PROGRAMMING IN HASKELL

Chapter 15 – Lazy Evaluation

Introduction

Expressions in Haskell are evaluated using a simple technique called lazy evaluation, which:

- Avoids doing unnecessary evaluation;
- Ensures termination whenever possible;
- Supports programming with infinite lists;
- Allows programs to be more modular.

Evaluating Expressions

```
square n = n * n
```

Example:

```
    square (1+2)
=
    square 3
=
    3 * 3
=
    9
```

Apply + first.

Another evaluation order is also possible:

```
  square (1+2)
=
  (1+2) * (1+2)
=
  3 * (1+2)
=
  3 * 3
=
  9
```

Apply square first.

Any way of evaluating the <u>same</u> expression will give the <u>same</u> result, provided it terminates.

Evaluation Strategies

There are two main strategies for deciding which reducible expression (redex) to consider next:

- Choose a redex that is innermost, in the sense that does not contain another redex;

- Choose a redex that is outermost, in the sense that is not contained in another redex.

Termination

```
infinity = 1 + infinity
```

Example:

```
    fst (0, infinity)
=
    fst (0, 1 + infinity)
=
    fst (0, 1 + (1 + infinity))
=
    ⋮
```

Innermost evaluation.

```
    fst (0, infinity)
=
    0
```

Outermost evaluation.

Note:

- Outermost evaluation may give a result when innermost evaluation fails to terminate;

- If any evaluation sequence terminates, then so does outermost, with the same result.

Number of Reductions

Innermost:

```
  square (1+2)
=
  square 3
=
  3 * 3
=
  9
```

3 steps.

Outermost:

```
  square (1+2)
=
  (1+2) * (1+2)
=
  3 * (1+2)
=
  3 * 3
=
  9
```

4 steps.

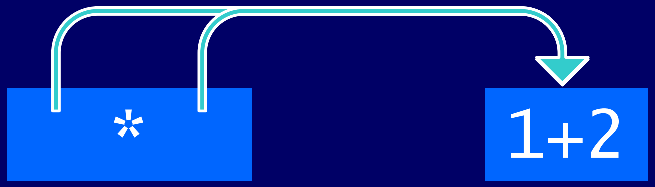
Note:

- The outmost version is inefficient, because the argument 1+2 is duplicated when square is applied and is hence evaluated twice.
- Due to such duplication, outermost evaluation may require more steps than innermost.
- This problem can easily be avoided by using pointers to indicate sharing of arguments.

Example:

```
square (1+2)
```

=



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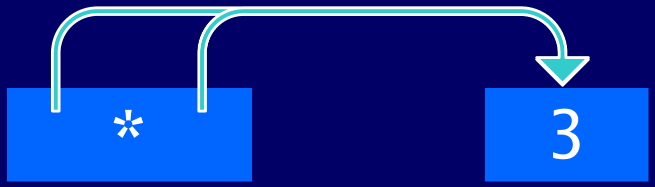


=

9

Shared argument evaluated once.

This gives a new evaluation strategy:

$$\boxed{\text{lazy evaluation}} = \boxed{\begin{array}{c}\text{outermost evaluation}\\ +\\ \text{sharing of arguments}\end{array}}$$

Note:

- Lazy evaluation ensures <u>termination</u> whenever possible, but <u>never</u> requires more steps than innermost evaluation and sometimes fewer.

Infinite Lists

```
ones = 1 : ones
```

Example:

```
  ones

= 1 : ones

= 1 : (1 : ones)

= 1 : (1 : (1 : ones))

= ⋮
```

An infinite list of ones.

What happens if we select the first element?

Innermost:

```
  head ones
=
  head (1:ones)
=
  head (1:(1:ones))
=
  ⋮
```

Does not terminate.

Lazy:

```
  head ones
=
  head (1:ones)
=
  1
```

Terminates in 2 steps!

Note:

- In the lazy case, only the <u>first</u> element of ones is produced, as the rest are not required.
- In general, with <u>lazy</u> evaluation expressions are only evaluated as <u>much as required</u> by the context in which they are used.
- Hence, ones is really a <u>potentially</u> infinite list.

Modular Programming

Lazy evaluation allows us to make programs more modular by separating control from data.

```
> take 5 ones
[1,1,1,1,1]
```

The data part ones is only evaluated as much as required by the control part take 5.

