Статистическая физика

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$$\delta E = \delta Q + \delta A = TdS - PdV$$

– для квазиравновесных процессов.

$C_V(T,V)$, $P(T,V)$ – всё, что нужно знать о газе. Но они зависимы. (есть связь)

Задача – находить св-ва тел, исходя из микроскопических параметров.

В 1 см^3 $N \sim 10^{19}$ частиц.

$\Psi(q_1, \dots q_s, t)$ – пусть мы её знаем, – мало пользы от неё.

Система не бывает замкнутой $\Psi(q, Q, t)$.

$\psi_E(q)$ – вер-сть находится в этом состоянии $W(E) = \left|\int \psi_E^*(q)\Psi(q,Q,t)\,dQ\,dq\right|^2$

Вводим макро- и микроскопич. описание

E, x - макроскоп. параметры

Стат. вес $\Gamma(E,x)$ - это число макроскопических сост-й, соотв. этим макроскоп. параметрам

Идеальный газ

$$\sum \frac{\vec{p}^2}{2m} = E \qquad s = 3N$$

- это ур-е сферы радиуса $\sqrt{2mE}$ в пр-ве размерности s

$$\Gamma \sim R^{s-1} \sim E^{\frac{s}{2}-\frac{1}{2}}$$

$S = \ln \Gamma$ - статист. энтропия

$\boxed{1}$ $\boxed{2}$ Для 2-х тел $\Gamma = \Gamma_1 \cdot \Gamma_2 \Rightarrow S = S_1 + S_2$

Постулат: все микроскопические состояния равновероятны. (Больцман).

Стат. вес равновесного сост. $\gg$ ст. веса неравновесного сост.

Равновесие: $\frac{\partial S}{\partial x} > 0$ - переход к равновесию

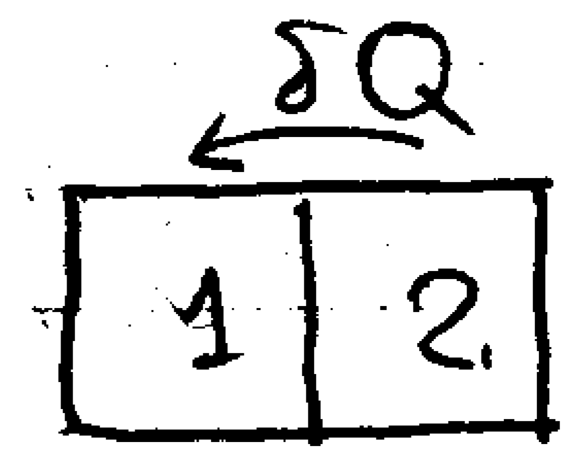
$\downarrow$

$\frac{\partial S}{\partial x} = 0.$

Температура.

$x \equiv E_1$

$E_2 = E - E_1$.



$E_1 + E_2 = E = const$

К Равновесию:

$$S(E, E_1) = S_1(E_1) + S_2(E - E_1).$$

$$\delta S = \frac{\partial S_1}{\partial E_1} \cdot dE_1 + \frac{\partial S_2}{\partial E_2} dE_2 = \qquad \frac{\partial S_i}{\partial E_i} = \beta_i$$

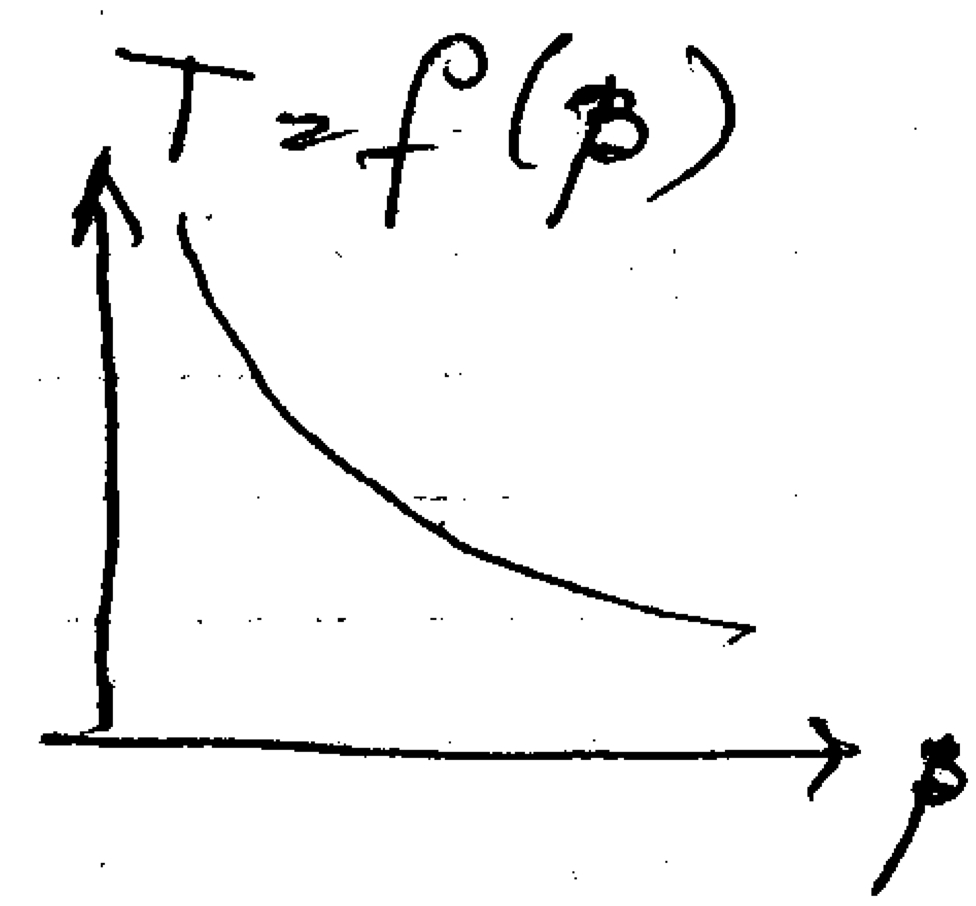
$$= \beta_1 dE_1 - \beta_2 dE_1 = (\beta_1 - \beta_2) dE_1 \geq 0$$

$$(\beta_1 - \beta_2)\, \delta Q_{2\to 1} > 0 \Rightarrow$$

Если $\beta_1 > \beta_2$, $\delta Q > 0$

Если $\beta_1 < \beta_2$, $\delta Q < 0$

Гипотеза: $\beta = \frac{1}{T}$.



$f(\beta)$ универсальна.

Равновесие: $\beta_1 = \beta_2$.

Пример. Ид. газ.

$$S = \ln \Gamma = \frac{3N}{2} \ln E + const.$$

$$\beta = \frac{\partial S}{\partial E} = \frac{3N}{2E} \qquad E = \frac{3}{2}\frac{N}{\beta} = \frac{3}{2} NkT$$

$$\longrightarrow \quad kT = \frac{1}{\beta}.$$

$kT \doteq T$, $[T] = $ Дж.

$k = 1{,}38 \cdot 10^{-23}$ Дж/К $= \frac{1}{11600} \frac{эВ}{К}$

$\frac{\partial S}{\partial E} = \frac{1}{T} \sim dE = TdS = \delta Q$

$\Rightarrow S$ - энтропия

$dQ = kT^\circ dS = T^\circ d(kS)$

$S' = kS$.

У нас энтропия будет безразмерно,

$[T] = $ Дж,

Равновесие: $T_1(E_{10}) = T_2(E_{20})$.

$\sum_{E_1} \Gamma(E, E_1) = \Gamma(E)$

$\Gamma(E, E_{10}) = \Gamma_{max}(E, E_1)$.

Точность δE, число слагаемых $\approx \frac{E}{\delta E}$

$\frac{E}{\delta E} \sim N$

Оценка снизу: $\Gamma > \Gamma_{max}$

Сверху: $\Gamma < \Gamma_{max} \cdot \frac{E}{\delta E}$

$\ln \Gamma_{max} < S < \ln \Gamma_{max} + \underbrace{\ln N}_{\ll \ln \Gamma_{max}}$

$\Rightarrow S \approx \ln \Gamma_{max}$

$\Rightarrow$ энтропию можно складывать

$$W(E_1) = \begin{cases} E < E_1 < E + \delta E : & 1/\Gamma \\ E_1 \notin (E, E+\delta E) : & 0 \end{cases}$$

— микроканоническое распр.

для замкнутой система,

$$dw = A \cdot \delta(E - H(q,p))\, dp\, dq$$

$$q = q_1, \ldots q_s$$
$$p = q_1, \ldots q_s,$$

$$S = \ln \Gamma = -\ln w$$

$$(S = -k \ln w)$$

Обобщение: $\quad S = -\langle \ln w \rangle = -\sum_i w_i \ln w_i$

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Еще пример.

ε ———
0 ——— — два уровня энергии,

N частиц, $N, L \gg 1$

$$E = L\varepsilon. \qquad \Gamma = C_N^L = \frac{N!}{L!(N-L)!}$$

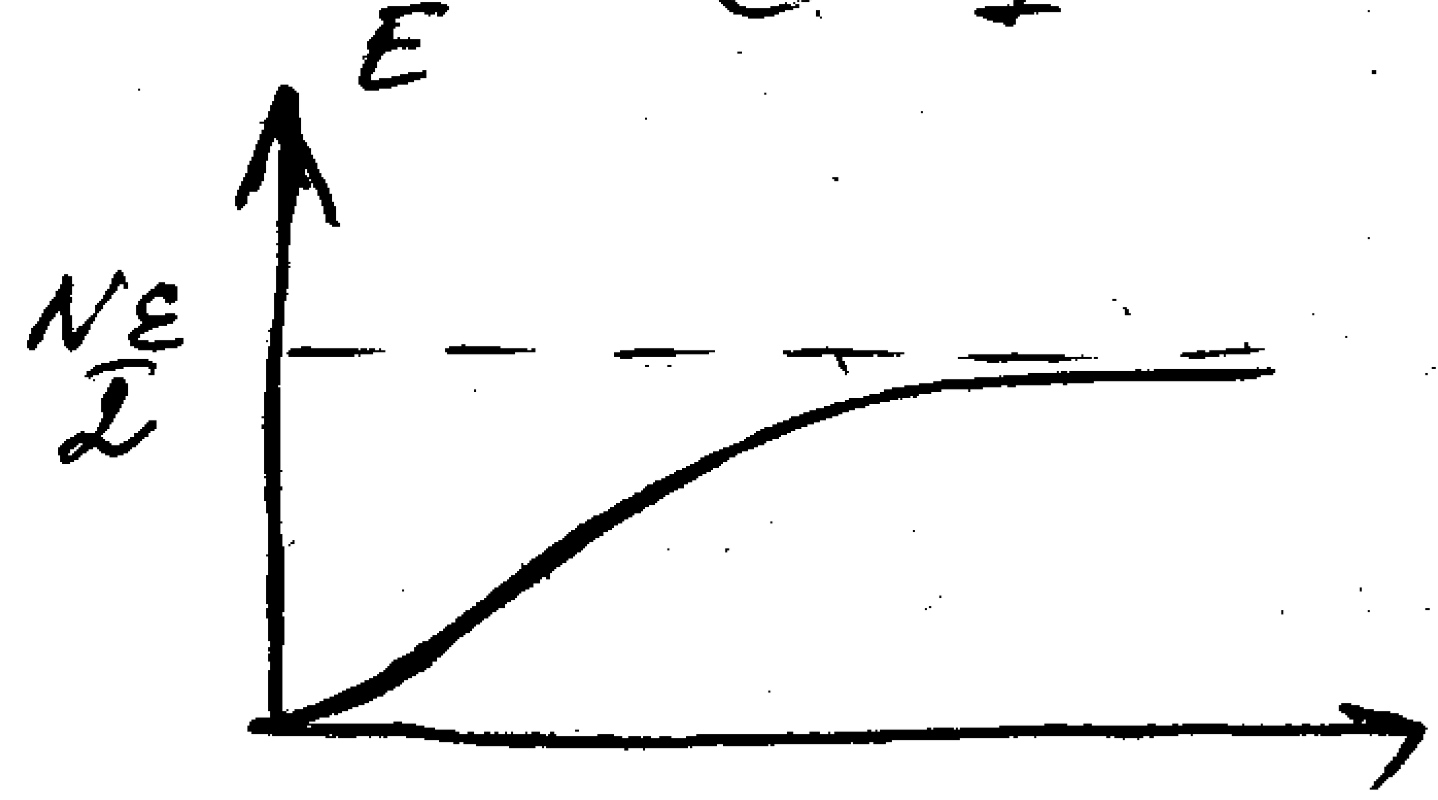
$$S = \ln \Gamma = \ln N! - \ln L! - \ln (N-L)!$$

$$\frac{\partial S}{\partial E} = \beta. \qquad \frac{\partial}{\partial L} \ln L! = \ln L$$

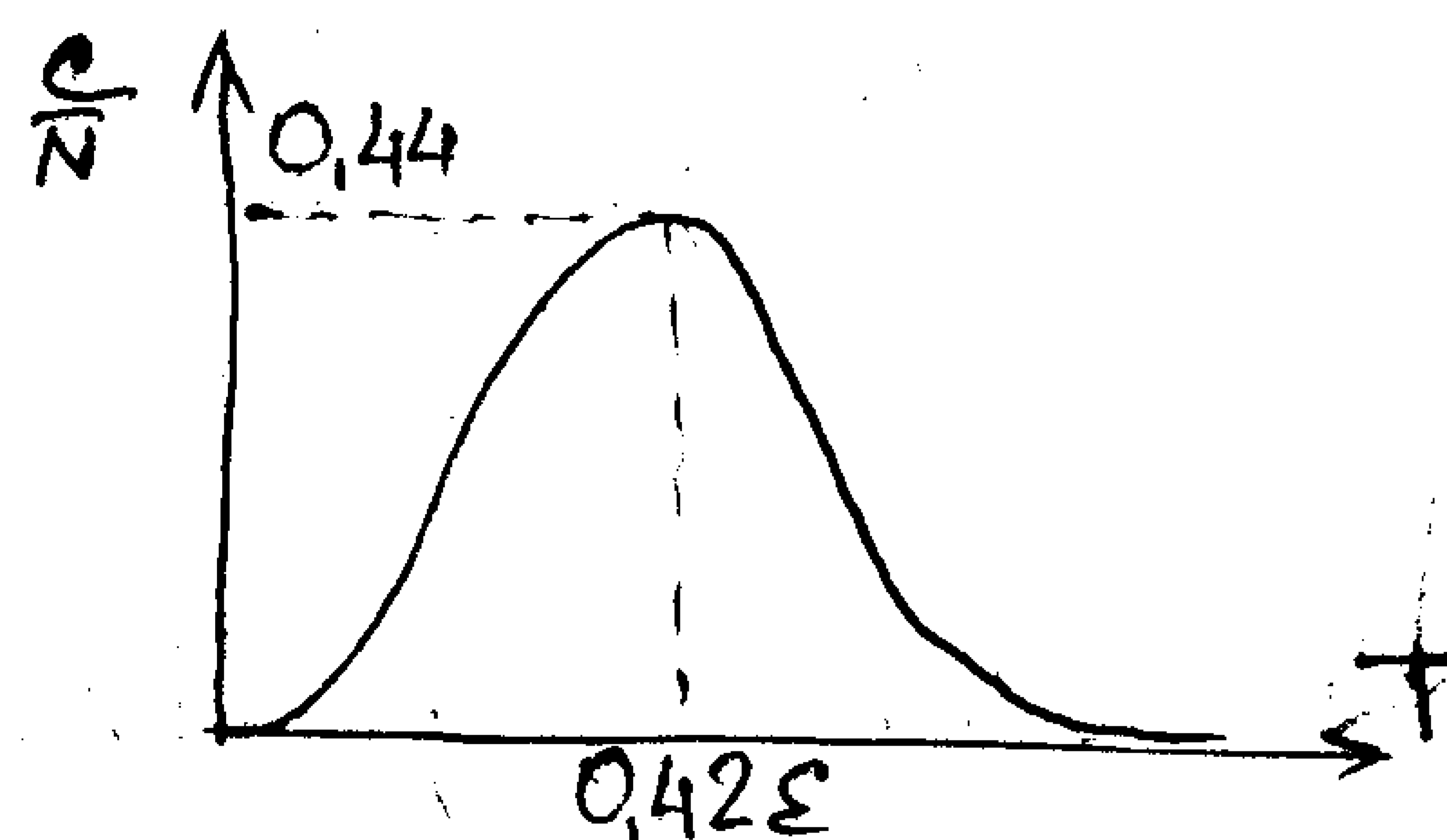
$$\frac{\partial S}{\partial E} = \frac{1}{\varepsilon} \frac{\partial S}{\partial L} = \frac{1}{\varepsilon}\left(-\ln L + \ln(N-L)\right) = \frac{1}{T}$$

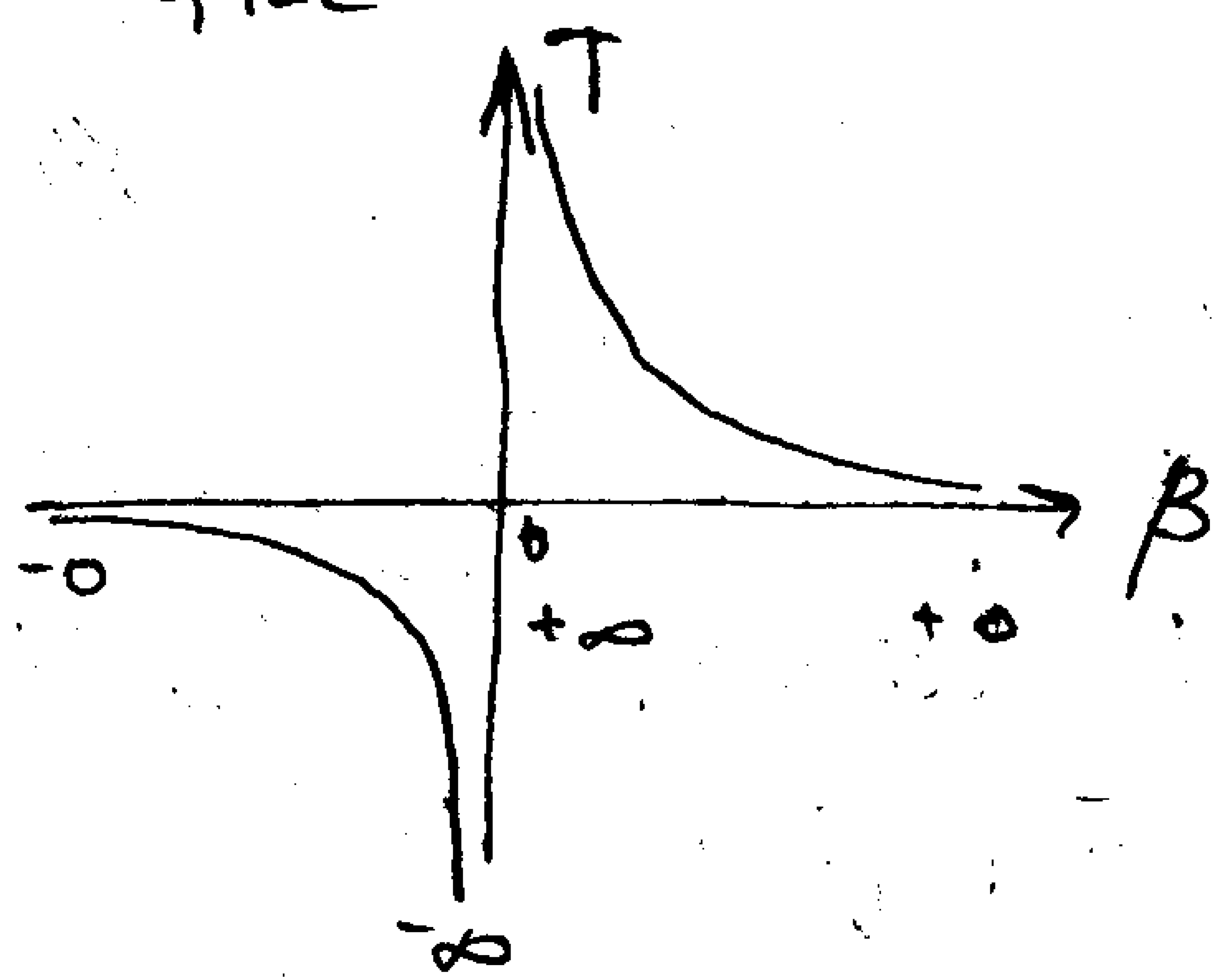
$$\frac{L}{N-L} = e^{-\frac{\varepsilon}{T}}$$ — получим известное соотношение.

$$L = \frac{N}{e^{\frac{\varepsilon}{T}}+1}, \quad E = \frac{N\varepsilon}{e^{\frac{\varepsilon}{T}}+1}$$



$$C = \frac{dE}{dT} = \frac{N\varepsilon\cdot\left(+\frac{\varepsilon}{T^2}\cdot e^{\frac{\varepsilon}{T}}\right)}{\left(e^{\frac{\varepsilon}{T}}+1\right)^2} = \left(\frac{\varepsilon}{T}\right)^2 N\cdot\frac{1}{\left(e^{\frac{\varepsilon}{T}}+1\right)}$$





Тело с отриц. температурой горячее, чем с положит.

Обобщенные силы в термодинамическо[й] системе.

$S(x,\lambda)$. λ – внешние параметры

$TdS = \delta Q$, $dE = TdS + \delta A$, $\delta A = -PdV$ для газа

$\langle E\rangle = \sum w_k E_k$, $\frac{\partial E_k}{\partial \lambda} = \Lambda_k$

$\left(\frac{\partial \langle E\rangle}{\partial \lambda}\right)_S = \sum w_k \Lambda_k = \langle\Lambda\rangle$, w_k не меняются

$dS = (\beta_1 - \beta_2) dE_{1-2} = 0$

$d\langle E\rangle = TdS + \langle\Lambda\rangle d\lambda$

$\beta_1 = \beta_2$.

1. Ид. газ. $\Gamma = A E^{\frac{3N}{2}} V^N$

$S = \frac{3N}{2}\ln E + N\ln V + S_0$

$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V = \frac{3}{2}\frac{N}{E} \Rightarrow E = \frac{3}{2}NT$

$\frac{\partial S}{\partial V} = 0 = \frac{3N}{2}\frac{1}{E}\cdot\frac{\partial E}{\partial V} + \frac{N}{V}$

$\left(\frac{\partial E}{\partial V}\right)_S = -P = -\frac{2E}{3V} = -\frac{NT}{V}$

2. $\uparrow \vec{\mathcal{H}}$ N, μ $\mathcal{E}_\uparrow = -\mu\mathcal{H}$, $\mathcal{E}_\downarrow = \mu\mathcal{H}$,

$N_\uparrow + N_\downarrow = N$

$\uparrow\downarrow$ $E = N_\uparrow \mathcal{E}_\uparrow + N_\downarrow \mathcal{E}_\downarrow = -(N_\uparrow - N_\downarrow)\mu\mathcal{H}$

$\mathcal{H} \leftarrow \lambda$ какая обобщ. сила?

$? \leftarrow \Lambda$ $\Gamma = C_N^{N_\uparrow}$; $S = const$, $N_{\uparrow\downarrow} = const$.

$$\Lambda = \left(\frac{\partial E}{\partial \mathcal{H}}\right)_S = -\mu(N_\uparrow - N_\downarrow) = -\mathcal{M}$$

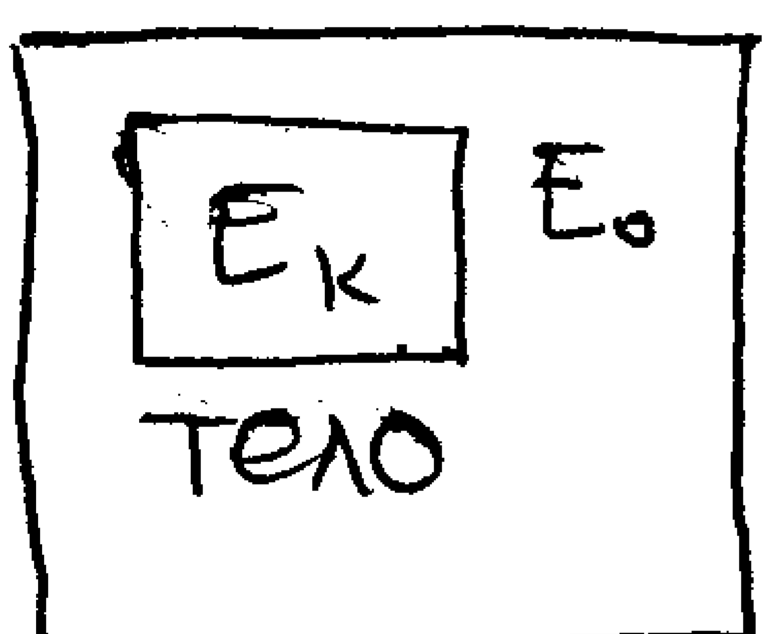
$$dE = TdS - \mathcal{M}\, d\mathcal{H}$$

$$\frac{N_\uparrow}{N_\downarrow} = e^{\frac{2\mu\mathcal{H}}{T}}$$

Если мы меняем $\mathcal{H}$ при $S = const$ — меняется T! Даже $T < 0$.

Канонические распределения

Распределение Гиббса



$$E_k + E_0 = E_{полн} \qquad V = const$$

$$W(|k\rangle) - ?$$

$$E_0 = E_{полн} - E_k$$

термостат $\Gamma_{полн} = \Gamma_{тела}\Gamma_{терм} = \Gamma(|k\rangle)\Gamma_0(E_0) =$

$$= 1 \cdot \Gamma_0(E_{полн} - E_k) = e^{S_0(E_{полн} - E_k)}$$

$$dE = TdS, \quad V = const, \quad \frac{\partial S}{\partial E} = \frac{1}{T}, \quad \frac{\partial^2 S}{\partial E^2} = -\frac{1}{T^2}\left(\frac{\partial T}{\partial E}\right)_V =$$

$$= -\frac{1}{C_V T^2}$$

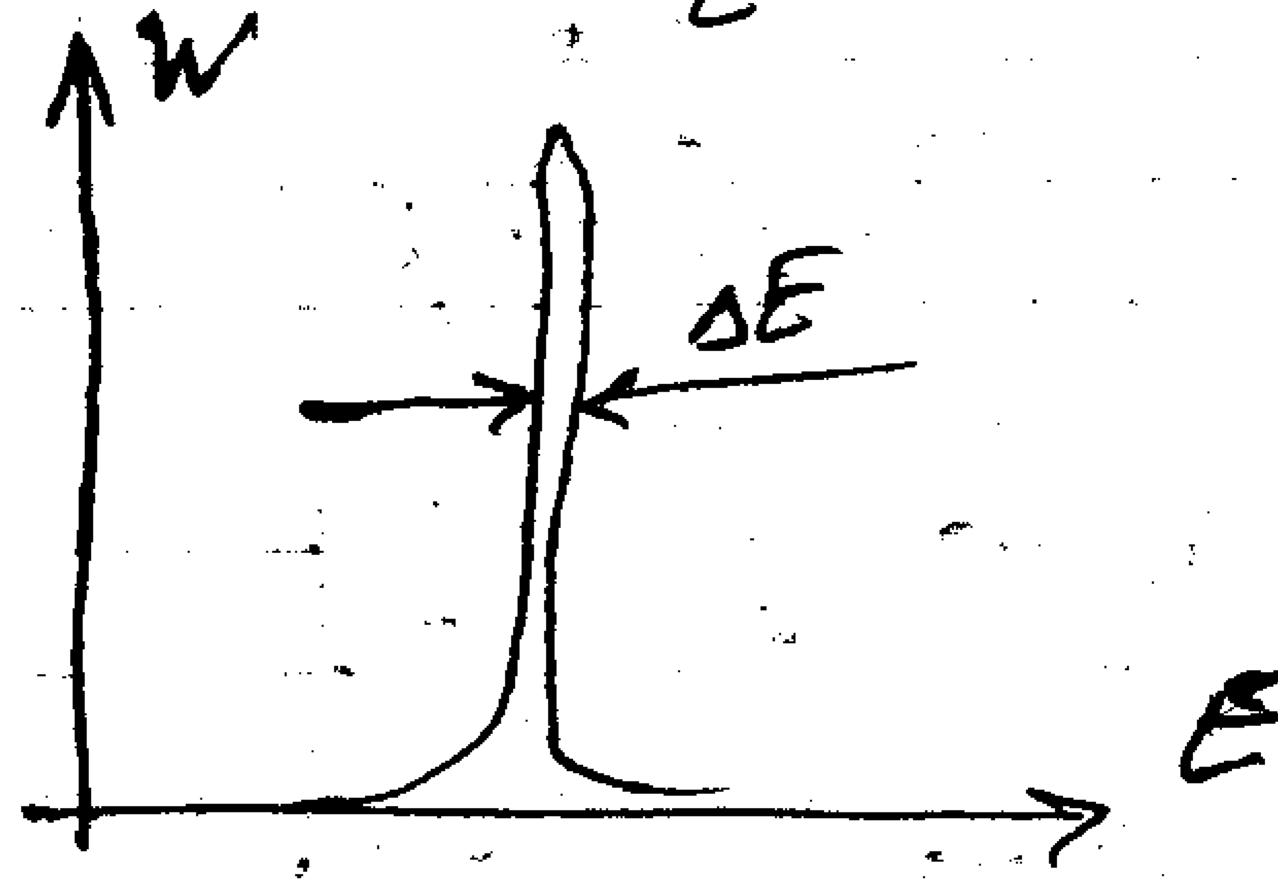
$$S_0(E_{полн} - E_k) = S_0(E_{полн}) - E_k \cdot \frac{1}{T_0} + \cancel{\frac{E_k^2}{2T_0^2 C_{V0}}}, \quad C_{V0} \to \infty$$

$$W(|k\rangle) = Const \cdot e^{-\frac{E_k}{T_0}}$$

$$w(|k\rangle) = \frac{1}{Z} e^{-\frac{E_k}{T_0}}$$

$$Z = \sum_{|k\rangle} e^{-\frac{E_k}{T_0}}$$ – стат. сумма

$$W(E) = \Gamma(E_k = E) \cdot w(|k\rangle) = \frac{1}{Z} e^{S(E) - \frac{E}{T}}$$



$$Z = \sum e^{-\beta E_k}$$

$$\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\sum E_k \frac{e^{-\beta E_k}}{Z} =$$

$$= -\sum E_k w(|k\rangle) = -\langle E\rangle$$

$$\langle E\rangle = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$$

$$\langle E^2\rangle = \sum W_k E_k^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

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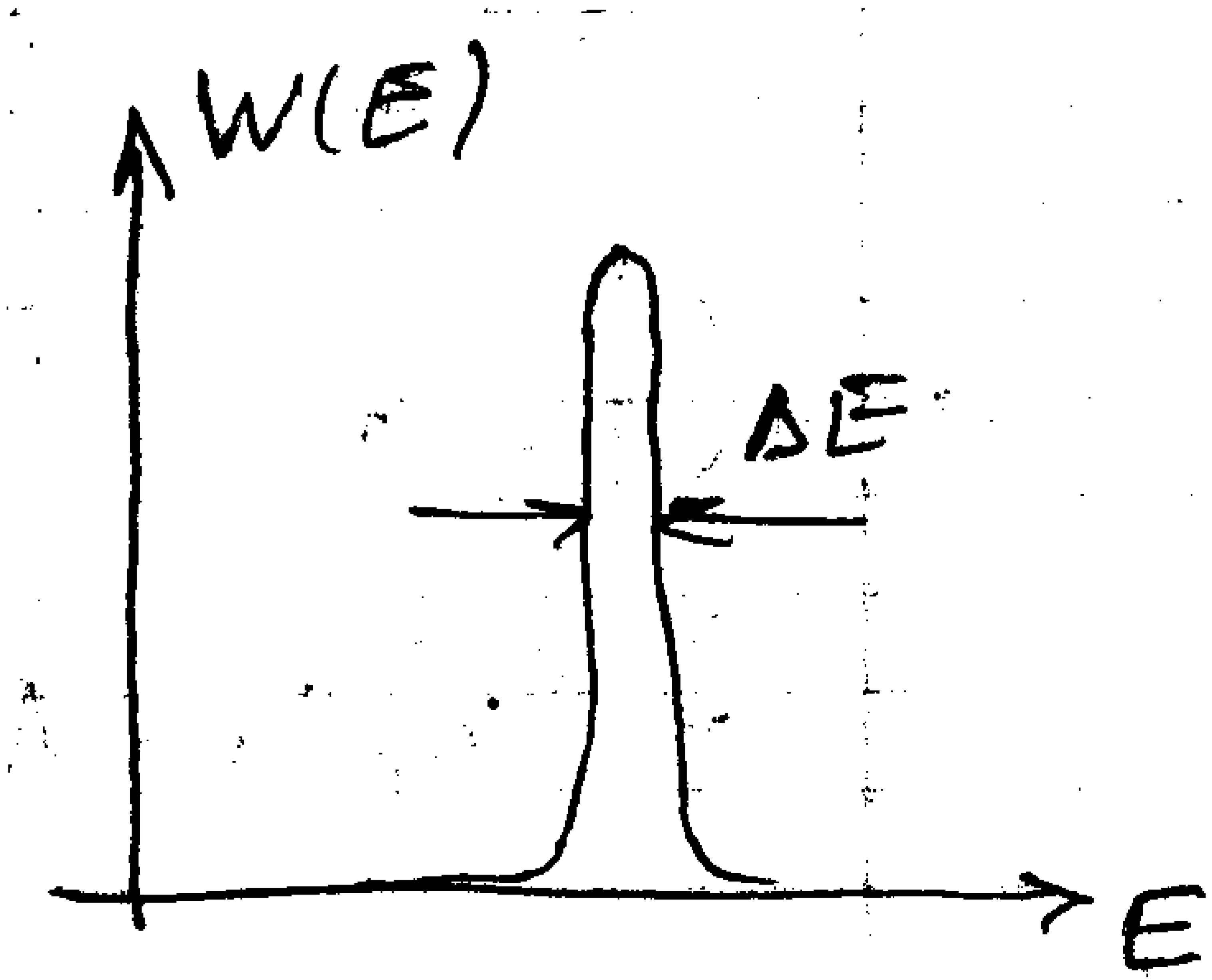


$$\frac{\Delta \mathcal{E}}{E} \sim \frac{\sqrt{T^2 N}}{TN} = \frac{1}{\sqrt{N}}$$ $\Rightarrow$ отклонение мало

$\Rightarrow$ будем писать не $\bar{E}, \langle E\rangle$, а $\mathcal{E}$.

Температура тела д.б. равна темп. в термостате, но есть флуктуации.

$\Delta T \sim C_V^{-1}\Delta E, \quad \frac{\Delta T}{T} \sim \frac{1}{\sqrt{N}}$

$S = -\langle \ln w_k \rangle = -\sum_{|k\rangle} w_k \ln w_k \ominus, \; \{ w_k = \frac{1}{Z} e^{-\beta E_k} \Rightarrow$

$\ominus -\sum_{|k\rangle}' (-\ln Z - \beta E_k) \cdot w_k = \ln Z + \beta E = \ln Z + \frac{E}{T}$

$\ln Z = S - \frac{E}{T} = -\frac{F}{T}$

$Z = e^{-F/T}$

$F = E - TS$

$F = -T\ln Z$

$dF = \left(\frac{\partial F}{\partial T}\right)_\lambda dT + \left(\frac{\partial F}{\partial \lambda}\right)_T d\lambda$

$\left(\frac{\partial F}{\partial T}\right)_\lambda = -T \frac{1}{Z}\underbrace{\frac{\partial Z}{\partial T}}_{\frac{\partial Z}{\partial \beta}\cdot\frac{\partial \beta}{\partial T}} - \ln Z = -\frac{E}{T} + \frac{F}{T} = \frac{F-E}{T} = -S$

$\left(\frac{\partial F}{\partial \lambda}\right)_T = -T \cdot \frac{1}{Z}\frac{\partial Z}{\partial \lambda} = -\frac{T}{Z}\cdot \sum_{|k\rangle} e^{-\beta E_k}\frac{\partial E_k}{\partial \lambda}(-\beta) =$

$= \sum w_k \Lambda_k = \Lambda \Rightarrow \quad dF = -S\,dT + \Lambda\, d\lambda$

$dF = -S\,dT + \Lambda\, d\lambda$

$Z = \sum_{|k\rangle} \langle k| e^{-\beta \hat{H}} |k\rangle = \mathrm{Tr}\left(e^{-\beta\hat{H}}\right).$

$\hat{\rho} = \frac{1}{Z} e^{-\beta\hat{H}}$ – статистический оператор

Двухуровневая система

——— g_2, ε_2 g - кратность вырождения

——— g_1, ε_1

Z - ?

$$Z = \left(g_1 e^{-\varepsilon_1/T} + g_2 e^{-\varepsilon_2/T}\right)^N$$

$$F = -T \cdot N \ln\left(g_1 e^{-\varepsilon_1/T} + g_2 e^{-\varepsilon_2/T}\right),$$

Идеальный больцмановский газ.

Энергия взаимод. $U \ll \langle \frac{p^2}{2m} \rangle$ - идеальный газ.

Квантовая тождественность.

$p_{хар}$ - характ. имп.

$\frac{V p_{хар}^3}{(2\pi\hbar)^3}$ - число состояний.

Если $\frac{V p_{хар}^3}{(2\pi\hbar)^3} \gg N$; газ наз. больцман.

$$\frac{V}{N} \gg \left(\frac{2\pi\hbar}{p_{хар}}\right)^3, \quad \frac{V}{N} = a^3, \quad \frac{2\pi\hbar}{p_{хар}} = \lambda_{хар}$$

$$\Rightarrow \quad a^3 \gg \lambda_{хар}^3$$

„тело" - одна молекула.

