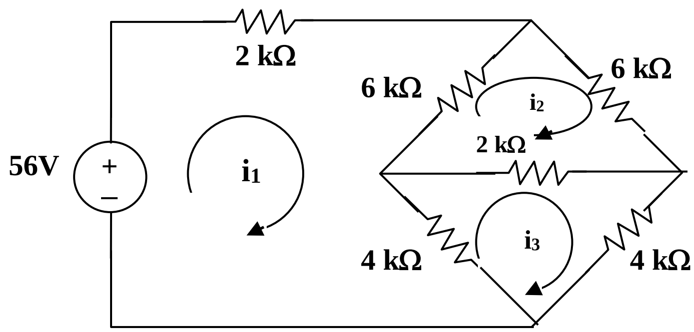
Now $6I_1 - 2I_2 = 0.012 + 3[-2I_1 + 4I_2 = -0.009]$ leads to,

$10I_2 = 0.012 - 0.027 = -0.015$ or $I_2 = -1.5$ mA and $I_1 = (-0.003+0.012)/6 = 1.5$ mA.

Thus,

$I_x = I_1 - I_2 = (1.5+1.5)$ mA = **3 mA**.

Solution 3.40



Assume all currents are in mA and apply mesh analysis for mesh 1.

$$-56 + 12i_1 - 6i_2 - 4i_3 = 0 \text{ or } 6i_1 - 3i_2 - 2i_3 = 28 \qquad (1)$$

for mesh 2,

$$-6i_1 + 14i_2 - 2i_3 = 0 \text{ or } -3i_1 + 7i_2 - i_3 = 0 \qquad (2)$$

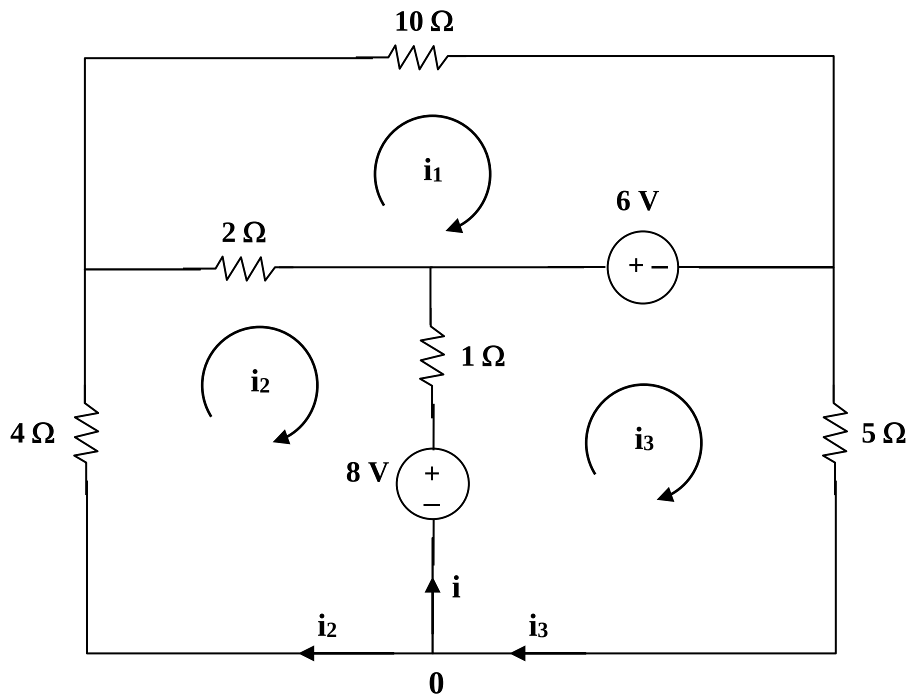
for mesh 3,

$$-4i_1 - 2i_2 + 10i_3 = 0 \text{ or } -2i_1 - i_2 + 5i_3 = 0 \qquad (3)$$

Solving (1), (2), and (3) using MATLAB, we obtain,

$$\boldsymbol{i_o} = i_1 = \mathbf{8\ mA}.$$

Solution 3.41



For loop 1,

$$6 = 12i_1 - 2i_2 \longrightarrow 3 = 6i_1 - i_2 \qquad (1)$$

For loop 2,

$$-8 = -2i_1 + 7i_2 - i_3 \qquad (2)$$

For loop 3,

$$-8 + 6 + 6i_3 - i_2 = 0 \longrightarrow 2 = -i_2 + 6i_3 \qquad (3)$$

We put (1), (2), and (3) in matrix form,

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234, \quad \Delta_2 = \begin{vmatrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & 2 & 6 \end{vmatrix} = 240$$

$$\Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38$$

At node 0, $i + i_2 = i_3$ or $i = i_3 - i_2 = \dfrac{\Delta_3 - \Delta_2}{\Delta} = \dfrac{-38 - 240}{-234} =$ **1.188 A**

Solution 3.42

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Determine the mesh currents in the circuit of Fig. 3.88.

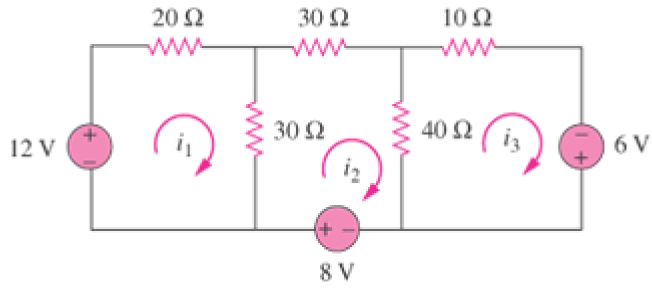


Figure 3.88

Solution

For mesh 1,

$$-12+50I_1-30I_2=0 \quad\longrightarrow\quad 12=50I_1-30I_2 \qquad (1)$$

For mesh 2,

$$-8+100I_2-30I_1-40I_3=0 \quad\longrightarrow\quad 8=-30I_1+100I_2-40I_3 \qquad (2)$$

For mesh 3,

$$-6+50I_3-40I_2=0 \quad\longrightarrow\quad 6=-40I_2+50I_3 \qquad (3)$$

Putting eqs. (1) to (3) in matrix form, we get

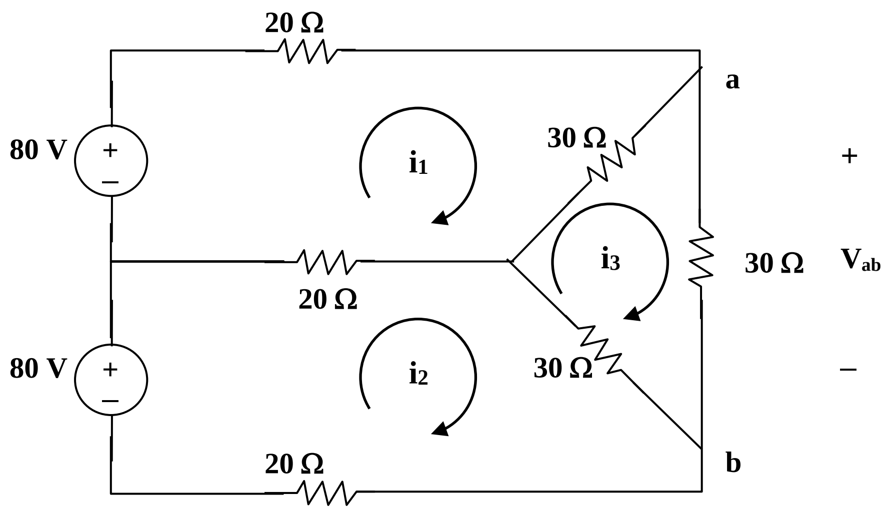
$$\begin{pmatrix} 50 & -30 & 0 \\ -30 & 100 & -40 \\ 0 & -40 & 50 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 12 \\ 8 \\ 6 \end{pmatrix} \quad\longrightarrow\quad AI=B$$

Using Matlab,

$$I=A^{-1}B=\begin{pmatrix} 0.48 \\ 0.40 \\ 0.44 \end{pmatrix}$$

i.e. I_1 = **480 mA**, I_2 = **400 mA**, I_3 = **440 mA**

Solution 3.43



For loop 1,

$$80 = 70i_1 - 20i_2 - 30i_3 \longrightarrow 8 = 7i_1 - 2i_2 - 3i_3 \qquad (1)$$

For loop 2,

$$80 = 70i_2 - 20i_1 - 30i_3 \longrightarrow 8 = -2i_1 + 7i_2 - 3i_3 \qquad (2)$$

For loop 3,

$$0 = -30i_1 - 30i_2 + 90i_3 \longrightarrow 0 = i_1 + i_2 - 3i_3$$

(3)

Solving (1) to (3), we obtain $i_3 = 16/9$

$$I_o = i_3 = 16/9 = \mathbf{1.7778\ A}$$

$$V_{ab} = 30i_3 = \mathbf{53.33\ V}.$$

Solution 3.44

Use mesh analysis to obtain i_o in the circuit of Fig. 3.90.

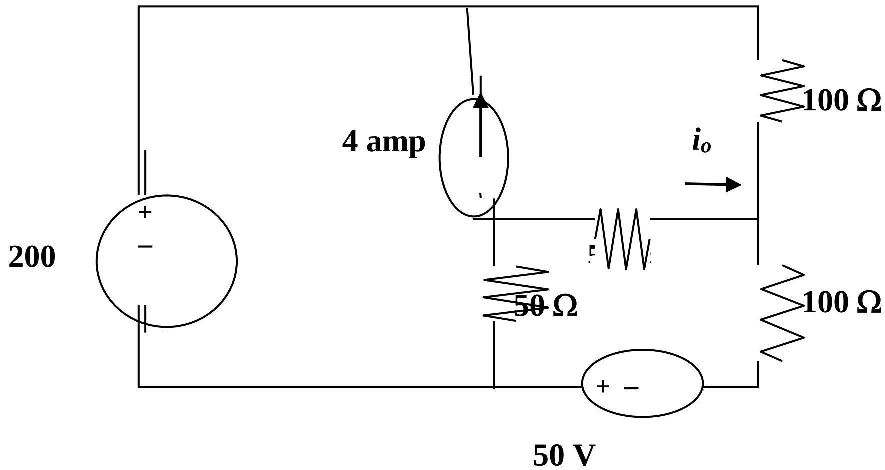
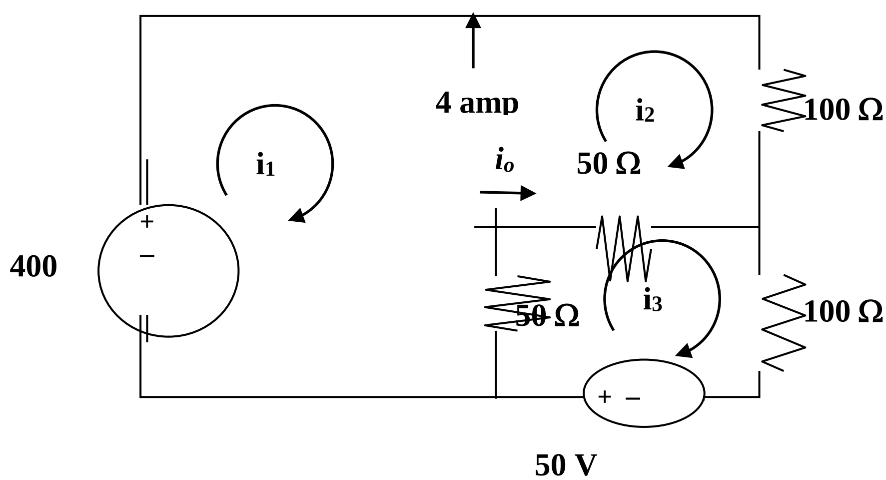


Figure 3.90
For Prob. 3.44.

Step 1. We need to redraw the circuit using a supermesh. Next we identify our unknown loop currents. Then we write our mesh equations and write the equation incorporating the current from the current source.



Supermesh (loop 1 and 2), $-400 + 100i_2 + 50(i_2-i_3) + 50(i_1-i_3) = 0$; loop 3 produces $50(i_3-i_1) + 50(i_3-i_2) + 100i_3 - 50 = 0$; and $i_2 - i_1 = 4$. We have three equations and three unknowns. Finally we note that $i_o = i_3 - i_2$.

Step 2. Since we need i_2 and i_3 let us use the constraint equation, $i_1 = i_2–4$, to allow us to solve for i_2 and i_3.

Using the first two equations we get,

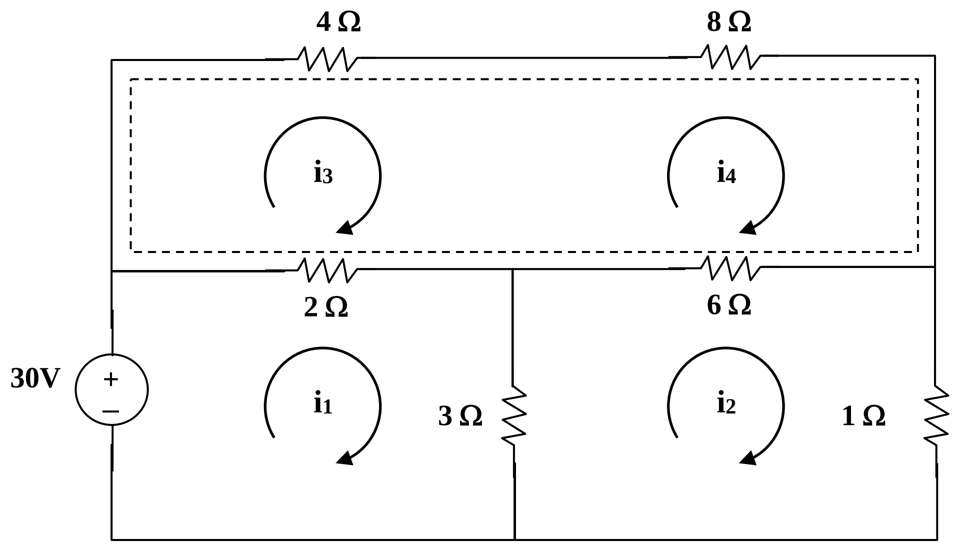
$100i_2 + 50(i_2–i_3) + 50((i_2–4)–i_3) = 400$ or $200i_2 – 100i_3 = 600$ and
$50(i_3–(i_2–4)) + 50(i_3–i_2) + 100i_3 = 50$ or $–100i_2 + 200i_3 = –150$. Thus,

$$\begin{bmatrix} 200 & -100 \\ -100 & 200 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 600 \\ -150 \end{bmatrix} or \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \frac{\begin{bmatrix} 200 & 100 \\ 100 & 200 \end{bmatrix}}{40,000 - 10,000}\begin{bmatrix} 600 \\ -150 \end{bmatrix}.$$

Finally, i_2 = [(120,000–15,000)/30,000] = 3.5 amps and i_3 = [(60,000–30,000)/30,000] = 1 amp. Now we get,

$$i_o = i_3 – i_2 = 1–3.5 = \mathbf{–2.5\ amps}.$$

Solution 3.45



For loop 1, $30 = 5i_1 - 3i_2 - 2i_3$ (1)

For loop 2, $10i_2 - 3i_1 - 6i_4 = 0$ (2)

For the supermesh, $6i_3 + 14i_4 - 2i_1 - 6i_2 = 0$ (3)

But $i_4 - i_3 = 4$ which leads to $i_4 = i_3 + 4$ (4)

Solving (1) to (4) by elimination gives $i = i_1 =$ **8.561 A**.

Solution 3.46

Calculate the mesh currents i_1 and i_2 in Fig. 3.92.

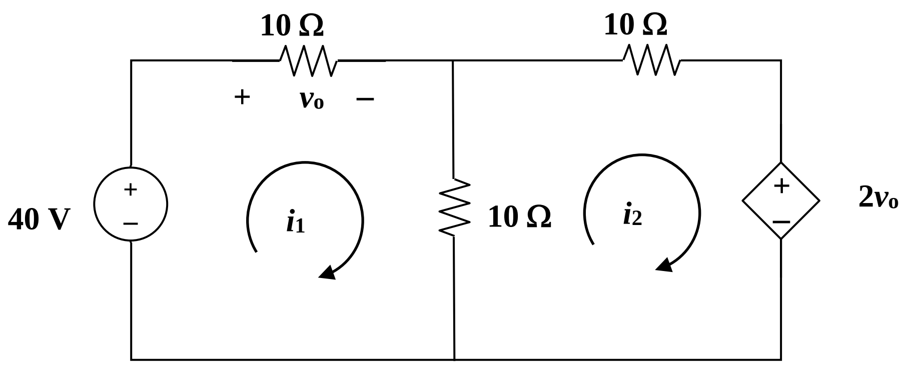


Figure 3.92
For Prob. 3.46.

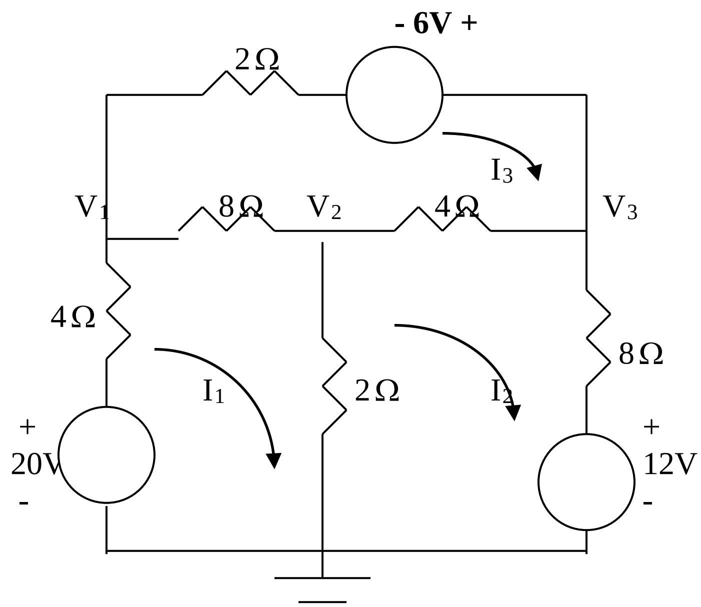
Step 1. Loop 1 $-40 + 10i_1 + 10(i_1 - i_2) = 0$ and for loop 2 $10(i_2 - i_1) + 10\,i_2 + 2v_o = 0$.

