

Matlab 求解微分方程作业

贾博方

222021321132005

Using the Runge-Kutta method

1. $dy/dt = 1 + t - y, \quad y(0) = 0; \quad (y(t) = t)$

```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 0;
```

```
h = 0.1;
```

```
[x,y] = RungeKutta4(x0, xn, y0, h);
```

```
plot(x,y,'r.')
```

```
hold on
```

```
function [x,y] = RungeKutta4(x0, xn, y0, h)
```

```
n = (xn-x0)/h;
```

```
x = zeros(n+1,1);
```

```
y = zeros(n+1,1);
```

```
x(1) = x0;
```

```
y(1) = y0;
```

```
for i = 1:n
```

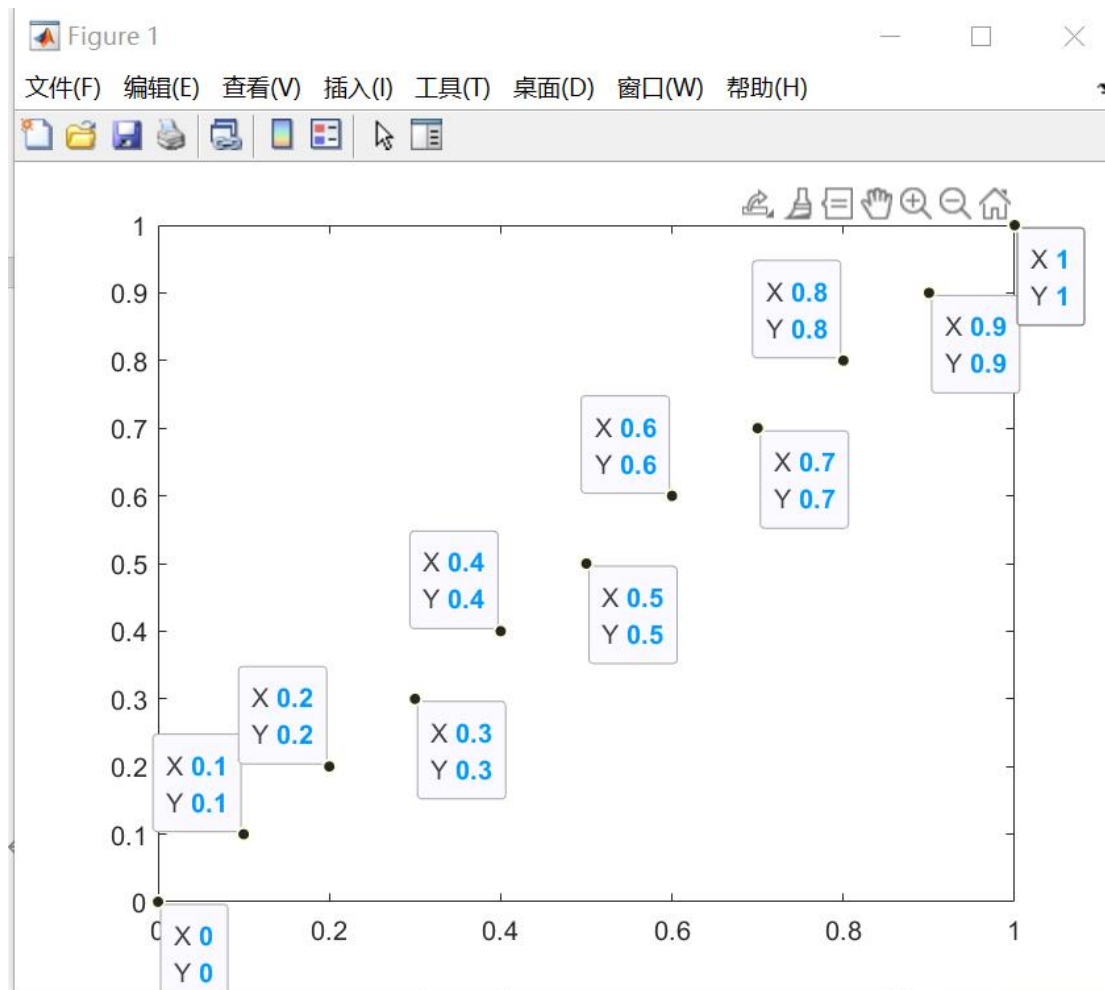
```
    x(i+1) = x(i)+h;
```

```

K1 = fun(x(i), y(i));
K2 = fun(x(i)+h/2, y(i)+K1*h/2);
K3 = fun(x(i)+h/2, y(i)+K2*h/2);
K4 = fun(x(i)+h, y(i)+K3*h);
y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
end
end

function f = fun(x,y)
f = 1+x-y;
end

```



```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 0;
```

```
m0 = 0;
```

```
mn = 1;
```

```
q0 = 0;
```

```
h = 0.1;
```

```
p = 0.025;
```

```
[x,y] = RungeKutta4(x0, xn, y0, h);
```

```
plot(x,y, 'r')
```

```
hold on;
```

```
[m,q] = RungeKutta(m0, mn, q0, p);
```

```
plot(m,q, 'b')
```

```
hold on;
```

```
function [x,y] = RungeKutta4(x0, xn, y0, h)
```

```
n = (xn-x0)/h;
```

```
x = zeros(n+1,1);
```

```
y = zeros(n+1,1);
```

```
x(1) = x0;
```

```
y(1) = y0;
```

```
for i = 1:n
```

```
    x(i+1) = x(i)+h;
```

```
    K1 = gun(x(i), y(i));
```

```
    K2 = gun(x(i)+h/2, y(i)+K1*h/2);
```

```
    K3 = gun(x(i)+h/2, y(i)+K2*h/2);
```

```
    K4 = gun(x(i)+h, y(i)+K3*h);
```

```
    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
```

```
end
```

