Assignment 3

1) $K = \begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix}$

$x = hello \Rightarrow [(h,e);(l,l),(o,a)] = [(7,4),(11,11),(14,0)]$

$(7,4)\begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} = (125, 92) = (21, 14) \rightarrow v, o$

$(11,11)\begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} = (19, 5) \rightarrow t, f$

$(14,0)\begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} = (24, 8) \rightarrow y, i$

$y = votfyi$

2) $K = \begin{pmatrix} 11 & 8 \\ 3 & 7 \end{pmatrix}$

$\det K = \begin{vmatrix} 11 & 8 \\ 3 & 7 \end{vmatrix} = 11 \cdot 7 - 3 \cdot 8 = 77 - 24 = 53$

$53 \,\%\, 26 = 1$

$\gcd(1, 26) = 1$ ✓

$\Rightarrow \exists\, K^{-1}$

$K^{-1} = (\det K)^{-1} \begin{pmatrix} 7 & -8 \\ -3 & 11 \end{pmatrix}$

$(\det K)^{-1} = 1$, $\quad 1 \cdot 1 = 1 \cdot 1 = [1]_{26}$

$K^{-1} = \begin{pmatrix} 11 & -8 \\ -3 & 7 \end{pmatrix} = \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix}$

$y = *i\,y\,j \rightarrow 23, 8, 24, 9$

$(23, 8) \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix} = (345, 502) = (7, 8)$

$(24, 9) \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix} = (375, 531) = (11, 11)$

$\pi = hill$