Домашня контрольна
"Процес Пуассона та елементи страхової математики"
Семенов Богуслав КА-85

Дано:

Процес страхового ризику $U(t)=U_0+ct-\sum_{i=1}^{N(t)} X_i, t\geq 0$

Розподіл страхових виплат: $X_i \sim 0.5\,\delta_1+0.5\,\delta_{10}$

Для випадку 1: $U_0=1, \lambda=1$

Для випадку 2: $U_0=10, \lambda=5$

Попередні розрахунки

$$EX=E(0.5\,\delta_1+0.5\,\delta_{10})=0.5\,E\,\delta_1+0.5\,E\,\delta_{10}=0.5*1+0.5*10=5.5$$

$$F_X(x)=\begin{cases} 0, & x\leq 1\\ 0.5, & 1<x\leq 10\\ 1, & x>10\end{cases}$$

$$\bar{F}_X(x)=\begin{cases} 1, & x\leq 1\\ 0.5, & 1<x\leq 10\\ 0, & x>10\end{cases}$$

Випадок 1:

1. $c>\lambda\mu=\lambda EX=1\cdot 5.5=5.5$
2. $c=2\lambda\mu=2\cdot 5.5=11$
3. $\varphi(u)=\varphi(0)+\frac{\lambda}{c}\int_0^u \varphi(u-y)\bar{F}_X(y)\,dy, \forall u\geq 0$

$$\varphi(u)\overset{NPC}{=}1-\frac{\lambda\mu}{c}+\frac{\lambda}{c}\int_0^u \varphi(u-y)\bar{F}_X(y)\,dy=1-\frac{5.5}{11}+\frac{1}{11}\int_0^u \varphi(u-y)\bar{F}_X(y)\,dy=0.5+\frac{1}{11}\int_0^u \varphi(u-y)\bar{F}_X(y)\,dy$$

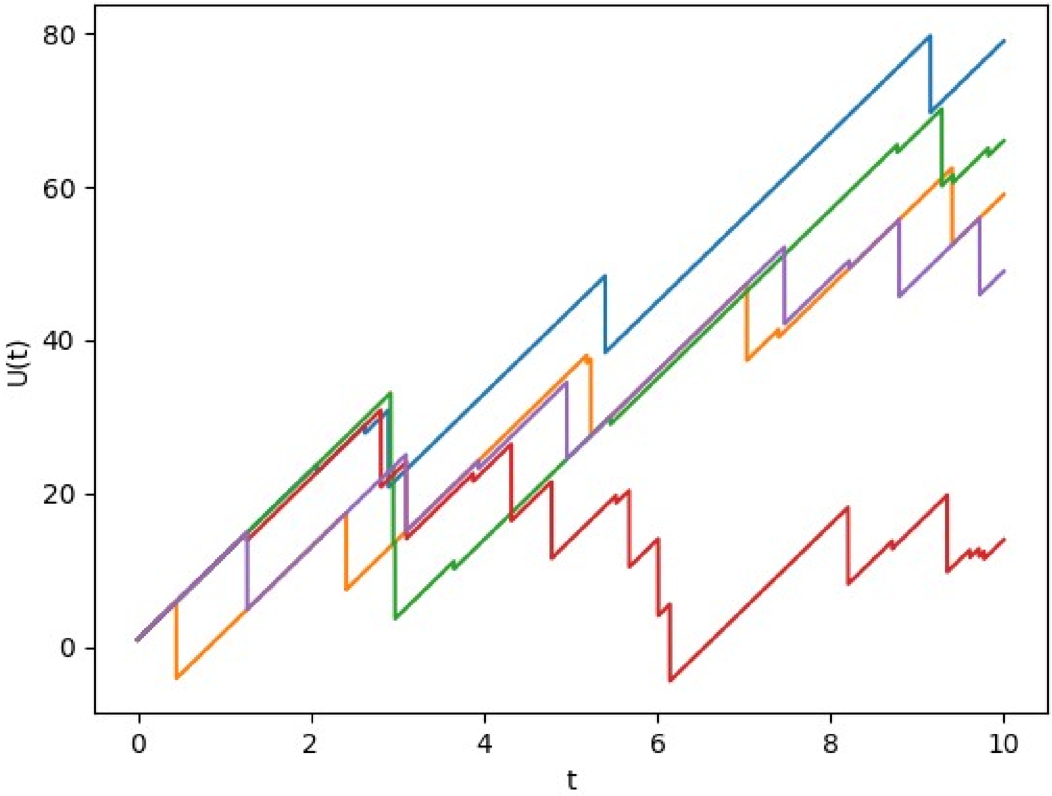
4. $\Phi(p)=L\{\varphi(u)\}=\dfrac{1-\frac{\lambda\mu}{c}}{p[1-\frac{\lambda}{c}L\{\bar{F}_X(x)\}]}$