end

```
function [m,q] = RungeKutta(m0, mn, q0, p)
```

```
k = (mn-m0)/p;
```

```
m = zeros(k+1,1);
```

```
q = zeros(k+1,1);
```

```
m(1) = m0;
```

```
q(1) = q0;
```

```
for j = 1:k
```

```
    m(j+1) = m(j)+p;
```

```
    K1 = gun(m(j), q(j));
```

```
    K2 = gun(m(j)+p/2, q(j)+K1*p/2);
```

```
    K3 = gun(m(j)+p/2, q(j)+K2*p/2);
```

```
    K4 = gun(m(j)+p, q(j)+K3*p);
```

```
    q(j+1) = q(j)+p/6*(K1+2*K2+2*K3+K4);
```

```
end
```

```
end
```

```
function f = fun(x,y)
```

```
f = 1+x-y;
```

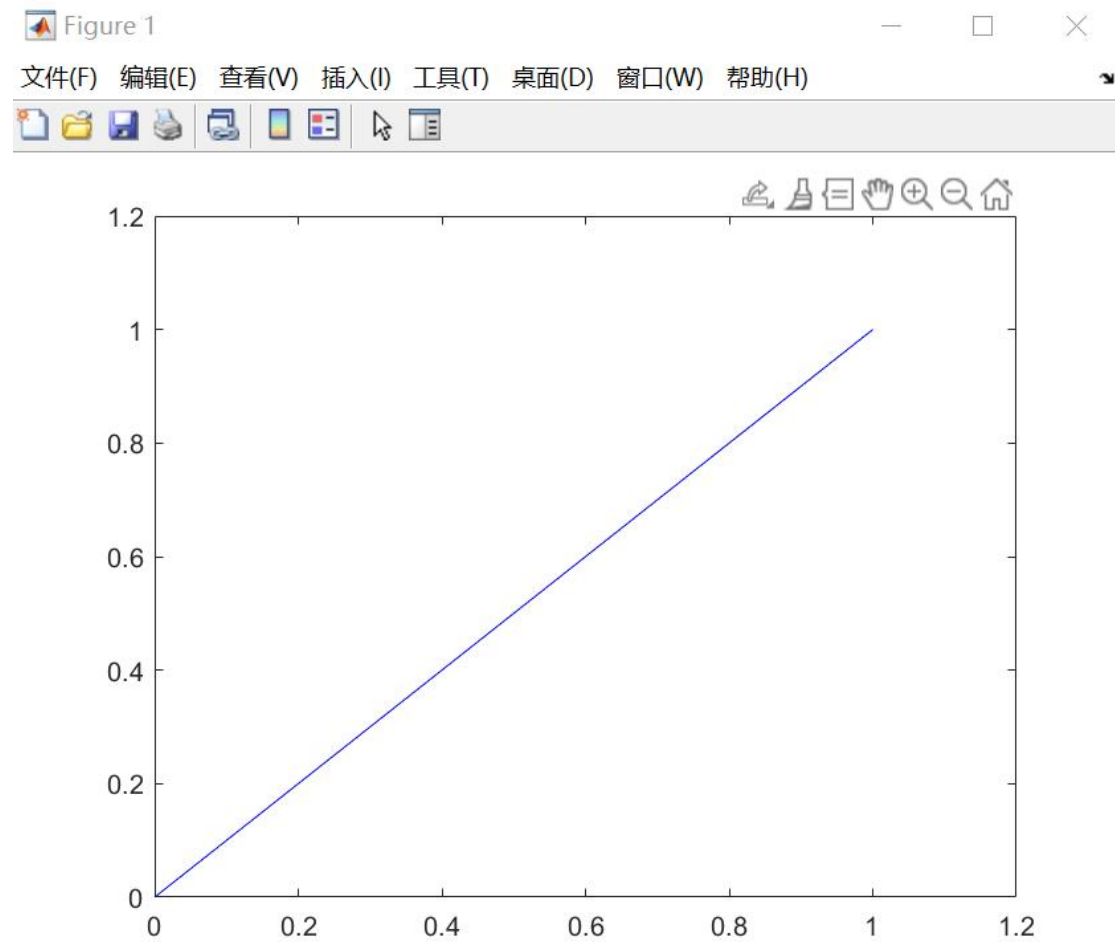
```
end
```

```
function g = gun(m,q)
```

```
g = 1+m-q;
```

end

曲线重合



2. $dy/dt=2ty, \quad y(0)=2; \quad (y(t)=2e^{t^2})$

```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 2;
```

```
h = 0.1;
```

```
[x,y] = RungeKutta4(x0, xn, y0, h);
```

```
plot(x,y,'r.')
```

```
hold on
```

```
function [x,y] = RungeKutta4(x0, xn, y0, h)
```

```
n = (xn-x0)/h;
```

```
x = zeros(n+1,1);
```

```
y = zeros(n+1,1);
```

```
x(1) = x0;
```

```
y(1) = y0;
```

```
for i = 1:n
```

```
    x(i+1) = x(i)+h;
```

```
    K1 = fun(x(i), y(i));
```

```
    K2 = fun(x(i)+h/2, y(i)+K1*h/2);
```

```
    K3 = fun(x(i)+h/2, y(i)+K2*h/2);
```

```
    K4 = fun(x(i)+h, y(i)+K3*h);
```

```
    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
```

