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Optimal Solution for Travelling Salesman Problem using Heuristic Shortest Path Algorithm with Imprecise Arc Length

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Abstract. The shortest path problem is a popular problem in graph theory. It is about finding a path with minimum length between a specified pair of vertices. In any network the weight of each edge is usually represented in a form of crisp real number and subsequently the weight is used in the calculation of shortest path problem using deterministic algorithms. However, due to failure, uncertainty is always encountered in practice whereby the weight of edge of the network is uncertain and imprecise. In this paper, a modified algorithm which utilized heuristic shortest path method and fuzzy approach is proposed for solving a network with imprecise arc length. Here, interval number and triangular fuzzy number in representing arc length of the network are considered. The modified algorithm is then applied to a specific example of the Travelling Salesman Problem (TSP). Total shortest distance obtained from this algorithm is then compared with the total distance obtained from traditional nearest neighbour heuristic algorithm. The result shows that the modified algorithm can provide not only on the sequence of visited cities which shown to be similar with traditional approach but it also provides a good measurement of total shortest distance which is lesser as compared to the total shortest distance calculated using traditional approach. Hence, this research could contribute to the enrichment of methods used in solving TSP.

INTRODUCTION

A graph network $G = (V, E)$ is usually used to represent a shortest path problem where V is a set of nodes and E represents a set of edges or arcs. The shortest path problem involved optimization in finding the shortest route from a specific starting node to a final destination node. This problem occurs in many applications in various fields such as scheduling [1], transportation [2], routing [3] and communications [4]. Generally, a weight is often associated with the edge in the network to represent cost, time, length or distance where the length of a path is equal to the sum of the arc lengths on the path in which more than one path between a specified pair of nodes might exist. Many researchers had studied extensively on how to solve the shortest path problem since 1950's [5, 6, 7]. Several most popular algorithms associated to the deterministic version are developed namely Dijkstra's algorithm [8], Floyd-Warshall algorithm [9], Bellman-Ford algorithm [10] and Genetic algorithm [11]. On the other hand, the Traveling Salesman Problem (TSP) is a combinatorial optimization problem of finding the shortest path that goes through every vertex exactly once and returns to the start. Here, the starting node and the destination node are the same node. Here, the minimum cost or distance of travelling all nodes once is studied since 1800s. Among popular algorithm to solve the TSP are exact algorithm [12], heuristic algorithm [13] and approximation algorithm [14]. However, the element of uncertainty is exists in almost all real life problem including TSP especially in the construction of edge weight. Practically, the edge weight may not be a fixed real number but it may come in an imprecise way. However, only a few cases where the weight of each edge of the network is represented as deterministic value. A research involving probability theory had been used to attack the problem [ref?] but fuzzy approaches gives a powerful tool in dealing this problem, therefore the research is then shifted to the use of fuzzy approaches in solving the TSP [15, 16, 17]. Sengupta and Pal [16] used ranking method for the interval arcs in the asymmetric TSP where each pair of nodes is considered independent of the direction of the journey. This study is an extension of Sengupta and Pal [16] whereby the TSP is solved by using shortest path algorithm which is developed based on a heuristic method and fuzzy approach for symmetric TSP network where distance between each pair of node is considered independent of the direction of the destination.

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DEFINITIONS AND PRELIMINARIES

The impreciseness of the weight of each edge of the network can be represented as interval number and triangular fuzzy number as defined below.

Definition 2.1: Interval Number [23]

An interval number is defined as $A = [a_L, a_R] = \{a: a_L \leq a \leq a_R\}$ where a_L and a_R are the real numbers that denote the left end point and the right end point of the interval A.

Another way to represents an interval number in terms of midpoint and width is $A = \langle M(A), W(A)\rangle$, where $M(A) =$ midpoint of $A = \frac{a_R + a_L}{2}$ and $W(A)$= half width of $A = \frac{a_R - a_L}{2}$. According to Kaufmann and Gupta [23], the addition of two interval numbers by consider the set of intervals $A = [a_L, a_R]$ and $B = [b_L, b_R]$, then the addition of two interval numbers $A = [a_L, a_R]$ and $B = [b_L, b_R]$ is given by $A \oplus B = [a_L + b_L, a_R + b_R]$.

