FinalProjectFall2021

December 7, 2021

1 Final Project

Financial Data Science II

Fall 2021

Group Members: * Junichi Koganemaru * Bohang Wei * Kaiyue Liao * Peiwen Xiang * Renjia Guo

1.1 Guidelines

You should submit both your `.ipynb` file and a `.pdf` version of the notebook via Canvas.

We should be able to take your code, and easily run it. It can load standard Python packages, of course, but it should otherwise be self-contained, not reliant upon any other code.

This is to be completed in your groups. All group members should participate in, and understand, all aspects of the analysis.

Different groups are not allowed to interact or in any way communicate regarding their analyses.

We are not looking for a formal "report," but **explain what you are doing** and **state any conclusions in practical terms.** For example, do not just say "The p-value is 0.02." What does that mean in practical terms?

Visualizations will be graded, in part, on their readability. Make fonts sufficiently large. Do not label axes with things such as "CSCORE_B." Use readable names.

You can email me with questions. If it is relevant to the entire class, I will ask you to post it on the Discussion Board. Please do not post questions directly to the Discussion Board regarding this, however.

1.2 The Data

We will continue using the Fannie Mae data set used in the Homeworks 2 and 3. Here we will use a different subset. Just as in Homework 2, it only contains loans for which there is a current unpaid balance, i.e. for which `CURRENT_UPB` is greater than zero.

This data file is available as `project.csv` on Canvas.

1.3 Your Objective

Imagine that you are constructing a simulation model that will generate realistic-looking loans of the type shown in this data set. We will focus here on one particular aspect of this: Simulating credit scores.

Our focus will be on the column `CSCORE_B`.

1.4 Specific Steps

1. There are four missing values in `CSCORE_B`. Fill them in by using the following procedure: Cluster the loans, and then determine which cluster each of the loans with the missing value belong to. Replace the missing credit score value with the average credit score of those observations in the same cluster. Of course, the procedure cannot use `CSCORE_B` as one of the variables used to cluster loans. You can use any clustering procedure of your choice.
2. Visualize the distribution of `CSCORE_B`. Comment on its shape.
3. Find a parametric model that fits to the distribution of `CSCORE_B`. You can consider any distribution you would like, as long as it is parametric. Justify your choice using visualization and AIC. As in any real-world problem, you should not expect the fit to be "perfect." Get the best-fitting parametric distribution that you can. Report the MLE of the parameters for this fit.
4. Visualize how the distribution of `CSCORE_B` varies over different property types, i.e., for different values of `PROP`. Does it seem that the distribution changes across the different values of `PROP`? (Just to be clear, when I say "the distribution changes" I mean any aspect of the distribution changes. For example, going from a Normal(10,2) distribution to a Normal(12,2) is a "change" in distribution.)
5. Is there strong evidence that the distribution of `CSCORE_B` varies across the different values of `PROP`? Test a relevant hypothesis, and report the p-value. This test should be performed using a likelihood ratio test.

1.5 Grading

Each of the five steps listed above will be assessed and scored based on three elements: (1) Appropriate choice of method(s) of analysis and/or visualization, (2) correct implmentation of those method(s), and (3) valid interpretation of the results. Each of these three elements will be given equal weight, and the total points allocated to each step are as follows: Step 1, 18 points. Step 2, 6 points. Step 3, 15 points. Step 4, 6 points. Step 5, 18 points. This is a total of 63 points.

In addition, the notebooks will be assessed based on the overall quality of the visualizations (on a scale of 0 to 6 points) on the overall quality of practical explanations of results (on a scale of 0 to 6 points), and on overall organization of the notebook, i.e., can we find where the different steps are, etc., or is it a mess (on a scale of 0 to 5 points).

Hence, there are total of 80 points.

1.6 Peer Assessment

This is everyone's least favorite part, but I have little choice but to incoporate some component of peer feedback, i.e. assessing how much your groupmates contributed to the work. Without this, I have little basis for avoiding problems where students let the work fall on their groupmates.

Any negative information found through this process will be investigated by the instructor.

I do reserve the right to adjust individual students' grades based on lack of engagement with the project.

The easy way to avoid any problems is to make sure that everyone is engaged and participating. Please advise me of any issues as soon as possible.

2 Step 1: Filling in missing credit scores

2.1 Importing packages

[ ]:
```
#Importing standard packages
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
```

[ ]:
```
#Installing nonstandard packages
import sys
!{sys.executable} -m pip install umap-learn
!{sys.executable} -m pip install gowers
!{sys.executable} -m pip install tabulate
```

```
Requirement already satisfied: umap-learn in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (0.5.2)
Requirement already satisfied: numpy>=1.17 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from umap-
learn) (1.20.0)
Requirement already satisfied: scipy>=1.0 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from umap-
learn) (1.6.2)
Requirement already satisfied: pynndescent>=0.5 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from umap-
learn) (0.5.5)
Requirement already satisfied: scikit-learn>=0.22 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from umap-
learn) (0.24.1)
Requirement already satisfied: tqdm in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from umap-
learn) (4.59.0)
Requirement already satisfied: numba>=0.49 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from umap-
learn) (0.53.1)
Requirement already satisfied: llvmlite<0.37,>=0.36.0rc1 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from
numba>=0.49->umap-learn) (0.36.0)
Requirement already satisfied: setuptools in
```

```
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from
numba>=0.49->umap-learn) (52.0.0.post20210125)
Requirement already satisfied: joblib>=0.11 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from
pynndescent>=0.5->umap-learn) (1.0.1)
Requirement already satisfied: threadpoolctl>=2.0.0 in
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-packages (from scikit-
learn>=0.22->umap-learn) (2.1.0)
ERROR: Could not find a version that satisfies the requirement gowers
ERROR: No matching distribution found for gowers
Collecting tabulate
  Downloading tabulate-0.8.9-py3-none-any.whl (25 kB)
Installing collected packages: tabulate
Successfully installed tabulate-0.8.9
```

[ ]:
```
import warnings
warnings.filterwarnings('ignore')
```

[ ]:
```
!export PATH=/Library/TeX/texbin:$PATH
```

2.2 Exploratory Data Analysis

2.2.1 Loading data

We first load in the data from project.csv.

[ ]:
```
#Loading data
loandata = pd.read_csv('project.csv', sep = '|')

loandata.columns = ("POOL_ID", "LOAN_ID", "ACT_PERIOD", "CHANNEL", "SELLER",␣
 ↪"SERVICER",
                        "MASTER_SERVICER", "ORIG_RATE", "CURR_RATE",␣
 ↪"ORIG_UPB", "ISSUANCE_UPB",
                        "CURRENT_UPB", "ORIG_TERM", "ORIG_DATE", "FIRST_PAY",␣
 ↪"LOAN_AGE",
                        "REM_MONTHS", "ADJ_REM_MONTHS", "MATR_DT", "OLTV",␣
 ↪"OCLTV",
                        "NUM_BO", "DTI", "CSCORE_B", "CSCORE_C", "FIRST_FLAG",␣
 ↪"PURPOSE",
                        "PROP", "NO_UNITS", "OCC_STAT", "STATE", "MSA", "ZIP",␣
 ↪"MI_PCT",
                        "PRODUCT", "PPMT_FLG", "IO", "FIRST_PAY_IO",␣
 ↪"MNTHS_TO_AMTZ_IO",
                        "DLQ_STATUS", "PMT_HISTORY", "MOD_FLAG",␣
 ↪"MI_CANCEL_FLAG", "Zero_Bal_Code",
                        "ZB_DTE", "LAST_UPB", "RPRCH_DTE", "CURR_SCHD_PRNCPL",␣
 ↪"TOT_SCHD_PRNCPL",
```

```
                         "UNSCHD_PRNCPL_CURR", "LAST_PAID_INSTALLMENT_DATE",␣
↪"FORECLOSURE_DATE",
                         "DISPOSITION_DATE", "FORECLOSURE_COSTS",␣
↪"PROPERTY_PRESERVATION_AND_REPAIR_COSTS",
                         "ASSET_RECOVERY_COSTS",␣
↪"MISCELLANEOUS_HOLDING_EXPENSES_AND_CREDITS",
                         "ASSOCIATED_TAXES_FOR_HOLDING_PROPERTY",␣
↪"NET_SALES_PROCEEDS",
                         "CREDIT_ENHANCEMENT_PROCEEDS",␣
↪"REPURCHASES_MAKE_WHOLE_PROCEEDS",
                         "OTHER_FORECLOSURE_PROCEEDS",␣
↪"NON_INTEREST_BEARING_UPB", "PRINCIPAL_FORGIVENESS_AMOUNT",
                         "ORIGINAL_LIST_START_DATE", "ORIGINAL_LIST_PRICE",␣
↪"CURRENT_LIST_START_DATE",
                         "CURRENT_LIST_PRICE", "ISSUE_SCOREB", "ISSUE_SCOREC",␣
↪"CURR_SCOREB",
                         "CURR_SCOREC", "MI_TYPE", "SERV_IND",␣
↪"CURRENT_PERIOD_MODIFICATION_LOSS_AMOUNT",
                         "CUMULATIVE_MODIFICATION_LOSS_AMOUNT",␣
↪"CURRENT_PERIOD_CREDIT_EVENT_NET_GAIN_OR_LOSS",
                         "CUMULATIVE_CREDIT_EVENT_NET_GAIN_OR_LOSS",␣
↪"HOMEREADY_PROGRAM_INDICATOR",
                         "FORECLOSURE_PRINCIPAL_WRITE_OFF_AMOUNT",␣
↪"RELOCATION_MORTGAGE_INDICATOR",
                         "ZERO_BALANCE_CODE_CHANGE_DATE",␣
↪"LOAN_HOLDBACK_INDICATOR", "LOAN_HOLDBACK_EFFECTIVE_DATE",
                         "DELINQUENT_ACCRUED_INTEREST",␣
↪"PROPERTY_INSPECTION_WAIVER_INDICATOR",
                         "HIGH_BALANCE_LOAN_INDICATOR", "ARM_5_YR_INDICATOR",␣
↪"ARM_PRODUCT_TYPE",
                         "MONTHS_UNTIL_FIRST_PAYMENT_RESET",␣
↪"MONTHS_BETWEEN_SUBSEQUENT_PAYMENT_RESET",
                         "INTEREST_RATE_CHANGE_DATE", "PAYMENT_CHANGE_DATE",␣
↪"ARM_INDEX",
                         "ARM_CAP_STRUCTURE", "INITIAL_INTEREST_RATE_CAP",␣
↪"PERIODIC_INTEREST_RATE_CAP",
                         "LIFETIME_INTEREST_RATE_CAP", "MARGIN",␣
↪"BALLOON_INDICATOR",
                         "PLAN_NUMBER", "FORBEARANCE_INDICATOR",␣
↪"HIGH_LOAN_TO_VALUE_HLTV_REFINANCE_OPTION_INDICATOR",
                         "DEAL_NAME", "RE_PROCS_FLAG", "ADR_TYPE", "ADR_COUNT",␣
↪"ADR_UPB")
```

Then we take at the dimensions of the data and examine the first first rows.

[ ]:
```
#Examining shape of data
loandata.shape
```

[ ]: (4999, 108)

[ ]:
```
loandata.head()
```

[ ]:
```
   POOL_ID    LOAN_ID  ACT_PERIOD CHANNEL  \
0     1443   33229873       82021       R
1     1473   40837377       82021       C
2     1474   41378070       82021       C
3     5228  125559711       82021       R
4     5124   94867529       82021       R

                                 SELLER                              SERVICER  \
0                                 Other                                 Other
1                                 Other          Pingora Loan Servicing, LLC
2  Truist Bank (formerly SunTrust Bank)  Truist Bank (formerly SunTrust Bank)
3                                 Other                                 Other
4                                 Other                                 Other

  MASTER_SERVICER  ORIG_RATE  CURR_RATE  ORIG_UPB  …  MARGIN  \
0      FANNIE MAE      3.875      3.875  105000.0  …     NaN
1      FANNIE MAE      4.625      4.625  238000.0  …     NaN
2      FANNIE MAE      3.750      3.750  409000.0  …     NaN
3      FANNIE MAE      3.090      3.090  314000.0  …     NaN
4      FANNIE MAE      4.250      4.250  232000.0  …     NaN

   BALLOON_INDICATOR  PLAN_NUMBER  FORBEARANCE_INDICATOR  \
0                  N          NaN                      7
1                  N          NaN                      7
2                  N          NaN                      7
3                  N          NaN                      7
4                  N          NaN                      7

   HIGH_LOAN_TO_VALUE_HLTV_REFINANCE_OPTION_INDICATOR        DEAL_NAME  \
0                                                  N    CAS 2016 C03 G2
1                                                  N    CAS 2017 C03 G1
2                                                  N    CAS 2017 C04 G2
3                                                  N    CAS 2021 R01 G1
4                                                  N    CAS 2020 R01 G1

   RE_PROCS_FLAG  ADR_TYPE  ADR_COUNT  ADR_UPB
0            NaN         7        NaN      NaN
1            NaN         7        NaN      NaN
2            NaN         7        NaN      NaN
3            NaN         7        NaN      NaN
```

```
4              NaN          7          NaN       NaN

[5 rows x 108 columns]
```

