Inhibitor Petri Nets

– addendum to CDC –

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1 Inhibitor Nets. Why ?

Two workers (robots) P_1 and P_2 want to produce certain goods in two steps: P_1 completes the first step, then stores the goods, and P_2 takes the goods and completes the second step. Suppose P_1 is allowed to take a break whenever he wants, but P_2 is allowed to do it only when no goods are stored.

The PN in Figure 1 is not a sound model of this problem because P_2 can

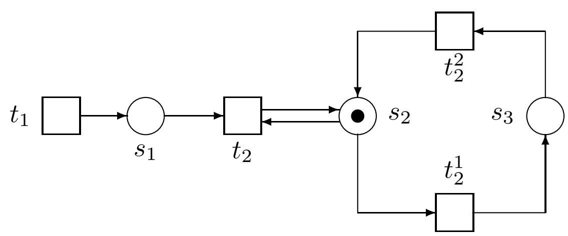

Figure 1

take a break when s_1 still holds goods. The meaning of the elements in the net are:

- t_1 – P_1;
- goods are stored in s_1;
- t_2 – P_2;
- t_2^1 – P_2 takes a break;
- t_2^2 – P_2 back to work.

Definition 1 An *Inhibitor Petri Net*, abbreviated IPN, is a pair $\gamma = (\Sigma, I)$, where Σ is a PN and $I \subseteq S \times T$ such that $F \cap I = \emptyset$.

Elements of I are represented as in Figure 2.

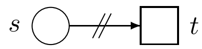

Figure 2

Definition 2 Let $\gamma = (\Sigma, I)$ be an inhibitor Petri net, M a marking of γ and $t \in T$.

(1) $M[t\rangle_{\gamma,i}$ if:

  (a) $M[t\rangle_{\Sigma}$;

  (b) $M(s) = 0$, for all $s \in S$ such that $(s, t) \in I$.

(2) $M[t\rangle_{\gamma,i}M'$ if $M[t\rangle_{\gamma,i}$ and $M[t\rangle_{\Sigma}M'$.

The IPN in Figure 3 is a sound model of the problem above.

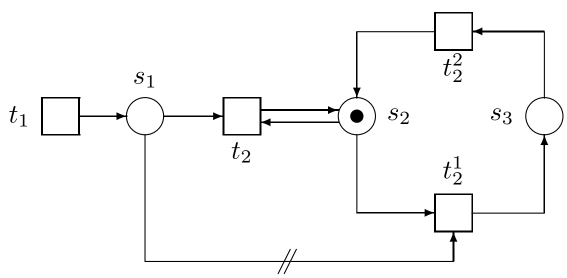

Figure 3

2 Inhibitor Nets and 1-inhibitor Nets

Definition 3 A *k-inhibitor Petri net* ($k \geq 0$), abbreviated k-IPN, is an inhibitor Petri net $\gamma = (\Sigma, I)$ such that $|\{s \in S|(s, t) \in I\}| \leq k$, for all $t \in T$.

Lemma 1 Let $\gamma = (\Sigma, I, M_0)$ be a marked inhibitor Petri net and $cod(I) = \{t_1, \ldots, t_r\}$, $r \geq 1$. Then, there exists an 1-inhibitor Petri net $\gamma' = (\Sigma', I', M_0')$ such that:

(1) $S' = S \cup \{s_{t_1}, \ldots, s_{t_r}\}$ ($s_{t_i} \notin S$ for any $1 \leq i \leq r$);

(2) $T' = T$;

(3) $I' = \{(s_{t_i}, t_i) | 1 \leq i \leq r\}$;

(4) $M'_0 = (\underbrace{M_0}_{S}, \underbrace{\beta_1}_{s_{t_1}}, \ldots, \underbrace{\beta_r}_{s_{t_r}})$, where,

$$\beta_i = \sum_{s \in S,\ (s,t_i) \in I} M_0(s),$$

for all $1 \leq i < r$;

