Homework 1

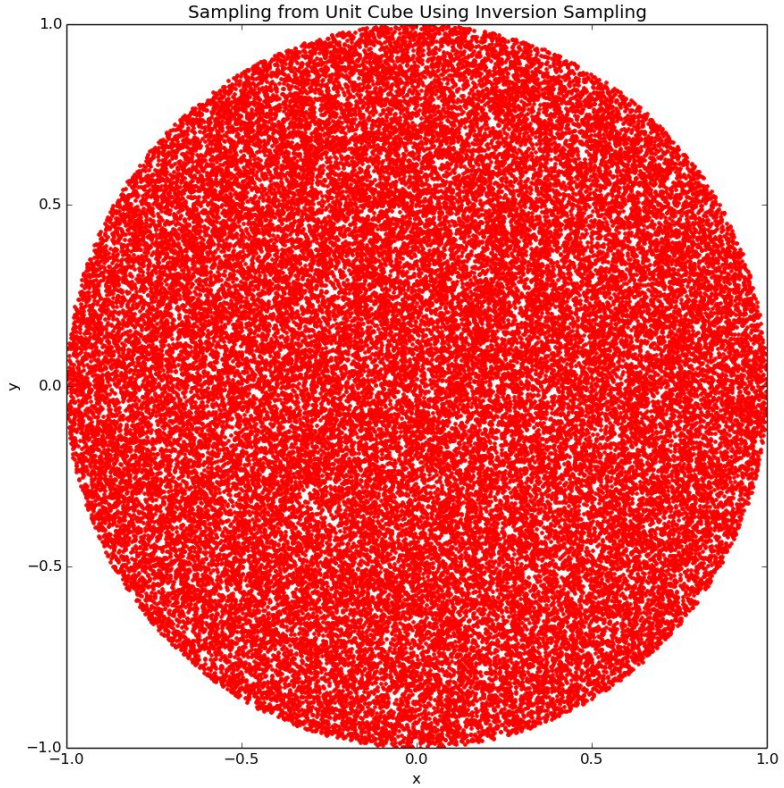
QUESTION 1) Source code is given in *InversionSampling.ipynb* file

In this question, we are asked to draw uniform samples from a circular region using inversion method.

- theta values are drawn from a uniform density and mapped into the interval of [0,2π].
- r^2 values also drawn from a uniform density. Then, in order to apply inversion method, their square roots are taken and used as r.

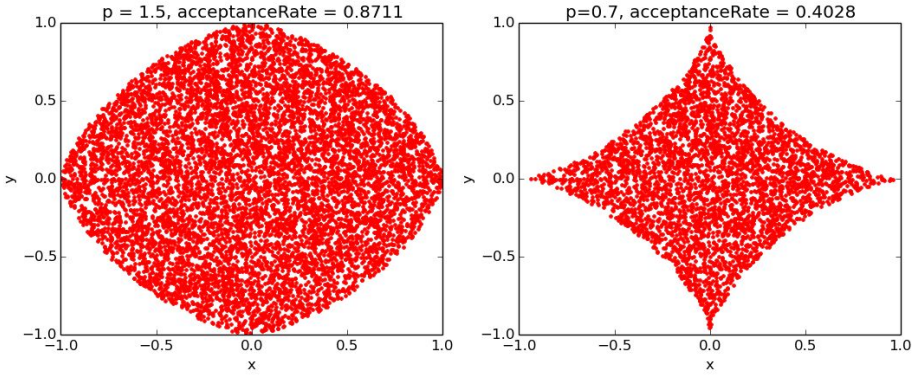
At this point, we have N many (r, theta) tuples. Application of polar to cartesian transformation is done by the *p2c(rad,thet)*.

After running for 50000 samples, the following illustration is generated:

