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PhD topic proposal
Generating and teleporting entanglement for quantum networks

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1 Description of the immediate research area and its problems

In the rapidly advancing fields of quantum communication and quantum computing, the efficient transfer of secure quantum information is of great importance. A key quantum resource is entanglement, which facilitates processes such as quantum teleportation and entanglement swapping. However, conducting these experiments over long distances through optical fibers presents significant challenges due to transmission losses. To mitigate this, photons must be generated at wavelengths compatible with existing fiber-optic networks, particularly in the O and C near-infrared bands (1310 nm and 1550 nm, respectively), where transmission losses are minimal.

These advances contribute to the broader goal of realizing quantum networks, which require robust capabilities for generating and characterizing quantum coherence and entanglement. Such networks rely on quantum interconnects, which convert quantum states between physical systems in a reversible manner, enabling the distribution of entanglement and the teleportation of quantum states across network nodes [1]. Additionally, free-space communication methods are being explored for applications in metropolitan areas [2]. To maximize the efficiency of these telecom networks for multiple users, the available bandwidth is divided into many frequency channels using Dense Wavelength Division Multiplexing (DWDM).

The focus of this thesis is to implement a Sagnac interferometer source of polarization-entangled photons centered around 1560 nm, designed to be sufficiently broadband to accommodate multiple DWDM frequency channels. These photons will be generated via Spontaneous Parametric Down-Conversion [3] (SPDC) from a 50 mm periodically poled lithium niobate (PPLN) nonlinear crystal located at the center of the interferometer. The source will first be implemented and characterized in the laboratory through techniques such as quantum state tomography, Clauser-Horne-Shimony-Holt (CHSH) inequality measurements [4], Hanbury-Brown and Twiss (HBT) interferometry [5], and Hong-Ou-Mandel (HOM) interferometry [6]. Subsequently, quantum teleportation [7] and entanglement swapping [8] experiments will be conducted in collaboration with the Jožef Stefan Institute, which will develop an identical entanglement source with our assistance. To ensure stability, the pump laser wavelength will be locked to an absorption line in Rubidium gas using atomic spectroscopy via the $^{87}Rb\ D_2$ transition [9]. These efforts are foundational for the development of quantum repeaters, a critical component for the future global quantum internet. Quantum repeaters may enable the distribution of entanglement over arbitrarily long distances by overcoming exponential loss scaling, even in low-loss fiber networks where attenuation is approximately 0.18 dB/km. Without such repeaters, entanglement distribution is limited to distances of only a few hundred kilometers. This thesis aims to achieve not only the first realization of a high-yield entanglement source at non-degenerate frequencies in Slovenia but also to demonstrate quantum teleportation and entanglement swapping. The entanglement will be generated by bi-directionally pumping the PPLN crystal in the center of the Sagnac interferometer, introducing photon indistinguishability essential for these protocols.

Key words: Quantum Entanglement, Quantum Key Distribution, Entanglement Swapping

2 Overview of related research and relevant literature

Sebastian Neumann paper [**?**],

First go over how to use SPDC for entanglement generation, advantages, disadvantages, types, ...

Progress up to 2000s, 2010s, 2020, 2024, present.

What is the work based on? Neumanns paper, introduce Bennik and Boyd for parameters,.... Introduce HBT, HOM, Teleportation, Entanglement Swapping, Zeillinger, Rainer, others,

How to then generate entanglement

How to stabilize the 780 laser to the Rubidium Gas Cell, some references for locking and such which we plan to use

3 Research goals

We will use a Continous Wave (CW) 780 nm Toptica DLPro laser to pump a 50 mm nonlinear PPLN crystal which is phase matched for Type-0 SPDC (e $\rightarrow$ e + e, e meaning extraordinary polarization) for generating entangled photons.

Goal 0 - Build a Sagnac source of entanglement and reach the currently know state of the art in performance metrics such as brightness and heralding.

