

- (1) **(The Busy Beaver problem)** This is a very interesting and fun problem! Start by watching the following videos:

- <https://www.youtube.com/watch?v=DILF8usqp7M>
- <https://www.youtube.com/watch?v=CE8UhcYJSOI>
- <https://www.youtube.com/watch?v=ZiTeuZSDB0U>

You may also read <http://www.logique.jussieu.fr/~michel/tmi.html>

Can you produce the first few busy beavers? Compete with your friends!

- (2) Play with the TM simulator at <http://turingmaschine.klickagent.ch>

First, observe and try to understand how the multi-tape TMs work, then how the same operations are done on one tape.

Can you see how to design a TM that on input 1^n produces 1^{n^2} using 2 or 1 tape(s)?

Solution

For 1^{n^2} we run the multiplication TM to compute $n \times n$ (TM would run on on 1^n and 1^n).

- (3) Recall the language $\{0^{2^n} \mid n \geq 0\} = \{0, 00, 0000, 00000000, 0^{16}, 0^{32}, 0^{64}, \dots\}$ from the lecture. The language L consisting of all strings of 0's whose length is a power of 2.

The formal description of the presented TM that recognizes includes:

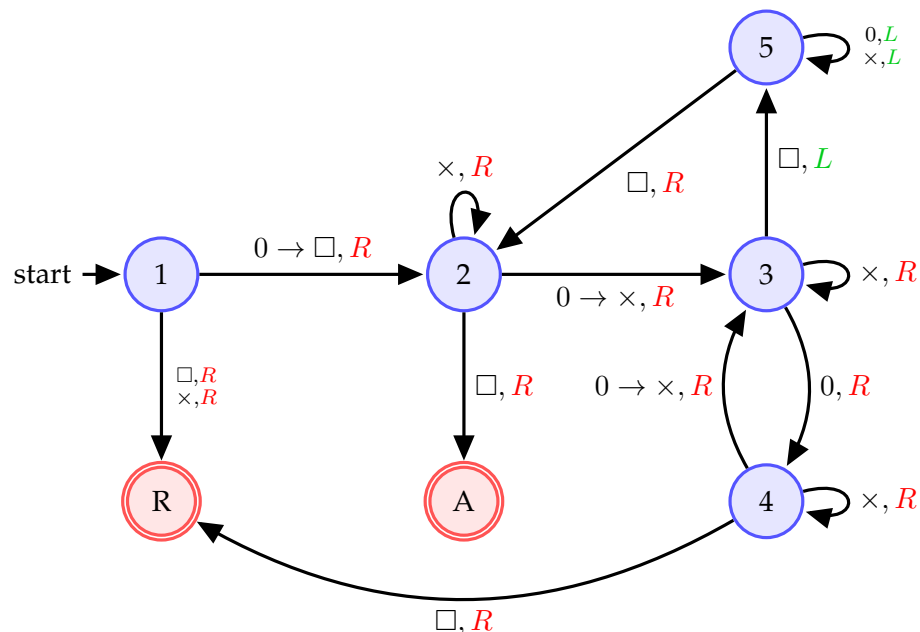
- $Q = \{1, 2, 3, 4, 5, A, R\}$
- $\Sigma = \{0\}$
- $\Gamma = \{0, \times, \square\}$
- The start, accept and reject states are 1, A and R, respectively.
- δ is given by the state diagram

Notation:

$a \rightarrow b, R$: on reading a on the tape replace it with b , then move right.

L : move left.

a, R : $a \rightarrow a, R$



Trace this TM on the following inputs:

$0, 0^2, 0^3, 0^4, 0^5, 0^6, 0^7, 0^8, 0^9$

Solution

For 0 we get the following sequence of *configurations*:

$$10\Box \rightarrow \Box 2\Box \rightarrow \Box\Box A$$

For 00 we get:

$$100\Box \rightarrow \Box 20\Box \rightarrow \Box \times 3\Box \rightarrow \Box 5 \times \Box \rightarrow 5\Box \times \Box \rightarrow \Box 2 \times \Box \rightarrow \Box \times 2\Box \rightarrow \Box \times \Box A$$

For the rest use JFLAP or the associated Python script, as the sequences are longer.

(4) You are given a TM where:

- $Q = \{q, p, q_{\text{accept}}, q_{\text{reject}}\}$
- $q_{\text{start}} = q$
- $\Sigma = \{0, 1\}$
- $\Gamma = \{0, 1, \Box\}$
- δ is given by the following table:

State	Tape symbol	Transition
q	0	$(q, 0, R)$
q	1	$(p, 0, R)$
q	\Box	(q, \Box, R)
p	0	$(q, 0, L)$
p	1	$(q_{\text{accept}}, 1, R)$
p	\Box	$(q, 0, L)$

For example (second row in the table), if the TM is in state q and the currently read symbol is 1 then the TM changes its state to state p , writes 0 (replaces 1 with 0) and then moves to the right.

