Тема 6 „Понятие о производной"

1. Найти производную выражения:

а) $\boxed{(\sin x \cdot \cos x)'} = (\sin x)' \cdot \cos x +$
$+ (\cos x)' \cdot \sin x = \cos^2 x - \sin^2 x = \boxed{\cos 2x}$.

в) $\boxed{\left(\ln (2x+1)^3\right)'} = \left(u(g(f(x)))\right)' =$
$= \frac{1}{(2x+1)^3} \cdot \left((2x+1)^3\right)'$

$\left((2x+1)^3\right)' = 3(2x+1)^2 \cdot (2x+1)'$

$(2x+1)' = 2$

$\left(\ln (2x+1)^3\right)' = \frac{2 \cdot 3 \cancel{(2x+1)^2}^{1} \cdot 1}{\cancel{(2x+1)^3}_{2x+1}} =$

$= \boxed{\frac{6}{2x+1}}$.

с) $\boxed{\left(\sqrt{\sin^2(\ln(x^3))}\right)'} = \left(s(h(u(g(f(x)))))\right)' =$
$= \frac{1}{2\sqrt{\sin^2(\ln(x^3))}} \cdot \left(\sin^2(\ln(x^3))\right)'$

$\left(\sin^2(\ln(x^3))\right)' = 2\sin(\ln(x^3)) \cdot$
$\cdot \left(\sin(\ln(x^3))\right)'$

$\left(\sin(\ln(x^3))\right)' = \cos(\ln(x^3)) \cdot (\ln(x^3))'$

$$(\ln(x^3))' = \frac{1}{x^3} \cdot (x^3)'$$

$$(x^3)' = 3x^2$$

$$\left(\sqrt{\sin^2(\ln(x^3))}\right)' = \frac{\cancel{2} \cdot 3\cancel{x^2} \cdot \cos(\ln(x^3)) \cdot \sin(\ln(x^3))}{\cancel{2} \cdot \sqrt{\sin^2(\ln(x^3))} \cdot \cancel{x^3}_{x}} =$$

$$= \frac{3 \cdot \cos(\ln(x^3)) \cdot \sin(\ln(x^3))}{x \sqrt{\sin^2(\ln(x^3))}} =$$

$$= \frac{3 \cdot \cos(\ln(x^3)) \cdot \sin(\ln(x^3)) \cdot \sqrt{\sin^2(\ln(x^3))}}{x \cdot \sin^2(\ln(x^3))} =$$

$$= \boxed{\frac{3 \cdot \cos(\ln(x^3)) \cdot \sqrt{\sin^2(\ln(x^3))}}{x \cdot \sin(\ln(x^3))}}$$

d) $$\boxed{\left(\frac{x^4}{\ln(x)}\right)'} = \frac{(x^4)' \cdot \ln(x) - (\ln(x))' \cdot x^4}{\ln^2(x)} =$$

$$= \frac{4x^3 \cdot \ln x - \frac{1}{x} \cdot x^4}{\ln^2(x)} = \frac{4x^3 \cdot \ln x - x^3}{\ln^2(x)} =$$

$$= \boxed{\frac{x^3(4\ln x - 1)}{\ln^2(x)}}$$

2. Найти выражение производной функции и её значение в точке:

$$f(x) = \cos(x^2 + 3x), \quad x_0 = \sqrt{\pi}$$

$$f'(\sqrt{\pi}) = (\cos(x^2 + 3x))' = -\sin(x^2+3x) \cdot$$

$$\cdot (x^2+3x)' = -\sin(x^2+3x)\cdot(2x+3)\Big|_{x=\sqrt{\pi}} =$$

$$= -\sin(\pi + 3\sqrt{\pi})\cdot(2\sqrt{\pi}+3) \approx -5{,}38$$

3. Найти значение производной функции в точке:

$$f(x) = \frac{x^3 - x^2 - x - 1}{1 + 2x + 3x^2 - 4x^3}, \quad x_0 = 0.$$

$$f'(0) = \left(\frac{x^3 - x^2 - x - 1}{1+2x+3x^2-4x^3}\right)' = \left(\frac{f}{g}\right)' =$$

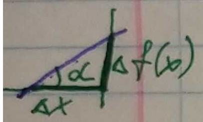
$$= \frac{f'g - g'f}{g^2} =$$

$$\boxed{\begin{aligned} f' &= (x^3 - x^2 - x - 1)' = 3x^2 - 2x - 1 \\ g' &= (1 + 2x + 3x^2 - 4x^3)' = 2 + 6x - 12x^2 \end{aligned}}$$

$$= \frac{(3x^2-2x-1)(1+2x+3x^2-4x^3) - (2+6x-12x^2)(x^3-x^2-x-1)}{(1+2x+3x^2-4x^3)^2}\Bigg|_{x=0} =$$

$$= \frac{-1\cdot 1 - 2\cdot(-1)}{1} = \frac{-1+2}{1} = 1.$$

4. Найти угол наклона касательной к графику функции в точке:



$$f(x) = \sqrt{3x} \cdot \ln x \, , \; x_0 = 1 .$$

$$tg\alpha = \frac{\Delta f(x)}{\Delta x} = f'(x_0) .$$

$$f'(1) = \left(\sqrt{3x} \cdot \ln x\right)' = \left(\sqrt{3x}\right)' \cdot \ln x + \left(\ln x\right)' \cdot \sqrt{3x} = \frac{\sqrt{3}}{2\sqrt{x}} \cdot \ln x + \frac{\sqrt{3x}}{x}\Bigg|_{x=1} =$$

$$= \frac{\sqrt{3}}{2\sqrt{1}} \cdot \underbrace{\ln 1}_{=0} + \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$f'(1) = \sqrt{3} \Rightarrow tg\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3} .$$