Problem

Let X and Y be jointly continous random variables such that X$\sim Unif(0,1)(that\ is\ , f_X(x) = 1\ for\ 0 < x < 1)$ and the conditional probability density function of Y given X is:

$$f_{Y|X}(y|x) = \sqrt{x} * e^{-\sqrt{x}*y}, y > 0.$$

Find $\mathbb{E}[\mathrm{Y}|\mathrm{X} = \mathrm{x}]$ for each $\mathrm{x} \in (0,1)$, and calculate $\mathbb{E}[\mathrm{Y}] + 9$.

Solution

We compute

$$\mathbb{E}[\mathrm{Y}|\mathrm{X} = \mathrm{x}] = \int_0^\infty y * f_{Y|X}(y|x)dy = \int_0^\infty y * \sqrt{x} * e^{-\sqrt{x}*y}\, dy = \frac{1}{\sqrt{x}}.$$

Then,

$$\mathbb{E}[\mathrm{Y}] = \int_0^1 \mathbb{E}[\mathrm{Y}|\mathrm{X} = \mathrm{x}] * f_X(x)\, dx = \int_0^1 \frac{1}{\sqrt{x}} dx = 2.$$

Thus,

$$\mathbb{E}[\mathrm{Y}] + 9 = 2 + 9 = \mathbf{11}$$

Answer: 11