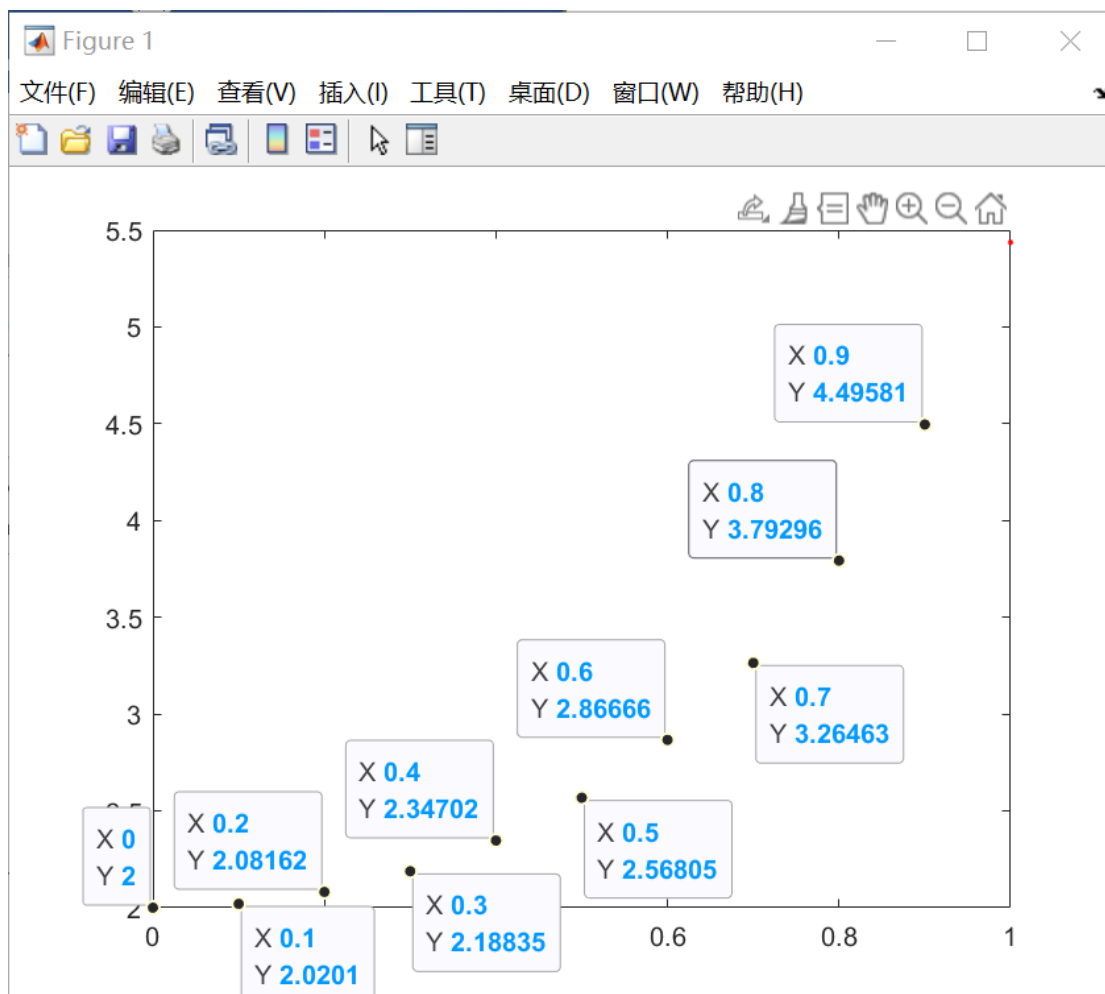
```
end
```

```
end
```

```
function f = fun(x,y)
```

```
f = 2*x*y;
```

```
end
```



```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 2;
```

```
m0 = 0;
```

```
mn = 1;
```

```
q0 = 2;
```

```
h = 0.1;
```



```

p = 0.025;

[x,y] = RungeKutta4(x0, xn, y0, h);

plot(x,y, 'r')

hold on;

[m,q] = RungeKutta(m0, mn, q0, p);

plot(m,q, 'b')

hold on;

function [x,y] = RungeKutta4(x0, xn, y0, h)

n = (xn-x0)/h;

x = zeros(n+1,1);

y = zeros(n+1,1);

x(1) = x0;

y(1) = y0;

for i = 1:n

    x(i+1) = x(i)+h;

    K1 = gun(x(i), y(i));

    K2 = gun(x(i)+h/2, y(i)+K1*h/2);

    K3 = gun(x(i)+h/2, y(i)+K2*h/2);

    K4 = gun(x(i)+h, y(i)+K3*h);

    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);

```

end

end

```
function [m,q] = RungeKutta(m0, mn, q0, p)
```

```
k = (mn-m0)/p;
```

```
m = zeros(k+1,1);
```

```
q = zeros(k+1,1);
```

```
m(1) = m0;
```

```
q(1) = q0;
```

```
for j = 1:k
```

```
    m(j+1) = m(j)+p;
```

```
    K1 = gun(m(j), q(j));
```

```
    K2 = gun(m(j)+p/2, q(j)+K1*p/2);
```

```
    K3 = gun(m(j)+p/2, q(j)+K2*p/2);
```

```
    K4 = gun(m(j)+p, q(j)+K3*p);
```

```
    q(j+1) = q(j)+p/6*(K1+2*K2+2*K3+K4);
```

```
end
```

```
end
```

```
function f = fun(x,y)
```