Goal 1 - Once we have a working source, we need to characterize it using Quantum State Tomography (QST), CHSH, HBT, and HOMI for different pairs of DWDM channels (references).

Goal 2 - might need to introduce extra filtering – 100 GHz not enough -> Filtering cavities Long-term stability, locking, ...? Demonstrate Quantum Teleportation within the lab, and Entanglement swapping (we will use the same Φ^+ or Φ^- Bell state, then perform a Bell measurement on each part of the pair) with IJS, IJS Reactor, Beyond Semiconductor, where to do long distance stuff (need fibers) other distant parties,

Goal 3 - Free space application using OAM modes, try to do entanglement in this regime?? topological photons

Why use CW instead of Pulsed Wave (PW)? Advantages of Continuous Wave compared to Pulsed Laser entanglement sources:

4 Outline of research and research methods

An existing Dark Fiber (DF) will be used for network testing once all of the local tests have been made, including QST, HBT, and HOMI. The DFs location is currently undisclosed.

5 Expected results and original contributions to science

6 Contents and introduction

Today I will be speaking to you about a part of my thesis, which is generating and teleporting entanglement for quantum networks. I will begin with a bit of theory, where I will explain the basics of the Spontaneous Parametric Downconversion (SPDC). Here I will speak a bit about Phase Matching and how we try to optimize it in the lab. I will also speak a bit about how we plan on Distributing Entanglement, so Quantum Teleportation and Entanglement Swapping. Then I will speak about the present state that we are dealing with in the lab and how far we have gone so far.

7 Motivation

My project is in the scope of the SiQUID project. The goal of it is to demonstrate that a slovenian quantum network is possible to make, train young researchers in the field of Quantum Technologies, as well as creating the sources of entanglement needed for entanglement based Quantum Key Distribution. The main goal for me in this regard would be to create a bright source of polarization entanglemed photon pairs using a Sagnac interferometer. The reason we chose a Sagnac in this case is due to its vibrational stability, and it's a convenient way to do so as it allows for a compact design. A possible downside to this is that the alignment of this type of interferometer is a bit tedious, as you have to perfectly overlap two beams within the interferometer, and in the crystal, and there is coupling between the various degrees of freedom that we have available to us.

8 Theory

8.1 Spontaneous Parametric Downconversion

Before talking about entanglement, we need to know how to generate photon pairs. A good way to do this is SPDC. SPDC is a process in which a pump photon of frequency ω_p gets "downconverted" to two photons of lower frequency, usually denoted as ω_s and ω_i. These output photons can have the same frequency, and thus this can be a degenerate process, or they can have different frequencies, being non-degenerate. When we started this entire design process, we wanted to have degenerate frequencies, as we would be able to separate the two photons easily based on their polarization. This design utilited a type-II process, meaning that we would be able to separate the two photons on the output of the PBS. Now we have decided to go for a type-0 process, thus altering the design a bit.

8.2 Phase Matching

What is Phase Matching? Phase matching refferrs to fixing the relative phase between two or more frequencies of light as they propagate through the crystal. Phase matching are the conditions at which some process will happen. In the case of SPDC the Phase Matching conditions are energy conservation ($\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$), and momentum conservation ($k_p = k_s + k_i$). The efficiency of this process is also dictated by how well we satisfy these two conditions - being slightly off from $\Delta k = 0$ could result in large losses of photon generation efficiency.

8.3 Crystals

An important step is to choose the correct material for the job. In this case, no ordinary material can be used, as the phase matching condition $k_1 + k_2 = k_3 = \frac{n\omega_3}{c} = \frac{n\omega_1}{c} + \frac{n\omega_2}{c}$ is impossible to satisfy in most cases as $n_1(\omega_1) < n_2(\omega_2) < n_3(\omega_3)$ for $\omega_1 < \omega_2 < \omega_3$. For this, we choose a birefringent material, which has a fast and a slow axis.