Next we convert some of the columns to the appropriate datatypes.

```
[ ]: #converting variables to approprate datatype
loandata['POOL_ID'] = loandata['POOL_ID'].astype("category")
loandata['LOAN_ID'] = loandata['LOAN_ID'].astype("category")
loandata['ZIP'] = loandata['ZIP'].astype("category")
loandata['ACT_PERIOD'] = loandata['ACT_PERIOD'].astype("datetime64[ns]")
loandata['ORIG_DATE'] = loandata['ORIG_DATE'].astype("datetime64[ns]")
loandata['FIRST_PAY'] = loandata['FIRST_PAY'].astype("datetime64[ns]")
loandata['MATR_DT'] = loandata['MATR_DT'].astype("datetime64[ns]")
loandata['FIRST_PAY_IO'] = loandata['FIRST_PAY_IO'].astype("datetime64[ns]")
loandata['Zero_Bal_Code'] = loandata['Zero_Bal_Code'].astype("category")
loandata['ZB_DTE'] = loandata['ZB_DTE'].astype("datetime64[ns]")
loandata['RPRCH_DTE'] = loandata['RPRCH_DTE'].astype("datetime64[ns]")
loandata['LAST_PAID_INSTALLMENT_DATE'] = loandata['LAST_PAID_INSTALLMENT_DATE'].
 ↪astype("datetime64[ns]")
loandata['FORECLOSURE_DATE'] = loandata['FORECLOSURE_DATE'].
 ↪astype("datetime64[ns]")
loandata['DISPOSITION_DATE'] = loandata['DISPOSITION_DATE'].
 ↪astype("datetime64[ns]")
loandata['ORIGINAL_LIST_START_DATE'] = loandata['ORIGINAL_LIST_START_DATE'].
 ↪astype("datetime64[ns]")
loandata['CURRENT_LIST_START_DATE'] = loandata['CURRENT_LIST_START_DATE'].
 ↪astype("datetime64[ns]")
loandata['ZERO_BALANCE_CODE_CHANGE_DATE'] =␣
 ↪loandata['ZERO_BALANCE_CODE_CHANGE_DATE'].astype("datetime64[ns]")
loandata['LOAN_HOLDBACK_EFFECTIVE_DATE'] =␣
 ↪loandata['LOAN_HOLDBACK_EFFECTIVE_DATE'].astype("datetime64[ns]")
loandata['INTEREST_RATE_CHANGE_DATE'] = loandata['INTEREST_RATE_CHANGE_DATE'].
 ↪astype("datetime64[ns]")
loandata['PAYMENT_CHANGE_DATE'] = loandata['PAYMENT_CHANGE_DATE'].
 ↪astype("datetime64[ns]")
```

2.2.2 Handling missing values

Since there are missing values in the dataset, we first seek to identify the columns with all missing values or "significantly many" missing values. We decided that any column containing more than 66/4999 missing values qualifies as having "significantly many" missing values.

```
[ ]: #Identifying columns with all missing values or mostly missing values
na_columns = []
for vals in loandata.columns:
    if (loandata[vals].isna().all()) or (loandata[vals].isna().sum() > 66):
```

```
        na_columns.append(vals)
```

We were able to identify 61/108 of such columns.

[ ]:
```
len(na_columns)
```

[ ]:
```
61
```

We then proceed to drop these columns from the original dataset. We also drop columns where there is only 1 unique value.

[ ]:
```
#Dropping identified columns
X = loandata.drop(columns = na_columns)

#Dropping columns with 1 unique value
nunique = X.nunique()
cols_to_drop = nunique[nunique == 1].index
X = X.drop(cols_to_drop, axis=1)
```

[ ]:
```
loandata.shape
```

[ ]:
```
(4999, 108)
```

In total, so far we've dropped 68/108 columns.

[ ]:
```
X.shape
```

[ ]:
```
(4999, 40)
```

These are the remaining columns.

[ ]:
```
#remaining columns
X.columns
```

[ ]:
```
Index(['POOL_ID', 'LOAN_ID', 'CHANNEL', 'SELLER', 'SERVICER', 'ORIG_RATE',
       'CURR_RATE', 'ORIG_UPB', 'ISSUANCE_UPB', 'CURRENT_UPB', 'ORIG_TERM',
       'ORIG_DATE', 'FIRST_PAY', 'LOAN_AGE', 'REM_MONTHS', 'ADJ_REM_MONTHS',
       'MATR_DT', 'OLTV', 'OCLTV', 'NUM_BO', 'CSCORE_B', 'FIRST_FLAG',
       'PURPOSE', 'PROP', 'NO_UNITS', 'OCC_STAT', 'STATE', 'MSA', 'ZIP',
       'DLQ_STATUS', 'PMT_HISTORY', 'MOD_FLAG', 'ISSUE_SCOREB', 'SERV_IND',
       'CUMULATIVE_CREDIT_EVENT_NET_GAIN_OR_LOSS',
       'RELOCATION_MORTGAGE_INDICATOR', 'HIGH_BALANCE_LOAN_INDICATOR',
       'FORBEARANCE_INDICATOR', 'DEAL_NAME', 'ADR_TYPE'],
      dtype='object')
```

Next, we seek to identify the datapoints where the Borrower Credit Score at Origination is missing.

[ ]:
```
missing = X[ X['CSCORE_B'].isnull() == True ]
```

We've identified 4 rows (as expected) with missing credit scores.

[ ]:
```
missing.head()
```

[ ]:
```
      POOL_ID   LOAN_ID CHANNEL                  SELLER  \
1772     5123  94259369       R  Wells Fargo Bank, N.A.
1933     1398  26909202       R  Wells Fargo Bank, N.A.
3529     1484  43103805       R                   Other
4759     5123  94576786       R                   Other

                    SERVICER  ORIG_RATE  CURR_RATE  ORIG_UPB  ISSUANCE_UPB  \
1772  Wells Fargo Bank, N.A.      3.750      3.750   69000.0      36772.61
1933  Wells Fargo Bank, N.A.      4.625      4.625  180000.0     176231.93
3529                   Other      3.625      3.625  120000.0     119070.60
4759                   Other      4.000      4.000  149000.0      73153.42

      CURRENT_UPB  …                                       PMT_HISTORY  \
1772     20602.03  …  000000000000000000000000000000000000000000000000
1933    152852.86  …  000000000000000000000000000000000000000000000000
3529    108745.27  …  000000000000000000000000000000000000000000000000
4759     52627.71  …  000000000000000000000000000000000000000000000000

     MOD_FLAG ISSUE_SCOREB  SERV_IND  \
1772        N        638.0         N
1933        N        781.0         N
3529        N        806.0         N
4759        N          NaN         N

      CUMULATIVE_CREDIT_EVENT_NET_GAIN_OR_LOSS  RELOCATION_MORTGAGE_INDICATOR  \
1772                                       0.0                              N
1933                                       0.0                              N
3529                                       0.0                              N
4759                                       0.0                              N

     HIGH_BALANCE_LOAN_INDICATOR  FORBEARANCE_INDICATOR        DEAL_NAME  \
1772                           N                      7    CAS 2019-HRP1
1933                           N                      7  CAS 2015 C01 G2
3529                           N                      7  CAS 2017 C05 G1
4759                           N                      7    CAS 2019-HRP1

      ADR_TYPE
1772         7
1933         7
3529         7
4759         7

[4 rows x 40 columns]
```

We realized that these 4 rows might have other missing values, so we decided to identify the other columns that might contain missing values.

[ ]:
```
missing_columns = []
for vals in missing.columns:
    if (missing[vals].isna().any()):
        missing_columns.append(vals)
```

We found that in addition to the Borrower Credit Score at Origination, the Number of Borrowers and the Borrower Credit Score At Issuance are also missing in some of the rows.

[ ]:
```
missing_columns
```

[ ]:
```
['NUM_BO', 'CSCORE_B', 'ISSUE_SCOREB']
```

Next we took at the correlation between these two columns and the column containing the the Borrower Credit Score at Origination. We found both columns have positive correlation and one column has high correlation, so instead of dropping the two columns we decided to fill in the missing values with appropriate values instead.

We found that in the original dataset, the column containing the number of borrowers only contains 1 missing value and the column containing the Borrower Credit Score At Issuance has 57 missing values.

We decided to only fill in the values in the 4 rows that we identified above, and then drop any row that contains missing values in the end.

For the column containing the number of borrowers, we decided to fill in the missing number of borrowers with -1. For the column containing the Borrower Credit Score At Issuance, we decided to fill in the missing value with the average of the column in the original dataframe.

This could introduce some bias when we cluster later, but since we're only modifying 2/40 columns, and 2/4901 rows (after dropping the other rows), we don't think this will result in anything too significant.

[ ]:
```
print("Correlation between Borrower Credit Score at Origination and Number of␣
 ↪Borrowers: " + str(X['CSCORE_B'].corr(X['NUM_BO'])))
print("Correlation between Borrower Credit Score at Origination and Borrower␣
 ↪Credit Score at Issuance: " + str(X['CSCORE_B'].corr(X['ISSUE_SCOREB'])))
```

```
Correlation between Borrower Credit Score at Origination and Number of
Borrowers: 0.0823690995560387
Correlation between Borrower Credit Score at Origination and Borrower Credit
Score at Issuance: 0.6943425775597027
```

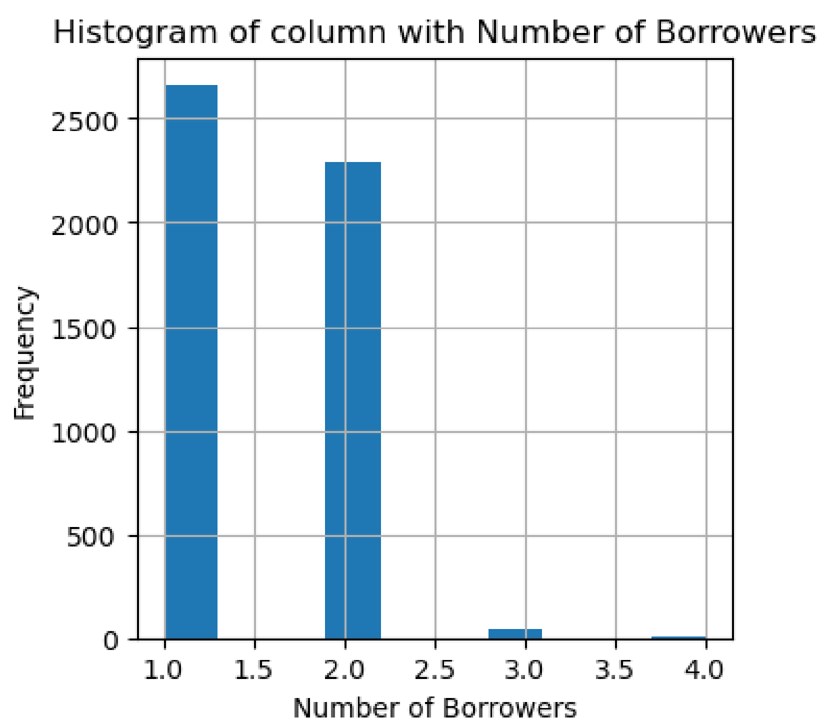
[ ]:
```
print("Number of missing values in the column with number of borrowers: " +␣
 ↪str(X['NUM_BO'].isnull().sum()))
print("Number of missing values in the column with Borrower Credit Score at␣
 ↪Issuance: " + str(X['ISSUE_SCOREB'].isnull().sum()))
```

```
Number of missing values in the column with number of borrowers: 1
Number of missing values in the column with Borrower Credit Score at Issuance:
57
```

[ ]:
```python
plt.figure(figsize=(4, 4), dpi=100)
X['NUM_BO'].hist()
plt.xlabel("Number of Borrowers")
plt.ylabel("Frequency")
plt.title('Histogram of column with Number of Borrowers')

print(X['NUM_BO'].describe())
```

```
count    4998.000000
mean        1.478191
std         0.522291
min         1.000000
25%         1.000000
50%         1.000000
75%         2.000000
max         4.000000
Name: NUM_BO, dtype: float64
```

