**Министерство образования и науки РФ Санкт-Петербургский политехнический университет Петра Великого Институт компьютерных наук и технологий Высшая школа программной инженерии**

**ЛАБОРАТОРНАЯ РАБОТА № 3**

по дисциплине «Вычислительная математика»

Выполнил

студент гр. 3530904/80004 Мащенко Б.Б.

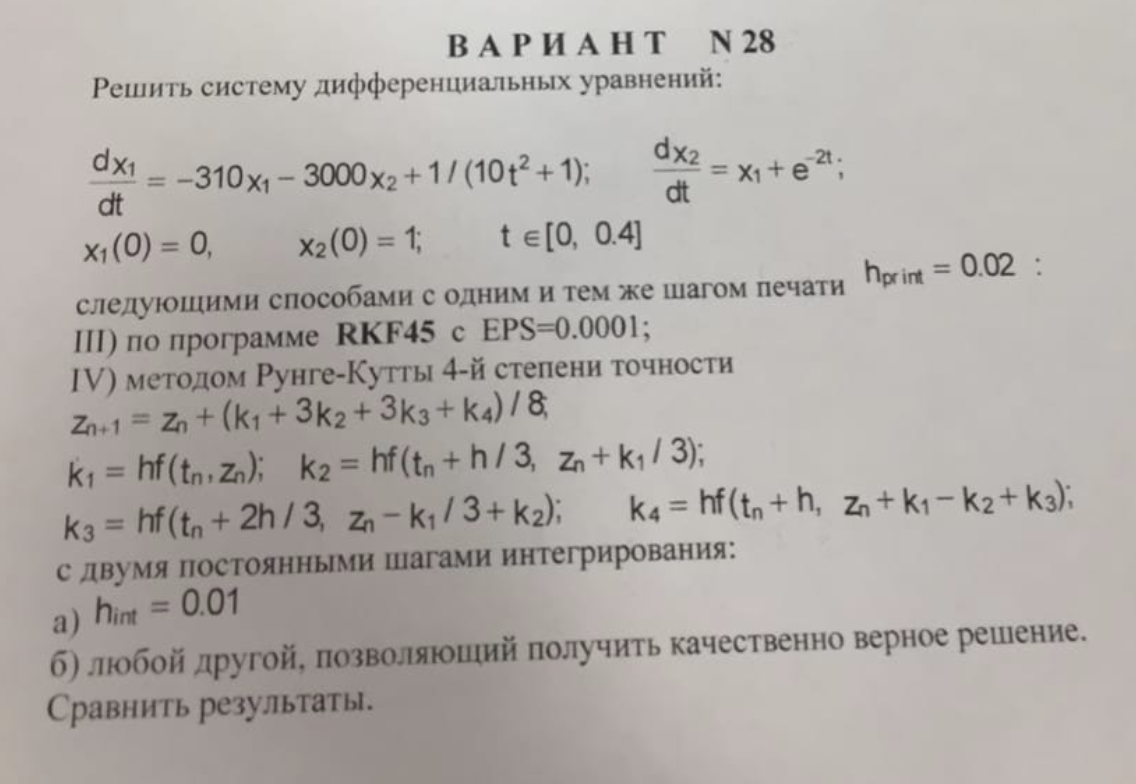
Руководитель,

к.т.н, доцент Леонтьева Т.В.

Санкт-Петербург

2020

Формулировка задания



Выполнение работы

Была реализована в соответствии с условиями задачи функция sourceFunction(double, double \*, double \*). Затем система дифференциальных уравнений была посчитана с использованием функции RKF45, также методом Рунге-Кутта 4 степени точности два раза, с h = 0.01 и h = 0.00923.