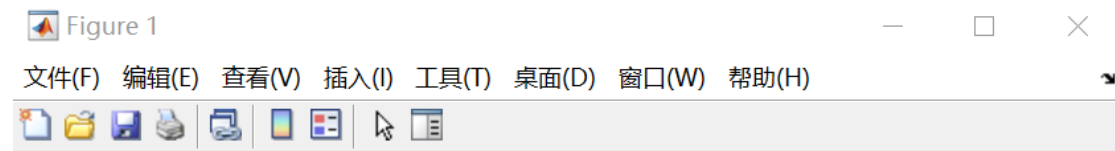
```
f = 2*x*y;
```

```
end
```

```
function g = gun(m,q)
```

```
g = 2*m*q;
```

```
end
```



3. $dy/dt = 1 + y^2 - t^2$, $y(0) = 0$; ($y(t) = t$)

```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 0;
```

```
h = 0.1;
```

```
[x,y] = RungeKutta4(x0, xn, y0, h);
```

```
plot(x,y,'r.')
```

```
hold on
```

```
function [x,y] = RungeKutta4(x0, xn, y0, h)
```

```
n = (xn-x0)/h;
```

```
x = zeros(n+1,1);
```

```
y = zeros(n+1,1);
```

```
x(1) = x0;
```

```
y(1) = y0;
```

```
for i = 1:n
```

```
    x(i+1) = x(i)+h;
```

```
    K1 = fun(x(i), y(i));
```

```
    K2 = fun(x(i)+h/2, y(i)+K1*h/2);
```

```
    K3 = fun(x(i)+h/2, y(i)+K2*h/2);
```

```
    K4 = fun(x(i)+h, y(i)+K3*h);
```

```
    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
```

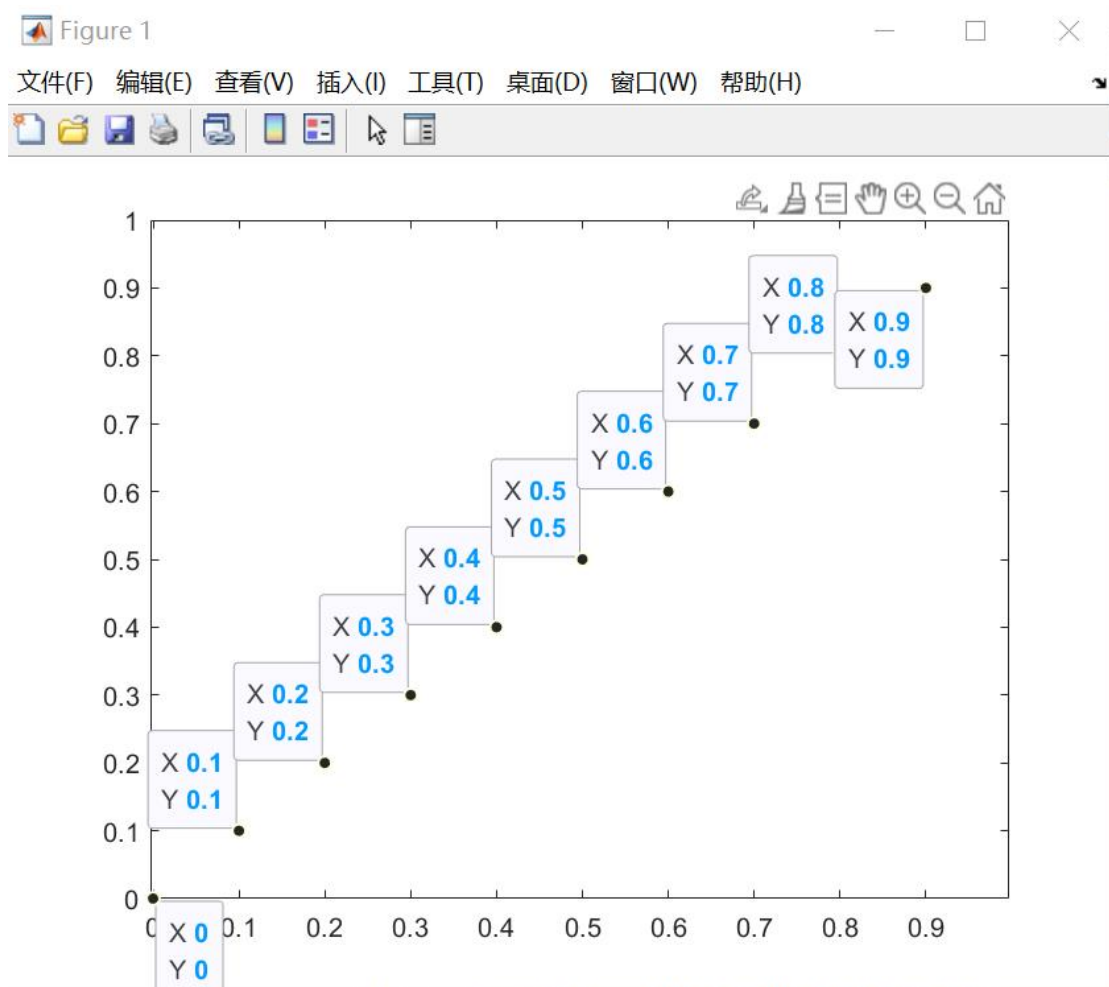
```
end
```

```
end
```

```
function f = fun(x,y)
```

```
f = 1+y^2-x^2;
```