(5) each reachable marking M' in γ' is of the form

$$M' = (\underbrace{M}_{S}, \underbrace{\alpha_1}_{s_{t_1}}, \ldots, \underbrace{\alpha_r}_{s_{t_r}}),$$

where M is reachable in γ and

$$\alpha_i = \sum_{s \in S,\ (s,t_i) \in I} M(s),$$

for all $1 \leq i \leq r$;

(6) if $M \in \mathbf{N}^S$ then, for every $w \in T^*$,

$$M_0[w\rangle_{\gamma,i} M \quad \Leftrightarrow \quad M'_0[w\rangle_{\gamma',i}(M, \alpha_1, \ldots, \alpha_r),$$

where $\alpha_1, \ldots, \alpha_r$ are as in (5);

(7) $FiringSequences(\gamma) = FiringSequences(\gamma')$.

Proof. Let $\gamma = (\Sigma, I, M_0)$ be an inhibitor net, $\Sigma = (S, T, F, W)$, and

$$cod(I) = \{t_1, \ldots, t_r\} \quad (r \geq 1).$$

The basic idea to construct γ' is:

- if transition t tests places $s_1, \ldots, s_k$ ($(s_i, t) \in I$, for all $1 \leq i \leq k$), then we consider a new place s_t such that for every marking M' of γ' the following holds:

  $$M'(s_t) = 0 \quad \Leftrightarrow \quad (\forall 1 \leq i \leq k)(M'(s_i) = 0).$$

  Then, it will be enough for t to test only s_t (in γ').

The definition of γ' is:

- $S' = S \cup \{s_{t_1}, \ldots, s_{t_r}\}$ (a new place s_{t_i} is associated to each transition $t_i \in cod(I)$, for any $1 \leq i \leq r$);

- $T' = T$;

- $F' = F \cup \bigcup_{t \in cod(I)} \{(s_t, t') | \exists s' \in S: \ (s', t) \in I \ \wedge \ (s', t') \in F\}$
$\qquad \cup \bigcup_{t \in cod(I)} \{(t', s_t) | \exists s' \in S: \ (s', t) \in I \ \wedge \ (t', s') \in F\};$

- $$W'(x,y) = \begin{cases} W(x,y), & \text{if } (x,y) \in F \\ W(s',t'), & \text{if } \exists t \in cod(I),\ \exists t' \in T',\ \exists s' \in S: \\ & (s',t) \in I,\ (s',t') \in F,\ (x,y) = (s_t, t') \\ W(t',s'), & \text{if } \exists t \in \ cod(I),\ \exists t' \in T',\ \exists s' \in S: \\ & (s',t) \in I,\ (t',s') \in F,\ (x,y) = (t', s_t); \end{cases}$$

- $I' = \{(s_t, t) | t \in cod(I)\};$

- $M'_0 = (M_0, \beta_1, \ldots, \beta_r)$, where

$$\beta_i = \sum_{s \in S,\ (s,t_i) \in I} M_0(s),$$

for all $1 \leq i \leq r$.

γ' is an 1-inhibitor net and satisfies (1)-(7). $\square$

3 Reachability, Coverability, Boundedness and Liveness for Inhibitor Petri Nets

Let $A = (Q, q_0, q_f, C, x_0, I)$ be a CM. Define an 1-inhibitor Petri net as follows:

- associate a place s_u to each $u \in Q \cup C$;
- associate a transition t to each instruction $I(q, c, q')$, as follows:

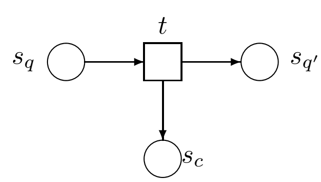

Figure 4

- associate two transitions t' and t'' to each instruction $I(q, c, q', q'')$, as follows:

A configuration $\sigma = (q, x)$ of A is simulated by the marking M given by:

$$\begin{array}{lcll} M_\sigma(s_q) & = & 1, & \\ M_\sigma(s_{q'}) & = & 0, & \forall q' \in Q - \{q\}, \\ M_\sigma(s_c) & = & x(c), & \forall c \in C. \end{array}$$

