

Space Complexity

(Not examinable)

Dr Kamal Bentahar

School of Computing, Electronics and Mathematics
Coventry University

Week 11 – 2/4/2019

We also want to measure the amount of **memory** used by computation.

Space complexity

The **space complexity** of a TM that always halts is the maximum number of tape cells $f(n)$ that a TM \mathcal{M} scans on any input of length n .

We say that \mathcal{M} “**runs in space** $f(n)$ ” if its space-complexity is $f(n)$.

Space-complexity classes: **PSPACE** and **NSPACE**

Let $f : \mathbb{N} \rightarrow \mathbb{R}^+$ be a function.

Definitions

SPACE($f(n)$) = $\{L \mid L \text{ is a language decided by an } O(f(n)) \text{ space DTM}\}$

NSPACE($f(n)$) = $\{L \mid L \text{ is a language decided by an } O(f(n)) \text{ space NDTM}\}$

- **DTM**: Deterministic Turing Machine.
- **NDTM**: Nondeterministic Turing Machine.

Savitch Theorem

On the face of it, simulating NDTMs using DTMs seems to require an exponential increase in time...

For **space complexity**, “Savitch theorem” states that an “NDTM that uses $f(n)$ space” can be converted to a “DTM that uses $O((f(n))^2)$ ” only!

This is surprising!

Savitch's Theorem

For any function $f : \mathbb{N} \rightarrow \mathbb{R}^+$, where $f(n) \geq n$,

$$\text{NSPACE}(f(n)) \subset \text{SPACE}(f^2(n))$$

PSPACE vs NPSPACE

Definitions

PSPACE: class of languages that are decidable in **polyspace** on a DTM

$$\mathbf{PSPACE} = \mathbf{SPACE}(1) \cup \mathbf{SPACE}(n) \cup \mathbf{SPACE}(n^2) \cup \dots$$

NPSPACE: class of languages that are decidable in **polyspace** on a NDTM

$$\mathbf{NPSPACE} = \mathbf{NSPACE}(1) \cup \mathbf{NSPACE}(n) \cup \mathbf{NSPACE}(n^2) \cup \dots$$

By Savitch theorem, we have the surprising result:

$$\mathbf{PSPACE} = \mathbf{NPSPACE}$$

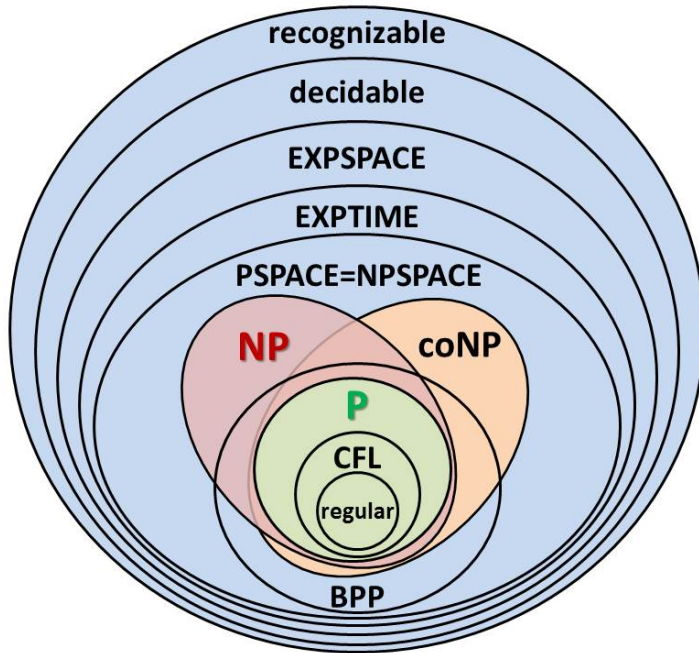
Space
Complexity

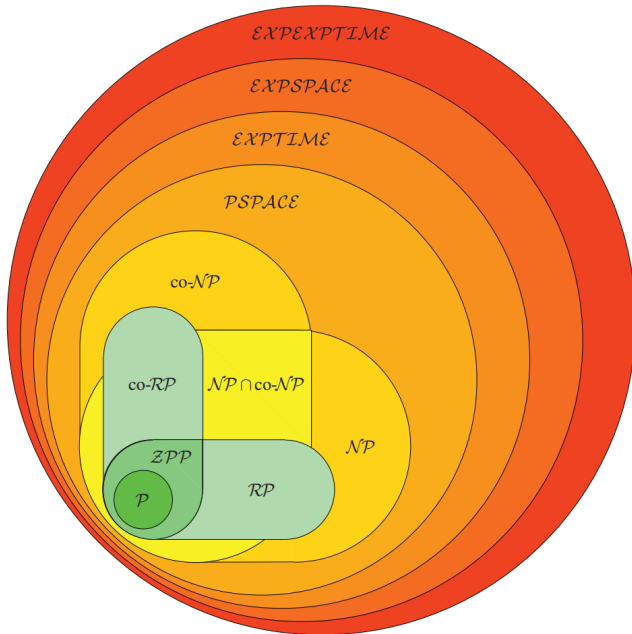
Space
complexity
PSPACE & NPSPACE

Savitch
Theorem

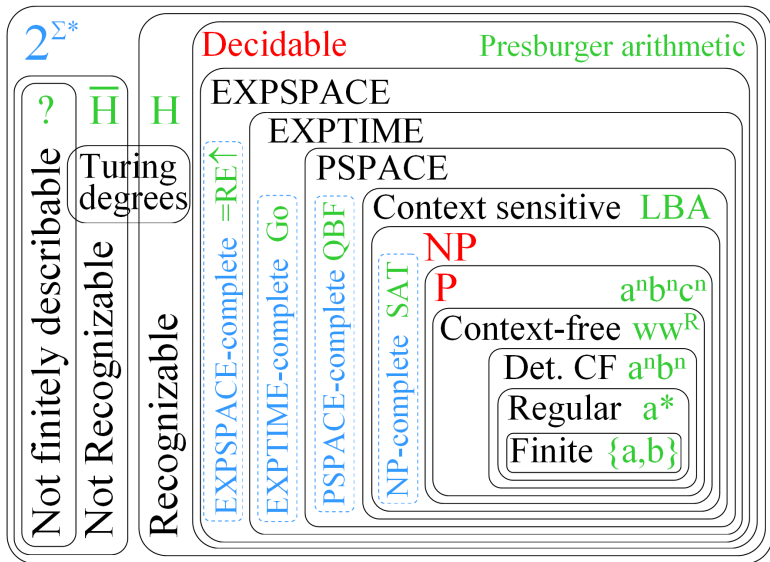
PSPACE vs
NPSPACE

Complexity
Union





The Extended Chomsky Hierarchy



Space Complexity

Space complexity
PSPACE & NSPACE

Savitch Theorem

PSPACE vs NSPACE

Complexity Union