We now have two equations but three unknowns so we need a constraint equation or $v_o = 10i_1$.

Step 2. We now have $20i_1 - 10i_2 = 40$ and $-10i_1 + 20i_2 + 2(10i_1) = 0 = 10i_1 + 20i_2$ or $i_1 = -2i_2$ which leads to $20(-2i_2) - 10i_2 = -50i_2 = 40$ or $i_2 =$ **–800 mA**. Now we get $i_1 = -2(-0.8) =$ **1.6 A**.

Solution 3.47

First, transform the current sources as shown below.



For mesh 1,

$$-20+14I_1-2I_2-8I_3=0 \quad\longrightarrow\quad 10=7I_1-I_2-4I_3 \qquad (1)$$

For mesh 2,

$$12+14I_2-2I_1-4I_3=0 \quad\longrightarrow\quad -6=-I_1+7I_2-2I_3 \qquad (2)$$

For mesh 3,

$$-6+14I_3-4I_2-8I_1=0 \quad\longrightarrow\quad 3=-4I_1-2I_2+7I_3 \qquad (3)$$

Putting (1) to (3) in matrix form, we obtain

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 10 \\ -6 \\ 3 \end{pmatrix} \quad\longrightarrow\quad AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{bmatrix} 2 \\ 0.0333 \\ 1.8667 \end{bmatrix} \quad\longrightarrow\quad \underline{I_1=2.5,\ I_2=0.0333, I_3=1.8667}$$

But

$$I_1 = \frac{20 - V}{4} \quad \longrightarrow \quad V_1 = 20 - 4I_1 = \mathbf{10\ V}$$

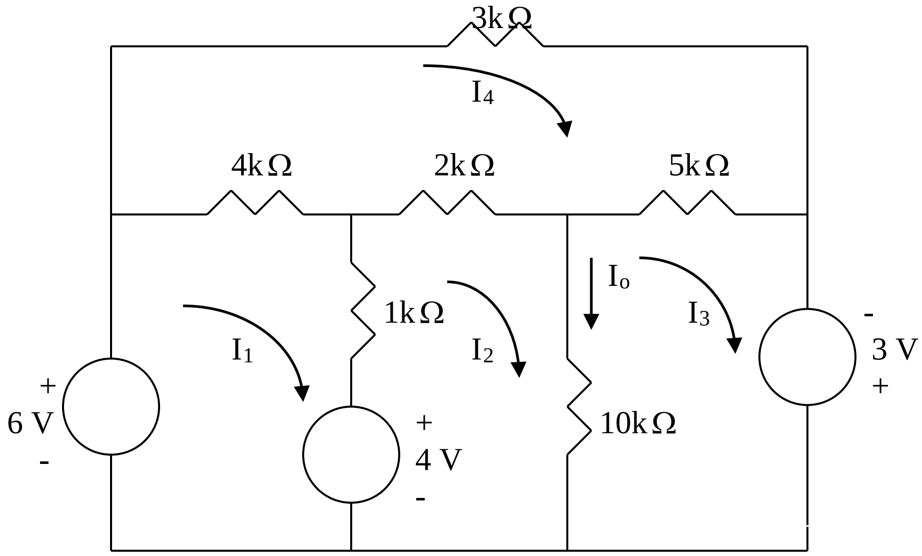
$$V_2 = 2(I_1 - I_2) = \mathbf{4.933\ V}$$

Also,

$$I_2 = \frac{V_3 - 12}{8} \quad \longrightarrow \quad V_3 = 12 + 8I_2 = \mathbf{12.267\ V}.$$

Solution 3.48

We apply mesh analysis and let the mesh currents be in mA.



For mesh 1,

$$-6+8+5I_1-I_2-4I_4=0 \longrightarrow 2=5I_1-I_2-4I_4 \qquad (1)$$

For mesh 2,

$$-4+13I_2-I_1-10I_3-2I_4=0 \longrightarrow 4=-I_1+13I_2-10I_3-2I_4 \qquad (2)$$

For mesh 3,

$$-3+15I_3-10I_2-5I_4=0 \longrightarrow 3=-10I_2+15I_3-5I_4 \qquad (3)$$

For mesh 4,

$$-4I_1-2I_2-5I_3+14I_4=0 \qquad (4)$$

Putting (1) to (4) in matrix form gives

$$\begin{pmatrix} 5 & -1 & 0 & -4 \\ -1 & 13 & -10 & -2 \\ 0 & -10 & 15 & -5 \\ -4 & -2 & -5 & 14 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}=\begin{pmatrix} 2 \\ 4 \\ 3 \\ 0 \end{pmatrix} \longrightarrow AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{pmatrix} 3.608 \\ 4.044 \\ 3.896 \\ 3 \end{pmatrix} \; 0.148$$

The current through the 10kΩ resistor is $I_o = I_2 - I_3$ = **148 mA**.

Solution 3.49

Find $\boldsymbol{v_o}$ and $\boldsymbol{i_o}$ in the circuit of Fig. 3.94.

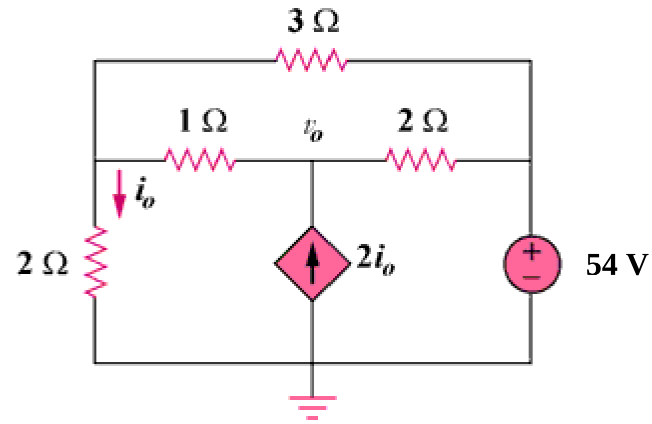
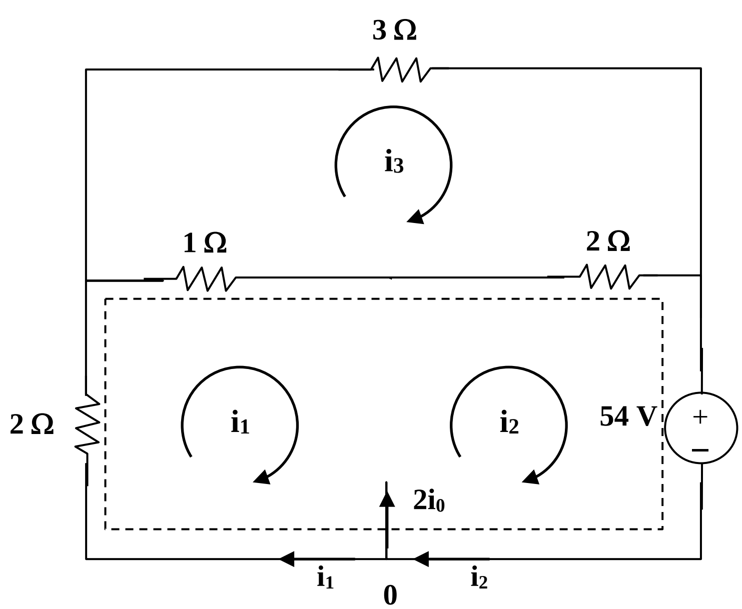


Figure 3.94
For Prob. 3.49.

Step 1. First we note that we have three unknown loop currents but we can only write two mesh equations (one is a supermesh). So we will need a constraint equation or $i_o = -i_1$ and $i_2 - i_1 = 2i_o = -2i_1 = i_2 - i_1$ or $i_2 = -i_1$. Now we have three equations and three unknowns.



(a)

The supermesh gives us $2i_1 + 1(i_1-i_3) + 2(i_2-i_3) + 54 = 0$ and loop 3 produces $1(i_3-i_1) + 3i_3 + 2(i_3-i_2) = 0$. Finally $v_o = 2(i_2-i_3) + 54 = -2(i_1+i_3) + 54$.

Step 2. $3i_1 + 2i_2 – 3i_3 = –54 = (3–2)i_1 – 3i_3$ or $i_1 – 3i_3 = –54$.

Next $–i_1 – 2i_2 + 6i_3 = 0 = (–1+2)i_1 + 6i_3 = i_1 + 6i_3 = 0$. This leads to $i_1 = –6i_3$ and $–6i_3 – 3i_3 = –54$ or $i_3 = 6$ and $i_1 = –36$ or i_o = **36 A**. Finally, $v_o = –2(–36+6) + 54 = 60 + 54 =$ **114 V**.

Solution 3.50

Use mesh analysis to find the current $\boldsymbol{i_o}$ in the circuit in Fig. 3.95.

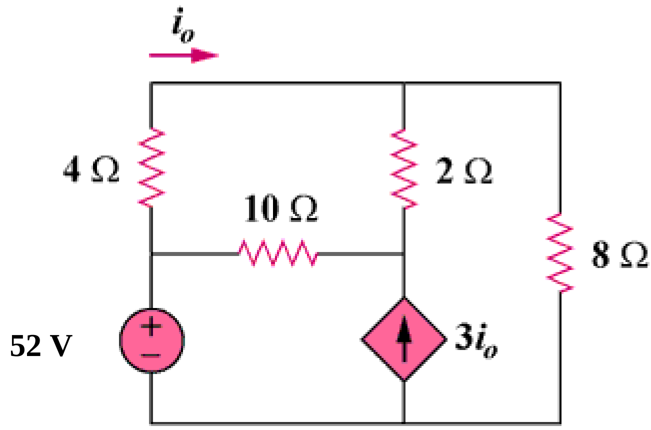
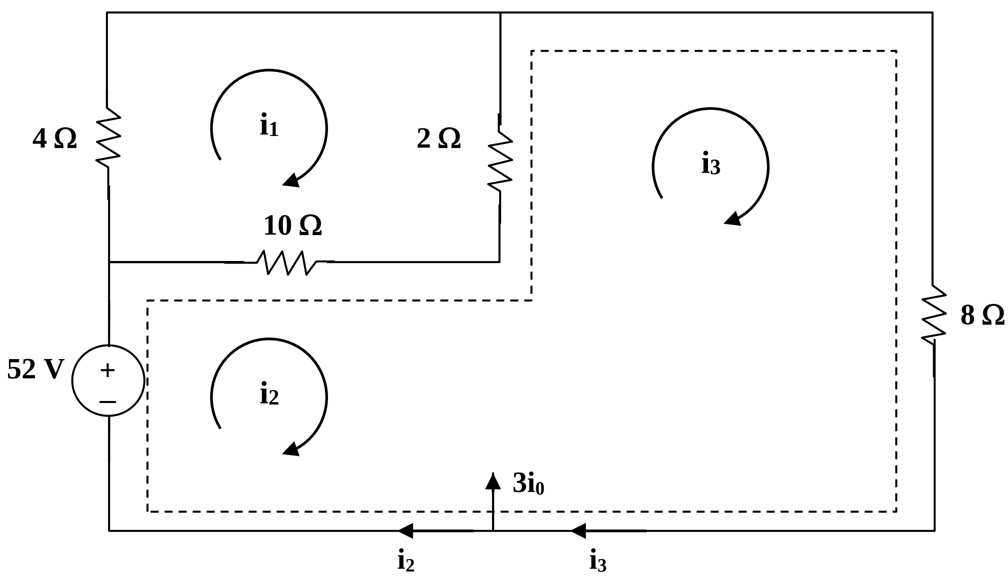


Figure 3.95
For Prob. 3.50.

Step 1. We note that we have three unknown loop currents but only two mesh equations (one is a supermesh). So we need a two constraint equations, one for i_2 and i_3 and i_o i_1.



For loop 1, $\quad 16i_1 - 10i_2 - 2i_3 = 0$ which leads to $8i_1 - 5i_2 - i_3 = 0 \qquad (1)$

For the supermesh, $-52 + 10i_2 - 10i_1 + 10i_3 - 2i_1 = 0$

or $\qquad -6i_1 + 5i_2 + 5i_3 = 26 \qquad (2)$

Also, $3i_0 = i_3 – i_2$ and $i_0 = i_1$ which leads to $3i_1 = i_3 – i_2$ (3)

Using $i_3 = 3i_1 + i_2$ we get $8i_1 – 5i_2 – 3i_1 –i_2 = 0$ or $5i_1 = 6i_2$ or $i_1 = 1.2i_2$ and $–6i_1 + 5i_2 + 5i_3 = 26$ becomes $–6(1.2i_2) + 5i_2 + 5(3.6i_2+i_2) = 26 = (–7.2+5+23)i_2$ or $i_2 = 1.25$; $i_1 = 1.2x1.25 = 1.5$; and $i_3 = 3x1.5+1.25 = 5.75$. Finally,

i_o = **1.5 A**.

Solution 3.51

Apply mesh analysis to find v_o in the circuit in Fig. 3.96.

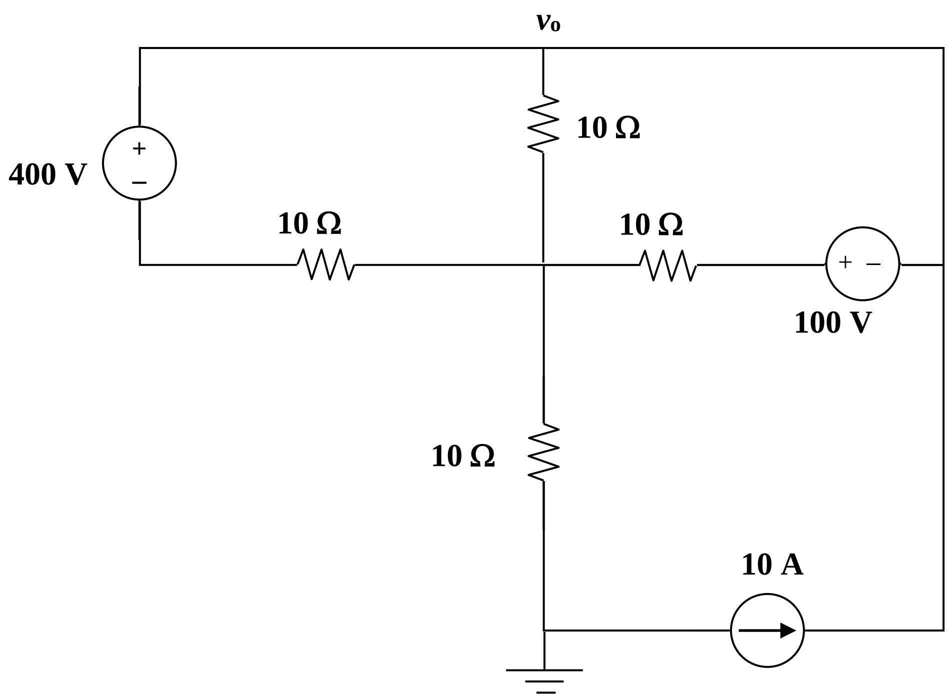
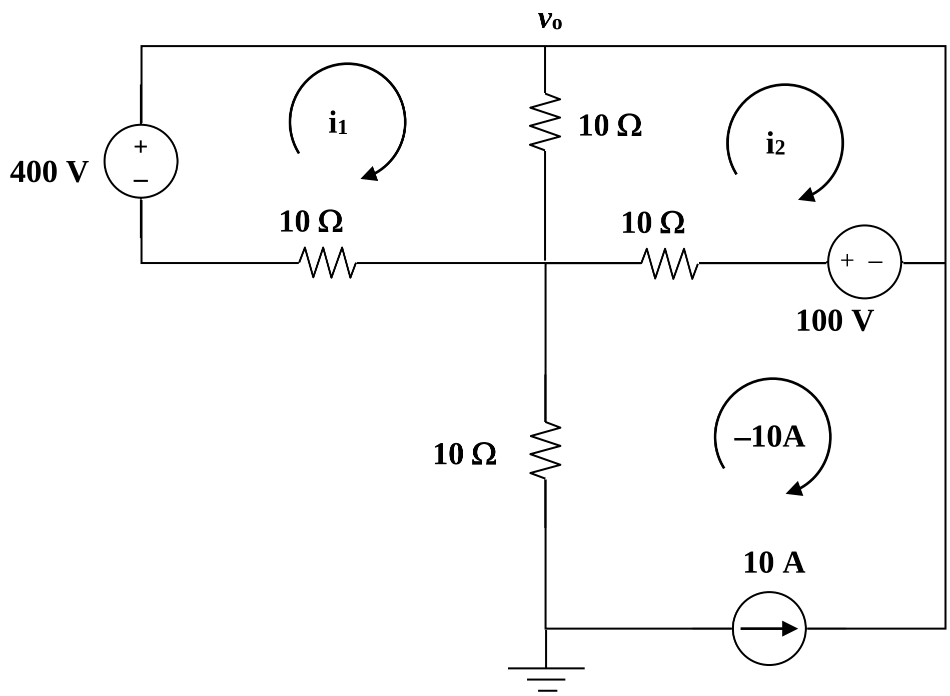


Figure 3.96
For Prob. 3.51.

Solution continued on the next page...

