Problem

A certain disease has an incidence rate of 0.7% (i.e., the probability a person is infected). A test is carried out to determine who has the disease, but it is not fully accurate. If the false negative rate (i.e., the probability a person tests negative but is infected) is 2.5% and the false positive rate (i.e., the probability a person tests positive but is not infected) is 1.5%, provide the tenths digit of the probability (%) that a person who tests positive is actually infected (i.e., the accuracy of the test).

Solution

This exercise requires Baye's Theorem which says that given 2 events A and B where $\bar{A}$ is not A:

$$P(A|B) = \frac{P(B|A)*P(A)}{P(A)*P(B|A) + P(\bar{A})*P(B|\bar{A})}$$

Adapted to our exercise we have:

$$P(\text{Infected}|\text{Positive}) = \frac{P(\text{Positive}|\text{Infected})*P(\text{Infected})}{P(\text{Infected})*P(\text{Positive}|\text{Infected}) + P(\text{Not infected})*P(\text{Positive}|\text{Not infected})}$$

We were given:

- P(Infected) = 0.7%
- P(Negative|Infected) = 2.5%
- P(Positive|Not Infected) = 1.5%

We calculate:

- P(Not infected) = 100% – P(Infected) = 100% – 0.7% = 99.3%
- P(Positive|Infected) = 100% – P(Negative|Infected) = 100% – 2.5% =97.5%
- P(Negative|Not infected) = 100% – P(Positive|Not Infected) = 100% – 1.5% = 98.5%

By plugging in the values in the Baye's Theorem we get:

$$P(\text{Infected}|\text{Positive}) = \frac{97.5\%*0.7\%}{0.7\%*97.5\%+99.3\%*1.5\%} = 0.31423 = 31.423\%$$

For this probability the tens number is **3**.

Answer: 3

Second solution

Here I provided a second solution as working with numbers might be better for some people to learn. Let's assume a population of 100000 people, although the number is irrelevant.

Let's create a table that will help solve the excercise:

	Positive test outcome	Negative test outcome	
Infected	682.5	17.5	700
Not infected	1489.5	97810.5	99300

	2172	97828	

The number of infected people is:

$$100000*P(Infected) = 100000*0.7\% = 700$$

The number of people not infected is:

$$100000-700 = 99300$$

The number of infected people who test negative is:

$$P(Negative|Infected)*700 = 2.5\%*700 = 17.5$$

Do not stumble because you can't have 17.5 people in reality. You only care about the corectness of your calculations.

The number of infected people who tested positive is:

$$700 - 17.5 = 682.5$$

The number of people who are not infected but test positive is:

$$P(Positive|Not\ infected)*99300 = 1.5\%*99300 = 1489.5$$

The number of people who are not infected and test negative is:

$$99300 - 1489.5 = 97810.5$$

We can now calculate how many people test positive and how many negative:

$$\text{Number of people who tested positive} = 682.5 + 1489.5 = 2172$$

$$\text{Number of people who tested negative} = 17.5 + 97810.5 = 97828$$

So the probability that an infected person tests positive is:

$$P(Infected|Positive) = \frac{\text{Number of infected people who tested positive}}{\text{Number of people who tested positive}} = \frac{682.5}{2172} = 0.31423 = 31.423\%$$

For this probability the tens number is **3**.

Answer: 3