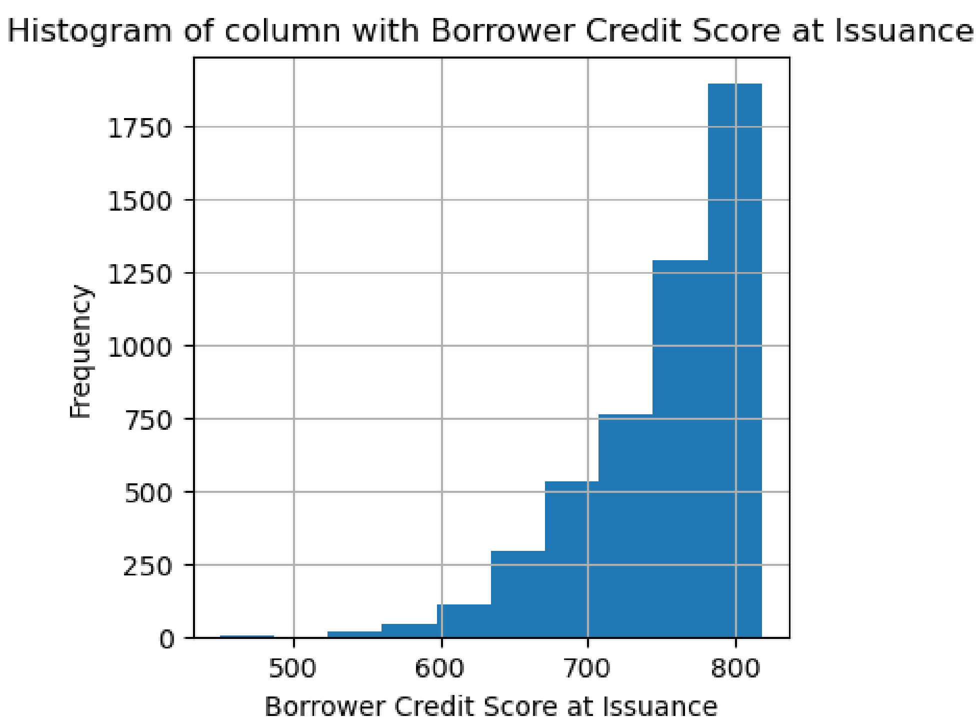


[ ]:
```python
plt.figure(figsize=(4, 4), dpi=100)
X['ISSUE_SCOREB'].hist()
```

```
plt.xlabel("Borrower Credit Score at Issuance")
plt.ylabel("Frequency")
plt.title('Histogram of column with Borrower Credit Score at Issuance')

print(X['ISSUE_SCOREB'].describe())
```

```
count    4942.000000
mean      752.033590
std        53.856117
min       450.000000
25%       721.000000
50%       768.000000
75%       794.000000
max       818.000000
Name: ISSUE_SCOREB, dtype: float64
```




```
[ ]: #Filling in missing number of borrowers with -1
X['NUM_BO'] = X['NUM_BO'].fillna(-1)

#Filling in missing Borrower Credit Score at Issuance with the mean of the␣
 ↪entire column
mean = X['ISSUE_SCOREB'].mean()
```

```
X.loc[X['LOAN_ID'] == 94576786, 'ISSUE_SCOREB'] = X.loc[X['LOAN_ID'] ==␣
↪94576786, 'ISSUE_SCOREB'].fillna(mean)
```

After filling in the missing values, we double check to see if this was done correctly.

```
[ ]: missing2 = X[ X['LOAN_ID'].isin([94259369,26909202, 43103805, 94576786]) ]
```

```
[ ]: missing2['NUM_BO']
```

```
[ ]: 1772    2.0
     1933    2.0
     3529    2.0
     4759   -1.0
     Name: NUM_BO, dtype: float64
```

```
[ ]: missing2['ISSUE_SCOREB']
```

```
[ ]: 1772    638.00000
     1933    781.00000
     3529    806.00000
     4759    752.03359
     Name: ISSUE_SCOREB, dtype: float64
```

Next we drop a few more ID columns, and also any row (other than the 4 rows that we identified above) that contains missing values.

```
[ ]: X = X.drop(columns = ['POOL_ID', 'ZIP', 'ORIG_DATE'])
     X = X.dropna(subset = X.columns.difference(['CSCORE_B']), how = 'any')
```

In total, we've dropped 71/108 columns and also 98/4999 rows.

```
[ ]: X.shape
```

```
[ ]: (4901, 37)
```

Next we store the filtered data without the Loan ID associated to each loan and also the Borrower Credit Score at Origination. This is the dataset that we will be performing clustering on.

```
[ ]: CreditScoreB = X['CSCORE_B']
     X_withoutID = X.drop(columns = ['LOAN_ID', 'CSCORE_B'])
```

```
[ ]: X_withoutID.head()
```

```
[ ]:   CHANNEL                                SELLER  \
     0       R                                 Other
     1       C                                 Other
     2       C  Truist Bank (formerly SunTrust Bank)
     3       R                                 Other
```

```
4       R                                 Other

                                   SERVICER  ORIG_RATE  CURR_RATE  ORIG_UPB  \
0                                     Other      3.875      3.875  105000.0
1           Pingora Loan Servicing, LLC      4.625      4.625  238000.0
2  Truist Bank (formerly SunTrust Bank)      3.750      3.750  409000.0
3                                     Other      3.090      3.090  314000.0
4                                     Other      4.250      4.250  232000.0

   ISSUANCE_UPB  CURRENT_UPB  ORIG_TERM                      FIRST_PAY  …  \
0     103725.97     91736.60        360 1970-01-01 00:00:00.000052015  …
1     234629.13    211659.69        360 1970-01-01 00:00:00.000082016  …
2     406027.37    370177.64        360 1970-01-01 00:00:00.000122016  …
3     309175.28    309175.28        360 1970-01-01 00:00:00.000122020  …
4     230000.00    223310.96        360 1970-01-01 00:00:00.000072019  …

                                  PMT_HISTORY  MOD_FLAG  ISSUE_SCOREB  \
0  000000000000000000000000000000000000000000000000         N         807.0
1  000000000000000000000000000000000000000000000000         N         618.0
2  000000000000000000000000000000000000000000000000         N         776.0
3  XXXXXXXXXXXXXXXXXXXXXXXXXXXX0000000000000000000         N         817.0
4  000000000000000000000000000000000000000000000000         N         805.0

  SERV_IND  CUMULATIVE_CREDIT_EVENT_NET_GAIN_OR_LOSS  \
0        N                                       0.0
1        N                                       0.0
2        N                                       0.0
3        N                                       0.0
4        N                                       0.0

   RELOCATION_MORTGAGE_INDICATOR  HIGH_BALANCE_LOAN_INDICATOR  \
0                              N                            N
1                              N                            N
2                              N                            N
3                              N                            N
4                              N                            N

  FORBEARANCE_INDICATOR        DEAL_NAME ADR_TYPE
0                     7  CAS 2016 C03 G2        7
1                     7  CAS 2017 C03 G1        7
2                     7  CAS 2017 C04 G2        7
3                     7  CAS 2021 R01 G1        7
4                     7  CAS 2020 R01 G1        7

[5 rows x 35 columns]
```

[ ]:
```
X_withoutID.shape
```

[ ]: (4901, 35)

2.2.3 Data Visualization

Before performing clustering, we perform nonlinear dimension reduction with UMAP and TSNE w.r.t. to the Gower's metric.

[ ]:
```
import gower
```

[ ]:
```
#Calculating distance matrix w.r.t. Gowers similarity metric
dist_matrix = gower.gower_matrix(X_withoutID)
```

[ ]:
```
dist_matrix.shape
```

[ ]: (4901, 4901)

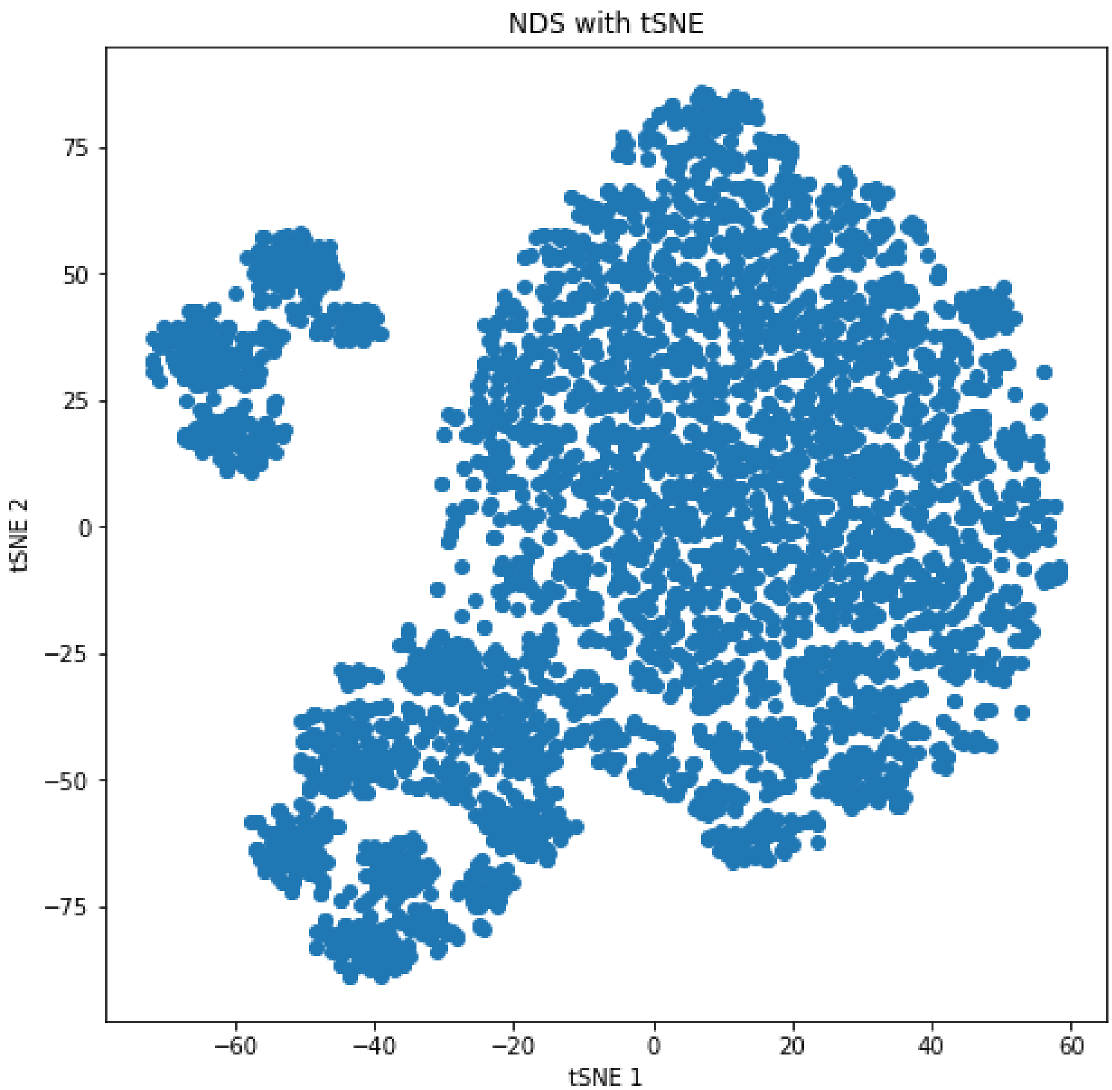
[ ]:
```
#Using tSNE and UMAP for nonlinear dimension reduction
from sklearn.manifold import TSNE
import umap.umap_ as umap
```

[ ]:
```
X_tsne = TSNE(n_components=2, metric = 'precomputed').fit_transform(dist_matrix)
X_umap = umap.UMAP(metric = 'precomputed').fit_transform(dist_matrix)
```

```
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-
packages/sklearn/manifold/_t_sne.py:691: FutureWarning: 'square_distances' has
been introduced in 0.24 to help phase out legacy squaring behavior. The 'legacy'
setting will be removed in 1.1 (renaming of 0.26), and the default setting will
be changed to True. In 1.3, 'square_distances' will be removed altogether, and
distances will be squared by default. Set 'square_distances'=True to silence
this warning.
  warnings.warn(
/Users/junichikoganemaru/opt/anaconda3/lib/python3.8/site-
packages/umap/umap_.py:1780: UserWarning: using precomputed metric;
inverse_transform will be unavailable
  warn("using precomputed metric; inverse_transform will be unavailable")
```