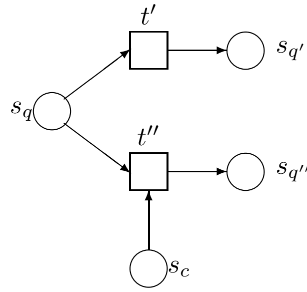

Figure 5 æ

Let M_0 be the marking corresponding to the initial configuration. Let J be the set of pairs (s_c, t').

The net $\gamma = (\Sigma, J, M_0)$ such defined is an 1-inhibitor Petri net. Moreover:

(*) $\sigma = (q, x)$ is reachable in A from $\sigma_0 = (q_0, x_0)$ iff M_σ is reachable in γ from M_0.

Define now 3 new nets obtained from γ as follows.

Let Σ_1 be the net in Figure 6, where $t_1, \ldots, t_r$ are new transitions associated to places $s \in S - \{q_f\}$ (one transition to each place).

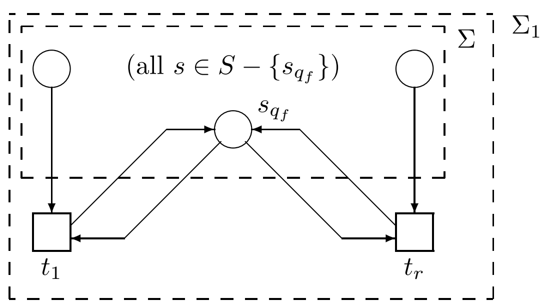

Figure 6

Let Σ_2 be the net in Figure 7, where s^* is a new place.

Let Σ_3 be the net in Figure 8, where t^* is a new transition.

Let $\gamma_i = (\Sigma_i, J, M_0^i)$, where

$$M_0^1 = M_0^3 = M_0$$

and

$$M_0^2(y) = \begin{cases} M_0(y), & y \in S \\ 0, & y = s^* \end{cases}$$

γ_i is an 1-inhibitor net. Moreover:

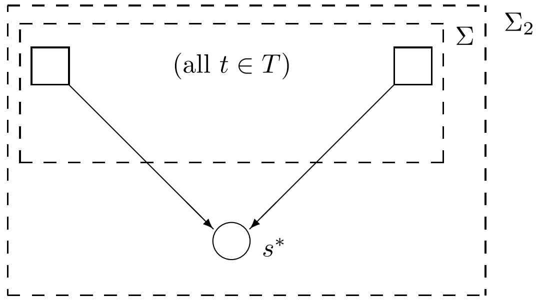

Figure 7

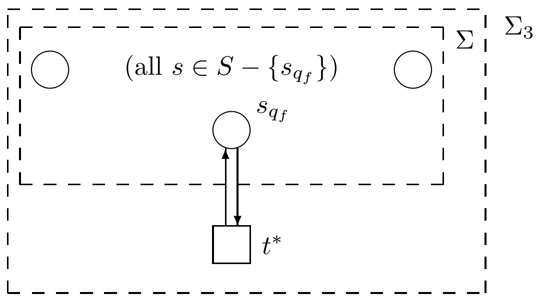

Figure 8

(1) A marking M such that $M(s_{q_f}) = 1$ is reachable in γ iff A halts;

(2) The marking $M(y) = \begin{cases} 1, & y = s^* \\ 0, & \text{otherwise} \end{cases}$ is coverable in γ_1 iff A halts;

(3) γ_2 is bounded iff A halts;

(3) t^* (in γ_3) is live iff A halts.

Theorem 1 The reachable, coverability, boundedness and liveness problem are all undecidable for inhibitor Petri nets.

Theorem 2 The family of languages generated by (labeled) inhibitor Petri nets equals the family of type 0 Chomsky languages (or, the family of languages accepted by Turing machines).