

Problem

Consider a two-dimensional function, $f(x, y) = x^2 + y^2$. Determine the gradient of the function at point (2,3) and calculate the sum of the rates of change in the x and y directions at the same point.

Solution

The gradient of this function would be a vector containing the partial derivatives with respect to x and y.

$$\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

For our function $f(x, y)$ the partial derivatives are:

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

Therefore the gradient of $f(x, y)$:

$$\nabla f(x, y) = (2x, 2y)$$

This gradient vector tells us the rate of change of the function in the x direction and the y direction at any given point (x,y). Our point is (2,3):

$$\nabla f(2,3) = (4, 6)$$

This tells us that the rate of change of the function is 4 in the x direction and 6 in the y direction at the point (2,3). So, the sum requested is:

$$4 + 6 = 10$$

Answer: 10