3(1). $y = \frac{x^2}{y(1+x^3)}$

$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

$dy = \frac{x^2}{y(1+x^3)}dx$

$y\,dy = \frac{x^2}{1+x^3}dx$

$\int \frac{x^2}{1+x^3}dx$

$\int \frac{1}{3t}dt$

$\frac{1}{3}\int \frac{1}{t}dt$

$\frac{1}{3}\int \frac{1}{1+x^2}dt$

$\frac{1}{3}ln\,(|1+x^3|)+c$

$\int y\,dy = \int \frac{x^2}{1+x^2}dx$

$\frac{y^2}{2}+c = \frac{1}{3}ln\,(|1+x^3|)+c$

$\frac{y^2}{2} = \frac{1}{3}ln\,(|1+x^3|)+c$

$y^2 = \frac{2}{3}ln\,(|1+x^3|)+c$

$y = \sqrt{\frac{2}{3}ln\,(|1+x^3|)}+c$

$dx = \frac{1}{t}dt$

$= \frac{1}{3x^2}dt$

$\int \frac{x^2}{1+x^3} + \frac{1}{3x^2}$

$\int \frac{1}{3(1+x^2)}$ $t = (1+x^2)$

4(2). $\frac{dy}{dx} = \frac{x+3y}{x-y}$

$(x-y)dy = (x+3y)dx \;\rightarrow\; (x-y)dy - (x+3y)dx = 0$

misal :

y = ux

dy = udx +udu

$(x-ux)(udx+udu) - (x+3ux)dx = 0$

$uxdx - uxdu - u^2xdx - u^2xdu - xdx - 3uxdx = 0$

$uxdx - u^2xdx - xdx - 3uxdx + uxdu - u^2xdu = 0$

$(ux - u^2xdx - 3ux)dx + (ux - u^2xdu)du = 0$

$x(u - u^2 - 1 - 3u)dx + x(u - u^2)du = 0$

$(-u^2 - 2u - 1)dx + (u - u^2)du = 0$

$(-u^2 - 2u - 1)dx(\frac{1}{(-u^2-2u-1)}) + (u - u^2)du(\frac{1}{(-u^2-2u-1)}) = 0(\frac{1}{(-u^2-2u-1)})$

$dx + \frac{u-u^2}{(-u^2-2u-1)}du = 0$

$\int \frac{u^2+u}{u^2+2+1}du$

$\int \frac{u^2-u+3u+1-(-3u+1)}{u^2+2+1}du$

$\int \frac{u^2-u+3u+1}{u^2+2+1} + \frac{3u+1}{u^2+2+1}du$

$\int \frac{u^2+2u+1}{u^2+2+1} + \frac{3u+1}{u^2+2+1}du$

$\frac{3x+1}{x^2+2x+1}$

$\frac{3x+1}{(x+1)^2}$

$\frac{a}{(x+1)} + \frac{b}{(x+1)^2}$

$\frac{3x+1}{(x+1)^2} \times (x+1)^2 = \frac{a}{(x+1)} + \frac{b}{(x+1)^2} \times (x+1)^2$

$\int 1 + \frac{3u+1}{u^2+2+1} du$

$u + \int \frac{3u+1}{u^2+2+1} du$

$u + \int \frac{3}{u+1} - \frac{2}{(u^2+1)^2} du$

$u - \int \frac{3}{u+1} du - \int \frac{2}{(u+1)^2} du$

$u - 3ln\,(|u+1|) - \frac{2}{u+1} + c$

$3x + 1 = (x+1)a + b$

$3x + 1 = Ax + (A+B)$

$A = 3\,, \ B = -2$

$\frac{3}{x+1} + \frac{-2}{(x+1)^2}$

1(3). $y' + 2y = e^{-x}$

$g(x) = 2$

$h(x) = e^{-x}$

$u(x) = e^{\int 2dx}$

$= e^{2x}$

$h(x) = e^{-x}$

$y = \frac{1}{e^{2x}} \int e^{-x} \cdot e^{2x} dx$

$y = \frac{1}{e^{2x}} \int e^x dx$

$y = \frac{1}{e^{2x}} (e^x + c)$

$y = \frac{e^x}{e^{2x}} + \frac{c}{e^{2x}}$

$y = \frac{1}{e^x} + \frac{c}{e^{2x}}$