Crystal Size Before periodic poling you could only have a limited size of the nonlinear material along the propagation direction. This means that you would only be able to generate broad signals out of the crystals and at lower intensities, as the intensity and bandwidth of the process goes proportionally to $I \propto \mathrm{sinc}^2\left(\frac{L\Delta k}{2}\right)$. Today we have periodic poling available to us which allows us to create oppositely oriented fields inside the crystal. This allows for higher photon generation rates, longer crystals and more narrow bandwidths.

8.4 Types of processes

There are three different types of phase matching.

- Type-0 is one in which a photon of some linear polarization, say $|H\rangle$ gets downconverted into two photons of lower energies of the same polarization - o $\rightarrow$ o + o. This process used to be physically impossible to produce as the phase matching condition $k_1 + k_2 = k_3 = \frac{n\omega_3}{c} = \frac{n\omega_1}{c} + \frac{n\omega_2}{c}$ is impossible to achieve in most cases.
- Type-I is similar to type-0, except that the produced two photons are orthogonally polarized to the input photon - o $\rightarrow$ e + e
- In type-II instead you get two orthogonal polarizations out from the crystal. e $\rightarrow$ e + o

8.5 Phase Matching Temperature

The first important step is to determine the correct Phase Matching Temperature for our crystal. For different types of poling periods these temperatures will vary, and in general, the lower the poling period the higher the Phase Matching Temperature. The opposite seems to be true for the temperature bandwidth, which would be the acceptable temperature range for the material. Similarly, one should not heat up certain crystals too much as this could destroy the Periodic Poling properties of the material. For Periodically Poled Lithium Niobate (PPLN), the safe operating regime up to around 200 °C.

8.6 Bandwidths

Next would be the bandwidth of the process. In general, you would like it to be as narrow as possible - especially for certain applications as entanglement swapping, and there are other methods to achieve this which I won't go into now. In general, type-II processes are much more narrow than type-0 ones. In the plot I have on the slide, there's about a factor of 120 in bandwidth difference. This is why Type-II was the primary choice for our group. Unfortunately, this also comes at the price of reduced intensity. The difference in intensity was around 25 times lower compared to the type-0 crystal which led to us using it in the end design.

8.7 Different Designs

First figure Schematic of one method to produce and select the polarization-entangled state from the down-conversion crystal. The extra birefringent crystals Cl and C2, along with the half wave plate HWPO, are used to compensate the birefringent walk-off effects from the production crystal. By appropriately setting half wave plate HWP1 and quarter wave plate QWP1, one can produce all four of the orthogonal EPR-Bell states. Each polarizer P1 and P2 consisted of two stacked polarizing beam splitters preceded by a rotatable half wave plate.

Interesting thing is that here with the use of various compensating crystals and wave plates they can create any Bell state.

2nd figure Similar to 1, but doesn't require realignment, also much brighter.

3rd figure Shows a use of noncollinear phase matching at room temperature (not really, it's 32 °C), robust source of high brightness

4th figure They present a source of polarization-entangled photon pairs based on time-reversed Hong-Ou-Mandel interference. By superimposing four pair-creation possibilities on a polarization beam splitter, pairs of identical photons are separated into two spatial modes without the usual requirement for wavelength distinguishability or noncollinear emission angles. Our source yields high-fidelity polarization entanglement and high pair-generation rates without any requirement for active interferometric stabilization, which makes it an ideal candidate for a variety of applications, in particular those requiring indistinguishable photons.