Результат работы

Вывод программы:

result for RKF45 with H = 0.02 and EPSILON = 0.0001

T = 0.00, FLAG = 2, X = (0.000000; 1.000000)

T = 0.02, FLAG = 2, X = (-8.591703; 0.865187)

T = 0.04, FLAG = 2, X = (-7.226865; 0.726118)

T = 0.06, FLAG = 2, X = (-6.081972; 0.611497)

T = 0.08, FLAG = 2, X = (-5.138191; 0.516987)

T = 0.10, FLAG = 2, X = (-4.359148; 0.438969)

T = 0.12, FLAG = 2, X = (-3.715446; 0.374478)

T = 0.14, FLAG = 2, X = (-3.182716; 0.321086)

T = 0.16, FLAG = 2, X = (-2.740971; 0.276803)

T = 0.18, FLAG = 2, X = (-2.374065; 0.240002)

T = 0.20, FLAG = 2, X = (-2.068600; 0.209346)

T = 0.22, FLAG = 2, X = (-1.813542; 0.183743)

T = 0.24, FLAG = 2, X = (-1.600060; 0.162296)

T = 0.26, FLAG = 2, X = (-1.420774; 0.144271)

T = 0.28, FLAG = 2, X = (-1.269593; 0.129066)

T = 0.30, FLAG = 2, X = (-1.141678; 0.116187)

T = 0.32, FLAG = 2, X = (-1.032957; 0.105229)

T = 0.34, FLAG = 2, X = (-0.940042; 0.095861)

T = 0.36, FLAG = 2, X = (-0.860298; 0.087810)

T = 0.38, FLAG = 2, X = (-0.791467; 0.080852)

T = 0.40, FLAG = 2, X = (-0.731654; 0.074803)

T = 0.42, FLAG = 2, X = (-0.679426; 0.069513)

T = 0.44, FLAG = 2, X = (-0.633524; 0.064857)

T = 0.46, FLAG = 2, X = (-0.592875; 0.060733)

T = 0.48, FLAG = 2, X = (-0.556706; 0.057057)

T = 0.50, FLAG = 2, X = (-0.524308; 0.053760)

T = 0.52, FLAG = 2, X = (-0.495061; 0.050784)

T = 0.54, FLAG = 2, X = (-0.468556; 0.048081)

T = 0.56, FLAG = 2, X = (-0.444385; 0.045613)

T = 0.58, FLAG = 2, X = (-0.422180; 0.043348)

T = 0.60, FLAG = 2, X = (-0.401733; 0.041257)

T = 0.62, FLAG = 2, X = (-0.382801; 0.039319)

T = 0.64, FLAG = 2, X = (-0.365157; 0.037515)

T = 0.66, FLAG = 2, X = (-0.348702; 0.035829)

T = 0.68, FLAG = 2, X = (-0.333287; 0.034249)

T = 0.70, FLAG = 2, X = (-0.318764; 0.032761)

T = 0.72, FLAG = 2, X = (-0.305093; 0.031359)

T = 0.74, FLAG = 2, X = (-0.292177; 0.030032)

T = 0.76, FLAG = 2, X = (-0.279914; 0.028775)

T = 0.78, FLAG = 2, X = (-0.268296; 0.027582)

T = 0.80, FLAG = 2, X = (-0.257257; 0.026448)

T = 0.82, FLAG = 2, X = (-0.246719; 0.025367)

T = 0.84, FLAG = 2, X = (-0.236694; 0.024337)

T = 0.86, FLAG = 2, X = (-0.227131; 0.023353)

T = 0.88, FLAG = 2, X = (-0.217971; 0.022413)

T = 0.90, FLAG = 2, X = (-0.209233; 0.021515)

T = 0.92, FLAG = 2, X = (-0.200878; 0.020655)

T = 0.94, FLAG = 2, X = (-0.192855; 0.019832)

T = 0.96, FLAG = 2, X = (-0.185189; 0.019044)

T = 0.98, FLAG = 2, X = (-0.177849; 0.018288)

T = 1.00, FLAG = 2, X = (-0.170788; 0.017564)

---------------------------------------

punct a) runge-kyte H\_int = 0.01

T = 0.00, X = (4.750974; 0.898453)

T = 0.02, X = (18.905053; 0.703006)

T = 0.04, X = (44.029348; 0.497185)

T = 0.06, X = (90.184354; 0.242732)

T = 0.08, X = (176.338350; -0.127445)

T = 0.10, X = (338.310372; -0.735912)

T = 0.12, X = (643.785677; -1.810879)