$$L\{\bar{F}_X(p)\}=\int_0^{+\infty} e^{-pt}\bar{F}_X(t)\,dt=\int_0^1 e^{-pt}\,dt+\int_1^{10} e^{-pt}0.5\,dt+\int_{10}^{+\infty} e^{-pt}0\,dt=\int_0^1 e^{-pt}\,dt+\int_1^{10} e^{-pt}0.5\,dt=$$

$$=\frac{1-e^{-p}}{p}+\frac{0.5e^{-p}-e^{-10p}}{p}=\frac{1-0.5e^{-p}-0.5e^{-10p}}{p}$$

$$\Phi(p)=\frac{0.5}{p\left[1-\frac{1-0.5e^{-p}-0.5e^{-10p}}{11p}\right]}=\frac{0.5}{p-\frac{1-0.5e^{-p}-0.5e^{-10p}}{11}}$$

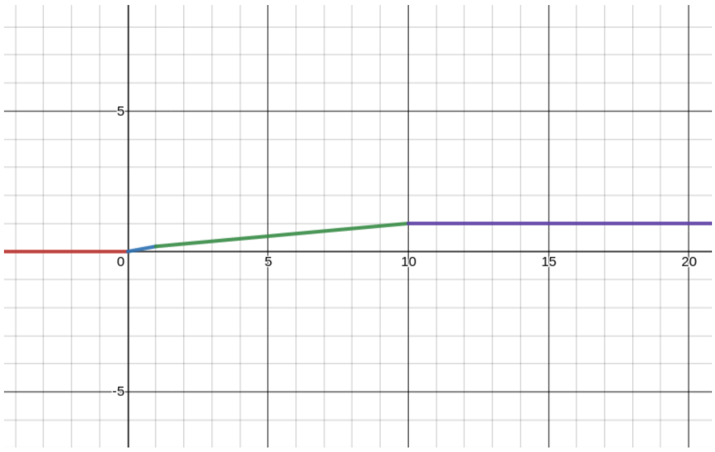
5.



6. Процент провалених 50 %

7. $F_{\tilde{X}}(x)=\frac{1}{EX}\int_0^x \bar{F}_X(t)dt=\frac{2}{11}\int_0^x \bar{F}_X(t)dt=$

$$=\begin{cases} 0, & x\le 0 \\ \frac{2x}{11}, & 0<x\le 1 \\ \frac{(1+x)}{11}, & 1<x\le 10 \\ 1, & x>10 \end{cases}$$



Процент провалених 45.3690 %

8. $\Phi^{-1}(p)=L^{-1}\{\Phi(p)\}=\varphi(u)$

$$P\{провалених\}=1-\varphi(u)=1-\Phi^{-1}(p)$$

Процент провалених 45.2 %

Випадок 2:

1. $c>\lambda\mu=\lambda EX=5\cdot 5.5=27.5$
2. $c=1.05\lambda\mu=1.05\cdot 27.5=28.875$
3. $\varphi(u)=\varphi(0)+\frac{\lambda}{c}\int_0^u \varphi(u-y)\bar{F}_X(y)dy, \forall u\geq 0$

$$\varphi(u)\overset{NPC}{=}1-\frac{\lambda\mu}{c}+\frac{\lambda}{c}\int_0^u \varphi(u-y)\bar{F}_X(y)dy=1-\frac{27.5}{28.875}+\frac{5}{28.875}\int_0^u \varphi(u-y)\bar{F}_X(y)dy=$$

