Sensitivity of
Nozzle Guide Vane Flow Capacity
to Geometric Changes

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Thanks

Abstract

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Chapter 1

Capacity - definition and derivation

1.1 Reasons for considering capacity

The capacity of an internal combustion engine is an intuitive concept. It is simple to derive from the engine's geometry, it has tangible units of volume, and it provides an heuristic for the engine's size, performance, and air mass flow rate.

The design of turbomachinery invites an analagous concept to that of IC engine capacity, but a definition is not so obvious. Mass flow rate through an engine's nozzle guide vane is a function of the NGV's geometry and of its boundary conditions. If the mass flow rate can be quantified in a way which mitigates the boundary conditions, then the effects of an NGV's geometry on its mass flow rate may be isolated. A particular NGV will thus have a mass flow rate capacity, just as a particular IC cylinder has a volumetric capacity.

Defining a geometric mass flow rate capacity allows for experimental testing of NGVs without recreating the extreme boundary conditions to which real NGVs are exposed. Setting the correct pressure ratio is sufficent, as the subsequent derivation will show. This expedites the testing of different NGV geometries' effects on capacity.

The predictability of NGV capacity affects the design of every downstream tur-

bine stage. If the NGV mass flow rate differs from its expected value due to a poor prediction, all subsequent turbine stages will be sized for an incorrect mass flow rate. The resulting errors in flow velocity and pressure will compound with each additional stage, leading to increasingly incorrect specification of turbine sizes and turning angles.

A strong understanding of capacity predictability should allow engine-makers to pre-empt geometric changes that happen to NGVs during service, such as erosion and cooling hole blockage. It should also account for geometric uncertainties arising from the manufacturing process. While this study advocates for improved accuracy of capacity predictions, emphasis is placed on how these predictions are limited by the unpredictability of real-world manufacture and service.

1.2   Mass flow rate through an NGV

In one dimension, an engine nozzle may be modelled as a compressible flow from an upstream reservoir of total pressure p_0, accelerating to velocity v and density ρ through a nozzle of cross-sectional area A. Mass flow rate through the nozzle is thus

$$\dot{m} = \rho A v \tag{1.1}$$

where density may be expressed as a function of *pressure ratio*, the ratio of the nozzle pressure to the total pressure

$$\rho = \frac{p_0}{RT_0} \left(\frac{p}{p_0} \right)^{\frac{1}{\gamma}} \tag{1.2}$$

and velocity is given by the compresssible form of Bernouilli's equation as

$$v = \sqrt{2 \left(\frac{\gamma}{\gamma - 1} \right) \left[\frac{p_0}{\rho_0} - \frac{p}{\rho} \right]} \tag{1.3}$$

The above equations combine to express mass flow rate through the nozzle as

$$\dot{m} = A \frac{p_0}{RT_0} \left(\frac{p}{p_0} \right)^{\frac{1}{\gamma}} \sqrt{2 \left(\frac{\gamma}{\gamma - 1} \right) \left[\frac{p_0}{\left(\frac{p_0}{RT_0} \right)} - \frac{p}{\left(\frac{p_0}{RT_0} \right) \left(\frac{p}{p_0} \right)^{\frac{1}{\gamma}}} \right]} \tag{1.4}$$

which simplifies to

$$\dot{m} = A\frac{p_0}{RT_0}\left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}\sqrt{2\left(\frac{\gamma}{\gamma-1}\right)\left[RT_0 - RT_0\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{1.5}$$

$$\dot{m} = \frac{p_0}{\sqrt{T_0}}A\sqrt{\frac{\gamma}{R}}\left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}\sqrt{\left(\frac{2}{\gamma-1}\right)\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{1.6}$$

1.3   NGV capacity in 1 dimension

Capacity is defined as

$$\Gamma = \frac{\sqrt{T_0}}{p_0}\,\dot{m} \tag{1.7}$$

This provides an expression of mass flow rate independent of upstream total pressure p_0 and upstream total temperature T_0. The expression is purely a function of throat area A and pressure ratio $\frac{p}{p_0}$:

$$\Gamma = A\sqrt{\frac{\gamma}{R}}\left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}\sqrt{\left(\frac{2}{\gamma-1}\right)\left[1 - \left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{1.8}$$

