Think about the following problems using pen-and-paper and "quick code experiments." Try to develop your solutions from theoretical analysis and experimental observations.

(1) Add the missing arithmetic operators $(+, -, \times, /)$ and parentheses to the following expression to make it true:

$$3 \quad 1 \quad 3 \quad 6 \quad = \quad 8.$$

(2) You have a 5 litre jug and a 3 litre jug, and an unlimited supply of water, but no measuring cups.

How would you come up with exactly 4 litre of water?

(3) Clever Bear walked one mile due south. Then he changed direction and walked one mile due east. Then he turned again to the left and walked one mile due north... he was surprised to find himself exactly at the point he started from!

What is the colour of Clever Bear?

(4) There are 100 closed lockers in a hallway. A man begins by opening all one hundred lockers. Next, he closes every second locker. Then he goes to every third locker and closes it if it is open or opens it if it is closed.

He continues like this until his 100^{th} pass in the hallway, in which he only changes the state of locker number 100.

How many lockers will be left open at the end?

(5) Little Alice has 10 pockets and £44 in £1 coins.

She wants to put her coins in her pockets so distributed that each pocket contains a different number of pounds. Can she do so?

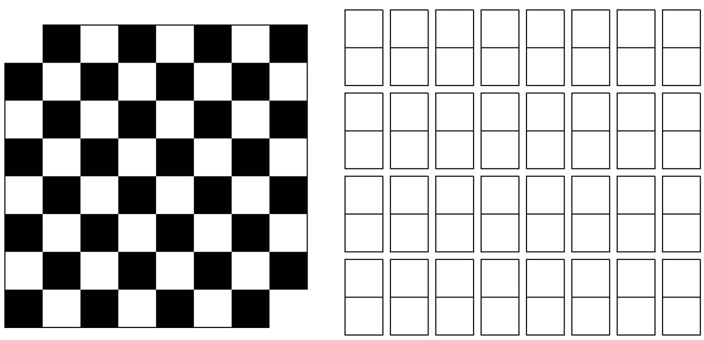
(6) We are next to a building of 100 floors. We have two identical eggs, and we are wondering what is the highest floor from which we can drop them without them breaking.

How do we find out using the minimal number of drops (in the worst case scenario).

How about if we had 3 eggs? 4 eggs? etc.

What is the optimal number of eggs to minimize the number of trials?

(7) Below is an 8×8 chess board in which two diagonally opposite corners have been cut off.



You are given plenty of dominoes, such that each domino can cover exactly two squares.

Can you cover the entire board with dominoes? (No dominoes are allowed to overlap or be partly outside the board.)

Can you *prove* your answer is correct? (Show an example solution if this is possible, or show that it is impossible.)

If there are any symbols or terminology you do not recognize then please let us know.

(1) Give the truth table for the following propositions

Expression	**Meaning**
$a \wedge b$	a and b
$a \vee b$	a or b
$a \oplus b$	a xor b
$\neg a$ (or $\bar{a}$)	not a
$a \implies b$	a implies b, or: if a then b
$a \iff b$	a and b are equivalent, or: "a if and only if b"

It is usual to apply these "bit-wise" to the bits of integers, e.g. $\texttt{0011} \oplus \texttt{0101} = \texttt{0110}$.

(2) Recall that $\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of **natural numbers**, and $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of **integers**.

Consider the following set definitions

- $A = \{a \in \{1, 2, 3, 4\} \mid (a < 2) \vee (a > 3)\}$
- $B = \{a \in \mathbb{N} \mid a < 9\}$
- $C = \{a \in \mathbb{N} \mid a > 2 \wedge a < 7\}$
- $D = \{i \in \mathbb{Z} \mid i^2 \leq 9\}$

a) Give an explicit enumeration for each set, i.e. write down the elements in the form $\{x_1, x_2, \ldots\}$.

b) What is the cardinality of each set?

c) Which of these sets are subsets of at least one other set?

(3) If the set A is $\{1, 3, 4\}$ and the set B is $\{3, 5\}$, write down:

Expression	**Meaning**
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B
$A - B$	A minus B
$A \times B$	Cartesian product of A and B: set of all possible pairs (a, b) where $a \in A$ and $b \in B$
2^B (or $\mathcal{P}(B)$)	power set of B: set of all subsets of B

(4) Draw the (undirected) graph $G = (V, E)$, where

$$\begin{aligned} V &= \{1, 2, 3, 4, 5\} \\ E &= \{(1, 2), (1, 4), (2, 3), (2, 4), (3, 5), (1, 5)\} \end{aligned}$$

a) Is the graph connected?

b) What about the graph $G' = (V', E')$, where $V' = \{1, 2, 3, 4\}$ and $E' = \{(1, 3), (2, 4)\}$?

(5) Draw the graph $G = (V, E)$, where $V = \{1, \ldots, 5\}$ and

$$E = \{(a, b) \mid a, b \in V \wedge (a < b < a + 3)\}.$$

(6) Express the following expressions using O-notation

- $x + 5$
- 2016
- $543x + x^3 + 13$
- $2784x + 132 \times 1074$
- $x^{578} + 4685 + 2^x$
- $x^2 + x(\log x)^2 + 35$
- $x + x \log^2 x + 35$
- $2016^x + x^x + x!$
- $x^{86754} + x!$