Problem

A small bakery produces two types of cakes: chocolate cakes and vanilla cakes. Each chocolate cake requires 1 cup of flour, 1 cup of sugar, and 1 egg. Each vanilla cake requires 1 cup of flour, 2 cups of sugar, and 2 eggs. The bakery has 8 cups of flour, 9 cups of sugar, and 8 eggs available. A chocolate cake sells for $6, and a vanilla cake sells for $13. The bakery wants to maximize its profit. How many chocolate cakes will it make?

Solution

Let x = number of chocolate cakes

Let y = number of vanilla cakes

Objective function:

$$\max_{x,y} \mathrm{Z} = 6\mathrm{x} + 13\mathrm{y}$$

Constraints:

Flour constraint: $x+y \leq 8$

Sugar constraint: $x+2y \leq 9$

Egg constraint: $x+2y \leq 8$

Non-negativity constraint: $x \geq 0$, $y \geq 0$

The sugar and egg constraints are almost similar, except that one has an upper limit of 9, and the other has an upper limit of 8. Therefore, we only need to select the one with the lower upper limit, which is the one with the 8:

$$x+y \leq 8$$

$$x+2y \leq 8$$

Finding the corner points:

For constraint 1:

For x = 0, y = 8

For y = 0, x = 8

The points are (0, 8) & (8, 0)

For constraint 2:

For x = 0, y = 4

For y = 0, x = 8

The points are (0, 4) & (8, 0)

Now remember that we have to select the corner points for the feasible region:

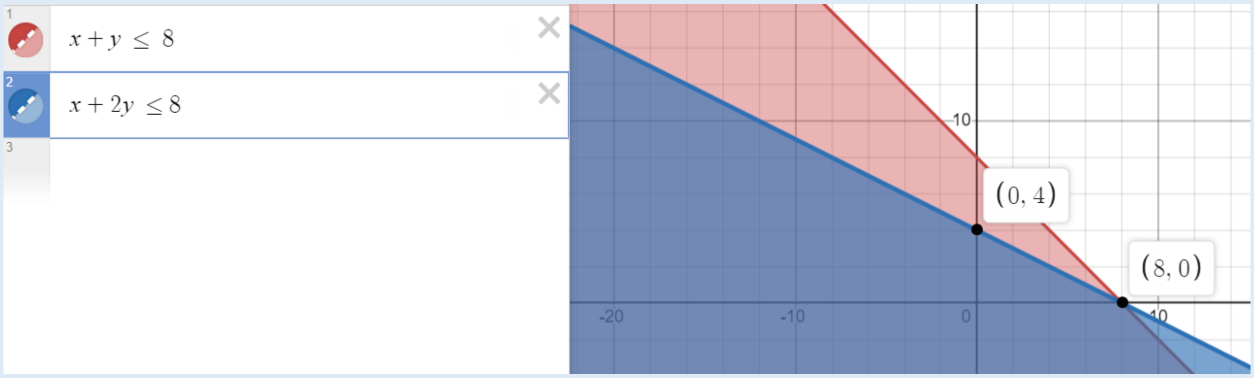

Figure 1. Feasible region (https://www.desmos.com/calculator)

And because x ≥ 0 and y ≥ 0, we are looking at quadrant 1.

We can tell the corner points are (0, 4) & (8, 0).

Hence:

$$Z_1 = 6 * 0 + 13 * 4 = 52$$

$$Z_2 = 6 * 8 + 13 * 0 = 48$$

$$Z_1 \geq Z_2$$

Therefore, the bakery will make 0 chocolate cakes and 4 vanilla cakes. So, the answer is **0**.

Answer: 0