Definition 2.3: Triangular Fuzzy Number [23]

A triangular fuzzy number is represented by a triplet $A = \langle m, \alpha, \beta\rangle$ with membership function

$$\mu_A = \begin{cases} 1 - \dfrac{m - x}{\alpha} & \text{for } m - \alpha < x \leq m \\ 1 - \dfrac{x - m}{\beta} & \text{for } m < x < m + \beta \\ 0 & \text{otherwise} \end{cases}$$

where midpoint, $m \in \mathbb{R}$, α, β are the left and right hand spread with $\alpha, \beta > 0$. According to Kaufmann and Gupta [21], the addition of triangular fuzzy numbers $A = \langle m_1, \alpha_1, \beta_1\rangle$ and $B = \langle m_2, \alpha_2, \beta_2\rangle$ is given by $A \oplus B = \langle m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2\rangle$.

For example, consider a data distance between two point as an interval number of [16, 19]. The equivalent representation of the interval number in term of midpoint and width by using Definition 2.1 is $\langle 17.5, 1.5\rangle$. If $\alpha = 1$ and $\beta = 2$ and let $m = 18$, therefore the equivalent triangular fuzzy number for [16, 19] can be represented as $\langle 18, 2, 1\rangle$.

ACCEPTABILITY INDEX FOR INTERVAL NUMBERS AND TRIANGULAR FUZZY NUMBERS

Many researchers have been made an effort for solving the problem of comparing imprecise or uncertain quantities. The comparison of interval numbers schemes is starting from an extensive research paper done by Moore [24]. In this literature, we found two transitive order relations defined over intervals $A = [a_L, a_R]$ and $B = [b_L, b_R]$. Based on Sengupta and Pal [16], the first one as an extension of '<' on the real line as $A < B$, if only if $a_R < b_L$. Then the other as an extension of the concept of set inclusion i.e. $A \subseteq B$ if only if $a_L \geq b_L$ and $a_R \leq b_R$. Then the order relation is extended by using acceptability index that was introduced by Sengupta and Pal [25]. The preliminaries definition related to an acceptability index for interval numbers are as follows:

Definition 2.4: The Acceptability Index ($\mathcal{A}$-index) for Interval Numbers [23]

The acceptability index defines as 'total dominance' and 'partial dominance' of two interval numbers where the proposition '$A = \langle M_1, W_1\rangle$ is inferior to $B = \langle M_2, W_2\rangle$' is given by $\mathcal{A}(A \prec B) = \frac{M_2 - M_1}{W_1 + W_2}$.

Definition 2.5: If $\mathcal{A}(A \prec B) \geq 1$ then A is said to be 'totally dominating' over B in the sense of minimization and B is said to be 'totally dominating' over A in the sense of maximization. Then denoted this by $A \prec B$, i.e., minimum $\{A, B\} = A$.

Definition 2.6: If $0 < \mathcal{A}(A \prec B) < 1$ then A is said to be 'partially dominating' over B in the sense of minimization and B is said to be 'partially dominating' over A in the sense of maximization. Then denoted this by $A \prec_P B$, i.e., minimum $\{A, B\} = A$.

Numerical example for acceptability index for an interval numbers are given below:

Example 1 Let $A = [16,19] = \langle 17.5,1.5\rangle$ and $B = [29,30] = \langle 29.5,0.5\rangle$. Then

$\mathcal{A}(A \prec B) = \frac{29.5 - 17.5}{0.5 + 1.5} = 6 > 1$.

So in minimization, A is totally dominating over B.