Step 1. First we identify the unknown loop currents and write the mesh equations.



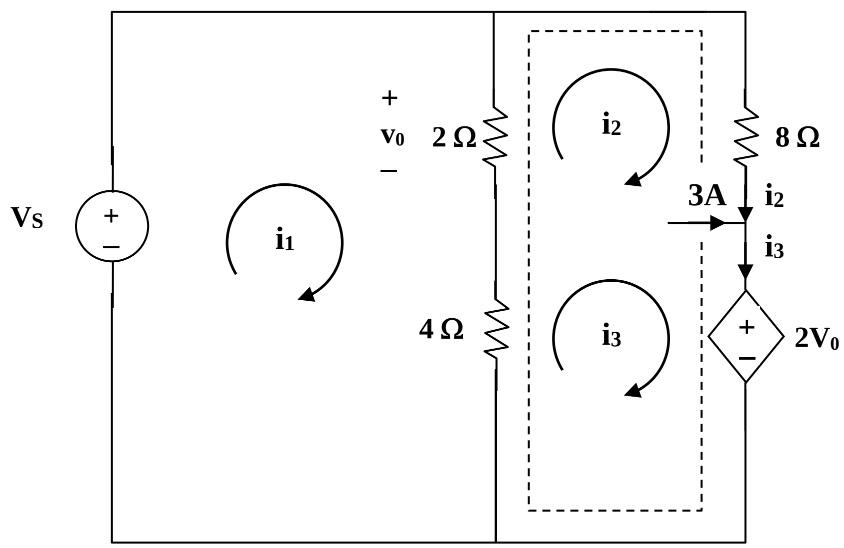
For loop 1 we get $-400 + 10(i_1 - i_2) + 10i_1 = 0$ or $20i_1 - 10i_2 = 400$.
For loop 2 we get $10(i_2 - i_1) - 100 + 10(i_2 - (-10)) = 0$ or $-10i_1 + 20i_2 = 0$.

Finally we need v_o. Clearly $v_o = 10(i_1 - i_2) + 10(10)$.

Step 2. From $-10i_1 + 20i_2 = 0$ we obtain $i_1 = 2i_2$ and from $20i_1 - 10i_2 = 400$ we obtain $40i_2 - 10i_2 = 30i_2 = 400$ or $i_2 = 13.333$ amps and $i_1 = 26.67$ amps. Now,

$v_o = 133.33 + 100 =$ **233.3 volts**.

Solution 3.52



For mesh 1,

$2(i_1 - i_2) + 4(i_1 - i_3) - 12 = 0$ which leads to $3i_1 - i_2 - 2i_3 = 6$ (1)

For the supermesh, $2(i_2 - i_1) + 8i_2 + 2v_0 + 4(i_3 - i_1) = 0$

But $v_0 = 2(i_1 - i_2)$ which leads to $-i_1 + 3i_2 + 2i_3 = 0$
(2)

For the independent current source, $i_3 = 3 + i_2$ (3)

Solving (1), (2), and (3), we obtain,

$i_1 =$ **3.5 A**, $i_2 =$ **-0.5 A**, $i_3 =$ **2.5 A**.

Solution 3.53

Applying mesh analysis leads to;

$-12 + 4kI_1 - 3kI_2 - 1kI_3 = 0$ (1)

$-3kI_1 + 7kI_2 - 4kI_4 = 0$

$-3kI_1 + 7kI_2 = -12$ (2)

$-1kI_1 + 15kI_3 - 8kI_4 - 6kI_5 = 0$

$-1kI_1 + 15kI_3 - 6k = -24$ (3)

$I_4 = -3mA$ (4)

$-6kI_3 - 8kI_4 + 16kI_5 = 0$

$-6kI_3 + 16kI_5 = -24$ (5)

Putting these in matrix form (having substituted $I_4 = 3mA$ in the above),

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ -1 & 0 & 15 & -6 \\ 0 & 0 & -6 & 16 \end{bmatrix} k \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ -24 \\ -24 \end{bmatrix}$$

ZI = V

Using MATLAB,

```
>> Z = [4,-3,-1,0;-3,7,0,0;-1,0,15,-6;0,0,-6,16]

Z =

     4    -3    -1     0
    -3     7     0     0
    -1     0    15    -6
     0     0    -6    16

>> V = [12,-12,-24,-24]'

V =

    12
   -12
   -24
   -24
```

We obtain,

```
>> I = inv(Z)*V

    I =
```

1.6196 mA
–1.0202 mA
–2.461 mA
3 mA
–2.423 mA

Solution 3.54

Let the mesh currents be in mA. For mesh 1,

$$-12+10+2I_1-I_2=0 \quad \longrightarrow \quad 2=2I_1-I_2 \qquad (1)$$

For mesh 2,

$$-10+3I_2-I_1-I_3=0 \quad \longrightarrow \quad 10=-I_1+3I_2-I_3 \qquad (2)$$

For mesh 3,

$$-12+2I_3-I_2=0 \quad \longrightarrow \quad 12=-I_2+2I_3 \qquad (3)$$

Putting (1) to (3) in matrix form leads to

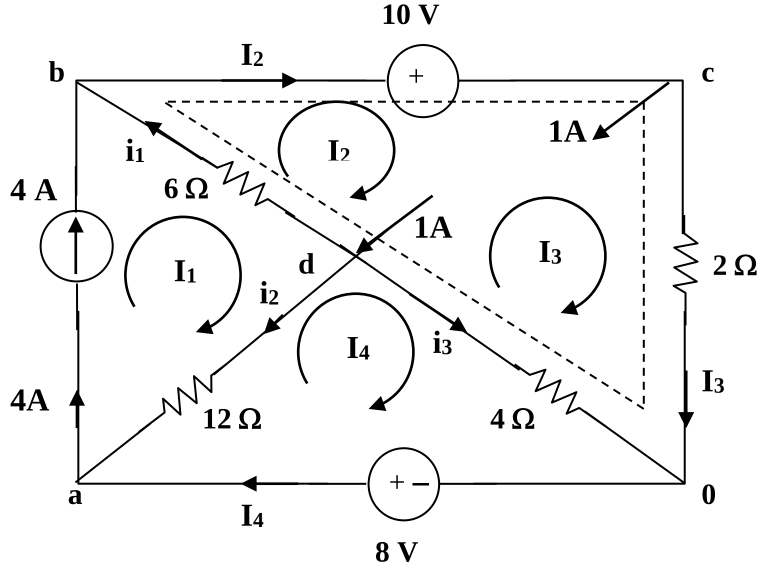
$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}=\begin{pmatrix} 2 \\ 10 \\ 12 \end{pmatrix} \quad \longrightarrow \quad AI=B$$

Using MATLAB,

$$I=A^{-1}B=\begin{bmatrix} 5.25 \\ 8.5 \\ 10.25 \end{bmatrix} \quad \longrightarrow \quad \underline{I_1=5.25\,\text{mA},\ I_2=8.5\,\text{mA}, I_3=10.25\,\text{mA}}$$

I_1 = **5.25 mA**, I_2 = **8.5 mA**, and I_3 = **10.25 mA**.

Solution 3.55



It is evident that $I_1 = 4$ (1)

For mesh 4, $12(I_4 – I_1) + 4(I_4 – I_3) – 8 = 0$ (2)

For the supermesh $6(I_2 – I_1) + 10 + 2I_3 + 4(I_3 – I_4) = 0$
or $-3I_1 + 3I_2 + 3I_3 – 2I_4 = -5$ (3)

At node c, $I_2 = I_3 + 1$ (4)

Solving (1), (2), (3), and (4) yields, $I_1 = 4A$, $I_2 = 3A$, $I_3 = 2A$, and $I_4 = 4A$

At node b, $i_1 = I_2 – I_1 =$ **-1A**

At node a, $i_2 = 4 – I_4 =$ **0A**

At node 0, $i_3 = I_4 – I_3 =$ **2A**

Solution 3.56

Determine $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$ **i**n the circuit of Fig. 3.101.

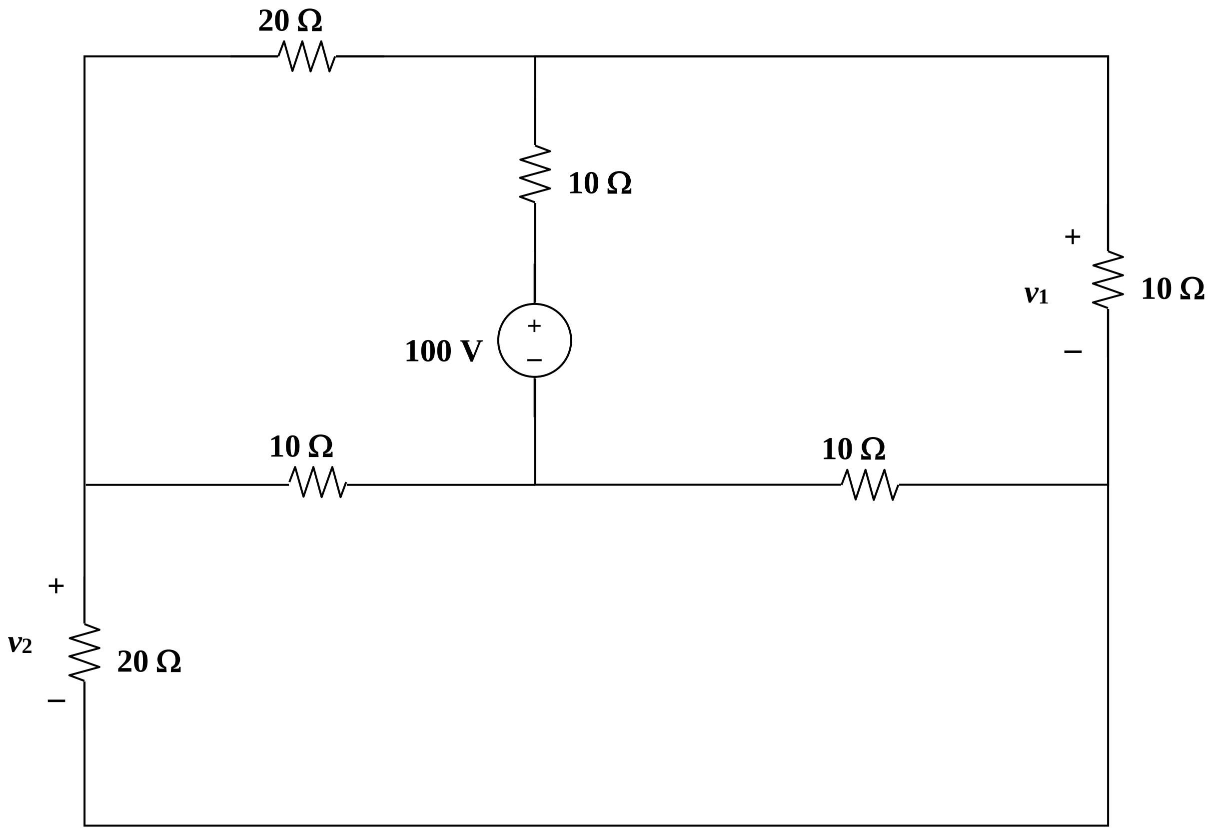
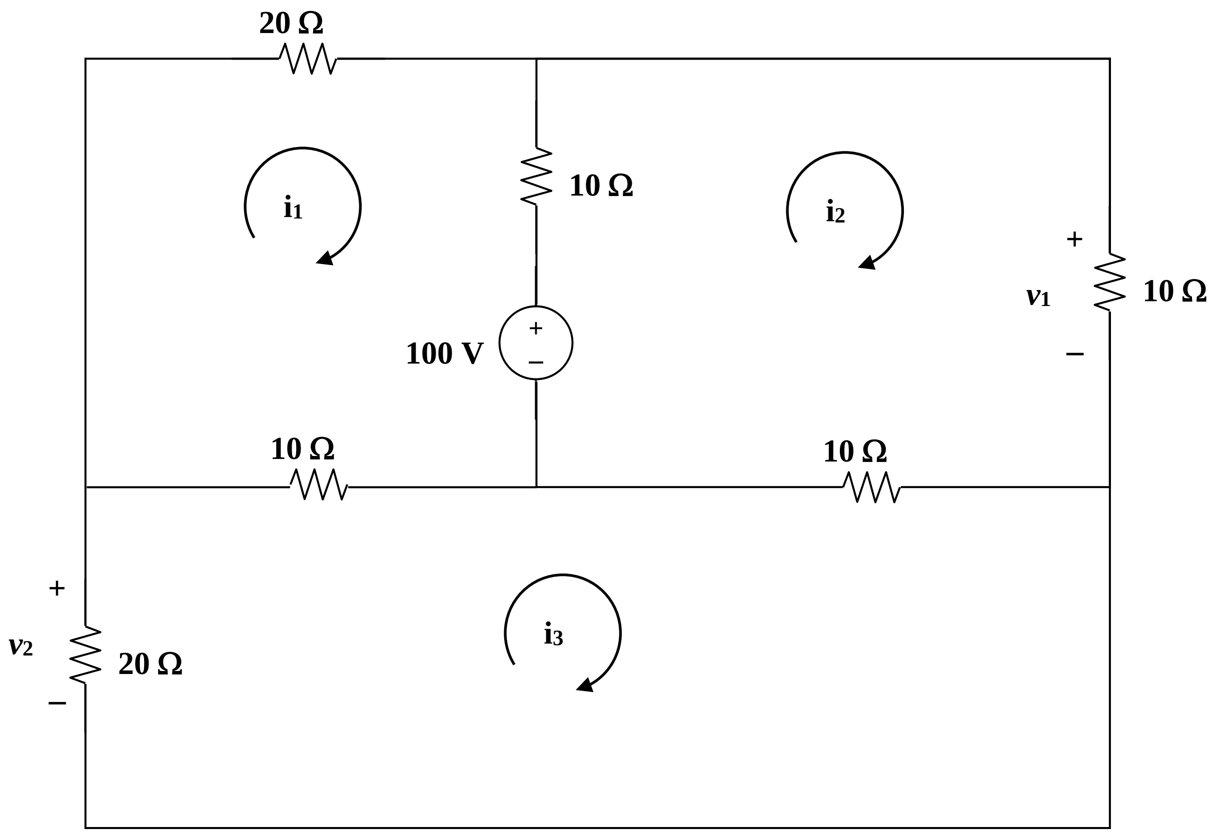


Figure 3.101
For Prob. 3.56.

Step 1. First we redraw the circuit and establish the unknown loop currents. Next we write the three mesh equations and put them into matrix form.

We will have a three by three matrix which we can invert and solve for the unknown loop currents. Finally we can solve for v_1 (= $10i_2$) and v_2 (= $20i_3$).

$20i_1 + 10(i_1 – i_2) + 100 + 10(i_1 – i_3) = 0$ or $40i_1 – 10i_2 – 10i_3 = –100$
$–100 + 10(i_2 – i_1) + 10i_2 + 10(i_2 – i_3) = 0$ or $–10i_1 + 30i_2 – 10i_3 = 100$
$20i_3 + 10(i_3 – i_1) + 10(i_3 – i_2) = 0$ or $–10i_1 – 10i_2 + 40i_3 = 0$ which leads to,



Step 2.

$$\begin{bmatrix} 40 & -10 & -10 \\ -10 & 30 & -10 \\ -10 & -10 & 40 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -100 \\ 100 \\ 0 \end{bmatrix}$$ using MATLAB we get,

```
>> R=[40,-10,-10;-10,30,-10;-10,-10,40]

R =

   40  -10  -10
  -10   30  -10
  -10  -10   40

>> V=[-100;100;0]
```

V =

-100
100
0

```
>> I=inv(R)*V
```

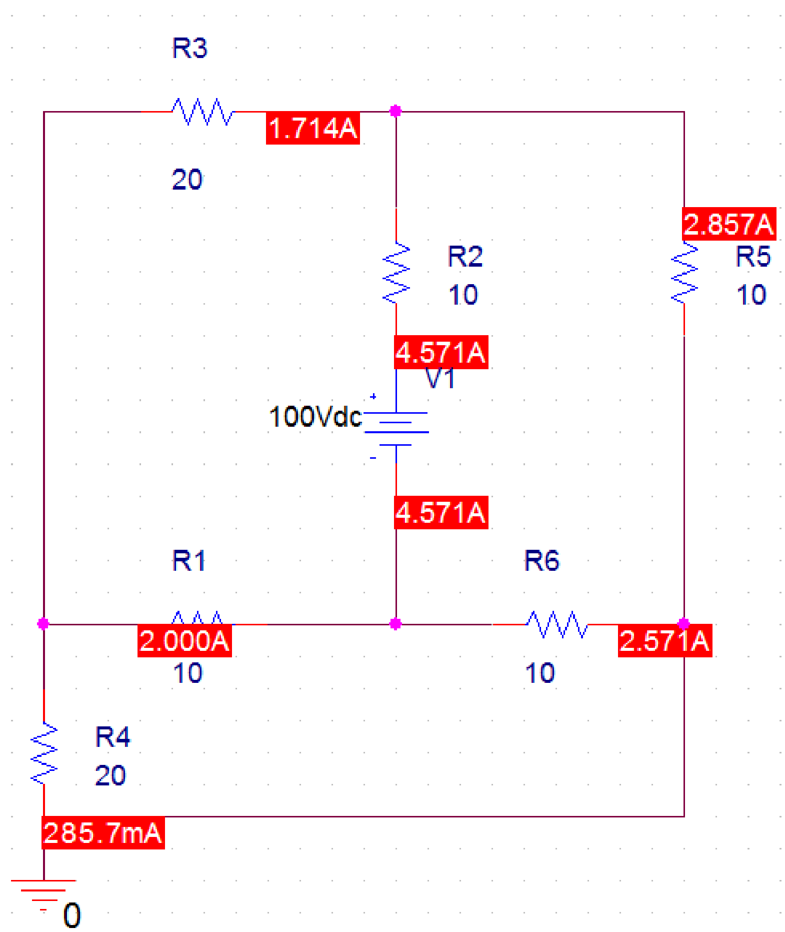
I =

-1.7143
2.8571
0.2857

Thus, $i_1 = -1.7143$ A, $i_2 = 2.8571$ A and $i_3 = 0.2857$ A which leads to,

$$v_1 = 10i_2 = 28.57 \text{ V and } v_2 = -20i_3 = -5.714 \text{ V.}$$

Checking with PSpice we get,



Solution 3.57

In the circuit in Fig. 3.102, find the values of R, V_1, and V_2 given that i_o = 20 mA.