9 Entanglement

If we have two systems, $|\psi_i\rangle$ and $|\phi_i\rangle$ described by two separate Hilbert spaces, the state of those two systems can be described as a tensor product $|\psi_i\rangle \otimes |\phi_i\rangle$ of their state spaces. You can write this as a Schmidt decomposition: $|\xi\rangle = \sum_{i,j} a_{ij} |\psi_i\rangle |phi_i\rangle \to \sum_i b_i |\psi'_i\rangle |\phi'_i\rangle$. If $b_i \neq 0$ and $b_{i\neq j} = 0$ the state is said to be separable. If more than one $b_i \neq 0$ then $|\xi\rangle$ is said to be entangled, and the states can no longer be described without their comprising states. An example of entangled states are Bell states, they are also maximally entangled:

$$
\begin{aligned}
|\Psi^-\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|V\rangle - |V\rangle|H\rangle) \\
|\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|V\rangle + |V\rangle|H\rangle) \\
|\Phi^-\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|H\rangle - |V\rangle|V\rangle) \\
|\Phi^+\rangle &= \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle) \quad (9.1)
\end{aligned}
$$

9.1 Why do we care about entanglement?

Entanglement sources have many applications. Some notable ones are Quantum Computation, Quantum Imaging, and Quantum Sensing (to be explained). As we wish to distribute these entangled states over long distances, due to fiber losses it isn't viable to do so over distances larger than a couple hundred kilometers.

Table 1: Relevant fiber loss. *Source: Thorlabs*

λ [nm]	430	532	780	1310	1550	1900
Loss [dB/km]	50	30	12	0.32	0.18	5

Example: Loss in fiber for 1550/1560 nm 200 km of fiber $\rightarrow$ -36 dB $\rightarrow$ 10^4 loss. Start with 1 W, end up with 0.0001 W. It would be desirable to have a more robust way to transport photons from A to B. In our case, we generate a pair of $|V\rangle$ polarized photons in each of the branches of the Sagnac interferometer, or in the case of type-II we get $|H\rangle + |V\rangle$, totalling to 4 photons being created "at the same time". This leads us to the entangled state for type-II/0 SPDC:

$$|\Psi_p\rangle = \frac{1}{\sqrt{2}}(a_H^\dagger(\omega_p) + a_V^\dagger(\omega_p))|0\rangle$$

$$|\Psi_{\text{Type-2}}\rangle = \frac{1}{\sqrt{2}}(a_H^\dagger(\omega_s)a_V^\dagger(\omega_i) + a_V^\dagger(\omega_i)a_H^\dagger(\omega_s))|0\rangle \qquad |\Psi_{\text{Type-0}}\rangle = \frac{1}{\sqrt{2}}(a_H^\dagger(\omega_s)a_H^\dagger(\omega_i) + a_V^\dagger(\omega_i)a_V^\dagger(\omega_s))|0\rangle$$

as one branch of the phons will be rotated in polarization due to a wave plate being in one of the branches of the Sagnac interferometer. This leads us to one of the Bell States mentioned above. The reason why this is important is that if the conditions for this to happen are satisfied, you can no longer predict the result of the polarization measurement. You can no longer tell which photon is which and from where it came from. But the measurements will be perfectly anti-correlated in polarization.

9.2 Distributing Entanglement

Due to condisderable losses in fibers it is not feasible to transport entangled photons over distances greater than a few hundred kilometers. Thus we must find a different solution to long distance Entanglement Dirstribution. This could be solved with quantum repeater, but currently one does not exist, or is difficult to create. That being said, it is reasonable to establish an entanglement swapping network beforehand.

9.3 Quantum Teleportation

The basis of Quantum Teleportation is that the sender and receiver share an initial entangled pair - in our case polarization entangled photons of the sort described above, in one of the 4 Bell States so that the photons are just as likely to be horizontally or vertically polarized relative to the pump, then perform a Bell State Measurement (BSM) on the receiving entangled photon and an initial state which we would like to teleport to the receiver. By entangling the initial state photon with one of the other entangled photons, and measuring the type of entanglement that they share, we can then send that information to B through a classical signal change B so that it has the same polarization as X, all without ever knowing their polarizations.

