Умножение матриц.

$$\underbrace{\begin{pmatrix} 2 & 3 & -1 \end{pmatrix}}_{1\times 3}\underbrace{\begin{pmatrix} 2 \\ -5 \\ -6 \end{pmatrix}}_{3\times 1} = 2\cdot 2 + 3\cdot(-5) + (-1)\cdot(-6) =$$

$$= 4 - 15 + 6 = \underbrace{-5}_{1\times 1}$$

$$2\times 5 \cdot 5\times 3 = 2\times 3$$

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 1 & -1 & 0 \\ 2 & -2 & 1 \end{pmatrix}}_{3\times 3}\underbrace{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}}_{3\times 1} = \underbrace{\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}}_{3\times 1}$$

$$\underbrace{\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}}_{2\times 2}\underbrace{\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}}_{2\times 2} = \begin{pmatrix} 5 & 5 \\ 0 & -5 \end{pmatrix}$$

$$\underbrace{i\begin{pmatrix} \quad \end{pmatrix}}_{m\times k}\underbrace{\begin{array}{c} j \\ \begin{pmatrix} \quad \end{pmatrix} \end{array}}_{k\times n} = \underbrace{\begin{pmatrix} ij \end{pmatrix}}_{m\times n}$$

Задачка 1

$$A\cdot B \neq B\cdot A$$

$$\begin{pmatrix} x & y \end{pmatrix}\underbrace{\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}} = \begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} 2x+3y \\ 4x+5y \end{pmatrix} =$$

$$= \underbrace{2x^2 + 3xy} + \underbrace{4xy + 5y^2}$$

$$(x\ y)\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= a_{11}x^2 + (a_{12}+a_{21})xy + a_{22}y^2$$

$$(x\ y)\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = a_{11}x^2 + 2a_{12}xy + a_{22}y^2$$

$A^T = A$ - симметр. матрица.

$A^T = -A$ - кососимм.

$$\Rightarrow A = \begin{pmatrix} 0 & a_{12} & \dots & a_{1n} \\ -a_{12} & \ddots & & \\ -a_{1n} & \dots & & 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}^T = \begin{pmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & & a_{n2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & & a_{nn} \end{pmatrix}$$

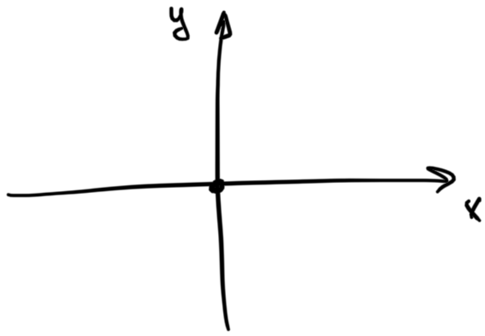
Кривые второго порядка

$$\underbrace{a_{11}x^2 + 2a_{12}xy + a_{22}y^2} + \underbrace{a_1 x + a_2 y} + \underbrace{a_0} = 0$$

$$(x\ y)\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + (a_1\ a_2)\begin{pmatrix} x \\ y \end{pmatrix} + a_0 = 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$$

$x, y \in \mathbb{R}$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0, \quad x, y \in \mathbb{C}$$

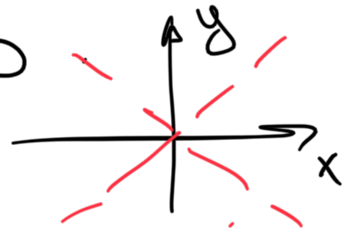
$\mathbb{C} \sim \mathbb{R}^2$

$\mathbb{C}^2 \sim \mathbb{R}^4$

$$\frac{x^2}{a^2} - \frac{(iy)^2}{b^2} = 0$$

$$\left(\frac{x}{a} - \frac{iy}{b}\right)\left(\frac{x}{a} + \frac{iy}{b}\right) = 0$$

$$\frac{x}{a} = \pm iy/b$$



$$(x\ y)\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + (a_1\ a_2)\begin{pmatrix} x \\ y \end{pmatrix} + a_0 = 0$$

1) Поворот на α: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$$(AB)^T = B^T A^T$$

$$\underset{1\times 2}{(x \;\; y)} = \underset{1\times 2}{(x_1 \;\; y_1)} \underset{2\times 2}{\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}}$$

$$(x_1 \;\; y_1) \underbrace{\overbrace{\overset{T^T}{\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}} \overset{A}{\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}} \overset{T}{\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}}}}_{A'=} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} +$$

$$+ \; (a_1 a_2) \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + a_0 = 0$$

$$\underbrace{2a_{12}\cos 2\alpha + (a_{22} - a_{11})\sin 2\alpha}_{} = 0$$

элемент, который стоит на месте 12 матрицы A'

$$\Rightarrow \quad \operatorname{tg} 2\alpha = \frac{2a_{12}}{a_{11} - a_{22}}$$