Example 2 Let $A = [16,19] = \langle 17.5,1.5\rangle$ and $C = [18,19] = \langle 18.5,0.5\rangle$. Then

$\mathcal{A}(A \prec C) = \frac{18.5-17.5}{0.5+1.5} = 0.5 < 1.$
So in minimization, A is partially dominating over C with level of satisfaction 0.5.

Nayeem and Pal [23] extended the acceptability index from interval number to triangular fuzzy number which it is originally introduced by Sengupta and Pal [25]. So, the preliminaries definition related to an acceptability index for triangular fuzzy number is applied in our study as follows:

Definition 2.7: The Acceptability Index ($\mathcal{A}$-index) for triangular Fuzzy Numbers [23]
The acceptability index ($\mathcal{A}$-index) of the proposition '$A = \langle a, \alpha, \beta \rangle$ is preferred to $B = \langle b, \gamma, \delta \rangle$' is given by

$$\mathcal{A}(A \prec B) = \frac{b-a}{\beta+\gamma}$$

Definition 2.8: If $\mathcal{A}(A \prec B) \geq 1$ then A is said to be 'totally dominating' over B in case of minimization and B is said to be 'totally dominating' over A in the case of maximization. Then is denoted this by $A \prec B$, i.e., minimum $\{A, B\} = A$.
Definition 2.9: If $0 < \mathcal{A}(A \prec B) < 1$ then, A is said to be 'partially dominating' over B in the case of minimization and B is said to be 'partially dominating' over A in the case of maximization. Then is denoted this by $A \prec_P B$, such that minimum.$\{A, B\} = A$.
Numerical example for acceptability index for triangular fuzzy numbers are as follows:
Example 1 Let $A = \langle 18,2,1 \rangle$ and $B = \langle 30,1,2 \rangle$ be two triangular fuzzy numbers. Then $\mathcal{A}(A \prec B) = \frac{30-18}{1+1} = 6 > 1$. So in minimization, A is totally dominating over B.
Example 2 Let $A = \langle 18,2,1 \rangle$ and $C = \langle 19,1,1 \rangle$. Then $\mathcal{A}(A \prec C) = \frac{19-18}{1+1} = 0.5 < 1$. So in minimization, A is partially dominating over C with level of satisfaction 0.5.

METHODOLOGY

In this section, the development of the algorithm for Traveling Salesman Problem (TSP) with imprecise arc length using heuristic shortest path method is presented. The algorithm is developed based on Nayeem and Pal (2005), Sengupta and Pal (2009) and Kumar and Kaur (2011).
The notations used in this study are as follows:

- N: The set of all nodes in a network.
- P: The set of nodes with permanent labels.
- T: The set of nodes with temporary labels.
- $N_{P(j)}$: The set of minimum arc length(s) expressed in interval number or triangular fuzzy number from starting node to node *j* (*j*=2,3,…n).
- e_i: The distance between starting node to node *i*.
- e_{ij}: The distance between node *i* and *j*.
- $\tilde{e}_i$: The fuzzy distance between starting node to node *i*.
- $\tilde{e}_{ij}$: The fuzzy distance between node *i* and *j*.

The algorithm can be described as follows:

Step 1: Consider the data distance of a network in a form of symmetric matrix. Represent the matrix in a form of interval number using midpoint and half width while triangular fuzzy numbers in a form of $\langle m, \alpha, \beta \rangle$ where m is mean and α is left-spreads and β is right-spreads.
Consider the following graph network with 3 nodes and 3 arc lengths.