Распределение частиц.

$$W_k = \frac{1}{z} e^{-\frac{\varepsilon_k}{T}}, \quad \varepsilon = \frac{p^2}{2m}$$

$$z = \int e^{-\frac{p^2}{2mT}} \cdot g \frac{V d^3p}{(2\pi\hbar)^3} = V \cdot \left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}$$

Если $\varepsilon = \varepsilon_0 + \frac{p^2}{2m}$, $z = V\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} e^{-\varepsilon_0/T}$

Стат. сумма N молекул Z - ?

$$Z = \frac{z^N}{N!}$$

$$N! \approx \left(\frac{N}{e}\right)^N \sqrt{2\pi N}$$ ← выкидываем

$$Z = \left(\frac{ze}{N}\right)^N \frac{1}{\sqrt{2\pi N}}$$

$$F = -TN \ln \frac{ze}{N} = TN \ln N - TN \ln ze =$$

$$= -TN \left(\ln \frac{V}{N} + \frac{3}{2} \ln \frac{mT}{2\pi\hbar^2}\right) - NT$$

$$\frac{V p_{хар}^3}{(2\pi\hbar)^3} \gg N \sim \left\{\frac{p^2}{2m} \sim T\right\} \sim \frac{(mT)^{3/2}}{(2\pi\hbar)^3} \gg \frac{N}{V}$$

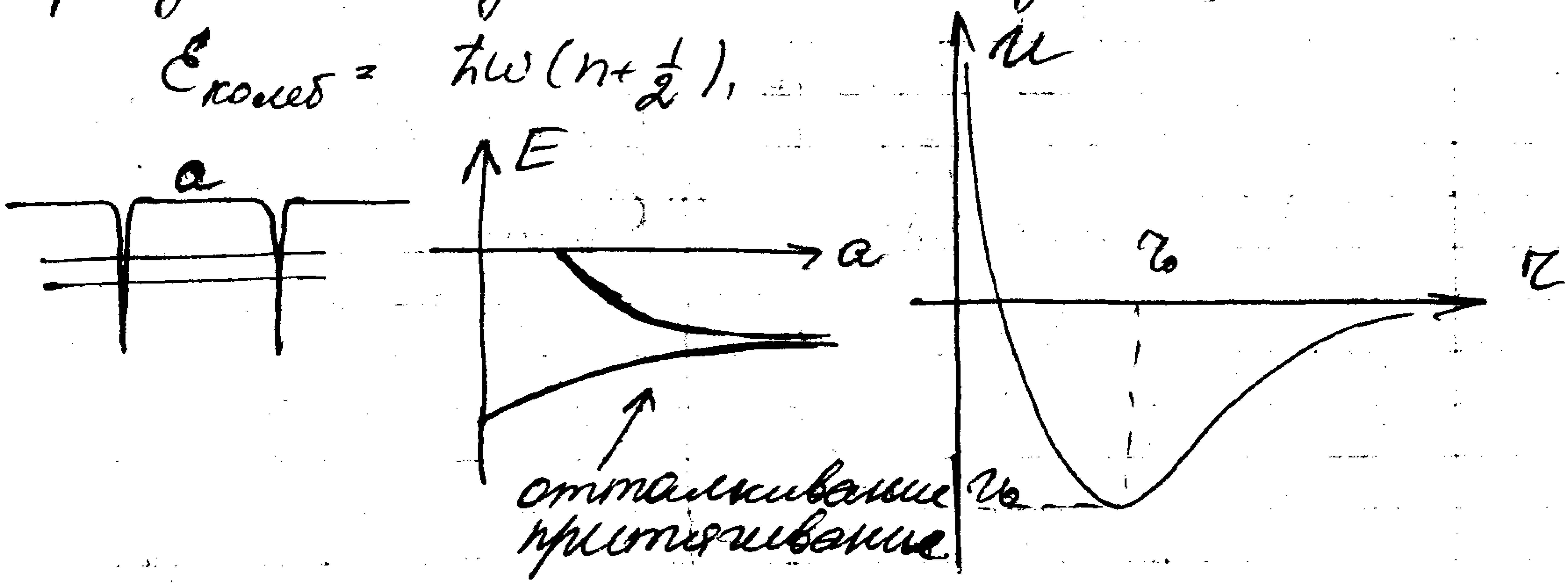
$$\varepsilon = \varepsilon_0 + \frac{p^2}{2m} + \varepsilon_{вращ} + \varepsilon_{колеб}$$

$$\varepsilon_{вращ} = \frac{\hbar^2 l(l+1)}{2J}, \quad l = 0, 1 \ldots$$

$g = 2l+1$ - кратность

Формула не для всех молекул,

$$\mathcal{E}_{колеб} = \hbar\omega\left(n+\frac{1}{2}\right),$$



Атомная система единиц: $m_e = \hbar = |e| = 1$

$a_Б = 1 = 10^{-8}$ см, $\mathcal{E}_0 \approx 26$ эВ, $J \sim M r_0^2 \sim M \sim 10^4$

$U_0 \sim 1$, $r_0 \sim 1$, $U''(r_0) \sim 1$

$$\omega \sim \sqrt{\frac{U''(r_0)}{M}} \sim 10^{-2}$$

$$\mathcal{E}_{дисс} = \mathcal{E}_0 - \frac{\hbar\omega}{2},$$

$$T_{комн} \approx \frac{1}{40}\ эВ.$$

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1 а.е. = 30 эВ.

$$\mathcal{E} = \mathcal{E}_0^0 + \frac{p^2}{2m} + \frac{\hbar^2 l(l+1)}{2J} + \hbar\omega\left(n+\frac{1}{2}\right)$$

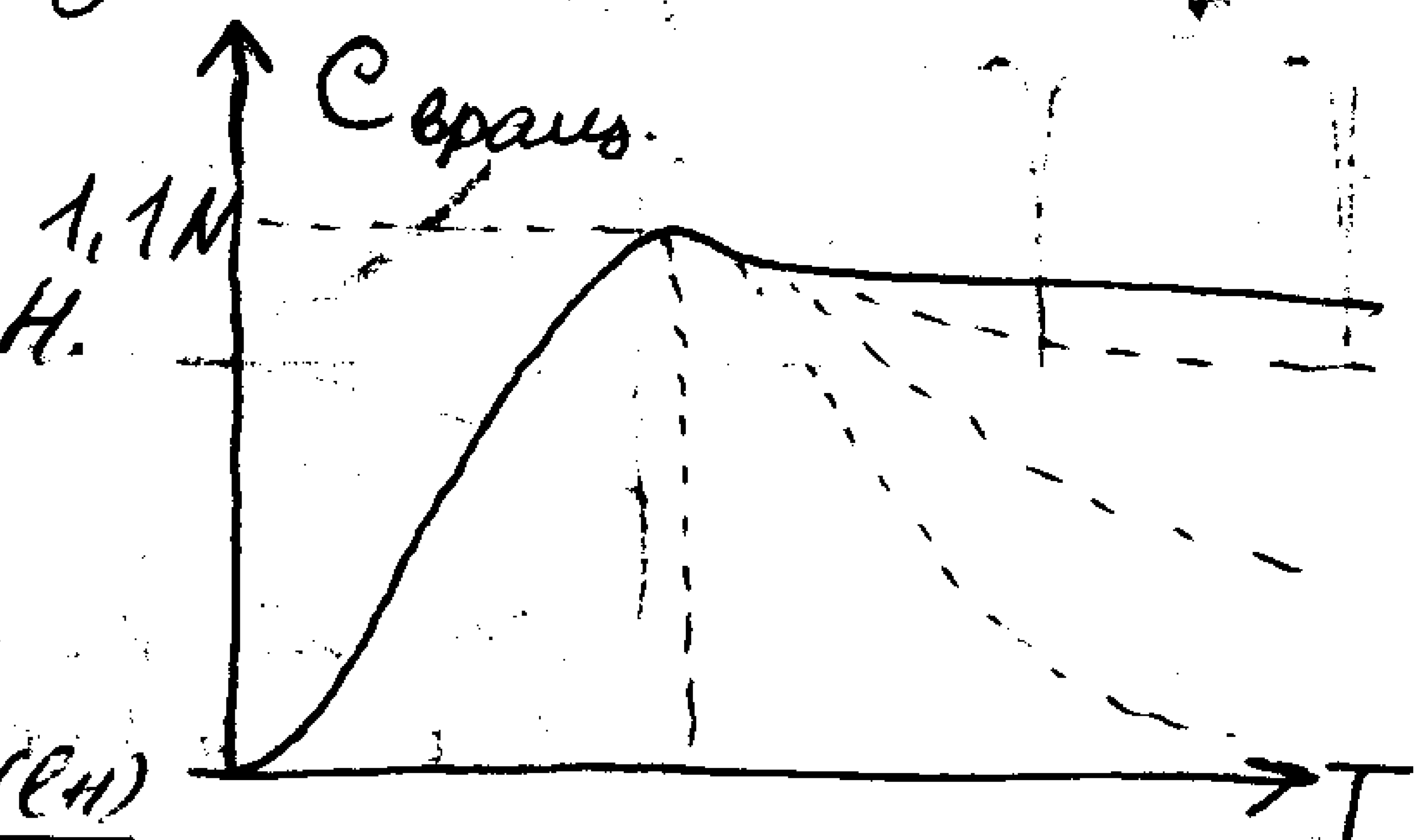
$\Delta\mathcal{E} \sim 1 \ll T$ $\quad \frac{1}{M} \sim 10^{-4}$ $\quad \frac{1}{\sqrt{M}} \sim 10^{-2}$

$E_{пост} = \frac{3}{2}NT$ $\quad C_V = \frac{3}{2}N$

$$E_{вращ} = \frac{\hbar^2 l(l+1)}{2J}, \quad g = 2l+1$$

$$Z_{вр} = \sum_{l=0}^{\infty} (2l+1) e^{-\frac{\hbar^2 l(l+1)}{2JT}}$$

Если $T \lessapprox \frac{\hbar^2}{2J} = T_{вр}$, достаточно 2-х, 3-х слагаемых



$\frac{\hbar^2}{2J} = T_{вр} \sim 80K$ для H.

Если $T \gg \frac{\hbar^2}{2J}$, то

$$Z_{вр} \approx \int_0^{\infty} (2l+1) e^{-\frac{\hbar^2 l(l+1)}{2JT}} dl = \left\langle x = \frac{\hbar^2}{2JT} l(l+1), \; dx = \frac{\hbar^2}{2JT}(2l+1)dl \right\rangle$$

$$= \int_0^{\infty} \frac{2JT}{\hbar^2} \cdot e^{-x} dx = \frac{2JT}{\hbar^2}$$

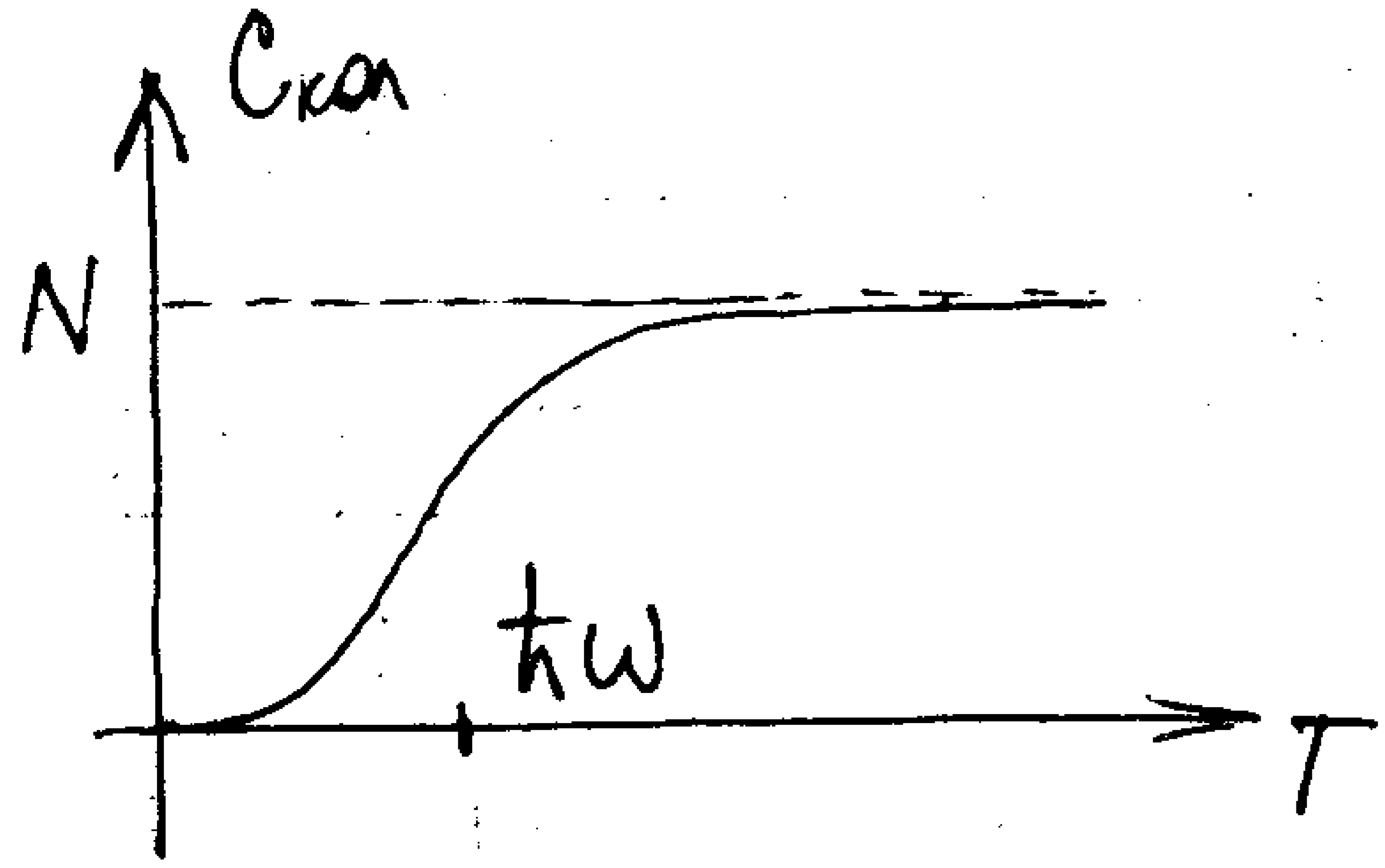
$$T \gg \mathcal{E}_{вр}(l+1) - \mathcal{E}_{вр}(l) \sim \hbar\omega_{вр}$$

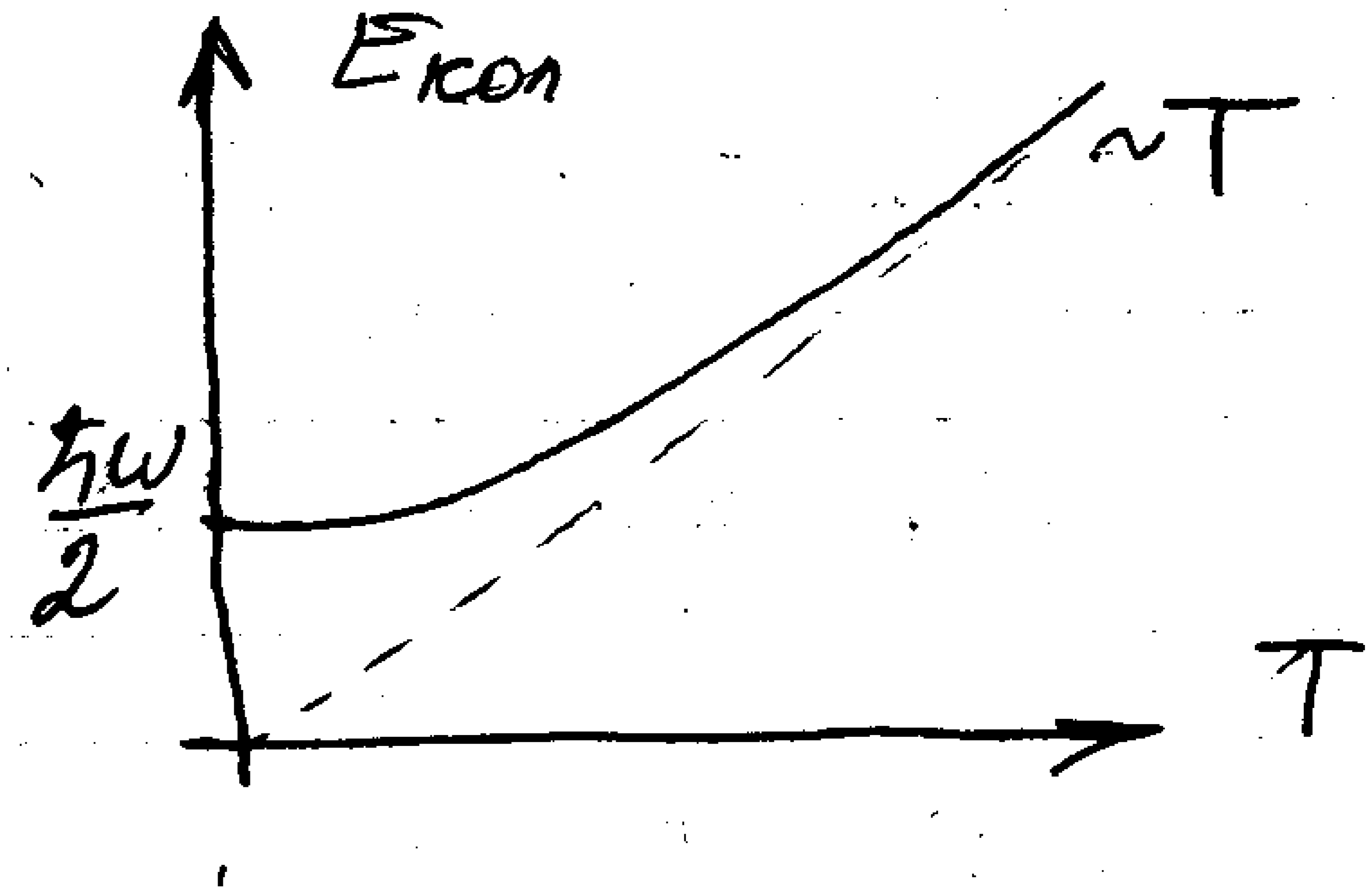
$$\cancel{\Omega_{вр}} = F_{вр} = -NT \ln Z_{вр} = -NT \ln \frac{2JT}{\hbar^2},$$

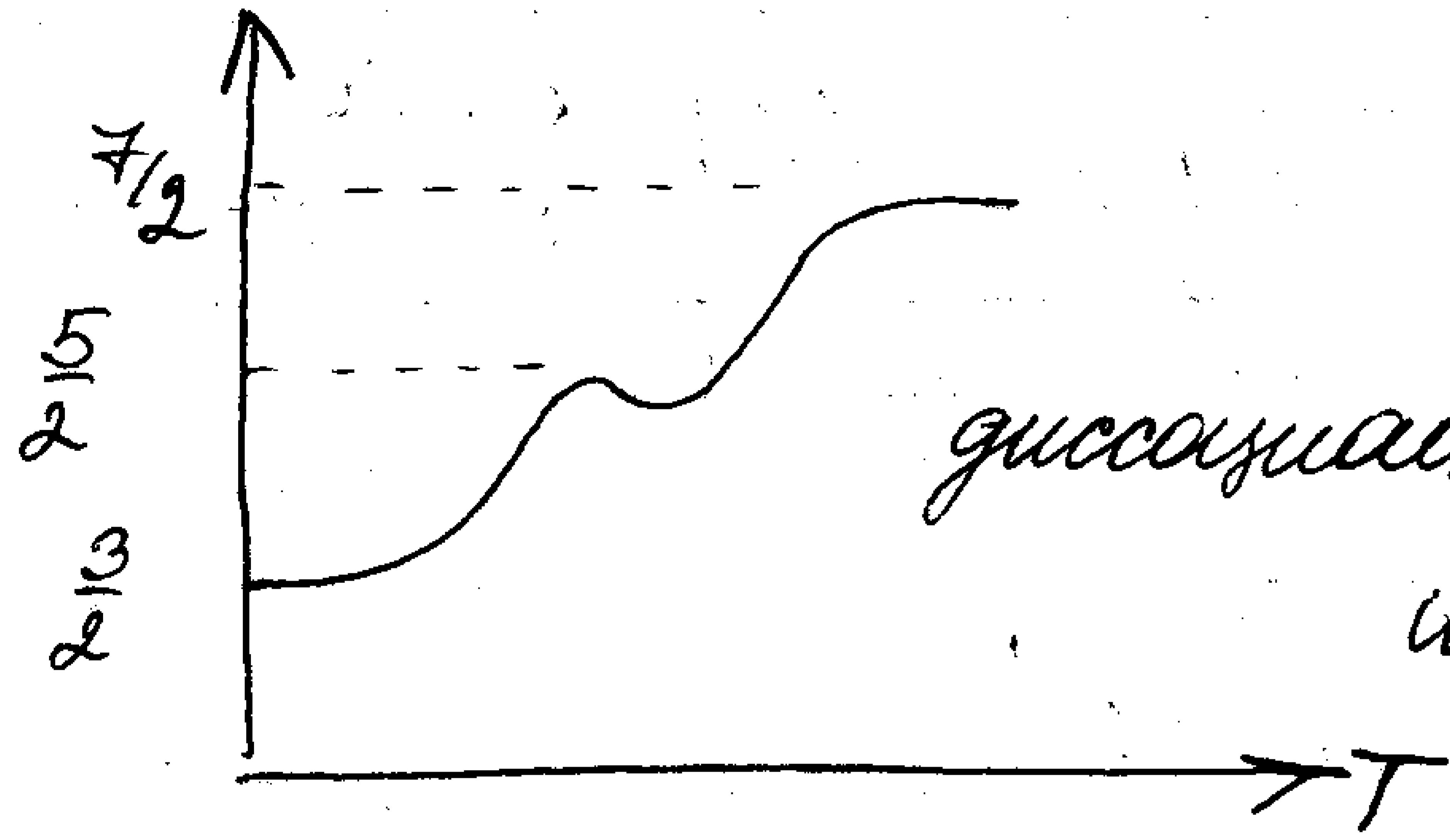
$$S_{вр} = -\frac{\partial F_{вр}}{\partial T}, \quad C_{вр} = N$$

$$Z_{кол} = \sum_{n=0}^{\infty} e^{-\frac{\hbar\omega(n+\frac{1}{2})}{T}} = \frac{e^{-\frac{\hbar\omega}{2T}}}{1 - e^{-\frac{\hbar\omega}{T}}} = \frac{1}{2 sh \frac{\hbar\omega}{2T}}$$

$$F_{кол} = NT \ln\left(2 sh \frac{\hbar\omega}{2T}\right)$$







диссоциация → ионизация → излучение → e^+e^-

$$\mathcal{E} = \mathcal{E}_{кин} + U(\vec{r})$$

$$W \sim e^{-\frac{\mathcal{E}}{T}} = e^{-\frac{\mathcal{E}_{кин}}{T}} \cdot e^{-\frac{U}{T}}$$

Если имеется магнитное поле,

$$Z = \frac{1}{(2\pi\hbar)^3}\int e^{-\frac{H(\vec{p},\vec{r})}{T}} d^3p\, dV$$

$$H = \frac{1}{2m}\left(\bar{p} - \frac{e}{c}\bar{A}\right)^2 + U(\bar{r}) \quad , \bar{p} - \frac{e}{c}\bar{A} = \bar{P} = m\bar{v}$$

$$Z = \frac{1}{(2\pi\hbar)^3}\int e^{-\frac{P^2}{2mT} - \frac{U}{T}} d^3P\, dV$$ – не зависит от $\vec{H}$

⇒ магнитных свойств у классического газа нет.

$$\mathcal{E}_{n,p_z} = \frac{p_z^2}{2m} + \hbar\omega\left(n+\frac{1}{2}\right), \quad \omega = \frac{e\mathcal{H}}{mc},$$

$$Z_\perp = g \sum_n e^{-\frac{\hbar\omega(n+1/2)}{T}} = \frac{g}{2\,\mathrm{sh}\,\frac{\hbar\omega}{2T}} =$$

$$Z = Z_\parallel \cdot Z_\perp \qquad = \frac{g}{2\,\mathrm{sh}\,\frac{e\hbar\mathcal{H}}{2mcT}}$$

При $T\to\infty$ зависимость от $\mathcal{H}$ пропадает: $\mathrm{sh}\,x \approx x$, $g \sim \mathcal{H}$

$$Z_\perp = \frac{\tilde{g}\mathcal{H}}{2\,\mathrm{sh}\,\frac{\mu\mathcal{H}}{T}}, \quad \mu = \frac{e\hbar}{2mc},$$

$$F_\perp = -NT\ln Z_\perp = -NT\left(\ln\mathcal{H} - \ln\left(\mathrm{sh}\,\frac{\mu\mathcal{H}}{T}\right)\right)+\ldots$$

$$\mathcal{M} = -\frac{\partial F}{\partial\mathcal{H}} = -\frac{\partial F_\perp}{\partial\mathcal{H}} = NT\left(\frac{1}{\mathcal{H}} - \frac{\mu}{T}\cdot\frac{\mathrm{ch}\,\frac{\mu\mathcal{H}}{T}}{\mathrm{sh}\,\frac{\mu\mathcal{H}}{T}}\right).$$

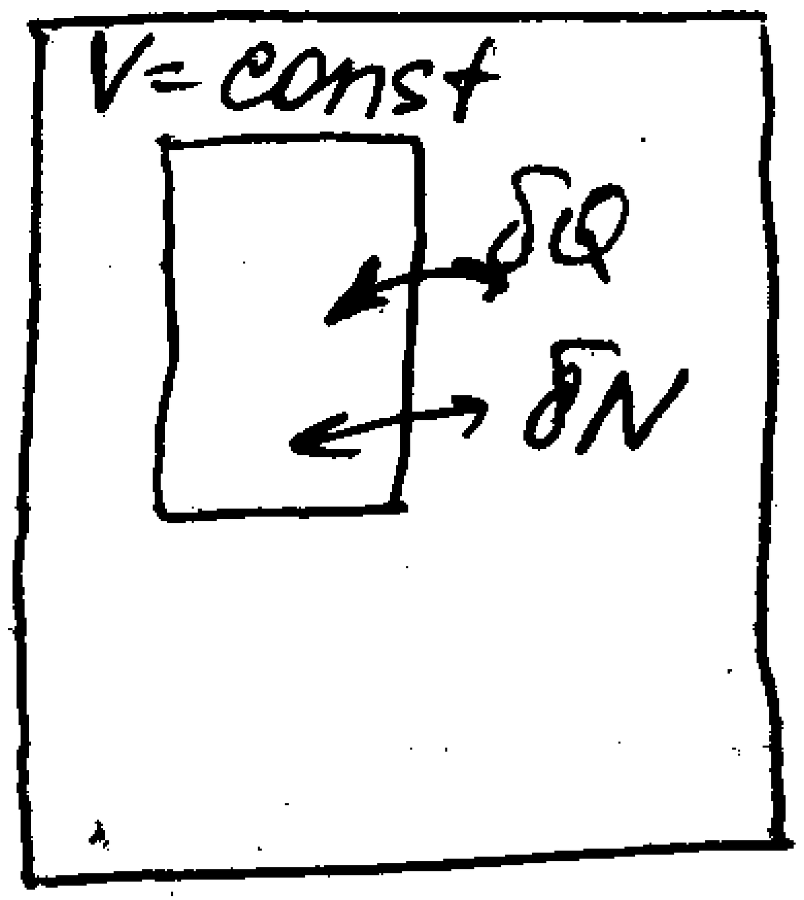
$$\mathrm{cth}\,x = \frac{1}{x} + \frac{x}{3} + \ldots$$

Если $\frac{\mu\mathcal{H}}{T} \ll 1$, то

$$\mathcal{M} = NT\cdot\frac{\mu^2\mathcal{H}}{3T^2} = -\frac{N\mu^2\mathcal{H}}{3T}$$

$$\Rightarrow \chi = -\frac{N\mu^2}{3TV}$$

Системы с переменным числом частиц.

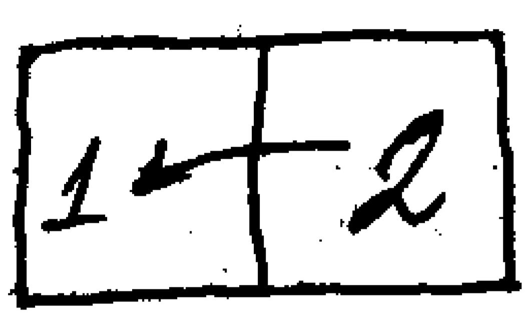


N частиц в сост. i:

$$W(|N,i\rangle) \sim \Gamma_0(N_{полн}-N, E_{полн}-E) = e^{S_0(N_{полн}-N,\, E_{полн}-E)}$$

$$TdS - pdV + \mu dN = dE$$

μ - химический потенциал



Обмен теплом и част.

$$F = F_1 + F_2 = F_1(N_1, \dots) + F_2(N_2, \dots)$$

$N_2 + N_1 = N$, $F(N, T, \lambda)$, Равновесие:

$$\frac{\partial F}{\partial N_1} = 0 = \frac{\partial F_1}{\partial N_1} - \frac{\partial F_2}{\partial N_2}, \quad \frac{\partial F_i}{\partial N_i} = \mu_i$$

$$\Rightarrow \quad \mu_1 = \mu_2.$$

$$W(E) \sim e^{S - \frac{E}{T_0}} = e^{-\frac{F(T_0)}{T_0}}$$

W_{max} при $F_{min} \Rightarrow \left(\frac{\partial F}{\partial x}\right)_{T,N} = 0.$

$\delta F \le 0 \Rightarrow \delta F = \delta N_1(\mu_1 - \mu_2) \le 0$

$\Rightarrow$ частицы идут в сторону меньшего потенциала.

$$dF = -SdT - PdV + \mu dN$$

$$R = F + PV$$

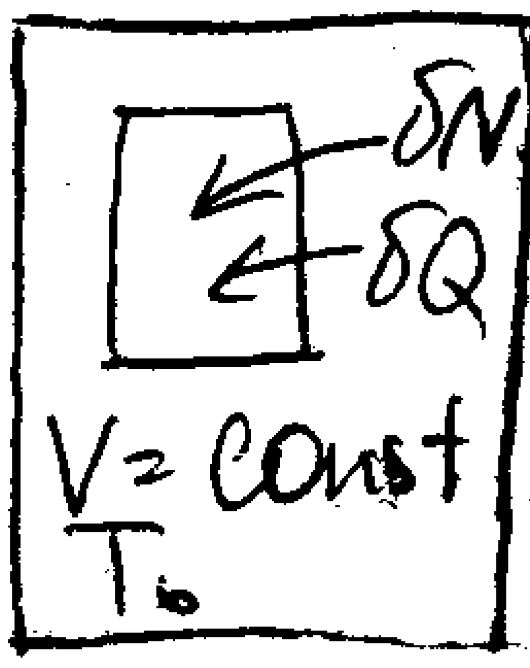
$d\Phi = -SdT + VdP + \mu dN$

$\Phi(T,P,N) = N\varphi(T,P)$,

$\mu = \varphi$.

4.03.02

$dE = TdS - PdV + \mu dN$



$w(N,|k\rangle) \sim \Gamma_0(E_{полн}-E,\ N_{полн}-N)$

$\Gamma(N,|k\rangle) = 1$

$S_0(E_{полн}-E,\ N_{полн}-N) = S_0(E_{полн},\ N_{полн}) - \frac{E}{T_0} + \frac{\mu N}{T_0}$

$\Rightarrow\ w(N,|k\rangle) = \frac{1}{Q} e^{+\frac{\mu_0 N}{T_0} - \frac{E}{T_0}}$

$Q = \sum_{N,|k\rangle} e^{\frac{\mu N - E_{N,k}}{T}} = \sum_N e^{\frac{\mu N}{T}} Z_N$

$w(N,E) = \Gamma(N,E)\, w(N,|k\rangle) =$, $E_k = E$.

$= \frac{1}{Q} e^{S + \frac{\mu N}{T} - \frac{E}{T}}$, $Q(T,\mu,\lambda)$

Ω-потенциал.

$\Omega = -T \ln Q$

$\frac{\partial\Omega}{\partial T} = -\ln Q + \frac{\cancel{T}}{Q}\sum e^{\frac{\mu N - E_{N,k}}{T}} \cdot \frac{\mu N - E_{N,k}}{T^{\cancel{2}}} =$

$= w(N,|k\rangle)$

$$= \frac{\Omega}{T} + \frac{\mu\langle N\rangle - \langle E\rangle}{T}$$

$$S = -\sum w \ln w = \sum w \left(\ln Q - \frac{\mu N}{T} + \frac{E}{T}\right) =$$

$$= \ln Q - \frac{\mu\langle N\rangle}{T} + \frac{\langle E\rangle}{T} = -\frac{\partial\Omega}{\partial T}$$

$$\frac{\partial\Omega}{\partial\mu} = -\frac{T}{Q}\frac{\partial Q}{\partial\mu} = -\frac{T}{Q}\sum e^{\frac{\mu N - E}{T}}\frac{N}{T} = -\langle N\rangle$$

$$\frac{\partial\Omega}{\partial\lambda} = \langle\Lambda\rangle$$

$$d\Omega = -S\,dT + \Lambda\, d\lambda - N\, d\mu$$

$$\Omega = F - \mu N$$

$$\Phi = N\varphi(T,P) = N\mu.$$

$$\Omega = F - \Phi = -PV$$

Энтропия неравновесного больцмановского газа.

$S = -\sum w \ln w$, число сост-й в i-й группе — G_i, — например $\frac{dV\,d^3p}{(2\pi\hbar)^3}$,

$G_i \gg 1$, N_i частиц, $N_i \gg 1$.

$N_i \ll G_i$

$f_i = \frac{N_i}{G_i}$ - число заполнения.

$\Gamma_i = C_{G_i}^{N_i}$; $\Gamma = \prod_i \Gamma_i$

$S = \sum_i \ln \Gamma_i$

$$\Gamma_i = C_{G_i}^{N_i} = \frac{G_i!}{N_i!(G_i - N_i)!} \approx \frac{G_i^{N_i}}{N_i!} = \left(\frac{eG_i}{N_i}\right)^{N_i}$$

$$S = \sum \ln \Gamma_i = \sum N_i \ln \frac{eG_i}{N_i} = \sum G_i f_i \ln \frac{e}{f_i}$$

$$S_{\text{пост}} = \int \frac{dV\, d^3p}{(2\pi\hbar)^3} \cdot f(\vec{r},\vec{p}) \ln \frac{e}{f(\vec{r},\vec{p})}$$

— в пределе.

$$\sum_i N_i = N, \quad \sum_i E_i N_i = E$$

Найдем экстремум S при усл.

$$\frac{\partial}{\partial f_i} \left\{ \sum G_i f_i \ln \frac{e}{f_i} - \alpha \sum G_i f_i - \beta \sum E_i G_i f_i \right\} = 0.$$

$$\ln \frac{e}{f_i} - 1 - \alpha - \beta E_i = 0.$$

$$\Rightarrow f_i = e^{-\alpha - E_i \beta} = e^{-\frac{\mu}{T} - \frac{E_i}{T}}$$

Химическое равновесие

$2H_2O + O_2 \rightleftarrows 2H_2O.$

~~Равновесного~~ $\sum \nu_i A_i = 0.$ - запись реак.

ν_i - стехиометрические коэфф-ты.

$T = const,\ V = const.$

δN_0 - число актов реакции

$\delta N_i = -\nu_i \, dN_0.$

$$\delta F = \sum \left(\frac{\partial F}{\partial N_i}\right)_{T,V} \delta N_i = -\delta N_0 \sum \nu_i \mu_i$$

Условие равновесия: $\delta F = 0,\ \sum \nu_i \mu_i = 0.$

$$\sum \nu_i \mu_i = 0$$

Ид. газ:

$$F_i = -T \ln \frac{z_i^{N_i}}{N_i!} \ ; \ \mu_i = \frac{\partial F}{\partial N_i} = -T \ln \frac{z_i}{N_i}$$

$$\mu_i = -T \ln \frac{V}{N_i} + \varphi_i(T)$$

$$\sum \nu_i \left(-T \ln \frac{V}{N_i} + \varphi_i\right) = 0.$$

$$\prod_i \left(\frac{N_i}{V}\right)^{\nu_i} = e^{\sum \frac{\nu_i (\varphi_i T)}{T}} = K_\nu(T)$$

Пример

$A \rightleftarrows A^+ + e \quad , \ N_0 = N_A + N_{A^+}$

$N_e = N_{A^+}$

$$\mu_A = -T \ln \frac{Z_A}{N_A} \quad , \mu_A = \mu_{A^+} + \mu_e .$$

$$\alpha = \frac{N_{A^+}}{N_0}$$

$$\frac{Z_A}{N_A} = \frac{Z_{A^+} Z_e}{N_{A^+} N_e}$$

$$Z_A = V \left(\frac{m_A T}{2\pi \hbar^2} \right)^{3/2} e^{-\frac{\varepsilon_0 A}{T}}$$

$$\frac{N_e^2}{N_A} = V \left(\frac{m_e T}{2\pi \hbar^2} \right)^{3/2} e^{-I/T} \quad , I = \varepsilon_{0A^+} + \varepsilon_{0e} - \varepsilon_{0A}$$

$$\frac{\alpha^2}{1-\alpha} = \frac{V}{N_0} \left(\frac{m_e T}{2\pi \hbar^2} \right)^{3/2} e^{-I/T}$$

$$\frac{\alpha^2}{1-\alpha} = G(T) e^{-I/T}$$

Стат. физика.

$\sum \nu_i A_i = 0$

$T = const$, $V = const$

$\delta F = -\delta N_0 \sum \nu_i \mu_i \overset{\text{равновесие}}{=} 0$

$\mu_i = -T \ln \frac{Z_i}{N_i}$ идеальный газ.

$\mu_i = -T \ln \frac{V}{N_i} + \varphi_i(T)$

$\sum \nu_i \mu_i = 0 \Rightarrow \prod_i \left(\frac{N_i}{V}\right)^{\nu_i} = e^{-\sum \nu_i \varphi_i / T} = K_V(T)$

Если $V \neq const$, $P = const$

Вводят $P_i = \frac{N_i T}{V}$

Закон действующих масс

$\mu_i = T \ln P_i + \chi_i(T)$

Равновесие: $\prod_i P_i^{\nu_i} = K_p(T) = e^{-\sum \chi_i \nu_i / T}$

Выделение тепла в хим. реакции.

$\delta Q = \delta E$, $E = F + TS = F - T\frac{\partial F}{\partial T} = -\left(\frac{\partial}{\partial T} \frac{F}{T}\right) \cdot T^2$

$\delta E = -T^2 \frac{\partial}{\partial T} \frac{\delta F}{T}$

$\delta F = -\delta N_0 \sum \nu_i \left(-T \ln \frac{V}{N_i} + \varphi_i(T)\right) = +T^2 \frac{\partial}{\partial T} \ln K_V(T)$

Теорема Нернста

$T \to 0 \Rightarrow S \to 0.$

– это если нижнее сост. невырожденно.