$$=\frac{11}{231}+\frac{40}{231}\int_0^u \varphi(u-y)\bar{F}_X(y)dy$$

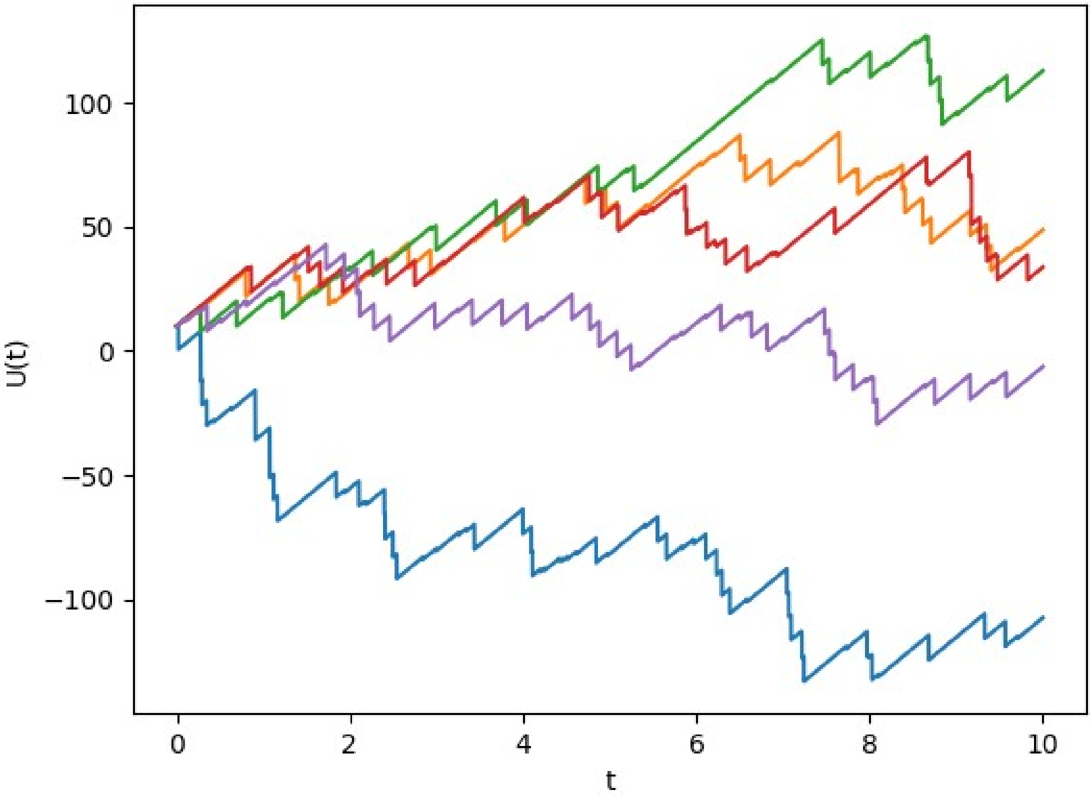
4. $\Phi(p)=L\{\varphi(u)\}=\dfrac{1-\dfrac{\lambda\mu}{c}}{p[1-\dfrac{\lambda}{c}L\{\bar{F}_X(x)\}]}$

$$L\{\bar{F}_X(p)\}=\int_0^{+\infty} e^{-pt}\bar{F}_X(t)dt=\int_0^1 e^{-pt}dt+\int_1^{10} e^{-pt}0.5dt+\int_{10}^{+\infty} e^{-pt}0dt=\int_0^1 e^{-pt}dt+\int_1^{10} e^{-pt}0.5dt=$$

$$=\frac{1-e^{-p}}{p}+\frac{0.5e^{-p}-e^{-10p}}{p}=\frac{1-0.5e^{-p}-0.5e^{-10p}}{p}$$

$$\Phi(p)=\frac{\dfrac{11}{231}}{p[1-\dfrac{11}{231}\dfrac{1-0.5e^{-p}-0.5e^{-10p}}{p}]}=\frac{\dfrac{11}{231}}{p-40\dfrac{1-0.5e^{-p}-0.5e^{-10p}}{231}}$$

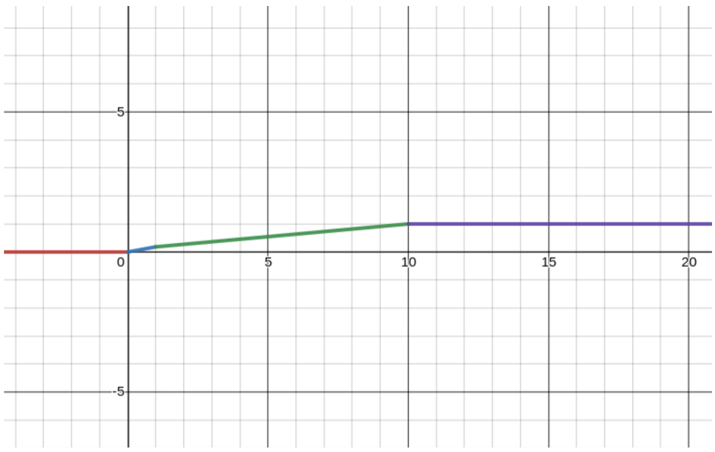
5.



6. Процент провалених 80 %

7. $F_{\tilde{X}}(x) = \frac{1}{EX}\int_0^x \bar{F}_X(t)\,dt = \frac{2}{11}\int_0^x \bar{F}_X(t)\,dt =$

$$= \begin{cases} 0, & x \le 0 \\ \frac{2x}{11}, & 0 < x \le 1 \\ \frac{(1+x)}{11}, & 1 < x \le 10 \\ 1, & x > 10 \end{cases}$$



Процент провалених 86.714 %

8. $\Phi^{-1}(p) = L^{-1}\{\Phi(p)\} = \varphi(u)$

$$P\{провалених\} = 1 - \varphi(u) = 1 - \Phi^{-1}(p)$$

Процент провалених 86.84 %

Код: https://github.com/Bogusik/ruin-theory