T = 0.14, X = (1220.702947; -3.780933)

T = 0.16, X = (2310.922712; -7.454015)

T = 0.18, X = (4371.693238; -14.355712)

T = 0.20, X = (8267.483734; -27.368757)

T = 0.22, X = (15632.668353; -51.942019)

T = 0.24, X = (29557.225122; -98.376209)

T = 0.26, X = (55883.135063; -186.145241)

T = 0.28, X = (105655.386420; -352.066269)

T = 0.30, X = (199755.904262; -665.746141)

T = 0.32, X = (377664.573387; -1258.784829)

T = 0.34, X = (714023.047065; -2379.988133)

T = 0.36, X = (1349950.697798; -4499.754201)

T = 0.38, X = (2552251.336580; -8507.429266)

T = 0.40, X = (4825350.916406; -16084.433332)

T = 0.42, X = (9122929.752879; -30409.700916)

T = 0.44, X = (17248039.691221; -57493.404945)

T = 0.46, X = (32609575.625111; -108698.528487)

T = 0.48, X = (61652479.461915; -205508.211308)

T = 0.50, X = (116561719.494439; -388539.014444)

T = 0.52, X = (220374501.400598; -734581.623540)

T = 0.54, X = (416645542.164647; -1388818.428583)

T = 0.56, X = (787720478.584485; -2625734.885601)

T = 0.58, X = (1489284030.230765; -4964280.059860)

T = 0.60, X = (2815677620.041437; -9385592.027841)

T = 0.62, X = (5323390500.758312; -17744634.965372)

T = 0.64, X = (10064535165.846144; -33548450.517351)

T = 0.66, X = (19028261798.261574; -63427539.293648)

T = 0.68, X = (35975307462.656815; -119917691.509782)

T = 0.70, X = (68015815671.889908; -226719385.541956)

T = 0.72, X = (128592401504.957901; -428641338.320169)

T = 0.74, X = (243120009095.589417; -810400030.290190)

T = 0.76, X = (459648767196.615417; -1532162557.294794)

T = 0.78, X = (869023450481.356445; -2896744834.911722)

T = 0.80, X = (1642997461066.560059; -5476658203.530140)

T = 0.82, X = (3106292074829.199707; -10354306916.073296)

T = 0.84, X = (5872833453974.183594; -19576111513.224228)

T = 0.86, X = (11103325748920.156250; -37011085829.711761)

T = 0.88, X = (20992225244052.382812; -69974084146.820068)

T = 0.90, X = (39688425852036.750000; -132294752840.102203)

T = 0.92, X = (75035930126507.156250; -250119767088.337799)

T = 0.94, X = (141864805395427.843750; -472882684651.407349)

T = 0.96, X = (268213147700730.843750; -894043825669.085083)

T = 0.98, X = (507090482371694.625000; -1690301607905.631348)

T = 1.00, X = (958717943233985.375000; -3195726477446.603027)

---------------------------------------

punct b) runge-kyte H\_int = 0.00928333

T = 0.00, X = (0.820759; 0.917422)

T = 0.02, X = (2.253954; 0.773099)

T = 0.04, X = (3.448726; 0.652622)

T = 0.06, X = (4.445262; 0.551971)

T = 0.07, X = (5.276945; 0.467809)

T = 0.09, X = (5.971511; 0.397361)

T = 0.11, X = (6.552003; 0.338325)

T = 0.13, X = (7.037568; 0.288784)

T = 0.15, X = (7.444112; 0.247149)

T = 0.17, X = (7.784856; 0.212099)

T = 0.19, X = (8.070784; 0.182533)

T = 0.20, X = (8.311026; 0.157540)

T = 0.22, X = (8.513170; 0.136361)

T = 0.24, X = (8.683523; 0.118366)

T = 0.26, X = (8.827329; 0.103028)

T = 0.28, X = (8.948949; 0.089914)

T = 0.30, X = (9.052009; 0.078659)

T = 0.32, X = (9.139526; 0.068963)

T = 0.33, X = (9.214010; 0.060574)

T = 0.35, X = (9.277550; 0.053282)