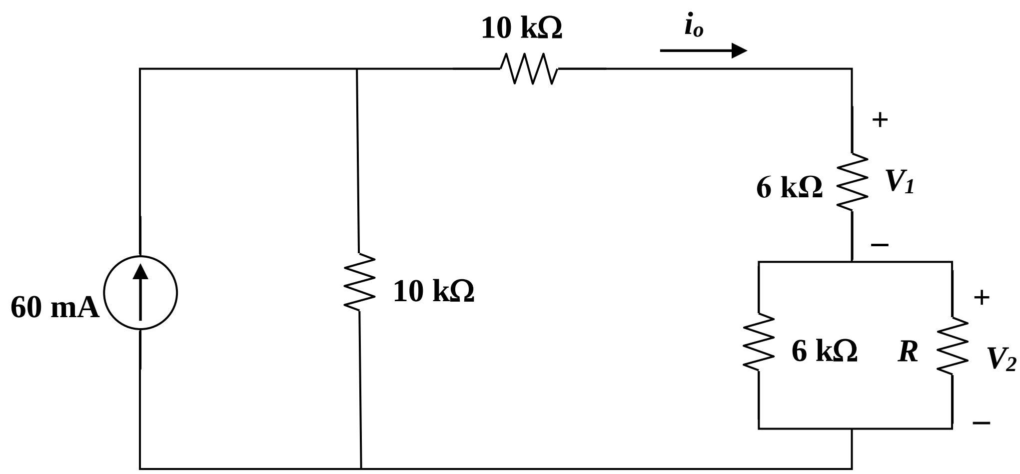


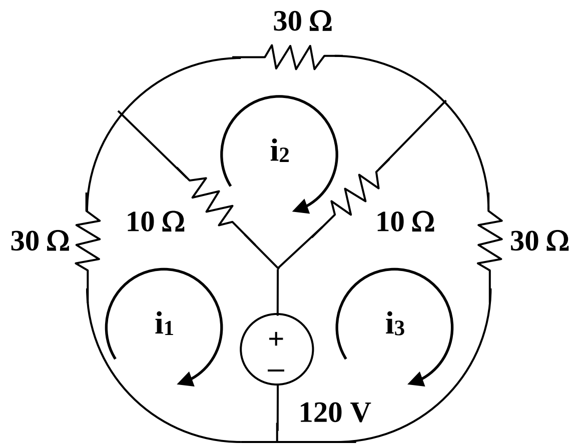
Figure 3.102
For Prob. 3.57.

Step 1. Since $i_o = 0.02$ A, $V_1 = 6{,}000 \times 0.02$. By current division we get $V_2/R = 0.02[6k/(6k+R)]$ and $0.04 \times 10{,}000 = 0.02[10k + 6k + 6k \times R/(6k+R)]$. We can now solve for R, V_1, and V_2.

Step 2. $400 = 200 + 120 + 120R/(6k+R)$ or $R/(6k+R) = 80/120 = (2/3)$ or $1.5R = 6k + R$ or R = **12 kΩ**. V_1 = **120 V**. Now we can find V_2.

$V_2 = R\{0.02[6k/(6k+12k)]\} = 12k\{120/18k\}$ = **80 V**.

Solution 3.58



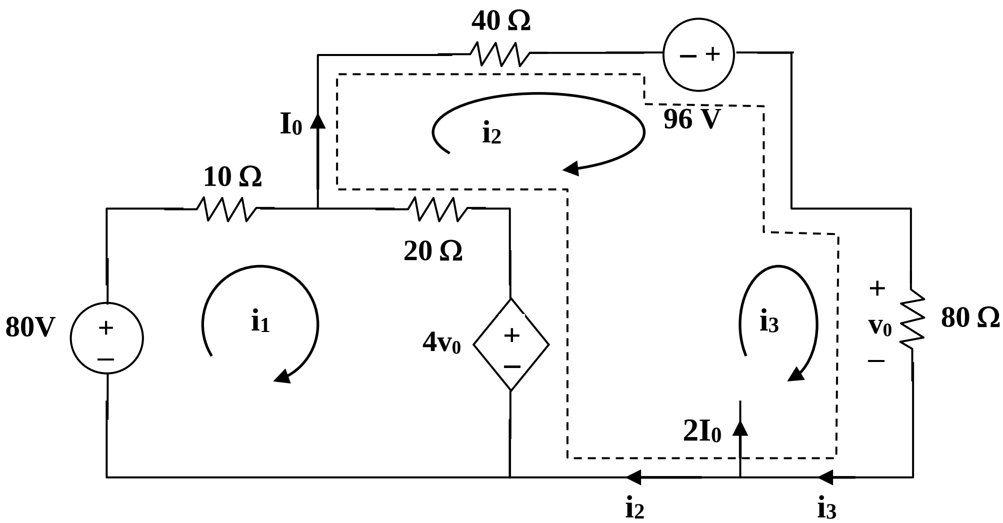
For loop 1, $120 + 40i_1 - 10i_2 = 0$, which leads to $-12 = 4i_1 - i_2$ (1)

For loop 2, $50i_2 - 10i_1 - 10i_3 = 0$, which leads to $-i_1 + 5i_2 - i_3 = 0$ (2)

For loop 3, $-120 - 10i_2 + 40i_3 = 0$, which leads to $12 = -i_2 + 4i_3$ (3)

Solving (1), (2), and (3), we get, i_1 = **-3A**, i_2 = **0**, and i_3 = **3A**

Solution 3.59



For loop 1, $-80 + 30i_1 - 20i_2 + 4v_0 = 0$, where $v_0 = 80i_3$

or $4 = 1.5i_1 - i_2 + 16i_3$ (1)

For the supermesh, $60i_2 - 20i_1 - 96 + 80i_3 - 4\,v_0 = 0$, where $v_0 = 80i_3$

or $4.8 = -i_1 + 3i_2 - 12i_3$ (2)

Also, $2I_0 = i_3 - i_2$ and $I_0 = i_2$, hence, $3i_2 = i_3$

(3)

From (1), (2), and (3), $\begin{bmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 4.8 \\ 0 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 3 & -2 & 32 \\ -1 & 3 & -12 \\ 0 & 3 & -1 \end{vmatrix} = 5,\ \Delta_2 = \begin{vmatrix} 3 & 8 & 32 \\ -1 & 4.8 & -12 \\ 0 & 0 & -1 \end{vmatrix} = -22.4,\ \Delta_3 = \begin{vmatrix} 3 & -2 & 8 \\ -1 & 3 & 4.8 \\ 0 & 3 & 0 \end{vmatrix} = -67.2$$

$I_0 = i_2 = \Delta_2/\Delta = -28/5 =$ **-4.48 A**

$v_0 = 8i_3 = (-84/5)80 =$ **-1.0752 kvolts**

Solution 3.60

Calculate the power dissipated in each resistor in the circuit in Fig. 3.104.

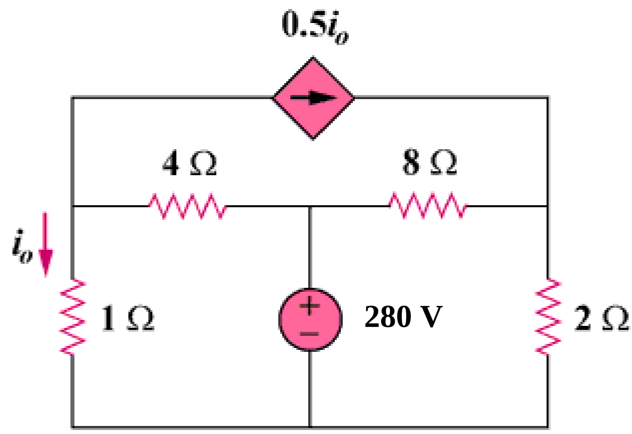
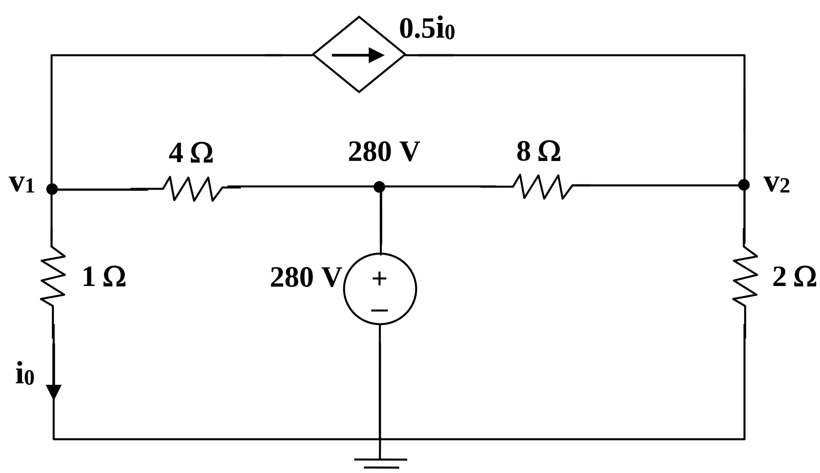


Figure 3.104
For Prob. 3.60.

Step 1. First we identify all of the unknown nodes of which we find two. Next we write two nodal equations. Since we have three unknowns but only two equations we need a constraint equation, $i_o = (v_1–0)/1 = v_1$.



At node 1, $[(v_1–0)/1] + [(v_1–280)/4] + 0.5i_o = 0$ and at node 2, $[(v_2–280)/8] – 0.5i_o + [(v_2–0)/2] = 0$. Finally $P_1 = (v_1)^2/1$; $P_4 = (v_1–280)^2/4$; $P_8 = (v_2–280)^2/8$; and $P_2 = (v_2)^2/2$.

Step 2. $(1+0.25)v_1 + 0.5v_1 = 1.75v_1 = 70$ or $v_1 = 40$ V. $(0.125+0.5)v_2 = 35+20 = 55$ or $v_2 = 55/0.625 = 88$ V. Finally,

P_1 = **1.6 kW**; P_4 = **14.4 kW**; P_8 = **4.608 kW**; and P_2 = **3.872 kW**.

Solution 3.61

Calculate the current gain $\boldsymbol{i_o}/\boldsymbol{i_s}$ in the circuit of Fig. 3.105.

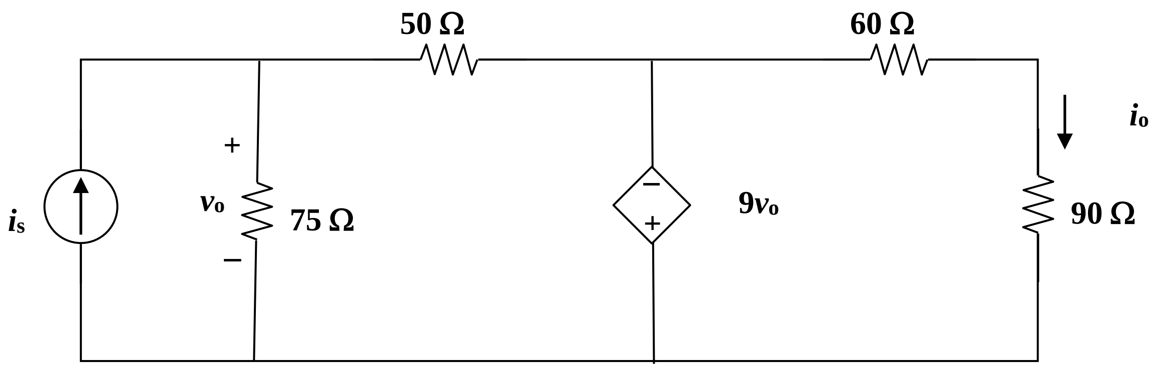
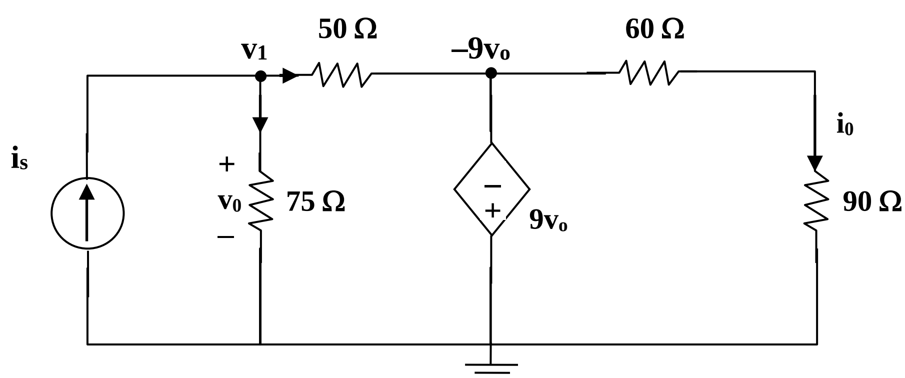


Figure 3.105
For Prob. 3.61.

Step 1. Since we wish to calculate the gain of this circuit we need to find i_o in terms of i_s. We can do this by using nodal analysis. First we identify the unknown nodes of which there is really only one. We can then write one nodal equation but we end up with two unknowns so we need a constraint equation, $v_o = v_1$.

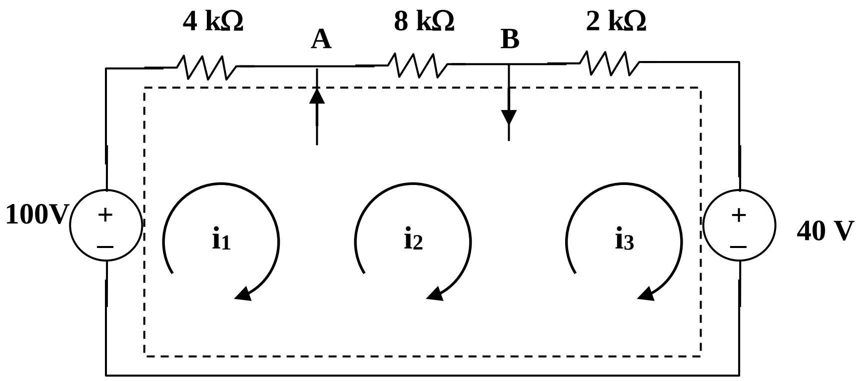


At node 1 we get, $-i_s + [(v_1-0)/75] + [(v_1-(-9v_o))/50] = 0$. Finally we can find $i_o = (-9v_o-0)/150 = -3v_o/50$.

Step 2. $(0.013333+0.2)v_1 = i_s$ or $v_1 = 4.688i_s$ and $i_o = -3(4.688i_s)/50 = -0.2813i_s$ which leads to,

$$i_o/i_s = \mathbf{-0.2813}.$$

Solution 3.62



We have a supermesh. Let all R be in kΩ, i in mA, and v in volts.

For the supermesh, $-100 + 4i_1 + 8i_2 + 2i_3 + 40 = 0$ or $30 = 2i_1 + 4i_2 + i_3$ (1)

At node A, $i_1 + 4 = i_2$ (2)

At node B, $i_2 = 2i_1 + i_3$ (3)

Solving (1), (2), and (3), we get i_1 = **2 mA**, i_2 = **6 mA**, and i_3 = **2 mA**.

Solution 3.63

Find v_x, and i_o in the circuit shown in Fig. 3.107.

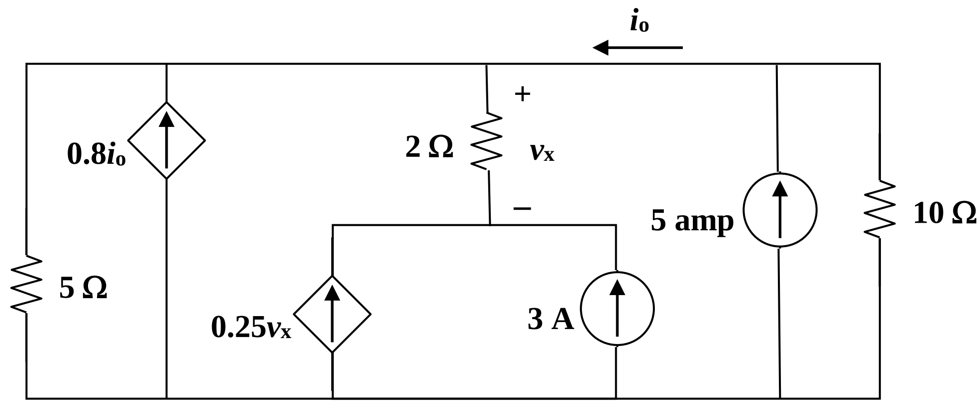
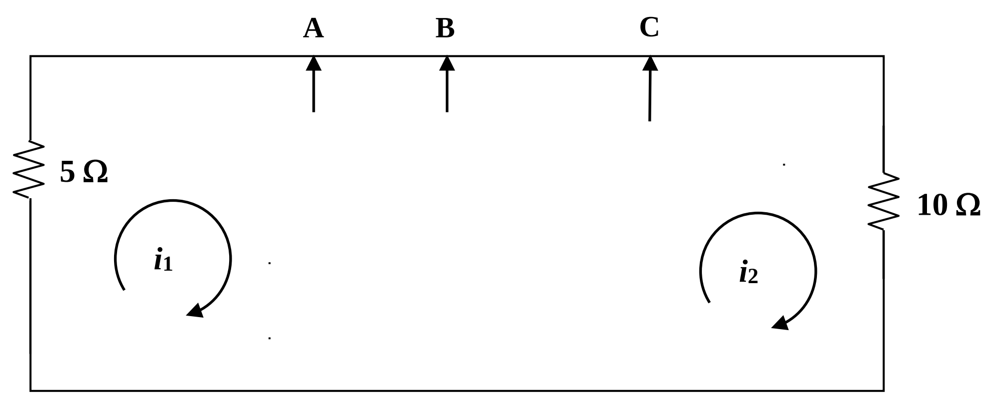


Figure 3.107
For Prob. 3.63.