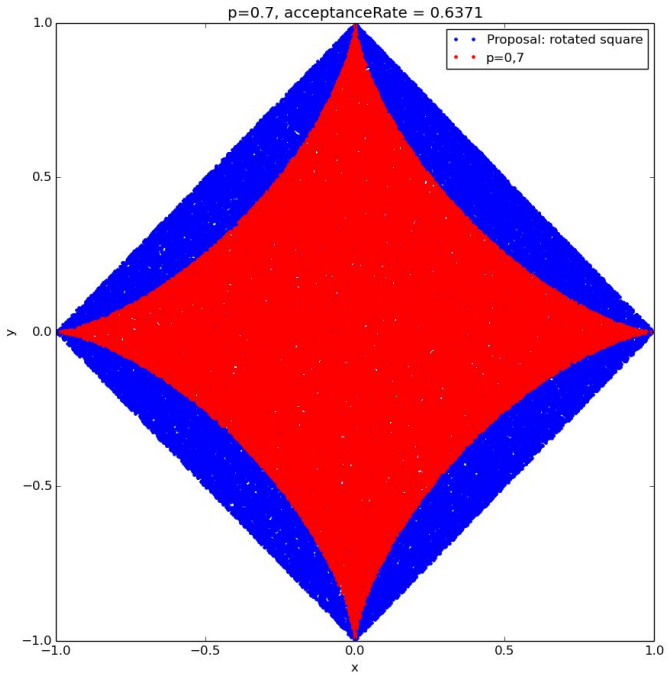


QUESTION 2) Source code is given in *RejectionSampling.ipynb* file

Using the samples generated with the previous method, we applied rejection sampling corresponding to the p-norm regions, given in the question. The implementation of the rejection functionality is in *pNorm(p, x, y)* method. Running with 10000 samples, the following illustration is generated (acceptance rates are given in the titles):



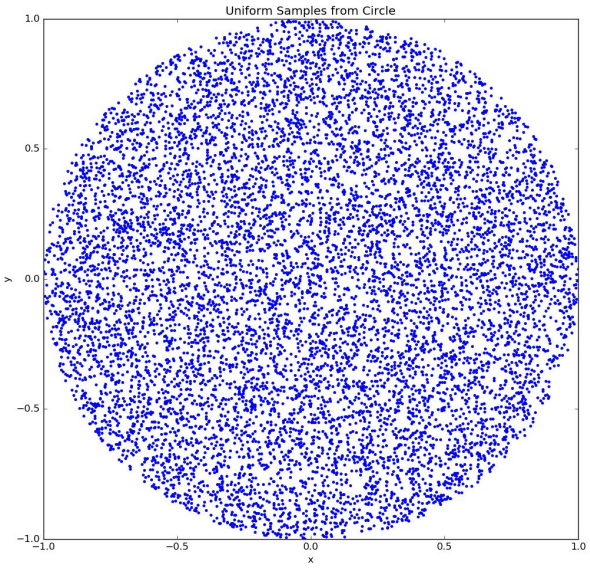
In order to improve the acceptance rate, another proposal is tried. A rotated square that contain the desired area is modeled and rejection sampling is conducted only on that area. In the following illustration, blue dots represent the rejected samples. We can see the increase in the acceptance rate numerically from the title as well: (Source code is in *RejectionSamplingWithDifferentProposal.ipynb*)



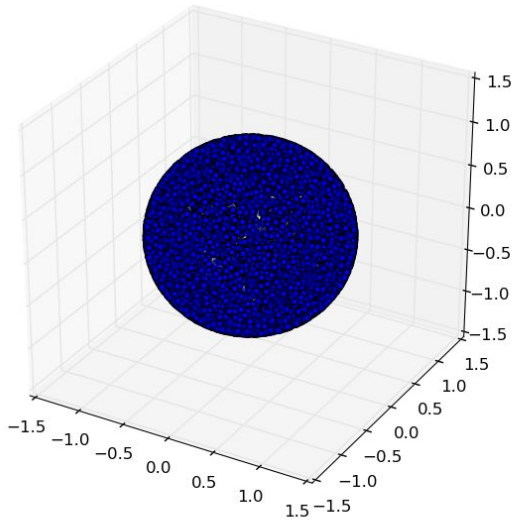
BONUS QUESTION 1) Source code is given in *GeneralizedInversionMethod.ipynb* file

The direction is obtained using n-variate gaussians with 0 mean and identity covariance matrix. After directions are taken from such normal distribution, they are normalized so that we are left with finding the distance to the origin. Again, inversion method is used. First, we sampled **r^dim** from uniform distribution and then we took their 1/n th power. After multiplying these r values with the dimension vectors, we obtained the samples. A couple of example illustrations are given below:

