HP-1 Engine Preliminary Analysis Report: Parts B & C

This report details the analysis and corrected calculations for Task B (Maximum Allowable TSFC) and Task C (Minimum Specific Thrust) based on the requirements outlined in the HP-1 design problem manual [1].

1 Task B: Maximum Allowable TSFC

The maximum allowable Thrust Specific Fuel Consumption (TSFC) for the cruise segment is determined using the Breguet range equation. Key parameters and corrections based on the manual are:

- Cruise Mach Number: $M = 0.83$
- Cruise Altitude: $h = 11\,\text{km}$ ($T = 216.65\,\text{K}$, $a = 295.07\,\text{m/s}$)
- Cruise Speed: $V = M \times a = 0.83 \times 295.07\,\text{m/s} = 244.91\,\text{m/s}$
- Cruise Segment Range: $R = 10\,650\,\text{km}$ (from Table 1 [1])
- Start of Cruise Weight: $W_{\text{start}} = 1\,577\,940\,\text{N}$ (approx. $160\,850\,\text{kg}$) [1]
- Cruise Fuel: $50\,240\,\text{kg}$ (from Table 1 [1])
- End of Cruise Weight: $W_{\text{end}} = W_{\text{start}} - (\text{Cruise Fuel} \times g) = 160\,850\,\text{kg} - 50\,240\,\text{kg} = 110\,610\,\text{kg}$
- Cruise Weight Fraction: $W_{\text{end}}/W_{\text{start}} = 110610/160850 = 0.6877$
- Assumed Lift-to-Drag Ratio: $L/D = 17.0$ (User assumption, requires verification)
- Installation Loss Factor: $\Phi_{\text{loss}} = 0.02$ [1]

The Breguet range equation is:

$$R = \frac{V}{S_{\text{installed}}} \left(\frac{L}{D}\right) \ln\left(\frac{W_{\text{start}}}{W_{\text{end}}}\right)$$

Rearranging for installed TSFC ($S_{\text{installed}}$ in $\text{kg}/(\text{N}\,\text{s})$):

$$S_{\text{installed}} = \frac{V}{R} \left(\frac{L}{D}\right) \ln\left(\frac{W_{\text{start}}}{W_{\text{end}}}\right)$$

$$S_{\text{installed}} = \frac{244.91\,\text{m/s}}{10\,650 \times 10^3\,\text{m}} (17.0) \ln\left(\frac{1}{0.6877}\right)$$

$$S_{\text{installed}} = (2.2996 \times 10^{-5}\,\text{s}^{-1})(17.0)(0.3744) = 1.4620 \times 10^{-4}\,\text{kg}/(\text{N}\,\text{s})$$

The uninstalled TSFC (S) is:

$$S = \frac{S_{\text{installed}}}{1 - \Phi_{\text{loss}}} = \frac{1.4620 \times 10^{-4}\,\text{kg}/(\text{N}\,\text{s})}{1 - 0.02} = \frac{1.4620 \times 10^{-4}\,\text{kg}/(\text{N}\,\text{s})}{0.98}$$

$$S = 1.4918 \times 10^{-4}\,\text{kg}/(\text{N}\,\text{s}) = 14.92\,\text{mg}/(\text{N}\,\text{s})$$

Result Task B: The maximum allowable uninstalled TSFC is $14.92\,\text{mg}/(\text{N}\,\text{s})$. This value is realistic for modern high-bypass turbofan engines at cruise conditions. The original calculation result ($17.08\,\text{mg}/(\text{N}\,\text{s})$) differed primarily due to using incorrect initial assumptions for cruise weight fraction and range.

2 Task C: Minimum Specific Thrust

The minimum required specific thrust ($F/\dot{m}_{\text{air}}$) is determined by the required engine thrust at the start of cruise and the maximum possible air mass flow through the engine inlet.