T = 0.37, X = (9.331885; 0.046915)

T = 0.39, X = (9.378465; 0.041327)

T = 0.41, X = (9.418496; 0.036396)

T = 0.43, X = (9.452987; 0.032024)

T = 0.45, X = (9.482776; 0.028124)

T = 0.46, X = (9.508565; 0.024628)

T = 0.48, X = (9.530939; 0.021476)

T = 0.50, X = (9.550390; 0.018619)

T = 0.52, X = (9.567327; 0.016016)

T = 0.54, X = (9.582094; 0.013633)

T = 0.56, X = (9.594980; 0.011441)

T = 0.58, X = (9.606228; 0.009415)

T = 0.59, X = (9.616041; 0.007534)

T = 0.61, X = (9.624593; 0.005783)

T = 0.63, X = (9.632029; 0.004145)

T = 0.65, X = (9.638474; 0.002609)

T = 0.67, X = (9.644031; 0.001163)

T = 0.69, X = (9.648791; -0.000200)

T = 0.71, X = (9.652831; -0.001490)

T = 0.72, X = (9.656217; -0.002712)

T = 0.74, X = (9.659005; -0.003872)

T = 0.76, X = (9.661245; -0.004976)

T = 0.78, X = (9.662979; -0.006027)

T = 0.80, X = (9.664245; -0.007030)

T = 0.82, X = (9.665076; -0.007987)

T = 0.84, X = (9.665501; -0.008902)

T = 0.85, X = (9.665546; -0.009778)

T = 0.87, X = (9.665234; -0.010616)

T = 0.89, X = (9.664585; -0.011418)

T = 0.91, X = (9.663619; -0.012188)

T = 0.93, X = (9.662351; -0.012925)

T = 0.95, X = (9.660799; -0.013633)

T = 0.97, X = (9.658976; -0.014312)

T = 0.98, X = (9.656894; -0.014964)

Вывод

Сравнив результаты расчета по методу Рунге-Кутты четвертой степени точности с результатами расчетов с помощью программы RKF45 можно отметить, что результат сильно зависит от шага интегрирования. При большом шаге система оказывается неустойчивой и не дает ожидаемой погрешности вычислений, при шаге интегрирования, вычисленной по формуле h|λк|макс = 2.785 система ведёт себя более единообразно.

Исходный код программы:

Основной класс программы.

#include **<cstdio>**

#include **<cmath>**

#include **<iostream>**

#include **"Apollo/rkf45.h"**

**void** sourceFunction(**double**, **double** \*, **double** \*);

**int** main() {

**int** N = 2;

**int** FLAG = 1;

**double** WORK[15];

**int** I\_WORK[30];

**double** X0[2] = {0, 1};

**double** T = 0;

**double** TOUT = 0;

**double** R\_EPSILON = 0.0001;

**double** A\_EPSILON = 0.0001;

**double** h = 0.02;

std::cout << std::endl << **"result for RKF45 with H = 0.02 and EPSILON = 0.0001"** << std::endl << std::endl;

**while** (TOUT < 1.00001) {

RKF45(sourceFunction, N, X0, &T, &TOUT, &R\_EPSILON, &A\_EPSILON, &FLAG, WORK, I\_WORK);

printf(**"T = %.2f,\t FLAG = %d, \t X = (%.6f; %.6f)\n"**, T, FLAG, X0[0], X0[1]);

TOUT += h;

}

std::cout << std::endl << **"---------------------------------------"** << std::endl;

**double** K1[2];

**double** K2[2];

**double** K3[2];

**double** K4[2];

**double** kTemporary[2];

**double** Zn[2];

std::cout << std::endl << **"punct a) runge-kyte H\_int = 0.01"** << std::endl << std::endl;

X0[0] = 0;

X0[1] = 1;

T = 0;

h = 0.01;

**for** (**int** i = 0; T < 1.0001; i++) {

sourceFunction(T, X0, K1);

K1[0] \*= h;

K1[1] \*= h;

kTemporary[0] = X0[0] + K1[0] / 3;

kTemporary[1] = X0[1] + K1[1] / 3;

sourceFunction(T + h / 3, kTemporary, K2);

