常微与偏微课程作业

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题目1.

$$y'' + 3y = t^3 - 1$$

解答.

$$r^2 + 3 = 0$$

$$r1 = +\sqrt{3}i \quad r2 = -\sqrt{3}i$$

$$\Phi(t) = A0 + A1t + A2t^2 + A3t^3$$

$$\Phi'(t) = A1 + 2A2t + 3A3t^2$$

$$\Phi''(t) = 2A2 + 6A3t$$

$$(2A2 + 3A0) + (6A3 + 3A1)t + 3A2t^2 + 3A3t^3 = t^3 - 1$$

$$2A2 + 3A0 = -1$$

$$6A3 + 3A1 = 0$$

$$3A2 = 0$$

$$3A3 = 1$$

$$A0 = -\frac{1}{3} \quad A1 = -\frac{2}{3} \quad A2 = 0 \quad A3 = \frac{1}{3}$$

$$\Phi(t) = -\frac{1}{3} - \frac{2}{3}t + \frac{1}{3}t^3$$

$$y(t) = C1\cos\sqrt{3}t + C2\sin\sqrt{3}t - \frac{1}{3} - \frac{2}{3}t + \frac{1}{3}t^3$$

题目2.

$$y'' - y = t^2 e^t$$

解答.

$$r^2 - 1 = 0$$

$$r1 = +1 \quad r2 = -1$$

$$\Phi(t) = t(A0 + A1t + A2t^2)e^t$$

$$\Phi'(t) = [A0 + (2A1 + A0)t + \Phi 3A2 + A1\Psi t^2 + A2t^3]e^t$$

$$\Phi''(t) = [(2A1 + 2A0) + (6A2 + 4A1 + A0)t + \Phi 6A2 + A1)t^2 + A2t^3]e^t$$

$$6A2 = 1$$

$$6A2 + 4A1 = 0$$

$$2A1 + 2A0 = 0$$

$$A0 = \frac{1}{4} \quad A1 = -\frac{1}{4} \quad A2 = \frac{1}{6}$$

$$\Phi(t) = t(\frac{1}{4} - \frac{1}{4}t + \frac{1}{6}t^2)e^t$$

$$y(t) = C1e^t + C2e^{-t} + t(\frac{1}{4} - \frac{1}{4}t + \frac{1}{6}t^2)e^t$$

题目3.

$$y'' + y' + y = t^2 + t + 1$$

解答.

$$r^2 + r + 1 = 0$$

$$r1 = \frac{-1 + \sqrt{3}i}{2} \quad r2 = \frac{-1 - \sqrt{3}i}{2}$$

$$\Phi(t) = A0 + A1t + A2t^2$$

$$\Phi'(t) = A1 + 2A2t$$

$$\Phi''(t) = 2A2$$

$$2A2 + A1 + A0 = 1$$

$$2A2 + A1 = 1$$

$$A2 = 1$$

$$A0 = 0 \quad A1 = -1 \quad A2 = 1$$

$$\Phi(t) = -t + t^2$$

$$y(t) = e^{-\frac{1}{2}t}(\cos\frac{\sqrt{3}}{2}t + \sin\frac{\sqrt{3}}{2}t) - t + t^2$$

题目4.

$$y'' + 4y = t\sin 2t$$

解答.

$$r^2 + 4 = 0$$

$$r1 = 2i \quad r2 = -2i$$

$$\Phi(t) = t(A0 + A1t)e^{2it}$$

$$\Phi'(t) = (A0 + 2A1t + 2A0it + 2A1it^2)e^{2it}$$

$$\Phi''(t) = (2A1 + 4A0i + 8A1it - 4A0 - 4A1t^2)e^{2it}$$

$$2A1 + 4A0i + 8A1it = t$$

$$8A1i = 1$$

$$2A1 + 4A0i = 0$$

$$A0 = \frac{1}{16} \quad A1 = \frac{1}{8i}$$

$$\Phi(t) = t(\frac{1}{16} + \frac{1}{8i}t)e^{2it}$$

$$y(t) = C1e^2it + C2e^{-2it} + t(\frac{1}{16} + \frac{1}{8i}t)e^{2it}$$

题目5.

$$y'' + y' - 6y = \sin t + e^2t$$

解答.

$$r^2 + r - 6 = 0$$

$$r1 = 2 \quad r2 = -3$$

$$\Phi(t) = (A0 + A1t)e^{it}$$

$$\Phi'(t) = A1e^{it} + i(A0 + A1t)e^{it}$$

$$\Phi''(t) = iA1e^{it} - (A0 + A1t)e^{it}$$

$$(A1 - A0 + A0i + A1i - 6A0) + (A1it - 7A1t) = 1$$

$$A1t(-7 + i) = 0$$

$$(A1 - 7A0) + (A0 + A1)i = 1$$

$$A0 = \frac{1}{-7 + i} \quad A1 = 0$$

$$\Phi(t) = \frac{1}{-7 + i}e^{it}$$

$$y1(t) = C1e^2t + C2e^{-3t} + \frac{1}{-7 + i}e^{it}$$

$$\phi(t) = t(A0 + A1t)e^{2t}$$

$$\phi'(t) = (A0 + 2A1t + 2A0t + 2A1t^2)e^{2t}$$

$$\phi''(t) = (2A1 + 4A0 + 6A1t + 4A0t + 4A1t^2)e^{2t}$$

$$2A1 + 5A0 + 8A1t = t$$

$$8A1 = 1$$

$$2A1 + 5A0 = 0$$

$$A0 = -\frac{1}{20} \quad A1 = \frac{1}{8}$$

$$\phi(t) = t(-\frac{1}{20} + \frac{1}{8}t)e^{2t}$$

$$y2(t) = C1e^2t + C2e^{-3t} + t(-\frac{1}{20} + \frac{1}{8}t)e^{2t}$$

$$y(t) = y1(t) + y2(t) = 2C1e^2t + 2C2e^{-3t} + \frac{1}{-7 + i}e^{it} + t(-\frac{1}{20} + \frac{1}{8}t)e^{2t}$$