Solution

Step 1. First we need to redraw the circuit to reflect the unknown currents.



For the supermesh, $5i_1 + 10i_2 = 0$.

At A, $-i_1 - 0.8i_o + I_{AB} = 0$. At B, $-I_{AB} - 0.25v_x - 3 + I_{BC} = 0$. Finally, at C, $-I_{BC} - 5 + i_2 = 0$.

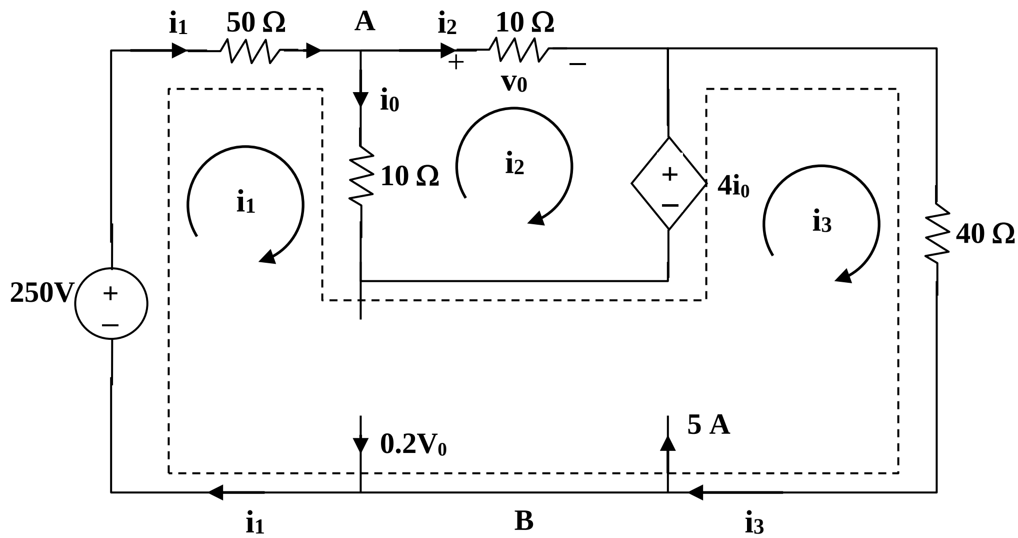
Our constraint equations are $v_x = 2(-0.25v_x - 3)$ and $i_o = -I_{BC}$. We now have 5 unknowns and 5 equations.

Step 2. From $v_x = 2(-0.25v_x-3)$ we get $v_x = -6/1,5 =$ **-4 volts**.

From $5i_1 + 10i_2 = 0$, $i_o = -I_{BC}$, and $-I_{BC} - 5 + i_2 = 0$ we get $i_2 = 5 - i_o$ and $i_1 = -2i_2 = 2i_o - 10$.

From $-i_1 - 0.8i_o + I_{AB} = 0$, $-I_{AB} - 0.25v_x - 3 + I_{BC} = 0$ or $I_{AB} = -2 - i_o$, and $i_o = -I_{BC}$ or $I_{BC} = -i_o$, $i_1 = 2i_o - 10$, and $v_x = -4$ we get
$-(2i_o - 10) - 0.8i_o + (-2-i_o) = 0$ or
$-(2i_o - 10) - 0.8i_o + (-2 - i_o) = 0 = -3.8i_o + 10 - 2$ or $i_o =$ **2.105 A**.

Solution 3.64



For mesh 2, $20i_2 - 10i_1 + 4i_0 = 0$ (1)

But at node A, $i_o = i_1 - i_2$ so that (1) becomes $i_1 = (16/6)i_2$ (2)

For the supermesh, $-250 + 50i_1 + 10(i_1 - i_2) - 4i_0 + 40i_3 = 0$

or $28i_1 - 3i_2 + 20i_3 = 125$ (3)

At node B, $i_3 + 0.2v_0 = 2 + i_1$ (4)

But, $v_0 = 10i_2$ so that (4) becomes $i_3 = 5 + (2/3)i_2$ (5)

Solving (1) to (5), $i_2 = 0.2941$ A,

$v_0 = 10i_2 =$ **2.941 volts**, $i_0 = i_1 - i_2 = (5/3)i_2 =$ **490.2mA**.

Solution 3.65

For mesh 1,

$$-12 + 12I_1 - 6I_2 - I_4 = 0 \text{ or}$$

$$12 = 12I_1 - 6I_2 - I_4 \qquad (1)$$

For mesh 2,

$$-6I_1 + 16I_2 - 8I_3 - I_4 - I_5 = 0 \qquad (2)$$

For mesh 3,

$$-8I_2 + 15I_3 - I_5 - 9 = 0 \text{ or}$$

$$9 = -8I_2 + 15I_3 - I_5 \qquad (3)$$

For mesh 4,

$$-I_1 - I_2 + 7I_4 - 2I_5 - 6 = 0 \text{ or}$$

$$6 = -I_1 - I_2 + 7I_4 - 2I_5 \qquad (4)$$

For mesh 5,

$$-I_2 - I_3 - 2I_4 + 8I_5 - 10 = 0 \text{ or}$$

$$10 = -I_2 - I_3 - 2I_4 + 8I_5 \qquad (5)$$

Casting (1) to (5) in matrix form gives

$$\begin{pmatrix} 12 & -6 & 0 & 1 & 0 \\ -6 & 16 & -8 & -1 & -1 \\ 0 & -8 & 15 & 0 & -1 \\ -1 & -1 & 0 & 7 & -2 \\ 0 & -1 & -1 & -2 & 8 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 9 \\ 6 \\ 10 \end{pmatrix} \longrightarrow \quad AI = B$$

Using MATLAB we input:
Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]
and V=[12;0;9;6;10]

This leads to

```
>> Z=[12,-6,0,-1,0;-6,16,-8,-1,-1;0,-8,15,0,-1;-1,-1,0,7,-2;0,-1,-1,-2,8]

Z =

   12   -6    0   -1    0
   -6   16   -8   -1   -1
    0   -8   15    0   -1
   -1   -1    0    7   -2
    0   -1   -1   -2    8

>> V=[12;0;9;6;10]
```

```
V =

    12
     0
     9
     6
    10

>> I=inv(Z)*V

I =

    2.1701
    1.9912
    1.8119
    2.0942
    2.2489
```

Thus,

I = [2.17, 1.9912, 1.8119, 2.094, 2.249] A.

Solution 3.66

The mesh equations are obtained as follows.

$$-12 + 24 + 30I_1 - 4I_2 - 6I_3 - 2I_4 = 0$$

or

$$30I_1 - 4I_2 - 6I_3 - 2I_4 = -12 \qquad (1)$$

$$-24 + 40 - 4I_1 + 30I_2 - 2I_4 - 6I_5 = 0$$

or

$$-4I_1 + 30I_2 - 2I_4 - 6I_5 = -16 \qquad (2)$$

$$-6I_1 + 18I_3 - 4I_4 = 30 \qquad (3)$$

$$-2I_1 - 2I_2 - 4I_3 + 12I_4 - 4I_5 = 0 \qquad (4)$$

$$-6I_2 - 4I_4 + 18I_5 = -32 \qquad (5)$$

Putting (1) to (5) in matrix form

$$\begin{bmatrix} 30 & -4 & -6 & -2 & 0 \\ -4 & 30 & 0 & -2 & -6 \\ -6 & 0 & 18 & -4 & 0 \\ -2 & -2 & -4 & 12 & -4 \\ 0 & -6 & 0 & -4 & 18 \end{bmatrix} \mathrm{I} = \begin{bmatrix} -12 \\ -16 \\ 30 \\ 0 \\ -32 \end{bmatrix}$$

ZI = V

Using MATLAB,

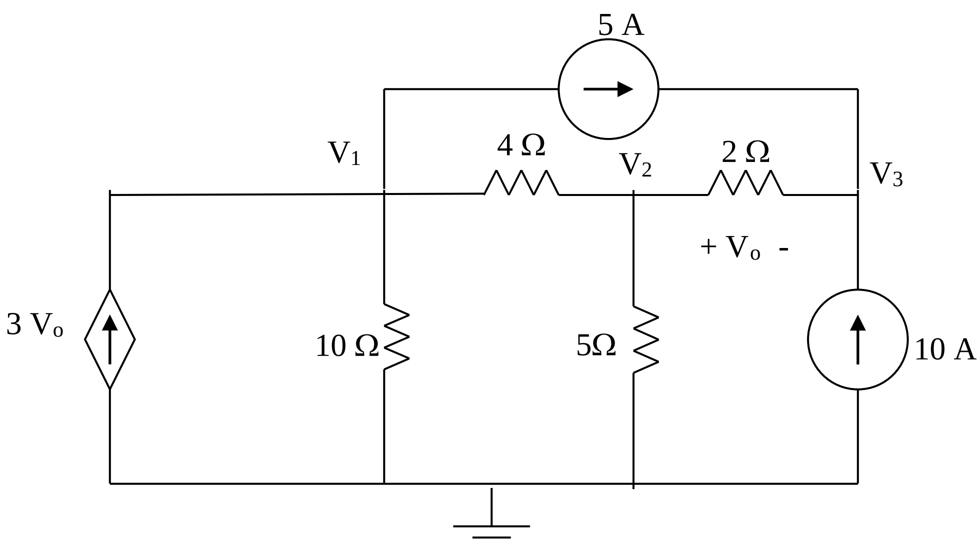
```
>> Z = [30,-4,-6,-2,0;
-4,30,0,-2,-6;
-6,0,18,-4,0;
-2,-2,-4,12,-4;
0,-6,0,-4,18]
```

```
Z =

    30    -4    -6    -2     0
    -4    30     0    -2    -6
    -6     0    18    -4     0
    -2    -2    -4    12    -4
     0    -6     0    -4    18

>> V = [-12,-16,30,0,-32]'

V =

   -12
   -16
    30
     0
   -32

>> I = inv(Z)*V

I =
```

-0.2779 A
-1.0488 A
1.4682 A
-0.4761 A
-2.2332 A

Solution 3.67

Consider the circuit below.



$$\begin{bmatrix} 0.35 & -0.25 & 0 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} -5+3V_o \\ 0 \\ 15 \end{bmatrix}$$

Since we actually have four unknowns and only three equations, we need a constraint equation.

$$V_o = V_2 - V_3$$

Substituting this back into the matrix equation, the first equation becomes,

$$0.35V_1 - 3.25V_2 + 3V_3 = -5$$

This now results in the following matrix equation,

$$\begin{bmatrix} 0.35 & -3.25 & 3 \\ -0.25 & 0.95 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix} V = \begin{bmatrix} -5 \\ 0 \\ 15 \end{bmatrix}$$

Now we can use MATLAB to solve for V.

```
>> Y=[0.35,-3.25,3;-0.25,0.95,-0.5;0,-0.5,0.5]

Y =

    0.3500   -3.2500    3.0000
   -0.2500    0.9500   -0.5000
         0   -0.5000    0.5000

>> I=[–5,0,15]'

I =

    –5
     0
     15

>> V=inv(Y)*I

V =

–410.5262
–194.7368
–164.7368
```

$V_o = V_2 – V_3 = –77.89 + 65.89 =$ **–30 V**.

Let us now do a quick check at node 1.

–3(–30) + 0.1(–410.5) + 0.25(–410.5+194.74) + 5 = 90–41.05–102.62+48.68+5 = 0.01; essentially zero considering the accuracy we are using. The answer checks.

Solution 3.68

Although there are many ways to work this problem, this is an example based on the same kind of problem asked in the third edition.

Problem

Find the voltage V_o in the circuit of Fig. 3.112.

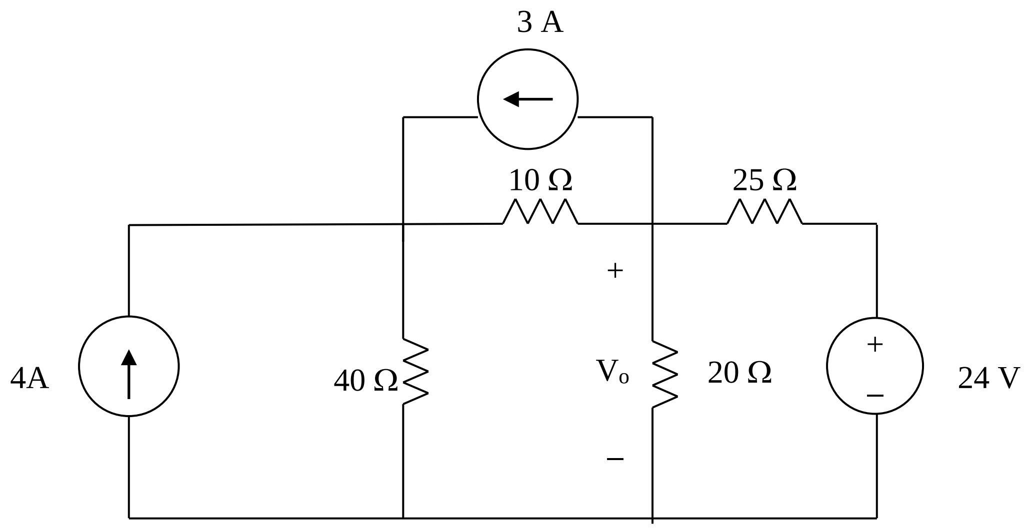
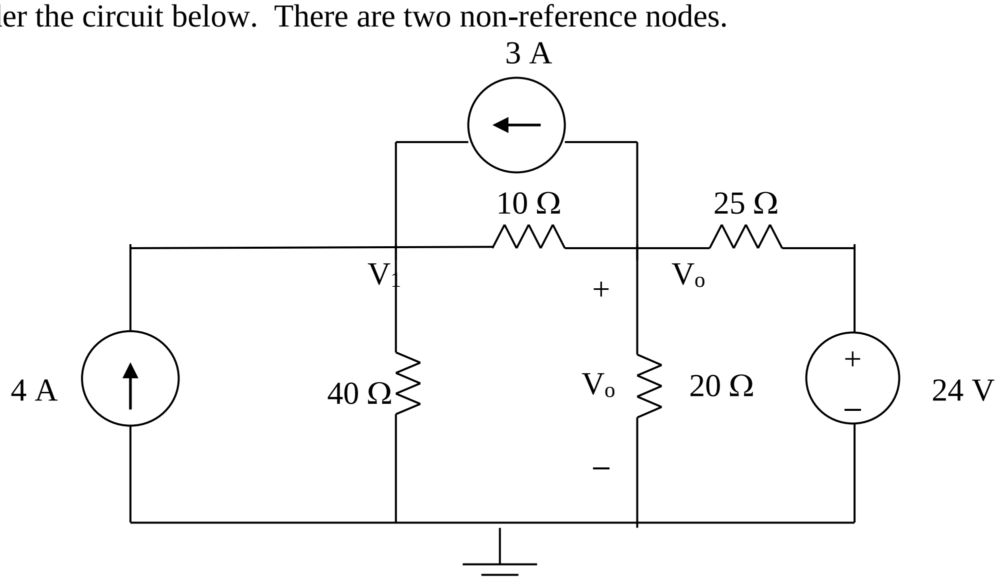


Figure 3.112
For Prob. 3.68.