2D



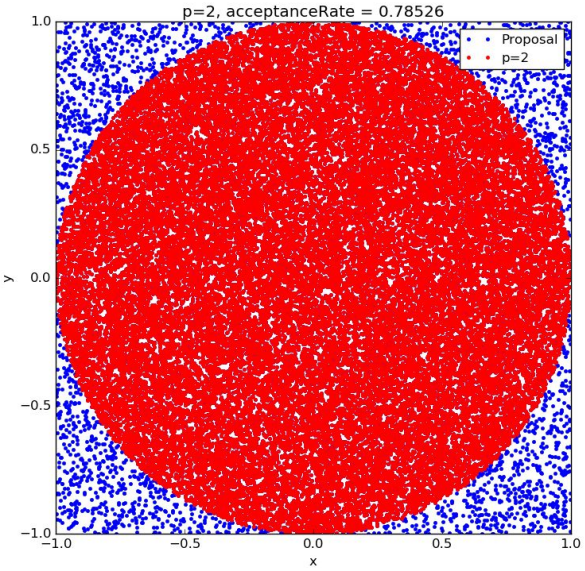
3D



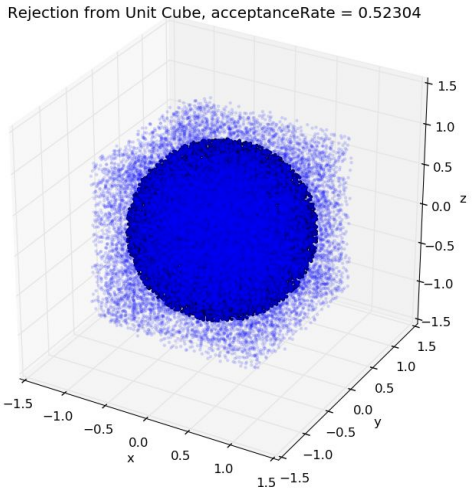
BONUS QUESTION 2) Source code is given in *GeneralizedRejectionSampling.ipynb* file

An n-cube with the edge length of 2 was created using random samples. This acted as the proposal, in this question. Then, the samples within the space bounded by the n-cube is evaluated for rejection sampling. For this question, 2D and 3D cases are illustrated as examples which are given below. Transparent points in the 3D depiction show the rejected samples.

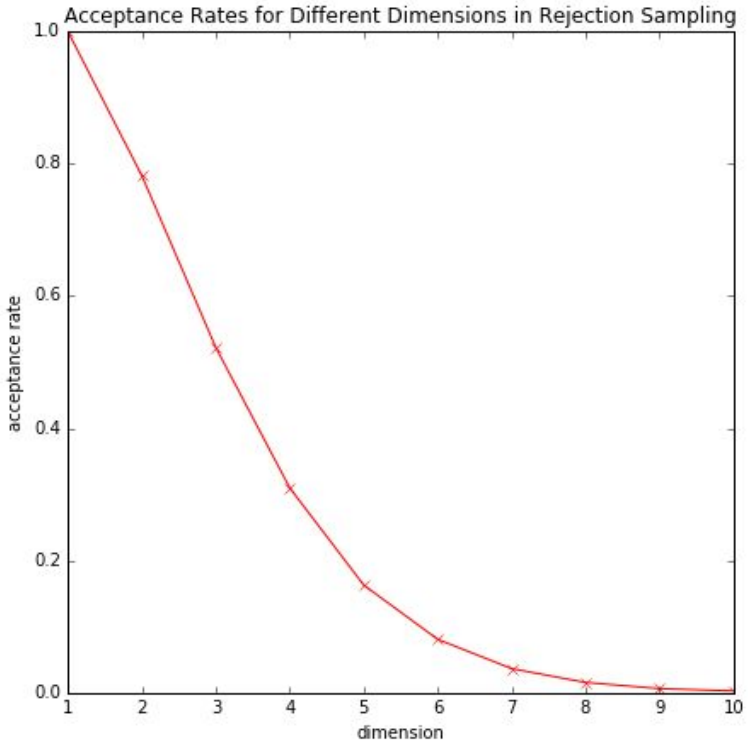
2D



3D



As can be seen from the titles, the acceptance rate drops significantly from 2D to 3D. It actually makes sense because as the dimension grows, most of the space bounded by n-cube accumulates in the corners. As the dimension grows, acceptance rate continues to decrease. The acceptance rates of the tests conducted with different numbers of dimensions are given in the following graph:



These rates are also verified using the volume ratios:

For example:

- **2D**: Area ratio: π/4 = 0.7854, my estimation: 0.78585
- **3D:** Volume ratio: 4π/24 = 0.5236, my estimation: 0.52336
- **10D:** Volume ratio: 0.0025, my estimation: 0.0026