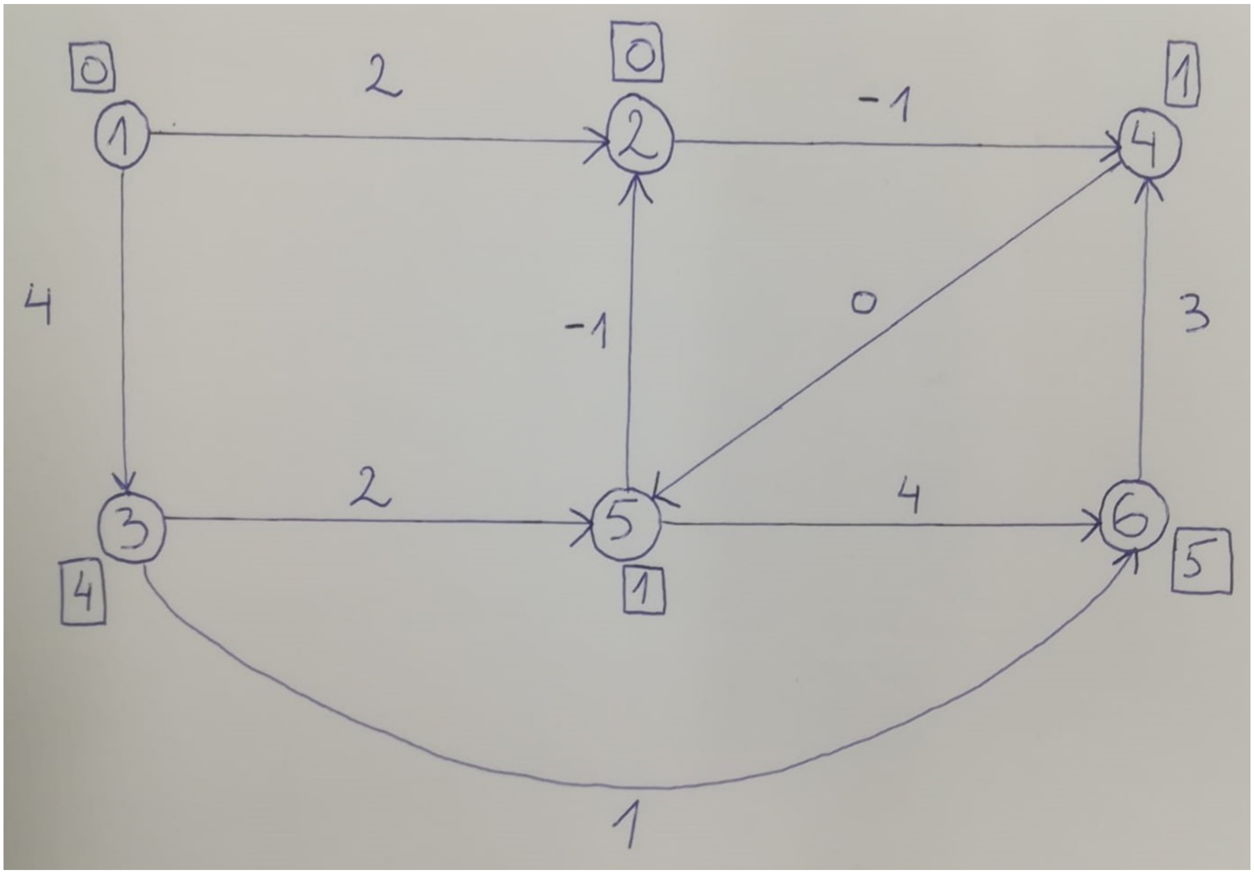
Problem

Given this map, calculate the minimum travel cost from point 1 to all points.



Solution

This involves using the Bellman-Ford Algorithm. In short, you want to calculate the cost associated with reaching one point given a chosen path and apply the same concept to all other points, but the catch is you cannot use one route, say 1→2, more than once.

The way to go about it is to first envision a box above each point where you will insert and update the cost associated with reaching that point. At first, assume that the values are infinity for all points. Since we start from 1, let's say that we take the route 1→2. At this point, you might want to make a list of all direct routes and start to cross the ones you used (e.g., the route 1→2 would now be crossed). The cost of taking this route from point 1 to 2 is 2. So now that 2 is less than infinity, you update the cost associated with reaching that point.

Moving on, let's say we take the route 2→4. We now take the cost of reaching point 2 and add to it the cost of taking the route leading to 4. The result is 1. Now update the box above point 4 with this cost of 1 associated with reaching point 4. Next, let's go to 5. We take route 4→5, add to the initial cost of reaching 4 the cost of reaching 5 by starting from 4. This is 1 + 0, and so the cost of reaching 5 is 1. You only care about the cost of reaching the point where you are plus the cost of traveling to another point. In the beginning, notice that I didn't add anything in addition to the route's cost when I started from 1 and went to 2. That is because 1 was the starting point, and so the cost of arriving at 1 is 0. By now you should know what to do to arrive at all points and accurately calculate the minimum cost of doing so. The caveat is that the costs in the box update. For example, let's resume from 5, where we were.

Let's now go to 2 by using route 5→2. We now calculate the cost of reaching 2, which is the cost of reaching 5, that is 1, plus the cost of the route, which is -1. This gives us 0. But wait a second, we already have a 2 in the box. Now what? Well, you update the value. Now it is 0 because there is such a route that I can take to point 2 that would cost me nothing. Now you are equipped with everything to complete the map. At the end, add all the values in the boxes above each point. That is the minimum cost of reaching all points.

Bear in mind that this is a simplified example and in order to be sure of reaching the most optimal value you have to repeat the process but by taking the routes in a different order. If a map has n points then the number of iterations needed to ensure optimal values is n-1.

In this case, all the iterations will give the same result of 11 so **11** is the optimal value.

Answer: 11