Solution

Consider the circuit below. There are two non-reference nodes.



$$\begin{bmatrix} 0.125 & -0.1 \\ -0.1 & 0.19 \end{bmatrix} V = \begin{bmatrix} +4+3 \\ -3+24/25 \end{bmatrix} = \begin{bmatrix} 7 \\ -2.04 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Y=[0.125,-0.1;-0.1,0.19]

Y =

   0.1250   -0.1000
  -0.1000    0.1900

>> I=[7,-2.04]'

I =

   7.0000
  -2.0400

>> V=inv(Y)*I

V =

  81.8909
  32.3636
```

Thus, V_o = **32.36 V**.

We can perform a simple check at node V_o,

3 + 0.1(32.36–81.89) + 0.05(32.36) + 0.04(32.36–24) =
3 – 4.953 + 1.618 + 0.3344 = – 0.0004; answer checks!

Solution 3.69

For the circuit in Fig. 3.113, write the node voltage equations by inspection.

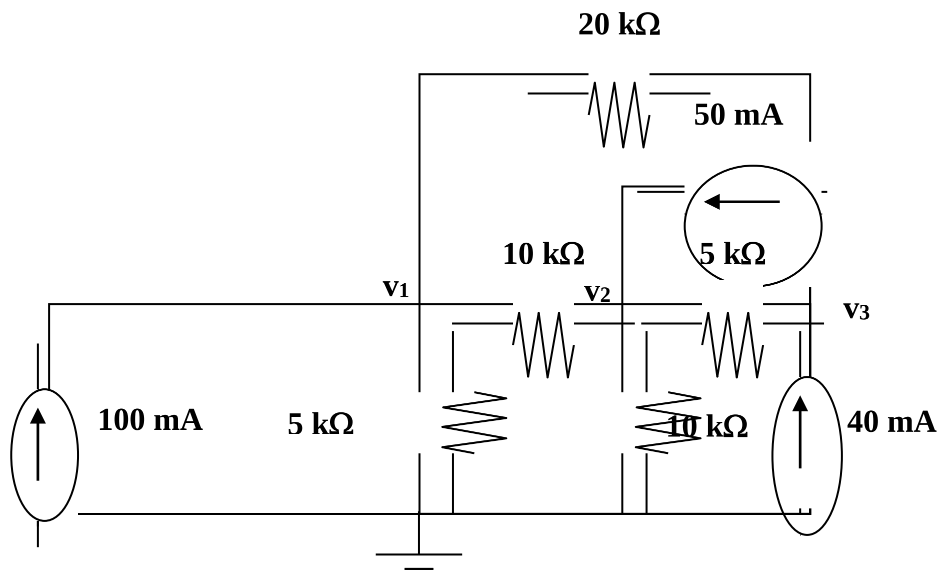


Figure 3.113
For Prob. 3.69.

Step 1. Assume that all conductance's are in mS, all currents are in mA, and all voltages are in volts.

$G_{11} = (1/5) + (1/10) + (1/20) = 0.35$, $G_{22} = (1/10) + (1/10) + (1/5) = 0.4$,
$G_{33} = (1/5) + (1/20) = 0.25$, $G_{12} = –1/10 = –0.1$, $G_{13} = –0.05$,
$G_{21} = –0.1$, $G_{23} = –0.2$, $G_{31} = –0.05$, $G_{32} = –0.2$

$i_1 = 100$, $i_2 = 50$, and $i_3 = 30–50 = –10$.

Step 2. The node-voltage equations are:

$$\begin{bmatrix} 0.35 & -0.1 & -0.05 \\ -0.1 & 0.4 & -0.2 \\ -0.05 & -0.2 & 0.25 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \\ -10 \end{bmatrix}$$

Solution 3.70

$$\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} V = \begin{bmatrix} 4I_x + 20 \\ -4I_x - 7 \end{bmatrix}$$

With two equations and three unknowns, we need a constraint equation,

$I_x = 2V_1$, thus the matrix equation becomes,

$$\begin{bmatrix} -5 & 0 \\ 8 & 5 \end{bmatrix} V = \begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

This results in $V_1 = 20/(–5) =$ **–4 V** and
$V_2 = [–8(–4) – 7]/5 = [32 – 7]/5 =$ **5 V**.

Solution 3.71

$$\begin{bmatrix} 9 & -4 & -5 \\ -4 & 7 & -1 \\ -5 & -1 & 9 \end{bmatrix} I = \begin{bmatrix} 30 \\ -15 \\ 0 \end{bmatrix}$$

We can now use MATLAB solve for our currents.

```
>> R=[9,–4,–5;–4,7,–1;–5,–1,9]

R =

   9   –4   –5
  –4    7   –1
  –5   –1    9

>> V=[30,–15,0]'

V =

  30
  –15
   0

>> I=inv(R)*V

I =
```

6.255 A
1.9599 A
3.694 A

Solution 3.72

By inspection, write the mesh-current equations for the circuit in Fig. 3.116.

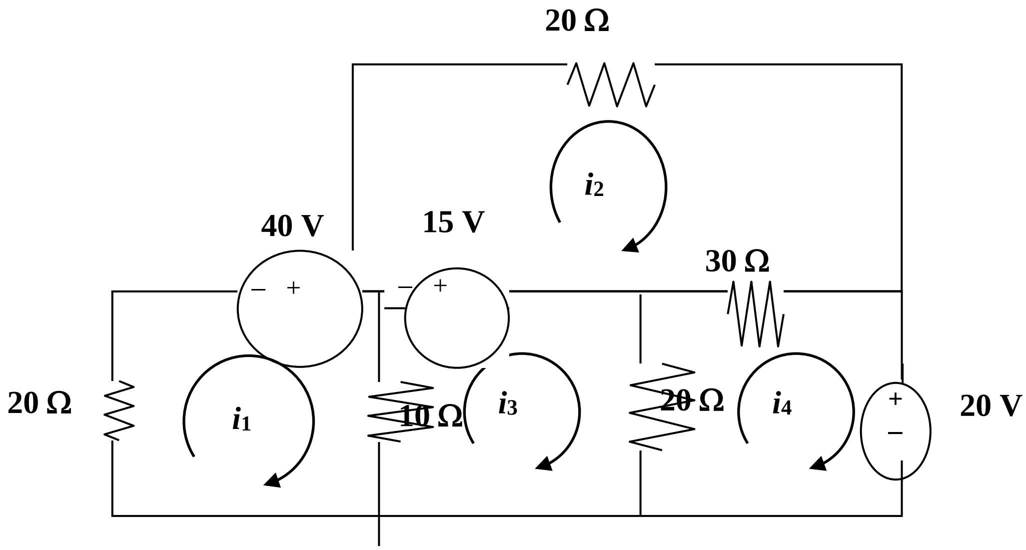


Figure 3.116
For Prob. 3.72.

Step 1. First we write the resistance equations by inspection.

R_{11} = 20 + 10 = 30, R_{22} = 20 + 30 = 50, R_{33} = 10 + 20 = 30, R_{44} = 20 + 30 = 50, R_{12} = 0, R_{13} = –10, R_{14} = 0, R_{21} = 0, R_{23} = 0, R_{24} = –30, R_{31} = –10, R_{32} = 0, R_{34} = –20, R_{41} = 0, R_{42} = –30, R_{43} = –20, we note that $R_{ij} = R_{ji}$ for all i not equal to j. Finally v_1 = 40; v_2 = –15; v_3 = 15; and v_4 = –20.

Step 2. Hence the mesh-current equations are:

$$\begin{bmatrix} 30 & 0 & -10 & 0 \\ 0 & 20 & 0 & -30 \\ -10 & 0 & 30 & -20 \\ 0 & -30 & -20 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 40 \\ -15 \\ 15 \\ -20 \end{bmatrix}$$

Solution 3.73

Write the mesh-current equations for the circuit in Fig. 3.117.

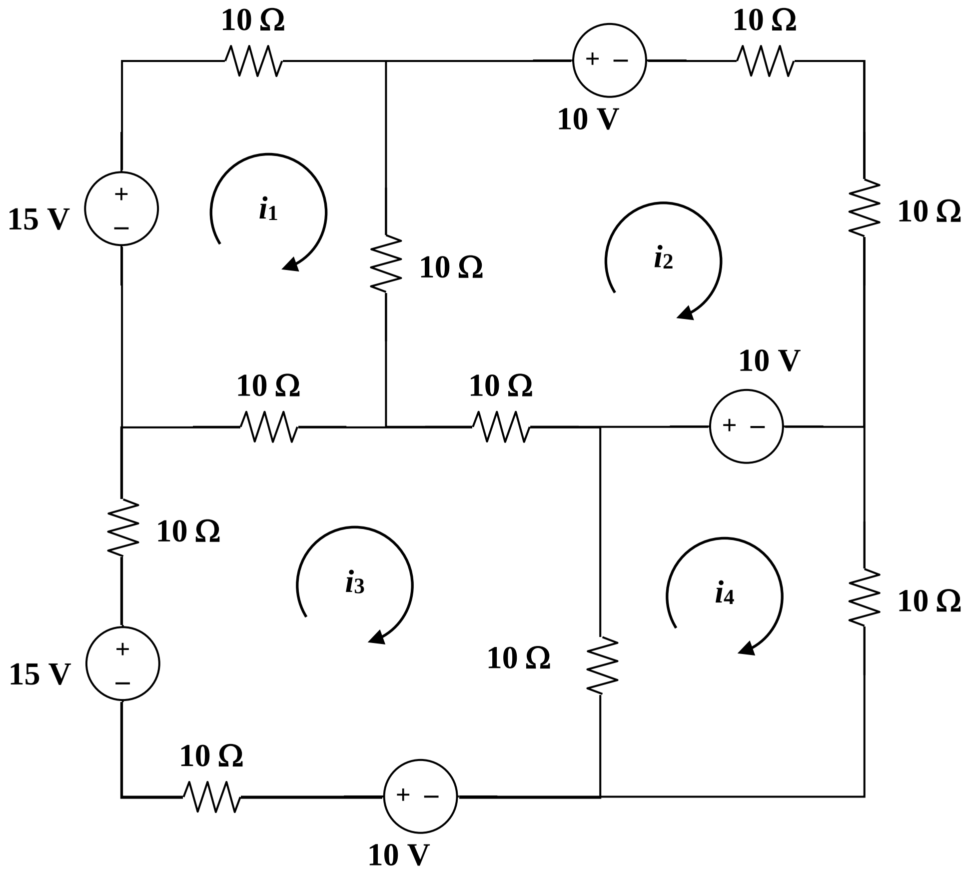


Figure 3.117
For Prob. 3.73.

Solution

Loop 1. $-15 + 10i_1 + 10(i_1-i_2) + 10(i_1-i_3) = 0$ or $30i_1 - 10i_2 - 10i_3 = 15$

Loop 2. $10(i_2-i_1) + 10 + 20i_2 - 10 + 10(i_2-i_3) = 0$ or $-10i_1 + 40i_2 - 10i_3 = 0$

Loop 3. $-10 + 20i_3 - 15 + 10(i_3-i_1) + 10(i_3-i_2) + 10(i_3-i_4) = 0$ or
$-10i_1 - 10i_2 + 50i_3 - 10i_4 = 25$

Loop 4. $10(i_4-i_3) + 10 + 10i_4 = 0$ or $-10i_3 + 20i_4 = -10$

Thus, $$\begin{bmatrix} 30 & -10 & -10 & 0 \\ -10 & 40 & -10 & 0 \\ -10 & -10 & 50 & -10 \\ 0 & 0 & -10 & 20 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 25 \\ -10 \end{bmatrix}.$$

Solution 3.74

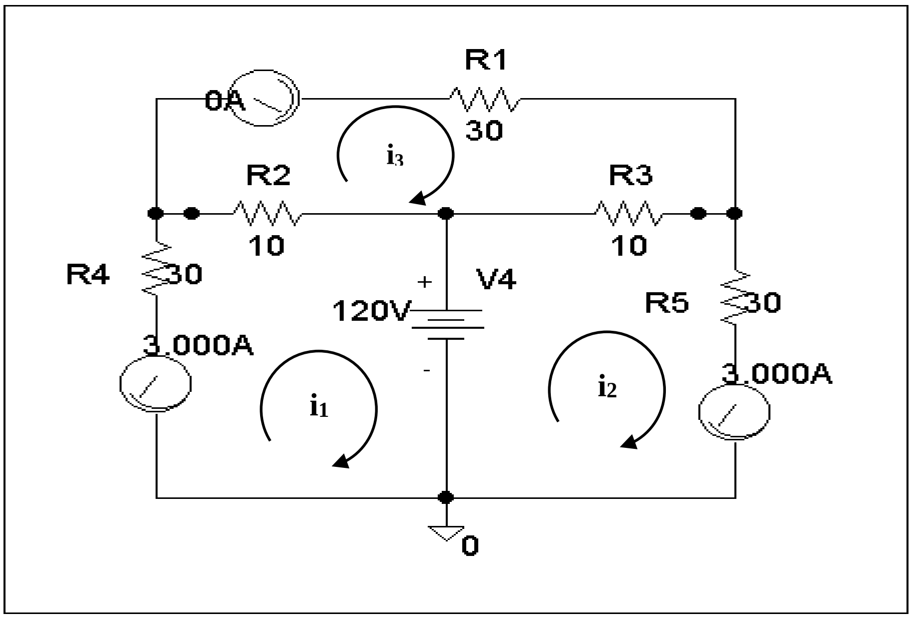
$R_{11} = R_1 + R_4 + R_6$, $R_{22} = R_2 + R_4 + R_5$, $R_{33} = R_6 + R_7 + R_8$,
$R_{44} = R_3 + R_5 + R_8$, $R_{12} = -R_4$, $R_{13} = -R_6$, $R_{14} = 0$, $R_{23} = 0$,
$R_{24} = -R_5$, $R_{34} = -R_8$, again, we note that $R_{ij} = R_{ji}$ for all i not equal to j.

$$\text{The input voltage vector is} = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$$

$$\begin{bmatrix} R_1 + R_4 + R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_2 + R_4 + R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6 + R_7 + R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3 + R_5 + R_8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ -V_4 \end{bmatrix}$$

Solution 3.75

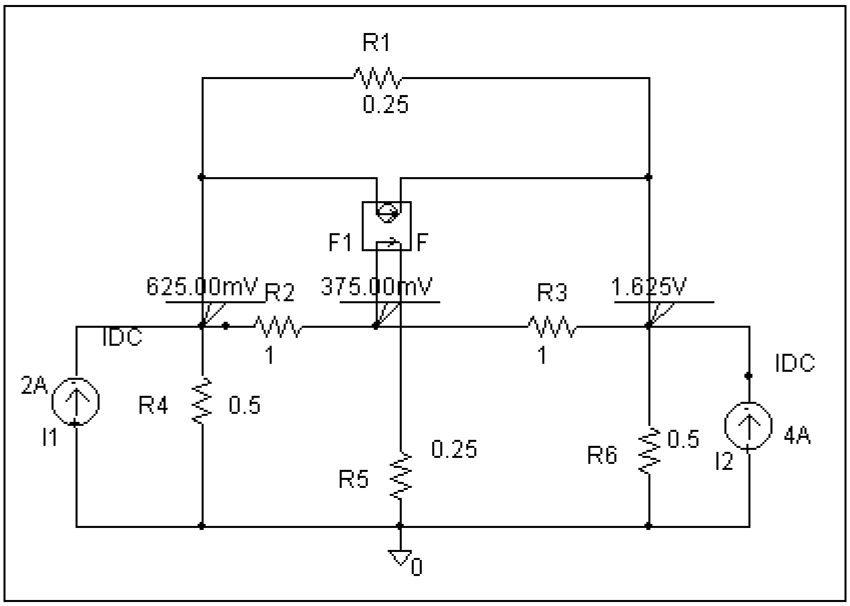
```
* Schematics Netlist *

R_R4          $N_0002 $N_0001  30
R_R2          $N_0001 $N_0003  10
R_R1          $N_0005 $N_0004  30
R_R3          $N_0003 $N_0004  10
R_R5          $N_0006 $N_0004  30
V_V4          $N_0003 0 120V
v_V3          $N_0005 $N_0001 0
v_V2          0 $N_0006 0
v_V1          0 $N_0002 0
```



Clearly, i_1 = **–3 amps**, i_2 = **0 amps**, and i_3 = **3 amps**, which agrees with the answers in Problem 3.44.

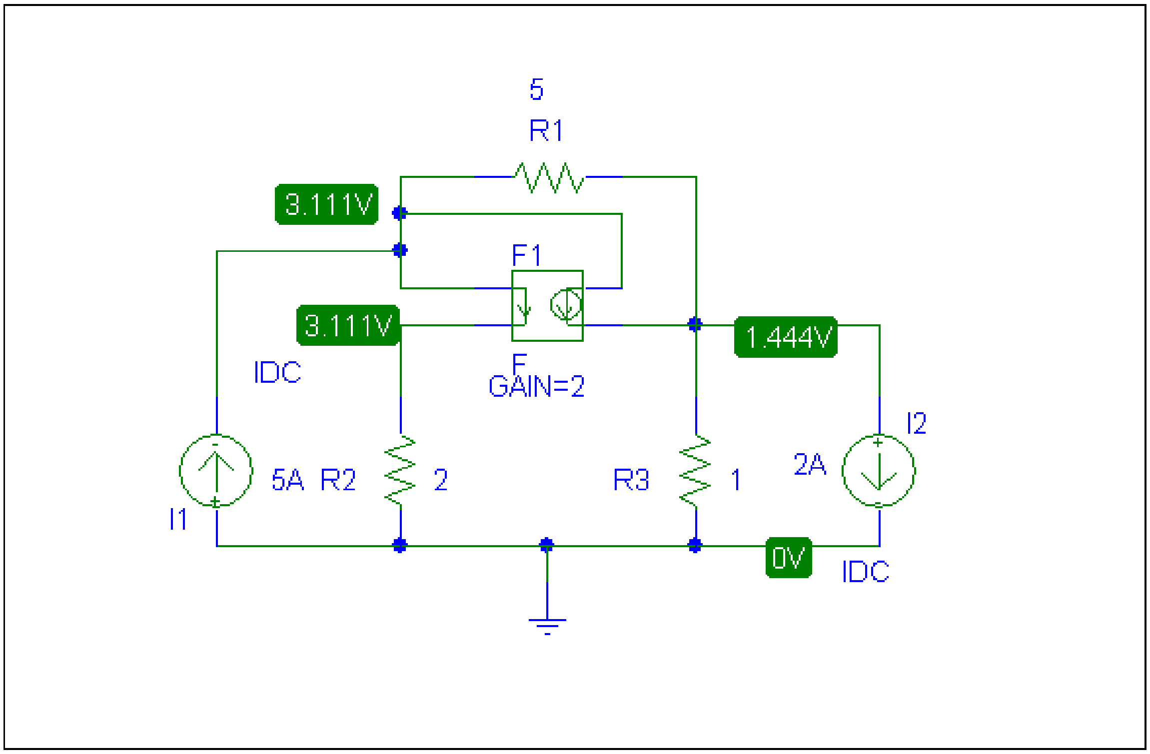
Solution 3.76