Комп. моделирование.

$$dw = A\,\delta(E - \varepsilon_1 - \dots - \varepsilon_N)\, d^2 v_1 \dots d^2 v_N$$

$$\varepsilon_i = \frac{m v_i^2}{2}$$ — плоский газ.

$$d^2 v_1 \sim d\varepsilon_1 \Rightarrow dw = A\,\delta(\cdot)\, d\varepsilon_1 \dots d\varepsilon_N$$

$$\varepsilon_i = (E - \varepsilon_1) x_i$$

$$dw = d\varepsilon_1 \cdot A \int \delta(\cdot)\, d\varepsilon_2 \dots d\varepsilon_N = d\varepsilon_1 A \int \delta((E-\varepsilon_1)(1 - x_2 - \dots x_N)) \cdot$$

$$\cdot (E - \varepsilon_1)^{N-1} = d\varepsilon_1 A (E - \varepsilon_1)^{N-2}$$

$$\Rightarrow \frac{dw}{d\varepsilon_1} = A \cdot (E - \varepsilon_1)^{N-2} \approx A e^{-\frac{\varepsilon_1}{E}(N-2)}, \quad \varepsilon_1 \ll E$$

Квантовые газы

$$\frac{p^2}{2m} \gg u, \quad \frac{V (mT)^{3/2}}{(2\pi\hbar)^3} \gg N$$ больцман. газ.

$$\frac{p^2}{2m} \sim T$$ Кван. газ: $$\frac{V \cdot p_{хар}^3}{(2\pi\hbar)^3} \lesssim N$$

$$\frac{V}{N} \lesssim \frac{(2\pi\hbar)^3}{p_{хар}^3} \sim \lambda_{хар}^3$$ — пакеты частиц перекрываются.

Ферми- и бозе- газы

↓ полуцелый спин   ↓ целый спин.

$$W_{N,|i\rangle} = \frac{1}{Q} e^{\frac{N\mu - E_{N,i}}{T}}$$

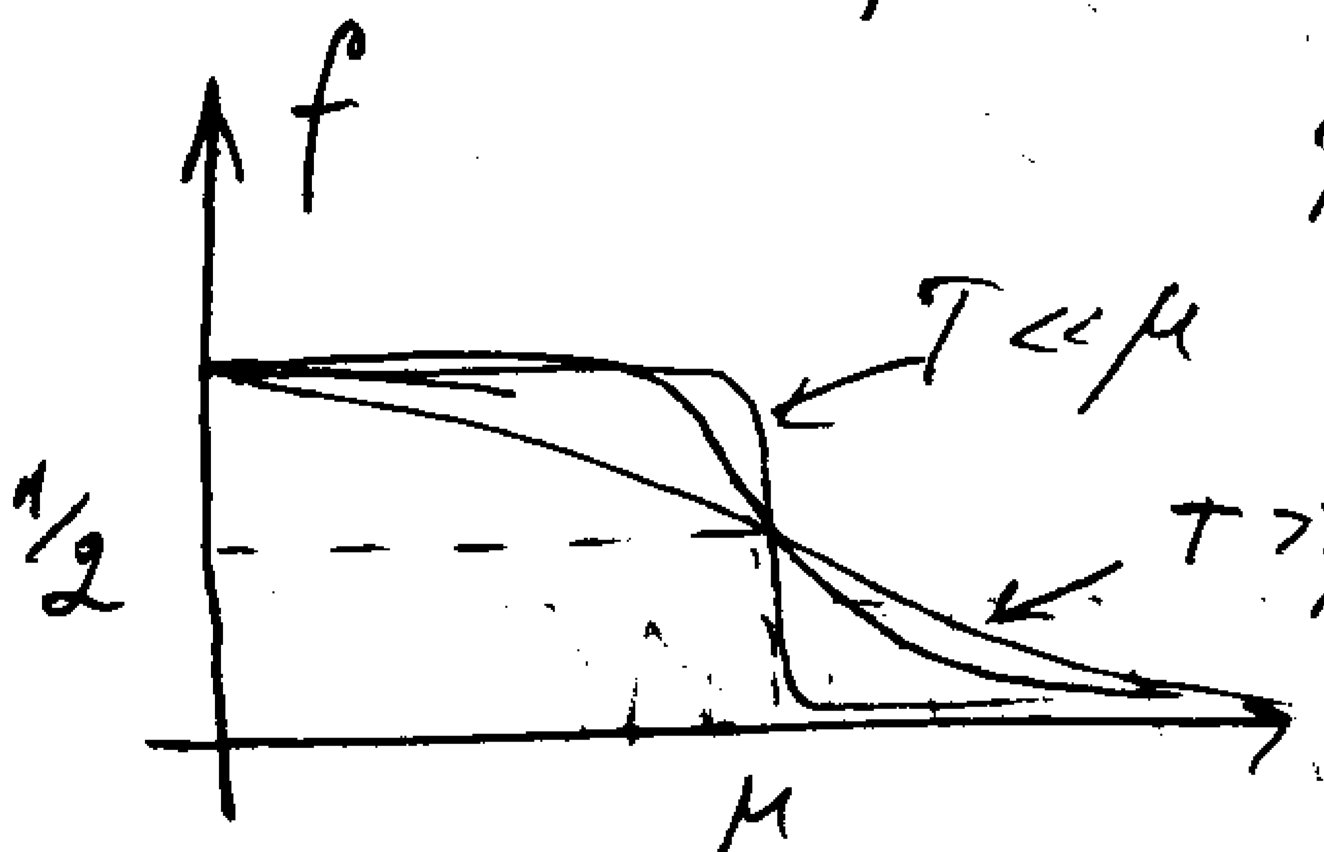
„тело" - совокупность частиц в сост. $|i\rangle$

а) $N=0,\ E=0$ б) $N=1,\ E=\varepsilon$

$$w_0 = \frac{1}{q},\quad w_1 = \frac{e^{\frac{\mu-\varepsilon}{T}}}{q},\quad q = 1+e^{\frac{\mu-\varepsilon}{T}}$$

$$w_0 = \frac{1}{e^{\frac{\mu-\varepsilon}{T}}+1},\quad w_1 = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}$$

$$\langle N\rangle = f = 0\cdot w_0 + 1\cdot w_1 = w_1$$



Распределение Ферми-Дирака

$$\int f\, d\Gamma = N \Rightarrow \mu(V,T,N)$$

$$\int \frac{2V\, d^3p}{\left(e^{\frac{\varepsilon-\mu}{T}}+1\right)(2\pi\hbar)^3} = N$$

Ферми. газ при $T=0$.

$s=\pm 1/2$

$$\frac{2\cdot V\cdot \frac{4}{3}\pi p_0^3}{(2\pi\hbar)^3} = N,\quad p_0 = \hbar\left(\frac{N}{V}\right)^{1/3}(3\pi^2)^{1/3}$$ - гран. импульс.

$$\varepsilon_0 = \frac{p^2}{2m}$$ - гран. энергия.

$$d^3p = 4\pi p^2\, dp = 4\pi p\, d\frac{p^2}{2} \sim \sqrt{\varepsilon}\, d\varepsilon,\quad N = \int\frac{2V\, d^3p}{(2\pi\hbar)^3} = \frac{2}{3}A\varepsilon_0^{3/2}$$

$$E = A\int \varepsilon\sqrt{\varepsilon}\,d\varepsilon = \frac{2}{5}A\,\varepsilon_0^{5/2} = \frac{3}{5}N\varepsilon_0$$

$\varepsilon_0 \sim 1$ а.е ~ 30 эВ, $T_{комн} \sim 1/40$ эВ.

$$P = -\left(\frac{\partial E}{\partial V}\right)_S = -\left(\frac{\partial E}{\partial V}\right)_{T=0}$$

$$p_0 \sim V^{-1/3}, \quad \varepsilon_0 \sim V^{-2/3}, \quad E \sim V^{-2/3}$$

$$\Rightarrow P = +\frac{2}{3}\frac{E}{V} = \frac{2}{5}\frac{N\varepsilon_0}{V}$$

$$P_{атом} = \frac{N}{V}T$$

$$\frac{P}{P_{атом}} \sim \frac{N}{V}\varepsilon_0 \Big/ \left(\frac{N}{V}\right)_{атм} T \sim \frac{10^{22}\,30}{10^{19}\,1/40} \sim 10^6$$

15.03.02. $f(\varepsilon) = \dfrac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}$, $\int f\,d\Gamma = N$.

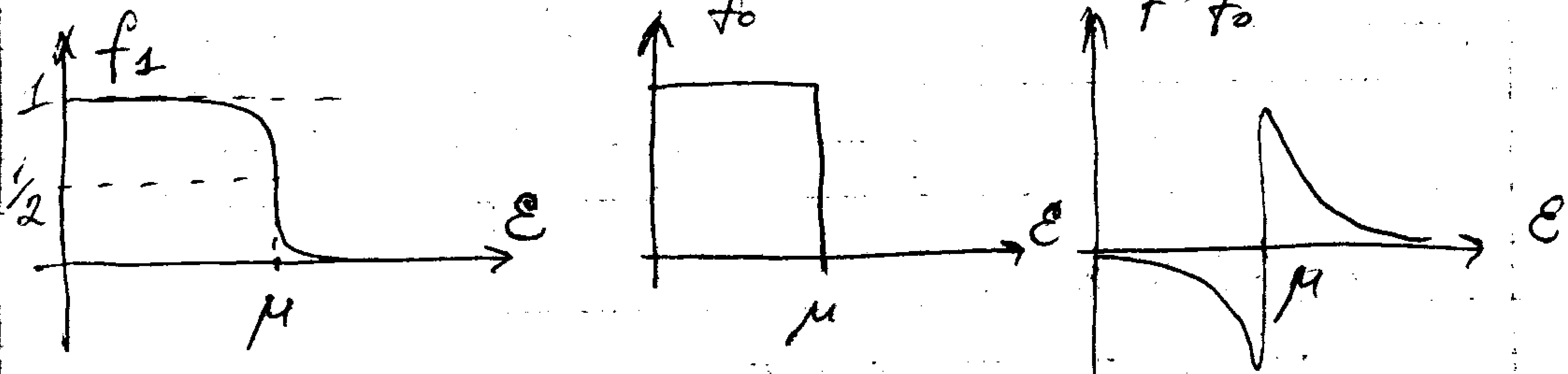
$$\int f\cdot\varepsilon\,d\Gamma = E, \quad I = \int \mathcal{F}(\varepsilon)f(\varepsilon)\,d\varepsilon$$

$$I_0 = \int_0^{\mu} \mathcal{F}(\varepsilon)\,d\varepsilon$$

$$I - I_0 = -\int_0^{\mu} \mathcal{F}(\varepsilon)(1-f(\varepsilon))\,d\varepsilon +$$

$$+\int_{\mu}^{\infty} \mathcal{F}(\varepsilon)f(\varepsilon)\,d\varepsilon = \left\langle \frac{\varepsilon-\mu}{T} = x, \ \varepsilon = \mu - Tx \atop \varepsilon = \mu + Tx \right\rangle =$$

$$= T\int_0^{\infty} \frac{\mathcal{F}(\mu+Tx)}{e^x+1}\,dx - T\int_0^{\mu/T} \frac{\mathcal{F}(\mu-Tx)}{e^x+1}\,dx =$$



$$-T\cdot F(\mu-T)+T\cdot F(\mu+T)\approx 2T^2F'(\mu).$$

$$1-\frac{1}{e^{-x}+1}=\frac{1}{e^{x}+1}\Rightarrow$$

$$\approx T\int_0^\infty \frac{dx}{e^x+1}\left(\mathcal{F}(\mu+Tx)-\mathcal{F}(\mu-Tx)\right)=$$

$$=\int_0^\infty \frac{x\,dx}{e^x+1}\cdot 2T^2\mathcal{F}'(\mu)=\frac{\pi^2T^2}{6}\mathcal{F}'(\mu).$$

Найти c e^--газа при $T\to 0$.

$$\sigma=\frac{ne^2\tau}{m}$$

$$dE=TdS-PdV+\mu dN \qquad ; \quad d\Gamma=\frac{2Vd^3p}{(2\pi\hbar)^3}=B\sqrt{\mathcal{E}}\,d\mathcal{E}.$$

$$N=B\int_0^\infty f(\mathcal{E})\sqrt{\mathcal{E}}\,d\mathcal{E}=B\Big(\int_0^\mu\sqrt{\mathcal{E}}\,d\mathcal{E}+$$

$$+\frac{\pi^2T^2}{6}\cdot\frac{1}{2\sqrt{\mu}}\Big)=\frac{2}{3}B\mu^{3/2}\left(1+\frac{\pi^2T^2}{8\mu^2}\right)$$

$$E=B\int_0^\infty \mathcal{E}f(\mathcal{E})\cdot\sqrt{\mathcal{E}}\,d\mathcal{E}=\frac{2}{5}B\mu^{5/2}+$$

$$+B\frac{\pi^2T^2}{6}\cdot\frac{3}{2}\sqrt{\mu}=\frac{2}{5}B\mu^{5/2}\left(1+\frac{5\pi^2T^2}{8\mu^2}\right)$$

(0)-приближение $\mu = \mathcal{E}_0$

$$N = \frac{2}{3} B \mathcal{E}_0^{3/2} \Rightarrow \mathcal{E}_0 = \left(\frac{3N}{2B}\right)^{2/3}$$

(1) приближение

$$N = \frac{2}{3} B \mu^{3/2} \left(1 + \frac{\pi^2 T^2}{8 \mathcal{E}_0^2}\right) \Rightarrow$$

$$\mu = \left(\frac{3N}{2B}\right)^{2/3} \cdot \left(1 - \frac{\cancel{2}\pi^2 T^2}{\cancel{3} \cdot 8 \mathcal{E}_0^2}\right)$$

$$E = \frac{2}{5} B \cdot \mathcal{E}_0^{5/2} \left(1 - \frac{\pi^2 T^2}{16 \cancel{\mu_0}^2}\right)^{5/2} \left(1 + \frac{5\pi^2 T^2}{8 \mathcal{E}_0^2}\right) =$$

$$= \cancel{\frac{2B}{5} \mathcal{E}_0^{5/2}} = \frac{3}{5} N \mathcal{E}_0 \left(1 + \frac{5\pi^2 T^2}{12 \mathcal{E}_0^2}\right)$$

$$C_v = \frac{\pi^2}{2} \frac{T}{\mathcal{E}_0} N$$

Насколько e^--газ идеален?

$$U \sim \frac{e^2}{r}$$

идеальность:

$$E_k \sim \frac{\hbar^2}{2mr^2}$$

$$\frac{e^2}{r} \ll \frac{\hbar^2}{2mr^2}$$

$$r \ll \frac{\hbar^2}{me^2} = a_Б$$

В металле $r \sim a_Б$ – условие не выполнено.

НО! Газ e^- и дырок достаточно разрежен.
Идеален e^--газ внутри атома, внутри белого карлика.

$$d\Omega = -SdT - PdV - \cancel{\mu dN}\, Nd\mu$$

$$S = -\left(\frac{\partial \Omega}{\partial T}\right)_{V,\mu}$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_{V,N}$$

„Тело" $N = 0, 1$
$\varepsilon = 0, \varepsilon$

$$Q = \sum{}' e^{\frac{\mu N}{T} - \frac{E_{N,k}}{T}} = \sum_{N,|k\rangle} e^{\frac{N_k(\mu - \varepsilon_k)}{T}} =$$

$$= \prod_{|k\rangle} \left(1 + e^{\frac{\mu - \varepsilon_k}{T}}\right)$$

$$\Omega = -T\ln Q = -T \sum \ln\left(1 + e^{\frac{\mu - \varepsilon_k}{T}}\right) =$$

$$= -TB\int_0^\infty \ln\left(1 + e^{\frac{\mu-\varepsilon}{T}}\right)\sqrt{\varepsilon}\, d\varepsilon = -TB\frac{2}{3}\varepsilon^{2/3} \cdot$$

$$\cdot \ln(\ldots)\Big|_0^\infty + TB\frac{2}{3}\int \frac{\varepsilon^{3/2}\, d\varepsilon}{1 + e^{\frac{\mu-\varepsilon}{T}}} e^{\frac{\mu - \varepsilon}{T}} \left(-\frac{1}{T}\right) =$$

$$= -\frac{2}{3}\int \frac{\varepsilon^{3/2}\, d\varepsilon}{e^{\frac{\varepsilon - \mu}{T}} + 1} = -\frac{2}{3}E$$

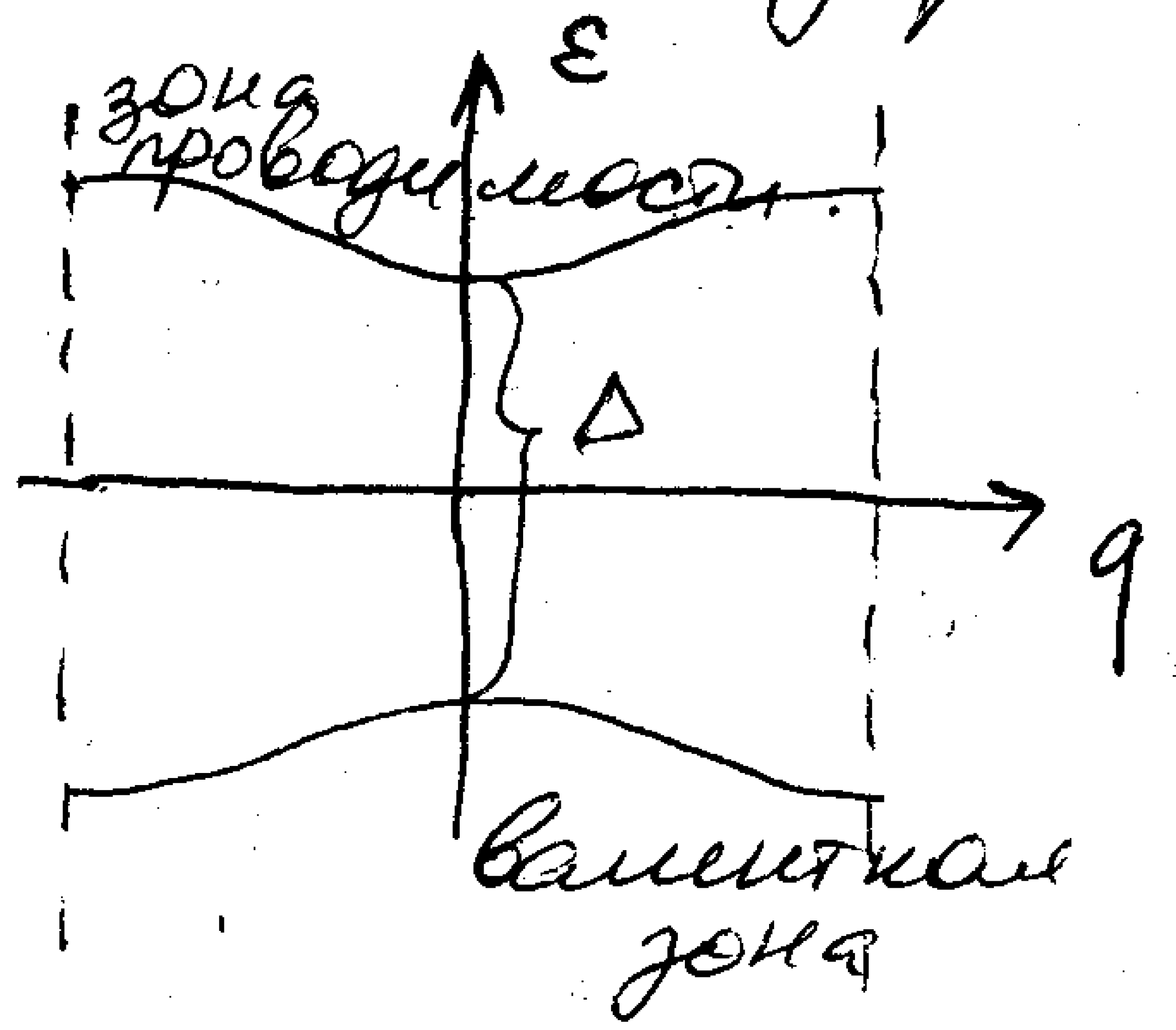
$$\Omega = -\frac{2}{3}\left(\frac{2}{5}B\mu^{5/2} + B\frac{\pi^2 T^2}{4}\sqrt{\mu}\right),$$

$$S = B\frac{\pi^2 T}{3}\sqrt{\mu}$$

$$N = -\frac{\partial \Omega}{\partial \mu} = \frac{2}{3} B \mu^{3/2}$$

$$C_V = T \frac{\pi^2}{3} \mu^{1/2} B = \frac{\pi^2}{2} \frac{TN}{\mu}$$

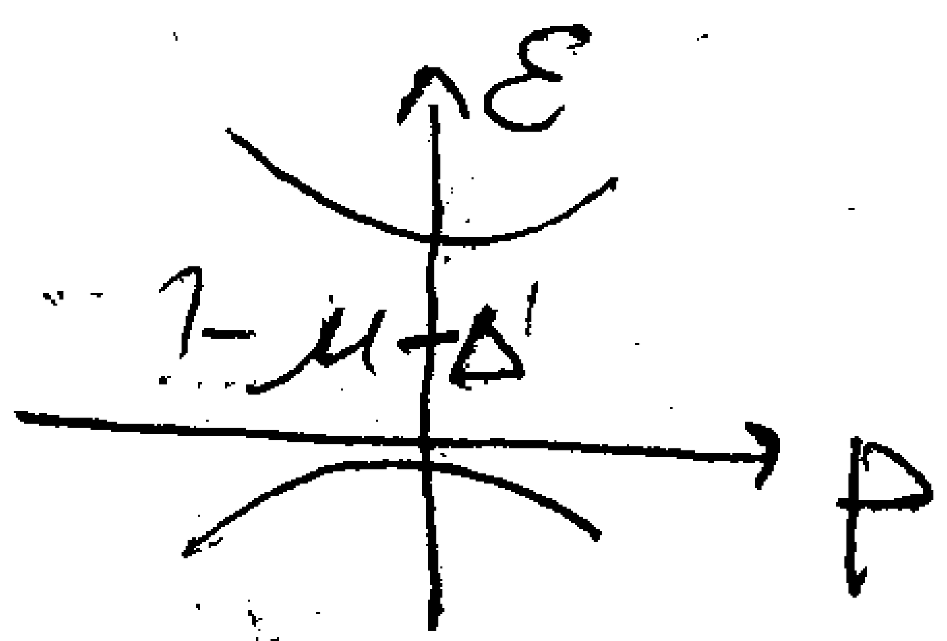
Полупроводники.



$\Delta \sim 10$ эВ – диэлектрик.

$\Delta \sim 1$ эВ – полупроводник.

$$\varepsilon_{val} = -\varepsilon_{hole}(p)$$



$$\varepsilon_{hole}(p) = -\frac{p^2}{2m_{hole}}$$

$$\varepsilon_{cond} = \Delta + \varepsilon_e(p), \quad \varepsilon_e(p) = \frac{p^2}{2m_e}$$

Примесей нет

$$N_e = \int f(\varepsilon)\, d\Gamma_e = \int \frac{2V d^3p}{\left(e^{\frac{\varepsilon_e - \mu + \Delta}{T}} + \cancel{1}\right)(2\pi\hbar)^3} =$$

$$\varepsilon_e - \mu \gg T$$

$$= \frac{2V}{(2\pi\hbar)^3} \int d^3p\, e^{-\frac{p^2}{2m_e T} + \frac{\mu-\Delta}{T}} = 2V e^{\frac{\mu-\Delta}{T}} \left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_h = \frac{2V}{(2\pi\hbar)^3} \int d^3p \left(1 - \frac{1}{e^{\frac{\varepsilon_{val} - \mu}{T}} + 1}\right) =$$

$$= \frac{2V}{(2\pi\hbar)^3}\int \frac{d^3p}{e^{\frac{\mu-\varepsilon_{val}}{T}}+1} = \frac{2V}{(2\pi\hbar)^3}\int \frac{d^3p}{e^{\frac{+\mu+\cancel{\Delta}+p^2/2m_h}{T}}+\cancel{1}} =$$

$$= 2Ve^{-\mu/T}\left(\frac{m_hT}{2\pi\hbar^2}\right)^{3/2}$$

$N_e = N_h$ — нейтральность.

$$N_e = N_h = \sqrt{N_eN_h} = 2V\left(\frac{T}{2\pi\hbar^2}\right)^{3/2}(m_em_h)^{3/4}e^{-\Delta/2T}$$

$$\mu = \frac{\Delta}{2} + \frac{3}{2}T\ln\frac{m_e}{m_h}$$

18.03.02.

$$\sigma = \frac{N_ee^2\tau_e}{Vm_e} + \frac{N_he^2\tau_h}{Vm_h} \sim e^{-\frac{\Delta}{2T}}$$

Распределение Бозе-Эйнштейна

$S = 0, 1, 2, \ldots$:

Идеальный газ, бозоны

$|k\rangle$ — для одной частицы

$W(N_k)$. „тело" - совокупность частиц в сост. $|k\rangle$

$N_k = 0, 1, \ldots$

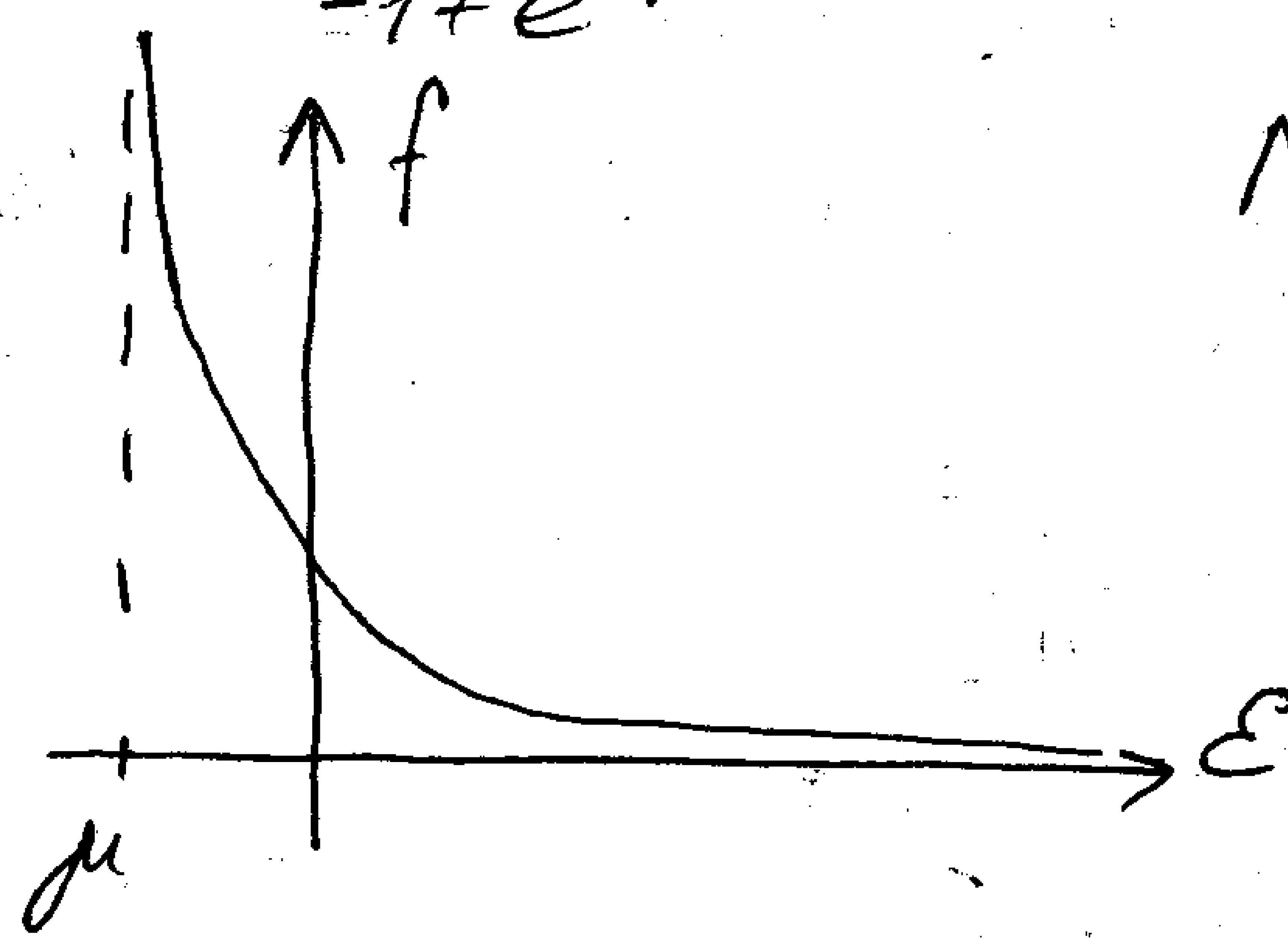
$$W = \frac{1}{q}e^{\frac{N_k(\mu-\varepsilon_k)}{T}}, \quad q = \sum_{N_k=0}^{\infty} e^{\frac{N_k(\mu-\varepsilon_k)}{T}} = \frac{1}{1-e^{\frac{\mu-\varepsilon_k}{T}}}$$

$$\langle N_k\rangle = f(\mathcal{E}) = T\frac{\partial}{\partial\mu}\ln q$$

$$\frac{T}{q}\frac{\partial q}{\partial\mu} = T\sum_{N_k=0}^{\infty}\frac{N_k}{T}\frac{e^{N_k\left(\frac{\mu-\mathcal{E}_k}{T}\right)}}{q} = \langle N_k\rangle$$

$$\langle N_k\rangle = -T\frac{\partial}{\partial\mu}\ln\left(1-e^{\frac{\mu-\mathcal{E}}{T}}\right) = \frac{1}{-1+e^{\frac{\mathcal{E}-\mu}{T}}}$$

$$f(\mathcal{E}) = \frac{1}{-1+e^{\frac{\mathcal{E}-\mu}{T}}}$$



$$N = \int f(\mathcal{E})\,d\Gamma$$

$$\Rightarrow \mu(V,T,N)$$

$$f_{\text{Больцм}} = e^{\frac{\mu-\mathcal{E}}{T}}$$

Конденсация Бозе-Эйнштейна

$s=0$.

$$f(\mathcal{E}) = \frac{1}{e^{\frac{\mathcal{E}-\mu}{T}}-1}$$

$$d\Gamma = \frac{V\,d^3p}{(2\pi\hbar)^3} = B\,V\sqrt{\mathcal{E}}\,d\mathcal{E}$$

$$N = BV\int_0^\infty \frac{\sqrt{\mathcal{E}}\,d\mathcal{E}}{e^{\frac{\mathcal{E}-\mu}{T}}-1} =$$

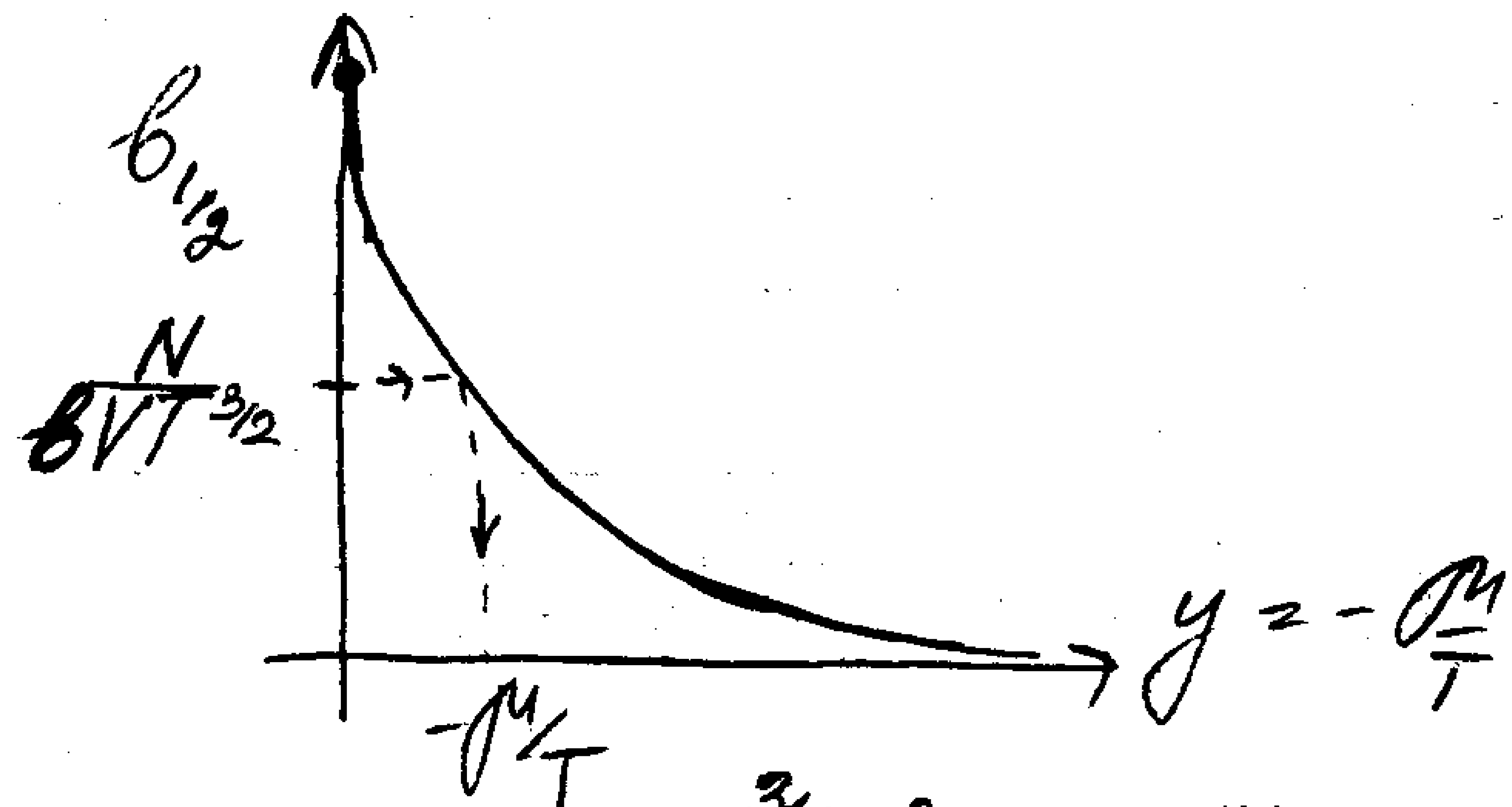
$$= BVT^{3/2}\int_0^\infty \frac{\sqrt{x}\,dx}{e^{x-\frac{\mu}{T}}-1}$$

$$b_a(y) = \int_0^\infty \frac{x^a\,dx}{e^{x+y}-1}$$

$$\frac{d}{dy} b_a(y) = \int_0^\infty x^a dx \, \frac{\partial}{\partial x} \frac{1}{e^{x+y}-1} = \frac{x^a}{e^{x+y}-1} \Big|_0^\infty -$$

$$-a \int_0^\infty dx \cdot x^{a-1} \frac{1}{e^{x+y}-1} = -a\, b_{a-1}(y).$$

$$N = BT^{3/2} V \, b_{1/2}\left(-\frac{\mu}{T}\right) \quad , \mu < 0$$



$T\downarrow, \ \mu \nearrow$

$$N_{\varepsilon=0} = \frac{1}{e^{-\frac{\mu}{T}}-1} \approx \frac{T}{(-\mu)}$$

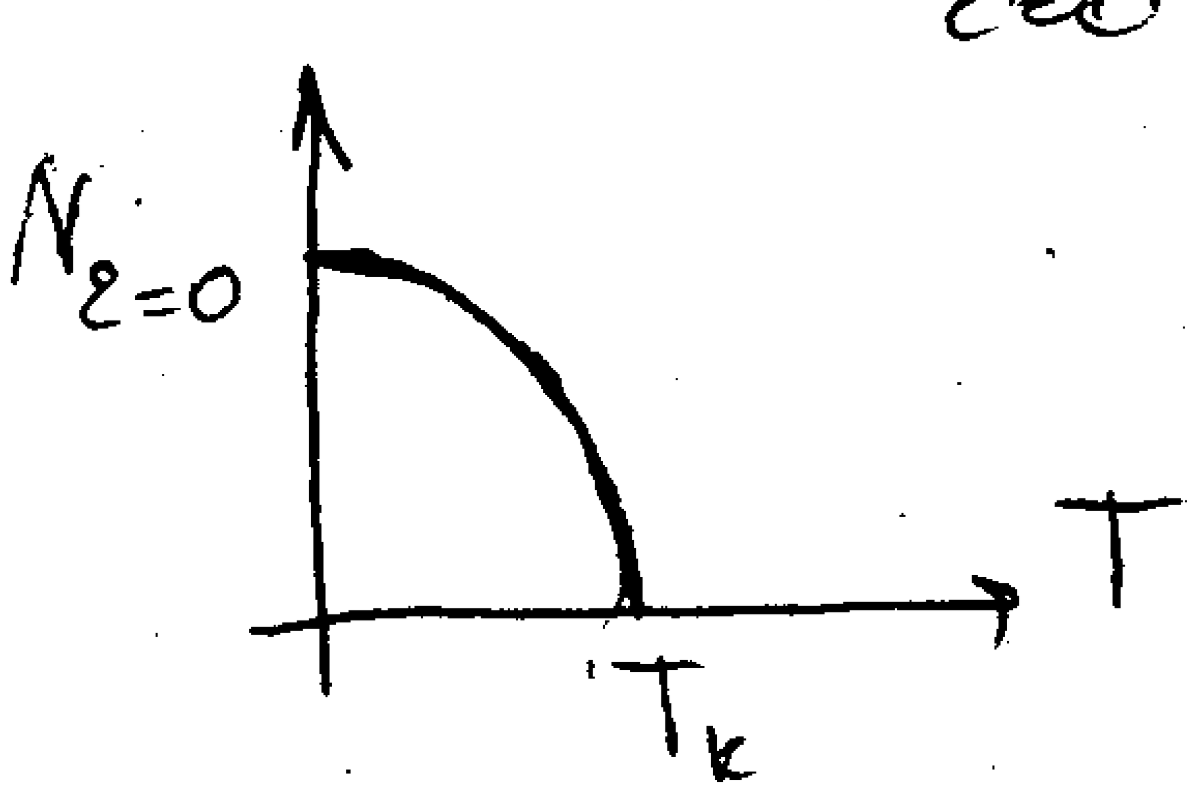
$$N = BVT^{3/2} b_{1/2}(-\mu/T) + N_{\varepsilon=0}.$$

T_k : $\quad BT_k^{3/2} b_{1/2}(0) = N$

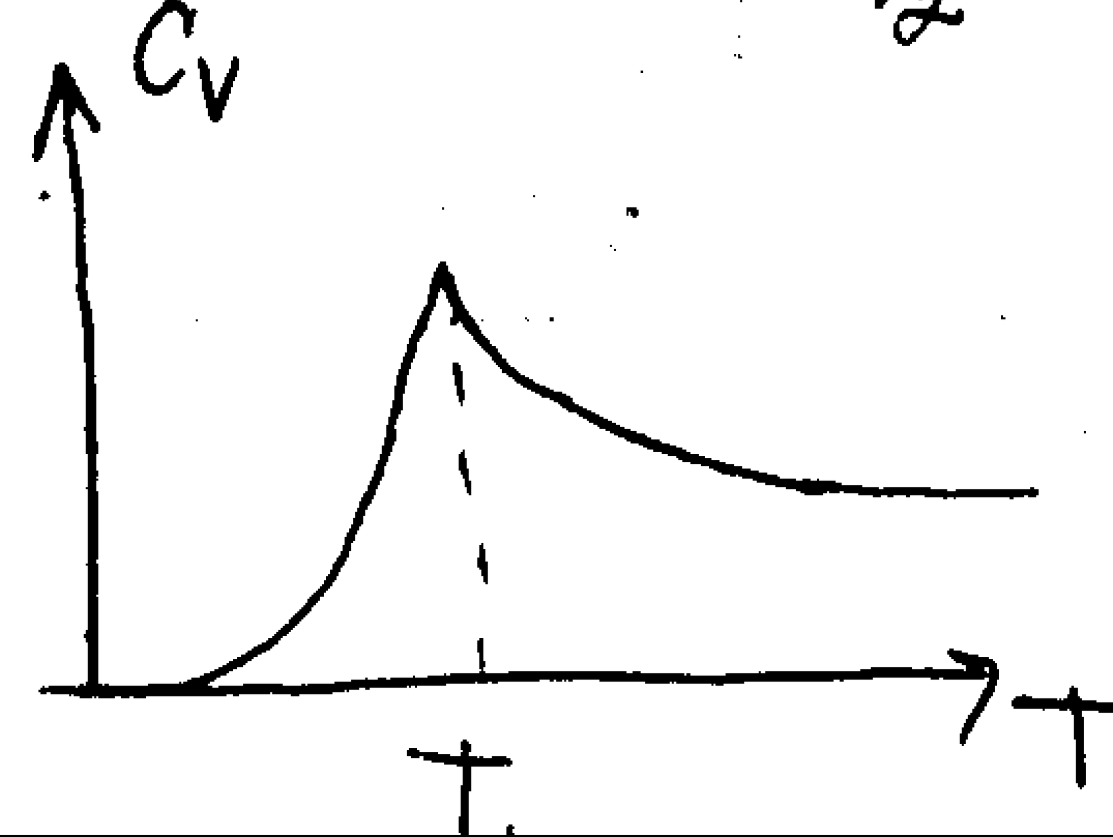
$T > T_k$: $\mu < 0 \quad N_{\varepsilon=0} \approx 0 \quad , N, V \to \infty , \frac{N}{V} = const$

$T < T_k \quad N = BVT^{3/2} b_{1/2}(0) + N_{\varepsilon=0}$

$$\Rightarrow N_{\varepsilon=0} = N - BVT^{3/2} b_{1/2}(0) = N\left(1 - \frac{T^{3/2}}{T_k^{3/2}}\right)$$



$$E = BVT^{5/2} b_{3/2}\left(-\frac{\mu}{T}\right)$$



^4He

$E = BT^{5/2} V b_{3/2}(0)$, $P = \frac{2}{3}\frac{E}{V} = BT^{5/2} b_{3/2}(0)$

$E = \frac{3}{2} NT = \frac{3}{2} PV$

Газ фотонов.

