Splint

Bohaz

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We wish to obtain the coefficients of the cubic equations between the small interval $[0, h]$

$$y(\delta) = C_0 + C_1\delta + C_2\delta^2 + C_3\delta^3 \tag{1}$$

where $y(0) = y_a$ and $y(h) = y_b$
Spline provides us with second derivatives $y''(0) = y_a''$ and $y''(h) = y_b''$.
Thus immediately, we have

$$\boxed{C_0 = y_a} \tag{2}$$

For C_2 and C_3 We linearly interpolate y'' in the interval

$$y''(\delta) = y_a'' + \frac{y_b'' - y_a''}{h}\delta \tag{3}$$

compare cofficient with

$$y''(\delta) = 2C_2 + 6C_3\delta \tag{4}$$

yields

$$\boxed{C_2 = \frac{1}{2}y_a''} \tag{5}$$

$$\boxed{C_3 = \frac{y_b'' - y_a''}{6h}} \tag{6}$$

Now consider

$$y_b = y(h) = C_0 + C_1h + C_2h^2 + C_3h^3 \tag{7}$$

$$y_b = y_a + C_1h + \frac{1}{2}y_a''h^2 + \frac{y_b'' - y_a''}{6h}h^3 \tag{8}$$

after rearrangement and simplification, we obtain

$$\boxed{C_1 = \frac{y_b - y_a}{h} - \frac{h}{6}(2y_a'' + y_b'')} \tag{9}$$