```
* Schematics Netlist *

I_I2          0 $N_0001 DC 4A
R_R1          $N_0002 $N_0001  0.25
R_R3          $N_0003 $N_0001  1
R_R2          $N_0002 $N_0003  1
F_F1          $N_0002 $N_0001 VF_F1 3
VF_F1         $N_0003 $N_0004 0V
R_R4          0 $N_0002  0.5
R_R6          0 $N_0001  0.5
I_I1          0 $N_0002 DC 2A
R_R5          0 $N_0004  0.25
```



Clearly, v_1 = **625 mVolts**, v_2 = **375 mVolts**, and v_3 = **1.625 volts**, which agrees with the solution obtained in Problem 3.27.

Solution 3.77



As a check we can write the nodal equations,

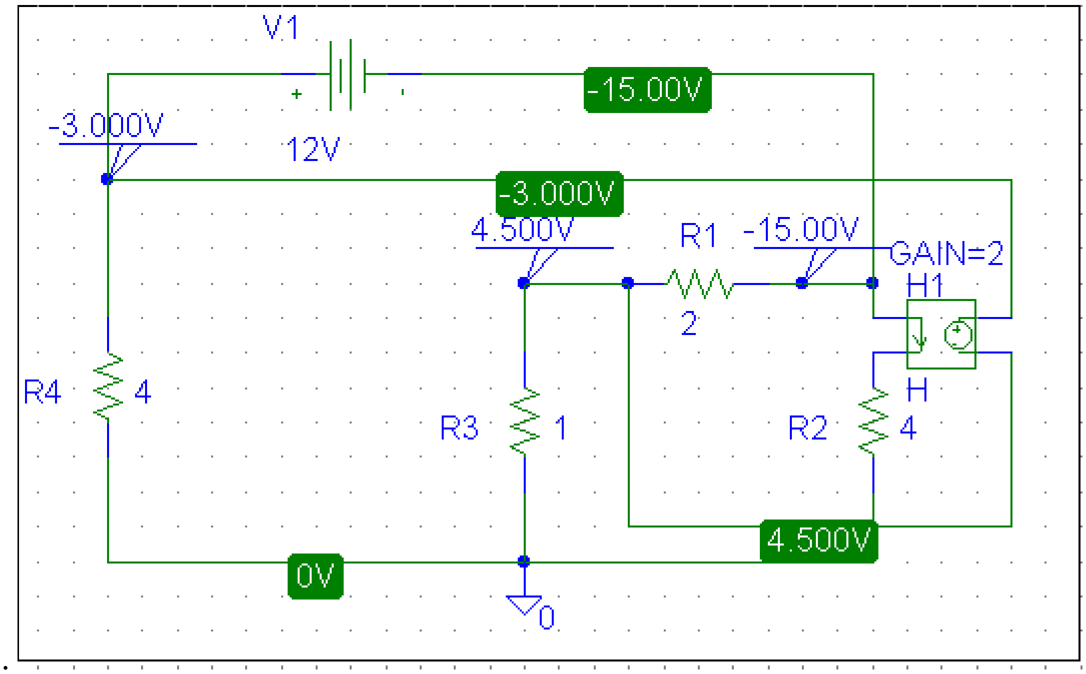
$$\begin{bmatrix} 1.7 & -0.2 \\ -1.2 & 1.2 \end{bmatrix} V = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Solving this leads to V_1 = **3.111 V** and V_2 = **1.4444 V**. The answer checks!

Solution 3.78

The schematic is shown below. When the circuit is saved and simulated the node voltages are displayed on the pseudo components as shown. Thus,

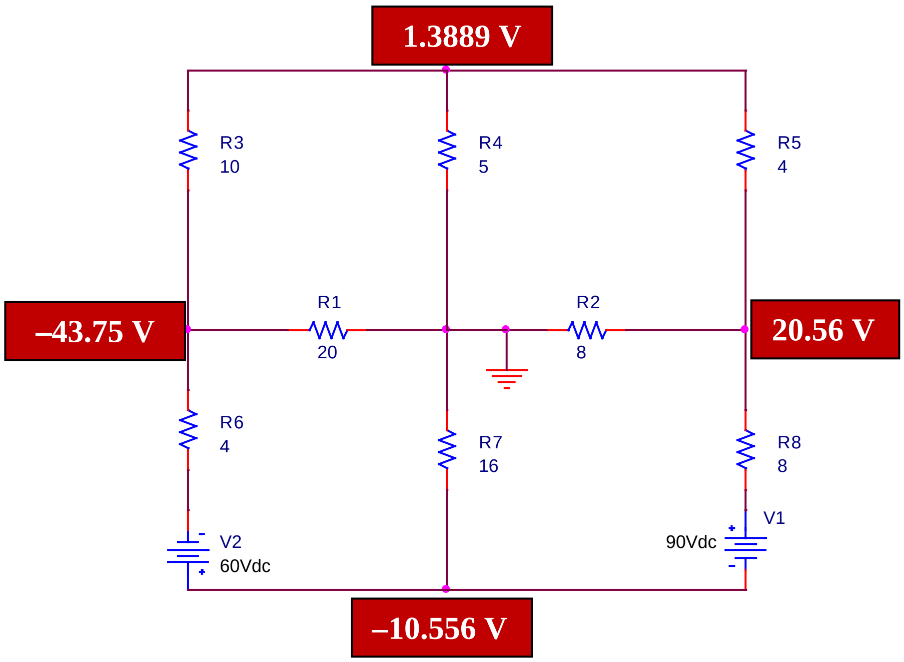
$\underline{V_1 = -3\text{V}, \quad V_2 = 4.5\text{V}, V_3 = -15\text{V},}$



Solution 3.79

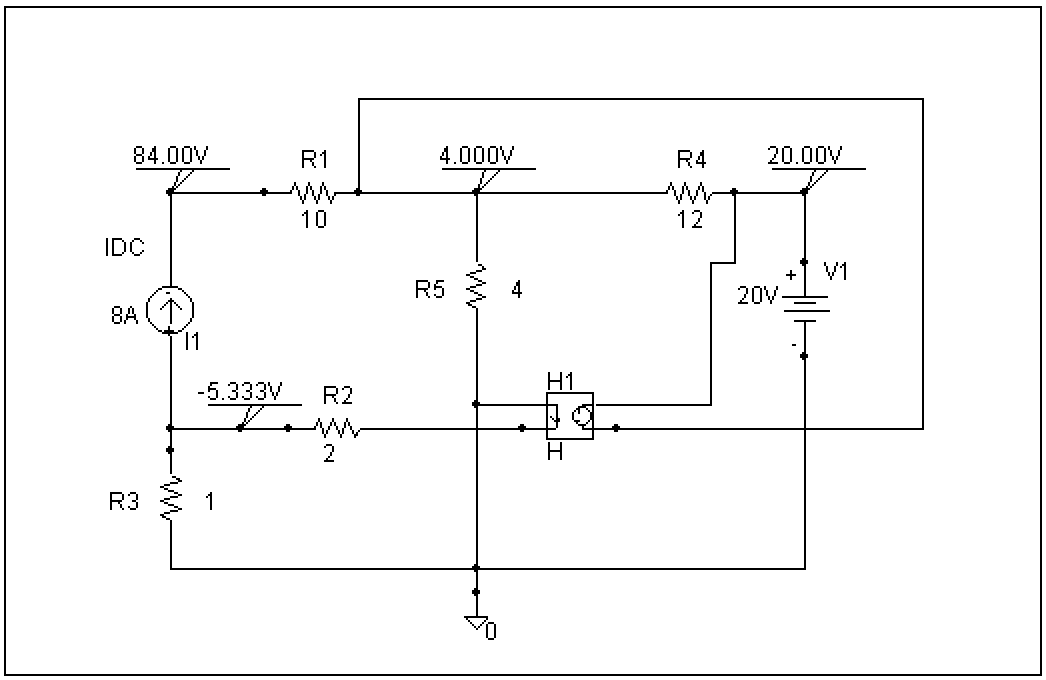
The schematic is shown below. When the circuit is saved and simulated, we obtain the node voltages as displayed. Thus,

V_a = **–10.556 volts**; V_b = **20.56 volts**; V_c = **1.3889 volts**; and V_d = **–43.75 volts**.



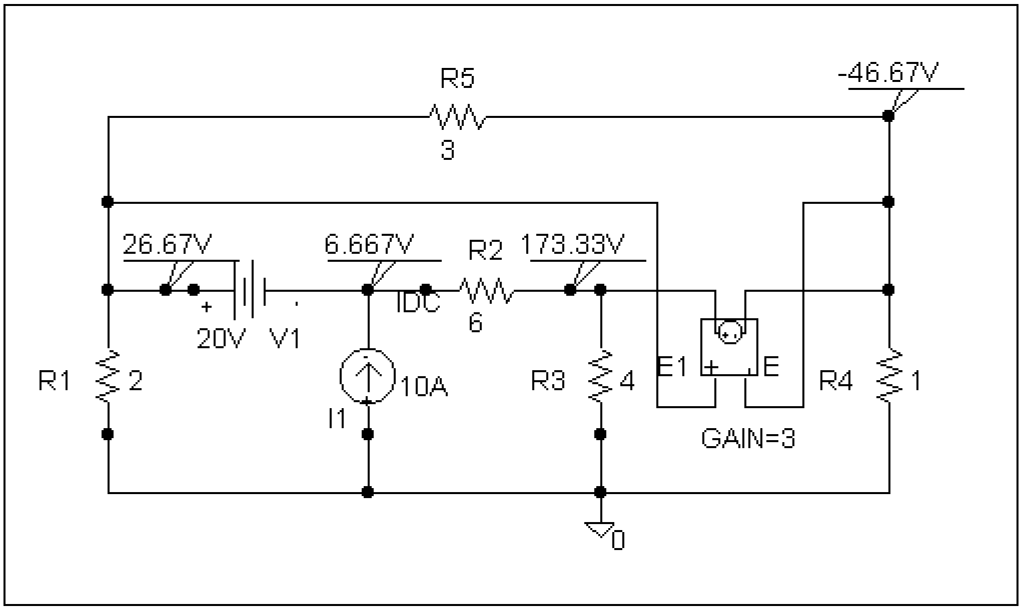
Solution 3.80

```
* Schematics Netlist *

H_H1          $N_0002 $N_0003 VH_H1 6
VH_H1         0 $N_0001 0V
I_I1          $N_0004 $N_0005 DC 8A
V_V1          $N_0002 0 20V
R_R4          0 $N_0003  4
R_R1          $N_0005 $N_0003  10
R_R2          $N_0003 $N_0002  12
R_R5          0 $N_0004  1
R_R3          $N_0004 $N_0001  2
```



Clearly, v_1 = **84 volts**, v_2 = **4 volts**, v_3 = **20 volts**, and v_4 = **-5.333 volts**

Solution 3.81

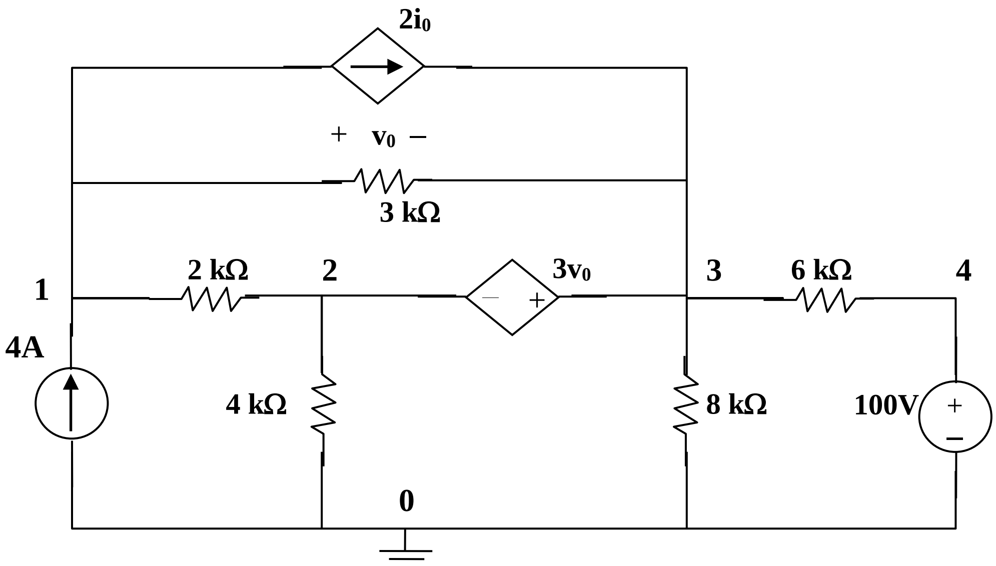


Clearly, v_1 = **26.67 volts**, v_2 = **6.667 volts**, v_3 = **173.33 volts**, and v_4 = **–46.67 volts** which agrees with the results of Example 3.4.

This is the netlist for this circuit.