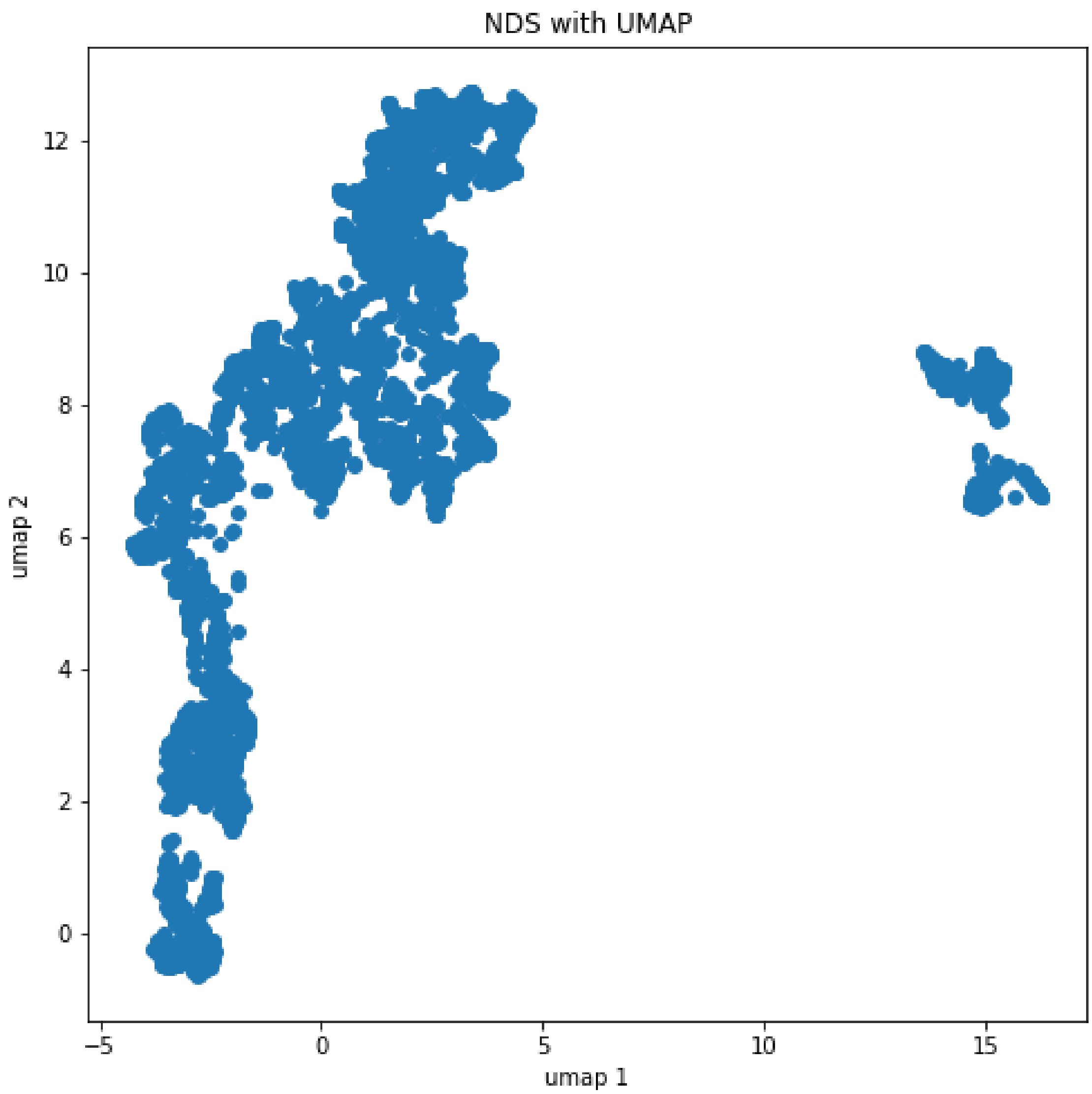
[ ]:
```
plt.figure(figsize=(8,8))
plt.scatter(X_tsne [:,0],X_tsne [:,1])
plt.title("NDS with tSNE")
plt.xlabel('tSNE 1')
plt.ylabel('tSNE 2')
```

[ ]: Text(0, 0.5, 'tSNE 2')



[ ]:
```
plt.figure(figsize=(8,8))
plt.scatter(X_umap[:,0],X_umap[:,1])
plt.title("NDS with UMAP")
plt.xlabel('umap 1')
plt.ylabel('umap 2')
```

[ ]: Text(0, 0.5, 'umap 2')



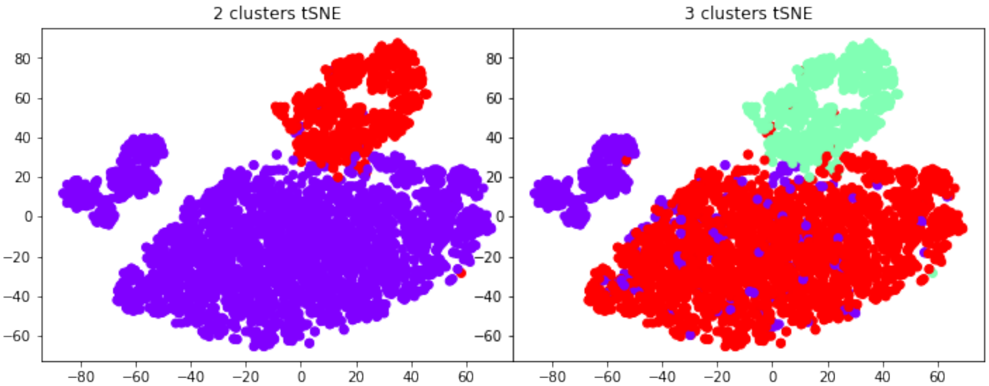
2.3 Clustering

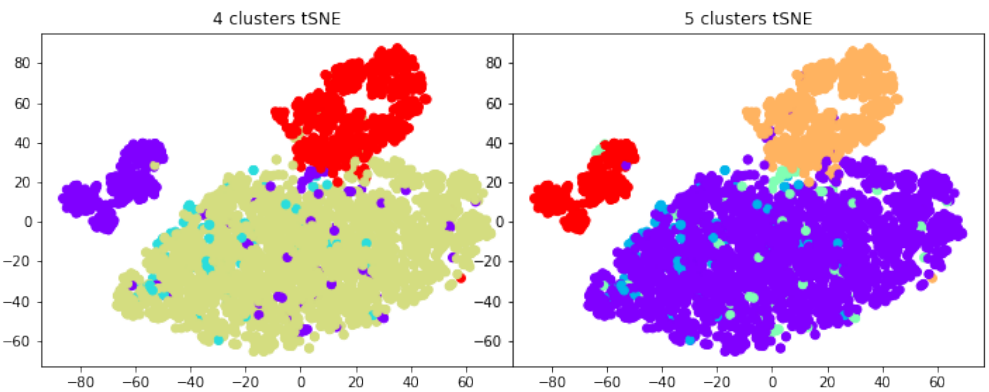
We decided to go with a type of hierarchical clustering for our filtered dataset. We try to visualize the dataset with different number of clusers with UMAP and TSNE.

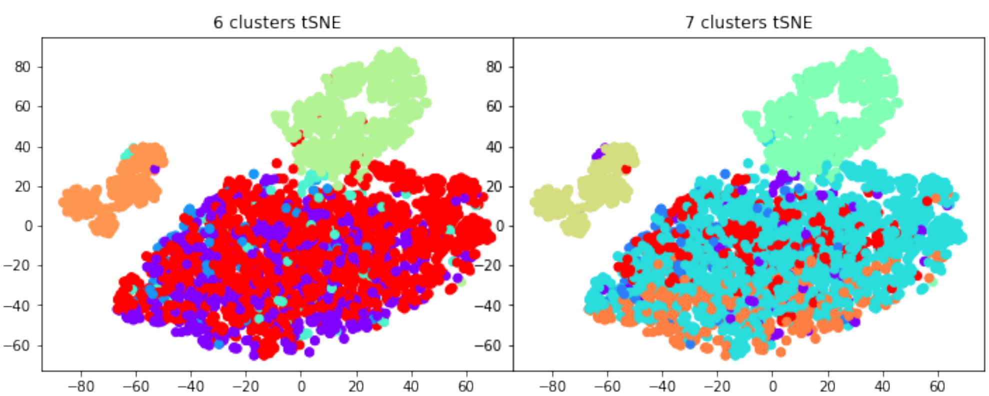
[ ]:
```
#Clustering using AgglomerativeClustering
from sklearn.cluster import AgglomerativeClustering
```

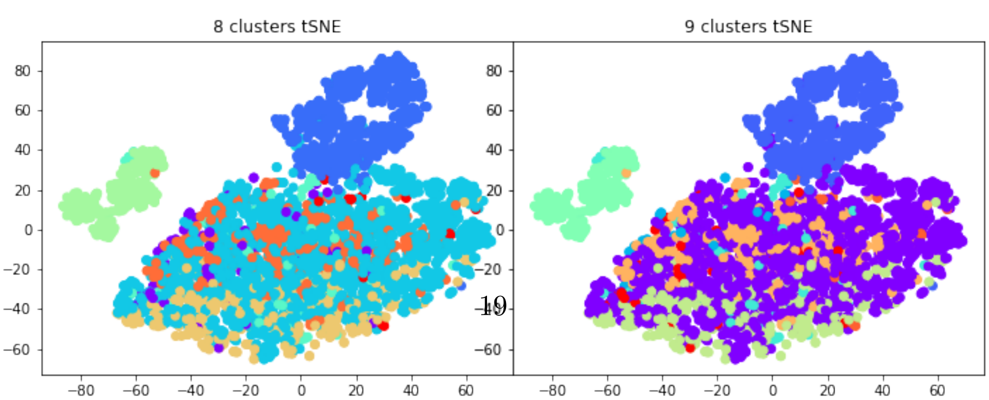
[ ]:
```
#Comibing plots with differet number of clusters, visualizing with tSNE
fig = plt.figure
fig, axs = plt.subplots(4,2, figsize=(12, 24))
fig.subplots_adjust(hspace = .5, wspace=.001)
axs = axs.ravel()
```

```
for k in range(2,10):
    clustering_Agglomerative = AgglomerativeClustering(n_clusters = k,␣
↪affinity= 'precomputed', linkage = 'complete').fit(dist_matrix)
    axs[k-2].scatter(X_tsne[:,0],X_tsne[:,1], c = clustering_Agglomerative.
↪labels_, cmap='rainbow')
    axs[k-2].set_title(str(k)+' clusters tSNE')
```


2 clusters tSNE
3 clusters tSNE
80
60
40
20
0
−20
−40
−60
−80
−60
−40
−20
0
20
40
60


4 clusters tSNE
5 clusters tSNE

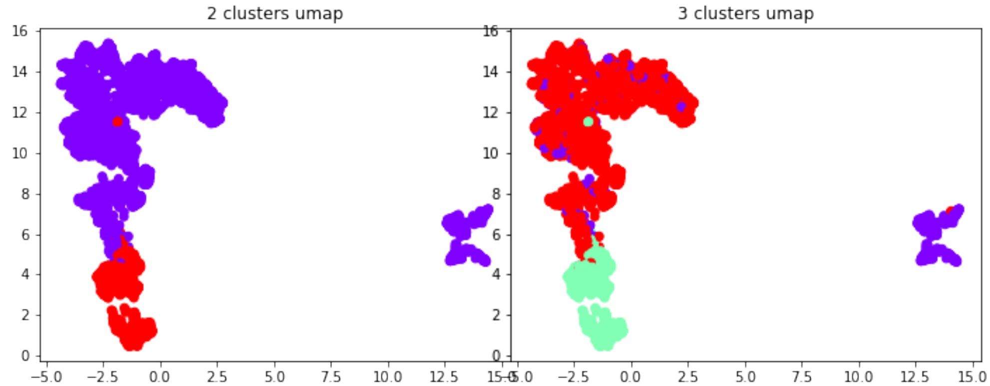

6 clusters tSNE
7 clusters tSNE

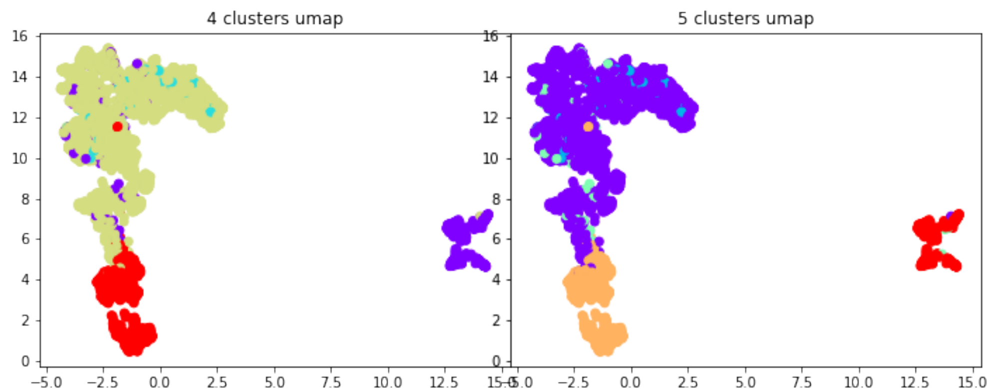

8 clusters tSNE
9 clusters tSNE

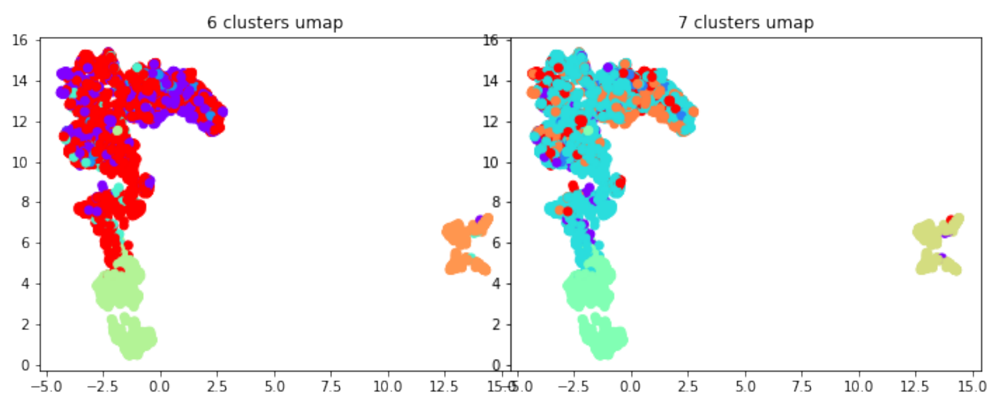
[ ]:

