Problem

Consider the matrix A= $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

Calculate the sum of A's rank and the dimension of A's null space.

Solution

To determine the rank of matrix A, we can use row reduction (Gaussian elimination) to put the matrix into row-echelon form.

Performing row operations:

$$R_2 \to R_2 - 2R_1$$

$$R_3 \to R_3 - 3R_1$$

Results in:

$$A \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now, the matrix is in row-echelon form. We see that the first row is non-zero, so the rank of matrix A is 1.

To find the dimension of the null space of A, we need to solve the homogeneous system of linear equations represented by the matrix equation Ax=0.

$$Ax=0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the row-echelon form of A, we have:

$$x_1 + 2x_2 + 3x_3 = 0$$

The system only has one independent equation. We can choose x_2=t and x_3=s, where t and s are parameters. Then, x_1=−2t−3s.

So, the general solution is:

$$x = t \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

The dimension of the null space (also known as the kernel) of A is two since there are two free parameters, t and s.

The rank of matrix A is 1.

The dimension of the null space of A is 2.

The sum of the two is 1+2 = **3**.

Answer: 3