```
* Schematics Netlist *

R_R1         0 $N_0001  2
R_R2         $N_0003 $N_0002  6
R_R3         0 $N_0002  4
R_R4         0 $N_0004  1
R_R5         $N_0001 $N_0004  3
I_I1         0 $N_0003 DC 10A
V_V1         $N_0001 $N_0003 20V
E_E1         $N_0002 $N_0004 $N_0001 $N_0004 3
```

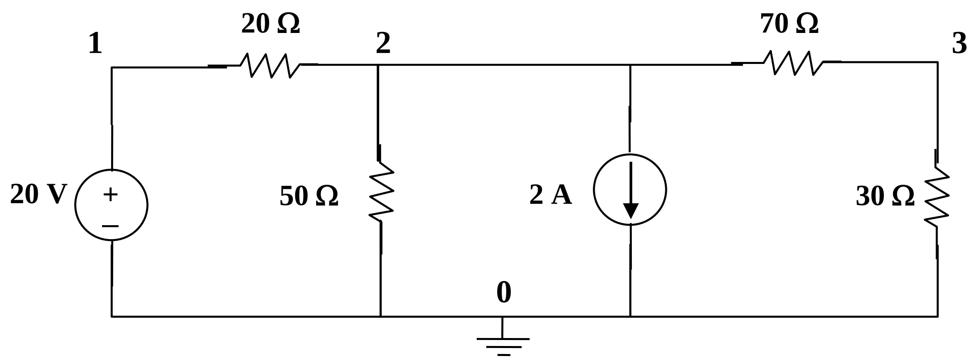
Solution 3.82



This network corresponds to the Netlist.

Solution 3.83

The circuit is shown below.



When the circuit is saved and simulated, we obtain v_2 = **–12.5 volts**

Solution 3.84

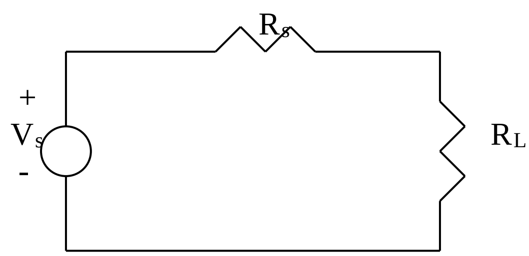
From the output loop, $v_0 = 50i_0x20x10^3 = 10^6i_0$ (1)

From the input loop, $15x10^{-3} + 4000i_0 – v_0/100 = 0$ (2)

From (1) and (2) we get, i_0 = **2.5 μA** and v_0 = **2.5 volt**.

Solution 3.85

The amplifier acts as a source.



For maximum power transfer,

$$R_L = R_s = \underline{\underline{9\Omega}}$$

Solution 3.86

Let v_1 be the potential across the 2 k-ohm resistor with plus being on top. Then,

Since $i = [(0.047–v_1)/1k]$
$[(v_1–0.047)/1k] – 400[(0.047–v_1)/1k] + [(v_1–0)/2k] = 0$ or

$401[(v_1–0.047)] + 0.5v_1 = 0$ or $401.5v_1 = 401x0.047$ or
$v_1 = 0.04694$ volts and $i = (0.047–0.04694)/1k = 60$ nA

Thus,

$v_0 = –5000x400x60x10^{-9} =$ **–120 mV**.

Solution 3.87

For the circuit in Fig. 3.123, find the gain $\boldsymbol{v_o/v_s}$.

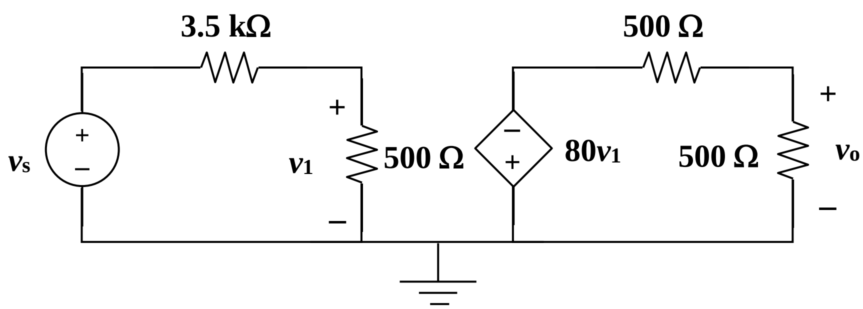


Figure 3.123
For Prob. 3.87.

Step 1. We can solve this using mesh analysis with two unknown mesh currents.

For the loop on the left we get, $-v_s + 3{,}500i_1 + 500i_1 = 0$ and $v_1 = 500i_1$.
For the loop on the right we get, $80v_1 + 500i_2 + 500i_2$ and $v_o = 500i_2$.

Step 2. $i_1 = v_s/4{,}000$ and $v_1 = 500v_s/4{,}000 = v_s/8$. $i_2 = -80(v_s/8)/1{,}000$ and $v_o = 500(-10v_s)/1{,}000 = -5v_s$. Therefore,

$$v_o/v_s = \mathbf{-5}.$$

Solution 3.88

Determine the gain v_o/v_s of the transistor amplifier circuit in Fig. 3.124.

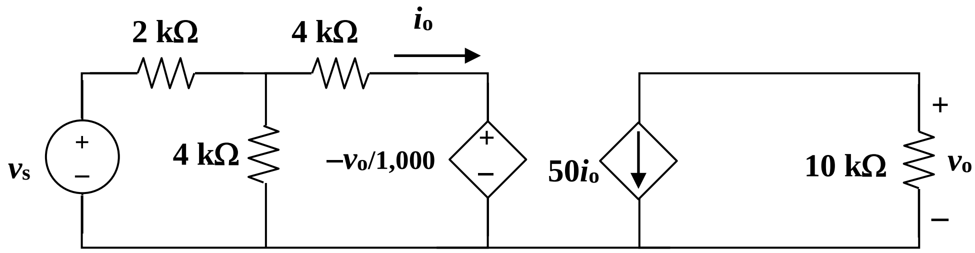


Figure 3.124
For Prob. 3.88.

Solution

Step 1. The loop on the right gives us $v_o = -10k(50i_o)$. We have two loops in the left hand circuit which produces $-v_s + 2ki_1 + 4k(i_1 - i_o) = 0$ and $4k(i_o - i_1) + 4ki_o - (v_o/1000) = 0$.

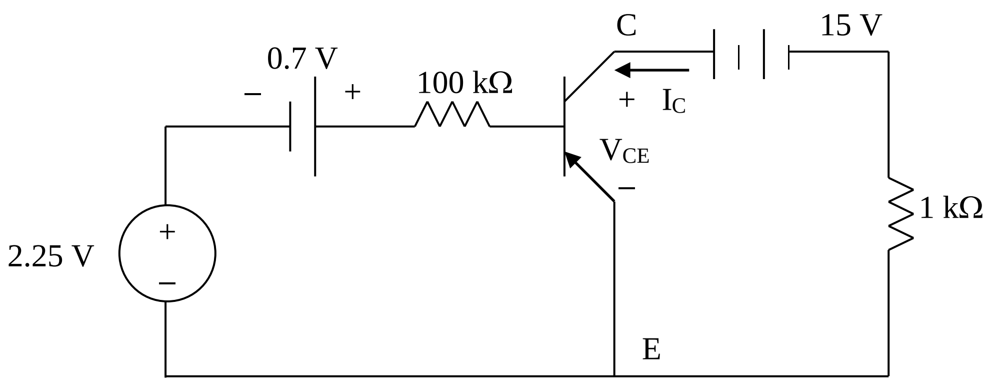
Step 2. $8ki_o - 4ki_1 - (-500ki_o/1000) = 0$ or $8.5ki_o = 4ki_1$ or $i_1 = 2.125i_o$ and $(2k+4k)i_1 - 4ki_o = v_s = 2.125(6k)i_o - 4ki_o = 8.75ki_o$ or $i_o = v_s/8.75k$.

Now we have $v_o = -500kv_s/8.75k = -57.14v_s$ or

$$v_o/v_s = \mathbf{-57.14}.$$

Solution 3.89

Consider the circuit below.



For the left loop, applying KVL gives

$$-2.25 - 0.7 + 10^5 I_B + V_{BE} = 0 \text{ but } V_{BE} = 0.7 \text{ V means } 10^5 I_B = 2.25 \text{ or}$$

$$I_B = \mathbf{22.5\ \mu A}.$$

For the right loop, $-V_{CE} + 15 - I_C x10^3 = 0$. Addition ally, $I_C = \beta I_B = 100x22.5x10^{-6} =$ 2.25 mA. Therefore,

$$V_{CE} = 15\text{–}2.25x10^{-3}x10^3 = \mathbf{12.75\ V}.$$

Solution 3.90

Calculate $\boldsymbol{v_s}$ for the transistor in Fig. 3.126, given that $\boldsymbol{v_o}$ = 6 V, β = 90, V_{BE} = 0.7V.

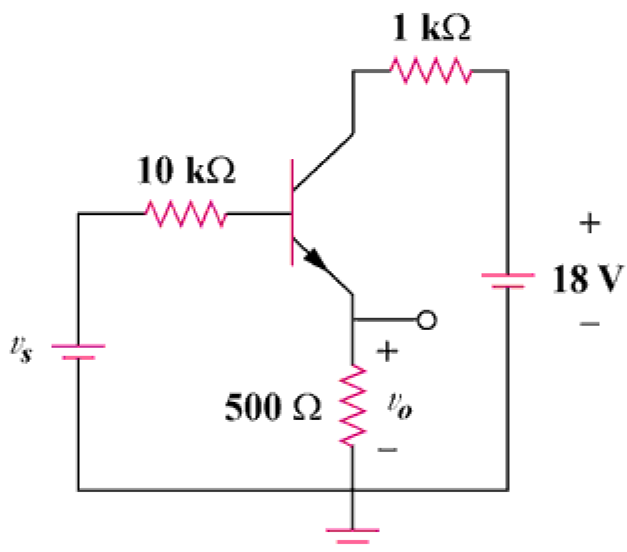
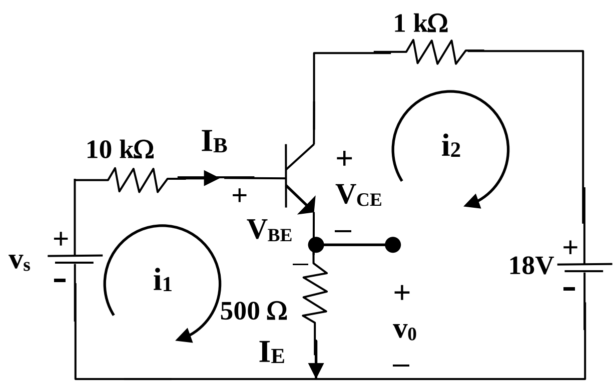


Figure 3.126
For Prob. 3.90.



For loop 1, $-v_s + 10k(I_B) + V_{BE} + I_E\,(500) = 0 = -v_s + 0.7 + 10{,}000I_B + 500(1 + \beta)I_B$

which leads to $v_s - 0.7 = 10{,}000I_B + 500(91)I_B = 55{,}500I_B$

But, $v_0 = 500I_E = 500\text{x}91I_B = 6$ which leads to $I_B = 1.318680\text{x}10^{-4}$

Therefore, $v_s = 0.7 + 55{,}500I_B =$ **8.019 volts**.

Solution 3.91

For the transistor circuit of Fig. 3.127, find $\boldsymbol{I_B}$, $\mathbf{V}_{\boldsymbol{CE}}$, and $\boldsymbol{v_o}$. Take $\beta = 150$, $V_{BE} = 0.7$V.

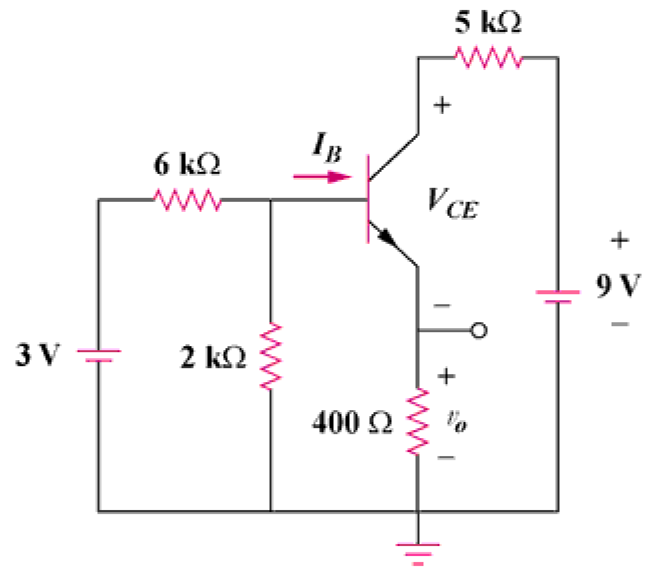
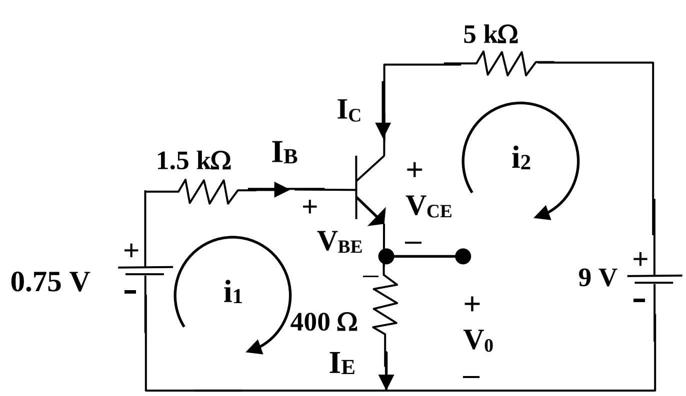


Figure 3.127
For Prob. 3.91.

Solution

We first determine the Thevenin equivalent for the input circuit.

$R_{Th} = 6||2 = 6x2/8 = 1.5$ kΩ and $V_{Th} = 2(3)/(2+6) = 0.75$ volts



For loop 1, $-0.75 + 1.5kI_B + V_{BE} + 400I_E = 0 = -0.75 + 0.7 + 1{,}500I_B + 400(1 + \beta)I_B$ or $(1{,}500 + 400\text{x}151)I_B = 61{,}900I_B = 0.05$ or

$$I_B = 0.05/61{,}900 = \mathbf{0.8078\ \mu A}.$$

$$v_0 = 400I_E = 400(1 + \beta)I_B = 400(151)0.8078 = \mathbf{48.49\ mV}$$

For loop 2, $-400I_E - V_{CE} - 5kI_C + 9 = 0$, but, $I_C = \beta I_B$ and $I_E = (1 + \beta)I_B$

$$V_{CE} = 9 - 5k\beta I_B - 400(1 + \beta)I_B = 9 - 0.60585 - 0.04879 = 9 - 0.6546 =$$

$$V_{CE} = \mathbf{8.345\ volts}.$$

Solution 3.92

Using Fig. 3.28, design a problem to help other students better understand transistors. Make sure you use reasonable numbers!

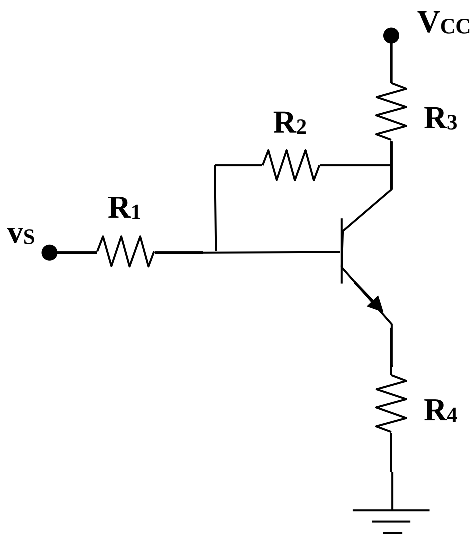


Figure 3.28
For Prob. 3.92.

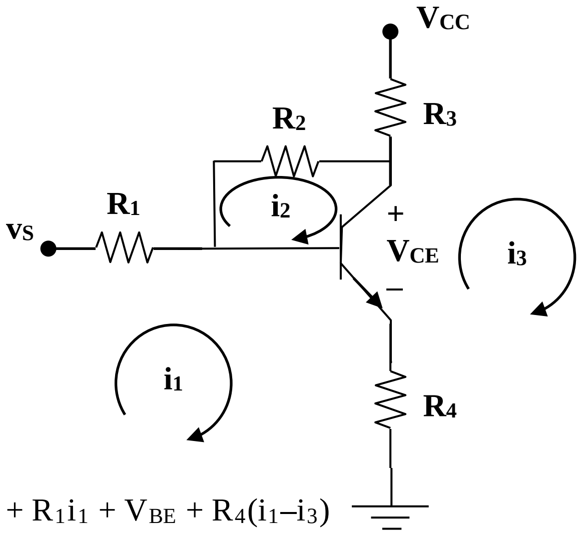
Although there are many ways to work this problem, this is just one example that could qualify as a solution.

Problem

Given the circuit shown in Fig. 3.28 and R_1 = 100 kΩ, R_2 = 1 kΩ, R_3 = 1 kΩ, R_4 = 100 Ω, β= 100, V_{CC} = 30 V, v_s = 20 V, and V_{BE} = 0.7. Determine V_{CE}.

Solution continued on the next page...

Solution



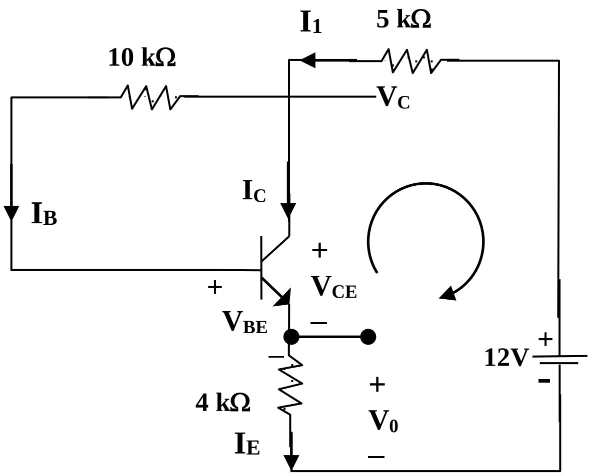
Loop 1, $-v_s + R_1i_1 + V_{BE} + R_4(i_1-i_3)$
Loop 2, $R_2i_2 + V_{CB} = 0$
Loop 3, $R_4(i_3-i_1) - V_{CE} + R_3i_3 + V_{CC} = 0$.

We also have some constraint equations, $I_B = i_1 - i_2$, $I_C = i_2 - i_3 = \beta I_B$, and $V_{CE} = V_{BE} + V_{CB}$.

$100.1ki_1 - 0.1ki_3 = 20-0.7 = 19.3$
$1ki_2 + V_{CB} = 0$
$-0.1ki_1 + 1.1ki_3 - V_{CE} = -30$
$i_2 - i_3 = 100(i_1-i_2)$ or $100i_1 - 101i_2 + i_3 = 0$ or $i_3 = -100i_1 + 101i_2$
$V_{CB} = V_{CE} - 0.7$

Substituting for values of i_3 and V_{CB} we get
$100.1ki_1 + 10ki_1 - 10.1ki_2 = 19.3$ or $110.1ki_1 - 10.1ki_2 = 19.2$ or $i_1 = 0.091735i_2 + 0.17439/k$
$1ki_2 + V_{CE} = 0.7$ or $i_2 = -[(V_{CE})/k] + 0.7/k$
$-0.1ki_1 - 110ki_1 + 111.1ki_2 - V_{CE} = -30$
$= -110.1ki_1 + 111.1ki_2 - V_{CE} = -10.1ki_2 - 19.2 + 111.1ki_2 - V_{CE} = 101ki_2 - 19.2 - V_{CE}$
$= -101V_{CE} + 70.7 - 19.2 - V_{CE} = -30$ or $70.7 + 30 - 19.2 = 102V_{CE}$ or
$V_{CE} = 81.5/102 =$ **799 mV**.

Solution continued on the next page...



$$I_1 = I_B + I_C = (1 + \beta)I_B \quad \text{and} \quad I_E = I_B + I_C = I_1$$

Applying KVL around the outer loop,

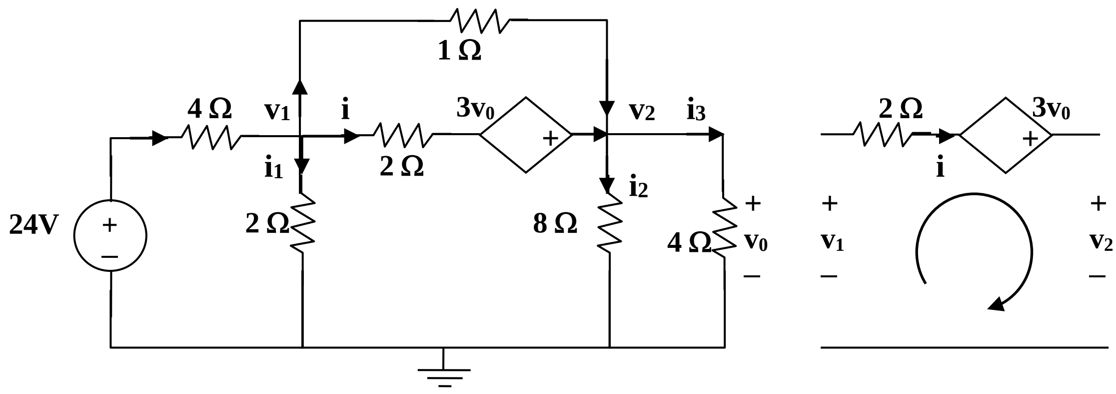
$$4kI_E + V_{BE} + 10kI_B + 5kI_1 = 12$$

$$12 - 0.7 = 5k(1 + \beta)I_B + 10kI_B + 4k(1 + \beta)I_B = 919kI_B$$

$$I_B = 11.3/919k = 12.296\ \mu A$$

Also, $12 = 5kI_1 + V_C$ which leads to $V_C = 12 - 5k(101)I_B =$ **5.791 volts**

Solution 3.93



(a) (b)

From (b), $-v_1 + 2i - 3v_0 + v_2 = 0$ which leads to $i = (v_1 + 3v_0 - v_2)/2$

At node 1 in (a), $((24 - v_1)/4) = (v_1/2) + ((v_1 + 3v_0 - v_2)/2) + ((v_1 - v_2)/1)$, where $v_0 = v_2$

or $24 = 9v_1$ which leads to $v_1 =$ **2.667 volts**

At node 2, $((v_1 - v_2)/1) + ((v_1 + 3v_0 - v_2)/2) = (v_2/8) + v_2/4$, $v_0 = v_2$

$v_2 = 4v_1 =$ **10.66 volts**

Now we can solve for the currents, $i_1 = v_1/2 =$ **1.333 A**, $i_2 =$ **1.333 A**, and

$i_3 =$ **2.6667** A.