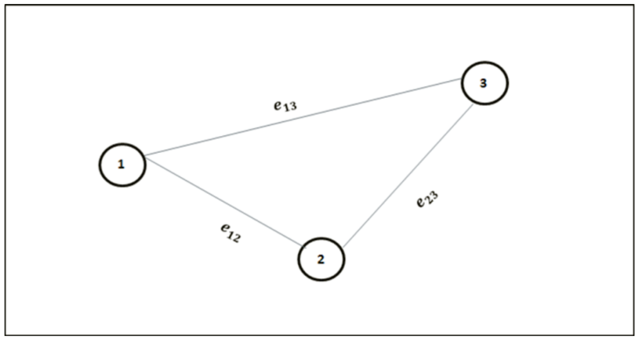

FIGURE 1. Graph Network with 3 nodes and 3 arc lengths

The symmetric matrices representing data distance in Figure 1are as follow:

$$\begin{bmatrix} [0,0] & [29,30] & [16,19] \\ [29,30] & [0,0] & [18,19] \\ [16,19] & [18,19] & [0,0] \end{bmatrix} \quad \begin{bmatrix} \langle 0,0\rangle & \langle 29.5,0.5\rangle & \langle 17.5,1.5\rangle \\ \langle 29.5,0.5\rangle & \langle 0,0\rangle & \langle 18.5,0.5\rangle \\ \langle 17.5,1.5\rangle & \langle 18.5,0.5\rangle & \langle 0,0\rangle \end{bmatrix} \quad \begin{bmatrix} \langle 0,0,0\rangle & \langle 30,1,2\rangle & \langle 18,2,1\rangle \\ \langle 30,1,2\rangle & \langle 0,0,0\rangle & \langle 19,1,1\rangle \\ \langle 18,2,1\rangle & \langle 19,1,1\rangle & \langle 0,0,0\rangle \end{bmatrix}$$

(a) (b) (c)

FIGURE 2. Types of data distance in a form of symmetric matrices using (a) Interval number (b) Interval number using midpoint and width (c) Triangular fuzzy number

Starting Step 2 onwards, the example is referring to the network in Figure 1 with corresponding data distance as in Figure 2(b).

Step 2: Set a Starting Node
Consider Figure 1. Here, we set node 1 as starting node and label as permanent label with membership value in a form of interval number in terms of midpoint and width as $e_1 = \langle 0,0\rangle$

Step 3: Establish the set P as the set of nodes with permanent label and T as the set of nodes with temporary label. Consider Figure 1. Here, P and T are firstly initialized as P $= \{1\}$ and T $= \{2,3\}$.

Step 4: Find the set of minimum arc length(s) $N_{P(j)}$ in a form of interval number and triangular fuzzy number. In this step, the set of minimum arc length(s) is calculated such that
$N_{P(j)} =$ minimum $\{e_i \oplus e_{ij} / \ i \in \mathrm{P}\}, j \neq 1, j = \{2,3,\dots,n\}$.
If minimum value occurs which correspond to the value of j, then label node j as [$N_{P(j)}, j$]. But, if minimum value occurs which correspond to more than one value of j, then return back to the previous iteration by assuming both paths as optimum and consider it as permanent labels.
Consider Figure 1. The set of minimum arc length(s) in a form of interval number for the first iteration is determined as follows:
From Step 3, P $= \{1\}$ and T $= \{2,3\}$ is established and by putting $i = 1$ and $j = 2,3$ in Step 4, then the value of $N_{P(1)}$ is

$$\begin{aligned} N_{P(2)} &= \text{minimum } \{e_1 \oplus e_{12}, e_1 \oplus e_{13}\} \\ &= \text{minimum } \{\langle 0,0\rangle \oplus \langle 29.5,0.5\rangle, \langle 0,0\rangle \oplus \langle 17.5,1.5\rangle\} \\ &= \text{minimum } \{\langle 29.5,0.5\rangle, \langle 17.5,1.5\rangle\} \end{aligned}$$

In this step, Definition 2.4 is applied to find the minimum distance from node 1 to 3, i.e., $\mathcal{A}(\langle 34,1\rangle \prec \langle 165,1\rangle) = \frac{29.5-17.5}{0.5+1.5} = 6 > 1$. Since the minimum value corresponding to e_{13} occurs at $j = 3$, then label the second visited node as $[\langle 17.5,1.5\rangle, 3]$.