end



```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 0;
```

```
m0 = 0;
```

```
mn = 1;
```

```
q0 = 0;
```

```

h = 0.1;

p = 0.025;

[x,y] = RungeKutta4(x0, xn, y0, h);

plot(x,y,'r')

hold on;

[m,q] = RungeKutta(m0, mn, q0, p);

plot(m,q,'b')

hold on;

function [x,y] = RungeKutta4(x0, xn, y0, h)

n = (xn-x0)/h;

x = zeros(n+1,1);

y = zeros(n+1,1);

x(1) = x0;

y(1) = y0;

for i = 1:n

    x(i+1) = x(i)+h;

    K1 = gun(x(i), y(i));

    K2 = gun(x(i)+h/2, y(i)+K1*h/2);

    K3 = gun(x(i)+h/2, y(i)+K2*h/2);

    K4 = gun(x(i)+h, y(i)+K3*h);

```

```

        y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
end
end

function [m,q] = RungeKutta(m0, mn, q0, p)
k = (mn-m0)/p;
m = zeros(k+1,1);
q = zeros(k+1,1);
m(1) = m0;
q(1) = q0;
for j = 1:k
    m(j+1) = m(j)+p;
    K1 = gun(m(j), q(j));
    K2 = gun(m(j)+p/2, q(j)+K1*p/2);
    K3 = gun(m(j)+p/2, q(j)+K2*p/2);
    K4 = gun(m(j)+p, q(j)+K3*p);
    q(j+1) = q(j)+p/6*(K1+2*K2+2*K3+K4);
end
end

function f = fun(x,y)
f = 1+y^2-x^2;
end

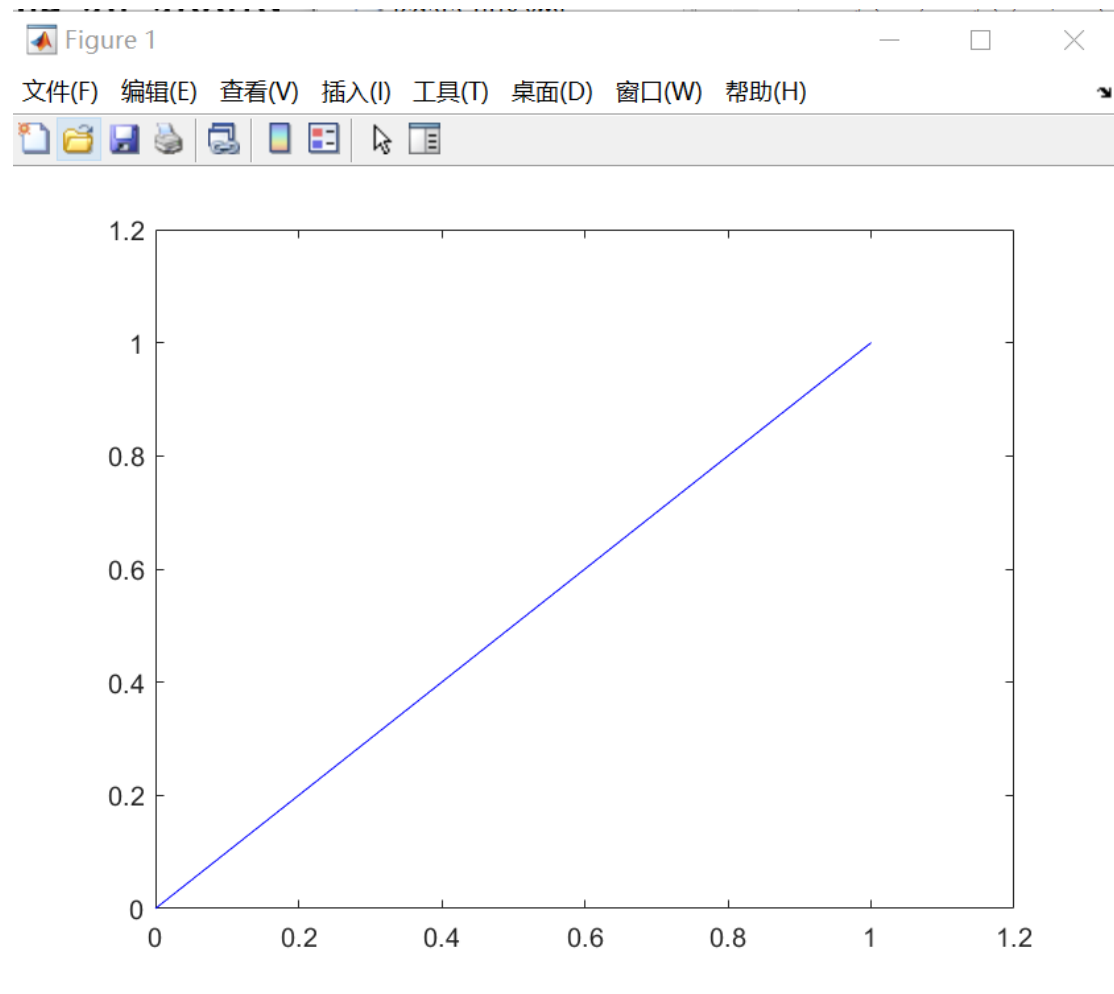
```

```
function g = gun(m,q)
```

```
g = 1+q^2-m^2;
```

```
end
```

曲线重合



4. $dy/dt = te^{-y} + t/(1+t^2)$, $y(0)=0$; ($y(t)=\ln(1+t^2)$)

```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```