BSM A BSM is a coincidence measurement between different detectors. The simplest one would be the $|\Psi^{-}\rangle$ of 9 measurement. In it you only have a 50:50 beam splitter and two detectors. The only time both photons will fall on their own separate detector is when either both of them get transmitted, or both get reflected. This can be improved and the 2nd state can be measured if a PBS is added one of the branches after the BS. Thus we'd have a 50% complete BSM. By adding another PBS in the other branch we can increase the completeness to 75%. In order to reach a complete BSM, one must use nonlinear elements[‡].

Then we report the result of the BSM via a classical communications channel to the receiver. This same procedure does not work if the entangled pair does not exist. The amazing thing is that this should work regardless of the distance between the entangled states, and doesn't vialate causality, as the cassical message is being sent at most at the speed of causality to the receiver.

It is also important to notice that the Bell-state measurement does not reveal any information on the properties of any of the particles. This is the very reason why quantum teleportation using coherent two-particle superpositions works, while any measurement on one-particle superpositions would fail. The fact that no information whatsoever is gained on either particle is also the reason why quantum teleportation escapes the verdict of the no-cloning theorem. After successful teleportation particle 1 is not available in its original state any more, and therefore particle 3 is not a clone but is really the result of teleportation.

[‡]PhysRevLett.86.1370

9.4 Entanglement Swapping

The principle behind Entanglement swapping can be described in the exact same way as Quantum Teleportation. The only difference is now that the arbitrary state which we're trying to teleport is part of an entangled pair. This means that if the same process of Quantum Teleportation is performed by Alice, Bob, Charlie and Dora, where Alice and Bob share one entangled pair, and Charlie and Dora share another. Now the teleported state will be the state of one of the entangled photons, thus swapping the entanglement between Charlie and Bob.

10 Present State

10.1 Parameters

10.2 Building a Sagnac interferometer

References

[1] H. J. Kimble, "The quantum internet," *Nature*, vol. 453, no. 7198, p. 1023–1030, Jun. 2008.

[2] A. Kržič, S. Sharma, C. Spiess, U. Chandrashekara, S. Töpfer, G. Sauer, L. J. González-Martín del Campo, T. Kopf, S. Petscharnig, T. Grafenauer, R. Lieger, B. Ömer, C. Pacher, R. Berlich, T. Peschel, C. Damm, S. Risse, M. Goy, D. Rieländer, A. Tünnermann, and F. Steinlechner, "Towards metropolitan free-space quantum networks," *npj Quantum Information*, vol. 9, no. 1, p. 1–9, Sep. 2023.

[3] J. Catalano, "Spontaneous parametric down-conversion and quantum entanglement," jul 2014, supervisor: Andres La Rosa. [Online]. Available: https://pdxscholar.library.pdx.edu/cgi/viewcontent.cgi?article=1588&context=honorstheses

[4] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, "Proposed experiment to test local hidden-variable theories," *Physical Review Letters*, vol. 23, no. 15, p. 880–884, Oct. 1969.

[5] R. H. Brown and R. Twiss, "Lxxiv. a new type of interferometer for use in radio astronomy," *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, vol. 45, no. 366, p. 663–682, Jul. 1954.

[6] C. K. Hong, Z. Y. Ou, and L. Mandel, "Measurement of subpicosecond time intervals between two photons by interference," *Physical Review Letters*, vol. 59, no. 18, p. 2044–2046, Nov. 1987.

[7] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," *Nature*, vol. 390, no. 6660, p. 575–579, Dec. 1997.

[8] T. Jennewein, G. Weihs, J.-W. Pan, and A. Zeilinger, "Experimental nonlocality proof of quantum teleportation and entanglement swapping," *Physical Review Letters*, vol. 88, no. 1, p. 017903, Dec. 2001.

[9] T. Metger, "Saturated absorption spectroscopy of molecular iodine for laser locking," jul 2017, supervisor: Christoph Fischer. [Online]. Available: https://ethz.ch/content/dam/ethz/special-interest/phys/quantum-electronics/tiqi-dam/documents/semester_theses/semesterthesis-Tony_Metger