Step 5: Find the fuzzy shortest path from starting node to destination node.
In this step, the destination node (node n) is selected and labelled as[$N_{P(n)}, j$]. One can check the label of node j for each iteration in order to find the fuzzy shortest path from the starting node to the destination node. Consider Figure 1. The numerical solution for second iteration is as follows:
Following Step 4, since the minimum distance corresponding to $j = 3$ occurs and $e_3 = \langle 17.5,1.5\rangle$, thus we established P $= \{1,3\}$ and T $= \{2\}$. Next, let $i = 3$ and $j = 2$, then the value of $N_{P(3)}$ is calculated.

$$\begin{aligned} N_{P(3)} &= \text{minimum } \{e_3 \oplus e_{32}\} \\ &= \text{minimum } \{\langle 17.5,1.5\rangle \oplus \langle 18.5,0.5\rangle\} \\ &= \text{minimum } \{\langle 36,2\rangle\} \\ &= \langle 36,2\rangle \end{aligned}$$

Since the minimum value occurs which correspond to e_{32} at $j = 2$, then node 3 is labelled as$[\langle 36,2\rangle, 2]$. Before proceed to the next step, we can find the fuzzy shortest path from starting node to destination node by set destination node as node 3 and label as$\left[\, N_{P(3)}, 2\right]$. By checking the label of node j for each iteration, we obtained the fuzzy shortest path from node $1-3-2$ with shortest distance 36 KM.

Step 6: Find the Fuzzy Shortest Path from Destination Node and Return to the Starting Node.
Therefore if the set of temporary label $\mathrm{T} = \{\ \emptyset\ \}$, we repeated the same procedure in Step 3 until Step 4 by letting $\mathrm{T} = \{\ 1\ \}$ and set the destination node $[\ N_{P(n)}\ , j\ ] = e_j$ where e_j is the distance from starting node to destination node. Consider Figure 1. The numerical solution for the third iteration is as follows:
From Step 5, since the destination node $[\langle 36,2\rangle, 2]$ and the minimum value occurs which correspond at $= 2$, then the value of $e_2 = \langle 36,2\rangle$ and we establish $\mathrm{P} = \{1,3,2\}$ and $\mathrm{T} = \{1\}$. So putting $i = 3$ and $j = 2$, then the value of $N_{P(4)}$ is

$$\begin{aligned} N_{P(4)} &= \text{minimum } \{e_2 \oplus e_{21}\} \\ &= \text{minimum } \{\langle 36,2\rangle \oplus \langle 29.5,0.5\rangle\} \\ &= \text{minimum } \{\langle 65.5,2.5\rangle\} \\ &= \langle 65.5,2.5\rangle \end{aligned}$$

Since the minimum value occurs corresponding e_{21} at $j = 1$, then node number 4 is labelled as $[\langle 65.5,2.5\rangle, 1]$. Here, the destination node number 4 is represented as $[\langle 65.5,2.5\rangle, 1]$and subsequently the minimum value which corresponds to $j = 1$ is obtained as $N_{P(4)} = \langle 65.5,2.5\rangle$. Thus, the shortest path is represented as $1-3-2-1$ with total fuzzy shortest distance 65.5 KM. The same procedure is used to find the shortest path and shortest distance for a network with arc length(s) in triangular fuzzy number.

NUMERICAL EXAMPLE

In order to illustrate the algorithm, we consider a road network as undirected graph shown in Figure 2 in which there are 10 nodes and 45 arcs. Since it is undirected graph, therefore $e_{ij} = e_{ji}\ \forall i, j = 1,2,...,10$.