- 1) Draw the state diagram of this TM.
- 2) Describe the property of an input string that makes this TM halt, i.e. go into the accept or reject states.
- 3) Identify a string that makes it halt from the list below.

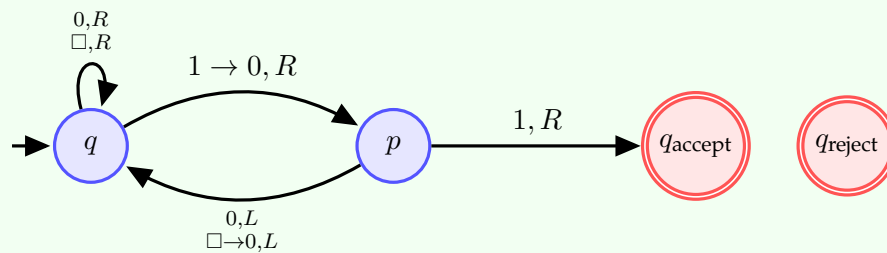
0000 0100 1010 0110

- 4) Simulate this TM on the input 1010110, and identify which one of the following configurations is valid.

00000000 $q\Box$ 00000 $p10$ 10 q 10110 000000 $p0$

Solution

1) State diagram



2) Accepts strings that contain 11.

If a string is not accepted it *loops* (goes on and on without stopping)

Rejects nothing!

3) 0110

4) 00000

10

(5) A non-deterministic TM with start state q_0 has the following transition function:

	0	1	□
q_0	$\{(q_1, 0, R)\}$	$\{(q_1, 0, R)\}$	$\{(q_1, 0, R)\}$
q_1	$\{(q_1, 1, R), (q_2, 0, L)\}$	$\{(q_1, 1, R), (q_2, 1, L)\}$	$\{(q_1, 1, R), (q_2, \square, L)\}$
q_2	$\{(q_{\text{accept}}, 0, R)\}$	$\{(q_2, 1, L)\}$	$\{(q_{\text{reject}}, \square, R)\}$

1) Draw the state diagram of this TM.

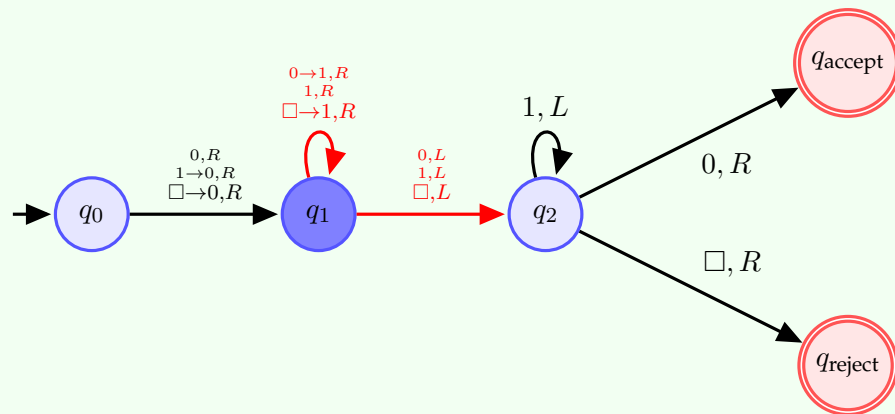
2) Simulate all sequences of 5 moves, starting from initial configuration q_01010 .3) Find, in the list below, one of the configurations reachable from the initial configuration in **exactly** 5 moves.

q_20110 $0q_{\text{accept}}110$ $011111q_1$ $0111q_21$

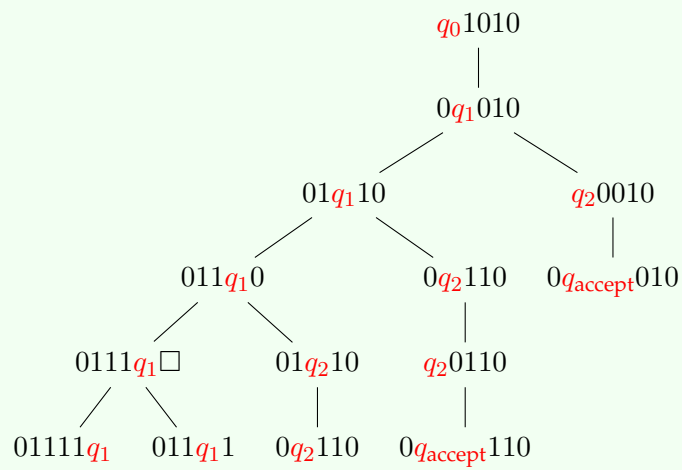
JFLAP does not handle non-deterministic TMs.

Solution

1) State diagram



2)



3) $0q_{\text{accept}}110$