A scale constant σ is defined as

$$\sigma = \sqrt{\frac{2\gamma}{R(\gamma-1)}} \tag{1.9}$$

for a compact expression of capacity as a function of throat area A and pressure ratio r:

$$\Gamma(A,r) = \sigma A\sqrt{r^{\frac{2}{\gamma}}\left(1 - r^{\frac{\gamma-1}{\gamma}}\right)} \tag{1.10}$$

This expression has its maximum value at the critical pressure ratio

$$r_c = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{1-\gamma}} \tag{1.11}$$

At lower ratios, the nozzle is choked and mass flow rate cannot increase further. Choked capacity is given by

$$\Gamma_c(A) = \sigma A\sqrt{\left(\frac{\gamma+1}{2}\right)^{\frac{2}{1-\gamma}} - \left(\frac{\gamma+1}{2}\right)^{\frac{1+\gamma}{1-\gamma}}} \tag{1.12}$$

It is shown that the capacity of one-dimensional nozzle flow is a function of only the flow's minimum area, provided the flow is choked and the ratio of specific heats is assumed constant.

The following chapters will analyse the capacity of two-dimensional and three-dimensional nozzle flows, presenting and discussing analytical techniques for applying the 1D capacity equation to 2D and 3D data. In such cases, capacity will be quantified using equation 1.7.

Chapter 2

Geometric throat area

A 1-dimensional supersonic nozzle is equivalent to a single streamline of variable cross-sectional area. It is possible to solve for the flow conditions throughout the nozzle, provided area is specified as a function of position along the nozzle. The mass flow rate capacity is a function of only the flow's minimum area, as in equation 1.12.

A 2-dimensional nozzle may be modelled as a group of adjacent streamlines, each of which may have dissimilar area functions. The streamlines' minimum areas may not correspond to a straight line across the narrowest part of the nozzle.

This is illustrated in Figure 2.1a by considering the division of a 2-dimensional flow into a finite number of streams of finite width. Each stream has an individual line of minimum width where sonic conditions exist. Collectively these lie on a line distinct from the overall passage line of minimum width. These two definitions of the effective throat line are marked.

If this concept is extended to infinitesimal streamlines (and the flow is isentropic) each streamline will experience sonic conditions at its point of minimum area, combining to form the 2-dimensional sonic line. The effective throat area of a 2-dimensional nozzle is thus the sum of its streamlines' throat areas. This is distinct from the sonic line length, and may be expressed by the integral

$$\int_1^2 \hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{r}} \, dL \tag{2.1}$$

where $\hat{v}$ is the unit vector of local flow velocity, $\hat{r}$ is the unit vector perpendicular to the local sonic line, and the integral is performed on the scalar infinitessimal dL over the length of the sonic line, as illustrated in Figure 2.1b.
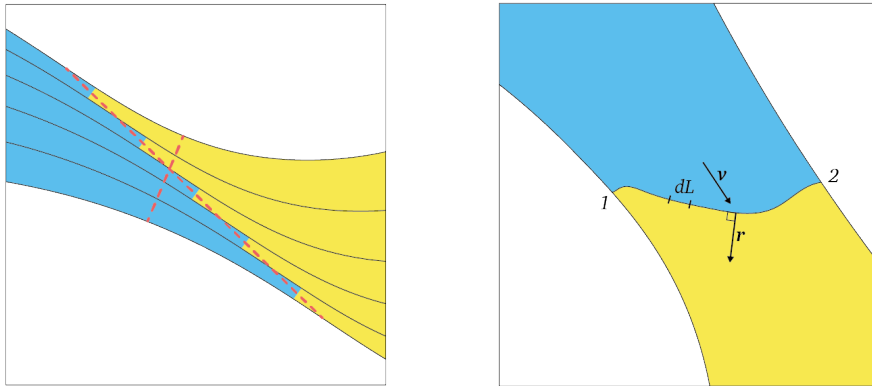
(a) Different minimum area points of dissimilar streams

(b) Integral for 2D nozzle equivalent throat area

Figure 2.1

–Review any definitions of minimum area from the literature. I haven't got any from the first group of papers from David.

2.1   1D vs 2D capacity uncertainty

– A family of 2D NGVs were run at various PRs in CFD. The point is to see how their capacity varies when plotted against various definitions of throat area. The usefulness of each definition will be discussed.

– Present boundary conditions

– Present mesh

– When the "throat area" of the 2D family is plotted against their capacity at a choking PR, it shows a pretty good proportional relationship. This is despite some obvious differences in the shape of the M1 line.