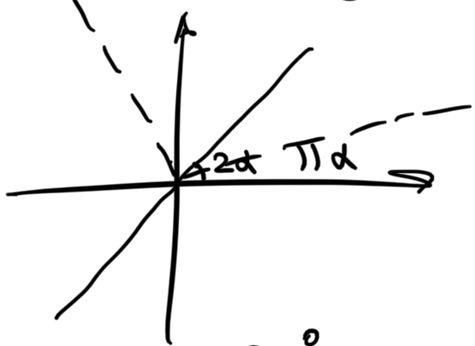
1) $a_{11} = a_{22} \Rightarrow \alpha = \pi/4$

2) $a_{11} \neq a_{22} \Rightarrow \quad \cos 2\alpha = \pm \dfrac{1}{\sqrt{1 + \operatorname{tg}^2 2\alpha}}$,

знак берём такой же как $\operatorname{tg} 2\alpha$

$$\cos\alpha = \sqrt{\frac{\cos 2\alpha + 1}{2}}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$



α и $\alpha + 90^\circ$

№ 807 (1) $5x^2+4xy+8y^2-32x-56y+80=0$

1) Поворот

$$\operatorname{tg}2\alpha=\frac{4}{5-8}=-4/3,\quad \cos 2\alpha=-\sqrt{\frac{1}{1+\operatorname{tg}^2 2\alpha}}=$$

$$=-\sqrt{\frac{1}{1+16/9}}=-3/5$$

$$\cos\alpha=\sqrt{\frac{-3/5+1}{2}}=\frac{1}{\sqrt5},\quad \sin\alpha=2/\sqrt5\ \Rightarrow$$

$$T=\frac{1}{\sqrt5}\begin{pmatrix}1&-2\\2&1\end{pmatrix}$$

$$A'=T^TAT=\frac{1}{\sqrt5}\begin{pmatrix}1&2\\-2&1\end{pmatrix}\begin{pmatrix}5&2\\2&8\end{pmatrix}T=$$

$$=\frac{1}{\sqrt5}\begin{pmatrix}9&18\\-8&4\end{pmatrix}\cdot\frac{1}{\sqrt5}\begin{pmatrix}1&-2\\2&1\end{pmatrix}=\frac{1}{5}\begin{pmatrix}45&0\\0&20\end{pmatrix}=$$

$$=\begin{pmatrix}9&0\\0&4\end{pmatrix}$$

$$(a_1'\ a_2')=(a_1\ a_2)T=(-32\ -56)\cdot\frac{1}{\sqrt5}\begin{pmatrix}1&-2\\2&1\end{pmatrix}=$$

$$=-8/\sqrt5\,(4\ 7)\begin{pmatrix}1&-2\\2&1\end{pmatrix}=-8/\sqrt5\,(18\ -1)$$

Ур-е кривой в „новой" с.к.:

$$9x_1^2+4y_1^2-\frac{8\cdot18}{\sqrt5}x_1+\frac{8}{\sqrt5}y_1+80=0$$

2) Пар-ный перенос

Выделяем полные квадраты

$$9\left(x_1^2-\frac{8\cdot 2}{\sqrt{5}}x_1+\frac{64}{5}\right)+4\left(y_1^2+\frac{2}{\sqrt{5}}y_1+\frac{1}{5}\right)-$$

в квадрат

$$-\frac{9\cdot 64}{5}-\frac{4}{5}+80=0$$

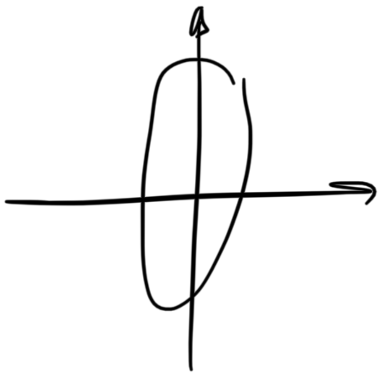
$$9\left(x_1-\frac{8}{\sqrt{5}}\right)^2+4\left(y_1+1/\sqrt{5}\right)^2=36$$

Замена $\begin{cases} x_2 = x_1 - 8/\sqrt{5} \\ y_2 = y_1 + 1/\sqrt{5} \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \frac{1}{\sqrt{5}}\begin{pmatrix} 8 \\ -1 \end{pmatrix}$

$$9x_2^2+4y_2^2=36$$

$$\frac{x_2^2}{4}+\frac{y_2^2}{9}=1$$

!