$S = 1$.

$\mathcal{E} = pc$

Число γ не фикс.

$A \to A + \gamma \Rightarrow \mu_A = \mu_A + \mu_\gamma \Rightarrow \mu_\gamma = 0$

$f(\mathcal{E}) = \frac{1}{e^{\frac{\mathcal{E}}{T}} - 1} =$, $\mathcal{E} = \hbar\omega$

$= \frac{1}{e^{\frac{\hbar\omega}{T}} - 1}$; $N = \int \frac{V\, d^3p}{(2\pi\hbar)^3} \frac{1}{e^{\frac{\mathcal{E}}{T}} - 1} =$

$= \frac{2V}{(2\pi)^3} \int \frac{d^3k}{e^{\frac{\hbar\omega}{T}} - 1} = \frac{2V}{(2\pi)^3} \cdot 4\pi \int \frac{k^2 dk}{e^{\frac{\hbar\omega}{T}} - 1} =$

$= \frac{2V \cdot 4\pi}{(2\pi)^3 c^3} \int_0^\infty \frac{\omega^2 d\omega}{e^{\frac{\hbar\omega}{T}} - 1} = \frac{VT^3}{\pi^2 c^3 \hbar^3} b_2(0)$

$\frac{V}{N} \sim \lambda^3 \sim \frac{1}{k^3} = \frac{\hbar^3 c^3}{\hbar^3 \omega^3} = \frac{\hbar^3 c^3}{T^3}$

$$b_a(0) = \int_0^\infty \frac{x^a\,dx}{e^x-1} = \int_0^\infty x^a e^{-x}(1+e^{-x}+e^{-2x}+\ldots)\,dx =$$

$$= \sum_{n=1}^\infty \left(\frac{z}{n}\right)^a e^{-z}\frac{dz}{n} = \sum_{n=1}^\infty \frac{1}{n^{a+1}}\int_0^\infty z^a e^{-z}\,dz =$$

$$= \zeta(a+1)\,\Gamma(a+1) \qquad , \quad \zeta(a) = \sum_{n=1}^\infty \frac{1}{n^a}$$

Газ фотонов. 22.03.02.

$$\varepsilon = cp = \hbar\omega = \hbar\cdot ck \quad , \quad \mu = 0$$

$$dE = \frac{2V\,d^3k}{(2\pi)^3}\cdot\frac{\hbar\omega}{e^{\frac{\hbar\omega}{T}}-1} = \frac{2V\,\hbar\omega^2\,d\omega\cdot 4\pi}{8\pi^3 c^3(e^{\frac{\hbar\omega}{T}}-1)} =$$

$$= \frac{V\hbar\omega^3\,d\omega}{\pi^2 c^3(e^{\frac{\hbar\omega}{T}}-1)} \quad , \quad x = \frac{\hbar\omega}{T}$$

$$E = \frac{V\hbar T^4}{\hbar^4\pi^2c^3}\int_0^\infty \frac{x^3\,dx}{e^x-1} = \frac{VT^4}{\hbar^3\pi^2c^3}\cdot\frac{\pi^4}{15} = \frac{VT^4\pi^2}{15\hbar^3c^3}$$

$$P = \frac{1}{3}\frac{E}{V} \qquad , \quad v_z = c\cdot\cos\theta$$

$$I = \int \varepsilon\, n(\varepsilon)\cdot v_z\, d\varepsilon\, d\Omega = \int_0^{\pi/2} \cos\theta\sin\theta\; 2\pi\, d\theta\;\cdot$$

$$\cdot\; c\,\varepsilon\, n(\varepsilon)\,d\varepsilon = \pi c \int \varepsilon\, n(\varepsilon)\,d\varepsilon$$

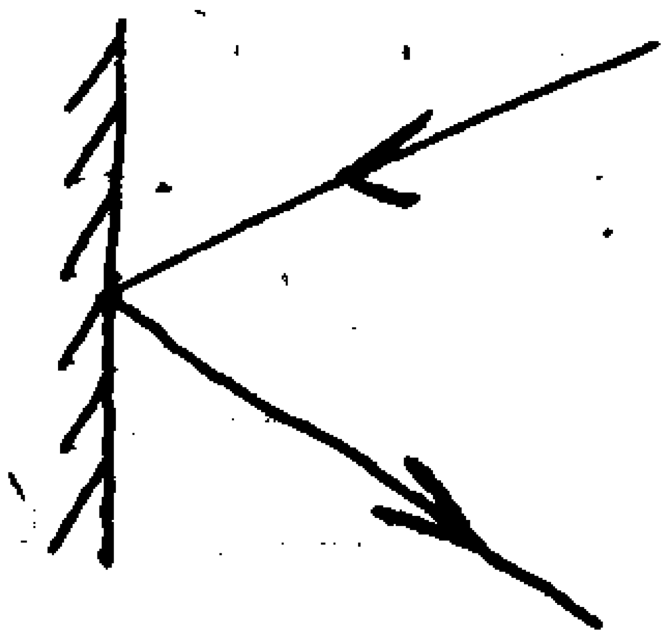
$$E = V\int \varepsilon\, n(\varepsilon)\,d\varepsilon\,d\Omega = V\int \varepsilon\cdot n(\varepsilon)\,d\varepsilon\cdot 4\pi.$$

$$\Longrightarrow\quad I = \frac{c}{4}\cdot\frac{E}{V}$$

$$I = \frac{\pi^2 T^4}{60\hbar^3 c^2} = \tilde{\sigma} T^4$$

$$I = \sigma T^4 \quad , T - в\ ^\circ K$$

$$\sigma = \frac{\pi^2 k^4}{60\hbar^3 c^2} = 5{,}67\cdot 10^{-8}\ \frac{Вт}{м^2 K^4}$$



I - падает

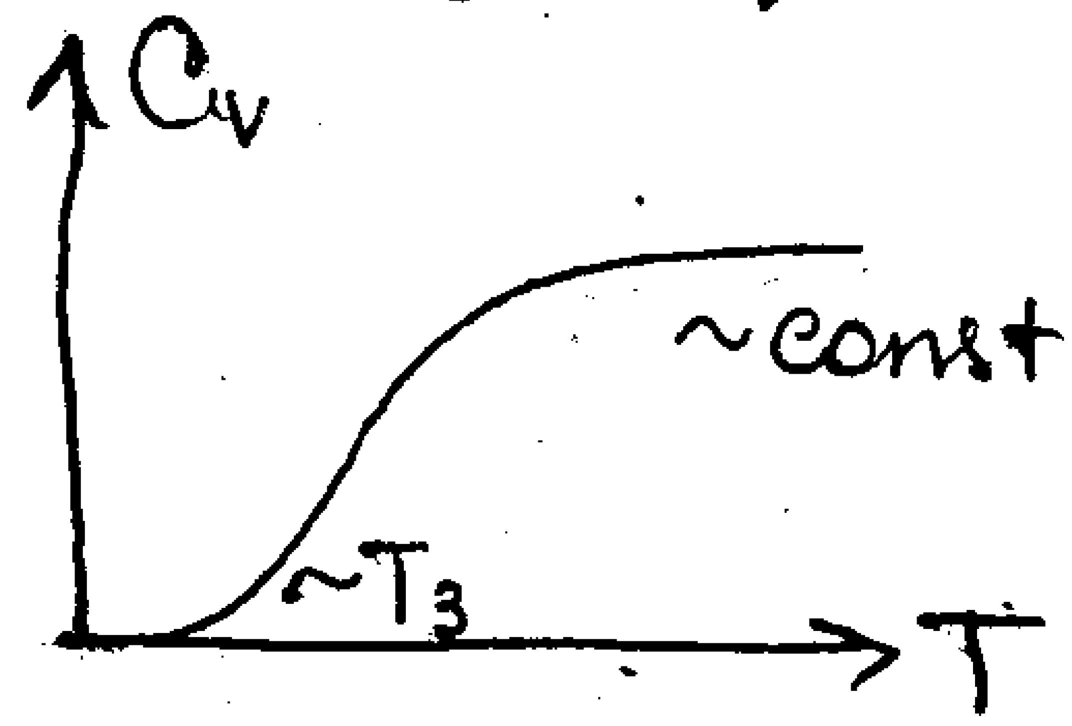
aI - поглощение

aI - излучение

Кирхгоф

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V \sim T^3$$

$T \to \infty$:



$$C = \left(\frac{3}{2} + \frac{3}{2}\right) N = 3N$$

– закон Дюлонг и Пти

Число волн $\frac{3V\, d^3k}{(2\pi)^3}$, $\omega_t = c_t k$

$\omega_l = c_l k$

$$\sum_{l,t,t} \frac{V d^3 k}{(2\pi)^3} = \frac{4\pi\, d\omega \cdot V\omega^2}{(2\pi)^3}\left(\frac{1}{c_l^3} + \frac{2}{c_t^3}\right) =$$

$$\frac{1}{c_l^3} + \frac{2}{c_t^3} = \frac{3}{u^3}$$ – определение.

$$= \frac{3}{2}\frac{V\omega^2 d\omega}{\pi^2 u^3}$$ – число колебаний.

$$\int_0^{\omega_{max}} \frac{3V\omega^2 d\omega}{2\pi^2 u^3} = 3N = \frac{V\omega_{max}^3}{2\pi^3 u^3}$$

$$\Rightarrow \omega_{max} = \left(6\pi^2 \frac{N}{V}\right)^{1/3} u$$

$$\hbar\omega_{max} = T_D$$

$$E = \int_0^{\omega_{max}} \frac{\hbar\omega \cdot 3V \cdot \omega^2 d\omega}{2\pi^2 u^3} \left(\frac{1}{e^{\frac{\hbar\omega}{T}}-1} + \frac{1}{2}\right) =$$

$$= \frac{3V\hbar}{2\pi^2 u^3} \cdot \frac{1}{2} \cdot \frac{\omega_{max}^4}{2} + \frac{3\hbar V T^4}{2\pi^2 u^3 \hbar^4} \cdot \int_0^{T_D/T} \frac{x^3 dx}{e^x - 1} =$$

$$= 3N\left(\frac{3}{8} T_D + T\,D\left(\frac{T_D}{T}\right)\right). \qquad D(x) = \frac{3}{x^3}\int_0^x \frac{x^3 dx}{e^x - 1}$$

$$E = \begin{cases} 3NT, & T \gg T_D \\ 3N \cdot \frac{\pi^4}{5} \frac{T^4}{T_D^3} + \frac{9}{8} N T_D, & T \ll T_D \end{cases}$$

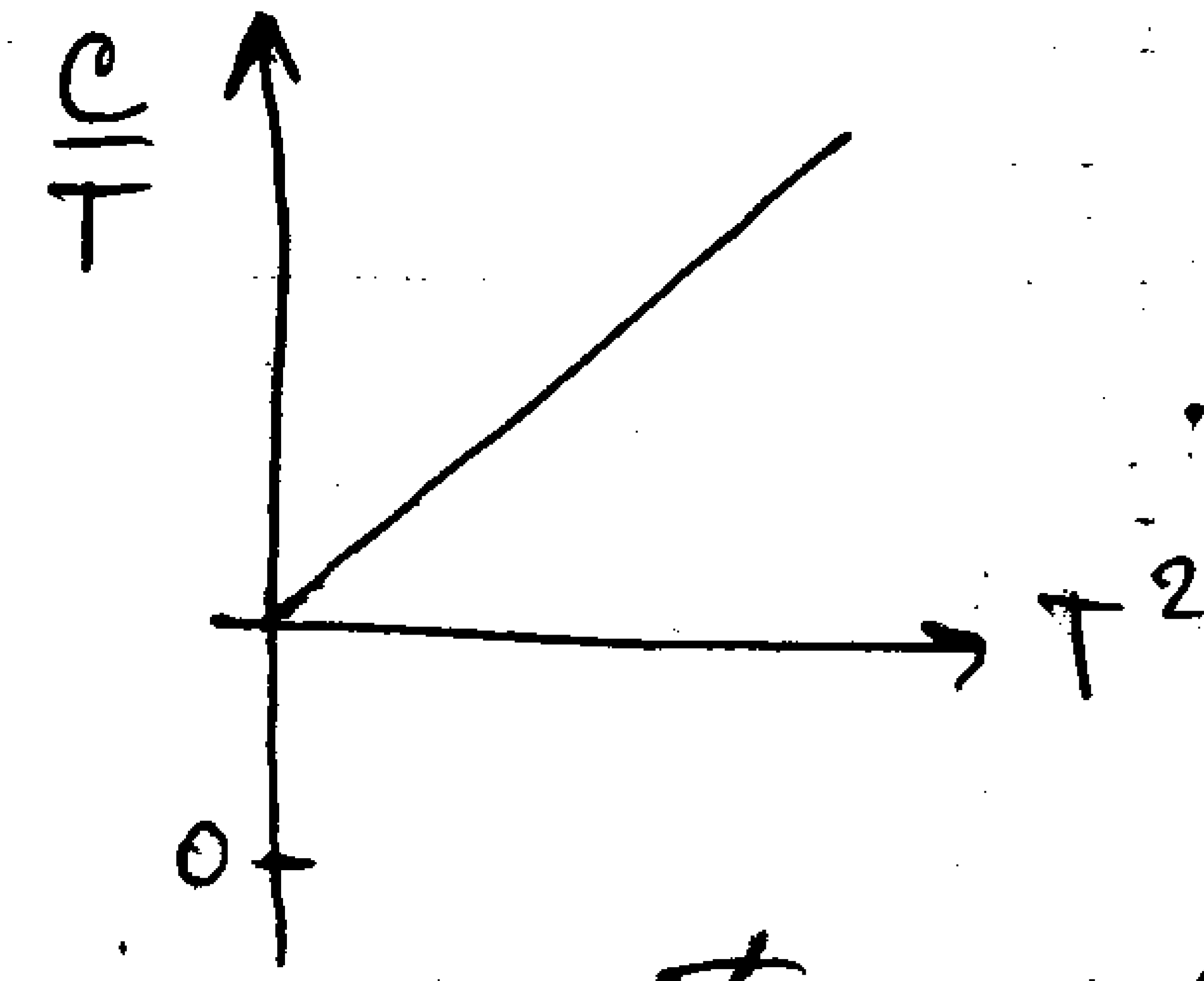
$$C_V = \begin{cases} 3N, & T \gg T_D \\ \frac{12}{5}\pi^4 \frac{T^3}{T_D^3} N, & T \ll T_D \end{cases}$$

T_D: Pb : 100K

алмаз : 2000K

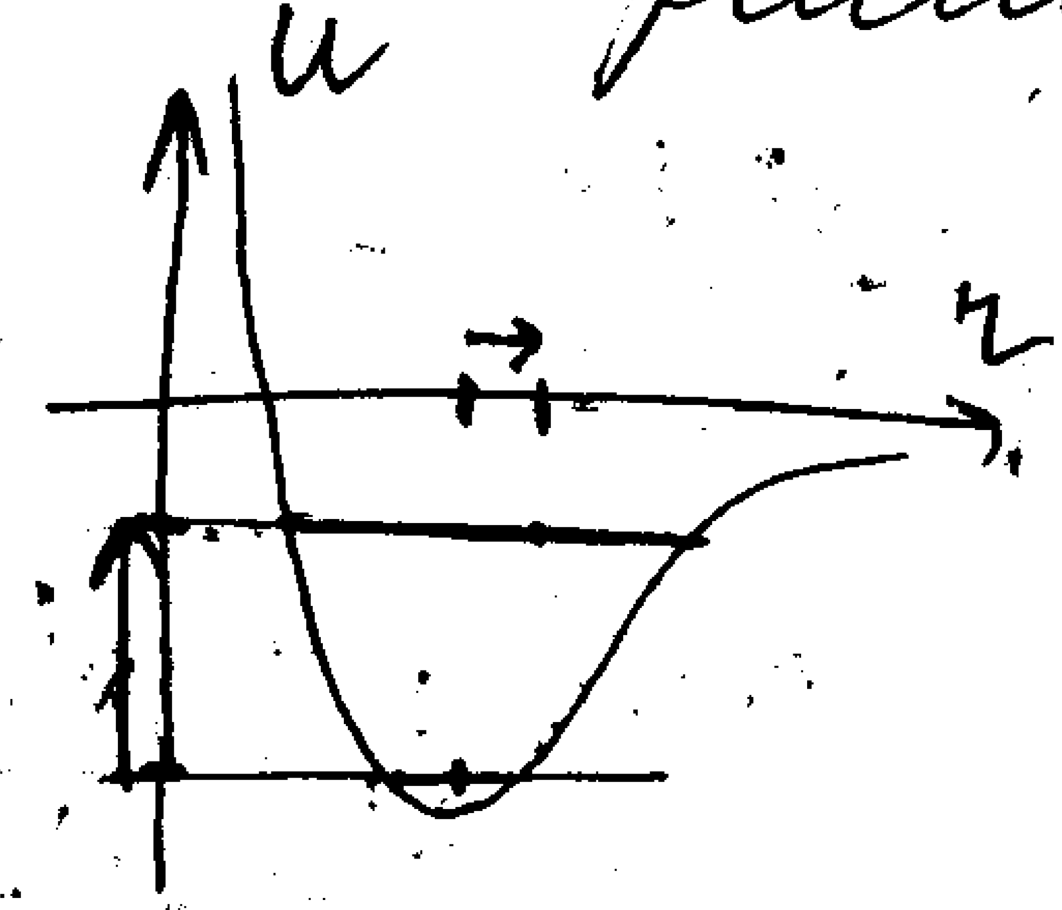
фононы – звуковые фотоны

металл: $c = aT + bT^3$.



$$\frac{C}{T} = a + bT^2$$

Тепловое расширение.



$i-1 \quad i \quad i+1$

$$x = x_i - x_{i+1}$$

$$U = \frac{kx^2}{2} - \frac{bx^3}{3}$$

$$f_{i,i-1} = -\frac{\partial U}{\partial x_i} = -kx + bx^2$$

$$\langle f \rangle = b\langle x^2 \rangle = \frac{b\varepsilon}{k}$$

$$\delta a \sim k\langle f \rangle \sim b\varepsilon.$$

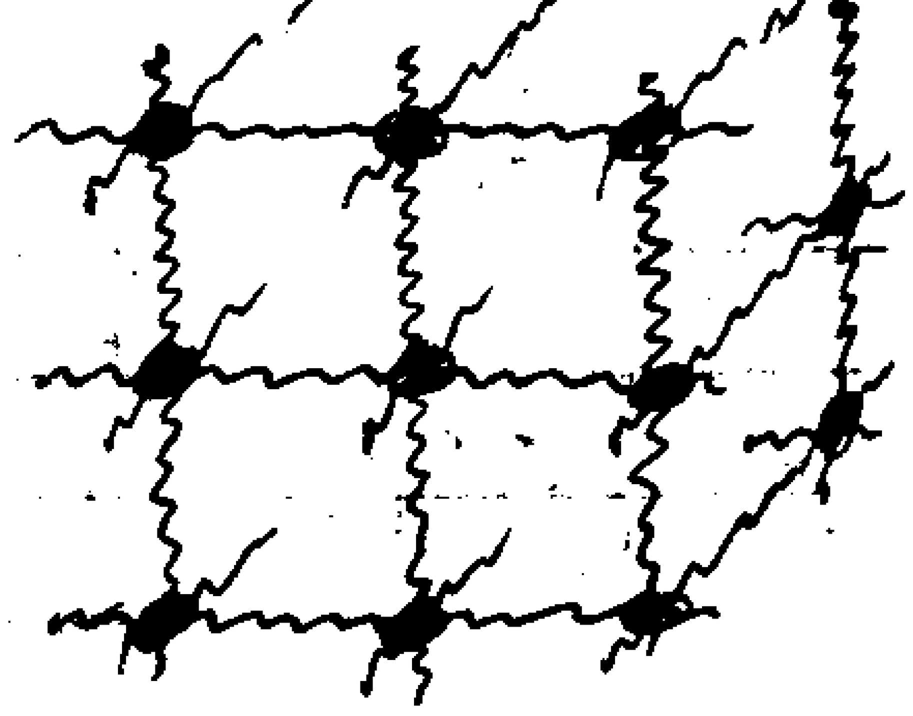
29.03.02

$$N \quad \frac{k\langle x^2 \rangle}{2} = \frac{E}{2} \Rightarrow$$

$$\langle f \rangle = \frac{bE}{kN}$$

$$\langle \delta x \rangle = \frac{\langle f \rangle}{k}$$

$$\delta l = N\langle \delta x \rangle = \frac{bE}{k^2 N} = \frac{bE}{k^2}$$



$Na^3 = V$

$P = \frac{\langle f \rangle}{a^2} \approx \frac{bE}{3kNa^2} \sim \frac{ba}{k} \cdot \frac{E}{V}$

a.e.: $a \sim 1$; $k = u'(r) \sim 1$

$b = 2u^{4}(r) \sim 1 \Rightarrow P \approx \frac{E}{V}$

$$P \sim \frac{E}{V}.$$

$$\alpha = \frac{1}{3}\frac{\partial V}{\partial T} = \frac{1}{3K}\left(\frac{\partial P}{\partial T}\right)_V$$

$T \gg T_D$ $\qquad E \sim NT$, $P \sim \frac{NT}{V}$

$$\frac{\partial P}{\partial T} \sim \frac{N}{V} \sim \frac{1}{a^3}$$

$\Rightarrow \alpha \sim \frac{1}{Ka^3} \sim 1$ $\qquad [\alpha] = \frac{1}{T} \sim \frac{1}{kT^\circ}$

30 эВ $\sim 3 \cdot 10^5$ K°

$\Rightarrow \alpha \sim 3 \cdot 10^{-6} \frac{1}{K}$

$F = \Omega + \mu N$ $\qquad \mu = 0$ $\qquad \Omega = -T \ln Q.$

$$Q = \prod_{\{k\}} q_k = \prod_{\{k\}} \sum_{n=0}^{\infty} e^{-\frac{\hbar\omega_k}{T}(n+\frac{1}{2})} = \prod_a \frac{e^{-\frac{\hbar\omega_a}{2T}}}{1 - e^{-\frac{\hbar\omega_a}{T}}}$$

$$F = \sum_a \left(\frac{\hbar\omega_a}{2} + T \ln\left(1 - e^{-\frac{\hbar\omega_a}{T}}\right)\right)$$

$$P_{ph} = -\frac{\partial F_{ph}}{\partial V} = -\sum_a \left(\frac{\hbar}{2} \frac{\partial \omega_a}{\partial V} + \frac{T e^{-\frac{\hbar\omega_a}{T}}}{1 - e^{-\frac{\hbar\omega_a}{T}}} \frac{\hbar}{T} \frac{\partial \omega_a}{\partial V} \right)$$

$$= -\hbar \sum_a \underbrace{\left(\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega_a}{T}} - 1} \right)}_{n_a} \frac{\partial \omega_a}{\partial V} = -\hbar \sum_a n_a \frac{\partial \omega_a}{\partial V}$$

$$\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V = \frac{1}{V} \cdot 3\alpha V \cdot = 3\alpha$$

$$\frac{1}{K} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \Rightarrow 3\alpha V = \frac{V}{K}\left(\frac{\partial P}{\partial T}\right)_V$$

$$\alpha = \frac{1}{3K}\left(\frac{\partial P_{ph}}{\partial T}\right)_V$$

$$\left(\frac{\partial P_{ph}}{\partial T}\right) = \hbar \sum_a \frac{\partial \omega_a}{\partial V} \frac{\partial n_a}{\partial T}$$

$$E_{ph} = \sum_a \hbar \omega_a n_a$$

$$C_{ph} = \frac{\partial E}{\partial T} = \sum \hbar \omega_a \frac{\partial n_a}{\partial T}$$

Если $\frac{\partial \omega_a}{\partial V} = \gamma \omega_a$, $\alpha \sim C$

При $T \ll T_D$ $\alpha \sim T^3$

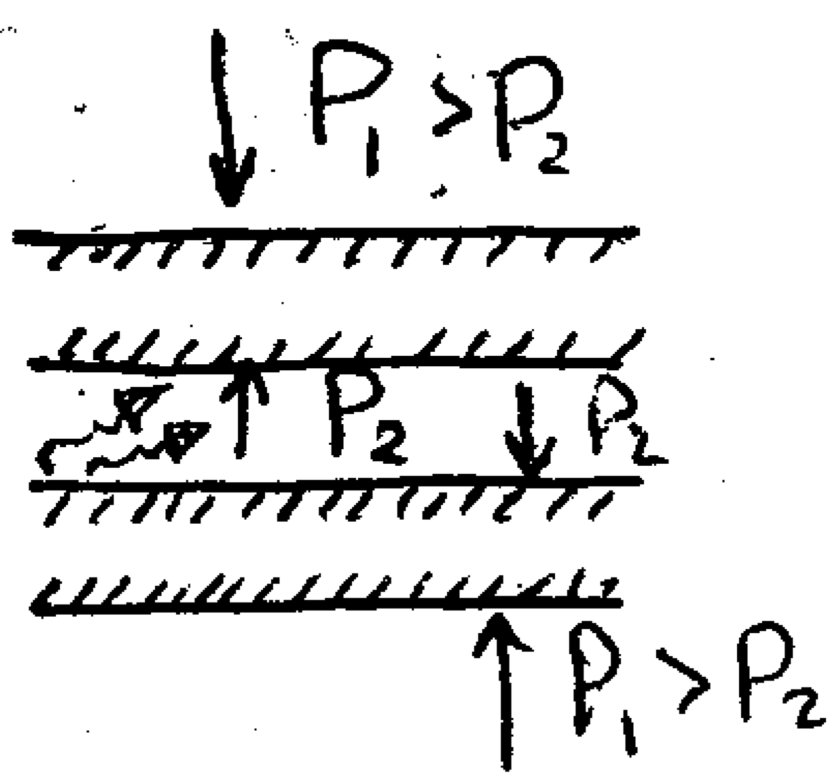
$T \gg T_D$ $\alpha \sim const$

$\alpha \sim C$ — Закон Грюнайзена.

Силы Казимира
(молек. притяжение).

Фотонный газ

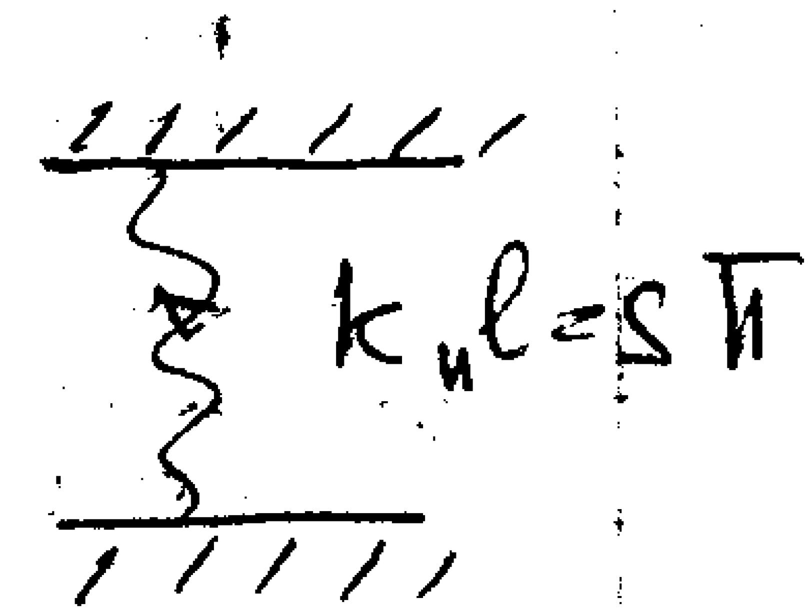
$$P = -\hbar \sum_a n_a \frac{\partial \omega_a}{\partial V} \quad , \quad n_a = \frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega_a}{T}} - 1}$$



Два проводника притягиваются.

$$\omega \sim \frac{1}{V^{1/3}} \qquad \frac{\partial \omega}{\partial V} = \frac{\omega}{3V} \Rightarrow$$

$$\Rightarrow P = \frac{E}{3V}$$



$$\omega = c\sqrt{\vec{k}_\perp^2 + \left(\frac{\pi s}{l}\right)^2} = \omega(\vec{k}_\perp, s)$$

$$P = -\hbar c \frac{\partial}{\partial l} \sum_s \int \omega(\vec{k}_\perp, s) \frac{d^2 k_\perp}{(2\pi)^2} =$$

$$= -\frac{\hbar c \cdot 2\pi}{(2\pi)^2 c^2} \frac{\partial}{\partial l} \sum \int_{\frac{\pi s}{l}}^{\infty} \omega^2 d\omega \cdot R(\omega) = \qquad R(\omega) = \begin{cases} 1, & \omega \to 0 \\ 0, & \omega \to \infty \end{cases}$$

при $\omega \to \infty$ фотоны проходят сквозь металлы

$$= + \frac{\hbar}{2\pi c} \sum_s \omega_s^2 \frac{\partial \omega_s}{\partial l} R(\omega_s) =$$

$$\frac{\partial \omega_s}{\partial l} = -\frac{\omega_s}{l}$$

$$= \frac{-\hbar}{2\pi c l} \sum_{s=1}^{\infty} \omega_s^3 R(\omega_s)$$

$$P_1 = P(\ell \to L) = -\frac{\hbar}{2\pi c} \cdot \frac{1}{L} \sum_s \omega_s^3 R(\omega_s) =$$

$$\omega = \frac{\pi s}{cL}$$

$$= \frac{-\hbar}{2\pi \cancel{L} c} \cdot \frac{cL}{\pi} \int_0^\infty \omega^3 R(\omega) d\omega$$

$$\int_0^\infty f(x)dx = \frac{1}{2} f(0) + \sum_{S=1}^{\infty} f(S) + \frac{1}{12} f'(0) - \frac{1}{720} f'''(0) + \frac{1}{5040} f^{V}(0)$$

$$(\omega^3)'|_{\omega=0} = 0$$

$$(\omega^3)'''|_{\omega=0} = 6$$

$$(\omega^3)^{V}|_0 = 0.$$

$$P_1 - P_2 = \frac{\hbar}{2\pi c \ell} \frac{\pi^3}{\ell^3} \cdot \frac{6}{720} = \frac{\pi^2 \hbar}{240 \ell^4}$$

1.04.02.

$$\mathcal{E} = U(\vec{r}) + \frac{p^2}{2m}$$

$$f(\mathcal{E}) = \frac{1}{e^{\frac{\mathcal{E}-\mu}{T}} \pm 1}$$

S

G_i – сост. в i-й группе

N_i частиц в i-й гр.

$$C_{G_i}^{N_i}$$

Больцм. газ: $N_i \ll G_i$

Ферми-газ $N_i \not\ll G_i$

$$\Gamma_i = C_{G_i}^{N_i} = \frac{G_i!}{N_i!(G_i-N_i)!} \qquad \Gamma = \prod_i \Gamma_i$$

$$S_i = \ln \Gamma_i \quad , \quad f_i = N_i / G_i$$

$$S = \sum_i G_i \left(f_i \ln f_i + (1-f_i)\ln(1-f_i)\right)$$

Бозе-газ: N_i G_i

$$\Gamma_i = C^{N_i}_{N_i+G_i-1}$$

Неидеальные газы.