```

xn = 1;

y0 = 0;

h = 0.1;

[x,y] = RungeKutta4(x0, xn, y0, h);

plot(x,y, 'r.')

hold on

function [x,y] = RungeKutta4(x0, xn, y0, h)

n = (xn-x0)/h;

x = zeros(n+1,1);

y = zeros(n+1,1);

x(1) = x0;

y(1) = y0;

for i = 1:n

    x(i+1) = x(i)+h;

    K1 = fun(x(i), y(i));

    K2 = fun(x(i)+h/2, y(i)+K1*h/2);

    K3 = fun(x(i)+h/2, y(i)+K2*h/2);

    K4 = fun(x(i)+h, y(i)+K3*h);

    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);

end

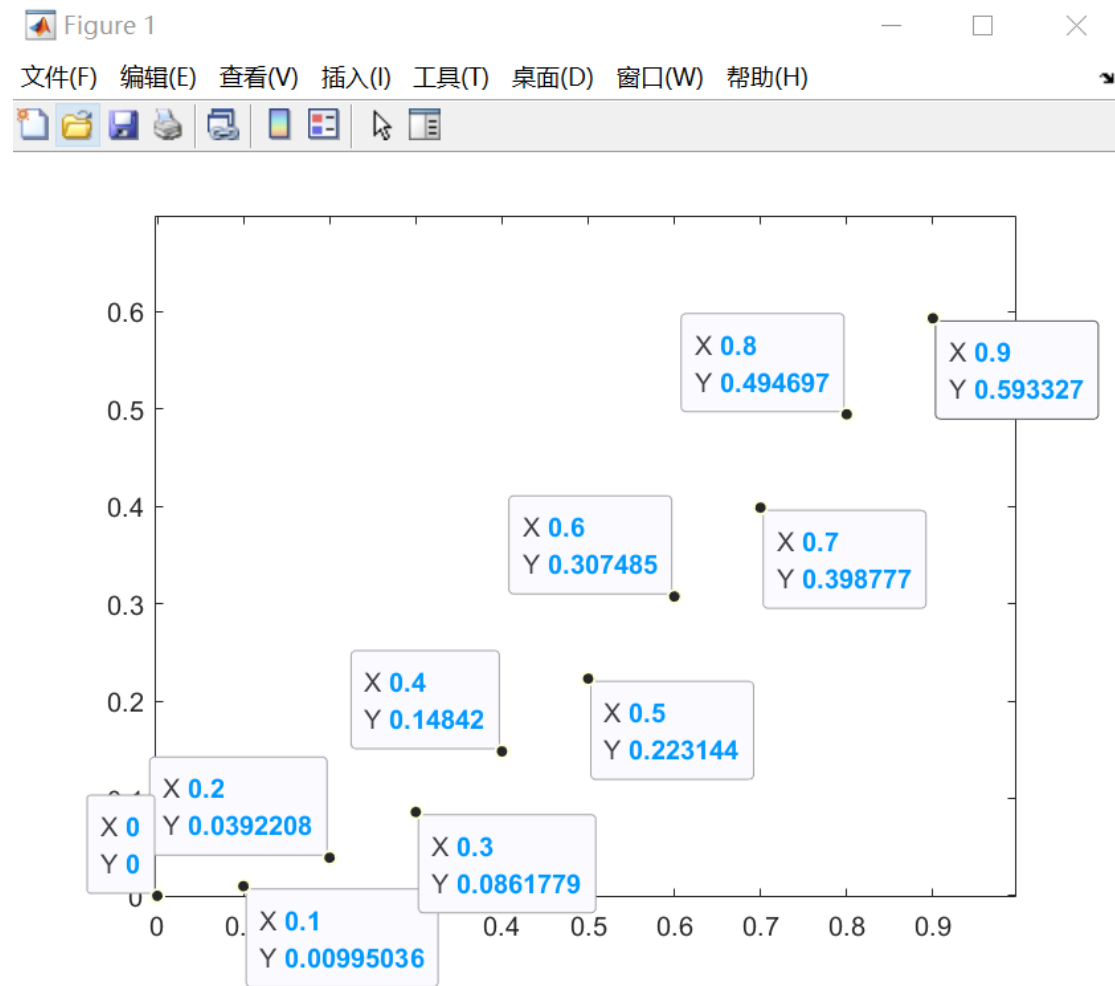
end

```

```
function f = fun(x,y)

f = x*exp(-y)+x/(1+x^2);

end
```



```
clc,clear

set(0,'defaultfigurecolor','w')

x0 = 0;

xn = 1;

y0 = 0;

m0 = 0;
```

```

mn = 1;

q0 = 0;

h = 0.1;

p = 0.025;

[x,y] = RungeKutta4(x0, xn, y0, h);

plot(x,y, 'r')

hold on;

[m,q] = RungeKutta(m0, mn, q0, p);

plot(m,q, 'b')

hold on;

function [x,y] = RungeKutta4(x0, xn, y0, h)

n = (xn-x0)/h;

x = zeros(n+1,1);

y = zeros(n+1,1);

x(1) = x0;

y(1) = y0;

for i = 1:n

    x(i+1) = x(i)+h;

    K1 = gun(x(i), y(i));