3) Повёрнем на 90°.

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$$

т.е. $\begin{cases} x_2 = -y_3 \\ y_2 = x_3 \end{cases}$

$\Rightarrow \dfrac{x_3^2}{9}+\dfrac{y_3^2}{4}=1$ – канон. ур-е

Формула перехода:

$$\begin{pmatrix} x \\ y \end{pmatrix} = 1/\sqrt{5} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 1/\sqrt{5} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \left(\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + \frac{1}{\sqrt{5}} \begin{pmatrix} 8 \\ -1 \end{pmatrix} \right) =$$

$$= 1/\sqrt{5} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + 1/5 \begin{pmatrix} 10 \\ 15 \end{pmatrix} =$$

$$= 1/\sqrt{5} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} =$$

$$= 1/\sqrt{5} \begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Канон. с.к:

$$O' = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$e_1' = 1/\sqrt{5} \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad e_2' = \frac{1}{\sqrt{5}} \begin{pmatrix} -1 \\ -2 \end{pmatrix}$$

№ 807 (3) $\quad x^2 - 4xy + 4y^2 + 4x - 3y - 7 = 0$

1) ....

$$5y_1^2 + \sqrt{5}x_1 - 2\sqrt{5}y_1 - 7 = 0$$

2) $5\left(y_1^2 - \frac{2}{\sqrt{5}}y_1 + 1/5\right) - 1 + \sqrt{5}x_1 - 7 = 0$

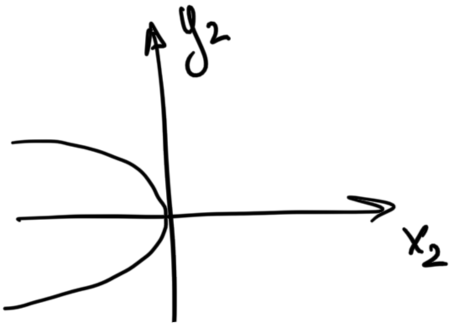
$$5(y_1 - 1/\sqrt{5})^2 = -\sqrt{5}x_1 + 8$$

$$5(y_1 - 1/\sqrt{5})^2 = -\sqrt{5}\left(x_1 - \frac{8}{\sqrt{5}}\right) \quad !$$

Замена: $\begin{cases} y_2 = y_1 - 1/\sqrt{5} \\ x_2 = x_1 - 8/\sqrt{5} \end{cases}$

$\Rightarrow \quad 5y_2^2 = -\sqrt{5}\,x_2$

$y_2^2 = -\frac{1}{\sqrt{5}}\,x_2$



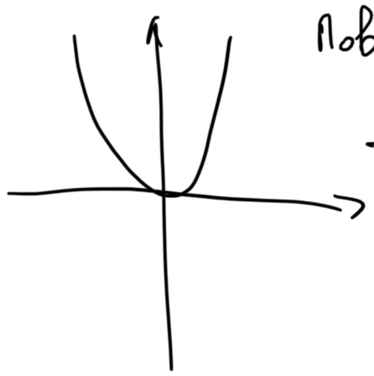
3) Поворот $180°$:

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} \Rightarrow y_3^2 = \frac{1}{\sqrt{5}}\,x_3$$

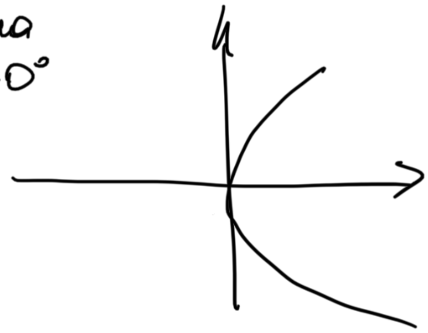
$$\begin{pmatrix} x \\ y \end{pmatrix} = T \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$F(5,0)$ в „новой" с.к

$$\Rightarrow \quad \begin{pmatrix} x \\ y \end{pmatrix} = T \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$



Поворот на $270°$ $\longrightarrow$



$2xy + 2x - 3y + 5 = 0$

1) $\alpha = \pi/4$ $\qquad T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$$A' = T^T A T = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} T =$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow 2xy \rightarrow x_1^2 - y_2^2$$