K2[0] \*= h;

K2[1] \*= h;

kTemporary[0] = X0[0] - K1[0] / 3 + K2[0];

kTemporary[1] = X0[1] - K1[1] / 3 + K2[1];

sourceFunction(T + 2 \* h / 3, kTemporary, K3);

K3[0] \*= h;

K3[1] \*= h;

kTemporary[0] = X0[0] + K1[0] - K2[0] + K3[0];

kTemporary[1] = X0[1] + K1[1] - K2[1] + K3[1];

sourceFunction(T + h, kTemporary, K4);

K4[0] \*= h;

K4[1] \*= h;

Zn[0] = X0[0] + (K1[0] + 3 \* K2[0] + 3 \* K3[0] + K4[0]) / 8;

Zn[1] = X0[1] + (K1[1] + 3 \* K2[1] + 3 \* K3[1] + K4[1]) / 8;

X0[0] = Zn[0];

X0[1] = Zn[1];

**if** (!(i % 2)) {

printf(**"T = %.2f,\t X = (%.6f; %.6f)\n"**, T, Zn[0], Zn[1]);

}

T += h;

}

h = 2.785 / 300;

std::cout << std::endl << **"---------------------------------------"** << std::endl;

std::cout << std::endl << **"punct b) runge-kyte H\_int = "** << h << std::endl << std::endl;

X0[0] = 0;

X0[1] = 1;

T = 0;

**for** (**int** i = 0; T < 1.0001; i++) {

sourceFunction(T, X0, K1);

K1[0] \*= h;

K1[1] \*= h;

kTemporary[0] = X0[0] + K1[0] / 3;

kTemporary[1] = X0[1] + K1[1] / 3;

sourceFunction(T + h / 3, kTemporary, K2);

K2[0] \*= h;

K2[1] \*= h;

kTemporary[0] = X0[0] - K1[0] / 3 + K2[0];

kTemporary[1] = X0[1] - K1[1] / 3 + K2[1];

sourceFunction(T + 2 \* h / 3, kTemporary, K3);

K3[0] \*= h;

K3[1] \*= h;

kTemporary[0] = X0[0] + K1[0] - K2[0] + K3[0];

kTemporary[1] = X0[1] + K1[1] - K2[1] + K3[1];

sourceFunction(T + h, kTemporary, K4);

K4[0] \*= h;

K4[1] \*= h;

Zn[0] = X0[0] + (K1[0] + 3 \* K2[0] + 3 \* K3[0] + K4[0]) / 8;

Zn[1] = X0[1] + (K1[1] + 3 \* K2[1] + 3 \* K3[1] + K4[1]) / 8;

X0[0] = Zn[0];

X0[1] = Zn[1];

**if** (!(i % 2)) {

printf(**"T = %.2f,\t X = (%.6f; %.6f)\n"**, T, Zn[0], Zn[1]);

}

T += h;

}

}

**void** sourceFunction(**double** T, **double** \*x, **double** \*dx) {

dx[0] = -310 \* x[0] - 3000 \* x[1] + 1

/ (10 \* pow(T, 2) + 1);

dx[1] = x[0] + exp(-2 \* T);

}

**QUANC8**

**void** quanc8(**double** (\*FUN)(**double**), **double** A, **double** B, **double** ABSERR,

**double** RELERR, **double** \*RESULT, **double** \*ERREST, **int** \*NOFUN, **double** \*FLAG)

{

**double** QRIGHT[32],F[17],X[17],FSAVE[9][31],XSAVE[9][31];

**int** LEVMIN,LEVMAX,LEVOUT,NOMAX,NOFIN,LEV,NIM,J,I;

**double** W0,W1,W2,W3,W4,COR11,AREA,X0,F0,STONE,STEP;

**double** QLEFT,QNOW,QDIFF,QPREV,TOLERR,ESTERR;

LEVMIN=1;

LEVMAX=30;

LEVOUT=6;

NOMAX=5000;

NOFIN=NOMAX-(8\*(LEVMAX-LEVOUT+pow(2,(**double**)(LEVOUT+1))));