    K2 = gun(x(i)+h/2, y(i)+K1*h/2);

```

```

    K3 = gun(x(i)+h/2, y(i)+K2*h/2);
    K4 = gun(x(i)+h, y(i)+K3*h);
    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
end
end

function [m,q] = RungeKutta(m0, mn, q0, p)
k = (mn-m0)/p;
m = zeros(k+1,1);
q = zeros(k+1,1);
m(1) = m0;
q(1) = q0;
for j = 1:k
    m(j+1) = m(j)+p;
    K1 = gun(m(j), q(j));
    K2 = gun(m(j)+p/2, q(j)+K1*p/2);
    K3 = gun(m(j)+p/2, q(j)+K2*p/2);
    K4 = gun(m(j)+p, q(j)+K3*p);
    q(j+1) = q(j)+p/6*(K1+2*K2+2*K3+K4);
end
end

function f = fun(x,y)

```

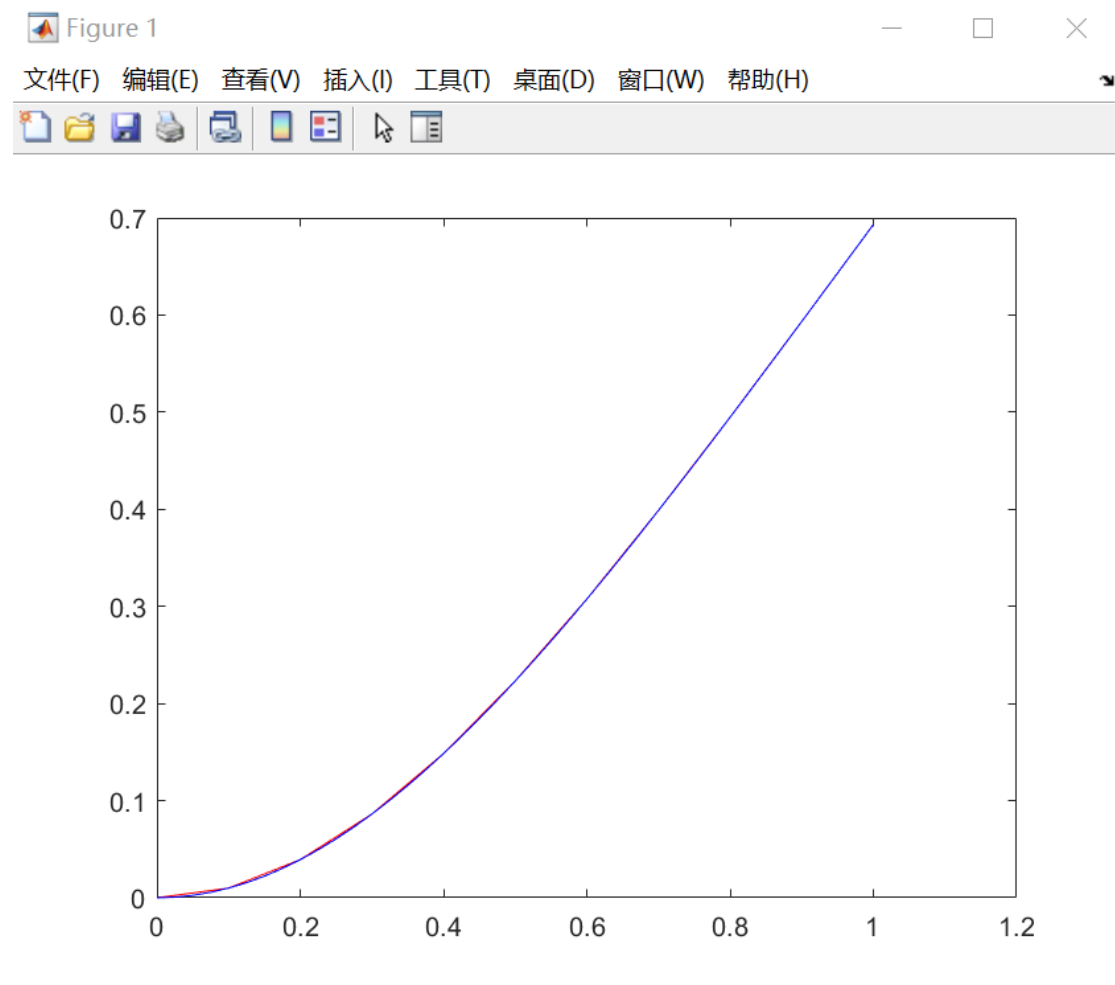
```
f = x*exp(-y)+x/(1+x^2);
```

```
end
```

```
function g = gun(m,q)
```

```
g = m*exp(-q)+m/(1+m^2);
```

```
end
```



5. $\frac{dy}{dt} = -1 + 2t + \frac{y^2}{(1+t^2)^2}$, $y(0)=1$; $(y(t)=1+t^2)$

```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```

x0 = 0;

xn = 1;

y0 = 1;

h = 0.1;

[x,y] = RungeKutta4(x0, xn, y0, h);

plot(x,y, 'r.')

hold on


function [x,y] = RungeKutta4(x0, xn, y0, h)

n = (xn-x0)/h;

x = zeros(n+1,1);

y = zeros(n+1,1);

x(1) = x0;

y(1) = y0;

for i = 1:n

    x(i+1) = x(i)+h;

    K1 = fun(x(i), y(i));

    K2 = fun(x(i)+h/2, y(i)+K1*h/2);

    K3 = fun(x(i)+h/2, y(i)+K2*h/2);

    K4 = fun(x(i)+h, y(i)+K3*h);