Классич. газ.

$$U = \sum_{i<j} u(\vec{r}_i - \vec{r}_j)$$

$$Z = \int \frac{d^3p_1 \cdots d^3p_N}{(2\pi\hbar)^{3N} N!} \, dV_1 \cdots dV_N \, e^{-\sum p_i^2/2mT} e^{-U/T} =$$

$$= Z_{ид} \cdot \frac{1}{V^N} \int dV_1 \cdots dV_N \, e^{-U/T}$$

$$u_{ij} \ll T \quad , \quad \frac{N\tau}{V} \ll 1 \qquad \tau \sim r^3$$

Тройные столкновения не учитываем.

$$\frac{1}{V^N}\int dV_N\int dV_{N-1}\cdots\int dV_2\, e^{-\beta\sum_{j=3}^{N}U(\vec{r}_2-\vec{r}_j)}\int dV_1\, e^{-\beta\sum_{j=2}^{N}U(\vec{r}_1-\vec{r}_j)}$$

$$\int dV_1\left[1-\left(1-e^{-\beta\sum_{j=2}^{N}U_{1j}}\right)\right] = V-2B(N-1)$$

$$\int\left(1-e^{-\frac{U_{12}}{T}}\right)dV_1 = 2B$$

$$\frac{1}{V^N}\big(V-2B(N-1)\big)\big(V-2B(N-2)\big)\dots(V-2B)V =$$

$$=\prod_{j=0}^{N-1}\left(1-\frac{2B}{V}j\right)$$

$$F=-T\ln Z = F_{ug} - T\sum_{j=1}^{N-1}\ln\left(1-\frac{2B}{V}j\right) =$$

$B \ll V$ $$= F_{ug} + \sum_{j=1}^{N}\frac{2B}{V}j =$$

$$= F_{ug} + \frac{2BT}{V}\frac{N(N-1)}{2} = F_{ug} + \frac{TBN^2}{V}$$

$$P = -\frac{\partial F}{\partial V} = \frac{NT}{V} + \frac{BN^2T}{V^2}$$

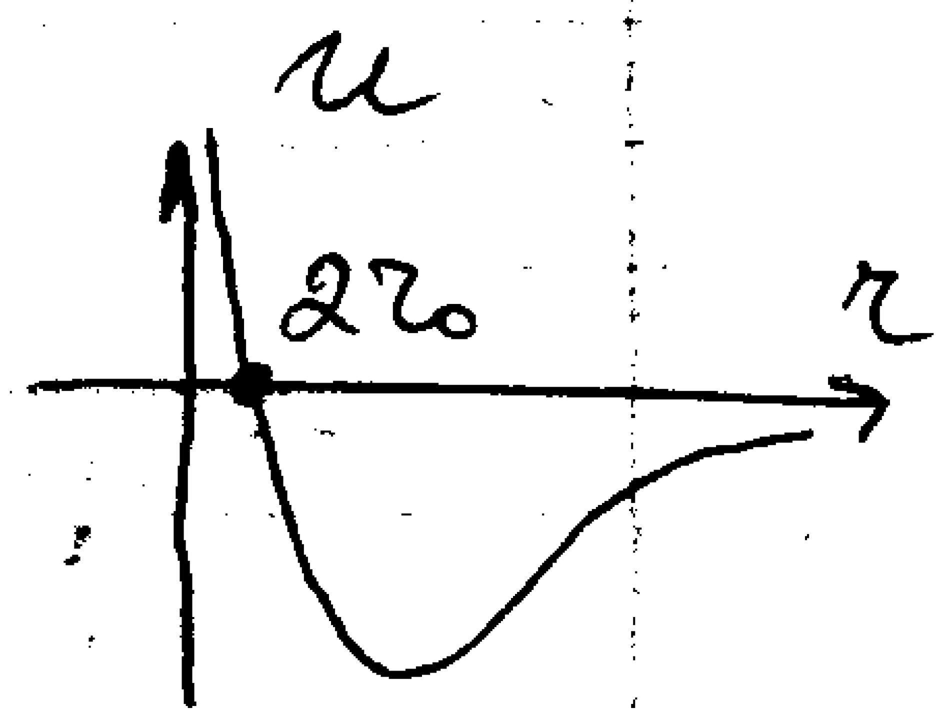
$$P = \frac{NT}{V} + \frac{BN^2T}{V^2}$$

$$P = T\sum_{n=1}^{\infty} B_n(T)\left(\frac{N}{V}\right)^n, \quad B_1 \equiv 1, \quad B_2 \equiv B, \ldots$$

Вириальные коэффициенты.

$$B = \frac{1}{2}\int \left(1 - e^{-\frac{u}{T}}\right) dV = \frac{4\pi}{2}\int_0^{\infty}\left(1 - e^{-\frac{u(r)}{T}}\right) r^2 dr =$$

$$= 2\pi\left(\int_0^{2r_0} + \int_{2r_0}^{\infty}\right) =$$



$$= 2\pi\cdot\frac{(2r_0)^3}{3} + \frac{2\pi}{T}\int_{2r_0}^{\infty} u(r)\, r^2 dr = b - \frac{a}{T}$$

$$F = F_{ug} + \frac{N^2 T}{V}\left(b - \frac{a}{T}\right)$$

$$P = -\frac{\partial F}{\partial V} = \frac{NT}{V} + \frac{N^2 T}{V^2}\left(b - \frac{a}{T}\right) =$$

$$= P_{ug} + \frac{N^2}{V^2} bT - \frac{N^2 a}{V^2}$$

$$E = F + TS = E_{ug} - \frac{N^2 a}{V}$$

Газ Ван-дер-Ваальса.

$$P = \frac{NT}{V}\cdot\left(1 + \frac{Nb}{V}\right) - \frac{N^2 a}{V^2}$$

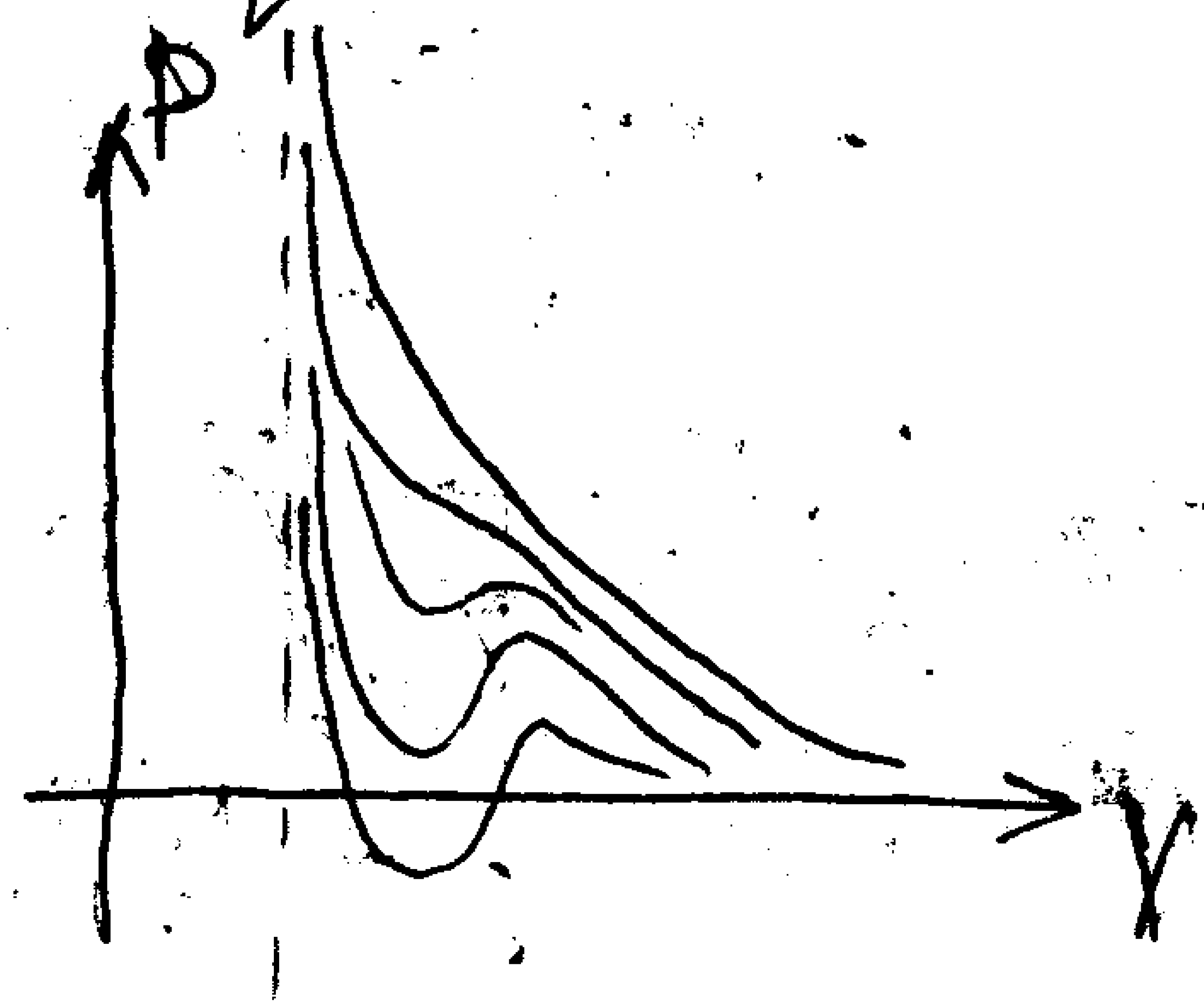
$$\simeq\left(1 - \frac{Nb}{V}\right)^{-1}$$

$$\frac{N^2 T b}{V^2} \longrightarrow -NT\ln\left(1 - \frac{Nb}{V}\right)$$

$$F = F_{ид} - NT\ln\left(1-\frac{Nb}{V}\right) - \frac{N^2a}{V}$$

$$P = P_{ид} + \frac{N^2Tb}{V^2\left(1-\frac{Nb}{V}\right)} - \frac{N^2a}{V^2} =$$

$$= \frac{NT}{V-Nb} - \frac{N^2a}{V^2}$$

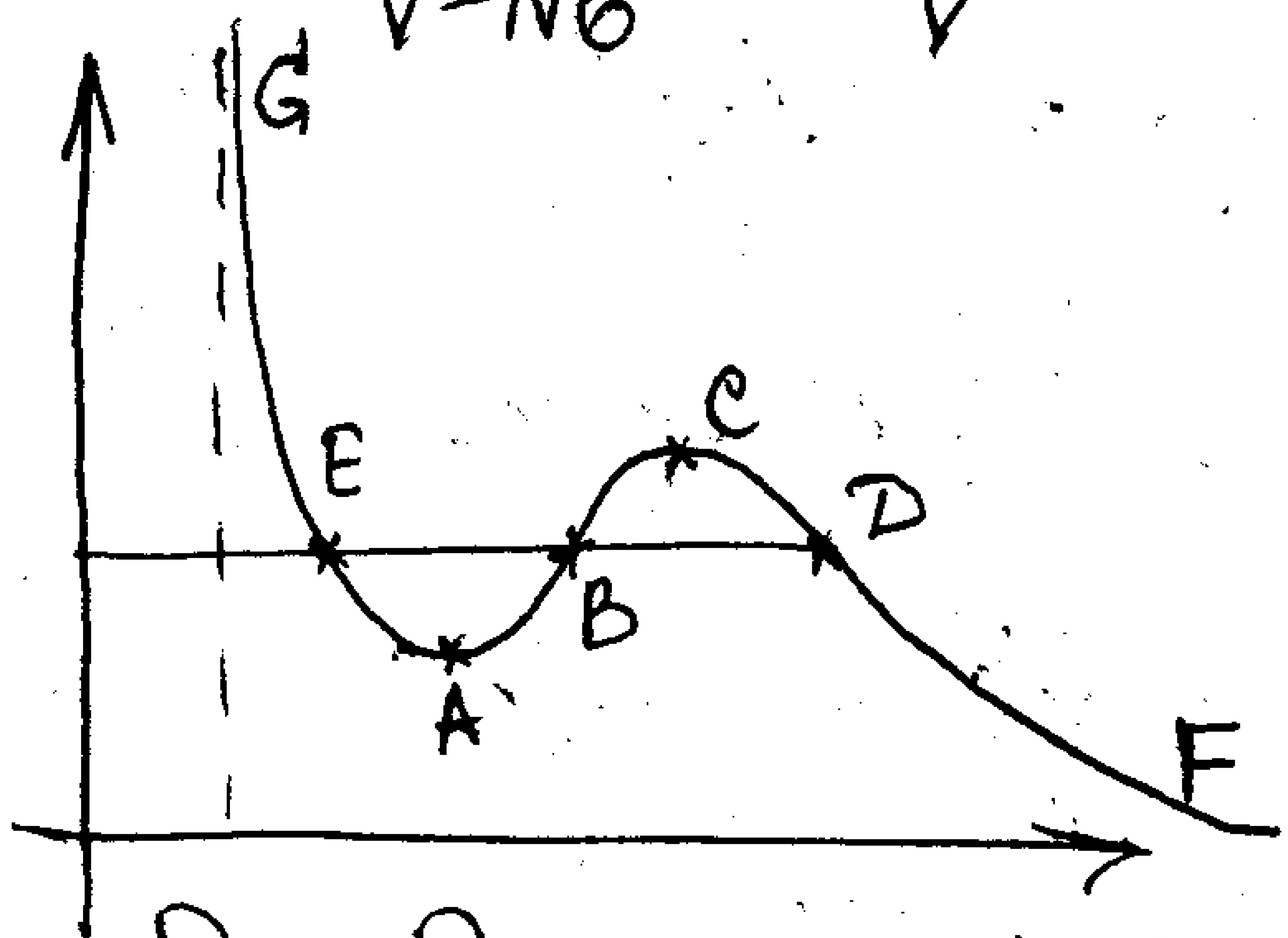


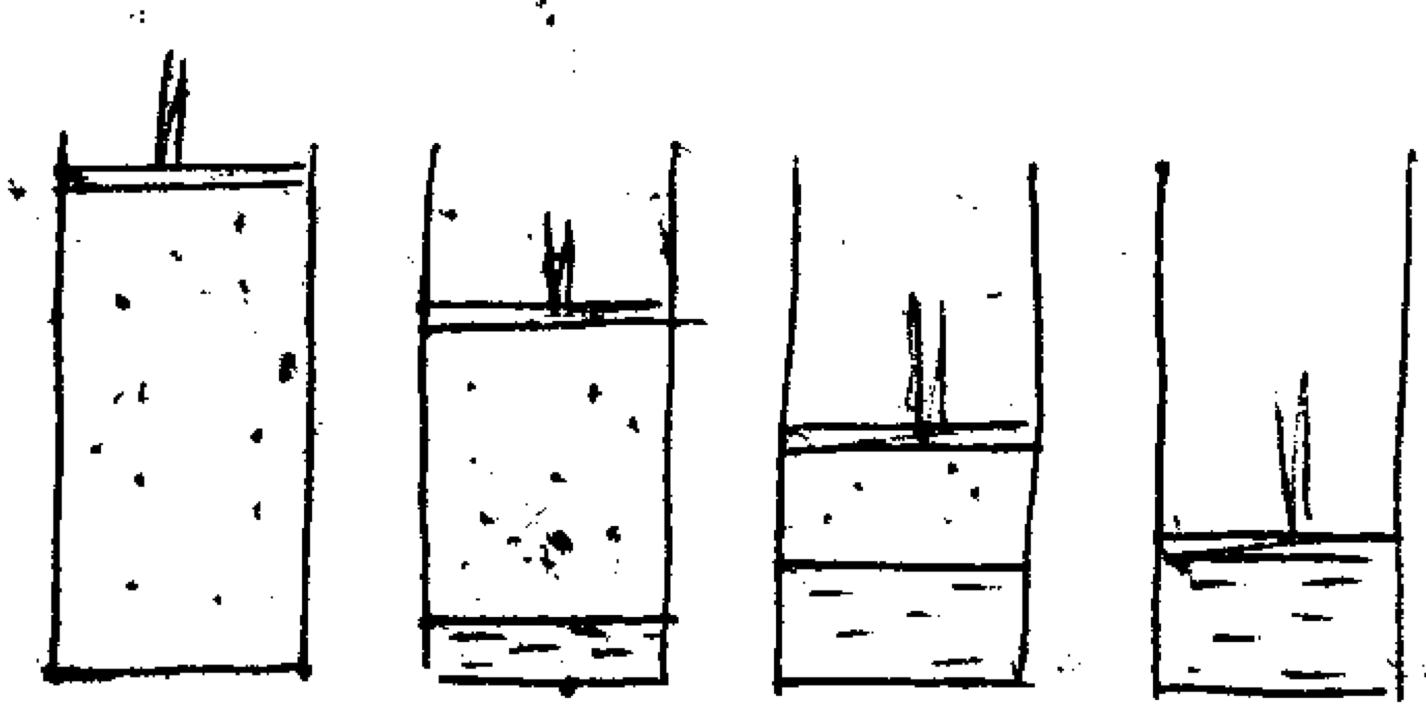
$P_{ж} = P_{п}$

$\mu_{ж} = \mu_{п}$

5.04.02.

$$P = \frac{NT}{V-Nb} - \frac{Na}{V}$$





Равновесное сжатие

$P_{п} = P_{ж}$ $\mu_{п} = \mu_{ж}$

$$\begin{cases} N_{п} + N_{ж} = N \\ N_{п}v_{п} + N_{ж}v_{ж} = V \end{cases}$$

$$\oint\limits_{EABCDBE} V\,dP = \int d\Phi = \Phi_B - \Phi_E = N_п \mu_п - N_ж \mu_ж =$$

$$= N(\mu_п - \mu_ж) = 0$$

Плазма.

$$\frac{e^2}{a} \ll T \qquad V a^{-3} = N$$

Экранировка

вокруг $\oplus$ $\quad n_e \sim n_{e0}\, e^{\frac{e^2}{rT}}$

$$n_i \sim n_{i0}\, e^{-\frac{e^2}{rT}}$$

$$\rho \sim e(n_i - n_e) \sim e(\delta n_e - \delta n_i) \sim \frac{n e^3}{rT}$$

$$n_{e0} = n_{i0} = n = \frac{N}{V}, \quad r^3 \rho \sim e$$

$$r^3 \cdot n \cdot \frac{e^2}{rT} \sim 1 \Rightarrow r \sim \sqrt{\frac{T}{ne^2}}$$ радиус Дебая

$$\delta E \approx -N\frac{e^2}{r} \sim -\frac{Ne^2 \cdot \sqrt{ne^2}}{\sqrt{T}} \sim \frac{-|e|^3 N^{3/2}}{\sqrt{VT}}$$

$$\delta E = -\frac{A N^{3/2}}{\sqrt{VT}}$$

$$\delta C = \left(\frac{\partial \delta E}{\partial T}\right)_V = \frac{A N^{3/2}}{2V^{1/2} T^{3/2}}; \quad \delta S = \int_\infty^T \frac{\delta C_v}{T}\, dT =$$

$$= -\frac{AN^{3/2}}{3V^{1/2}T^{3/2}}$$

$$\delta F = \delta E - T\delta S = \frac{2AN^{3/2}}{3V^{1/2}T^{1/2}}$$

$$\delta P = -\left(\frac{\partial \delta F}{\partial V}\right)_T = -\frac{AN^{3/2}}{3V^{3/2}T^{1/2}}$$

$$e_j = z_j e \quad , n_{j0} \qquad \sum z_j n_{j0} = 0$$

$$\Delta\varphi = -4\pi e \sum_j z_j n_j = -4\pi e \sum z_j n_{j0} e^{-\frac{e_j \varphi}{T}} =$$

$$= -4\pi e \sum z_j \left(n_{j0} - n_{0j}\frac{e z_j \varphi}{T}\right) = \frac{4\pi e^2}{T} \sum z_j^2 n_{j0} \varphi$$

$$\Delta\varphi - \varkappa^2\varphi = 0 \qquad \varkappa^2 = \frac{4\pi e^2}{T} \sum z_j^2 n_{j0}$$

$$\varphi = \frac{c e^{-\varkappa r}}{r} + \cancel{\frac{c' e^{\varkappa r}}{r}} \quad r \to 0: \quad \varphi \to \frac{e z_k}{r}$$

$$\Rightarrow \varphi = \frac{e z_k}{r} e^{-\varkappa r}$$

$$\frac{1}{2} z_k e \left(\varphi - \frac{z_k e}{r}\right)_{r \to 0} = -\frac{1}{2} z_k^2 e^2 \varkappa$$

$$\cancel{\delta E = -\frac{N}{2} z_k^2 e^2 \varkappa}$$

$$\delta E = -\frac{\varkappa e^2}{2}\sum z_k^2 n_{k0}$$

$$r^2 \sim \frac{T}{ne^2}$$

$$\Rightarrow n\cdot r^3 \sim \left(T/e^2_{/a}\right)^{3/2} \gg 1$$ — число экранирующих частиц.

$$\varkappa = \frac{1}{r_D}$$

Вырожденная плазма

$\hbar\vec{k}_1,\ \hbar\vec{k}_2$

$$\Psi = c\left(e^{i\bar{k}_1\bar{r}_1 + i\bar{k}_2\bar{r}_2} - e^{i\bar{k}_1\bar{r}_2 + i\bar{k}_2\bar{r}_1}\right)$$

$$dW = |\Psi|^2 dV_1 dV_2 = c^2\left(2 - e^{i\bar{k}_1(\bar{r}_1-\bar{r}_2)+i\bar{k}_2(\bar{r}_2-\bar{r}_1)} - \text{к.с.}\right)dV_1 dV_2 =$$

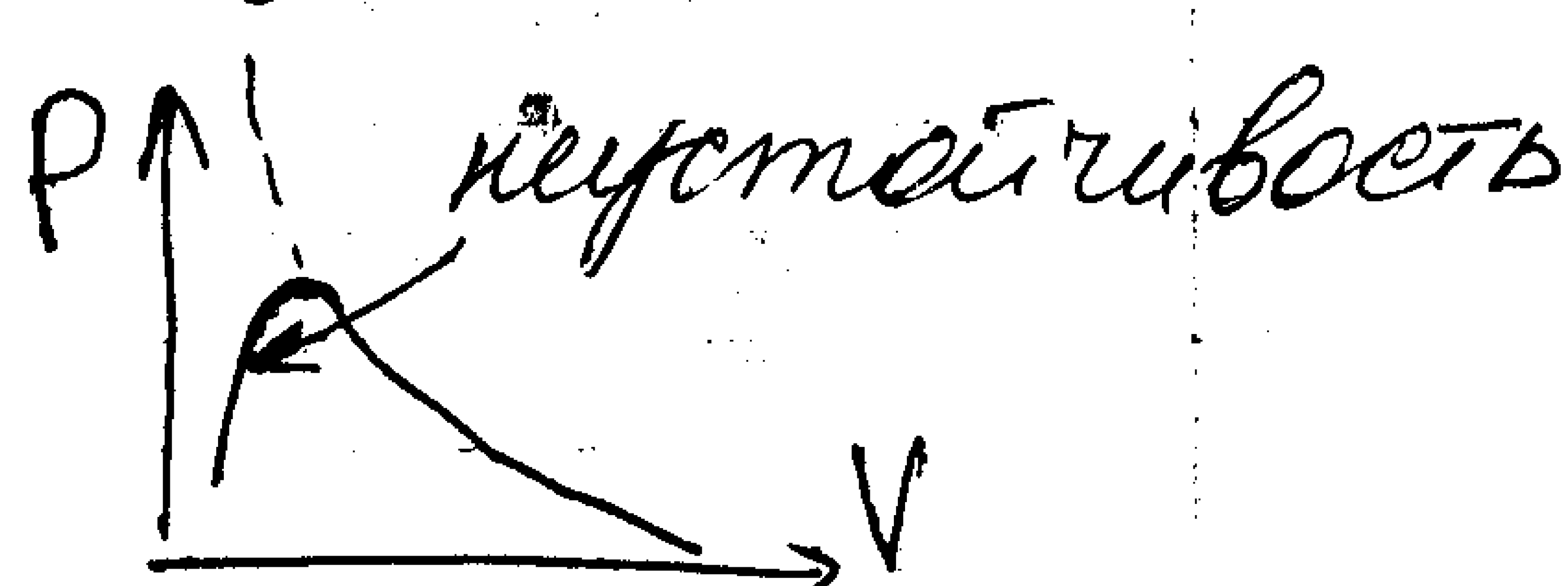
$$= 2c^2\left(1-\cos(\bar{k}_1-\bar{k}_2)(\bar{r}_1-\bar{r}_2)\right)dV_1 dV_2 \simeq$$

~ 0 кроме $|\vec{r}_1-\vec{r}_2| \sim \lambda \sim \dfrac{2\pi}{|\bar{k}_1-\bar{k}_2|}$

$$\lambda_{хар} \sim n^{-1/3}$$

$$\Rightarrow \delta E_{обм} \sim -N\frac{e^2}{\lambda_{хар}} \sim -Ne^2 n^{1/3}$$

$$\delta P \sim -\frac{AN^{3/2}}{V^{3/2}T^{1/2}}$$



Пространственная дисперсия статической диэлектрической проницаемости.

$\varepsilon(\vec{k})$ $(\omega = 0)$

$$\Delta\varphi - \varkappa^2\varphi = -4\pi\rho$$

$$\varphi_k = \int \varphi e^{-i\bar{k}\bar{r}} dV,$$

$$\rho_{\bar{k}} = \int \rho e^{-i\bar{k}\bar{r}} dV$$

$$\bar{E}_{\bar{k}} = -ik\varphi_{\bar{k}} \; ; \; \bar{D}_k :$$

$$-\varepsilon(\bar{k}) i\bar{k} \bar{D}_k = -4\pi\rho_k$$

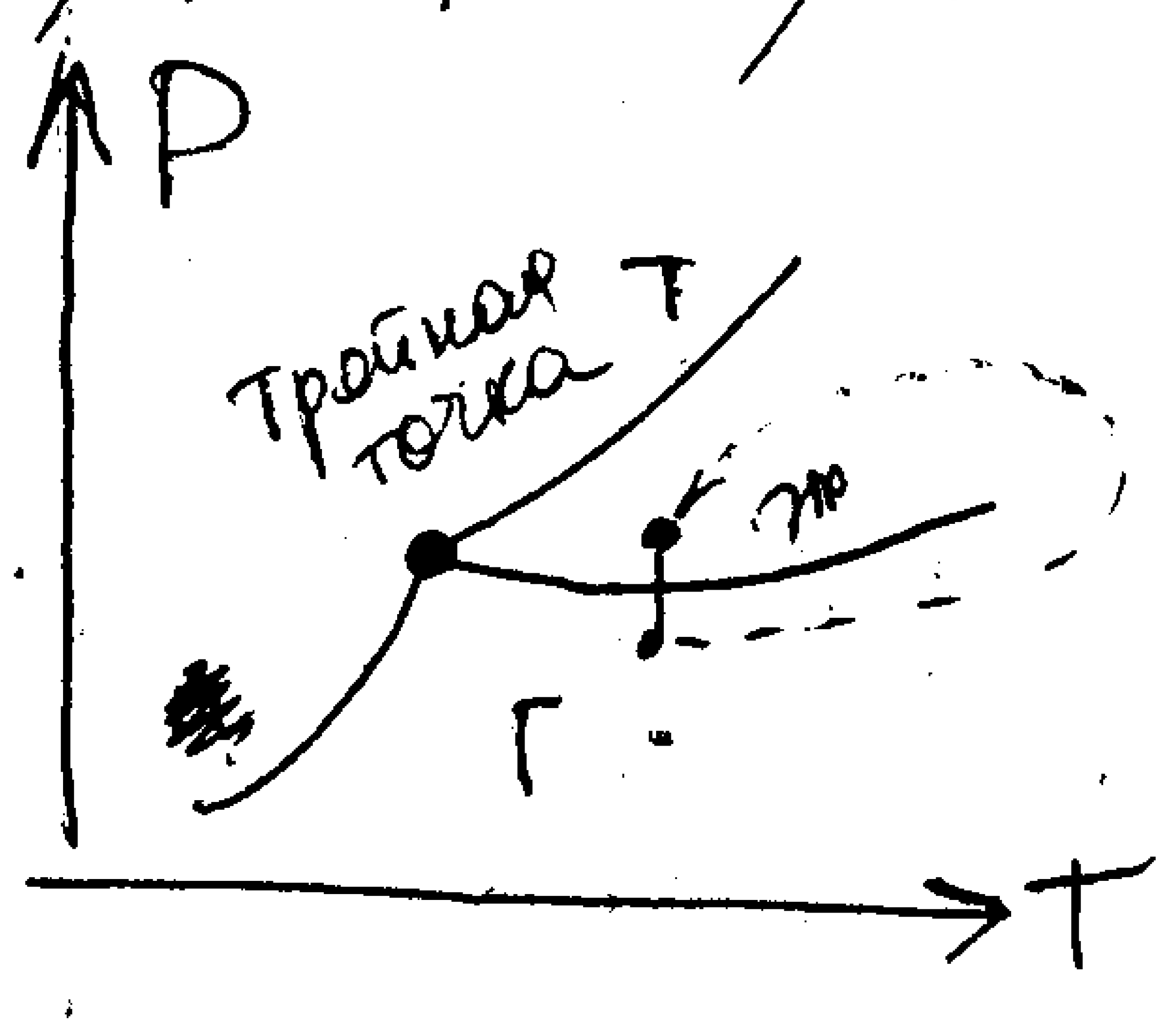
$$D_k = \varepsilon(\bar{k})\bar{E}_{\bar{k}}$$

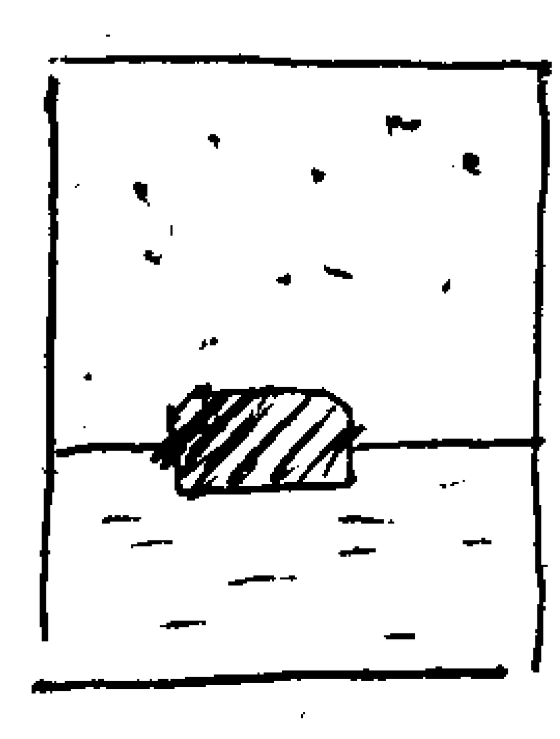
$$\varepsilon(\bar{k}) = 1 + \frac{\varkappa^2}{k^2}$$

12.04.02

Г, ж, Т.

$$\mu_Г(P,T) = \mu_ж(P,T) = \mu_Т(P,T)$$





$$\begin{cases} N_Г + N_{ж} + N_Т = N \\ N_Г v_Г + N_{ж} v_{ж} + N_Т v_Т = V \\ N_Г \varepsilon_Г + N_{ж} \varepsilon_{ж} + N_Т \varepsilon_Т = E \end{cases}$$

$$d\mu = -s\,dT + v\,dP$$

$$= \left(\frac{\partial \mu_Г}{\partial T}\right)_P dT + \left(\frac{\partial \mu_Г}{\partial P}\right)_T dP = \frac{\partial \mu_{ж}}{\partial T} dT + \frac{\partial \mu_{ж}}{\partial P} dP$$

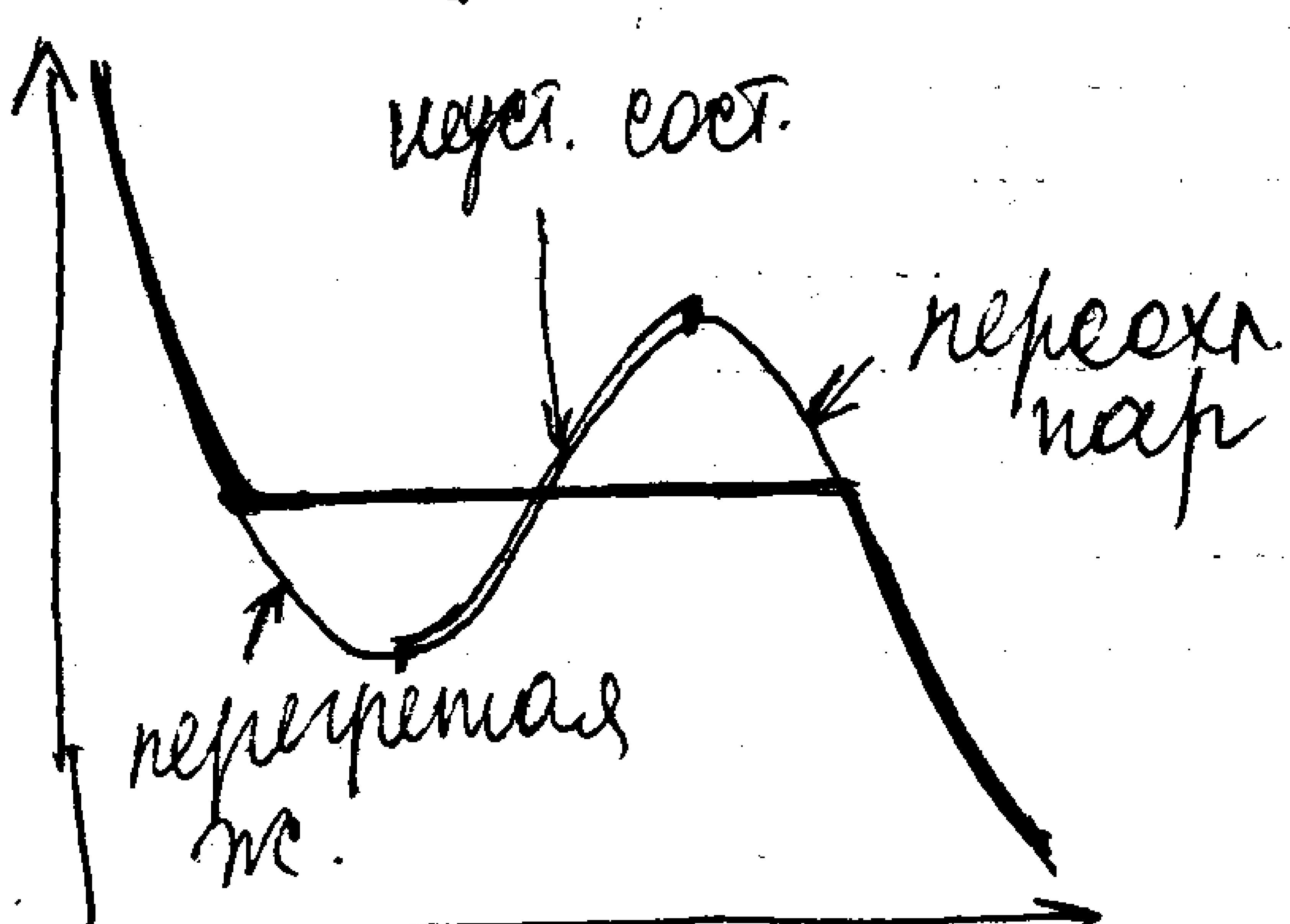
$$-s_Г\,dT + v_Г\,dP = -s_{ж}\,dT + v_{ж}\,dP$$

$$\frac{dP}{dT} = \frac{s_Г - s_{ж}}{v_Г - v_{ж}}$$

$$q = T(s_Г - s_{ж}) = w_Г - w_{ж}$$

$$dw = T\,ds + v\,dP$$

$$\frac{dP}{dT} = \frac{q}{T\Delta v}$$ – уравнение Клайперона-Клаузиуса



Фазовые переходы II рода

$-\mu \mathcal{H}$, $\varepsilon_{\uparrow\downarrow} = \mp \mu \mathcal{H}$, $N_{\uparrow\downarrow} = \dfrac{e^{-\frac{\mu \mathcal{H}}{T}}}{e^{\frac{\mu \mathcal{H}}{T}} + e^{-\frac{\mu \mathcal{H}}{T}}} \cdot N$

$$\mathcal{M} = \mu (N_\uparrow - N_\downarrow) = MV$$

$$\frac{N_\uparrow}{N_\downarrow} = e^{2\mu H/T} \qquad M = \frac{N}{V}\mu\, th \frac{\mu H}{T}$$

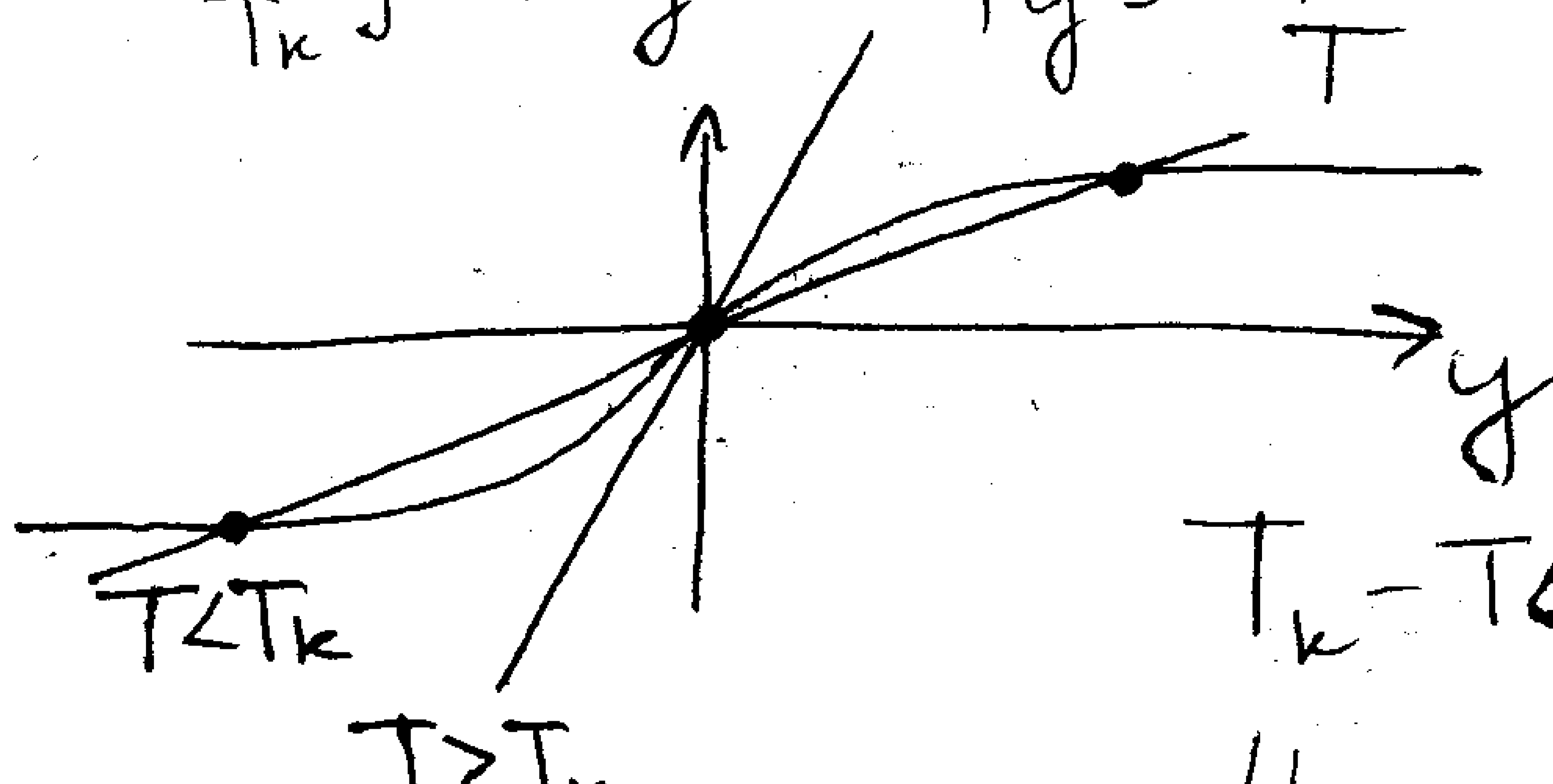
$$N_\uparrow + N_\downarrow = N$$

$$\mathcal{H} \to \mathcal{H} + \beta M$$

$$M = \frac{N\mu}{V}\, th \frac{\mu(\mathcal{H} + \beta M)}{T}$$

$$\mathcal{H} = 0 \qquad T_k = \beta \mu^2 \frac{N}{V}$$

$$\frac{T}{T_k} y = th\, y, \quad y = \frac{\mu\beta M}{T} \qquad M = \frac{Ty}{\mu\beta}$$



$$T_k - T \ll T_k, \quad y \ll 1$$

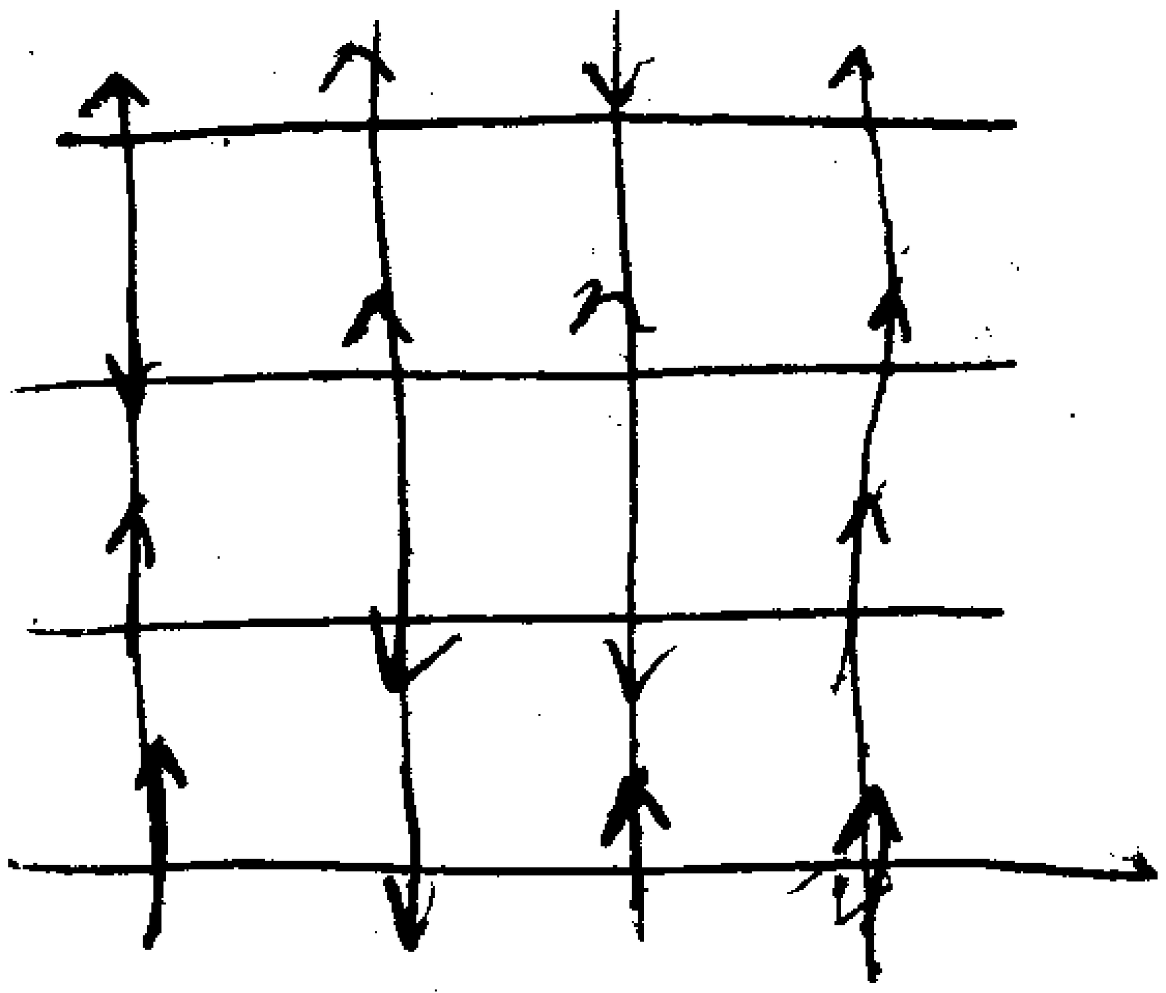
$$th\, y \approx y - \frac{y^3}{3} \Rightarrow$$

$$\frac{T}{T_k} y = y\left(1 - \frac{y^2}{3}\right)$$

$$\Rightarrow \quad y = 0$$

$$y = \pm\sqrt{3\left(1 - \frac{T}{T_k}\right)}$$

Модель Изинга



Молекулярное поле

$$N_\downarrow = N - N_\uparrow$$

$$\frac{\partial}{\partial N_\uparrow} F(T, N_\uparrow) = 0$$

$$S = \ln \Gamma = \ln C_N^{N_\uparrow} = \ln \frac{N!}{N_\uparrow!}$$

$$E = N_\uparrow q \cdot \frac{N_\uparrow}{N} J = \frac{q}{N} N_\uparrow (N - N_\uparrow)$$

$$\frac{qJ}{N}(N - 2N_\uparrow) - T \ln \frac{N - N_\uparrow}{N_\uparrow} = 0$$

$$\frac{N_\uparrow}{N_\downarrow} = \exp(-qJ \; N_\uparrow - N_\downarrow$$

$$M = \frac{N_\uparrow}{V} \mu h \ldots$$

$$Z = 2\left(1 + e^{-\frac{J}{T}}\right)^{N-1}$$

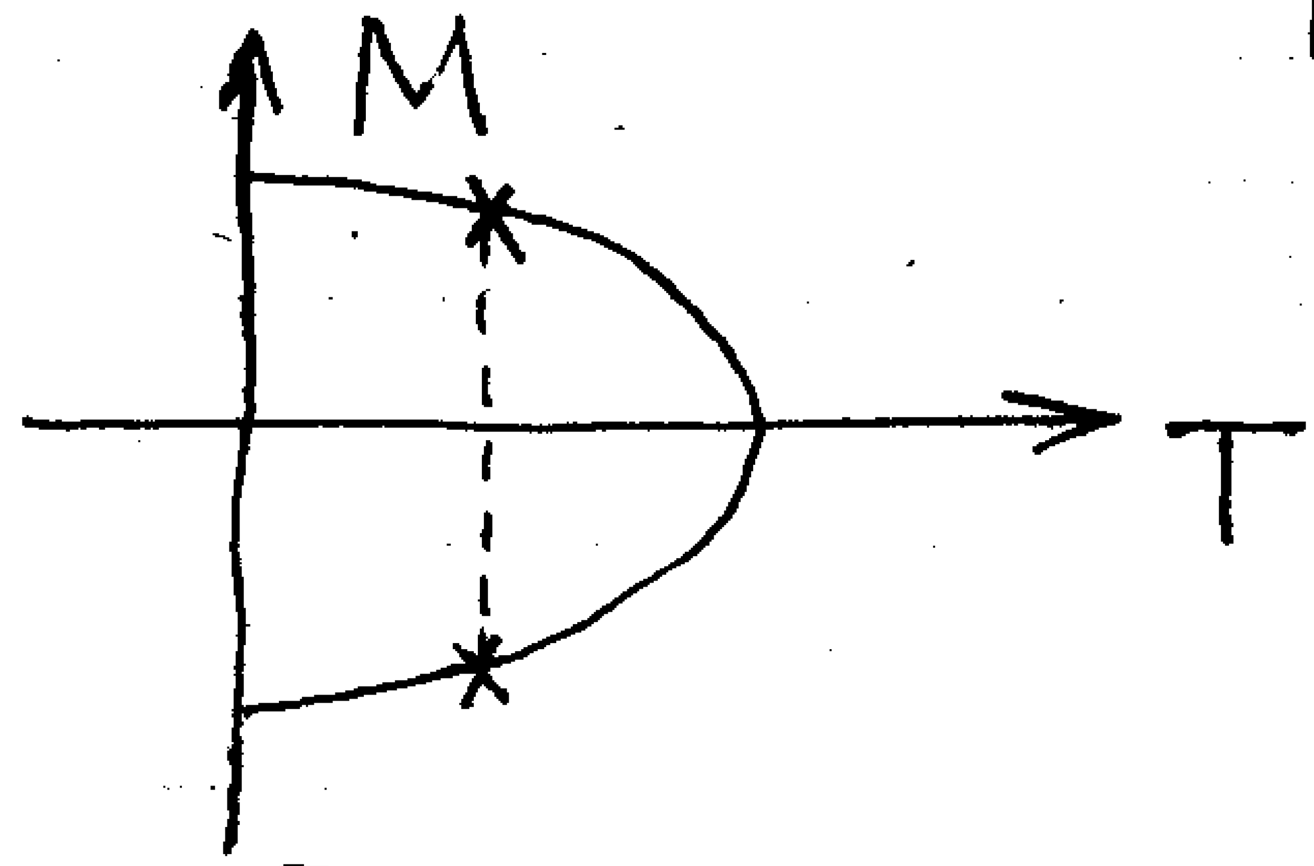
$$F = (N-1) \ln\left(1 + e^{-\frac{J}{T}}\right) + \ln 2$$

15.04.