W0=3956.0/14175.0;

W1=23552.0/14175.0;

W2=-3712.0/14175.0;

W3=41984.0/14175.0;

W4=-18160.0/14175.0;

\*FLAG=0.0;

\*RESULT=0.0;

COR11=0.0;

\*ERREST=0.0;

AREA=0.0;

\*NOFUN=0;

**if**(A == B)

**return**;

LEV=0;

NIM=1;

X0=A;

X[16]=B;

QPREV=0.0;

F0=FUN(X0);

STONE=(B-A)/16.0;

X[8]=(X0+X[16])/2.0;

X[4]=(X0+X[8])/2.0;

X[12]=(X[8]+X[16])/2.0;

X[2]=(X0+X[4])/2.0;

X[6]=(X[4]+X[8])/2.0;

X[10]=(X[8]+X[12])/2.0;

X[14]=(X[12]+X[16])/2.0;

**for** (J=2; J<=16; J=J+2){

F[J]=FUN(X[J]);

}

\*NOFUN=9;

trenta:

X[1]=(X0+X[2])/2.0;

F[1]=FUN(X[1]);

**for** (J=3; J<=15; J=J+2){

X[J]=(X[J-1]+X[J+1])/2.0;

F[J]=FUN(X[J]);

}

\*NOFUN=\*NOFUN+8;

STEP=(X[16]-X0)/16.0;

QLEFT=(W0\*(F0+F[8])+W1\*(F[1]+F[7])+W2\*(F[2]+F[6])+W3\*(F[3]+F[5])

+W4\*F[4])\*STEP;

QRIGHT[LEV+1]=(W0\*(F[8]+F[16])+W1\*(F[9]+F[15])+W2\*(F[10]+F[14])

+W3\*(F[11]+F[13])+W4\*F[12])\*STEP;

QNOW=QLEFT+QRIGHT[LEV+1];

QDIFF=QNOW-QPREV;

AREA=AREA+QDIFF;

ESTERR=fabs(QDIFF)/1023.0;

**if** (ABSERR > (RELERR\*fabs(AREA))\*(STEP/STONE))

TOLERR=ABSERR;

**else**

TOLERR=(RELERR\*fabs(AREA))\*(STEP/STONE);

**if** (LEV < LEVMIN)

**goto** cinquanta;

**if** (LEV >= LEVMAX)

**goto** sessantadue;

**if** (\*NOFUN > NOFIN)

**goto** sessanta;

**if** (ESTERR <= TOLERR)

**goto** settanta;

cinquanta:

NIM=2\*NIM;

LEV=LEV+1;

**for** (I=1; I<=8; I++) {

FSAVE[I][LEV]=F[I+8];

XSAVE[I][LEV]=X[I+8];

}

QPREV=QLEFT;

**for** (I=1; I<=8; I++) {

J=-I;

F[2\*J+18]=F[J+9];

X[2\*J+18]=X[J+9];

}

**goto** trenta;

sessanta:

NOFIN=2\*NOFIN;

LEVMAX=LEVOUT;

\*FLAG=\*FLAG+((B-X0)/(B-A));

**goto** settanta;

sessantadue:

\*FLAG=\*FLAG+1.0;

settanta:

\*RESULT=\*RESULT+QNOW;

\*ERREST=\*ERREST+ESTERR;

COR11=COR11+QDIFF/1023.0;

**while** (NIM%2 != 0) {

NIM=NIM/2;

LEV=LEV-1;

}

NIM=NIM+1;

**if** (LEV <= 0)

**goto** ottanta;

QPREV=QRIGHT[LEV];

X0=X[16];

F0=F[16];

**for** (I=1; I<=8; I++) {

F[2\*I]=FSAVE[I][LEV];

X[2\*I]=XSAVE[I][LEV];

}

**goto** trenta;

ottanta:

\*RESULT=\*RESULT+COR11;

**if** (\*ERREST == 0.0)

**return**;

**while** (fabs(\*RESULT)+(\*ERREST) == fabs(\*RESULT))

\*ERREST=2.0\*(\*ERREST);

**return**;

}