    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);

end

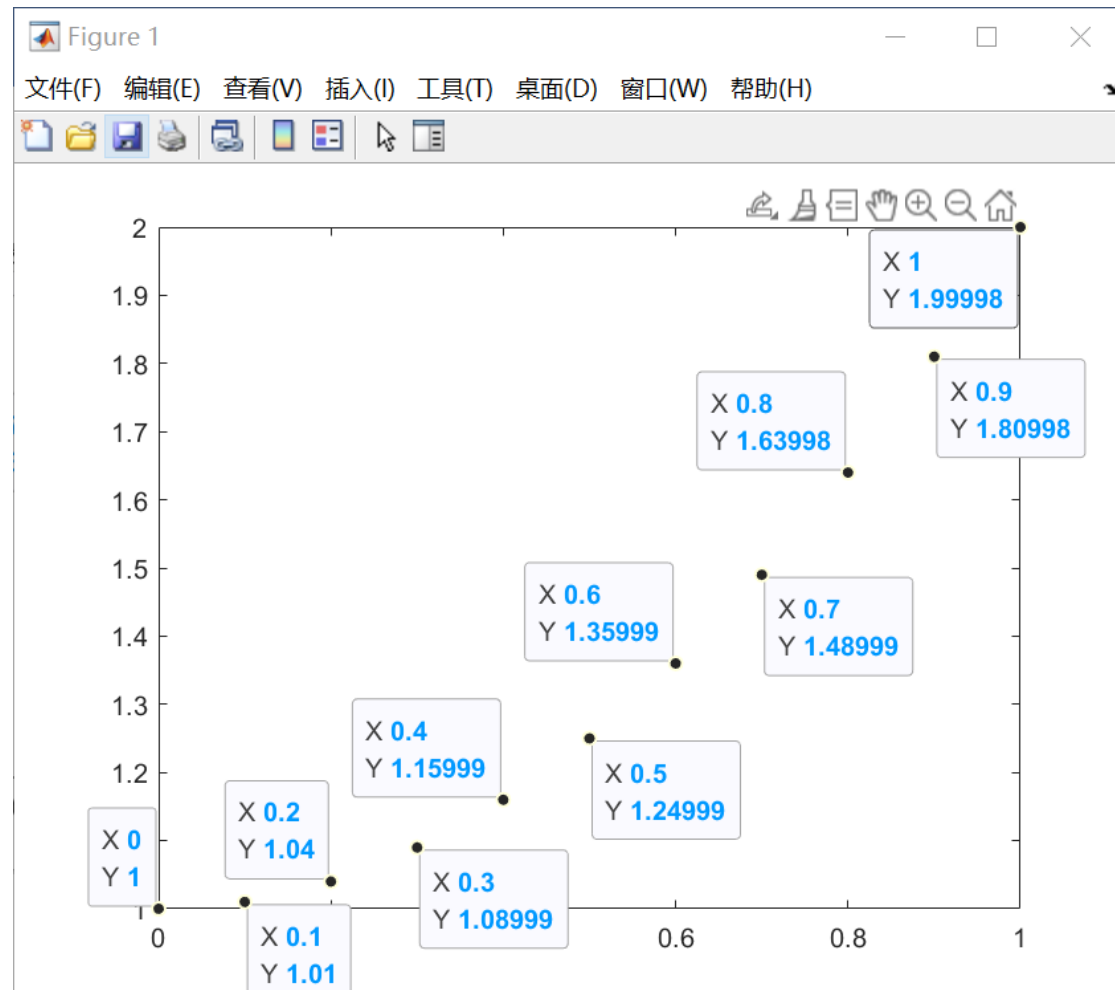
```

```
end
```

```
function f = fun(x,y)
```

```
f = -1+2*x+y^2/((1+x^2)^2);
```

```
end
```



```
clc,clear
```

```
set(0,'defaultfigurecolor','w')
```

```
x0 = 0;
```

```
xn = 1;
```

```
y0 = 1;
```

```
m0 = 0;
```

```

mn = 1;

q0 = 1;

h = 0.1;

p = 0.025;

[x,y] = RungeKutta4(x0, xn, y0, h);

plot(x,y, 'r')

hold on;

[m,q] = RungeKutta(m0, mn, q0, p);

plot(m,q, 'b')

hold on;

function [x,y] = RungeKutta4(x0, xn, y0, h)

n = (xn-x0)/h;

x = zeros(n+1,1);

y = zeros(n+1,1);

x(1) = x0;

y(1) = y0;

for i = 1:n

    x(i+1) = x(i)+h;

    K1 = gun(x(i), y(i));

    K2 = gun(x(i)+h/2, y(i)+K1*h/2);

```



```

    K3 = gun(x(i)+h/2, y(i)+K2*h/2);
    K4 = gun(x(i)+h, y(i)+K3*h);
    y(i+1) = y(i)+h/6*(K1+2*K2+2*K3+K4);
end
end

function [m,q] = RungeKutta(m0, mn, q0, p)
k = (mn-m0)/p;
m = zeros(k+1,1);
q = zeros(k+1,1);
m(1) = m0;
q(1) = q0;
for j = 1:k
    m(j+1) = m(j)+p;
    K1 = gun(m(j), q(j));
    K2 = gun(m(j)+p/2, q(j)+K1*p/2);
    K3 = gun(m(j)+p/2, q(j)+K2*p/2);
    K4 = gun(m(j)+p, q(j)+K3*p);
    q(j+1) = q(j)+p/6*(K1+2*K2+2*K3+K4);
end
end

function f = fun(x,y)

```

```
f = -1+2*x+y^2/((1+x^2)^2);
```

```
end
```

```
function g = gun(m,q)
```

```
g = -1+2*m+q^2/((1+m^2)^2);
```

```
end
```