$\pm \mu \mathcal{H}$; $\quad N_\uparrow + N_\downarrow = N$

$$F = qJ\,\frac{N_\uparrow N_\downarrow}{N} - T\ln C_N^{N_\uparrow} \quad (\mathcal{H} \equiv 0)$$

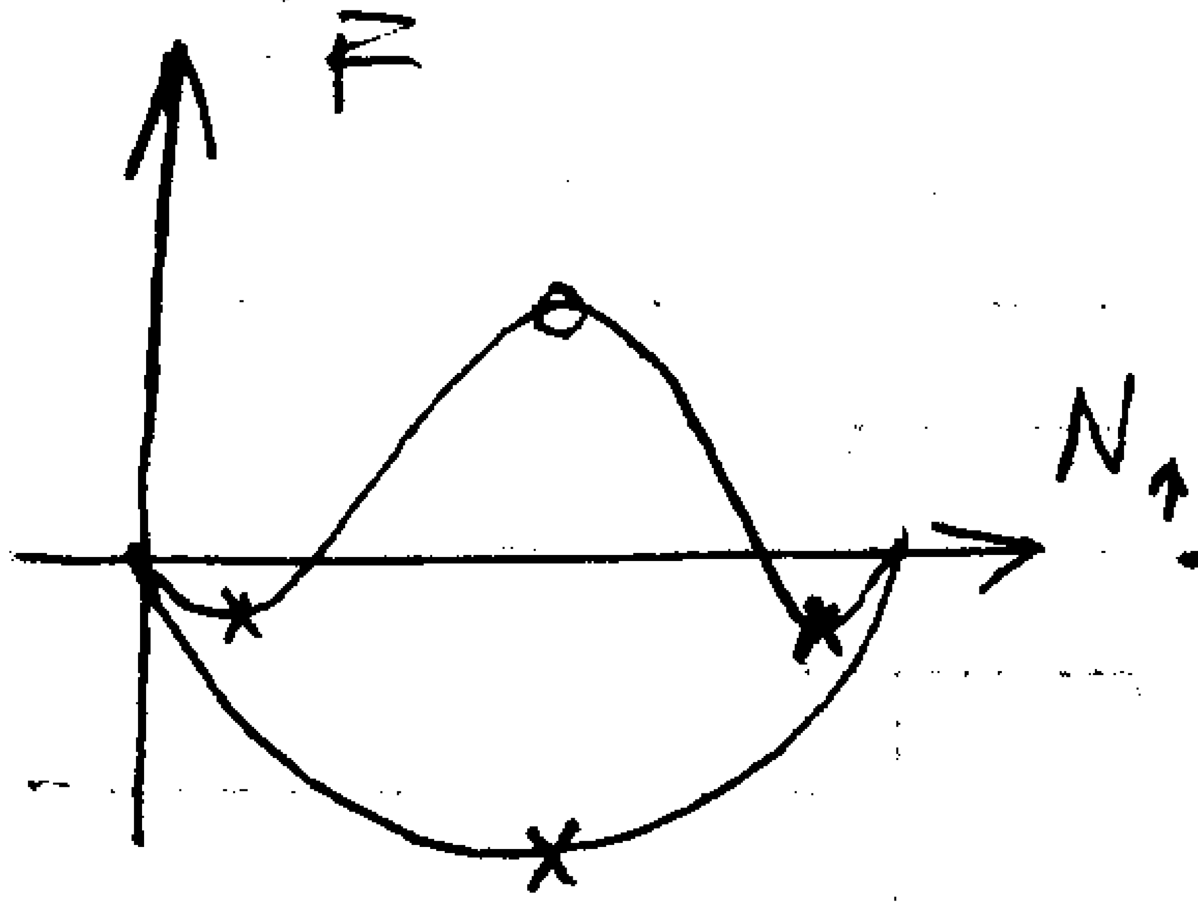
$$\mathcal{M} = \mu\,(N_\uparrow - N_\downarrow)/V$$

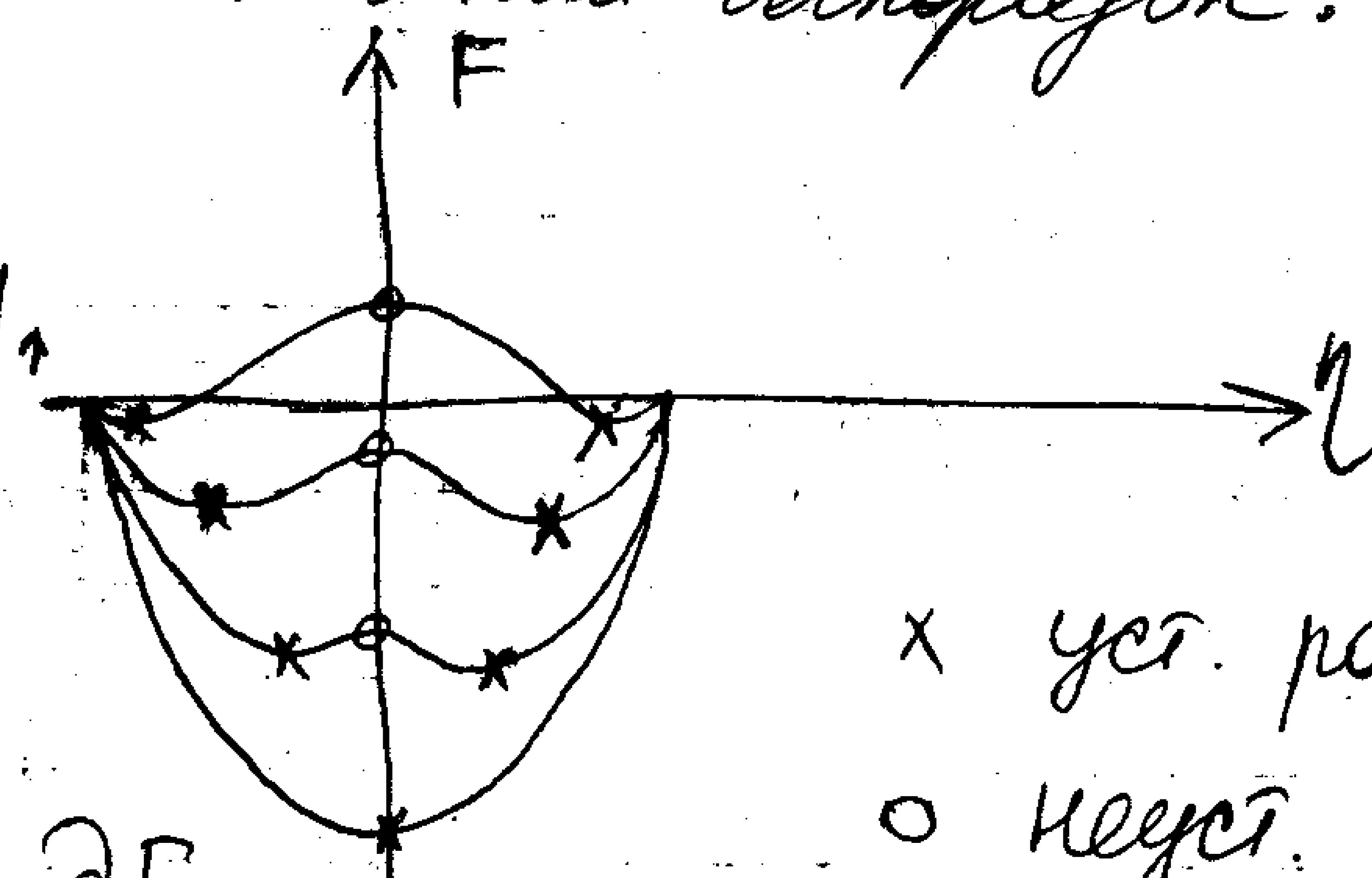


$$\eta = \frac{N_\uparrow - N_\downarrow}{N}$$

полный порядок: $\eta = \pm 1$

полный беспорядок: $\eta = 0$





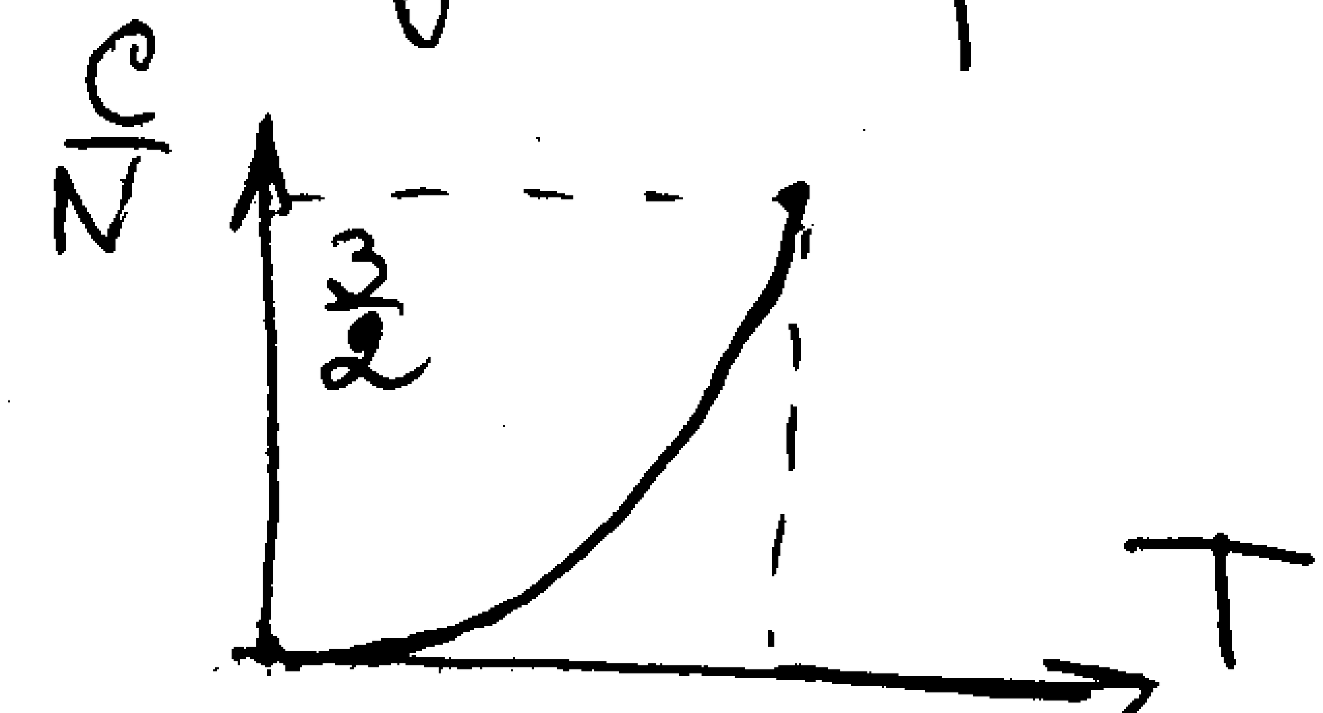
× уст. равнов.

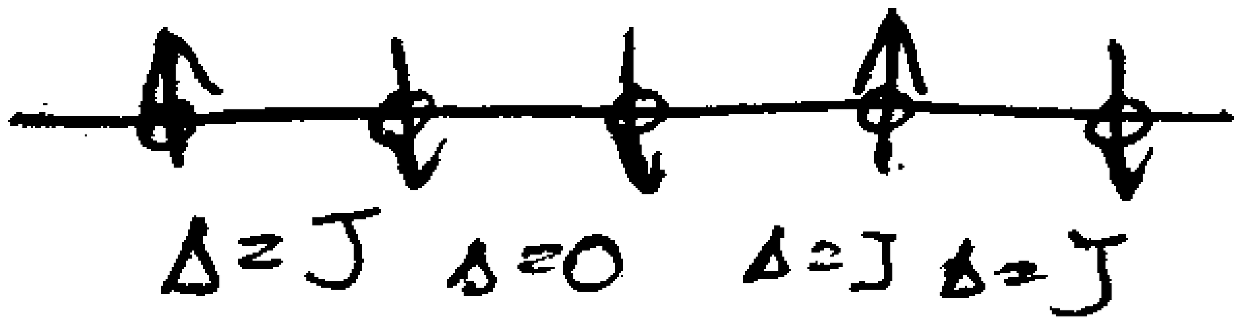
○ неуст. равн.

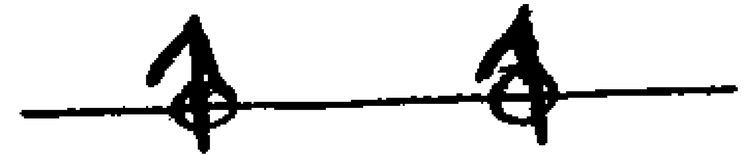
Равновесие: $\dfrac{\partial F}{\partial N_\uparrow} = 0$

$$E = qJ\,\frac{N_\uparrow N_\downarrow}{N} \sim y^2 \sim M^2 \quad , \quad y^2 = 3\,\frac{T_k - T}{T}$$

$$C = \frac{dE}{dT} = \begin{cases} \frac{3}{2}N & T < T_k \\ 0 & T > T_k \end{cases}$$







$$\frac{1-\sigma_k\sigma_{k+1}}{2}J.$$

$\sigma_k \backslash \sigma_{k+1}$	-1	1
-1	$e^{-\frac{\mu H}{T}}$	$e^{-\frac{J}{T}}$
1	$e^{-\frac{J}{T}}$	$e^{\frac{\mu H}{T}}$

$= \Lambda_{\sigma_k\sigma_{k+1}}$

$$Z=\sum_{\sigma_1,\ldots\sigma_N=\pm1}\Lambda_{\sigma_1\sigma_2}\Lambda_{\sigma_2\sigma_3}\ldots\Lambda_{\sigma_{N-1}\sigma_N}\Lambda_{\sigma_N\sigma_1}=$$

$$=\sum_{\sigma_1}\left(\Lambda^N\right)_{\sigma_1\sigma_1}=\operatorname{Tr}\Lambda^N.$$ В диаг. виде:

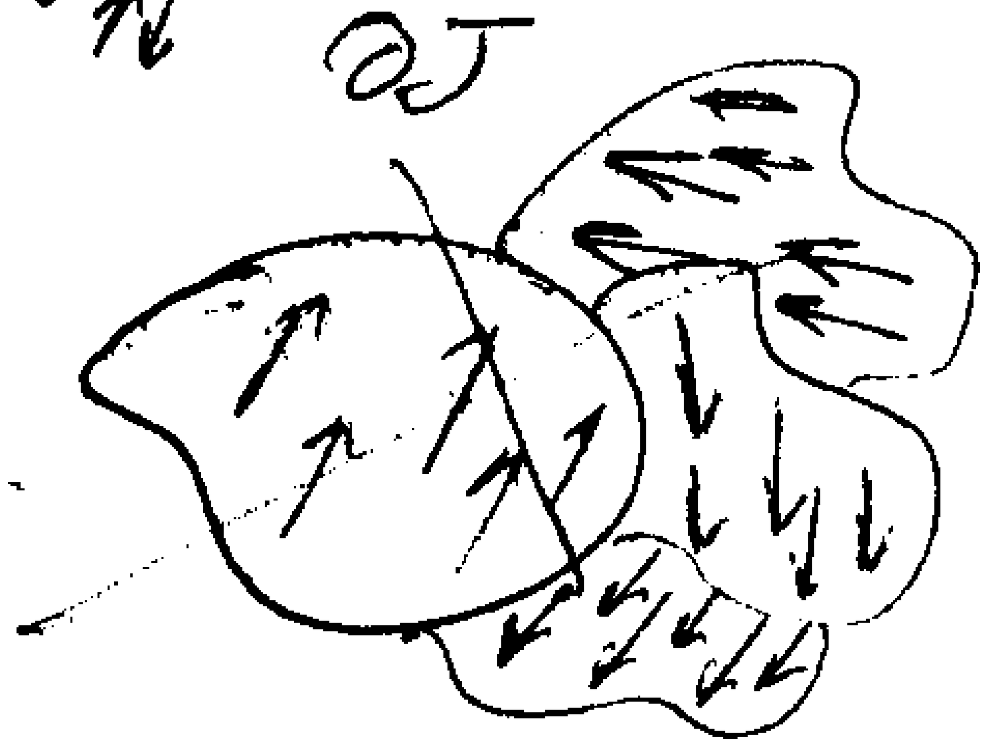
$$\lambda^2-2\lambda\,\mathrm{ch}\frac{\mu H}{T}+1-e^{-\frac{2J}{T}}=0.$$

$$\lambda_{1,2}=\mathrm{ch}\frac{\mu H}{T}\pm\sqrt{\mathrm{sh}^2\frac{\mu H}{T}+e^{-\frac{2J}{T}}}$$

$$Z=\lambda_1^N+\lambda_2^N=\lambda_1^N\left(1+\left(\frac{\lambda_2}{\lambda_1}\right)^N\right)$$

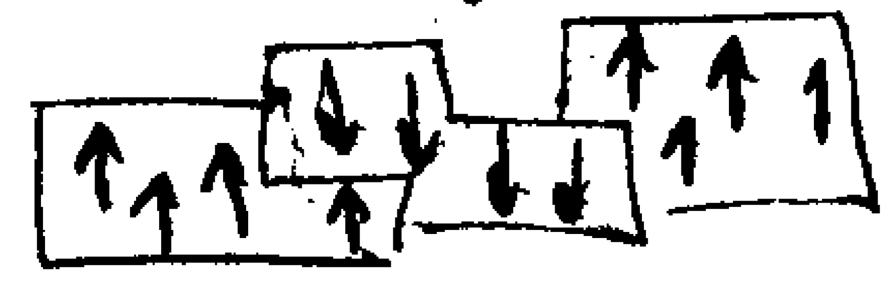
$$F=-TN\ln\lambda_1-T\cancel{\ln\left(1+\left(\frac{\lambda_2}{\lambda_1}\right)^N\right)}$$

$$N_N=\frac{\partial F}{\partial J}$$



Порядок – граница мала

Беспорядок – гр. велика.



$\Gamma \lesssim 3^L$, $E = JL$, L - число узлов

$F \sim JL - T\ln\Gamma = (J - T\ln 3)L$

Нужен $\min F$

$T < J/\ln 3 \quad L = 0$

$T > J/\ln 3 \quad L = \infty$

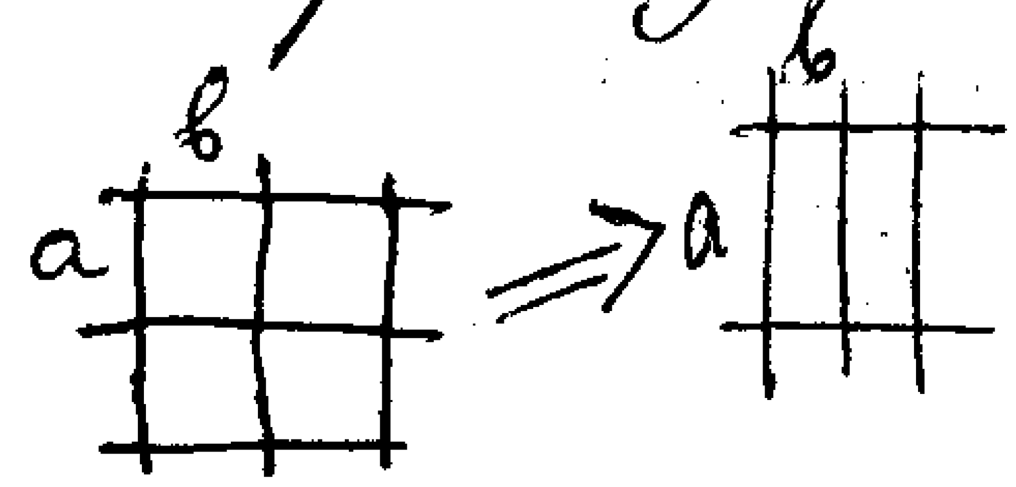
$\Rightarrow T_k \sim \dfrac{J}{\ln 3} \quad \left(= \dfrac{J}{\ln(1+\sqrt{2})}\right)$

Фазовые переходы: I р: $\Delta S, \Delta V \neq 0$

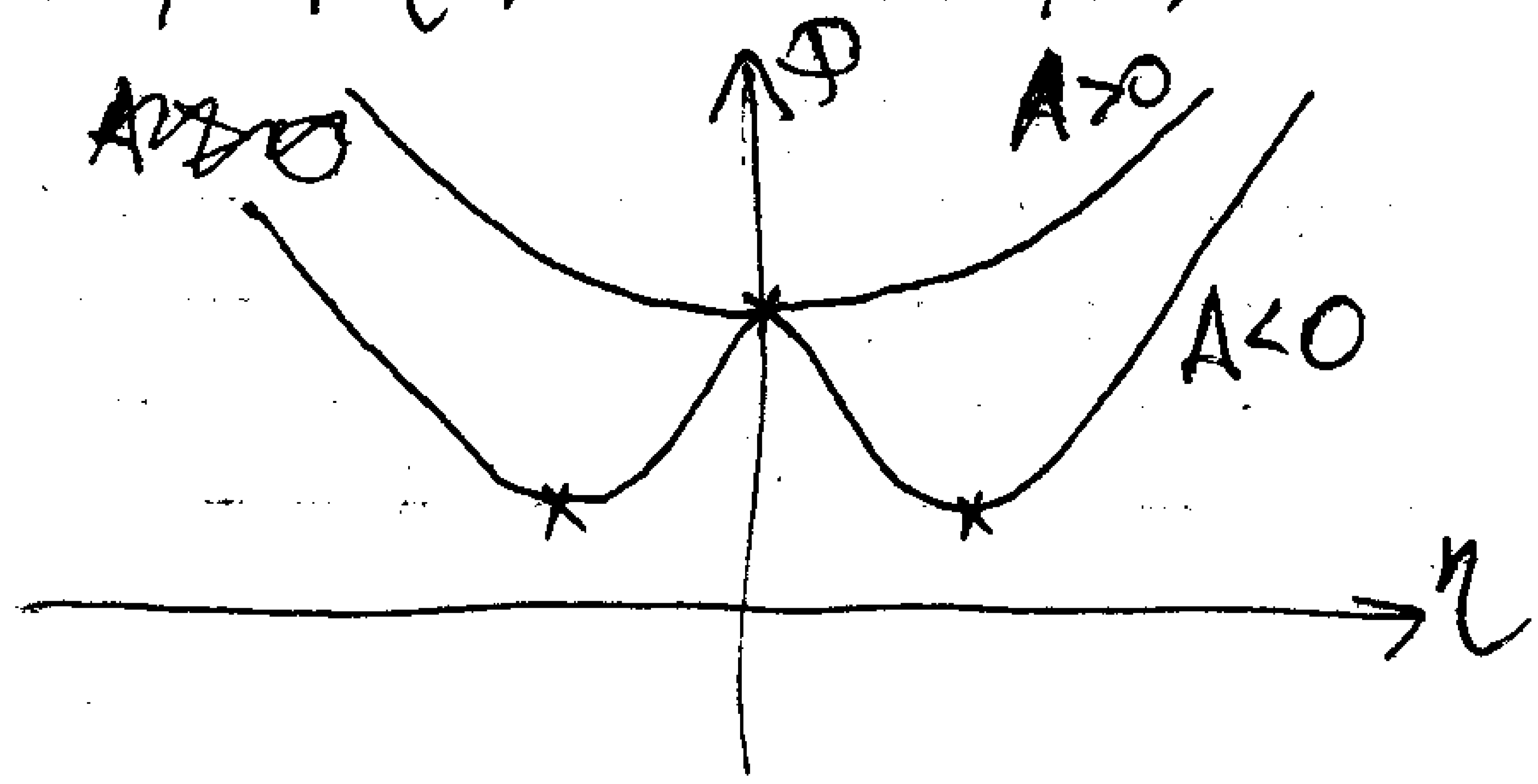
II р: $\Delta S, \Delta V = 0$

$\Delta C_p \neq 0$.

$\eta = \dfrac{a-b}{a}$



$\Phi(T,P,\eta) = \Phi_0(T,P) + A(T,P)\eta^2 + B(T,P)\eta^4 + \ldots$

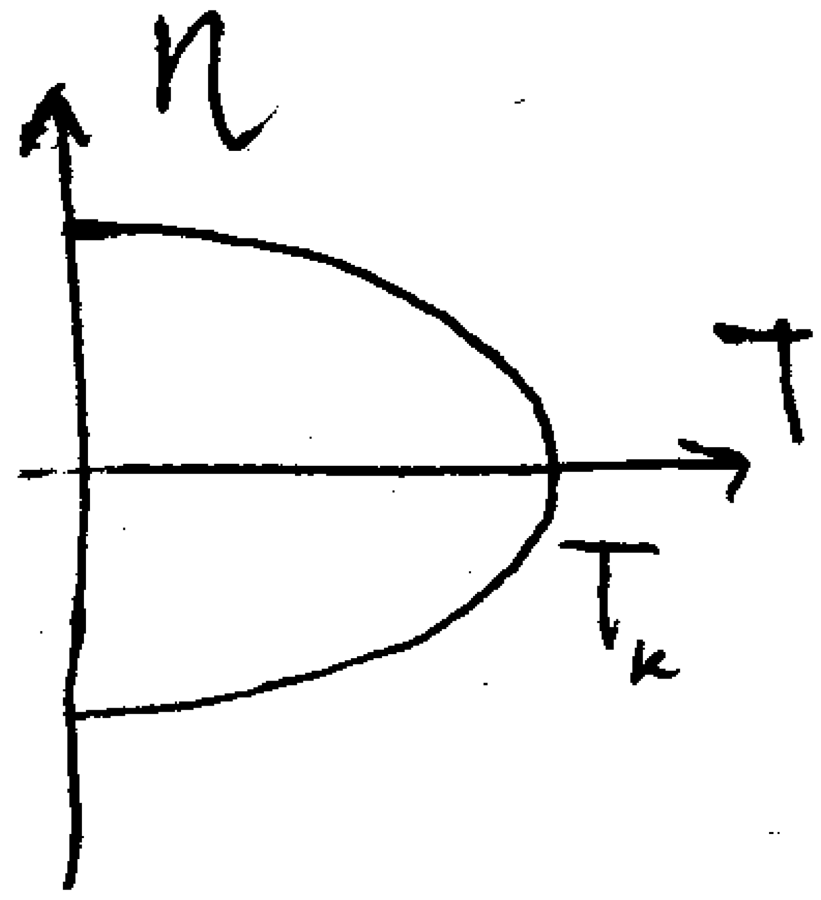


$A = a(P)(T_* - T_k)$

$\dfrac{\partial\Phi}{\partial\eta} = 0 \Rightarrow$

$2a(T_* - T_k)\eta + 4B\eta^3 = 0$

$\eta_0 = 0, \quad \bar{\eta}^2_{1,2} = \dfrac{2B}{a(T_k - T)}$



$$S = -\frac{\partial \Phi}{\partial T} = -a\eta^2 + \overset{0}{\frac{\partial \Phi}{\partial \eta}} \cdot \frac{\partial \eta}{\partial T} = \begin{cases} 0 & T > T_k \\ -\dfrac{a^2(T_k - T)}{2B} & T < T_k \end{cases}$$

$$C = T\frac{\partial S}{\partial T} = \begin{cases} 0, & T > T_k \\ \dfrac{a^2 T}{2B}, & T < T_k \end{cases} \qquad \Delta C = \frac{a^2 T_k}{2B}$$

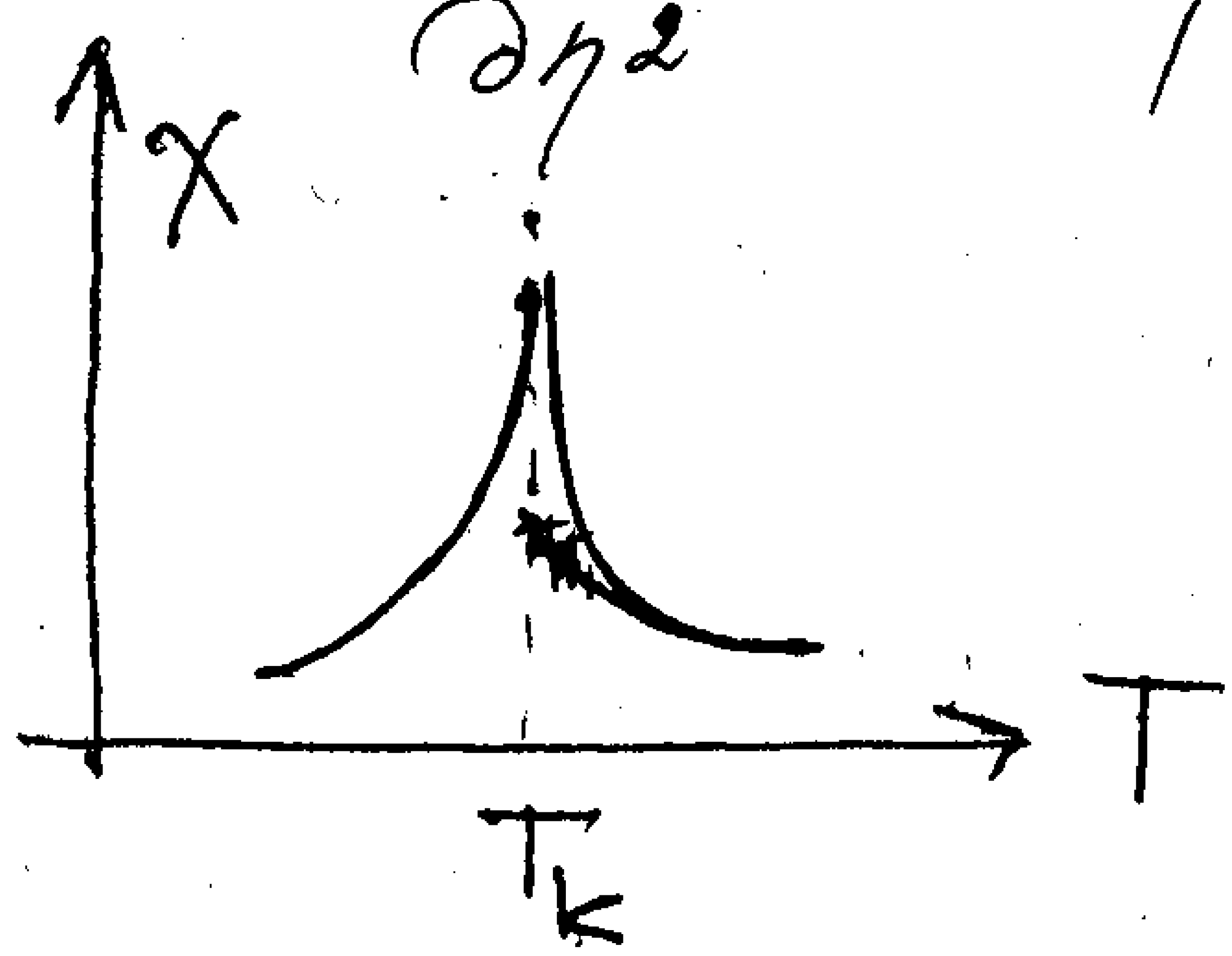
$$\tilde{\Phi} = \Phi - \eta h$$, h – поле.

$$\frac{\partial \tilde{\Phi}}{\partial \eta} = 0 \; ; \; \frac{\partial \Phi}{\partial \eta} = h \Rightarrow \qquad \eta h \sim \mu H$$

$$\eta(h, T)$$

$$\chi = \left(\frac{\partial \eta}{\partial h}\right)_{h=0} \; ; \quad \frac{\partial^2 \Phi}{\partial \eta^2} \cdot \frac{\partial \eta}{\partial h} = 1.$$

$$\chi = 1 \Big/ \frac{\partial^2 \Phi}{\partial \eta^2} = \begin{cases} \dfrac{1}{2a(T - T_k)} & T > T_k \\ \dfrac{1}{4a(T_k - T)} & T < T_k \end{cases}$$

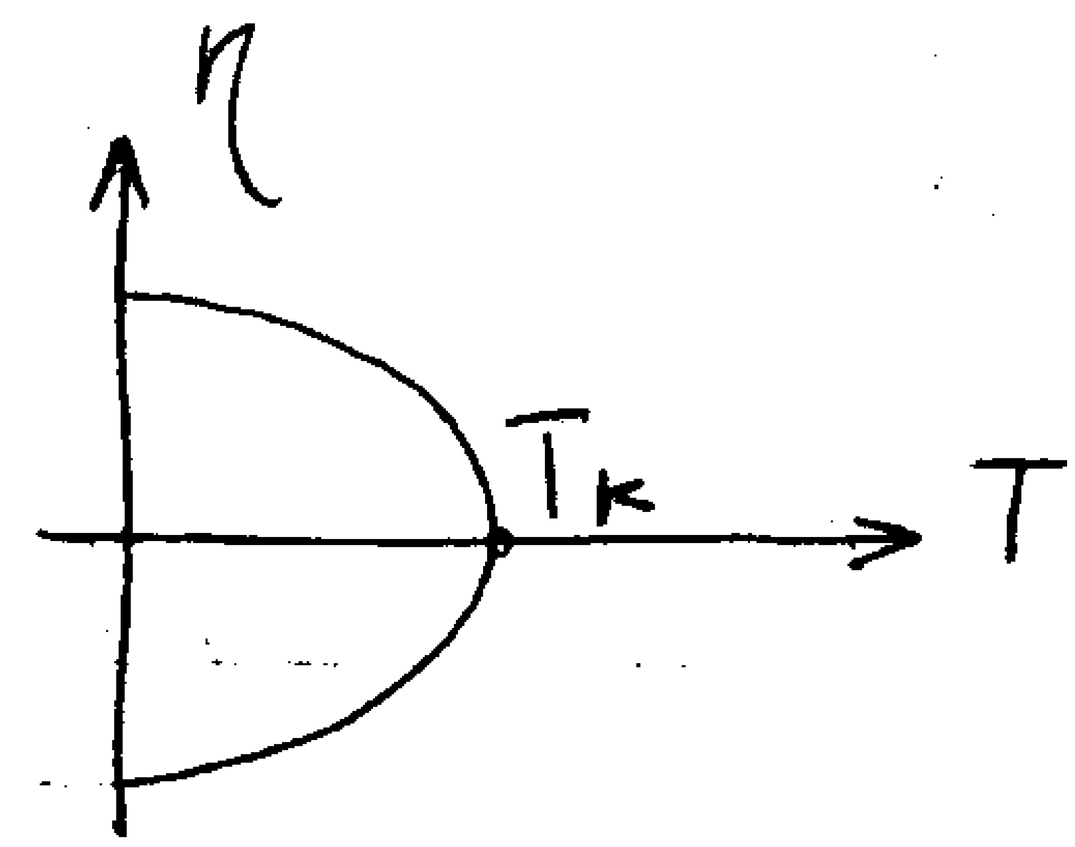


19.04.02.

$$C \sim |T-T_k|^{-\alpha} \sim |\tau|^{-\alpha}$$

$$\tau = \frac{T-T_k}{T_k}$$

α – крит. индекс.



$$\eta \sim (-\tau)^{\beta}$$

$$\chi \sim |\tau|^{-\gamma}$$

$$C = T\frac{\partial^2 \Phi}{\partial T^2}$$

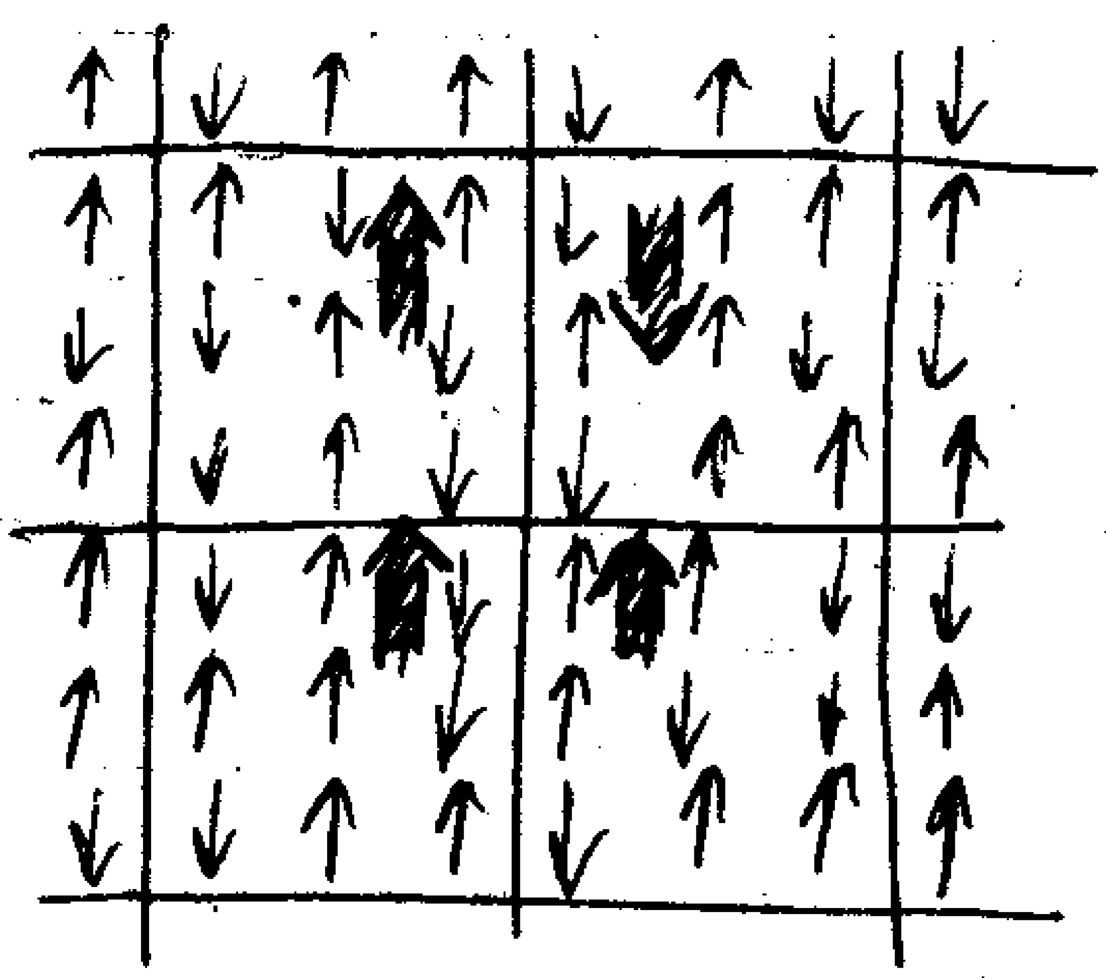
$$\chi = \frac{1}{\frac{\partial^2 \Phi}{\partial \eta^2}}$$

теор. Ландау:

$\alpha = 0,\ \beta = \frac{1}{2},\ \gamma = 1$

Масштабная инвариантность.