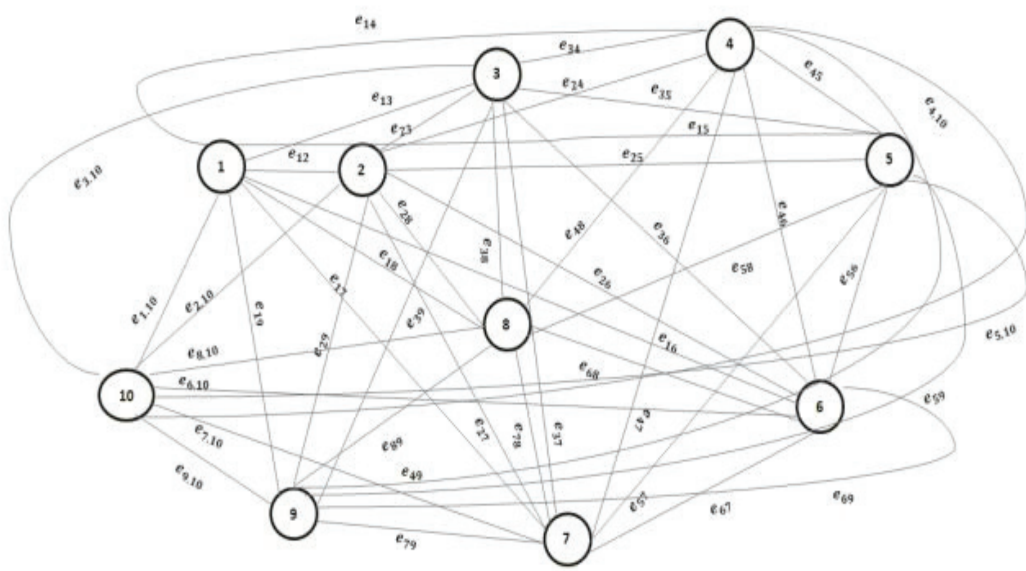

FIGURE 3. A road network

We represent all arcs in a form of interval number as in Table 1.

TABLE 1. The arc lengths of the network

Arc	Length	Arc	Length
e_{11}	<0,0>	e_{16}	<581.5,0.5>
e_{12}	<34,1>	e_{17}	<392.5,0.5>
e_{13}	<165,1>	e_{18}	<124.5,0.5>
e_{14}	<351.5,2>	e_{19}	<118.5,1.5>
e_{15}	<393,2>	$e_{1,10}$	<137,2>

A run of the algorithm shows that the expected shortest path is $1 \to 2 \to 8 \to 9 \to 10 \to 3 \to 4 \to 5 \to 6 \to 7 \to 1$ and the length of this path is 1804.05km. As for triangular fuzzy number, the total length of the same path is 1807km. Similarly, we run the nearest neighbour heuristic algorithm to the same set of data with data distance as crisp value and the result shows that a similar sequence of nodes is obtained with a total distance of 1812km. Comparison of the results between these two algorithms is depicted in Table 2.

TABLE 2. Comparison of sequence of nodes using fuzzy heuristic shortest path and nearest neighbour heuristic

Algorithm	Sequence of nodes	Total distance (km)
1) Fuzzy heuristic shortest path with imprecise arc length as	$1 \to 2 \to 8 \to 9 \to 10 \to 3 \to 4 \to 5 \to 6 \to 7 \to 1$	
a) interval number		1804.05
b) triangular fuzzy number		1807.0
2) Nearest neighbour heuristic	$1 \to 2 \to 8 \to 9 \to 10 \to 3 \to 4 \to 5 \to 6 \to 7 \to 1$	1812.0

The result shows that the fuzzy heuristic algorithm provides sequence of visited nodes which shown to be similar with traditional approach. Further analysis on the performance of the algorithm need to be conducted to show the effectiveness of the algorithm when dealing with a dense and imprecise data distance. However this study provides an alternative method in solving a TSP and also provides a platform to further investigate the TSP which specifically involve uncertainty in their information.

CONCLUSION

In this study, shortest path algorithm is developed based on heuristic method which involved imprecise information of data distance to solve a well-known Travelling Salesman Problem. The main contribution of this paper is that it provides not only alternative way of solving the TSP with imprecise information but also provides a platform of further research to be investigated within this domain. Besides, enhancement on the analysis part of this paper is also crucial.

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