2.1 Required Uninstalled Thrust per Engine

- Start of Cruise Weight: $W = 1\,577\,940\,\text{N}$ [1]
- Lift-to-Drag Ratio: $L/D = 17.0$ (Assumed)
- Drag: $D = W/(L/D) = 1\,577\,940\,\text{N}/17.0 = 92\,820\,\text{N}$
- Required Rate of Climb: $P_s = 1.5\,\text{m/s}$ [1]
- Cruise Speed: $V = 244.91\,\text{m/s}$
- Required Installed Thrust (Total): $T_{\text{installed}} = D + \frac{W \times P_s}{V} = 92\,820\,\text{N} + \frac{1\,577\,940\,\text{N} \times 1.5\,\text{m/s}}{244.91\,\text{m/s}} = 92\,820\,\text{N} + 9665\,\text{N} = 102\,485\,\text{N}$
- Required Uninstalled Thrust (Total): $T_{\text{uninstalled, total}} = \frac{T_{\text{installed}}}{1-\Phi_{\text{loss}}} = \frac{102\,485\,\text{N}}{1-0.02} = 104\,577\,\text{N}$
- Required Uninstalled Thrust per Engine (2 engines): $T_{\text{uninstalled, eng}} = T_{\text{uninstalled, total}}/2 = 104\,577\,\text{N}/2 = 52\,288\,\text{N}$

This required thrust (52 288 N/engine) is higher than the originally calculated 48 744 N/engine due to the use of the correct start-of-cruise weight.

2.2 Maximum Inlet Mass Flow

The maximum mass flow per unit area is given by the manual [1]:

$$\frac{\dot{m}}{A} = 231.8\frac{\delta_0}{\sqrt{\theta_0}} \quad (\text{kg/s/m}^2)(\text{kg/s/m}^2)^{(\text{kg/s/m}^2)(\text{kg/s/m}^2)}$$

Where $\theta_0 = T_{t0}/T_{SL}$ and $\delta_0 = P_{t0}/P_{SL}$. At $M = 0.83$, $h = 11\,\text{km}$ ($T = 216.65\,\text{K}$, $P = 22\,631.7\,\text{Pa}$):

- $T_{t0} = T(1 + \frac{\gamma-1}{2}M^2) = 216.65(1 + 0.2 \times 0.83^2) = 246.49\,\text{K}$
- $P_{t0} = P(1 + \frac{\gamma-1}{2}M^2)^{\gamma/(\gamma-1)} = 22631.7(1 + 0.2 \times 0.83^2)^{3.5} = 35\,611\,\text{Pa}$
- $\theta_0 = T_{t0}/288.15\,\text{K} = 246.49/288.15 = 0.8554$
- $\delta_0 = P_{t0}/101\,325\,\text{Pa} = 35611/101325 = 0.3514$

$$\frac{\dot{m}}{A} = 231.8\frac{0.3514}{\sqrt{0.8554}} = 231.8 \times \frac{0.3514}{0.9249} = 88.01\,\text{kg/s/m}^2$$

2.3 Minimum Specific Thrust Calculation

The minimum uninstalled specific thrust is $F_{\text{uninstalled}}/\dot{m}_{\text{air, max}} = T_{\text{uninstalled, eng}}/(A_{\text{inlet}} \times (\dot{m}/A)_{\max})$.

The manual states a maximum allowable inlet diameter of 2.2 m [1]. Calculations for larger diameters are included for comparison but violate this constraint.

Table 1: Recalculated Minimum Specific Thrust vs. Inlet Diameter

Diameter (m)	Inlet Area (m^2)	Max Mass Flow (kg/s)	Min Spec. Thrust (N s/kg)	Feasible
2.2	3.801	334.5	**156.3**	**Yes**
2.5	4.909	432.0	121.0	No
2.75	5.940	522.8	99.0	No
3.0	7.069	622.1	84.1	No
3.25	8.296	730.1	71.6	No
3.5	9.621	846.7	61.7	No

Result Task C: Considering the constraint $D_{\text{inlet}} \leq 2.2\,\text{m}$, the minimum allowable uninstalled specific thrust at the start of cruise is 156.3 N s/kg. The specific thrust values decrease with increasing inlet diameter (mass flow) for the fixed thrust requirement. The recalculated values differ from the original calculations due to the corrected thrust requirement and the application of the specific mass flow formula from the manual.

References

[1] Provided ‘Gas Turbine Design Problem HP-1‘ file detailing the HP-1 Aircraft Design Problem.

[2] Elements of Propulsion, Gas Turbines and Rockets 2nd edition.