(Скейлинг)



$$\tau(\lambda \vec{r}) = \lambda^{\Delta_\tau} \cdot \tau(\vec{r})$$

$$\Phi(\lambda \vec{r}) = \lambda^{\Delta_\Phi} \Phi(\vec{r})$$

Δ_τ, Δ_Φ – аномальные размерности.

$$C(\lambda \vec{r}) \sim |\tau(\lambda \vec{r})|^{-\alpha}$$

$$\lambda^{\Delta_c} \cdot C(\vec{r}) \sim \lambda^{-\alpha \cdot \Delta_\tau} |\tau(\vec{r})|$$

$$\Rightarrow \Delta_c = -\alpha \Delta_\tau \qquad \Delta_\chi = -\gamma \Delta_\tau$$

$$\Delta_\eta = \beta \Delta_\tau$$

$$\Delta_c = \Delta_\varphi - 2\Delta_\tau$$

$$\Delta_\chi = -\Delta_\varphi + 2\Delta_\eta$$

$$\Delta_c + \Delta_\chi = 2\Delta_\eta - 2\Delta_\tau$$

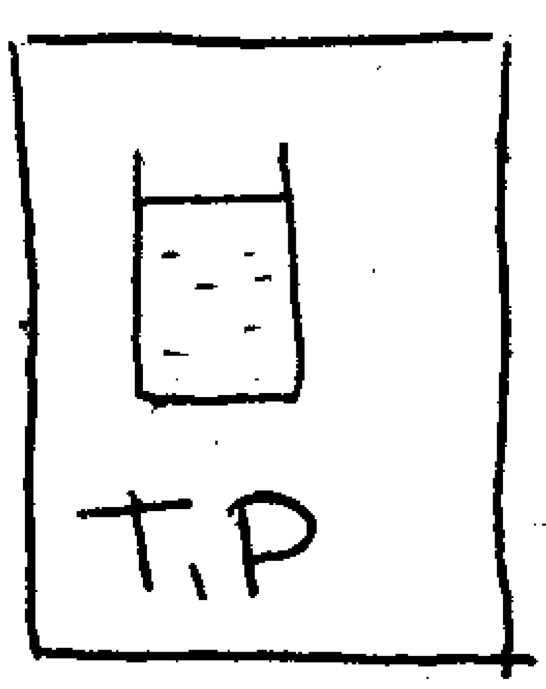
$$-\alpha\Delta_\tau - \gamma\Delta_\tau = 2\beta\Delta_\tau - 2\Delta_\tau$$

$$\Rightarrow \alpha + 2\beta + \gamma = 2$$

Теория подобия $\alpha \approx 0,\ \beta = \frac{1}{3},$ $\gamma = \frac{4}{3}$

Флуктуации.

Квазистатические флукт.-ии.



t_1 − релаксация внутри тела

t_2 − −"− тело-термостат.

$t_1 \ll t_2$

$$w(E,V) = A\exp\{S(E,V) + S_0(E_{полн} - E, V_{полн} - V)\}$$

$$dE = TdS - PdV$$

$$S_0(-) = S_0(E_п, V_п) - \frac{E}{T_0} - \frac{P_0V}{T_0}$$

$$w \sim e^{S - \frac{E + P_0V}{T_0}}$$

$$E = \langle E\rangle + \Delta E\ ;\quad V = \langle V\rangle + \Delta V,$$

$$S = \langle S\rangle + \Delta S.$$

$$w \sim e^{\Delta S - \frac{\Delta E + P_0 \Delta V}{T_0}}$$

Выразим всё ч/з $\Delta V, \Delta S$.

$$\Delta E = \left(\frac{\partial E}{\partial V}\right)_S \Delta V + \left(\frac{\partial E}{\partial S}\right)_V \Delta S = -P\Delta V + T\Delta S$$

$$\Delta E = -P\Delta V + T\Delta S + \frac{1}{2}\left(\Delta V \frac{\partial}{\partial V} + \Delta S \frac{\partial}{\partial S}\right)^2 E =$$

$$= -P\Delta V + T\Delta S + \frac{1}{2}\left(\Delta V \frac{\partial}{\partial V} + \Delta S \frac{\partial}{\partial S}\right)(-P\Delta V + T\Delta S) =$$

$$= -P\Delta V + T\Delta S + \frac{1}{2}(-\Delta P \Delta V + \Delta T \Delta S)$$

$$\Rightarrow w \sim e^{\frac{\Delta P \Delta V - \Delta T \Delta S}{2T_0}}$$

$\Delta V, \Delta T$

$$\Delta P = \left(\frac{\partial P}{\partial V}\right)_T \Delta V + \left(\frac{\partial P}{\partial T}\right)_V \Delta T \quad \Big| \Delta V$$

$$\Delta S = \left(\frac{\partial S}{\partial V}\right)_T \Delta V + \left(\frac{\partial S}{\partial T}\right)_V \Delta T \quad \Big| -\Delta T$$

$$\exp\left\{+\frac{1}{2T}\left[\left(\frac{\partial P}{\partial V}\right)_T \Delta V^2 - \frac{C_V}{T}\Delta T^2\right]\right\}$$

$$dF = -SdT - \cancel{\ } - PdV$$

$$\frac{\partial S}{\partial V} = \frac{\partial P}{\partial T}$$

$$e^{-\frac{x^2}{2a^2} - \frac{y^2}{2b^2}}, \quad \langle x^2 \rangle = a^2$$

$$\langle y^2 \rangle = b^2$$

$$\langle xy \rangle = 0$$

$$\langle \Delta T^2 \rangle = \frac{T}{C_V}, \qquad \langle \Delta V^2 \rangle = -T\left(\frac{\partial V}{\partial P}\right)_T$$

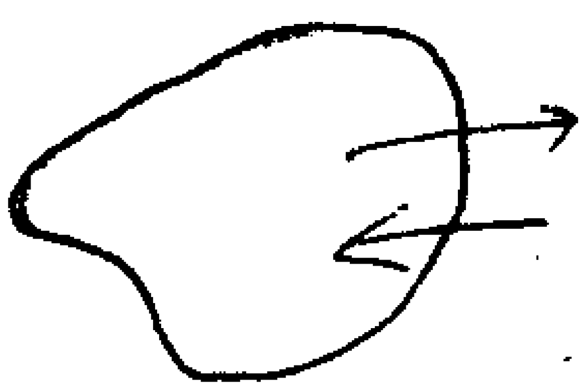
$$\langle \Delta V \Delta T \rangle = 0$$

$$\langle \Delta f^2 \rangle = \left\langle \left(\frac{\partial f}{\partial T}\Delta T + \frac{\partial f}{\partial V}\Delta V \right)^2 \right\rangle = \left(\frac{\partial f}{\partial T}\right)^2 \frac{T}{C_V} -$$

$$- \left(\frac{\partial f}{\partial V}\right)^2 T \left(\frac{\partial V}{\partial P}\right)_T$$

$$\langle \Delta S^2 \rangle = C_p, \qquad \langle \Delta P^2 \rangle = -T\left(\frac{\partial P}{\partial V}\right)_S$$

Флуктуации числа частиц.



δN $\qquad \Delta V = 0, \quad \langle \Delta N^2 \rangle \neq 0.$

$$n = \frac{N}{V}, \qquad \langle \Delta n^2 \rangle = \left(-\frac{N}{V^2}\Delta V\right)^2$$

$$\langle \Delta n^2 \rangle = \left\langle \frac{\Delta N^2}{V^2} \right\rangle$$

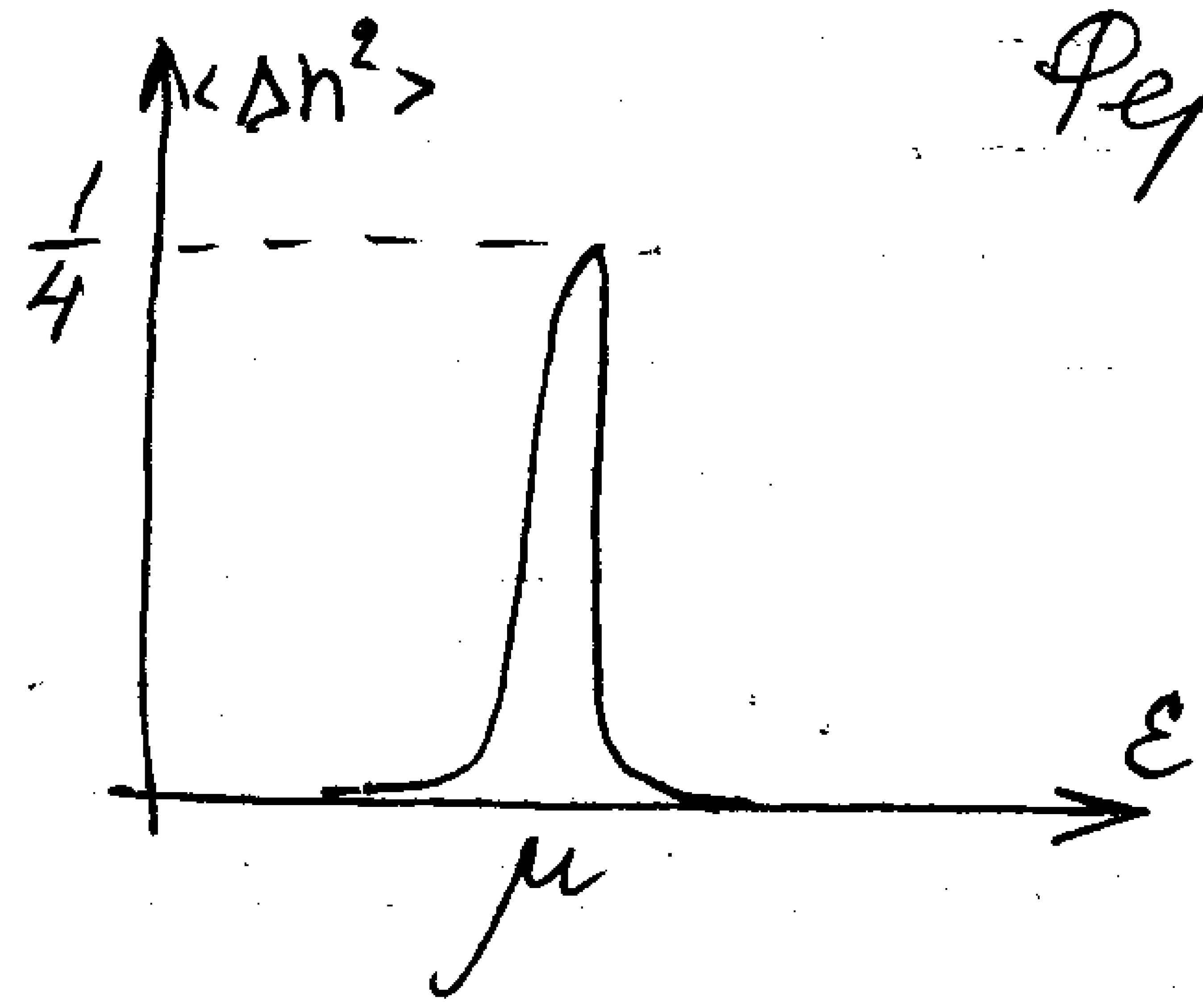
$$\Rightarrow \quad \langle \Delta N^2 \rangle = \frac{N^2}{V^2}\langle \Delta V^2 \rangle = -T\frac{N^2}{V^2}\left(\frac{\partial V}{\partial P}\right)_T$$

$$dW \sim e^{\frac{\mu N}{T} - \frac{E}{T}}$$

$$W(N, |k\rangle) = \frac{1}{Q} e^{\frac{\mu N}{T} - \frac{E_{N,k}}{T}}$$

$$\langle \Delta N^2 \rangle = +T\frac{\partial N}{\partial \mu} \qquad \langle N(|k\rangle) \rangle = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}$$

$$\langle N^2(|k\rangle) \rangle = \frac{e^{\frac{\varepsilon-\mu}{T}}}{\left(e^{\frac{\varepsilon-\mu}{T}}+1\right)^2} = n(1-n)$$



Ферми-газ

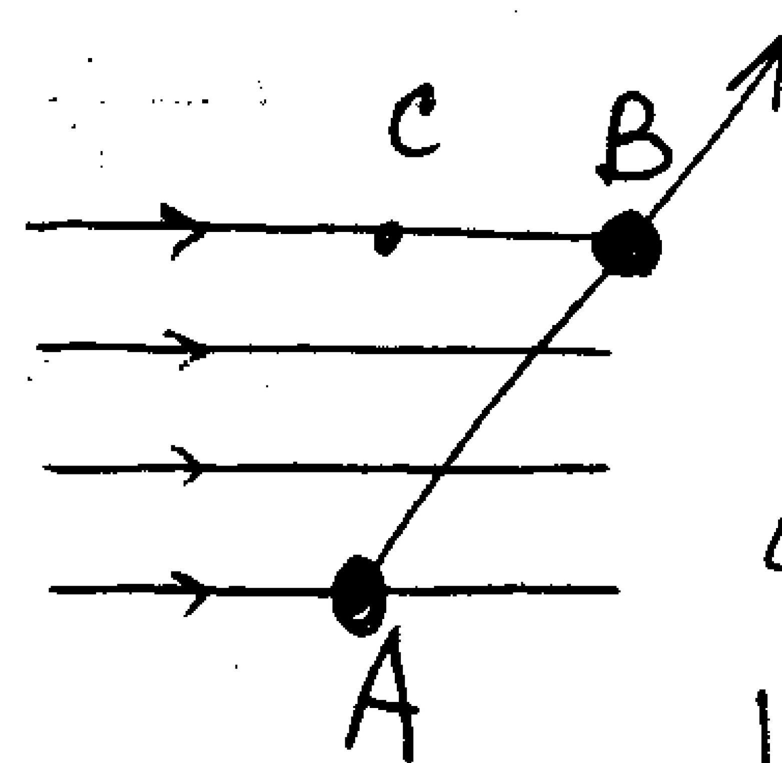
Бозе-газ

$\langle \Delta n^2 \rangle = n + n^2$

$n \ll 1 \Rightarrow \Delta n \sim \sqrt{n}$

$n \gg 1 \Rightarrow \Delta n \sim n$

Рассеяние света



$AB - CB = \frac{\lambda}{2}$ — нет света

$\vec{E} = \vec{E}_0 \cos\omega t$

$V \ll \lambda^3$

$\vec{P} = \frac{\varepsilon - 1}{4\pi} \vec{E}$, $\vec{d} = \vec{P} V$

$\delta \vec{d} = \frac{\delta \varepsilon}{4\pi} V \vec{E}$

$I = \frac{2}{3c^3} \langle (\delta \ddot{d})^2 \rangle = \frac{2}{3c^3} \cdot \omega^4 \langle \delta d^2 \rangle =$

$= \frac{2\omega^4}{3c^3} \frac{V^2}{16\pi^2} E^2 \langle \delta \varepsilon^2 \rangle$

$\langle \delta \varepsilon^2 \rangle$ — ? , $\varepsilon(\rho, T)$

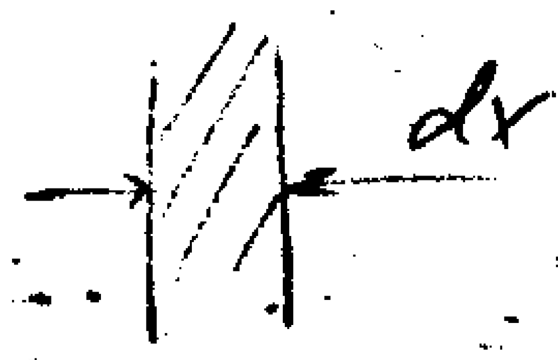
$\langle \delta \varepsilon^2 \rangle = \left(\frac{\partial \varepsilon}{\partial \rho}\right)^2 \langle \Delta \rho^2 \rangle = \left(\frac{\partial \varepsilon}{\partial \rho}\right)^2 \cdot \frac{\rho^2}{V^2} \langle \Delta V^2 \rangle =$

$$= -\left(\frac{\partial \varepsilon}{\partial \rho}\right)^2 \frac{\rho^2}{V^2} T \left(\frac{\partial V}{\partial P}\right)_T = \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T^2 \cdot \frac{\rho^2}{V^2} T \left(\frac{\partial \rho}{\partial P}\right)_T \cdot \frac{V}{\rho} \Rightarrow$$

$$\mathcal{I}_V = \frac{2\omega^4}{3c^3} \cdot \frac{V}{16\pi^2} \cdot E^2 \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T^2 \cdot \rho \cdot \left(\frac{\partial \rho}{\partial P}\right)_T$$

$\Rightarrow \mathcal{I}_V \sim V$, уже $V \not< \lambda^3$

$$S = \frac{c}{4\pi} E^2$$



$V = dx$

$$\frac{dS}{dx} = -\frac{\omega^4}{2\pi c^4} \cdot \rho T \left(\frac{\partial \varepsilon}{\partial \rho}\right)^2 \cdot \left(\frac{\partial \rho}{\partial P}\right)_T = -h$$

h - коэф. экстинции

26.04.02.

$V = dx \cdot 1\,см^2$

$$\mathcal{I} = \frac{2}{3c^3} \left(\frac{\delta \varepsilon}{4\pi}\right)^2 E^2 V = \frac{2}{3c^4} \frac{\langle (\delta\varepsilon)^2 \rangle}{4\pi} \frac{c}{4\pi} E^2 V$$

$$\langle \delta\varepsilon^2 \rangle = \left(\frac{\partial \varepsilon}{\partial \rho} \frac{\rho}{V}\right)^2 \langle \Delta V^2 \rangle = -\left(\frac{\partial \varepsilon}{\partial \rho} \rho\right)^2 \frac{T}{V^2} \left(\frac{\partial V}{\partial P}\right)_T$$

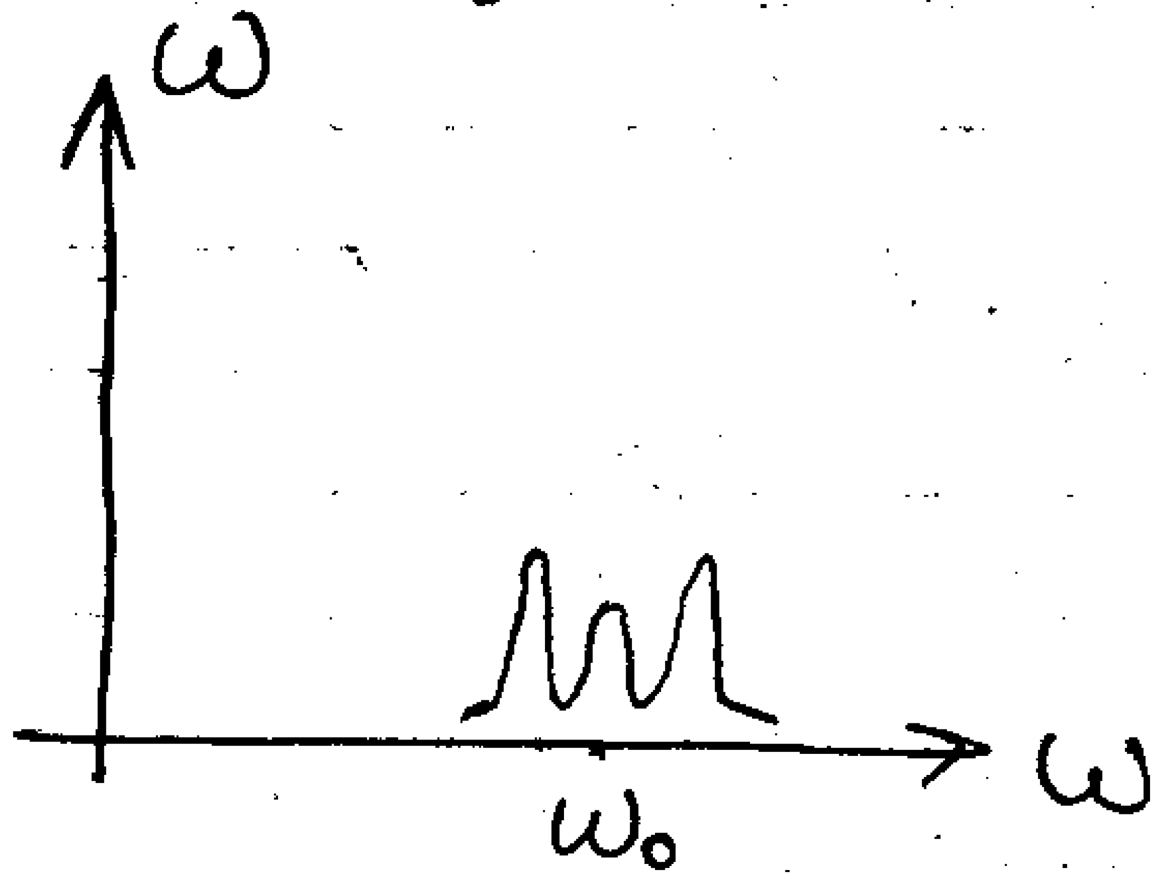
$$-\left(\frac{\partial V}{\partial P}\right)_T = \frac{V}{P} = \frac{V^2}{NT} \qquad (PV = NT).$$

$$\varepsilon - 1 \sim \rho \qquad \frac{\partial \varepsilon}{\partial \rho} = \frac{\varepsilon - 1}{\rho} = \frac{n^2 - 1}{\rho} = \frac{2(n-1)}{\rho} \qquad n \approx 1.$$

$$\Rightarrow \langle \delta\varepsilon^2 \rangle = \frac{4(n-1)^2}{N} \qquad h = \frac{4(n-1)^2 V}{6\pi c^4 N}$$

$$\mathcal{I} = h \cdot \frac{cE^2}{4\pi} dx = -dS, \quad S = S_0 e^{-hx}$$

Тонкая структура спектра Рэлеевского рассеяния.



Дублет Мандельштама - Бриллюэна.

ΔP, $S = const$ $\quad v_s$

ΔS, $P = const$ $\quad v \approx 0$.

Падает волна $\vec{E} \sim \vec{E}_0 \cos(\vec{k}\vec{r} - \omega t)$

Рассеянная волна $\vec{E} \sim \delta\varepsilon \cdot \vec{E}$

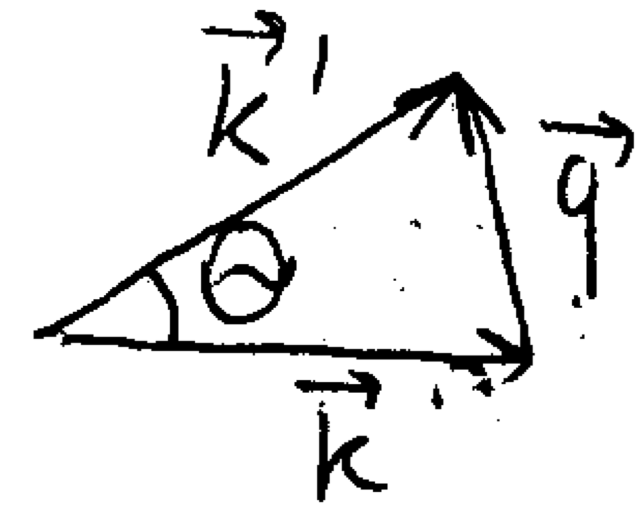
$\delta\varepsilon \sim \cos(\vec{q}\vec{r} - \Omega t)$

$\vec{E}_{расс} \sim \cos(\vec{k}\vec{r} - \omega t)\cos(\vec{q}\vec{r} - \Omega t) \sim$

$\sim \cos((\vec{k}+\vec{q})r - (\Omega+\omega)t) + \cos((\vec{k}-\vec{q})r + (\Omega-\omega)t)$

$\Rightarrow$ 1) $\omega' = \omega + \Omega$ $\qquad$ 2) $\omega' = \omega - \Omega$

$\vec{k}' = \vec{k} + \vec{q}$ $\qquad$ $\vec{k}' = \vec{k} - \vec{q}$



$q \lesssim k$, $\quad k = \frac{\omega n}{c}$

$q = \frac{\Omega}{v_s}$

$\Rightarrow \Omega \ll \omega$, $\omega' \approx \omega$, $k' \approx k$

$$q = 2k\sin\frac{\theta}{2} \quad ; \quad \omega' = \omega \pm q v_s = \omega \pm 2k v_s \sin\frac{\theta}{2} =$$

$$= \omega \pm 2\omega \frac{n v_s}{c} \sin\frac{\theta}{2}$$

Флуктуации параметра пор-ка в ф. п. II р.

$$\eta \qquad \Omega(V, T, \mu, \eta) = V\,\Phi(T, \mu, \eta)$$

$$w(\eta) \sim e^{-\frac{\Omega}{T}} \qquad \frac{\partial \Omega}{\partial \eta} = 0 \text{ — в т. равновесия}$$

$$\Omega = \Omega_0 + \frac{1}{2}\frac{\partial^2 \Omega}{\partial \eta^2}(\delta\eta)^2$$

$$w \sim e^{-\frac{1}{2T}\cdot\frac{\partial^2 \Omega}{\partial \eta^2}(\delta\eta)^2}$$

$$\Phi = \Phi_0 + at\eta^2 + B\eta^4 \qquad t = T - T_k$$

$$2at\eta + 4B\eta^3 = 0 \Rightarrow \eta_0^2 = \begin{cases} 0 & t > 0 \\ \frac{a|t|}{2B}, & t < 0 \end{cases}$$

$$\frac{\partial^2 \Omega}{\partial \eta^2} \simeq Va|t|$$

$$\langle \delta\eta^2 \rangle = \frac{T}{\frac{\partial^2 \Omega}{\partial \eta^2}} \sim \frac{T}{Va|t|}$$

Орнштейн, Цернике.

$$\Omega = \int [\Phi + g(\nabla\eta)^2]\,dV$$

$$a|t|\,\delta\eta^2 \sim g\,\frac{\delta\eta^2}{R^2} \qquad \nabla\eta \sim \frac{\delta\eta}{R}$$

$$\Rightarrow R \sim \left(\frac{g}{a|t|}\right)^{1/2} \qquad (V \to R^3).$$

$$\langle \delta\eta^2 \rangle \sim \frac{T_k}{a|t|R^3} \sim \frac{T_k}{a|t|} \cdot \frac{a^{3/2}|t|^{3/2}}{g^{3/2}} \sim \frac{T_k\, a^{1/2}|t|^{1/2}}{g^{3/2}}$$

$$\cancel{\sigma \sim R \cdot \frac{\delta\eta^2}{R^2} \sim \frac{T_k\, a^{1/2}|t|^{1/2}}{g^{3/2}} \cdot \frac{a^{1/2}|t|^{1/2}}{g^{1/2}}}$$

$$\cancel{\sigma \sim a|t|\,\delta\eta^2 R \sim a|t|\,T_k}$$

Условие применимости теории

$$\langle \delta\eta^2 \rangle \ll \eta_0^2 \qquad T_k \frac{a^{1/2}|t|^{1/2}}{g^{3/2}} \ll \frac{a|t|}{B}$$

$$|t| \gg \frac{T_k^2 B^2}{g^3 a}$$

$$\frac{|t|}{T_k} \gg \frac{T_k B^2}{a g^3}$$ Критерий Гинзбурга-Леванюка

Зависимость флуктуаций от времени.

Броуновское движение

$$m\dot{\vec{v}} = -\alpha\vec{v} + f_{\text{случ}}$$

$$\text{м.б. } \alpha = 6\pi\eta a.$$

$$\left\langle \frac{mv^2}{2} \right\rangle = \frac{3}{2}T$$

$$\langle v_x(t_1) v_x(t_2) \rangle = \varphi(t_1, t_2) = \varphi(t_1 - t_2) \ominus$$

– автокорреляционная ф.

$$\ominus\ \varphi(t_2 - t_1).$$

$$\langle f_{сл}(t) f_{сл}(t + \delta t) \rangle = 0 \quad , \text{если } \delta t \geq \Delta t.$$

$$\frac{d\varphi(t_1 - t_2)}{dt_1} = \langle \dot{v}_x(t_1) v_x(t_2) \rangle = -\left\langle \left(\frac{\alpha}{m} v_x(t_1) + f_{сл}(t_1) \right) \right.$$

$$\left. v_x(t_2) \right\rangle = -\frac{\alpha}{m}\varphi \quad , \text{ если } t_1 > t_2.$$

$$\frac{\alpha}{m} = \frac{1}{\tau} \Rightarrow \varphi(t) = c\, e^{-t/\tau} \quad t > 0.$$

$$\varphi(t) = c\, e^{-|t|/\tau} = \frac{T}{m} e^{-|t|/\tau}$$

$$x = x_0 + \int_0^t v_x(t)\, dt \quad , x_0 = 0.$$

$$\langle x^2(t) \rangle = \int_0^t dt_1 \int_0^t dt_2 \langle v_x(t_1) v_x(t_2) \rangle .$$

$$= \int_0^t \int_0^t \varphi(t_1 - t_2)\,dt_1\,dt_2 - \int_0^t dt_1 \int_{\infty}^{\infty} dt_3\, \varphi(t_3) =$$

$$= t \cdot \frac{T}{m} \cdot 2\tau = \underline{2t\,T\alpha}$$

$$r^2 = 6\mathcal{D}t, \quad \mathcal{D} = \frac{T\tau}{m} \sim v_T^2 \tau$$

$$\langle \dot{v}_x(t_1) v_x(t_2) \rangle = \frac{d}{dt_1} \varphi(t_1 - t_2) \ominus$$

$$\langle v_x(t_1) \dot{v}_x(t_2) \rangle = -\frac{d}{dt_1} (\varphi(t_1 - t_2))$$

$$\ominus \frac{T}{m\tau} e^{-\frac{|t|}{\tau}} \operatorname{sgn} t$$

29.04.02.

$$\varphi(t_1 - t_2) = \langle v_x(t_1) v_x(t_2) \rangle = \frac{T}{m} e^{-\frac{|t_1 - t_2|}{\tau}}$$

$$m\dot{v}_x + \alpha v_x = f_{сл}, \quad \tau = \frac{m}{\alpha},$$

$$\langle \dot{v}_x(t_1) v_x(t_2) \rangle = \frac{T}{m\tau} e^{-\frac{|t_1 - t_2|}{\tau}} \operatorname{sgn}(t_2 - t_1)$$

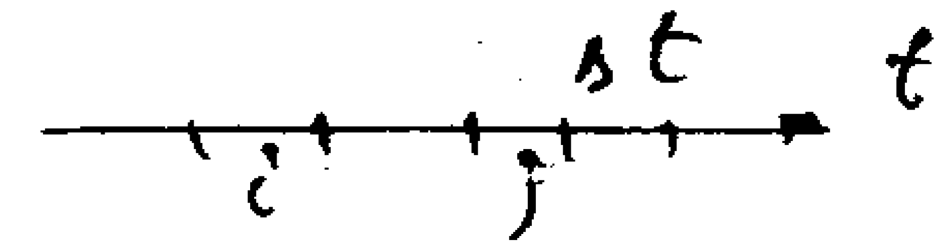
Корреляционная ф. случайных сил.

$$\langle f_{сл}(t_1) f_{сл}(t_2) \rangle = m^2 \left(\frac{d}{dt_1} + \frac{1}{\tau}\right)\left(\frac{d}{dt_2} + \frac{1}{\tau}\right)\varphi$$

$$\left(\frac{d}{dt_{1,2}} + \frac{1}{\tau}\right)\varphi(t_1 - t_2) = 0 \quad \text{при } t_1 \gtrless t_2$$

$$\ominus\ m^2 \cdot \frac{T}{m} \cdot \frac{1}{\tau}\, 2\delta(t_1 - t_2) = 2\alpha T \delta(t_1 - t_2)$$

$$\langle f_{сл\,i} f_{сл\,j} \rangle = 0, \quad i \neq j$$



$$\Delta p = -\alpha v \Delta t + \Delta p_{сл}, \qquad \langle \Delta p_{сл} \rangle = 0$$

$$\langle (\bar{p} + \Delta\bar{p})^2 \rangle \xrightarrow{t} \langle \bar{p}^2 \rangle$$

$$2\vec{p}\Delta\vec{p} + \langle \Delta p^2 \rangle = 0 \qquad \Delta t \ll \tau$$

$$\langle (\Delta\vec{p}_{сл})^2 \rangle - \overbrace{\langle 2\alpha\vec{v}\Delta\vec{p}_{сл} \rangle}^{\to 0} + \alpha^2 v^2 \Delta t^2 +$$

$$+ 2\vec{p}(\cancel{\Delta\vec{p}_{сл}} - \alpha\vec{v}\Delta t) = 0$$

$$\frac{\alpha^2 v^2 \Delta t^2}{mv \cdot \alpha v \Delta t} = \frac{\Delta t}{\tau} \ll 1$$