– Why are there still small deviations? That is, can I present a better definition of throat area that fits even better? Possible topics:

– Most likely it will follow the 1D rule if it is taken to be that streamline's M1 point. But not certain.

– the relationship between the minA line and the M1 line

– the total length of each M1 line

– a plot of capacity vs M1 line length

– Why aren't there larger deviations? That is, can I derive a sensible geometric parameter and show that it isn't changing significantly?

– Is this all really at design PR? What if the M1 lines were obtained at more choked PRs but I forgot to say so? What does the correlation look like at other PRs?

2.2   2D vs 3D capacity uncertainty

– 3D CFD data was shared by Rolls-Royce for comparison with the 2D data. The point is to see why 3D is still so different from 2D, ie what are we failing so badly to predict with 2D?

– Discuss the most important ways in which the 3D capacity trends are different from the 2D ones. From a superficial look it appears that the 3D trends never choke. What could this mean about their having a M1 line at all?

– What does the correlation look like at other PRs?

– Review 3D effects from the literature.

Chapter 3

Trailing edge

3.1 Uncertainties in the trailing edge shape (SS erosion) - how hard do they make it to predict capacity

– Uncertanties occur in the suction-side traling edge because of various factors like erosion through cooling failure (for example in desert environments) and manufacturing variations. If variations and life-cycle changes happen to the trailing edge, what effect on capacity can we expect, and why?

–Review the literature on what sort of erosion is expected, and show that itâĂŹs justified to look at a 2D midspan slice as I have done, because this the hottest bit of the part and more erosion would happen here anyway.

–2D CFD was run for 1 NGV with varying amounts of erosion simulated on the TE SS flange.

–Present boundary conditions

–Present mesh with larger and better pictures, and show how the mesh is altered as erosion amount varies

–Show capacity trend for all the data as an initial summary, then show capacity vs erosion amount for a couple of relevant PRs (perhaps design and fully choked)

–2D CFD predicts a maximum 2 percent change in capacity (when the flange is completely gone)

–What is changing in the flow as a result of the erosion, and is it driving capacity change via the expected route of throat area changes (whatever we've decided is the best definition of throat area in the previous chapter)

3.2 Centred-ejection option (PS cutbacks) (and could it reduce "loss"?)

–2D CFD was done on an alternative design of TE featuring centred coolant ejection. This shape of TE was also incrementally cut back on the pressure side so as to more resemble the existing TE design.

–Present boundary conditions

–Present mesh with larger and better pictures, and show how the mesh is altered as the design is altered

–Plot capacity trends for the centred-ejection designs vs the baseline

–Is there better capacity predictability because the effects of erosion are less pronounced due to the thicker flanges?

–Review the literature to decide what is "loss". –Jie Gao TE paper uses total pressure coefficient $Cpt = (p1t - p2t)/(p2t - p2)$

–Discuss whether this "loss" may be worse or better if a centred-ejection design is used, still perhaps plotting more than one definition of loss

–When using this configuration, is there an optimal blowing rate that might re-energise the base region? Plot blowing rate vs loss, remembering that the graph I currently have is erroneously for the other type of TE, not for the centred-ejection kind

–Compare my centred-ejection TE with a "naturally-formed by erosion" centred-ejection TE from the literature

Chapter 4

Leading edge

–It is sometimes necessary to add extra cooling holes on the suction-side at the LE.

–Coolant introduced into this part of the flow has complex interactions with the mainstream, affecting NGV capacity

–Review some relevant literature on the effects of film cooling on capacity

4.1   sensitivity of capacity to cooling holes on the leading suction side

–2D CFD was done on an XWB NGV with no coolant features apart from the addition of a single cooling row on the upstream suction side. The position of this featrure was varied, thus varying whereabouts in the flow the coolant was introduced.

–Present boundary conditions

–Present mesh, showing how the variable location coolant row is introduced

–Review the literature for why correctly modelling turbulence is important for capturing the mainstream/coolant mixing, and thus why two turbulence models are compared.

–Plot capacity changes versus hole position and discuss its causes, mentioning the literature reviewed in the chapter intro (plus the two definitions of capacity where coolant is concerned - which one is used in earlier chapters??) A maximum 0.5 percent change in capacity resulted from an unrealistically drastic shifting of a 2D cooling slot along the vaneâĂŹs suction side.

4.2 Consideration of corrections necessary to interpret 2D result

–Different:

–Mass flow rate between a 2D slot and a 3D row of holes

–Mixing between coolant and mainstream

–Angle of ejection