

Problem

Let $f(x, y): \mathbb{R} \rightarrow \mathbb{R}^2$. For some $c \in \mathbb{R}$:

$$f(x, y) = \begin{cases} ce^{-x^3y}, & \text{for } x > 1 \text{ and } y > 0 \\ 0, & \text{otherwise} \end{cases}$$

For what value of c is f a joint density function?

Solution

For f to be joint density, $c \geq 0$ so that $f \geq 0$ and

$$1 = \int_0^\infty \int_1^\infty ce^{-x^3y} dy dx = c \int_1^\infty \left. \frac{e^{-x^3y}}{-x^3} \right|_0^\infty dx = c \int_1^\infty \frac{1}{x^3} dx = c - \left. \frac{1}{2x^2} \right|_1^\infty = \frac{c}{2}$$

Hence $c = 2$

Answer: 2