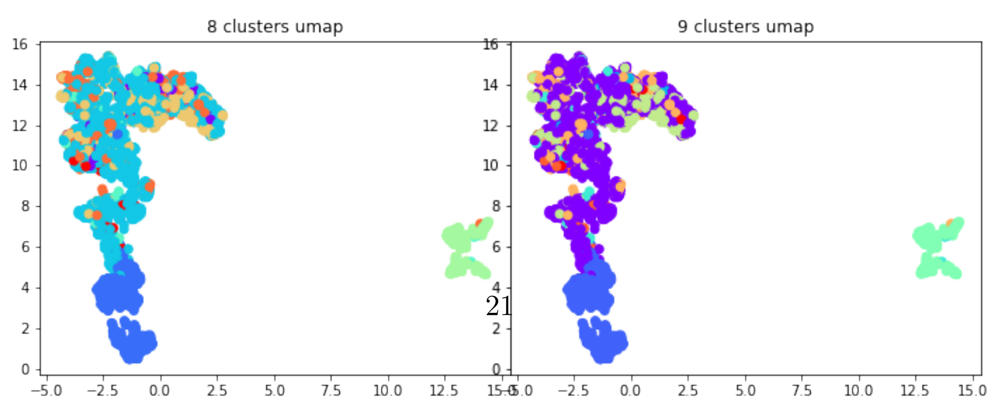
```
#Comibing plots with differet number of clusters, visualizing with UMAP
fig = plt.figure
fig, axs = plt.subplots(4,2, figsize=(12, 24))
fig.subplots_adjust(hspace = .5, wspace=.001)
axs = axs.ravel()

for k in range(2,10):
    clustering_Agglomerative = AgglomerativeClustering(n_clusters = k,␣
 ↪affinity= 'precomputed', linkage = 'complete').fit(dist_matrix)
    axs[k-2].scatter(X_umap[:,0],X_umap[:,1], c = clustering_Agglomerative.
 ↪labels_, cmap='rainbow')
    axs[k-2].set_title(str(k)+' clusters umap')
```


2 clusters umap
3 clusters umap


4 clusters umap
5 clusters umap

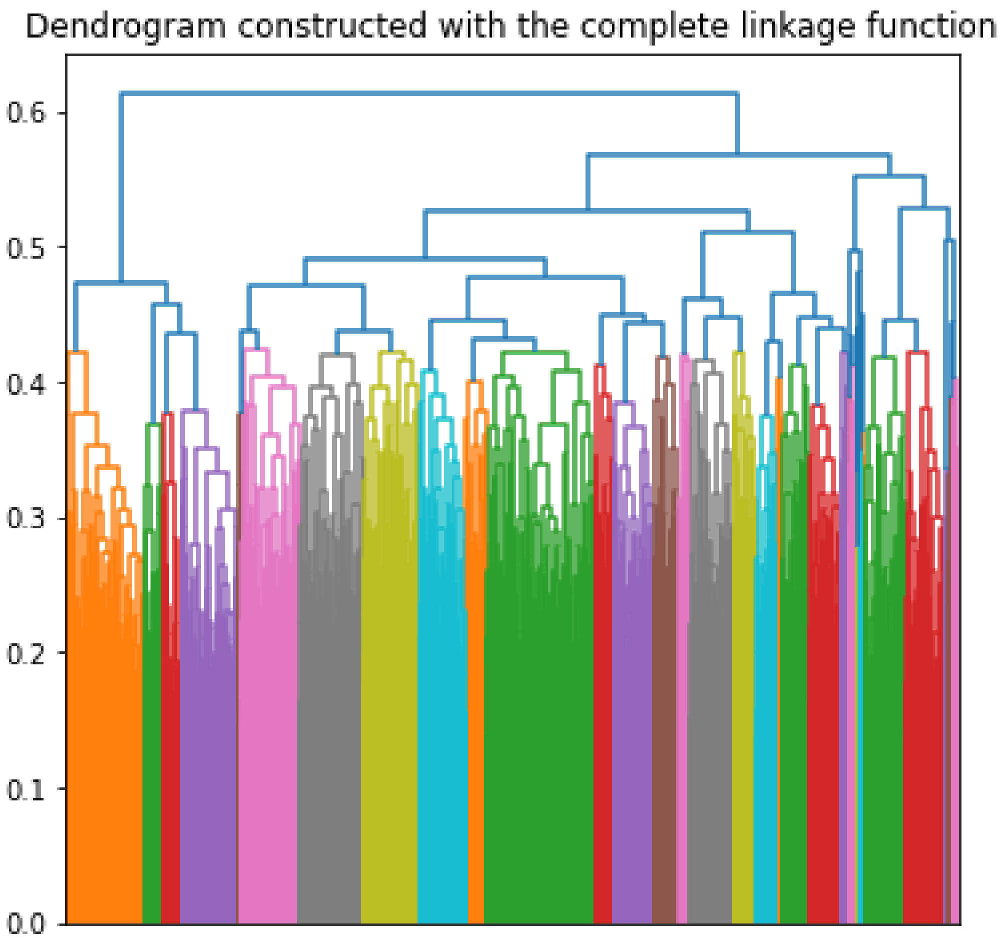

6 clusters umap
7 clusters umap


8 clusters umap
9 clusters umap

Next we try to select a reasonable number of clusters to work with. To do so we first visualize the dendrogram constructed from the complete linkage function.

[ ]:
```
from scipy.cluster import hierarchy
from scipy.spatial.distance import squareform
```

[ ]:
```
condensed_dist_matrix = squareform(dist_matrix)
Z = hierarchy.linkage(condensed_dist_matrix, 'complete')
plt.figure(figsize=(6,6))
plt.title("Dendrogram constructed with the complete linkage function")
dn = hierarchy.dendrogram(Z, no_labels = True)
```



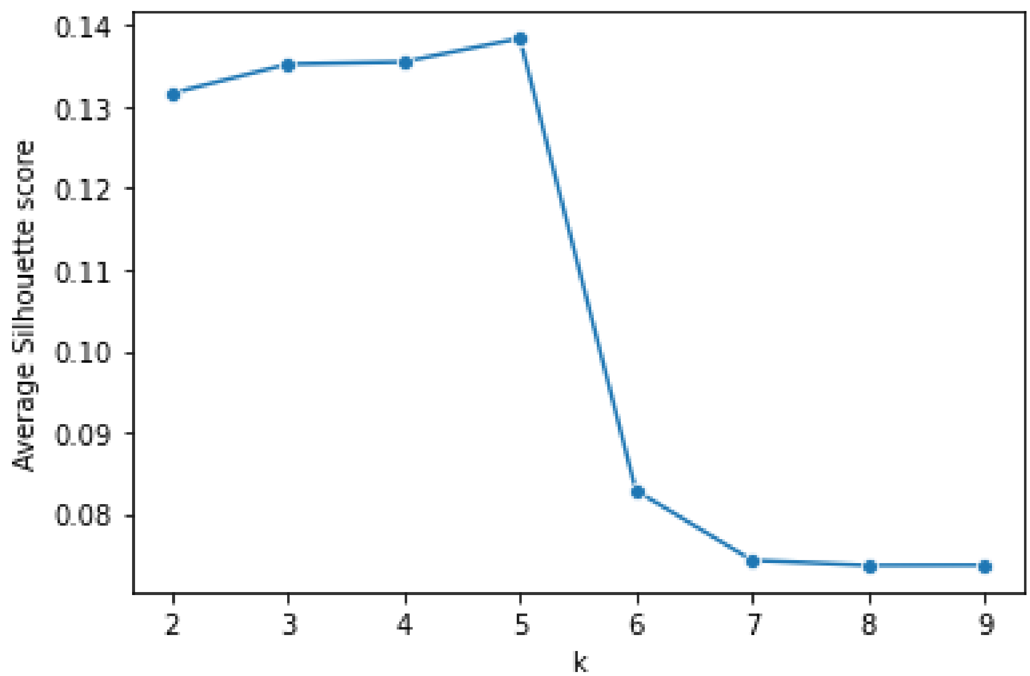
The dendrogram seems to suggest that cutting somewhere between 0.5 and 0.6 would be a good choice. The result should be around 4 or 5 clusters. To cross validate this we also take a look at the Silhouette score of the clusters.

```
[ ]: from sklearn.metrics import silhouette_score
```

```
[ ]: silhouette_scores = []

     for k in range(2,10):
         clustering_Agglomerative = AgglomerativeClustering(n_clusters = k,␣
      ↪affinity= 'precomputed', linkage = 'complete').fit(dist_matrix)
         silhouette_scores.append(silhouette_score(dist_matrix,␣
      ↪clustering_Agglomerative.labels_, metric = 'precomputed'))

     sns.lineplot(x = range(2,10), y = silhouette_scores, marker = 'o')
     plt.xlabel('k')
     plt.ylabel('Average Silhouette score')
     plt.show()
```



Here we compare the average Silhouette scores of each cluster even though the clustering method we chose isn't centroid based.

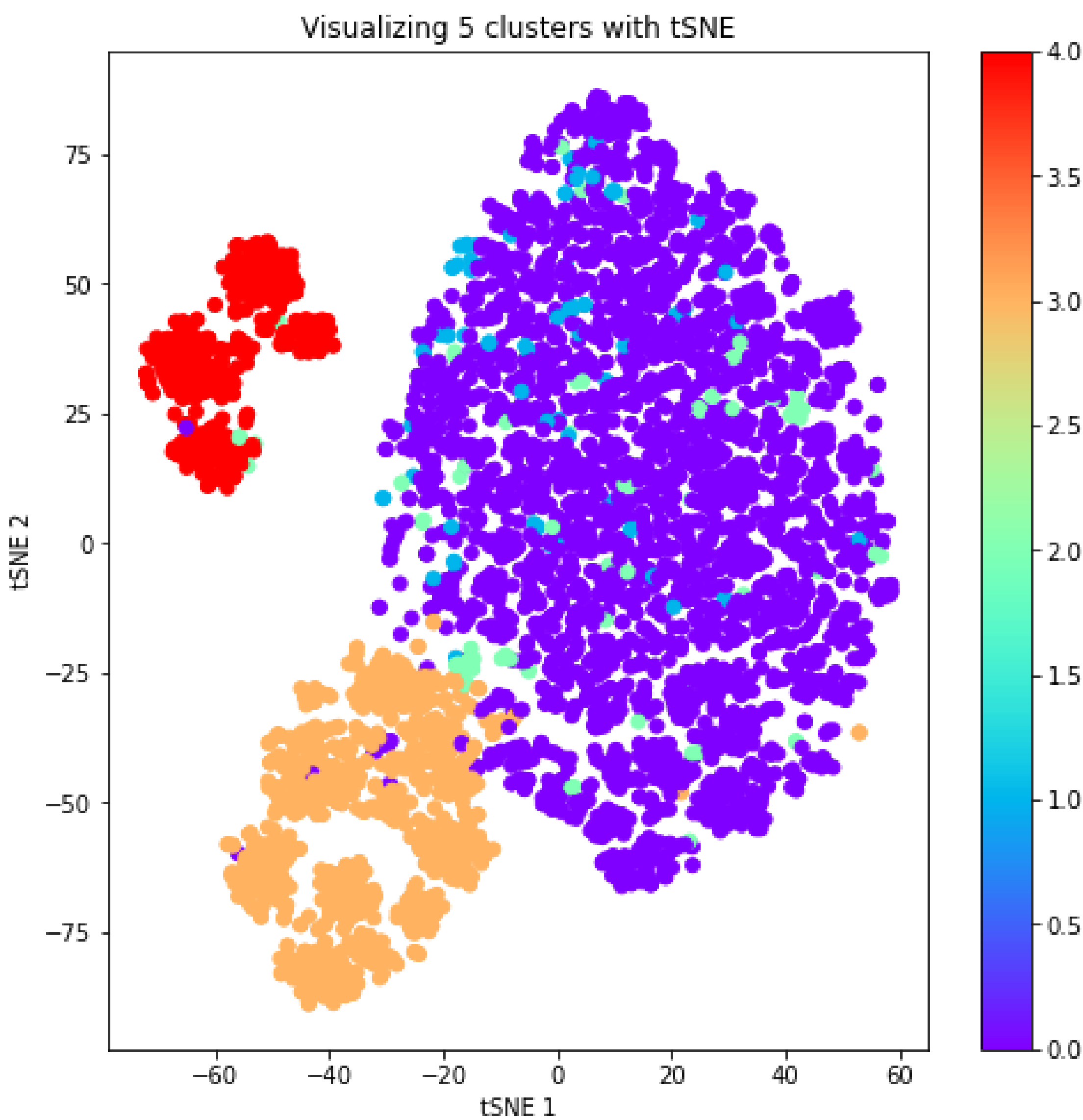
Based on both the dendrogram and the average Silhouette scores computed above, we decided on working with 5 clusters.

```
[ ]: clustering = AgglomerativeClustering(n_clusters = 5, affinity= 'precomputed',␣
      ↪linkage = 'complete').fit(dist_matrix)
```

Here we try to visualize the 5 clusters using tSNE and UMAP.