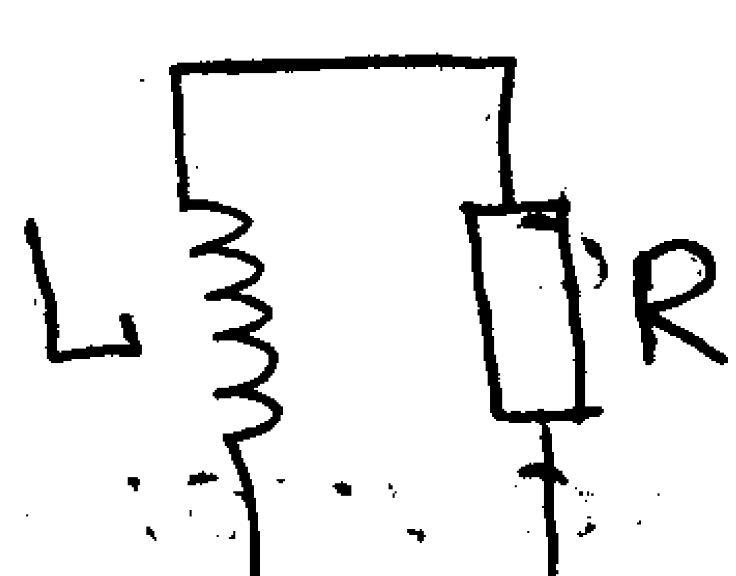
$$\Rightarrow \langle \Delta p_{сл}^2 \rangle = 2\alpha m v^2 \Delta t$$

$$\frac{mv^2}{2} = \frac{3}{2}kT \Rightarrow \langle \Delta p_{сл}^2 \rangle = 6\alpha T \cdot \Delta t$$

$$\vec{f}_{сл} = \frac{\Delta\vec{p}_{сл}}{\Delta t} \Rightarrow f_{сл\,x}(t_i) f_{сл\,x}(t_j) = \frac{2\alpha T}{\Delta t}\delta_{ij}$$

$$\sum_i \frac{2\alpha T}{\Delta t} \cdot \Delta t\, \delta_{ij} = 2\alpha T$$

$$\frac{L I^2}{2} = \frac{T}{2}$$



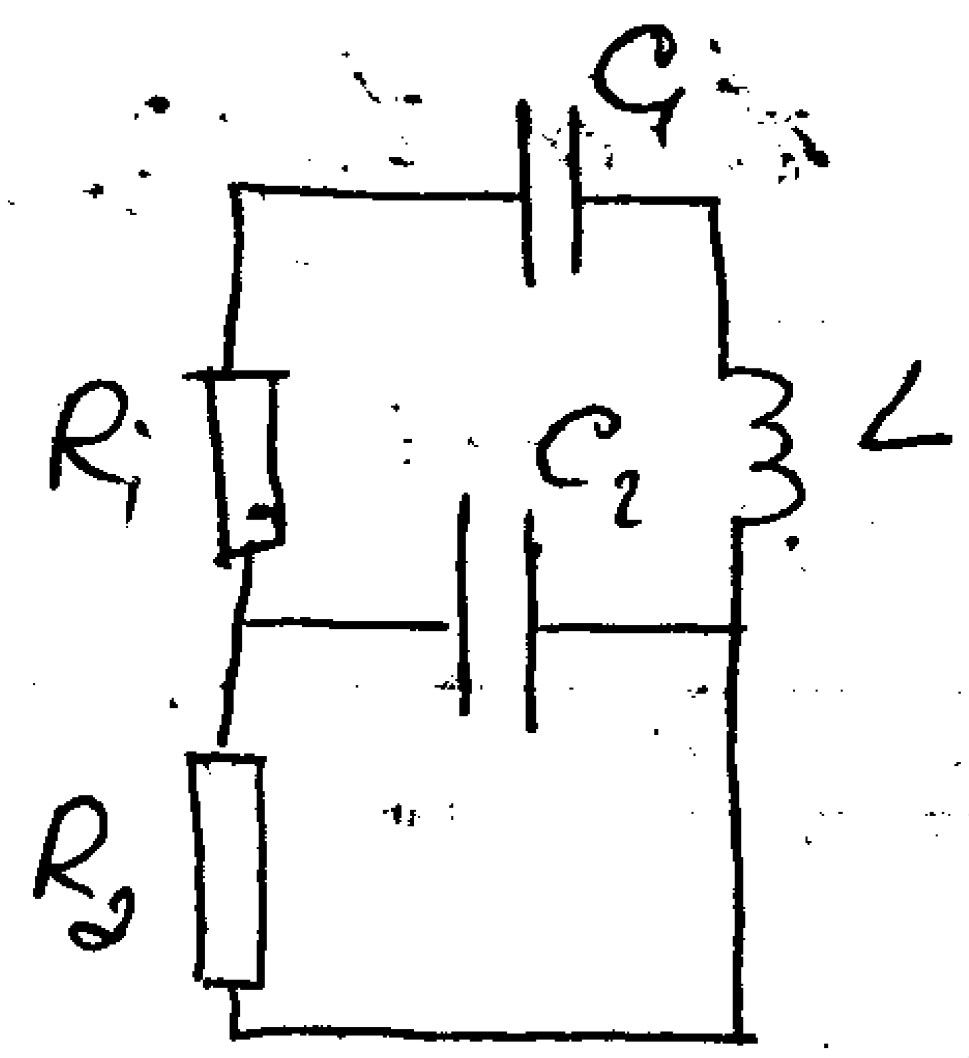
$$L\frac{dI}{dt} + IR = \mathcal{E}_{сл}, \qquad \langle \mathcal{E}_{сл} \rangle = 0$$

$$\langle I(t_1) I(t_2) \rangle = \frac{T}{L} e^{-\frac{|t_1 - t_2|}{\tau}}$$

$(t_1 - t_2) \geq$

$$\tau = \frac{L}{R},$$

$$\langle \mathcal{E}_{сл}(t_1)\mathcal{E}_{сл}(t_2) \rangle = 2RT\delta(t_1 - t_2).$$



$x(t)$, $-t_m<t<t_m$

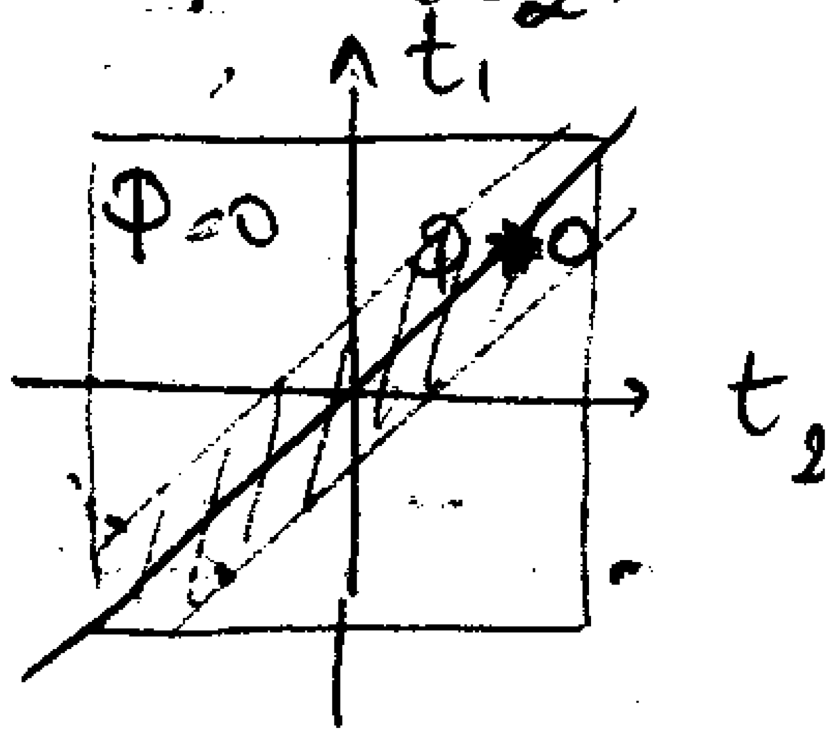
$t_m\to\infty$:

$$x(t)=\int_{-\infty}^{\infty} x_\omega e^{i\omega t}\frac{d\omega}{2\pi}$$

$$x(\omega)=\int_{-t_m}^{t_m} x(t)e^{-i\omega t}dt$$

$$\langle x_{\omega_1} x_{\omega_2}\rangle = \int_{-t_m}^{t_m}\int_{-t_m}^{t_m}\langle x(t_1)x(t_2)\rangle e^{i\omega_1 t_1+i\omega_2 t_2}dt_1\,dt_2 =$$

$\langle x(t_1)x(t_2)\rangle=\Phi(t_1-t_2)$, при $|t_1-t_2|\gg\tau$ $\Phi\to0$



$t_2=t_1+t_3$

$$=\int_{-t_m}^{t_m}\int_{-\infty}^{\infty}\underbrace{\Phi(t_3)e^{i\omega_2 t_3}dt_3}_{(x^2)_\omega}\,dt_1\,e^{i(\omega_1+\omega_2)t_1}=$$

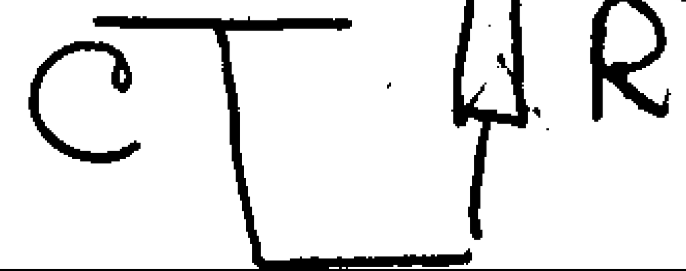
$$= 2\pi\delta(\omega_1+\omega_2)(x^2)_\omega$$

$(\mathcal{E}^2)_\omega=2RT$

$(f_{\text{сл}}^2)_\omega=2\alpha T$

Т. Винера-Хинчина

(флуктуационно-диссипационная Т)



$\langle q(t)q(t')\rangle$ — ?

$$\frac{q}{C} + \dot{q}R = \mathcal{E}$$

$$q\left(-i\omega R + \frac{1}{C}\right) = \mathcal{E}_\omega$$

$$\langle q_\omega q_{\omega'} \rangle = \frac{\langle \mathcal{E}_\omega \mathcal{E}_{\omega'} \rangle}{\left(\frac{1}{C} - i\omega R\right)\left(\frac{1}{C} - i\omega' R\right)}$$

$$(q^2)_\omega = \frac{2RT}{\left(\frac{1}{C} - i\omega R\right)\left(\frac{1}{C} + i\omega R\right)}$$

$$\langle q(t_1) q(t_2) \rangle = \int_{-\infty}^{\infty} \frac{2RT\, C^2\, e^{-i\omega(t_1 - t_2)}}{(1 - i\omega RC)(1 + i\omega RC)} \cdot \frac{d\omega}{2\pi} =$$

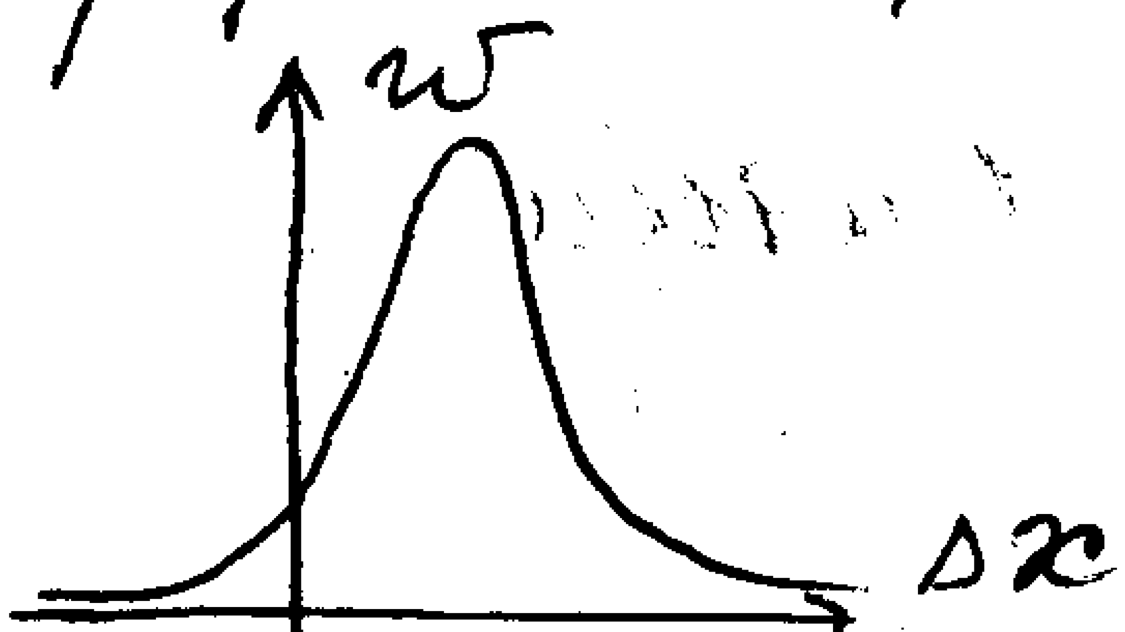
$$= CT e^{-\frac{|t_1 - t_2|}{\tau}}$$

Формула Найквиста

Кинетика

3.04.

Ф. распр. $f(x,t)$, Δt, $\langle \Delta x^2 \rangle \sim \Delta t$



w - вер-сть сместиться за время Δt на Δx.

$$f(x, t+\Delta t) = \int f(x-\Delta x, t)\, w(\Delta x)\, d\Delta x$$

$$f(x,t) + \frac{\partial f}{\partial t}\Delta t = \int \left[f(x,t) - \Delta x \frac{\partial}{\partial x}(f w) + \frac{1}{2}\Delta x^2 \frac{\partial^2 (f w)}{\partial x^2} \right] d\Delta x$$

$$\Delta t \frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}\left(f \int \Delta x\, w(\Delta x, x)\, d\Delta x\right) +$$

$$+ \frac{1}{2}\frac{\partial^2}{\partial x^2}\left(f \int \Delta x^2\, w(\Delta x, x)\, d\Delta x\right).$$

$$\frac{\langle \Delta x \rangle}{\Delta t} = v \qquad \frac{\langle \Delta x^2 \rangle}{2\Delta t} = D.$$

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}(vf) + \frac{\partial^2}{\partial x^2}(Df)$$

$$\frac{\partial f}{\partial t} + \frac{\partial j}{\partial x} = 0,$$

$$j = vf - \frac{\partial}{\partial x}(Df)$$

$$v = \mathrm{const},\ D = \mathrm{const},\ \frac{\partial f}{\partial t} = 0,$$

$$j(x=0) = 0.$$

$$\frac{\partial j}{\partial x} = 0 \Rightarrow j = 0 \Rightarrow vf - D\frac{\partial f}{\partial x} = 0$$

$$f = C e^{\frac{vx}{D}}$$

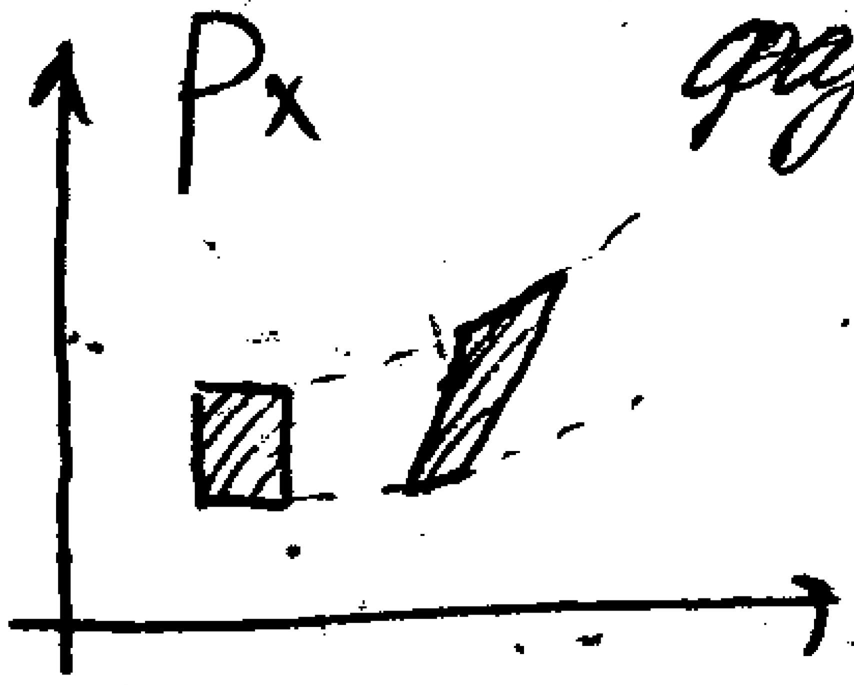
Кинетическое уравнение

$$f(\vec{r}, \vec{p}, t)$$

$$d\Gamma = d^3r\, d^3p.$$

$$\int f\, d^3p = n(\vec{r}, t)$$

$$\int \vec v f(\vec r,\vec p,t)\, d^3p = \vec j = n\vec v$$



фаз. объем сохр.

$$\frac{df}{dt}=0.$$

$$\frac{\partial f}{\partial t} + \dot{\vec r}\frac{\partial f}{\partial \vec r} + \dot{\vec p}\frac{\partial f}{\partial \vec p}=0.$$

$$\frac{\partial f}{\partial t} + \vec v\frac{\partial f}{\partial \vec r} + \vec F\frac{\partial f}{\partial \vec p}=0.$$

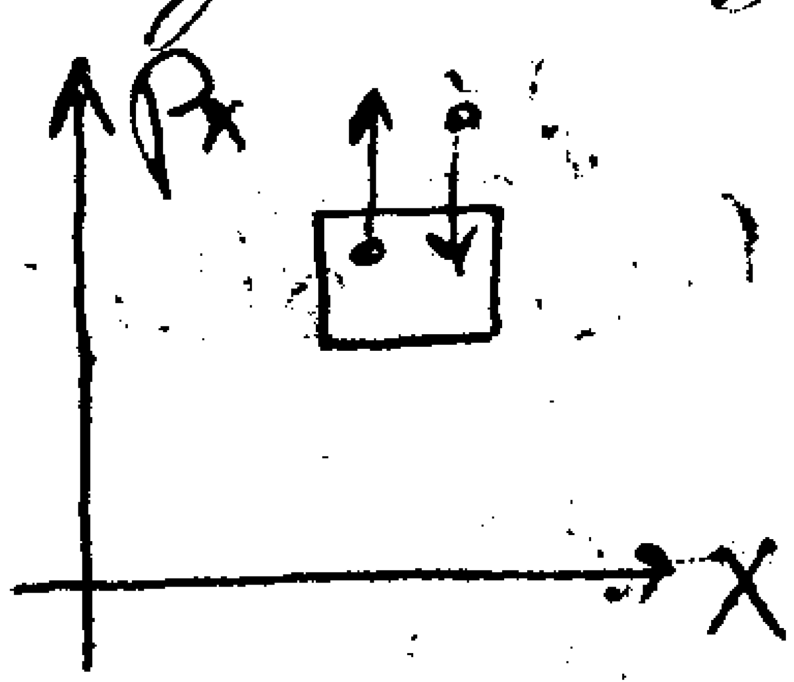
Напр., $\vec F = e\vec E + \frac{e}{c}[\vec v\times\vec{\mathcal{H}}]$

$t=0 \quad f_0(\vec r_0,\vec p)$, $\quad \vec r = \vec r_0 + \vec v t$

$$f(\vec r,\vec p,t) = f_0(\vec r - \vec v t, \vec p)$$

Взаимодействие: короткодейств. (газ)

дальнодейств. (плазма)



$$\frac{\partial f}{\partial t}+\vec v\frac{\partial f}{\partial \vec r}+\vec F\frac{\partial f}{\partial \vec p} = I_{\text{ухода}} - I_{\text{прихода}}$$

$$\rho_a = z_a e_a \int f_a\, d^3p$$

$$\mathrm{div}\,\vec E = 4\pi\rho$$

Колебания. $\qquad f=f_0+\delta f$; $\delta f, \vec E$ – малы

Движение ионов не учитываем.

$$\frac{\partial \delta f}{\partial t} + \vec v\frac{\partial \delta f}{\partial \vec r} + e\vec E\frac{\partial (f_0+\cancel{\delta f})}{\partial \vec p}=0$$ линеаризация

$$\mathrm{div}\,\vec E = 4\pi e\int (f_0+\delta f)\,d^3p - z_i e n_i$$

$$\delta f, \bar{E} \sim e^{ikz - i\omega t}$$

$$i(kv_x - \omega)\delta f + eE\frac{\partial f_0}{\partial p_x} = 0$$

$$\delta f = -i\frac{eE}{\omega - kv_x}\frac{\partial f_0}{\partial p_x}$$

$$ikE = 4\pi e\int \delta f\, d^3p$$

$$ikE = -ieE\cdot 4\pi e\int\frac{d^3p}{\omega - kv_x}\frac{\partial f_0}{\partial p_x}, \quad kv \ll \omega$$

$$\frac{1}{\omega - kv_x} = \frac{1}{\omega}\left(1 + \frac{kp_x}{m\omega} + \left(\frac{kp_x}{m\omega}\right)^2 + \left(\frac{kp_x}{m\omega}\right)^3 + \dots\right)$$

$$\int\frac{dp_x\, d^2p_\perp}{\omega - kv_x}\frac{\partial f}{\partial p_x} = \frac{1}{\omega}\Bigg[\underbrace{\int_{-\infty}^{\infty} df_0}_{0} + \underbrace{\int p_x\frac{\partial f_0}{\partial p_x}dp_x}_{-\frac{kn}{m\omega}} + 0 +$$

$$+ \underbrace{\int p_x^3\frac{\partial f_0}{\partial p_x}dp_x}_{-\frac{3n\langle p_x^2\rangle k^3}{m^3\omega^3}}\Bigg] \Rightarrow$$

$$\frac{4\pi ne^2}{m\omega^2}\left(1 + \frac{3k^2\langle v_x^2\rangle}{\omega^2}\right) = 1$$

$$\frac{\omega_p^2}{\omega^2}\left(1 + \frac{3k^2\langle v_x^2\rangle}{\omega^2}\right) = 1.$$

$$w^2 \approx w_p^2\left(1+\frac{3k^2\langle v_x^2\rangle}{\omega_p^2}\right).$$

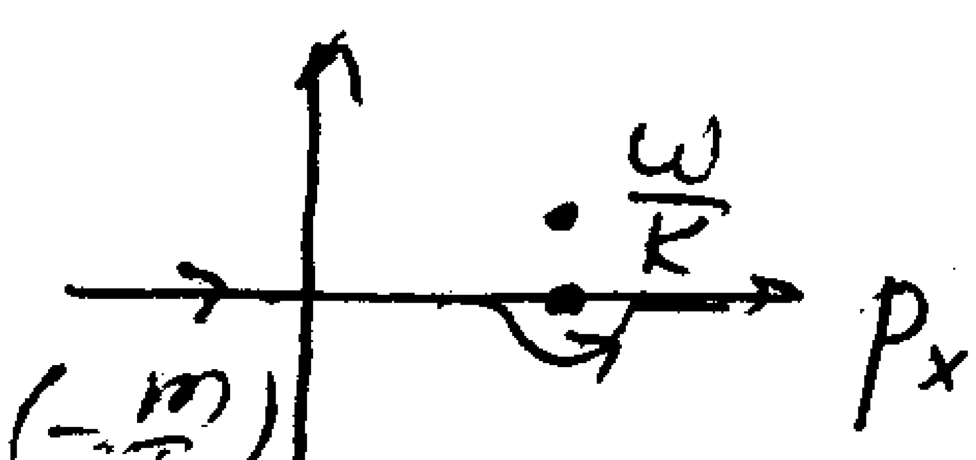
$$w = w_p + \frac{k^2 v^2}{2w_p}$$

$$\int \frac{dp_x\, dp_\perp^2}{\omega - k v_x}\cdot\frac{df}{dp_x} = \frac{1}{w}\left(-\frac{kn}{m\omega} - \frac{3n\langle p_x^2\rangle k^3}{m^3\omega^3}\right).$$

Далее $v_x = \frac{\omega}{k}$

Нарастание колебаний $e^{-i\omega t+\delta t}$, $t \to \infty$

$\omega \to \omega + i\delta$



$$\int \frac{dp_x \frac{\partial f_0}{\partial p_x}}{\omega - kp_x/m} = \int + \pi i \frac{\partial f}{\partial p_x}\Big|_{p_x=\frac{m\omega}{k}} \cdot \left(-\frac{m}{k}\right)$$

$$\frac{\partial f_0}{\partial p_x} = \frac{\partial f_0}{\partial \varepsilon}\cdot\frac{\partial \varepsilon}{\partial p_x} = \frac{f}{T} v_x$$

17.05. 1

$R = R_0 + aT$

При высоких T число фононов $\sim \frac{T}{\hbar\omega}$,

$T(x)$ – плавная зависимость.

$f_0 = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}$, $f = f_0 + \delta f$

$dE = TdS + \mu dN \Rightarrow TdS = dE - \mu dN = (\varepsilon-\mu)dN$

Квазичастицы с $\epsilon = v_0(|\bar p| - |\bar p_0|)$.

Поток тепла $q_x = \int v_x(\varepsilon-\mu)(\cancel{f_0}+\delta f)\,d^3p$, $\frac{\partial}{\partial t} = 0$

$\cancel{\frac{\partial f}{\partial t}} + \bar v \frac{\partial f}{\partial \bar r} + \cancel{F \frac{\partial f}{\partial \bar p}} = \mathcal{J}$ общий вид кинет. ур-я.

$v_x \frac{\partial f}{\partial x} = \frac{-\delta f}{\tau}$

$z = \frac{\varepsilon-\mu}{T}$, $f_0(z)$

$\frac{\partial f_0}{\partial x} = \frac{\partial f_0}{\partial z}\frac{\partial}{\partial x}\frac{\varepsilon-\mu}{T} = \frac{\partial f_0}{\partial z}\left(-\frac{\varepsilon-\mu}{T^2}\frac{\partial T}{\partial x}\right)$

$\frac{\partial f_0}{\partial \varepsilon} = \frac{\partial f_0}{\partial z}\cdot\frac{\partial}{\partial \varepsilon}\frac{\varepsilon-\mu}{T} = \frac{1}{T}\frac{\partial f_0}{\partial z}$

$\frac{\partial f_0}{\partial x} = -\frac{\partial f_0}{\partial \varepsilon}\cdot\frac{\varepsilon-\mu}{T}\frac{\partial T}{\partial x}$

$$\mu = \mu_0 + \frac{T^2}{\mu_0} \cdot k$$

$$\frac{\partial z}{\partial x} = -\frac{\varepsilon-\mu}{T^2}\frac{\partial T}{\partial x} - \frac{\partial \mu}{\partial x \cdot T}$$

$$\frac{T}{\mu} \Big/ \frac{\varepsilon-\mu}{T} \sim \frac{T}{\mu} \ll 1.$$

$$v_x \frac{\varepsilon-\mu}{T}\frac{\partial T}{\partial x}\frac{\partial f_0}{\partial \varepsilon} = -\frac{\delta f}{\tau}$$

$$q = \int v_x^2 \tau (\varepsilon-\mu)^2 \frac{\partial f_0}{\partial \varepsilon} \cdot \frac{1}{T}\frac{\partial T}{\partial x} d^3p \qquad d^3p = A\sqrt{\varepsilon}\, d\varepsilon.$$

$$\int f_0 d^3p = A\int_0^{\mu_0} \sqrt{\varepsilon}\, d\varepsilon = \frac{2}{3}A\mu^{3/2} = n.$$

$$v_x^2 \to \frac{v^2}{3} = \frac{2}{3}\frac{\varepsilon}{m}, \quad \varepsilon = \frac{mv^2}{2}$$

$$\frac{\partial f_0}{\partial \varepsilon} \approx -\delta(\varepsilon-\mu)$$

$$\int_0^\infty F(\varepsilon)\frac{\partial f_0}{\partial \varepsilon} d\varepsilon = \mathcal{F} f_0\Big|_0^\infty - \int_0^\infty \mathcal{F}'(\varepsilon) f_0(\varepsilon) d\varepsilon =$$

$$= -\int_0^\mu \mathcal{F}'(\varepsilon) d\varepsilon - \frac{\pi^2 T^2}{6}\mathcal{F}''(\mu) = -\mathcal{F}(\mu) - \frac{\pi^2}{6}T^2\mathcal{F}''(\mu)$$

$$\mathcal{F}(0) = 0. \quad ; \quad q = -\frac{dT}{dx}\frac{\tau}{T}\frac{2A}{3m}2\mu^{3/2}\frac{\pi^2 T^2}{6} = -æ\frac{d}{d}$$

$$\mathcal{F} \sim \varepsilon\sqrt{\varepsilon}\cdot(\varepsilon-\mu)^2 \qquad , \; æ = \frac{\pi^2 \tau T n}{3m}$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$\frac{\varkappa}{\sigma} = \frac{\pi^2}{3}\frac{T}{e^2}$$ – число Лоренца

Закон Видемана-Франца.

$$-v_x \frac{\partial f}{\partial \varepsilon}\frac{\varepsilon-\mu}{T}\frac{\partial T}{\partial x} + eE\frac{\partial f_0}{\partial p_x} = -\frac{\delta f}{\tau}$$

$$j_x = e\int v_x\,\delta f\cdot d^3p = -e^2E\tau\int v_x \frac{\partial f_0}{\partial p_x}d^3p + e\frac{\tau}{T}\frac{\partial T}{\partial x}.$$

$$\int v_x^2(\varepsilon-\mu)\frac{\partial f_0}{\partial \varepsilon}d^3p = \sigma E - \alpha\frac{dT}{dx}.$$

$$q_x = -\varkappa\frac{\partial T}{\partial x} + eE\tau\int(\varepsilon-\mu)v_x\frac{\partial f_0}{\partial p_x}d^3p = -\varkappa\frac{\partial T}{\partial x} + \beta E$$

$$\beta = \alpha T$$

$$\alpha = \frac{e\tau}{T}\int\frac{2\varepsilon}{3m}(\varepsilon-\mu)\frac{\partial f_0}{\partial \varepsilon}\frac{3n}{2\mu^{3/2}}\sqrt{\varepsilon}\,d\varepsilon =$$

$$= \frac{ne\tau}{m\mu^{3/2}}\frac{\partial^2}{\partial\varepsilon^2}\left[(\varepsilon-\mu)\varepsilon^{3/2}\right]\Big|_{\varepsilon=\mu}\cdot\frac{\pi^2T}{6} = \frac{ne\tau\pi^2T}{2m\mu^{1/2}}$$

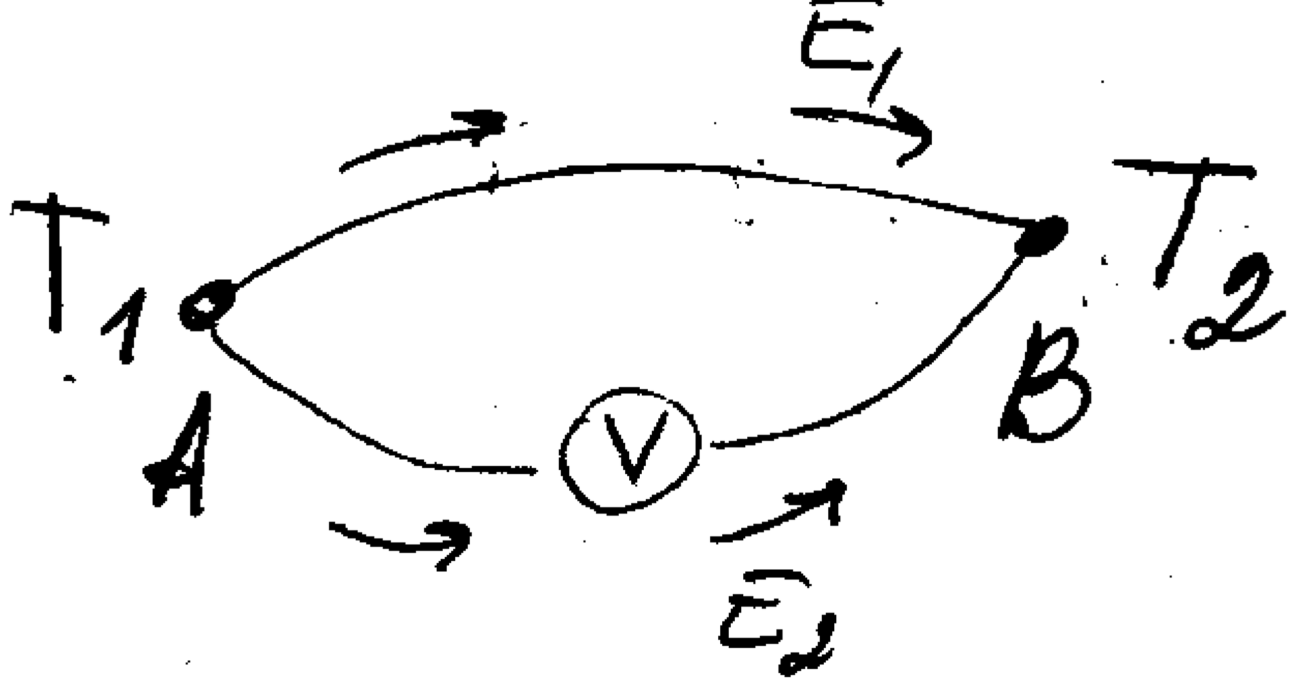
$$j = \sigma E - \alpha\frac{\partial T}{\partial x}$$

$$q = \varkappa\frac{\partial T}{\partial x} + \beta E$$

$$E = \frac{\alpha}{\sigma}\frac{\partial T}{\partial x} = Q\frac{\partial T}{\partial x}$$

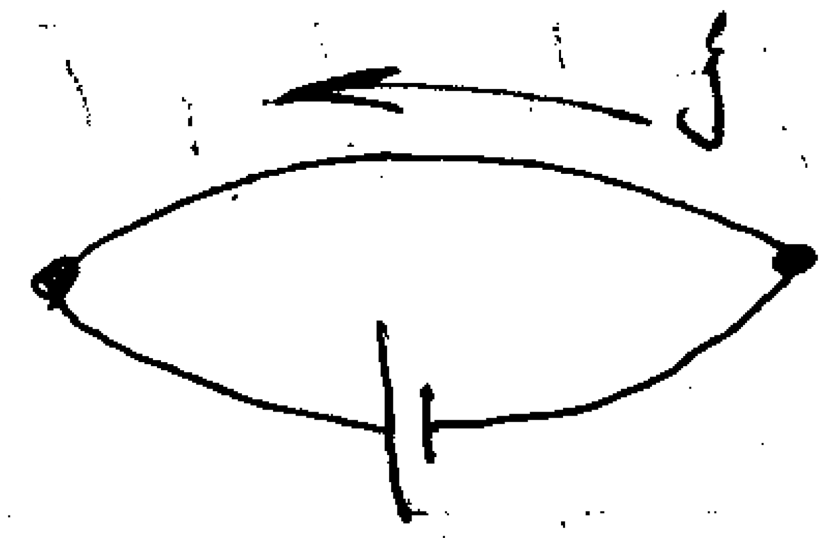
$$q = -\varkappa\frac{\partial T}{\partial x} + \underset{\text{мало}}{\beta Q}\frac{\partial T}{\partial x}$$

Термо-ЭДС 24.05



Эффект Пельтье

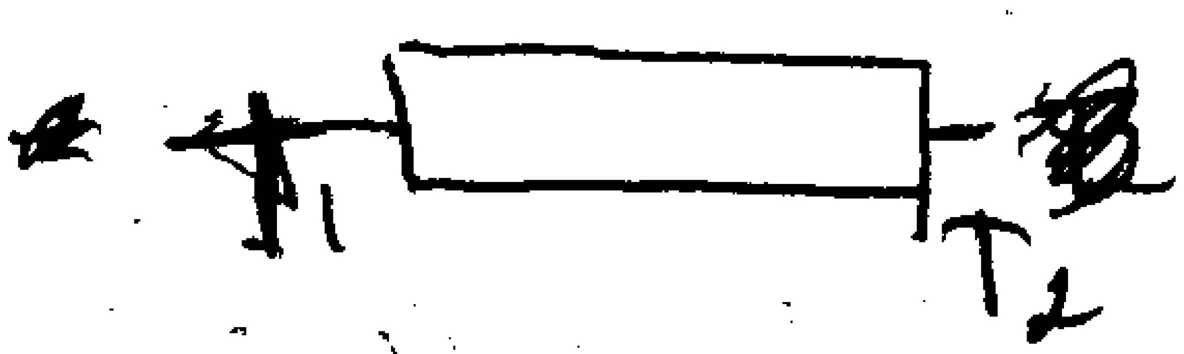
$$q = \beta E = \frac{\beta}{\sigma} j$$



$$\frac{\beta}{\sigma} = \Pi$$

$$\frac{\alpha}{\sigma} = Q \qquad \Pi = TQ$$

$$j = \varkappa \frac{dT}{dx}\frac{1}{T(x)} - \varkappa \frac{dT}{dx}\cdot\frac{1}{T(x+dx)} = \varkappa\left(\frac{1}{T}\frac{dT}{dx}\right)^2$$



$$jE = \sigma E^2 - \alpha \frac{dT}{dx} E.$$

Квантовое кинетическое ур-е

$$\int f(\vec r, \vec p, t)\, d^3p = n(\vec r) \qquad \psi(x, q, t)$$

$$dw = dx \int |\psi(x, q, t)\, \psi^*(x, q, t)\, dq$$ матрица плотности

$$\rho(x, x, t) = w(x, t)$$

$$H(\hat p, \hat x) + \mathcal{H}(q, p_q)$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\hat H \psi$$

$$\frac{\partial \rho(x_1, x_2, t)}{\partial t} = \int \frac{\partial \psi_1}{\partial t}\psi_2^* + \psi_1 \frac{\partial \psi_2^*}{\partial t} =$$

$$= \frac{i}{\hbar}(\hat H_2 - \hat H_1)\rho(x_1, x_2, t)$$

$$\hat{H} = \frac{p^2}{2m} + U(x)$$

$$f(x,p,t) = \int \rho\left(x - \frac{\hbar\xi}{2}, x + \frac{\hbar\xi}{2}, t\right) e^{ip\xi} \frac{d\xi}{2\pi}$$

$$\frac{x_1 + x_2}{2} = x, \quad x_2 - x_1 = \hbar\xi$$

$$\rho(x_1, x_2, t) = \int f\left(\frac{x_1 + x_2}{2}, p, t\right) e^{-i\frac{p(x_2 - x_1)}{\hbar}} dp$$

$$\frac{\partial\rho}{\partial t} = \frac{i}{\hbar}\left(\hat{H}_2 - \hat{H}_1\right)\rho(x_1, x_2, t)$$

$$\int f(x,p,t)\,dx = \tilde{\rho}(p, p, t) = \hat{w}(p,t)$$

$$\psi(x) = a(x) e^{ik(x)x}, \quad \text{при} \quad |x - x_0| \lesssim l$$

$$k(x) \approx k(x_0), \quad w(x) = |a(x)|^2$$

$$\rho(x_1, x_2) = a(x_1) a(x_2) e^{ik_1 x_1 - ik_2 x_2}$$

$$a(x) \approx a(x_0)$$

$$\rho\left(x - \frac{\hbar\xi}{2}, x + \frac{\hbar\xi}{2}\right)_l \approx a\left(x - \frac{\hbar\xi}{2}\right) a\left(x + \frac{\hbar\xi}{2}\right) e^{-ik(x_0)\hbar\xi}$$

$$f(x,p,t) = |a(x)|^2 \int e^{i(p - \hbar k(x_0))\xi} d\xi =$$

$$= |a(x)|^2 \tilde{\delta}^{-l}(p - \hbar k(x_0))$$

$$\psi(x) = \frac{1}{\pi^{1/4}\sigma^{1/2}} e^{-\frac{(x - x_0)^2}{2\sigma^2} + ipx/\hbar}$$