Without using lazy evaluation the control and data parts would need to be <u>combined</u> into one:

```
replicate :: Int → a → [a]
replicate 0 _ = []
replicate n x = x : replicate (n-1) x
```

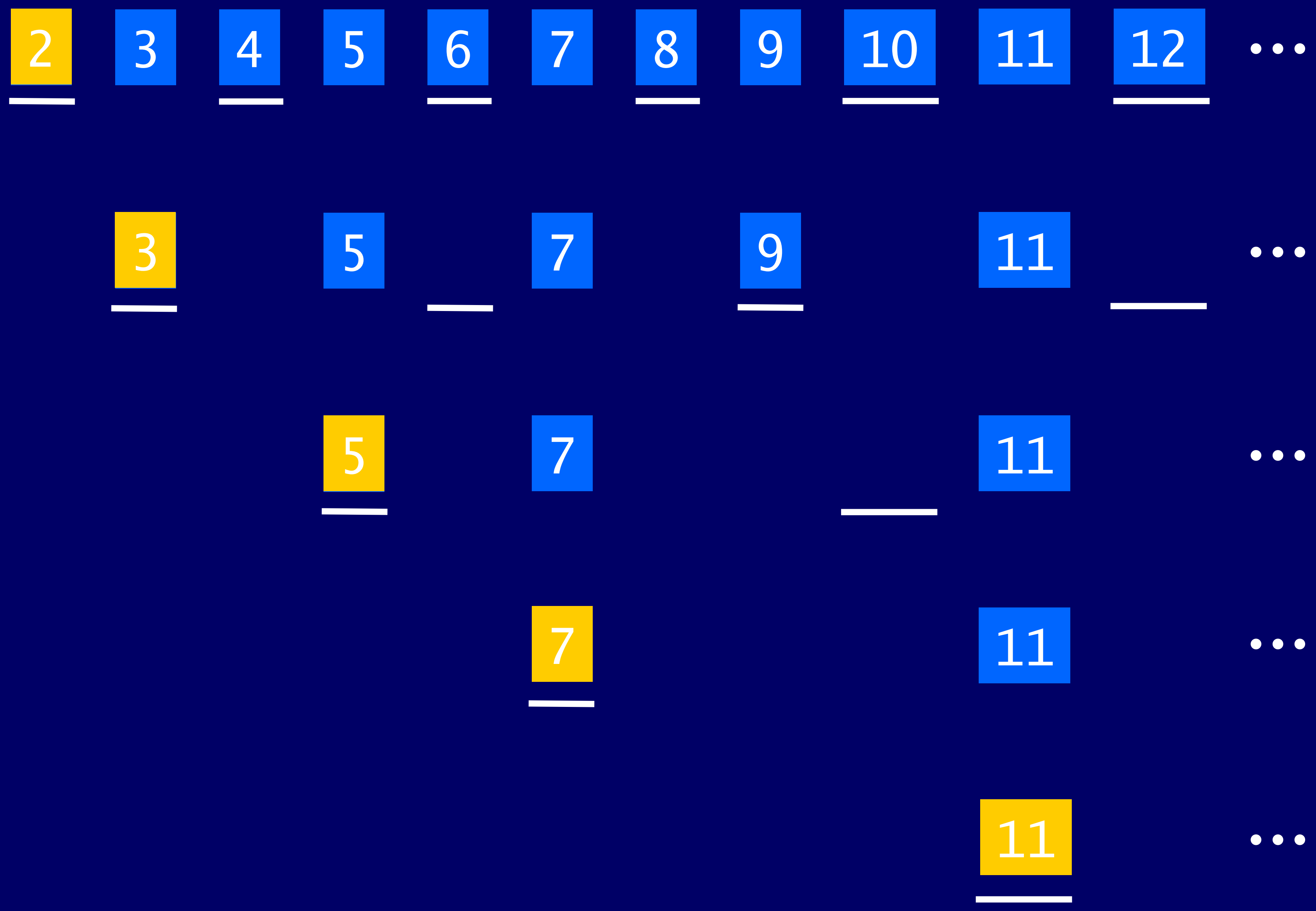
Example:

```
> replicate 5 1
[1,1,1,1,1]
```

Generating Primes

To generate the <u>infinite</u> sequence of primes:

1. Write down the infinite sequence 2, 3, 4, …;
2. Mark the first number p as being prime;
3. Delete all multiples of p from the sequence;
4. Return to the second step.



2 3 4 5 6 7 8 9 10 11 12 ...
3 5 7 9 11 ...
5 7 11 ...
7 11 ...
11 ...

This idea can be directly translated into a program that generates the infinite list of primes!

```
primes :: [Int]
primes = sieve [2..]
```

```
sieve :: [Int] → [Int]
sieve (p:xs) =
    p : sieve [x | x ← xs, mod x p /= 0]
```

Examples:

```
> primes
[2,3,5,7,11,13,17,19,23,29,31,37,41,43,…
```

```
> take 10 primes
[2,3,5,7,11,13,17,19,23,29]
```

```
> takeWhile (< 10) primes
[2,3,5,7]
```

We can also use primes to generate an (infinite?) list of twin primes that differ by precisely two.

```
twin :: (Int,Int) → Int
twin (x,y) = y == x+2
```

```
twins :: [(Int,Int)]
twins = filter twin (zip primes (tail primes))
```

```
> twins
[(3,5),(5,7),(11,13),(17,19),(29,31),…
```

Exercise

(1) The Fibonacci sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, …

starts with 0 and 1, with each further number being the sum of the previous two. Using a list comprehension, define an expression

```
fibs :: [Integer]
```

that generates this infinite sequence.