[ ]:
```
plt.figure(figsize=(8,8))
plt.scatter(X_tsne [:,0],X_tsne [:,1], c = clustering.labels_, cmap='rainbow')
plt.title("Visualizing 5 clusters with tSNE")
plt.xlabel('tSNE 1')
plt.ylabel('tSNE 2')
plt.colorbar()
```

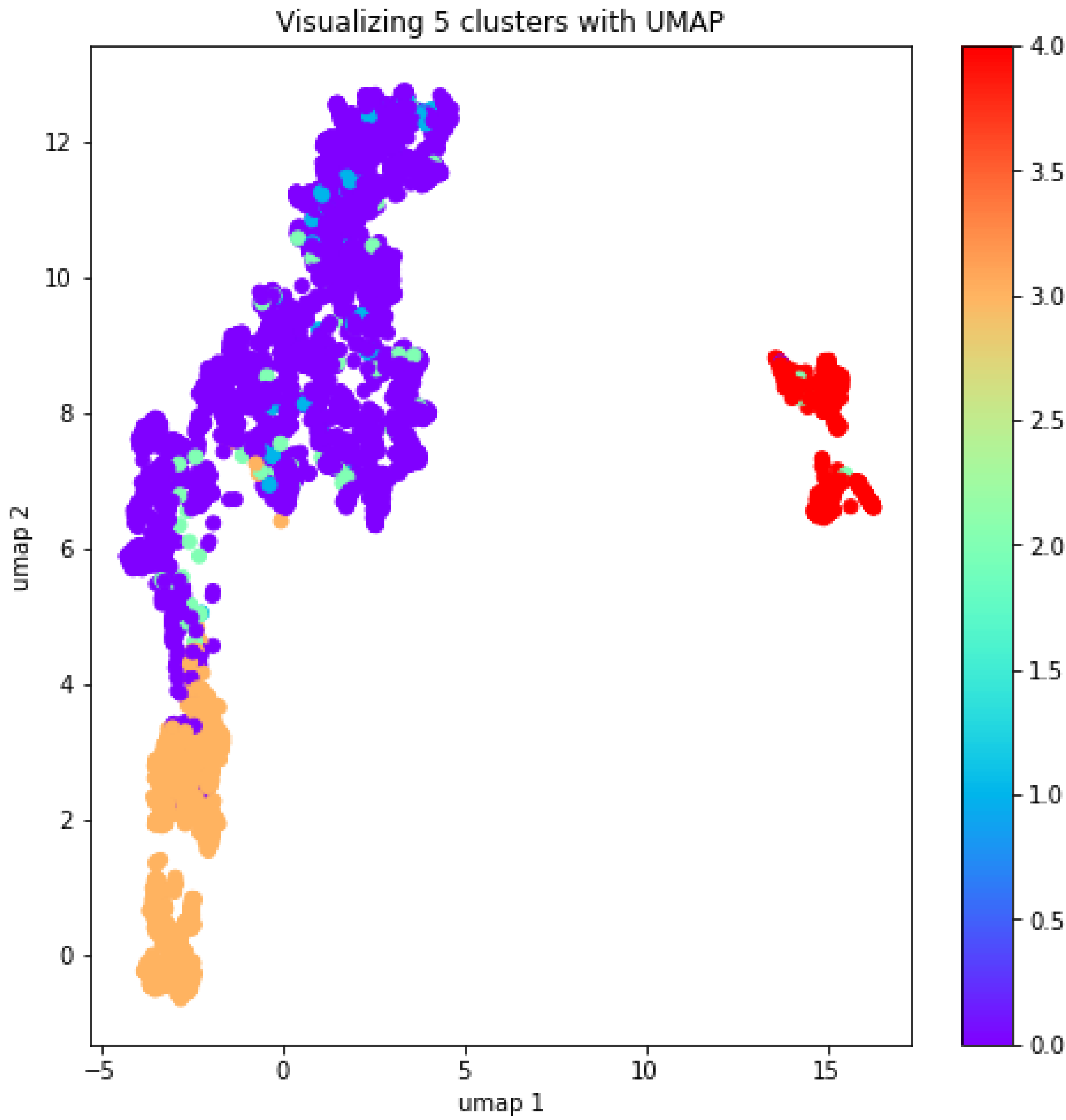
[ ]: <matplotlib.colorbar.Colorbar at 0x7fdb231732b0>



[ ]:
```
plt.figure(figsize=(8,8))
plt.scatter(X_umap [:,0],X_umap [:,1], c = clustering.labels_, cmap='rainbow')
plt.title("Visualizing 5 clusters with UMAP")
```

```
plt.xlabel('umap 1')
plt.ylabel('umap 2')
plt.colorbar()
```

[ ]: <matplotlib.colorbar.Colorbar at 0x7fdb2b5a8bb0>



2.4 Replacing Credit Score value with Clustering results

Next we proceed to replace the missing credit scores with the clustering results.

```
[ ]: #Adding cluster labels in the filtered data frame
X['ClusterId'] = pd.Series(clustering.labels_, X.index)
```

```
[ ]: X.head()
```

```
[ ]:      LOAN_ID CHANNEL                                 SELLER  \
    0   33229873       R                                  Other
    1   40837377       C                                  Other
    2   41378070       C  Truist Bank (formerly SunTrust Bank)
    3  125559711       R                                  Other
    4   94867529       R                                  Other

                                   SERVICER  ORIG_RATE  CURR_RATE  ORIG_UPB  \
    0                                 Other      3.875      3.875  105000.0
    1            Pingora Loan Servicing, LLC      4.625      4.625  238000.0
    2  Truist Bank (formerly SunTrust Bank)      3.750      3.750  409000.0
    3                                 Other      3.090      3.090  314000.0
    4                                 Other      4.250      4.250  232000.0

       ISSUANCE_UPB  CURRENT_UPB  ORIG_TERM  … MOD_FLAG  ISSUE_SCOREB  SERV_IND  \
    0     103725.97     91736.60        360  …        N         807.0         N
    1     234629.13    211659.69        360  …        N         618.0         N
    2     406027.37    370177.64        360  …        N         776.0         N
    3     309175.28    309175.28        360  …        N         817.0         N
    4     230000.00    223310.96        360  …        N         805.0         N

       CUMULATIVE_CREDIT_EVENT_NET_GAIN_OR_LOSS RELOCATION_MORTGAGE_INDICATOR  \
    0                                      0.0                             N
    1                                      0.0                             N
    2                                      0.0                             N
    3                                      0.0                             N
    4                                      0.0                             N

       HIGH_BALANCE_LOAN_INDICATOR  FORBEARANCE_INDICATOR        DEAL_NAME  \
    0                            N                      7  CAS 2016 C03 G2
    1                            N                      7  CAS 2017 C03 G1
    2                            N                      7  CAS 2017 C04 G2
    3                            N                      7  CAS 2021 R01 G1
    4                            N                      7  CAS 2020 R01 G1

       ADR_TYPE ClusterId
    0         7         0
    1         7         0
    2         7         0
    3         7         4
    4         7         0
```

```
[5 rows x 38 columns]
```

[ ]:
```
X.shape
```

[ ]:
```
(4897, 38)
```

We isolate the rows with the missing credit scores and also split the original dataframe into 5 separate dataframes based on the clustering.

[ ]:
```
toreplace = X[ X['LOAN_ID'].isin([94259369,26909202, 43103805, 94576786]) ]
```

[ ]:
```
cluster0 = X[X['ClusterId'] == 0]
cluster1 = X[X['ClusterId'] == 1]
cluster2 = X[X['ClusterId'] == 2]
cluster3 = X[X['ClusterId'] == 3]
cluster4 = X[X['ClusterId'] == 4]
```

Here we take note of the sizes of each cluster. We see that the first cluster is significantly larger than the rest, and the next two clusters is significantly smaller than the rest.

[ ]:
```
print(cluster0.shape)
print(cluster1.shape)
print(cluster2.shape)
print(cluster3.shape)
print(cluster4.shape)
```

```
(3344, 38)
(82, 38)
(75, 38)
(952, 38)
(448, 38)
```

We found that using our approach, the first and last loans were grouped into to the fourth cluster and the second and third loans were grouped into the first cluster.

[ ]:
```
toreplace
```

[ ]:
```
       LOAN_ID CHANNEL                 SELLER                SERVICER  \
1772  94259369       R  Wells Fargo Bank, N.A.  Wells Fargo Bank, N.A.
1933  26909202       R  Wells Fargo Bank, N.A.  Wells Fargo Bank, N.A.
3529  43103805       R                   Other                   Other
4759  94576786       R                   Other                   Other

      ORIG_RATE  CURR_RATE  ORIG_UPB  ISSUANCE_UPB  CURRENT_UPB  ORIG_TERM  \
1772      3.750      3.750   69000.0      36772.61     20602.03        180
1933      4.625      4.625  180000.0     176231.93    152852.86        360
3529      3.625      3.625  120000.0     119070.60    108745.27        360
4759      4.000      4.000  149000.0      73153.42     52627.71        180
```

```
      … MOD_FLAG  ISSUE_SCOREB  SERV_IND  \
1772  …         N     638.00000        N
1933  …         N     781.00000        N
3529  …         N     806.00000        N
4759  …         N     752.03359        N

      CUMULATIVE_CREDIT_EVENT_NET_GAIN_OR_LOSS RELOCATION_MORTGAGE_INDICATOR  \
1772                                      0.0                             N
1933                                      0.0                             N
3529                                      0.0                             N
4759                                      0.0                             N

      HIGH_BALANCE_LOAN_INDICATOR  FORBEARANCE_INDICATOR        DEAL_NAME  \
1772                            N                      7    CAS 2019-HRP1
1933                            N                      7  CAS 2015 C01 G2
3529                            N                      7  CAS 2017 C05 G1
4759                            N                      7    CAS 2019-HRP1

      ADR_TYPE ClusterId
1772         7         3
1933         7         0
3529         7         0
4759         7         3

[4 rows x 38 columns]
```

We then calculated the average credit score of the clusters and then filled in the missing values, as instructed in the prompt.

```
[ ]: mean0 = cluster0['CSCORE_B'].mean()
     mean3 = cluster3['CSCORE_B'].mean()

     X_filled = X

     X_filled.loc[X_filled['LOAN_ID'] == 94259369, 'CSCORE_B'] = X_filled.
      ↪loc[X_filled['LOAN_ID'] == 94259369, 'CSCORE_B'].fillna(mean3)
     X_filled.loc[X_filled['LOAN_ID'] == 26909202, 'CSCORE_B'] = X_filled.
      ↪loc[X_filled['LOAN_ID'] == 26909202, 'CSCORE_B'].fillna(mean0)
     X_filled.loc[X_filled['LOAN_ID'] == 43103805, 'CSCORE_B'] = X_filled.
      ↪loc[X_filled['LOAN_ID'] == 43103805, 'CSCORE_B'].fillna(mean0)
     X_filled.loc[X_filled['LOAN_ID'] == 94576786, 'CSCORE_B'] = X_filled.
      ↪loc[X_filled['LOAN_ID'] == 94576786, 'CSCORE_B'].fillna(mean3)
```

```
[ ]: print(mean0)
     print(mean3)
```

```
754.0032914422502
757.2589473684211
```

We double check to see if we filled in the credit scores successfully.

[ ]:
```
X_filled['CSCORE_B'].isnull().any()
```

[ ]:
```
False
```

3 Step 2: Visualizing the distribution of the Borrower Credit Score at Origination

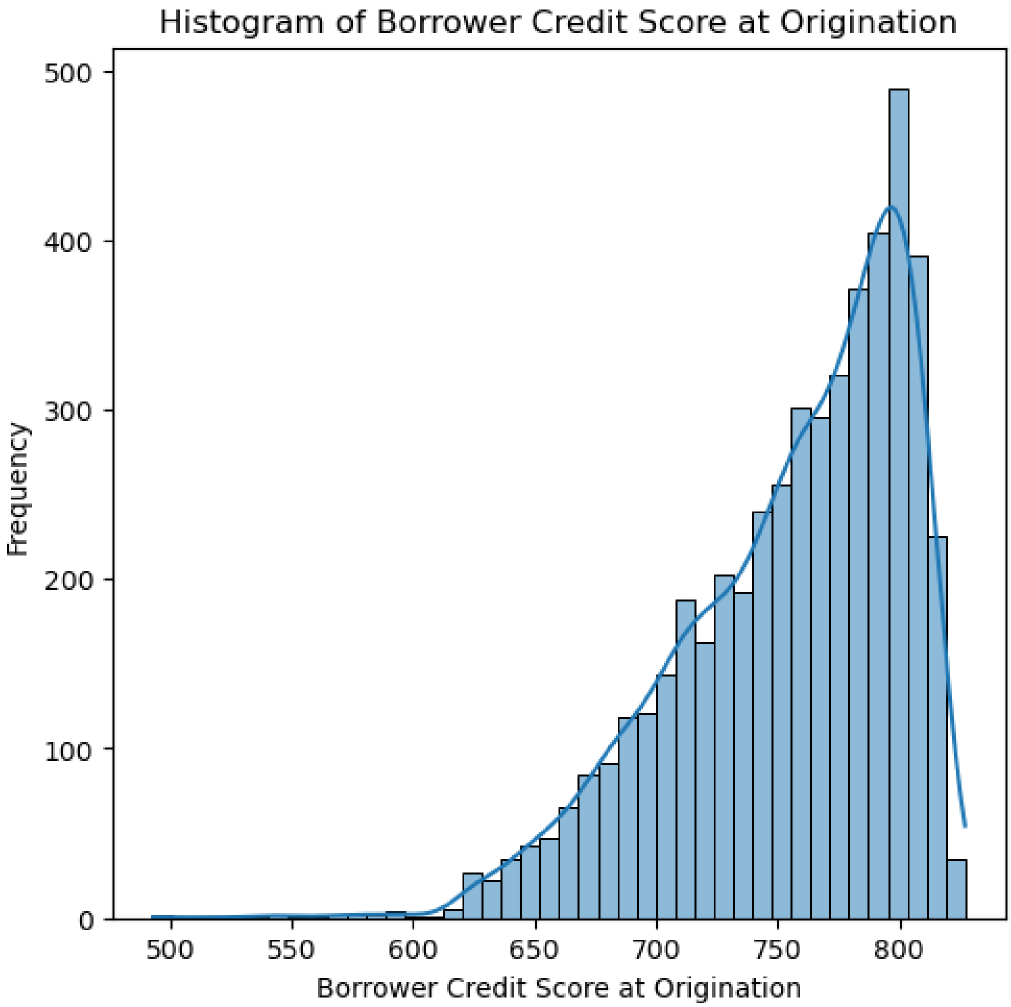
3.1 Visualizing the distribution using seaborn

After filling in the missing credit scores, we visualize the distribution of the credit scores with a histogram plot. We fit a nonparametric smooth with KDE to help with the visualization.

[ ]:
```
print(X_filled['CSCORE_B'].describe())
plt.figure(figsize = (6,6), dpi=100)
sns.histplot(data=X_filled['CSCORE_B'], stat = 'count', kde = True)
plt.title("Histogram of Borrower Credit Score at Origination")
plt.xlabel("Borrower Credit Score at Origination")
plt.ylabel("Frequency")
```

```
count    4901.000000
mean      755.288211
std        47.271554
min       493.000000
25%       725.000000
50%       766.000000
75%       794.000000
max       827.000000
Name: CSCORE_B, dtype: float64
```

[ ]:
```
Text(0, 0.5, 'Frequency')
```



We notice that the distribution is left-skewed and the Borrower Credit Scores at Originiation are concentrated around 800.

4 Step 3: Identifying a parametric model that fits the distribution of the Borrower Credit Score at Origination

4.1 Visualizing the distributions of different parametric fits

[ ]:
```
import scipy.stats
from tabulate import tabulate
```

We use the .fit() function from scipy.stats to fit the credit score data to the distributions we've encountered in lectures. We then plot each fit against the original histogram.

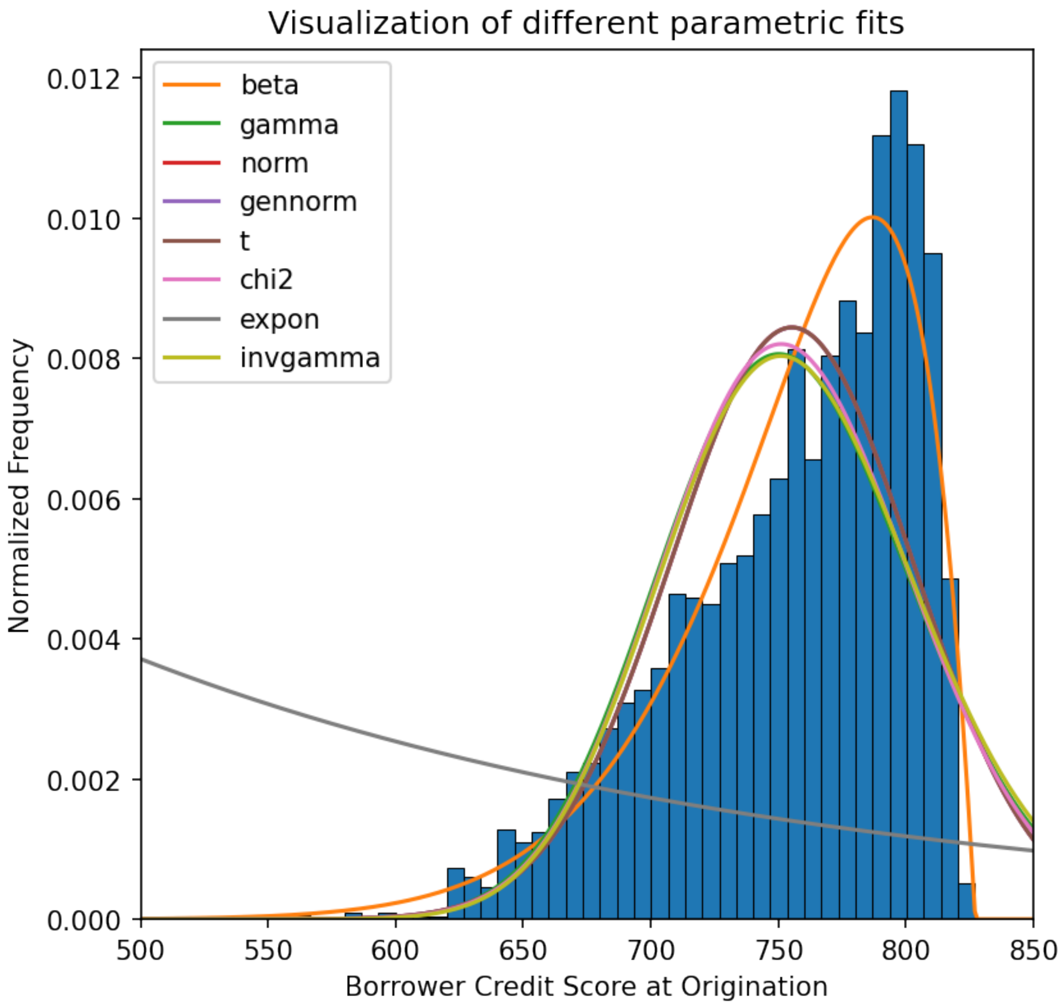
We note that in our implementation we normalized the histogram by setting density = True; this avoids rescaling each pdf from scipy.stats.

[ ]:
```
plt.figure(figsize=(6, 6), dpi=150)
N = len(X_filled['CSCORE_B'].values.flatten())
x = np.arange(N)
y = X_filled['CSCORE_B']
h = plt.hist(y.values.flatten(), bins=50, linewidth=0.5, edgecolor = 'black',␣
 ↪density = True)

sample_distributions = ['beta', 'gamma', 'norm', 'gennorm', 't', 'chi2',␣
 ↪'expon', 'invgamma']
for distributions in sample_distributions:
    distribution = getattr(scipy.stats, distributions)
    parameters = distribution.fit(y)
    args = parameters[:-2]
    floc = parameters[-2]
    fscale = parameters[-1]
    if len(args) > 0:
        pdf = distribution.pdf(x, *args, loc=floc, scale=fscale)
    else:
        pdf = distribution.pdf(x, loc=floc, scale=fscale)

    plt.plot(pdf, label=distributions)
    plt.xlim(500,850)

plt.xlabel("Borrower Credit Score at Origination")
plt.ylabel("Normalized Frequency")
plt.title("Visualization of different parametric fits")
plt.legend( prop={"size":10})
plt.show()
```



4.2 AIC value of each fit

From the visualization we notice that the parametric fits with the beta seems to capture the left-skewness of the original distribution the best, and the parametric fit with the exponential distribution fit the worst. We validate this by computing the AIC of each fitted parametric model.

[ ]:
```
AIC_results=[]

for distributions in sample_distributions:
    np.seterr(divide = 'ignore')
    distribution = getattr(scipy.stats, distributions)
    parameters = distribution.fit(y)
    args = parameters[:-2]
    floc = parameters[-2]
    fscale = parameters[-1]
    if len(args) > 0:
```

```
        aic = -2*np.sum(distribution.logpdf(y, *args, loc = floc, scale =␣
↪fscale)) + 2*(len(parameters))
        rs ={          "Distribution":distributions,
                       "distribution parameters":args,
                       "location":floc,
                       "scale":fscale,
                       "AIC": aic}
        AIC_results.append(rs)
    if len(args) == 0:
        aic = -2*np.sum(distribution.logpdf(y, loc = floc, scale = fscale)) +␣
↪2*(len(parameters))
        rs ={          "Distribution":distributions,
                       "distribution parameters":args,
                       "location":floc,
                       "scale":fscale,
                       "AIC": aic}
        AIC_results.append(rs)

AIC_df = pd.DataFrame(AIC_results).sort_values(by = ['AIC'])
```

[ ]:

```
#this table might be hard to read in the exported pdf, but in the Jupyter␣
↪notebook the table looks quite nice
print(tabulate(AIC_df, headers='keys', tablefmt='psql', showindex=False))
```

```
+----------------+------------------------------------------+------------+------
--------+---------+
| Distribution   | distribution parameters                  |   location |
scale |     AIC |
|----------------+------------------------------------------+------------+------
--------+---------|
| beta           | (17.734212702067985, 2.0140878739300003) |   122.093  |
705.088   | 50265.1 |
| norm           | ()                                       |   755.288  |
47.2667  | 51707.1 |
| t              | (121721.78219990141,)                    |   755.289  |
47.2664  | 51709   |
| gennorm        | (2.0005800863015124,)                    |   755.283  |
66.8553  | 51709.1 |
| chi2           | (414.579797372979,)                      |    52.8342 |
1.69244 | 51945   |
| gamma          | (143.9960393295234,)                     |   158.692  |
4.13667 | 51997   |
| invgamma       | (375.59554551199284,)                    |  -210.105  |
361948       | 52060.9 |
| expon          | ()                                       |   493      |
262.288   | 64397.7 |
+----------------+------------------------------------------+------------+------
```

```
--------+---------+
```

We note that the fitted model using the beta distribution has the smallest AIC value amongst the 8 models, and the fitted model using the exponential distribution has the highest AIC value. This is consistent with what we observed above.

Therefore based on the visualization and the AIC values, we decided to model the credit scores using the beta distribution.

4.3 MLE of the parameters

The pdf of the beta distribution is given by

$$f(x;\alpha,\beta,\mu,\sigma) = \frac{1}{\sigma}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{x-\mu}{\sigma}\right)^{\alpha-1}\left(1-\frac{x-\mu}{\sigma}\right)^{\beta-1}.$$

The MLE of the four parameters

$$\hat{\theta}_{MLE} = (\hat{\alpha},\hat{\beta},\hat{\mu},\hat{\sigma})$$

is given by

$$\hat{\theta}_{MLE} = \underset{\theta=(\alpha,\beta,\mu,\sigma)\in\mathbb{R}^4}{\operatorname{argmax}} L(\theta),$$

where

$$L(\theta) = \prod_{i=1}^{N} f(X_i;\theta) = \prod_{i=1}^{N} f(X_i;\alpha,\beta,\mu,\sigma).$$

In our setup, $N = 4901$, the number of credit scores we are considering.

In general, there is no known closed form expression of the MLEs for the beta distribution for arbitrary shape parameters, so we approximate them numerically instead. Using the .fit() function from scipy.stats, we find that the MLE is given by the following.

[ ]:
```python
parameters = scipy.stats.beta.fit(y)
alpha_MLE = parameters[0]
beta_MLE =parameters[1]
mu_MLE = parameters[2]
sigma_MLE = parameters[3]

print("MLE of alpha: " + str(alpha_MLE))
print("MLE of beta: " + str(beta_MLE))
print("MLE of mu: " + str(mu_MLE))
print("MLE of sigma: " + str(sigma_MLE))
```

```
MLE of alpha: 17.734212702067985
MLE of beta: 2.0140878739300003
MLE of mu: 122.09282878984381
MLE of sigma: 705.0881369452222
```

5 Step 4: Visualizing the borrower credit score at origination over different property types

5.1 Splitting the filtered dataframe by property type

[ ]:
```
X_filled['PROP'].value_counts()
```

[ ]:
```
SF    3277
PU    1079
CO     464
MH      52
CP      29
Name: PROP, dtype: int64
```

[ ]:
```
X_SF = X_filled[ X_filled['PROP'] == "SF"]
X_PU = X_filled[ X_filled['PROP'] == "PU"]
X_CO = X_filled[ X_filled['PROP'] == "CO"]
X_MH = X_filled[ X_filled['PROP'] == "MH"]
X_CP = X_filled[ X_filled['PROP'] == "CP"]
```

[ ]:
```
df_dict = {
    "All": X_filled,
    "Single-family home": X_SF,
    "Planned Urban Development": X_PU,
    "Condominium": X_CO,
    "Manufactured home": X_MH,
    "Co-operative": X_CP
}
```

5.2 Fitting the splitted credit scores over property type using the skewed norm distribution

[ ]:
```
beta_results=[]

for prop, df in df_dict.items():
        y = df['CSCORE_B']
        parameters = scipy.stats.beta.fit(y)
        alpha_MLE = parameters[0]
        beta_MLE = parameters[1]
        mu_MLE = parameters[2]
        sigma_MLE = parameters[3]
        rs ={          "Property Type": prop,
                       "Counts": len(df['CSCORE_B']),
                       "MLE for alpha": alpha_MLE,
                       "MLE for beta": beta_MLE,
                       "MLE for mu": mu_MLE,
                       "MLE for sigma": sigma_MLE
```

```
                    }
        beta_results.append(rs)

beta_df = pd.DataFrame(beta_results)
```

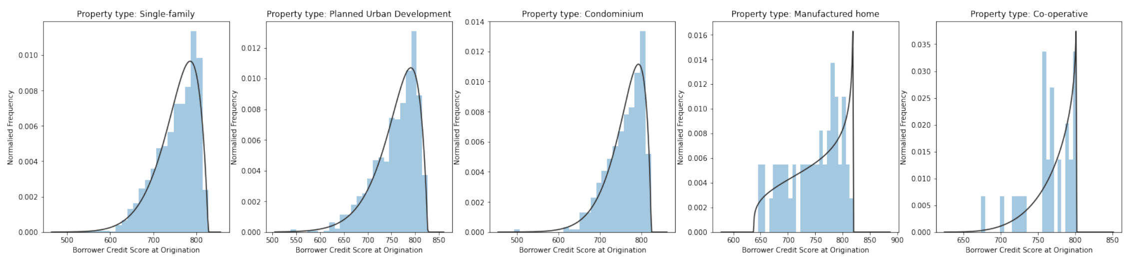
5.3 Final results

```
[ ]: fig, axs = plt.subplots(1, 5, figsize=(27.5, 5.5))
sns.distplot(X_SF ['CSCORE_B'], fit = scipy.stats.beta, bins = 25, kde = False,␣
 ↪ax=axs[0])
sns.distplot(X_PU ['CSCORE_B'], fit = scipy.stats.beta, bins = 25, kde = False,␣
 ↪ax=axs[1])
sns.distplot(X_CO ['CSCORE_B'], fit = scipy.stats.beta, bins = 25, kde = False,␣
 ↪ax=axs[2])
sns.distplot(X_MH ['CSCORE_B'], fit = scipy.stats.beta, bins = 25, kde = False,␣
 ↪ax=axs[3])
sns.distplot(X_CP ['CSCORE_B'], fit = scipy.stats.beta, bins = 25, kde = False,␣
 ↪ax=axs[4])

axs[0].set(xlabel = "Borrower Credit Score at Origination", ylabel = "Normalied␣
 ↪Frequency", title = "Property type: Single-family")
axs[1].set(xlabel = "Borrower Credit Score at Origination", ylabel = "Normalied␣
 ↪Frequency", title = "Property type: Planned Urban Development")
axs[2].set(xlabel = "Borrower Credit Score at Origination", ylabel = "Normalied␣
 ↪Frequency", title = "Property type: Condominium")
axs[3].set(xlabel = "Borrower Credit Score at Origination", ylabel = "Normalied␣
 ↪Frequency", title = "Property type: Manufactured home")
axs[4].set(xlabel = "Borrower Credit Score at Origination", ylabel = "Normalied␣
 ↪Frequency", title = "Property type: Co-operative")
print(tabulate(beta_df, headers='keys', tablefmt='psql', showindex=False))
#this table might be hard to read in the exported pdf, but in the Jupyter␣
 ↪notebook the table looks quite nice
```

```
+---------------------------+----------+-----------------+----------------+-----
---------+-----------------+
| Property Type             |   Counts |   MLE for alpha |   MLE for beta |
MLE for mu |   MLE for sigma |
|---------------------------+----------+-----------------+----------------+-----
---------+-----------------|
| All                       |     4901 |         17.7342 |        2.01409 |
122.093  |         705.088 |
| Single-family home        |     3277 |         13.8572 |        1.98798 |
237.643  |         589.604 |
| Planned Urban Development |     1079 |         24.5124 |        1.95576 |
-84.4683 |         910.772 |
| Condominium               |      464 |         18.7054 |        1.78287 |
```

```
97.9206 |          726.607 |
| Manufactured home           |       52 |          1.2604  |        0.76409  |
637.636  |          182.364 |
| Co-operative                |       29 |          4.57278 |        0.858519 |
617.289  |          184.711 |
+----------------------------+----------+-----------------+----------------+-----
---------+-----------------+
```



Based on the visualization of the parametric fits and the MLE for the three parameters, it seems like the distribution changes across different property types.

6 Step 5: Hypothesis testing

6.1 Mathematical setup

We model the bowerrer's credit scores at origination by independent random variables $\{X_i\}_{i=1}^{4901}$ where the X_i's are drawn from the beta distribution. If X_i is drawn from the beta distribution, then we write $X_i \sim beta(\theta_i)$, where $\theta_i = (\alpha_i, \beta_i, \mu_i, \sigma_i)$.

Recall that under certain regularity conditions, the MLE is a conisistent estimator for θ. So practically speaking, if we assume that $X_i \sim beta(\theta_0)$ for all $i \in \{1, 2, ..., N\}$, we can approximate θ_0 with $\hat{\theta}_{MLE} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{N} f(X_i; \theta)$, where $f(x; \theta)$ is the pdf of the beta distrbution.

If the distribution of parameter scores does not change over the various property types, then the three parameters for the parametric fit should be constant. Therefore we propose the following test by partitioning the credit scores by the 5 different property types.

We again model the bowerrer's credit scores at origination by independent random variables $\{X_i\}_{i=1}^{4901}$ where the X_i's are drawn from the beta distribution, but we assume in addition that the parameter θ in $beta(\theta)$ is constant for credit scores of loans of the same property type.

Recall that these are the MLE parameters found using the .fit() function over credit scores of various property types.

[ ]:
```python
#this table might be hard to read in the exported pdf, but in the Jupyter␣
↪notebook the table looks quite nice
print(tabulate(beta_df, headers='keys', tablefmt='psql', showindex=False))
```

```
+----------------------------+----------+-----------------+----------------+-----
---------+-----------------+
```

```
| Property Type              |   Counts |   MLE for alpha |   MLE for beta |
MLE for mu |   MLE for sigma |
|----------------------------+----------+-----------------+----------------+-----
---------+-----------------|
| All                        |     4901 |        17.7342  |       2.01409  |
122.093  |          705.088 |
| Single-family home         |     3277 |        13.8572  |       1.98798  |
237.643  |          589.604 |
| Planned Urban Development  |     1079 |        24.5124  |       1.95576  |
-84.4683 |          910.772 |
| Condominium                |      464 |        18.7054  |       1.78287  |
97.9206 |          726.607 |
| Manufactured home          |       52 |         1.2604  |       0.76409  |
637.636  |          182.364 |
| Co-operative               |       29 |         4.57278 |       0.858519 |
617.289  |          184.711 |
+----------------------------+----------+-----------------+----------------+-----
---------+-----------------+
               Property Type  Counts  MLE for alpha  MLE for beta  MLE for mu  \
0                        All    4901      17.734213      2.014088  122.092829
1         Single-family home    3277      13.857189      1.987978  237.643390
2  Planned Urban Development    1079      24.512438      1.955759  -84.468349
3                Condominium     464      18.705377      1.782865   97.920585
4          Manufactured home      52       1.260396      0.764090  637.635602
5               Co-operative      29       4.572777      0.858519  617.289067

   MLE for sigma
0     705.088137
1     589.604050
2     910.771506
3     726.606650
4     182.364398
5     184.710933
```

We define $\Theta_0 = \{\theta = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5) \in \mathbb{R}^{20} \mid \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5, \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5, \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5, \sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5\}$ and we wish to test

$$H_0 : \theta \in \Theta_0 \quad \text{versus} \quad H_1 : \theta \in \mathbb{R}^{20} \setminus \Theta_0.$$

The likelihood function $L(\theta)$ is given by

$$L(\theta) = \prod_{i=1}^{4901} f(X_i, \theta_i).$$

We use the likelihood ratio test by constructing the test statistic

$$T = -2 \log \left(\frac{\max_{\theta \in \Theta_0} L(\theta)}{\max_{\theta \in \Theta_0^c} L(\theta)} \right).$$

We note that under H_0, we're assuming that all the independent random variables are drawn from the beta distribution with the same parameter θ_0. By approminating θ_0 with the MLE, we find that $\underset{\theta \in \Theta_0}{\operatorname{argmax}} L(\theta) \approx \hat{\theta}_{MLE}$ where

```
[ ]: %%latex
\begin{multline}
\mathbb{R}^{20} \ni \hat{\theta}_{MLE} = (17.7342,17.7342,17.7342,17.7342,17.
↪7342,2.01409,2.01409,2.01409,2.01409,2.01409, \\ 122.093,122.093,122.093,122.
↪093,122.093,705.088,705.088,705.088,705.088,705.088)
\end{multline}
```

$$
\begin{multline}
\mathbb{R}^{20} \ni \hat{\theta}_{MLE} = (17.7342, 17.7342, 17.7342, 17.7342, 17.7342, 2.01409, 2.01409, 2.01409, 2.01409, 2.01409, \\ 122.093, 122.093, 122.093, 122.093, 122.093, 705.088, 705.088, 705.088, 705.088, 705.088) \quad (1)
\end{multline}
$$

Here the MLE parameters found by using .fit() on all the data points.

Since we freeze the other parameters as constants when we compute the MLEs, we next note that under our additional hypothesis that the parameter θ in $beta(\theta)$ is constant for credit scores of loans of the same property type, we find that $\underset{\theta \in \Theta_0^c}{\operatorname{argmax}} L(\theta) \approx \hat{\theta}_{MLE,5}$, where

```
[ ]: %%latex
\begin{multline}
\mathbb{R}^{20} \ni  \hat{\theta}_{MLE, 5} = (13.8572,  24.5124, 18.7054, 1.
↪2604, 4.57278,1.98798, 1.95576,   1.78287, 0.76409, 0.858519, \\ 237.643,␣
↪-84.4683, 97.9206,  637.636, 617.289, 589.604, 910.772, 726.607, 182.364, ␣
↪184.711)
\end{multline}
```

$$
\begin{multline}
\mathbb{R}^{20} \ni \hat{\theta}_{MLE,5} = (13.8572, 24.5124, 18.7054, 1.2604, 4.57278, 1.98798, 1.95576, 1.78287, 0.76409, 0.858519, \\ 237.643, -84.4683, 97.9206, 637.636, 617.289, 589.604, 910.772, 726.607, 182.364, 184.711) \quad (2)
\end{multline}
$$

Here the MLE parameters are found by using .fit() on the data points partitioned by the different property types.

Furthermore, we note that since $\dim(\Theta_0) = 4$ (this is easy to see as Θ_0 can naturally be written as the nullspace of a rank 4 matrix), by Wilk's theorem we have $T \sim \chi^2_{20-4} = \chi^2_{16}$ under H_0.

The p-value for this test is then $\mathbb{P}(\chi^2_{16} > T)$.

6.2 Implementation

Next we implemenet the test by calculating T for our observed data and the p-value for this test.

```
[ ]: from scipy.stats import chi2
```

We calculate T via

$$T = -2 \log \left(\frac{L(\hat{\theta}_{MLE})}{L(\hat{\theta}_{MLE,5})} \right) = -2 \left(\log L(\hat{\theta}_{MLE}) - \log L(\hat{\theta}_{MLE,5}) \right)$$

where

$$\log L(\theta) = \sum_{i=1}^{4901} \log f(x; \theta).$$

[ ]:
```
alpha, b, mu, sigma = scipy.stats.beta.fit(X_filled['CSCORE_B'])

alpha_1, b_1, mu_1, sigma_1 = scipy.stats.beta.fit(X_SF['CSCORE_B'])
alpha_2, b_2, mu_2, sigma_2 = scipy.stats.beta.fit(X_PU['CSCORE_B'])
alpha_3, b_3, mu_3, sigma_3 = scipy.stats.beta.fit(X_CO['CSCORE_B'])
alpha_4, b_4, mu_4, sigma_4 = scipy.stats.beta.fit(X_MH['CSCORE_B'])
alpha_5, b_5, mu_5, sigma_5 = scipy.stats.beta.fit(X_CP['CSCORE_B'])


logL_H0 = np.sum(scipy.stats.beta.logpdf(X_filled['CSCORE_B'], alpha, b,  loc =␣
 ↪mu, scale = sigma))

logL_H1 = np.sum(scipy.stats.beta.logpdf(X_SF['CSCORE_B'], alpha_1, b_1, loc =␣
 ↪mu_1, scale = sigma_1)) + \
       np.sum(scipy.stats.beta.logpdf(X_PU['CSCORE_B'], alpha_2, b_2, loc =␣
 ↪mu_2, scale = sigma_2)) + \
       np.sum(scipy.stats.beta.logpdf(X_CO['CSCORE_B'], alpha_3, b_3, loc =␣
 ↪mu_3, scale = sigma_3)) + \
       np.sum(scipy.stats.beta.logpdf(X_MH['CSCORE_B'], alpha_4, b_4, loc =␣
 ↪mu_4, scale = sigma_4)) + \
       np.sum(scipy.stats.beta.logpdf(X_CP['CSCORE_B'], alpha_5, b_5, loc =␣
 ↪mu_5, scale = sigma_5))

T = -2* (logL_H0 - logL_H1)
print(T)
```

```
72.75724408860697
```

Using T, we can calulate $p = \mathbb{P}(\chi^2_{16} > T)$ as follows.

[ ]:
```
p = 1 - chi2.cdf(T,16)
print(p)
```

```
3.2660627535818776e-09
```

Since $p \ll 0.01$, this suggests that there is very strong evidence against the null hypothesis H_0, that the distribution stays constant across the various property types. This coincides with our intuition from part 4 that the distribution seems to vary across different property types.

7 References

1. Visualizing distributions of data. https://seaborn.pydata.org/tutorial/distributions.html
2. Fitting empirical distribution to theoretical ones with Scipy (Python)? https://stackoverflow.com/questions/6620471/fitting-empirical-distribution-to-theoretical-ones-with-scipy-python
3. scipy.stats.beta. https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.beta.html
4. Lecture notes by Professor Schafer.
5. All of Statistics by Wasserman.